Neutrosophic Sets and Systems

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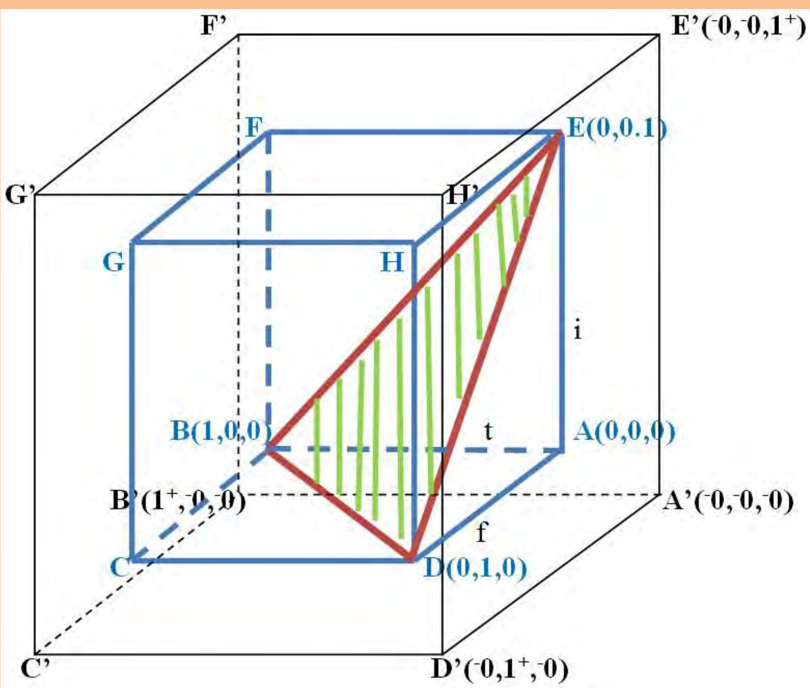

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"Neutrosophic Sets and Systems" has been created for publications on advanced studies in neutrosophy, neutrosophic set, neutrosophic logic, neutrosophic probability, neutrosophic statistics that started in 1995 and their applications in any field, such as the neutrosophic structures developed in algebra, geometry, topology, etc.

The submitted papers should be professional, in good English, containing a brief review of a problem and obtained results.

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e. notions or ideas supporting neither <A> nor <antiA>). The <neutA> and <antiA> ideas together are referred to as <nonA>.

Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only).

According to this theory every idea <A> tends to be neutralized and balanced by <antiA> and <nonA> ideas - as a state of equilibrium.

In a classical way <A>, <neutA>, <antiA> are disjoint two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well.

Neutrosophic Set and *Neutrosophic Logic* are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and respectively intuitionistic fuzzy logic). In neutrosophic logic a proposition has a degree of truth (T), a degree of indeterminacy (I), and a degree of falsity (F), where T, I, F are standard or non-standard subsets of $]^-0, 1^+[$.

Neutrosophic Probability is a generalization of the classical probability and imprecise probability.

Neutrosophic Statistics is a generalization of the classical statistics.

What distinguishes the neutrosophics from other fields is the <neutA>, which means neither <A> nor <antiA>.

<neutA>, which of course depends on <A>, can be indeterminacy, neutrality, tie game, unknown, contradiction, ignorance, imprecision, etc.

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Multi-attribute Decision Making based on Rough Neutrosophic Variational Coefficient Similarity Measure

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Abstract: The purpose of this study is to propose new similarity measures namely rough variational coefficient similarity measure under the rough neutrosophic environment. The weighted rough variational coefficient similarity measure has been also defined. The weighted rough variational coefficient similarity measures between the rough ideal alternative and each alternative are calculated to find the best alternative. The ranking order of all the alternatives can be determined by using the numerical values of similarity measures. Finally, an illustrative example has been provided to show the effectiveness and validity of the proposed approach. Comparisons of decision results of existing rough similarity measures have been provided.

Keywords: Neutrosophic set, Rough neutrosophic set; Rough variation coefficient similarity measure; Decision making.

1 Introduction

In 1965, L. A. Zadeh grounded the concept of degree of membership and defined fuzzy set [1] to represent/manipulate data with non-statistical uncertainty. In 1986, K. T. Atanassov [2] introduced the degree of non-membership as independent component and proposed intuitionistic fuzzy set (IFS). F. Smarandache introduced the degree of indeterminacy as independent component and defined the neutrosophic set [3, 4, 5]. For purpose of solving practical problems, Wang et al. [6] restricted the concept of neutrosophic set to single valued neutrosophic set (SVNS), since single value is an instance of set value. SVNS is a subclass of the neutrosophic set. SVNS consists of the three independent components namely, truth-membership, indeterminacy-membership and falsity-membership functions.

The concept of rough set theory proposed by Z. Pawlak [7] is an extension of the crisp set theory for the study of intelligent systems characterized by inexact, uncertain or insufficient information. The hybridization of rough set theory and neutrosophic set theory produces the rough neutrosophic set theory [8, 9], which was proposed by Broumi, Dhar and Smarandache [8, 9]. Rough neutrosophic set theory is also a powerful mathematical tool to deal with incompleteness.

Literature review reflects that similarity measures play an important role in the analysis and research of clustering analysis, decision making, medical diagnosis, pattern recognition, etc. Various similarity measures [10, 11, 12, 13, 14, 15, 16, 17, 18] of SVNSs and hybrid SVNSs are available in the literature. The concept of similarity measures in rough neutrosophic environment [19, 20, 21] has been recently proposed.

Pramanik and Mondal [19] proposed cotangent similarity measure of rough neutrosophic sets. In the same study [19],Pramanik and Mondal established its basic properties and provided its application to medical diagnosis. Pramanik and Mondal [20] also proposed cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis. The same authors [21] also studied Jaccard similarity measure and Dice similarity measures in rough neutrosophic environment and provided their applications to multi attribute decision making. Mondal and Pramanik [22] presented tri-complex rough neutrosophic similarity measure and its application in multi-attribute decision making. Together with F. Smarandache and S. Pramnik, K. Mondal [23] presented hypercomplex rough neutrosophic similarity measure and its application in multi-attribute decision making. Mondal, Pramanik, and Smarandache [24] presented several trigonometric Hamming similarity measures of rough neutrosophic sets and their applications in multi attribute decision making problems.

Different methods for multiattribute decision making (MADM) and multicriteria decision making (MCDM) problems are available in the literature in different environment such as crisp environment [25, 26, 27, 28, 29], fuzzy environment [30, 31], intuitionistic fuzzy environment [32, 33, 34, 35, 36, 37, 38, 39, 40], neutrosophic environment [41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62], interval neutrosophic environment [63, 65, 66, 67, 68], neutrosophic soft expert environment [69], neutrosophic bipolar environment [70, 71], neutrosophic soft environment [72, 73, 74, 75, 76], neutrosophic hesitant fuzzy environment [77, 78, 79], rough neutrolsophic environment [80, 81], etc. In neutrosophic environment Biswas, Pramanik and Giri [82] studied hybrid vector similarity measure and its application in multi-attribute decision making. Getting motivation from the work of Biswas, Pramanik and Giri [82], for hybrid vector similarity measure in neutrosophic envionment, we extend the concept in rough neutrosophic environment.

In this paper, a new similarity measurement is proposed, namely rough variational coefficient similarity measure under rough neutrosophic environment. A numerical example is also provided.

Rest of the paper is structured as follows. Section 2 presents neutrosophic and rough neutrosophic preliminaries. Section 3 discusses various similarity measures and varional coefficient similarity measure in crisp environment. Section 4 presents various similarity measures and variational similarity measure for single valued neutrosophic sets. Section 5 presents variational coefficient similarity measure and weighted variational coefficient similarity measure for rough neutrosophic sets and establishes their basic properties. Section 6 is devoted to present multi attribute decision making based on rough neutrosophic variational coefficient similarity measure. Section 7 demonstrates the application of rough variational coefficient similarity measures to investment problem Finally, section 8 concludes the paper with stating the future scope of research.

2 Neutrosophic preliminaries

Definition 2.1 [3, 4, 5] Neutrosophic set

Let X be a space of points (objects) with generic element in X denoted by x. Then a neutrosophic set A in X is denoted by $A=\{x(T_A(x), I_A(x), F_A(x)): x \in X\}$ where, $T_A(x)$ is the truth membership function, $I_A(x)$ is the indeterminacy membership function and $F_A(x)$ is the falsity membership function. The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]^-0, 1^+[$. There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ i.e. $^-0 \le T_A(x)+I_A(x)+F_A(x) \le 3^+$.

Definition 2.2 [6] (**Single-valued neutrosophic set**).

Let X be a universal space of points (objects), with a generic element $x \in X$. A single-valued neutrosophic set (SVNS) $N \subset X$ is denoted by

$N=\int_x \langle T_N(x), I_N(x), F_N(x)\rangle / x, \forall x \in X$, when X is continuous;

$N=\sum_{i=1}^{m} \langle T_N(x), I_N(x), F_N(x)\rangle / x, \forall x \in X$, when X is discrete.

SVNS is characterized by a true membership function $T_N(x)$, a falsity membership function $F_N(x)$ and an indeterminacy function $I_N(x)$ ith $T_N(x)$, $F_N(x)$, $I_N(x)$ $\in$ [0, 1] for all $x \in X$. For each $x \in X$, of a SVNS N $0 \le T_N(x)+I_N(x)+F_N(x) \le 3$.

2.1 Some operational rules and properties of SVNSs

Let $N_A=\langle T_A, I_A, F_A\rangle$ and $N_B=\langle T_B, I_B, F_B\rangle$ be two SVNSs in *X*. Then the following operations are defined as follows:

I. Complement: $N_{A^c}=\langle F_A, 1\text{-}I_A, T_A\rangle \ \forall x \in X$.

II. Addition: $N_A \oplus N_B=\langle T_A+T_B-T_AT_B, I_AI_B, F_AF_B\rangle$

III. Multiplication:

$N_A \otimes N_B=\langle T_AT_B, I_A+I_B\text{-}I_AI_B, F_A+F_B\text{-}F_AF_B\rangle$

IV. Scalar Multiplication:

$\lambda N_A=\langle 1-(1-T_A)^\lambda, I_A^\lambda, F_A^\lambda\rangle$ for $\lambda>0$.

V. $\langle N_A\rangle^\lambda=\langle (T_A)^\lambda, 1-(1-I_A)^\lambda, 1-(1-F_A)^\lambda\rangle$ for $\lambda>0$.

Definition 2.3 [6]

Complement of a SVNS N is denoted by N^c and is defined by

$T_{N^c}(x)=F_N(x)$; $I_{N^c}(x)=1-I_N(x)$; $F_{N^c}(x)=T_N(x)$

Definition 2.4 [6]

A SVNS N_A is contained in the other SVNS N_B, denoted as $N_A \subseteq N_B$, if and only if

$T_{N_A}(x) \le T_{N_B}(x)$; $I_{N_A}(x) \ge I_{N_B}(x)$; $F_{N_A}(x) \ge F_{N_B}(x) \ \forall x \in X$

Definition 2.5 [6]

Two SVNSs N_A and N_B are equal, i.e. $N_A= N_B$, if and only if $N_A \supseteq N_B$ and $N_A \subseteq N_B$

Definition 2.6 [6]

Union of two SVNSs N_A and N_B is a SVNS N_C, written as $N_C = N_A \cup N_B$. Its truth membership, indeterminacy-membership and falsity membership functions are related to those of N_A and N_B by

$T_{N_C}(x)=\max(T_{N_A}(x), T_{N_B}(x))$; $I_{N_C}(x)=\min(I_{N_A}(x), I_{N_B}(x))$; $F_{N_C}(x)=\min(F_{N_A}(x), F_{N_B}(x))$ for all x in X.

Definition 2.7 [6] Intersection of two SVNSs N_A and N_B is a SVNS N_D, written as $N_D = N_A \cap N_B$, whose truth membership, indeterminacy-membership and falsity membership functions are related to those of N_A and N_B by $T_{N_C}(x)=\min(T_{N_A}(x), T_{N_B}(x))$; $I_{N_C}(x)=\max(I_{N_A}(x), I_{N_B}(x))$; $F_{N_C}(x)=\max(F_{N_A}(x), F_{N_B}(x))$ for all x in X.

Definition 2.8 Rough Neutrosophic Sets [8, 9]

Let Z be a non-null set and R be an equivalence relation on Z. Let P be neutrosophic set in Z with the membership function T_P indeterminacy function I_P and non-membership function F_P. The lower and the upper approximations of P in the approximation (Z, R) denoted by $\underline{N}(P)$ and $\overline{N}(P)$ are respectively defined as follows:

$$\underline{N}(P)=\left\langle <x, T_{\underline{N}(P)}(x), I_{\underline{N}(P)}(x), F_{\underline{N}(P)}(x)> / z\in[x]_R, x\in Z\right\rangle,$$

$$\overline{N}(P)=\left\langle <x, T_{\overline{N}(P)}(x), I_{\overline{N}(P)}(x), F_{\overline{N}(P)}(x)> / z\in[x]_R, x\in Z\right\rangle,$$

Here, $T_{\underline{N}(P)}(x)=\wedge_z\in[x]_R\, T_P(z)$, $I_{\underline{N}(P)}(x)=\wedge_z\in[x]_R\, I_P(z)$, $F_{\underline{N}(P)}(x)=\wedge_z\in[x]_R\, F_P(z)$, $T_{\overline{N}(P)}(x)=\vee_z\in[x]_R\, T_P(z)$, $I_{\overline{N}(P)}(x)=\vee_z\in[x]_R\, I_P(z)$, $F_{\overline{N}(P)}(x)=\vee_z\in[x]_R\, F_P(z)$

So, $0\leq \sup T_{\underline{N}(P)}(x)+\ \sup I_{\underline{N}(P)}(x)+\ \sup F_{\underline{N}(P)}(x)\ \leq 3$

$0\leq \sup T_{\overline{N}(P)}(x)+\ \sup I_{\overline{N}(P)}(x)+\ \sup F_{\overline{N}(P)}(x)\ \leq 3$

Here $\vee$ and $\wedge$ denote "max" and "min'' operators respectively. $T_P(z)$, $I_P(z)$ and $F_P(z)$ denote respectively the membership, indeterminacy and non-membership function of z with respect to P. It is easy to see that $\underline{N}(P)$ and $\overline{N}(P)$ are two neutrosophic sets in Z.

Thus NS mappings $\underline{N}$, $\overline{N}$: $N(Z) \rightarrow N(Z)$ are, respectively, referred to as the lower and the upper rough NS approximation operators, and the pair $(\underline{N}(P), \overline{N}(P))$ is called the rough neutrosophic set [8, 9] in (Z, R).

From the above definition, it is seen that $\underline{N}(P)$ and $\overline{N}(P)$ have constant membership on the equivalence classes of R. if $\underline{N}(P) = \overline{N}(P)$ i.e. $T_{\underline{N}(P)}(x)= T_{\overline{N}(P)}(x)$, $I_{\underline{N}(P)}(x)= I_{\overline{N}(P)}(x)$ and $F_{\underline{N}(P)}(x)= F_{\overline{N}(P)}(x), \forall x\in Z$.

P is said to be a definable neutrosophic set in the approximation (Z, R). It can be easily proved that zero neutrosophic set (0_N = (0, 1, 1)) and unit neutrosophic sets (1_N = (1, 0, 0)) are definable neutrosophic sets.

Definition 2.9 [8, 9]

If $N(P)=(\underline{N}(P), \overline{N}(P))$ is a rough neutrosophic set in (Z, R) , the rough complement [8, 9] of $N(P)$ is the rough neutrosophic set denoted by $\sim N(P)=(\underline{N}(P)^c, \overline{N}(P)^c)$ where $\underline{N}(P)^c$, $\overline{N}(P)^c$ are the complements of neutrosophic sets of $\underline{N}(P)$, $\overline{N}(P)$ respectively.

$$\underline{N}(P)^c=\left\langle <x, F_{\underline{N}(P)}(x), 1-I_{\underline{N}(P)}(x), T_{\underline{N}(P)}(x)> / x\in Z\right\rangle \text{ and}$$

$$\overline{N}(P)^c=\left\langle <x, F_{\overline{N}(P)}(x), 1-I_{\overline{N}(P)}(x), T_{\overline{N}(P)}(x)> / x\in Z\right\rangle$$

Definition 2.10 [8, 9]

If $N(P_1)$ and $N(P_2)$ are the two rough neutrosophic sets of the neutrosophic set P respectively in Z, then the following definitions [8, 9] hold:

$$N(P_1)=N(P_2)\Leftrightarrow \underline{N}(P_1)=\underline{N}(P_2)\wedge \overline{N}(P_1)=\overline{N}(P_2)$$

$$N(P_1)\subseteq N(P_2)\Leftrightarrow \underline{N}(P_1)\subseteq \underline{N}(P_2)\wedge \overline{N}(P_1)\subseteq \overline{N}(P_2)$$

$$N(P_1)\cup N(P_2)=<\underline{N}(P_1)\cup \underline{N}(P_2), \overline{N}(P_1)\cup \overline{N}(P_2)>$$

$$N(P_1)\cap N(P_2)=<\underline{N}(P_1)\cap \underline{N}(P_2), \overline{N}(P_1)\cap \overline{N}(P_2)>$$

$$N(P_1)+N(P_2)=<\underline{N}(P_1)+\underline{N}(P_2), \overline{N}(P_1)+\overline{N}(P_2)>$$

$$N(P_1).N(P_2)=<\underline{N}(P_1).\underline{N}(P_2), \overline{N}(P_1).\overline{N}(P_2)>$$

If N, M, L are the rough neutrosophic sets in (Z, R), then the following proposition are stated from definitions [8, 9].

Proposition 1 [8, 9]

1. $\sim(\sim N)=N$
2. $N\cup M=M\cup N, N\cap M=M\cap N$
3. $(L\cup M)\cup N=L\cup(M\cup N)$, $(L\cap M)\cap N=L\cap(M\cap N)$
4. $(L\cup M)\cap N=(L\cup M)\cap(L\cup N)$, $(L\cap M)\cup N=(L\cap M)\cup(L\cap N)$

Proposition 2 [8, 9]

De Morgan's Laws are satisfied for rough neutrosophic sets .

