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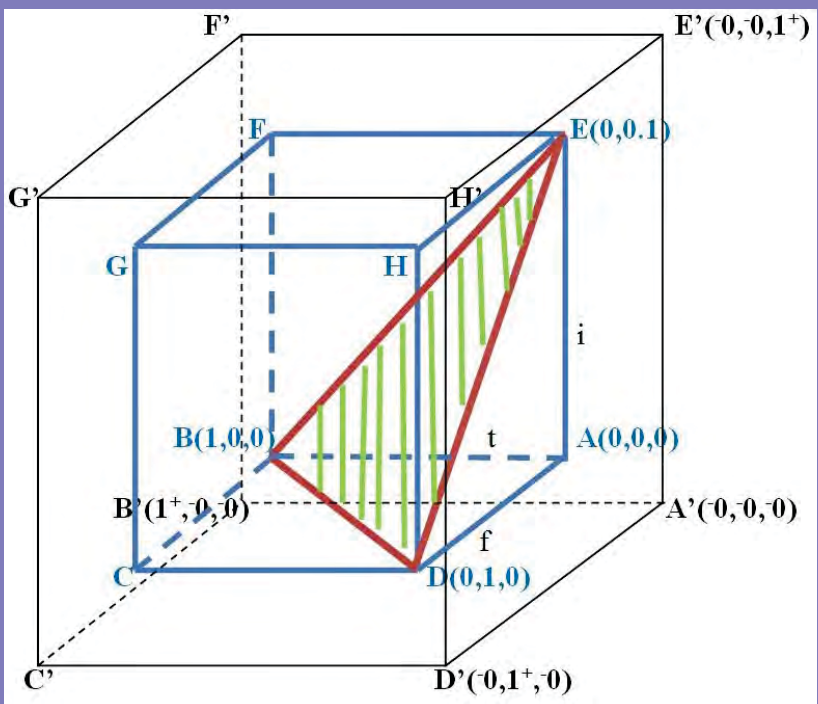

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*"*Neutrosophic Sets and Systems*"* has been created for publications on advanced studies in neutrosophy, neutrosophic set, neutrosophic logic, neutrosophic probability, neutrosophic statistics that started in 1995 and their applications in any field, such as the neutrosophic structures developed in algebra, geometry, topology, etc.

The submitted papers should be professional, in good English, containing a brief review of a problem and obtained results.

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e. notions or ideas supporting neither <A> nor <antiA>). The <neutA> and <antiA> ideas together are referred to as <nonA>.

Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only).

According to this theory every idea <A> tends to be neutralized and balanced by <antiA> and <nonA> ideas - as a state of equilibrium.

In a classical way <A>, <neutA>, <antiA> are disjoint two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well.

Neutrosophic Set and *Neutrosophic Logic* are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and respectively intuitionistic fuzzy logic). In neutrosophic logic a proposition has a degree of truth (T), a degree of indeterminacy (I), and a degree of falsity (F), where T, I, F are standard or non-standard subsets of $]^-0, 1^+[$.

Neutrosophic Probability is a generalization of the classical probability and imprecise probability.

Neutrosophic Statistics is a generalization of the classical statistics.

What distinguishes the neutrosophics from other fields is the <neutA>, which means neither <A> nor <antiA>.

<neutA>, which of course depends on <A>, can be indeterminacy, neutrality, tie game, unknown, contradiction, ignorance, imprecision, etc.

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NEUTROSOPHIC PROJECT EVALUATION AND REVIEW TECHNIQUES

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Abstract: One of the most important and challenging jobs that any manager can take in the management of a large scale project that requires coordinating numerous activities throughout the organization. Initially, the activity times are static within the CPM technique and probabilistic within the PERT technique. Since neutrosophic set is the generalization of fuzzy set and intuitionistic fuzzy set, a new method of project evaluation and review technique for a project network in neutrosophic environment is proposed in this paper. Considering single valued neutrosophic number as the time of each activity in the project network, neutrosophic expected task time, neutrosophic variance, neutrosophic critical path and the neutrosophic total expected time for completing the project network are calculated here. The main concept of Neutrosophic Project Evaluation and Review Technique(NPERT) method is to solve the ambiguities in the activity times of a project network easily than other existing methods like classical PERT, Fuzzy PERT etc.The proposed method is explained by an illustrative example and the results are discussed here.

Keywords: Neutrosophic set, Single Valued Neutrosophic Numbers, Neutrosophic critical path, Neutrosophic expected task times, Neutrosophic variance.

1. Introduction

The success of any large-scale project is very much dependent upon the quality of the planning, scheduling, and controlling of various phases of the project. Unless some type of planning and coordinating tool is used, the number of phases does not need to be very large before management starts losing control. Project Evaluation and Review Technique (PERT) is the best project management tool used to schedule, organize and coordinate the tasks in such type of large-scale project[3]. It is originally designed to plan a manufacturing project by employing a network of interrelated activities, coordinating optimum cost and time. It also emphasizes the relationship between the time of each activity, the costs associated with each phase, and the resulting time and cost for the anticipated completion of the entire project (Harry, 2004). PERT is also an integrated project man-agement system to manage the complexities of major manufacturing projects and the time deadlines created by defence industry projects. Most of these management systems were developed following World War II, and each has its advantages. PERT was first developed in 1958 by the U.S. Navy Special Projects office on the Polaris missile system. Existing integrated planning on such a large scale was deemed inadequate, so the Navy pulled in the Lockheed Aircraft Corporation and the management consulting firm of Booz, Allen, and Hamilton. Traditional techniques such as line of balance, Gantt charts and other systems were eliminated and PERT evolved as a means to deal with various time periods and it takes to finish the critical activities of an overall project. All defence contractors adopted PERT to manage the massive one-time projects associated with the industry after 1960. Smaller businesses awarded defence related government contracts, found it necessary to use PERT[9]. A typical PERT network consists of activities and events. An event is the completion of one program component at a particular time. An activity is defined as the time and resources required to move from one event to another. Therefore, when events and activities are clearly defined, progress of a program is easily monitored, and the path of the project proceeds toward termination. PERT mandates that each preceding event be completed before succeeding events and thus the final project can be considered complete. The critical path is a combination of events and activities. Slack time is defined as the difference between the total expected activity time for the project and the actual time for the entire project. Slack time is the spare time experienced in the

PERT (Ghaleb, 2001). PERT plays a major role whenever uncertainity occurs in activity times of a project network[3]. Several researchers are introduced and discussed the concept of PERT/CPM in various situations[1,2,5,6,7,10,14,16,17,18,19]. Neutrosophic sets have been introduced as a generalization of crisp sets, fuzzy sets, and intuitionistic fuzzy sets to represent uncertain, inconsistent and incomplete information about real world problems. Elements of neutrosophic set are characterized by a truth-membership, falsity-membership and indeterminacy membership functions[11,12]. Neutrosophic set theory is applied in multi attribute decision making[15]. The subtraction and division of neutrosophic numbers were discussed in [13]. In this paper, new algorithm for finding project evaluation and review technique(NPERT) by neutro-sophic numbers for a given network is introduced in a better way than other existing methods. Neutrosophic critical path and their variance of a project network are calculated here. The neutrosophic expected task times for completing the project and the probability of time for completing the project within a expected period of time are also derived.

2.Preliminaries

Some basic definitions in neutrosophic set and neutrosophic numbers which are very useful in the construction of NPERT presented here.

Definition 2.1. [8]. Let E be a universe. A neutrosophic set A in E is characterized by a truth-membership function T_A, a indeterminacy-membership function I_A and a falsity-membership function F_A. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard elements of [0,1]. It can be written as

$$A = \{< x, (T_A(x), I_A(x), F_A(x)) >: x \in E; T_A(x), I_A(x), F_A(x) \in]\, 0; 1^+[\ \}$$

There is no restriction on the sum of $T_A(x)$ $I_A(x)$ and $F_A(x)$. So, $0 \leq \quad T_A(x) + I_A(x) + F_A(x) \leq 3^+$:

Definition 2.2. [8]. Let E be a universe. A single valued neutrosophic set A, which can be used in real scientific and engineering applications, in E is characterized by a truth-membership function T_A, an indeterminacy-membership function I_A and a falsity-membership function F_A. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard elements of [0,1]. It can be written as

$$A = \{< x; (T_A(x), I_A(x), F_A(x)) >: x \in E, T_A(x), I_A(x), F_A(x) \in [0; 1]\ \}$$

Definition 2.3. [4]. Let $\tilde{a} =< (a_1, b_1, c_1); \tilde{w}_a, \tilde{u}_a, \tilde{y}_a >$, and $\tilde{b} =< (a_2, b_2, c_2); \tilde{w}_b, \tilde{u}_b, \tilde{y}_b >$ be two single valued triangular neutrosophic numbers and $\gamma \neq 0$ be any real number. Then,

1. $\tilde{a} + \tilde{b} =< (a_1 + a_2, b_1 + b_2, c_1 + c_2); \tilde{w}_a \wedge \tilde{w}_b, \tilde{u}_a \vee \tilde{u}_b, \tilde{y}_a \vee \tilde{y}_b >$
2. $\tilde{a} - \tilde{b} =< (a_1 - c_2, b_1 - b_2, c_1 - a_2); \tilde{w}_a \wedge \tilde{w}_b, \tilde{u}_a \vee \tilde{u}_b, \tilde{y}_a \vee \tilde{y}_b >$
3. $$\tilde{a}\tilde{b} = \begin{cases} < (a_1a_2, b_1b_2, c_1c_2); \tilde{w}_a \wedge \tilde{w}_b, \tilde{u}_a \vee \tilde{u}_b, \tilde{y}_a \vee \tilde{y}_b > (c_1 > 0; c_2 > 0) \\ < (a_1c_2, b_1b_2, c_1a_2); \tilde{w}_a \wedge \tilde{w}_b, \tilde{u}_a \vee \tilde{u}_b, \tilde{y}_a \vee \tilde{y}_b > (c_1 < 0; c_2 > 0) \\ < (c_1c_2, b_1b_2, a_1a_2); \tilde{w}_a \wedge \tilde{w}_b, \tilde{u}_a \vee \tilde{u}_b, \tilde{y}_a \vee \tilde{y}_b > (c_1 < 0; c_2 < 0) \end{cases}$$
4. $$\frac{\tilde{a}}{\tilde{b}} = \begin{cases} < (a1/c2, b1/b2, c1/a2); \tilde{w}_a \wedge \tilde{w}_b, \tilde{u}_a \vee \tilde{u}_b, \tilde{y}_a \vee \tilde{y}_b > (c1 > 0; c2 > 0) \\ < (c1/c2, b1/b2, c1/c2); \tilde{w}_a \wedge \tilde{w}_b, \tilde{u}_a \vee \tilde{u}_b, \tilde{y}_a \vee \tilde{y}_b > (c_1 < 0; c_2 > 0) \\ < (c1/a2, b1/b2, a1/c2);); \tilde{w}_a \wedge \tilde{w}_b, \tilde{u}_a \vee \tilde{u}_b, \tilde{y}_a \vee \tilde{y}_b > (c_1 < 0; c_2 < 0) \end{cases}$$
5. $$\gamma\tilde{a} = \begin{cases} < (\gamma a1, \gamma b1, \gamma c1); \tilde{w}_a, \tilde{u}_a, \tilde{y}_a > (\gamma > 0) \\ < (\gamma c1, \gamma b1, \gamma a1); \tilde{w}_a, \tilde{u}_a, \tilde{y}_a > (\gamma > 0) \end{cases}$$
6. $\tilde{a}^{-1} =< (1/c1, 1/b1, 1/a1); \tilde{w}_a, \tilde{u}_a, \tilde{y}_a > (\tilde{a} \neq 0)$

Definition 2.4. [8]. Let $\tilde{A}_1 =< T_1, I_1, F_1 >$ be a single valued neutrosophic number. Then, the score function $s(\tilde{A}_1)$, accuracy function $a(\tilde{A}_1)$, and certainty function $c(\tilde{A}_1)$ of an single valued neutrosophic numbers are defind

1. $s(\tilde{A}_1) = (T_1 + 1 - I_1 + 1 - F_1)/3$
2. $a(\tilde{A}_1) = T_1 - F_1$
3. $c(\tilde{A}_1) = T_1$

Definition 2.5. [13]. Let $A = (t_1, i_1, f_1)$ and $B = (t_2, i_2, f_2)$ be two single-valued neutrosophic numbers, where $t_1, i_1, f_1, t_2, i_2, f_2 \in [0; 1]$, and $0 \le t_1, i_1, f_1 \le 3$ and $0 \le t_2, i_2, f_2 \le 3$. The division of neutrosophic numbers A and B is defined as follows:

$A \oslash B = (t_1, i_1, f_1) \oslash (t_2, i_2, f_2) = (\frac{t_1}{t_2}, \frac{i_1 - i_2}{1 - i_2}, \frac{f_1 - f_2}{1 - f_2})$, where $t_1, i_1, f_1, t_2, i_2, f_2 \in [0; 1]$, with the restriction that $t_2 \neq 0$, $i_2 \neq 1$ and $f_2 \neq 1$

Similarly, the division of neutrosophic numbers only partially works, i.e. when $t_2 \neq 0$, $i_2 \neq 1$ and $f_2 \neq 1$. In the same way, the restriction that $(\frac{t_1}{t_2}, \frac{i_1 - i_2}{1 - i_2}, \frac{f_1 - f_2}{1 - f_2}) \in ([0, 1], [0,1], [0, 1])$ is set when the traditional case occurs, when the neutrosophic number components t, i, f are in the interval [0, 1].

3 NPERT Analysis

NPERT computations are the same as those of NCPM[8]. The main difference is that instead of activity duration we use neutrosphic expected time for the activity. Activity times are represented by a neutrosphic probability distribution. This neutrosphic probability distribution is based on three different time estimates are as follows:

1. Neutrosophic Optimistic Time [$t_o^N = (t^T_o, t^I_o, t^F_o)^N$]:

In this time, each and every activity of a network is going well without any disturbance like shortage of money, labour and raw materials etc., and the project will be completed within a period of expected time.

2. Neutrosophic Pessimistic Time [$t_p^N = (t^T_p, t^I_p, t^F_p)^N$]:

In this time, most of the activities in a project are disturbed when the work is going. So, the project will not be completed in an expected period of time. The time of completion of the project will go more than the expected time.

3. Neutrosophic Most likely Time [$t_m^N = (t^T_m, t^I_m, t^F_m)^N$]:

In this time, some of the activities are disturbed when the work is going. So the time of completion will extend slightly more than the expected time

Also the neutrosophic expected mean time [$t_e^N = (t_e^T, t_e^I, t_e^F)^N$ and the neutrosophic variance $\sigma^{2^N} = (\sigma^{T^2}, \sigma^{I^2}, \sigma^{F^2})^N$ of the project network are given as follows:

$$t_e^N = \frac{1}{6}[t_0^N + 4t_m^N + t_p^N] \text{ and } \sigma^{2^N} = (\frac{t_p^N - t_0^N}{6})$$

The neutrosophic earliest and latest start time as well as neutrosophic earliest and latest finish time of each activity for finding neutrosophic critical path of a project are derived by using forward pass and backward pass calculations as follows:

(a). Forward Pass Calculations:

Here, we start from the initial node 1 with starting time zero in increasing order and end at final node n. At each node, neutrosophic earliest start and finish times are calculated by considering $E(\mu_i)^N$.

Step:1 Set $E(\mu_1)^N =< 0, 0, 0 >$, i=1 for the initial node.

Step:2 Set neutrosophic earliest start for each activity as $ES(\mu_{ij})^N = E(\mu_i)$ for all activities (i,j) that start at node i.

Step:3 Compute neutrosophic earliest finish for each activity as $EF\ (\mu_{ij})^N = E(\mu_{ij})^N + (t_{ij})^N = E(\mu_i)^N + (t_{ij})^N$, for all activities (i,j) that start at node i and move on to next node.

Step:4 If node j > i, compute neutrosophic earliest occurance for each node j using $E(\mu_j)^N = \max\{EF\ (\mu_{ij})^N\} = \max\{E(\mu_i)^N + (t_{ij})^N\}$, for all immediate predecessor activities.

Step:5 If j=n(final node), then the neutrosophic earliest finish time for the project is given by $E(\mu_n)^N = \max\{EF\ (\mu_{ij})^N\} = \max\{E(\mu_{n-1})^N + (t_{ij})^N\}$:

(b). Backward Pass Calculations:

Here, we start from last(final) node n of the network in decreasing order and end at initial node 1. At every node, neutrosophic latest finish and start times for each activity are calculated by considering $L(\mu_j)^N$.

Step:1 $L(\mu_n)^N = E(\mu_n)^N$, j=n.

Step:2 Set neutrosophic latest finish time for each activity as $LF\ (\mu_{ij})^N = L(\mu_j)$ that ends at node j.

Step:3 Compute neutrosophic latest occurence time for all activities ends at j as $LS(\mu_{ij})^N = LF(\mu_{ij})^N\ (t_{ij})^N = L(\mu_j)^N\ (t_{ij})^N$, for all activities (i,j) that start at node i and move on to next node.

Step:4 If node i < j, compute neutrosophic latest occurence time for each node i, using $L(\mu_i)^N = \min\{LS(\mu_{ij})^N\} = \min\{L(\mu_i)^N - (t_{ij})^N\}$ by proceeding backward process from node j to node 1.

Step:5 If j=1(initial node),then we have $L(\mu_1)^N = \min\{LS(\mu_{ij})^N\} = \min\{L(\mu_2)^N - (t_{ij})^N\}$:

From the above calculations, an activity (i,j) will be critical if it satisfies the following conditions:
(i). $E(\mu_i)^N = L(\mu_i)^N$ and $E(\mu_j)^N = L(\mu_j)^N$
(ii). $E(\mu_j)^N - E(\mu_i)^N = L(\mu_j)^N - L(\mu_i)^N = (t_{ij})^N$.
An activity which does not satisfies the above conditions is called non critical activity.

3.1 Neutrosophic Float or Slack of an activity and event:

The neutrosophic time of an activity which makes delay in its completion time without affecting the total project completion time is called neutrosophic float. Neutrosophic event float and neutrosophic activity float are two types of neutrosophic floats.

1. Neutrosophic Event float:
The neutrosophic float of an event is the difference between its neutrosophic latest and earliest time. (i.e). Neutrosophic event float = $L(\mu_i)^N - E(\mu_i j)^N$

2. Neutrosophic activity float:
Neutrosophic total float and neutrosophic free float are two types of neutrosophic activity floats.They are calculated as follows:

a. Neutrosophic total float(NTF)
Neutrosophic total float is the positive difference between the neutrosophic earliest finish(start) time and neutrosophic latest finish(start) time of an activity.
Neutrosophic total float (NTF) = LF $(\mu_{ij})^N$ - EF $(\mu_{ij})^N$ (or) LS$(\mu_{ij})^N$ - ES$(\mu_{ij})^N$

b. Neutrosophic free float(NFF)
The delay in neutrosophic time of an activity which does not cause delay in its immediate successor activities is called neutrosophic free float of an activity.
Neutrosophic free float(NFF) = (E$(\mu_j)^N$ - E$(\mu_i)^N$) - $(t_{ij})^N$

3.2 Neutrosophic Project Evaluation an Review Technique Algorithm:

In this section, NPERT algorithm for a project network is established. This algorithm is used to find the neutrosophic critical path, neutrosophic expected time to complete each activity in a project and to calculate the probability of the neutrosophic expected time to compete the total project within a given period of time when it is not able to find best solution using existing methods in uncertain environment.

Step:1 Calculate neutrosophic earliest and latest work time for every activity using forward pass and backward pass calculations.

Step:2 Using neutrosophic earliest and latest work time of every activity, determine neutrosophic critical path for the given network.

Step:3 Calculate neutrosophic expected time of every activity for the given network.

Step:4 Calculate neutrosophic expected variance.

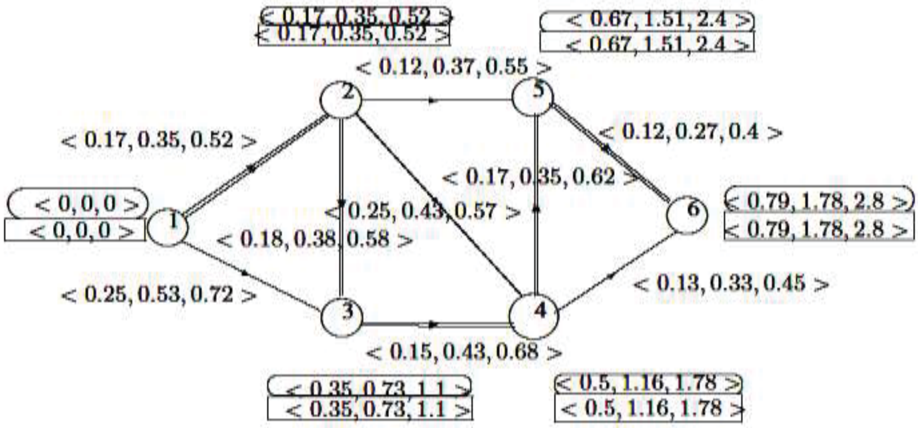
Step:5 Calculate neutrosophic total float of every activity.

Step:6 Find neutrosophic standard normal $Z^N = \frac{D^N - E(\mu_c^N)}{\sigma^N}$ using the neutrosophic critical path.

Step:7 Finally estimate the probability of completing the project within a due date.

Illustrative Example:
The neutrosophic estimate times for all activities in a project network are given in the following table.

Task	Neutro. optimistic time (t_o^N)	Neutro. Most likely time(t_m^N)	Neutro. Pessimistic time(t_p^N)
A(1,2)	< 0.1, 0.7, 0.8 >	< 0.2, 0.3, 0.5 >	< 0.1, 0.2, 0.3 >
A(1,3)	< 0.2, 0.8, 0.9>	< 0.3, 0.5, 0.7 >	< 0.1, 0.4, 0.6 >
A(2,3)	< 0.1, 0.4, 0.7 >	< 0.2, 0.4, 0.6 >	< 0.2, 0.3, 0.4 >
A(2,4)	< 0.2, 0.6, 0.7 >	< 0.3, 0.4, 0.5 >	< 0.1, 0.4, 0.7 >
A(2,5)	< 0.1, 0.3, 0.6 >	< 0.1, 0.4, 0.5 >	< 0.2, 0.5, 0.7 >
A(3,4)	< 0.2, 0.5, 0.9 >	< 0.1, 0.4, 0.6 >	< 0.3, 0.5, 0.8 >
A(4,5)	< 0.1, 0.2, 0.4 >	< 0.2, 0.4, 0.7 >	< 0.1, 0.3, 0.5 >
A(4,6)	< 0.2, 0.4, 0.6 >	< 0.1, 0.3, 0.4 >	< 0.3, 0.4, 0.5 >
A(5,6)	< 0.1, 0.5, 0.7 >	< 0.1, 0.2, 0.3 >	< 0.2, 0.3, 0.5 >



Determine the following:

i. Neutrosophic earliest and neutrosophic latest expected times for each node

ii. Neutrosophic critical path

iii. Neutrosophic expected task times and their variance

iv. The probability that the project will be completed within $[D^N =] < 0.8, 2, 3 >$

Solution:

i(a).Neutrosophic earliest expected task times:

$E(\mu_1)^N = < 0, 0, 0 >$

$E(\mu_2)^N = < 0, 0, 0 > + < 0.17, 0.35, 0.52 >$

$= < 0.17, 0.35, 0.52 >$

$E(\mu_3)^N = \max\{< 0, 0, 0 > + < 0.25, 0.53, 0.72 >, < 0.17, 0.35, 0.52 > + < 0.18, 0.38, 0.58 >\}$

$= < 0.35, 0.73, 1.1 >$

Similarly,

$E(\mu_4)^N = < 0.5, 1.16, 1.78 >$

$E(\mu_5)^N = < 0.67, 1.51, 2.4 >$

$E(\mu_6)^N = < 0.79, 1.78, 2.8 >$

i(b). Neutrosophic latest expected task times:

$L(\mu_6)^N = < 0.79, 1.78, 2.8 >$

$L(\mu_5)^N = < 0.79, 1.78, 2.8 > - < 0:12, 0:27, 0:4 >$

$L(\mu_4)^N = \min\{< 0.79, 1.78, 2.8 > - < 0.13, 0.33, 0.45 >, < 0.69, 1.51, 2.4 > - < 0.17, 0.35, 0.62 >\}$

$= < 0.5, 1.16, 1.78 >$

Similarly,

$L(\mu_3)^N = < 0.35, 0.73, 1.1 >$

$L(\mu_2)^N = < 0.17, 0.35, 0.52 >$

$L(\mu_1)^N = < 0, 0, 0 >$

ii. From i(a) and i(b), we conclude that the neutrosophiic critical path is $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6$.

iii. From above neutrosophic critical path, the total neutrosophic expected task time for completing the project is < 0.79, 1.78, 2.8 >

Also, the neutrosophic variance for the critical activities are:

Task	$t_e N = \frac{1}{6}[t_o^N + 4t_m^N + t_p^N]$	$\sigma^{2^N} = (\frac{t_p^N \; t_o^N}{6})^2$
(1,2)	< 0.17, 0.35, 0.52 >	< 0, 0.0064, 0.0064 >
(2,3)	< 0.18, 0.38, 0.58 >	< 0.0004, 0.0004, 0.0025 >
(3,4)	< 0.15, 0.43, 0.68 >	< 0.0004, 0, 0.0004 >
(4,5)	< 0.17, 0.35, 0.62 >	< 0, 0.0004, 0.0004 >
(5,6)	< 0.12, 0.27, 0:4 >	< 0.0004, 0.0009, 0.0009 >
Total	< 0.79, 1.78, 2.8 >	< 0.0012, 0.0081, 0.0106 >

Table.1

Hence, from table(1), the neutrosophic expected mean time [E(μ_e^N)] = < 0.79, 1.78, 2.8 > and the neutrosophic expected vaiance [σ^{2^N}] =<0.0012, 0.0081, 0.0106>

So, σ^N = < 0.0346, 0.09, 0.103 > .

Now,

$$Z^N = \frac{D^N - E(\mu e^N)}{\sigma^N}$$

$$= \frac{<0.8,2,3> - <0.79,1.78,2.8>}{<0.0346,0.09,0.103>}$$

$$= \frac{<0.01,0.22,0.2>}{<0.0346,0.09,0.103>}$$

= <0.29,0.143,0.108>

= 0.6796

by score function in definition 2.4.

iv. From the table of area under normal curve, P ($Z^N \leq$ 0.6796) = 0.5 + 0.2517 = 0.7517.
Therefore, the requried probability that the project will be completed within the time < 0.8, 2, 3 > is 0.7517.

4 Applications

This method is very useful than other existing methods like PERT, fuzzy PERT and intuitionistic fuzzy PERT etc., whenver uncertainty occurs in various activities like planning, scheduling, developing, designing, testing, maintaining and advertising for the fields of administration, construction, manufacturing and marketing etc.,

5 Advantages

1. It gives the better accuracy than other methods for each and every activity of a project.

2. Due to its accuracy, it is easy to find the best optimum schedule for every project.

3. Also the level of performance of each and every activity will be increased by interrelating them.

4. Controlling each activity in a project become very simple.

Conclusion

In this paper, NPERT calculation with single valued neutrosophic numbers for finding the total neutrosophic expected task time for completing a project network is introduced. The proposed method helps the users to take right decisions in scheduling, organizing and completing the project within a minimum duration. Also, it helps to find the probability of neutrosophic estimate time of a project. Comparing with other existing meth-ods, this method gives better results and also the NPERT algorithm is explained by an example using a set of neutrosophic numbers as length of arcs in a network. The applications and advantages of proposed method are also given. In future, we have planned to use this NPERT method in various network models.

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On Some Similarity Measures of Single Valued Neutrosophic Rough Sets.

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Abstract. In this paper we have obtained the similarity measures between single valued neutrosophic rough sets by analyzing the concept of its distance between them and studied its properties. Further we have studied its similarity based on its membership degrees and studied its properties. We have also defined the cardinality of two single valued neutrosophic rough sets. A numerical example in medical diagnosis is given for the proposed similarity measure of the single valued neutrosophic rough sets which helps us to prove the usefulness and flexibility of the proposed method.

Keywords: Single valued neutrosophic rough sets, similarity measure, cardinality.

1 Introduction

Fuzzy sets are generalizations of classical (crisp) sets which is based on partial membership of the elements and this was proposed by Zadeh [32] in 1965.. In 1983,K. Atanassov [2] proposed the concept of Intuitionistic fuzzy set which is a generalization of fuzzy set theory and is based on the degree of membership and non-membership and is described in the real unit interval [0,1], whose sum also belongs to the same interval.

IFS has numerous applications in decision making problems, medical diagnosis etc. After the theory of IFS many theories have been developed which are suitable in their respective areas.

In 1995 Florentin Smarandache [27] proposed the concept of Neutrosophic logic which provides the main distinction of fuzzy and IFS. It is a logic which is based on degree of truth (T), degree of indeterminacy (I) and degree of falsity (F) and lies in the nonstandard unit interval $]0^-,1^+[$. Neutrosophic set theory deals with uncertainity factor i.e, indeterminacy factor which is independent of truth and falsity values. Neutrosophic theory is applicable to the fields which is related to indeterminacy factor i.e, in the field of image processing, medical diagnosis and decision making problem.

In 1982, Pawlak [18] introduced the concept of rough set which is based upon the approximation of sets known as lower and upper approximation of a set. These two lower and upper approximation operators based on equivalence relation.

Rough fuzzy sets, intuitionistic fuzzy rough sets, neutrosophic rough sets are introduced by combining the rough sets respectively with fuzzy, intuitionstic, neutrosophic sets. In particular rough neutrosophic set initiated by Broumi and Smarandache (2014) [5]. C. Antony Crispin Sweety & I. Arockiarani(2016)[1] studied the concept of neutrosophic rough set algebra[1]. Wang (2010) [30] proposed the concept of SVNS which is a very new hot research topic.

SVNS and rough sets both deals with inaccuracy information and both combined together to provide a new hybrid model of single valued neutrosophic rough set. Many authors [3,4,6,8,9,19,31] studied the concept of similarity and entropy between the two single valued neutrosophic sets which helps to identify whether two sets are identical or atleast to what degree they are identical by using the concept of distance formula and membership function. Similarity plays a vital role in many fields like computational intelligence, psychology and linguistics, medical diagnosis, multi-attribute decision making problems.

Smarandache.F introduced the "Neutrosophic Sets and Systems" and its applications have been spreaded in all directions at an amazing rate. Smarandache, F. & Pramanik, S. (Eds). (2016)[28] *New trends in neutrosophic theory and applications* emphases on theories, procedures, systems for decision making, medical diagnosis and also discussed the topic includes e-learning, graph theory and some more. Recently Smarandache, F. & Pramanik, S. (Eds). (2018) and Mondal, K., Pramanik, S., & Giri, B. C. (2018) [29,17] studies New trends in neutrosophic theory and applications, Fuzzy Multicriteria Decision Making Using Neutrosophic Sets which provides the innovative study and application papers from diverse viewpoints covering the areas of neutrosophic studies, such as decision making, graph theory, image processing, probability theory, topology, and some abstract papers.

Pramanik, S., Roy, R., Roy, T. K., & Smarandache, F. (2018)[24] studied multi-attribute decision making based on several trigonometric hamming similarity measures under interval rough neutrosophic environment. Pramanik, S., Roy, R., Roy, T. K. & Smarandache, F. (2017)[23] also proposed the concept of multi criteria decision making using correlation coefficient under rough neutrosophic environment. Pramanik, S., & Mondal, K. (2015)[20] Mondal, K., Pramanik, S., & Smarandache, F. (2016) [9] studied several trigonometric Hamming similarity measures of rough neutrosophic sets and their applications in decision making.

Medical diagnosis is the process of determining which disease or condition explains a person's symptoms and signs. Similarity measures plays a efficient role in analysing the medical diagnosis problem. S. Pramanik, and K. Mondal. (2015)[12] described the cotangent similarity measure of rough neutrosophic sets and its application to medical diagnosis. And also Pramanik, S., & Mondal, K. (2015)[13] studied Cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis.

In this paper Section 2 gives some basic definitions of rough sets, neutrosophic sets, SVNSs and single valued neutrosophic rough sets. Section. 3 provides the distance and cardinality of two single valued neutrosophic rough sets with suitable example. In Section.4, we investigate the similarity measure of two single valued neutrosophic rough sets based on distance formulae and membership degrees. Section 5 gives a numerical example in medical diagnosis for the proposed similarity measure of single valued neutrosophic rough sets. Section 6 concludes the paper.

2 Preliminaries

In this section we recall the basic definitions of rough sets, Neutrosophic sets and single valued neutrosophic rough sets which will be used in the rest of the paper.

2.1 Definition 2.1[5]

Let U be any non-empty set. Suppose R is an equivalence relation over U. For any non – null subset X of U, the sets $A_1(x) = \{X : [x]_R \subseteq X\}$ and $A_2(x) = \{X : [x]_R \cap X \neq \phi\}$ are called the lower approximation and upper approximation respectively of X where the pair S=(U,R) is called an approximation space. This equivalence relation R is called indiscernibility relation. The pair $A(X) = (A_1(X), A_2(X))$ is called the rough set of X in S. Here $[x]_R$ denotes the equivalence class of R containing X.

2.2 Definition 2.2[27]

Let X be an universe of discourse, with a generic element in X denoted by x, the neutrosophic (NS) set is an object having the form, $A = \{< x : \mu_A(x), \nu_A(x), \omega_A(x) >, x \in X\}$ where the functions

$\mu, \nu, \omega : X \to \left]^{-}0,1^{+}\right[$ define respectively the degree of membership (or truth) , the degree of indeterminacy, and the degree of non-membership (or falsehood) of the element $x \in X$ to the set A with the condition,

$${}^{-}0 \leq \mu_A(x) + \nu_A(x) + \omega_A(x) \leq 3^{+}$$

2.3 Definition 2.3[30]

Let U be a space of points (objects), with a generic element in U denoted by x . A single valued neutrosophic set (SVNS) A in U is characterized by a truth-membership function T_A , an indeterminacy- membership function I_A and a falsity membership function F_A , where $\forall x \in U$, $T_A(x), I_A(x), F_A(x) \in [0,1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ A SVNS A can be expressed as $A = \{< x : T_A(x), I_A(x), F_A(x) >, x \in U\}$

2.4 Definition 2.4[7]

A SVNS R in $U \times U$ is referred to as a single valued neutrosophic relation (SVNR) in U, denoted by

$$R = \{< (x, y) : T_R(x, y), I_R(x, y), F_R(x, y) > / (x, y) \in U \times U\}$$

where $T_R : U \times U \to [0,1]$, $I_R : U \times U \to [0,1]$ *and* $F_R : U \times U \to [0,1]$ represent the truth – membership function, indeterminacy-membership function and falsity-membership function of R respectively. Based on a SVNR, Yang et al.[4] gave the notion of single valued neutrosophic rough set as follows.

Let $\widetilde{R}$ be a SVNR in U, the tuple $(U, \widetilde{R})$ is called a single valued neutrosophic approximation space $\forall \widetilde{A} \in SVNS(U)$, the lower and upper approximations of $\widetilde{A}$ with respect to $(U, \widetilde{R})$, denoted by $\underline{\widetilde{R}}(\widetilde{A})$ *and* $\overline{\widetilde{R}}(\widetilde{A})$ are two SVNS's whose membership functions are defined as $\forall x \in U$,

$$T_{\underline{\widetilde{R}}(\widetilde{A})}(x) = \wedge_{y \in U} (F_{\widetilde{R}}(x, y) \vee T_{\widetilde{A}}(y)),$$

$$I_{\underline{\widetilde{R}}(\widetilde{A})}(x) = \vee_{y \in U} ((1 - I_{\widetilde{R}}(x, y) \wedge I_{\widetilde{A}}(y)),$$

$$F_{\underline{\widetilde{R}}(\widetilde{A})}(x) = \vee_{y \in U} (T_{\widetilde{R}}(x, y) \wedge F_{\widetilde{A}}(y)),$$

$$T_{\overline{\widetilde{R}}(\widetilde{A})}(x) = \vee_{y \in U} (T_{\widetilde{R}}(x, y) \wedge T_{\widetilde{A}}(y)),$$

$$I_{\overline{\widetilde{R}}(\widetilde{A})}(x) = \wedge_{y \in U} (I_{\widetilde{R}}(x, y) \vee I_{\widetilde{A}}(y)),$$

$$F_{\overline{\widetilde{R}}(\widetilde{A})}(x) = \wedge_{y \in U} (F_{\widetilde{R}}(x, y) \vee F_{\widetilde{A}}(y)).$$

The pair $(\underline{\widetilde{R}}(\widetilde{A}), \overline{\widetilde{R}}(\widetilde{A}))$ is called a single valued neutrosophic rough set of $\widetilde{A}$ with respect to $(U, \widetilde{R})$. $\underline{\widetilde{R}}$ *and* $\overline{\widetilde{R}}$ are referred to as single valued neutrosophic lower and upper approximation operators respectively.

3 Distance between two single valued neutrosophic rough sets

In this section we define the distance between two single valued neutrosophic rough sets of $\widetilde{A}$ and $\widetilde{B}$ with respect to $(U, \widetilde{R}_1)$ *and* $(U, \widetilde{R}_2)$ in the universe $U = \{x_1, x_2, x_3, ... x_n\}$.

3.1 Definition 3.1

Let us consider two single valued neutrosophic rough sets of $\widetilde{A}$ and $\widetilde{B}$ with respect to $(U,\widetilde{R}_1)$ *and* $(U,\widetilde{R}_2)$ in the universe $U=\{x_1, x_2, x_3, ... x_n\}$. Here $\underline{\widetilde{R}}$ *and* $\overline{\widetilde{R}}$ are referred to as the single valued neutrosophic lower and upper approximation operators respectively. Throughout this section $\widetilde{A}$ and $\widetilde{B}$ denote the single valued neutrosophic rough sets with respect to $(U,\widetilde{R}_1)$ *and* $(U,\widetilde{R}_2)$.

(i) The Hamming distance of two single valued neutrosophic rough sets $\widetilde{A}$ and $\widetilde{B}$ with respect to its lower approximation:

$$d_{\underline{N}}(\widetilde{A},\widetilde{B})=\sum_{i=1}^{n}\{|T_{\underline{\widetilde{R}}(\widetilde{A})}(x_i)-T_{\underline{\widetilde{R}}(\widetilde{B})}(x_i)|+|I_{\underline{\widetilde{R}}(\widetilde{A})}(x_i)-I_{\underline{\widetilde{R}}(\widetilde{B})}(x_i)|+|F_{\underline{\widetilde{R}}(\widetilde{A})}(x_i)-F_{\underline{\widetilde{R}}(\widetilde{B})}(x_i)|\} \quad (1)$$

(ii) The Hamming distance of two single valued neutrosophic rough sets $\widetilde{A}$ and $\widetilde{B}$ with respect to its upper approximation:

$$d_{\overline{N}}(\widetilde{A},\widetilde{B})=\sum_{i=1}^{n}\{|T_{\overline{\widetilde{R}}(\widetilde{A})}(x_i)-T_{\overline{\widetilde{R}}(\widetilde{B})}(x_i)|+|I_{\overline{\widetilde{R}}(\widetilde{A})}(x_i)-I_{\overline{\widetilde{R}}(\widetilde{B})}(x_i)|+|F_{\overline{\widetilde{R}}(\widetilde{A})}(x_i)-F_{\overline{\widetilde{R}}(\widetilde{B})}(x_i)|\} \quad (2)$$

(iii) The normalized Hamming distance of $\widetilde{A}$ and $\widetilde{B}$ with respect to its lower approximation:

$$l_{\underline{N}}(\widetilde{A},\widetilde{B})=\frac{1}{3n}\sum_{i=1}^{n}\{|T_{\underline{\widetilde{R}}(\widetilde{A})}(x_i)-T_{\underline{\widetilde{R}}(\widetilde{B})}(x_i)|+|I_{\underline{\widetilde{R}}(\widetilde{A})}(x_i)-I_{\underline{\widetilde{R}}(\widetilde{B})}(x_i)|+|F_{\underline{\widetilde{R}}(\widetilde{A})}(x_i)-F_{\underline{\widetilde{R}}(\widetilde{B})}(x_i)|\} \quad (3)$$

(iv) The normalized Hamming distance of $\widetilde{A}$ and $\widetilde{B}$ with respect to its upper approximation:

$$l_{\overline{N}}(\widetilde{A},\widetilde{B})=\frac{1}{3n}\sum_{i=1}^{n}\{|T_{\overline{\widetilde{R}}(\widetilde{A})}(x_i)-T_{\overline{\widetilde{R}}(\widetilde{B})}(x_i)|+|I_{\overline{\widetilde{R}}(\widetilde{A})}(x_i)-I_{\overline{\widetilde{R}}(\widetilde{B})}(x_i)|+|F_{\overline{\widetilde{R}}(\widetilde{A})}(x_i)-F_{\overline{\widetilde{R}}(\widetilde{B})}(x_i)|\} \quad (4)$$

(v) The Euclidian distance of two single valued neutrosophic rough sets $\widetilde{A}$ and $\widetilde{B}$ with respect to its lower approximation:

$$e_{\underline{N}}(\widetilde{A},\widetilde{B})=\sqrt{\sum_{i=1}^{n}(T_{\underline{\widetilde{R}}(\widetilde{A})}(x_i)-T_{\underline{\widetilde{R}}(\widetilde{B})}(x_i))^2+(I_{\underline{\widetilde{R}}(\widetilde{A})}(x_i)-I_{\underline{\widetilde{R}}(\widetilde{B})}(x_i))^2+(F_{\underline{\widetilde{R}}(\widetilde{A})}(x_i)-F_{\underline{\widetilde{R}}(\widetilde{B})}(x_i))^2} \quad (5)$$

(vi) The Euclidian distance of two single valued neutrosophic rough sets $\widetilde{A}$ and $\widetilde{B}$ with respect to its upper approximation:

$$e_{\overline{N}}(\widetilde{A},\widetilde{B})=\sqrt{\sum_{i=1}^{n}(T_{\overline{\widetilde{R}}(\widetilde{A})}(x_i)-T_{\overline{\widetilde{R}}(\widetilde{B})}(x_i))^2+(I_{\overline{\widetilde{R}}(\widetilde{A})}(x_i)-I_{\overline{\widetilde{R}}(\widetilde{B})}(x_i))^2+(F_{\overline{\widetilde{R}}(\widetilde{A})}(x_i)-F_{\overline{\widetilde{R}}(\widetilde{B})}(x_i))^2} \quad (6)$$

(vii) The normalized Euclidian distance of two single valued neutrosophic rough sets $\widetilde{A}$ and $\widetilde{B}$ with respect to its lower approximation:

$$q_{\underline{N}}(\widetilde{A},\widetilde{B})=\sqrt{\frac{1}{3n}\sum_{i=1}^{n}(T_{\underline{\widetilde{R}}(\widetilde{A})}(x_i)-T_{\underline{\widetilde{R}}(\widetilde{B})}(x_i))^2+(I_{\underline{\widetilde{R}}(\widetilde{A})}(x_i)-I_{\underline{\widetilde{R}}(\widetilde{B})}(x_i))^2+(F_{\underline{\widetilde{R}}(\widetilde{A})}(x_i)-F_{\underline{\widetilde{R}}(\widetilde{B})}(x_i))^2} \quad (7)$$

(viii) The normalized Euclidian distance of two single valued neutrosophic rough sets $\widetilde{A}$ and $\widetilde{B}$ with respect to its upper approximation:

$$q_{\overline{N}}(\widetilde{A},\widetilde{B})=\sqrt{\frac{1}{3n}\sum_{i=1}^{n}(T_{\overline{\widetilde{R}}(\widetilde{A})}(x_i)-T_{\overline{\widetilde{R}}(\widetilde{B})}(x_i))^2+(I_{\overline{\widetilde{R}}(\widetilde{A})}(x_i)-I_{\overline{\widetilde{R}}(\widetilde{B})}(x_i))^2+(F_{\overline{\widetilde{R}}(\widetilde{A})}(x_i)-F_{\overline{\widetilde{R}}(\widetilde{B})}(x_i))^2} \quad (8)$$

Now for equations (1) – (8) the following conditions holds:

(a) $0\le d_{\underline{N}}(\widetilde{A},\widetilde{B})\le 3n$, $0\le d_{\overline{N}}(\widetilde{A},\widetilde{B})\le 3n$ (9)

(b) $0\le l_{\underline{N}}(\widetilde{A},\widetilde{B})\le 1$, $0\le l_{\overline{N}}(\widetilde{A},\widetilde{B})\le 1$ (10)

(c) $0\le e_{\underline{N}}(\widetilde{A},\widetilde{B})\le \sqrt{3n}$, $0\le e_{\overline{N}}(\widetilde{A},\widetilde{B})\le \sqrt{3n}$ (11)

(d) $0\le q_{\underline{N}}(\widetilde{A},\widetilde{B})\le 1$, $0\le q_{\overline{N}}(\widetilde{A},\widetilde{B})\le 1$ (12)

Example 3.2

Let $U=\{x_1, x_2, x_3\}$ be the universe and $\widetilde{R}_1,\widetilde{R}_2\in SVNS(U\times U)$ is given in Table 1 and Table 2

Let $\widetilde{A}=\{<x_1,(0.3,0.4,0.5)>,<x_2,(0,1,0.3)>,<x_3,(0.4,0.3,0.6)>\}$

$\widetilde{B}=\{<x_1,(0.2,0.8,0.1)>,<x_2,(1,0.3,1)>,<x_3,(0.5,0.3,0)>\}$ are SVNS's in U.

