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Born: Balcesti, Valcea district, Romania**.**
Date of Birth: 12.10.1954
Academic degree:

- PhD, PostDocs, is an Emeritus Professor of Mathematics at the University of New Mexico, United States.
- He got his MSc in Mathematics and Computer Science from the University of Craiova, Romania, Ph. D in Mathematics from the State University of Kishinev.
- Postdoctoral in Applied Mathematics from Okayama University of Sciences, Japan .And The Guangdong University of Technology, Guangzhou, China.

His scientific achievements:

- He is the founder of neutrosophy (generalization of dialectics), neutrosophic set, logic, probability and statistics since 1995.
- He published hundreds of papers and books on neutrosophic physics, superluminal and instantaneous physics, unmatter, quantum paradoxes, absolute theory of relativity, redshift and blueshift due to the medium gradient and refraction index besides the Doppler effect, paradoxism, outerart, neutrosophy as a new branch of philosophy, Law of Included Multiple-Middle, Law of Included Infinitely-Many-Middles, multispace and multistructure, HyperSoft set, TreeSoft Set, IndetermSoft Set and IndetermHyperSoft Set, SuperHyperGraph,

SuperHyperTopology, SuperHyperAlgebra, SuperHyperFunction, Neutrosophic SuperHyperAlgebra, degree of dependence and independence between neutrosophic components, refined neutrosophic set, neutrosophic over-under-off-set (with degrees of membership/ indeterminacy/nonmembership less than 0 and bigger than1), plithogenic set / logic / probability / statistics, symbolic plithogenic algebraic structures, neutrosophic triplet and duplet structures, quadruple neutrosophic structures, extension of algebraic structures to NeutroAlgebra and AntiAlgebra, NeutroGeometry and AntiGeometry, NeutroTopology and AntiTopology, Refined Neutrosophic Topology, Refined Neutrosophic Crisp Topology, Dezert-Smarandache Theory.

- He reviewed many peers and many books in international journals.
- He presented papers and plenary lectures to tenths of international conferences and seminars around the world.
- He also published many books of poetry, dramas, children's stories, literary translations, essays, a novel, folklore collections, traveling memories, and art albums.
- He wrote articles and books in four languages: English, Romanian, French, and Spanish.
- His books were translated to Arabic, Chinese, Russian, Spanish, Greek, Portuguese, Italian, German, Serbo-Croatian, and Turkish, see
  https://fs.unm.edu/LiteratureLibrary.htm and
  https://fs.unm.edu/ScienceLibrary.htm .
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In The Name of Allah, The Most Beneficent, The Most Merciful

So Exalted is Allah, the True Sovereign. Do not be hasty with the Quran before its inspiration is concluded to you, and say, "My Lord, increase me in knowledge"

(Allah Almighty is Truthful)

My dear colleagues

Searching for the best is the path of human interest at all times, and in our endeavor to provide the best and keep pace with the great scientific development witnessed in our contemporary world, we present to you this humble work entitled:

(*Neutrosophic linear models and algorithms to find their optimal solution)*

Science is the basis for managing life's affairs and human activities, and living without knowledge is a kind of wandering and a kind of loss. Using scientific methods helps us understand the foundations of choice, decision-making, and adopting the right solutions when there are many solutions and many options.

Through this work, we present a study of linear models using the concepts of neutrosophic science, the science that was built on the basis that there is no absolute truth, there is no confirmed data, issues cannot be limited to right and wrong only. There is a third state between error and right, an indeterminate, undetermined, uncertain state. It is indeterminacy. Neutrosophic science gave each issue three dimensions, namely (T, I, F), correctness in degrees, indeterminacy in degrees, and error in degrees. It was founded by the American philosopher and mathematician Florentin Smarandache, in 1995 and came as a generalization of fuzzy logic that was founded by the scientist Lotfi. A. Zadeh, in 1965. Neutrosophics is a new science. It studies the different spectrums that a person can imagine in a single issue, which gives a more accurate description of the data of the issue under study and thus accurate results that leave no room for coincidence that help in making decisions that suit all the circumstances experienced by the work environment of the system under study. In our quest to search for the best. In this book, we present a study of linear models and algorithms to find the optimal solution for them using the concepts of neuroscientific science. We know that the linear programming method is one of the important methods of operations research, the science that was the product of the great scientific development that our contemporary world is witnessing. The name operations research is given to the group of scientific methods used. In analyzing problems and searching for optimal solutions, it is a science whose applications have achieved widespread success in various fields of life. The characteristic that characterizes this science is the development of mathematical models, tools and techniques that have the ability to express the concepts of efficiency and scarcity in a well-defined mathematical model for a specific situation. It has the ability to use scientific methods to solve complex dilemmas in managing large systems in factories, institutions and companies and helps decision makers in them to make optimal scientific decisions for the workflow. These issues were addressed according to classical logic, but the ideal solution was a specific value appropriate to the circumstances in which the data was collected. It does not take into account the changes that may occur in the work environment. To obtain more accurate results and enjoy a margin of freedom, we present in this book a study of neutrosophic linear models and algorithms to find the optimal solution for them. What is meant by neutrosophic models are models in which the data are neutrosophic values, that is, variables such as in the objective function, which expresses profit if the model is a maximization model, and expresses a cost if the model is a minimization model, which in turn is affected by environmental conditions.

We take it in the form $Nc_j = c_j \ \pm \varepsilon_j$, where ε_{ij} is the indeterminacy, and it takes one of the forms $\varepsilon_j \in [\lambda_{j1}, \lambda_{j2}]$ or $\varepsilon_j \in \{\lambda_{j1}, \lambda_{j2}\}$ or otherwise, which is any neighborhood of the value c_{ij} that we obtain while adding The data on the issue then becomes the cost (or profit) matrix $Nc_j = [c_j \ \pm \varepsilon_j]$, and also the fixed values that represent the right side of the constraint swings, which express the available capabilities of capital, time, raw materials, etc., and they are also affected. In environmental conditions, we take it from the form $Nb_j = b_j + \delta_j$, where δ_j is the indeterminacy of the required quantities. It can take one of the forms

$\delta_j \in [\mu_{i1}, \mu_{i2}]$ or $\delta_j \in \{\mu_{i1}, \mu_{i2}\}$, and the same situation applies to examples of variables in constraints that express quantities. The raw materials consumed in the production process are taken from the form $Na_{ij} = a_{ij} + \gamma_{ij}$, where γ_{ij} is the indeterminacy of the quantities necessary for the raw material i to produce one unit of product j. It can take one of the forms $\gamma_{ij} \in [\varphi_{ij1}, \varphi_{ij2}]$, or $\gamma_{ij} \in \{\varphi_{ij1}, \varphi_{ij2}\}$, which helps us obtain more accurate results and gives companies a margin of freedom.

This book includes eight chapters:

Chapter I: Study of neutrosophic linear equations.

Chapter II: Neutrosophic Linear Models.

Chapter III: The graphical method for finding the optimal solution for neutrosophic linear models.

Chapter IV: The simplex direct neutrosophic algorithm for finding the optimal solution for linear models.

Chapter V: The modified simplex neutrosophic algorithm to find the optimal solution for linear models.

Chapter VI: The simplex algorithm with a synthetic basis to find the optimal solution for linear models.

Chapter VII: Neutrosophic Dual Linear Models and the Binary Algorithm.

Chapter VIII: Some applications to neutrosophic linear models.

We hope to God Almighty that this work will achieve the desired benefit from its preparation.

Authors

Florentin Smarandache and Maissam Ahmad Jdid

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Chapter I

Study of neutrosophic linear equations

Chapter I

Study of Neutrosophic linear equations

Introduction:

Since the research and studies using neutrosophics produced more accurate results compared to research employing classical logic, we considered necessary to reformulate the systems of linear equations and some of the methods for solving them using the concepts of neutrosophic science, given the significance of the linear programming method as one of the methods of operations research.

1.1. Systems of linear equations according to classical logic:

The propositions of linear equations in which the number of equations equals m and the number of variables equals n are given according to classical logic in the following general form:

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$

$$\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m$$

And written in the following matrix form:

$$A.X = B$$

Where:

$$A = \begin{bmatrix} a_{11} & a_{12} \dots a_{1n} \\ a_{21} & a_{22} \dots a_{2n} \\ \dots \dots \dots & \dots \dots \dots \dots \\ a_{m1} & a_{m2} \dots a_{mn} \end{bmatrix} \quad B = \begin{bmatrix} b_1 \\ b_2 \\ .. \\ b_m \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ .. \\ x_n \end{bmatrix}$$

Where a_{ij} and b_i are real numbers for all values of

$i = 1,2,\dots,m$ and $j = 1,2,\dots,n$.

Three cases of linear equation systems were identified and distinguished.

First case:

The number of variables and equations is the same, i.e., $m = n$.

Second case:

There are more equations than variables, i.e., $m > n$

Third case:

The number of equations is less than the number of variables, i.e., $m < n$.

The systems of linear equations that follow will be presented using the ideas of neutrosophic science. In this case, the real numbers a_{ij} and b_i will be treated as neutrosophic numbers, or as indefinite values of the form Nb_i and Na_{ij}. Perfectly determined, they can be any neighborhood of the real numbers a_{ij} and b_i, expressed in any of the following forms:

$Na_{ij} = a_{ij} + \varepsilon_{ij}$ and $Nb_i = b_i + \mu_i$ where $\varepsilon_{ij} \in [\lambda_{1ij}, \lambda_{2ij}]$ or $\varepsilon_{ij} \in \{\lambda_{1ij}, \lambda_{2ij}\}$ or otherwise, then the systems of neutrosophic linear equations is written in the form below.

1.2. Systems of neutrosophic linear equations where the number of equations equals m and the number of variables equals n:

General form:

$$Na_{11}x_1 + Na_{12}x_2 + \cdots + Na_{1\mathrm{n}}x_n = Nb_1$$

$$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2\mathrm{n}}x_n = Nb_2$$

$$\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots .$$

$$Na_{\mathrm{m}n}x_1 + Na_{\mathrm{m}n}x_2 + \cdots + Na_{mn}x_m = Nb_m$$

In the following matrix form**:**

$$NA.\mathrm{X} = NB$$

Where:

$$NA = \begin{bmatrix} Na_{11} & Na_{12} \dots Na_{1n} \\ Na_{21} & Na_{22} \dots Na_{2n} \\ \dots \dots \dots \dots \dots \dots \dots & \\ Na_{m1} & Na_{m2} \dots Na_{mn} \end{bmatrix} \quad NB = \begin{bmatrix} Nb_1 \\ Nb_2 \\ .. \\ Nb_m \end{bmatrix} \quad \mathrm{X} = \begin{bmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \\ .. \\ \mathrm{x}_\mathrm{n} \end{bmatrix}$$

We analyze the preceding equation systems in accordance with the three previously mentioned cases to determine their general solution.

First case:

The number of variables and equations is the same, i.e., $m = n$.

We write the systems of equations as follows:

$$Na_{11}x_1 + Na_{12}x_2 + \cdots + Na_{1\mathrm{n}}x_n = Nb_1$$

$$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2\mathrm{n}}x_n = Nb_2$$

$$\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots$$

$$Na_{n1}x_1 + Na_{n2}x_2 + \cdots + Na_{nn}x_n = Nb_n$$

Or, in matrix form:

$$NA\,.X = NB$$

Where:

$$NA = \begin{bmatrix} Na_{11} & Na_{12} \dots Na_{1n} \\ Na_{21} & Na_{22} \dots Na_{2n} \\ \dots\dots\dots\dots\dots\dots\dots \\ Na_{n1} & Na_{n2} \dots Na_{nn} \end{bmatrix} \quad NB = \begin{bmatrix} Nb_1 \\ Nb_2 \\ .. \\ Nb_n \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ .. \\ x_n \end{bmatrix}$$

The matrix is a square matrix whose determinant is $\Delta_N = |NA|$

Here we distinguish two cases:

1- $\Delta_N = 0$. Two cases follow from this case:

a. If $\Delta_N = 0$ and $\Delta_{N_{x_j}} \neq 0$ where $\Delta_{N_{x_j}}$ is the determinant resulting from the determinant of the matrix of Δ_N after replacing the column containing of the unknown x_j with the column of constants, then the systems have no solution.

b. If $\Delta_N = 0$ and $\Delta_{N_{x_j}} = 0$, this means that the systems of equations are not linearly independent, meaning that some are linearly related to each other. In order to handle this case, we eliminate one of the two equations that are linearly related; as a result, there are now m' equations instead of two, where $m' = m - 1$ and $m' < n$,which is the same as the second case that will be addressed later.

c. When $\Delta_N \neq 0$, that is, the systems of equations are linearly independent and the systems have a single solution, that can be found in multiple ways. We investigate the Gauss-Jordan method in this study because it serves as the foundation for the direct simplex algorithm that we employ to find the best solution for linear models.

1.3. Gauss- Jordan method for solving systems of neutrosophic linear equations where $m = n$:

We express the equations in the following matrix form to make the method's mathematical foundation more clear:

$$\begin{bmatrix} Na_{11} & Na_{12} \dots Na_{1n} \\ Na_{21} & Na_{22} \dots Na_{2n} \\ \dots \dots \dots \dots \dots \dots \dots & \\ Na_{n1} & Na_{n2} \dots Na_{nn} \end{bmatrix} . \begin{bmatrix} x_1 \\ x_2 \\ .. \\ x_n \end{bmatrix} = \begin{bmatrix} Nb_1 \\ Nb_2 \\ .. \\ Nb_n \end{bmatrix} \quad (1)$$

Or in the following abbreviated form:

$$NA\,.X = NB \quad (2)$$

Since $\Delta_N = |NA| \neq 0$, this means that the matrix NA has an inverse i.e., NA^{-1}. We multiply both sides of equation (2) by $NA^{-1}$and we find:

$$NA^{-1}.(NA\,.X) = NA^{-1}.NB$$

Hence, we get:

$$I.\mathrm{X} = NB'$$

Which is written in the following detailed form:

$$\begin{bmatrix} 1 & 0 & 0 \dots 0 \\ 0 & 1 & 0 \dots 0 \\ \dots \dots \dots \dots \dots & & \\ 0 & 0 & 0 \dots 1 \end{bmatrix} . \begin{bmatrix} x_1 \\ x_2 \\ .. \\ x_n \end{bmatrix} = \begin{bmatrix} Nb'_1 \\ Nb'_2 \\ .. \\ Nb'_n \end{bmatrix} \quad (3)$$

This process is the basis of the Gauss- Jordan method for solving a system of linear equations. In order to convert Figure (1) to Figure (2), we follow the following steps:

1- We express Figure (1) in the following table:

Variables / Equations	x_1	x_2	….	x_n	NB
1	Na_{11}	Na_{12}	….	Na_{1n}	Nb_1
2	Na_{21}	Na_{22}	…	Na_{2n}	Nb_2
….	…	…	…	…	…
n	Na_{n1}	Na_{n2}	…	Na_{nn}	Nb_n

Table No. (1): Table of equations

2- We convert the matrix NA to the unit matrix I by processing the rows of the table so that we make all non-diagonal elements in all its rows equal to zero and the diagonal elements equal to one. The steps below are used to eliminate the variable x_s from the equation t:

a- To make x_s equal to one, we divide all the elements of row t by Na_{ts}. This causes x_s to equal one and modifies the other expressions.

b- We set all elements of the column with x_s (except row t) equal to zero.

c- We calculate the rest of the elements of the new table from the following two relation:

$$\left.\begin{aligned} Na'_{ij} &= \left(Na_{ij} - Na_{is}\frac{Na_{tj}}{Na_{ts}}\right) = \frac{Na_{ij}Na_{ts} - Na_{is}Na_{tj}}{Na_{ts}} \\ Nb'_i &= \left(Nb_i - Na_{is}\frac{Nb_t}{Na_{ts}}\right) = \frac{Nb_iNa_{ts} - Na_{is}Nb_t}{Na_{ts}} \end{aligned}\right] \quad (4)$$

The element Na_{ts} is called the pivot element.

Following the previous process, the following table is produced:

Variables / Equations	Nx_1	Nx_2	….	Nx_n	NB'
1	1	0	….	0	Nb'_1
2	0	1	…	0	Nb'_2
….	…	…	…	…	…
n	0	0	…	1	Nb'_n

Table No. (2) Final solution table

The linear equation systems are expressed in the following matrix form:

$$I.NX = NB'$$

$$\begin{bmatrix} 1 & 0 & 0 \dots 0 \\ 0 & 1 & 0 \dots 0 \\ \dots & \dots & \dots \dots \dots \\ 0 & 0 & 0 \dots 1 \end{bmatrix} . \begin{bmatrix} x_1 \\ x_2 \\ .. \\ x_n \end{bmatrix} = \begin{bmatrix} Nb'_1 \\ Nb'_2 \\ .. \\ Nb'_n \end{bmatrix} \quad \Longrightarrow$$

$$\begin{bmatrix} Nx_1 \\ Nx_2 \\ .. \\ Nx_n \end{bmatrix} = \begin{bmatrix} Nb'_1 \\ Nb'_2 \\ .. \\ Nb'_n \end{bmatrix}$$

$$\Longrightarrow Nx_1 = Nb'_1 , Nx_2 = Nb'_2 , \dots , Nx_n = Nb'_n$$

Second case:

There are more equations than variables, i.e., $m > n$.

In this case, we form a new system from the set of equations in which the number of equations is equal to the number of variables by excluding a number of equations of $\mathrm{m} - \mathrm{n}$. Then, to make sure the equations that were excluded are satisfied; we solve the new systems in the same way as we solved the first case.

Third case: There are fewer equations than there are variables, i.e., $m < n$.

We are presented with a set of equations of the following form in this particular case.

$$\left.\begin{matrix} Na_{11}x_1 + Na_{12}x_2 + \cdots + Na_{1\mathrm{m}}x_m + \cdots + Na_{1\mathrm{n}}x_n = Nb_1 \\ Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2\mathrm{m}}x_m + \cdots + Na_{2\mathrm{n}}x_n = Nb_2 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{\mathrm{mm}}x_m + \cdots + Na_{mn}x_n = Nb_m \end{matrix}\right] \quad (5)$$

Which is written in the following matrix form:

$$NA = \begin{bmatrix} Na_{11} & Na_{12} \dots Na_{1m} \dots\dots Na_{1n} \\ Na_{21} & Na_{22} \dots Na_{2m} \dots\dots Na_{1n} \\ \dots\dots\dots\dots & \dots\dots\dots\dots\dots\dots\dots\dots \\ Na_{m1} & Na_{m2} \dots Na_{mm} \dots\dots Na_{mn} \end{bmatrix} NB = \begin{bmatrix} Nb_1 \\ Nb_2 \\ .. \\ Nb_n \end{bmatrix} X = \begin{bmatrix} x_1 \\ x_2 \\ .. \\ x_m \\ .. \\ .. \\ x_n \end{bmatrix}$$

In the following brief form:

$$NA_{(m.n)} \,. X_{(n.1)} = NB_{(m.1)} \quad (6)$$

1- We divide the matrix $NA_{(m.n)}$ into two matrices:
   a- A square matrix of rank $(m.m)$ which we denote $NC_{(m.m)}$.
   b- And a rectangular matrix of rank $(m.n-m)$ which we denote $ND_{(m.n-m)}$
2- The column matrix $X_{(n.1)}$ is divided into two matrices $X'_{(m.1)}$ and $X''_{(n-m.1)}$.

Subsequently, the equation systems (5) are expressed in the matrix form shown below:

$$\left[NC_{(m.m)}, ND_{(m.n-m)}\right] . \begin{bmatrix} X'_{(m.1)} \\ X''_{(n-m.1)} \end{bmatrix} = NB_{(m.1)} \quad (7)$$

$$NC_{(m.m)}.X'_{(m.1)} + ND_{(m.n-m)}.X''_{(n-m.1)} = NB_{(m.1)}$$

We find that:

$$NC_{(m.m)}.X'_{(m.1)} = NB_{(m.1)} - ND_{(m.n-m)}.X''_{(n-m.1)} \quad (8)$$

Assuming that $|NC| \neq 0$, we multiply both sides in relation (8) by NC^{-1} and we find:

$$NC^{-1}.NC.X' = NC^{-1}.(NB - ND.X'')$$

$$I.X' = NC^{-1}.NB - NC^{-1}.ND.X'') \quad (9)$$

Assuming that $NC^{-1}.NB = NB'$ and $NC^{-1}.ND = ND'_{(m.n-m)}$ we find that:

$$\begin{bmatrix} 1 & 0 & 0 \dots 0 \\ 0 & 1 & 0 \dots 0 \\ \dots & \dots & \dots \dots \dots \\ 0 & 0 & 0 \dots 1 \end{bmatrix} . \begin{bmatrix} x_1 \\ x_2 \\ .. \\ x_m \end{bmatrix}$$

$$= \begin{bmatrix} Nb'_1 \\ Nb'_2 \\ .. \\ Nb'_m \end{bmatrix} - \begin{bmatrix} Nd'_{11} & Nd'_{12} \dots Nd'_{1(n-m)} \\ Nd'_{21} & Nd'_{22} \dots Nd'_{1(n-m)} \\ \dots & \dots \\ Nd'_{m1} & Nd'_{m2} \dots Nd'_{m(n-m)} \end{bmatrix} . \begin{bmatrix} x_{m+1} \\ x_{m+2} \\ \dots \\ x_n \end{bmatrix} \quad (10)$$

Which can be converted as follows into a set of linear equations:

$$Nx_1 = Nb'_1 - (Nd'_{11}x_{m+1} + Nd'_{12}x_{m+2} + \cdots + Nd'_{1(n-m)}x_n)$$
$$Nx_2 = Nb'_2 - (Nd'_{21}x_{m+1} + Nd'_{22}x_{m+2} + \cdots + Nd'_{2(n-m)}x_n)$$
$$\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots$$
$$Nx_m = Nb'_m - (Nd'_{m1}x_{m+1} + Nd'_{m2}x_{m+2} + \cdots + Nd'_{m(n-m)}x_n)$$

This means that we were able to calculate m in terms of $(n-m)$, $x_{m+1}, x_{m+2}, \dots, x_n$. We note that the values of the variables $x_1, x_2, \dots, x_m$, it relates to the values taken by the variables $x_{m+1}, x_{m+2}, \dots, x_n$, or in other words, what we give to the variables $x_{m+1}, x_{m+2}, \dots, x_n$, and that for every proposition of values such as $\beta_{m+1}, \beta_{m+2}, \dots, \beta_n$ for these variables we get a set of values for the variables $x_1, x_2, \dots, x_m$ is:

$$Nx_1 = Nb'_1 - (Nd'_{11}\beta_{m+1} + Nd'_{12}\beta_{m+2} + \cdots + Nd'_{1(n-m)}\beta_n)$$
$$Nx_2 = Nb'_2 - (Nd'_{21}\beta_{m+1} + Nd'_{22}\beta_{m+2} + \cdots + Nd'_{2(n-m)}\beta_n)$$
$$\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots$$
$$Nx_m = Nb'_m - (Nd'_{m1}\beta_{m+1} + Nd'_{m2}\beta_{m+2} + \cdots + Nd'_{m(n-m)}\beta_n)$$

Thus, we obtain a solution that includes all the variables of proposition (5)

Here is how the solution is structured:

$$(\beta_1, \beta_2, \dots, \beta_m, \beta_{m+1}, \beta_{m+2}, \dots, \beta_n)$$

But since the variables $x_{m+1}, x_{m+2}, \dots, x_n$ can take an infinite number of qualitative values (even if they are restricted by certain conditions), we obtain an infinite number of corresponding values for the variables $x_1, x_2, \dots, x_m$.

Thus, the set of equations (5) has an infinite number of acceptable solutions of the following form if $|NC| \neq 0$:

$$(x_1, x_2, \dots, x_m, x_{m+1}, x_{m+2}, \dots, x_n)$$

Thus, we obtain a solution that includes all variables of the proposition, which is the ordered solution:

$$(\beta_1, \beta_2, \dots, \beta_m, \beta_{m+1}, \beta_{m+2}, \dots, \beta_n)$$

1.4. Basic solutions of the neutrosophic linear equations:

Since proposition (5) has an infinite number of acceptable solutions, we will try to limit ourselves to a limited number by setting the variables $x_{m+1}, x_{m+2}, \dots, x_n$ equal to zero. Then proposition (9) takes the following form:

$$\begin{bmatrix} 1 & 0 & 0 \dots 0 \\ 0 & 1 & 0 \dots 0 \\ \dots & \dots & \dots \dots \dots \\ 0 & 0 & 0 \dots 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ .. \\ x_m \end{bmatrix} = \begin{bmatrix} Nb'_1 \\ Nb'_2 \\ .. \\ Nb'_m \end{bmatrix} \qquad (11)$$

We get:

$$x_1 = Nb'_1 \quad , x_2 = Nb'_2 \ , \dots, x_m = Nb'_m$$

So, the complete solution is:

$$(Nb'_1, Nb'_2, \dots, Nb'_m, 0,0, \dots ,0)$$

Because it can be attributed to the rule with single normal vectors in the space R^m, we refer to this solution as the basic solution:

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ .. \\ 0 \end{bmatrix} \quad e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ .. \\ 0 \end{bmatrix} \quad \dots \quad \dots \quad e_m = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ .. \\ m \end{bmatrix}$$

The set of vectors $e_1, e_2, \dots, e_m$ form a rule because they are linearly independent, and the vector NB' can be expressed using the factorials $x_1, x_2, \dots, x_m$ as follows:

$$NB' = e_1x_1 + e_2x_2 + \cdots + e_mx_m$$

We call the variables$x_1, x_2, \dots, x_m$, basic variables and we call other variables $x_{m+1}, x_{m+2}, \dots, x_n$

free or non-basic variables because they take qualitative values.

The process of selecting the variables $x_1, x_2, \dots, x_m$ as basic variables is a random process, since we can form other basic solutions, if we know that the available possibilities to obtain basic solutions are:

$$C_n^m = \frac{n!}{m!\,(n-m)!}$$

There is a finite number of infinitely possible solutions.

Example 1:

The two linear equations below have a joint solution.

$$2x_1 + 7x_2 + 3x_3 + 2x_4 = [2,5]$$
$$3x_1 + 9x_2 + 4x_3 + x_4 = [3,7]$$
$$x_1 + 5x_2 + 3x_3 + 4x_4 = [4,8]$$

In the set of equations, the number of variables is $n = 4$ and the number of equations is $m = 3$. Therefore, the number of basic variables is equal to 3 and the number of non-basic free variables is $n - m = 1$. The number of possible solutions is calculated from the relation:

$$C_n^m = \frac{n!}{m!\,(n-m)!}$$

i.e.,

$$C_4^3 = \frac{4!}{3!\,(4-3)!} = 4$$

We write as follows:

$(x_1, x_2, x_3, 0)$, $(x_1, x_2, 0, x_4)$, $(x_1, 0, x_3, x_4)$, $(0, x_2, x_3, x_4)$

To obtain these solutions, we write the systems of equations in the following form:

$$2x_1 + 7x_2 + 3x_3 = [2,5] - 2x_4$$
$$3x_1 + 9x_2 + 4x_3 + x_4 = [3,7] - x_4$$
$$x_1 + 5x_2 + 3x_3 = [4,8] - 4x_4$$

The previous proposition is written in the following matrix form:

$$\begin{bmatrix} 2 & 7 & 3 \\ 3 & 9 & 4 \\ 1 & 5 & 3 \end{bmatrix} . \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} [2,5] \\ [3,7] \\ [4,8] \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} . [x_4]$$

$$C = \begin{bmatrix} 2 & 7 & 3 \\ 3 & 9 & 4 \\ 1 & 5 & 3 \end{bmatrix} \quad X' = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad NB = \begin{bmatrix} [2,5] \\ [3,7] \\ [4,8] \end{bmatrix} \quad D = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} \quad X'' = [x_4]$$

We calculate the determinant $|C|$.