1. $\sim(N(P_1)\cup N(P_2))=(\sim(N(P_1))\cap(\sim N(P_2))$
2. $\sim(N(P_1)\cap N(P_2))=(\sim(N(P_1)\cup(\sim N(P_2))$

Proposition 3 [8, 9]

If P_1 and P_2 are two neutrosophic sets in U such that $P_1\subseteq P_2$ *then* $N(P_1)\subseteq N(P_2)$

1. $N(P_1\cap P_2)\subseteq N(P_1)\cap N(P_2)$
2. $N(P_1\cup P_2)\supseteq N(P_1)\cup N(P_2)$

Proposition 4 [8, 9]

1. $\underline{N}(P)=\sim\overline{N}(\sim P)$
2. $\overline{N}(P)=\sim\underline{N}(\sim P)$
3. $\underline{N}(P)\subseteq\overline{N}(P)$

3 Similarity measures and variational coefficient similarity measure in crisp environment

The vector similarity measure is one of the important tools for the degree of similarity between objects. However, the Jaccard, Dice, and cosine similarity measures are often used for this purpose. Jaccard [83] , Dice [84] , and cosine [85] similarity measures between two vectors are stated below.

Let X = (x_1, x_2, …, x_n) and Y = (y_1, y_2, …, y_n) be two n-dimensional vectors with positive co-ordinates.

Definition 3.1 [83]

Jeccard index of two vectors (measuring the "similarity" of these vectors) can be defined as follows:

$$J(X, Y) = \frac{X.Y}{\|X\|^2+\|Y\|^2-X.Y} = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2+\sum_{i=1}^{n} y_i^2-\sum_{i=1}^{n} x_i y_i} \quad (1)$$

where $\|X\|^2 = \sum_{i=1}^{n} x_i^2$ and $\|Y\|^2 = \sum_{i=1}^{n} y_i^2$ are the Euclidean norm of X and Y, $X.Y=\sum_{i=1}^{n} x_i y_i$ is the inner product of the vector X and Y.

Proposition 5 [83]

Jaccard index satisfies the following properties:

1. $0\leq$ J(X, Y) ≤ 1

2. J(X, Y) = J(Y, X)
3. J(X, Y) = 1, for $X = Y$ i.e, $x_i = y_i$ (i = 1, 2, …, n) for every $x_i \in X$ and $y_i \in Y$

Definition 3.2 [84]

The Dice similarity measure can be defined as follows:

$$E(X, Y) = \frac{2\, X.Y}{\|X\|^2 + \|Y\|^2} = \frac{2\sum_{i=1}^{n} x_i\, y_i}{\sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{n} y_i^2} \quad (2)$$

Proposition 6 [84]

The Dice similarity measure satisfies the following properties:

1. 0 ≤ E(X, Y) ≤ 1
2. E(X, Y) = E(Y, X)
3. J(X, Y) = 1, for $X = Y$ i.e, $x_i = y_i$ (i = 1, 2, …, n) for every $x_i \in X$ and $y_i \in Y$.

Definition 3.3 [85]

The cosine similarity measure between two vectors X and Y is the inner product of these two vectors divided by the product of their lengths and can be defined as follows:

$$C(X, Y) = \frac{X.Y}{\|X\|.\|Y\|} = \frac{\sum_{i=1}^{n} x_i\, y_i}{\sqrt{\sum_{i=1}^{n} x_i^2}\sqrt{\sum_{i=1}^{n} y_i^2}} \quad (3)$$

Proposition 7 [85]

The cosine similarity measure satisfies the following properties

1. 0 ≤ C(X, Y) ≤ 1
2. C(X, Y) = C(Y, X)
3. C(X, Y) = 1, for $X = Y$ i.e, $x_i = y_i$ (i = 1, 2, …, n) for every $x_i \in X$ and $y_i \in Y$.

These three formulas are similar in the sense that they take values in the interval [0, 1]. Jaccard and Dice similarity measures are undefined when $x_i = 0$, and $y_i = 0$ for i = 1, 2, …, n and cosine similarity measure is undefined when $x_i = 0$ or $y_i = 0$ for i = 1, 2, …, n.

Definition 3.4 [86]

Variational co-efficient similarity measure can be defined as follows:

$$V(X, Y) = \lambda \frac{2\, X.Y}{\|X\|^2 + \|Y\|^2} + (1\text{-}\lambda)\frac{X.Y}{\|X\|.\|Y\|}$$

$$= \lambda \frac{2\sum_{i=1}^{n} x_i\, y_i}{\sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{n} y_i^2} + (1-\lambda)\frac{\sum_{i=1}^{n} x_i\, y_i}{\sqrt{\sum_{i=1}^{n} x_i^2}\sqrt{\sum_{i=1}^{n} y_i^2}} \quad (4)$$

Proposition 8 [86]

Variational co-efficient similarity measure satisfies the following properties:

1. 0 ≤ V(X, Y) ≤ 1
2. V(X, Y) = V(Y, X)
3. V(X, Y) = 1, for $X = Y$ i.e, $x_i = y_i$ (i = 1, 2, …, n) for every $x_i \in X$ and $y_i \in Y$.

4. Various similarity measures for single valued neutrosophic sets.

Assume $N_A = \langle T_A, I_A, F_A \rangle$ and $N_B = \langle T_B, I_B, F_B \rangle$ be two SVNSs in a universe of discourse $X = (x_1, x_2, …, x_n)$. $T_A, I_A, F_A \in [0,1]$ for any $x_i \in X$ in N_A or $T_B, I_B, F_B \in [0,1]$ for any $x_i \in X$ in N_B can be considered as a vector representation with three elements. Let $w_i \in [0,1]$ be the weight of each element x_i for i = 1, 2, …, n such that $\sum_{i=1}^{n} w_i = 1$, then Jaccard, Dice and cosine similarity measures can be presented as follows:

Definition 4.1[10] Jaccard similarity measure between $N_A = \langle T_A, I_A, F_A \rangle$ and $N_B = \langle T_B, I_B, F_B \rangle$ can be defined as follows:

$Jac(N_A, N_B)$ =

$$\frac{1}{n}\sum_{i=1}^{n} \frac{(T_A(x_i)\,T_B(x_i) + I_A(x_i)\,I_B(x_i) + F_A(x_i)\,F_B(x_i))}{\left\langle \begin{array}{l} [(T_A(x_i))^2 + (I_A(x_i))^2 + (F_A(x_i))^2] \\ + [(T_B(x_i))^2 + (I_B(x_i))^2 + (F_B(x_i))^2] \\ - \{T_A(x_i)\,T_B(x_i) + I_A(x_i)\,I_B(x_i) \\ + F_A(x_i)\,F_B(x_i)\} \end{array} \right\rangle} \quad (5)$$

Proposition 9 [10]

Jaccard similarity measure satiefies the following properites:

1. $0 \le Jac(N_A, N_B) \le 1$;
2. $Jac(N_A, N_B) = Jac(N_B, N_A)$;
3. $Jac(N_A, N_B) = 1$; if $N_A = N_B$ i.e., $T_A(x_i) = T_B(x_i)$, $I_A(x_i) = I_B(x_i)$, and $F_A(x_i) = F_B(x_i)$, for every x_i (i = 1, 2, …, n) in X.

Definition 4.1.1 [10] Weighted Jaccard similarity measure between $N_A = \langle T_A, I_A, F_A \rangle$ and $N_B = \langle T_B, I_B, F_B \rangle$ can be defined as follows:

$Jac_w(N_A, N_B)$=

$$\sum_{i=1}^{n} w_i \frac{(T_A(x_i)\,T_B(x_i) + I_A(x_i)\,I_B(x_i) + F_A(x_i)\,F_B(x_i))}{\left\langle \begin{array}{l} [(T_A(x_i))^2 + (I_A(x_i))^2 + (F_A(x_i))^2] \\ + [(T_B(x_i))^2 + (I_B(x_i))^2 + (F_B(x_i))^2] \\ - \{T_A(x_i)\,T_B(x_i) + I_A(x_i)\,I_B(x_i) \\ + F_A(x_i)\,F_B(x_i)\} \end{array} \right\rangle} \quad (6)$$

Proposition 10 [10]

Weighted Jaccard similarity measure satisfies the following properties:

1. $0 \le Jac_w(N_A, N_B) \le 1$;
2. $Jac_w(N_A, N_B) = Jac_w(N_B, N_A)$;
3. $Jac_w(N_A, N_B) = 1$; if $N_A = N_B$ i.e., $T_A(x_i) = T_B(x_i)$, $I_A(x_i) = I_B(x_i)$, and $F_A(x_i) = F_B(x_i)$, for every x_i (i = 1, 2, …, n) in X.

Definition 4.2 [11]

Dice similarity measure between $N_A=\langle T_A, I_A, F_A\rangle$ and $N_B=\langle T_B, I_B, F_B\rangle$ is defined as:

$Dic(N_A, N_B)$ =

$$\frac{1}{n}\sum_{i=1}^{n}\frac{2\begin{bmatrix}T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)\\+F_A(x_i)F_B(x_i)\end{bmatrix}}{\left\langle\begin{matrix}\left[(T_A(x_i))^2+(I_A(x_i))^2+(F_A(x_i))^2\right]\\+\left[(T_B(x_i))^2+(I_B(x_i))^2+(F_B(x_i))^2\right]\end{matrix}\right\rangle} \quad (7)$$

Proposition 11 [11]

Dice similarity measure satisfies the following properties:

1. $0 \le Dic(N_A, N_B) \le 1$;
2. $Dic(N_A, N_B) = Dic(N_B, N_A)$;
3. $Dic(N_A, N_B) = 1$; if $N_A = N_B$ i.e., $T_A(x_i) = T_B(x_i)$, $I_A(x_i) = I_B(x_i)$, and $F_A(x_i) = F_B(x_i)$, for every x_i (i = 1, 2, …, n) in X.

Definition 4.2.1 [11]

Weighted Dice similarity measure between $N_A=\langle T_A, I_A, F_A\rangle$ and $N_B=\langle T_B, I_B, F_B\rangle$ can be defined as follows:

$Dic_w(N_A, N_B)$ =

$$\sum_{i=1}^{n} w_i\frac{2\begin{bmatrix}T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)\\+F_A(x_i)F_B(x_i)\end{bmatrix}}{\left\langle\begin{matrix}\left[(T_A(x_i))^2+(I_A(x_i))^2+(F_A(x_i))^2\right]\\+\left[(T_B(x_i))^2+(I_B(x_i))^2+(F_B(x_i))^2\right]\end{matrix}\right\rangle} \quad (8)$$

Proposition 12 [11]

Weighted Dice similarity measure

1. $0 \le Dic_w(N_A, N_B) \le 1$;
2. $Dic_w(N_A, N_B) = Dic_w(N_B, N_A)$;
3. $Dic_w(N_A, N_B) = 1$; if $N_A = N_B$ i.e., $T_A(x_i) = T_B(x_i)$, $I_A(x_i) = I_B(x_i)$, and $F_A(x_i) = F_B(x_i)$, for every x_i (i = 1, 2, …, n) in X.

Definition 4.3 [12]

Cosine similarity measure between $N_A=\langle T_A, I_A, F_A\rangle$ and $N_B=\langle T_B, I_B, F_B\rangle$ can be defined as follows:

$$Cos(N_A, N_B) = \frac{1}{n}\sum_{i=1}^{n}\frac{(T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i))}{\sqrt{(T_A(x_i))^2+(I_A(x_i))^2+(F_A(x_i))^2}\sqrt{(T_B(x_i))^2+(I_B(x_i))^2+(F_B(x_i))^2}} \quad (9)$$

Proposition 13 [12]

Cosine similarity measure satisfies the following properties:

1. $0 \le Cos_w(N_A, N_B) \le 1$;
2. $Cos_w(N_A, N_B) = Cos_w(N_B, N_A)$
3. $Cos_w(N_A, N_B) = 1$; if $N_A = N_B$ i.e., $T_A(x_i) = T_B(x_i)$, $I_A(x_i) = I_B(x_i)$, and $F_A(x_i) = F_B(x_i)$, for every x_i (i = 1, 2, …, n) in X.

Definition 4.3.1 [12]

Weighted cosine similarity measure between $N_A=\langle T_A, I_A, F_A\rangle$ and $N_B=\langle T_B, I_B, F_B\rangle$ can be defined as follows:

$Cos_w(N_A, N_B)$=

$$\sum_{i=1}^{n} w_i\frac{(T_A(x_i)T_B(x_i)+I_A(x_i)I_B(x_i)+F_A(x_i)F_B(x_i))}{\sqrt{(T_A(x_i))^2+(I_A(x_i))^2+(F_A(x_i))^2}\sqrt{(T_B(x_i))^2+(I_B(x_i))^2+(F_B(x_i))^2}} \quad (10)$$

Proposition 14 [12]

Weighted cosine similarity measure satisfies the following properties:

1. $0 \le Cos_w(N_A, N_B) \le 1$;
2. $Cos_w(N_A, N_B) = Cos_w(N_B, N_A)$
3. $Cos_w(N_A, N_B) = 1$; if $N_A = N_B$ i.e., $T_A(x_i) = T_B(x_i)$, $I_A(x_i) = I_B(x_i)$, and $F_A(x_i) = F_B(x_i)$, for every x_i (i = 1, 2, …, n) in X.

Jaccard and Dice similarity measures between two neutrosophic sets $N_A=\langle T_A, I_A, F_A\rangle$ and $N_B = \langle T_B, I_B, F_B\rangle$ are undefined when $T_A(x_i) = I_A(x_i) = F_A(x_i) = 0$ and $T_B(x_i) = I_B(x_i) = F_B(x_i) = 0$ for all i = 1, 2, …, n. Similarly the cosine formula for two neutrosophic sets $N_A=\langle T_A, I_A, F_A\rangle$ and $N_B=\langle T_B, I_B, F_B\rangle$ is undefined when $T_A(x_i) = I_A(x_i) = F_A(x_i) = 0$ or $T_B(x_i) = I_B(x_i) = F_B(x_i) = 0$ for all i = 1, 2, …, n.

5 Variational similarity measures for rough neutrosophic sets

The notion of rough neutrosophic set (RNS) is used as vector representations in 3D-vector space. Assume that $X = (x_1, x_2,\ldots, x_n)$ and $Y = (y_1, y_2, \ldots, y_n)$ be two n-dimensional vectors with positive co-ordinates. Jaccard, Dice, cosine and cotangent similarity measures between two vectors are stated as follows.

Definition 5.1 [21] Jaccard similarity measure under rough neutrosophic environment

Assume that $A = \left\langle\left(\underline{T}_A(x_i), \underline{I}_A(x_i), \underline{F}_A(x_i)\right), \left(\overline{T}_A(x_i), \overline{I}_A(x_i), \overline{F}_A(x_i)\right)\right\rangle$ and $B = \left\langle\left(\underline{T}_B(x_i), \underline{I}_B(x_i), \underline{F}_B(x_i)\right), \left(\overline{T}_B(x_i), \overline{I}_B(x_i), \overline{F}_B(x_i)\right)\right\rangle$ in $X = (x_1, x_2,\ldots, x_n)$ be any two rough neutrosophic sets. Jacard similarity measure [21] between rough neutrosophic sets A and B can be defined as follows:

$Jac_{RNS}(A, B)$ =

$$\frac{1}{n}\sum_{i=1}^{n}\frac{(\delta T_A(x_i)\,\delta T_B(x_i)+\delta I_A(x_i)\,\delta I_B(x_i)+\delta F_A(x_i)\,\delta F_B(x_i))}{\left\langle \begin{array}{l}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\ +\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\\ -[\delta T_A(x_i)\,\delta T_B(x_i)+\delta I_A(x_i)\,\delta I_B(x_i)\\ +\delta F_A(x_i)\,\delta F_B(x_i)]\end{array}\right\rangle} \quad (11)$$

Proposition 15 [21]

Jaccard similarity measure [21] between A and B satisfies the following properties:

1. $0 \le Jac_{RNS}(A,B) \le 1$;
2. $Jac_{RNS}(A,B) = Jac_{RNS}(B,A)$;
3. $Jac_{RNS}(A,B)=1$; iff $A = B$
4. If C is a RNS in Y and $A \subset B \subset C$ then,

$Jac_{RNS}(A, C) \le Jac_{RNS}(A, B)$, and $Jac_{RNS}(A, C) \le Jac_{RNS}(B, C)$

Definition 5.1.1 [21]

If we consider the weights of each element x_i, weighted rough Jaccard similarity measure [21] between rough neutrosophic sets A and B can be defined as follows:

$Jac_{WRNS}(A, B) =$

$$\sum_{i=1}^{n} w_i\frac{(\delta T_A(x_i)\,\delta T_B(x_i)+\delta I_A(x_i)\,\delta I_B(x_i)+\delta F_A(x_i)\,\delta F_B(x_i))}{\left\langle \begin{array}{l}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\ +\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\\ -[\delta T_A(x_i)\,\delta T_B(x_i)+\delta I_A(x_i)\,\delta I_B(x_i)\\ +\delta F_A(x_i)\,\delta F_B(x_i)]\end{array}\right\rangle} \quad (12)$$

$w_i \in [0,1]$, $i = 1, 2,\dots, n$ and $\sum_{i=1}^{n} w_i = 1$. If we take $w_i = \frac{1}{n}$, $i = 1, 2,\dots, n$, then $Jac_{WRNS}(A, B) = Jac_{RNS}(A,B)$

Proposition 16 [21]

The weighted rough Jaccard similarity [21] measure between two rough neutrosophic sets A and B also satisfies the following properties:

1. $0 \le Jac_{WRNS}(A,B) \le 1$;
2. $Jac_{WRNS}(A,B) = Jac_{WRNS}(B,A)$;
3. $Jac_{WRNS}(A,B)=1$; iff $A = B$
4. If C is a WRNS in Y and $A \subset B \subset C$ then, $Jac_{WRNS}(A, C) \le Jac_{WRNS}(A, B)$, and $Jac_{WRNS}(A, C) \le Jac_{WRNS}(B, C)$

Definition 5.2 [21] **Dice similarity measure under rough neutrosophic environment**

In this section, Dice similarity measure and the weighted Dice similarity measure for rough neutrosophic sets have been stated due to Pramanik and Mondal [21].