$\widetilde{R}_1$	x_1	x_2	x_3
x_1	(0,0.6,0.4)	(1,0,0.4)	(0.3,0.7,0.2)
x_2	(0,0.1,0.5)	(0.5,0,0.4)	(0.3,0.4,0.8)
x_3	(1,0,0.6)	(0.6,1,1)	(0,0,1)

Table 1: SVNR $\widetilde{R}_1$

$\widetilde{R}_2$	x_1	x_2	x_3
x_1	(0,0,1)	(0.2,0.1,0.6)	(1,0,0.5)
x_2	(0,0.1,0.3)	(0.5,0.4,1)	(0.5,1,0)
x_3	(1,1,0)	(0.4,1,1)	(1,0,0)

Table 2: SVNR $\widetilde{R}_2$

According to Definition 2.4 , we have

$$T_{\underline{\widetilde{R}}(\widetilde{A})}(x_1)=\wedge_{y\in U}(F_{\widetilde{R}}(x_1,y)\vee T_{\widetilde{A}}(y))=0.4$$

$$I_{\underline{\widetilde{R}}(\widetilde{A})}(x_1)=\vee_{y\in U}((1-I_{\widetilde{R}}(x_1,y)\wedge I_{\widetilde{A}}(y))=1$$

$$F_{\underline{\widetilde{R}}(\widetilde{A})}(x_1)=\vee_{y\in U}(T_{\widetilde{R}}(x_1,y)\wedge F_{\widetilde{A}}(y))=0.3,$$

$$T_{\overline{\widetilde{R}}(\widetilde{A})}(x_1)=\vee_{y\in U}(T_{\widetilde{R}}(x_1,y)\wedge T_{\widetilde{A}}(y))=0.3$$

$$I_{\overline{\widetilde{R}}(\widetilde{A})}(x_1)=\wedge_{y\in U}(I_{\widetilde{R}}(x_1,y)\vee I_{\widetilde{A}}(y))=0.6$$

$$F_{\overline{\widetilde{R}}(\widetilde{A})}(x_1)=\wedge_{y\in U}(F_{\widetilde{R}}(x_1,y)\vee F_{\widetilde{A}}(y))=0.4$$

Hence,

$$\underline{\widetilde{R}}(\widetilde{A})(x_1)=(0.4,1,0.3) \quad and \quad \overline{\widetilde{R}}(\widetilde{A})(x_1)=(0.3,0.6,0.4)$$

Similarly we can obtain,

$$\underline{\widetilde{R}}(\widetilde{A})(x_2)=(0.4,1,0.3) \quad and \quad \overline{\widetilde{R}}(\widetilde{A})(x_2)=(0.3,0.4,0.4)$$
$$\underline{\widetilde{R}}(\widetilde{A})(x_3)=(0.6,0.4,0.5) \quad and \quad \overline{\widetilde{R}}(\widetilde{A})(x_3)=(0.3,0.3,0.6)$$
$$\underline{\widetilde{R}}(\widetilde{B})(x_1)=(0.5,0.8,0.2) \quad and \quad \overline{\widetilde{R}}(\widetilde{B})(x_1)=(0.5,0.3,0.5)$$
$$\underline{\widetilde{R}}(\widetilde{B})(x_2)=(0.3,0.8,0.5) \quad and \quad \overline{\widetilde{R}}(\widetilde{B})(x_2)=(0.5,0.4,0)$$
$$\underline{\widetilde{R}}(\widetilde{B})(x_3)=(0.2,0.3,0.4) \quad and \quad \overline{\widetilde{R}}(\widetilde{B})(x_3)=(0.5,0.3,0)$$

Then the distance between $\widetilde{A}$ *and* $\widetilde{B}$ will be as follows :

$$d_{\underline{N}}(\widetilde{A},\widetilde{B})=\sum_{i=1}^{n}\{|T_{\underline{\widetilde{R}}(\widetilde{A})}(x_i)-T_{\underline{\widetilde{R}}(\widetilde{B})}(x_i)|+|I_{\underline{\widetilde{R}}(\widetilde{A})}(x_i)- I_{\underline{\widetilde{R}}(\widetilde{B})}(x_i)|+|F_{\underline{\widetilde{R}}(\widetilde{A})}(x_i)- F_{\underline{\widetilde{R}}(\widetilde{B})}(x_i)|\}$$
$$=\sum_{i=1}^{n}\{|T_{\underline{\widetilde{R}}(\widetilde{A})}(x_i)-T_{\underline{\widetilde{R}}(\widetilde{B})}(x_i)|+|I_{\underline{\widetilde{R}}(\widetilde{A})}(x_i)- I_{\underline{\widetilde{R}}(\widetilde{B})}(x_i)|+|F_{\underline{\widetilde{R}}(\widetilde{A})}(x_i)- F_{\underline{\widetilde{R}}(\widetilde{B})}(x_i)|\}$$
$$= 1.5$$
$$d_{\underline{N}}(\widetilde{A},\widetilde{B})=1.5$$

Similarly the other distances will be,

$$d_{\overline{N}}(\widetilde{A},\widetilde{B})=2$$
$$l_{\underline{N}}(\widetilde{A},\widetilde{B})=0.1666 \quad , \quad l_{\overline{N}}(\widetilde{A},\widetilde{B})=0.2222$$
$$e_{\underline{N}}(\widetilde{A},\widetilde{B})=0.5745 \quad , \quad e_{\overline{N}}(\widetilde{A},\widetilde{B})=0.86023$$
$$q_{\underline{N}}(\widetilde{A},\widetilde{B})=0.1916 \quad , \quad q_{\overline{N}}(\widetilde{A},\widetilde{B})=0.30916$$

3.3 Definition 3.3 (Cardinality)

The cardinality of a single valued neutrosophic rough set of $\widetilde{A}$ with respect to $(U,\widetilde{R})$ is denoted as $\underline{\widetilde{R}}[c]$ *and* $\overline{\widetilde{R}}[c]$, where $\underline{\widetilde{R}}[c]=[\underline{\widetilde{R}}(c^l),\underline{\widetilde{R}}(c^u)]$ is known as single valued neutrosophic lower approximation cardinality and, $\overline{\widetilde{R}}[c]=[\overline{\widetilde{R}}(c^l),\overline{\widetilde{R}}(c^u)]$ is known as single valued neutrosophic upper approximation cardinality.

Here $\underline{\widetilde{R}}(c^l)$, $\underline{\widetilde{R}}(c^u)$ denotes minimum and maximum cardinality of a single valued neutrosophic rough set with respect to lower approximation and is defined as ,

$$\underset{\sim}{\underline{R}}(c^l)=\sum_{i=1}^{n}T_{\underset{\sim}{\underline{R}}(A)}(x_i) \quad \text{and} \quad \underset{\sim}{\underline{R}}(c^u)=\sum_{i=1}^{n}\{T_{\underset{\sim}{\underline{R}}(A)}(x_i)+(1-I_{\underset{\sim}{\underline{R}}(A)}(x_i))\} \tag{13}$$

Here $\overline{\overline{R}}(c^l)$, $\overline{\overline{R}}(c^u)$ denotes minimum and maximum cardinality of a single valued neutrosophic rough set with respect to upper approximation and is defined as ,

$$\overline{\overline{R}}(c^l)=\sum_{i=1}^{n}T_{\overline{\overline{R}}(A)}(x_i) \quad \text{and} \quad \overline{\overline{R}}(c^u)=\sum_{i=1}^{n}\{T_{\overline{\overline{R}}(A)}(x_i)+(1-I_{\overline{\overline{R}}(A)}(x_i))\} \tag{14}$$

Example 3.4

Let us consider the single valued neutrosophic rough set of $\widetilde{B}$ from Example 3.2 we have the following cardinality,

$$\underset{\sim}{\underline{R}}(c^l)=\sum_{i=1}^{n}T_{\underset{\sim}{\underline{R}}(A)}(x_i)$$

$$=\sum_{i=1}^{3}T_{\underset{\sim}{\underline{R}}(A)}(x_i)$$

$$\underset{\sim}{\underline{R}}(c^l)=1$$

$$\underset{\sim}{\underline{R}}(c^u)=\sum_{i=1}^{n}\{T_{\underset{\sim}{\underline{R}}(A)}(x_i)+(1-I_{\underset{\sim}{\underline{R}}(A)}(x_i))\}$$

$$=\sum_{i=1}^{n}\{T_{\underset{\sim}{\underline{R}}(A)}(x_i)+(1-I_{\underset{\sim}{\underline{R}}(A)}(x_i))\}$$

$$\underset{\sim}{\underline{R}}(c^u)=2.1$$

$$\underset{\sim}{\underline{R}}[c]=[\underset{\sim}{\underline{R}}(c^l),\underset{\sim}{\underline{R}}(c^u)]=[1,2.1]$$

Similarly we can obtain,

$$\overline{\overline{R}}[c]=[\overline{\overline{R}}(c^l),\overline{\overline{R}}(c^u)]=[1.5,3.5]$$

4 Similarity measure between two single valued neutrosophic rough sets:

In this section we have defined the similarity measure between two single valued neutrosophic rough sets by the following two methods .

(i) Distance based similarity measure
(ii) Membership degree based similarity measure

A similarity measure between two single valued neutrosophic rough sets is a function defined as $S:\underline{N}(U)^2\to[0,1]$ *and* $\overline{N}(U)^2\to[0,1]$ which satisfies the following properties.

(i) $S_{\underline{N}}(\widetilde{A},\widetilde{B})\in[0,1]$ *and* $S_{\overline{N}}(\widetilde{A},\widetilde{B})\in[0,1]$

(ii) $S_{\underline{N}}(\widetilde{A},\widetilde{B})=1\Leftrightarrow\widetilde{A}=\widetilde{B}$ *and* $S_{\overline{N}}(\widetilde{A},\widetilde{B})=1\Leftrightarrow\widetilde{A}=\widetilde{B}$ (15)

(iii) $S_{\underline{N}}(\widetilde{A},\widetilde{B})=S_{\underline{N}}(\widetilde{B},\widetilde{A})$ *and* $S_{\overline{N}}(\widetilde{A},\widetilde{B})=S_{\overline{N}}(\widetilde{B},\widetilde{A})$

(iv) $\widetilde{A}\subset\widetilde{B}\subset\widetilde{C}\Rightarrow S_{\underline{N}}(\widetilde{A},\widetilde{C})\le S_{\underline{N}}(\widetilde{A},\widetilde{B})\wedge S_{\underline{N}}(\widetilde{B},\widetilde{C})$ *and* $S_{\overline{N}}(\widetilde{A},\widetilde{C})\le S_{\overline{N}}(\widetilde{A},\widetilde{B})\wedge S_{\overline{N}}(\widetilde{B},\widetilde{C})$

where $S_{\underline{N}}(\widetilde{A},\widetilde{B})$ and $S_{\overline{N}}(\widetilde{A},\widetilde{B})$ denotes the similarity measure of two single valued neutrosophic rough sets with respect to lower and upper approximation respectively.

4.1 Distance based similarity measure:

In general similarity measure or similarity function is a real-valued function that quantifies the similarity between two objects. It is the inverse of distance metrics.
Using the distance formulae it is generally defined as,

$$S^1(A,B)=\frac{1}{1+d(A,B)} \qquad (16)$$

For example if we consider the Euclidian distance of two single valued neutrosophic rough sets of $\widetilde{A}$ and $\widetilde{B}$ with respect to its lower approximation then it's associated similarity can be calculated as,

$$S^1{}_{\underline{N}}(\widetilde{A},\widetilde{B})=\frac{1}{1+e_{\underline{N}}(\widetilde{A},\widetilde{B})}$$

Example 4.1.1

From Example 3.2 the similarity measure can be calculated as,

$$S^1{}_{\underline{N}}(\widetilde{A},\widetilde{B})=\frac{1}{1+e_{\underline{N}}(\widetilde{A},\widetilde{B})}=0.6351$$

Proposition 4.1.2

The distance based similarity measure $S^1{}_{\underline{N}}$ *and* $S^1{}_{\overline{N}}$ with respect to lower and upper approximation of two single valued neutrosophic rough sets of $\widetilde{A}$ *and* $\widetilde{B}$ satisfies the following properties.

(i) $0\le S^1{}_{\underline{N}}(\widetilde{A},\widetilde{B})\le 1 \quad and \quad 0\le S^1{}_{\overline{N}}(\widetilde{A},\widetilde{B})\le 1$

(ii) $S^1{}_{\underline{N}}(\widetilde{A},\widetilde{B})=1\Leftrightarrow \widetilde{A}=\widetilde{B} \quad and \quad S^1{}_{\overline{N}}(\widetilde{A},\widetilde{B})=1\Leftrightarrow \widetilde{A}=\widetilde{B}$ (17)

(iii) $S^1{}_{\underline{N}}(\widetilde{A},\widetilde{B})=S^1{}_{\underline{N}}(\widetilde{B},\widetilde{A}) \quad and \quad S^1{}_{\overline{N}}(\widetilde{A},\widetilde{B})=S^1{}_{\overline{N}}(\widetilde{B},\widetilde{A})$

(iv) $\widetilde{A}\subset\widetilde{B}\subset\widetilde{C} \Rightarrow S^1{}_{\underline{N}}(\widetilde{A},\widetilde{C})\le S^1{}_{\underline{N}}(\widetilde{A},\widetilde{B})\wedge S^1{}_{\underline{N}}(\widetilde{B},\widetilde{C}) \quad and \quad S^1{}_{\overline{N}}(\widetilde{A},\widetilde{C})\le S^1{}_{\overline{N}}(\widetilde{A},\widetilde{B})\wedge S^1{}_{\overline{N}}(\widetilde{B},\widetilde{C})$

Proof:

The results (i) – (iii) holds trivially from definition. It is enough to prove only (iv).
Let us consider three single valued neutrosophic rough sets $\widetilde{A}$, $\widetilde{B}$ and $\widetilde{C}$ with respect to $(U,\widetilde{R})$ in the universe $U=\{x_1,x_2,x_3,.......x_n\}$. Let $\widetilde{A}\subset\widetilde{B}\subset\widetilde{C}$ then we have

$$T_{\underline{\widetilde{R}}(\widetilde{A})}(x)\le T_{\underline{\widetilde{R}}(\widetilde{B})}(x)\le T_{\underline{\widetilde{R}}(\widetilde{C})}(x);I_{\underline{\widetilde{R}}(\widetilde{A})}(x)\ge I_{\underline{\widetilde{R}}(\widetilde{B})}(x)\ge I_{\underline{\widetilde{R}}(\widetilde{C})}(x)\, and$$
$$F_{\underline{\widetilde{R}}(\widetilde{A})}(x)\ge F_{\underline{\widetilde{R}}(\widetilde{B})}(x)\ge F_{\underline{\widetilde{R}}(\widetilde{C})}(x)\,\forall x\in U$$

Now

$|T_{\underline{R}(\widetilde{A})}(x)- T_{\underline{R}(\widetilde{B})}(x)|\leq|T_{\underline{R}(\widetilde{A})}(x)- T_{\underline{R}(\widetilde{C})}(x)|\, and$

$|T_{\underline{R}(\widetilde{B})}(x)- T_{\underline{R}(\widetilde{C})}(x)|\leq|T_{\underline{R}(\widetilde{A})}(x)- T_{\underline{R}(\widetilde{C})}(x)|$ *will hold.*

Similarly,

$|I_{\underline{R}(\widetilde{A})}(x)- I_{\underline{R}(\widetilde{B})}(x)|\geq|I_{\underline{R}(\widetilde{A})}(x)- I_{\underline{R}(\widetilde{C})}(x)|\, and$

$|I_{\underline{R}(\widetilde{B})}(x)- I_{\underline{R}(\widetilde{C})}(x)|\geq|I_{\underline{R}(\widetilde{A})}(x)- I_{\underline{R}(\widetilde{C})}(x)|$ *and also*

$|F_{\underline{R}(\widetilde{A})}(x)- F_{\underline{R}(\widetilde{B})}(x)|\geq|F_{\underline{R}(\widetilde{A})}(x)- F_{\underline{R}(\widetilde{C})}(x)|\, and$

$|F_{\underline{R}(\widetilde{B})}(x)- F_{\underline{R}(\widetilde{C})}(x)|\geq|F_{\underline{R}(\widetilde{A})}(x)- F_{\underline{R}(\widetilde{C})}(x)|$ *holds*

Thus

$d_{\underline{N}}(\widetilde{A},\widetilde{B})\leq d_{\underline{N}}(\widetilde{A},\widetilde{C}) \Rightarrow S^1{}_{\underline{N}}(\widetilde{A},\widetilde{B})\geq S^1{}_{\underline{N}}(\widetilde{A},\widetilde{C})$ *and*

$d_{\underline{N}}(\widetilde{B},\widetilde{C})\leq d_{\underline{N}}(\widetilde{A},\widetilde{C}) \Rightarrow S^1{}_{\underline{N}}(\widetilde{B},\widetilde{C})\geq S^1{}_{\underline{N}}(\widetilde{A},\widetilde{C})$

$$\Rightarrow S^1{}_{\underline{N}}(\widetilde{A},\widetilde{C})\leq S^1{}_{\underline{N}}(\widetilde{A},\widetilde{B})\wedge S^1{}_{\underline{N}}(\widetilde{B},\widetilde{C})$$

This is true for all the distance functions defined in equations (1) to (8)
Hence the result.

4.2 Similarity measure based on membership degrees

Another similarity measure of $S^2{}_{\underline{N}}\, and\, S^2{}_{\overline{N}}$ between two single valued neutrosophic rough sets of $\widetilde{A}$ *and* $\widetilde{B}$ with respect to lower and upper approximation will be defined as follows:

$$S^2{}_{\underline{N}}(\widetilde{A},\widetilde{B})=\frac{\sum_{i=1}^{n}\{\min\{T_{\underline{R}(\widetilde{A})}(x_i),T_{\underline{R}(\widetilde{B})}(x_i)\}+\min\{I_{\underline{R}(\widetilde{A})}(x_i),I_{\underline{R}(\widetilde{B})}(x_i)\}+\min\{F_{\underline{R}(\widetilde{A})}(x_i),F_{\underline{R}(\widetilde{B})}(x_i)\}\}}{\sum_{i=1}^{n}\{\max\{T_{\underline{R}(\widetilde{A})}(x_i),T_{\underline{R}(\widetilde{B})}(x_i)\}+\max\{I_{\underline{R}(\widetilde{A})}(x_i),I_{\underline{R}(\widetilde{B})}(x_i)\}+\max\{F_{\underline{R}(\widetilde{A})}(x_i),F_{\underline{R}(\widetilde{B})}(x_i)\}\}}$$

$$S^2{}_{\overline{N}}(\widetilde{A},\widetilde{B})=\frac{\sum_{i=1}^{n}\{\min\{T_{\overline{R}(\widetilde{A})}(x_i),T_{\overline{R}(\widetilde{B})}(x_i)\}+\min\{I_{\overline{R}(\widetilde{A})}(x_i),I_{\overline{R}(\widetilde{B})}(x_i)\}+\min\{F_{\overline{R}(\widetilde{A})}(x_i),F_{\overline{R}(\widetilde{B})}(x_i)\}\}}{\sum_{i=1}^{n}\{\max\{T_{\overline{R}(\widetilde{A})}(x_i),T_{\overline{R}(\widetilde{B})}(x_i)\}+\max\{I_{\overline{R}(\widetilde{A})}(x_i),I_{\overline{R}(\widetilde{B})}(x_i)\}+\max\{F_{\overline{R}(\widetilde{A})}(x_i),F_{\overline{R}(\widetilde{B})}(x_i)\}\}}$$

Example 4.2.1

From Example 3.2 the similarity measure can be calculated as,

$$S^2{}_{\underline{N}}(\widetilde{A},\widetilde{B})=\frac{\sum_{i=1}^{n}\{\min\{T_{\underline{R}(\widetilde{A})}(x_i),T_{\underline{R}(\widetilde{B})}(x_i)\}+\min\{I_{\underline{R}(\widetilde{A})}(x_i),I_{\underline{R}(\widetilde{B})}(x_i)\}+\min\{F_{\underline{R}(\widetilde{A})}(x_i),F_{\underline{R}(\widetilde{B})}(x_i)\}\}}{\sum_{i=1}^{n}\{\max\{T_{\underline{R}(\widetilde{A})}(x_i),T_{\underline{R}(\widetilde{B})}(x_i)\}+\max\{I_{\underline{R}(\widetilde{A})}(x_i),I_{\underline{R}(\widetilde{B})}(x_i)\}+\max\{F_{\underline{R}(\widetilde{A})}(x_i),F_{\underline{R}(\widetilde{B})}(x_i)\}\}}$$

$S^2{}_{\underline{N}}(\widetilde{A},\widetilde{B})=0.7115$

$$S^2_{\overline{N}}(\widetilde{A},\widetilde{B})=\frac{\sum_{i=1}^{n}\{\min\{T_{\overline{R}(\widetilde{A})}(x_i),T_{\overline{R}(\widetilde{B})}(x_i)\}+\min\{I_{\overline{R}(\widetilde{A})}(x_i),I_{\overline{R}(\widetilde{B})}(x_i)\}+\min\{F_{\overline{R}(\widetilde{A})}(x_i),F_{\overline{R}(\widetilde{B})}(x_i)\}\}}{\sum_{i=1}^{n}\{\max\{T_{\overline{R}(\widetilde{A})}(x_i),T_{\overline{R}(\widetilde{B})}(x_i)\}+\max\{I_{\overline{R}(\widetilde{A})}(x_i),I_{\overline{R}(\widetilde{B})}(x_i)\}+\max\{F_{\overline{R}(\widetilde{A})}(x_i),F_{\overline{R}(\widetilde{B})}(x_i)\}\}}$$

$$S^2_{\overline{N}}(\widetilde{A},\widetilde{B})=0.5349$$

Proposition 4.2.2

The membership degree based similarity measure $S^2_{\underline{N}}$ *and* $S^2_{\overline{N}}$ with respect to lower and upper approximation of two single valued neutrosophic rough sets of $\widetilde{A}$ *and* $\widetilde{B}$ satisfies the following properties.

$$\begin{aligned}
&\text{(i) } 0\le S^2_{\underline{N}}(\widetilde{A},\widetilde{B})\le 1 \quad and \quad 0\le S^2_{\overline{N}}(\widetilde{A},\widetilde{B})\le 1\\
&\text{(ii) } S^2_{\underline{N}}(\widetilde{A},\widetilde{B})=1\Leftrightarrow \widetilde{A}=\widetilde{B} \quad and \quad S^2_{\overline{N}}(\widetilde{A},\widetilde{B})=1\Leftrightarrow \widetilde{A}=\widetilde{B} \qquad (18)\\
&\text{(iii) } S^2_{\underline{N}}(\widetilde{A},\widetilde{B})=S^2_{\underline{N}}(\widetilde{B},\widetilde{A}) \quad and \quad S^2_{\overline{N}}(\widetilde{A},\widetilde{B})=S^2_{\overline{N}}(\widetilde{B},\widetilde{A})\\
&\text{(iv) } \widetilde{A}\subset\widetilde{B}\subset\widetilde{C} \Rightarrow S^2_{\underline{N}}(\widetilde{A},\widetilde{C})\le S^2_{\underline{N}}(\widetilde{A},\widetilde{B})\wedge S^2_{\underline{N}}(\widetilde{B},\widetilde{C}) \quad and \quad S^2_{\overline{N}}(\widetilde{A},\widetilde{C})\le S^2_{\overline{N}}(\widetilde{A},\widetilde{B})\wedge S^2_{\overline{N}}(\widetilde{B},\widetilde{C})
\end{aligned}$$

Proof: The results (i) – (iii) holds trivially from definition. It is enough to prove only (iv).

Let us consider three single valued neutrosophic rough sets $\widetilde{A}$, $\widetilde{B}$ and $\widetilde{C}$ with respect to $(U,\widetilde{R})$ in the universe $U=\{x_1,x_2,x_3,.......x_n\}$. Let $\widetilde{A}\subset\widetilde{B}\subset\widetilde{C}$ then we have

$$T_{\underline{R}(\widetilde{A})}(x)\le T_{\underline{R}(\widetilde{B})}(x)\le T_{\underline{R}(\widetilde{C})}(x);\, I_{\underline{R}(\widetilde{A})}(x)\ge I_{\underline{R}(\widetilde{B})}(x)\ge I_{\underline{R}(\widetilde{C})}(x)\, and$$

$$F_{\underline{R}(\widetilde{A})}(x)\ge F_{\underline{R}(\widetilde{B})}(x)\ge F_{\underline{R}(\widetilde{C})}(x)\,\forall\, x\in U$$

Now,

$$T_{\underline{R}(\widetilde{A})}(x)+I_{\underline{R}(\widetilde{A})}(x)+F_{\underline{R}(\widetilde{B})}(x)\ge T_{\underline{R}(\widetilde{A})}(x)+I_{\underline{R}(\widetilde{A})}(x)+F_{\underline{R}(\widetilde{C})}(x)\, and$$

$$T_{\underline{R}(\widetilde{B})}(x)+I_{\underline{R}(\widetilde{B})}(x)+F_{\underline{R}(\widetilde{A})}(x)\le T_{\underline{R}(\widetilde{C})}(x)+I_{\underline{R}(\widetilde{C})}(x)+F_{\underline{R}(\widetilde{A})}(x)$$

$$S^2_{\underline{N}}(\widetilde{A},\widetilde{B})=\frac{T_{\underline{R}(\widetilde{A})}(x)+I_{\underline{R}(\widetilde{A})}(x)+F_{\underline{R}(\widetilde{B})}(x)}{T_{\underline{R}(\widetilde{B})}(x)+I_{\underline{R}(\widetilde{B})}(x)+F_{\underline{R}(\widetilde{A})}(x)}\ge\frac{T_{\underline{R}(\widetilde{A})}(x)+I_{\underline{R}(\widetilde{A})}(x)+F_{\underline{R}(\widetilde{C})}(x)}{T_{\underline{R}(\widetilde{C})}(x)+I_{\underline{R}(\widetilde{C})}(x)+F_{\underline{R}(\widetilde{A})}(x)}=S^2_{\underline{N}}(\widetilde{A},\widetilde{C})$$

Similarly we have,

$$T_{\underline{R}(\widetilde{B})}(x)+I_{\underline{R}(\widetilde{B})}(x)+F_{\underline{R}(\widetilde{C})}(x)\ge T_{\underline{R}(\widetilde{A})}(x)+I_{\underline{R}(\widetilde{A})}(x)+F_{\underline{R}(\widetilde{C})}(x)\, and$$

$$T_{\underline{R}(\widetilde{C})}(x)+I_{\underline{R}(\widetilde{C})}(x)+F_{\underline{R}(\widetilde{A})}(x)\ge T_{\underline{R}(\widetilde{C})}(x)+I_{\underline{R}(\widetilde{C})}(x)+F_{\underline{R}(\widetilde{B})}(x)$$

$$S^2_{\underline{N}}(\widetilde{B},\widetilde{C})=\frac{T_{\underline{R}(\widetilde{B})}(x)+I_{\underline{R}(\widetilde{B})}(x)+F_{\underline{R}(\widetilde{C})}(x)}{T_{\underline{R}(\widetilde{C})}(x)+I_{\underline{R}(\widetilde{C})}(x)+F_{\underline{R}(\widetilde{B})}(x)}\ge\frac{T_{\underline{R}(\widetilde{A})}(x)+I_{\underline{R}(\widetilde{A})}(x)+F_{\underline{R}(\widetilde{C})}(x)}{T_{\underline{R}(\widetilde{C})}(x)+I_{\underline{R}(\widetilde{C})}(x)+F_{\underline{R}(\widetilde{A})}(x)}=S^2_{\underline{N}}(\widetilde{A},\widetilde{C})$$

$$\Rightarrow S^2_{\underline{N}}(\widetilde{A},\widetilde{C})\le S^2_{\underline{N}}(\widetilde{A},\widetilde{B})\wedge S^2_{\underline{N}}(\widetilde{B},\widetilde{C})$$

Hence the proof.

5 **Applications to Medical Diagnosis**:

In this section we present some real life applications of the similarity measure of single valued neutrosophic rough sets. Many real life practical problems consist of more uncertainty and incomplete information. To deal this problem effectively , rough neutrosophic set helps to deal with uncertainty and incompleteness.

Let us consider a medical diagnosis problem for the illustration of the proposed approach. Medical diagnosis is the process of determining which disease or condition explains a person's symptoms and signs. Diagnosis is a challenging one which consists of uncertainties and many signs & symptoms are non-specific. To handle this way of problem, rough neutrosophic set provided a good way in which several possible explanations are compared and contrasted must be perfomed by the method of similarity measure. So similarity measure helps to identify whether two sets are identical or atleast to what degree they are identical by using the concept of distance formula and membership function.

Let us consider the same example which we have discussed in earlier Section 3 in Example 3.2 and apply that example to medical diagnosis problem, let $U=\{x_1, x_2, x_3\}$ be the universe of patients. Consider the same two SVNS's A and B with respect to SVNR's $\widetilde{R}_1, \widetilde{R}_2 \in SVNS\,(U \times U)$ respectively which is given in Table 1 and Table 2. Let D = {Viral fever, Malaria, Typhoid} be the set of diseases and also $\widetilde{R}_1, \widetilde{R}_2$ denotes the relation between the patients and diseases of the SVNS's A and B respectively.

Hence, this section provides relative study among similarity measures proposed in this paper. The comparision study of similarity measures based on different distances formulae and membership degree is given in Table 3 in detail.

$$\underline{\widetilde{R}}(\widetilde{A})(x_1)=(0.4,1,0.3) \quad and \quad \overline{\widetilde{R}}(\widetilde{A})(x_1)=(0.3,0.6,0.4)$$

Similarly we can obtain,

$$\underline{\widetilde{R}}(\widetilde{A})(x_2)=(0.4,1,0.3) \quad and \quad \overline{\widetilde{R}}(\widetilde{A})(x_2)=(0.3,0.4,0.4)$$

$$\underline{\widetilde{R}}(\widetilde{A})(x_3)=(0.6,0.4,0.5) \quad and \quad \overline{\widetilde{R}}(\widetilde{A})(x_3)=(0.3,0.3,0.6)$$

$$\underline{\widetilde{R}}(\widetilde{B})(x_1)=(0.5,0.8,0.2) \quad and \quad \overline{\widetilde{R}}(\widetilde{B})(x_1)=(0.5,0.3,0.5)$$

$$\underline{\widetilde{R}}(\widetilde{B})(x_2)=(0.3,0.8,0.5) \quad and \quad \overline{\widetilde{R}}(\widetilde{B})(x_2)=(0.5,0.4,0)$$

$$\underline{\widetilde{R}}(\widetilde{B})(x_3)=(0.2,0.3,0.4) \quad and \quad \overline{\widetilde{R}}(\widetilde{B})(x_3)=(0.5,0.3,0)$$

Table 3: Similarity values

Similarity measure based on	$S_{\underline{N}}(\widetilde{A},\widetilde{B})$	$S_{\overline{N}}(\widetilde{A},\widetilde{B})$
Hamming distance	0.4	0.3333
Normalized hamming distance	0.8572	0.8182
Euclidian distance	0.6351	0.5376
Normalized euclidian distance	0.8392	0.7638
Membership degree	0.7115	0.5349

In Table 3 $S_{\underline{N}}(\widetilde{A},\widetilde{B})$, $S_{\overline{N}}(\widetilde{A},\widetilde{B})$ denotes the similarity lower and upper approximation measure of the two single valued neutrosophic rough sets respectively. In practical it represents the lower and upper approximation similarity measures between patients and diseases of two single valued neutrosophic rough sets.That is through hamming distance the similarity lower and upper measure between patients and diseases of two single valued neutrosophic rough sets A and B will be 0.4 and 0.3333 respectively.

Table 3 represents that each method has its own way to calculate the similarity measure and also any method can be preferrable to calculate the similarity measure between two single valued neutrosophic rough sets.

6 Conclusion

Single valued neutrosophic set (SVNS) is an instance of NS and it is an extension of fuzzy set and IFS. Compare to previous traditional models like fuzzy set, IFS, NS, crisp set , it provides more precise, compatible and flexible in comparison. By combining the concept of SVNS with rough set a new hybrid model of single valued neutrosophic rough set was introduced and now-a-days it is a very new hot research topic. In this paper we have defined the notion of similarity between two single valued neutrosophic rough sets based on distance formulae and membership degrees. We have also studied some properties on them and proved some prepositions and a numerical example is given in medical diagnosis for the proposed similarity measure concept.

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Blockchain Single and Interval Valued Neutrosophic Graphs

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Abstract. Blockchain Technology (BCT) is a growing and reliable technology in various fields such as developing business deals, economic environments, social and politics as well. Without having a trusted central party this technology, gives the guarantee for safe and reliable transactions using Bitcoin or Ethereum. In this paper BCT has been considered using Bitcoins. Also Blockchain Single and Interval Valued Neutrosophic Graphs have been proposed and applied in transaction of Bitcoins. Also degree, total degree, minimum and maximum degree have been found for the proposed graphs. Further, comparative analysis is done with advantages and limitations of different types of Blockchain graphs.

Keywords: Blockchian Technology, Bitcoins, Fuzzy Graph, Neutrosophic Graphs, Properties

1. Introduction

A completely peer-to-peer form of electronic cash will permit payments through online and direct transaction can be done from one participant to another without facing any financial organization. If a central party wants to avoid double-spending then the main gain will be lost even though digital signatures contribute part of the solution. This issue was the reason of bargain a solution to this problem based on peer-to-peer network. For direct transaction of two willing parties without having a trusted third party, an electronic system using cryptographic proof (signaling code) can be used. Fuzzy logic is introduced by Zadeh to deal uncertainty of the problem. Fuzzy graphs are playing an important role in network where impreciseness exists on the vertices and egdes. Yeh and Banh also proposed the fuzzy graph independently and examined various connectedness theories [1-4].

The universal problems namely sustainable development or transformation of assets can be dealt effectively by Block chain technology than the existing financial systems. The financial sector acquires in various operative costs for the smooth and effective functioning of the entire system. These costs consist of time and money needed for investment in framework, electricity cost spent for operation and from Automated Teller Machines, consumption of water and gas by the employees and wastage production.

Also there is no possibility of creating fiat currency without costs. In order to give assurance in a regular basis in the quality standards for the bank notes in circulation, the used ones are shredded. To find an overview of the overall cost of an existing financial system, the cost for the production of coins and noted will be included. Whereas in BCT, one needs only to connect to the network and do not obtain the electricity cost for any source. Also the production of the crypto currency (a digitalized currency, where encoding method is applied to control the production of currency and funds transference verification) [5-7].

Platforms of Central banking, improvement of business processing, automotive ownership, sharing of health information, deals and voting can be potentially replaced by Block chain Technology. BCT plays an important role, in political components namely governmental interference, control leadership and taxation. Also BCT is very useful in Exchange rates of currency market growth and monetary as an economic component. BCT is very helpful in social components namely environmental situation, culture, behavior of the customer and demand. In the same way, BCT has a potential action in modern technologies and tendency [8-9].

BCT permits an emerging set of participants to continue with a secure and alter–proof ledger for all the activities without having a third trusted party. Here, transactions are not actually documented but instead, every participant keeps a provincial copy of the ledger which is a related listing of blocks and they comprise agreed transactions [19]. Nagoorgani and Radha introduced the concept of degree of fuzzy vertex. A crypto currency is nothing a Bitcoin which is a universal payment system and also the initial decentralized digital currency since the system works without a single administrator or central bank. Bitcoins made as a payment for a process called mining and can be exchanged for different currencies. Nakamoto and Satoshi were introduced the concept of Blockchain and applied as an important component of Bitcoin where it act as a public ledger for all the transactions. To solve double-spending problem, Blockchain for the Bitcoin has been an appropriate choice without the help of trusted third part as a central server. Block chain transactions will be done on the interchangeable ledger data saved at every node [42].

A Blockchain network can be seen as a reliable computer whose private states are auditable by anyone. A ledger of transactions may call as a Blockchain. Generally a physical ledger will be maintained by a centralized party whereas in Blockchain is a distributed ledger which locates on the device of every participants. Bitcoins are believable and best used [40, 45]. A Fuzzy Set (FS) can be described mathematically by assigning a value, a grade of membership to every desirable person in the universe of discourse. This grade of membership associates a degree for the participant is either identical or appropriate to the approach performed by FS. A fuzzy subset of FS, X is a function from membership to non-membership and is defined by $\eta : X \to [0,1]$ continuous rather than unexpected. Fuzzy relationships are popular and essential in the fields of computer chains, decision making, neural network, expert systems etc. Direct relationship and also indirect relationship also will be considered in graph theory.

Model of relation is nothing but a graph which is a comfortable way of describing information about the connection between two objects. In graph, points and relations are defined by vertices and edges respectively. While impreciseness exists in the statement of the phenomenon or in the communication or both, fuzzy graph model can be designed for getting an optimized output. Maximizing the Utility of the application is always done by the researchers during the constructing of a model with a key characteristics reliability, complexity and impreciseness. Among these, impreciseness is a considerable one in maximizing the utility of the technique. This situation can be described by fuzzy sets, introduced by Lotfi. A. Zadeh [24, 25].

Zadeh formulated, grade of membership in order to handle with impreciseness. Atannasov introduced intuitionistic fuzzy set by including the grade of non-membership in FS as a separate element. Samarandache introduced Neutrosophic set (NS) by finding the membership degree of indeterminacy, it can be viewed from the logical point of view as a self-ruling component to handle with uncertain, undetermined and unpredictable data which are exist in the real world problem. The NSs are defined by the membership functions of truth, indeterminacy and falsity whose values take from the real standard interval. Wang et al. proposed the theoretical concept of single-valued Neutrosophic sets (SVNS) and Interval valued Neutrosophic Sets (IVNSs) as well [26-34].

If uncertainty exists in vertices or edges set or both then the structure turns into a fuzzy graph. It can be established by taking the set of vertices and edges as FS, in the same way one can model any other types of fuzzy graphs [21-15, 32].Graph theory defines the relationship between various individuals and has got many number of applications in different fields namely database theory, modern discipline and technology, neural networks, data scooping cluster analysis, knowledge systems image capturing and control theory. Handling Indeterminacy on the object or edge or both cannot be handled fuzzy graphs and hence Neutrosophic graphs have been introduced. [44, 47-48].

A new perspective for neutrosophic theory and its applications also proposed [49]. There are many methods have been proposed under single valued neutrosophic, interval valued neutrosophic and neutrosophic environments by colloborating with other methods such as TOPSIS, DEMATEL, VIKOR. Also all these hybrid and extension methods applied in the process of decision making. Further, NS-cross entropy, hyperbolic sine similarity measure, hybrid binary algorithm similarity measure method and single-valued co-neutrosophic graphs play an important role in decision making. In fuzzy graph all the edges are represented by fuzzy numbers and that may be interval valued fuzzy number also. Whereas in neutrososphic graph the edges are represented by single valued neutrosophic numbers [50-62].

The remaining part of the paper is organized as follows. In section 2, review of literature is presented. In section 3, basic concepts related to the presented work is given. In section 4, Blockchain single valued and interval valued neutroosphic graphs are proposed and applied for Bitcoin transaction. Also degree, total degree, minimum and maximum degree have been found. In section 5, qualitative analysis has been done with the limitations and advantages of various types of graphs. In section 6, conclusion of the paper is given with the future work.

2. Review of Literature

[Yeh and Bang 1] proposed fuzzy relations, fuzzy graphs and applied them in cluster analysis. [Satoshi 2] presented a solution for the problem of double-sending using a peer-to-peer network. [Leroy 3] portrayed the evolution and proof of linguistic care of an accumulator back-end. [Dey et al.4] have done a vertex colouring of a fuzzy graph. [Dey et al. 5] applied the concept of fuzzy graph in light control in traffic control management. [Ober et al. 6] proposed a model and obscurity of the Bitcoin transaction graph. [Decker and Wattenhofer 7] examined about knowledge reproduction in the network of Bitcoin. [Fleder et al. 8] linked bit coin public keys to real people and commented about the public transaction graph and hence done a graph analysis scheme to find and compiled activity of known as well as unknown users.

[Stanfill and Wholey 9] proposed a transactional graph on the basis of computation with error management. [Ye 10] proposed aggregation operators under simplified neutrosophic environment and applied them in a decision making problem. [Biswas et al. 11] introduced a new methodology for dealing unknown weight information and applied in a decision making problem. [Biswas et al. 12] proposed grey relational analysis based on entropy under single valued neutrosophic setting and applied in a decision making process with multi attribute.

[Mondal and Pramanik 13] introduced a model for clay-brick selection based on grey relational analysis for neutrosophic decision making. [Mondal and Pramanik 14] proposed neutrosophic tangent similarity measure and applied in multiple attribute decision making. [Biswas et al. 15] introduced cosine similarity measure with trapezoidal fuzzy neutrosophic numbers and applied in a decision making problem. [Broumi et al. 16] introduced an extended TOPSIS methodology using interval neutrosophic uncertain linguistic variables. [Greaves and Au 17] investigated the prognostic power of Blockchain network using lineaments on the future price of Bitcoin. [Pilkington 18] clarified the main ethics behind block chain technique and few of its application of cutting edge.

[Bonneau et al. 19] Analyzed invisibility problems in Bitcoin and contribute an evaluation plan for private- enlarging proposals and contributed a new intuition on language disintermediation protocols. [Smarandache and Pramanik 20] introduced a new direction for neutrosophic theory and applications. [Biswas et al. 21] proposed TOPSIS methodology under single-valued neutrosophic setting for multi-attribute group decision making. [Biswas et al. 22] proposed aggregation operators for triangular fuzzy neutrosophic set information and used for a decision making problem. [Biswas et al. 23] introduced a ranking method based on value and ambiguity index using single-valued trapezoidal neutrosophic numbers and its application to decision making problem. [Eyal et al. 24] designed a block chain protocol called Bitcoin –next generation. [Broumi et al. 25] introduced operational laws on interval valued neutrosophic graphs.

[Broumi et al. 26] proposed the formulas to find degree, size and order of a single valued neutrosophic graphs. [Pramanik et al. 27] proposed hybrid similarity measures under neutrosophic environment and applied them in decision making problem. [Dalapati et al. 28] introduced IN-cross entropy for interval neutrosophic set environment and applied in multi attribute group decision making process. [Broumi et al. 29] proposed uniform single valued neutrosophic graphs. [Cocco et al. 30] paid attention at the threats and opportunities of carrying out Blockchain mechanism across banking. [Jeoseph et al. 31] reviewed the approval and future use of block chain technology.

[Chan and Olmsted 32] proposed a design for prevailing transactions from Ethereum into a graph database namely leveraging graph computer. [Illgner 33] proposed a blockchian to fix all Blockchains. [Swan and Filippi 34] explained about the philosophy of Bockchain technology. [Banuelos et al. 35] proposed an advanced method to implement business developments on top of commodity Blockchain technology. [Dinh et al. 36] surveyed the case of the art targeting on private Blockchain where the parties are authenticated. [Desai 37] analysed industry application and have legal perspectives for Blockchain technology. [Jain et al. 38] analyzed asymmetrical associations using fuzzy graph and finding hidden connections in Facebook. [Raikwar et al. 39] proposed a framework of Blockchain for insurance processes.

[Ramkumar 40] proposed Blockchain integrity framework. [Hill 41] presented a review on Blockchain [Arockiaraj and Charumathi 42] introduced the Blockchain fuzzy graph and its concepts and properties. [Halaburda 43] answered for the question, Blockchain transformation without the Blockchain. [Gupta and Sadoghi 44] explained about Blockchain process in detailed manner. [Ramkumar 45] accomplished large scale measure in Blockchian. [Asraf et al. 46] proposed Dombi fuzzy graphs. [Marapureddy 47] introduced fuzzy graph for the semi group. [Quek et al. 48] introduced a few of the results for complex Neutrosophic sets on graph theory. [Smarandache and Pramanik 49] introduced a new perspective to neutrosophic theory and its applications.

[Basset et al. 50] proposed an extended neutrosophic AHP-SWOT analysis for critical planning and decision making. [Basset et al. 51] proposed association rule mining algorithm to analyze big data. [Basset et al. 52] introduced Group ANP-TOPSIS framework under hybrid neutrosophic setting for supplier selection problem. [Basset et al. 53] presented a hybrid approach of neutrosophic sets and DEMATEL method to enhance the criteria for supplier selection. The same authors presented a series of article[63-69]. ([Pramanik et al. 54] proposed NS-cross entropy under single valued neutrosophic environment and applied in a MAGDM problem. [Biswas et al. 55] proposed neutrosophic TOPSIS method and solved group decision making problem.

[Pramanik and Mallick 56] proposed VIKOR method using trapezoidal neutrosophic numbers and solved MAGDM problem using proposed method. [Biswas et al. 57] solved MADM problem by introducing distance measure using interval trapezoidal neutrosophic numbers. [Biswas et al. 58] introduced TOPSIS strategy for solving MADM problem with trapezoidal numbers. [Biswas et al. 59] solved MAGDM problem using expected value of neutrosophic trapezoidal numbers. [Mondal et al. 60] introduced hyperbolic sine similarity measure based MADM strategy under single valued neutrosophic environment. [Mondal et al. 61] proposed hybrid binary algorithm similarity measure under single valued neutrosophic set assessments for MAGDM problem. [Dhavaseelan et al. 62] proposed single-valued co-neutrosophic graphs.

The above literature survey motivated to propose Blockchain single and interval valued Neutrosophic Graphs and applied them in Blockchain technology using Bitcoins.

3. Basic Concepts

Some basic concepts needed for the proposed concepts and their application, are listed below.

3.1 Bitcoins [40]

Bitcoin is the digital currency and worldwide payment system and are believable and best used when,

- There are a series of transaction
- Need to be recorded
- Need to be verified with respect to purity of the information and the order of the events.

3.2 Blockchain [42]

A Blockchain is a network and can be seen as a reliable computer whose private states are auditable by anyone. It can also be defined as follows.

- Cryptographic approach for modeling an unalterable append-only public ledger
- It includes a methodology for obtaining an open general agreement on each entry
- Ledger entries are mappings of the states of processes by the Blockchain network.

Uses of Blockchain

- A uniform approach to execute a variety of application processes
- Reliable and efficient Low upward approaches for stakeholders/users namely states with query application and audit correctness of changes of states.

3.3 Graph [46]

A mathematical system $G=(V,E)$ is called a graph, where a vertex set is $V=V(G)$ and an edge set is $E=E(G)$. In this paper, undirected graph has been considered and hence every edge is considered as an unordered pair of different vertices.

3.4 Fuzzy Graph [47]

Consider a non-empty finite set V , λ be a fuzzy subsets on V and δ be a fuzzy subsets on $\mathrm{V}\times\mathrm{V}$ .A fuzzy graph is a pair $\mathrm{G}=(\lambda,\delta)$ over the set V if $\delta(a,b)\le\min\{\lambda(a),\lambda(b)\}$ for all $(a,b)\in\mathrm{V}\times\mathrm{V}$ where λ is a fuzzy vertex and δ is a fuzzy edge. Where:

1. A fuzzy subset is a mapping $\lambda:\mathrm{V}\to[0,1]$ of V .
2. A fuzzy relation is a mapping $\delta:\mathrm{V}\times\mathrm{V}\to[0,1]$ on λ of V if $\delta(a,b)\le\min\{\lambda(a),\lambda(b)\}$
3. If $\delta(a,b)=\min\{\lambda(a),\lambda(b)\}$ then G is a strong fuzzy graph.