We find:

$$|C| = \begin{vmatrix} 2 & 7 & 3 \\ 3 & 9 & 4 \\ 1 & 5 & 3 \end{vmatrix} = -3 \neq 0$$

We determine the matrix's reciprocal to identify the solutions:

$$C = \begin{bmatrix} 2 & 7 & 3 \\ 3 & 9 & 4 \\ 1 & 5 & 3 \end{bmatrix}$$

We find:

$$C^{-1}=\begin{bmatrix}\frac{-7}{3} & 2 & \frac{-1}{3}\\ \frac{5}{3} & -1 & \frac{-1}{3}\\ -2 & 1 & 1\end{bmatrix}$$

We compensate in the relation:

$$NC^{-1}.NC.X' = NC^{-1}.(NB - ND.X'')$$

We get*:*

$$\begin{bmatrix}\frac{-7}{3} & 2 & \frac{-1}{3}\\ \frac{5}{3} & -1 & \frac{-1}{3}\\ -2 & 1 & 1\end{bmatrix}.\begin{bmatrix}2 & 7 & 3\\ 3 & 9 & 4\\ 1 & 5 & 3\end{bmatrix}$$

$$=\begin{bmatrix}\frac{-7}{3} & 2 & \frac{-1}{3}\\ \frac{5}{3} & -1 & \frac{-1}{3}\\ -2 & 1 & 1\end{bmatrix}.\left(\begin{bmatrix}[2,5]\\ [3,7]\\ [4,8]\end{bmatrix}-\begin{bmatrix}2\\ 1\\ 4\end{bmatrix}.[x_4]\right)$$

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}Nx_1\\ Nx_2\\ Nx_3\end{bmatrix}$$

$$=\begin{bmatrix}\frac{-7}{3} & 2 & \frac{-1}{3}\\ \frac{5}{3} & -1 & \frac{-1}{3}\\ -2 & 1 & 1\end{bmatrix}.\begin{bmatrix}[2,5]\\ [3,7]\\ [4,8]\end{bmatrix}-\begin{bmatrix}\frac{-7}{3} & 2 & \frac{-1}{3}\\ \frac{5}{3} & -1 & \frac{-1}{3}\\ -2 & 1 & 1\end{bmatrix}.\begin{bmatrix}2\\ 1\\ 4\end{bmatrix}.[x_4]$$

Which can be transformed into the following systems of equations:

$$Nx_1=\begin{bmatrix}\left[0,\frac{-1}{3}\right]\\ -\left[1,\frac{4}{3}\right]\\ [3,5]\end{bmatrix}-\begin{bmatrix}\frac{-16}{3}\\ 1\\ 1\end{bmatrix}.[x_4]$$

Setting the free variable x_4 equal to zero we get:

$$Nx_1 = \begin{bmatrix} \left[0, \frac{-1}{3}\right] \\ -\left[1, \frac{4}{3}\right] \\ [3,5] \end{bmatrix}$$

i.e.,

$$x_1 = \left[0, \frac{-1}{3}\right], x_2 = -\left[1, \frac{4}{3}\right], x_3 = [3,5]$$

Thus, we obtain the first neutrosophic basic solution, which is:

$$(x_1, x_2, x_3, 0) = \left(\left[0, \frac{-1}{3}\right], -\left[1, \frac{4}{3}\right], [3,5], 0\right)$$

We obtain other basic solutions in the same way.

Dissolved basic solutions:

The basic solution is a degenerate and invalid solution if we obtain a value of zero for the variables that we have chosen as a basis.

1.5. Gauss- Jordan method for solving a set of linear equations where $m < n$:

The following are the basic steps of the Gaussian-Jordan method, which are based on the previously mentioned mathematical principles:

1- We write the systems of equations (5) in the following matrix form:

$$I.X' + NC^{-1}.D.X'' = NC^{-1}.NB = NB'$$

$$[I\ , NC^{-1}.ND]\,.\begin{bmatrix} X' \\ X'' \end{bmatrix} = NB' \qquad (12)$$

Which is written in the following detailed form:

$$\begin{bmatrix} 1 & 0 & 0 \dots 0 & Nd'_{11} & Nd'_{12} \dots Nd'_{1(n-m)} \\ 0 & 1 & 0 \dots 0 & Nd'_{21} & Nd'_{22} \dots Nd'_{1(n-m)} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 \dots 1 & Nd'_{m1} & Nd'_{m2} \dots Nd'_{m(n-m)} \end{bmatrix} . \begin{bmatrix} x_1 \\ x_2 \\ .. \\ x_n \end{bmatrix} = \begin{bmatrix} Nb'_1 \\ Nb'_2 \\ .. \\ Nb'_m \end{bmatrix} \quad (13)$$

The transition from Figure (5) to Figure (12) is done the same steps we mentioned in the previous paragraph, but this method does not give us a basic solution unless we set the free variables equal to zero. If we do that, we only get the first solution. To obtain all solutions, we perform the following steps:

a. We organize the following table:

Variables / Equations	x_1	x_2	….	x_m	x_{m+1}	x_{m+2}	….	x_n	NB
1	a_{11}	a_{12}	….	a_{1m}	a_{1m+1}	a_{1m+2}	….	a_{1n}	Nb'_1
2	a_{21}	a_{22}	…	a_{2m}	a_{2m+1}	a_{2m+2}	….	a_{2n}	Nb'_2
….	…	…	…	…	…	…	…	…	…
m	a_{m1}	a_{m2}	…	a_{mm}	a_{mm+1}	a_{mm+2}	…	a_{mn}	Nb'_m

Table No. (3) The first table for the Gauss- Jordan method

b. We find the identity matrix $I_{m\times m}$ by processing the rows of the previous table in the same way as explained in the previous paragraph. To do this the specify variables that are entered in the base and let them be $x_1, x_2, \dots, x_m$. As a result of this processing, we obtain the following table:

Variables / Equations	x_1	x_2	….	x_m	x_{m+1}	x_{m+2}	….	x_n	NB'
1	1	0	….	0	Nd'_{11}	Nd'_{12}	….	Nd'_{1n-m}	Nb'_1
2	0	1	…	0	Nd'_{21}	Nd'_{22}	….	Nd'_{2n-m}	Nb'_2
….	…	…	…	…	…	…	…	…	…
m	0	0	…	1	Nd'_{m1}	Nd'_{22}	…	Nd'_{mn-m}	Nb'_2

Table No. (4): Table of the first basic solution

c. Setting all the free variables in Table (4) equal to zero, we obtain the following first basic solution:

$$(Nb'_1, Nb'_2, \dots, Nb'_m, 0,0, \dots, 0)$$

a. To obtain a second basic solution, we replace one of the basic variables, say x_m, with one of the non-basic variables x_{m+1}, by selecting the appropriate pivot element, and here it is Nd'_{m1}. We work to delete x_{m+1} from all equations except equation m. In this equation, we set the coefficient of this variable to one. We use the two relations (4) to carry out the necessary computations. We solve the subsequent second basic solution:

$$(Nb'_1, Nb'_2, \dots, Nb'_{m-1}, 0, Nb'_{m+1}, 0, \dots, 0)$$

We repeat the steps mentioned in step (d) to get additional basic solutions.

1.6. Non-negative basic solutions of systems of neutrosophic linear equations:

Certain basic solutions are unsatisfactory because they defy the condition if all or some of the variables are required to be non-negative. Among the basic solutions, we must search for positive basic solutions in such a situation.

The Gauss-Jordan method was developed to directly obtain positive solutions because it is difficult to apply the method described in the example, particularly when there are many variables.

The new method was called the simplex method, which is carried out according to the following steps:

1.7. The simplex method for finding non-negative basic solutions to a system of linear equations where m < n:

In the systems of equations (5):

1- By multiplying the equation with the negative second side by (-1), we are able to make all elements of the constant's column NB on the second side of the equations non-negative.
2- We put the coefficients of the new systems in a table.
3- We form a rule consisting of m variables by selecting the variable that we want to enter into the rule, for example, x_s, then we calculate the index.

$$\theta = Min\left[\frac{Nb_i}{Na_{is}}\right] = \frac{Nb_t}{Na_{ts}} > 0; \quad Na_{is} > 0, Nb_i > 0$$

We call the element Na_{ts} the pivot element, we delete the variable x_s from all equations according to the Gauss- Jordan method, except for the equation t, in which its coefficient is equal to one. We repeat the previous step until we form a base consisting of m variables.
4- Setting the non-basic variables equal to zero we obtain the following non-negative basic solution:

$$(Nb'_1, Nb'_2, \dots, Nb'_{m-1}, 0, Nb'_{m+1}, 0, \dots, 0)$$

5- We designate one of the variables as a basic variable, find the pivot element, and then carry out the same steps as for the variablex_s to obtain additional new non-negative basic solutions. We continue working until we have all of the non-negative basic solutions after we find a new one.

We explain the above using the following example:

Example 2:

$$x_1 - 3x_4 + 2x_5 = -[1,3]$$
$$x_2 + 2x_4 - 3x_5 = [2,8]$$

We multiply the first equation by (-1) until the condition

$Nb_i > 0$ is met, and we obtain the following new systems:

$$-x_1 - 3x_3 - 2x_5 = [1,3]$$

$$x_2 + 2x_4 - 3x_5 = [2,8]$$

The stopping criterion is when we cannot find a free column that we have not use for switching that contains a positive element (at least one), This means that all the elements of the free columns that were not used during the swap are negative values.

In the systems of equations, the number of variables is $n = 5$ and the number of equations is

$m = 2$ Therefore, the number of basic variables is equal to 2 and the number of non-basic free variables is $n - m = 3$. The number of possible solutions is calculated from the relation:

$$C_n^m = \frac{n!}{m!\,(n-m)!}$$

i.e.,

$$C_5^2 = \frac{5!}{2!\,(5-2)!} = 10$$

We write as follows:

$$(x_1, x_2, 0,0,0)\,, (x_1, 0, x_3, 0,0), (x_3, 0,0, x_4, 0), (x_1, 0,0,0, x_5),$$
$$(0, x_2, x_3, 0,0)(0, x_2, 0, x_4, 0), (0, x_2, 0,0, x_5), (0,0, x_3, x_4, 0),$$
$$(0,0, x_3, 0, x_5), (0,0,0, x_4, x_5)$$

To obtain these solutions, we organize the following table:

Variables / Equations	x_1	x_2	x_3	x_4	x_5	NB
1	-1	0	0	3	-2	[1,3]
2	0	1	0	2	-3	[2,8]

Table No. (5): The first table for the simplex method

To find a basic solution to the set of equations, we select a variable, for example x_4, to be a basic variable, and to determine the appropriate anchor element, we calculate the index:

$$\theta = Min\left[\frac{Nb_i}{Na_{is}}\right] = Min\left[\frac{[1,3]}{3}, \frac{[2,8]}{2}\right] = \frac{[1,3]}{3}$$

That is, the pivot is $a_{14} = 3$. The following table is produced after the necessary computations are made to remove the variable x_4 from the two equations:

Variables / Equations	x_1	x_2	x_3	x_4	x_5	NB'
x_4	$-\frac{1}{3}$	0	0	1	$-\frac{2}{3}$	$\left[\frac{1}{3}, 1\right]$
2	$\frac{2}{3}$	1	0	0	$\frac{5}{3}$	$\left[\frac{4}{3}, 6\right]$

Table No. (6) The second table for the simplex method

We choose another variable to be a basic variable. We note that the variable x_2 is ready to be a basic variable, and thus we get the following table:

Variables / Equations	x_1	x_2	x_3	x_4	x_5	NB'
x_4	$-\frac{1}{3}$	0	0	1	$-\frac{2}{3}$	$\left[\frac{1}{3}, 1\right]$
x_2	$\frac{2}{3}$	1	0	0	$\frac{5}{3}$	$\left[\frac{4}{3}, 6\right]$

Table No. (7): Final solution table

Thus, we obtain a base consisting of the variables $x_2\ , x_4$. We set the free variables equal to zero, and we obtain the following non-negative neutrosophic basic solution:

$$\left(0, \left[\frac{4}{3}, 6\right], 0, \left[\frac{1}{3}, 1\right], 0\right)$$

We repeat the steps we took to find the previous solution to get additional solutions.

Conclusion:

In this research, we have presented a study of the sets of neutrosophic linear equations as a basis for neutrosophic linear programming, and the Gauss-Jordan method, which is considered the mathematical basis for the simplex method to find positive basic solutions that can be used when there are constraints on some, or all, of the variables to be positive values, which in turn is the basis for the method direct simplex to find the optimal solution for linear models. Using the examples that we have presented on systems of neutrosophic equations, we have arrived at basic neutrosophic solutions that express indeterminate values. Such propositions can be used in cases where the data provided to the systems operating according to these systems of equations are subject to change. Here we can benefit from the margin of freedom offered by neutrosophic values.

Chapter II

Neutrosophic Linear Models

Introduction.

2–1- Basic formulas of neutrosophic linear models.

2-1-1-The general formula for the neutrosophic linear model.

2-1-2- The canonical neutrosophic formula for the linear model.

2-1-3- The standard neutrosophic formula for the linear model.

2-1-4- The symmetrical formula of the neutrosophic linear model.

2-2- How to move from one formula to another.

2-3- Examples of the above.

Conclusion.

Chapter II

Neutrosophic Linear Models

Introduction:

In this chapter, we present the formulas of neutrosophic linear mathematical models, that we mean linear models containing in their mathematical relation neutrosophic values, whether in the objective function relation or in the constraint relation, which takes into account all the changes that may occur in the operating environment of the system represented by the model, which it ensures that the facility has a safe workflow, meaning that we will take the variables in the objective function as neutrosophic values, i.e. $Nc_j = c_j \pm \varepsilon_j$

Also, the values that express the available capabilities are neutrosophic values, i.e., $Nb_i = b_i \pm \delta_i$ and , $Na_{ij} = a_{ij} \pm \mu_{ij}$ where $(j = 1,2, \dots, n , i = 1,2, \dots, m)$ are undefined values that have a margin of freedom and are taken according to the nature of the situation represented by the linear model, then We present the basic formulas of linear models using the following study:

2–1- Basic formulas of neutrosophic linear models:

Neutrosophic linear models can be classified according to the following formulas**:**

2-1-1-The general formula for the neutrosophic linear model:

The general neutrosophic formula for the linear mathematical model is given in abbreviated form as follows**:**

$$NZ = \sum_{j=1}^{n} (c_j \pm \varepsilon_j) x_j \longrightarrow Max\ or\ Min$$

Constraints:

$$\sum_{j=1}^{n} Na_{ij} x_j \begin{pmatrix} \geq \\ \leq \\ = \end{pmatrix} b_i \pm \delta_i \quad ; \quad i = 1,2,\dots,m$$

$$x_j \geq 0$$

Where $c_j + \varepsilon_j$, $b_i \pm \delta_i$, a_{ij}, $j = 1,2,\dots,n$, $i = 1,2,\dots,m$ are constants having set or interval values according to the nature of the given problem, x_j are decision variables.

It is given in the following detailed form:

Find:

$$NZ = Nc_1 x_1 + Nc_2 x_2 + \cdots + Nc_n x_n \longrightarrow (Max\ or\ Min)$$

Constraints:

$$Na_{\mathrm{i}1} x_1 + Na_{\mathrm{i}2} x_2 + \cdots + Na_{in} x_n \begin{pmatrix} \geq \\ \leq \\ = \end{pmatrix} Nb_{\mathrm{i}} \quad i = 1,2,\dots,m$$

$$x_1, x_2, \dots, x_n \geq 0$$

Linear models can also be expressed using matrices, and therefore the neutrosophic linear model given in the general form can be written using matrices as follows:

Find:

$$NZ = NC\,\mathrm{X} \longrightarrow (Max\ or\ Min)$$

Constraints:

$$NA\,X\begin{pmatrix}\geq\\ \leq\\ =\end{pmatrix}NB$$

$$X \geq 0$$

Where:

$$NA = \begin{bmatrix} Na_{11} & Na_{12} \dots Na_{1n} \\ Na_{21} & Na_{22} \dots Na_{2n} \\ \dots \dots \dots & \dots \dots \dots \dots \\ Na_{m1} & Na_{m2} \dots Na_{mn} \end{bmatrix} \quad NB = \begin{bmatrix} Nb_1 \\ Nb_2 \\ .. \\ Nb_m \end{bmatrix} \quad NC = \begin{bmatrix} Nc_1 \\ Nc_2 \\ .. \\ Nc_n \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ .. \\ x_n \end{bmatrix}$$

2-1-2- The canonical neutrosophic formula for the linear model:

The linear program is set canonical if all the variables are constrained to be non-negative and if all the constraints are given in the form of inequalities must be placed in the form

(≤ is less than or equal to). The neutrosophic canonical form is written in the following abbreviated form:

$$NZ = \sum_{j=1}^{n} (c_j \pm \varepsilon_j) x_j \longrightarrow Max$$

Constraints:

$$\sum_{j=1}^{n} Na_{ij} x_j \leq b_i \pm \delta_i \quad ; \quad i = 1,2, \dots, m$$

$$x_j \geq 0$$

It is given in the following detailed form:

Find:

$$NZ = Nc_1 x_1 + Nc_2 x_2 + \cdots + Nc_n x_n \longrightarrow Max$$

Constraints:

$$Na_{11}x_1 + Na_{12}x_2 + \cdots + \mathrm{N}a_{1n}x_n \leq Nb_1$$

$$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n \leq Nb_2$$

$$\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$

$$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n \leq Nb_m$$

$$x_1, x_2, \ldots, x_n \geq 0$$

Linear models can also be expressed using matrices, and therefore the neutrosophic linear model given in the canonical form can be written using matrices as follows:

Find:

$$NZ = NC\,\mathrm{X} \longrightarrow Max$$

Constraints:

$$NA\,\mathrm{X} \leq NB$$

$$\mathrm{X} \geq 0$$

Where:

$$NA = \begin{bmatrix} Na_{11} & Na_{12} \ldots Na_{1n} \\ Na_{21} & Na_{22} \ \ldots Na_{2n} \\ \ldots \ldots \ldots & \ldots \ldots \ldots \ldots \\ Na_{m1} & Na_{m2} \ldots Na_{mn} \end{bmatrix} \quad NB = \begin{bmatrix} Nb_1 \\ Nb_2 \\ \cdot\cdot \\ Nb_m \end{bmatrix} \quad NC = \begin{bmatrix} Nc_1 \\ Nc_2 \\ \cdot\cdot \\ Nc_n \end{bmatrix} \quad \mathrm{X} = \begin{bmatrix} x_1 \\ x_2 \\ \cdot\cdot \\ x_n \end{bmatrix}$$

2-1-3- The standard neutrosophic formula for the linear model:

The standard form plays an important role in finding a solution to linear programming problems, as the issue of searching for a solution to a linear programming problem has been transformed into the process of searching for a solution to a set of linear equations consisting of n equations with $n + m$ unknowns, and solving this sentence is useful if it is possible, i.e. If it fulfills the conditions of non-negative $x_j \geq 0$, then the optimal solution for the linear model is the ideal values of the variables that fulfill the constraints and give the objective function the greatest or smallest possible value according to the text of the problem being solved. The standard neutrosophic formula is given in the following abbreviated form:

Find:

$$NZ = \sum_{j=1}^{n} (c_j \pm \varepsilon_j) x_j \longrightarrow (Max \;\; or \; Min)$$

Constraints:

$$\sum_{j=1}^{n} Na_{ij} x_j = b_i \pm \delta_i \quad ; \;\; i = 1,2, \dots, m$$

$$x_j \geq 0$$

It is given in the following detailed form:
Find:

$$NZ = Nc_1 x_1 + Nc_2 x_2 + \cdots + Nc_n x_n \longrightarrow (Max \;\; or \; Min)$$

Constraints:

$$Na_{11} x_1 + Na_{12} x_2 + \cdots + \mathrm{N}a_{1n} x_n = Nb_1$$

$$Na_{21} x_1 + Na_{22} x_2 + \cdots + Na_{2n} x_n = Nb_2$$

$$\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots$$

$$Na_{m1} x_1 + Na_{m2} x_2 + \cdots + Na_{mn} x_n = Nb_m$$

$$x_1, x_2, \dots, x_n \geq 0$$

Linear models can also be expressed using matrices, and therefore the neutrosophic linear model given in the standard form can be written using matrices as follows:

Find:

$$NZ = NC\,X \longrightarrow (Max \;\; or \; Min)$$

Constraints:

$$NA\,X = NB$$

$$X \geq 0$$

Where:

$$NA = \begin{bmatrix} Na_{11} & Na_{12} \dots Na_{1n} \\ Na_{21} & Na_{22} \dots Na_{2n} \\ \dots \dots \dots \dots \dots \dots \dots & \\ Na_{m1} & Na_{m2} \dots Na_{mn} \end{bmatrix} \; NB = \begin{bmatrix} Nb_1 \\ Nb_2 \\ .. \\ Nb_m \end{bmatrix} \quad NC = \begin{bmatrix} Nc_1 \\ Nc_2 \\ .. \\ Nc_n \end{bmatrix} \; X = \begin{bmatrix} x_1 \\ x_2 \\ .. \\ x_n \end{bmatrix}$$

Here we note that all constraints are of the equality type, except for the non-negative constraints, which remain inequalities. Also, the right side of each equality constraint must be non-negative, and all decision variables must be non-negative. The objective function in the standard Neutrosophic form can be a maximization function or a minimization function.

2-1-4- The symmetrical formula of the neutrosophic linear model:

We say of a linear program that it is in the symmetrical form if all variables are constrained to be non-negative and if all constraints are given in the form of inequalities , the inequalities of the constraints of the maximization problem must be in the form ($\leq$) less than or equal to, while the inequalities of the constraints in the minimization problem must be In the form ($\geq$) is greater than or equal to, then we write the neutrosophic symmetric formula in one of the following two forms:

First figure:

The neutrosophic symmetric formula for the linear mathematical model is given in the abbreviated form as follows:

$$NZ = \sum_{j=1}^{n} (c_j \pm \varepsilon_j) x_j \longrightarrow Max$$

Constraints:

$$\sum_{j=1}^{n} Na_{ij} x_j \leq b_i \pm \delta_i \quad ; \; i = 1,2, \dots , m$$

$$x_j \geq 0$$

It is given in the following detailed form:

Find:

$$NZ = Nc_1 x_1 + Nc_2 x_2 + \cdots + Nc_n x_n \longrightarrow Max$$

Constraints:

$$Na_{11} x_1 + Na_{12} x_2 + \cdots + \mathrm{N}a_{1n} x_n \leq Nb_1$$

$$Na_{21} x_1 + Na_{22} x_2 + \cdots + Na_{2n} x_n \leq Nb_2$$

$$\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots$$

$$Na_{m1} x_1 + Na_{m2} x_2 + \cdots + Na_{mn} x_n \leq Nb_m$$

$$x_1, x_2, \dots , x_n \geq 0$$

Using matrices as follows:

Find:

$$NZ = NC \, \mathrm{X} \longrightarrow Max$$

Constraints:

$$NA\,\mathrm{X} \leq NB$$

$$\mathrm{X} \geq 0$$

Where:

$$NA = \begin{bmatrix} Na_{11} & Na_{12} \ldots Na_{1n} \\ Na_{21} & Na_{22} \ldots Na_{2n} \\ \ldots \ldots \ldots \ldots & \ldots \ldots \ldots \\ Na_{m1} & Na_{m2} \ldots Na_{mn} \end{bmatrix} NB = \begin{bmatrix} Nb_1 \\ Nb_2 \\ .. \\ Nb_m \end{bmatrix} \quad NC = \begin{bmatrix} Nc_1 \\ Nc_2 \\ .. \\ Nc_n \end{bmatrix} \quad \mathrm{X} = \begin{bmatrix} x_1 \\ x_2 \\ .. \\ x_n \end{bmatrix}$$

Second form:

The summary is as follows:

The neutrosophic symmetric formula for the linear mathematical model is given in the abbreviated form as follows:

$$NZ = \sum_{j=1}^{n} (c_j \pm \varepsilon_j) x_j \longrightarrow Min$$

Constraints:

$$\sum_{j=1}^{n} Na_{ij} x_j \geq b_i \pm \delta_i \quad ; \quad i = 1,2, \ldots, m$$

$$x_j \geq 0$$

It is given in the following detailed form:
Find:

$$NZ = Nc_1 x_1 + Nc_2 x_2 + \cdots + Nc_n x_n \longrightarrow Min$$

Constraints:

$$Na_{11} x_1 + Na_{12} x_2 + \cdots + \mathrm{N}a_{1n} x_n \geq Nb_1$$

$$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n \geq Nb_2$$

$$\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$

$$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n \geq Nb_m$$

$$x_1, x_2, \ldots, x_n \geq 0$$

Using matrices as follows:

Find:

$$NZ = NC\,\mathrm{X} \longrightarrow Min$$

Constraints:

$$NA\,\mathrm{X} \geq NB$$

$$\mathrm{X} \geq 0$$

Where:

$$NA = \begin{bmatrix} Na_{11} & Na_{12} \ldots Na_{1n} \\ Na_{21} & Na_{22} \ldots Na_{2n} \\ \ldots \ldots \ldots \ldots \ldots \ldots \ldots & \\ Na_{m1} & Na_{m2} \ldots Na_{mn} \end{bmatrix} \quad NB = \begin{bmatrix} Nb_1 \\ Nb_2 \\ .. \\ Nb_m \end{bmatrix} \quad NC = \begin{bmatrix} Nc_1 \\ Nc_2 \\ .. \\ Nc_n \end{bmatrix} \quad \mathrm{X} = \begin{bmatrix} x_1 \\ x_2 \\ .. \\ x_n \end{bmatrix}$$

2-2- How to move from one formula to another:

Some presentation of the formulas of the neutrosophic linear models. It should be noted that we can move from one formula to another by following the following elementary transformations:

- Converting the minimum value of the objective function f(x) to a maximum value by multiplying it by (-1) we get (- ($f(x)$)).
- If the inequalities were of the form (greater than or equal to) they will be converted to the form (less than

or equal to) by multiplying both sides by (-1), and vice versa.