Suppose that

$A = \left\langle (\underline{T}_A(x_i), \underline{I}_A(x_i), \underline{F}_A(x_i)), (\overline{T}_A(x_i), \overline{I}_A(x_i), \overline{F}_A(x_i)) \right\rangle$ and

$B = \left\langle (\underline{T}_B(x_i), \underline{I}_B(x_i), \underline{F}_B(x_i)), (\overline{T}_B(x_i), \overline{I}_B(x_i), \overline{F}_B(x_i)) \right\rangle$ be any two rough neutrosophic sets in $X = (x_1, x_2,\dots, x_n)$. Dice similarity measure between rough neutrosophic sets A and B can be defined as follows:

$DIC_{RNS}(A, B)=$

$$\frac{1}{n}\sum_{i=1}^{n}\frac{2\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]}{\left\langle \begin{array}{l}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\ +\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\end{array}\right\rangle} \quad (13)$$

Proposition 17 [21]

Dice similarity measure [21] satisfies the following properties.

1. $0 \le DIC_{RNS}(A,B) \le 1$;
2. $DIC_{RNS}(A,B) = DIC_{RNS}(B,A)$;
3. $DIC_{RNS}(A,B)=1$; iff $A = B$
4. If C is a RNS in Y and $A \subset B \subset C$ then,

$DIC_{RNS}(A, C) \le DIC_{RNS}(A, B)$, and $DIC_{RNS}(A, C) \le DIC_{RNS}(B, C)$,

For proofs of the above mentioned four properties, see [21].

Definition 5.2.1

If we consider the weights of each element x_i, a weighted rough Dice similarity measure between rough neutrosophic sets A and B can be defined as follows:

$DIC_{WRNS}(A, B) =$

$$\sum_{i=1}^{n} w_i\frac{2\left[\begin{array}{l}\delta T_A(x_i)\,\delta T_B(x_i)+\delta I_A(x_i)\,\delta I_B(x_i)\\ +\delta F_A(x_i)\,\delta F_B(x_i)\end{array}\right]}{\left\langle \begin{array}{l}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\ +\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\end{array}\right\rangle} \quad (14)$$

$w_i \in [0,1]$, $i = 1, 2,\dots, n$ and $\sum_{i=1}^{n} w_i = 1$. If we take $w_i = \frac{1}{n}$, $i = 1, 2,\dots, n$, then $DIC_{WRNS}(A, B) = DIC_{RNS}(A,B)$

Proposition 18 [21]

The weighted rough Dice similarity [21] measure between two rough neutrosophic sets A and B also satisfies the following properties:

1. $0 \le DIC_{WRNS}(A,B) \le 1$;
2. $DIC_{WRNS}(A,B) = DIC_{WRNS}(B,A)$;
3. $DIC_{WRNS}(A,B)=1$; iff $A = B$
4. If C is a RNS in Y and $A \subset B \subset C$ then,

$DIC_{WRNS}(A, C) \le DIC_{WRNS}(A, B)$, and

$DIC_{WRNS}(A, C) \le DIC_{WRNS}(B, C)$.

For proofs of the above mentioned four properties, see [21].

Definition 5.3 [20]

Cosine similarity measure can be defined as the inner product of two vectors divided by the product of their lengths. It is the cosine of the angle between the vector representations of two rough neutrosophic sets. The cosine similarity measure is a fundamental measure used in information technology. Pramanik and Mondal [20] defined cosine similarity measure between rough neutrosophic sets in 3-D vector space.

Assume that $A=\langle(\underline{T}_A(x_i),\underline{I}_A(x_i),\underline{F}_A(x_i)),(\overline{T}_A(x_i),\overline{I}_A(x_i),\overline{F}_A(x_i))\rangle$ and $B=\langle(\underline{T}_B(x_i),\underline{I}_B(x_i),\underline{F}_B(x_i)),(\overline{T}_B(x_i),\overline{I}_B(x_i),\overline{F}_B(x_i))\rangle$ in $X = (x_1, x_2,\ldots, x_n)$ be any rough neutrosophic sets. Pramanik and Mondal [20] defined cosine similarity measure between rough neutrosophic sets A and B as follows:

C_{RNS}(A, B) =

$$\frac{1}{n}\sum_{i=1}^{n}\frac{\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)+\delta F_A(x_i)\delta F_B(x_i)}{\sqrt{\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]}\sqrt{\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]}} \quad (15)$$

Here, $\delta T_A(x_i)=\frac{\underline{T}_A(x_i)+\overline{T}_A(x_i)}{2}$, $\delta T_B(x_i)=\frac{\underline{T}_B(x_i)+\overline{T}_B(x_i)}{2}$, $\delta I_A(x_i)=\frac{\underline{I}_A(x_i)+\overline{I}_A(x_i)}{2}$, $\delta I_B(x_i)=\frac{\underline{I}_B(x_i)+\overline{I}_B(x_i)}{2}$, $\delta F_A(x_i)=\frac{\underline{F}_A(x_i)+\overline{F}_A(x_i)}{2}$, $\delta F_B(x_i)=\frac{\underline{F}_B(x_i)+\overline{F}_B(x_i)}{2}$

Proposition 19 [20]

Let A and B be rough neutrosophic sets. Cosine similarity measure [20] between A and B satisfies the following properties.

1. $0\le C_{RNS}(A,B)\le 1$;
2. $C_{RNS}(A,B)=C_{RNS}(B,A)$;
3. $C_{RNS}(A,B)=1$; iff $A = B$
4. If C is a RNS in Y and $A\subset B\subset C$ then, $C_{RNS}(A, C) \le C_{RNS}(A, B)$, and $C_{RNS}(A, C) \le C_{RNS}(B, C)$.

Definition 5.3.1 [20]

If we consider the weights of each element x_i, a weighted rough cosine similarity measure between rough neutrosophic sets A and B can be defined as follows:

$$C_{WRNS}(A,B)=\sum_{i=1}^{n}w_i\frac{\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)+\delta F_A(x_i)\delta F_B(x_i)}{\sqrt{\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]}\sqrt{\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]}} \quad (16)$$

$w_i\in[0,1]$, i = 1, 2,…, n and $\sum_{i=1}^{n}w_i=1$. If we take $w_i=\frac{1}{n}$, i = 1, 2,…, n, then $C_{WRNS}(A, B) = C_{RNS}(A, B)$

Proposition 20 [20]

The weighted rough cosine similarity measure [20] between two rough neutrosophic sets A and B also satisfies the following properties:

1. $0\le C_{WRNS}(A,B)\le 1$;
2. $C_{WRNS}(A,B)=C_{WRNS}(B,A)$;
3. $C_{WRNS}(A,B)=1$; iff $A = B$
4. If C is a WRNS in Y and $A\subset B\subset C$ then, $C_{WRNS}(A, C) \le C_{WRNS}(A, B)$, and $C_{WRNS}(A, C) \le C_{WRNS}(B, C)$.

For proofs of the above mentioned four properties, see [20].

Definition 5.4 [19] **Cotangent similarity measures of rough neutrosophic sets**

Assume that $A=\langle(\underline{T}_A(x_i),\underline{I}_A(x_i),\underline{F}_A(x_i)),(\overline{T}_A(x_i),\overline{I}_A(x_i),\overline{F}_A(x_i))\rangle$ and $B=\langle(\underline{T}_B(x_i),\underline{I}_B(x_i),\underline{F}_B(x_i)),(\overline{T}_B(x_i),\overline{I}_B(x_i),\overline{F}_B(x_i))\rangle$ in $X = (x_1, x_2,\ldots, x_n)$ be any two rough neutrosophic sets. Pramanik and Mondal [19] defined cotangent similarity measure between rough neutrosophic sets A and B as follows:

$COT_{RNS}(A, B)$ =

$$\frac{1}{n}\sum_{i=1}^{n}\left\langle\cot\left(\frac{\pi}{12}\left(\begin{array}{l}3+|\delta T_A(x_i)-\delta T_B(x_i)|\\+|\delta I_A(x_i)-\delta I_B(x_i)|\\+|\delta I_A(x_i)-\delta I_B(x_i)|\end{array}\right)\right)\right\rangle \quad (17)$$

Here, $\delta T_A(x_i)=\frac{\underline{T}_A(x_i)+\overline{T}_A(x_i)}{2}$, $\delta T_B(x_i)=\frac{\underline{T}_B(x_i)+\overline{T}_B(x_i)}{2}$, $\delta I_A(x_i)=\frac{\underline{I}_A(x_i)+\overline{I}_A(x_i)}{2}$, $\delta I_B(x_i)=\frac{\underline{I}_B(x_i)+\overline{I}_B(x_i)}{2}$, $\delta F_A(x_i)=\frac{\underline{F}_A(x_i)+\overline{F}_A(x_i)}{2}$, $\delta F_B(x_i)=\frac{\underline{F}_B(x_i)+\overline{F}_B(x_i)}{2}$

Proposition 21 [19]

Cotangent similarity measure satisfies the following properties:

1. $0\le COT_{RNS}(A,B)\le 1$;
2. $COT_{RNS}(A,B)=COT_{RNS}(B,A)$;
3. $COT_{RNS}(A,B)=1$; iff $A = B$
4. If C is a RNS in Y and $A\subset B\subset C$ then, $COT_{RNS}(A, C) \le COT_{RNS}(A, B)$, and $COT_{RNS}(A, C) \le COT_{RNS}(B, C)$.

Definition 5.4.1

If we consider the weights of each element x_i, a weighted rough cotangent similarity measure [19] between rough neutrosophic sets A and B can be defined as follows:

$COT_{WRNS}(A, B)$ =

$$\sum_{i=1}^{n}w_i\left\langle\cot\left(\frac{\pi}{12}\left(\begin{array}{l}3+|\delta T_A(x_i)-\delta T_B(x_i)|\\+|\delta I_A(x_i)-\delta I_B(x_i)|\\+|\delta I_A(x_i)-\delta I_B(x_i)|\end{array}\right)\right)\right\rangle \quad (18)$$

$w_i\in[0,1]$, i = 1, 2,…, n and $\sum_{i=1}^{n}w_i=1$. If we take $w_i=\frac{1}{n}$, i = 1, 2,…, n, then $COT_{WRNS}(A, B) = COT_{RNS}(A, B)$

Proposition 22 [19]

The weighted rough cosine similarity measure between two rough neutrosophic sets A and B also satisfies the following properties:

1. $0\le COT_{WRNS}(A,B)\le 1$;
2. $COT_{WRNS}(A,B)=COT_{WRNS}(B,A)$;
3. $COT_{WRNS}(A,B)=1$; iff $A = B$
4. If C is a WRNS in Y and $A\subset B\subset C$ then, $COT_{WRNS}(A, C) \le COT_{WRNS}(A, B)$, and $COT_{WRNS}(A, C) \le COT_{WRNS}(B, C)$

Definition 5.5 (Variational co-efficient similarity measure between rough neutrosophic sets)

Let $A = \langle (\underline{T}_A(x_i), \underline{I}_A(x_i), \underline{F}_A(x_i)), (\overline{T}_A(x_i), \overline{I}_A(x_i), \overline{F}_A(x_i)) \rangle$ and $B = \langle (\underline{T}_B(x_i), \underline{I}_B(x_i), \underline{F}_B(x_i)), (\overline{T}_B(x_i), \overline{I}_B(x_i), \overline{F}_B(x_i)) \rangle$ be two rough neutrosophic sets. Variational co-efficient similarity measure between rough neutrosophic sets can be presented as follows:

$Var_{RNS}(A, B) =$

$$\frac{1}{n}\left[\lambda\sum_{i=1}^{n}\frac{2\begin{Bmatrix}\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)\\+\delta F_A(x_i)\delta F_B(x_i)\end{Bmatrix}}{\begin{Bmatrix}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\+\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\end{Bmatrix}} + (1-\lambda)\sum_{i=1}^{n}\frac{\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)+\delta F_A(x_i)\delta F_B(x_i)}{\sqrt{\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]}}\right] \quad (19)$$

Here, $\delta T_A(x_i) = \frac{\underline{T}_A(x_i)+\overline{T}_A(x_i)}{2}$, $\delta T_B(x_i) = \frac{\underline{T}_B(x_i)+\overline{T}_B(x_i)}{2}$,

$\delta I_A(x_i) = \frac{\underline{I}_A(x_i)+\overline{I}_A(x_i)}{2}$, $\delta I_B(x_i) = \frac{\underline{I}_B(x_i)+\overline{I}_B(x_i)}{2}$,

$\delta F_A(x_i) = \frac{\underline{F}_A(x_i)+\overline{F}_A(x_i)}{2}$, $\delta F_B(x_i) = \frac{\underline{F}_B(x_i)+\overline{F}_B(x_i)}{2}$

Proposition 23

The variational co-efficient similarity measure $Var_{RNS}(A, B)$ between two rough neutrosophic sets A and B, satisfies the following properties:

1. $0 \le Var_{RNS}(A,B) \le 1$;
2. $Var_{RNS}(A,B) = Var_{RNS}(B,A)$;
3. $Var_{RNS}(A,B)=1$; if $A = B$ i.e., $\delta T_A(x_i) = \delta T_B(x_i)$, $\delta I_A(x_i) = \delta I_B(x_i)$, and $\delta F_A(x_i) = \delta F_B(x_i)$, for every x_i (i = 1, 2, …, n) in X.

Proof.

(1.) It is obvious that $Var_{RNS}(A,B) \ge 0$. Thus it is required to prove that $Var_{RNS}(A,B) \le 1$.

From rough neutrosophic dice similarity measure it can be witten that

$$0 \le \frac{1}{n}\sum_{i=1}^{n}\frac{2\begin{bmatrix}\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)\\+\delta F_A(x_i)\delta F_B(x_i)\end{bmatrix}}{\left\langle\begin{matrix}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\+\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\end{matrix}\right\rangle} \le 1 \quad (20)$$

and from rough neutrosophic cosine similarity measure it can be written that

$$0 \le \frac{1}{n}\sum_{i=1}^{n}\frac{\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)+\delta F_A(x_i)\delta F_B(x_i)}{\sqrt{\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]}} \le 1 \quad (21)$$

Combining Eq.(20) and Eq.(21) , we obtain

$Var_{RNS}(A, B) =$

$$\frac{1}{n}\left[\lambda\sum_{i=1}^{n}\frac{2\begin{Bmatrix}\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)\\+\delta F_A(x_i)\delta F_B(x_i)\end{Bmatrix}}{\begin{Bmatrix}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\+\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\end{Bmatrix}} + (1-\lambda)\sum_{i=1}^{n}\frac{\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)+\delta F_A(x_i)\delta F_B(x_i)}{\sqrt{\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right].\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]}}\right] \quad (22)$$

$\le \lambda + (1-\lambda) = 1$

Thus, $0 \le Var_{RNS}(A,B) \le 1$;

(2.) $Var_{RNS}(A, B) =$

$$\frac{1}{n}\left[\lambda\sum_{i=1}^{n}\frac{2\begin{Bmatrix}\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)\\+\delta F_A(x_i)\delta F_B(x_i)\end{Bmatrix}}{\begin{Bmatrix}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\+\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\end{Bmatrix}} + (1-\lambda)\sum_{i=1}^{n}\frac{\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)+\delta F_A(x_i)\delta F_B(x_i)}{\sqrt{\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]}}\right]$$

$$= \frac{1}{n}\left[\lambda\sum_{i=1}^{n}\frac{2\begin{Bmatrix}\delta T_B(x_i)\delta T_A(x_i)+\delta I_B(x_i)\delta I_A(x_i)\\+\delta F_B(x_i)\delta F_A(x_i)\end{Bmatrix}}{\begin{Bmatrix}\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\\+\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\end{Bmatrix}} + (1-\lambda)\sum_{i=1}^{n}\frac{\delta T_B(x_i)\delta T_A(x_i)+\delta I_B(x_i)\delta I_A(x_i)+\delta F_B(x_i)\delta F_A(x_i)}{\sqrt{\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]}}\right]$$

$= Var_{RNS}(B,A)$

(3.) If $A = B$ i.e., $\delta T_A(x_i) = \delta T_B(x_i)$, $\delta I_A(x_i) = \delta I_B(x_i)$, and $\delta F_A(x_i) = \delta F_B(x_i)$, for every x_i (i = 1, 2, …, n) in X,

$Var_{RNS}(A, A) =$

$$\frac{1}{n}\left[\begin{array}{l}\lambda\sum_{i=1}^{n}\frac{2\left\{\begin{array}{l}\delta T_A(x_i)\delta T_A(x_i)+\delta I_A(x_i)\delta I_A(x_i)\\+\delta F_A(x_i)\delta F_A(x_i)\end{array}\right\}}{\left\{\begin{array}{l}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\+\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\end{array}\right\}}\\+(1-\lambda)\sum_{i=1}^{n}\frac{\delta T_A(x_i)\delta T_A(x_i)+\delta I_A(x_i)\delta I_A(x_i)+\delta F_A(x_i)\delta F_A(x_i)}{\sqrt{\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]}}\end{array}\right]$$

$$=\frac{1}{n}\left[n\lambda+n(1-\lambda)\right]=1$$

These results show the completion of the proofs of the three properties.