3.5 Blockchain Fuzzy Graph (BCFG) [42]

The pair $\mathrm{G}=(\lambda,\delta)$ is a BCFG, where λ is a fuzzy vertex set and δ is symmetric on λ such that $\delta(a,b)\le\min\{\lambda(a),\lambda(b)\}$, $\forall\, a,b\in\mathrm{V}$ with the following criterion.

1. If $i\neq j$ then $\sum\left[\delta(a_i,b_j)\le\min\left(\lambda(a_i),\lambda(b_j)\right)\right]=1$
2. If $i\neq j$ then $\sum\left[\delta(a_i,b_j)\le\max\left(\lambda(a_i),\lambda(b_j)\right)\right]=1$
3. If $i=j$ then $\sum\left[\delta(a_i,b_j)\le\min\left(\lambda(a_i),\lambda(b_j)\right)\right]=0$

3.6 Single Valued Neutrosophic Graph (SVNG) [26]

A pair $G=(R,S)$ is SVNG with elemental set V . Where:

1. Grade of truth, indeterminacy and falsity memberships of $a_i \in \mathbf{V}$ are defined by $T_R:\mathrm{V}\to[0,1]$, $I_R:\mathrm{V}\to[0,1]$ and $F_R:\mathrm{V}\to[0,1]$ respectively and $0\le T_R(a_i)+I_R(a_i)+F_R(a_i)\le 3, \forall a_i\in\mathbf{V}, i=1,2,3,\ldots,n$
2. The above three memberships of the edge $(a_i,b_j)\in\mathbf{E}$ are denoted by $T_S:\mathrm{E}\subseteq\mathrm{V}\times\mathrm{V}\to[0,1]$, $I_S:\mathrm{E}\subseteq\mathrm{V}\times\mathrm{V}\to[0,1]$ and $F_S:\mathrm{E}\subseteq\mathrm{V}\times\mathrm{V}\to[0,1]$ respectively and are defined by
   - $T_S(\{a_i,b_j\})\le\min[T_R(a_i),T_R(b_j)]$
   - $I_S(\{a_i,b_j\})\ge\max[I_R(a_i),I_R(b_j)]$
   - $F_S(\{a_i,b_j\})\ge\max[F_R(a_i),F_R(b_j)]$

where $0\le T_S(\{a_i,b_j\})+I_S(\{a_i,b_j\})+F_S(\{a_i,b_j\})\le 3, \forall\{a_i,b_j\}\in\mathbf{E}(i,j=1,2,\ldots,n)$.

Also Rand S are the single valued Neutrosophic vertex and edge set of V and **E** respectively. S is symmetric on R.

3.7 Interval Valued Neutrosophic Graph (IVNG) [25]

A pair $G=(R,S)$ is IVNG, where $R=\langle[T_R^L,T_R^U],[I_R^L,I_R^U],[F_R^L,F_R^U]\rangle$, is an IVN set on V and $S=\langle[T_S^L,T_S^U],[I_S^L,I_S^U],[F_S^L,F_S^U]\rangle$ is an IVN edge set on **E** satisfying the following conditions:

1. Here the lower and upper memberships functions of $a_i\in\mathbf{V}$ are defined by $T_R^L:\mathrm{V}\to[0,1]$, $T_R^U:\mathrm{V}\to[0,1]$, $I_R^L:\mathrm{V}\to[0,1]$, $I_R^U:\mathrm{V}\to[0,1]$ and $F_R^L:\mathrm{V}\to[0,1]$, $F_R^U:\mathrm{V}\to[0,1]$ respectively and $0\le T_R(a_i)+I_P(a_i)+F_P(a_i)\le 3, \forall a_i\in\mathbf{V}, i=1,2,3,\ldots,n$
2. And the same for edge $(a_i,b_j)\in\mathbf{E}$ are denoted by $T_S^L:\mathrm{V}\times\mathrm{V}\to[0,1]$, $T_S^U:\mathrm{V}\times\mathrm{V}\to[0,1]$ $I_S^L:\mathrm{V}\times\mathrm{V}\to[0,1]$, $I_S^U:\mathrm{V}\times\mathrm{V}\to[0,1]$ and $F_S^L:\mathrm{V}\times\mathrm{V}\to[0,1]$, $F_S^U:\mathrm{V}\times\mathrm{V}\to[0,1]$ respectively and are defined by
   - $T_S^L(\{a_i,b_j\})\le\min[T_R^L(a_i),T_R^L(b_j)]$
   - $T_S^U(\{a_i,b_j\})\le\min[T_R^U(a_i),T_R^U(b_j)]$
   - $I_S^L(\{a_i,b_j\})\ge\max[I_R^L(a_i),I_R^L(b_j)]$
   - $I_S^U(\{a_i,b_j\})\ge\max[I_R^U(a_i),I_R^U(b_j)]$
   - $F_S^L(\{a_i,b_j\})\ge\max[F_R^L(a_i),F_R^L(b_j)]$
   - $F_S^U(\{a_i,b_j\})\ge\max[F_R^U(a_i),F_R^U(b_j)]$

where $0\le T_S(\{a_i,b_j\})+I_S(\{a_i,b_j\})+F_S(\{a_i,b_j\})\le 3, \forall\{a_i,b_j\}\in\mathbf{E}(i,j=1,2,\ldots,n)$.

Also R and S are the interval valued Neutrosophic vertex and edge set of V and **E** respectively. S is symmetric on R.

4. Proposed Concepts

In this section, Blockchian single valued neutrosophic graph is proposed and applied in Blockchain technology with Bitcoin transaction.

4.1 Blockchain Single Valued Neutrosophic Graph (BCSVNG)

A pair $G=(R,S)$ is BCSVNG with elemental set V . Where:

1. $T_R:\mathrm{V}\to[0,1]$, $I_R:\mathrm{V}\to[0,1]$ and $F_R:\mathrm{V}\to[0,1]$ and $0\le T_R(x_i)+I_R(x_i)+F_R(x_i)\le 3, \forall x_i\in\mathbf{V}, i=1,2,3,\ldots,n$
2. $T_S:\mathrm{E}\subseteq\mathrm{V}\times\mathrm{V}\to[0,1]$, $I_S:\mathrm{E}\subseteq\mathrm{V}\times\mathrm{V}\to[0,1]$ and $F_S:\mathrm{E}\subseteq\mathrm{V}\times\mathrm{V}\to[0,1]$ are defined by

Case (i): If $i \neq j$ then

$$\sum\left[T_S\left(x_i, y_j\right) \leq \min\left[T_R\left(x_i\right), T_R\left(y_j\right)\right]\right] = 1$$

$$\sum\left[I_S\left(x_i, y_j\right) \geq \max\left[I_R\left(x_i\right), I_R\left(y_j\right)\right]\right] = 1$$

$$\sum\left[F_S\left(x_i, y_j\right) \geq \max\left[F_R\left(x_i\right), F_R\left(y_j\right)\right]\right] = 1$$

Case (ii): If $i = j$ then the above values are 0.

Where, $0 \leq T_S\left(\left\{x_i, y_j\right\}\right) + I_S\left(\left\{x_i, y_j\right\}\right) + F_S\left(\left\{x_i, y_j\right\}\right) \leq 3, \forall \left\{x_i, y_j\right\} \in \mathbf{E}(i, j = 1, 2, ..., n)$

Also R is a single valued Neutrosophic vertex of V and S is a single valued Neutrosophic edge set of $\mathbf{E}$. S is a symmetric single valued Neutrosophic relation on R.

4.1.1 Blockchain Single Valued Neutrosophic Graph in Bitcoin Transaction

Let us consider there are 4 persons in the Blockchain and everyone is doing a transaction using Bitcoin and they are saving 40% and investing the remaining 60% in Bitcoin.

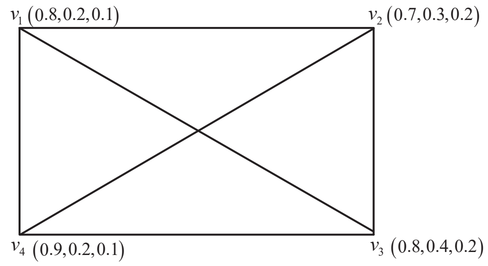

Figure 1: BCSVNG

Party 1: investing 20 lakhs and doing 3 transactions
Party 2: investing 15 lakhs and doing 3 transactions
Party 3: investing 10 lakhs and doing 3 transactions
Party 4: investing 5.5 lakhs and doing 3 transactions
For example, assume that the party-1 (v_1) has the total amount of 20 lakhs, from this he is saving 40% and invest the remaining 60% as Bitcoins for his crypto currencies.

The following are the transactions of Party-1:

Transaction 1: Party-1 to Party-2 : (v_1 to v_2)

$(0.7, 0.3, 0.2) \times 12,00,000$

$$= \left\langle \left(1-\left(1-T_R\right)^k\right), \left(1-\left(1-T_R\right)^k\right), \left(1-\left(1-T_R\right)^k\right) \right\rangle$$ **,** $k > 0$ (any arbitrary number) **[10]**

$$= \left\langle \left(1-\left(1-0.7\right)^{12,00,000}\right), \left(1-\left(1-0.3\right)^{12,00,000}\right), \left(1-\left(1-0.2\right)^{12,00,000}\right) \right\rangle$$

$$= \left\langle \left(1-\left(1-0.7\right)^{12,00,000}\right), \left(1-\left(1-0.3\right)^{12,00,000}\right), \left(1-\left(1-0.2\right)^{12,00,000}\right) \right\rangle$$

$$= \langle 1,1,1 \rangle$$

Similarly for other transactions namely

Transaction 2: Party-1 to Party-3 : (v_1 to v_3)
Transaction 3: Party-1 to Party-4 : (v_1 to v_4)

All the possible transaction are listed out in Table 1 with the sum value of each row.

			$(0.8,0.2,0.1)$	$(0.7,0.3,0.2)$	$(0.8,0.4,0.2)$	$(0.9,0.2,0.1)$	sum
			v_1	v_2	v_3	v_4	
	$(0.8,0.2,0.1)$	v_1	0	$(0.4,0.38,0.3)$	$(0.3,0.41,0.4)$	$(0.3,0.21,0.3)$	$(1,1,1)$
	$(0.7,0.3,0.2)$	v_2	$(0.4,0.38,0.3)$	0	$(0.4,0.37,0.3)$	$(0.2,0.25,0.4)$	$(1,1,1)$
	$(0.8,0.4,0.2)$	v_3	$(0.3,0.41,0.4)$	$(0.4,0.37,0.3)$	0	$(0.3,0.54,0.3)$	$(1,1,1)$
	$(0.9,0.2,0.1)$	v_4	$(0.3,0.21,0.3)$	$(0.2,0.25,0.4)$	$(0.3,0.54,0.3)$	0	$(1,1,1)$
sum			$(1,1,1)$	$(1,1,1)$	$(1,1,1)$	$(1,1,1)$	

Table 1: Transaction Table for BCSVNG

Where sum= $\sum(v_i, v_j)$

4.1.2 Properties of Blockchain Single Valued Neutrosophic Graph

In this section, degree, total degree, minimum and maximum degrees are found for Blockchain Single Valued Neutrosophic Graph.

(i). Degree of Single Valued Neutrosophic Graph (SVNG)

$d(v_1)=(d_T(v_1),d_I(v_1),d_F(v_1))$ **[26]**

$=(1,1,1)$

Where, $d_T(v_1)=T_S(v_1,v_2)+T_S(v_1,v_3)+T_S(v_1,v_4)$ $=0.4+0.3+0.3=1$

$d_I(v_1)=I_S(v_1,v_2)+I_S(v_1,v_3)+I_S(v_1,v_4)$ $=0.38+0.41+0.21=1$

$d_F(v_1)=F_S(v_1,v_2)+F_S(v_1,v_3)+F_S(v_1,v_4)$ $=0.3+0.4+0.3=1$

Similarly $d(v_2)=(d_T(v_2),d_I(v_2),d_F(v_2))=(1,1,1)$

$d(v_3)=(d_T(v_3),d_I(v_3),d_F(v_3))=(1,1,1)$

$d(v_4)=(d_T(v_4),d_4(v_4),d_F(v_4))=(1,1,1)$

And $\sum d(v_1)=\left(2\sum_{v_1\neq v_j} T_S(v_1,v_j),2\sum_{v_1\neq v_j} I_S(v_1,v_j),2\sum_{v_1\neq v_j} F_S(v_1,v_j)\right)$

$=(2(1),2(1),2(1))=(2,2,2)$

(ii). Total Degree of SVNG

$td(v_i)=(td_T(v_i),td_I(v_i),td_F(v_i))$ **[26]**

Where $td_T(v_i)=\sum T_S(v_i,v_j)+T_R(v_i)$

$td_T(v_1)=\sum T_S(v_1,v_j)+T_R(v_1)$ $=1+0.8=1.8$

$td_I(v_1)=\sum I_S(v_1,v_j)+I_R(v_1)$ $=1+0.2=1.2$

$td_F(v_1)=\sum F_S(v_1,v_j)+F_R(v_1)$ $=1+0.1=1.1$

Therefore, $td(v_1)=(td_T(v_1),td_I(v_1),td_F(v_1))=(1.8,1.2,1.1)$

Similarly, $td(v_2)=(td_T(v_2),td_I(v_2),td_F(v_2))=(1.7,1.3,1.2)$

$$td(v_3)=(td_T(v_3),td_I(v_3),td_F(v_3))=(1.8,1.4,1.2)$$
$$td(v_4)=(td_T(v_4),td_I(v_4),td_F(v_4))=(1.9,1.2,1.1)$$

(iii). Minimum degree of SVNG

It is $\xi(\mathrm{G})=(\xi_T(\mathrm{G}),\xi_I(\mathrm{G}),\xi_F(\mathrm{G}))$, where
$\xi_T(\mathrm{G})=\min\{d_T(v)/v\in \mathrm{V}\}$, $\xi_I(\mathrm{G})=\min\{d_I(v)/v\in \mathrm{V}\}$ and $\xi_F(\mathrm{G})=\min\{d_F(v)/v\in \mathrm{V}\}$ **[15]**

For the Fig. 1,
$\xi_T(\mathrm{G})=\min\{d_T(v)/v\in \mathrm{V}\}=1$
$\xi_I(\mathrm{G})=\min\{d_I(v)/v\in \mathrm{V}\}=1$
$\xi_F(\mathrm{G})=\min\{d_F(v)/v\in \mathrm{V}\}=1$

(iv). Maximum degree of SVNG

It is defined by $\eta(\mathrm{G})=(\eta_T(\mathrm{G}),\eta_I(\mathrm{G}),\eta_F(\mathrm{G}))$, where
$\eta_T(\mathrm{G})=\max\{d_T(v)/v\in \mathrm{V}\},\eta_T(\mathrm{G})=\max\{d_T(v)/v\in \mathrm{V}\},\eta_F(\mathrm{G})=\max\{d_F(v)/v\in \mathrm{V}\}$ **[26]**
For the Fig. 1,
$\eta_T(\mathrm{G})=\max\{d_T(v)/v\in \mathrm{V}\}=1$
$\eta_I(\mathrm{G})=\max\{d_I(v)/v\in \mathrm{V}\}=1$
$\eta_F(\mathrm{G})=\max\{d_F(v)/v\in \mathrm{V}\}=1$
For the Fig. 1,
$\eta_T(\mathrm{G})=\max\{d_T(v)/v\in \mathrm{V}\}=\eta_T(\mathrm{G})=\max\{d_T(v)/v\in \mathrm{V}\}=\eta_F(\mathrm{G})=\max\{d_F(v)/v\in \mathrm{V}\}=1$

4.2 Blockchain Interval Valued Neutrosophic Graph (BCIVNG)

A pair $\mathrm{G}=(R,S)$ is BCIVNG, where $R=\langle[T_R^L,T_R^U],[I_R^L,I_R^U],[F_R^L,F_R^U]\rangle$, is an IVN set on V and $S=\langle[T_S^L,T_S^U],[I_S^L,I_S^U],[F_S^L,F_S^U]\rangle$ is an IVN edge set on **E** satisfying conditions 1 and 2 as in the definition of IVNG and with the following criterions.

Case (i): If $i\neq j$ then

$$\sum\left[T_S^L(x_i,y_j)\leq \min\left[T_R^L(x_i),T_R^L(y_j)\right]\right]=0.5$$

$$\sum\left[T_S^U(x_i,y_j)\leq \min\left[T_R^U(x_i),T_R^U(y_j)\right]\right]=0.5$$

$$\sum\left[I_S^L(x_i,y_j)\geq \max\left[I_R^L(x_i),I_R^L(y_j)\right]\right]=0.5$$

$$\sum\left[I_S^U(x_i,y_j)\geq \max\left[I_R^U(x_i),I_R^U(y_j)\right]\right]=0.5$$

$$\sum\left[F_S^L(x_i,y_j)\geq \max\left[F_R^L(x_i),F_R^L(y_j)\right]\right]=0.5$$

$$\sum\left[F_S^U(x_i,y_j)\geq \max\left[F_R^U(x_i),F_R^U(y_j)\right]\right]=0.5$$

Case (ii): If $i=j$ then the above six values are 0.
Where $0\leq T_S(\{x_i,y_j\})+I_S(\{x_i,y_j\})+F_S(\{x_i,y_j\})\leq 3,\forall\{x_i,y_j\}\in \mathbf{E}(i,j=1,2,...,n)$

Also R is an interval valued Neutrosophic vertex of V and S is an interval valued Neutrosophic edge set of **E** . S is a symmetric interval valued Neutrosophic relation on R.

4.2.1 Blockchain Interval Valued Neutrosophic Graph in Bitcoin Transaction

Let us consider there are 3 persons in the Blockchain and everyone is doing a transaction using Bitcoin and they are saving 40% and investing the remaining 60% in Bitcoin.

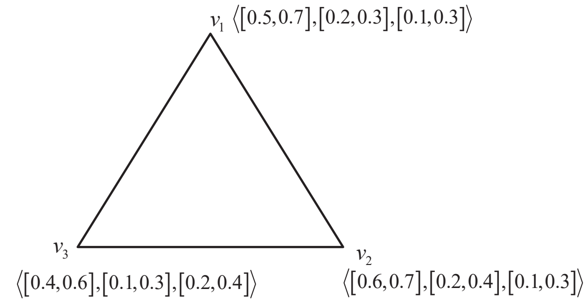

Figure 2: BCIVNG

Party 1: investing 20 lakhs and doing 2 transactions
Party 2: investing 15 lakhs and doing 2 transactions
Party 3: investing 10 lakhs and doing 2 transactions

For example, assume that the party-1 (v_1) has the total amount of 20 lakhs, from this he is saving 40% and invest the remaining 60% as Bitcoins for his crypto currencies.

The following are the transactions of Party-1:

Transaction 1: Party-1 to Party-2: (v_1 to v_2)

$$\langle [0.6,0.7],[0.2,0.4],[0.1,0.3] \rangle \times 12,00,000$$

$$= \left\{ \left[1-\left(1-T_R^L\right)^k, 1-\left(1-T_R^U\right)^k \right], \left[\left(I_R^L\right)^k, \left(I_R^U\right)^k \right], \left[\left(F_R^L\right)^k, \left(F_R^U\right)^k \right] \right\} \textbf{[25]}$$

$$= \left\{ \left[1-(1-0.6)^{12,00,000}, 1-(1-0.7)^{12,00,000} \right], \left[(0.2)^{12,00,000}, (0.4)^{12,00,000} \right], \left[(0.1)^{12,00,000}, (0.3)^{12,00,000} \right] \right\}$$

$$= \left\{ \left[1-(0.4)^{12,00,000}, 1-(0.3)^{12,00,000} \right], \left[(0.2)^{12,00,000}, (0.4)^{12,00,000} \right], \left[(0.1)^{12,00,000}, (0.3)^{12,00,000} \right] \right\}$$

$$= \{ [1-0, 1-0], [0,0], [0,0] \}$$

$$= \{ [1,1], [0,0], [0,0] \}$$

Transaction 2: Party-1 to Party-3: (v_1 to v_3) $= \{ [1,1], [0,0], [0,0] \}$

Table 1represent all possible transactions from one vertex to all other verteices. Here edge represents the transaction and the vertex represents the parties.

		$\langle[0.5,0.7],[0.2,0.3],[0.1,0.3]\rangle$	$\langle[0.6,0.7],[0.2,0.4],[0.1,0.3]\rangle$	$\langle[0.4,0.6],[0.1,0.3],[0.2,0.4]\rangle$	$\sum(v_i,v_j)$
		v_1	v_2	v_3	
$\langle[0.5,0.7],[0.2,0.3],[0.1,0.3]\rangle$	v_1	0	$\langle[0.217,0.283],[0.211,0.289],[0.302,0.313]\rangle$	$\langle[0.281,0.282],[0.198,0.199],[0.208,0.209]\rangle$	$(1,1,1)$
$\langle[0.6,0.7],[0.2,0.4],[0.1,0.3]\rangle$	v_2	$\langle[0.217,0.283],[0.211,0.289],[0.302,0.313]\rangle$	0	$\langle[0.28,0.283],[0.197,0.198],[0.208,0.209]\rangle$	$(1,1,1)$
$\langle[0.4,0.6],[0.1,0.3],[0.2,0.4]\rangle$	v_3	$\langle[0.281,0.282],[0.198,0.199],[0.208,0.209]\rangle$	$\langle[0.217,0.283],[0.302,0.313],[0.292,0.302]\rangle$	0	$(1,1,1)$
$\sum(v_i,v_j)$		$(1,1,1)$	$(1,1,1)$	$(1,1,1)$	

Table 2: Transaction Table for BCIVNG

From table 1 and table 2 it is observed that sum of all single /interval valued Neutrosophic edges of a particular Neutrosophic vertex is equal to (1, 1, 1). Hence the proposed method is an optimized one to deal indeterminacy of the data in Bitcoin transaction.

5. Comparative Analysis (Qualitative)

Blockchain approach has been applied in various fields as a growing technique. Here the advantages and limitations are listed out for Blockchian crisp, fuzzy and Neutrosophic graphs. This analysis will be very useful to understand the concept of Blockchian under different environments.

Type of Blockchain Graph	Advantages	Limitations
Blockchian Crisp Graph	• Faster Process with purity and detectable • Process clarity • Data will be permanent	• Unable to handle uncertainties • Size of the network will decide the security level
Blockchain Fuzzy Graph	• Can handle uncertainty exists in the vertex and edge sets • Invariable data	• Incapable to handle indeterminacy of the data and interval data • Large network will give more level of security
Blockchain Interval Valued Fuzzy Graph	• Can able to deal with data in terms of range	• Inadequate to handle undetermined data
Blockchain Single Valued Neutrosophic Graph	• Able to handle indeterminacy of the data	• Unfit to handle interval data
Blockchain Interval Valued	• Capable to handle interval	• Unsuited to handle criterion

Neutrosophic Graph	data as the participant's decision is always lie in a range.	insufficient information of the weights.

6. Conclusion

Reliability and assurance of the dealing is very important for any business transaction. Blockchain technology is such a technology and recently it is widely applied in many fields. In any field uncertainty is unavoidable one as the human behavior always uncertainty in nature. Also indeterminacy does not deal in any area field of mathematics whereas Neutrosophic set deals indeterminacy and hence an optimized solution can be obtain for any problem. In this paper Blockchain network has been used in terms of Bitcoin transaction where the vertex and edges have been considered as single and interval valued Neutrosophic sets. Also the degree, total degree, minimum and maximum degree have been found for the proposed Blockchain single valued Neutrosophic graph. In addition to this, contingent study has been done for various types of Blockchain graphs.

Notes

Compliance with Ethical Standards

Conflict of Interest

The authors declare that they have no conflict of interest

Ethical Approval

The article does not contain any studies with human participants or animal performed by any of the authors

Informed Consent

Informed consent was obtained from all individual participants included in the study

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Neutrosophic ideal of Subtraction Algebras

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Abstract: The notion of neutrosophic ideal in subtraction algebras is introduced, and several properties are investigated. Also we give conditions for a neutrosophic set to be a neutrosophic ideal. Characterization of neutorsophic ideal are discussed.

Keywords: Subtraction algebra,Neutrosophic set,Neutrosophic ideal

1 Introduction

The concept of Neutrosophic set, first introduced by Smarandache [17],is a powerful general formal framework that generalizes the concept of fuzzy set and intuitionistic fuzzy set. Recently, many researchers have been involved in extending the concepts and results of abstract algebra to the broader framework of the neutrosophic set theory[2, 3, 4, 5, 19]. Smarandache[17] and Wang et al.[18] introduced the concept of a single valued neutrosophic set as a subclass of the neutrosophic set and specified the definition of a neutrosophic set to make more applicable the theory to real life problems. In 1992, B. M. Schein have considered systems of the form $(\Phi; \circ, \backslash)$ [16], where Φ is a set of functions closed under the composition "$\circ$" of functions (and hence $(\Phi; \circ)$ is a function semigroup) and the set theoretic subtraction "$\backslash$" (and hence $(\Phi; \backslash)$ is a subtraction algebra in the sense of [1]). Jun et al. introduced the concept of ideal in subtraction algebras and continued studying on ideals in subtraction algebras[6, 8, 9, 14]. K. J. Lee and C. H. Park [11] introduced the concept of a fuzzy ideal in subtraction algebras and investigated some conditions for a fuzzy set to be a fuzzy ideal in subtraction algebras. Since then many reseachers worked in this area[7, 10, 12, 13].

In this paper, we apply the notion of neutrosophic sets in subtraction algebras. Also, we introduce the notion of neutrosophic ideal and give some conditions for a neutrosophic set to be a neutrosophic ideal in substraction algebras. Finally,we showed that neutrosophic image and neutrosophic inverse image of neutrosophic ideal are both neutrosophic ideal under certain conditions

2 Preliminaries

We review some definitions and properties that are necessary for this paper.

Definition 2.1. [1] An algebra $(X, -)$ is called a subtraction algebra if a single binary operation $-$ satisfies the following identities: for any $x, y, z \in X$,

(SA1) $x - (y - x) = x$,

(SA2) $x-(x-y)=y-(y-x)$,

(SA3) $(x-y)-z=(x-z)-y$,

We introduced an order relation X on a subtraction algebras: $a \leq b \Leftrightarrow a-b=0$, where $0=a-a$ is an element that does not depend on the choice of $a \in X$.

Proposition 2.2. [9] Let $(X,-)$ be a subtraction algebra. Then we have the following axioms:

(SP1) $(x-y)-y=x-y$,

(SP2) $x-0=x$ and $0-x=0$,

(SP3) $(x-y)-x=0$,

(SP4) $x-(x-y) \leq y$,

(SP5) $(x-y)-(y-x)=x-y$,

(SP6) $x-(x-(x-y))=x-y$,

(SP7) $(x-y)-(z-y) \leq x-z$,

(SP8) $x \leq y$ if and only if $x=y-w$ for some $w \in X$,

(SP9) $x \leq y$ implies $x-z \leq y-z$ and $z-y \leq z-x$ for all $z \in X$,

(SP10) $x, y \leq z$ implies $x-y=x \wedge (z-y)$,

(SP11) $(x \wedge y)-(x \wedge z) \leq x \wedge (y-z)$, for all $x, y, z \in X$.

Definition 2.3. [9] A nonempty subset A of a subtraction algebra X is called an ideal of X, denoted by $A \lhd X$, if it satisfies:

(SI1) $a-x \in A$ for all $a \in A$ and $x \in X$,

(SI2) for all $a, b \in A$, whenever $a \vee b$ exists in X then $a \vee b \in A$.

Proposition 2.4. [9] Let X be a subtraction algebra and let $x, y \in X$. If $w \in X$ is an upper bound for x and y, then the element

$$x \vee y := w-((w-y)-x)$$

is a least upper bound for x and y.

Definition 2.5. [11] A fuzzy set μ in X is called a fuzzy ideal of X if it satisfies:

(SFI1) $\mu(x-y) \geq \mu(x)$,

(SFI2) $\exists\, x \vee y \Rightarrow \mu(x \vee y) \geq \min\{\mu(x), \mu(y)\})$ for all $x, y \in X$.

We give some preliminaries about single valued neutrosophic sets and set operations, which will be called neutrosophic sets, for simplicity.

Definition 2.6. [18] Let X be a space of points (objects), with a generic element in X denoted by x. A single valued neutrosophic set A on X is characterized by truth-membership function t_A, indeterminacy-membership function i_A and falsity-membership function f_A. For each point x in $X, t_A(x), i_A(x), f_A(x) \in [0,1]$. A neutrosophic set A can be written as denoted by a mapping defined as $A : X \to [0,1] \times [0,1] \times [0,1]$ and

$$A = \{< x, t_A(x), i_A(x), f_A(x) >, x \in X\}$$

for simplicity.

Definition 2.7. [15, 18] Let A and B be two neutrosophic sets on X. Then

(1) A is contained in B, denoted as $A \subseteq B$, if and only if $\mathscr{N}_A(x) \leq \mathscr{N}_B(x)$. i.e., $t_A(x) \leq t_B(x), i_A(x) \leq i_B(x)$ and $f_A(x) \geq f_B(x)$. Two sets A and B is called equal, i.e., $A = B$ iff $A \subseteq B$ and $B \subseteq A$.

(2) the union of A and B is denoted by $C = A \cup B$ and defined as $\mathscr{N}_C(x) = \mathscr{N}_A(x) \vee \mathscr{N}_B(x)$ where $\mathscr{N}_A(x) \vee \mathscr{N}_B(x) = (t_A(x) \vee t_B(x), i_A(x) \vee i_B(x), f_A(x) \wedge f_B(x))$, for each $x \in X$. i.e., $t_C(x) = \max\{t_A(x), t_B(x)\}, i_C(x) = \max\{i_A(x), i_B(x)\}$ and $f_C(x) = \min\{f_A(x), f_B(x)\}$.

(3) the intersection of A and B is denoted by $C = A \cap B$ and defined as $\mathscr{N}_C(x) = \mathscr{N}_A(x) \wedge \mathscr{N}_B(x)$ where $\mathscr{N}_A(x) \wedge \mathscr{N}_B(x) = (t_A(x) \wedge t_B(x), i_A(x) \wedge i_B(x), f_A(x) \vee f_B(x))$, for each $x \in X$.i.e., $t_C(x) = \min\{t_A(x), t_B(x)\}, i_C(x) = \min\{i_A(x), i_B(x)\}$ and $f_C(x) = \max\{f_A(x), f_B(x)\}$.

(4) the complement of A is denoted by A^c and defined as $\mathscr{N}_A^c(x) = (f_A(x), 1 - i_A(x), t_A(x))$, for each $x \in X$.

Definition 2.8. [4] Let $g : X_1 \to X_2$ be a function and A, B be the neutrosophic sets of X_1 and X_2, respectively. Then the image of a neutrosophic set A is a neutrosophic set of X_2 and it is defined as follows:$\forall y \in X_2$

$$\begin{aligned} g(A)(y) &= (t_{g(A)}(y), i_{g(A)}(y), f_{g(A)}(y)) \\ &= (g(t_A)(y), g(i_A)(y), g(f_A)(y)), \end{aligned}$$

where

$$g(t_A)(y) = \begin{cases} \bigvee t_A(x) & \text{if } x \in g^{-1}(y), \\ 0 & \text{otherwise,} \end{cases}$$

$$g(i_A)(y) = \begin{cases} \bigvee i_A(x) & \text{if } x \in g^{-1}(y), \\ 0 & \text{otherwise,} \end{cases}$$

$$g(f_A)(y) = \begin{cases} \bigwedge t_A(x) & \text{if } x \in g^{-1}(y), \\ 0 & \text{otherwise,} \end{cases}$$

And the preimage of a neutrosophic set B is a neutrosophic set of X_1 and it is defined as follows:

$$\begin{aligned} g^{-1}(B)(x) &= (t_{g^{-1}(B)}(x), i_{g^{-1}(B)}(x), f_{g^{-1}(B)}(x)) \\ &= (t_B(g(x)), i_B(g(x)), f_B(g(x))) \\ &= B(g(x)), \forall x \in X_1. \end{aligned}$$

Definition 2.9. [4] Let $A = \{< x, t_A(x), i_A(x), f_A(x) >, x \in X\}$ be a neutrosophic set on X and $\alpha \in [0,1]$. Define the α-level sets of A as follows: $(t_A)_\alpha = \{x \in X \mid t_A(x) \geq \alpha\}, (i_A)_\alpha = \{x \in X \mid i_A(x) \geq \alpha\}$, and $(f_A)^\alpha = \{x \in X \mid f_A(x) \leq \alpha\}$.

3 Neutrosophic ideals

In what follows, let X be a subtraction algebra unless otherwise specified.

Definition 3.1. A neutrosophic set A of X is called a neutrosophic ideal of X if the following conditions are true: $\forall x, y \in X$,

(SNI1) $\mathscr{N}_A(x-y) \geq \mathscr{N}_A(x)$ i.e., $t_A(x-y) \geq t_A(x), i_A(x-y) \geq i_A(x)$ and $f_A(x-y) \leq f_A(x)$;

(SNI2) $\exists x \vee y \Rightarrow \mathscr{N}_A(x \vee y) \geq \mathscr{N}_A(x) \wedge \mathscr{N}_A(y)$,i.e., $t_A(x \vee y) \geq t_A(x) \wedge t_A(y), i_A(x \vee y) \geq i_A(x) \wedge i_A(y)$ and $f_A(x \vee y) \leq f_A(x) \vee f_A(y)$ whenever there exists $x \vee y$.

Proposition 3.2. *If a neutrosophic set A of X satisfies*

$$(\forall x, a, b \in X)\left(\mathscr{N}_A(x-((x-a)-b)) \geq \mathscr{N}_A(a) \wedge \mathscr{N}_A(b) \right) \tag{3.1}$$

then A is a neutrosophic ideal of X.

Proof. Let $A = \{< x, t_A(x), i_A(x), f_A(x) >, x \in X\}$ be a neutrosophic set of X that satisfies (3.1). By (SP2) and (SP3) we have $(x-y)-(((x-y)-x)-x) = (x-y)-(0-x) = (x-y)-0 = x-y$. From this we get

$$\begin{aligned} t_A(x-y) &= t_A((x-y)-(((x-y)-x)-x)) \geq t_A(x) \wedge t_A(x) = t_A(x), \\ i_A(x-y) &= i_A((x-y)-(((x-y)-x)-x)) \geq i_A(x) \wedge i_A(x) = i_A(x), \\ f_A(x-y) &= f_A((x-y)-(((x-y)-x)-x)) \leq f_A(x) \vee f_A(x) = f_A(x). \end{aligned}$$

Now suppose $x \vee y$ exists for $x, y \in X$. If we take $w = x \vee y$, we have $x \vee y = w-((w-x)-y)$ by Proposition 2.4. It follows from (3.1) that

$$\begin{aligned} t_A(x \vee y) &= t_A(w-((w-x)-y)) \geq t_A(x) \wedge t_A(y), \\ i_A(x \vee y) &= i_A(w-((w-x)-y)) \geq i_A(x) \wedge i_A(y), \\ f_A(x \vee y) &= f_A(w-((w-x)-y)) \leq f_A(x) \vee f_A(y). \end{aligned}$$

Hence A is a neutrosophic ideal of X. □

Proposition 3.3. *For every neutrosophic ideal A of X, we have the following inequality:*

$$(\forall x \in X)\, (\mathscr{N}_A(0) \geq \mathscr{N}_A(x)). \tag{3.2}$$

Proof. Let $A = \{< x, t_A(x), i_A(x), f_A(x) >, x \in X\}$ be a neutrosophic ideal of X. Putting $y = x$ in (SNI1), then

$$t_A(0) = t_A(x-x) \geq t_A(x), i_A(0) = i_A(x-x) \geq i_A(x), f_A(0) = f_A(x-x) \leq f_A(x).$$

Hence (3.2) is valid. □

Proposition 3.4. *Let A be a neutrosophic set of X such that*

(SNI3) $(\forall x \in X) \quad (\mathscr{N}_A(0) \geq \mathscr{N}_A(x))$, (SNI4) $(\forall x, y, z \in X) \quad (\mathscr{N}_A(x-z) \geq \mathscr{N}_A((x-y)-z) \wedge \mathscr{N}_A(y).)$

Then we have the following implication:

$$(\forall a, x \in X)(x \leq a \Rightarrow \mathscr{N}_A(x) \geq \mathscr{N}_A(a)). \tag{3.3}$$

Proof. Let $a, x \in X$ be such that $x \leq a$. Then

$$\begin{aligned} t_A(x) &= t_A(x-0) \geq t_A((x-a)-0) \wedge t_A(a) = t_A(0) \wedge t_A(a) = t_A(a), \\ i_A(x) &= i_A(x-0) \geq i_A((x-a)-0) \wedge i_A(a) = i_A(0) \wedge i_A(a) = i_A(a), \\ f_A(x) &= f_A(x-0) \leq f_A((x-a)-0) \vee f_A(a) = f_A(0) \vee f_A(a) = f_A(a). \end{aligned}$$

Hence $\mathscr{N}_A(x) \geq \mathscr{N}_A(a)$. □

Theorem 3.5. *If a neutrosophic set A in X satisfies (SNI3) and (SNI4), then A is a neutrosophic ideal of X.*

Proof. Let A be aneutrosophic in X satisfying (SNI3) and (SNI4), and let $x, y \in X$. Then $x - y \leq x$ by (SP3). It follows from Proposition 3.4 that

$$\mathscr{N}_A(x-y) \geq \mathscr{N}_A(x),$$

i.e., (SNI1) is valid. Also, we have

$$\mathscr{N}_A(x \vee y) \geq \mathscr{N}_A(x)$$

whenever $x \vee y$ exists in X by using Proposition 3.4 and so

$$\mathscr{N}_A(x \vee y) \geq \mathscr{N}_A(x) \wedge \mathscr{N}_A(y).$$

Thus (SNI2) is valid. Therefore $\mathscr{N}_A$ is a neutrosophic ideal of X. □

Proposition 3.6. *A necessary and sufficient condition for a neutrosophic set A of X to be a neutrosophic ideal of X is that t_A ,i_A and $1 - f_A$ are fuzzy ideals of X.*

Proof. Assume that $A = \{< x, t_A(x), i_A(x), f_A(x) >, x \in X\}$ is a neutrosophic ideal of X. For any $x, y \in X$, we have $t_A(x-y) \geq t_A(x), i_A(x-y) \geq i_A(x)$ and $f_A(x-y) \leq f_A(x)$. Thus

$$(1 - f_A)(x-y) \geq (1 - f_A(x)).$$

Now suppose $x \vee y$ exists for $x, y \in X$. We have $t_A(x \vee y) \geq t_A(x) \wedge, t_A(y), i_A(x \vee y) \geq i_A(x) \wedge i_A(y)$ and $f_A(x \vee y) \leq f_A(x) \vee f_A(y)$. Thus

$$(1 - f_A)(x \vee y) \geq (1 - f_A(x)) \wedge (1 - f_A(y)).$$

Hence t_A, i_A and $1 - f_A$ are fuzzy ideal of X.

Conversely,assume that t_A, i_A and $1 - f_A$ are fuzzy ideal of X and $x, y \in R$. Then $t_A(x-y) \geq t_A(x), i_A(x-y) \geq i_A(x)$ and $1 - f_A(x-y) \geq (1 - f_A(x))$. Thus

$$f_A(x-y) = 1 - (1 - f_A(x-y)) \leq 1 - (1 - f_A(x)) = f_A(x).$$

It follows that $\mathscr{N}_A(x-y) \geq \mathscr{N}_A(x) \wedge \mathscr{N}_A(y)$. Suppose $x \vee y$ exists for $x, y \in X$, we have $t_A(x \vee y) \geq t_A(x) \wedge t_A(y), i_A(x \vee y) \geq i_A(x) \wedge i_A(y)$ and $(1-f_A)(x \vee y) \geq 1-f_A(x) \wedge 1-f_A(y)$. Thus

$$f_A(x \vee y) \leq f_A(x) \vee f_A(y).$$

It follows that

$$\mathscr{N}_A(x \vee y) \geq \mathscr{N}_A(x) \wedge \mathscr{N}_A(y).$$

Hence A is a neutrosophic ideal of X. □

Theorem 3.7. *A is a neutrosophic ideal of X if and only if for all $\alpha \in [0,1]$, the α-level sets of A, $(t_A)_\alpha$,$(i_A)_\alpha$ and $(f_A)^\alpha$ are ideals of X.*

Proof. Assume that $A = \{< x, t_A(x), i_A(x), f_A(x) >, x \in X\}$ is a neutrosophic ideal of X. Let $x \in X$, $a \in (t_A)_\alpha, a \in (i_A)_\alpha$ and $a \in (f_A)^\alpha$. Then $t_A(a) \geq \alpha, i_A(a) \geq \alpha$, and $f_A(a) \leq \alpha$. By Definition 3.1(SNI1), we have

$$t_A(a-x) \geq t_A(a) \geq \alpha, i_A(a-x) \geq i_A(a) \geq \alpha, f_A(a-x) \leq f_A(a) \leq \alpha.$$

Hence $a-x \in (t_A)_\alpha$, $a-x \in (t_A)_\alpha$ and $a-x \in (t_A)_\alpha$. Let $a, b \in (t_A)_\alpha$ $a, b \in (i_A)_\alpha$ and $a, b \in (f_A)^\alpha$ and assume that there exists $a \vee b$. Then $t_A(a) \geq \alpha$ and $t_A(b) \geq \alpha$, which imply from Definition 3.1(SNI2) that

$$t_A(a \vee b) \geq t_A(a) \wedge t_A(b) \geq \alpha, i_A(a \vee b) \geq i_A(a) \wedge i_A(b) \geq \alpha, f_A(a \vee b) \leq f_A(a) \vee f_A(b) \leq \alpha.$$

and so that $a \vee b \in (t_A)_\alpha$,$a \vee b \in (i_A)_\alpha$ and $a \vee b \in (f_A)^\alpha$. Therefore $(t_A)_\alpha$,$(i_A)_\alpha$ and $(f_A)^\alpha$ are ideals of X. Conversely, assume that $t_A(x-y) < t_A(x)$ for some $x, y \in X$. Then

$$t_A(x-y) < \alpha < t_A(x)$$

for some $\alpha \in (0,1]$. This implies that $x \in (t_A)_\alpha$ but $x-y \notin (t_A)_\alpha$. This is contradiction. Therefore $t_A(x-y) \geq t_A(x)$ for all $x, y \in X$. Similary $i_A(x-y) \geq i_A(x)$. If $f_A(x-y) > f_A(x)$ for all $x, y \in X$. Then

$$t_A(x-y) > \alpha > f_A(x)$$

for some $\alpha \in (0,1]$. This implies that $x \in (f_A)^\alpha$ but $x-y \notin (f_A)^\alpha$. This is contradiction. Therefore $f_A(x-y) \leq f_A(x)$ for all $x, y \in X$. Suppose that $x \vee y$ exists such that $t_A(x \vee y) < t_A(x) \wedge t_A(y)$ for some $x, y \in X$, Then

$$t_A(x \vee y) < \alpha < t_A(x) \wedge t_A(y)$$

for some $\alpha \in (0,1]$. It follows that $x, y \in (t_A)_\alpha$ and $x \vee y \notin (t_A)_\alpha$. This is contradiction. Therefore $t_A(x \vee y) \geq t_A(x) \wedge t_A(y)$ for all $x, y \in X$. Similary $i_A(x \vee y) \geq i_A(x) \wedge i_A(y)$. If $x \vee y$ exists such that $f_A(x \vee y) > f_A(x) \wedge t_A(y)$ for some $x, y \in X$, Then

$$f_A(x \vee y) > \alpha > f_A(x) \vee f_A(y)$$

for some $\alpha \in (0,1]$. It follows that $x, y \in (f_A)^\alpha$ and $x \vee y \notin (f_A)^\alpha$. This is contradiction. Therefore $f_A(x \vee y) \leq f_A(x) \vee f_A(y)$ for all $x, y \in X$. Hence A is a neutrosophic ideal of X. □

Theorem 3.8. *Let A and B are neutrosophic ideals of X. Then $A \cap B$ is a neutrosophic ideal of X.*

Proof. Suppse that $A = \{< x, t_A(x), i_A(x), f_A(x) >, x \in X\}$ and $B = \{< x, t_B(x), i_B(x), f_B(x) >, x \in X\}$ are neutrosophic ideals of X and let $x, y \in X$. By Definition 3.1 , we have

$$\begin{aligned} t_{A\cap B}(x-y) &= t_A(x-y) \wedge t_B(x-y) \geq t_A(x) \wedge t_B(x) = t_{A\cap B}(x), \\ i_{A\cap B}(x-y) &= i_A(x-y) \wedge i_B(x-y) \geq i_A(x) \wedge i_B(x) = i_{A\cap B}(x), \\ f_{A\cap B}(x-y) &= f_A(x-y) \vee f_B(x-y) \leq f_A(x) \vee f_B(x) = f_{A\cap B}(x). \end{aligned}$$

Now suppose $x \vee y$ exists for $x, y \in X$. By Definition 3.1, we have

$$\begin{aligned} t_{A\cap B}(x \vee y) &= t_A(x \vee y) \wedge t_B(x \vee y) \\ &\geq (t_A(x) \wedge t_A(y)) \wedge (t_B(x) \wedge t_B(y)) \\ &= (t_A(x) \wedge t_B(x)) \wedge (t_A(y) \wedge t_B(y)) \\ &= t_{A\cap B}(x) \wedge t_{A\cap B}(y). \end{aligned}$$

Similary we get $i_{A\cap B}(x \vee y) \geq i_{A\cap B}(x) \wedge i_{A\cap B}(y)$. Also we obtain

$$\begin{aligned} f_{A\cap B}(x \vee y) &= f_A(x \vee y) \vee f_B(x \vee y) \\ &\leq (f_A(x) \vee f_A(y)) \vee (f_B(x) \vee f_B(y)) \\ &= (f_A(x) \vee f_B(x)) \vee (f_A(y) \vee f_B(y)) \\ &= f_{A\cap B}(x) \vee f_{A\cap B}(y). \end{aligned}$$

Hence A is a neutrosophic ideal of X. □

Theorem 3.9. *Let A be a neutrosophic ideal of X. Then the set*

$$K := \{x \in X \mid \mathscr{N}_A(x) = \mathscr{N}_A(0)\}$$

is an ideal of X.