- The equality constraint can be converted into two inequalities of different direction.
- If the left side of an (inequality) constraint is given in absolute value, it can be converted into two regular inequalities.
- Constraint inequalities of the type (greater than or equal to) are converted to an equality constraint by subtracting an appropriate positive variable (i.e., artificial variable) from the left side of the inequality and this variable is entered into the objective function with zero coefficient.
- Constraint inequalities of the type (less than or equal to) are converted into an equality constraint by adding an appropriate positive variable (i.e., slack variable) to the left-hand side of the inequality and then this variable is entered into the objective function with zero coefficient.
- If one of the decision variables x is not constrained by the non-negative condition (that is, it can be negative, positive or zero), then it can be expressed as the difference between two non-negative variables x', x'' as follows $x = x' - x''$ and $x', x'' \geq 0$

2-3- Examples of the above:

The linear models in all examples are given in detailed form:

Example 1:

Call the following neutrosophic linear programming in its general form:

$$Min\ NL = (3 \pm \varepsilon_1)x_1 - (3 \pm \varepsilon_2)x_2 + (7 \pm \varepsilon_3)x_3$$

Constraints:

$$x_1 + x_2 + 3x_3 \leq 40 \pm \delta_1$$
$$x_1 + 9x_2 - 7x_3 \geq 50 \pm \delta_2$$
$$5x_1 + 3x_2 = 20 \pm \delta_3$$
$$|5x_2 + 8x_3| \leq 100 \pm \delta_4$$
$$x_1, x_2 \geq 0$$

Where ε_j It is indeterminate and could be

$$\varepsilon_j \in [\lambda_{1j}, \lambda_{2j}] \text{ or } \varepsilon_j \in \{\lambda_{1j}, \lambda_{2j}\} ; j = 1,2,3.$$

Also, the values that express the available possibilities δ_i are neutrosophic values. This means that
It is indeterminate and could be

$$\delta_i \in [\mu_{1i}, \mu_{2i}] \text{ or } \delta_i \in \{\mu_{1i}, \mu_{2i}\} ; i = 1,2,3,4$$

To convert the above problem into the neutrosophic canonical form, we perform the following transformations:

• The objective function is a function of minimization that we turn into a function of maximization:

$$Min\ NL = (3 \pm \varepsilon_1)x_1 - (3 \pm \varepsilon_2)x_2 + (7 \pm \varepsilon_3)x_3$$

Transformed into:

$$Max\ NZ = -(3 \pm \varepsilon_1)x_1 + (3 \pm \varepsilon_2)x_2 - (7 \pm \varepsilon_3)x_3$$

• The second constraint is given (greater than or equal to) is converted into (less than or equal) by multiplying both sides by (-1) we get:

$$-x_1 - 9x_2 + 7x_3 \leq -(50 \pm \delta_2)$$

• Third constraint $5x_1 + 3x_2 = 20 \pm \delta_3$ transformed into two entries:

$$5x_1 + 3x_2 \leq 20 \pm \delta_3$$
$$5x_1 + 3x_2 \geq 20 \pm \delta_3$$

Then we turn the constraint $5x_1 + 3x_2 \geq 20 \pm \delta_3$ into:

$$-5x_1 - 3x_2 \leq -(20 \pm \delta_3)$$

• The constraint $|5x_2 + 8x_3| \leq 100 \pm \delta_4$ is equivalent to the two inequalities:

$$5x_2 + 8x_3 \leq 100 \pm \delta_4$$
$$-5x_2 - 8x_3 \leq 100 \pm \delta_4$$

• The variable x_3 is not restricted by the non-negative constraint, so it is replaced by the following assumption

$x_3 = x_3' - x_3''$ where $x_3', x_3'' \geq 0$.

The canonical neutrosophic form becomes:

$$Max\ NZ = -(3 \pm \varepsilon_1)x_1 + (3 \pm \varepsilon_2)x_2 - (7 \pm \varepsilon_3)(x_3' - x_3'')$$

Constraints:

$$x_1 + x_2 + 3(x_3' - x_3'') \leq 40 \pm \delta_1$$
$$-x_1 - 9x_2 + 7(x_3' - x_3'') \leq -(50 \pm \delta_2)$$
$$5x_1 + 3x_2 \leq 20 \pm \delta_3$$
$$-5x_1 - 3x_2 \leq -(20 \pm \delta_3)$$
$$5x_2 + 8(x_3' - x_3'') \leq 100 \pm \delta_4$$
$$-5x_2 - 8(x_3' - x_3'') \leq 100 \pm \delta_4$$

$$x_1, x_2, x_3', x_3'' \geq 0$$

Example 2:

A factory produces four types of products $S_1, S_2,, S_3, S_4$. For this purpose, the following raw materials are used M_1, M_2, M_3 . The factory management wants to study the optimal organization of production during a period of time (for example, a month) and determine the monthly production for each product in order to achieve a maximum profit, bearing in mind that the profit is directly proportional to the number of units sold of the products. The available quantities of raw materials needed for each product and the profit have been showed in the following table:

Products / Materials	Product Type				Available Quantities
	S_1	S_2	S_3	S_4	
M_1	1.5	1	2.4	1	$3000 \pm \delta_1$
M_2	1	5	1	3.5	$9000 \pm \delta_2$
M_3	1.5	3	3.5	1	$7000 \pm \delta_3$
win one product	$4 \pm \varepsilon_1$	$8 \pm \varepsilon_2$	$5 \pm \varepsilon_3$	$6 \pm \varepsilon_4$	

Let us suppose $x_1, x_2,\ x_3, x_4$ are the numbers of units produced from the types Products, during the production period (a month for example), and accordingly, the consumed quantity of the raw material M_1 in the production of the four varieties will be:

$$1.5x_1 + x_2 + 2.4x_3 + x_4$$

And it must not exceed $3000 \pm \delta_1$ from the available quantity, that is:

$$1.5x_1 + x_2 + 2.4x_3 + x_4 \leq 3000 \pm \delta_1 \qquad (1)$$

Likewise, the amount of raw material M_2 consumed in the production of the four types is:

$$x_1 + 5x_2 + x_3 + 3.5x_4 \leq 9000 \pm \delta_2 \qquad (2)$$

And the amount consumed of the raw material M_3 in the production of the four types is:

$$1.5x_1 + 3x_2 + 3.5x_3 + x_4 \leq 7000 \pm \delta_3 \qquad (3)$$

In addition, the produced quantities must be non-negative, i.e.:

$$x_1, x_2, x_3, x_4 \geq 0 \qquad (4)$$

And they are what is called non-negative conditions.

Thus, we have identified all the constraints imposed on the variables of the problem.

We now define the objective function. If quantified units x_1, x_2, x_3, x_4 of species are produced in order, then the profit during the productive period will be:

$$NZ = (4 \pm \varepsilon_1)x_1 + (8 \pm \varepsilon_2)x_2 + (5 \pm \varepsilon_3)x_3 + (6 \pm \varepsilon_4)x_4$$

It represents the objective function. Therefore, the mathematical model of the problem is:

$$Max\ NZ = (4 \pm \varepsilon_1)x_1 + (8 \pm \varepsilon_2)x_2 + (5 \pm \varepsilon_3)x_3 + (6 \pm \varepsilon_4)x_4$$

Constraints:

$$1.5x_1 + x_2 + 2.4x_3 + x_4 \leq 3000 \pm \delta_1$$
$$x_1 + 5x_2 + x_3 + 3.5x_4 \leq 9000 \pm \delta_2$$
$$1.5x_1 + 3x_2 + 3.5x_3 + x_4 \leq 7000 \pm \delta_3$$
$$x_1, x_2, x_3, x_4 \geq 0$$

We have obtained a neutrosophical canonical linear model using the appropriate transformations, which can be written in the following neutrosophical standard form:

$$Max\ NZ = (4 \pm \varepsilon_1)x_1 + (8 \pm \varepsilon_2)x_2 + (5 \pm \varepsilon_3)x_3 + (6 \pm \varepsilon_3)x_4 + 0y_1 + 0y_2 + 0y_3$$

Constraints:

$$1.5x_1 + x_2 + 2.4x_3 + x_4 + y_1 = 3000 \pm \delta_1$$
$$x_1 + 5x_2 + x_3 + 3.5x_4 + y_2 = 9000 \pm \delta_2$$
$$1.5x_1 + 3x_2 + 3.5x_3 + x_4 + y_3 = 7000 \pm \delta_3$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Conclusion:

The indeterminacy that we added to the data described by the linear model provides us with neutrosophical linear models that simulate reality and take into account most of the changes that could occur in the operating environment of the system represented by the linear mathematical model, and enable us to continue studying linear programming topics such as finding accompanying programs that need to be developed. The mathematical model in the symmetrical form, solving linear models using the simplex method that requires developing models in the standard form, and other linear programming topics.

Chapter III
The graphical method for finding the optimal solution for neutrosophic linear models

Chapter III

The graphical method for finding the optimal solution for neutrosophic linear models

Introduction:

After introducing the linear models and their different formulas based on the concepts of neutrosophic science, in this chapter we present the neutrosophic graphical method that we use to solve the neutrosophic linear models.

The graphical method represents the model graphically and is one of the simplest methods for solving linear programming problems. However, it is not sufficient to solve all linear programming problems, as linear programming problems often contain a large number of variables, and the use of the graphical method is limited to the following cases:

- The number of unknowns is n = 1, or n = 2, or n = 3.
- In linear models whose constraints are equal constraints, if the number of unknowns and the number of equations meet one of the following conditions: n – m = 1 or n – m = 2 or n – m = 3.

Here we can transform the model into a function of one variable, or two variables, or three variables, by using the non-negative constraints that the variables of the linear model have.

In this research, we present a reformulation of the graphical method for solving linear models using neutrosophics, as well as the graphical method for solving linear models where the constraints are equal, and the difference between the number of unknowns and the number of constraints is equal to one, two, or three.

3.1. Graphical method for solving linear models

We find the optimal solution by following the steps below:

1. We determine the half-planes defined by the inequalities of the constraints by drawing the straight lines resulting from the transformation of the inequalities of the constraints. To do this, we specify two points that fulfill the constraint, and connect the two points to obtain the straight line that corresponds to the constraint. This straight line divides the plane into two halves to determine the half-plane that satisfies the constraint. We select a point at the top of the mapping from one of the two half-planes. We substitute the coordinates of this point into the inequality. If it is satisfied, then the region in which this point is located is the solution region. If it is not satisfied, then the opposite region is the solution region.
2. We define the common solution region, i.e., the region resulting from the intersection of the halves of the planes defined by constraint inequalities. This region must be non-empty so that we can proceed with the solution.
3. To represent the objective function, we note that its relation contains three unknowns, Z, x_1, x_2. Therefore, we need to know a value for Z that is unknown to us. Here we assume a value, let it be $Z_1 = 0$, draw the equation of the objective function Z_1 specify another value, let it be Z_2, and represent the equation. We get a line that is parallel to the first line, and if we continue like this, we get a series of parallel lines that represent the objective function.

4. We draw ray $\vec{C} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$ where c_1 is coefficient of x_1 and c_2 is coefficient of x_2 in the objective function statement, and the direction of its increasing function is the direction of ray $\vec{C} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$, and the direction of its decreasing function is the opposite direction. This ray, i.e., the drawing is done according to the type of objective function (maximization or minimization). To put it more clearly, we find the optimal solution point by drawing the line representing Z_1 parallel to itself towards the ray $\vec{C} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$ to find the maximum value of the objective function (and reversing this direction to find the smallest value), until it passes through the last point of the common solution region and this point is the optimal solution point, which is located at the boundaries of the common solution region and any other displacement, no matter how small, takes it out of it.

3.2. Graphical method for finding the optimal solution for neutrosophic linear models

From the definition of neutrosophic linear models, we find that we can apply the same previous steps to obtain the optimal solution, which is a neutrosophic value suitable for all conditions. We illustrate the above with the following example:

Example 1

A company produces two types of products A_1 ,A_2 and uses three types of raw materials B_1 ,B_2 , B_3 in the production process; the available quantities of each of the raw materials are B_i ; $i = 1,2,3$, the quantity required to produce one unit of each products A_j ; $j = 1,2$, and the profit derived from one unit of each of the products A_1 ,A_2 is shown in the following table:

products / raw materials	A_1	A_2	available quantities
B_1	6	4	36
B_2	2	3	12
B_3	5	0	10
profit	[6,8]	[2,4]	

Table Issue data

Requirement

Determine the quantities that must be produced of each product A_j ; $j = 1,2$,for the company to achieve maximum profit:

Solution:

Suppose x_j is the quantity produced from the product, where $j = 1,2$, then we can formulate the following neutrosophic linear mathematical model:

$$Z = [6,8]x_1 + [2,4]x_2 \rightarrow Max$$

Constraints:

$$6x_1 + 4x_2 \leq 36 \quad (1)$$

$$2x_1 + 3x_2 \leq 12 \quad (2)$$

$$5x_1 \leq 15$$

$$x_1, x_2 \geq 0$$

The previous model is a linear neutrosophic model because the coefficients of the variables in the objective function are undetermined. To find the optimal solution for the previous model, we will apply the graphical method according to the following steps:

The first constraint

We draw the straight line representing the first constraint:

$$6x_1 + 4x_2 = 36$$

We impose:

$$x_1 = 0 \Rightarrow 4x_2 = 36 \Rightarrow x_2 = 9$$

We get the first point: $A(0,9)$.

We impose:

$$x_2 = 0 \Rightarrow 6x_1 = 36 \Rightarrow x_1 = 6$$

We get the second point: $B(6,0)$

We take a point at the top of the designation from one of the two halves of the resulting plane after having drawn the straight line through the two points $A(0,9)$ and $B(6,0)$. Let it be the point $O(0,0)$ and substitute it in the inequality of the first entry. We find that the inequality is satisfied i.e., the half of the plane to which the point $O(0,0)$ belongs is half of the solution plane of the first-constraint inequality.

We proceed in the same way for the second and third constraints and obtain the following graphical representation: Figure No. (1):

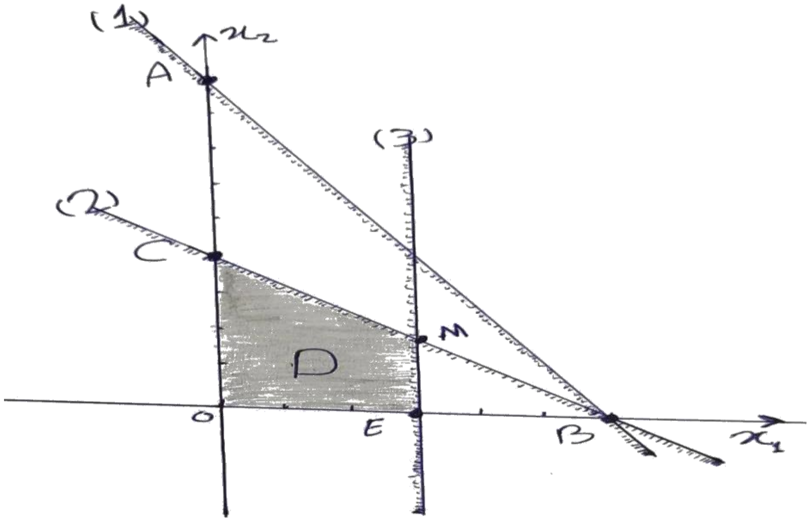

Figure No. (1) Graphic representation of the limitations of the linear model in Example 1

After we have shown the constraints, we notice that the common solution area is bounded by the polygon whose vertices are the points, $(0,0)$, $E(3,0)$, M and $C(0,4)$

The point M is the intersection point, and the second and third constraints coordinates are obtained by solving the following two equations:

$$2x_1 + 3x_2 = 12$$

$$5x_1 = 15$$

We find: $M(3,2)$

Substituting the coordinates of the vertex points into the objective function expression, we get:

$$Z_O = 0$$

$$Z_E \in [12,16]$$

$$Z_M \in [22,32]$$

$$Z_C \in [8,16]$$

This means that the highest value of function Z is reached at point (3,2),i.e., the company must produce three units of the first product and two units of the second product, then it will achieve the maximum profit.

$$Max\ Z = Z_M \in [22,32]$$

Note:

The process of substitution of the objective function by the coordinates of the points of the vertices of the common solution area is possible when the number of points is small, since we can easily substitute them in the objective function, and the point that gives the best value for the objective function is the optimal solution, but when there is a large number of constraints, we get a large number from the vertical points located on the perimeter of the common solution region. In this case, the method of determining the coordinates of all these points and substituting them into the objective function becomes impractical. Therefore, we resort to the representation of the objective function and the determination of the optimal solution point as we have already mentioned.

3.3. Non-negative constraints for optimal solution of some neutrosophic linear models using the graphical method

Example 2

Find the optimal solution for the following linear neutrosophic model:

$$Z = x_1 - x_2 - 3x_3 + x_4 + [2,5]x_5 - x_6 + 2x_7 - [10,15] \to Max$$

Constraints:

$$x_1 - x_2 + x_3 = 5 \qquad (1)$$

$$2x_1 - x_2 - x_3 - x_4 = -11 \qquad (2)$$

$$x_1 + x_2 - x_5 = -4 \qquad (3)$$

$$x_2 + x_6 = 6 \qquad (4)$$

$$2x_1 - 3x_2 - x_6 + 2x_7 = 8 \qquad (5)$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0$$

Solution:

We note that the number of constraints is $m = 5$ and the number of variables is $n = 7$, which means that $n - m = 2$. Therefore, based–on the non-negative constraints, find the optimal solution for the previous model using the graphical method according to the following steps:

1- We calculate five variables in terms of only two variables.
2- Since the variables of the linear model satisfy the non-negative constraints, then we obtain from the variables that we calculated five inequalities of the type greater than or equal to.
3- The objective function with only two variables is obtained by substituting the five variables.
4- We write the new model, which is a linear model with two variables, so that the optimal solution can be found graphically.

We apply the previous steps to Example 2.
We find:

$$x_3 = 5 - x_1 + x_2 \qquad (1)'$$

$$x_4 = 3x_1 - 2x_2 + 6 \qquad (2)'$$

$$x_5 = x_1 + x_2 + 4 \qquad (3)'$$

$$x_6 = 6 - x_2 \qquad (4)'$$

$$x_7 = 7 - x_1 + x_2 \qquad (5)'$$

Substituting in the objective function, we get:

$$Z = [1,4]x_1 + [3,6]x_2 + [8,25]$$

Since $x_3, x_4, x_5, x_6, x_7 \geq 0$ from (1)', (2)', (3)', (4)', (5)', we get the following set of constraints:

$$5 - x_1 + x_2 \geq 0$$

$$-3x_1 + 2x_2 - 3 \geq 0$$

$$x_1 + x_2 + 4 \geq 0$$

$$6 - x_2 \geq 0$$

$$7 - x_1 + x_2 \geq 0$$

Neutrosophic linear mathematical model:

Find:

$$Z = [1,4]x_1 + [3,6]x_2 + [8,25] \rightarrow Max$$

Constraints:

$$5 - x_1 + x_2 \geq 0$$

$$3x_1 - 2x_2 + 6 \geq 0$$

$$x_1 + x_2 + 4 \geq 0$$

$$6 - x_2 \geq 0$$

$$7 - x_1 + x_2 \geq 0$$

$$x_1, x_2 \geq 0$$

The model has two variables, so the optimal solution can be found graphically by following the same steps indicated in Example (1).

The required graphic representation is found in Figure No. (2):

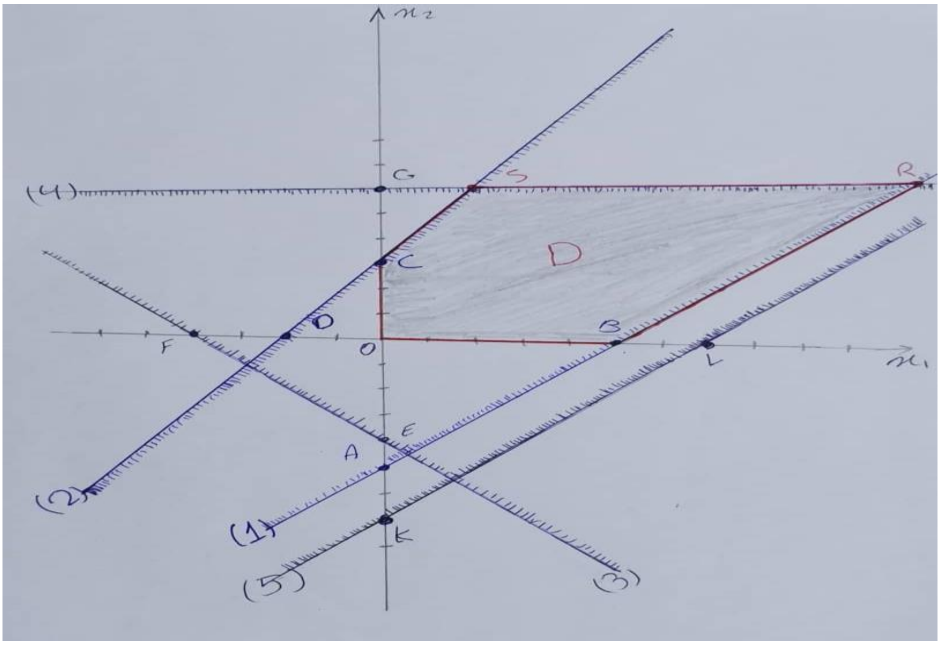

Figure No. (2): Graphical representation of the constraints of the linear model in Example 2

Region D is the region of joint solutions and is defined by the polygon $OBRSC$, where,$O(0,0)$, $(5,0)$, $C(0,3)$, and for the two points R , S we find:

Point R is the point of intersection of the first and fourth entries.

We obtain its coordinates by solving the set of equations:

$$5 - x_1 + x_2 = 0$$

$$6 - x_2 = 0$$

We get: $R(11,6)$

Point S is the point of intersection of the second and fourth entries.

We obtain its coordinates by solving the set of equations:

$$3x_1 - 2x_2 + 6 = 0$$

$$6 - x_2 = 0$$

We get: $S(2,6)$

Since the optimal solution is located at one of the vertices of the common solution region, we substitute the coordinates of these points with the objective function:

At point $O(0,0)$

$$Z_O = 0$$

At point $B(5,0)$

$$Z_B = [13,45]$$

At point $R(11,6)$

$$Z_R \in [37,105]$$

At point $S(2,6)$

$$Z_S \in [28,69]$$

At point $C(0,3)$

$$Z_C \in [17,43]$$

The greatest value of the objective function is at the point $R(11,6)$ that is $x_1 = 11$ and $x_2 = 6$.

We calculate the values of the remaining variables by (1)', (2)', (3)', (4)', (5)'.

We find: $x_3 = 0$, $x_4 = 27$, $x_5 = 21$, $x_6 = 0$, $x_7 = 2$.

Substituting in the objective function of the original model we obtain the maximum value of the Z function, which is:

$$MaxZ \in [68,126]$$

Important Notes:

1- A vertical point in space R^n is covered by the graphical solution. The ideal solution pertains to a vertical point, which is the outcome of several lines or planes intersecting, therefore the number of non-existent components is at least $n - m$ components.

2- Certain conditions that are irrelevant to the solution process might be included in the model.

3- When one of the sides of the common solution area that passes through the ideal solution point is parallel to the straight-line Z=0, the ideal solution can be a single point or an infinite number of points. Thus, when the objective function is represented by a straight line, this line will apply to the parallel side, and all of the infinitely many points on that side will be perfect solutions.

4- We say that the objective function has an endless number of acceptable solutions that offer us greater values of Z if the region of acceptable solutions is open in terms of growing the function Z, meaning that we cannot stop at a particular perfect solution.

5- The situation in which there is no perfect (acceptable) solution and the zone of alternatives is an empty set (the problem is impossible to solve) when the conditions conflict.

Conclusion:

This chapter covered both the graphical method and a method that is rarely covered in references on classical operations research: using non-negative constraints to graphically find the optimal solution for some neutrosophic linear models. However, it should be noted that there are some neutrosophic linear models that have two variables. In these cases, it may be difficult to reach the common solution region or to determine the optimal solution once the common solution has been found, so it is preferable to use the simplex neutrosophic method. The researcher must choose the best approach for the model he wishes to solve because the main objective is to arrive at the best solution.

Chapter IV

The simplex direct neutrosophic algorithm for finding the optimal solution for linear models

Introduction.

4-1- The neutrosophic linear models set in the symmetrical form and of the Max type.

4-2- The neutrosophic linear models are in symmetric form and are of type Min.

Conclusion.

Chapter IV

The simplex direct neutrosophic algorithm for finding the optimal solution for linear models

Introduction:

Linear programming is the method that helps in selecting decisions and approving the best program for independent activities, taking into account the available resources. Linear programming is used in solving problems in which the goal is specific, such as securing a maximum profit, securing a minimum cost, or saving the greatest time or effort … Etc., noting that the problem of linear programming consisting of a linear function and knowledge of a set of inequalities or equations (constraints) is characterized by the presence of a large number of acceptable non-negative solutions, and what is required is to find the optimal solution from among these solutions. To reach this solution, we rely on the information that we presented it in the first chapter when we studied non-negative solutions to the system of neutrosophic linear equations. Then we used the simplex method, which is the mathematical basis for the direct simplex algorithm used to find the optimal solution for the linear models that we will present in this chapter.

Direct simplex algorithm for solving neutrosophic linear models:

The direct simplex algorithm consists of three stages:

a- The stage of converting the imposed model into an equivalent systematic form.

b- The stage of converting the regular form into a basic form to obtain the non-negative basic solutions.
c- The stage of searching for the optimal solution required from among the non-negative basic solutions.

In this chapter, we will use the direct simplex algorithm to find the optimal solution for the neutrosophic linear models, which were presented in the second chapter of this book, and here we distinguish the following cases:

1- The neutrosophic linear models are in symmetric form and are of type Max.
2- The neutrosophic linear models are in symmetric form and are of type Min.
3- The neutrosophic linear models are given in the general form.

Using the direct simplex method to find the optimal solution:

4-1- The neutrosophic linear models set in the symmetrical form and of type Max:

The neutrosophic linear model of type Max is written in the symmetrical form, as we mentioned in the second chapter, in the following form:

Find:

$$NZ = Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n \longrightarrow Max$$

Constraints:

$$Na_{11}x_1 + Na_{12}x_2 + \cdots + \mathrm{N}a_{1n}x_n \leq Nb_1$$

$$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n \leq Nb_2$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n \leq Nb_m$$

$$x_1, x_2, \ldots, x_n \geq 0$$

The study is carried out according to the following steps:

1- We write the model in standard form; we get:

Find:

$$NZ = Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n + 0.y_1 + 0.y_2 + \cdots + 0.y_m \longrightarrow Max$$

Constraints:

$$Na_{11}x_1 + Na_{12}x_2 + \cdots + \mathrm{N}a_{1n}x_n + y_1 = Nb_1$$

$$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n + y_2 = Nb_2$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n + y_m = Nb_m$$

$$x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_m \geq 0$$

2- We convert the model to the basic form. Here we notice that the additional variables serve as an initial base from which we start searching for the optimal solution. Therefore, we arrange the model information in the following table:

Variables / **basic**	x_1	x_2	……	x_n	y_1	y_1	...	y_m	**Available quantities**
y_1	Na_{11}	Na_{12}	...	Na_{1n}	1	0	...	0	Nb_1
y_2	Na_{21}	Na_{22}	...	Na_{2n}	0	1	...	0	Nb_2
...	...	...	...	...	...	...	...	...	...
y_m	Na_{m1}	Na_{m2}	...	Na_{mn}	0	0	...	1	Nb_m
objective function	Nc_1	Nc_2	...	Nc_n	0	0	0	0	$Z - 0$

Table No. (1): Basic information of the model

We have a first base consisting of the variables $y_1, y_2, \ldots, y_m$, then the variables $x_1, x_2, \ldots, x_n$are non-base variables and we move to the next step:

3- We determine the appropriate variable from the equations and insert it into the rule by studying examples of the variables in the row of the objective function Z. Since the objective function is a maximization function, we select the largest positive values in the row of the objective function. In other words, we take:

$$Max(Nc_1, Nc_2, \ldots, Nc_n) = Nc_s$$

For example, let it be Nc_s corresponding to the variable x_s.