Definition 5.6 (Weighted variational co-efficient similarity measure between rough neutrosophic sets)

Let $A=\left\langle(\underline{T}_A(x_i),\underline{I}_A(x_i),\underline{F}_A(x_i)),(\overline{T}_A(x_i),\overline{I}_A(x_i),\overline{F}_A(x_i))\right\rangle$ and $B=\left\langle(\underline{T}_B(x_i),\underline{I}_B(x_i),\underline{F}_B(x_i)),(\overline{T}_B(x_i),\overline{I}_B(x_i),\overline{F}_B(x_i))\right\rangle$ be any two rough neutrosophic sets. Rough variational co-efficient similarity measure between rough neutrosophic setsA and B in 3-D vector space can be presented as follows:

Var_{WRNS}(A, B) =

$$\left[\begin{array}{l}\lambda\sum_{i=1}^{n}w_i\frac{2\left\{\begin{array}{l}\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)\\+\delta F_A(x_i)\delta F_B(x_i)\end{array}\right\}}{\left\{\begin{array}{l}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\+\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\end{array}\right\}}\\+(1-\lambda)\sum_{i=1}^{n}w_i\frac{\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)+\delta F_A(x_i)\delta F_B(x_i)}{\sqrt{\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right].\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]}}\end{array}\right] \quad (23)$$

If $w=\left[\frac{1}{n},\frac{1}{n},...,\frac{1}{n}\right]^T$, then Eq.(23) is reduced to Eq.(19).

Proposition 24

The weighted variational co-efficient similarity measure also satisfies the following properties:

1. $0\le Var_{WRNS}(A,B)\le 1$;
2. $Var_{WRNS}(A,B)=Var_{WRNS}(B,A)$;
3. $Var_{WRNS}(A, B)$ = 1; if A = B i.e., $\delta T_A(x_i)=\delta T_B(x_i)$, $\delta I_A(x_i)=\delta I_B(x_i)$, and $\delta F_A(x_i)=\delta F_B(x_i)$, for every x_i (i = 1, 2, …, n) in X.

Proof:

(1.) It is obvious that $Var_{WRNS}(A,B)\ge 0$. Thus it is required to prove that $Var_{WRNS}(A,B)\le 1$.

From rough neutrosophic weighted dice similarity measure, it can be written that

$$0\le\frac{1}{n}\sum_{i=1}^{n}w_i\frac{2\left[\begin{array}{l}\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)\\+\delta F_A(x_i)\delta F_B(x_i)\end{array}\right]}{\left\langle\begin{array}{l}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\+\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\end{array}\right\rangle}\le 1 \quad (24)$$

and from rough neutrosophic weighted cosine similarity measure it can be written that

$$0\le\frac{1}{n}\sum_{i=1}^{n}w_i\frac{\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)+\delta F_A(x_i)\delta F_B(x_i)}{\sqrt{\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]}}\le 1 \quad (25)$$

Combining Eq.(24) and Eq.(25), we obtain

Var_{WRNS}(A, B) =

$$\left[\begin{array}{l}\lambda\sum_{i=1}^{n}w_i\frac{2\left\{\begin{array}{l}\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)\\+\delta F_A(x_i)\delta F_B(x_i)\end{array}\right\}}{\left\{\begin{array}{l}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\+\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\end{array}\right\}}\\+(1-\lambda)\sum_{i=1}^{n}w_i\frac{\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)+\delta F_A(x_i)\delta F_B(x_i)}{\sqrt{\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]}}\end{array}\right] \quad (26)$$

$$\le\lambda+(1-\lambda)=1$$

Thus, $0\le Var_{WRNS}(A,B)\le 1$;

(2.) $Var_{WRNS}(A, B)$ =

$$\left[\begin{array}{l}\lambda\sum_{i=1}^{n}w_i\frac{2\left\{\begin{array}{l}\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)\\+\delta F_A(x_i)\delta F_B(x_i)\end{array}\right\}}{\left\{\begin{array}{l}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\+\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\end{array}\right\}}\\+(1-\lambda)\sum_{i=1}^{n}w_i\frac{\delta T_A(x_i)\delta T_B(x_i)+\delta I_A(x_i)\delta I_B(x_i)+\delta F_A(x_i)\delta F_B(x_i)}{\sqrt{\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]}}\end{array}\right]$$

$$=\left[\begin{array}{l}\lambda\sum_{i=1}^{n}w_i\frac{2\left\{\begin{array}{l}\delta T_B(x_i)\delta T_A(x_i)+\delta I_B(x_i)\delta I_A(x_i)\\+\delta F_B(x_i)\delta F_A(x_i)\end{array}\right\}}{\left\{\begin{array}{l}\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\\+\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\end{array}\right\}}\\+(1-\lambda)\sum_{i=1}^{n}w_i\frac{\delta T_B(x_i)\delta T_A(x_i)+\delta I_B(x_i)\delta I_A(x_i)+\delta F_B(x_i)\delta F_A(x_i)}{\sqrt{\left[(\delta T_B(x_i))^2+(\delta I_B(x_i))^2+(\delta F_B(x_i))^2\right]\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]}}\end{array}\right]$$

$$=Var_{WRNS}(B,A)$$

(3.) If A = B i.e.,

$\delta T_A(x_i) = \delta T_B(x_i)$, $\delta I_A(x_i) = \delta I_B(x_i)$, and $\delta F_A(x_i) = \delta F_B(x_i)$, for every x_i (i = 1, 2, …, n) in X,

$Var_{WRNS}(A, A) =$

$$\left[\lambda \sum_{i=1}^{n} w_i \frac{2\left\{\begin{array}{l}\delta T_A(x_i)\delta T_A(x_i)+\delta I_A(x_i)\delta I_A(x_i)\\+\delta F_A(x_i)\delta F_A(x_i)\end{array}\right\}}{\left\{\begin{array}{l}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\+\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\end{array}\right\}} + (1-\lambda)\sum_{i=1}^{n} w_i \frac{\begin{array}{l}\delta T_A(x_i)\delta T_A(x_i)+\delta I_A(x_i)\delta I_A(x_i)\\+\delta F_A(x_i)\delta F_A(x_i)\end{array}}{\sqrt{\begin{array}{l}\left[(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2\right]\\\sqrt{(\delta T_A(x_i))^2+(\delta I_A(x_i))^2+(\delta F_A(x_i))^2}\end{array}}}\right]$$

$$= \left[\lambda \sum_{i=1}^{n} w_i + (1-\lambda)\sum_{i=1}^{n} w_i\right] = 1$$

These results show the completion of the proofs of the three proiperties.

6. Multi attribute decision making based on rough neutrosophic variational coefficient similarity measure

In this section, a rough variational co-efficient similarity measure is employed to multi-attribute decision making in rough neutrosophic environment. Assume that $A = \{A_1, A_2,..., A_m\}$ be the set of alternatives and $C = \{C_1, C_2,..., C_n\}$ be the set of attributes in a multi-attribute decision making problem. Assune that w_j be the weight of the attribute C_j provided by the decision maker such that each $w_i \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$ However, in real situation decision maker may often face difficulty to evaluate alternatives over the attributes due to vague or incomplete information about alternatives in a decision making situation. Rough neutrosophic set can be used in MADM to deal with incomplete information of the alternatives. In this paper, the assessment values of all the alternatives with respect to attributes are considered as the rough neutrosophic values (see Table 1).

Table1: Rough neutrosophic decision matrix

$D_{RNS} = \langle \underline{d}_{ij}, \overline{d}_{ij} \rangle_{m\times n} =$

	C_1	C_2	.	C_n
A_1	$\langle \underline{d}_{11}, \overline{d}_{11} \rangle$	$\langle \underline{d}_{12}, \overline{d}_{12} \rangle$	...	$\langle \underline{d}_{1n}, \overline{d}_{1n} \rangle$
A_2	$\langle \underline{d}_{21}, \overline{d}_{21} \rangle$	$\langle \underline{d}_{22}, \overline{d}_{22} \rangle$	...	$\langle \underline{d}_{2n}, \overline{d}_{2n} \rangle$
.	...	...	...	...
.	...	...	...	...
A_m	$\langle \underline{d}_{m1}, \overline{d}_{m1} \rangle$	$\langle \underline{d}_{m2}, \overline{d}_{m2} \rangle$	...	$\langle \underline{d}_{mn}, \overline{d}_{mn} \rangle$

(27)

Here $\langle \underline{d}_{ij}, \overline{d}_{ij} \rangle$ is the rough neutrosophic number for the i-th alternative and the j-th attribute.

Definition 6.1: Transforming operator for SVNSs [80]

The rough neutrosophic decision matrix (27) can be transformed to single valued neutrosophic decision matrix whose ij-th element α_{ij} can be presented as follows:

$$\alpha_{ij} = \left\langle \frac{\underline{d}_{ij}+\overline{d}_{ij}}{2} \right\rangle_{m\times n}, \text{ for } i = 1, 2, 3,..., m;$$

j = 1, 2, 3, ..., n. . (28)

Step1. Determine the neutrosophic relative positive ideal solution

In multi-criteria decision-making environment, the concept of ideal point has been used to help identify the best alternative in the decision set.

Definition 6.2 [51].

Let H be the collection of two types of attributes, namely, benefit type attribute (P) and cost type attribute (L) in the MADM problems. The relative positive ideal neutrosophic solution (RPINS) $Q_S^+ = [\delta q_S^+, \delta q_S^+, ..., \delta q_S^+]$ is the solution of the decision matrix $D_S = \langle \delta T_{ij}, \delta I_{ij}, \delta F_{ij} \rangle_{m\times n}$ where, every component of Q_S^+ has the following form:

for benefit type attribute, every component of Q_S^+ has the following form:

$$q_S^+ = \langle \delta T_j^+, \delta I_j^+, \delta F_j^+ \rangle = \left\langle \max_i \{\delta T_{ij}\}, \min_i \{\delta I_{ij}\}, \min_i \{\delta F_{ij}\} \right\rangle \text{ for } j \in P \quad (29)$$

and for cost type attribute, every component of Q_S^+ has the following form

$$q_S^+ = \langle \delta T_j^+, \delta I_j^+, \delta F_j^+ \rangle = \left\langle \min_i \{\delta T_{ij}\}, \max_i \{\delta I_{ij}\}, \max_i \{\delta F_{ij}\} \right\rangle \text{ for } j \in L \quad (30)$$

Step 2. Determine the weighted variational co-efficient similarity measure between ideal alternative and each alternative.

The variational co-efficient similarity measure between ideal alternative Q_S^+ and each alternative A_i for i = 1, 2, …, m can be determined by the following equation as follows:

$Var_{WRNS}(Q_S^+, D_S) =$

$$\left[\lambda \sum_{i=1}^{n} w_i \frac{2\left\{\delta T_j^+.\delta T_{ij}+\delta I_j^+.\delta I_{ij}+\delta F_j^+.\delta F_{ij}\right\}}{\left\{\begin{array}{l}\left[(\delta T_j^+)^2+(\delta I_j^+)^2+(\delta F_j^+)^2\right]+\\\left[(\delta T_{ij})^2+(\delta I_{ij})^2+(\delta F_{ij})^2\right]\end{array}\right\}} + (1-\lambda)\sum_{i=1}^{n} w_i \frac{\left\{\delta T_j^+.\delta T_{ij}+\delta I_j^+.\delta I_{ij}+\delta F_j^+.\delta F_{ij}\right\}}{\sqrt{\left[(\delta T_j^+)^2+(\delta I_j^+)^2+(\delta F_j^+)^2\right]}\sqrt{(\delta T_{ij})^2+(\delta I_{ij})^2+(\delta F_{ij})^2}}\right] \quad (31)$$

Step3. Rank the alternatives.

According to the values obtained from Eq.(31), the ranking order of all the alternatives can be easily determined. Highest value indicates the best alternative.

Step 4. End.

7 Numerical example

In this section, rough neutrosophic MADM regarding investment problem is considered to demonstrate the applicability and the effectiveness of the proposed approach. However, investment problem is not easy to solve. It not only requires oodles of patience and discipline, but also a great deal of research and a sound understanding of the market, mathematical tools, among others. Suppose an investment company wants to invest a sum of money in the best option. Assume that there are four possible alternatives to invest the money: (1) A_1 is a computer company; (2) A_2 is a garment company; (3) A_3 is a telecommunication company; and (4) A_4 is a food company. The investment company must take a decision based on the following three criteria: (1) C_1 is the growth factor; (2) C_2 is the environmental impact; and (3) C_3 is the risk factor. The four possible alternatives are to be evaluated under the attribute by the rough neutrosophic assessments provided by the decision maker. These assessment values are given in the rough neutrosophic decision matrix (see the table 2).

Table2. Rough neutrosophic decision matrix

$$D=\left\langle \underline{N}_{ij}(P), \overline{N}_{ij}(P)\right\rangle_{4\times 3}=$$

$$\begin{array}{c|ccc} & C_1 & C_2 & C_3 \\ \hline A_1 & \left\langle \begin{matrix}(0.1,0.2,0.2),\\(0.3,0.2,0.2)\end{matrix}\right\rangle & \left\langle \begin{matrix}(0.6,0.4,0.3),\\(0.8,0.2,0.3)\end{matrix}\right\rangle & \left\langle \begin{matrix}(0.3,0.2,0.3),\\(0.5,0.2,0.1)\end{matrix}\right\rangle \\ A_2 & \left\langle \begin{matrix}(0.2,0.4,0.3),\\(0.4,0.2,0.3)\end{matrix}\right\rangle & \left\langle \begin{matrix}(0.6,0.3,0.3),\\(0.8,0.1,0.1)\end{matrix}\right\rangle & \left\langle \begin{matrix}(0.1,0.4,0.3),\\(0.3,0.2,0.3)\end{matrix}\right\rangle \\ A_3 & \left\langle \begin{matrix}(0.3,0.2,0.3),\\(0.5,0.2,0.1)\end{matrix}\right\rangle & \left\langle \begin{matrix}(0.5,0.2,0.3),\\(0.7,0.2,0.1)\end{matrix}\right\rangle & \left\langle \begin{matrix}(0.0,0.2,0.4),\\(0.2,0.2,0.2)\end{matrix}\right\rangle \\ A_4 & \left\langle \begin{matrix}(0.0,0.4,0.4),\\(0.2,0.2,0.2)\end{matrix}\right\rangle & \left\langle \begin{matrix}(0.5,0.4,0.4),\\(0.7,0.2,0.2)\end{matrix}\right\rangle & \left\langle \begin{matrix}(0.2,0.3,0.3),\\(0.4,0.1,0.1)\end{matrix}\right\rangle \end{array} \quad (32)$$

The known weight information is given as follows: $W = [w_1, w_2, w_3]^T = [0.3, 0.3, 0.4]$ and $\sum_{i=1}^{3} w_i = 1$.

Step1. Determine the types of criteria.

First two types i.e. C_1 and C_2 of the given criteria are benefit type criteria and the last one criterion i.e. C_3 is the cost type criteria.

Step2. Determine the relative neutrosophic positive ideal solution

Using Eq. (29), Eq.(30), the relative positive ideal neutrosophic solution for the given matrix defined in Eq.(32) can be obtained as:

$$Q_S^{+}=[(0.4,0.2,0.2),(0.7,0.2,0.2),(0.1,0.3,0.3)]$$

Step3. Determine the weighted variational similarity measure

The weighted variational co-efficient similarity measure is determined by using Eq.(28), Eq.(31) and Eq.(32). The results obtained for different values of have been shown in the Table-3.

Table-3. Results of rough variational similarity measure for different values of , $0 \le \lambda \le 1$

Similarity measure method	Values of s	Measure values	Ranking order
$Var_{WRNS}(Q_S^{+}, D_S)$	0.10	0.8769; 0.9741; 0.9917; 0.8107	$A_3 > A_2 > A_1 > A_4$
	0.25	0.8740, 0.9739 0.9905 0.8078	$A_3 > A_2 > A_1 > A_4$
	0.50	0.8692; 0.9735; 0.9887; 0.8028	$A_3 > A_2 > A_1 > A_4$
	0.75	0.8643; 0.9730; 0.9868; 0.7979	$A_3 > A_2 > A_1 > A_4$
	0.90	0.8614; 0.9728; 0.9857; 0.7949	$A_3 > A_2 > A_1 > A_4$

Step 4. Rank the alternatives.

According to the different values of , the results obtained in Table-3 reflects that A_3 is the best alternative.

8. Comparisons of different rough similarity measure with rough variation similarity measure

In this section, four existing rough similarity measures - namely: rough cosine similarity measure, rough dice similarity measure, rough cotangent similarity measure and rough Jaccard similarity measure - have been compared with proposed rough variational co-efficient similarity measure for different values of λ. The comparison results are listed in the Table 3 and Table 4.

Table-4. Results of existing rough neutrosophic similarity measure methods.

Rough similarity measure methods	Values of s	Measure values	Ranking order
$JAC_{WRNS}(Q_S^+, D_S)$ [21]	...	0.7870, 0.9471; 0.9739; 0.6832	$A_3 > A_2 > A_1 > A_4$
$DIC_{WRNS}(Q_S^+, D_S)$ [21]	...	0.8595; 0.9726; 0.9873; 0.7929	$A_3 > A_2 > A_1 > A_4$
$C_{WRNS}(Q_S^+, D_S)$ [20]	...	0.8788; 0.9738; 0.9920; 0.9132	$A_3 > A_2 > A_4 > A_1$
$COT_{WRNS}(Q_S^+, D_S)$ [19]	...	0.8472; 0.9358; 0.9643; 0.8103	$A_3 > A_2 > A_1 > A_4$

Conclusion

In this paper, we have proposed rough variational coefficient similarity measures. We also proved some of their basic properties. We have presented an application of rough neutrosophic variational coefficient similarity measure for a decision making problem on investment. The concept presented in the paper can be applied to deal with other multi attribute decision making problems in rough neutrosophic environment.

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Regular Single Valued Neutrosophic Hypergraphs

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Abstract. In this paper, we define the regular and totally regular single valued neutrosophic hypergraphs, and discuss the order and size along with properties of regular and totally regular single valued neutrosophic hypergraphs. We also extend work on completeness of single valued neutrosophic hypergraphs.

Keywords: Single valued neutrosophic hypergraphs, regular single valued neutrosophic hypergraphs and totally regular single valued neutrosophic hypergraphs.