Proof. Let A be a neutrosophic ideal of X and $a \in K$. Then $\mathscr{N}_A(a) = \mathscr{N}_A(0)$. By (SNI1), we have

$$\mathscr{N}_A(a-x) \geq \mathscr{N}_A(a) = \mathscr{N}_A(0)$$

for $x \in X$. It follows from (3.2) that $\mathscr{N}_A(a-x) = \mathscr{N}_A(0)$ so that $a - x \in K$. Let $a, b \in K$ and assume that there exists $a \vee b$. By means of (SNI2), we know that

$$\mathscr{N}_A(a \vee b) \geq \min\{\mathscr{N}_A(a), \mathscr{N}_A(b)\} = \mathscr{N}_A(0).$$

Thus $\mathscr{N}_A(a \vee b) = \mathscr{N}_A(0)$ by (3.2), and so $a \vee b \in K$. Therefore K is an ideal of X. □

Theorem 3.10. *Let $g : X_1 \to X_2$ be a homomorphism. Then the image $f(A)$ of a neutrosophic ideal A of X_1 is a neutrosophic ideal of X_2.*

Proof. For any $y_1, y_2 \in f(X_1)$, Consider the set

$$S = \{a_1 - a_2 \mid a_1 \in g^{-1}(y_1), a_2 \in g^{-1}(y_2)\}.$$

If $x \in S$ then $x = x_1 - x_2$ for $x_1 \in g^{-1}(y_1)$ and $x_2 \in g^{-1}(y_2)$ and so

$$f(x) = f(x_1 - x_2) = f(x_1) - f(x_2) = y_1 - y_2,$$

that is, $x = x_1 - x_2 \in f^{-1}(y_1 - y_2)$. It follows that

$$\begin{aligned}
g(t_A)(y_1 - y_2) &= \bigvee_{x \in f^{-1}(y_1 - y_2)} t_A(x) \geq t_A(x_1 - x_2) \geq t_A(x_1)\\
g(i_A)(y_1 - y_2) &= \bigvee_{x \in f^{-1}(y_1 - y_2)} i_A(x) \geq i_A(x_1 - x_2) \geq i_A(x_1)\\
g(f_A)(y_1 - y_2) &= \bigwedge_{x \in f^{-1}(y_1 - y_2)} f_A(x) \leq f_A(x_1 - x_2) \leq f_A(x_1).
\end{aligned}$$

Then

$$\begin{aligned}
g(A)(y_1 - y_2) &= (g(t_A)(y_1 - y_2), g(i_A)(y_1 - y_2), g(f_A)(y_1 - y_2)))\\
&= (\bigvee_{x \in f^{-1}(y_1 - y_2)} t_A(x), \bigvee_{x \in f^{-1}(y_1 - y_2)} i_A(x), \bigwedge_{x \in f^{-1}(y_1 - y_2)} f_A(x))\\
&\geq (t_A(x_1 - x_2), i_A(x_1 - x_2), f_A(x_1 - x_2))\\
&\geq (t_A(x_1), i_A(x_1), f_A(x_1)).
\end{aligned}$$

Cnsequently,

$$\begin{aligned}
g(A)(y_1 - y_2) &\geq (\bigvee_{x_1 \in f^{-1}(y_1)} t_A(x_1), \bigvee_{x_1 \in f^{-1}(y_1)} i_A(x_1), \bigwedge_{x_1 \in f^{-1}(y_1 - y_2} f_A(x_1))\\
&= (g(t_A)(y_1), g(i_A)(y_1), g(f_A)(y_1))\\
&= g(A)(y_1).
\end{aligned}$$

If $y_1 \vee y_2$ exist for any $y_1, y_2 \in f(X_1)$. We first consider the set

$$T = \{a_1 \vee a_2 \mid a_1 \in g^{-1}(y_1), a_2 \in g^{-1}(y_2)\}.$$

If $x \in T$ then $x = x_1 \vee x_2$ for $x_1 \in g^{-1}(y_1)$ and $x_2 \in g^{-1}(y_2)$ and so

$$f(x) = f(x_1 \vee x_2) = f(x_1) \vee f(x_2) = y_1 \vee y_2,$$

that is, $x = x_1 \vee x_2 \in f^{-1}(y_1 \vee y_2)$. It follows that

$$\begin{aligned}
g(t_A)(y_1 \vee y_2) &= \bigvee_{x \in f^{-1}(y_1 \vee y_2)} t_A(x) \geq t_A(x_1 \vee x_2),\\
g(i_A)(y_1 \vee y_2) &= \bigvee_{x \in f^{-1}(y_1 \vee y_2)} i_A(x) \geq i_A(x_1 \vee x_2),\\
g(f_A)(y_1 \vee y_2) &= \bigwedge_{x \in f^{-1}(y_1 \vee y_2)} f_A(x) \leq f_A(x_1 \vee x_2).
\end{aligned}$$

Then

$$\begin{aligned} g(A)(y_1 \vee y_2) &= (g(t_A)(y_1 \vee y_2), g(i_A)(y_1 \vee y_2), g(f_A)(y_1 \vee y_2))) \\ &= (\bigvee_{x \in f^{-1}(y_1 \vee y_2)} t_A(x), \bigvee_{x \in f^{-1}(y_1 \vee y_2)} i_A(x), \bigwedge_{x \in f^{-1}(y_1 \vee y_2)} f_A(x)) \\ &\geq (t_A(x_1 \vee x_2), i_A(x_1 \vee x_2), f_A(x_1 \vee x_2)) \\ &\geq (t_A(x_1) \wedge t_A(x_2), i_A(x_1) \wedge i_A(x_2), f_A(x_1) \vee f_A(x_2)) \\ &= (t_A(x_1), i_A(x_1), f_A(x_1)) \wedge (t_A(x_2), i_A(x_2), f_A(x_2)). \end{aligned}$$

Cnsequently,

$$\begin{aligned} g(A)(y_1 - y_2) &\geq (\bigvee_{x_1 \in f^{-1}(y_1)} t_A(x_1), \bigvee_{x_1 \in f^{-1}(y_1)} i_A(x_1), \bigwedge_{x_1 \in f^{-1}(y_1)} f_A(x_1)) \\ &\wedge (\bigvee_{x_2 \in f^{-1}(y_2)} t_A(x_2), \bigvee_{x_2 \in f^{-1}(y_2)} i_A(x_2) \bigwedge_{x_2 \in f^{-1}(y_2} f_A(x_1)) \\ &= (g(t_A)(y_1), g(i_A)(y_1), g(f_A)(y_1)) \wedge (g(t_A)(y_2), g(i_A)(y_2), g(f_A)(y_2)) \\ &= g(A)(y_1) \wedge g(A)(y_2). \end{aligned}$$

Hence $g(A)$ is a neutrosophic ideal of $f(X_1)$. □

Theorem 3.11. *Let $g : X_1 \to X_2$ be a homomorphism. Then the preimage $f^{-1}(B)$ of a neutrosophic ideal B of X_2 is a neutrosophic ideal of X_1.*

Proof. Let $B = \{< x, t_B(x), i_B(x), f_B(x) >, x \in X_2\}$ be a neutrosophic ideal of X_2 and $x, y \in X_1$. Then

$$\begin{aligned} g^{-1}(B)(x - y) &= (t_B(g(x - y)), i_B(g(x - y)), f_B(g(x - y)) \\ &= (t_B(g(x) - g(y)), i_B(g(x) - g(y)), f_B(g(x) - g(y)) \\ &\geq (t_B(g(x)), i_B(g(x)), f_B(g(x)) \\ &= g^{-1}(B)(x). \end{aligned}$$

Now suppse $x \vee y$ exists for $x, y \in X_1$. Then

$$\begin{aligned} g^{-1}(B)(x \vee y) &= (t_B(g(x \vee y)), i_B(g(x \vee y)), f_B(g(x \vee y)) \\ &= (t_B(g(x) \vee g(y)), i_B(g(x) \vee g(y)), f_B(g(x) \vee g(y)) \\ &\geq t_B(g(x)) \wedge i_B(g(y)), i_B(g(x) \wedge i_B(g(y)), f_B(g(x) \vee f_B(g(y))) \\ &= (t_B(g(x)), i_B(g(x)), f_B(g(x)) \wedge (t_B(g(y)), i_B(g(y)), f_B(g(y)) \\ &= g^{-1}(B)(x) \wedge g^{-1}(B)(y) \end{aligned}$$

Hence $g^{-1}(B)$, is a neutrosophic ideal of X_1. □

4 conclusions

F.Smarandache introduced the concept of neutrosophic sets, which can be seen as a new mathematical tool for dealing with uncertainty. In this paper, we apply the notion of neutrosophic sets in subtraction algebras.

Also, we introduce the notion of neutrosophic ideal and give some conditions for a neutrosophic set to be a neutrosophic ideal in substraction algebras. Finally,we showed that neutrosophic image and neutrosophic inverse image of neutrosophic ideal are both neutrosophic ideal under certain conditions Based on these results,we could apply neutrosophic sets to other types of ideals in subtraction algebra. Also,we believe that such a results applied for other algebraic structure.

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Energy and Spectrum Analysis of Interval Valued Neutrosophic Graph using MATLAB

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Abstract. In recent time graphical analytics of uncertainty and indeterminacy has become major concern for data analytics researchers. In this direction, the mathematical algebra of neutrosophic graph is extended to interval-valued neutrosophic graph. However, building the interval-valued neutrosophic graphs, its spectrum and energy computation is addressed as another issues by research community of neutrosophic environment. To resolve this issue the current paper proposed some related mathematical notations to compute the spectrum and energy of interval-valued neutrosophic graph using the MATAB.

Keywords: Interval valued neutrosophic graphs. Adjacency matrix. Spectrum of IVNG. Energy of IVNG. Complete-IVNG.

1 Introduction

The handling uncertainty in the given data set is considered as one of the major issues for the research communities. To deal with this issue the mathematical algebra of neutrosophic set is introduced [1]. The calculus of neutrosophic sets (NSs)[1, 2] given a way to represent the uncertainty based on acceptation, rejection and uncertain part, independently. It is nothing but just an extension of fuzzy set [3], intuitionistic fuzzy set [4-6], and interval valued fuzzy sets [7] beyond the unipolar fuzzy space. It characterizes the uncertainty based on a truth-membership function (T), an indeterminate-membership function (I) and a falsity-membership function(F) independently of a defined neutrosophic set via real a standard or non-standard unit interval$]^{-}0, 1^{+}[$. One of the best suitable example is for the neutrosophic logic is win/loss and draw of a match, opinion of people towards an event is based on its acceptance, rejection and uncertain values. These properties of neutrosophic set differentiate it from any of the available approaches in fuzzy set theory while measuring the indeterminacy. Due to which mathematics of single valued neutrosophic sets (abbr. SVNS) [8] as well as interval valued neutrosophic sets (abbr.IVNS) [9-10] is introduced for precise analysis of indeterminacy in the given interval. The IVNS represents the acceptance, rejection and uncertain membership functions in the unit interval [0, 1] which helped a lot for knowledge processing tasks using different classifier [11], similarity method [12-14] as well as multi-decision making process [15-17] at user defined weighted method [18-24]. In this process a problem is addressed while drawing the interval-valued neutrosophic graph, its spectrum and energy analysis. To achieve this goal, the current paper tried to focus on introducing these related properties and its analysis using MATLAB.

2 Literature Review

There are several applications of graph theory which is a mathematical tool provides a way to visualize the given data sets for its precise analysis. It is utilized for solving several mathematical problems. In this process, a problem is addressed while representing the uncertainty and vagueness exists in any given attributes (i.e. vertices) and their corresponding relationship i.e edges. To deal with this problem, the properties of fuzzy graph [25-26] theory is extended to intuitionistic fuzzy graph [28-30], interval valued fuzzy graphs [31] is studied with applications [32—33]. In this case a problem is addressed while measuring with indeterminacy and its situation. Hence, the neutrosophic graphs and its properties is introduced by Smaranadache [34-37] to characterizes them using their truth, falsity, and indeterminacy membership-values (T, I, F) with its applications [38-40]. Broumi et al. [41] introduced neutrosophic graph theory considering (T, I, F) for vertices and edges in the graph specially termed as "Single valued neutrosophic graph theory (abbr. SVNG)" with its other properties [42-44]. Afterwards several researchers studied the neutrosophic graphs and its applications [65, 68]. Broumi et al. [50] utilized the

SVNGs to find the shortest path in the given network subsequently other researchers used it in different fields [51-53, 59-60, 65]. To measure the partial ignorance, Broumi et al. [45] introduced interval valued-neutrosophic graphs and its related operations [46-48] with its application in decision making process in various extensions[49, 54, 57 61, 62, 64,73-84].

Some other researchers introduced antipodal single valued neutrosophic graphs [63, 65], single valued neutrosophic digraph [68] for solving multi-criteria decision making. Naz et al.[69] discussed the concept of energy and laplacian energy of SVNGs. This given a major thrust to introduce it into interval-valued neutrosophic graph and its matrix. The matrix is a very useful tool in representing the graphs to computers, matrix representation of SVNG, some researchers study adjacency matrix and incident matrix of SVNG. Varol et al. [70] introduced single valued neutrosophic matrix as a generalization of fuzzy matrix, intuitionistic fuzzy matrix and investigated some of its algebraic operations including subtraction, addition, product, transposition. Uma et al. [66] proposed a determinant theory for fuzzy neutrosophic soft matrices. Hamidiand Saeid [72] proposed the concept of accessible single-valued neutrosophic graphs.

It is observed that, few literature have shown the study on energy of IVNG. Hence this paper, introduces some basic concept related to the interval valued neutrosophic graphs are developed with an interesting properties and its illustration for its various applications in several research field.

3 Preliminaries

This section consists some of the elementary concepts related to the neutrosophic sets, single valued neutrosophic sets, interval-valued neutrosophic sets, single valued neutrosophic graphs and adjacency matrix for establishing the new mathematical properties of interval-valued neutrosophic graphs. Readers can refer to following references for more detail about basics of these sets and their mathematical representations [1, 8, 41].

Definition 3.1:[1] Suppose ξ be a nonempty set. A neutrosophic set (abbr.NS) N in$\xi$is an object taking the form N_{NS}= {<x: $T_N(k)$, $I_N(k)$, $F_N(k)$>, k∈ ξ} (1)

Where $T_N(k):\xi \to]^-0,1^+[$, $I_N(k):\xi \to]^-0,1^+[$,$F_N(k):\xi \to]^-0,1^+[$ are known as truth-membership function, indeterminate –membership function and false-membership unction, respectively. The neutrosophic sets is subject to the following condition:

$$^-0 \leq T_N(k)+I_N(k)+F_N(k) \leq 3^+ \quad (2)$$

Definition 3.2:[8]Suppose ξ be a nonempty set. A single valued neutrosophic sets N (abbr. SVNs) inξ is an object taking the form:

$$N_{SVNs}=\{<k:T_N(k), I_N(k), F_N(k)>, k\in \xi\} \quad (3)$$

where $T_N(k)$, $I_N(k)$, $F_N(k) \in [0, 1]$ are mappings. $T_N(k)$denote the truth-membership function of an element x ∈ ξ, $I_N(k)$denote the indeterminate –membership function of an element k ∈ ξ.$F_N(k)$denote the false–membership function of an element k ∈ ξ. The SVNs subject to condition

$$0 \leq T_N(k)+I_N(k)+F_N(k) \leq 3 \quad (4)$$

Example 3.3: Let us consider following example to understand the indeterminacy and neutrosophic logic:

In a given mobile phone suppose 100 calls came at end of the day.

1. 60 calls were received truly among them 50 numbers are saved and 10 were unsaved in mobile. In this case these 60 calls will be considered as truth membership i.e. 0.6.

2. 30 calls were not-received by mobile holder. Among them 20 calls which are saved in mobile contacts were not received due to driving, meeting, or phone left in home, car or bag and 10 were not received due to uncertain numbers. In this case all 30 not received numbers by any cause (i.e. driving, meeting or phone left at home) will be considered as Indeterminacy membership i.e. 0.3.

3. 10 calls were those number which was rejected calls intentionally by mobile holder due to behavior of those saved numbers, not useful calls, marketing numbers or other cases for that he/she do not want to pick or may be blocked numbers. In all cases these calls can be considered as false i.e. 0.1 membership value.

The above situation can be represented as (0.6, 0.3, 0.1) as neutrosophic set.

Definition 3.4: [10] Suppose ξ be a nonempty set. An interval valued neutrosophic sets N (abbr.IVNs) in $\xi$is an object taking the form:

$$N_{IVNs}=\{<\text{k}:\tilde{T}_N(k),\, \tilde{I}_N(k),\tilde{F}_N(k)>,\text{k}\in \xi>\} \quad (5)$$

Where $\tilde{T}_N(k)$, $\tilde{I}_N(k)$, $\tilde{F}_N(k) \subseteq int[0,1]$ are mappings. $\tilde{T}_N(k)$=[$T_N^L(k)$, $T_N^U(k)$] denote the interval truth-membership function of an element k$\in \xi$.$\tilde{I}_N(k)$=[$I_N^L(x)$, $I_N^U(k)$] denote the interval indeterminate-membership function of an element k$\in \xi$.$\tilde{F}_N(k)$=[$F_N^L(k)$, $F_N^U(k)$] denote the false-membership function of an element k$\in \xi$.

Definition 3.4: [10]For every two interval valued-neutrosophic sets A and B in ξ, we define

$$(\text{N} \cup \text{M})\ (\text{k})= ([T_C^L(\text{k}), T_C^U(\text{k})],\ [\ I_C^L(\text{k}), I_C^U(\text{k})],\ [\ F_C^L(\text{k}), F_C^U(\text{k})]\ ) \text{ for all k} \in \xi \quad (6)$$

Where

T_C^L(k)= T_N^L(k) ∨ T_M^L(k), T_C^U(k)= T_N^U(k)∨ T_M^U(k)

I_C^L(k)= I_N^L(k)∧ I_M^L(k), I_C^U(k)= I_N^U(k)∧ I_M^U(k)

F_C^L(k)= F_N^L(k)∧ F_M^L(k), F_C^U(k)= F_N^U(k)∧ F_M^U(k)

Definition 3.5: [41]A pair G=(V,E) is known as single valued neutrosophic graph (abbr.SVNG) if the following holds:

1. V= $\{k_i$:i=1,..,n} such as T_1:V→ [0,1] is the truth-membership degree, I_1:V→[0,1] is the indeterminate –membership degree and F_1:V→[0,1]is the false membership degree of k_i ∈ V subject to condition

$$0 \leq T_1(k_i)+I_1(k_i)+F_1(k_i) \leq 3 \quad (7)$$

2. E={(k_i, k_j): $(k_i, k_j) \in V \times V$} such as T_2:$V \times V \rightarrow [0,1]$ is the truth-memebership degree, I_2:$V \times V \rightarrow [0,1]$ is the indeterminate –membership degree and F_2:$V \times V \rightarrow [0,1]$ is the false-memebership degree of (k_i,k_j) ∈ E defined as

$$T_2(k_i,k_j) \leq T_1(k_i) \wedge T_1(k_j) \quad (8)$$

$$I_2(k_i, k_j) \geq I_1(k_i) \vee I_1(k_j) \quad (9)$$

$$F_2(k_i, k_j) \geq F_1(k_i) \vee F_1(k_j) \quad (10)$$

Subject to condition $0 \leq T_2(k_i k_2)+I_2(k_1 k_2)+F_2(k_i k_j) \leq 3\ \forall\ (k_i, k_j) \in$ E. (11)

The Fig. 1 shows an illustration of SVNG.

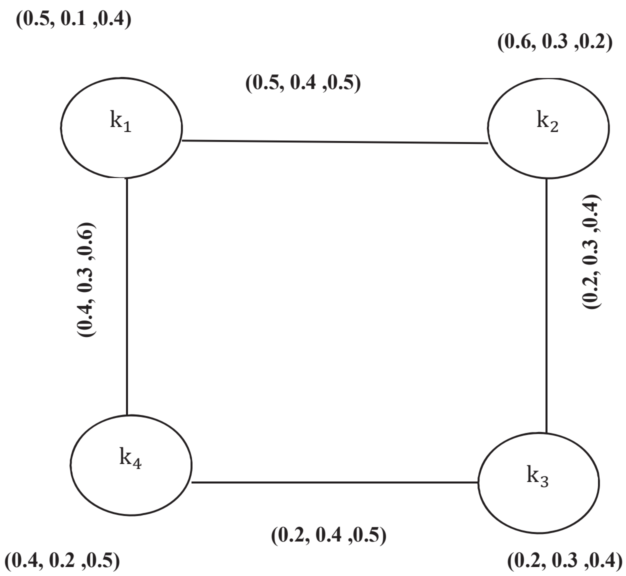

Fig. 1. An illustration of single valued neutrosophic graph

Definition 3.6[41]. A single valued neutrosophic graph G=(N, M) of G^*= (V, E) is termed strong single valued neutrosophic graph if the following holds:

$T_M(k_ik_j)= T_N(k_i) \wedge T_N(k_j)$ (12)

$I_M(k_ik_j) = I_N(k_i) \vee I_N(k_i)$ (13)

$F_M(k_ik_j)= F_N(k_i) \vee F_N(k_j)$ (14)

$\forall$ (k_i , k_j) ∈ E.

Where the operator ∧denote minimum and the operator ∨denote the maximum

Definition 3.8[41]. A single valued neutrosophic graph G=(N, M) of G^*= (V, E) is termed complete single valued neutrosophic graph if the following holds:

$T_M(k_ik_j)= T_N(k_i) \wedge T_N(k_j)$ (15)

$I_M(k_ik_j) = I_N(k_i) \vee I_N(k_i)$ (16)

$F_M(k_ik_j)= F_N(k_i) \vee F_N(k_j)$ (17)

$\forall k_i$, k_j ∈V.

Definition 3.9:[70] The Eigen value of a graph G are the Eigen values of its adjacency matrix.

Definition 3.10:[70]The spectrum of a graph is the set of all Eigen values of its adjacency matrix

$$\lambda_1 \geq \lambda_2 \ldots \geq \lambda_n \quad (18)$$

Definition 3.11:[70]The energy of the graph G is defined as the sum of the absolute values of its eigenvalues and denoted it by E(G):

$$E(G)=\sum_{i=1}^{n}|\lambda_i| \quad (19)$$

4.Some Basic Concepts of Interval Valued Neutrosophic Graphs

Throughout this paper, we abbreviate G^*=(V, E) as a crisp graph, and G=(N, M) an interval valued neutrosophic graph.In this section we have defined some basic concepts of interval valued neutrosophic graphs and discuses some of their properties.

Definition 4.1:[45] A pair G=(V,E) is called an interval valued neutrosophic graph (abbr.IVNG) if the following holds:

1. V= {k_i:i=1,..,n} such as T_1^L:V→ [0,1] is the lower truth-membership degree,T_1^U:V→ [0,1] is the upper truth-membership degree,I_1^L:V→ [0,1] is the lower indeterminate-membership degree,I_1^U:V→ [0,1] is the upper indterminate-membership degree, and F_1^L:V→ [0,1] is the lower false-membership degree,F_1^U:V→ [0,1] is the upper false-membership degree,of v_i ∈ V subject to condition
$$0 \leq T_1^U(k_i)+I_1^U(k_i)+F_1^U(k_i) \leq 3 \quad (20)$$
2. E={(k_i, k_j): $(k_i, k_j) \in V \times V$} such as $T_2^L: V \times V \rightarrow [0,1]$ is the lower truth-memebership degree, as $T_2^U: V \times V \rightarrow [0,1]$ is the upper truth-memebership degree, $I_2^L: V \times V \rightarrow [0,1]$ is the lower indeterminate-memebership degree, $I_2^U: V \times V \rightarrow [0,1]$ is the upper indeterminate-memebership degree and $F_2^L: V \times V \rightarrow [0,1]$ is the lower false-memebership degree, $F_2^U: V \times V \rightarrow [0,1]$ is the upper false-memebership degree of (k_i,k_j) ∈ E defined as
$$T_2^L(k_i,k_j)\leq T_1^L(k_i) \wedge T_1^L(k_j) ,T_2^U(k_i,k_j)\leq T_1^U(k_i) \wedge T_1^U(k_j) \quad (21)$$
$$I_2^L(k_i,k_j)\geq I_1^L(k_i) \vee I_1^L(k_j) ,I_2^U(k_i,k_j)\geq I_1^U(k_i) \vee I_1^U(k_j) \quad (22)$$
$$F_2^L(k_i,k_j)\geq F_1^L(k_i) \vee F_1^L(k_j) ,F_2^U(k_i,k_j)\geq F_1^U(k_i) \vee F_1^U(k_j) \quad (23)$$

Subject to condition $0 \leq T_2^U(k_ik_2)+I_2^U(k_1k_2)+F_2^U(k_ik_j) \leq 3 \ \forall (k_i, k_j) \in E.$ (24)

Example 4.2. Consider a crisp graph G^* such that V= $\{k_1, k_2, k_3\}$, E=$\{k_1k_2, k_2k_3, k_3k_4\}$. Suppose N be an interval valued neutrosophic subset of V and suppose M an interval valued neutrosophic subset of E denoted by:

	k_1	k_2	k_3
T_N^L	0.3	0.2	0.1
T_N^U	0.5	0.3	0.3
I_N^L	0.2	0.2	0.2
I_N^U	0.3	0.3	0.4
F_N^L	0.3	0.1	0.3
F_N^U	0.4	0.4	0.5

	k_1k_2	k_2k_3	k_3k_1
T_M^L	0.1	0.1	0.1
T_M^U	0.2	0.3	0.2
I_M^L	0.3	0.4	0.3
I_M^U	0.4	0.5	0.5
F_M^L	0.4	0.4	0.4
F_M^U	0.5	0.5	0.6

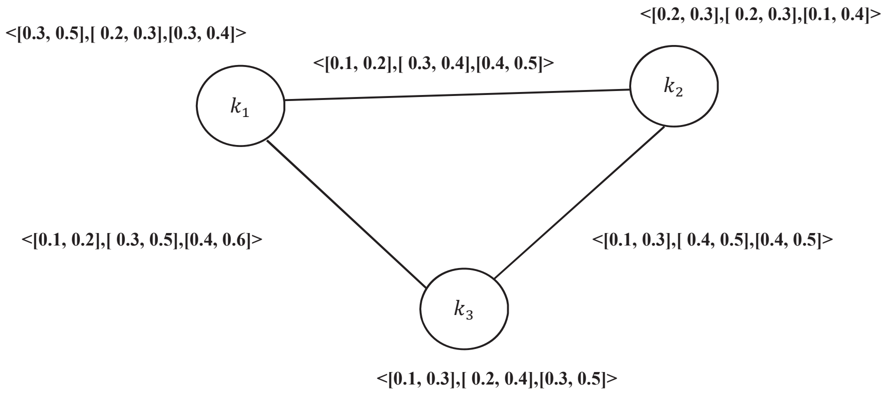

Fig. 2.Example of an interval valued neutrosophic graph

Definition 4.3A graph G=(N , M) is termed simple interval valued neutrosophic graph if it has neither self lops nor parallel edges in an interval valued neutrosophic graph.

Definition 4.4 The degree d(k) of any vertex k of an interval valued neutrosophic graph G=(N, M) is defined as follow:

$$d(v)= [\, d_T^L(k), d_T^U(k)],[d_I^L(k), d_I^U(k)],[d_F^L(k), d_F^U(k)] \quad (25)$$

Where

$d_T^L(k)=\sum_{k_i \neq k_j} T_M^L\,(k_ik_j)$ known as the degree of lower truth-membership vertex

$d_T^U(k)=\sum_{k_i \neq k_j} T_M^U\,(k_ik_j)$ known as the degree of upper truth-membership vertex

$d_I^L(k)=\sum_{k_i \neq k_j} I_M^L\,(k_ik_j)$ known as the degree of lower indterminate-membership vertex

$d_I^U(k)=\sum_{k_i \neq k_j} I_M^U\,(k_ik_j)$ known as the degree of upperindeterminate-membership vertex

$d_F^L(k)=\sum_{k_i \neq k_j} F_M^L\,(k_ik_j)$ known as the degree of lower false-membership vertex

$d_F^U(k)=\sum_{k_i \neq k_j} F_M^U\,(k_ik_j)$ known as the degree of upperfalse-membership vertex

Example 4.5 Consider an IVNG G=(N, M) presented in Fig. 4 with vertices set V=$\{k_i: i = 1,..,4\}$ and edges set E =$\{k_1k_4, k_4k_3, k_3k_2, k_2k_1\}$.

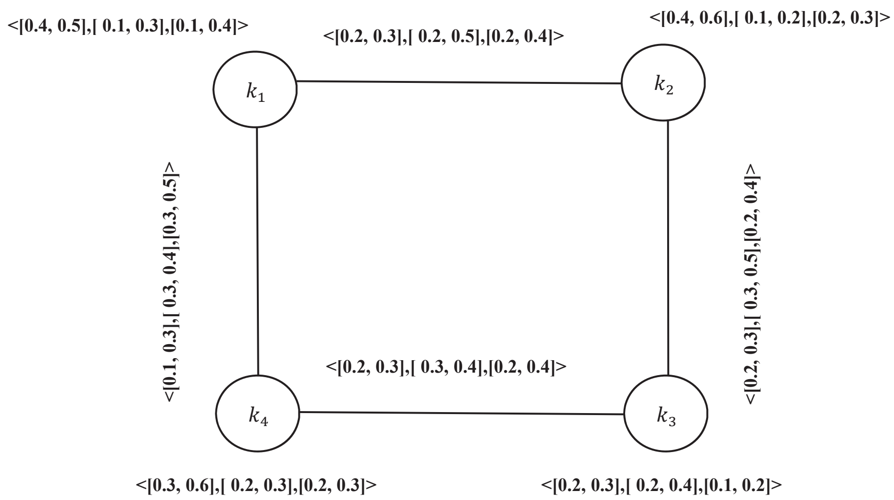

Fig. 4.Illutstrationof an interval valued neutrosophic graph

The degree of each vertex $k_i$is given as follows:

$d(k_1)$= ([0.3, 0.6], [0.5, 0.9], [0.5, 0.9]),

$d(k_2)$= ([0.4, 0.6], [0.5, 1.0], [0.4, 0.8]),

$d(k_3)$= ([0.4, 0.6], [0.6, 0.9], [0.4, 0.8]),

$d(k_4)$= ([0.3, 0.6], [0.6, 0.8], [0.5, 0.9]).

Definition 4.6. A graph G=(N, M) is termed regular interval valued neutrosophic graph if d(k)=r=([r_{1L}, r_{1U}], [r_{2L}, r_{2U}], [r_{3L}, r_{3U}]), ∀ k ∈ V.

(i.e.) if each vertex has same degree r, then G is said to be a regular interval valued neutrosophic graph of degree r.

Definition 4.7. A graph G=(N,M) is termed irregular interval valued neutrosophic graph if the degree of some vertices are different than other.

Example 4.8 Let us Suppose, G is a regular interval-valued neutrosophic graph as portrayed in Fig. 5 having vertex set V={k_1, k_2, k_3, k_4} and edge sets E={k_1k_2,k_2k_3, k_3k_4 ,k_4k_1} as follows.

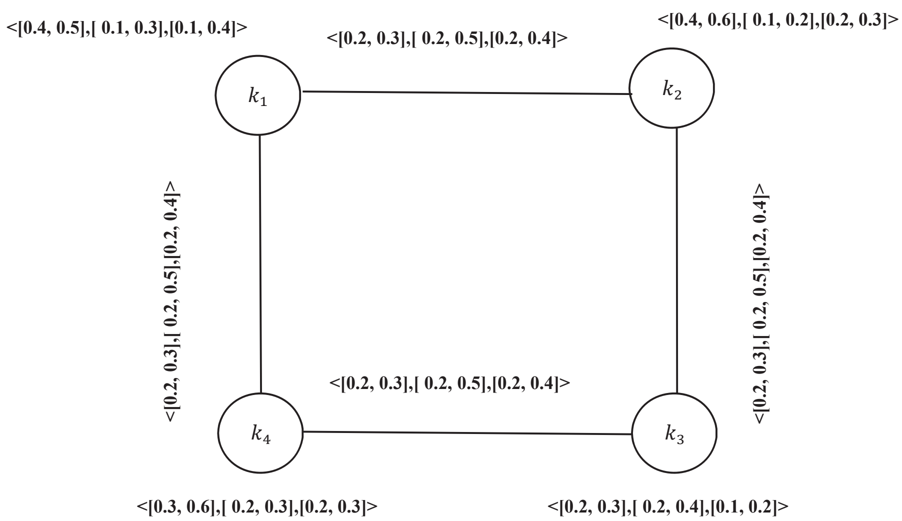

Fig.5 .Regular IVN-graph.

In the Fig. 5. All adjacent vertices k_1k_4 , k_4k_3 , k_3k_2 , k_2k_1 have the same degree equal <[0.4,0.6],[0.4,1],[0.4,0.8]>. Hence, the graph G is a regular interval valued neutrosophic graph.

Definition 4.9 A graph G= (N, M) on G^* is termed strong interval valued neutrosophic graph if the following holds:

$$T_M^L(k_i, k_j) = T_N^L(k_i) \wedge T_N^L(k_j)$$
$$T_M^U(k_i, k_j) = T_N^U(k_i) \wedge T_N^U(k_j)$$
$$I_M^L(k_i, k_j) = I_N^L(k_i) \vee I_N^L(k_j)$$
$$I_M^U(k_i, k_j) = I_N^U(k_i) \vee I_N^U(k_j)$$
$$F_M^L(k_i, k_j) = F_N^L(k_i) \vee F_N^L(k)$$
$$F_M^U(k_i, k_j) = F_N^U(k_i) \vee F_N^U(k_j) \; \forall \, (k_i, k_j) \in E \qquad (26)$$

Example 4.10. Consider the strong interval valued neutrosophic graph G=(N, M) in Fig. 6 with vertex set N ={k_1, k_2, k_3, k_4}and edge set M={k_1k_2, k_2k_3, k_3k_4, k_4k_1} as follows:

	k_1	k_2	k_3
T_N^L	0.3	0.2	0.1
T_N^U	0.5	0.3	0.3
I_N^L	0.2	0.2	0.2
I_N^U	0.3	0.3	0.4
F_N^L	0.3	0.1	0.3
F_N^U	0.4	0.4	0.5

	k_1k_2	k_2k_3	k_3k_1
T_M^L	0.2	0.1	0.1
T_M^U	0.3	0.3	0.3
I_M^L	0.2	0.2	0.2
I_M^U	0.3	0.4	0.4
F_M^L	0.3	0.3	0.3
F_M^U	0.4	0.4	0.5

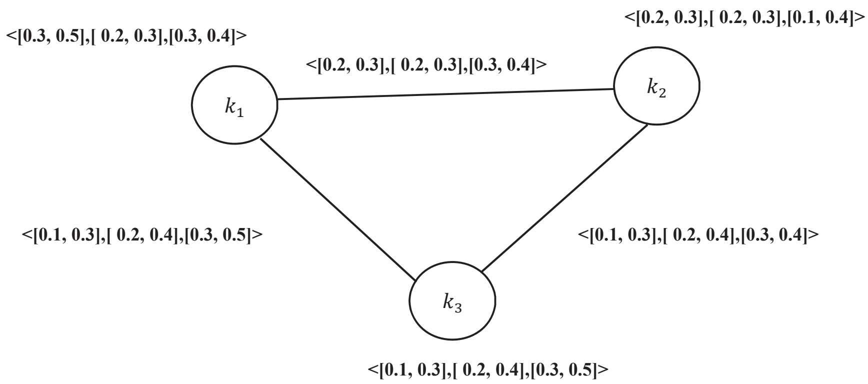

Fig.6.Illustration of strong IVNG

Proposition 4.11 For every $k_i, k_j \in V$, we have

$T_M^L(k_i, k_j) = T_M^L(k_j, k_i)$ and $\quad T_M^U(k_i, k_j) = T_M^U(k_j, k_i)$
$I_M^L(k_i, k_j) = I_M^L(k_j, k_i)$ *and* $\quad I_M^U(k_i, k_j) = I_M^U(k_j, k_i)$
$F_M^L(k_i, k_j) = F_M^L(k_j, k_i)$ and $\quad F_M^U(k_i, k_j) = F_M^U(k_j, k_i)$ (27)

Proof. Suppose G =(N, M) be an interval valued neutrosophic graph, suppose $\boldsymbol{k_i}$ is a neigbourhood of $\boldsymbol{k_j}$ in G.Then , we have

$\boldsymbol{T_M^L(k_i, k_j)}$ =min [$\boldsymbol{T_N^L(k_i), T_N^L(k_j)}$] and $\boldsymbol{T_M^U(k_i, k_j)}$ =min [$\boldsymbol{T_N^U(k_i), T_N^U(k_j)}$]

$\boldsymbol{I_M^L(k_i, k_j)}$ =max [$\boldsymbol{I_N^L(k_i), I_N^L(k_j)}$] and $\boldsymbol{I_M^U(k_i, k_j)}$ =max[$\boldsymbol{I_N^U(k_i), I_N^U(k_j)}$]

$\boldsymbol{F_M^L(k_i, k_j)}$ =max [$\boldsymbol{F_N^L(k_i), F_N^L(k_j)}$] and $\boldsymbol{F_M^U(k_i, k_j)}$ =max [$\boldsymbol{F_N^U(k_i), F_N^U(k_j)}$]

Similarly we have also for

$\boldsymbol{T_M^L(k_j, k_i)}$ =min [$\boldsymbol{T_N^L(k_j), T_N^L(k_i)}$] and $\boldsymbol{T_M^U(k_j, k_i)}$ =min [$\boldsymbol{T_N^U(k_j), T_N^U(k_i)}$]

$\boldsymbol{I_M^L(k_j, k_i)}$ =max [$\boldsymbol{I_N^L(k_j), I_N^L(k_i)}$] and $\boldsymbol{I_M^U(k_j, k_i)}$ =max[$\boldsymbol{I_N^U(k_j), I_N^U(k_i)}$]

$\boldsymbol{F_M^L(k_j, k_i)}$ =max [$\boldsymbol{F_N^L(k_j), F_N^L(k_i)}$] and $\boldsymbol{F_M^U(k_j, k_i)}$ =max [$\boldsymbol{F_N^U(k_j), F_N^U(k_i)}$]

Thus

$\boldsymbol{T_M^L(k_i, k_j)} = \boldsymbol{T_M^L(k_j, k_i)}$ and $\boldsymbol{T_M^U(k_i, k_j)} = \boldsymbol{T_M^U(k_j, k_i)}$
$\boldsymbol{I_M^L(k_i, k_j)} = \boldsymbol{I_M^L(k_j, k_i)}$ *and* $\boldsymbol{I_M^U(k_i, k_j)} = \boldsymbol{I_M^U(k_j, k_i)}$
$\boldsymbol{F_M^L(k_i, k_j)} = \boldsymbol{F_M^L(k_j, k_i)}$ and $\boldsymbol{F_M^U}(\boldsymbol{k_i}, k) = \boldsymbol{F_M^U(k_j, k_i)}$

Definition 4.12 The graph G= (N, M) is termed an interval valued neutrosophic graph if the following holds

$\boldsymbol{T_M^L(k_i, k_j)}$ =min [$\boldsymbol{T_N^L(k_i), T_N^U(k_j)}$] and $\boldsymbol{T_M^U(k_i, k_j)}$ =min [$\boldsymbol{T_N^U(k_i), T_N^U(k_j)}$]

$\boldsymbol{I_M^L(k_i, k_j)}$ =max [$\boldsymbol{I_N^L(k_i), I_N^L(k_j)}$] and $\boldsymbol{I_M^U(k_i, k_j)}$ =max[$\boldsymbol{I_N^U(k_i), I_N^U(k_j)}$]

$\boldsymbol{F_M^L(k_i, k_j)}$ =max [$\boldsymbol{F_N^L(k_i), F_N^L(k_j)}$] and $\boldsymbol{F_M^U(k_i, k_j)}$ =max [$\boldsymbol{F_N^U(k_i), F_N^U(k_j)}$] $\forall \boldsymbol{k_i}, \boldsymbol{k_j} \in V$ (28)

Example 4.13. Consider the complete interval valued neutrosophic graph G=(N, M) portrayed in Fig. 7 with vertex set A ={k_1, k_2, k_3, k_4}and edge set E={k_1k_2,k_1k_3 ,k_2k_3, k_1k_4, k_3k_4 ,k_2k_4}as follows

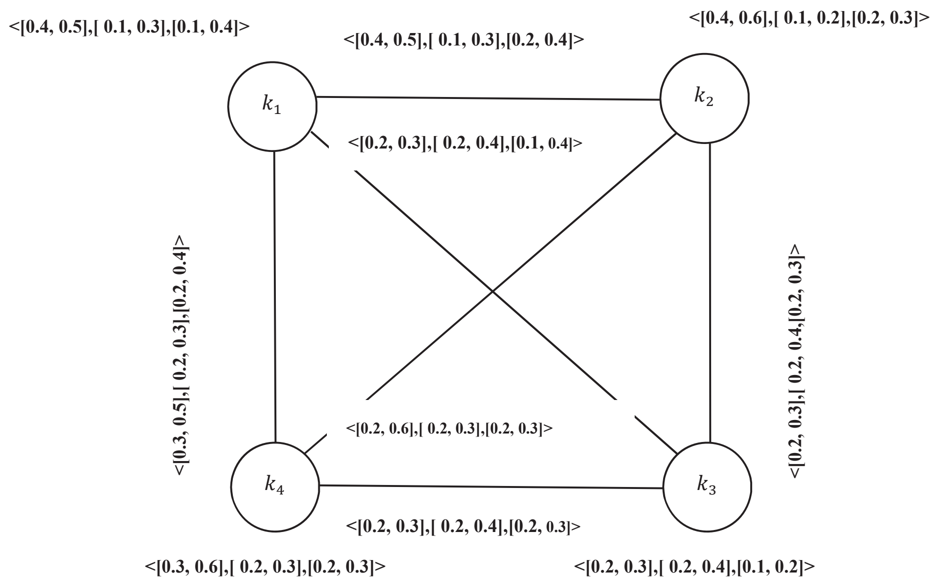

Fig.7 .Illustration of complete IVN-graph

In the following based on the extension of the adjacency matrix of SVNG [69], we defined the concept of adjacency matrix of IVNG as follow:

Definition 4.14: The adjacency matrix M(G) of IVNG G= (N, M) is defined as a square matrix M(G)=$[a_{ij}]$, with a_{ij}=<$\tilde{T}_M(k_i,k_j)$,$\tilde{I}_M^L(k_i,k_j)$,$\tilde{F}_M^L(k_i,k_j)$>, where

$\tilde{T}_M(k_i,k_j)$= [$T_M^L(k_i,k_j)$,$T^U(k_i,k_j)$] denote the strength of relationship

$\tilde{I}_M(k_i,k_j)$= [$I_M^L(k_i,k_j)$,$I_M^U(k_i,k_j)$] denote the strength of undecided relationship

$\tilde{F}_M(k_i,k_j)$=[$F_M^L(k_i,k_j)$,$F_M^U(k_i,k_j)$] denote the strength of non-relationship between k_i and k_j (29)

The adjacency matrix of an IVNG can be expressed as sixth matrices, first matrix contain the entries as lower truth-membership values, second contain upper truth-membership values, third contain lower indeterminacy-membership values, forth contain upper indeterminacy-membership, fifth contains lower non-membership values and the sixth contain the upper non-membership values, i.e.,

$$M(G)=<[T_M^L(k_i,k_j),T_M^U(k_i,k_j)]\,,\,[I_M^L(k_i,k_j),I_M^U(k_i,k_j)]\,,\,[F_M^L(k_i,k_j),F_M^U(k_i,k_j)]> , \quad (30)$$

From the Fig. 1, the adjacency matrix of IVNG is defined as:

$$\boldsymbol{M_G}=\begin{bmatrix} \mathbf{0} & <[\mathbf{0.1},\mathbf{0.2}],[\mathbf{0.3},\mathbf{0.4}],[\mathbf{0.4},\mathbf{0.5}]> & <[\mathbf{0.1},\mathbf{0.2}],[\mathbf{0.3},\mathbf{0.5}],[\mathbf{0.4},\mathbf{0.6}]> \\ <[\mathbf{0.1},\mathbf{0.2}],[\mathbf{0.3},\mathbf{0.4}],[\mathbf{0.4},\mathbf{0.5}]> & \mathbf{0} & <[\mathbf{0.1},\mathbf{0.3}],[\mathbf{0.4},\mathbf{0.5}],[\mathbf{0.4},\mathbf{0.5}]> \\ <[\mathbf{0.1},\mathbf{0.2}],[\mathbf{0.3},\mathbf{0.5}],[\mathbf{0.4},\mathbf{0.6}]> & <[\mathbf{0.1},\mathbf{0.3}],[\mathbf{0.4},\mathbf{0.5}],[\mathbf{0.4},\mathbf{0.5}]> & \mathbf{0} \end{bmatrix}$$

In the literature, there is no Matlab toolbox deals with neutrosophic matrix such as adjacency matrix and so on. Recently Broumi et al [58] developed a Matlab toolbox for computing operations on interval valued neutrosophicmatrices.So, we can inputted the adjacency matrix of IVNG in the workspace Matlab as portrayed in Fig. 8.