Thus, we have determined the pivot column. This means that the variable x_s will enter the base to determine the variable that will exit from the base, and therefore the pivot line. We calculate the following indicator:

$$\theta \in Min\left[\frac{Nb_i}{Na_{is}}\right] = \frac{b_{tN}}{Na_{ts}} > 0; \quad Na_{is} > 0, Nb_i > 0$$

The element located at the intersection of the pivot column with the pivot row is the pivot element.

- We divide the pivot row by the pivot element, we get:

$$\frac{Na_{t1}}{Na_{ts}}, \frac{Na_{t2}}{Na_{ts}}, \ldots, \frac{Na_{ts-1}}{Na_{ts}}, 1, \frac{Na_{ts+1}}{Na_{ts}}, \ldots, \frac{Na_{tn}}{Na_{ts}}, \ldots, \frac{Nb_t}{Na_{ts}}$$

- We make all the elements of the pivot column equal to zeros, except for the pivot element, which is equal to one.
- We perform the appropriate calculations to calculate the current of the new table using the following relations:

$$Na'_{ij} = Na_{ij} - Na_{tj}\frac{Na_{is}}{Na_{ts}} = \frac{Na_{ij}Na_{ts} - Na_{tj}Na_{is}}{Na_{ts}}$$

$$Nb'_{i} = Nb_{i} - Nb_{t}\frac{Na_{is}}{Na_{ts}} = \frac{Nb_{i}Na_{ts} - Nb_{t}Na_{is}}{Na_{ts}}$$

$$Nc'_{j} = Nc_{j} - Nc_{s}\frac{Na_{tj}}{Na_{ts}} = \frac{Nc_{j}Na_{ts} - Nc_{s}Na_{tj}}{Na_{ts}}$$

We get the following table:

Variables / basic	x_1	x_2	…	x_{s-1}	x_s	…	x_n	y_1	y_1	…	y_m	Available quantities
y_1	Na'_{11}	Na'_{12}	…	Na'_{1s-1}	0	…	Na'_{1n}	1	0	…	0	Nb'_1
y_2	Na_{21}	Na_{22}	…	Na'_{2s-1}	0	…	Na_{2n}	0	1	…	0	Nb'_2
…	…	…	…	…	0	…	…	…	…	…	…	…
x_s	$\frac{Na_{t1}}{Na_{ts}}$	$\frac{Na_{t2}}{Na_{ts}}$	…	$\frac{Na_{ts-1}}{Na_{ts}}$	1	…	$\frac{Na_{tn}}{Na_{ts}}$	0	0	…	0	$\frac{Nb_t}{Na_{ts}}$
…	…	…	…	…	0	…	…	…	…	…	…	…
y_m	Na_{m1}	Na_{m2}	…	Na'_{ms-1}	0	…	Na_{mn}	0	0	…	1	Nb'_m
objective function	Nc'_1	Nc'_2	…	Nc'_{s-1}	0	…	Nc'_n	0	0	0	0	NZ'

Table No. (2) The first step in the simplex direct neutrosophic algorithm

- We apply the stopping criterion of the Simplex algorithm to the objective function row in Table No. (2).

Stopping criterion:

Since the objective function is of the maximize type, the objective function row in the table must not contain any positive value, (but if the objective function is of the minimization type, the objective function row in the new table must not contain any negative value), in If the criterion is not met, we return to step No. (3) and repeat the same steps until the stopping criterion is met and we obtain the required optimal solution. Thus, we obtain new non-negative neutrosophic basic solutions and non-basic (free) solutions equal to zero. The ideal solution is written in the following form:

$$\left(Nb_1^{'}, Nb_2^{'}, \dots, Nb_m^{'}, 0,0, \dots, 0\right)$$

The following table represents the final solution if the basic solutions are: $(x_1, x_2, \dots, x_m)$

variables / basic	x_1	x_2	…	x_m	x_{m+1}	…	x_n	y_1	y_2	…	y_m	Available quantities
x_1	1	0	…	0	Na'_{1m+1}	…	Na'_{1n}	$N\beta_{11}$	$N\beta_{11}$	…	$N\beta_{1m}$	Nb'_1
x_2	0	1	…	0	Na'_{2m+1}	…	Na_{2n}	$N\beta_{21}$	$N\beta_{22}$	…	$N\beta_{2m}$	Nb'_2
…	…	…	…	0	0	…	…	…	…	…	…	…
x_m	0	0	…	1	$Na'_{m,m+1}$	…	Na_{mn}	$N\beta_{m1}$	$N\beta_{m2}$	…	$N\beta_{mm}$	Nb'_m
objective function	0	0	…	0	Nc'_{m+1}	…	Nc'_n	Nq_1	Nq_2	…	Nq_m	NZ'

Table No. (3) The final solution in the simplex direct neutrosophic algorithm

Where $N\beta_{ii}$ and Nq_1 are the examples of the additional variables in the constraints and in the objective function after performing the aforementioned iterative operations, the optimal solution is:

$$x_1 = Nb'_1, x_2 = Nb'_2, \dots, x_m = Nb'_m$$

Which gives the maximum value of the following objective function:

$$NZ' = Nc_1 Nb'_1 + Nc_2 Nb'_2 + \dots + Nc_m Nb'_m$$

We explain the above using the following example:

Example1:

Problem Statements Classical Values:

A company produces two types of products A, B using four raw materials F_1, F_2, F_3, F_4. The quantities needed from each of these materials to produce one unit of each of the two producers A, B, the available quantities of the raw materials, and the profit returned from one unit of both products are shown in the following table:

Products / Raw Materials	Required quantity per unit		Available quantities of the raw materials
	A	***B***	
F_1	2	3	19
F_2	2	1	13
F_3	0	3	15
F_4	3	0	18
Profit Returned per unit	7	5	

Table No. (4) Classic data for the issue

Required:

Finding the optimal production plan that makes the company's profit from the producers A, B as large as possible.

We symbolize the quantities produced from the product A with the symbol x_1, and from the product B with the symbol x_2 , after building the appropriate mathematical model and solving it, we conclude that $x_1 = 5, x_2 = 3$, and hence the maximum profit $MaxZ = 50$ of monetary unit.

Problem Statements neutrosophic Values:

A company produces two types of products A, B using four raw materials F_1, F_2, F_3, F_4. The quantities needed from each of these materials to produce one unit of each of the two producers A, B, the available quantities of the raw materials, and the profit returned from one unit of both products are shown in the following table:

Raw Materials \ Products	Required quantity per unit		Available quantities of the raw materials
	A	B	
F_1	2	3	[14,20]
F_2	2	1	[10,16]
F_3	0	3	[12,18]
F_4	3	0	[15,21]
Profit Returned per unit	[5,8]	[3,6]	

Table No. (5) Neutrosophic data for the issue

Required:

Finding the optimal production plan that makes the company's profit from the producers A, B as large as possible.

Symbolize the quantities produced from the product A with the symbol x_1, and from the product B with the symbol x_2 , the problem will be reformulated from the neutrosophic perspective as follow:

$$NZ = [5,8]\, x_1 + [3,6]\, x_2 \longrightarrow Max$$

Constraints:

$$2x_1 + 3x_2 + y_1 = [14,20]$$
$$2x_1 + x_2 + y_2 = [10,16]$$
$$3x_2 + y_3 = [12,18]$$
$$3x_1 + y_4 = [15,21]$$

$$x_1, x_2 \geq 0$$

The above program needs to reformulated to an equivalent form by adding slack variables:

$$NZ = [5,8]\, x_1 + [3,6]\, x_2 + 0y_1 + 0y_2 + 0y_3 + 0y_4 \longrightarrow Max$$

Constraints:

$$2x_1 + 3x_2 + y_1 = [14,20]$$
$$2x_1 + x_2 + y_2 = [10,16]$$
$$3x_2 + y_3 = [12,18]$$
$$3x_1 + y_4 = [15,21]$$
$$x_1, x_2, y_1, y_2, y_3, y_4 \geq 0$$

We arrange the previous information in the following table:

Variables / basic	x_1	x_1	y_1	y_2	y_3	y_4	Available quantities
y_1	2	3	1	0	0	0	[14,20]
y_2	2	1	0	1	0	0	[10,16]
y_3	0	3	0	0	1	0	[12,18]
y_4	3	0	0	0	0	1	[15,21]
objective function	[5,8]	[3,6]	0	0	0	0	$NZ - 0$

Table No. (6): The first step in the simplex method

- We note that the additional variables form an initial base consisting of the variables (y_1, y_2, y_3, y_4). Then we consider the variables (x_1, x_2) are non-basic variables and we move to the next step:
- We determine the appropriate variable from the equations and insert it into the rule by studying the examples of the variables included in the expression for the objective function NZ. Since the objective function is a maximization function, we choose the variable with the largest positive examples from the last row in the table, that is, from the row of the objective function. In other words, we take

$$\text{Max}([5,8], [3,6]) = [5,8]$$

It is clear that versus to the column of x_1 , meaning that the variable x_1 should be placed instead of one of the basic variables. Now to demonstrate which basic variables should be ejected, the following calculation has been done:

$$\theta \in Min\left[\frac{[14,20]}{2},\frac{[10,16]}{2},\frac{[15,21]}{3}\right]=\frac{[15,21]}{3}=[5,7]$$

The value of θ indicates that the row versus to the variable y_4, and the element positioned in the cross row/column is 3 which is the pivot element, divide the elements of the row versus to y_4 yields:

$$\frac{3}{3},\frac{0}{3},\frac{0}{3},\frac{0}{3},\frac{0}{3},\frac{1}{3},\frac{[15,21]}{3}=[5,7]$$

Then we make all the elements of the pivot column equal to zero, except for the pivot element, which is equal to one. We perform the appropriate calculations using the following relations:

$$Na'_{ij}=Na_{ij}-Na_{tj}\frac{Na_{is}}{Na_{ts}}=\frac{Na_{ij}Na_{ts}-Na_{tj}Na_{is}}{Na_{ts}}$$

$$Nb'_{i}=Nb_{i}-Nb_{t}\frac{Na_{is}}{Na_{ts}}=\frac{Nb_{i}Na_{ts}-Nb_{t}Na_{is}}{Na_{ts}}$$

$$Nc'_{j}=Nc_{j}-Nc_{s}\frac{Na_{tj}}{Na_{ts}}=\frac{Nc_{j}Na_{ts}-Nc_{s}Na_{tj}}{Na_{ts}}$$

We obtain the following table:

variables / basic	x_1	x_2	y_1	y_2	y_3	y_4	Available quantities
y_1	0	3	1	0	0	$\frac{-2}{3}$	[4,6]
y_2	0	1	0	1	0	$\frac{-2}{3}$	[0,4]
y_3	0	3	0	0	1	0	[12,18]
x_1	1	0	0	0	0	$\frac{1}{3}$	[5,7]
objective function	0	[3,6]	0	0	0	$[\frac{-8}{3},\frac{-5}{3}]$	$NZ - [25,56]$

For No. (7), the second step is the simplex method

It still there is a non-negative value in the row of the objective function (i.e., [3,6]) which is versus to the x_2 column, this leads to the fact that the variable x_2 should be entered into the basic variables. Now, the question is: which basic variable should be ejected? Track the following calculation to answer this question,

$$\theta \in \text{Min}\left[\frac{[4,6]}{3},\frac{[0,4]}{1},\frac{[12,18]}{3}\right] = \frac{[4,6]}{3} = [\frac{4}{3},2]$$

which is versus to the slack variable y_1, the pivot element equal 3, hence the row versus to y_1 should be divided by 3 , the required calculations yield the following table:

Variables / basic	x_1	x_2	y_1	y_2	y_3	y_4	Available quantities
x_2	0	1	$\frac{1}{3}$	0	0	$\frac{-2}{9}$	$[\frac{4}{3},2]$
y_2	0	0	$\frac{1}{3}$	1	0	$\frac{-4}{9}$	$[\frac{-4}{3},2]$
y_3	0	0	-1	0	1	$\frac{2}{3}$	[8,12]
x_1	1	0	0	0	0	$\frac{1}{3}$	[5,7]
objective function	0	0	[-2, -1]	0	0	$[\frac{-6}{9},-1]$	$NZ - [29,68]$

Table No. (8) Final solution

It is clear from the row of the objective function that all the elements are either zero or neutrosophic negative numbers, this means that we have reached to the optimal solution is:

$$x_1^* \in [5,7], x_2^* \in \left[\frac{4}{3},2\right], y_2^* \in \left[\frac{-4}{3},2\right], y_3^* \in [8,12], y_1^* = y_4^* = 0$$

Substitute the above optimal solution into the objective maximum function, yields:

$$MaxNZ \in [5,8].[5,7] + [3,6].\left[\frac{4}{3},2\right] = [25,56] + [4,12] = [29,68]$$

which is identical to the result in the previous tableau.

Substituting the optimal solution into the constraints we find:

$$2[5,7] + 3\left[\frac{4}{3},2\right] + 0 = [14,20]$$

$$2[5,7] + \left[\frac{4}{3},2\right] + \left[\frac{-4}{3},2\right] = [10,16]$$

$$3\left[\frac{4}{3},2\right] + [8,12] = [12,18]$$

$$3[5,7] + 0 = [15,21]$$

We observe that the optimal solution satisfies all constraints.

We summarize the previous results in the following table:

Classical logic								
issue data						results		
c_1	c_2	b_1	b_2	b_3	b_4	x_1	x_2	$Max\ Z$
7	5	19	13	15	18	5	3	50
Neutrosophic logic								
issue data						results		
c_{1N}	c_{2N}	b_{1N}	b_{2N}	b_{3N}	b_{4N}	x_{1N}	x_{2N}	$Max\ NZ$
[5,8]	[3,6]	[14,20]	[10,16]	[12,18]	[15,21]	[5,7]	$[\frac{4}{3},2]$	[29,68]

Table No. (9) Comparison between the results of solving the problem, classical data, and neutrosophical data

4-2- The neutrosophic linear models are in symmetric form and are of type Min.

The neutrosophic linear model of type Min is written in the symmetrical form, as we mentioned in the second chapter, in the following form:

Find:

$$NL = Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n \longrightarrow Min$$

Constraints:

$$Na_{11}x_1 + Na_{12}x_2 + \cdots + \mathrm{N}a_{1n}x_n \geq Nb_1$$

$$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n \geq Nb_2$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n \geq Nb_m$$

$$x_1, x_2, \dots, x_n \geq 0$$

We can search for the optimal solution by following one of the following methods:

- The optimal solution for the previous linear model can be found by converting the objective function to a function of the maximization type, as we mentioned in the second chapter, by multiplying the line of the objective function by (-1), then we write the model in the standard form. Here we notice that there is no ready-made initial rule because the variables All additional ones are preceded by a minus sign, meaning we must first search for an initial solution and then improve this solution until we reach the optimal solution by following the same previous steps.
- We can also find the dual model, and it will certainly be like a symmetry of the maximization type, and then we find the optimal solution for it as we did previously, or by using the dual algorithm to solve the model and the dual model, which we will present in the seventh chapter of this book.
- In such models, it is preferable to use the synthetic simplex algorithm, which will be presented in Chapter 6 of this book.
- Also, we can find the solution without making any change in the objective function, but we make one modification to the aforementioned steps, which is that when we want to determine the anchor column, we select the most negative element, and its column is the anchor column, and we follow the solution as we mentioned above, and the stopping criterion here is that all The line elements of the objective function are either positive or zero.

1- **Neutrosophic linear models are given in the general form**:

The neutrosophic linear model is written in the following general form:

Find:

$$NZ = Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n \longrightarrow (Max\ or\ Min)$$

Constraints:

$$Na_{i1}x_1 + Na_{i2}x_2 + \cdots + Na_{in}x_n \begin{pmatrix} \geq \\ \leq \\ = \end{pmatrix} Nb_i \quad i = 1,2, \dots, m$$

$$x_1, x_2, \dots, x_n \geq 0$$

In a first step, we write the model in the standard form, and to write it in the basic form, we rarely have additional variables. Here we notice that some of the additional variables are suitable to be a basic variable, and others are not. Also, if there are some constraints of the type of equality, then there are no corresponding additional variables. As a result, there are no basic variables. A ready-made prototype, and we need to build an initial base from which to start searching for the optimal solution. Also, here it is preferable to use a simplex with an artificial base that will be presented in the sixth chapter of this book.

Important Notes:

In the final table for the model of the maximization type, if some of the examples corresponding to the free variables in the objective function line are positive, this means that we have not reached the required ideal solution, and in this case, we must delete the free variables corresponding to the positive value. We return to step (2) and do the necessary. We repeat this.

Operations until we get a objective function line that contains nothing but zeros or negative (positive) numbers. Or we reach one of the following cases:

a- There is no ideal solution because the solution region is open in the direction of increasing NZ, and we infer this from the absence of a positive element in the fulcrum.

b- There is an infinite number of optimal solutions because the levels of the objective function NZ are parallel to one of the sides or surfaces of the common solution region. We infer this from the appearance of a zero in the last row corresponding to one of the free variables in the table of the last optimal solution. Then we can obtain another optimal solution by changing the variable. with one of the basic variables, we will get another basic solution as a result.

c- If there is no optimal solution, this happens because the constraints conflict with each other. We infer this from the absence of any positive element except for the constant Nb_i in one of the lines. This means that the right side takes a negative value while the left side takes a positive value, and this only happens. When restrictions conflict.

d- After obtaining the optimal solution, we must verify that it satisfies all the imposed constraints and gives us the same value for the objective function, by substituting the objective function and the constraints.

Conclusion:

We conclude from the previous study and the results shown in Table (9) that when using neutrosophic data we obtain areas of any indeterminate values, and this indeterminacy is more accurate, simulates reality, and takes into account most of the changes that may occur in the operating environment of the system represented by the linear mathematical model, while the values that We obtain when solving according to classical data, they are specific values and do not take into account the change that may occur in the operating environment of the system represented by the mathematical model. Therefore, neutrosophical data provide us with a more general and comprehensive study than the study using known classical data, i.e. working using classical data It is no longer sufficient at the present time, because the development of science and the instability in the status of the facility's work environment has placed before us a large number of cases that need quick and accurate treatment to avoid losses that the facility may be exposed to, which cannot be treated using classical data, and here comes the role of data. Neutrosophy, which provides us with more comprehensiveness in interpreting the results and helps us in obtaining the required accurate results. On the one hand, we focus on the necessity of choosing the appropriate algorithm to solve the model under study from among the algorithms that we will present in this book to save effort and time in searching for the optimal solution.

Chapter V

Modified Neutrosophic Simplex algorithm to find the optimal solution for linear models

Chapter V

Modified Neutrosophic Simplex algorithm to find the optimal solution for linear models

Introduction:

In this chapter, we present the modified neutrosophic simplex algorithm, which was introduced to address the difficulty we faced when applying the direct simplex algorithm, which is the large number of calculations required to be performed in each step of the solution, which requires a lot of time and effort.

5-1- Steps of the modified simplex neutrosophic algorithm:

We explain the steps of the modified simplex algorithm using the following neutrosophic linear mathematical model:

$$Max\,Z = Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n$$

$$\begin{gathered} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \le Nb_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \le Nb_2 \\ a_{31}x_1 + a_{32}x_2 + \cdots + a_{3n}x_n \le Nb_3 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \le Nb_m \end{gathered}$$

$$x_1, x_2, \dots, x_n \ge 0$$

To find the optimal solution for this linear neutrosophic model using the modified simplex algorithm.

1- We write the neutrosophic linear model in standard form, we get the following model:

$$\max Z = c_{1N}x_1 + c_{2N}x_2 + \cdots + c_{nN}x_n + 0y_1 + 0y_2 + \cdots + 0y_m$$

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + y_1 = b_{1N}$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + y_2 = b_{2N}$$
$$a_{31}x_1 + a_{32}x_2 + \cdots + a_{3n}x_n + y_3 = b_{3N}$$
$$\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n + y_m = b_{mN}$$

$$x_1, x_2, \dots, x_n, y_1, y_2, y_3, \dots, y_m \geq 0$$

2- We convert the regular linear model to the basal form and place the coefficients in a short table whose first column includes the basic variables and whose top row includes the non- basic variables only.

a. We define the pivot column, which is the column corresponding to the largest positive value in the objective function row because the objective function is a maximization function (but if the objective function is a minimization function, it is the column corresponding to the most negative values). Let this column be the column of the variable x_s.

b. We define the pivot row; The pivot row is determined with following indicator:

$$\theta = min\left[\frac{b_{iN}}{a_{is}}\right] = \frac{b_{tN}}{a_{ts}} > 0; \quad a_{is} > 0, b_{iN} > 0$$

Let this row be the line of the base variable y_t

Then the pivot element is the element resulting from the intersection of the fulcrum column and the pivot row, i.e., We explain the second step in the following table:

Non-Basic variables / Basic Variables	x_1	x_2	…..	x_s	….	x_n	Nb_i
y_1	a_{11}	a_{12}	……	a_{1s}	…..	a_{1n}	Nb_1
y_2	a_{21}	a_{22}	…….	a_{1s}	….	a_{2n}	Nb_2
……..	…….	……..	…….	……	…..	……	…….
y_t	a_{t1}	a_{t2}	……	a_{ts}	……	a_{tn}	Nb_t
……	……	……	….	…..	…..	……	…….
y_m	a_{m1}	a_{m2}	…….	a_{1s}	….	a_{mn}	Nb_m
NZ	Nc_1	Nc_2	……	Nc_s	…..	Nc_n	$NZ - Nc_o$

Table No. 1: Anchor element table

We calculate the new elements corresponding to the pivot row and the pivot column according to the following steps:

1. We put opposite the pivot element $a_{ts}$the reciprocal of $\frac{1}{a_{ts}}$.
2. We calculate the elements of the row corresponding to the pivot row (except the pivot row element) by dividing the elements of the pivot row by the anchor element a_{ts}
3. We calculate all the elements of the column opposite the fulcrum (except the fulcrum element) by dividing the elements of the fulcrum column by the fulcrum element a_{ts} and then multiplying them by(-1)
4. We calculate the other elements from the following relation:

$$a'_{ij} = a_{ij} - a_{tj}\frac{a_{is}}{a_{ts}} = \frac{a_{ij}a_{ts} - a_{tj}a_{is}}{a_{ts}} \qquad (1)$$

$$Nb'_i = Nb_i - Nb_t\frac{a_{is}}{a_{ts}} = \frac{Nb_i a_{ts} - Nb_t a_{is}}{a_{ts}} \qquad (2)$$

$$Nc'_j = Nc_j - Nc_s\frac{a_{tj}}{a_{ts}} = \frac{Nc_j a_{ts} - Nc_s a_{tj}}{a_{ts}} \qquad (3)$$

We obtain the following table:

Non-Basic Variables / Basic Variables	x_1	x_2	…..	y_t	….	x_n	Nb'_i
y_1	a'_{11}	a'_{12}	……	$\frac{-a_{1s}}{a_{ts}}$	…..	a'_{1n}	Nb'_1
y_2	a'_{21}	a'_{22}	…….	$\frac{-a_{2s}}{a_{ts}}$	….	a'_{21}	Nb'_2
……..	…….	……..	…….	……	…..	……	…….
x_s	$\frac{a_{t1}}{a_{ts}}$	$\frac{a_{t2}}{a_{ts}}$	……	$\frac{1}{a_{ts}}$	……	$\frac{a_{tn}}{a_{ts}}$	$\frac{Nb_t}{a_{ts}}$
……	……	……	….	…..	…..	……	…….
y_m	a'_{m1}	a'_{m1}	…….	$\frac{-a_{ms}}{a_{ts}}$	….	a'_{mn}	Nb'_m
NZ	Nc'_1	Nc'_2	……	$\frac{-Nc_s}{a_{ts}}$	…..	Nc'_n	$NZ - Nc'_0$

Table No. 2: The first stage in searching for the optimal solution

We apply the stopping criterion of the Simplex algorithm to the objective function row in Table (2) below:

Since the objective function is of the maximize type, the objective function row in the table must not contain any positive value, (but if the objective function is of the minimize type, the objective function row in the new table must not contain any negative value), in the event that the Criterion: We return to step No. (3) and repeat the same steps until the stopping criterion is met and we obtain the desired ideal solution.

We explain the above using the following example:

Example:

A company produces two types of products A, B using four raw materials F_1, F_2, F_3, F_4. The quantities needed from each of these materials to produce one unit of each of the two producers A, B, the available quantities of the raw materials, and the profit returned from one unit of both products are shown in the following table:

Raw Materials \ Products	Required quantity per unit		Available quantities of the raw materials
	A	***B***	
F_1	2	3	[14,20]
F_2	2	1	[10,16]
F_3	0	3	[12,18]
F_4	3	0	[15,21]
Profit Returned per unit	[5,8]	[3,6]	

Table No. 3: Issue data

Required:

Finding the optimal production plan that makes the company's profit from the producers A, B as large as possible.

Symbolize the quantities produced from the product A with the symbol x_1, and from the product B with the symbol x_2 , the problem will be reformulated from the neutrosophic perspective as follow:

$$\max Z \in [5,8]\, x_1 + [3,6]\, x_2$$

Constraints:

$$2x_1 + 3x_2 \leq [14,20]$$
$$2x_1 + x_2 \leq [10,16]$$
$$3x_2 \leq [12,18]$$
$$3x_1 \leq [15,21]$$

$$x_1 \geq 0, x_2 \geq 0$$

We apply the modified simplex algorithm:

1- The standard form of the previous linear model is:

$$\max Z \in [5,8]\, x_1 + [3,6]\, x_2 + 0y_1 + 0y_2 + 0y_3 + 0y_4$$

Constraints:

$$2x_1 + 3x_2 + y_1 = [14,20]$$
$$2x_1 + x_2 + y_2 = [10,16]$$
$$3x_2 + y_3 = [12,18]$$
$$3x_1 + y_4 = [15,21]$$

$$x_1, x_2 \geq 0, y_1, y_2, y_3, y_4 \geq 0$$

2- We organize the previous information in the following modified simplex table:

Non-basic var. / Basic var.	x_1	x_1	b_i
y_1	2	3	[14,20]
y_2	2	1	[10,16]
y_3	0	3	[12,18]
y_4	3	0	[15,21]
objective function	[5,8]	[3,6]	$Z-0$

Table No.4: Simplex table according to the modified Neutrosophic simplex algorithm

We know $[a,b] \leq [c,d]$ if $a \leq c$ and $b \leq d$,Therefore.

It is clear that $\max([5,8],[3,6]) = [5,8]$ versus to the column of x_1 , meaning that the variable x_1 should be placed instead of one of the basic variables.

Now to demonstrate which basic variables should be ejected, the following calculation has been done:

$$\theta \in min\left[\frac{[14,20]}{2}, \frac{[10,16]}{2}, \frac{[15,21]}{3}\right] = \frac{[15,21]}{3} = [5,7]$$

The value of θ indicates that the row versus to the variable y_4, and the element positioned in the cross row/column is 3 where is the pivot element, divide the elements of the row versus to y_4 yields.

3- We calculate the elements of the new table using relations (1), (2), (3), we obtain the following table:

Non-basic var. / Basic var.	x_1	x_2	b_i'
y_1	$\frac{-2}{3}$	**3**	[4,6]
y_2	$\frac{-2}{3}$	1	[0,4]
y_3	0	3	[12,18]
x_1	$\frac{1}{3}$	0	[5,7]
objective function	$\left[\frac{-8}{3},\frac{-5}{3}\right]$	[3,6]	$Z-[25,56]$

Table No.5: Table of the first step in searching for the optimal solution

4- We apply a stopping criterion in the algorithm. We find:

It still there is a non-negative value in the row of the objective function (i.e., [3,6]).