1 Introduction

The notion of neutrosophic sets (NSs) was proposed by Smarandache [8] as a generalization of the fuzzy sets [14], intuitionistic fuzzy sets [12], interval valued fuzzy set [11] and interval-valued intuitionistic fuzzy sets [13] theories. The neutrosophic set is a powerful mathematical tool for dealing with incomplete, indeterminate and inconsistent information in real world. The neutrosophic sets are characterized by a truth-membership function (*t*), an indeterminacy-membership function (*i*) and a falsity membership function (*f*) independently, which are within the real standard or nonstandard unit interval $]^-0 , 1^+[$. In order to conveniently use NS in real life applications, Wang et al. [9] introduced the concept of the single-valued neutrosophic set (SVNS), a subclass of the neutrosophic sets. The same authors [10] introduced the concept of the interval valued neutrosophic set (IVNS), which is more precise and flexible than the single valued neutrosophic set. The IVNS is a generalization of the single valued neutrosophic set, in which the three membership functions are independent and their value belong to the unit interval [0, 1]. More works on single valued neutrosophic sets, interval valued neutrosophic sets and their applications can be found on *http://fs.gallup.unm.edu/NSS/*.
Hypergraph is a graph in which an edge can connect more than two vertices, hypergraphs can be applied to analyse architecture structures and to represent system partitions, Mordesen J.N and P.S Nasir gave the definitions for fuzzy hypergraphs. Parvathy. R and M. G. Karunambigai's paper introduced the concepts of Intuitionistic fuzzy hypergraphs and analyse its components, Nagoor Gani. A and Sajith Begum. S defined degree, order and size in intuitionistic fuzzy graphs and extend the properties. Nagoor Gani. A and Latha. R introduced irregular fuzzy graphs and discussed some of its properties.

Regular intuitionistic fuzzy hypergraphs and totally regular intuitionistic fuzzy hypergraphs are introduced by Pradeepa. I and Vimala. S in [0]. In this paper we extend regularity and totally regularity on single valued neutrosophic hypergraphs.

2 Preliminaries

Definition 2.1 Let X be a space of points (objects) with generic elements in X denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by truth membership function $T_A(x)$, indeterminacy membership function $I_A(x)$ and a falsity membership function $F_A(x)$. For each point $x \in X$; $T_A(x), I_A(x), F_A(x) \in [0, 1]$.

Definition 2.2 Let A be a SVNS on X then support of A is denoted and defined by, $Supp(A) = \{ x : x \in X , T_A(x) > 0 , I_A (x) > 0 , F_A (x) > 0 \}$.

Definition 2.3 A hypergraph is an ordered pair *H = (X, E)*, where

(1) $X = \{x_1, x_2, \ldots, x_n\}$ be a finite set of vertices.

(2) $E = \{E_1, E_2, \ldots, E_m\}$ be a family of subsets of X.

(3) E_j for $j = 1,2,3,\ldots,m$ and $\bigcup_j (E_j) =$ X.

The set X is called set of vertices and E is the set of edges(or hyper edges).

Definition 2.4 The single valued neutrosophic hypergraph is an ordered pair *H = (X, E),* where

(1) $X = \{x_1, x_2, \ldots, x_n\}$ be a finite set of vertices.

(2) $E = \{E_1, E_2, \ldots, E_m\}$ be a family of SVNSs of X.

(3) $E_j \neq O = (0, 0, 0)$ for $j = 1, 2, 3, \ldots, m$ and $\bigcup_j Supp(E_j) = X$.

The set *X* is called set of vertices and *E* is the set of SVN-edges(or SVN-hyperedges).

Proposition 2.5 The single valued neutrosophic hypergraph is the generalization of fuzzy hypergraphs and intuitionistic fuzzy hypergraphs.

3 Regular and totally regular SVNHGs

Definition 3.1 The open neighbourrhood of a vertex *x* in single valued neutrosophic hypergraphs (SVNHGs) is the set of adjacent vertices of *x*, excluding that vertex and is denoted by *N(x)*.

Definition 3.2 The closed neighbourhood of a vertex *x* in single valued neutrosophic hypergraphs (SVNHGs) is the set of adjacent vertices of *x*, including that vertex and is denoted by *N[x]*.

Example 3.3 Consider a single valued neutrosophic hyper graphs *H = (X, E),* where *X = {a, b, c, d, e}* and *E = {P, Q, R, S},* which are defined by

P = {(a, .1, .2, .3), (b, .4, .5, .6)}

Q = {(c, .1, .2, .3), (d, .4, .5, .6), (e, .7, .8, .9)}

R = {(b, .1, .2, .3), (c, .4, .5, .6)}

S = {(a, .1, .2, .3), (d, .4, .5, .6)}

Then the open neighbourhood of a vertex *a* contain *b* and *d*. The closed neighbourhood of a vertex *b* contain *b*, *a* and *c*.

Definition 3.4 Let *H = (X, E)* be a SVNHG, the open neighbourhood degree of a vertex *x,* which is denoted and defined by $deg(x) = (deg_T(x), deg_I(x), deg_F(x))$, where

$$deg_T(x) = \sum_{x \in X} T_E(x)$$

$$deg_I(x) = \sum_{x \in X} I_E(x)$$

$$deg_F(x) = \sum_{x \in X} F_E(x)$$

Example 3.5 Consider a single valued neutrosophic hypergraphs *H = (X, E)* where, *X = {a, b, c, d, e}* and *E = {P, Q, R, S},* which are defined by

P = {(a, .1, .2, .3), (b, .4, .5, .6)}

Q = {(c, .1, .2, .3), (d, .4, .5, .6), (e, .7, .8, .9)}

R = {(b, .1, .2, .3), (c, .4, .5, .6)}

S = {(a, .1, .2, .3), (d, .4, .5, .6)}

Then the open neighbourhood of a vertex *a* is *b* and *d*, and therefore the open neighbourhood degree degree of a vertex *a* is (.8, 1, 1.2).

Definition 3.6 Let *H = (X, E)* be a SVNHG, the closed neighbourhood degree of a vertex *x,* which is denoted and defined by

$$deg[x] = (deg_T[x], deg_I[x], deg_F[x])$$

where

$$deg_T[x] = deg_T(x) + T_E(x)$$

$$deg_I[x] = deg_I(x) + I_E(x)$$

$$deg_F[x] = deg_F(x) + F_E(x)$$

Example 3.7 Consider a single valued neutrosophic hypergraphs *H = (X, E)*, where *X = {a, b, c, d, e}* and *E = {P, Q, R, S},* which is defined by

P = {(a, .1, .2, .3), (b, .4, .5, .6)}

Q = {(c, .1, .2, .3), (d, .4, .5, .6), (e, .7, .8, .9)}

$R = \{(b, .1, .2, .3), (c, .4, .5, .6)\}$

$S = \{(a, .1, .2, .3), (d, .4, .5, .6)\}$

The closed neighbourhood of a vertex *b* contain *b*, *a* and *c*, hence the closed neighbourhood degree of a vertex *a* is (.9, .1.2, 1.5).

Definition 3.8 Let $H = (X, E)$ be a SVNHG, then H is said to be an *n*-regular SVNHG if all the vertices have the same open neighbourhood degree $n = (n_1, n_2, n_3)$.

Definition 3.9 Let $H = (X, E)$ be a SVNHG, then H is said to be an *m*-totally regular SVNHG if all the vertices have the same closed neighbourhood degree $m = (m_1, m_2, m_3)$.

Proposition 3.10 A regular SVNHG is the generalization of regular fuzzy hypergraphs and regular intuitionistic fuzzy hypergraphs.

Proposition 3.11 A totally regular SVNHG is the generalization of totally regular fuzzy hypergraphs and totally regular intuitionistic fuzzy hypergraphs.

Example 3.12 Consider a single valued neutrosophic hypergraphs $H = (X, E)$, where $X = \{a, b, c, d\}$ and $E = \{P, Q, R, S\}$, which are defined by

$P = \{(a, .8, .2, .3), (b, .8, .2, .3)\}$

$Q = \{(b, .8, .2, .3), (c, .8, .2, .3)\}$

$R = \{(c, .8, .2, .3), (d, .8, .2, .3)\}$

$S = \{(d, .8, .2, .3), (a, .8, .2, .3)\}$

Here the open neighbourhood degree of every vertex is (1.6, .4, .6), hence H is regular SVNHG and closed neighbourhood degree of every vertex is (2.4, .6, .9). Hence H is both regular and totally regular SVNHG.

Theorem 3.13 Let $H = (X, E)$ be a SVNHG which is both regular and totally regular SVNHG then E is constant.

Proof: Suppose H is an *n*-regular and an *m*-totally regular SVNHG. Then

$$deg(x) = n = (n_1, n_2, n_3)$$

$$deg[x] = m = (m_1, m_2, m_3)$$

for all $x \in E_i$. Consider the $deg[x] = m$, hence by definition $deg(x) + E_i(x) = m$ this implies that $E_i(x) = m - n$ for all x in E_i. Therefore E is constant.

Remark 3.14 The converse of above theorem need not to be true in general.

Example 3.15 Consider a SVNHG $H = (X, E)$, where $X = \{a, b, c, d\}$ and $E = \{P, Q, R, S\}$, *which is* defined by

$P = \{(a, .8, .2, .3), (b, .8, .2, .3)\}$

$Q = \{(b, .8, .2, .3), (d, .8, .2, .3)\}$

$R = \{(c, .8, .2, .3), (d, .8, .2, .3)\}$

$S = \{(d, .8, .2, .3), (a, .8, .2, .3)\}$

Here E is constant but $deg(a) = (1.6, .4, .6)$ and $deg(d) = (2.4, .6, .9)$. Therefore $deg(a)$ and $deg(d)$ are not equal, hence H is not regular SVNHG. Also $deg[a] = (2.4, .6, .9)$ and $deg[d] = (3.2, .8, 1.2)$, thus $deg[a]$ and $deg[d]$ are not equal, hence H is not totally regular SVNHG, we conclude that H is neither regular and nor totally regular SVNHG.

Theorem 3.16 Let $H = (X, E)$ be a SVNHG, then E is costant on X if and only if following are equivalent

(1) H is regular SVNHG.

(2) H is totally regular SVNHG.

Proof: Suppose $H = (X, E)$ be a SVNHG and E is constant in H, then $E_i(x) = c = (c_1, c_2, c_3)$ for all $x \in E_i$. Suppose H is an n-regular SVNHG, then $deg(x) = n = (n_1, n_2, n_3)$ for all $x \in E_i$. Consider $deg[x] = deg(x) + E_i(x) = n + c$ for all $x \in E_i$. Hence H is totally regular SVNHG. Next suppose that H is an *m*-totally regular SVNHG, then $deg[x] = m = (m_1, m_2, m_3)$ for all $x \in E_i$, that is $deg(x) + E_i(x) = m$ for all $x \in E_i$. This implies that $deg(x) = m - c$ for all $x \in E_i$. Thus H is regular SVNHG.

Conversely: Suppose contrary E is not constant, that is $E_i(x)$ and $E_i(y)$ not equal for some x and y in X. Let $H = (X, E)$ be an n-regular SVNHG, then $deg(x) = n = (n_1, n_2, n_3)$ for all $x \in E_i$. Consider

$$deg[x] = deg(x) + E_i(x) = n + E_i(x)$$

$$deg[y] = deg(y) + E_i(y) = n + E_i(y)$$

since $E_i(x)$ and $E_i(y)$ are not equal for some x and y in X, hence $deg[x]$ and $deg[y]$ are not equal, thus H is not totally regular SVNHG, which contradict to our assumption. Next let H be totally regular SVNHG, then $deg[x] = deg[y]$, that is

$$deg(x) + E_i(x) = deg(y) + E_i(y)$$

$$deg(x) - deg(y) = E_i(y) - E_i(x)$$

since RHS of above equation is nonzero, hence LHS of above equation is also nonzero, thus $deg(x)$ and $deg(y)$ are not equals, so H is not regular SVNHG, which is again contradict to our assumption, thus our supposition was wrong, hence E must be constant, this completes the proof.

Definition 3.17 Let $H = (X, E)$ be a regular SVNHG, then the order of SVNHG H, which is denoted and defined by $O(H) = (p, q, r)$, where

$$p = \sum_{x \in X} T_{E_i}(x)$$

$$q = \sum_{x \in X} I_{E_i}(x)$$

$$r = \sum_{x \in X} F_{E_i}(x)$$

for every x ∈ X and the size of regular SVNHG, which is denoted and defined by $S(H) = \sum_{i=1}^{n}(S_{E_i})$, where $S(E_i) = (a, b, c)$, which is defined by

$$a = \sum_{x \in E_i} T_{E_i}(x)$$

$$b = \sum_{x \in E_i} I_{E_i}(x)$$

$$c = \sum_{x \in E_i} F_{E_i}(x).$$

Example 3.18 Consider the SVNHG $H = (X, E)$, where $X = \{a, b, c, d\}$ and $E = \{P, Q, R, S\}$, which is defined by

$P = \{(a, .8, .2, .3), (b, .8, .2, .3)\}$

$Q = \{(b, .8, .2, .3), (c, .8, .2, .3)\}$

$R = \{(c, .8, .2, .3), (d, .8, .2, .3)\}$

$S = \{(d, .8, .2, .3), (a, .8, .2, .3)\}$

Here the order and the size of H are given (3.2, .8, 1.2) and (6.4, 1.6, 2.4), respectively.

Proposition 3.19 The size of an n-regular SVNHG $H = (X, E)$ is nk/2, where |X|= k.

Proposition 3.20 Let $H = (X, E)$ be an m-totally regular SVNHG, then $2S(H) + O(H) = mk$, where $|X| = k$.

Corollary 3.21 Let $H = (X, E)$ be an n-regular and an m-totally regular SVNHG, then $O(H) = k(m - n)$, where $|X| = k$.

Proposition 3.22 The dual of an n-regular and an m-totally regular SVNHG $H= (X, E)$ is again an n-regular and an m-totally regular SVNHG.

Definition 3.23 The SVNHG is said to be complete SVNHG if for every x in X, $N(x) = \{x: x \text{ in } X\text{-}\{x\}\}$, that is $N(x)$ contains all remaining vertices of X except x.

Example 3.24 Consider a single valued neutrosophic hypergraphs $H = (X, E)$, where $X = \{a, b, c, d\}$ and $E = \{P, Q, R\}$, which is defined by

$P = \{(a, .4, .6, .3), (c, .8, .2, .3)\}$

$Q = \{(a, .8, .8, .3), (b, .8, .2, .1), (d, .8, .2, .1)\}$

$R = \{(c, .4, .9, .9), (d, .7, .2, .1), (b, .4, .2, .1)\}$

Here $N(a) = \{b, c, d\}$, $N(b) = \{a, c, d\}$, $N(c) = \{a, b, d\}$, $N(d) = \{a, b, c\}$. Hence H is complete SVNHG.

Remark 3.25 In a complete SVNHG $H = (X, E)$ the cardinali-ty of $N(x)$ is same for every vertex.

Theorem 3.26 Every complete SVNHG $H = (X, E)$ is both regular and totally regular if is constant in H.

Proof: Let $H = (X, E)$ be a complete SVNHG, suppose E is constant in H, so $\forall x \in E_i$, $E_i(x) = c = (c_1, c_2, c_3)$, since SVNHG is complete, then by definition for every vertex x in X, $N(x) = \{ x : x \text{ in } X\text{-}\{x\}\}$, the open neighbourhood degree of every vertex is same. Hence $deg(x) = n = (n_1, n_2, n_3)$ for all x ∈ E_i. Hence complete SVNHG is regular SVNHG. Also $deg[x] = deg(x) + E_i(x) = n + c$ for all x ∈ E_i. Thus H is totally regular SVNHG.

Remark 3.27 Every complete SVNHG is totally regular even if E is not constant.

Definition 3.28 The SVNHG is said to be k-uniform if all the hyperedges have same cardinality.

Example 3.29 Consider a SVNHG $H = (X, E)$, where $X = \{a, b, c, d\}$ and $E = \{P, Q, R\}$, which is defined by

$P = \{(a, .8, .2, .3), (b, .7, .5, .3)\}$

$Q = \{(b, .8, .1, .8), (c, .8, .4, .2)\}$

$R = \{(c, .8, .1, .4), (d, .8, .9, .5)\}$

4 Conclusion

Theoretical concepts of graphs and hypergraphs are highly utilized by computer science applications. The SVNHG are more flexible than fuzzy hypergraphs and intuitionistic fuzzy hypergraphs. The concepts of SVNHGs can be applied in various areas of engineering and computer science. In this paper, we defined the concept of regular and totally regular SVNHGs. We plan to extend our research work to regular and totally regular on Bipolar SVNHGs, regular and totally regular on interval valued neutrosophic hypergraphs, irregular and totally irregular on SVNHGs, irregular and totally irregular on bipolar SVNHGs.

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Triangular Dense Fuzzy Neutrosophic Sets

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Abstract. In this study, we introduce the concept of denser property in fuzzy membership function used in neutrosophic sets. We present several new definitions and study their properties. Defuzzification methods over neutrosophic triangular dense fuzzy sets and neutrosophic triangular intuitionistic dense fuzzy sets are then given. Finally practical applicability of the methods have been discussed with graphical implications in recent times.

Keywords: Dense fuzzy set; Triangular dense fuzzy neutrosophic set; defuzzification method.