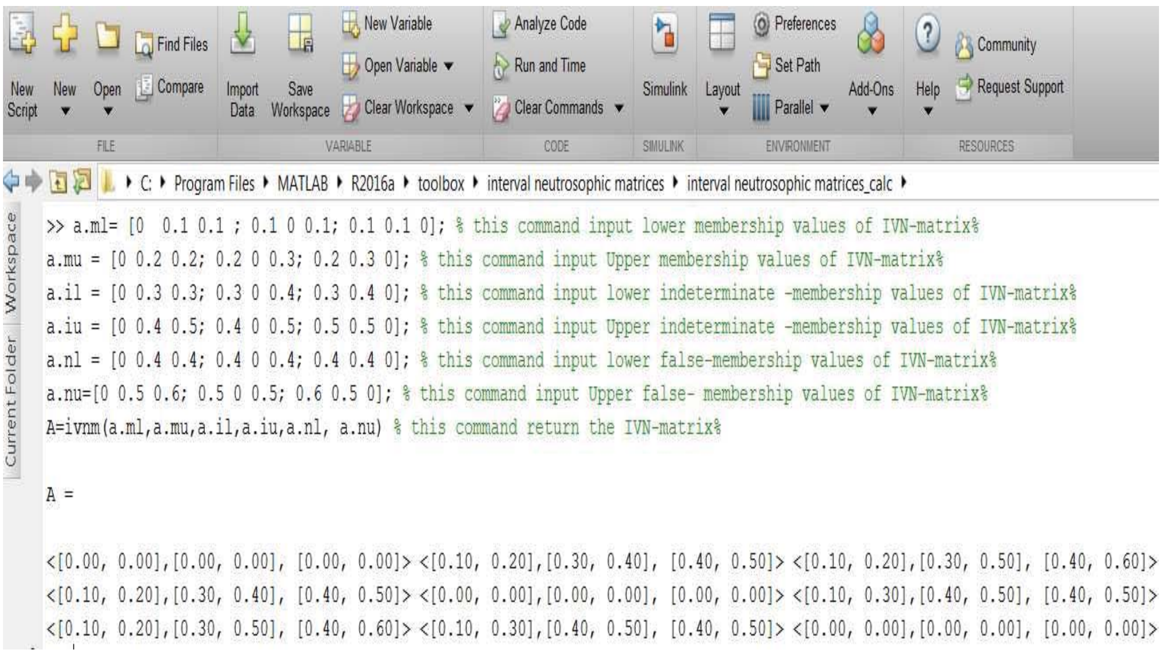

Fig. .8 Screen shot of Workspace MATLAB

Definition 4.15: The spectrum of adjacency matrix of an IVNG M(G) is defined as

$$<\tilde{R}, \tilde{S}, \tilde{Q}>=<[\tilde{R}^L, \tilde{R}^U],[\tilde{S}^L, \tilde{S}^U],<[\tilde{Q}^L, \tilde{Q}^U]> \quad (31)$$

Where $\tilde{R}^L$ is the set of eigenvalues of $M(T_M^L(k_i, k_j))$, $\tilde{R}^U$ is the set of eigenvalues of $M(T_B^U(k_i, k_j))$, $\tilde{S}^L$ is the set of eigenvalues of $M(I_M^L(k_i, k_j))$, $\tilde{S}^U$ is the set of eigenvalues of $M(I_M^U(k_i, k_j))$, $\tilde{Q}^L$ is the set of eigenvalues of $M(F_M^L(k_i, k_j))$ and $\tilde{Q}^U$ is the set of eigenvalue of $M(F_B^U(k_i, k_j))$ respectively.

Definition 4.16: The energy of an IVNG G= (N,M) is defined as

$$\mathbf{E(G)=<E(\tilde{T}_M(k_i, k_j)), E(\tilde{I}_M^L(k_i, k_j)), E(\tilde{F}_M^L(k_i, k_j))>} \quad (32)$$

Where

$$E(\tilde{T}_M(k_i, k_j)) = [E(T_M^L(k_i k_j)), E(T_M^U(k_i k_j))] = [\sum_{\substack{i=1 \\ \lambda_i^L \in \tilde{R}^L}}^{n} |\lambda_i^L|, \sum_{\substack{i=1 \\ \lambda_i^U \in \tilde{R}^U}}^{n} |\lambda_i^U|]$$

$$E(\tilde{I}_M(k_i, k_j)) = [E(I_M^L(k_i k_j)), E(I_M^U(k_i k_j))] = [\sum_{\substack{i=1 \\ \delta_i^L \in \tilde{S}^L}}^{n} |\delta_i^L|, \sum_{\substack{i=1 \\ \delta_i^U \in \tilde{S}^U}}^{n} |\delta_i^U|]$$

$$E(\tilde{F}_M(k_i, k_j)) = [E(F_M^L(k_i k_j)), E(F_M^U(k_i k_j))] = [\sum_{\substack{i=1 \\ \zeta_i^L \in \tilde{Q}^L}}^{n} |\zeta_i^L|, \sum_{\substack{i=1 \\ \zeta_i^U \in \tilde{Q}^U}}^{n} |\zeta_i^U|]$$

Definition 4.17: Two interval valued neutrosophic graphs G_1 and G_2 are termed equienergetic, if they have the same number of vertices and the same energy.

Proposition4.18: If an interval valued neutrosophic G is both regular and totally regular, then the eigen values are balanced on the energy.

$$\sum_{i=1}^{n} \pm\lambda_i^L = 0, \sum_{i=1}^{n} \pm\lambda_i^U = 0, \sum_{i=1}^{n} \pm\delta_i^L = 0, \sum_{i=1}^{n} \pm\delta_i^U = 0, \sum_{i=1}^{n} \pm\zeta_i^L = 0 \text{ and} \sum_{i=1}^{n} \pm\zeta_i^U = 0. \quad (33)$$

4.19. MATLAB program for findingspectrum of an interval valued neutrosophic graph

To generate the MATLAB program for finding the spectrum of interval valued neutrosophic graph. The program termed "Spec.m" is written as follow:

```
Function SG=Spec(A);
% Spectrum of an interval valued neutrosophic matrix A
% "A" have to be an interval valued neutrosophic  matrix - "ivnm" object:
a.ml=eig(A.ml);             % eigenvalues of lower membership of ivnm%
a.mu=eig(A.mu);             % eigenvalues of upper membership of ivnm%
a.il=eig(A.il);             % eigenvalues of lower rindeterminate-membership of ivnm%
a.iu=eig(A.iu);             % eigenvalues of upper indterminate- membership of ivnm%
a.nl=eig(A.nl);             % eigenvalues of lower false-membership of ivnm%
a.nu=eig(A.nu);             % eigenvalues of upper false-membership of ivnm%
SG=ivnm(a.ml,a.mu,a.il,a.iu,a.nl,a.nu);
```

4.20. MATLAB program for finding energy of an interval valued neutrosophic graph

To generate the MATLAB program for finding the energy of interval valued neutrosophic graph. The program termed "ENG.m"iswritten as follow:

```
function EG=ENG(A);
% energy of an interval valued neutrosophic matrix A
% "A" have to be an interval valued neutrosophic  matrix - "ivnm" object:
a.ml=sum(abs(eig(A.ml)));
a.mu=sum(abs(eig(A.mu)));
a.il=sum(abs(eig(A.il)));
a.iu=sum(abs(eig(A.iu)));
a.nl=sum(abs(eig(A.nl)));
a.nu=sum(abs(eig(A.nu)));
EG=ivnm(a.ml,a.mu,a.il,a.iu,a.nl,a.nu);
```

Example4.21: The spectrum and the energy of an IVNG, illustrated in Fig. 6, are given below:

Spec($T_M^L(k_ik_j)$)={ -0.10, -0.10,0.20}, Spec($T_M^U(k_ik_j)$)={-0.30,-0.17,0.47}
Spec($I_M^L(k_ik_j)$)={-0.40,-0.27,0.67}, Spec($I_M^U(k_ik_j)$)={-0.53,-0.40,0.93]}
Spec($F_M^L(k_ik_j)$)={-0.40,-0.40,0.80}, Spec($F_M^U(k_ik_j)$)={ -0.60,-0.47,1.07}

Hence,
Spec(G)={<[-0.10, -0.30], [-0.40, -0.53],[-0.40, -0.60]>, <[-0.10, -0.17], [-0.27, -0.40],[-0.40, -0.47]>, <[0.20, 0.47], [0.67, 0.93],[0.80, 1.07]>}
Now ,
E($T_M^L(k_ik_j)$)=0.40, E($T_M^U(k_ik_j)$)=0.94

E($I_M^L(k_ik_j)$)=1.34,E($I_M^U(k_ik_j)$)=1.87

E($F_M^L(k_ik_j)$)=1.60,E($F_M^U(k_ik_j)$)=2.14

Therefore
E(G)= <[0.40, 0.94],[1.34, 1.87], [1.60, 2.14]>
Based on toolbox MATLAB developed in [58], the readers can run the program termed “Spec.m”, for computing the spectrum of graph, by writing in command window “Spec (A)” as described below:

```
>> Spec(A) % this command return the spectrum of IVN-matrix%
Warning! The created new object is NOT an interval valued neutrosophic matrix

ans =

<[-0.10, -0.30],[-0.40, -0.53], [-0.40, -0.60]>
<[-0.10, -0.17],[-0.27, -0.40], [-0.40, -0.47]>
<[0.20, 0.47],[0.67, 0.93], [0.80, 1.07]>
```

Similarly, the readers can also run the program termed “ENG.m”, for computing the energy of graph, by writing in command window “ENG (A) as described below:

```
>> ENG(A) % this command return the Energy of IVN-matrix%
Warning! The created new object is NOT an interval valued neutrosophic matrix

ans =

<[0.40, 0.94],[1.34, 1.87], [1.60, 2.14]>
```

In term of the number of vertices and the sum of interval truth-membership, interval indeterminate-membership and interval false-membership, we define the upper and lower bounds on energy of an IVNG.

Proposition 4.22. Suppose G= (N, M) be an IVNG on n vertices and the adjacency matrix of G.then

$$\sqrt{2\sum_{1\le ij\le n}\left(T_M^L(k_ik_j)\right)^2+n(n-1)|T^L|^{2/N}}\le E\left(T_M^L(k_ik_j)\right)\le\sqrt{2n\sum_{1\le ij\le n}\left(T_M^L(k_ik_j)\right)^2} \quad (34)$$

$$\sqrt{2\sum_{1\le ij\le n}\left(T_M^U(k_ik_j)\right)^2+n(n-1)|T^U|^{2/N}}\le E\left(T_M^U(k_ik_j)\right)\le\sqrt{2n\sum_{1\le ij\le n}\left(T_M^U(k_ik_j)\right)^2} \quad (35)$$

$$\sqrt{2\sum_{1\le ij\le n}\left(I_M^L(k_ik_j)\right)^2+n(n-1)|I^L|^{2/N}}\le E\left(I_M^L(k_ik_j)\right)\le\sqrt{2n\sum_{1\le ij\le n}\left(I_M^L(k_ik_j)\right)^2} \quad (36)$$

$$\sqrt{2\sum_{1\le ij\le n}\left(I_M^U(k_ik_j)\right)^2+n(n-1)|I^U|^{2/N}}\le E\left(I_M^U(k_ik_j)\right)\le\sqrt{2n\sum_{1\le ij\le n}\left(I_M^U(k_ik_j)\right)^2} \quad (37)$$

$$\sqrt{2\sum_{1\le ij\le n}\left(F_M^L(k_ik_j)\right)^2+n(n-1)|F^L|^{2/N}}\le E\left(F_M^L(k_ik_j)\right)\le\sqrt{2n\sum_{1\le ij\le n}\left(F_M^L(k_ik_j)\right)^2} \quad (38)$$

$$\sqrt{2\sum_{1\le ij\le n}\left(F_M^U(k_ik_j)\right)^2+n(n-1)|F^U|^{2/N}}\le E\left(F_M^U(k_ik_j)\right)\le\sqrt{2n\sum_{1\le ij\le n}\left(F_M^U(k_ik_j)\right)^2} \quad (39)$$

Where $|\boldsymbol{T^L}|$,$|\boldsymbol{T^U}|$,$|\boldsymbol{I^L}|$,$|\boldsymbol{I^U}|$,$|\boldsymbol{F^L}|$and $|\boldsymbol{F^U}|$ are the determinant of $\boldsymbol{M(T_M^L(k_i,k_j))}$, $\boldsymbol{M(T_M^U(k_i,k_j))}$, $\boldsymbol{M(I_M^L(k_i,k_j))}$, $\boldsymbol{M(I_M^U(k_i,k_j))}$, $\boldsymbol{M(F_M^L(k_i,k_j))}$ and$\boldsymbol{M(F_M^U(k_i,k_j))}$,respectively.

Proof: proof is similar as in Theorem 3.2 [69]

Conclusion

This paper introduces some basic operations on interval-valued neutrosophic set to increase its utility in various fields for multi-decision process. To achieve this goal, a new mathematical algebra of interval-valued neutrosophic graphs, its energy as well as spectral computation is discussed with mathematical proof using MATLAB. In the near future, we plan to extend our research to interval valued neutrosophic digraphs and developed the concept of domination in interval valued-neutrosophic graphs. Same time the author will focus on handling its necessity for knowledge representation and processing tasks [85-87].

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A Neutrosophic Technique Based Efficient Routing Protocol For MANET Based On Its Energy And Distance

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Abstract. In the last decade, characterizing the energy in MANET based on its acceptation, rejection and uncertain part is addressed as one of the major issues by the researchers. An efficient energy routing protocol for MANET is another issue. To resolve these issues current paper focuses on utilizing the properties of neutrosophic technique. The essential idea of the protocol is to choose an energy efficient route with respect to neutrosophic technique. In this neutrosophic set, we have three components such as (T, I, F). Each parameter such as energy and distance is taken from these neutrosophic sets to determine the efficient energy route in MANET. After taking a brief survey about energy efficient routing for MANET using various methods, we are trying to implement the neutrosophic set technique to find the efficient energy route for MANET which provides the better energy route in uncertain situations. The comparative analysis between vague set MANET and neutrosophic MANET for the values of energy functions and distance functions is done by using Matlab and the result is discussed graphically

Keywords: MANET; neutrosophic set; energy efficient routing protocol; granular Computing.

1 Introduction

Wireless networking technologies play a vital role for giving rise to many new applications in internet world. Mobile ad-hoc network (MANET) is one of the most leading fields for research and development of wireless network. Now a days, wireless ad-hoc network has become one of the most vibrant and active field of communication and networks due to the popularity of mobile devices. Also, mobile or wireless network has become one of the indeed requirement for the users around the world. In this network, there are no groundwork stations or mobile switching centres and other structures of these types. The topology of Mobile ad-hoc network (MANET) changes dynamically. Each node is within others node's radio range via wireless networks. In the present era, nearly everyone has a mobile phone and most of it are smart phones. These devices are very cheaper and more powerful which make Mobile ad-hoc network (MANET) as the speed-growing network [1, 26, 36, 37]. Because of frequent braking of communication links, the nodes in mobile ad-hoc networks are free to move to anywhere. Also, a node in Mobile ad-hoc network (MANET) performs complete access to send data from one node to the other very fast and provides accurate services. Mobile ad-hoc network (MANET) is user friendly network which is easy to add or remove from the network. In this, each node contains some energy with limited battery capacity. The energy has been lost very speed in ad-hoc networks by transforming the data from one node to another node and also over all network's lifetime. Therefore the energy efficient routing indicates that the selecting route requires high energy and shortest distance. In this regard recently one of the authors has utilized implications of weighted concept lattice [31] and its implications using three-way neutrosophic environment [32-33] at different threshold [25] beyond the fuzzy logic [40]. It is shown that the computing paradigm of neutrosophic logic provides an authorization to deal with indeterminacy in the given network when compared to any other approaches available in fuzzy logic. Hence the current paper focused on introducing the concept of neutrosophic logic for analyzing the energy efficient routing protocol in Mobile ad-hoc network (MANET). Neutrosophic set was introduced by Florentin Smarandache [34] in 1995. Neutrosophic set is the generalization of fuzzy set, intuitionistic set fuzzy set, classical set and paraconsistent set etc. In intuitionistic fuzzy sets[2], the uncertainty is dependent on the degree of belongingness and degree of non-belongingness. In case of neutrosophy theory, the indeterminacy factor is independent of truth and falsity membership-values. Also neutrosophic sets are more general than IFS, because there are no conditions between the degree of truth, degree of indeterminacy and degree of falsity. In 2005, Wang et.al [38] introduced single valued neutrosophic sets which can be used in real world applications. In this case, a problem is addressed while dealing with efficient route in routing protocol based on its distance or energy. To shoot this problem, the current paper introduces a method to characterize the energy efficient route in Mobile ad-hoc network (MANET) based on its acceptation, rejection and uncertain part. In the same time the analysis of the proposed method is compared with one of the existing methods to validate the results. The motivation is to discover the precise and efficient path based on its maximal acceptance, minimum rejection, and minimal indeterminacy. The objective is to provide an optimal routing in Mobile ad-hoc network (MANET) in minimal energy utilization when compared to vague set [18]. One of the significant outputs of the proposed method is that it deals with uncertainty independent from truth and false membership-values.

The remaining part of the paper is organized as follows: Section 2 provides preliminaries about each of the set theories. Section 3 provides proposed method with its comparative analysis in Section 4. Section 5 provides conclusions and future research.

2 Overview of Mobile ad-hoc networks[28]

Mobile Ad Hoc networking (MANET) can be classified into first, second and third generations. The first generation of mobile ad-hoc network came up with “packet radio” networks (PRNET) in 1970s and it has evolved to be a robust, reliable, operational experimental network. The PRNET used a combination of ALOHA and channel access approaches CSMA for medium access, and a distance-vector routing to give packet-switched networking to mobile field elements in an infrastructure less, remote environment. The second generation evolved in early 1980’s when SURAN (Survivable Adaptive Radio Networks) significantly improved upon the radios, scalability of algorithms, and resilience to electronic attacks. During this period include GloMo (Global Mobile Information System) and NTDR (Near Term Digital Radio)were developed. The aim of GloMo was to give office-environment Ethernet-type multimedia connectivity anytime, anywhere, in handheld devices. Channel access approaches were in the CSMA/CA and TDMA molds, and several novel routing and topology control schemes were developed. The NTDR used clustering and link- state routing, and self-organized into a two-tier ad hoc network. Now used by the US Army, NTDR is the only “real” (non-prototypical) ad hoc network in use today. The third generation evolved in 1990’s also termed as commercial network with the advent of Notebooks computers, open source software and equipments based on RF and infrared. IEEE 802.11 subcommittee adopted the term “ad hoc networks.” The development of routing within the Mobile ad-hoc networking (MANET) working group and the larger community forked into reactive (routes on- demand) and proactive (routes ready-to-use) routing protocols 141. The 802.1 1 subcommittee standardized a medium access protocol that was based on collision avoidance and tolerated hidden terminals, making it usable, if not optimal, for building mobile ad hoc network prototypes out of notebooks and 802.11 PCMCIA cards. HIPERLAN and Bluetooth were some other standards that addressed and benefited ad hoc networking. With the increase of portable devices with wireless communication, ad-hoc networking plays an important role in many applications such as commercial, military and sensor networks, data networks etc., Mobile ad-hoc networks allow users to access and exchange information regardless of their geographic position or proximity to infrastructure. Since Mobile ad-hoc networking (MANET) has no static infrastructure, it offers an advantageous decentralized character to the network. Decentralization makes the networks more flexible and more robust.

3 Preliminaries

Definition of Fuzzy Set:

Fuzzy set was introduced by Zadeh in 1965 [40] and it gives new trend in application of mathematics. Every value of the fuzzy set consisting of order pair one is true membership and another one is false membership which lies between 0 and 1. Several authors [30, 39, 21-23, 27, 29] used fuzzy set theory in ad-hoc network and wireless sensor network to solve routing problems. The logic in fuzzy set theory is vastly used in all fields of mathematics like networks, graphs, topological space …etc.

Definition:[9]Intuitionistic Fuzzy Set:

Intuitionistic Fuzzy Sets are the extension of usual fuzzy sets. All outcomes which are applicable for fuzzy sets can be derived here also. Almost all the research works for fuzzy sets can be used to draw information of IFSs.

Further, there have been defined over IFSs not only operations similar to those of ordinary fuzzy sets, but also operators that cannot be defined in the case of ordinary fuzzy sets.

Definition:[17,24] Adroit System:

Adroit system [17, 24] is a computer program that efforts to act like a human effect in a particular subject area to give the solution to the particular unpredictable problem. Sometimes, adroit systems are used instead of human minds. Its main parts are knowledge based system and inference engine. In that the software is the knowledge based system which can be solved by artificial intelligence technique to find efficient route. The second part is inference engine which processes data by using rule based knowledge.

Definition:[34] Neutrosophic Set:

A neutrosophic set is a triplet which contains a truth membership function, a false membership function and indeterminacy function. Many authors extended this neutrosophic theory in different fields of mathematics such as decision making, optimization, graph theory etc.,[3-16, 42-52]. In particular, with the best knowledge, this is the first time to calculate efficient energy protocol for MANET based on the neutrosophic technique.

Let U be the universe. The neutrosophic set A in U is characterized by a truth-membership function T_A, a indeterminacy-membership function I_A and a falsity-membership function F_A. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard elements of [0,1]. It can be written as

$$A_{NS} = \{< T_A(x), I_A(x), F_A(x) >, x \in U, T_A(x), I_A(x), F_A(x) \in \left]^{-}0, 1^{+}\right[,\} \quad (1)$$

There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and F (x). So $0^- \leq T_A(x)+I_A(x)+F_A(x) \leq 3^+$.

Definition:[35] Let U be a universe of discourse and A the neutrosophic set A ⊂ U. Let $T_A(x), I_A(x), F_A(x)$ be the functions that describe the degree of membership, indeterminate membership and non-membership respectively of a generic element x ∈ U with respect to the neutrosophic set A. A single valued neutrosophic overset (SVNOV) A on the universe of discourse U is defined as:

$$A_{SVNOV} = \{< T_A(x), I_A(x), F_A(x) >, x \in U, T_A(x), I_A(x), F_A(x) \in [0, \Omega]\} \quad (2)$$

where $T_A(x), I_A(x), F_A(x)$: U→$[0, \Omega]$, $0<1<\Omega$ and Ω is called overlimit. Then there exists at least one element in A such that it has at least one neutrosophic component >1and no element has neutrosophic component <0

Definition:[35]Let U be a universe of discourse and the neutrosophic set A⊂ U. Let $T_A(x), I_A(x), F_A(x)$ be the functions that describe the degree of membership, indeterminate membership and non-membership respectively of a generic element x ∈ U with respect to the neutrosophic set A. A single valued neutrosophic underset (SVNU) A on the universe of discourse U is defined as:

$$A_{SVNU} = \{< T_A(x), I_A(x), F_A(x) >, x \in U, T_A(x), I_A(x), F_A(x) \in [\Psi, 1]\} \quad (3)$$

where $T_A(x), I_A(x), F_A(x)$: U→$[\Psi, 1]$, $\Psi<0<1$ and Ψ is called lowerlimit.Then there exists at least one element in A such that it has at least one neutrosophic component<0 and no element has neutrosophic component >1

Definition:[35] Let U be a universe of discourse and the neutrosophic set A⊂ U. Let $T_A(x), I_A(x), F_A(x)$ be the functions that describe the degree of membership, indeterminate membership and non-membership respectively of a generic element x ∈ U with respect to the neutrosophic set A. A single valued neutrosophic offset (SVNOF) A on the universe of discourse U is defined as:

$$A_{SVNOF} = \{< T_A(x), I_A(x), F_A(x) >, x \in U, T_A(x), I_A(x), F_A(x) \in [\Psi, \Omega]\} \quad (4)$$

where $T_A(x), I_A(x), F_A(x)$: U→$[\Psi, 1]$, $\Psi<0<1<\Omega$ and Ψ is called underlimit while Ω is called overlimit. Then there exist some elements in A such that at least one neutrosophic component >1, and at least another neutrosophic component < 0

Example 1: Let A={(x_1,<1.2, 0.4,0.1>),(x_2 ,<0.2, 0.3,-0.7>)},since $T(x_1)$=1.2 >1 , $F(x_2)$= - 0.7 <0

Definition:[35]The complement of a single valued neutrosophic overset/ underset/offset A is denoted by C(A) and is defined by

$$C(A) = \{(x, < F_A(x), \Psi + \Omega - I_A(x), T_A(x) >, x \in U\} \quad (5)$$

Definition:[35] The intersection of two single valued neutrosophic overset/ underset/offset A and B is a single valued neutrosophic overset/ underset/offset denoted C and is denoted by $C = A \cap B$ and is defined by

$$C = A \cap B = \{(x, < \min(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \max(F_A(x), F_B(x))), x \in U\} \quad (6)$$

Definition:[35] The union of two single valued neutrosophic overset/ underset/offset A and B is a single valued neutrosophic overset/ underset/offset denoted C and is denoted by $C = A \cup B$ and is defined by

$$C = A \cup B = \{(x, < \max(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \min(F_A(x), F_B(x))), x \in U\} \quad (7)$$

The following table 1, describe the neutrosophic oversets, neutrosophic undersets, neutrosophic offsets and Single valued neutrosophic sets

Types of neutrosophic sets	Ψ **(under limit)**	Ω **(overlimit)**
neutrosophic oversets	0	$1 < \Omega$
neutrosophic undersets	$\Psi < 0$	1
neutrosophic offsets	$\Psi < 0$	$1 < \Omega$
Single valued neutrosophic sets	0	1

Table 1. Some type of neutrosophic sets

It can be observed that, the algebra of neutrosophic set provides an independent way to deal with indeterminacy beyond the truth and false membership-values of a vague set. However characterizing the distance of routing protocol in MANET based on its truth, falsity and indeterminacy membership-values is complex problem. To deal with this problem, one of the algorithms is proposed in the next section with an illustrative example.

4 PROPOSED PROTOCOL

In this section, a method is proposed to characterize the efficient routing path in MANET based on the neutrosophic technique using energy and distance. In this proposed protocol, energy function may be low, medium and high and also in a similar way distance may be short, medium and long. To represent these levels a neutrosophic set based membership function μ, Indeterminacy σ and non-membership γ is defined in this paper.

All these energy membership functions $E_L, E_M \, and \, E_H$ and distance membership functions $D_S, D_M \, and \, D_L$ are given in Table 2 and Table3.

Linguistic value	**Notation**	**Neutrosophic range**	**Basic value**
Low	E_L	(E_L^+, E_L^0, E_L^-)	(0,0.9,1.8)
Medium	E_M	(E_M^+, E_M^0, E_M^-)	(1.8,2.7,3.5)
High	E_H	(E_H^+, E_H^0, E_H^-)	(3.5,4.4,5)

Table 2. A neutrosophic set based representation of energy function

Linguistic value	**Notation**	**Neutrosophic range**	**Basic value**
Short	Ds	(DL+,DL0,DL-)	(0,9,17)
Medium	DM	(DM+,DM0,DM-)	(17,26,34)
Large	DL	(DH+,DH0,DH-)	(34,42,50)

Table 3. A neutrosophic set based representation of distance function

The a single valued neutrosophic overset/ underset/offset are characterized by three memberships, the truth-membership, indeterminacy-membership and false membership functions as described in definitions above.

It gives an interpretation of membership grades. Low, medium and high of the energy and distance functions are written as follows:

Neutrosophic Energy function values:

$\mu(E_L)$ = (0.3, 0.7, 1.2), $\mu(E_M)$ = (1.4, 2.3, 3), $\mu(E_H)$ = (3.2, 4, 4.8)

$\sigma(E_L)$ = (0.4, 0.9, 1.4), $\sigma(E_M)$ = (1.3, 2.6, 3.2), $\sigma(E_H)$ = (3.4, 3.8, 4.6)

$\gamma(E_L)$ = (0.2, 0.6, 1.4), $\gamma(E_M)$ = (1.2, 2.5, 3.4), $\gamma(E_H)$ = (3, 4.2, 4.5)

Neutrosophic Distance function values:

$\mu(D_s)$ = (0.2, 4, 10), $\mu(D_M)$ = (12, 20, 32), = (30, 38, 44)

$\sigma(D_s)$ = (0.5, 5, 12), $\sigma(D_M)$ = (10, 23, 30), $\sigma(D_L)$ = (29, 41, 49)

$\gamma(D_s)$ = (0.3, 6, 8), $\gamma(D_M)$ = (14, 21, 28), $\gamma(D_L)$ = (32, 40, 47)

Recently, several authors tried to deduce the neutrosophic values in various fields [40]. The current paper tried most suitable and ideal solution deduced by considering true members function μ for the better solution. These neutrosophic values are used for efficient route selection in MANET which is given below in table 3. By comparing different routes of the MANET, rating of the route is calculated by the Eq. 8 as given below:

$$NR_{i,j} = {meanof \quad \mu(E_i)} \Big/ {meanof \quad \mu(D_i)} \qquad (8)$$

From the rating of different route given in Table 4, each value of $NR_{i,j}$ is a neutrosophic route having different values which determine the nature of the route in MANET.

S.No	Neutrosophic possible route
1	If Energy is $\mu(E_L)$ and (Distance is $\mu(D_s)$,then the route is R1.
2	If Energy is $\mu(E_L)$ and (Distance is $\mu(D_M)$, then the route is R2.
3	If Energy is $\mu(E_L)$ and (Distance is $\mu(D_L)$,then the route is R3.
4	If Energy is $\mu(E_M)$ and (Distance is $\mu(D_s)$,then the route is R4.
5	If Energy is $\mu(E_M)$ and (Distance is $\mu(D_M)$,then the route is R5.
6	If Energy is $\mu(E_M)$ and (Distance is $\mu(D_L)$,then the route is R6.
7	If Energy is $\mu(E_H)$ and (Distance is $\mu(D_s)$,then the route is R7.
8	If Energy is $\mu(E_H)$ and (Distance is $\mu(D_M)$,then the route is R8.
9	If Energy is $\mu(E_H)$ and (Distance is $\mu(D_L)$,then the route is R9.

Table 4. A neutrosophic technique based efficient route selection

Route number	Neutrosophic Rating of route	Enlightenment of Rating
R1	0.154929	Good
R2	0.034375	Bad
R3	0.019642	Very Bad
R4	0.471830	Excellent
R5	0.104687	Poor
R6	0.059821	Very poor
R7	0.873239	Very excellent
R8	0.19375	Very good
R9	0.110714	Medium

Table 5. Enlightenment of rating of different routes in Neutrosophic Technique

Hence, each neutrosophic route has a specific rating in MANET. Table 4 provides a way to defined different neutrosophic routes by considering various energy functions and as well as distance functions. Following that the sequences of different routes based on their rating is given in Table 5. The decreasing order according to rating on the routes is R3 < R2< R5 < R9 < R1 <R8 < R4 < R7.Table 5 represents that based on neutrosophic ordering defined by the proposed method route R7 is one of the best energy efficient route among them for the given MANET.

5 Comparative Analysis

While comparing vague set and neutrosophic set, vague set is equivalent to intuitionistic fuzzy set because both of them having only truth and false membership functions. Also neutrosophic set is the generalization of fuzzy and intuitionistic fuzzy sets. Hence the results obtained by using neutrosophic set is better than the results obtained by using vague set. In this section, the comparative analysis among neutrosophic and vague set based routing protocol is discussed. The membership values of energy and distance functions of vague set Manet and neutrosophic set Manet are given in Table 6. Comparison between Vague set rating of route (VSR) and Neutrosophic rating of route (NRR) are given in Table 7.

Table 6. Membership values of energy and distance function

Notation	Base Value of Energy function		Notation	Base Value of Distance function	
	VM	NM		VM	NM
EL	(0,1.8)	(0,0.9,1.8)	Ds	(0, 17)	(0,9,17)
EM	(1.8, 3.5)	(1.8,2.7,3.5)	DM	(17, 34)	(17,26,34)
EH	(3.5, 5)	(3.5,4.4,5)	DL	(34, 50)	(34,42,50)

Table 7. Comparison between Vague Set Rating of route(VSR) and Neutrosophic Rating of route (NRR):

Route number	Vague Set Rating of route(VSR)	Neutrosophic Rating of route (NRR)	Enlightenment of Rating	
			VSR	NRR
R1	0.011842	0.154929	Very Bad	Good
R2	0.021176	0.034375	Bad	Bad
R3	0.105882	0.019642	Satisfactory	Very Bad
R4	0.059211	0.471830	Medium	Excellent
R5	0.105882	0.104687	Less Good	Poor
R6	0.529412	0.059821	Good	Very poor

R7	0.1	0.873239	Very good	Very excellent
R8	0.178824	0.19375	Excellent	Very good
R9	0.894118	0.110714	Very excellent	Medium

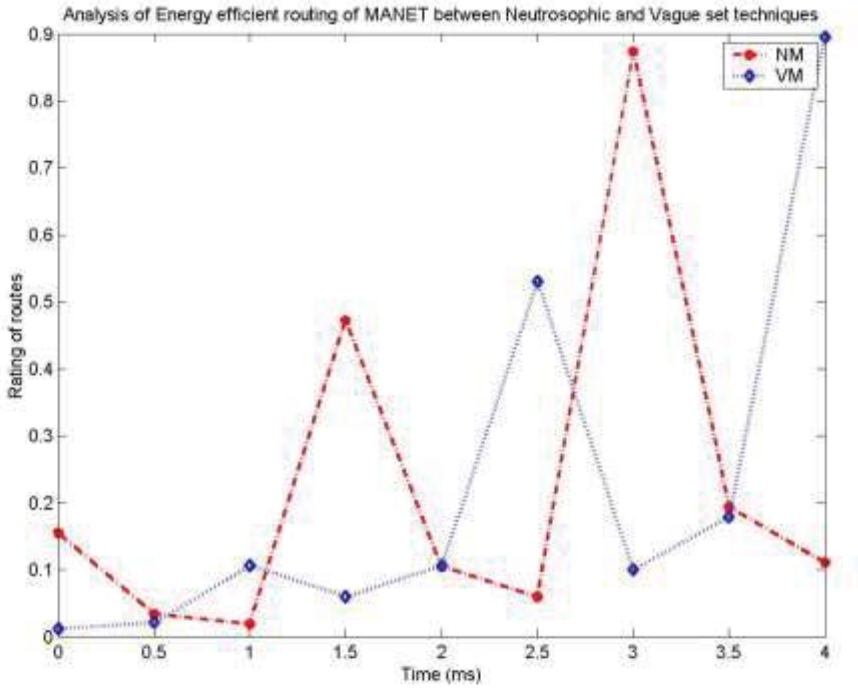

Figure. 1: The comparison of energy efficient MANET using neutrosophic and vague set

The graph for rating of routes of MANET using neutrosophic set and vague set techniques are plotted in Figure 1 for the values of energy functions and distance functions by using Table 6 and Table 7 with the help of Matlab software. It provides an information that, the efficient energy routing of mobile ad-hoc network using neutrosophic set technique(NM) is much better than the efficient energy routing of MANET using vague set technique(VM) in uncertain environment. However, the proposed method is focused on static environment in case the node and data set changes at each interval of time then the proposed unable to represent the case precisely. To deal with dynamic environment author will focus in near future to introduce the extensive properties of neutrosophic set and its applications to wireless ad-hoc network(WANET), flying ad-hoc network(FANET) and vehicular ad-hoc network(VANET).

Conclusion and future work

This paper utilizes properties of single valued neutrosophic for finding an efficient routing protocol for MANET based on distance and energy. In this regard, several algorithms are proposed to characterize it based on truth and falsity membership-values of a defined vague set. However the current paper aimed at dealing with uncertainty in routing protocol of MANET based on its truth, falsity and indeterminacy membership-values, indeterminacy. It is shown that the proposed method provides a precise representation and selection of energy efficient routing protocol when compared to vague sets as shown in Table 6 and 7. In future, the authors will focuses on investigating the energy efficient routes for WANET, FANET, VANET for dynamic environment

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Neutrosophic Nano ideal topolgical structure

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Abstract: This paper addressed the concept of Neutrosophic nano ideal topology which is induced by the two literature, they are nano topology and ideal topological spaces. We defined its local function, closed set and also defined and give new dimnesion to codense ideal by incorporating it to ideal topological structures. we investigate some properties of neutrosophic nano topology with ideal.

Keywords: neutrosophic nano ideal, neutrosophic nano local function, topological ideal, neutrosophic nano topological ideal.

1 Introduction and Preliminaries

In 1983, K. Atanassov [1] proposed the concept of IFS(intuitionstic fuzzy set) which is a generalization of FS(fuzzy set) [17], where each element has true and false membership degree. Smarandache [15] coined the concept of NS (neutrosophic set) which is new dimension to the sets. Neutrosophic set is classified into three independently related functions namely, membership, indeterminacy function and non-membership function. Lellis Thivagar [8], introduced the new notion of neutrosophic nano topology, which consist of upper, lower approximation and boundary region of a subset of a universal set using an equivalence class on it. There have been wide range of studies on neutrosophic sets, ideals and nano ideals [9, 10, 11,12,13,14]. Kuratowski [7] and Vaidyanathaswamy [16] introduced the new concept in topological spaces, called ideal topological spaces and also local function in ideal topological space was defined by them. Afterwards the properties of ideal topological spaces studied by Hamlett and Jankovic[5,6].

In this paper, we introduce the new concept of neutrosophic nano ideal topological structures, which is a generalized concept of neutrosophic nano and ideal topological structure. Also defined the codense ideal in neutrosophic nano topological structure.
We recall some relevant basic definitions which are useful for the sequel and in particular, the work of M. L. Thivagar [8], Parimala et al [9], F. Smarandache [15].

Definition 1.1. Let U be universe of discourse and R be an indiscernibility relation on U. Then U is divided into disjoint equivalence classes. The pair (U, R) is said to be the approximation space. Let F be a NS in U with the true μ_F, the indeterminancy σ_F and the false function ν_F. Then,

(i) The lower approximation of F with respect to equivalence class R is the set denoted by $\overline{N}(F)$ and defined as follows
$\overline{N}(F) = \left\{ \left\langle a, \mu_{\overline{R}(F)}(a), \sigma_{\overline{R}(F)}(a), \nu_{\overline{R}(F)}(a) \right\rangle | y \in [a]_R, a \in U \right\}$

(ii) The higher approximation of F with respect to equivalence class R is the set is denoted by $\underline{N}(F)$ and defined as follows, $\underline{N}(F) = \left\{ \left\langle a, \mu_{\underline{R}(F)}(a), \sigma_{\underline{R}(F)}(a), \nu_{\underline{R}(F)}(a) \right\rangle | y \in [a]_R, a \in U \right\}$

(iii) The boundary region of F with respect to equivalence class R is the set of all objects is denoted by $B(F)$ and defined by$B(F) = \overline{N}(F) - \underline{N}(F)$.

where,

$$\mu_{\overline{R}(F)}(a) = \bigcup_{y_1 \in [a]_R} \mu_F(y_1), \ \sigma_{\overline{R}(F)}(a) = \bigcup_{y_1 \in [a]_R} \sigma_F(y_1),$$
$$\nu_{\overline{R}(F)}(a) = \bigcap_{y_1 \in [a]_R} \nu_F(y_1). \ \mu_{\underline{R}(F)}(a) = \bigcap_{y_1 \in [a]_R} \mu_F(y_1),$$
$$\sigma_{\underline{R}(F)}(a) = \bigcap_{y_1 \in [a]_R} \sigma_F(y_1), \ \nu_{\underline{R}(F)}(a) = \bigcap_{y_1 \in [a]_R} \nu_F(y_1).$$

Definition 1.2. Let U be a nonempty set and the neutrosophic sets X and Y in the form $X = \{\langle a, \mu_X(a), \sigma_X(a), \nu_X(a) \rangle,$ and $Y = \{\langle a, \mu_Y(a), \sigma_Y(a), \nu_Y(a) \rangle, a \in U\}$. Then the following statements hold:

(i) $0_N = \{\langle a, 0, 0, 1 \rangle, a \in U\}$ and $1_N = \{\langle a, 1, 1, 0 \rangle, a \in U\}$.

(ii) $X \subseteq y$ if and only if $\mu_X(a) \leq \mu_Y(a), \sigma_X(a) \leq \sigma_Y(a), \nu_X(a) \geq \nu_Y(a)$ for all $a \in U$.

(iii) $X = Y$ if and only if $X \subseteq Y$ and $Y \subseteq X$.

(iv) $X^C = \{\langle a, \nu_X(a), 1 - \sigma_X(a), \mu_X(a) \rangle, a \in U\}$.

(v) $X \cap Y$ if and only if $\mu_X(a) \wedge \mu_X(a), \sigma_X(a) \wedge \sigma_Y(a), \nu_Y(a) \vee \nu_Y(a)$ for all $a \in U$.

(vi) $X \cup Y$ if and only if $\mu_Y(a) \vee \mu_Y(a), \sigma_X(a) \vee \sigma_Y(a), \nu_X(a) \wedge \nu_Y(a)$ for all $a \in U$.

(vii) $X - Y$ if and only if $\mu_X(a) \wedge \nu_Y(a), \sigma_X(a) \wedge 1 - \sigma_Y(a), \nu_X(a) \vee \mu_Y(a)$ for all $a \in U$.

Definition 1.3. Let X be a non-empty set and I is a neutrosophic ideal (NI for short) on X if

(i) $A_1 \in I$ and $B_1 \subseteq A_1 \Rightarrow B_1 \in I$ [heredity],

(ii) $A_1 \in I$ and $B_1 \in I \Rightarrow A_1 \cup B_1 \in I$ [finite additivity].

2 Neutrosophic nano ideal topological spaces

In this section we introduce a new type of local function in neutrosophic nano topological space. Before that we shall consider the following concepts.

Neutrosophic nano ideal topological space(in short NNI) is denoted by $(U, \tau_{\mathcal{N}}(F), I)$, where $(U, \tau_{\mathcal{N}}(F), I)$ is a neutrosophic nano topological space(in short NNT) $(U, \tau_{\mathcal{N}}(F))$ with an ideal I on U

Definition 2.1. Let $(U, \tau_{\mathcal{N}}(F), I)$ be a NNI with an ideal I on U and $(.)^*_{\mathcal{N}}$ be a set of operator from $P(U)$ to $P(U) \times P(U)$ ($P(U)$ is the set of all subsets of U). For a subset $X \subset U$, the neutrosophic nano local function $X^*_{\mathcal{N}}(I, \tau_{\mathcal{N}}(F))$ of X is the union of all neutrosophic nano points (NNP, for short) $C(\alpha, \beta, \gamma)$ such that $X^*_{\mathcal{N}}(I, \tau_{\mathcal{N}}(F)) = \vee\{C(\alpha, \beta, \gamma) \in U : X \cap G \notin I \text{for all } G \in N(C(\alpha, \beta, \gamma))\}$. We will simply write $X^*_{\mathcal{N}}$ for $X^*_{\mathcal{N}}(I, \tau_{\mathcal{N}}(F))$.

Example 2.2. Let $(U, \tau_{\mathcal{N}}(F))$ be a neutrosophic nano topological space with an ideal I on U and for every $X \subseteq U$.

(i) If $I = \{0_\sim\}$, then $X^*_{\mathcal{N}} = \mathcal{N}cl(X)$,

(ii) If $I = P(U)$, then $X^*_{\mathcal{N}} = 0_\sim$.

Theorem 2.3. Let $(U, \tau_{\mathcal{N}}(F))$ be a NNT with ideals I, I' on U and X, B be subsets of U. Then

(i) $X \subseteq B \Rightarrow X^*_{\mathcal{N}} \subseteq B^*_{\mathcal{N}}$,

(ii) $I \subseteq I' \Rightarrow X^*_{\mathcal{N}}(I') \subseteq X^*_{\mathcal{N}}(I)$,

(iii) $X^*_{\mathcal{N}} = \mathcal{N}cl(X^*_{\mathcal{N}}) \subseteq \mathcal{N}cl(X)$ ($X^*_{\mathcal{N}}$ is a neutrosophic nano closed subset of $\mathcal{N}cl(X)$),

(iv) $(X^*_{\mathcal{N}})^*_{\mathcal{N}} \subseteq X^*_{\mathcal{N}}$,

(v) $X^*_{\mathcal{N}} \cup B^*_{\mathcal{N}} = (X \cup B)^*_{\mathcal{N}}$,

(vi) $X^*_{\mathcal{N}} - B^*_{\mathcal{N}} = (X - B)^*_{\mathcal{N}} - B^*_{\mathcal{N}} \subseteq (X - B)^*_{\mathcal{N}}$,

(vii) $V \in \tau_{\mathcal{N}}(F) \Rightarrow V \cap X^*_{\mathcal{N}} = V \cap (V \cap X)^*_{\mathcal{N}} \subseteq (V \cap X)^*_{\mathcal{N}}$ and

(viii) $J \in I \Rightarrow (X \cup J)^*_{\mathcal{N}} = X^*_{\mathcal{N}} = (X - J)^*_{\mathcal{N}}$.

Proof. (i) Let $X \subset B$ and $a \in X^*_{\mathcal{N}}$. Assume that $a \notin B^*_{\mathcal{N}}$. We have $G_{\mathcal{N}} \cap B \in I$ for some $G_{\mathcal{N}} \in G_{\mathcal{N}}(a)$. Since $G_{\mathcal{N}} \cap X \subseteq G_{\mathcal{N}} \cap B$ and $G_{\mathcal{N}} \cap B \in I$, we obtain $G_{\mathcal{N}} \cap X \in I$ from the definition of ideal. Thus, we have $a \notin X^*_{\mathcal{N}}$. This is a contradiction. Clearly, $X^*_{\mathcal{N}} \subseteq B^*_{\mathcal{N}}$.
(ii) Let $I \subseteq I'$ and $a \in X^*_{\mathcal{N}}(I')$. Then we have $G_{\mathcal{N}} \cap X \notin I'$ for every $G_{\mathcal{N}} \in G_{\mathcal{N}}(a)$. By hypothesis, we obtain $G_{\mathcal{N}} \cap X \notin I$. So $a \in X^*_{\mathcal{N}}(I)$.
(iii) Let $a \in X^*_{\mathcal{N}}$. Then for every $G_{\mathcal{N}} \in G_{\mathcal{N}}(a), G_{\mathcal{N}} \cap X \notin I$. This implies that $G_{\mathcal{N}} \cap X \neq 0_\sim$. Hence

$a \in \mathcal{N}cl(X)$.
(iv) From (iii), $(X^*_{\mathcal{N}})^*_{\mathcal{N}} \subseteq \mathcal{N}cl(X^*_{\mathcal{N}}) = X^*_{\mathcal{N}}$, since $X^*_{\mathcal{N}}$ is a neutrosophic nano closed set.
The proofs of the other conditions are also obvious.
Theorem 2.4. If $(U, \tau_{\mathcal{N}}(F), I)$ is a NNT with an ideal I and $X \subseteq X^*_{\mathcal{N}}$, then $X^*_{\mathcal{N}} = \mathcal{N}cl(X^*_{\mathcal{N}}) = \mathcal{N}cl(X)$.
Proof. For every subset X of U, we have $X^*_{\mathcal{N}} = \mathcal{N}cl(X^*) \subseteq \mathcal{N}cl(X)$, by Theorem 2.3. (iii) $X \subseteq X^*_{\mathcal{N}}$ implies that $\mathcal{N}cl(X) \subseteq Ncl(X^*_{\mathcal{N}})$ and so $X^*_{\mathcal{N}} = \mathcal{N}cl(X^*_{\mathcal{N}}) = \mathcal{N}cl(X)$.

Definition 2.5. Let $(U, \tau_{\mathcal{N}}(F))$ be a NNT with an ideal I on U. The set operator $\mathcal{N}cl^*$ is called a neutrosophic nano$*$-closure and is defined as $\mathcal{N}cl^*(X) = X \cup X^*_{\mathcal{N}}$ for $X \subseteq a$.

Theorem 2.6. The set operator $\mathcal{N}cl^*$ satisfies the following conditions:

(i) $X \subseteq \mathcal{N}cl^*(X)$,

(ii) $\mathcal{N}cl^*(0_\sim) = 0_\sim$ and $\mathcal{N}cl^*(1_\sim) = 1_\sim$,

(iii) If $X \subset B$, then $\mathcal{N}cl^*(X) \subseteq \mathcal{N}cl^*(B)$,

(iv) $\mathcal{N}cl^*(X) \cup \mathcal{N}cl^*(B) = \mathcal{N}cl^*(X \cup B)$.