This means that we have not yet reached the optimal solution, so we repeat the previous steps as follows:

Where is versus to the x_2 column, this leads to the fact that the variable x_2 should be entered into the basic variables. Now, the question is: which basic variable should be ejected?

Track the following calculation to answer this question,

And here too because $[a,b]\le[c,d]$ if $a\le c$ and $b\le d$,Therefore.

$$\theta\in\min\left[\frac{[4,6]}{3},\frac{[0,4]}{1},\frac{[12,18]}{3}\right]=\frac{[4,6]}{3}=[\frac{4}{3},2]$$

which is versus to the slack variable y_1, the pivot element equal 3, hence the row versus to y_1 should be divided by 3 , the required calculations yield the following tableau:

Non-basic var. / basic var.	x_1	x_2	b'_i
x_2	$\frac{-2}{9}$	$\mathbf{\frac{1}{3}}$	$[\frac{4}{3},2]$
y_2	$\frac{-4}{9}$	$\frac{-1}{3}$	$[\frac{-4}{3},2]$
y_3	$\frac{2}{3}$	$\mathbf{-1}$	$[8,12]$
x_1	1	0	$[5,7]$
objective function	$[-6,-1]$	$[-2,-1]$	$Z-[29,68]$

Table No. 6: Final solution table

We apply the algorithm stopping criterion. We find that the criterion has been met and thus we have reached the optimal solution.

The optimal solution for the linear model is:

$$x_1^* \in [5,7], x_2^* \in \left[\frac{4}{3},2\right], y_2^* \in \left[\frac{-4}{3},2\right], y_3^* \in [8,12], y_1^* = y_4^* = 0$$

The value of the objective function corresponds to:

$$\max Z \in [5,8].[5,7] + [3,6].\left[\frac{4}{3},2\right] = [25,56] + [4,12] = [29,68]$$

It is clear from the row of the objective function that all the elements are neutrosophic negative numbers, this means that we have reached to the optimal solution is:

$$x_1^* \in [5,7], x_2^* \in \left[\frac{4}{3},2\right], y_2^* \in \left[\frac{-4}{3},2\right], y_3^* \in [8,12], y_1^* = y_4^* = 0$$

Substitute the above optimal solution into the objective maximum function, yields:

This means that the company must produce quantity

$x_1^* \in [5,7]$,of product A and quantity $x_2^* \in \left[\frac{4}{3}, 2\right]$of product B, thereby achieving a maximum profit of:

$$\max Z \in [5,8].[5,7] + [3,6].\left[\frac{4}{3}, 2\right] = [25,56] + [4,12] = [29,68]$$

To compare between the modified Simplex method and the direct Simplex method, we solved the same example using the direct Simplex algorithm. Below are the solution tables:

Non-basic var. / Basic var.	x_1	x_1	y_1	y_2	y_3	y_4	b_i
y_1	2	3	1	0	0	0	[14,20]
y_2	2	1	0	1	0	0	[10,16]
y_3	0	3	0	0	1	0	[12,18]
y_4	3	0	0	0	0	1	[15,21]
objective function	[5,8]	[3,6]	0	0	0	0	$Z - 0$

Table No. 7: Simplex table according to the direct neutrosophic simplex algorithm

Non-basic var. / Basic var.	x_1	x_2	y_1	y_2	y_3	y_4	b_i
y_1	0	**3**	1	0	0	$\frac{-2}{3}$	[4,6]
y_2	0	1	0	1	0	$\frac{-2}{3}$	[0,4]
y_3	0	3	0	0	1	0	[12,18]
x_1	1	0	0	0	0	$\frac{1}{3}$	[5,7]
objective function	0	[3,6]	0	0	0	$[\frac{-8}{3}, \frac{-5}{3}]$	$Z - [25,56]$

Table No. 8: Table of the first step in searching for the optimal solution

Non-basic var. / basic var.	x_1	x_2	y_1	y_2	y_3	y_4	b_i
x_2	0	1	$\frac{1}{3}$	0	0	$\frac{-2}{9}$	$[\frac{4}{3}, 2]$
y_2	0	0	$\frac{1}{3}$	1	0	$\frac{-4}{9}$	$[\frac{-4}{3}, 2]$
y_3	0	0	-1	0	1	$\frac{2}{3}$	[8,12]
x_1	1	0	0	0	0	$\frac{1}{3}$	[5,7]
objective function	0	0	[-2, -1]	0	0	$[\frac{-6}{9}, -1]$	$Z - [29,68]$

Table No. 9: Final solution table

It is clear from the row of the objective function that all the elements are either zero or neutrosophic negative numbers, this means that we have reached to the optimal solution is:

$$x_1^* \in [5,7], x_2^* \in \left[\frac{4}{3}, 2\right], y_2^* \in \left[\frac{-4}{3}, 2\right], y_3^* \in [8,12], y_1^* = y_4^* = 0$$

Substitute the above optimal solution into the objective maximum function:

$$\max Z \in [5,8].[5,7] + [3,6].\left[\frac{4}{3}, 2\right] = [25,56] + [4,12] = [29,68]$$

Conclusion:

From the previous study, we note that we obtained the same optimal solution that was obtained when we used the direct simplex method, but with a much smaller number of calculations than the number we did in the direct simplex algorithm, as is clear by comparing the solution tables using the modified simplex method, Tables No. (4)., No. (5), No. (6), with solution tables using the direct simplex method, Tables No. (7), No. (8), No. (9). Therefore, to shorten time and effort, we focus on the necessity of using the modified Simplex method to find the optimal solution for linear models. Especially when the model contains a large number of variables and constraints.

Chapter VI

Finding a rule solution for linear models using artificial variables

Introduction.

6-1- Artificial base simplex algorithm.

6-2- Processing the model and all constraints of type equals.

6-3- Processing model constraints mixed.

Conclusion.

Chapter VI

Finding a rule solution for linear models using artificial variables

Introduction:

In this chapter, we present the simplex algorithm with neutrosophic artificial variables, which is preferred for use in linear models where there is no ready-made base where we con using to obtain the optimal solution. Therefore, artificial variables are added to the constraints, the number where is equal to the number of constraints that do not contain a base variable, and as a first step in the research. Regarding the optimal solution, we must get rid of all artificial variables and convert them to non-basic variables so that they take the value of zero and thus do not affect the ideal solution of the linear model. We explain the above using the following study:

6-1- Artificial base simplex algorithm:

The purpose of solving linear models is to select the optimal solution from the set of acceptable solutions. This is done based on a base solution that is improved using the direct simplex algorithm, consists of three basic stages:

1. The stage of converting the imposed model into an equivalent systematic form.
2. The stage of converting the regular form into a basic form to obtain the non-negative basic solutions.
3. The stage of searching for the ideal solution required from among the non-negative basic solutions.
4. Therefore, the process of searching for the optimal solution does not begin until after obtaining a base solution, but in

many linear models we face great difficulty in obtaining the base solution, so the simplex method with an artificial base was proposed, where a base is formed consisting of a set of artificial variables that are non-negative is added to constraints that do not contain a basic variable, thus obtaining the basic solution. Then we improve it using the direct simplex algorithm until we obtain the optimal solution. In this research, we will reformulate the simplex algorithm with an artificial basis to find the optimal solution for linear models for where it is difficult to obtain a basis solution, using the concepts of neutrosophic.

Text of the issue:

Find the optimal solution for the following neutrosophic linear model:

$$Max\,Z = NC_1x_1 + NC_2x_2 + \cdots + NC_nx_n + NC_0$$

Constraints:

$$\begin{gathered} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + \varepsilon_1 = Nb_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + \varepsilon_2 = Nb_2 \\ a_{31}x_1 + a_{32}x_2 + \cdots + a_{3n}x_n + \varepsilon_3 = Nb_3 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.. \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n + \varepsilon_m = Nb_m \end{gathered}$$

$$x_1, x_2, \dots, x_n \geq 0$$

Where:

$$NC_j = C_j \pm \varepsilon_j \;,\quad Nb_i = b_i \pm \delta_i,\quad a_{ij},$$

$$j = 1,2,\dots,n\,, i = 1,2,\dots,m$$

are constants having set or interval values according to the nature of the given problem, x_j are decision variables. It is worthy to mention that the coefficients subscribed by the index

N are of neutrosophic values. The objective function coefficients $NC_1, NC_2, \dots, NC_n$ have neutrosophic meaning are intervals of possible values: That is, $Nc_j \in [\lambda_{j1}, \lambda_{j2}]$, where $\lambda_{j1}, \lambda_{j2}$ are the upper and the lower bounds of the objective variables x_j respectively, $j = 1,2, \dots, n$. Also, we have the values of the right-hand side of the inequality constraints $Nb_1, Nb_2, \dots, Nb_m$ are regarded as neutrosophic interval values: $Nb_i \in [\mu_{i1}, \mu_{i2}]$, here, μ_{i1}, μ_{i2} are the upper and the lower bounds of the constraint $i = 1,2, \dots, m$.

In the previous model, we note that the number of variables is n and the number of constraints is m, and this model is in the standard form.

We move to the second stage, which is to find a basic solution. Here we use the simplex algorithm with an artificial base, where is represented with:

1- From the standard form, we form an artificial base form by adding to the left side of each of the constraint equations a non-negative artificial variable ε_i. Thus, we form a base consisting of the non-negative variables $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_m$

2- Since the artificial variables are introduced into constraints that were originally linear equations, these variables must take the value of zero so that the linear constraints are not affected.

3- Therefore, we must move all of them from the base until they become non-base variables, and to be able to make this transition, we use the direct simplex algorithm.

4- We introduce these variables into the objective function with the likes of M (where M is a sufficiently large positive number that is at least greater than any $|Nc_j|$) and preceded by a minus sign (because the objective function is a maximization function) so that we do not transfer them back to the base variables again. second.

5- We obtain the following basic form of the neutrosophic linear model:

$$Max\,Z = NC_1x_1 + NC_2x_2 + \cdots + NC_nx_n - M\varepsilon_1 - M\,\varepsilon_2 - \cdots - M\,\varepsilon_m + NC_0$$

Constraints:

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + \varepsilon_1 = Nb_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + \varepsilon_2 = Nb_2$$
$$a_{31}x_1 + a_{32}x_2 + \cdots + a_{3n}x_n + \varepsilon_3 = Nb_3$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n + \varepsilon_m = Nb_m$$

$$x_j \geq 0\ \ , \varepsilon_i > 0\ , Nb_i > 0\ ; j = 1,2,\ldots,n\ and\ i = 1,2,\ldots,m$$

6- After obtaining the basic solution, we use the direct simplex algorithm to improve this solution to reach the optimal solution. Therefore, we arrange the previous information in a table as follows:

Variables / Basic	x_1	x_2	....	x_n	ε_1	ε_2	....	ε_m	b_i
ε_1	a_{11}	a_{12}	....	a_{1n}	1	0	....	0	b_1
ε_2	a_{21}	a_{22}	...	a_{2n}	0	1	0	0	b_2
....	...	...	...	...	...	...	...	...	...
ε_m	a_{m1}	a_{m2}	...	a_{mn}	0	0	...	1	b_m
Objective function	NC_1	NC_2	....	NC_n	$-M$	$-M$	...	$-M$	$Z - NC_0$

Table No. (1) General data of the model

We get rid of the artificial variables. Here we study the constants b_i corresponding to the artificial variables and select the largest of them, let it be b_t corresponding to the variable ε_t and we consider its row to be the pivot row. Then we determine the pivot element in it by dividing the elements of the objective function row (elements NC_j) by the elements of the ε_t row and then we take the smallest positive ratio θ where:

$$\theta = \underset{j}{\text{Min}} \left[\frac{NC_j}{a_{tj}} > 0 \right] = \frac{NC_s}{a_{ts}}$$

Where $a_{tj} > 0$, then the pivot element is a_{ts}, and we exchange the variables x_s and ε_t,According to the direct neutrosophic Simplex algorithm instructions.

We repeat step (7) until we get rid of all the artificial variables and obtain a normal base consisting of the basic variables.

After getting rid of the artificial variables, we return to working according to the direct neutrosophic simplex algorithm.

6-2- Processing the model and all constraints of type equals:

We explain how to find the optimal solution for linear models whose constraints are all equal constraints using the simplex algorithm with a synthetic rule using the following example:

Example 1:

Find the ideal solution for the following linear model:

$$Max\, Z = -12x_1 + [6,9]x_2 + 3x_3$$

Constraints:

$$8x_1 - x_2 + 4x_3 = [4,6]$$

$$6x_1 - 3x_2 + 3x_3 = [-12,-9]$$

$$x_1, x_2, x_3 \geq 0$$

Solution:

1- We convert the model to the standard form, multiply the second equation by (-1) and we obtain the following model:

Find a rule solution for the following neutrosophic linear model:

$$Max\, Z = -12x_1 + [6,9]x_2 + 3x_3$$

Constraints:

$$8x_1 - x_2 + 4x_3 = [4,6]$$

$$-6x_1 + 3x_2 - 3x_3 = [9,12]$$

$$x_1, x_2, x_3 \geq 0$$

2- We add the artificial variables and enter them into the objective function with a capital letter M preceded by a minus sign. Here we take $M = 15$.

Find a rule solution for the following neutrosophic linear model:

$$Max\, Z = -12x_1 + [6,9]x_2 + 3x_3 - 15\varepsilon_1 - 15\varepsilon_2$$

Constraints:

$$8x_1 - x_2 + 4x_3 = [4,6]$$

$$-6x_1 + 3x_2 - 3x_3 = [9,12]$$

$$x_1, x_2, x_3, \varepsilon_1, \varepsilon_2 \geq 0$$

We arrange the previous information in the following table:

Basic \ Variables	x_1	x_2	x_3	ε_1	ε_2	b_i
ε_1	8	-1	4	1	0	[4,6]
ε_2	-6	3	-3	0	1	[9,12]
Objective function	-12	[6,9]	3	-15	-15	$Z-0$

Table No. (2) :Artificial base table

Since the rule is artificial, we study the constants $b_i$and find that the largest of them belongs to the interval [9,12] corresponding to the variable ε_2. Therefore, we divide the objective function row by the positive elements in the ε_2 row and calculate the index θ, and we find that:

$$\theta = \underset{j}{\text{Min}}\left[\frac{[6,9]}{3}\right] = \frac{[6,9]}{3}$$

Thus, the pivot element is (3) corresponding to x_2. Therefore, we replace x_2 with ε_2, then the variable x_2 becomes a base variable and ε_2 comes out of the base. We perform the necessary calculations and obtain the following table:

Basic \ Variables	x_1	x_2	x_3	ε_1	ε_2	b_i
ε_1	6	0	3	1	$\frac{1}{3}$	[7,10]
x_2	-2	1	-1	0	$\frac{1}{3}$	[3,4]
Objective function	[0,6]	0	[9,12]	-15	$[-18,-17]$	$Z-[18,36]$

Table No. (3): The first change table in the base

The artificial variable ε_1 is still present in the base, so we perform another substitution, adopting the pivot line as the line opposite it. to determine the pivot column, we calculate the index θ, we find:

$$\theta = \underset{j}{\text{Min}}\left[\frac{[0,6]}{6}, \frac{[9,12]}{3}\right] \in \frac{[0,6]}{6}$$

Thus, the pivot element is (6) corresponding to x_1, so we move x_1 to the base instead of ε_1, so we get the following table:

Variables / Basic	x_1	x_2	x_3	ε_1	ε_2	b_i
x_1	1	0	$\frac{1}{2}$	$\frac{3}{6}$	$\frac{1}{18}$	$\left[\frac{7}{6}, \frac{10}{6}\right]$
x_2	0	1	0	$\frac{1}{3}$	$\frac{4}{9}$	$\left[\frac{16}{3}, \frac{22}{3}\right]$
Objective function	0	0	9	$[-18, -15]$	$\left[-18, \frac{-50}{3}\right]$	$Z - [18,46]$

Table No. (4): The second change in the base

From the previous table, we note that the base variables x_1, x_2, and thus we have obtained an initial solution for the linear model, which It gives us the following rule solution:

$$\left(x_1 \in \left[\frac{7}{6}, \frac{10}{6}\right], x_2 \in \left[\frac{16}{3}, \frac{22}{3}\right], x_3 = 0, \varepsilon_1 = 0, \varepsilon_2 = 0\right)$$

But it is clear from the table that this solution is not the ideal solution because in the objective function row there is a positive value corresponding to the variable x_3. Therefore, we apply the direct simplex algorithm to improve the basic solution. We obtain the ideal solution from the following table:

Variables / Basic	x_1	x_2	x_3	ε_1	ε_2	b_i
x_3	2	0	1	1	$\frac{1}{9}$	$\left[\frac{7}{3}, \frac{10}{3}\right]$
x_2	0	1	0	$\frac{1}{3}$	$\frac{4}{9}$	$\left[\frac{16}{3}, \frac{22}{3}\right]$
objective function	-18	0	0	$[-27, -24]$	$\left[-19, \frac{-53}{3}\right]$	$Z - [39,76]$

Table No. (5): The optimal solution for the model

Optimal solution for the linear model:

$$x_1 = 0, x_2 \in \left[\frac{16}{3}, \frac{22}{3}\right], x_3 \in \left[\frac{7}{3}, \frac{10}{3}\right], \varepsilon_1 = 0, \varepsilon_2 = 0$$

In this solution, the objective function takes its greatest value, which is:

$$Z \in [39,76]$$

The solution can be verified by substituting the constraints and the objective function statement, we note that the values in the ideal solution of the previous linear model are neutrosophic values.

6-3- Processing model constraints mixed:

We illustrate how to find the optimal solution for linear models with mixed constraints using the simplex algorithm with a synthetic rule using the following example:

Example 2:

Find the ideal solution for the following linear model:

$$Min\, Z = -3x_1 + [8,10]x_2 + [0,6]x_3$$

Constraints:

$$x_1 - 2x_2 + x_3 \leq [3,7]$$

$$-4x_1 + x_2 + 2x_3 \geq [9,6]$$

$$2x_1 - x_3 = 1$$

$$x_1, x_2, x_3 \geq 0$$

Converting this model to standard form the problem becomes:

Find the ideal solution for the following linear model:

$$Min\, Z = -3x_1 + [8,10]x_2 + [0,6]x_3 + 0y_1 + 0y_2$$

Constraints:

$$x_1 - 2x_2 + x_3 + y_1 = [3,7]$$

$$-4x_1 + x_2 + 2x_3 - y_2 = [9,6]$$

$$2x_1 - x_3 = 1$$

$$x_1, x_2, x_3, y_1, y_2 \geq 0$$

The variable y_1 in the first constraint is a basic variable, and since there are no other basic variables, we add artificial variables to the second and third restrictions and enter them into the objective function in sufficiently positive times because the model is a minimization model, and thus we obtain the following basic form:

Find the ideal solution for the following linear model:

$$Min\, Z = -3x_1 + [8,10]x_2 + [0,6]x_3 + 0y_1 + 0y_2 + 12\varepsilon_1 + 12\varepsilon_2$$

Constraints:

$$x_1 - 2x_2 + x_3 + y_1 = [3,7]$$

$$-4x_1 + x_2 + 2x_3 - y_2 + \varepsilon_1 = [9,6]$$

$$2x_1 - x_3 + \varepsilon_2 = 1$$

$$x_1, x_2, x_3, y_1, y_2, \varepsilon_1, \varepsilon_2 \geq 0$$

We follow the same steps mentioned in example 1 to remove the artificial variables from the base and insert the basic variables. After obtaining the base solution, we use the direct simplex method to find the optimal solution.

Important Notes:

1- If the row ε_i does not include a positive element and $b_t > 0$, this indicates a conflict of constraints and the problem is unsolvable.

2- If we cannot find a positive ratio $\frac{NC_j}{a_{tj}}$, we calculate the largest negative ratio θ' where:

$$\theta' = Max\left[\frac{NC_j}{a_{tj}} < 0\right] = \frac{NC_s}{a_{ts}}$$

Where $a_{tj} > 0$, so a_{ts} is the pivot element and it is definitely a positive element.

Conclusion:

Through the previous study, we presented one of the important methods for finding the optimal solution for neutrosophic linear models, which is the synthetic simplex method that we resort to when we are unable to find a rule solution. We found that the optimal solution that we obtained is neutrosophic values, indeterminate values, perfectly defined, belonging to a field that represents its minimum. The smallest value that the objective function can take, and the highest alone represents the highest value of the objective function, which is proportional to the conditions surrounding the system's operating environment, which can be represented by the linear model.

Chapter VII

Neutrosophic Dual Linear Models and the Binary Algorithm

Chapter VII

Neutrosophic Conjugate Linear Models and the Dual Algorithm

Introduction:

In our practical life, we encounter many problems that are formulated in the form of linear mathematical models consisting of an objective function and a set of constraints in the form of equations or inequalities. The linear model is written in many formulas that are distinguished by the type of the objective function and the form of the constraints. The formulas of linear models are explained in the second chapter of this book, and we mentioned previously that each of these formulas has a use. For example, when we want to find the optimal solution for a linear model, we must first put it in the standard form. We mentioned that we use symmetric formulas in the dual theory, which is one of the most important theories in linear programming, and its basic idea is that for every Linear model there is a conjugate linear model, as solving the original linear model gives a solution to the dual model, and therefore when solving the linear programming model, we actually get solutions for two linear models. In this chapter, we present a study of the neutrosophic dual models and the binary simplex algorithm that works to find the optimal solution for the two models. The original and the dual ones at the same time. The importance of this algorithm is evident in that it is relied upon in several operations research topics such as, integer programming algorithms, some nonlinear programming algorithms, sensitivity analysis in linear programming.

7-1- Neutrosophic companion models:

7-1-1- The matrix form of the neutrosophic conjugate models:

To find the associated model for a given neutrosophic linear model using matrices, we put the neutrosophic linear model in the symmetrical form. As we found in the second chapter, the linear model is in the symmetrical form if all the variables are constrained to be non-negative and if all constraints are given in the form of inequalities (and the inequalities of the constraints of the maximization model must be they are written in the form (≤ less than or equal to) while the inequalities of the minimization model constraints must be in the form

(≥ greater than or equal to), then the linear model is written in the neutrospheres form in one of two cases:

The first case: The original model is symmetrical and of the maximization type:

Original model:

Find:

$$NZ = NC\,\mathrm{X} \longrightarrow Max$$

Constraints:

$$NA\,\mathrm{X} \le NB$$

$$\mathrm{X} \ge 0$$

Where:

$$NA = \begin{bmatrix} Na_{11} & Na_{12} \dots Na_{1n} \\ Na_{21} & Na_{22} \dots Na_{2n} \\ \dots\dots\dots\dots\dots\dots\dots & \\ Na_{m1} & Na_{m2} \dots Na_{mn} \end{bmatrix} \quad NB = \begin{bmatrix} Nb_1 \\ Nb_2 \\ \vdots \\ Nb_m \end{bmatrix} \quad NC = \begin{bmatrix} Nc_1 \\ Nc_2 \\ \vdots \\ Nc_n \end{bmatrix} \quad \mathrm{Y} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

The dual linear model:

Find:

$$NL = NB\,\mathrm{Y} \longrightarrow Min$$

Constraints:

$$NA^T\mathrm{Y} \geq NC$$

$$\mathrm{Y} \geq 0$$

Where:

$$NA^T = \begin{bmatrix} Na_{11} & Na_{21} \dots Na_{m1} \\ Na_{12} & Na_{22} \dots Na_{m2} \\ \dots \dots \dots \dots \dots \dots \dots & \\ Na_{1n} & Na_{2n} \dots Na_{mn} \end{bmatrix} NB = \begin{bmatrix} Nb_1 \\ Nb_2 \\ .. \\ Nb_m \end{bmatrix} NC = \begin{bmatrix} Nc_1 \\ Nc_2 \\ .. \\ Nc_n \end{bmatrix} \ \mathrm{Y} = \begin{bmatrix} y_1 \\ y_2 \\ .. \\ y_m \end{bmatrix}$$

The second case: The model is symmetrical and miniaturized:

Original model:

Find:

$$NZ = NC\,\mathrm{X} \longrightarrow Min$$

Constraints:

$$NA\mathrm{X} \geq NB$$

$$\mathrm{X} \geq 0$$

Where:

$$NA = \begin{bmatrix} Na_{11} & Na_{12} \dots Na_{1n} \\ Na_{21} & Na_{22} \dots Na_{2n} \\ \dots \dots \dots \dots \dots \dots \dots & \\ Na_{m1} & Na_{m2} \dots Na_{mn} \end{bmatrix} NB = \begin{bmatrix} Nb_1 \\ Nb_2 \\ .. \\ Nb_m \end{bmatrix} NC = \begin{bmatrix} Nc_1 \\ Nc_2 \\ .. \\ Nc_n \end{bmatrix} \mathrm{X} = \begin{bmatrix} x_1 \\ x_2 \\ .. \\ x_n \end{bmatrix}$$

The dual linear model:

Find:

$$NL = NB\, \text{Y} \longrightarrow Max$$

Constraints:

$$NA^T \text{Y} \leq NC$$

$$\text{Y} \geq 0$$

Where:

$$NA^T = \begin{bmatrix} Na_{11} & Na_{21} \dots Na_{m1} \\ Na_{12} & Na_{22} \dots Na_{m2} \\ \dots \dots \dots \dots \dots \dots \dots \\ Na_{1n} & Na_{2n} \dots Na_{mn} \end{bmatrix} NB = \begin{bmatrix} Nb_1 \\ Nb_2 \\ .. \\ Nb_m \end{bmatrix} NC = \begin{bmatrix} Nc_1 \\ Nc_2 \\ .. \\ Nc_n \end{bmatrix} \text{Y} = \begin{bmatrix} y_1 \\ y_2 \\ .. \\ y_m \end{bmatrix}$$

We summarize the process of finding neutrosophic dual models using matrices in the following steps:

1. We define a new non-negative variable for each constraint of the original model
2. We make the wind (cost) vector in the original model a column vector of constants in the companion model
3. We make the constants column vector in the original model the cost (profit) vector in the companion model
4. We transform a matrix of the parsimony of the variables of the constraints in the original model into the parsimony of the variables in the dual model
5. We reverse the direction of the constraint inequalities
6. We reverse the direction of the examples, that is, we change the increase to the maximum limit to a decrease to the minimum limit, and vice versa.

7-1-2- Finding neutrosophic dual models using the double table:

We previously found that we can write linear models in three forms:

The matrix form is as shown in the previous paragraph.

The following short form:

$$NZ = \sum_{j=1}^{n} (c_j \pm \varepsilon_j) x_j \longrightarrow (Max \;\; or \;\; Min)$$

Constraints:

$$\sum_{j=1}^{n} Na_{ij} x_j \begin{pmatrix} \geq \\ \leq \\ = \end{pmatrix} b_i \pm \delta_i \quad ; \;\; i = 1,2, \dots, m$$

$$x_j \geq 0$$

The detailed figure follows:

Find:

$$NZ = Nc_1 x_1 + Nc_2 x_2 + \cdots + Nc_n x_n \longrightarrow (Max \;\; or \;\; Min)$$

Constraints:

$$Na_{i1} x_1 + Na_{i2} x_2 + \cdots + Na_{in} x_n \begin{pmatrix} \geq \\ \leq \\ = \end{pmatrix} Nb_i \quad ; i = 1,2, \dots, m$$

$$x_1, x_2, \dots, x_n \geq 0$$

To find the dual model, we put the neutrosophic linear model in the symmetrical form, and here we distinguish two cases:

First case:

The original model is symmetrical and of the maximization type:

$$NZ = \sum_{j=1}^{n} (c_j \pm \varepsilon_j) x_j \longrightarrow Max$$

Constraints:

$$\sum_{j=1}^{n} Na_{ij}x_j \leq b_i \pm \delta_i \quad ; \quad i = 1,2,\dots,m$$

$$x_j \geq 0$$

Second case:

The model is symmetrical and miniaturized:

$$NL = \sum_{j=1}^{n} (c_j \pm \varepsilon_j) x_j \longrightarrow Min$$

Constraints:

$$\sum_{j=1}^{n} Na_{ij}x_j \geq b_i \pm \delta_i \quad ; \quad i = 1,2,\dots,m$$

$$x_j \geq 0$$

In both cases, we have $x_j \geq 0$, which are the decision variables, unknown values that we obtain after solving the linear model.