1. Introduction

For any kind of multi-attribute decision making (MADM) it is essential to have adequate crisp data, but in modern situations crisp data are inadequate. The data for which they used to rely more are basically imprecise, vague, inappropriate and piecewise untruth as a whole. Belnap [3] made an attempt to study with the four valued logic namely Truth (T), false (F), Unknown (U) and Contradiction (C). He used a bi-lattice where the four components were inter-related. Smarandache [13] founded and developed the neutrosophic set, neutrosophic logic, neutrosophic probability and neutrosophic statistics. Several researchers Ye [19], Biswas et al. [4,5], Mandal and Pramanik [10] etc. have discussed several ranking method based on current problems using neutrosophic sets (NS). Also, recently the multivalued power operator in NS has been veveloped by Peng at al.[12].Fuzzy set theory was first studied by Zadeh [21] but after few decades later the concept on hesitant fuzzy set has been grown by Torra[15]. Moreover, in intuitionistic fuzzy environment, numerous research articles have been studied by eminent practitioner. The concept on intuitionistic fuzzy sets (IFS) has been developed independently by Atanassov[1,2] and Dubois et al. [9]. Through its process,Wang et al. [17,18], Pei and Zheng [11] discussed new concepts on evidence based IFS and a novel approach for decision making respectively. However, the decision maker's (DM) are usually applying their appropriate membership grade values of the different attributes which are prior and experienced data. But in reality, the data predicted a day ago may not be useful for tomorrow and in many cases those grade values demanding changing values with the change of the dealing frequency among several monopoly enterprises or between the time gap also. Thus, it is troublesome to find the actual data (because most of the original data is in hidden and secret under some national or international law and orders). For instance, to find the information over flood victims in a particular place several opinions may come out. But the data accepted by the authorities usually vary the reality because of the limitations on governmental financial supports to be offered to the victims. However, the situation began to clear as the day passing on. To model the above situation, in this article, we first give some basic concepts on neutrosophic set (NS), and then we develop the NS under dense fuzzy environment. The fuzzy components under several compositions are discussed from the existing literature. Next, some extensions are made with proper justification.

The paper is organized as follows: Section 2 describes some basic concepts of NS for subsequent use. In section 3, we develop the NS under dense fuzzy environment. Section 4 deals with defuzzified values of NS. Section 5 improves NS assessment under dense fuzzy environment. Section 6 gives further implications of dense fuzzy in NS. Section 7 presents applications to show the practicality and feasibility of our method. Section 8 ends the paper with some concluding remarks..

2. Preliminaries

Here, we shall discuss some basic concepts and operations on neutrosophic set.

Definition 1.[4] Let X be a space of points (objects) with generic element x. Then a neutrosophic set (NS) A in X is characterize by a truth membership function T_A, an indeterminacy membership function I_A and a falsity membership function F_A. It is denoted by $N_s =< T_A, I_A, F_A >$ where the functions T_A, I_A and F_A are real standard or non-standard subsets of $]0^-, 1^+[$. That is $T_A : X \to]0^-, 1^+[$, $I_A : X \to]0^-, 1^+[$ and is $F_A : X \to]0^-, 1^+[$ satisfying the relation is $0^- \leq supT_A(x) + sup\, I_A(x) + supF_A(x) \leq 3^+$,

Definition 2. [13]The complementA^c of a NS A is defined as follows:

i) $T_{A^c}(x) = \{1^+\} - T_A(x)$,
ii) $I_{A^c}(x) = \{1^+\} - I_A(x)$;
iii) $F_{A^c}(x) = \{1^+\} - F_A(x)$

Definition 3. [13]A neutrosophic set A is contained in other neutrosophic set B i.e, $A \subseteq B$ if and only if the following results hold good for

$\forall x \in X$

i) $infT_A(x) \le infT_B(x)$, $supT_A(x) \le supT_B(x)$
ii) $infI_A(x) \ge infI_B(x)$, $supI_A(x) \ge supI_B(x)$
iii) $infF_A(x) \ge infF_B(x)$, $supF_A(x) \ge supF_B(x)$

Definition 4. [16]The complement N_s^c of a single valued NS is given by

i) $T_{N_s^c}(x) = F_{N_s^c}(x)$,
ii) ii) $I_{N_s^c}(x) = 1 - I_{N_s}(x)$;$F_{N_s^c}(x) = T_{N_s}(x)$.

Definition 5. [16]The union of two single valued neutrosophic sets A and B , denoted by $C = A \cup B$. Its truth membership , indeterminancy membership, and falsity membership functions are related to those of A and B as follows:

i) $T_C(x) = \max(T_A(x), T_B(x))$
ii) $I_C(x) = \max(I_A(x), I_B(x))$
$iii)$ $F_C(x) = \min(F_A(x), F_B(x))$,
$\forall x \in X$

Definition 6. The intersection of two single valued neutrosophic sets A and B , denoted by $C = A \cap B$. Its truth membership , indeterminancy membership, and falsity membership functions are related to those of A and B as follows:

$i)$ $T_C(x) = \min(T_A(x), T_B(x))$
$ii)$ $I_C(x) = \min(I_A(x), I_B(x))$
$iii)$ $F_C(x) = \max(F_A(x), F_B(x))$,
$\forall x \in X$.

Definition 7. The additionof two single valued neutrosophic sets A and B , denoted by $C = A \oplus B$. Its truth membership , indeterminancy membership, and falsity membership functions are related to those of A and B as follows:

$i)$ $T_C(x) = T_A(x) + T_B(x) - T_A(x)T_B(x)$
ii) $I_C(x) = I_A(x)I_B(x)$
iii) $F_C(x) = F_A(x)F_B(x)$,
$\forall x \in X$

Definition 8. The multiplication of two single valued neutrosophic sets A and B , denoted by $C = A \otimes B$. Its truth membership , indeterminancy membership, and falsity membership functions are related to those of A and B as follows:

i)$T_C(x) = T_A(x)T_B(x)$
ii) $I_C(x) = I_A(x) + I_B(x) - I_A(x)I_B(x)$
$iii)$ $F_C(x) = F_A(x) + F_B(x) - F_A(x)F_B(x)$,
$\forall x \in X$

Remark 1.Neutrosophic Cube describing IFS &NS. Jean Dezert [8] introduced the neutrosophic cube $A'B'C'D'E'F'G'H'$ to make a distinction between IFS and NS. For technical use, we take the classical interval [0,1] for the NS parameters T_A, I_A and F_A . Then the cube $ABCDEFGH$ is called technical / relative neutrosophic cube and its extension $A'B'C'D'E'F'G'H'$ is called the absolute neutrosophic cube. Now, we divide the technical neutrosophic cube into three disjoint regions. The observations from the following cube are

i) The equilateral triangle BDE, whose sides are equal to $\sqrt{2}$, it represents the geometrical locus of the points whose sum of the coordinates is 1. This triangle is known as Atanassov-Intuitionistic fuzzy set (A-IFS). Here, if q is a point on ΔBDE or inside of it then as in A-IFS, $t_q + i_q + f_q = 1$.
ii) The pyramid EABD[situated in the right side of the ΔEBD , including its faces ΔABD (base), ΔEBA and ΔEDA (lateral faces), but excluding its face ΔBDE] is the locus of the points whose sum of coordinates is less than 1. If p is point on EABD then $t_q + i_q + f_q < 1$ as in IFS with incomplete information.
iii) In the left side of ΔBDE in the cube there is the solid EFGCDEBD(excluding ΔBDE) which is the locus of points whose sum of their coordinates is greater than 1 as in the paraconsistent set . If a point r lies on EFGCDEBD, then $t_q + i_q + f_q > 1$.

Thus, we have a source which is capable to find only the degree of membership of an element; but it is unable to find the degree of non-membership. Another source is capable to find only the degree of non-membership of an element. Or, a source which only computes the indeterminacy. Putting these results we always have$t_q + i_q + f_q \neq 1$. Moreover, in information fusion, when dealing with indeterminate models (that is elements of the fusion space which are indeterminate/unknown, such as intersections we don't know if they are empty or not since we don't have enough information, similarly for complements of indeterminate elements etc) ; if we compute the believe in that element (truth), the disbelieve in that element(falsehood) and the indeterminacy part of that element, then the sum of these three components is strictly less than 1 (the difference to 1 is the missing information). This is shown in Fig. 1.

3 Triangular dense fuzzy environment

Here, we discuss the dense fuzzy environment in the all possible cases of NS.

Definition 9. [7]Let $\tilde{A}$ be the fuzzy number whose components are the elements of $\mathcal{R} \times N$, $\mathcal{R}$ being the set of real numbers and N being the set of natural numbers with the membership grade satisfying the functional relation$\mu : \mathcal{R} \times N \to [0,1]$. Now as $n \to \infty$ if $\mu(x,n) \to 1$ for some $x \in \mathcal{R}$then we call the set $\tilde{A}$ as dense fuzzy set. If $\tilde{A}$ is triangular then it is called TDFS. Now, if for some n, $\mu(x,n)$ attains the highest membership degree 1 then the set itself is called "Normalized Triangular Dense Fuzzy Set" or NTDFS. The graphical interpretation is shown in Fig. 2.

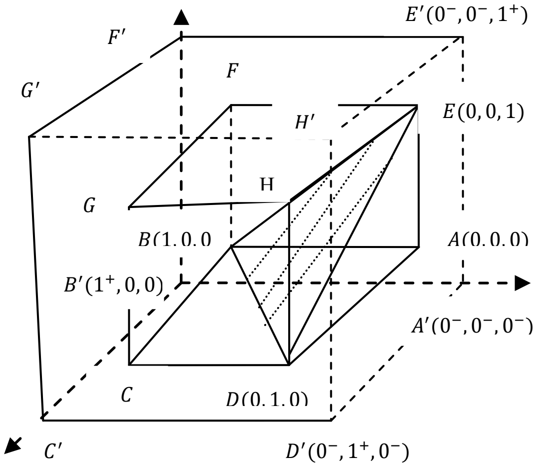

Fig-1: Geometric representation of **NeutrosophicCube**

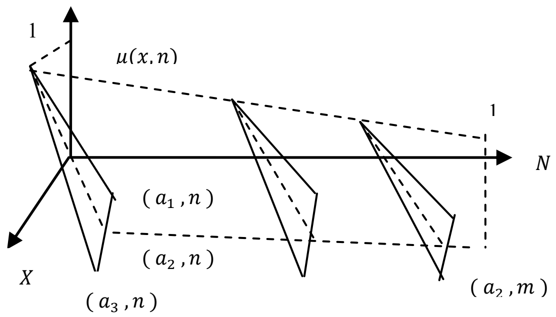

Fig.2: Membership function of NTDFS based on definition 9

Example 1. As per definitions (9) let us assume the TDFS as follows $\tilde{A} =< a_2\left(1-\frac{\rho_1}{1+n}\right), a_2, a_2\left(1+\frac{\sigma_1}{1+n}\right) >$, $for\ 0 < \rho_1, \sigma_1 < 1$(1)

The memberships function for $0 \le n$ is defined as follows:$\gamma_T(x,n) =$
lows:$\gamma_T(x,n) =$

$$\begin{cases} 0 \quad if \quad x < a_2\left(1-\frac{\rho_1}{1+n}\right) \ and \ x > a_2\left(1+\frac{\sigma_1}{1+n}\right) \\ \left\{\frac{x-a_2\left(1-\frac{\rho_1}{1+n}\right)}{\frac{\rho_1 a_2}{1+n}}\right\} \quad if \quad a_2\left(1-\frac{\rho_1}{1+n}\right) \le x \le a_2 \\ \left\{\frac{a_2\left(1+\frac{\sigma_1}{1+n}\right)-x}{\frac{\sigma_1 a_2}{1+n}}\right\} \quad if \quad a_2 \le x \le a_2\left(1+\frac{\sigma_1}{1+n}\right) \end{cases} \quad (2)$$

Similarly, here also we note that, the ordinary membership functions of falsehood and indeterminacy is given by

$$\gamma_F(x,n) = \begin{cases} 0 \quad if \quad x < b_2\left(1-\frac{\rho_2}{1+n}\right) \ and \ x > b_2\left(1+\frac{\sigma_2}{1+n}\right) \\ \left\{\frac{b_2-x}{\frac{\rho_2 b_2}{1+n}}\right\} \quad if \quad b_2\left(1-\frac{\rho_2}{1+n}\right) \le x \le b_2 \\ \left\{\frac{x-b_2}{\frac{\sigma_2 b_2}{1+n}}\right\} \quad if \quad b_2 \le x \le b_2\left(1+\frac{\sigma_2}{1+n}\right) \end{cases} \quad (3)$$

$$\gamma_I(x,n) = \begin{cases} 0 \quad if \quad x < c_2\left(1-\frac{\rho_3}{1+n}\right) \ and \ x > c_2\left(1+\frac{\sigma_3}{1+n}\right) \\ \left\{\frac{c_2-x}{\frac{\rho_3 c_2}{1+n}}\right\} \quad if \quad c_2\left(1-\frac{\rho_3}{1+n}\right) \le x \le c_2 \\ \left\{\frac{x-c_2}{\frac{\sigma_3 c_2}{1+n}}\right\} \quad if \quad c_2 \le x \le c_2\left(1+\frac{\sigma_3}{1+n}\right) \end{cases} \quad (4)$$

Definition 10: TDFS based on non-membership & indeterminate function

Let$\tilde{A}$ be the fuzzy number whose components are the elements of $\mathcal{R} \times N$ whose non-membership grade satisfying the functional relation $\vartheta : \mathcal{R} \times N \to [0,1]$. Now as $n \to \infty$ if $\vartheta(x,n) \to 0$ for some $x \in \mathcal{R}$then we call the set $\tilde{A}$ as dense fuzzy set. If we consider the fuzzy number $\tilde{A}$ of the form $\tilde{A} = \langle a_1, a_2, a_3 \rangle$ then we call it "Triangular Dense Fuzzy Set". Now, if for n= 0 in T, $\vartheta(x,n)$ attains the highest membership degree 1 then we can express this fuzzy number as "Normalized Triangular Dense Fuzzy Set" or NTDFS.

Example-2: Let the falsity set is given by

$\tilde{B} =< b_2(1-\rho_2)e^{-n}, b_2 e^{-n}, b_2(1+\sigma_2)e^{-n} >$

$for\, 0 < \rho_2, \sigma_2 < 1$ (5)

And its non-membership function for $0 \le n$ is defined as $\vartheta(x,n) =$

$$\begin{cases} 0 \; if \quad x < b_2(1-\rho_2)e^{-n} \; and \; x > b_2(1+\sigma_2)e^{-n} \\ \left\{\frac{b_2e^{-n} - x}{\rho_2 b_2 e^{-n}}\right\} \quad if \quad b_2(1-\rho_2)e^{-n} \le x \le b_2\, e^{-n} \\ \left\{\frac{x - b_2e^{-n}}{\sigma_2 b_2 e^{-n}}\right\} \; if \quad b_2\, e^{-n} \le x \le b_2(1+\sigma_2)e^{-n} \end{cases} \quad (6)$$

The graphical representation of non-membership function is given in Fig. 3.

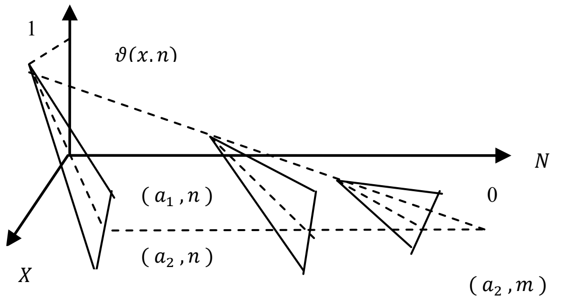

Fig. 3: Non membership function of NTDFS

Example-3: Let, the indeterminacy dense fuzzy set be of the form $\widetilde{C} =< c_2(1-\rho_3)e^{-n}, c_2e^{-n}, c_2(1+\sigma_3)e^{-n} >$ $for\; 0 < \rho_3, \sigma_3 < 1$ (7)

With the membership function

$$\pi(x,n) =$$

$$\begin{cases} 0 \quad if \quad x < c_2(1-\rho_3)e^{-n} \; and \; x > c_2(1+\sigma_3)e^{-n} \\ \left\{\frac{c_2e^{-n} - x}{\rho_3 c_2 e^{-n}}\right\} \quad if \quad c_2(1-\rho_3)e^{-n} \le x \le c_2\, e^{-n} \\ \left\{\frac{x - c_2e^{-n}}{\sigma_3 c_2 e^{-n}}\right\} \; if \quad c_2\, e^{-n} \le x \le c_2(1+\sigma_3)e^{-n} \end{cases}$$

(8)

Definition-11: Let $X \times N$ be a space of points (objects) with generic element (*x, n*). Then a neutrosophic set A in $X \times N$ is characterize by a truth membership function T_A, an indeterminacy membership function I_A and a falsity membership function F_A. The functions $T_A, I_A and F_A$ are real standard or non-standard subsets of $]0^-, 1^+[$.That is $T_A : X \times N \to]0^-, 1^+[$, $I_A : X \times N \to]0^-, 1^+[$ and is $F_A : X \times N \to]0^-, 1^+[$ having the property that, as $n \to \infty$ if $T_A(x,n) \to 1$, $I_A(x,n) \to 0 \leftarrow F_A(x,n)$ And satisfying the relation is $0^- \le supT_A(x,n) + supI_A(x,n) + supF_A(x,n) \le 3^+$,

Definition-12: A Neutrosophic set A in $X \times N$ is said to be Neutrosophic Intuitionistic Dense fuzzy Set if the elements of NS, that is the functional components $T_A, I_A and F_A$are taken from the real standard subsets of $[0,1]$. That is $T_A : X \times N \to [0,1]$, $I_A : X \times N \to [0,1]$ and is $F_A : X \times N \to [0,1]$ having the property that, as $n \to \infty$ if $T_A(x,n) \to 1$, $I_A(x,n) \to 0 \leftarrow F_A(x,n)$ satisfying the relation is $0 \le supT_A(x,n) + sup\; I_A(x,n) + supF_A(x,n) \le 3$,

Remark 2.The NS for dependency components(Ye [20])

Here we draw the simple Venn diagram for the NS with dependency components. We use this diagram to realize the overall assessment of the fuzzy components. According to probability theory, the overall score obtained from the fig.-4 is stated in (14). Note that, if $a_2 = b_2 = c_2$ holds in the fuzzy sets stated in (1), (5) and (7) then the common region of that set will be crisp one. This is shown in Fig-4

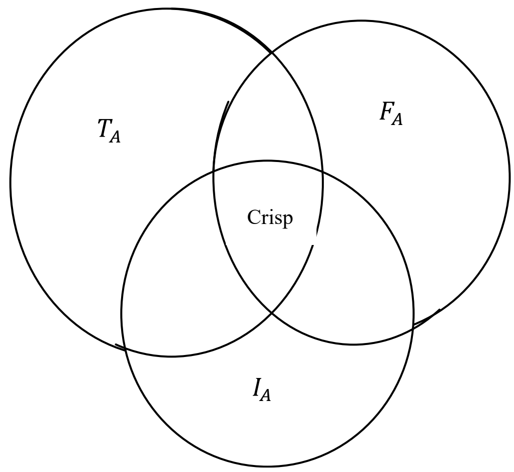

Fig.-4: Venn diagram of General Neutrosophic Set

4 Some basics over expected values of NS

Here we shall discuss over the ultimate score or expected defuzzified values for the proposed neutrosophic set$N_s =< T_A, I_A, F_A >$

i) When the components $T_A, I_A and F_A$ are independent Then as per [**Biswas et. al.[4]**] the total average expected score will be

$S(x) = \frac{1}{3}\{T_A(x) + I_A(x) + F_A(x)\}$(9)

And the truth favorite relative expected value (score) is given by

$S(x) = \frac{3\,T_A(x)}{T_A(x)+ I_A(x)+ F_A(x)}$ (10)

For information lacking in NS, this part can be divided into several sub cases.

a) The components $T_A, I_A and F_A$ all together constitute a positive skewed distribution.