(v) $\mathcal{N}cl^*(\mathcal{N}cl^*(X)) = \mathcal{N}cl^*(X)$.

Proof. The proofs are clear from Theorem 2.3 and the definition of $\mathcal{N}cl^*$.
Now, $\tau_{\mathcal{N}}(F)^*(I, \tau_{\mathcal{N}}(F)) = \{V \subset U : \mathcal{N}cl^*(U - V) = U - V\}$. $\tau_{\mathcal{N}}(F)^*(I, \tau_{\mathcal{N}}(F))$ is called neutrosophic nano$*$-topology which is finer than $\tau_{\mathcal{N}}(F)$ (we simply write $\tau_{\mathcal{N}}(F)^*$ for $\tau_{\mathcal{N}}(F)^*(I, \tau_{\mathcal{N}}(F))$). The elements of $\tau_{\mathcal{N}}(F)^*(I, \tau_{\mathcal{N}}(F))$ are called neutrosophic nano$*$-open (briefly, $\mathcal{N}*$-open) and the complement of an $\mathcal{N}*$-open set is called neutrosophic nano$*$-closed (briefly, $\mathcal{N}*$-closed). Here $\mathcal{N}cl^*(X)$ and $\mathcal{N}int^*(X)$ will denote the closure and interior of X respectively in $(U, \tau_{\mathcal{N}}(F)^*)$.
Remark 2.7. (i) We know from Example 2.2 that if $I = \{0_\sim\}$ then $X^*_{\mathcal{N}} = \mathcal{N}cl(X)$. In this case, $\mathcal{N}cl^*(X) = \mathcal{N}cl(X)$.
(ii) If $(U, \tau_{\mathcal{N}}(F), I)$ is a NNI with $I = \{0_\sim\}$, then $\tau_{\mathcal{N}}(F)^* = \tau_{\mathcal{N}}(F)$.
Definition 2.8. A basis $\beta(I, \tau_{\mathcal{N}}(F))$ for $\tau_{\mathcal{N}}(F)^*$ can be described as follows:
$\beta(I, \tau_{\mathcal{N}}(F)) = \{X - B : X \in \tau_{\mathcal{N}}(F), B \in I\}$.
Theorem 2.9. Let $(U, \tau_{\mathcal{N}}(F))$ be a NNT and I be an ideal on U. Then $\beta(I, \tau_{\mathcal{N}}(F))$ is a basis for $\tau_{\mathcal{N}}(F)^*$.

Proof. We have to show that for a given space $(U, \tau_{\mathcal{N}}(F))$ and an ideal I on U, $\beta(I, \tau_{\mathcal{N}}(F))$ is a basis for $\tau_{\mathcal{N}}(F)^*$. If $\beta(I, \tau_{\mathcal{N}}(F))$ is itself a neutrosophic nano topology, then we have $\beta(I, \tau_{\mathcal{N}}(F)) = \tau_{\mathcal{N}}(F)^*$ and all the open sets of $\tau_{\mathcal{N}}(F)^*$ are of simple form $X - B$ where $X \in \tau_{\mathcal{N}}(F)$ and $B \in I$.
Theorem 2.10. Let $(U, \tau_{\mathcal{N}}(F), I)$ be a NNT with an ideal I on U and $X \subseteq U$. If $X \subseteq X^*_{\mathcal{N}}$, then

(i) $\mathcal{N}cl(X) = \mathcal{N}cl^*(X)$,

(ii) $\mathcal{N}int(U - X) = \mathcal{N}int^*(U - X)$.

Proof. (i) Follows immediately from Theorem 2.4.
(ii) If $X \subseteq X^*_{\mathcal{N}}$, then $\mathcal{N}cl(X) = \mathcal{N}cl^*(X)$ by (i) and so $U - \mathcal{N}cl(X) = U - \mathcal{N}cl^*(X)$. Therefore, $\mathcal{N}int(U - X) = \mathcal{N}int^*(U - X)$.

Theorem 2.11. Let $(U, \tau_{\mathcal{N}}(F), I)$ be a NNT with an ideal I on U and $X \subseteq X$. If $X \subseteq X^*_{\mathcal{N}}$, then $X^*_{\mathcal{N}} = \mathcal{N}cl(X^*_{\mathcal{N}}) = n\text{-}cl(X) = \mathcal{N}cl^*(X)$.

Definition 2.12. A subset A of a neutrosophic nano ideal topological space $(U, \tau_{\mathcal{N}}(F), I)$ is $\mathcal{N}*$-dense in itself (resp. $\mathcal{N}*$-perfect) if $X \subseteq X^*_{\mathcal{N}}$ (resp. $X = X^*_{\mathcal{N}}$).

Remark 2.13. A subset X of a neutrosophic nano ideal topological space $(U, \tau_{\mathcal{N}}(F), I)$ is $\mathcal{N}^*$-closed if and only if $X^*_{\mathcal{N}} \subseteq X$.

For the relationship related to several sets defined in this paper, we have the following implication:

$$\mathcal{N}*\text{-dense in itself} \Leftarrow \mathcal{N}*\text{-perfect} \Rightarrow \mathcal{N}^*\text{-closed}$$

The converse implication are not satisfied asthe following shows.

Example 2.14. Let U be the universe, $X = \{P_1, P_2, P_3, P_4, P_5\} \subset U$, $U/R = \{\{P_1, P_2\}, \{P_3\}, \{P_4, P_5\}\}$ and $\tau_{\mathcal{N}}(F) = \{1_\sim, 0_\sim, \overline{\mathcal{N}}, \underline{\mathcal{N}}, B\}$ and the ideal $I = 0_\sim, 1_\sim$. For $X = \{< P_1, (.5, .4, .7) >, < P_2, (.6, .4, .5) >, < P_3, (.4, .5, .4) >, < P_4, (.7, .3, .4) >, < P_5, (.8, .5, .2) >\}$, $\underline{N}(X) = \{\frac{P_1, P_2}{.5, .4, .7}, \frac{P_3}{.4, .5, .4}, \frac{P_4, P_5}{.7, .3, .4}\}$, $\overline{N}(X) = \{\frac{P_1, P_2}{.6, .4, .5}, \frac{P_3}{.4, .5, .4}, \frac{P_4, P_5}{.8, .5, .2}\}$, $B(X) = \{\frac{P_1, P_2}{.6, .4, .5}, \frac{P_3}{.4, .5, .4}, \frac{P_4, P_5}{.4, .3, .7}\}$. If $I = 0_\sim$ then $X^*_{\mathcal{N}} = Ncl(a)$. Thus $X \subseteq X^*_{\mathcal{N}}$. Hence X is $\mathcal{N}^*$-dense but not $\mathcal{N}^*$-perfect.
If $I = 1_\sim$ then $X^*_{\mathcal{N}} = 0_\sim$. Thus $X \supseteq X^*_{\mathcal{N}}$. Hence $X^*_{\mathcal{N}}$ is $\mathcal{N}^*$-closed but not $\mathcal{N}^*$-perfect.

Lemma 2.15. Let $(U, \tau_{\mathcal{N}}(F), I)$ be a NNI and $X \subseteq U$. If X is $\mathcal{N}*$-dense in itself, then $X^*_{\mathcal{N}} = \mathcal{N}cl(X^*_{\mathcal{N}}) = \mathcal{N}cl(X) = \mathcal{N}cl^*(X)$.
Proof. Let X be $\mathcal{N}*$-dense in itself. Then we have $X \subseteq X^*_{\mathcal{N}}$ and using Theorem 2.11 we get$X^*_{\mathcal{N}} = \mathcal{N}cl(X^*_{\mathcal{N}}) = \mathcal{N}cl(X) = \mathcal{N}cl^*(X)$.

Lemma 2.16. If $(U, \tau_{\mathcal{N}}(F), I)$ is a NNT with an ideal I and $X \subseteq U$, then $X^*_{\mathcal{N}}(I, \tau_{\mathcal{N}}(F)) = X^*_{\mathcal{N}}(I, \tau_{\mathcal{N}}(F)^*)$ and hence $\tau_{\mathcal{N}}(F)^* = \tau_{\mathcal{N}}(F)^{**}$.

3 $\tau_{\mathcal{N}}(F)$-codence ideal

n this section we incorporated codence ideal [5] in ideal topological space and introduce similar concept in neutrosophic nano ideal topological spaces.

Definition 3.1. An ideal I in a space $(U, \tau_{\mathcal{N}}(F), I)$ is called $\tau_{\mathcal{N}}(F)$-codense ideal if $\tau_{\mathcal{N}}(F) \cap I = \{0_\sim\}$.
Following theorems are related to $\tau_{\mathcal{N}}(F)$-codense ideal.

Theorem 3.2. Let $(U, \tau_{\mathcal{N}}(F), I)$ be an NNI and I is $\tau_{\mathcal{N}}(F)$-codense with $\tau_{\mathcal{N}}(F)$. Then $U = U^*_{\mathcal{N}}$.
Proof. It is obvious that $U^*_{\mathcal{N}} \subseteq U$. For converse, suppose $a \in U$ but $a \notin U^*_{\mathcal{N}}$. Then there exists $G_x \in \tau_{\mathcal{N}}(F)(a)$ such that $G_x \cap U \in I$. That is $G_x \in I$, a contradiction to the fact that $\tau_{\mathcal{N}}(F) \cap I = \{0_\sim\}$. Hence $U = U^*_{\mathcal{N}}$.

Theorem 3.3. Let $(U, \tau_{\mathcal{N}}(F), I)$ be a NNI. Then the following conditions are equivalent:

(i) $U = U^*_{\mathcal{N}}$.

(ii) $\tau_{\mathcal{N}}(F) \cap I = \{0_{\sim}\}$.

(iii) If $J \in I$, then $\mathcal{N}int(J) = 0_{\sim}$.

(iv) For every $X \in \tau_{\mathcal{N}}(F)$, $X \subseteq X^*_{\mathcal{N}}$.

Proof. By Lemma 2.16, we may replace '$\tau_{\mathcal{N}}(F)$' by '$\tau_{\mathcal{N}}(F)^*$' in (ii), '$\mathcal{N}int(J) = 0_{\sim}$' by '$\mathcal{N}int^*(J) = 0_{\sim}$' in (iii) and '$X \in \tau_{\mathcal{N}}(F)$' by '$X \in \tau_{\mathcal{N}}(F)^*$' in (iv).

4 Conclusions

this paper, we introduced the notion of neutrosophic nano ideal topological structures and investigated some relations over neutrosophic nano topology and neutrosophic nano ideal topological structures and studied some of its basic properties. In future, it motivates to apply this concepts in graph structures.

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Constant single valued neutrosophic graphs with applications

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Abstract.In this paper, we introduced a new concept of single valued neutrosophic graph (SVNG) known as constant single valued neutrosophic graph (CSVNG). Basically, SVNG is a generalization of intuitionistic fuzzy graph (IFG). More specifically, we described and explored somegraph theoretic ideas related to the introduced concepts of CSVNG. An application of CSVNG in a Wi-Fi network system is discussed and a comparison of CSVNG with constant IFG is established showing the worth of the proposed work. Further, several terms like constant function and totally constant function are investigated in the frame-work of CSVNG and their characteristics are studied.

Keywords. Single valued neutrosophic graph. Constant single valued neutrosophic graph; constant function; totally constant function; Wi-Fi network.

1. Introduction

Dealing with uncertain situations and insufficient information requires some high potential mathematical tools. Graph theory is one of the mathematical tools which effectively deals with large data. If there are some of uncertainty factors, then fuzzy graph is the appropriate tool to be used. In addition to its ability of handling large data, graph theory has a special interest as it can be applied in several important areas including management sciences [19], social sciences [17], computer and information sciences [41], communication networks [18], description of group structures [39], database theory [26]and economics [25].

The concept of fuzzy set (FS) proposed by Zadeh [46] is among the famous toolsdealing with uncertain situations and insufficient information. After, Kaufmann [20] introduced the notion of fuzzy graph. A comprehensive study on fuzzy graphs is done by Rosenfeld [40]in which he shown some of their basic properties. The work in the field of graph theory is exemplary during the past decades as its concepts are applied in many real-life problems such as cluster analysis [14,6,45,30], slicing [30], for solving fuzzy intersecting equations [31,29], in some theory of data base [26], in networking problems [27], in the structure of a group [43, 32], in chemistry [44], in air trafficking [35], in the control of traffic [34] etc. The worth of FG lies in its capability of handling with uncertainties and it has done so far better but Atanassov [1] proposed that FSs only deals with one sided uncertainties which is not enough as human nature isn't limited to only yes type or no type problems. Hence the logic of intuitionistic fuzzy set (IFS) have been developed sufficient to deal with uncertainties of both yes and no types. Atanassov's IFS gave rise to the theory of IFG proposed by Parvathi and Karunambigai [36]. The structure of IFG is advanced and is applied successfully social networks [13], clustering [23], radio coverage network [21] and shortest path problems [32] etc. Furthermore, Parvathi et al [36-28] did some work on constant IFGs and operations of IFGs. The concept of intuitionistic fuzzy hypergraphs (IFHGs) was proposed by Parvathi et al. [37] which were applied in real life problems by Akram and Wieslaw [3]. NagoorGani and Shajitha [15] wrote about degree, order and size for IFGin 2010. Akram and Davvaz [2] gave the concept of strong IFG.

Smarandache in 1995 develop the neutrosophic logic which give rise to a novel theory of neutrosophic set (NS) [42] which give rise to the development of single/double and triple valued NSs [16,22,24]. Broumi et al initiated the concept of single-valued neutrosophic graph (SVNG) [7]. Work on the operations of SVNG can be found in [5]. Note on the degree, order and size of SVNG is present in [8].Recently, Broumi et al[47]introduced a single-valued neutrosophic techniques for analysis of WIFI connection. The hypergraph i.e. single-valued neutrosophic hyper graph is introduced in [4]. Neutrosophic sets and graphs have ben widely studied in recent decades. Various real life applications are discussed using neutrosophc techniques. For development in neutrosophic sets and graphs and their applications, one is refer to [9-12, 48-67,68-71].

In this paper, we introduced the concept of CSVNG and investigated some graph theoretic ideas related to this introduced concept. An application of CSVNG in a Wi-Fi network system is discussed and a comparison of CSVNG with constant IFG is established in order to show the worth of the proposed concept.

The rest of the paper is organized as follows. In Section 2, we recalled the necessary basic concepts and properties of IFG, CIFG and SVNG.In section 3, the concept of CSVNG is described and some related graph theoretic ideas are explored. In Section 4, we discussed the characteristic of CSVNGs, while section 5 deals with an application of CSVNGs in Wi-Fi network system. Finally, advantages and concluding remarks are discussed.

2 Preliminaries

This section is basically about some very basic definitions. The concepts of IFG, CIFG and SVNG are discussed and explained with the help of some examples. For undefined terms and notions, we refer to [5, 8, 35, 36].

Definition 1 [36]. A Pair $\mathrm{Ǥ} = (\grave{\mathrm{V}}, \tilde{\mathrm{E}})$ is said to be IFG if

(i) $\grave{\mathrm{V}} = \{\grave{v}_1, \grave{v}_2, \grave{v}_3, \ldots \grave{v}_n\}$ are the set of vertices such that $\dot{\mathrm{T}}_1 : \grave{\mathrm{V}} \longrightarrow [0, 1]$ and $\mathrm{F}_1 : \grave{\mathrm{V}} \longrightarrow [0, 1]$ represents the degree of membership and non-membership of the element $\grave{v}_i \in \grave{\mathrm{V}}$ respectively with a condition that $0 \leq \dot{\mathrm{T}}_1(\grave{v}_i) + \mathrm{F}_1(\grave{v}_i) \leq 1$ for all $\grave{v}_i \in \grave{\mathrm{V}}$, $(i \in I)$.

(ii) $\tilde{\mathrm{E}} \subseteq \grave{\mathrm{V}} \times \grave{\mathrm{V}}$ where $\dot{\mathrm{T}}_2 : \grave{\mathrm{V}} \times \grave{\mathrm{V}} \longrightarrow [0, 1]$ and $\mathrm{F}_2 : \grave{\mathrm{V}} \times \grave{\mathrm{V}} \longrightarrow [0, 1]$ represents the degree of membership and non-membership of the element $(\grave{v}_i, \grave{v}_j) \in \tilde{\mathrm{E}}$ such that $\dot{\mathrm{T}}_2(\grave{v}_i, \grave{v}_j) \leq \min\{\dot{\mathrm{T}}_1(\grave{v}_i), \dot{\mathrm{T}}_1(\grave{v}_j)\}$ and $\mathrm{F}_2(\grave{v}_i, \grave{v}_j) \leq \max\{\mathrm{F}_1(\grave{v}_i), \mathrm{F}_1(\grave{v}_j)\}$ with a condition $0 \leq \dot{\mathrm{T}}_2(\grave{v}_i, \grave{v}_j) + \mathrm{F}_2(\grave{v}_i, \grave{v}_j) \leq 1$ for all $(\grave{v}_i, \grave{v}_j) \in \tilde{\mathrm{E}}$ $(i \in I)$.

Example 1. Let $\mathrm{Ǥ} = (\grave{\mathrm{V}}, \tilde{\mathrm{E}})$ be an IFG where $\grave{\mathrm{V}} = \{\grave{v}_1, \grave{v}_2, \grave{v}_3\}$ be the set of vertices and $\tilde{\mathrm{E}} = \{\grave{v}_1\grave{v}_2, \grave{v}_1\grave{v}_3, \grave{v}_2\grave{v}_3\}$ be the set of edges. Then

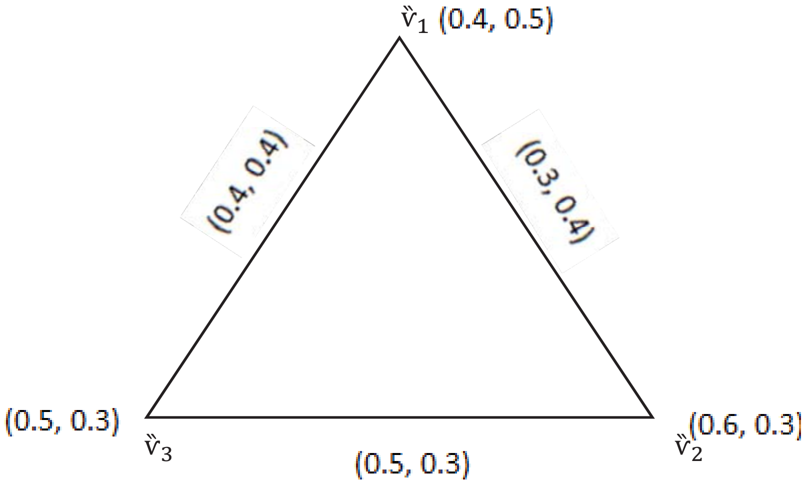

Figure 1 (IFG)

Definition 2 [28].A pair $\mathrm{Ǥ} = (\grave{\mathrm{V}}, \tilde{\mathrm{E}})$ is said to be Constant$-IFG$ of degree $(ƙ_i, ƙ_j)$ or $(ƙ_i, ƙ_j) - IFG$.If

$$ḍ_{\dot{\mathrm{T}}}(\grave{v}_i) = ƙ_i, ḍ\ (\grave{v}_j) = ƙ_j \forall \grave{v}_i, \grave{v}_j \in \grave{\mathrm{V}}.$$

Example 2. Let $\mathrm{Ǥ} = (\grave{\mathrm{V}}, \tilde{\mathrm{E}})$ be an IFG where $\grave{\mathrm{V}} = \{\grave{v}_1, \grave{v}_2, \grave{v}_3, \grave{v}_4\}$ be the set of vertices and $\tilde{\mathrm{E}} = \{\grave{v}_1\grave{v}_2, \grave{v}_2\grave{v}_3, \grave{v}_3\grave{v}_4, \grave{v}_4\grave{v}_1\}$ be the set of edges. Then

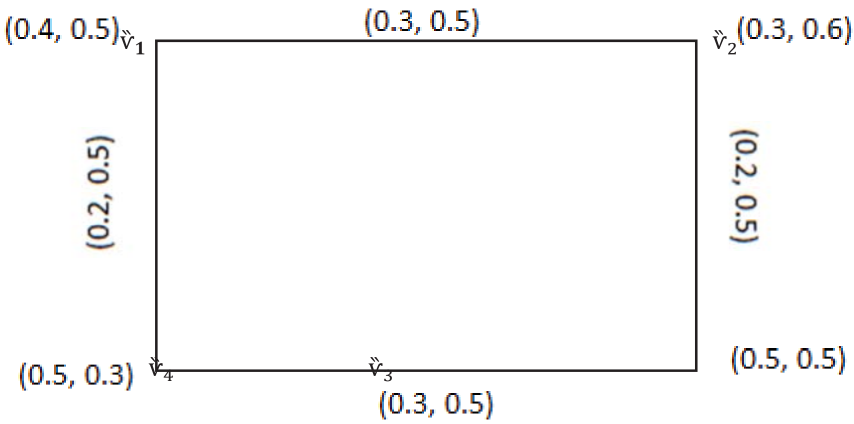

Figure 2 (Constant$-IFG$ of degree $(\Bbbk_i, \Bbbk_j)$)

The degree of $\grave{v}_1, \grave{v}_2, \grave{v}_3, \grave{v}_4$ is (0.5, 1.0).

Definition 3 [7]. A pair $G = (\grave{V}, \tilde{E})$ is said to be as $SVNG$ if

(i) $\grave{V} = \{\grave{v}_1, \grave{v}_2, \grave{v}_3, \ldots \grave{v}_n\}$ are the set of vertices such that $\dot{T}_1: \grave{V} \longrightarrow [0,1]$, $\hat{I}_1: \grave{V} \longrightarrow [0,1]$ and $F_1: \grave{V} \longrightarrow [0,1]$ denote the degree of membership, indeterminacy and non-membership of the element $\grave{v}_i \in \grave{V}$ respectively with a condition that $0 \leq T_1 + \hat{I}_1 + F_1 \leq 3$ for all $\grave{v}_i \in \grave{V}, (i \in I)$.

(ii) $\breve{E} \subseteq \grave{V} \times \grave{V}$ where $\dot{T}_2: \grave{V} \times \grave{V} \longrightarrow [0,1]$, $\hat{I}_2: \grave{V} \times \grave{V} \longrightarrow [0,1]$ and $F_2: \grave{V} \times \grave{V} \longrightarrow [0,1]$ denote the degree of membership, abstinence and non-membership of the element $(\grave{v}_i, \grave{v}_j) \in \tilde{E}$ such that $\dot{T}_2(\grave{v}_i, \grave{v}_j) \leq \min\{\dot{T}_2(\grave{v}_i), \dot{T}_2(\grave{v}_j)\}$, $\hat{I}_2(\grave{v}_i, \grave{v}_j) \geq \max\{\hat{I}_2(\grave{v}_i), \hat{I}_2(\grave{v}_j)\}$ and $F_2(\grave{v}_i, \grave{v}_j) \geq \max\{F_2(\grave{v}_i), F_2(\grave{v}_j)\}$ with a condition $0 \leq \dot{T}_2(\grave{v}_i, \grave{v}_j) + \hat{I}_2(\grave{v}_i, \grave{v}_j) \in +F_2(\grave{v}_i, \grave{v}_j) \leq 3$ for all $(\grave{v}_i, \grave{v}_j) \in \tilde{E}$, $(i \in I)$.

Example 3. Let $G = (\grave{V}, \tilde{E})$ be a SVNG where $\grave{V} = \{\grave{v}_1, \grave{v}_2, \grave{v}_3, \grave{v}_4\}$ be the set of vertices and $\tilde{E} = \{\grave{v}_1\grave{v}_2, \grave{v}_2\grave{v}_3, \grave{v}_3\grave{v}_4, \grave{v}_4\grave{v}_1\}$ be the set of edges. Then

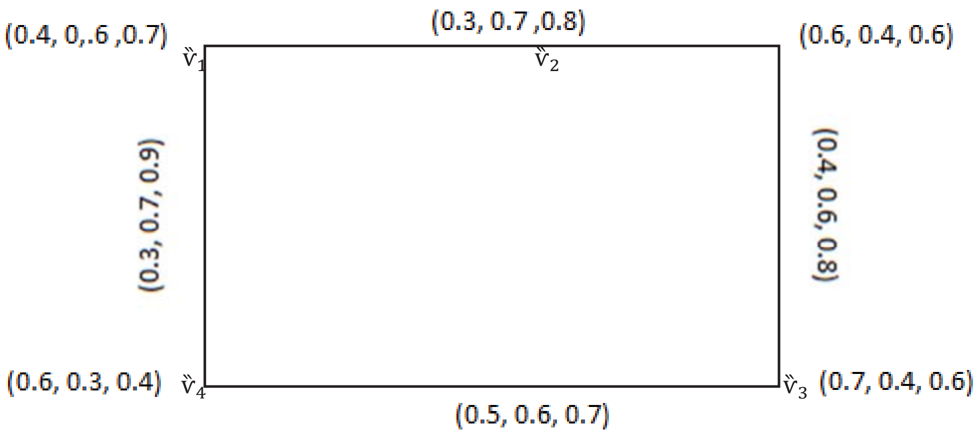

Figure 3 .SVNG

3 Constant single valued neutrosophic graph

In this section, the concept of CSVNG is introduced and supported with some examples. We discussed some related terms like completeness, total degree and constant function and exemplified them. Some results are also studied related to completeness and constant functions.

Definition 4. A pair $\text{G} = (\ddot{\text{V}}, \tilde{\text{E}})$ is said to be constant$-SVNG$ of degree $(\text{k}_i, \text{k}_j, \text{k}_k)$or $\left(\text{k}_i, \text{k}_j, \text{k}_k\right) - SVNG$.If $\text{d}_{\text{T}}(\ddot{\text{v}}_i) = \text{k}_i, \text{d}_{\text{I}}\left(\ddot{\text{v}}_j\right) = \text{k}_j$, and$\text{d}_{\text{F}}(\ddot{\text{v}}_k) = \text{k}_k \forall \ddot{\text{v}}_i, \ddot{\text{v}}_j, \ddot{\text{v}}_k \in \ddot{\text{V}}$.

Example 4. Let $\text{G} = (\ddot{\text{V}}, \tilde{\text{E}})$ be a SVNG where $\ddot{\text{V}} = \{\ddot{\text{v}}_1, \ddot{\text{v}}_2, \ddot{\text{v}}_3, \ddot{\text{v}}_4\}$ be the set of vertices and $\tilde{\text{E}} = \{\ddot{\text{v}}_1\ddot{\text{v}}_2, \ddot{\text{v}}_2\ddot{\text{v}}_3, \ddot{\text{v}}_3\ddot{\text{v}}_4, \ddot{\text{v}}_4\ddot{\text{v}}_1\}$ be the set of edges. Then CSVNG is shown in the below figure 4.

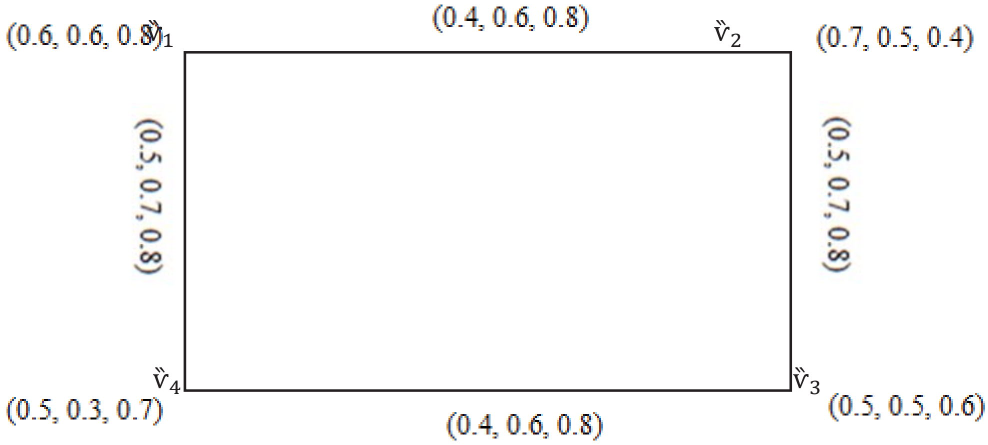

Figure 4 (Constant-SVNG of degree (k_i, k_j, k_k))

The degree of $\ddot{v}_1, \ddot{v}_2, \ddot{v}_3, \ddot{v}_4$ is $(0.9, 1.3, 1.6)$.

Remark 1. A complete $SVNG$ may not be a constant-$SVNG$.

Example 5. Consider a graph $G = (\ddot{V}, \tilde{E})$ where $\ddot{V} = \{\ddot{v}_1, \ddot{v}_2, \ddot{v}_3, \ddot{v}_4\}$ be the set of vertices and $\tilde{E} = \left\{\ddot{v}_1\ddot{v}_2, \ddot{v}_2\ddot{v}_3, \ddot{v}_2\ddot{v}_4, \ddot{v}_1\ddot{v}_3, \ddot{v}_3\ddot{v}_4, \ddot{v}_4\ddot{v}_1\right\}$ be the set of edges. Then

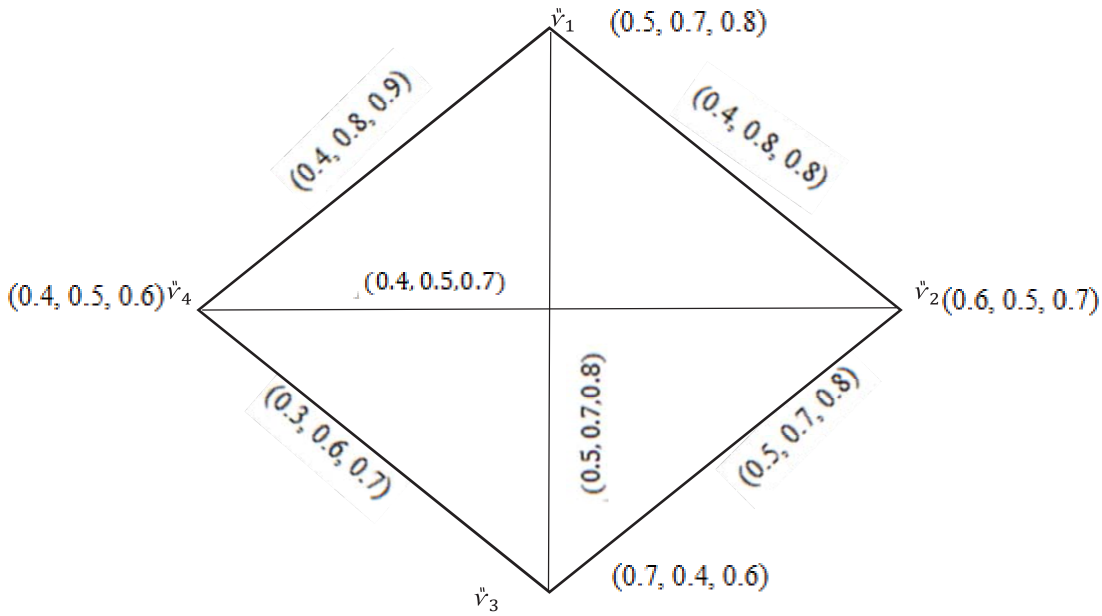

Figure 5 (G is complete but not Constant-$SVNG$)

Definition 5. The total degree of a vertex $\ddot{v}$ in a $SVNG$ is defined as

$$t\mathrm{d}(\ddot{v}) = \left[\sum_{\ddot{v}\in\tilde{E}} \mathrm{d}_{\dot{T}_2}(\ddot{v}) + \dot{T}_1(\ddot{v}), \sum_{\ddot{v}\in\tilde{E}} \mathrm{d}_{\hat{I}_2}(\ddot{v}) + \hat{I}_1(\tilde{v}), \sum_{\ddot{v}\in\tilde{E}} \mathrm{d}_{F_2}(\ddot{v}) + F_1(\ddot{v})\right]$$

If every vertex has the same total degree, then it is called $SVNG$ of total degree or totally constant $SVNG$.

Example 6. Consider a graph $G = (\ddot{V}, \tilde{E})$ where $\ddot{V} = \{\ddot{v}_1, \ddot{v}_2, \ddot{v}_3, \ddot{v}_4\}$ be the set of vertices and $\tilde{E} = \{\ddot{v}_1\ddot{v}_2, \ddot{v}_2\ddot{v}_3, \ddot{v}_3\ddot{v}_4, \ddot{v}_4\ddot{v}_1\}$ be the set of edges. Then

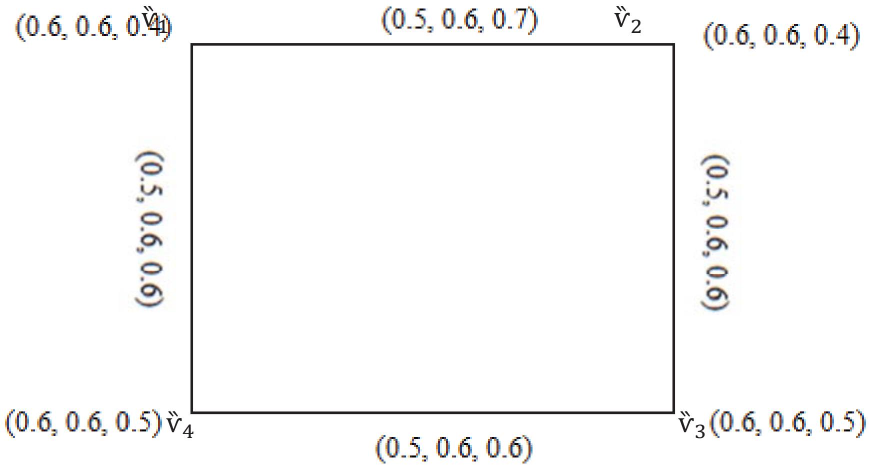

Figure 6 (SVNG)

Constant SVNG of total degree $(1.6, 1.8, 1.7)$.

Theorem 1. If G be a SVNG. Then $(\dot{T}_1, \hat{I}_1, F_1)$ is a constant function iff the following are equivalent.

(i) G is a constant SVNG.

(ii) G is a totally constant SVNG.

Proof Let $(\dot{T}_1, \hat{I}_1, F_1)$ be a constant function and $\dot{T}_1(\ddot{v}) = \dot{c}_1$, $\hat{I}_1(\ddot{v}) = \dot{c}_2$, and $F_1(\ddot{v}) = \dot{c}_3$ for all $\ddot{v}_i \in \ddot{V}$. Where $\dot{c}_1, \dot{c}_2$ and $\dot{c}_3$ are constants. Suppose that G is a $(k_i, k_j, k_k) - CSVNG$. Then $\mathrm{d}_{\dot{T}}(\ddot{v}_i) = k_1$, $\mathrm{d}_{\hat{I}}(\ddot{v}_i) = k_2$ and $\mathrm{d}_F(\ddot{v}_i) = k_3$ for all $\ddot{v}_i \in \ddot{V}$. Therefore, $t\mathrm{d}_{\dot{T}}(\ddot{v}_i) = \mathrm{d}_{\dot{T}}(\ddot{v}_i) + \dot{T}_1(\ddot{v}_i)$, $t\mathrm{d}_{\hat{I}}(\ddot{v}_i) = \mathrm{d}_{\hat{I}}(\ddot{v}_i) + \hat{I}_1(\ddot{v}_i)$ and $t\mathrm{d}_F(\ddot{v}_i) = \mathrm{d}_F(\ddot{v}_i) + F_1(\ddot{v}_i)$ for all $\ddot{v}_i \in \ddot{V}$, $t\mathrm{d}_{\dot{T}}(\ddot{v}_i) = k_1 + \dot{c}_1$, $t\mathrm{d}_{\hat{I}}(\ddot{v}_i) = k_2 + \dot{c}_2$ and $t\mathrm{d}_F(\ddot{v}_i) = k_3 + \dot{c}_3$ for all $\ddot{v}_i \in \ddot{V}$. Hence G is a totally constant SVNG.

Now, Assume that G is a $(\dot{T}_1, \hat{I}_1, F_1)$-totally constant SVNG. Then $t\mathrm{d}_{\dot{T}}(\ddot{v}_i) = r_1$, $t\mathrm{d}_{\hat{I}}(\ddot{v}_i) = r_2$ and $t\mathrm{d}_F(\ddot{v}_i) = r_3$ for all $\ddot{v}_i \in \ddot{V}$. $\mathrm{d}_{\dot{T}}(\ddot{v}_i) + \dot{T}_1(\ddot{v}_i) = r_1$, $\mathrm{d}_{\dot{T}}(\ddot{v}_i) + \dot{c}_1 = r_1$, $\mathrm{d}_{\dot{T}}(\ddot{v}_i) = r_1 - \dot{c}_1$, similarly $\mathrm{d}_{\hat{I}}(\ddot{v}_i) + \hat{I}_1(\ddot{v}_i) = r_2$, $\mathrm{d}_{\hat{I}}(\ddot{v}_i) = r_2 - \dot{c}_2$ and $\mathrm{d}_F(\ddot{v}_i) + F_1(\ddot{v}_i) = r_3$, $\mathrm{d}_F(\ddot{v}_i) = r_3 - \dot{c}_3$. Therefore, G is a constant SVNG. Hence (i) and (ii) are equivalent.

Conversely, assume that (i) and (ii) are equivalent That is G is a constant SVNG iff G is a totally constant SVNG. Assume $(\dot{T}_1, \hat{I}_1, F_1)$ is not a constant function. Then $\dot{T}_1(\ddot{v}_1) \neq \dot{T}_1(\ddot{v}_2)$, $\hat{I}_1(\ddot{v}_1) \neq \hat{I}_1(\ddot{v}_2)$ and $F_1(\ddot{v}_1) \neq F_1(\ddot{v}_2)$ for at least one pairof vertices $\ddot{v}_1, \ddot{v}_2 \in \ddot{V}$. Consider G be a $(k_i, k_j, k_k) - SVNG$. Then, $\dot{T}_1(\ddot{v}_1) = \dot{T}_1(\ddot{v}_2) = k_1$, $\hat{I}_1(\ddot{v}_1) = \hat{I}_1(\ddot{v}_2) = k_2$ and $F_1(\ddot{v}_1) = F_1(\ddot{v}_2) = k_3$. So, $t\mathrm{d}_{\dot{T}}(\ddot{v}_1) = \mathrm{d}_{\dot{T}}(\ddot{v}_1) + \dot{T}_1(\ddot{v}_1) = k_1 + \dot{T}_1(\ddot{v}_1)$, and $t\mathrm{d}_{\dot{T}}(\ddot{v}_2) = k_1 + \dot{T}_1(\ddot{v}_2)$. Similarly , $t\mathrm{d}_{\hat{I}}(\ddot{v}_1) = k_2 + \hat{I}_1(\ddot{v}_1)$, $t\mathrm{d}_{\hat{I}}(\ddot{v}_2) = k_2 + \hat{I}_1(\ddot{v}_2)$ and $t\mathrm{d}_F(\ddot{v}_1) = k_2 + F_1(\ddot{v}_1)$, $t\mathrm{d}_F(\ddot{v}_2) = k_2 + F_1(\ddot{v}_2)$ .Since $\dot{T}_1(\ddot{v}_1) \neq \dot{T}_1(\ddot{v}_2)$, $\hat{I}_1(\ddot{v}_1) \neq \hat{I}_1(\ddot{v}_2)$ and $F_1(\ddot{v}_1) \neq F_1(\ddot{v}_2)$. We have $t\mathrm{d}_{\dot{T}}(\ddot{v}_1) \neq t\mathrm{d}_{\dot{T}}(\ddot{v}_2)$, $t\mathrm{d}_{\hat{I}}(\ddot{v}_1) \neq t\mathrm{d}_{\hat{I}}(\ddot{v}_2)$ and $t\mathrm{d}_F(\ddot{v}_1) \neq t\mathrm{d}_F(\ddot{v}_2)$. Therefore, G is not totally constant SVNG which is contradiction to our supposition.

Now, consider G be a totally constant SVNG. Then, $t\mathrm{d}_{\dot{T}}(\ddot{v}_1) = t\mathrm{d}_{\dot{T}}(\ddot{v}_2)$, $\mathrm{d}_{\dot{T}}(\ddot{v}_1) + \dot{T}(\ddot{v}_1) = \mathrm{d}_{\dot{T}}(\ddot{v}_2) + \dot{T}(\ddot{v}_2)$, $\mathrm{d}_{\dot{T}}(\ddot{v}_1) - \mathrm{d}_{\dot{T}}(\ddot{v}_2) = \dot{T}(\ddot{v}_2) - \dot{T}(\ddot{v}_1)$ $(i.e. \neq 0)$ $\mathrm{d}_{\dot{T}}(\ddot{v}_1) \neq \mathrm{d}_{\dot{T}}(\ddot{v}_2)$. Similarly $\mathrm{d}_{\hat{I}}(\ddot{v}_1) \neq \mathrm{d}_{\hat{I}}(\ddot{v}_2)$ and $\mathrm{d}_F(\ddot{v}_1) \neq \mathrm{d}_F(\ddot{v}_2)$. , G is not constant which is contradiction to our assumption. Hence $(\dot{T}_1, \hat{I}_1, F_1)$ is constant function.

Example 7. Consider a graph $G = (\ddot{V}, \tilde{E})$ where $\ddot{V} = \{\ddot{v}_1, \ddot{v}_2, \ddot{v}_3, \ddot{v}_4\}$ be the set of vertices and $\tilde{E} = \{\ddot{v}_1\ddot{v}_2, \ddot{v}_2\ddot{v}_3, \ddot{v}_3\ddot{v}_4, \ddot{v}_4\ddot{v}_1\}$ be the set of edges. Then

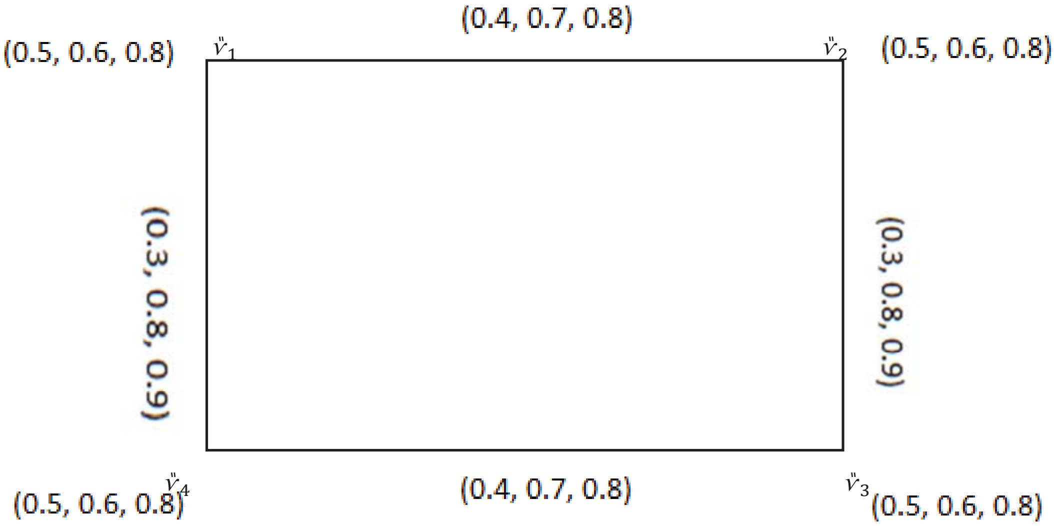

Figure 7. SVNG

$(\dot{T}_1, \hat{I}_1, F_1)$is a constant function, then G is constant and totally constant.

Theorem 2. Let G is constant and totally constant then $(\dot{T}_1, \hat{I}_1, F_1)$ is a constant function.

Proof. Assume that G be a (k_i, k_j, k_k) −constant and (r_1, r_2, r_3) −totally constant SVNG. Therefore, $d_{\dot{T}}(\ddot{v}_1) = k_1, d_{\hat{I}}(\ddot{v}_1) = k_2$and $d_F(\ddot{v}_1) = k_3$ for $\ddot{v}_1 \in \ddot{V}$ and $td_{\dot{T}}(\ddot{v}_1) = r_1, td_{\hat{I}}(\ddot{v}_1) = r_2$and $td_F(\ddot{v}_1) = r_3$ for all $\ddot{v} \in \ddot{V}$. $\dot{T}_1(\ddot{v}) + k_1 = r_1$for all $\ddot{v} \in \ddot{V}$. $\dot{T}_1(\ddot{v}) = r_1 - k_1$, for all $\ddot{v} \in \ddot{V}$. Hence $\dot{T}_1(\ddot{v}_1)$ is a constant function. Similarly $\hat{I}_1(\ddot{v}) = r_2 - k_2$ and $F_1(\ddot{v}) = r_3 - k_3$ for all $\ddot{v} \in \ddot{V}$.

Remark 2. Converse of the above theorem 2 is not true.

Example 8. Consider a graph $G = (\ddot{V}, \tilde{E})$ where $\ddot{V} = \{\ddot{v}_1, \ddot{v}_2, \ddot{v}_3, \ddot{v}_4\}$ be the set of vertices and $\tilde{E} = \{\ddot{v}_1\ddot{v}_2, \ddot{v}_2\ddot{v}_3, \ddot{v}_3\ddot{v}_4, \ddot{v}_4\ddot{v}_1\}$ be the set of edges. Then

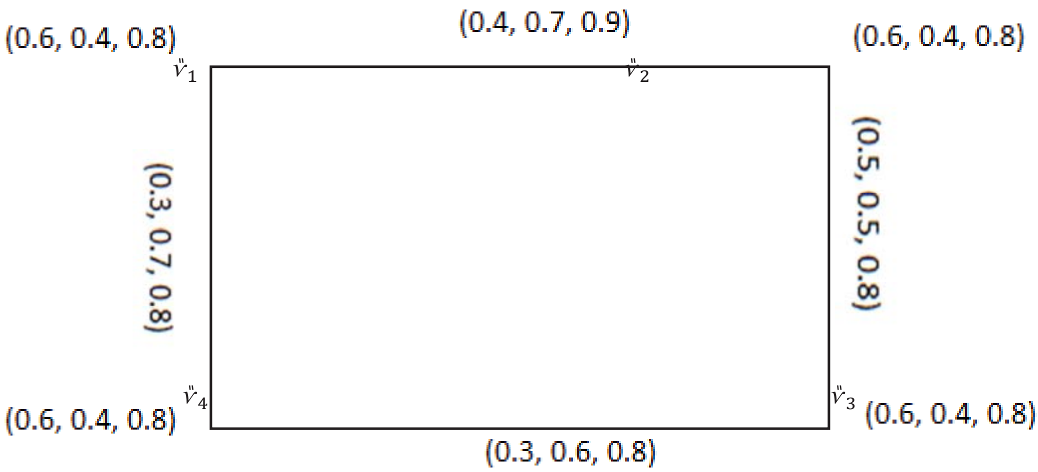

Figure 8. SVNG

$(\dot{T}_1, \hat{I}_1, F_1)$is a constant function But neither constant SVNG nor totally constant SVNG.