$Nc_j = c_j \pm \varepsilon_j$ and $Nb_i = b_i \pm \delta_i$ and $Na_{ij} = a_{ij} \pm \mu_{ij}$, where: ($j = 1,2,\dots,n$, $i = 1,2,\dots,m$) are the data of the issue under study, and they are neutrosophic values, unspecified values that enjoy a margin of freedom and are taken according to the nature of the situation represented by the linear model.

7-1-3- Constructing neutrosophic dual linear models using tables:

To build linear neutrosophic models using tables, we draw a double table for the original and dual models according to the following steps:

1. The coefficients of the objective function in the original model are the constants column in the companion model, and the constants column in the original model are the coefficients of the objective function in the companion model.
2. We invert the signs of the inequalities of the constraints (if they were in the original model of type (=), they become in the dual model of type =).
3. We change the objective from maximizing in the original model to minimizing in the dual model.
4. We place each constraint (row) in the original model corresponding to a column in the dual model, meaning there is one variable for each constraint in the original model.
5. The variables in the original model and the dual model satisfy the non-negative constraints.

We explain the above using the following two cases:

First case:

The original model is symmetrical and of the maximization type:

First case: The original model is symmetrical and of the maximization type

Find:

$$NZ = Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n \longrightarrow Max$$

Constraints:

$$Na_{11}x_1 + Na_{12}x_2 + \cdots + Na_{1n}x_n \le Nb_1$$
$$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n \le Nb_2$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n \le Nb_m$$
$$x_1, x_2, \ldots, x_n \ge 0$$

The binary table for the original model and the dual model is as follows:

		Original model			
		objective function / constants	$Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n$		***Max*** / **Constants column**
Dual vibrable	y_1	1	$Na_{11}x_1 + Na_{12}x_2 + \cdots + Na_{1n}x_n$	$\le$	Nb_1
	y_2	2	$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n$	$\le$	Nb_2
	…	…	… … … … … … … … … … … … … …	$\le$	…
	y_m	m	$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n$	$\le$	Nb_m
		Non-negative constraints	$x_1, x_2, \ldots, x_n$	$\ge$	0
		Dual model			
		Objective function / constraints	$Nb_1y_1 + Nb_2y_2 + \cdots + Nb_iy_m$		***Min*** / **Constants column**
		1	$Na_{11}y_1 + Na_{21}y_2 + \cdots + Na_{m1}y_m$	$\ge$	Nc_1
		2	$Na_{12}y_1 + Na_{22}y_2 + \cdots + Na_{m2}y_m$	$\ge$	Nc_2
		…	… … … … … … … … … … … … … …	$\ge$	…
		n	$Na_{1n}y_1 + Na_{2n}y_2 + \cdots + Na_{mn}y_m$	$\ge$	Nc_n
		Non-negative constraints	$y_1, y_2, \ldots, y_m$	$\ge$	0

Table No. (1) Objective follower of the maximization type

The second case: The original model is symmetrical and of the reduction type:

Find:

$$NL = Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n \longrightarrow Min$$

Constraints:

$$Na_{11}x_1 + Na_{12}x_2 + \cdots + Na_{1n}x_n \geq Nb_1$$
$$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n \geq Nb_2$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n \geq Nb_m$$
$$x_1, x_2, \ldots, x_n \geq 0$$

The binary table for the original model and the dual model is as follows:

		Original model			
		objective function / **constants**	$Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n$		***Min*** / **Constants column**
Dual vibrable	y_1	1	$Na_{11}x_1 + Na_{12}x_2 + \cdots + Na_{1n}x_n$	$\geq$	Nb_1
	y_2	2	$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n$	$\geq$	Nb_2
	…	…	… … … … … … … … … … … … … …	$\geq$	…
	y_m	m	$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n$	$\geq$	Nb_m
		Non-negative constraints	$x_1, x_2, \ldots, x_n$	$\geq$	0
		Dual model			
		objective function / **constants**	$Nb_1y_1 + Nb_2y_2 + \cdots + Nb_iy_m$		***Max*** / **Constants column**
		1	$Na_{11}y_1 + Na_{21}y_2 + \cdots + Na_{m1}y_m$	$\leq$	Nc_1
		2	$Na_{12}y_1 + Na_{22}y_2 + \cdots + Na_{m2}y_m$	$\leq$	Nc_2
		…	… … … … … … … … … … … … … …	$\leq$	…
		n	$Na_{1n}y_1 + Na_{2n}y_2 + \cdots + Na_{mn}y_m$	$\leq$	Nc_n
		Non-negative constraintS	$y_1, y_2, \ldots, y_m$	$\geq$	0

Table No. (2) objective follower in the original model of the reduce type

7-2- formulation of the binary neutrosophic algorithm.

The binary simplex algorithm (for the original and dual models) neutrosophic. Through this algorithm, we can find the two ideal solutions for both the original and dual models at the same time. Before starting the binary simplex algorithm, we must mention the modified simplex algorithm that we will use within the steps of the binary algorithm.

Modified simplex algorithm:

In the modified Simplex algorithm, after converting the regular linear model to the basic form, we place the coefficients in a short table whose first column includes the basic variables and whose top row includes the non-basic variables only. We define the pivot column, which is the column corresponding to the largest positive value in the objective function row if the objective function is a maximization function (but if the objective function is a minimization function, it is the column corresponding to the most negative values). Let this column be the column of the variable x_s. We define the pivot row. The pivot row is determined. Through the following indicator:

$$N\theta = min\left[\frac{Nb_i}{Na_{is}}\right] = \frac{Nb_t}{Na_{ts}} > 0; \quad Na_{is} > 0, Nb_i > 0$$

Let this line be the line of the base variable y_t, then the anchor element is the element resulting from the intersection of the anchor column and the anchor line, i.e., Na_{ts}. Then we calculate the new elements corresponding to the anchor line and the anchor column as follows:

1. We put opposite the pivot element Na_{ts} the reciprocal of $\frac{1}{Na_{ts}}$
2. We calculate the elements of the row corresponding to the pivot row (except the pivot element) by dividing the elements of the pivot row by the pivot element Na_{ts}
3. We calculate all the elements of the column opposite the pivot (except the pivot element) by dividing the elements of the pivot column by the pivot element Na_{ts} and then multiplying them by (-1)
4. We calculate the other elements from the following relation:

$$Nb'_i = Nb_i - Nb_t \frac{Na_{is}}{Na_{ts}} = \frac{Nb_i Na_{ts} - Nb_t Na_{is}}{Na_{ts}}$$

$$Na'_{ij} = Na_{ij} - Na_{tj} \frac{Na_{is}}{Na_{ts}} = \frac{Na_{ij} Na_{ts} - Na_{tj} Na_{is}}{Na_{ts}}$$

$$Nc'_j = Nc_j - Nc_s \frac{Na_{tj}}{Na_{ts}} = \frac{Nc_j Na_{ts} - Nc_s Na_{tj}}{Na_{ts}}$$

We apply the stopping criterion of the direct Simplex algorithm on the objective function row. If the objective function is of the maximize type, the objective function row in the table must not contain any positive value. (But if the objective function is of the minimization type, the objective function row in the new table must not be contains any negative value), if the criterion is not met, we repeat the same steps until the stopping criterion is met and we obtain the desired ideal solution.

7-2-1- Steps of the binary simplex algorithm:

a. We write the two models in basic form by adding or subtracting additional variables or using synthetic variables and isolating the non-restricting variables.

Basal form of the original model:

Find:

$NZ = Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n + 0u_1 + 0u_2 + \cdots + 0u_m \longrightarrow Max$

Constraints:

$$Na_{11}x_1 + Na_{12}x_2 + \cdots + Na_{1n}x_n + u_1 = Nb_1$$
$$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n + u_2 = Nb_2$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n + u_m = Nb_m$$
$$x_j \geq 0 \ ; \mathrm{j} = 1,2,\ldots,\mathrm{n}$$
$$u_i \geq 0 \ ; \mathrm{i} = 1,2,\ldots,\mathrm{m}$$

Here we do not require that $Nb_i \geq 0$.

Basic form of the dual model:

Find:

$NL = Nb_1y_1 + Nb_2y_2 + \cdots + Nb_iy_m + 0v_1 + 0v_2 + \cdots + 0v_n \longrightarrow Min$

Constraints:

$$Na_{11}y_1 + Na_{21}y_2 + \cdots + Na_{m1}y_m - v_1 = Nc_1$$
$$Na_{12}y_1 + Na_{22}y_2 + \cdots + Na_{m2}y_m - v_2 = Nc_2$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$Na_{1n}y_1 + Na_{2n}y_2 + \cdots + Na_{mn}y_m - v_n = Nc_n$$
$$y_i \geq 0 \ ; i = 1,2,\ldots,m$$
$$v_j \geq 0 \ ; \mathrm{j} = 1,2,\ldots,\mathrm{n}$$

Here we do not require that $Nc_j \geq 0$.

The two models have the same coefficients, and the matrix of instances of the dual model is the transpose of the matrix of instances of the original model. We write the two models in the following binary table:

		Original model			
		objective function / constants	$Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n + 0u_1 + 0u_2 + \cdots + 0u_m$		Max / Constants column
Dual vibrable	y_1	1	$Na_{11}x_1 + Na_{12}x_2 + \cdots + Na_{1n}x_n + u_1$	$=$	Nb_1
	y_2	2	$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n + u_2$	$=$	Nb_2
	…	…	… … … … … … … … … … … … … …	$=$	…
	y_m	m	$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n + u_m$	$=$	Nb_m
		Non-negative constraints	$x_1, x_2, \dots, x_n, u_1, u_2, \dots, u_m$	$\geq$	0
		Dual model			
		objective function / constants	$Nb_1y_1 + Nb_2y_2 + \cdots + Nb_iy_m + 0v_1 + 0v_2 + \cdots + 0v_n$		Min / Constants column
		1	$Na_{11}y_1 + Na_{21}y_2 + \cdots + Na_{m1}y_m - v_1$	$=$	Nc_1
		2	$Na_{12}y_1 + Na_{22}y_2 + \cdots + Na_{m2}y_m - v_2$	$=$	Nc_2
		…	… … … … … … … … … … … … … …	$=$	…
		n	$Na_{1n}y_1 + Na_{2n}y_2 + \cdots + Na_{mn}y_m - v_n$	$=$	Nc_n
		Non-negative constraints	$y_1, y_2, \dots, y_m, v_1, v_2, \dots, v_n$	$\geq$	0

Table No. (3) Standard format for the original and companion models

b. We place the variables and coefficients of the original model in the modified simplex table, and we place the variables of the dual model outside the table as follows:

			basic variables with a (-) sign in the dual model				
			$-v_1$	$-v_2$	…	$-v_n$	
		Non-basic vibrable / basic vibrable	x_1	x_2	…	x_n	Follow the objective of the dual model NB_i
Non-basic vibrable of the dual model	y_1	u_1	Na_{11}	Na_{12}	…	Na_{1n}	Nb_1
	y_2	u_2	Na_{21}	Na_{22}	…	Na_{2n}	Nb_2
	…	…	…	…	…	…	…
	y_m	u_m	Na_{m1}	Na_{m2}	…	Na_{mn}	Nb_m
		objective of the original model	Nc_1	Nc_2	…	Nc_n	$L - 0 \to Min$ $Z - 0 \to Max$

Table No. (4): The binary table for the original and dual models according to the modified Simplex algorithm

7-2-2- Binary simplex algorithm for the original and dual models:

From the modified simplex algorithm of the original model, we obtain the optimal solution of the original model when all the elements are in the last row (the objective function row of the original model) $Nc_j \le 0\ ; \mathrm{j} = 1,2,\dots,\mathrm{n}$ and at the same time all the elements are in the column The last (associated objective function column) $Nb_i \ge 0\ ; i = 1,2,\dots,m$ and we get the optimal solution for the dual model when all elements in the last column (associated objective function column) are $Nb_i \ge 0\ ; i = 1,2,\dots,m$ and at the same time the last row (the objective function row of the original model)

$Nc_j \le 0\ ; \mathrm{j} = 1,2,\dots,\mathrm{n}.$

(Because it will correspond to $Nc_j = -v_j$) which are the two conditions Same for both models. Therefore, when searching for the optimal solution for both models together, we must work to make all elements $Nb_i \geq 0 \ ; i = 1,2, \dots, m$ and to make all elements $Nc_j \leq 0 \ ; \mathrm{j} = 1,2, \dots, \mathrm{n}$, to achieve this we rely on one of the two models, put its variables and coefficients in a table, and place the dual model in an external frame of that table. In general, we find that the necessity of placing the two models in a short table does not allow us to get rid of the negative constants on the right side, and therefore the general case of the previous binary table can it must include negative constants $Nb_i < 0$, and the elements of the last row can include positive elements $Nc_j > 0$, so when searching for the optimal solution for the two models, we must work to address these elements based on one of the two models.

Depending on the original model, we do this in two stages:

First stage:

We make the constant Nb_i non-negative, which corresponds to obtaining a non-negative basic solution for the original model.

Second stage:

We make all elements of the objective function row non-positive (in the case of the objective function, maximization), and this corresponds to obtaining the optimal solution required for the original model.

Based on the dual model, we do this in two stages:

First stage:

We must make the elements of the dual model objective function column

$Nb_i \geq 0 \ ; \mathrm{i} = 1,2, \dots, \mathrm{m}$. The last row is non-negative

The second stage:

We must make the free constants for the dual model $-Nc_j$ non-positive, and this corresponds to obtaining the optimal solution for the dual model. We explain the above through the following example:

Find the optimal solution for both the following neutrosophic linear model and its dual using the binary algorithm

Example1:

Find:

$$[5,8]\, x_1 + [3,6]\, x_2 \longrightarrow Max$$

Constraints:

$$2x_1 + 3x_2 \leq [14,20]$$

$$2x_1 + x_2 \leq [10,16]$$

$$3x_2 \leq [12,18]$$
$$3x_1 \leq [15,21]$$

$$x_1 \geq 0, x_2$$

We form the binary table of the model and the dual model:

	Original model				
	objective function / constants	$[5,8]\ x_1 + [3,6]\ x_2$			**Max / Constants column**
Dual vibrable	y_1	1	$2x_1 + 3x_2$	$\leq$	$[14,20]$
	y_2	2	$2x_1 + x_2$	$\leq$	$[10,16]$
	y_3	3	$3x_2$	$\leq$	$[12,18]$
	y_4	4	$3x_1$	$\leq$	$[15,21]$
		Non-negative constraints	x_1, x_2	$\geq$	0
	Dual model				
		objective function / constants	$[14,20]y_1 + [10,16]y_2 + [12,18]y_3 + [15,21]y_4$		**Min / Constants column**
		1	$2y_1 + 2y_2 + 3y_4$	$\geq$	$[5,8]$
		2	$3y_1 + y_2 + 3y_4$	$\geq$	$[3,6]$
		Non-negative constraints	y_1, y_2, y_3, y_4	$\geq$	0

Table No. (5) The original model and its dual model

In the following table, we wrote the two models in standard form:

		Original model			
	objective function constants	$[5,8]\, x_1 + [3,6]\, x_2 + 0u_1 + 0u_2 + 0u_3 + 0u_4$			Max Constants column
Dual vibrable	y_1	1	$2x_1 + 3x_2 + u_1$	$=$	[14,20]
	y_2	2	$2x_1 + x_2 + u_2$	$=$	[10,16]
	y_3	3	$3x_2 + u_3$	$=$	[12,18]
	y_4	4	$3x_1 + u_4$	$=$	[15,21]
		Non-negative constraints	$x_1, x_2, u_1, u_2, u_3, u_4$	$\geq$	0
		Dual model			
		objective function constants	$[14,20]y_1 + [10,16]y_2 + [12,18]y_3 + [15,21]y_4 + 0v_1 + 0v_2$		Min Constants column
		1	$2y_1 + 2y_2 + 3y_4 - v_1$	$=$	[5,8]
		2	$3y_1 + y_2 + 3y_4 - v_2$	$=$	[3,6]
		Non-negative constraints	$y_1, y_2, y_3, y_4, v_1, v_2$	$\geq$	0

Table No. (6) Standard format for the original model and the dual model

We notice from the table that the standard form of the original model includes a ready-made base of additional variables u_1, u_2, u_3, u_4, but for the dual model there is no ready-made base. Therefore, we multiply the two restrictions by (-1) and we obtain the basic form of the dual model.

The following table shows the basic form of the original and dual models:

Dual vibrable					
		Original model			
		objective function / constants	$[5,8]\,x_1 + [3,6]\,x_2 + 0u_1 + 0u_2 + 0u_3 + 0u_4$		***Max*** / **Constants column**
	y_1	1	$2x_1 + 3x_2 + u_1$	$=$	$[14,20]$
	y_2	2	$2x_1 + x_2 + u_2$	$=$	$[10,16]$
	y_3	3	$3x_2 + u_3$	$=$	$[12,18]$
	y_4	4	$3x_1 + u_4$	$=$	$[15,21]$
		Non-negative constraints	$x_1, x_2, u_1, u_2, u_3, u_4$	$\geq$	0
		Dual model			
		objective function / constants	$[14,20]y_1 + [10,16]y_2 + [12,18]y_3 + [15,21]y_4 + 0v_1 + 0v_2$		***Min*** / **Constants column**
		1	$-2y_1 - 2y_2 - 3y_4 + v_1$	$=$	$-[5,8]$
		2	$-3y_1 - y_2 - 3y_4 + v_2$	$=$	$-[3,6]$
		Non-negative constraints	$y_1, y_2, y_3, y_4, v_1, v_2$	$\geq$	0

Table No. (7): The basic shape of the original model and the dual model

We put the two models in the modified Simplex algorithm table and we get the following table:

			According to the original model		
			$-v_1$	$-v_2$	
		Non-basic vibrable / basic vibrable	x_1	x_2	objective function NB_i
Non-basic vibrable Dual model	y_1	u_1	2	3	[14,20]
	y_2	u_2	2	1	[10,16]
	y_3	u_3	0	3	[12,18]
	y_4	u_4	3	0	[15,21]
		objective function Nc_i	[5,8]	[3,6]	$L-0$ / $Z-0$

			According to the dual model				
			u_1	u_2	u_3	u_4	
		Non-basic vibrable / basic vibrable	y_1	y_2	y_3	y_4	objective function Original model Nc_i
Non-basic vibrable	x_1	v_1	-2	-2	0	-3	$-[5,8]$
	x_2	v_2	-3	-1	-3	0	$-[3,6]$
		objective function Dual model NB_i	[14,20]	[10,16]	[12,18]	[15,21]	$Z-0$ / $L-0$

Table No. (8): The binary table for the original and dual models according to the modified Simplex algorithm

First stage:

1- For the original model:

Since the values in the constant's column are all positive, we study the values in the objective function row and determine the largest positive value. We find:

$$\max([5,8],[3,6]) = [5,8]$$

It is an expression of the variable x_1. This means that it will enter the base. To determine the element that will exit from the base, we calculate the index $N\theta$, where:

$$N\theta \in min\left[\frac{[14,20]}{2}, \frac{[10,16]}{2}, \frac{[15,21]}{3}\right] = \frac{[15,21]}{3} = [5,7]$$

We find that the pivot column is the column of the non-base variable x_1, meaning that the variable x_1 will enter the base instead of the variable u_4, and the pivot element is the element resulting from the intersection of the pivot row and the pivot column, which is (3)

We perform the switching between variables using a modified simplex algorithm.

2- For the dual model:

We study the elements of the objective function row. We notice that all the values are positive. Therefore, we study the elements of the constant's column. We find that they are all negative values. We choose the most negative of them, which is $(-[5,8])$ which is the row of the base variable v_1, so its row is the pivot row. To determine the pivot column and the pivot element, we calculate the index $N\theta'$where:

$$N\theta' \in Max\left[\frac{[14,20]}{-2}, \frac{[10,16]}{-2}, \frac{[15,21]}{-3}\right] = \frac{[15,21]}{-3}$$

So, the column of the non-base variable u_4 is the pivot column, meaning that the variable u_4 will enter the base instead of the variable v_1, and the pivot element is the element resulting from the intersection of the pivot row and the pivot column, which is (-3). We perform the switching between the variables using the modified simplex algorithm, from (1) and (2) We get the following double table:

			According to the original model		
			$-y_4$	$-v_2$	
		Non-basic vibrable / basic vibrable	u_4	x_2	objective function Original model NB_i
Non-basic vibrable	y_1	u_1	$\frac{-2}{3}$	3	[4,6]
	y_2	u_2	$\frac{-2}{3}$	1	[0,4]
	y_3	u_3	0	3	[12,18]
	v_1	x_1	$\frac{1}{3}$	0	[5,7]
		objective function Original model Nc_i	$\left[\frac{-8}{3}, \frac{-5}{3}\right]$	[3,6]	$L-[25,56]$ / $Z-[25,56]$

			According to the dual model				
			u_1	u_2	u_3	x_1	
		Non-basic vibrable / basic vibrable	y_1	y_2	y_3	v_1	objective function Original model Nc_i
Non-basic vibrable	u_4	y_4	$\frac{2}{3}$	$\frac{2}{3}$	0	$\frac{-1}{3}$	$\left[\frac{8}{3}, \frac{5}{3}\right]$
	x_2	v_2	-3	-1	-3	0	$-[3,6]$
		objective function Original model Nc_i	[4,6]	[0,4]	[12,18]	[5,7]	$Z-[25,56]$ / $L-[25,56]$

Table No. (9): The binary table for the first stage, the solution according to the original and dual models

Second phase:

We apply the stopping criterion of the algorithm

For the original model:

Since the values in the constant's column are all positive, we study the values in the objective function row. We notice that there is a positive value, which is [3,6], meaning that we have not yet reached the optimal solution. Therefore, we specify the pivot column, which is the column of the variable x_2 corresponding to the only positive value in the objective function row. [3,6] to determine the pivot row and the pivot element, we calculate the index $N\theta$, where:

$$N\theta \in \min\left[\frac{[4,6]}{3},\frac{[0,4]}{1},\frac{[12,18]}{3}\right]=\frac{[4,6]}{3}$$

It corresponds to the base element u_1, so its row is the pivot row and the pivot element is $(\mathbf{3})$. We swap between the variables using the modified simplex algorithm.

For the dual model:

We study the elements of the objective function row. We notice that all the values are positive. Therefore, we study the elements of the constant's column. We find that there is a single negative value, which is $(-[3,6])$, which is the line of the base variable v_2, so its row is the pivot row. To determine the pivot column and the pivot element, we calculate the index $N\theta'$ where:

$$N\theta' \in Max\left[\frac{[4,6]}{-3},\frac{[0,4]}{-1},\frac{[12,18]}{-3}\right]=\frac{[4,6]}{-3}$$

So, the column of the non-base variable y_1 is the pivot column, meaning that the variable y_1 will enter the base instead of the variable v_2, and the pivot element is the element resulting from the intersection of the pivot row and the pivot column, which is (-3). We perform the switching between the variables using the modified simplex algorithm, from (1) and (2) We get the following double table:

			According to the original model		
			$-y_4$	$-y_1$	
		Non-basic vibrable / **basic vibrable**	u_4	u_1	**objective function Dual model** NB_i
Non-basic vibrable	v_2	x_2	$\frac{-2}{9}$	$\frac{1}{3}$	$\left[\frac{4}{3}, 2\right]$
	y_2	u_2	$\frac{-4}{9}$	$\frac{-1}{3}$	$\left[\frac{4}{3}, 2\right]$
	y_3	u_3	$\frac{2}{3}$	-1	$[8,12]$
	v_1	x_1	1	0	$[5,7]$
		objective function Original model Nc_i	$[-6,-1]$	$[-2,-1]$	$L-[29,68]$ $Z-[29,68]$

			According to the dual model				
			x_2	u_2	u_3	x_1	
		Non-basic vibrable / **basic vibrable**	v_2	y_2	y_3	v_1	objective function Original model Nc_i
Non-basic vibrable	u_4	y_4	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{-2}{3}$	-1	$[-6,-1]$
	u_1	y_1	$\frac{-1}{3}$	$\frac{1}{3}$	1	0	$[-2,-1]$
		objective function Dual model NB_i	$\left[\frac{4}{3}, 2\right]$	$\left[\frac{4}{3}, 2\right]$	$[8,12]$	$[5,7]$	$Z-[29,68]$ $L-[29,68]$

Table No. (10): The binary algorithm table for the second stage

We apply the stopping criterion of the algorithm:

1- For the original model, we study the elements of the objective function row until the criterion for stopping the algorithm is met, which is the absence of any positive element
2- For the dual model, we also study the elements of the constants column until the criterion for stopping the algorithm is met, which is the absence of any negative element
3- We find that the criterion has been met and thus we have reached the optimal solution

The optimal solution of the original model is:

$$x_2^* \in \left[\frac{4}{3},2\right] \ ,u_2^* \in \left[\frac{4}{3},2\right] \ ,u_3^* \in [8,12]\,,x_1^* \in [5,7] \ ,u_1^* = u_4^* = 0$$

The value of the objective function corresponds to:

$$NZ^* = Max\,NZ \in [29,68]$$

The optimal solution of the dual model is:

$$y_1^* \in [1,2] \ ,y_4^* \in [1,6] \ ,v_2^* = y_2^* = y_3^* = v_1^* = 0$$

The value of the objective function corresponds to:

$$NL^* = MinNL \in [29,166]$$

We note that:

$$NZ^* = Max\,NZ \in [29,68] \leq NL^* = MinNL \in [29,166]$$

This solution is acceptable according to the following theory:

If $(x_1, x_2, \dots, x_n)$is an acceptable solution to the original model of type Max and $(y_1, y, \dots, y_m)$was an acceptable solution for the dual model of type Min, so the value of the objective function of the original model does not exceed the value of the objective function of the dual model for these two solutions, that is, it is

$$\sum_{j=1}^{n} Nc_j x_j \leq \sum_{i=1}^{m} Nb_i y_i$$

This is for all acceptable solutions for both models (including the optimal solution).