In this case (truth leads in the major part), the total score will be

$S(x) = w_1T_A(x) + w_2I_A(x) + w_3F_A(x)$, for $w_1 > w_2 > w_3$ and $w_1 + w_2 + w_3 = 1$ (11)

b) The components $T_A, I_A and F_A$ all together constitute a negative skewed distribution.

In this case (falsehood leads major part), the total score will be

$S(x) = w_1T_A(x) + w_2I_A(x) + w_3F_A(x)$, for $w_1 < w_2 < w_3$ and $w_1 + w_2 + w_3 = 1$ (12)

c) The components $T_A, I_A and F_A$ all together constitute a normal distribution.

In this case (truth and falsehood are symmetric) , the total score will be

$S(x) = w_1T_A(x) + w_2I_A(x) + w_3F_A(x)$, for $w_1 < w_2 > w_3$ and $w_1 + w_2 + w_3 = 1$ (13)

ii) When the components $T_A, I_A and F_A$ are dependent

a) If the components keep the positive sign then the ultimate score is given by

$S(x) = T_A(x) + I_A(x) + F_A(x) - T_A(x)I_A(x) - I_A(x)F_A(x) - T_A(x)F_A(x)+T_A(x)I_A(x)F_A(x)$ (14)

b) For the case of IFS, the effect of $I_A(x) \to \{0\}$ or unknown and the sign of $F_A(x)$ be negative and hence the ultimate score be

$S(x) = T_A(x) - F_A(x) + T_A(x)F_A(x)$ (15)

c) [Chen and Tan [6]] Since the expected values of $T_A(x) < 1\ and F_A(x) < 1$ so their product values $T_A(x)F_A(x) \ll 1$ and hence their effect can be ignored. In this case the score function be

$S(x) = T_A(x) - F_A(x)$ (16)

5 Improved NS assessments under dense fuzzy environment

Case-I : When all the membership functions keep their values of similar types for same number of observations/interactions or time durations. First of all, we shall discuss the fuzzy assessments under dense fuzzy developed by De and Beg [7]. The basic aim of the dense fuzzy model is that each and every fuzzy component reaches to singleton crisp value whenever we would like to experiencing with fuzzy data for a long period of time or interactions in practice. Here also we assume the learning experiences are conducted within the same elapsed time or interactions for all the fuzzy components of NS. Thus unlike Biswas et al. [4]; utilizing dense concept and $\alpha - cuts$ developed by De and Beg[7], the defuzzified value reduces from the formula $I(\widetilde{A}) = \frac{1}{2T}\iint_{\alpha=0,t=0}^{\alpha=1,t=T}\{L(\alpha,t) + R(\alpha,t)\}\, d\alpha dt = a_2\left\{1 + \frac{(\sigma-\rho)}{4T}Log(1+T)\right\}$ for time (T) dependent fuzzy membership function and that for frequency dependent membership function :

$I(\widetilde{A}) = \frac{1}{2N}\sum_{n=0}^{N} a_2\left\{2 + \frac{\sigma-\rho}{2(1+n)}\right\} = \frac{a_2}{2N}\left[2N + \frac{\sigma-\rho}{2}\left\{\frac{1}{1+0} + \frac{1}{1+1} + \frac{1}{1+2} + \cdots \ldots + \frac{1}{1+N}\right\}\right]$ and obtained as

5.1 For time dependent, (11) reduces to $I(S(x,t)) = \frac{1}{3}\{I[T_A(x,t) + I[\,I_A(x,t)] + I[F_A(x,t)]\}$

$\Rightarrow I(S) = \frac{1}{3}\Big[a_2\left\{1 + \frac{(\sigma_1-\rho_1)}{4T}Log(1+T)\right\} + b_2\left\{1 + \frac{(\sigma_2-\rho_2)}{4T}Log(1+T)\right\} + c_2\left\{1 + \frac{(\sigma_3-\rho_3)}{4T}Log(1+T)\right\}\Big]$ (17)

And that for frequency dependent, $3I(S(x,n)) = I[T_A(x,n) + I[\,I_A(x,n)] + I[F_A(x,n)]$

$\Rightarrow I(S) = \frac{1}{6N}\sum_{n=0}^{N} a_2\left\{2 + \frac{(\sigma_1-\rho_1)}{2(1+n)}\right\} + \frac{1}{6N}\sum_{n=0}^{N} b_2\left\{2 + \frac{(\sigma_2-\rho_2)}{2(1+n)}\right\} + \frac{1}{6N}\sum_{n=0}^{N} c_2\left\{2 + \frac{(\sigma_3-\rho_3)}{2(1+n)}\right\}$ (18)

5.2 For time dependent fuzzy components, (10) reduces to

$I(S) = 3a_2\left\{1 + \frac{(\sigma_1-\rho_1)}{4T}Log(1+T)\right\}/ \Big[a_2\left\{1 + \frac{(\sigma_1-\rho_1)}{4T}Log(1+T)\right\} + b_2\left\{1 + \frac{(\sigma_2-\rho_2)}{4T}Log(1+T)\right\} + c_2\left\{1 + \frac{(\sigma_3-\rho_3)}{4T}Log(1+T)\right\}\Big]$ (19)

And that for frequency dependent

$$I(S) = \frac{3}{2N}\sum_{n=0}^{N} a_2\left\{2 + \frac{(\sigma_1-\rho_1)}{2(1+n)}\right\}/$$

$[\frac{1}{2N}\sum_{n=0}^{N} a_2\left\{2 + \frac{(\sigma_1-\rho_1)}{2(1+n)}\right\} + \frac{1}{2N}\sum_{n=0}^{N} b_2\left\{2 + \frac{(\sigma_2-\rho_2)}{2(1+n)}\right\} + \frac{1}{2N}\sum_{n=0}^{N} c_2\left\{2 + \frac{(\sigma_3-\rho_3)}{2(1+n)}\right\}]$ (20)

Similarly for (11-13) we get

5.3 For time dependent, (11-13) reduces to $I(S(x,t)) = w_1I[T_A(x,t) + w_2I[\,I_A(x,t)] + w_3I[F_A(x,t)]$

$\Rightarrow I(S) = w_1a_2\left\{1 + \frac{(\sigma_1-\rho_1)}{4T}Log(1+T)\right\} + w_2b_2\left\{1 + \frac{(\sigma_2-\rho_2)}{4T}Log(1+T)\right\} + w_3c_2\left\{1 + \frac{(\sigma_3-\rho_3)}{4T}Log(1+T)\right\}$ with $w_1 + w_2 + w_3 = 1$

(21)

And that for frequency dependent,

$\Rightarrow I(S) = w_1 \frac{1}{2N}\sum_{n=0}^{N} a_2 \left\{2 + \frac{(\sigma_1-\rho_1)}{2(1+n)}\right\} + w_2 \frac{1}{2N}\sum_{n=0}^{N} b_2 \left\{2 + \frac{(\sigma_2-\rho_2)}{2(1+n)}\right\} + w_3 \frac{1}{2N}\sum_{n=0}^{N} c_2 \left\{2 + \frac{(\sigma_3-\rho_3)}{2(1+n)}\right\}$ with $w_1 + w_2 + w_3 = 1$ (22)

5.4 For frequency dependent fuzzy components(14) reduces to

$$I(S) = I(T_A) + I(I_A) + I(F_A) - I(T_A I_A) - I(I_A F_A) - I(T_A F_A) + I(T_A I_A F_A)$$

$$\Rightarrow I(S) = \frac{1}{2N}\sum_{n=0}^{N} a_2 \left\{2 + \frac{(\sigma_1-\rho_1)}{2(1+n)}\right\} + \frac{1}{2N}\sum_{n=0}^{N} b_2 \left\{2 + \frac{(\sigma_2-\rho_2)}{2(1+n)}\right\} + \frac{1}{2N}\sum_{n=0}^{N} c_2 \left\{2 + \frac{(\sigma_3-\rho_3)}{2(1+n)}\right\} - \frac{1}{2N}\sum_{n=0}^{N} a_2 b_2 \left[2 + \frac{\sigma_1+\sigma_2-\rho_1-\rho_2}{2(1+n)} + \frac{\rho_1\rho_2+\sigma_1\sigma_2}{6(1+n)^2}\right] - \frac{1}{2N}\sum_{n=0}^{N} b_2 c_2 \left[2 + \frac{\sigma_2+\sigma_3-\rho_2-\rho_3}{2(1+n)} + \frac{\rho_2\rho_3+\sigma_2\sigma_3}{3(1+n)^2}\right] - \frac{1}{2N}\sum_{n=0}^{N} a_2 c_2 \left[2 + \frac{\sigma_1+\sigma_3-\rho_1-\rho_3}{2(1+n)} + \frac{\rho_1\rho_3+\sigma_1\sigma_3}{6(1+n)^2}\right] + \frac{a_2 b_2 c_2}{2N}\sum_{n=0}^{N}\left[2 + \frac{\sigma_1-\rho_1}{1+n} + \frac{\rho_1+\sigma_2+\sigma_3-\sigma_1-\rho_2-\rho_3}{2(1+n)}\right] + \frac{a_2 b_2 c_2}{2N}\sum_{n=0}^{N}\left[\frac{\rho_1(\rho_2+\rho_3)+\sigma_1(\sigma_2+\sigma_3)}{2(1+n)^2} + \frac{\sigma_1\sigma_2\sigma_3-\rho_1\rho_2\rho_3}{(1+n)^3}\right] - \frac{a_2 b_2 c_2}{2N}\sum_{n=0}^{N}\left[\frac{\rho_1\rho_2+\rho_1\rho_3-\rho_2\rho_3+\sigma_1\sigma_2+\sigma_1\sigma_3-\sigma_2\sigma_3}{3(1+n)^2} + \frac{\sigma_1\sigma_2\sigma_3+\rho_1\rho_2\rho_3}{4(1+n)^3}\right] \quad (23)$$

(for detail, see appendix equation (A.1 - A.5)

Also, the results for time continuous fuzzy membership we have by simply integrating the above sum with respect to time and applying the dense rule and get

$$I(S) = a_2\left\{1 + \frac{(\sigma_1-\rho_1)}{4T}Log(1+T)\right\} + b_2\left\{1 + \frac{(\sigma_2-\rho_2)}{4T}Log(1+T)\right\} + c_2\left\{1 + \frac{(\sigma_3-\rho_3)}{4T}Log(1+T)\right\} - a_2 b_2\left[1 + \frac{\sigma_1+\sigma_2-\rho_1-\rho_2}{4T}Log(1+T) - \frac{\rho_1\rho_2+\sigma_1\sigma_2}{12\,T(1+T)}\right] - b_2 c_2\left[1 + \frac{\sigma_2+\sigma_3-\rho_2-\rho_3}{4T}Log(1+T) - \frac{\rho_2\rho_3+\sigma_2\sigma_3}{6\,T(1+T)}\right] - a_2 c_2\left[1 + \frac{\sigma_1+\sigma_3-\rho_1-\rho_3}{4T}Log(1+T) + \frac{\rho_1\rho_3+\sigma_1\sigma_3}{12T\,(1+T)}\right] + a_2 b_2 c_2\left[1 + \frac{\sigma_1-\rho_1}{2T}Log(1+T) + \frac{\rho_1+\sigma_2+\sigma_3-\sigma_1-\rho_2-\rho_3}{4T}Log(1+T) - \frac{\rho_1(\rho_2+\rho_3)+\sigma_1(\sigma_2+\sigma_3)}{4T(1+T)}\right] + a_2 b_2 c_2\left[\frac{\rho_1\rho_2+\rho_1\rho_3-\rho_2\rho_3+\sigma_1\sigma_2+\sigma_1\sigma_3-\sigma_2\sigma_3}{6T\,(1+T)} - \frac{\sigma_1\sigma_2\sigma_3-\rho_1\rho_2\rho_3}{4T(1+T)^2} + \frac{\sigma_1\sigma_2\sigma_3+\rho_1\rho_2\rho_3}{16T(1+T)^2}\right] \quad (24)$$

5.5 For time dependent fuzzy components, (15) reduces to

$$I(S) = I(T_A) + I(F_A) - I(T_A F_A) \Rightarrow I(S) = a_2\left\{1 + \frac{(\sigma_1-\rho_1)}{4T}Log(1+T)\right\} + b_2\left\{1 + \frac{(\sigma_2-\rho_2)}{4T}Log(1+T)\right\} - a_2 b_2\left[1 + \frac{\sigma_1+\sigma_2-\rho_1-\rho_2}{4T}Log(1+T) - \frac{\rho_1\rho_2+\sigma_1\sigma_2}{12\,T(1+T)}\right] \quad (25)$$

And that for frequency dependent fuzzy components,

$$\Rightarrow I(S) = \frac{1}{2N}\sum_{n=0}^{N} a_2\left\{2 + \frac{(\sigma_1-\rho_1)}{2(1+n)}\right\} + \frac{1}{2N}\sum_{n=0}^{N} b_2\left\{2 + \frac{(\sigma_2-\rho_2)}{2(1+n)}\right\} - \frac{1}{2N}\sum_{n=0}^{N} a_2 b_2\left[2 + \frac{\sigma_1+\sigma_2-\rho_1-\rho_2}{2(1+n)} + \frac{\rho_1\rho_2+\sigma_1\sigma_2}{6\,(1+n)^2}\right] \quad (26)$$

Case-II : When all the membership functions keep their values of different types for same number of observations/interactions or time durations. Here we shall take the membership functions of T_A, I_A and F_A as stated in (2), (6) and (8) .

Now the score function of (16) under time sensitive fuzzy numbers is given by, $S(x,t) = \begin{cases} \frac{x-a_2\left(1-\frac{\rho_1}{1+t}\right)}{\frac{\rho_1 a_2}{1+t}} - \frac{b_2e^{-t}-x}{b_2\rho_2 e^{-t}} if \quad u \le x \le v \text{ , say} \\ \frac{a_2\left(1+\frac{\sigma_1}{1+t}\right)-x}{\frac{\sigma_1 a_2}{1+t}} - \frac{x-b_2e^{-t}}{b_2\sigma_1 e^{-t}} if v \le x \le w \text{ say} \end{cases}$

$$\Rightarrow S(x,t) = \begin{cases} x\left[\frac{1+t}{\rho_1 a_2} + \frac{e^t}{\rho_2 b_2}\right] - \left(\frac{1+t}{\rho_1} + \frac{1}{\rho_2} - 1\right) if \quad u \le x \le v \text{ , say} \\ \left(\frac{1+t}{\sigma_1} + \frac{1}{\sigma_2} + 1\right) - x\left[\frac{1+t}{\sigma_1 a_2} + \frac{e^t}{\sigma_2 b_2}\right] if v \le x \le w \text{ say} \end{cases}$$

Using α −cuts and employing the index formula developed by De and Beg [7] we get, $\boldsymbol{I}\,(\boldsymbol{S}) = \frac{\mathbf{1}}{\mathbf{2T}} \iint_{\alpha=0,t=0}^{\alpha=1,t=T} \{L(\alpha,t) + R(\alpha,t)\}\, d\alpha dt$

$$= \frac{1}{2T}\int_0^T \left[\frac{\frac{1+t}{\rho_1}+\frac{1}{\rho_2}-1}{\frac{1+t}{\rho_1 a_2}+\frac{e^t}{\rho_2 b_2}} + \frac{\frac{1+t}{\sigma_1}+\frac{1}{\sigma_2}+1}{\frac{1+t}{\sigma_1 a_2}+\frac{e^t}{\sigma_2 b_2}} + \frac{1}{2}\left(\frac{1}{\frac{1+t}{\rho_1 a_2}+\frac{e^t}{\rho_2 b_2}} - \frac{1}{\frac{1+t}{\sigma_1 a_2}+\frac{e^t}{\sigma_2 b_2}}\right)\right] dt = \frac{a_2 b_2}{2T}\int_0^T \left[\frac{\left((1+t)\rho_2+\rho_1-\frac{\rho_1\rho_2}{2}\right)}{(1+t)\rho_2 b_2 + e^t \rho_1 a_2} + \frac{\left((1+t)\sigma_2+\sigma_1+\frac{\sigma_1\sigma_2}{2}\right)}{(1+t)\sigma_2 b_2 + e^t \sigma_1 a_2}\right] dt \qquad (27)$$

And that for discrete case, replacing *t* by *n* we write the expected score of the NS as

$$\boldsymbol{I}\,(\boldsymbol{S}\,) = \frac{a_2 b_2}{2N}\sum_{n=0}^{N} \left[\frac{\left((1+n)\rho_2+\rho_1-\frac{\rho_1\rho_2}{2}\right)}{(1+n)\rho_2 b_2 + e^n \rho_1 a_2} + \frac{\left((1+n)\sigma_2+\sigma_1+\frac{\sigma_1\sigma_2}{2}\right)}{(1+n)\sigma_2 b_2 + e^n \sigma_1 a_2}\right] \qquad (28)$$

Case-III: If we assume that the learning effects are not performed in same time duration / interactions for all fuzzy components then the above defuzzification formula (17) & (18) reduces to

$$\Rightarrow 3\, I(S) = a_2\left\{1 + \frac{(\sigma_1-\rho_1)}{4T_1} Log(1+T_1)\right\} + b_2\left\{1 + \frac{(\sigma_2-\rho_2)}{4T_2} Log(1+T_2)\right\} + c_2\left\{1 + \frac{(\sigma_3-\rho_3)}{4T_3} Log(1+T_3)\right\} (29)$$

And that for frequency dependent, $3I(S(x,m,n,p)) = I[T_A(x,m) + I[\,I_A(x,n)] + I[F_A(x,p)]$

$$\Rightarrow I(S) = \frac{1}{6M}\sum_{m=0}^{M} a_2\left\{2 + \frac{(\sigma_1-\rho_1)}{2(1+m)}\right\} + \frac{1}{6N}\sum_{n=0}^{N} b_2\left\{2 + \frac{(\sigma_2-\rho_2)}{2(1+n)}\right\} + \frac{1}{6P}\sum_{p=0}^{P} c_2\left\{2 + \frac{(\sigma_3-\rho_3)}{2(1+p)}\right\} \qquad (30)$$

The detailed discussion is made in Appendix B.