4 Characterization of constant SVNG on a cycle

This section is based on some important results on even (odd) cycles, bridges in SVNGs and cut vertex of even (odd) cycle. The stated results are supported with some examples. .

Theorem 3. If Ǥ is an SVNG where crisp graph Ǥ is an odd cycle. Then Ǥ is constant SVNG iff $(\dot{T}_2, \hat{I}_2, F_2)$ is a constant function.

Proof. Suppose $(\dot{T}_2, \hat{I}_2, F_2)$ is constant function $\dot{T}_2 = \dot{c}_1$, $\hat{I}_2 = \dot{c}_2$, and $F_2 = \dot{c}_3$ for all $(\ddot{v}_i, \ddot{v}_j) \in \tilde{E}$. Then $\text{ɗ}_{\dot{T}}(\ddot{v}_i) = 2\dot{c}_1$, $\text{ɗ}_{\hat{I}}(\ddot{v}_i) = 2\dot{c}_2$ and $\text{ɗ}_F(\ddot{v}_i) = 2\dot{c}_3$ for all $\ddot{v}_2 \in \ddot{V}$SoǤ is constant SVNG.

Conversely, assume that Ǥ is $(\text{ƙ}_1, \text{ƙ}_2, \text{ƙ}_3)$ −regular SVNG. If $\text{ɇ}_1, \text{ɇ}_2, \text{ɇ}_3 \ldots \text{ɇ}_{2n+1}$ be the edges of Ǥ in that order. If $\dot{T}_2(\text{ɇ}_1) = \dot{c}_1$, $\dot{T}_2(\text{ɇ}_2) = \text{ƙ}_1 - \dot{c}_1$, $\dot{T}_2(\text{ɇ}_3) = \text{ƙ}_1 - (\text{ƙ}_1 - \dot{c}_1) = \dot{c}_1$, $\dot{T}_2(\text{ɇ}_4) = \text{ƙ}_1 - \dot{c}_1$ and so on. Likewise, $\hat{I}_2(\text{ɇ}_1) = \dot{c}_2$, $\hat{I}_2(\text{ɇ}_2) = \text{ƙ}_2 - \dot{c}_2$, $\hat{I}_2(\text{ɇ}_3) = \text{ƙ}_2 - (\text{ƙ}_2 - \dot{c}_2) = \dot{c}_2$, $\hat{I}_2(\text{ɇ}_4) = \text{ƙ}_2 - \dot{c}_2$ and $F_2(\text{ɇ}_1) = \dot{c}_3$, $F_2(\text{ɇ}_2) = \text{ƙ}_3 - \dot{c}_3$, $F_2(\text{ɇ}_3) = \text{ƙ}_3 - (\text{ƙ}_3 - \dot{c}_3) = \dot{c}_3$, $F_2(\text{ɇ}_4) = \text{ƙ}_3 - \dot{c}_3$ and so on. Therefore

$$\dot{T}_2(\text{ɇ}_i) = \begin{Bmatrix} \dot{c}_1, if\ i\ is\ odd \\ \text{ƙ}_1 - \dot{c}_1, if\ i\ is\ even \end{Bmatrix}$$

Hence $\dot{T}_2(\text{ɇ}_1) = \dot{T}(\text{ɇ}_{2n+1}) = \dot{c}_1$. So, if ɇ_1 and ɇ_{2n+1} incident at a vertex $\ddot{v}_1$, then $\text{ɗ}_{\dot{T}}(\ddot{v}_1) = \text{ƙ}_1, \text{ɗ}(\text{ɇ}_1) + \text{ɗ}(\text{ɇ}_{2n+1}) = \text{ƙ}_1$, $\dot{c}_1 + \dot{c}_1 = \text{ƙ}_1$, $2\dot{c}_1 = \text{ƙ}_1$, $\dot{c}_1 = \frac{\text{ƙ}_1}{2}$.

Remark 3. The above theorem (3) is not true for totally constant SVNG.

Example 8. Consider a graph $Ǥ = (\ddot{V}, \tilde{E})$ where $\ddot{V} = \{\ddot{v}_1, \ddot{v}_2, \ddot{v}_3\}$ be the set of vertices and $\tilde{E} = \{\ddot{v}_1\ddot{v}_2, \ddot{v}_2\ddot{v}_3, \ddot{v}_3\ddot{v}_1\}$ be the set of edges. Then

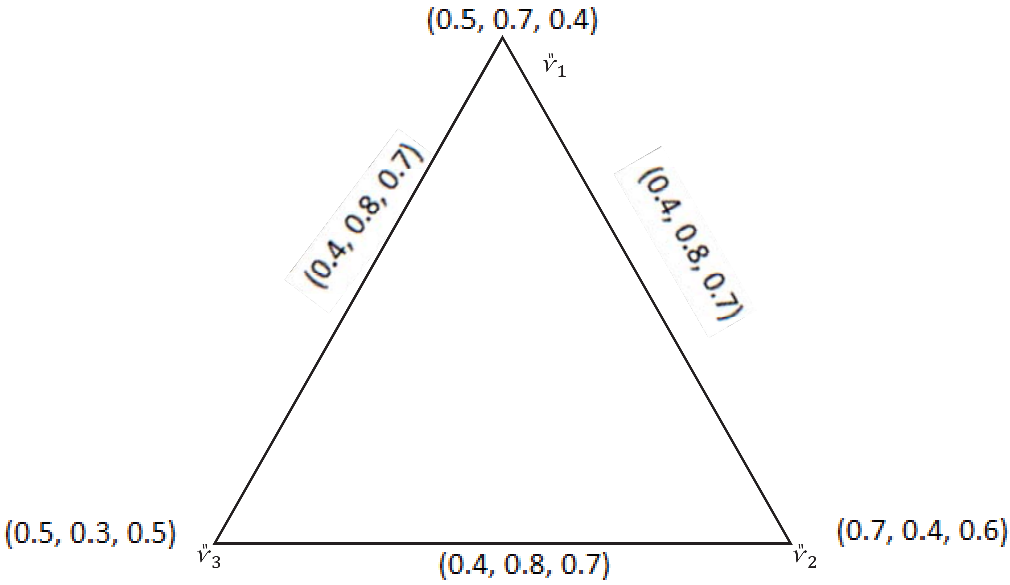

Figure 9. SVNG

$(\dot{T}_2, \hat{I}_2, F_2)$is constant function but not totally constant.

Theorem 4. If Ǥ is an SVNG where crisp graph Ǥ is an even cycle. Then Ǥ is constant SVNG iff either $(\dot{T}_2, \hat{I}_2, F_2)$ is a constant function or alternative edges have same membership, indeterminacy and non-membership values.

Proof. If $(\dot{T}_2, \hat{I}_2, F_2)$ is a constant function then Ǥ is constant SVNG. Conversely, assume that Ǥ is $(\text{ƙ}_1, \text{ƙ}_2, \text{ƙ}_3)$ −constant SVNG. If $\text{ɇ}_1, \text{ɇ}_2, \text{ɇ}_3 \ldots \text{ɇ}_{2n}$ be the edges of even cycle Ǥ in that order. By using the above theorem (3), $\dot{T}_2(\text{ɇ}_i) = \begin{Bmatrix} \dot{c}_1, if\ i\ is\ odd \\ \text{ƙ}_1 - \dot{c}_1, if\ i\ is\ even \end{Bmatrix}$, $\hat{I}_2(\text{ɇ}_i) = \begin{Bmatrix} \dot{c}_2, if\ i\ is\ odd \\ \text{ƙ}_2 - \dot{c}_2, if\ i\ is\ even \end{Bmatrix}$

And

$F_2(\text{ɇ}_i) = \begin{Bmatrix} \dot{c}_3, if\ i\ is\ odd \\ \text{ƙ}_3 - \dot{c}_3, if\ i\ is\ even \end{Bmatrix}$. If $\dot{c}_1 = \text{ƙ}_1 - \dot{c}_1$, the $(\dot{T}_2, \hat{I}_2, F_2)$ is constant function. If $\dot{c}_1 \neq \text{ƙ}_1 - \dot{c}_1$ then alternative edges have same membership, indeterminacy and non-membership values.

Remark 4. The above theorem (4) is not true for totally constant SVNG.

Example 9. Consider a graph $G = (\ddot{V}, \tilde{E})$ where $\ddot{V} = \{\ddot{v}_1, \ddot{v}_2, \ddot{v}_3, \ddot{v}_4\}$ be the set of vertices and $\tilde{E} = \{\ddot{v}_1\ddot{v}_2, \ddot{v}_2\ddot{v}_3, \ddot{v}_3\ddot{v}_4, \ddot{v}_4\ddot{v}_1\}$ be the set of edges. Then

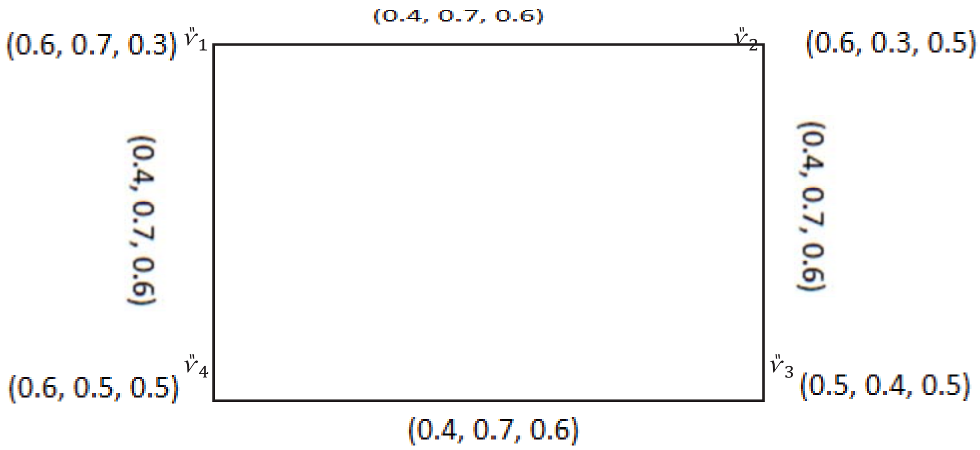

Figure 10.SVNG

$(\dot{T}_2, \hat{I}_2, F_2)$is constant function, then Ǥ is constant SVNG. But not totally constant SVNG.

Theorem 5. If Ǥ is constant SVNG is an odd cycle does not have SVN bridge. Hence it does not have SVN cut-vertex.

Proof. Suppose Ǥ is constant SVNG is an odd cycle of its crisp graph. Then $(\dot{T}_2, \hat{I}_2, F_2)$ is constant function. Therefore removal any edge does not reduce the strength of connectedness between any pair of vertex. Therefore Ǥ has no SVN edge and Hence there is no SVN cut vertex.

Remark 5. For totally constant the above theorem (5) is not true.

Example 10. Consider a graph $G = (\ddot{V}, \tilde{E})$ where $\ddot{V} = \{\ddot{v}_1, \ddot{v}_2, \ddot{v}_3\}$ be the set of vertices and $\tilde{E} = \{\ddot{v}_1\ddot{v}_2, \ddot{v}_2\ddot{v}_3, \ddot{v}_3\ddot{v}_1\}$ be the set of edges. Then

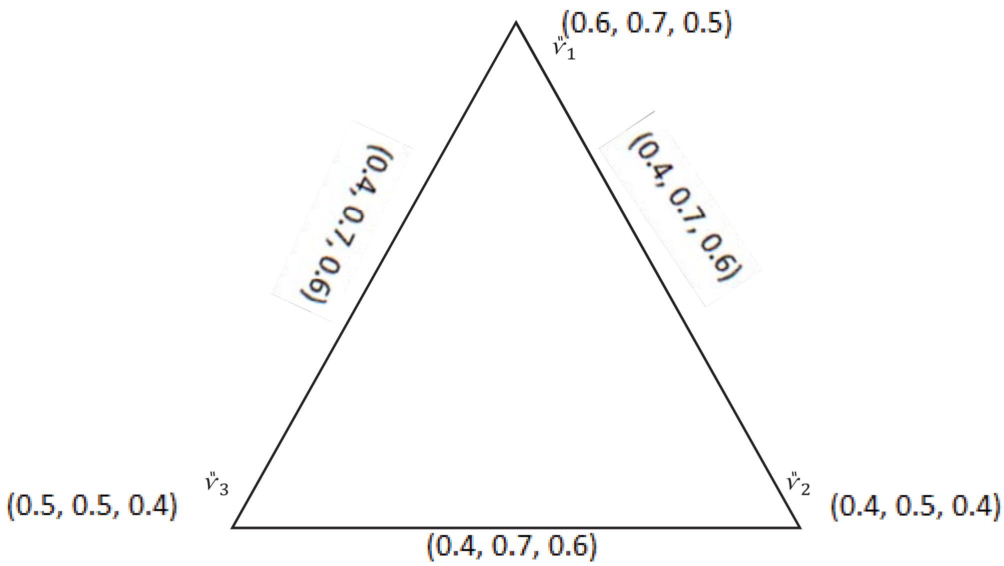

Figure 11 .SVNG

$(\dot{T}_2, \hat{I}_2, F_2)$is constant function, but neither SVN bridge nor SVN cut vertex.

Theorem 6. If G is constant SVNG is an even cycle of its crisp graph. Then either G does not have SVN bridge also it does not have SVN cut vertex.

Proof. Straightforward.

Remark 6. For totally constant the above theorem (6) is not true.

Example 11. Consider a graph $G = (\ddot{V}, \tilde{E})$ where $\ddot{V} = \{\ddot{v}_1, \ddot{v}_2, \ddot{v}_3, \ddot{v}_4\}$ be the set of vertices and $\tilde{E} = \{\ddot{v}_1\ddot{v}_2, \ddot{v}_2\ddot{v}_3, \ddot{v}_3\ddot{v}_4, \ddot{v}_4\ddot{v}_1\}$ be the set of edges. Then

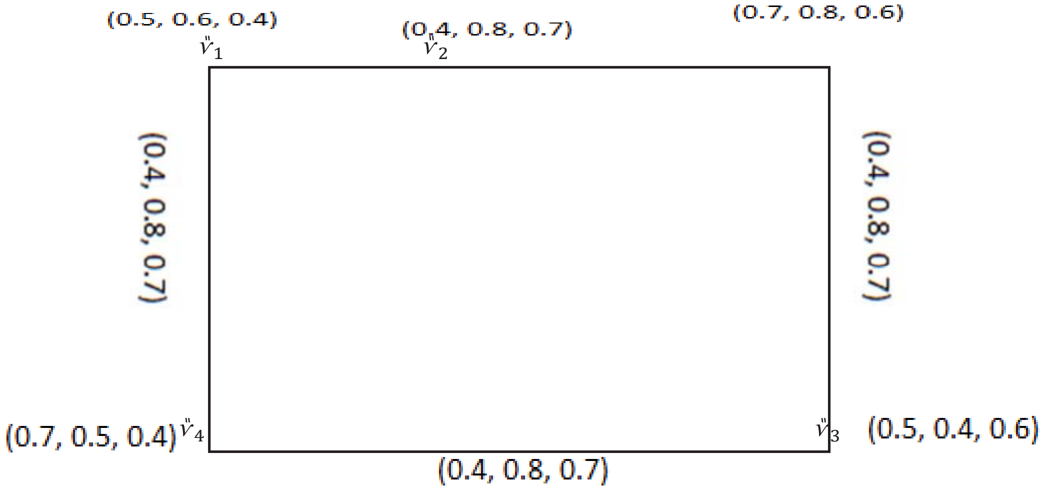

Figure 12.SVNG

$(\dot{T}_2, \hat{I}_2, F_2)$is constant function, but neither SVN bridge nor SVN cut vertex.

5 Application

In this section, we applied the concept of CSVNG to model a Wi-Fi system. It is discussed how the concept of CSVNGs is useful in modelling such network.

The Wi-Fi technology that is connected to the internet can be employed to deliver access to devices which are within the range of a wireless networks. The coverage extension can be as small area as few rooms to large as many square kilometres among two or more interconnected access points. The dependency of Wi-Fi range is on frequency band, radio power production and modulation techniques. Paralleled to traditional wired network security which is wired networking, simplified access is basic problem with wireless network security, it is essential that one either gain access to building (connecting/ relating into interior web tangibly), or a break through an exterior firewall. To facilitate Wi-Fi, one essentially require to be within the range of Wi-Fi linkage. The solid Wi-Fi hotspot device is the internal coin Wi-Fi which is designed to aid all internal setting owners. Make available 100 meters Wi-Fi signal range to outdoor and 30 meters to indoor. With the help of CSVNG this type of Wi-Fi linkage is deliberated and demonstrated.

The CSVNG is useful to a Wi-Fi network. The purpose for doing this is that there are three values in aCSVNG. The first one signifies connectivity, the second one defined the technical error of the device such as device is in range but changes between the connected and disconnected state and the third value indicates the disconnectivity.The notion of IFG only permits us to model two states such as connected and disconnected, a Wi-Fi system cannot be demonstrated using this confined structure of IFG. Though the CSVNG deliberate more than these two similarities.

An outdoor Wi-Fi co-ordination, comprises four vertices which characterise the Wi-Fi devices in such a way that there is a block between each two routers and collectively both routers have been giving signals to the block, given away in figure (13). The devices can provide signal to each block with the help of CSVNG persistently.

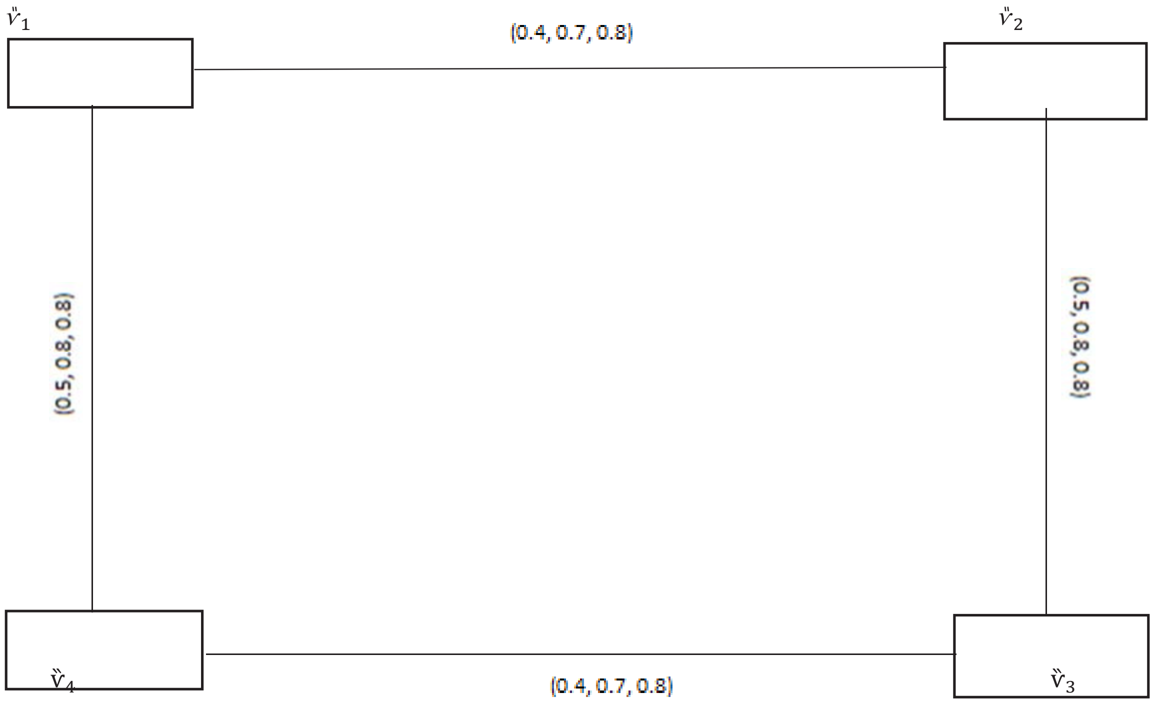

Figure 13.SVNG.

In figure 13, the four apexes denotes four different routers. The edge displays the signal strength of routers between each two routers. Each edge and apex take the single valued neutrosophic number form where the first value denotes the connectivity, the second one defined the technical error of the device, changes between the connected and disconnected state while the device is in range but, and the third value displays the disconnectivity. By using definition 4, the degree of every vertex is deliberated. In this situation which characterises that all router has been giving the same signal, so the degree of all routers is same. This also indicates that each router providing the same signal to the block. As a consequence, the concept of CSVNG displaying its importance, has been exercised to practical operations effectively.

Table 1 shows the degree of each vertex of figure 13.

vertex	Degree
$\ddot{v}_1$	(0.9, 1.5, 1.6)
$\ddot{v}_2$	(0.9, 1.5, 1.6)
$\ddot{v}_3$	(0.9, 1.5, 1.6)
$\ddot{v}_4$	(0.9, 1.5, 1.6)

Table 1 .vertex and its degree

Advantages:

The advantages of SVNGs over prevailing concepts of IFGs is due to the enhanced structure of SVNGs which allows us to deal with of more than two types ambiguous condition as it is done in the present situation of Wi-Fi system. While the IFG allow only to deal with two states connected and disconnected which means that IFGs cannot be employed to model the Wi-Fi system.

Conclusion:

The conception of CSVNG has been developed in this paper. With the help of examples, basic graph theoretic ideas such as degree of CSVNG, constant functions, totally CSVNG and characterization of CSVNG on a cycle are proved. That notion of CSVNG have been applied to a real-world problem of Wi-Fi system and the consequences are deliberated. A comparison of CSVNG with CIFG have showed the worth of CSVNGs. Further, in the proposed frame work, implementations in the field of engineering and computer sciences can be considered in near future.

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Neutrosophic $b^*g\alpha$-Closed Sets

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Abstract: This article introduces the concept of neutrosophic $b^*g\alpha$-closed sets, neutrosophic $b^*g\alpha$-border of a set, neutrosophic $b^*g\alpha$-frontier of a set in neutrosophic topological spaces and the properties of these sets are discussed. The connection between neutrosophic $b^*g\alpha$-border of a set and neutrosophic $b^*g\alpha$-frontier of a set in neutrosophic topological spaces are established.

Keywords: Neutrosophic $g\alpha$-closed sets, Neutrosophic $^*g\alpha$-closed sets, Neutrosophic $b^*g\alpha$-closed sets, Neutrosophic $b^*g\alpha$-border, Neutrosophic $b^*g\alpha$-frontier.

1 Introduction

Neutrosophic set initially proposed by Smarandache[8, 9] which is a generalization of Atanassov's[11] intuitionistic fuzzy sets and Zadeh's[12] fuzzy sets. Also it considers truth-membership function, indeterminacy-membership function and falsity-membership function. Since fuzzy sets and intuitionistic fuzzy sets fails to deal with indeterminacy-membership functions, Smarandache introduced the neutrosophic concept in various fields, including probability, algebra, control theory, topology, etc. Later Alblowi et al.,[20] introduced neutrosophic set based concepts in the neutrosophic field. These effective concepts has been applied by many researchers in the last two decades to propose many concepts in topology. Salama and Alblowi[3] proposed a new concept in neutrosophic topological spaces and it provides a brief idea about neutrosophic topology, which is a generalization of Coker's[6] intuitionistic fuzzy topology and Chang's[5] fuzzy topology.

Salama et al.,[4, 1, 2] introduced the generalization of neutrosophic sets, neutrosophic crisp sets and the neutrosophic closed sets in the field of neutrosophic topological spaces. Some neutrosophic continuous functions were introduced by Salama et al.,[2] as an initial continuous functions in neutrosophic topology. Further several researchers have defined some closed sets in neutrosophic topology, namely neutrosophic α-closed sets[10], neutrosophic αg-closed sets[7], neutrosophic b-closed sets[15], neutrosophic ω-closed sets[19], generalized neutrosophic closed sets[18] and neutrosophic $\alpha\psi$-closed sets[13] in neutrosophic topological spaces. Recently Iswarya and Bageerathi[16] proposed a new concept of neutrosophic frontier operator and neutrosophic semi-frontier operator in neutrosophic topological spaces, which provides the relationship between the operators of neutrosophic interior and neutrosophic closure. Vigneshwaran and Saranya[14] defined a new closed set as $b^*g\alpha$-closed sets in topological spaces, and it has been applied to define some topological functions as continuous functions, irresolute functions and homeomorphic functions with some separable axioms.

In this article, the notion of neutrosophic $b^*g\alpha$-closed sets in neutrosophic topological spaces are introduced and investigated their properties and the relation with other existing properties. The concept of neutrosophic $b^*g\alpha$-interior, neutrosophic $b^*g\alpha$-closure, neutrosophic $b^*g\alpha$-border and neutrosophic $b^*g\alpha$-frontier are introduced and discussed their properties. The connection between neutrosophic $b^*g\alpha$-border and neutrosophic $b^*g\alpha$-frontier in neutrosophic topological spaces are established with their related properties.

2 Preliminaries

In this section, we recall some of basic definitions which was already defined by various authors.

Definition 2.1. [3] Let X be a non empty fixed set. A neutrosophic set E is an object having the form $E = \{< x, mv(E(x)), iv(E(x)), nmv(E(x)) > \ \forall\ x \in X\}$, where $mv(E(x))$ represents the degree of membership, $iv(E(x))$ represents the degree of indeterminacy and $nmv(E(x))$ represents the degree of non-membership functions of each element $x \in X$ to the set E.

Remark 2.2. [3] A neutrosophic set $E = \{< x, mv(E(x)), iv(E(x)), nmv(E(x)) > \ \forall\ x \in X\}$ can be identified to an ordered triple $< mv(E), iv(E), nmv(E) >$ in $]^-0, 1^+[$ on X.

Definition 2.3. [3] Let E and F be two neutrosophic sets of the form, $E = \{< x, mv(E(x)), iv(E(x)), nmv(E(x)) > \ \forall\ x \in X\}$ and $F = \{< x, mv(F(x)), iv(F(x)), nmv(F(x)) > \ \forall\ x \in X\}$. Then,

i) $E \subseteq F$ if and only if $mv(E(x)) \leq mv(F(x))$, $iv(E(x)) \leq iv(F(x))$ and $nmv(E(x)) \geq nmv(F(x))$ $\forall\ x \in X$,

ii) $E = F$ if and only if $E \subseteq F$ and $F \subseteq E$,

iii) $\overline{E} = \{< x, nmv(E(x)), 1 - iv(E(x)), mv(E(x)) > \ \forall\ x \in X\}$,

iv) $E \cup F = \{x, max[mv(E(x)), mv(F(x))], min[iv(E(x)), iv(F(x))], min[nmv(E(x)), nmv(F(x))]$ $\forall\ x \in X\}$,

v) $E \cap F = \{x, min[mv(E(x)), mv(F(x))], max[iv(E(x)), iv(F(x))], max[nmv(E(x)), nmv(F(x))]$ $\forall\ x \in X\}$.

Definition 2.4. [3] A neutrosophic topology on a non empty set X is a family τ of neutrosophic subsets in X satisfying the following axioms:

i) $0_N, 1_N \in \tau$,

ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

iii) $\cup G_i \in \tau\ \forall\ \{G_i : i \in J\} \subseteq \tau$.

Then the pair (X, τ) or simply X is called a neutrosophic topological space.

Definition 2.5. [10] A neutrosophic set E in a neutrosophic topological space (X, τ) is called

i) a neutrosophic semiopen set (briefly NSOS) if $E \subseteq Ncl(Nint(E))$.

ii) a neutrosophic α-open set (briefly NαOS) if $E \subseteq Nint(Ncl(Nint(E)))$.

iii) a neutrosophic preopen set (briefly NPOS) if $E \subseteq Nint(Ncl(E))$.

iv) a neutrosophic regular open set (briefly NROS) if $E = Nint(Ncl(E))$.

v) a neutrosophic semipreopen or β-open set (briefly NβOS) if $E \subseteq Ncl(Nint(Ncl(E)))$.

A neutrosophic set E is called neutrosophic semiclosed (resp. neutrosophic α-closed, neutrosophic preclosed, neutrosophic regular closed and neutrosophic β-closed) (briefly NSCS, NαCS, NPCS, NRCS and NβCS) if the complement of E is a neutrosophic semiopen (resp. neutrosophic α-open, neutrosophic preopen, neutrosophic regular open and neutrosophic β-open).

Definition 2.6. [15] Let E be a subset of a neutrosophic topological space (X, τ). Then E is called a neutrosophic b(N_b.In brief)-closed set if $[Ncl(Nint(E))] \cup [Nint(Ncl(E))] \subseteq E$.

Definition 2.7. [17] Let E be a neutrosophic set in a neutrosophic topological space (X, τ). Then,

i) $Nint(E) = \bigcup\{F|\ F$ is a neutrosophic open set in (X, τ) and $F \subseteq E\}$ is called the neutrosophic interior of E;

ii) $Ncl(E) = \bigcap\{F|\ F$ is a neutrosophic closed set in (X, τ) and $F \supseteq E\}$ is called the neutrosophic closure of E.

3 Neutrosophic $b^*g\alpha$-closed sets

In this section, the new concept of neutrosophic $b^*g\alpha$-closed sets in neutrosophic topological spaces was defined and studied.

Definition 3.1. Let E be a subset of a neutrosophic topological space (X, τ). Then E is called

i) a neutrosophic $g\alpha$-open set($N_{g\alpha}OS$) if $V \subseteq N_\alpha int(E)$ whenever $V \subseteq E$ and V is a neutrosophic α-closed set in (X, τ).

ii) a neutrosophic $g\alpha$-closed set($N_{g\alpha}CS$) if $N_\alpha cl(E) \subseteq V$ whenever $E \subseteq V$ and V is a neutrosophic α-open set in (X, τ).

iii) a neutrosophic $^*g\alpha$-open set($N_{^*g\alpha}OS$) if $V \subseteq Nint(E)$ whenever $V \subseteq E$ and V is a neutrosophic $g\alpha$-closed set in (X, τ).

iv) a neutrosophic $^*g\alpha$-closed set($N_{^*g\alpha}CS$) if $Ncl(E) \subseteq V$ whenever $E \subseteq V$ and V is a neutrosophic $g\alpha$-open set in (X, τ).

Definition 3.2. Let E be a subset of a neutrosophic topological space (X, τ). Then E is called

i) a neutrosophic $b^*g\alpha$-open set($N_{b^*g\alpha}OS$) if $V \subseteq N_b int(E)$ whenever $V \subseteq E$ and V is a neutrosophic $^*g\alpha$-closed set in (X, τ).

ii) a neutrosophic $b^{*}g\alpha$-closed set($N_{b^{*}g\alpha}CS$) if $N_bcl(E) \subseteq V$ whenever $E \subseteq V$ and V is a neutrosophic $^{*}g\alpha$-open set in (X, τ).

Example 3.3. Let $X = \{p, q, r\}$ and the neutrosophic sets L and M are defined as,

$$L = \{< x, \left(\tfrac{p}{1/2}, \tfrac{q}{2/5}, \tfrac{r}{1/2}\right), \left(\tfrac{p}{3/10}, \tfrac{q}{3/10}, \tfrac{r}{1/5}\right), \left(\tfrac{p}{7/10}, \tfrac{q}{7/10}, \tfrac{r}{7/10}\right) > \ \forall\ x \in X\},$$

$$M = \{< x, \left(\tfrac{p}{7/10}, \tfrac{q}{7/10}, \tfrac{r}{7/10}\right), \left(\tfrac{p}{3/10}, \tfrac{q}{3/10}, \tfrac{r}{1/5}\right), \left(\tfrac{p}{1/2}, \tfrac{q}{3/5}, \tfrac{r}{1/2}\right) > \ \forall\ x \in X\}.$$

Then the neutrosophic topology $\tau = \{0_N, L, M, 1_N\}$, which are neutrosophic open sets in the neutrosophic topological space (X, τ).

If $N = \{< x, \left(\tfrac{p}{1/2}, \tfrac{q}{3/5}, \tfrac{r}{1/2}\right), \left(\tfrac{p}{4/5}, \tfrac{q}{4/5}, \tfrac{r}{9/10}\right), \left(\tfrac{p}{7/10}, \tfrac{q}{7/10}, \tfrac{r}{7/10}\right) > \ \forall\ x \in X\}$ and

$$E = \{< x, \left(\tfrac{p}{1/2}, \tfrac{q}{3/5}, \tfrac{r}{1/2}\right), \left(\tfrac{p}{3/10}, \tfrac{q}{3/10}, \tfrac{r}{3/10}\right), \left(\tfrac{p}{7/10}, \tfrac{q}{7/10}, \tfrac{r}{7/10}\right) > \ \forall\ x \in X\}.$$

Then the complement of L, M, N and E are,

$$\overline{L} = \{< x, \left(\tfrac{p}{7/10}, \tfrac{q}{7/10}, \tfrac{r}{7/10}\right), \left(\tfrac{p}{7/10}, \tfrac{q}{7/10}, \tfrac{r}{4/5}\right), \left(\tfrac{p}{1/2}, \tfrac{q}{2/5}, \tfrac{r}{1/2}\right) > \ \forall\ x \in X\},$$

$$\overline{M} = \{< x, \left(\tfrac{p}{1/2}, \tfrac{q}{3/5}, \tfrac{r}{1/2}\right), \left(\tfrac{p}{7/10}, \tfrac{q}{7/10}, \tfrac{r}{4/5}\right), \left(\tfrac{p}{7/10}, \tfrac{q}{7/10}, \tfrac{r}{7/10}\right) > \ \forall\ x \in X\},$$

$\overline{N} = \{< x, \left(\tfrac{p}{7/10}, \tfrac{q}{7/10}, \tfrac{r}{7/10}\right), \left(\tfrac{p}{1/5}, \tfrac{q}{1/5}, \tfrac{r}{1/10}\right), \left(\tfrac{p}{1/2}, \tfrac{q}{3/5}, \tfrac{r}{1/2}\right) > \ \forall\ x \in X\}$ and

$$\overline{E} = \{< x, \left(\tfrac{p}{7/10}, \tfrac{q}{7/10}, \tfrac{r}{7/10}\right), \left(\tfrac{p}{7/10}, \tfrac{q}{7/10}, \tfrac{r}{7/10}\right), \left(\tfrac{p}{1/2}, \tfrac{q}{3/5}, \tfrac{r}{1/2}\right) > \ \forall\ x \in X\}.$$

Hence N is a neutrosophic $^{*}g\alpha$-open set, $\overline{N}$ is a neutrosophic $^{*}g\alpha$-closed set, E is a neutrosophic $b^{*}g\alpha$-closed set, $\overline{E}$ is a neutrosophic $b^{*}g\alpha$-open set of a neutrosophic topological space (X, τ). Since $N_bcl(E)$ = $\{< x, \left(\tfrac{p}{1/2}, \tfrac{q}{3/5}, \tfrac{r}{1/2}\right), \left(\tfrac{p}{3/10}, \tfrac{q}{3/10}, \tfrac{r}{1/5}\right), \left(\tfrac{p}{7/10}, \tfrac{q}{7/10}, \tfrac{r}{7/10}\right) > \ \forall\ x \in X\}$, which is contained in N. That is $N_bcl(E) \subseteq N$.

Definition 3.4. Let E be a subset of a neutrosophic topological space (X, τ). Then $N_{b^{*}g\alpha}$-$int(E) = \bigcup\{F : F$ is neutrosophic $b^{*}g\alpha$-open set and $F \subset E\}$. The complement of $N_{b^{*}g\alpha}$-$int(E)$ is $N_{b^{*}g\alpha}$-$cl(E)$.

Remark 3.5. Let A be a subset of a neutrosophic topological space (X, τ), then $N_{b^{*}g\alpha}$-$int(A)$ is $N_{b^{*}g\alpha}$-open in (X, τ).

Theorem 3.6. *In the neutrosophic topological space (X, τ), if a subset E is a neutrosophic closed set then it is a neutrosophic $b^{*}g\alpha$-closed set.*

Proof. *Let $E \subseteq V$, where V is neutrosophic $^{*}g\alpha$-open in X. Since E is neutrosophic closed, $Ncl(E) = E$. But $N_bcl(E) \subseteq Ncl(E) = E$, which implies $N_bcl(E) \subseteq V$. Therefore E is neutrosophic $b^{*}g\alpha$-closed set.*

The converse of the above theorem need not be true. It can be seen by the following example.

Example 3.7. Let $X = \{p, q, r\}$ and the neutrosophic sets L and M are defined as,

$L = \{< x, \big(\frac{p}{1/2}, \frac{q}{2/5}, \frac{r}{1/2}\big), \big(\frac{p}{3/10}, \frac{q}{3/10}, \frac{r}{1/5}\big), \big(\frac{p}{7/10}, \frac{q}{7/10}, \frac{r}{7/10}\big) > \ \forall\ x \in X\},$

$M = \{< x, \big(\frac{p}{7/10}, \frac{q}{7/10}, \frac{r}{7/10}\big), \big(\frac{p}{3/10}, \frac{q}{3/10}, \frac{r}{1/5}\big), \big(\frac{p}{1/2}, \frac{q}{3/5}, \frac{r}{1/2}\big) > \ \forall\ x \in X\}.$

Then the neutrosophic topology $\tau = \{0_N, L, M, 1_N\}$ and the complement of neutrosophic sets L and M are defined as,

$\overline{L} = \{< x, \big(\frac{p}{7/10}, \frac{q}{7/10}, \frac{r}{7/10}\big), \big(\frac{p}{7/10}, \frac{q}{7/10}, \frac{r}{4/5}\big), \big(\frac{p}{1/2}, \frac{q}{2/5}, \frac{r}{1/2}\big) > \ \forall\ x \in X\},$

$\overline{M} = \{< x, \big(\frac{p}{1/2}, \frac{q}{3/5}, \frac{r}{1/2}\big), \big(\frac{p}{7/10}, \frac{q}{7/10}, \frac{r}{4/5}\big), \big(\frac{p}{7/10}, \frac{q}{7/10}, \frac{r}{7/10}\big) > \ \forall\ x \in X\}.$

If $E = \{< x, \big(\frac{p}{1/2}, \frac{q}{3/5}, \frac{r}{1/2}\big), \big(\frac{p}{3/10}, \frac{q}{3/10}, \frac{r}{3/10}\big), \big(\frac{p}{7/10}, \frac{q}{7/10}, \frac{r}{7/10}\big) > \ \forall\ x \in X\}.$

Then E is a neutrosophic $b^*g\alpha$-closed set but it is not a neutrosophic closed set of a neutrosophic topological space (X, τ). Since $Ncl(E) = \overline{M}$ which is not equal to the neutrosophic set E.

Theorem 3.8. *In the neutrosophic topological space (X, τ), if a subset E is a neutrosophic pre-closed set then it is a neutrosophic $b^*g\alpha$-closed set.*

Proof. *Let $E \subseteq V$, where V is neutrosophic $^*g\alpha$-open in X. Since E is neutrosophic pre-closed, $N_pcl(E)$ = E. But $N_bcl(E) \subseteq N_pcl(E) = E$, which implies $N_bcl(E) \subseteq V$. Therefore E is neutrosophic $b^*g\alpha$-closed set.*

Generally, the converse of the above theorem is not true. It can be seen by the following example.

Example 3.9. From Example 3.7. the neutrosophic set E is a neutrosophic $b^*g\alpha$-closed set but it is not a neutrosophic pre-closed set of a neutrosophic topological space (X, τ). Since $Ncl(Nint(E)) = \overline{M}$ which is not contained in the neutrosophic set E.

Theorem 3.10. *In the neutrosophic topological space (X, τ), if a subset E is a neutrosophic α-closed set then it is a neutrosophic $b^*g\alpha$-closed set.*

Proof. *Let $E \subseteq V$, where V is neutrosophic $^*g\alpha$-open in X. Since E is neutrosophic α-closed, $N_\alpha cl(E) = E$. But $N_bcl(E) \subseteq N_\alpha cl(E) = E$, which implies $N_bcl(E) \subseteq V$. Therefore E is neutrosophic $b^*g\alpha$-closed set.*

Generally, the converse of the above theorem is not true. It can be seen by the following example.

Example 3.11. From Example 3.7. the neutrosophic set E is a neutrosophic $b^*g\alpha$-closed set but it is not a neutrosophic α-closed set of a neutrosophic topological space (X, τ). Since $Ncl(Nint(Ncl(E))) = \overline{M}$ which is not contained in the neutrosophic set E.

Theorem 3.12. *In the neutrosophic topological space (X, τ), if a subset E is a neutrosophic $g\alpha$-closed set then it is a neutrosophic $b^*g\alpha$-closed set.*

Proof. *Let $E \subseteq V$, where V is neutrosophic $^*g\alpha$-open in X. Since every neutrosophic $^*g\alpha$-open set is neutrosophic α-open, V is neutrosophic α-open. Since E is neutrosophic $g\alpha$-closed in X, $N_\alpha cl(E) \subseteq V$. But $N_bcl(E) \subseteq N_\alpha cl(E) \subseteq V$, which implies $N_bcl(E) \subseteq V$. Therefore E is neutrosophic $b^*g\alpha$-closed.*

Generally, the converse of the above theorem is not true. It can be seen by the following example.

Example 3.13. From Example 3.7. the neutrosophic set E is a neutrosophic $b^*g\alpha$-closed set but it is not a neutrosophic $g\alpha$-closed set of a neutrosophic topological space (X,τ). Since N_α-open set $F = \{< x, \left(\frac{p}{5/10}, \frac{q}{5/10}, \frac{r}{5/10}\right), \left(\frac{p}{3/10}, \frac{q}{3/10}, \frac{r}{3/10}\right), \left(\frac{p}{7/10}, \frac{q}{7/10}, \frac{r}{7/10}\right) > \ \forall\ x \in X\}$.

Theorem 3.14. *In the neutrosophic topological space (X,τ), if a subset E is a neutrosophic $^*g\alpha$-closed set then it is a neutrosophic $b^*g\alpha$-closed set.*

Proof. *Let $E \subseteq V$, where V is neutrosophic $^*g\alpha$-open in X. Since every neutrosophic $^*g\alpha$-open set is neutrosophic $g\alpha$-open, V is neutrosophic $g\alpha$-open. Since E is neutrosophic $^*g\alpha$-closed in X, $Ncl(E) \subseteq V$. But $N_bcl(E) \subseteq Ncl(E) \subseteq V$, which implies $N_bcl(E) \subseteq V$. Therefore E is neutrosophic $b^*g\alpha$-closed.*

Generally, the converse of the above theorem is not true. It can be seen by the following example.

Example 3.15. From Example 3.7. the neutrosophic set E is a neutrosophic $b^*g\alpha$-closed set but it is not a neutrosophic $^*g\alpha$-closed set of a neutrosophic topological space (X,τ). Since $N_{g\alpha}$-open set $G = \{< x, \left(\frac{p}{7/10}, \frac{q}{7/10}, \frac{r}{7/10}\right), \left(\frac{p}{3/10}, \frac{q}{2/5}, \frac{r}{3/10}\right), \left(\frac{p}{1/2}, \frac{q}{3/5}, \frac{r}{1/2}\right) > \ \forall\ x \in X\}$.