7-3- Economic interpretation of the dual models:

We illustrate the economic interpretation of the dual model through the following example:

Example2:

A factory wants to transfer its products from two warehouses to three retail centres at the lowest possible cost. The following table shows the data provided by the factory official:

Sales centers / Stores	B_1	B_2	B_3	Available quantities
A_1	[1,3]	[2,4]	[0,3]	300
A_2	[4,6]	[1,4]	[1,5]	600
Quantities required	200	300	400	900 / 900

The plant manager's request was for a transportation plan with a minimum cost so that the distribution centres' orders could be met from the available quantities

The previous issue is a balanced transfer issue because

$$\sum_{i=1}^{2} a_i = \sum_{j=1}^{3} b_j = 900$$

To formulate the mathematical model

We assume x_{ij} the quantity transported from store i where $i = 1,2$, to distribution center j, where $j = 1,2,3$. Thus, we obtain the following linear model:

Find:

$$L \in [1,3]x_{11} + [2,4]x_{12} + [0,3]x_{13} + [4,6]x_{21} + [1,4]x_{22} + [1,5]x_{23} \longrightarrow Min$$

Constraints:

$$x_{11} + x_{12} + x_{13} \leq 300$$

$$x_{11} + x_{21} + x_{23} \leq 600$$

$$x_{11} + x_{21} \geq 200$$

$$x_{12} + x_{22} \geq 300$$

$$x_{13} + x_{23} \geq 400$$

$$x_{\mathrm{ij}} \geq 0 \ ; \ \ \mathrm{i} = 1,2 \ \ , \mathrm{j} = 1,2,3$$

We write the model in the following symmetrical form:

Since the objective function is a minimization function, all constraints must be of type greater than or equal to, so the model is written in the following symmetric form:

Find:

$$L \in [1,3]x_{11} + [2,4]x_{12} + [0,3]x_{13} + [4,6]x_{21} + [1,4]x_{22} + [1,5]x_{23} \longrightarrow Min$$

Constraints:

$$-x_{11} - x_{12} - x_{13} \geq -300$$

$$-x_{11} - x_{21} - x_{23} \geq -600$$

$$x_{11} + x_{21} \geq 200$$

$$x_{12} + x_{22} \geq 300$$

$$x_{13} + x_{23} \geq 400$$

$$x_{ij} \geq 0 \ ; \ \ i = 1,2 \ , j = 1,2,3$$

Forming the dual model, we obtain the following linear model:

Find:

$$Z = -300y_1 - 600y_2 + 200y_3 + 300y_4 + 400y_5 \longrightarrow Max$$

Constraints:

$$-y_1 + y_3 \leq [1,3]$$

$$-y_1 + y_4 \leq [2,4]$$

$$-y_1 + y_5 \leq [0,3]$$

$$-y_2 + y_3 \leq [4,6]$$

$$-y_2 + y_4 \leq [1,4]$$

$$-y_2 + y_5 \leq [1,5]$$

$$y_1, y_2, y_3, y_4, y_5 \geq 0$$

We formulate an appropriate text for the accompanying model based on the text of the original problem:

It is clear from the original model that the factory's goal is to transport all of its products at the lowest possible cost:

Text of the issue dual to the attached form:

A transport company submitted to a factory an offer that it would transport the entire quantity in the first warehouse, i.e., 300 units, at a price of y_1 monetary unit per unit, and transfer the entire quantity available in the second warehouse, 600, at a price of y_2 monetary units per unit. The company pledged that it would deliver $(200\,,300\,,400)$ units. To the three retail centers, respectively. These units are sold in these centers at a price of (y_3, y_4, y_5) monetary units, respectively. So that you can convince the official in the factory that if he accepts her offer, the cost of transportation in his factory will be less than the cost if he carries out the transportation process, so that he carries out the transportation process.

She used the constraints in the dual model and conducted the following discussion:

You pay the cost of transporting one unit from the first factory to the first sales center, an amount whose value belongs to the range [1,3], but if you use the transport company, the cost is $(y_3 - y_1)$, and we have from the first entry in the accompanying model

$$y_3 - y_1 \leq [1,3]$$

Here the official in the laboratory will notice that the transportation company's offer is an appropriate offer.

In the same way we discuss all the limitations of the dual model, the conclusion that the factory official will reach is that the cost of transportation on any route if the transportation company's offer is accepted is less than or equal to the cost that he would pay if he himself carried out the transportation process.

The transport company will adopt the values $(y_1, y_2, y_3, y_4, y_5)$,because it will achieve maximum profit through them, as the transport company's profit is calculated from the relation:

$$-300y_1 - 600y_2 + 200y_3 + 300y_4 + 400y_5$$

It is the same as the objective function of the dual model, meaning that the dual model represents the transportation company that is trying to maximize its profits

The basic theorem of association states that the optimal values of the model and the dual model are always equal. The manufacturer does not save any money because he will pay the transportation company the minimum cost of transportation, but it saves the trouble of solving the original model to determine the minimum cost of transportation, and for the transportation company, it has guaranteed the deal to transport the goods with the maximum profit.

Conclusion:

From the previous study, we arrived at a solution for the original and utility models at the same time, which are neutrosophic values from which we know the minimum and maximum profit that we can obtain, because the interpretation of the optimal solution for the original model is that it gives us the best production plan that makes the value of that production as large as possible, within Available capabilities. As for the optimal solution for the dual model, it gives us the best values for the prices of raw materials, which, if used without waste, will also give us the best production plan, and the result is the maximum profit.

Chapter VIII

Some applications to neutrosophic linear models

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Some applications to neutrosophic linear models

Introduction:

The linear programming method is one of the most used operations research methods in most practical fields. This method depends on converting the issue under study into a linear mathematical model, and then we find the optimal solution using special algorithms to solve linear models. This solution helps the decision makers responsible for managing the system. Which works according to this model to make sound decisions based on scientific foundations. The most important stage in linear programming is the stage of creating the linear model, that is, expressing the situation under study in mathematical relations. to formulate the linear model, the following basic elements must be present:

1. Determine the goal quantitatively, and it is expressed by the goal function, which is the function for which the maximum or minimum value is required. That is, we must be able to express the goal quantitatively, such as if the goal is to achieve the greatest profit or achieve the lowest cost.
2. Determine the constraints: The constraints that express the available resources must be specific, that is, the resources must be measurable, and expressed in a mathematical formula in the form of inequalities or equals.
3. Identifying the different alternatives: This element indicates that the problem should have more than one

solution so that linear programming can be applied, because if the problem had one solution, there would be no need to use linear programming, whose benefit is focused on helping to choose the best solution from among the acceptable solutions.

In this chapter, we present some problems that lead to neutrosophic linear models, that is, we will take some or all of the problem data as neutrosophic values.

8-1- Problem of the composition of mixtures:

By mixtures, we mean anything that is installed from a number of materials such as diets - medicine - any metal mixture- and here the stretch loop is to choose the materials that enter the composition of this mixture so that the cost of production is as little as possible, the goal of putting forward this model in the field of education is that the student can link between neutrosophic equations and linear inequations as well as the neutrosophic function and problems from real life.

General text of the problem:

We want to install a mixture of raw materials $A_1, A_2, \dots, A_n$and the price of one unit of each of them is equal to

$NC_1, NC_2, \dots, NC_n$ respectively, and the meal must include an amount of important elements

$B_1, B_2, \dots, B_m$ that the quantity of each element shall not be less than$B_1, B_2, \dots, B_m$

$Nb_1, Nb_2, .., Nb_m$ Unit in the order required to find the necessary amounts of each of the materials

$A_1, A_2, \dots, A_n$ Which must be included in the mixture so that its cost is as low as possible, knowing that the content of each of the materials $A_1, A_2, .., A_n$ of each of the elements $B_1, B_2, \dots, B_m$, is shown in the following table:

Materials / **Elements**	A_1	A_2	………	A_n	NB
B_1	a_{11}	a_{12}	……….	a_{1n}	Nb_1
B_2	a_{21}	a_{22}	……….	a_{2n}	Nb_2
……………….	……….	………….	……….	………….	
B_m	a_{m1}	a_{m2}	………..	a_{mn}	Nb_m

Table No. (1) Raw materials and elements for the problem of composition of mixtures

If and $Nc_j = c_j \pm \varepsilon_j \quad j = 1,2, \ldots, n$ where ε_j is indefinite and can be $\varepsilon_j = [\lambda_{1j}\ ,\lambda_{2j}]$ or $\varepsilon_j \epsilon \{\lambda_{1j}\ ,\lambda_{2j}\}$ or …

Also values that express the quantities of elements that must be available in the mixture and $Nb_i = b_i \pm \delta_i \quad i = 1,2, \ldots, m$ where δ_i is indefinite and can be, $\delta_i = [\mu_{1i}\ ,\mu_{2i}]$ or $\delta_i \epsilon \{\mu_{1i}\ ,\mu_{2i}\}$ or …

Building the Mathematical Model:

We symbolize the required amounts of each of the materials $A_1, A_2, \ldots, A_n$ be $x_1, x_2, \ldots, x_n$ and put all the information in the following table:

Materials / **Elements**	A_1	A_2	…	A_n	**Minimum amounts**
B_1	a_{11}	a_{12}	…	a_{1n}	Nb_1
B_2	a_{21}	a_{22}	…	a_{2n}	Nb_2
………	………	……	…	………	………
B_m	a_{m1}	a_{m2}	…	a_{mn}	Nb_m
Profit per unit	Nc_1	Nc_2	…	Nc_n	
Required amounts	x_1	x_2	…	x_n	

Table No. (2) General data on the issue of composition of mixtures

What is required in the problem is to determine a value for each of the variables $x_1, x_2, \ldots, x_n$ so that the objective function takes the smallest value, within the conditions imposed

Based on the data of the problem, the objective function is written in the following form:

$$NL = Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n$$

Mathematically formulate the terms we provide the following clarification:

Each unit of the material A_1 gives us a_{11} unit of the element B_1 , and thus we find that x_1 unit gives us $a_{11}x_1$ unit of the element B_1, and so we find for the rest of the materials and elements, and therefore the condition related to the element B_1 is as follows:

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \geq Nb_1$$

We follow in the same way for all materials and all elements we get the following mathematical model:

$$MinNL = Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n$$

Constraints:

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \geq Nb_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \geq Nb_2$$

$$\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \geq Nb_m$$

$$x_1, x_2, \dots, x_n \geq 0$$

We write in the following abbreviated form:

$$\text{Min}\, NL = \sum_{j=1}^{n} (c_j \pm \varepsilon_j)x_j$$

Constraints:

$$\sum_{j=1}^{n} a_{ij}x_j \geq b_i \pm \delta_i \ ; i = 1,2,\dots,m$$

$$x_j \geq 0$$

Where $c_j \pm \varepsilon_j$, $b_i \pm \delta_i$, a_{ij}, $j = 1,2,\dots,n$, $i = 1,2,\dots,m$

are constants having set or interval values according to the nature of the given problem, are decision variables

Example1:

A school wants to provide breakfast for students consisting four types of food that are A_1, A_2, A_3, A_4 and that the price of one unit of each of them respectively Nc_1, Nc_2, Nc_3, Nc_4 , and suppose that it is required that the meal includes a certain amount of important nutrients: proteinsB_1starch B_2 carbohydrates B_3 that the amount of protein in it is not less than Nb_1

unit and the amount of carbohydrates is Nb_2 unit and the amount of carbohydrates is Nb_3 unit, it is required to find the necessary amounts of substances that must be included into the meal so that its cost is as low as possible and contains at least the minimum number of nutrients required to be provided in the meal, if you know what each of the materials and the important nutrients they contain are required are shown in the following table:

where $Nc_j = c_j \pm \varepsilon_j$ and $j = 1,2,3,4$ where ε_j is indefinite and can be $\varepsilon_j = [\lambda_{1j}, \lambda_{2j}]$or $\varepsilon_j \epsilon \{\lambda_{1j}, \lambda_{2j}\}$ or …

Also values that express the amounts of nutrients that must be available in the meal$Nb_i = b_i \pm \delta_i$ and $i = 1,2,3$ where δ_i it is indefinite and can be $\delta_i = [\mu_{1i}, \mu_{2i}]$ or $\delta_i \epsilon \{\mu_{1i}, \mu_{2i}\}$ or …

We denote the required amounts of each of the materials A_1, A_2, A_3, A_4 with symbols x_1, x_2, x_3, x_4 respectively put the information contained in the text of the problem's table as follows:

Materials / Elements	A_1	A_2	A_3	A_4	Minimum amounts
B_1	a_{11}	a_{12}	a_{13}	a_{14}	Nb_1
B_2	a_{21}	a_{22}	a_{23}	a_{24}	Nb_2
B_3	a_{31}	a_{32}	a_{33}	a_{34}	Nb_3
Profit	Nc_1	Nc_2	Nc_3	Nc_4	
Required amounts	x_1	x_2	x_3	x_4	

Table No. (3): Basic data for building the linear model for example 1

Follow the objective function:

We find:

$$NL = Nc_1x_1 + Nc_2x_2 + Nc_3x_3 + Nc_4x_4$$

Nutrient conditions:

Nutrient protein requirement B_1

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 \geq Nb_1$$

Requirement of starch nutrient B_2

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 \geq Nb_2$$

Requirement of carbohydrate nutrient B_3

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 \geq Nb_3$$

Non-negative condition:

$$x_1, x_2, x_3, x_4 \geq 0$$

That's when the right mathematical model becomes.

Find:

$$MinNL = Nc_1x_1 + Nc_2x_2 + Nc_3x_3 + Nc_4x_4$$

Constraints:

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 \geq Nb_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 \geq Nb_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 \geq Nb_3$$

$$x_1, x_2, x_3, x_4 \geq 0$$

To obtain the optimal solution, we use the neutrosophic simplex method described in the research

8-2- Problem of product mixture:

What is meant by the mixture of products is the production of products that include in the composition of a number of raw materials in all production institutions for the workflow to be ideal and achieve the maximum profit must be based on a scientific study through which the quantities that must be produced from each product of the products are determined by using the available resources ideally so that the market need is secured and a profit is made. This model can be presented to students as an example of the use of linear models then students realize that pens and notebooks and benches, tables, transportation-------- and other products that they use in their daily lives have been manufactured using process research methods that depend on building a mathematical model and the optimal solution for this model is the ideal plan that the institution must adopt.

General text of the problem:

A production institution that can produce products $A_1, A_2, \dots, A_n$ and includes in its composition of raw materials, the quantities used of each $B_1, B_2, \dots, B_m$ of the raw materials in each of the products are shown in the following table:

Materials / Elements	A_1	A_2	...	A_n	NB
B_1	a_{11}	a_{12}	...	a_{1n}	Nb_1
B_2	a_{21}	a_{22}	...	a_{2n}	Nb_2
...	...	...	...	...	
B_m	a_{m1}	a_{m2}	...	a_{mn}	Nb_m

Table No. (4) Raw materials and elements for the product mix issue

The quantities available to the institution of these raw materials are $Nb_1, Nb_2, \dots, Nb_m$ where:

$Nb_i = b_i \pm \delta_i$ And where it is indefinite and can be or or ---- otherwise, it $i = 1,2, \dots, m\delta_i$

$\delta_i = [\mu_{1i}\ ,\mu_{2i}]$ $\delta_i \epsilon \{\mu_{1i}\ ,\mu_{2i}\}$ is required to find the amount of what must be produced from each of the products, knowing that the profit returned from one unit of each of the products is respectively ---- $j = 1,2, \dots, n\ \varepsilon_j$ where and

$$A_1, A_2, \dots, A_n NC_1, NC_2, \dots, NC_n\ Nc_j = c_j \pm \varepsilon_j$$

where it is indeterminate and can be

$\varepsilon_j = [\lambda_{1j}\ ,\lambda_{2j}]$ $\varepsilon_j \epsilon \{\lambda_{1j}\ ,\lambda_{2j}\}$- or other

Building the Mathematical Model:

We code the quantities produced from each of the products$A_1, A_2, \dots, A_n$ $Bx_1, x_2, \dots, x_n$ and put all the information in the following table:

Products / Materials	A_1	A_2	…	A_n	Available Quantities
B_1	a_{11}	a_{12}	…	a_{1n}	Nb_1
B_2	a_{21}	a_{22}	…	a_{2n}	Nb_2
…	…	…	…	…	…
B_m	a_{m1}	a_{m2}	…	a_{mn}	Nb_m
Profit	Nc_1	Nc_2	…	Nc_n	
Quantities produced	x_1	x_2	…	x_n	

Table No. (5) Neutrosophic data for the model

What is required in the problem is to determine a value for each of the variables$x_1, x_2, \dots, x_n$ so that the objective function takes the greatest value, within the imposed conditions.

Based on the data of the problem, the objective function is written in the following form:

$$NZ = Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n$$

Mathematically formulate the terms we provide the following clarification:

To produce one unit of the product A_1 , we need a_{11} unit of the material B_1, and thus we find that x_1 unit of the productA_1 needs A_1a_{11} unit of the materialB_1, and so we find for the rest of the products and materials, and therefore the condition related to the material is as follows:$B_1x_1A_1a_{11}x_1B_1B_1$

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq Nb_1$$

We follow in the same way for all products and all materials we get the following mathematical model:

Find the maximum value of the function

$$MaxNZ = Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n$$

Constraints:

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \le Nb_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \le Nb_2$$

… … … … … … … … … … … … ….

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \le Nb_m$$

$$x_1, x_2, \dots, x_n \ge 0$$

It shall be written in the following abbreviated form:

$$\mathrm{M}axNZ = \sum_{j=1}^{n} (c_j \pm \varepsilon_j) x_j$$

Constraints:

$$\sum_{j=1}^{n} a_{ij}x_j \le b_i \pm \delta_i \quad ; \quad i = 1,2,3, \dots, m$$

$$x_j \ge 0$$

Where $c_j \pm \varepsilon_j$, $b_i \pm \delta_i$, a_{ij}, $j = 1,2, \dots, n$, $i = 1,2, \dots, m$ are constants having set or interval values according to the nature of the given problem, x_j are decision variables

Example2:

A factory for manufacturing pens produces four types S_4 , S_3 , S_2 , S_1 and uses the following raw materials M_3 , M_2, M_1 for this. The factory management wants to study the optimal organization of production during a period of time (for example, a month) and determine the monthly production of each product to achieve a maximum profit, knowing that the profit is directly proportional to the number of units sold of products. We explain the available quantities of raw materials required for each product and the profit returned in the following table:

Materials / Elemants	S_1	S_2	S_3	S_4	Available Quantities
M_1	a_{11}	a_{12}	a_{13}	a_{14}	Nm_1
M_2	a_{21}	a_{22}	a_{23}	a_{24}	Nm_2
M_3	a_{31}	a_{32}	a_{33}	a_{34}	Nm_3

Table No. (6): Data in Example 2

To construct the mathematical model, we assume x $_1$ counting units produced from S_1

x_2 number of units produced fromS_2

x_3 number of units produced fromS_3

x_4 Number of units produced from S_4

During the productive period (for example, a month) we put the information in the following table:

Matereals / Elements	S_1	S_2	S_3	S_4	Available Quantities
M_1	a_{11}	a_{12}	a_{13}	a_{14}	Nm_1
M_2	a_{21}	a_{22}	a_{23}	a_{24}	Nm_2
M_3	a_{31}	a_{32}	a_{33}	a_{34}	Nm_3
Profit	Nc_1	Nc_2	Nc_3	Nc_4	
Quantities produced	x_1	x_2	x_3	x_4	

Table No. (7): Basic information for building the mathematical model

From the table we can see that the primary material condition M_1:

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 \leq Nm_1$$

Initial material requirement M_2:

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 \leq Nm_2$$

Initial material requirement M_3:

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 \leq Nm_3$$

In addition, the quantities produced must be non-negative, i.e.:

$$x_1, x_2, x_3, x_4 \geq 0$$

We now define the objective function If units of the same type are produced x_4 , x_3 , x_2 , x_1 respectively, the profit during the production period will be:

$$NZ = Nc_1x_1 + Nc_2x_2 + Nc_3x_3 + Nc_4x_4$$

Therefore, the mathematical model of the problem is:
Find the maximum value of the function

$$MaxNZ = Nc_1x_1 + Nc_2x_2 + Nc_3x_3 + Nc_4$$

Within the conditions

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 \leq Nm_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 \leq Nm_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 \leq Nm_3$$

$$x_1, x_2, x_3, x_4 \geq 0$$

We will apply the above to the model of optimal use of agricultural land, using the concepts of neutrosophic science. We will take data that is affected by the surrounding conditions, neutrosophic values.

Text of the issue:

Let us assume that we have n agricultural areas (plain or cultivated), the area of each of which is equal to $A_1, A_2, \cdots\cdots, A_n$, We want to plant it with m types of agricultural crops to secure the community's requirements for it. Knowing that we need of crop i the amount b_i , if the average productivity of one area in plain j of crop i is equal to

Na_{ij}tons/ha. Where $j = 1,2, \dots \dots, n$ and $i = 1,2, \dots \dots, m$, and the profit returned from one unit of crop i equal to Np_i, WhereNp_i is a neutrosophic value, an undefined non-specific value that designates a perfect and can be any neighbor of the value a_{ij}, also Np_iwhich can be any neighbor of p_i.

Required:

Determine the amount of area needed to be cultivated with each crop and in all regions to achieve the greatest possible profit and meet the needs of society.

Formulation of the mathematical model:

We symbolize by $\boldsymbol{x_{ij}}$ the amount of area in area j that must be cultivated with crop, and we place the data for the problem in the following table:

Regions / **Crops**	1	2	$\cdots$	n	**Order amount $\boldsymbol{b_i}$**	**profit amount $\boldsymbol{Np_i}$**
1	Na_{11} x_{11}	Na_{12} x_{12}	$\cdots$	Na_{1n} x_{1n}	b_1	Np_1
2	Na_{21} x_{21}	Na_{22} x_{22}	$\cdots$	Na_{2n} x_{2n}	b_2	Np_2
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots\cdots$	$\cdots$	$\cdots$
m	Na_{m1} x_{m1}	Na_{m2} x_{m2}	$\cdots$	Na_{mn} x_{mn}	b_m	Np_m
Available space $\boldsymbol{a_i}$	a_1	a_2	$\cdots$	a_n		

Table No. (8) Issue data

Then we find that the conditions imposed on the variables x_{ij} are:

1- Space restrictions:

The total area allocated to various crops in area **j** must be equal to a_j , that is, it must be:

$$x_{11} + x_{12} + \cdots \cdots + x_{m1} = a_1$$
$$x_{12} + x_{22} + \cdots \cdots + x_{m2} = a_2$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$x_{1n} + x_{2n} + \cdots\cdots + x_{mn} = a_n$$

2- Conditions for meeting community requirements:

The total production of crop i in all regions must not be less than the amountb_i , that is, it must be:

$$Na_{11}x_{11} + Na_{12}x_{12} + \cdots\cdots + Na_{1n}x_{1n} \geq b_1$$
$$Na_{21}x_{21} + Na_{22}x_{22} + \cdots\cdots + Na_{2n}x_{2n} \geq b_2$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$Na_{m1}x_{m1} + Na_{m2}x_{m2} + \cdots\cdots + Na_{mn}x_{mn} \geq b_m$$

Find the objective function:

We note that the profit resulting from the production of crop i only and from all regions is equal to the product of the profit times the quantity and i.e:

$$Np_i(Na_{i1}x_{i1} + Na_{i2}x_{i2} + \cdots\cdots + Na_{in}x_{in})$$

Thus, we find that the objective function, which expresses the total profit resulting from all crops, is equal to:

$$Z = Np_1\left(\sum_{j=1}^{n} Na_{1j}\ x_{1j}\right) + Np_2\left(\sum_{j=1}^{n} Na_{2j}\ x_{2j}\right) + \cdots\cdots + Np_m\left(\sum_{j=1}^{n} Na_{mj}\ x_{mj}\right) \to Max$$

From the above we get the following mathematical model:

Find the maximum value of

$$Z = Np_1\left(\sum_{j=1}^{n} Na_{1j}\ x_{1j}\right) + Np_2\left(\sum_{j=1}^{n} Na_{2j}\ x_{2j}\right) + \cdots\cdots + Np_m\left(\sum_{j=1}^{n} Na_{mj}\ x_{mj}\right) \to Max$$

Constraints:

$$x_{11} + x_{12} + \cdots\cdots + x_{m1} = a_1$$
$$x_{12} + x_{22} + \cdots\cdots + x_{m2} = a_2$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$x_{1n} + x_{2n} + \cdots\cdots + x_{mn} = a_n$$
$$a_{11}x_{11} + a_{12}x_{12} + \cdots\cdots + a_{1n}x_{1n} \geq b_1$$
$$a_{21}x_{21} + a_{22}x_{22} + \cdots\cdots + a_{2n}x_{2n} \geq b_2$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$a_{m1}x_{m1} + a_{m2}x_{m2} + \cdots\cdots + a_{mn}x_{mn} \geq b_m$$

$$x_{ij} \geq 0 \quad ; i = 1,2,\cdots\cdots,m \,, j = 1,2,\cdots\cdots,n$$

Example3:

Let us assume that we want to exploit four agricultural areas A_1, A_2, A_3, A_4, and the area of each of them, respectively, is 60,150,20,10, by planting them with the following crops: wheat, barley, cotton, tobacco, and beet, from which we need the following: 800, 200,600,1000,2500 Let us assume that the regions' productivity of these crops and their prices are given in the following table:

Regions / Crops	A_1	A_2	A_3	A_4	order	Price per ton
wheat	{4,6}	4	3	6	2500	{1400,1600}
barley	7	5	4	{3,5}	1000	{900,1100}
cotton	4	{9,11 }	8	5	600	{4500,6000}
tobacco	6	{2,4}	0	0	200	{4000,5000}
beet	3	{10,14}	10	6	800	{400,700}
Space	60	150	20	10		

Table No. (9) Example data

Required:

Formulate the mathematical model for this issue so that the production value is as large as possible

To formulate the mathematical model, we extract the following linear conditions:

Space restrictions:

$$x_{11} + x_{21} + x_{31} + x_{41} + x_{51} = 60$$

$$x_{12} + x_{22} + x_{32} + x_{42} + x_{52} = 150$$

$$x_{13} + x_{23} + x_{33} + x_{43} + x_{53} = 20$$

$$x_{14} + x_{24} + x_{34} + x_{44} + x_{54} = 10$$

Order restrictions:

$$\{4,6\}x_{11} + 4x_{12} + 3x_{13} + 6x_{14} \geq 2500$$

$$7x_{21} + 5x_{22} + 4x_{23} + \{3,5\}x_{24} \geq 1000$$

$$4x_{31} + \{9,11\}x_{32} + 8x_{33} + 5x_{34} \geq 600$$

$$6x_{41} + \{2,4\}x_{42} + 0x_{43} + 0x_{44} \geq 200$$

$$3x_{51} + \{10,14\}x_{52} + 10x_{53} + 6x_{54} \geq 800$$

Non-Negative restrictions:

$$x_{ij} \geq 0 \ ; i = 1,2,3,4,5 \quad and \quad j = 1,2,3,4$$

Objective function that expresses the value of production is:

$$\begin{aligned} Z = \ &\{1400,1600\}(\{4,6\}x_{11} + 4x_{12} + 3x_{13} + 6x_{14}) \\ &+ \{900,1100\}(7x_{21} + 5x_{22} + 4x_{23} \\ &+ \{3,5\}x_{24}) \\ &+ \{4500,6000\}(4x_{31} + \{9,11\}x_{32} + 8x_{33} \\ &+ 5x_{34}) \\ &+ \{4000,5000\}(6x_{41} + \{2,4\}x_{42} + 0x_{43} \\ &+ 0x_{44}) + \{400,700\}(3x_{51} + \{10,14\}x_{52} \\ &+ 10x_{53} + 6x_{54}) \rightarrow Max \end{aligned}$$

Mathematical model:

Find the maximum value of

$$Z = \{1400,1600\}(\{4,6\}x_{11} + 4x_{12} + 3x_{13} + 6x_{14}) + \{900,1100\}(7x_{21} + 5x_{22} + 4x_{23} + \{3,5\}x_{24}) + \{4500,6000\}(4x_{31} + \{9,11\}x_{32} + 8x_{33} + 5x_{34}) + \{4000,5000\}(6x_{41} + \{2,4\}x_{42} + 0x_{43} + 0x_{44}) + \{400,700\}(3x_{51} + \{10,14\}x_{52} + 10x_{53} + 6x_{54}) \rightarrow Max$$

Constraints:

$$x_{11} + x_{21} + x_{31} + x_{41} + x_{51} = 60$$

$$x_{12} + x_{22} + x_{32} + x_{42} + x_{52} = 150$$

$$x_{13} + x_{23} + x_{33} + x_{43} + x_{53} = 20$$

$$x_{14} + x_{24} + x_{34} + x_{44} + x_{54} = 10$$

$$\{4,6\}x_{11} + 4x_{12} + 3x_{13} + 6x_{14} \geq 2500$$

$$7x_{21} + 5x_{22} + 4x_{23} + \{3,5\}x_{24} \geq 1000$$

$$4x_{31} + \{9,11\}x_{32} + 8x_{33} + 5x_{34} \geq 600$$

$$6x_{41} + \{2,4\}x_{42} + 0x_{43} + 0x_{44} \geq 200$$

$$3x_{51} + \{10,14\}x_{52} + 10x_{53} + 6x_{54} \geq 800$$

$$x_{ij} \geq 0 \;;\; i = 1,2,3,4,5 \quad \text{and} \quad j = 1,2,3,4$$

The first issue:

An executive in one of the companies asked an expert in the science of operations research to help him obtain an optimal solution through which to achieve the lowest cost of transportation and operation of warehouses he wants to establish to expand the company's work and provided him with information through which the expert formulated the following issue: The text of the problem according to the concepts of neutrosophic science: A retail company plans to expand its activities in a specific area by establishing two new warehouses, the following table shows the potential locations, the number of customers and the possibility of meeting the demand for the sites where ($*$) has been placed in the event that the site can meet the customer's request and put ($\times$) the opposite and code Nc_{ij} For the cost of transferring one unit from site i to customer j he got the following table:

Customer / site	B_1	B_2	B_3	B_4
A_1	$*$ Nc_{11}	$*$ Nc_{12}	$\times$	$*$ Nc_{14}
A_2	$*$ Nc_{21}	$*$ Nc_{22}	$*$ Nc_{23}	$*$ Nc_{24}
A_3	$\times$	$*$ Nc_{32}	$*$ Nc_{33}	$*$ Nc_{34}
Customer orders	D_1	D_2	D_3	D_4

Table (10) Transportation cost in case of location selection

With the following information available for each of the candidate locations for warehouses

information / site	Operating cost per unit (monetary unit)	Initial Invested Capital (Monetary Unit)	Site Capacity
first	Np_1	k_1	A_1
second	Np_2	k_2	A_2
third	Np_3	k_3	A_3

Table (11) operation information

It is required to choose suitable locations for warehouses that make the total costs of investment, operation and transportation as small as possible.

Building the mathematical model: Each site has a fixed capital cost independent of the quantity stored in the warehouse referred to that site and also has a variable cost proportional to the quantity transported, and therefore the total cost of establishing and operating the warehouse is a non-linear function of the stored quantity and using binary integer variables can be formulated the issue of determining the location of the warehouse in a program with integers where we assume that the binary integer variable$\boldsymbol{\delta_i}$ Symbolizes the decision to choose the site or not to choose it in other words

$$\delta_i = \begin{cases} 1 & \text{if we chose the site } i \\ 0 & \text{otherwise} \end{cases}$$

Suppose that $\boldsymbol{x_{ij}}$ is the quantity transferred from site $\boldsymbol{i}$ to customer $\boldsymbol{j}$, so the constraint expressing the ability of the first site to meet the requests is as follows:

$$x_{11} + x_{12} + x_{14} \leq A_1\delta_1$$

When $\delta_1 = 1$, the first location with capacityA_1 is chosen. The quantity transported from the first site cannot exceed the capacity of that siteA_1 When $\delta_1 = 0$ the non-negative variables

$x_{11}, x_{12}, x_{14} = 0$ directly, indicating that it is not possible to ship from the first location

In a similar way, we obtain the following two constraints for the second and third signatories.

$$x_{21} + x_{22} + x_{23} + x_{24} \leq A_2\delta_2$$

$$x_{31} + x_{33} + x_{34} \leq A_3\delta_3$$

To choose exactly two locations, we need the following restriction:

$$\delta_1 + \delta_2 + \delta_3 = 2$$

As δ_1 can take one of the values of 0 or 1 only, the new constraint will force two variables from among the three variables, δ_i to be equal to one

The restrictions for customer requests can be written as follows:

first customer $x_{11} + x_{21} = D_1$

Second customer $x_{12} + x_{22} + x_{32} = D_2$

Third customer $x_{23} + x_{33} = D_3$

Forth customer $x_{14} + x_{24} + x_{34} = D_4$

To write the objective function, we note that the total cost of investment, operation and transportation for the first site is as follows:

$$k_1\delta_1 + Np_1(x_{11} + x_{12} + x_{14}) + Nc_{11}x_{11} + Nc_{12}x_{12} + Nc_{14}x_{14}$$

When we do not choose the first site, variable $\delta_1 = 0$ And that forces the variables

x_{11}، x_{12}، $x_{14} = 0$

In a similar way, the cost functions of the second and third sites can be written, and thus the full formulation of the issue of assigning the location of the warehouse is reduced to the following correct mixed program: Z is meant to be made minimal

$Z = k_1\delta_1 + Np_1(x_{11} + x_{12} + x_{14}) + Nc_{11}x_{11} + Nc_{12}x_{12} + Nc_{14}x_{14} + k_2\delta_2 + Np_2(x_{21} + x_{22} + x_{23} + x_{24}) + Nc_{21}x_{21} + Nc_{22}x_{22} + Nc_{23}x_{23} + Nc_{24}x_{24} + k_3\delta_3 + Np_3(x_{32} + x_{33} + x_{34}) + Nc_{32}x_{32} + Nc_{33}x_{33} + Nc_{34}x_{34}$

considering the following restrictions:

$$x_{11} + x_{12} + x_{14} \le A_1\delta_1$$

$$x_{21} + x_{22} + x_{23} + x_{24} \le A_2\delta_2$$

$$x_{31} + x_{33} + x_{34} \le A_3\delta_3$$

$$\delta_1 + \delta_2 + \delta_3 = 2$$

$$x_{11} + x_{21} = D_1$$

$$x_{12} + x_{22} + x_{32} = D_2$$

$$x_{23} + x_{33} = D_3$$

δ_i true variable for $i = 1,2,3$

$$x_{ij} \ge 0 \;\; ; \; i = 1,2,3 \;\; and \; j = 1,2,3,4$$

The second problem: The second request addressed by the executive was how I can choose the appropriate projects to operate a limited capital available in the company through a number of projects presented, through the information provided by the official in charge of managing the company, the expert formulated the following issue:

The issue of the capital budget: A company plans to disburse its capital during the T_j periods. Where:

$j = 1,2, \dots,$ n, and there isA_iA proposed project where $i = 1,2, - - -,$ m versus a limited capital B_j Available for investment in period j and when choosing any project i becomes in need of a certain capital in each period j we denote itNa_{ij}. It is a neutrosophic value, the value of each project is measured in terms of the liquidity flow corresponding to the project in each period minus the value of inflation, and this is called net present value (NPV), we denote it Nv_i Accordingly, the following table can be organized:

period / project	T_1	T_2	...	T_n
A_1	Na_{11}	Na_{12}	...	Na_{11}
A_2	Na_{21}	Na_{22}	...	Na_{2n}
...	...	...	...	...
A_m	Na_{m1}	Na_{m2}	...	Na_{mn}
Limited capital	B_1	B_2	...	B_n

Table (12) Return on Investment during Periods

What is required in this problem is to select the right projects that maximize the total value (NPV) of all selected projects. Formulation of the mathematical model:

Here we assume a binary integer variable $\boldsymbol{x_j}$It takes the value one if the project $\boldsymbol{j}$ is selected and takes the value zero if the project $\boldsymbol{j}$ is not selected

$$x_i = \begin{cases} 1 & \text{if we chose project } i \\ 0 & \text{otherwise} \end{cases}$$

Then the objective function is given by the following relation:

$$Z = \sum_{i=1}^{m} Nv_i \, x_i$$

Then the objective function is given by the following relation:

$$\sum_{i=1}^{m} Na_{ij} \, x_i \leq B_j \quad ; j = 1, \dots, n$$

Accordingly, we get the following mathematical model:

Find the maximum value of the function:

$$Z = \sum_{i=1}^{m} Nv_i \, x_i$$

considering the following restrictions:

$$\sum_{i=1}^{m} Na_{ij} \, x_i \leq B_j \quad ; j = 1, \dots, n$$

$\boldsymbol{x_i}$ A binary variable takes one of the values 0 or 1 for all values of $\boldsymbol{i = 1, \dots, m}$ in the previous two issues, we got models with integers that have special methods of solution. This research cannot be presented and we will present them in later research using the concepts of neutrosophic science

1- **Formulation of the problem and the construction of mathematical model according to neutrosophic values:**

The study concluded in the research [12] shows us how to construct neutrosophic linear models, (the linear model is a neutrosophic model if at least one of the likes of variables in the objective function or neutrosophic value constraints)

The text of the issue:

The company has n rank for inspectors and wants to assign the task of quality control to them, and K pieces should be audited daily during an S hour of work per day, in the following table we explain the full information about the inspectors and for all ranks:

About the Inspector / Inspector rank	Number of pieces checked (hour)	Accuracy (percent)	Inspector's remuneration (Monetary Unit per Hour)	Number of inspectors	The fine paid by the company for each fault to the inspector
1	NM_1	ND_1	G_1	A_1	R
2	NM_2	ND_2	G_2	A_2	R
…	…	…	…	…	…
n	NM_n	ND_n	G_n	A_n	R

Table (13) Information on inspectors using neutrosophic values

The number of pieces is a neutrosophic value $NM_j = M_j + \varepsilon_{\mathrm{j}}$ where ε_{j} is the indeterminacy on the number of pieces, it can take one of the shapes $[\lambda_{j1}, \lambda_{j2}]$ or $\{\lambda_{j1}, \lambda_{j2}\}$ or any value close to M_j as well as the precision, neutrosophic values

$ND_j = D_j + \delta_{\mathrm{j}}$

where δ_{j} is the indeterminacy on the precision that can take one of the shapes $[\mu_{j1}, \mu_{j2}]$ or $\{\mu_{j1}, \mu_{j2}\}$ or any value close to D_j.

Required: Formulate the appropriate mathematical model through which we can assign the optimal support to the inspectors so that the cost of inspection is as low as possible

Building the neutrosophic mathematical model:

To build the mathematical model, we impose

$x_{1,}x_2, - - --, x_n$ the number of inspectors of each rank on the order assigned to the inspection task, then the following inequality must be met:

$$x_j \leq A_j \quad ; \quad j = 1,2, --, n$$

Since the company needs to audit K piece daily within S working hour per day, the following set of restrictions must be met:

$$\sum_{j=1}^{n} S(NM_j)x_j \geq K$$

To obtain the objective function, we note that the company bears two types of costs during the inspection process, the inspector's fee and the fine corresponding to the error committed by the inspector for each piece then the target follower writes as follows:

$$NZ = S\sum_{j=1}^{n} G_j + (NM_j)R_j \left[\frac{100 - ND_j}{100}\right] \; x_j$$

Then the mathematical model is written as follows:

$$NZ = S\sum_{j=1}^{n} G_j + (NM_j)R_j \left[\frac{100 - ND_j}{100}\right] \; x_j \;\rightarrow \text{Min}$$

Constraints:

$$x_j \leq A_j \quad ; \quad j = 1,2,--,n$$

$$\sum_{j=1}^{n} S(NM_j)x_j \geq K$$

$$x_j \geq 0 \;;\; j = 1,2,----n$$

Example4:

A company has three ranks for inspectors and wants to assign the task of quality control to them, and 1500 pieces should be audited daily during 8 working hours per day, in the following table we explain the full information about the inspectors and for all ranks, in this example we will take the number of pieces checked by the inspectors from each rank as neutrosophic values

About the Inspector / Inspector rank	Number of pieces checked (hour)	Accuracy (percent)	Inspector's remuneration (Monetary Unit per Hour)	Number of inspectors	The fine paid by the company for each fault to the inspector
1	{15,16}	95	4	10	2
2	{10,11}	90	3	6	2
3	{25,26}	98	5	8	2

Table (14) Information on inspectors using neutrosophic values

Required: Formulate the appropriate mathematical model through which we can assign the optimal assignment to the inspectors so that the cost of inspection is as low as possible

To build the mathematical model, we impose $x_{1,}x_2, x_3$ as the number of inspectors from the three ranks in the order assigned to the inspection task, then the following inequality must be met:

$$x_1 \leq 10$$

$$x_2 \leq 6$$

$$x_3 \leq 8$$

Since the company needs to audit K pieces daily within S working hour per day, the following set of restrictions must be met:

$$\sum_{j=1}^{n} 8M_j x_j \geq 1500$$

That is:

$$8(M_1x_1 + M_2x_2 + M_3x_3) \geq 1500$$

From it we get the following restriction:

$$8\{15,16\}x_1 + 8\{10,11\}x_2 + 8\{25,26\}x_3 \geq 1500$$

To obtain the objective function, we note that the company bears two types of costs during the inspection process, the inspector's fee and the fine corresponding to the error committed by the inspector for each piece then the target follower writes as follows:

Then the cost of the inspector is calculated from j the hourly rank through the following relation:

$$NC_j = G_j + (NM_j)R_j\left(\frac{100 - D_j}{100}\right) \quad ; \quad j = 1,2,---,n$$

We get:

$$NC_1 = 4 + \{15,16\} \times 2 \times \left(\frac{100 - 95}{100}\right) = \{5.5,5.6\}$$

$$NC_2 = 3 + \{10,11\} \times 2 \times \left(\frac{100 - 90}{100}\right) = \{5,5.2\}$$

$$NC_3 = 5 + \{25,26\} \times 2 \times \left(\frac{100 - 98}{100}\right) = \{6,6.04\}$$

The total costs for all inspectors assigned to the task of quality control per hour shall be given by the following relation:

$$NTC_j = \sum_{j=1}^{n} \left[G_j + (NM_j)R_j\left(\frac{100 - D_j}{100}\right)\right] x_j$$

$$NTC_j = \{5.5,5.6\}x_1 + \{5,5.2\}x_2 + \{6,6.04\}x_3$$

substituting the following target phrase:

$$NZ = S\sum_{j=1}^{n} \left[G_j + (NM_j)R_j\left(\frac{100 - D_j}{100}\right)\right] x_j$$

We get:

$$NZ = \{44,44.8\}x_1 + \{40,41,6\}x_2 + \{48,48.32\}x_3$$

From the above, we can develop the following mathematical model:

We want to find:

$$Min(NZ) = \{44,44.8\}x_1 + \{40,41,6\}x_2 + \{48,48.32\}x_3$$

Constraints:

$$x_1 \leq 10$$

$$x_2 \leq 6$$

$$x_3 \leq 8$$

$$8\{15,16\}x_1 + 8\{10,11\}x_2 + 8\{25,26\}x_3 \geq 1500$$

$$x_j \geq 0 \ ; \ j = 1,2,3$$

Example 5:

A company has three ranks for inspectors and wants to assign the task of quality control to them, and 1500 pieces should be checked daily during 8working hours per day, in the following table we explain the full information about inspectors and for all ranks, in this example we will take the accuracy of inspection for each inspector as neutrosophic values in the form of areas whose minimum range is less accurate and the highest range is the highest accuracy that the inspector reaches by rank

About the Inspector / Inspector rank	Number of pieces checked (hour)	Accuracy (percent)	Inspector's remuneration (monetary unit per hour)	Number of inspectors	The fine paid by the company for each fault to the inspector
1	15	[95,97]	4	10	2
2	10	[90,92]	3	6	2
3	25	[98,99.5]	5	8	2

Table (15) Information on inspectors using neutrosophic values

Required: Formulate the appropriate mathematical model through which we can assign the optimal assignment to the inspectors so that the cost of inspection is as low as possible

To build the mathematical model, we impose $x_{1,}x_2, x_3$ the number of inspectors from the three ranks in the order assigned to the inspection task, then the following inequality must be met:

$$x_1 \leq 10$$

$$x_2 \leq 6$$

$$x_3 \leq 8$$

Since the company needs to audit K pieces daily within S working hour per day, the following set of restrictions must be met:

$$\sum_{j=1}^{n} 8M_j x_j \geq 1500$$

That is:

$$8(M_1x_1 + M_2x_2 + M_3x_3) \geq 1500$$

We get the following entry:

$$120x_1 + 80x_2 + 200x_3 \geq 1500$$

To obtain the objective function, we note that the company bears two types of costs during the inspection process, the inspector's fee and the fine corresponding to the error committed by the inspector for each piece then the target follower writes as follows:

Then the cost of the inspector is calculated from j the hourly rank through the following relation:

$$NC_j = G_j + M_j R_j \left(\frac{100 - ND_j}{100}\right) \quad ; \; j = 1,2,---,n$$

We get

$$NC_1 = 4 + 15 \times 2 \times \left(\frac{100 - [95,97]}{100}\right) = [4.9,5.5]$$

$$NC_2 = 3 + 10 \times 2 \times \left(\frac{100 - [90,92]}{100}\right) = [4.6,5]$$

$$NC_3 = 5 + 25 \times 2 \times \left(\frac{100 - [98,99.5]}{100}\right) = [5.25,6]$$

The total costs for all inspectors assigned to the task of quality control per hour shall be given by the following relation:

$$NTC_j = \sum_{j=1}^{n} \left[G_j + M_j R_j \left(\frac{100 - \mathrm{N}D_j}{100}\right)\right] x_j$$

$$NTC_j = [4.9,5.5]x_1 + [4.6,5]x_2 + [5.25,6]x_3$$

substituting the following target phrase:

$$NZ = S \sum_{j=1}^{n} \left[G_j + M_j R_j \left(\frac{100 - ND_j}{100}\right)\right] x_j$$

We get:

$$NZ = [39.2,44]x_1 + [36.8,40]x_2 + [42,48]x_3$$

From the above, we can develop the following mathematical model:

We want to find:

$$Min(NZ) = [39.2,44]x_1 + [36.8,40]x_2 + [42,48]x_3$$

Constraints:

$$x_1 \le 10$$

$$x_2 \leq 6$$

$$x_3 \leq 8$$

$$120x_1 + 80x_2 + 200x_3 \geq 1500$$

$$x_j \geq 0 \ ; \ j = 1,2,3$$

In the two examples! and 2 for the optimal solution we use the neutrosophic simplex method.

From the previous model, we notice that x_j takes a positive value only when $\delta_j = 1$, and in this case, the production of the product j is limited by the quantity d_j and the fixed production cost K_j is included in the goal function

The idea of indeterminacy is the basis of neutrosophic science represented here through the use of the binary integer variable because the optimal solution depends on the decision to produce a product or not to produce it, although we cannot guarantee the company a safe working environment in light of the great changes in the labor market through price strikes, resource availability or non-availability, and so on.

So it was necessary to reformulate this problem using neutrosophic values for the sales opportunity d_j and the cost of producing one unit of each product $C_j$and selling price P_j so that the sales opportunity becomes $d_j + \varepsilon_j$, production cost $C_j + \mu_j$ and selling price $P_j + \varphi_j$ where ε_j and μ_j and φ_j are the indeterminacy and is the change in the sales opportunity, cost and selling price respectively depending on the conditions of the work environment and takes one of the following forms:

$\varepsilon_j \in [\lambda_{j1}, \lambda_{j2}]$ Or $\varepsilon_j \in \{\lambda_{j1}, \lambda_{j2}\}$ … and $\mu_j \in [\nu_{j1}, \nu_{j2}]$ or

$\mu_j \in \{\nu_{j1}, \nu_{j2}\}$… and $\varphi_j \in [\theta_{j1}, \theta_{j2}]$ or $\varphi_j \in \{\theta_{j1}, \theta_{j2}\}$

Which are values close to the values d_j and C_j and can be any neighborhood to them.

Then the text of the problem becomes as follows:

The text of the problem according to neutrosophic science:

A company is planning to produce N product where the product j needs a fixed preparation cost or a fixed production cost K_j independent of the quantity produced, and needs a variable cost $C_j + \mu_j$ per production unit commensurate with the quantity produced, we suppose that each unit of the product j needs a_{ij} a unit of the supplier i where there is M supplier. Assuming that the product j that has a sales opportunity $d_j + \varepsilon_j$ is sold at the price of $P_j + \varphi_j$ monetary unit per unit and that only b_j unit of the supplier i is available where $i = 1,2,\dots,M$ the goal of the problem becomes to determine the optimal product mix that makes the net profit as great as possible.

Formulation of the mathematical model:

Determination of the cost:

From the text of the problem, we note that the total cost of production is the fixed cost in addition to the variable cost, which is a nonlinear function of the quantity produced,

But with the help of binary integer variables $\boldsymbol{\delta_j}$, the problem can be formulated in the form of a linear model with integers

We assume that the binary integer variable δ_j symbolizes the decision to produce the product j or not to produce it in other words

$$\delta_j = \begin{cases} 1 & j \text{ if production desicion was taken} \\ 0 & \text{otherwise} \end{cases}$$

Then the cost of producing one unit of the product becomes as follows $K_j\delta_j + (C_j + \mu_j)x_j$, where $\delta_j = 1$ if $x_j > 0$ and $\delta_j = 0$ if $x_j = 0$ and therefore the goal function becomes as follows:

$$Z = \sum_{j=1}^{N} (P_j + \varphi_j)\, x_j - \sum_{j=1}^{N} \left(K_j\delta_j + (C_j + \mu_j)x_j\right)$$

Restrictions of the problem:

A restriction on the supplier i is given in the following relation:

$$\sum_{j=1}^{N} a_{ij}\, x_j \le b_j \;\; ; i = 1,2, \dots, M$$

The restriction of the demand for the product j is given by the following relation:

$$x_j \le \left(d_j + \varepsilon_j\right)\delta_j \;\; ; j = 1,2, \dots, N$$

Mathematical model: Find the maximum value of the function:

$$Z = \sum_{j=1}^{N} (P_j + \varphi_j)\, x_j - \sum_{j=1}^{N} \left(K_j\delta_j + (C_j + \mu_j)x_j\right)$$

Within Restrictions

$$\sum_{j=1}^{N} a_{ij}\, x_j \le b_j \;\; ; i = 1,2, \dots, M$$

$$x_j \le \left(d_j + \varepsilon_j\right)\delta_j \;\; ; j = 1,2, \dots, N$$

$x_j \ge 0$ and $\delta_j = 1$ or $\delta_j = 0$

And or for all values

$$j = 1,2, \dots, N$$

From the previous model, we note that x_j takes a positive value only when $\delta_j = 1$ and in this case the production of the product j is limited by the quantity $d_j + \varepsilon_j$ and the fixed production cost K_j included in the goal function, by solving this model we get an optimal neutrosophic value for the goal function NZ^* through which we know the profit that the company can achieve in the best and worst conditions and enable the company to develop appropriate plans for the workflow in it.

Conclusion:

Through the previous study, we note that by using the linear programming method, we can provide the optimal solution to most of the problems that production companies can face by formulating the situation under treatment with a problem that can be converted into a linear model, the optimal solution of which helps decision-makers make optimal decisions for the workflow so that the greatest profit is achieved. It is possible, and to obtain solutions that have a margin of freedom, we can use the data, neutrosophic values, values that take into account all the conditions that the system represented by the linear model may experience.

References

1. Florentin Smarandache, Maissam Jdid, On Overview of Neutrosophic and Plithogenic Theories and Applications, Applied Mathematics and Data Analysis, Vo .2, No .1, 2023
2. L. A. ZADEH. Fuzzy Sets. Inform. Control 8 (1965).
3. Linear and Nonlinear Programming-DavidG, Luenbrgrr.YinyuYe - Springer Science + Business Media-2015.
4. Maissam Jdid, Operations Research, Faculty of Informatics Engineering, Al-Sham Private University Publications,2021. (Arabic version).
5. Bugaha J.S, Mualla.W, and others -Operations Research Translator into Arabic, The Arab Center for Arabization, Translation, Authoring and Publishing, Damascus,1998. (Arabic version).
6. Alali. Ibrahim Muhammad, Operations Research. Tishreen University Publications, 2004. (Arabic version).
7. Al Hamid. Mohammed Dabbas, Mathematical programming, Aleppo University, Syria, 2010. (Arabic version).
8. Maissam Jdid, Huda E Khalid, Mysterious Neutrosophic Linear Models, International Journal of Neutrosophic Science, Vol.18, No. 2, 2022.
9. Maissam Jdid, Florentin Smarandache, The Graphical Method for Finding the Optimal Solution for Neutrosophic linear Models and Taking Advantage of Non-negative Constraints to Find the Optimal Solution for Some Neutrosophic linear Models in Which the

Number of Unknowns is More than Three, Journal Neutrosophic Sets and Systems, NSS Vol.58,2023.

10. Maissam Jdid, AA Salama, Huda E Khalid ,Neutrosophic Handling of the Simplex Direct Algorithm to Define the Optimal Solution in Linear Programming ,International Journal of Neutrosophic Science, Vol.18,No. 1, 2022.
11. Maissam Jdid, Florentin Smarandache ,Neutrosophic Treatment of the Modified Simplex Algorithm to find the Optimal Solution for Linear Models , International Journal of Neutrosophic Science, Vol.23,No. 1, 2023.
12. Maissam Jdid, Florentin Smarandache, Neutrosophic Treatment of Duality Linear Models and the Binary Simplex Algorithm, Applied Mathematics and Data Analysis, Vo .2, No .2, 2023.
13. Maissam Jdid, Some Important Theories about Duality and the Economic Interpretation of Neutrosophic Linear Models and Their Dual Models, Applied Mathematics and Data Analysis, Vo .2, No .2, 2023.
14. Maissam Jdid, Florentin Smarandache, Optimal Agricultural Land Use: An Efficient Neutrosophic Linear Programming Method, Journal of Neutrosophic Systems with Applications, Vol. 10, 2023.
15. Maissam Jdid, The Use of Neutrosophic linear Programming Method in the Field of Education, Handbook of Research on the Applications of Neutrosophic Sets Theory and Their Extensions in Education, Chapter 15, IGI-Global,2023.
16. Maissam Jdid, Neutrosophic Mathematical Model of Product Mixture Problem Using Binary Integer Mutant,

Journal of Neutrosophic and Fuzzy Systems (JNFS), Vol .6, No .2, 2023.

17. Maissam Jdid, Florentin Smarandache, Said Broumi, Inspection Assignment Form for Product Quality Control, Journal of Neutrosophic Systems with Applications, Vol. 1, 2023.
18. Maissam Jdid, Florentin Smarandache, The Use of Neutrosophic Methods of Operation Research in the Management of Corporate Work, Journal of Neutrosophic Systems with Applications, Vol. 3, 2023.
19. Maissam Jdid, Neutrosophic linear models and algorithms to find their optimal solution, Biblio Publishing, ISBN, 978_1 _59973_778_2, (Arabic version). USA,2023.

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2022-2023