Theorem 3.16. *The union of any two neutrosophic $b^*g\alpha$-closed sets in (X,τ) is also a neutrosophic $b^*g\alpha$-closed set in (X,τ).*

Proof. *Let E and F be two neutrosophic $b^*g\alpha$-closed sets in (X,τ). Let V be a neutrosophic $^*g\alpha$-open set in X such that $E \subseteq V$ and $F \subseteq V$. Then we have, $E \cup F \subseteq V$. Since E and F are neutrosophic $b^*g\alpha$-closed sets in (X,τ), which implies $N_bcl(E) \subseteq V$ and $N_bcl(F) \subseteq V$. Now, $N_bcl(E \cup F) = N_bcl(E) \cup N_bcl(F) \subseteq V$. Thus, we have $N_bcl(E \cup F) \subseteq V$ whenever $E \cup F \subseteq V$, V is neutrosophic $^*g\alpha$-open set in (X,τ) which implies $E \cup F$ is a neutrosophic $b^*g\alpha$-closed set in (X,τ).*

Theorem 3.17. *Let E be a neutrosophic $b^*g\alpha$-closed subset of (X,τ). If $E \subseteq F \subseteq N_bcl(E)$, then F is also a neutrosophic $b^*g\alpha$-closed subset of (X,τ).*

Proof. *Let $F \subseteq V$, where V is neutrosophic $^*g\alpha$-open in (X,τ). Then $E \subseteq F$ implies $E \subseteq V$. Since E is neutrosophic $b^*g\alpha$-closed, $N_bcl(E) \subseteq V$. Also $F \subseteq N_bcl(E)$ implies $N_bcl(F) \subseteq N_bcl(E)$. Thus, $N_bcl(F) \subseteq V$ and so F is neutrosophic $b^*g\alpha$-closed.*

Theorem 3.18. *Let E be a neutrosophic $b^*g\alpha$-closed set in (X,τ). Then $N_bcl(E) - E$ has no non-empty neutrosophic $^*g\alpha$-closed set.*

Proof. *Let E be a neutrosophic $b^*g\alpha$-closed set in (X,τ), and F be a neutrosophic $^*g\alpha$-closed subset of $N_bcl(E) - E$. That is, $F \subseteq N_bcl(E) - E$, which implies that, $F \subseteq N_bcl(E) \cap \overline{E}$. That is $F \subseteq N_bcl(E)$ and $F \subseteq \overline{E}$, which implies $E \subseteq \overline{F}$, where $\overline{F}$ is a neutrosophic $^*g\alpha$-open set. Since E is neutrosophic $b^*g\alpha$-closed, $N_bcl(E) \subseteq \overline{F}$. That is $F \subseteq \overline{N_bcl(E)}$. Thus $F \subseteq N_bcl(E) \cap \overline{N_bcl(E)}$. Therefore $F = \phi$.*

4 Neutrosophic $b^*g\alpha$-Border

Definition 4.1. For any subset E of X, the neutrosophic $b^*g\alpha$-border of E is defined by

$$N_{b^*g\alpha}[Bd(E)] = E \setminus N_{b^*g\alpha}\text{-}int(E).$$

Theorem 4.2. *In the neutrosophic topological space (X,τ), for any subset E of X, the following statements are hold.*

i) $N_{b^*g\alpha}[Bd(\phi)] = N_{b^*g\alpha}[Bd(X)] = \phi$

ii) $E = N_{b^*g\alpha}\text{-}int(E) \cup N_{b^*g\alpha}[Bd(E)]$

iii) $N_{b^*g\alpha}\text{-}int(E) \cap N_{b^*g\alpha}[Bd(E)] = \phi$

iv) $N_{b^*g\alpha}\text{-}int(E) = E \setminus N_{b^*g\alpha}[Bd(E)]$

v) $N_{b^*g\alpha}\text{-}int(N_{b^*g\alpha}[Bd(E)]) = \phi$

vi) *E is $N_{b^*g\alpha}$-open iff $N_{b^*g\alpha}[Bd(E)] = \phi$*

vii) $N_{b^*g\alpha}[Bd(N_{b^*g\alpha}\text{-}int(E))] = \phi$

viii) $N_{b^*g\alpha}[Bd(N_{b^*g\alpha}[Bd(E)])] = N_{b^*g\alpha}[Bd(E)]$

ix) $N_{b^*g\alpha}[Bd(E)] = E \cap N_{b^*g\alpha}\text{-}cl(X \setminus E)$

Proof. *Statements i) to iv) are obvious by the definition of neutrosophic $b^*g\alpha$-border of E. If possible, let $x \in N_{b^*g\alpha}\text{-}int(N_{b^*g\alpha}[Bd(E)])$. Then $x \in N_{b^*g\alpha}[Bd(E)]$, since $N_{b^*g\alpha}[Bd(E)] \subseteq E$, $x \in N_{b^*g\alpha}\text{-}int(N_{b^*g\alpha}[Bd(E)]) \subseteq N_{b^*g\alpha}\text{-}int(E)$. Therefore $x \in N_{b^*g\alpha}\text{-}int(E) \cap N_{b^*g\alpha}[Bd(E)]$, which is the contradiction to iii). Hence v) is proved. E is neutrosophic $b^*g\alpha$-open iff $N_{b^*g\alpha}\text{-}int(E) = E$. But $N_{b^*g\alpha}[Bd(E)] = E \setminus N_{b^*g\alpha}\text{-}Int(E)$ implies $N_{b^*g\alpha}[Bd(E)] = \phi$. This proves vi) & vii). When $E = N_{b^*g\alpha}[Bd(E)]$, then the definition of neutrosophic $b^*g\alpha$-border of E becomes $N_{b^*g\alpha}[Bd(N_{b^*g\alpha}[Bd(E)])] = N_{b^*g\alpha}[Bd(E)] \setminus N_{b^*g\alpha}\text{-}int(N_{b^*g\alpha}[Bd(E)])$. By using vii), we get the proof of viii). Now, $N_{b^*g\alpha}[Bd(E)] = E \setminus N_{b^*g\alpha}\text{-}int(E) = E \cap (X \setminus N_{b^*g\alpha}\text{-}int(E)) = E \cap N_{b^*g\alpha}\text{-}cl(X \setminus E)$.*

5 Neutrosophic $b^*g\alpha$-Frontier

Definition 5.1. For any subset E of X, the neutrosophic $b^*g\alpha$-frontier of E is defined by

$$N_{b^*g\alpha}[Fr(E)] = N_{b^*g\alpha}\text{-}cl(E) \setminus N_{b^*g\alpha}\text{-}int(E).$$

Theorem 5.2. *In the Neutrosophic topological space (X,τ), for any subset E of X, the following statements will be hold.*

i) $N_{b^*g\alpha}[Fr(\phi)] = N_{b^*g\alpha}[Fr(X)] = \phi$

ii) $N_{b^*g\alpha}\text{-}int(E) \cap N_{b^*g\alpha}[Fr(E)] = \phi$

iii) $N_{b^*g\alpha}[Fr(E)] \subseteq N_{b^*g\alpha}\text{-}cl(E)$

iv) $N_{b^*g\alpha}\text{-}int(E) \cup N_{b^*g\alpha}[Fr(E)] = N_{b^*g\alpha}\text{-}cl(E)$

v) $N_{b^*g\alpha}\text{-}int(E) = E \setminus N_{b^*g\alpha}[Fr(E)]$

vi) *If E is $N_{b^*g\alpha}$-closed, then $E = N_{b^*g\alpha}\text{-}int(E) \cup N_{b^*g\alpha}[Fr(E)]$*

vii) $Fr(E) = Fr(N_{b^*g\alpha}[Fr(E)]$

viii) *If E is $N_{b^*g\alpha}$-open, then $E \cap N_{b^*g\alpha}[Fr(E)] = \phi$*

ix) $X = N_{b^*g\alpha}\text{-}cl(E) \cup N_{b^*g\alpha}\text{-}cl(X \setminus E)$

x) *If E is $N_{b^*g\alpha}$-open, then $N_{b^*g\alpha}[Fr(N_{b^*g\alpha}\text{-}int(E))] \subseteq N_{b^*g\alpha}[Fr(E)]$*

xi) *If E is $N_{b^*g\alpha}$-closed, then $N_{b^*g\alpha}[Fr(N_{b^*g\alpha}\text{-}cl(E))] \subseteq N_{b^*g\alpha}[Fr(E)]$*

xii) *If E is $N_{b^*g\alpha}$-open iff $N_{b^*g\alpha}[Fr(N_{b^*g\alpha}\text{-}int(E))] \cap N_{b^*g\alpha}\text{-}int(E) = \phi$*

Proof. *Statements i) to vii) are true by the definition of neutrosophic $b^*g\alpha$-frontier of E. By remark (3.5), If E is neutrosophic $b^*g\alpha$-open, $E = N_{b^*g\alpha}\text{-}int(E)$ and by statement - ii), $E \cap N_{b^*g\alpha}[Fr(E)] = \phi$. Hence viii) is proved. statement ix) is obvious. Since $N_{b^*g\alpha}\text{-}int(E)$ is $N_{b^*g\alpha}$-open, then $N_{b^*g\alpha}\text{-}int(E) = E$, which implies $N_{b^*g\alpha}[Fr(N_{b^*g\alpha}\text{-}int(E))] \subseteq N_{b^*g\alpha}[Fr(E)]$. Similarly, xi) can be proved. By remark(3.5) and by statement-ii), xii) is straight forward.*

6 Connection between Neutrosophic $b^*g\alpha$-Frontier and Neutrosophic $b^*g\alpha$-Border

Theorem 6.1. *In the neutrosophic topological space (X, τ), for any subset E of X, the following statements will be hold.*

i) $N_{b^*g\alpha}[Bd(E)] \setminus N_{b^*g\alpha}[Fr(E)] = \phi$

ii) $N_{b^*g\alpha}[Bd(E)] \subseteq N_{b^*g\alpha}[Fr(E)]$

iii) $N_{b^*g\alpha}[Fr(N_{b^*g\alpha}[Bd(E)])] = N_{b^*g\alpha}[Bd(E)]$

iv) $N_{b^*g\alpha}[Bd(N_{b^*g\alpha}[Fr(E)])] = N_{b^*g\alpha}[Fr(E)]$

v) *If E is neutrosophic $b^*g\alpha$- open, then $N_{b^*g\alpha}[Fr(E)] \cup N_{b^*g\alpha}[Bd(E)] = N_{b^*g\alpha}[Fr(E)]$*

vi) $N_{b^*g\alpha}[Fr(E)] \cap N_{b^*g\alpha}[Bd(E)] = N_{b^*g\alpha}[Bd(E)]$

vii) $\overline{N_{b^*g\alpha}[Fr(E)]} \cup \overline{N_{b^*g\alpha}[Bd(E)]} = \overline{N_{b^*g\alpha}[Bd(E)]}$

viii) $\overline{N_{b^*g\alpha}[Fr(E)]} \cap \overline{N_{b^*g\alpha}[Bd(E)]} = \overline{N_{b^*g\alpha}[Fr(E)]}$

Proof. *Statement i) to iv) are obvious by the definitions of Neutrosophic $b^*g\alpha$-Frontier and Neutrosophic $b^*g\alpha$-border of a set. Since E is Neutrosophic $b^*g\alpha$- open, then we have a statement from Neutrosophic $b^*g\alpha$-border of a set, $N_{b^*g\alpha}[Bd(E)] = \phi$, which implies $N_{b^*g\alpha}[Fr(E)] \cup \phi = N_{b^*g\alpha}[Fr(E)]$.Hence v) is proved. We know from statement - ii), $N_{b^*g\alpha}[Bd(E)] \subseteq N_{b^*g\alpha}[Fr(E)]$ which implies $N_{b^*g\alpha}[Fr(E)] \cap N_{b^*g\alpha}[Bd(E)] = N_{b^*g\alpha}[Bd(E)]$. It gives the proof of vi). By the above statement, $\overline{N_{b^*g\alpha}[Bd(E)]} = \overline{N_{b^*g\alpha}[Fr(E)] \cap N_{b^*g\alpha}[Bd(E)]}$, and by using De Morgan's law, $\overline{N_{b^*g\alpha}[Fr(E)] \cap N_{b^*g\alpha}[Bd(E)]} = \overline{N_{b^*g\alpha}[Fr(E)]} \cup \overline{N_{b^*g\alpha}[Bd(E)]}$, it gives the proof of vii). Similarly we can prove the statement viii).*

7 Conclusion

This article defined neutrosophic $b^*g\alpha$-closed sets in neutrosophic topological spaces and discussed some of their properties. Also neutrosophic $b^*g\alpha$-interior, neutrosophic $b^*g\alpha$-closure, neutrosophic $b^*g\alpha$-border and neutrosophic $b^*g\alpha$-frontier of a set were introduced and discussed their properties. The connection between neutrosophic $b^*g\alpha$-border of a set and neutrosophic $b^*g\alpha$-frontier of a set in neutrosophic topological spaces were established. This set can be used to derive few more new functions of neutrosophic $b^*g\alpha$-continuous and neutrosophic $b^*g\alpha$-homeomorphisms in neutrosophic topological spaces. In addition to this, it can be extended in the field of contra neutrosophic functions.

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Neutrosophic modifications of Simplified TOPSIS for Imperfect Information (nS-TOPSIS)

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Abstract: Technique for order performance by similarity to ideal solution (TOPSIS) is a Multi-Criteria Decision-Making method (MCDM), that consists on handling real complex problems of decision-making. However, real MCDM problems are often involves imperfect information such as uncertainty and inconsistency. The imperfect information is often manipulated through Neutrosophics theory, using certain degree of truth (T), falsity degree (F) and indeterminacy degree(I). and thus single-valued neutrosophic set (SVNs) had prodded a strong capacity to model such complex information. To overcome that kind of problems, In this paper, first, the authors simplify the popular TOPSIS method to a lite TOPSIS (S-TOPSIS), that gives the same result as standard version. Second, mapping S-TOPSIS to Neutrosophics Environment, investigating SVNS, called nS-TOPSIS, to deal with imperfect information in the real decision-making problems. Numerical examples show the contributions of proposed S-TOPSIS method to get the same results with standard TOPSIS with simple way of calculus, and how Neutrosophic environment manage the uncertain information using SVN.

Keywords: Technique for order performance by similarity to ideal solution (TOPSIS), MCDM, Single-Valued Neutrosophic set(SVNs), Neutrosophic Simplified TOPSIS(nS-TOPSIS).

1 Introduction

Technique for Order Preference by Similarity to Ideal Solution(TOPSIS) is a popular Multicriteria Decision Making (MCDM). TOPSIS was first introduced by Hwang and Yoon ([1]) to deal with structuring Multicriteria issues with crisp numerical values in real situation. However, real MCDM problems are often formulated under as set of indeterminate or inconsistent information. Thus, TOPSIS consists on many complicate steps of calculation. To deal with thoses problems, First, we introduce a lite version of TOPSIS method (S-TOPSIS) with guaranty of obtention of the same results simplifying many complicated steps of calculation. Thus, we introduce single valued neutrosophic set (SVNs) modifications of Simplified TOPSIS (nS-TOPSIS).

To manage information outcome from real problem, that are usually endowed with imperfection such as uncertainty, fuzziness and inconsistency, Smarandache ([2,3]) initiated a new notion, which is a generalization of the Intuitionistic $Fuzzy$ Set (IFS), called Neutrosophics Set (NS), which based on three values (truth (T), indeterminacy (I), and falsity (F) membership degrees). The main propriety of NS is that the sum of three values is 3 instead of 1 in the case of IFS. Although, the NS as introduced by Smarandache was a philosophical concept, unable to be used in real study cases. Many researchers are working on to produce mathematical property, theories, Arithmetic Operations, etc. On the one hand, Wang and al. ([5]) embodied Neutrosophic concept in a metric, called single-valued neutrosophic set (SVNs) as three values in one ($truth-membership\quad degree$, $indeterminacy-membership\quad degree$, and $falsity-membership\quad degree$). In addition, Broumi and al. ([4,6,7]) defined, in Neutrosophic space, similarity mesure and distances metric between SVNS values. the defined SVNS show stronge power to modelize imperfect information, such as uncertainty, imprecise, incomplete, and inconsistent information.

On the other hand, Other researchers are working on deploying Neutrosophic in MCDM field. Biswas ([8]) proposed extended TOPSIS Method to deal with real MCDM problems based on weighted Neutrosophic and aggregated SVNS operators

Ye [9,10] introduced two concepts, single valued neutrosophic cross-entropy of single valued neutrosophic and weighted correlation coefficient of SVNSs into multicriteria decision-making problems. Deli et al. [11] studied deploying Bipolar Neutrosophic Sets in Multi-Criteria Decision Making field

The remainder of the paper presents the preliminaries to build our Method, TOPSIS method and single valued neutrosophic set (SVNs). next Simplified-TOPSIS as first contribution was introduced. Then, hybrid methods Neutrosophic-TOPSIS and Neutrosophic-Simplified-TOPSIS are proposed to deal with real example. Results and discussions are presented at the end of this paper.

2 TOPSIS method

Consider a multi-attribute decision making problem that could be formulated as follow, $A = \{A_1, A_2, \cdots, A_n\}$ a set of m preferences, and $C = \{C_1, C_2, \cdots, C_n\}$ a set of n criteria. The relationships between preferences A_i and criteria C_j quantified by rating a_{ij} provided by decision maker. Weight vector W is a set of weights ω_i associated to criteria C_j. The all details described above could be reshaped on decision matrix bellow, denoted by D.

$$D = (a_{ij})_{m\times n} = \begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nm} \end{pmatrix} \text{(Decision Matrix)} \tag{2.1}$$

Technique for order performance by similarity to ideal solution (TOPSIS) method summarized as follow:

Step 1: Calculate normalized form of decision matrix r_{ij} dividing each element a_{ij} on the sum of whole column.

$$r_{ij} = a_{ij} / \left(\sum_{i=1}^{m} a_{ij}^2 \right)^{0.5} ;\ j = 1, 2, \cdots, n;\ i = 1, 2 \cdots, m \tag{2.2}$$

Step 2: Calculate also weighted form v_{ij} of matrix r_{ij} obtained from previous step, multiplying each element r_{ij} by its associated weight w_j.

$$v_{ij} = w_j r_{ij};\ j = 1, 2, \cdots, n;\ i = 1, 2 \cdots, m \tag{2.3}$$

Step 3: Based on the weighted decision matrix, we calculate positive ideal solution (POS) and negative ideal solution(NIS).

$$A^{+} = (v_1^{+}, v_2^{+}, \cdots, v_n^{+}) = \left\{ \begin{array}{l} (max_i \{v_{ij} | j \in B\}), \\ (min_i \{v_{ij} | j \in C\}) \end{array} \right\} \tag{2.4}$$

$$A^{-} = (v_1^{-}, v_2^{-}, \cdots, v_n^{-}) = \left\{ \begin{array}{l} (min_i \{v_{ij} | j \in B\}), \\ (max_i \{v_{ij} | j \in C\}) \end{array} \right\} \tag{2.5}$$

B quantify the benefit set, and C is the cost attribute set. **Step 4:** By subtracting each weighted element v_{ij} From POS and NIS, we got tow vectors of separation measures cited below.

$$S_i^{+} = \left\{ \sum_{j=1}^{n} (v_{ij} - v_j^{+})^2 \right\}^{0.5} ; \; i = 1, 2 \cdots, m \tag{2.6}$$

$$S_i^{-} = \left\{ \sum_{j=1}^{n} (v_{ij} - v_j^{-})^2 \right\}^{0.5} ; \; i = 1, 2 \cdots, m \tag{2.7}$$

Step 5: Using the both measures calculated in the previous step, we calculate the rating metric.

$$T_i = \frac{S_i^{-}}{(S_i^{+} + S_i^{-})}; \; i = 1, 2 \cdots, m \tag{2.8}$$

Once we calculate T_i that will be used to rank set of alternatives A_i.

2.1 Numerical example

Let consider the numerical example summarized by table Table-1. below, that contains alternatives with respect of criteria weights.

a_{ij}	C_1	C_2	C_3
ω_i	12/16	3/16	1/16
A_1	7	9	9
A_2	8	7	8
A_3	9	6	8
A_4	6	7	8

Table 1: Decision Matrix.

Table Table-2. is result of application of this formula $\sum_{i=1}^{n} a_{ij}$ on each column.

To determine Normalized matrix r_{ij} Table-3. each value is divide by $(\sum_{i=1}^{n} a_{ij}^2)^{1/2}$:

Weighted Decision matrix v_{ij} Table-4 is the multiplication of each column by w_j.

The table Table-5. below figure out the solution of the above MCDM problem listing furthermore, final rankings for decision matrix, separation metric from POS and NIS.

Preferences, in descending preference order, are ranked as $A_3 > A_1 > A_4 > A_2$ as showed in Table-5.

a_{ij}^2	C_1	C_2	C_3
ω_i	12/16	3/16	1/16
A_1	49	81	81
A_2	64	49	64
A_3	81	36	64
A_4	36	49	64
$\sum_{i=1}^{n} a_{ij}$	230	215	273

Table 2: Multiple decision matrix.

r_{ij}	C_1	C_2	C_3
ω_i	12/16	3/16	1/16
A_1	0.4616	0.6138	0.5447
A_2	0.5275	0.4774	0.4842
A_3	0.5934	0.4092	0.4842
A_4	0.3956	0.4774	0.4842
$\sum_{i=1}^{n} a_{ij}$	230	215	273

Table 3: Normalized decision matrix.

3 Simplified-TOPSIS method (our proposed method)

The Simplified-TOPSIS algorithmic consists on steps bellow :

Step 1: Structure the criteria of the decision-making problem under a hierarchy.

Let considere $C = \{C_1, C_2, \cdots, C_n\}$ is a set of Criteria, with $n \geq 2$, $A = \{A_1, A_2, \cdots, A_n\}$ is the set of Preferences (Alternatives), with $m \geq 1$, a_{ij} the score of preference i with respect to criterion j, and let ω_i weight of criteria C_i.

$$D = (a_{ij})_{m\times n} = \begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nm} \end{pmatrix} \quad \text{(Decision Matrix)} \tag{3.1}$$

Step 2: Calculation of the Weighted Decision Matrix v_{ij}.

Let v_{ij} Weighted Decision Matrix (WDM) that is obtained by multiplication of each column by its weight.

$$v_{ij} = w_j a_{ij};\ j = 1, 2, \cdots, n;\ i = 1, 2 \cdots, m \tag{3.2}$$

The difference between proposed method and standard TOPSIS section 2), the normalized step is ignored and WDM v_{ij} is calculated directly without normalization by multiplying a_{ij} with w_j.

Step 3: Determination of LIS and SIS.

The maximum (largest) ideal solution (LIS), as its name indicate, is the the set of maximums raws and smallest ideal solution (SIS) is the set of minimums raws.

$$A^+ = (v_1^+, v_2^+, \cdots, v_m^+) = (max_i \{v_{ij} | j = 1, 2, \cdots, n\}) \tag{3.3}$$

v_{ij}	$\boldsymbol{C_1}$	$\boldsymbol{C_2}$	$\boldsymbol{C_3}$
ω_i	12/16	3/16	1/16
A_1	0.3462	0.1151	0.0340
A_2	0.3956	0.0895	0.0303
A_3	0.4451	0.0767	0.0303
A_4	0.2967	0.0895	0.0303
v_{max}	0.4451	0.1151	0.0340
v_{min}	0.2967	0.0767	0.0303

Table 4: Weighted decision matrix.

Alternative	S_i^+	S_i^-	T_i
A_1	0.0989	0.0627	0.3880
A_2	0.0558	0.0997	0.6412
A_3	0.0385	0.1484	**0.7938**
A_4	0.1506	0.0128	0.0783

Table 5: Distance measure and ranking coefficient.

$$A^+ = (v_1^-, v_2^-, \cdots, v_m^-) = \left(\min_i \{v_{ij} | j = 1, 2, \cdots, n\}\right) \tag{3.4}$$

Step 4: Calculation of positive and negative solutions.

The positive and negative solution are the entropies of orders two of calculated using the formulas below respectively:

$$S_i^+ = \left\{\sum_{j=1}^{n} (v_{ij} - v_j^+)^2\right\}^{0.5} ; \ i = 1, 2 \cdots, m \tag{3.5}$$

$$S_i^- = \left\{\sum_{j=1}^{n} (v_{ij} - v_j^-)^2\right\}^{0.5} ; \ i = 1, 2 \cdots, m \tag{3.6}$$

Arrange preferences (set of alternatives A) based on value of sums of either alternative solutions $\left(S_i^+\right)$ or $\left(S_i^-\right)$. The choice of minimum or maximum depend on nature of problem, if the problem to be minimized or maximized

Step 5 (optional): Another step is missed in our Simplified TOPSIS is calculation of ranking measure T_i (relative closeness to the ideal solution), because of many reasons : first preferences can classified according to many aggregated measures calculated before, second, it's a way of normalization that can be changed by any form of normalization dividing by max, or normalized to $[0, 1]$ range, etc.

$$T_i = \frac{S_i^-}{(S_i^+ + S_i^-)}; \ i = 1, 2 \cdots, m \tag{3.7}$$

3.1 Numerical example

In order to check the consistency of our proposed method, the Simplified-TOPSIS method is applied on the same example (Decision Matrix presented in Table-1.) as classical TOPSIS.

a_{ij}	C_1	C_2	C_3
ω_i	12/16	3/16	1/16
A_1	7	9	9
A_2	8	7	8
A_3	9	6	8
A_4	6	7	8

Table 6: Decision matrix.

Weighed Decision Matrix is gotten (Table-2.).

$\omega_j a_{ij}$	C_1	C_2	C_3
ω_i	*12/16*	*3/16*	*1/16*
A_1	*84/16*	*27/16*	*9/16*
A_2	*96/16*	*21/16*	*8/16*
A_3	*108/16*	*18/16*	*8/16*
A_4	*72/16*	*21/16*	*8/16*

Table 7: Weighted decision matrix.

Next, we calculate the positive and negative solutions as follow :
S1+ = |84/16-108/16| + |27/16-27/16| + |9/16-9/16| = 1.5000
S2+ = |96/16-108/16| + |21/16-27/16| + |8/16-9/16| = 1.1875
S3+ = |108/16-108/16| + |18/16-27/16| + |8/16-9/16| = 0.6250
S4+ = |72/16-108/16| + |21/16-27/16| + |8/16-9/16| =2.6875
S1- = |84/16-72/16| + |27/16-18/16| + |9/16-8/16| =1.3750
S2- = |96/16-72/16| + |21/16-18/16| + |8/16-8/16| =1.6875
S3- = |108/16-72/16| + |18/16-18/16| + |8/16-8/16| =2.2500
S4- = |72/16-72/16| + |21/16-18/16| + |8/16-8/16| = 0.1875

By the end we got both sets of negative and positive solutions $(S3-, S2-, S1-, S4-)$ and $(S3+, S2+, S1+, S4+)$, before arranging preferences, we need to determine which solutions to use, that decision tacked based on the nature of problem, if we seek to minimize or maximize. The minimization of the solution, such as cost to pay, consists on the solution closer to the negative solution, while he maximization of the solution, such as price to sale, consists on the solution closer to the positive solution.

The optional ranking measure T_i confirm the same result.

$$T1 = (S1-)/[(S1-) + (S1+)] = 0.478261 \tag{3.8}$$

$$T2 = (S2-)/[(S2-) + (S2+)] = 0.586957 \tag{3.9}$$

$$T3 = (S3-)/[(S3-) + (S3+)] = 0.782609 \tag{3.10}$$

$$T4 = (S4-)/[(S4-) + (S4+)] = 0.065217 \tag{3.11}$$

The table (Table-8.) figure out all calculus did before

Alternative	S_i^+	S_i^-	T_i
A_1	1.5000	1.3750	0.478261
A_2	1.1875	1.6875	0.586957
A_3	0.6250	2.2500	0.782609
A_4	2.6875	0.1875	0.065217

Table 8: Distance measure and ranking coefficient.

By applying Simplified-TOPSIS, we get for T_3 (0.782609), T_2(0.586957), T_1(0.478261) and T_4(0.065217), and we got with classical TOPSIS T_3(0.7938), T_2(0.6412), T_1(0.3880) and T_4(0.0783). Hence the order obtained with our approach simplified-TOPSIS is the same of classical TOPSIS: T_3, T_2, T_1 and T_4, with little change in values between both approaches.

The both methods our simplified-TOPSIS and Standard TOPSIS produce the same results witt the same ranking $(T3, T2, T1 and then T4)$, with a little differences of ranking measures. For example, with Simplified-TOPSIS T_3 is 0.782609, and with TOPSIS T_3 is 0.7938, the same for all others(Simplified-TOPSIS : $T_2(0.586957)$, $T_1(0.478261)$ and $T_4(0.065217)$ and with TOPSIS : $T_2(0.6412)$, $T_1(0.3880)$ and $T_4(0.0783)$.

4 Standard TOPSIS in Neutrosophic [12]

Standard TOPSIS in Neutrosophic procedure can be summarized as follow :
Step 1: In order to apply neutrosophic TOPSIS algorithm, crisp number Decision Matrix need to be mapped to single valued neutrosophic environment, then, we got neutrosophic decision matrix

$$D = (d_{ij})_{\substack{1 \le i \le n \\ 1 \le j \le m}} = (T_{ij}, I_{ij}, F_{ij})_{\substack{1 \le i \le n \\ 1 \le j \le m}} \quad \text{(Neutrosophic Decision Matrix)} \tag{4.1}$$

Where T_{ij}, I_{ij} and F_{ij} are truth, indeterminacy and falsity membership scores respectively. i refer to preference A_i and j to creterion C_j.

And $w = (\omega_1, \omega_2, \cdots, \omega_n)$ with ω_i a single valued neutrosophic weight of criteria (so $\omega_i = (a_i, b_i, c_i)$).

Example 1:

To compare our method Neutrosophic Simplified TOPSIS (nS-TOPSIS : section 5) and standard NeutrosophicTOPSIS proposed by Biswas ([11]). we use Biswas's numercal example.

Let (DM_1, DM_2, DM_3, DM_4) fours decisions makers aims to select an alternative A_i $(A_1, A_2, A_3, A_4)$with respect six criteria$(C_1, C_2, C_3, C_4, C_5, C_6)$. The mapped weights of criteria and decision matrix in Neutrosophic environment are presented in tables Table-9. and Table-10. respectively.

	C_1	C_2	C_3
ω_i	$(0.755, 0.222, 0.217)$	$(0.887, 0.113, 0.107)$	$(0.765, 0.226, 0.182)$
	C_4	C_5	C_6
ω_i	$(0.692, 0.277, 0.251)$	$(0.788, 0.200, 0.180)$	$(0.700, 0.272, 0.244)$

Table 9: Criteria weights.

	C_1	C_2	C_3
A_1	$(0.864, 0.136, 0.081)$	$(0.853, 0.147, 0.: 092)$	$(0.800, 0.200, 0.150)$
A_2	$(0.667, 0.333, 0.277)$	$(0.727, 0.273, 0.219)$	$(0.667, 0.333, 0.277)$
A_3	$(0.880, 0.120, 0.067)$	$(0.887, 0.113, 0.064)$	$(0.834, 0.166, 0.112)$
A_4	$(0.667, 0.333, 0.277)$	$(0.735, 0.265, 0.195)$	$(0.768, 0.232, 0.180)$
	C_4	C_5	C_6
A_1	$(0.704, 0.296, 0.241)$	$(0.823, 0.177, 0.123)$	$(0.864, 0.136, 0.081)$
A_2	$(0.744, 0.256, 0.204)$	$(0.652, 0.348, 0.293)$	$(0.608, 0.392, 0.336)$
A_3	$(0.779, 0.256, 0.204)$	$(0.811, 0.189, 0.109)$	$(0.850, 0.150, 0.092)$
A_4	$(0.727, 0.273, 0.221)$	$(0.791, 0.209, 0.148)$	$(0.808, 0.192, 0.127)$

Table 10: Neutrosophic Decision Matrix.

Step 2: Weighted decision matrix in neutrosophic is gotten by applying aggregation operator of multiplication i. e. application of generalization of multiplication operator in Neutrosophic space.

$$D^w = D \otimes W = (d^w_{ij})_{\substack{1 \le i \le n \\ 1 \le j \le m}} = (T^w_{ij}, I^w_{ij}, F^w_{ij})_{\substack{1 \le i \le n \\ 1 \le j \le m}} \tag{4.2}$$

Step 3: Calculate of POS-SVNs (positive ideal solution in SVNs) and NIS-SVNs (negative ideal solution in SVNS) measures.

$$T^{w+}_j = \left\{ \left(max_i \left\{ T^{w_i}_{ij} \,|j \in B \right\} \right), \left(min_i \left\{ T^{w_i}_{ij} \,|j \in C \right\} \right) \right\} \tag{4.3}$$

$$Q^+_N = (d^{w+}_1, d^{w+}_2, \cdots, d^{w+}_n) \tag{4.4}$$

$$T^{w+}_j = \left\{ \left(max_i \left\{ T^{w_j}_{ij} \,|j \in B \right\} \right), \left(min_i \left\{ T^{w_j}_{ij} \,|j \in C \right\} \right) \right\} \tag{4.5}$$

$$I^{w+}_j = \left\{ \left(min_i \left\{ I^{w_j}_{ij} \,|j \in B \right\} \right), \left(max_i \left\{ I^{w_j}_{ij} \,|j \in C \right\} \right) \right\} \tag{4.6}$$

$$F^{w+}_j = \left\{ \left(min_i \left\{ F^{w_j}_{ij} \,|j \in B \right\} \right), \left(max_i \left\{ F^{w_j}_{ij} \,|j \in C \right\} \right) \right\} \tag{4.7}$$

$$Q^-_N = (d^{w-}_1, d^{w-}_2, \cdots, d^{w-}_n) \tag{4.8}$$

$$T^{w-}_j = \left\{ \left(min_i \left\{ T^{w_j}_{ij} \,|j \in B \right\} \right), \left(max_i \left\{ T^{w_j}_{ij} \,|j \in C \right\} \right) \right\} \tag{4.9}$$

$$I^{w-}_j = \left\{ \left(max_i \left\{ I^{w_j}_{ij} \,|j \in B \right\} \right), \left(min_i \left\{ I^{w_j}_{ij} \,|j \in C \right\} \right) \right\} \tag{4.10}$$

$$F^{w-}_j = \left\{ \left(max_i \left\{ F^{w_j}_{ij} \,|j \in B \right\} \right), \left(min_i \left\{ F^{w_j}_{ij} \,|j \in C \right\} \right) \right\} \tag{4.11}$$

Where B represents the benefit and C quantify the cost.

Step 4: Calculate length of each alternative from the POS-SVNs and NIS-SVNs calculated in previous step.

$$D_{Eu}^{i+}\left(d_{ij}^{wj}, d_{ij}^{w+}\right) = \sqrt{\frac{1}{3n}\sum_{j=1}^{n}\left\{\begin{array}{l}\left(T_{ij}^{wj}(x) - T_{ij}^{w+}(x)\right)^2 + \\ \left(I_{ij}^{wj}(x) - I_{ij}^{w+}(x)\right)^2 + \\ \left(F_{ij}^{wj}(x) - F_{ij}^{w+}(x)\right)^2\end{array}\right\}} \tag{4.12}$$

$$D_{Eu}^{i-}\left(d_{ij}^{wj}, d_{ij}^{w-}\right) = \sqrt{\frac{1}{3n}\sum_{j=1}^{n}\left\{\begin{array}{l}\left(T_{ij}^{wj}(x) - T_{ij}^{w-}(x)\right)^2 + \\ \left(I_{ij}^{wj}(x) - I_{ij}^{w-}(x)\right)^2 + \\ \left(F_{ij}^{wj}(x) - F_{ij}^{w-}(x)\right)^2\end{array}\right\}} \tag{4.13}$$

With $i = 1, 2 \cdots, m$

Step 5: Calculate the aggregated coefficient of closeness in Neutrosophic.

$$C_i^* = \frac{NS_i^-}{(NS_i^+ + NS_i^-)};\ i = 1, 2 \cdots, m \tag{4.14}$$

All values of aggregated coefficient of closeness are shown in the table Table-11. below.

Alternative	C_i^*
A_1	0.8190
A_2	0.1158
A_3	**0.8605**
A_4	0.4801

Table 11: Closeness Coefficient.

Using the associate values of aggregated coefficient of closeness C_i^* to preference A_i, in descending order, to rank alternatives. Hence, preferences could be ordered as follow $A_3 > A_1 > A_4 > A_2$. Then, the alternative A_3 is the best solution.

5 Neutrosophic-Simplified-TOPSIS (our proposed method)

Step 1: Construct Neutrosophic decision matrix.

As made for Standard Neutrosophic TOPSIS, let consider neutrosophic decision matrix and SVNs weighted criteria.

$$D = (d_{ij})_{\substack{1 \le i \le n \\ 1 \le j \le m}} = (T_{ij}, I_{ij}, F_{ij})_{\substack{1 \le i \le n \\ 1 \le j \le m}} \tag{5.1}$$

$$\begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \left(\begin{array}{cccc} d_{11} & d_{12} & \cdots & d_{1n} \\ d_{21} & d_{22} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ d_{m1} & \cdots & \cdots & d_{mn} \end{array}\right) \end{array}$$

Where T_{ij} denote truth, I_{ij} indeterminacy and N_{ij} falsity membership score of preference i knowing j in neutrosophic environment.

$w = (\omega_1, \omega_2, \cdots, \omega_n)$ with ω_i a single valued neutrosophic weight of criteria (so $\omega_i = (a_i, b_i, c_i)$).

Step 2: Calculate SVNs weighted decision matrix.

$$D^w = D \otimes W = (d_{ij}^w)_{\substack{1 \le i \le n \\ 1 \le j \le m}} = \omega_j \otimes d_{ij}^w = (T_{ij}^w, I_{ij}^w, F_{ij}^w)_{\substack{1 \le i \le n \\ 1 \le j \le m}} \tag{5.2}$$

$$\omega_j \otimes d_{ij} = \left(a_j T_{ij}, b_j + I_{ij} - b_j I_{ij}, c_j + F_{ij} - c_j F_{ij} \right) \tag{5.3}$$

Step 3: Calculate LNIS and SNIS metrics.

LNIS and SNIS are maximum (larger) and minimum (smaller) neutrosophic ideal solution respectively.

$$A_N^+ = (d_1^{w+}, d_2^{w+}, \cdots, d_n^{w+}) \tag{5.4}$$

$$d_j^{\omega+} = \left(T_j^{\omega+}, I_j^{\omega+}, F_j^{\omega+}\right) \tag{5.5}$$

$$T_j^{w+} = \left\{ \left(max_i \left\{ T_{ij}^{w_j} \,|j = 1, \cdots, n \right\} \right) \right\} \tag{5.6}$$

$$I_j^{w+} = \left\{ \left(min_i \left\{ I_{ij}^{w_j} \,|j = 1, \cdots, n \right\} \right) \right\} \tag{5.7}$$

$$F_j^{w+} = \left\{ \left(min_i \left\{ F_{ij}^{w_j} \,|j = 1, \cdots, n \right\} \right) \right\} \tag{5.8}$$

$$A_N^- = (d_1^{w-}, d_2^{w-}, \cdots, d_n^{w-}) \tag{5.9}$$

$$d_j^{\omega-} = \left(T_j^{\omega-}, I_j^{\omega-}, F_j^{\omega-}\right) \tag{5.10}$$

$$T_j^{w-} = \left\{ \left(min_i \left\{ T_{ij}^{w_j} \,|j = 1, \cdots, n \right\} \right) \right\} \tag{5.11}$$

$$I_j^{w-} = \left\{ \left(max_i \left\{ I_{ij}^{w_j} \,|j = 1, \cdots, n \right\} \right) \right\} \tag{5.12}$$

$$F_j^{w-} = \left\{ \left(max_i \left\{ F_{ij}^{w_j} \,|j = 1, \cdots, n \right\} \right) \right\} \tag{5.13}$$

Step 4: Determination of the distance measure of every alternative from the RNPIS and the RNNIS for SVNSs.

To perform that calculus, we need to introduce a new distance measure, in this paper we mapped Manhattan distance ([13]) to Neutrosophic environment (definition 1). The new proposed distance called Neutrosophic Manhattan distance that perform the difference between two single-valued neutrosophic(SVNs) measures.

Definition 1. Let $X_1 = (x_1, y_1, z_1)$ and $X_2 = (x_2, y_2, z_2)$ be a SVN numbers. Then the separation measure between X_1 and X_2 based on Manhattan distance is defined as follows:

$$D_{Manh}\left(X_1, X_2\right) = |x_1 - x_2| + |y_1 - y_2| + |z_1 - x_2| \tag{5.14}$$

The application of Neutrosophic Manhattan distance to calculate the separation from the maximum and minimum Neutrosophic ideal solution respectively are :

$$D_{Manh}^{j+}\left(d_{ij}^{wj}, d_{ij}^{w+}\right) = \left\{ \begin{array}{l} \left|T_{ij}^{wj}(x) - T_{ij}^{w+}(x)\right| + \\ \left|I_{ij}^{wj}(x) - I_{ij}^{w+}(x)\right| + \\ \left|F_{ij}^{wj}(x) - F_{ij}^{w+}(x)\right| \end{array} \right\} \tag{5.15}$$

with $j = 1, 2 \cdots, n$

$$NS_i^+ = \sum_{j=1}^{n} D_{Manh}^{j+}\left(d_{ij}^{wj}, d_{ij}^{w+}\right) \tag{5.16}$$

with $i = 1, 2 \cdots, m$

Similarly, the separation from the minimum neutrosophic ideal solution is:

$$D_{Manh}^{j-}\left(d_{ij}^{wj}, d_{ij}^{w-}\right) = \left\{ \begin{array}{l} \left|T_{ij}^{wj}(x) - T_{ij}^{w-}(x)\right| + \\ \left|I_{ij}^{wj}(x) - I_{ij}^{w-}(x)\right| + \\ \left|F_{ij}^{wj}(x) - F_{ij}^{w-}(x)\right| \end{array} \right\} \tag{5.17}$$

with $j = 1, 2 \cdots, n$

$$NS_i^- = \sum_{j=1}^{n} D_{Manh}^{j-}\left(d_{ij}^{wj}, d_{ij}^{w-}\right) \tag{5.18}$$

with $i = 1, 2 \cdots, m$

Preferences are ordered regarding to the values of NS_i^- or according to $1/NS_i^+$. In other words, the alternatives with the highest appraisal score is the best solution.

Step 5: Rank the alternatives according to Ranking coefficient NT_i.

Ranking coefficient is formulated as :

$$NT_i = \frac{NS_i^-}{(NS_i^+ + NS_i^-)};\ i = 1, 2 \cdots, m \tag{5.19}$$

A set of alternatives can now be ranked according to the descending order of the value of NT_i

5.1 Numerical example

Step 1. Formulate the MCDM problem in neutrosophic by building Neutrosophic decision matrix decision matrix and SVNs weights of criteria.

Let $A_i\,(A_1, A_2, A_3, A_4)$a set of alternative and $C_i\,(C_1, C_2, C, C_4, C_5, C_6)$ a set of criteria. Let considers the following neutrosophic weights of criteria (Table-12.) and neutrosophic decision matrix (Table-13.) respectively (used in above example 1).

Step 2: Calculation of SVNs Weighted Decision Matrix

$$D^w = (d_{ij}^w)_{\substack{1 \le i \le n \\ 1 \le j \le m}} = (T_{ij}^w, I_{ij}^w, F_{ij}^w)_{\substack{1 \le i \le n \\ 1 \le j \le m}} \tag{5.20}$$

	C_1	C_2	C_3
ω_i	$(0.755, 0.222, 0.217)$	$(0.887, 0.113, 0.107)$	$(0.765, 0.226, 0.182)$
	C_4	C_5	C_6
ω_i	$(0.692, 0.277, 0.251)$	$(0.788, 0.200, 0.180)$	$(0.700, 0.272, 0.244)$

Table 12: Criteria neutrosophic weights.

d_{ij}	C_1	C_2	C_3
A_1	$(0.864, 0.136, 0.081)$	$(0.853, 0.147, 0.092)$	$(0.800, 0.200, 0.150)$
A_2	$(0.667, 0.333, 0.277)$	$(0.727, 0.273, 0.219)$	$(0.667, 0.333, 0.277)$
A_3	$(0.880, 0.120, 0.067)$	$(0.887, 0.113, 0.064)$	$(0.834, 0.166, 0.112)$
A_4	$(0.667, 0.333, 0.277)$	$(0.735, 0.265, 0.195)$	$(0.768, 0.232, 0.180)$
	C_4	C_5	C_6
A_1	$(0.704, 0.296, 0.241)$	$(0.823, 0.177, 0.123)$	$(0.864, 0.136, 0.081)$
A_2	$(0.744, 0.256, 0.204)$	$(0.652, 0.348, 0.293)$	$(0.608, 0.392, 0.336)$
A_3	$(0.779, 0.256, 0.204)$	$(0.811, 0.189, 0.109)$	$(0.850, 0.150, 0.092)$
A_4	$(0.727, 0.273, 0.221)$	$(0.791, 0.209, 0.148)$	$(0.808, 0.192, 0.127)$

Table 13: Neutrosophic Decision Matrix.

$$d_{ij}^{w} = \Big(a_j T_{ij}, b_j + I_{ij} - b_j I_{ij}, c_j + F_{ij} - c_j F_{ij}\Big) \tag{5.21}$$

SVNs Weighted Decision Matrix is obtained by multiplication of weights of criteria with its associated column of neutrosophic decision matrix:

$$T_{11}^{\omega} = 0.864 \times 0.755 = 0.6523$$

$$I_{11}^{\omega} = 0.136 + 0.222 - 0.136 \times 0.222 = 0.328$$

$$F_{11}^{\omega} = 0.081 + 0.217 - 0.081 \times 0.217 = 0.280$$

d_{ij}^{w}	C_1	C_2	C_3
A_1	$(0.6523, 0.328, 0.28)$	$(0.7566, 0.2434, 0.1892)$	$(0.612, 0.381, 0.305)$
A_2	$(0.5036, 0.481, 0.434)$	$(0.6448, 0.3552, 0.3026)$	$(0.510, 0.484, 0.409)$
A_3	$(0.6644, 0.315, 0.269)$	$(0.787, 0.2132, 0.1642)$	$(0.638, 0.354, 0.274)$
A_4	$(0.5036, 0.481, 0.434)$	$(0.6519, 0.3481, 0.2811)$	$(0.588, 0.406, 0.329)$
	C_4	C_5	C_6
A_1	$(0.487, 0.491, 0.432)$	$(0.649, 0.342, 0.281)$	$(0.605, 0.371, 0.305)$
A_2	$(0.515, 0.462, 0.404)$	$(0.514, 0.478, 0.420)$	$(0.426, 0.557, 0.498)$
A_3	$(0.539, 0.462, 0.404)$	$(0.639, 0.351, 0.269)$	$(0.595, 0.381, 0.314)$
A_4	$(0.503, 0.474, 0.417)$	$(0.623, 0.367, 0.301)$	$(0.566, 0.412, 0.34)$

Table 14: Weighted Neutrosophic decision matrix.

Step 3: Determination of LNIS and SNIS.

	C_1	C_2	C_3
$d_j^{\omega+}$	$(0.664, 0.315, 0.269)$	$(0.887, 0.213, 0.264)$	$(0.638, 0.354, 0.274)$
	C_4	C_5	C_6
$d_j^{\omega+}$	$(0.539, 0.462, 0.404)$	$(0.649, 0.341, 0.294)$	$(0.605, 0.371, 0.305)$

Table 15: Maximum (large) Neutrosophic Ideal Solution(LNIS).

	C_1	C_2	C_3
$d_j^{\omega-}$	$(0.504, 0.481, 0.434)$	$(0.645, 0.355, 0.303)$	$(0.510, 0.484, 0.409)$
	C_4	C_5	C_6
$d_j^{\omega-}$	$(0.487, 0.491, 0.432)$	$(0.514, 0.478, 0.420)$	$(0.426, 0.557, 0.498)$

Table 16: Minimum (smaller) Neutrosophic Ideal Solution (SNIS).

	NS_i^+	NS_i^-	NT_i
A_1	*0,324*	*2,07*	*0,86459295*
A_2	*2,31*	*0,084*	*0,03521102*
A_3	*0,047*	*2,347*	***0,98021972***
A_4	*1,293*	*1,101*	*0,45987356*

Table 17: Neutrosophic Separation Measures and Neutrosophic Measure Ranking.

Step 4: Calculation of NS_i^+ and NS_i^- To calculate NS_i^+ and NS_i^-, we calculate sum of each line, and then subtracting from the LNIS and from SNIS respectively.

According to the obtained result (Table-17.), alternatives can be ranked as follow $A_3 > A_1 > A_4 > A_2$. Then the best preference is A_3. Using the same example, our proposed method neutrosophic-simplified-TOPSIS(nTOPSIS), we get similar result as neutrosophic-TOPSIS.

6 Conclusion

This paper aims to present tow new TOPSIS based approaches for MCDM. First one is Simplified TOPSIS (sTOPSIS) that simplify the TOPSIS calculation procedure. Second one, neutrosophic simplified-TOPSIS (nTOPSIS) extend the proposed method to neutrosophic environment, that use, instead of crisp number, the single valued neutrosophic(SVN). To formulate the both proposed method, many measures are defined such as Neutrosophic Manhattan Distance measure, that is used to calculate, distances from Maximum (larger) Neutrosophic Ideal Solution (LNIS) minimum neutrosophic ideal solutions, as two new defined measures.

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