6 Implication of Dense fuzzy in NS

In NS, the traditional concept of membership values of truth, falsehood and indeterminacy are either fixed or cannot be changed once it is assigned. But the present study reveals that such points are continuously changing because of learning experiences or flexibility of the human behavior and intentions. For instance, when we ask a question to a person about the life loss due to a certain train accident, then (s) he might be answered that 75% of the whole people died, 40% not died, and 30% is unknown. But, after few days later if the same question has been thrown to the same person, then obviously his/her answer might differ from the earlier statement. This may be 90%, 50% and 10% or 10%, 80%, 5%. Such kind of observation occurs due to information gathering from the society or learning experiences as soon as the time is passed/ increased frequency of human interactions within the locality/ society/ mass media etc. By this way before going to take governmental supports (Decision maker's accountability), on the basis of that prior information several enquiries committee will be formed and finally come to a concrete decision for establishing law and order in people's benefit. The earlier concepts on NS analyze the data based on first answer obtained from that person, but in our present study it analyzes the data subsequently obtained.

6.1 Procedure for the computation of defuzzified values of a Neutrosophic Sets(NSs)

Step-1: Find the NS involved in the different field of activities

Step-2: Find the appropriate membership components of that NSs

Step-3: Find the interior relationship among the different NSs

Step-4: Select the strategies involved in those NSs

a) If we are intending to find the joint performances then take their union using the definition 5.

b) If we are intending to find the common performances then take their intersection using the definition 6.
c) If the selection of one's NS might able to change the other's NS then to find the complexities involved take their multiplication using the definition 8.
d) If the selection of one's NS do not able to change the other's NS at all then to find the complexities involved take their addition using the definition 7.

Step-5: After getting the appropriate NS obtained from Step-4, use defuzzification rules which one you are going to assess.

7 Applications of dense fuzzy in NS (DFNS)

Several applications can be drawn from our day to day life problems (from science and engineering, sociology, philosophy, crime research, educational psychology etc.) The following are some major areas where the DFNS can be applied.

7.1 In any kind of decision making process

Example: Suppose, in a supply chain (SC) model the set of information $N_{ss} =< T_{As}, I_{As}, F_{As} >$ and $N_{sr} =< T_{Ar}, I_{Ar}, F_{Ar} >$ for both the supplier and retailer are available under dense fuzzy environment. First of all we have to obtain the bounds of each fuzzy components utilizing dense property; then check whether these sets are subsets of each other or not. Now applying congruency rule or similarity measures the score functions for the chain can be obtained and can be solved by the proposed defuzzification methods. Note that if these two sets are disjoint then the chain immediately gets breaking down and the decision maker will have to choose another model as well.

7.2 Psychological testing/ military selection

Example: Suppose in a psychological test there are five different attributes to be measured. The attributes are {Moral value, behavior, leadership, criminal offence, responsibility}. Among these attributes some of them have positively correlation, some of them are negatively correlation and rest of them has no relation. However we have to perform the membership functions of each attributes under dense fuzzy environment and exercising these every after some stipulated interval of time/ days. Take for instance, to gain best leadership quality, one might have to compromise with criminal activities and bad responsibility in many cases in practice. On the other hand, a person having good moral standards, (s)he might carry a good behavior and good leadership. Under these circumstances, whenever we wish to compute the total score, few of the score components of the NS became negative. Thus, to get the overall performance, the proposed defuzzification method can be applied and ranked accordingly.

7.3 Leadership assessment under social agenda

Example: Suppose an open problem on flood prone zone in a locality has been thrown before two political leaders having different ideologies. The problem itself contains three different parametric attributes, like {highly flood prone, unknown, no flood at all}. In this case, first set is an appropriate NS for the given attributes under dense fuzzy environment. Then, we ask to answer the question to the political leaders at different places and different interval of times. We usually notice that the scores obtained by them at different time are not the same. Thus, to have the actual score obtained by each of them, we might have to rely their views that were delivered at the final stage of assessment.

7.4 Assessing the age of a digital image

Let in a digital image be three parametric attributes to be measured. These attributes are {brightness, white, darkness}. First of all, with proper definitions, construct the membership functions of each of these attributes under dense fuzzy environment. Now changing time (with proper record), compute score values of the NS and compare with the values obtained from the specimen digital image each time. Continue this process until the expected value gets merged with the values of the specimen image. Finally, get the age, the time you have recorded last time.

7.5 Vulnerability/ risk assessment in disaster prone zone

Let the attributes under study for measuring the vulnerability or risk in a disaster management is {whether zone is disaster prone, insurance of life covered, valuation of the property}. Many times several attitudes may come whether a particular zone is disaster prone or not. The life involved in that area is known and hence the insurance covered is assessed but valuation of the property is quite unclear in practice. However if we think of birds/ animals like pet and firm animals' life insurance or beyond then this part also carry some information lack. Under these circumstances computing score values of the proposed NS at several years the actual risk can be measured. The existing research may be viewed in Takacs [14].

7.6 Economic, Cultural, Political, Climatic, Disease Mapping

With the help of NS we can map within Country/State even in the world also with respect to different socio economic parameters. For example, studying with {rich, mediocre, poor} in different countries of the world the economically sound/unsound such as developed, under developed, less developed countries can be identified globally and can map them accordingly. Similarly for peoples' cultural entity { true telling, no telling, lie telling} or { live to eat, no comments, eat to live} or { like dance, like song, both, none } can be measured at different time at different countries and then map accordingly. For political alliance taking 10 years data from different coun-try peoples' attitudes on {democracy, autocracy, idealism, materialism, and socialism} may be considered for better mapping of friendship. For mapping on different climatic zone the parameters like { hot, neither hot nor cold, cold} or { highly polluted, unknown, no pollution} and that for mapping of severe disease prone zone, taking 15-20 years data of public opinion on { Typhoid, Diarrhea, cholera, free of disease } etc. can be applied in developing NS.

7.7 Game theory

In this competitive world, behind any kind of activities there must have a hidden game. For each strategic player we may think of a NS having three or more fuzzy components. These components are changing for several reasons. For instance, sudden fall of share market, instant price hike of commodities, ammendment of Govt. policies etc. may cause the flexibility of fuzzy (non) membership grade. However, the players might have to change their plan within one day duration. Utilizing NS fuzzy cross product and algebraic properties the problem can be solved. To know the ultimate gain of each player our proposed defuzzification method can be applied.

7.8 Personality Test/ ability identification

For the identification of psychological ability or cognitive development, several possible attributes are taken for an individual. Then constructing different membership expected neutrosophic sets are drawn. Defuzzifying the given NS, we may easily measure the different abilities, which may guide to form a personality development.

Conclusion

In this study, we have explained the existing neutrosophic set under dense fuzzy environment. Traditionally, most of the researchers were experiencing with the (non) membership grade value directly from the study area, and took decisions through some aggregation rules or ranking scores. They do not feel the urge to defuzzify the NS earlier. Also, in some of the cases, fuzzy graph theory or fuzzy matrices have been developed to capture the decision theory. The concepts of human learning, the changing characteristics of the fuzzy membership with respect to time and the number of observations have been ignored by the founder thinkers of NS. We need to defuzzify the NS under dense fuzzy environment for the following reason:

a) The fuzzy elements in NS are obtained directly from field data only
b) The fuzzy flexibility may change with time elapsed and interactions covered
c) To know the individual value rather than membership degree, because a different value may carry the same degree, but the reverse is not true due to convexity property.
d) To realize the actual world rather than a hypothetical world.

Appendix A

To find the values of the following equation (15) $I(S) = I(T_A) + I(I_A) + I(F_A) - I(T_A I_A) - I(I_A F_A) - I(T_A F_A) + I(T_A I_A F_A)$ for the cases of general membership functions

We take the help of $\alpha - cuts$ of the fuzzy components.. Now as per De and Beg[7], the left and right $\alpha - cuts$ of the fuzzy set $\tilde{A} = \langle a_1, a_2, a_3 \rangle$ or $\tilde{A} =<$

$a_2\left(1-\frac{\rho_1}{1+n}\right), a_2, a_2\left(1+\frac{\sigma_1}{1+n}\right) >, \ for\ 0<\rho_1,\sigma_1<1$ is given by

$L^{-1}(\alpha,n)=a_2\left(1-\frac{\rho_1}{1+n}+\frac{\rho_1\alpha}{1+n}\right)$ and $R^{-1}(\alpha,n)=a_2\left(1+\frac{\sigma_1}{1+n}-\frac{\sigma_1\alpha}{1+n}\right)$

Now from the properties of $\alpha-cuts$ we have

$$(T_AF_A)_\alpha=(T_A)_\alpha(F_A)_\alpha$$

$$=\left\{a_2\left(1-\frac{\rho_1}{1+n}+\frac{\rho_1\alpha}{1+n}\right), a_2\left(1+\frac{\sigma_1}{1+n}-\frac{\sigma_1\alpha}{1+n}\right)\right\}\left\{b_2\left(1-\frac{\rho_2\alpha}{1+n}\right), b_2\left(1+\frac{\sigma_2\alpha}{1+n}\right)\right\}$$

$$=a_2b_2\left\{\left(1-\frac{\rho_1}{1+n}+\frac{\rho_1\alpha}{1+n}\right)\left(1-\frac{\rho_2\alpha}{1+n}\right),\left(1+\frac{\sigma_1}{1+n}-\frac{\sigma_1\alpha}{1+n}\right)\left(1+\frac{\sigma_2\alpha}{1+n}\right)\right\}$$

$$=a_2b_2\left[1-\frac{\rho_1}{1+n}+\alpha\left\{\frac{\rho_1\rho_2}{(1+n)^2}+\frac{\rho_1-\rho_2}{(1+n)}\right\}-\frac{\rho_1\rho_2\alpha^2}{(1+n)^2},\ 1+\frac{\sigma_1}{1+n}+\alpha\left\{\frac{\sigma_1\sigma_2}{(1+n)^2}+\frac{\sigma_2-\sigma_1}{(1+n)}\right\}-\frac{\sigma_1\sigma_2\alpha^2}{(1+n)^2}\right]$$

Note that, in above we assume the left and right $\alpha-cuts$ are increasing and decreasing functions of α respectively. If it is impossible to determine whether the above conditions hold or not, then the gross value of the $\alpha-cuts$ are given by

The left $\alpha-cut$ of $(T_AF_A)_\alpha=$

$$=Min\left\{a_2\left(1-\frac{\rho_1}{1+n}+\frac{\rho_1\alpha}{1+n}\right), a_2\left(1+\frac{\sigma_1}{1+n}-\frac{\sigma_1\alpha}{1+n}\right)\right\}\left\{b_2\left(1-\frac{\rho_2\alpha}{1+n}\right), b_2\left(1+\frac{\sigma_2\alpha}{1+n}\right)\right\}$$

$$=Min\,a_2b_2\left\{\left(1-\frac{\rho_1}{1+n}+\frac{\rho_1\alpha}{1+n}\right)\left(1-\frac{\rho_2\alpha}{1+n}\right),\left(1-\frac{\rho_1}{1+n}+\frac{\rho_1\alpha}{1+n}\right)\left(1+\frac{\sigma_2\alpha}{1+n}\right),\left(1+\frac{\sigma_1}{1+n}-\frac{\sigma_1\alpha}{1+n}\right)\left(1-\frac{\rho_2\alpha}{1+n}\right),\left(1+\frac{\sigma_1}{1+n}-\frac{\sigma_1\alpha}{1+n}\right)\left(1+\frac{\sigma_2\alpha}{1+n}\right)\right\}$$

And the right $\alpha-cut$ of $(T_AF_A)_\alpha=$

$$=Max\,a_2b_2\left\{\left(1-\frac{\rho_1}{1+n}+\frac{\rho_1\alpha}{1+n}\right)\left(1-\frac{\rho_2\alpha}{1+n}\right),\left(1-\frac{\rho_1}{1+n}+\frac{\rho_1\alpha}{1+n}\right)\left(1+\frac{\sigma_2\alpha}{1+n}\right),\left(1+\frac{\sigma_1}{1+n}-\frac{\sigma_1\alpha}{1+n}\right)\left(1-\frac{\rho_2\alpha}{1+n}\right),\left(1+\frac{\sigma_1}{1+n}-\frac{\sigma_1\alpha}{1+n}\right)\left(1+\frac{\sigma_2\alpha}{1+n}\right)\right\}$$

The same rule could be applied in other cases also.

Therefore, the index value is given by

$$I(T_AF_A)=\frac{1}{2N}\sum_{n=0}^{N}a_2b_2\left[1-\frac{\rho_1}{1+n}+\frac{1}{2}\left\{\frac{\rho_1\rho_2}{(1+n)^2}+\frac{\rho_1-\rho_2}{(1+n)}\right\}-\frac{\rho_1\rho_2}{3\,(1+n)^2}+\ 1+\frac{\sigma_1}{1+n}+\frac{1}{2}\left\{\frac{\sigma_1\sigma_2}{(1+n)^2}+\frac{\sigma_2-\sigma_1}{(1+n)}\right\}-\frac{\sigma_1\sigma_2}{3\,(1+n)^2}\right]$$

$$=\frac{1}{2N}\sum_{n=0}^{N}a_2b_2\left[2+\frac{\sigma_1+\sigma_2-\rho_1-\rho_2}{2(1+n)}+\frac{\rho_1\rho_2+\sigma_1\sigma_2}{6\,(1+n)^2}\right]$$

(A.1)

Similarly,

$$I(T_AI_A)=\frac{1}{2N}\sum_{n=0}^{N}a_2c_2\left[2+\frac{\sigma_1+\sigma_3-\rho_1-\rho_3}{2(1+n)}+\frac{\rho_1\rho_3+\sigma_1\sigma_3}{6\,(1+n)^2}\right]$$

(A.2)

Also to find, $I(F_AI_A)$ we write,

$$(I_AF_A)_\alpha=(I_A)_\alpha(F_A)_\alpha$$

$$=\left\{b_2\left(1-\frac{\rho_2\alpha}{1+n}\right), b_2\left(1+\frac{\sigma_2\alpha}{1+n}\right)\right\}\left\{c_2\left(1-\frac{\rho_3\alpha}{1+n}\right), c_2\left(1+\frac{\sigma_3\alpha}{1+n}\right)\right\}$$

$$=b_2c_2\left[\left(1-\frac{\rho_2\alpha}{1+n}\right)\left(1-\frac{\rho_3\alpha}{1+n}\right),\left(1+\frac{\sigma_2\alpha}{1+n}\right)\left(1+\frac{\sigma_3\alpha}{1+n}\right)\right]$$

$$=b_2c_2\left[1-\alpha\left(\frac{\rho_2+\rho_3}{1+n}\right)+\frac{\rho_2\rho_3\alpha^2}{(1+n)^2}, 1+\alpha\left(\frac{\sigma_2+\sigma_3}{1+n}\right)+\frac{\sigma_2\sigma_3\alpha^2}{(1+n)^2}\right]$$

Therefore

$$I(F_AI_A)=\frac{1}{2N}\sum_{n=0}^{N}b_2c_2\left[2+\frac{\sigma_2+\sigma_3-\rho_2-\rho_3}{2(1+n)}+\frac{\rho_2\rho_3+\sigma_2\sigma_3}{3\,(1+n)^2}\right]$$

(A.3)

Moreover to find the index value of $(T_AI_AF_A)$ we write, the

$$(T_AI_AF_A)_\alpha=(T_A)_\alpha(I_A)_\alpha(F_A)_\alpha$$

$$= a_2b_2c_2\left[\left(1-\frac{\rho_1}{1+n}+\frac{\rho_1\alpha}{1+n}\right),\left(1+\frac{\sigma_1}{1+n}-\frac{\sigma_1\alpha}{1+n}\right)\right]\left[1-\alpha\left(\frac{\rho_2+\rho_3}{1+n}\right)+\frac{\rho_2\rho_3\alpha^2}{(1+n)^2},1+\alpha\left(\frac{\sigma_2+\sigma_3}{1+n}\right)+\frac{\sigma_2\sigma_3\alpha^2}{(1+n)^2}\right]$$

$$= a_2b_2c_2\left[1-\frac{\rho_1}{1+n}+\alpha\left\{\frac{\rho_1-\rho_2-\rho_3}{(1+n)}+\frac{\rho_1(\rho_2+\rho_3)}{(1+n)^2}\right\}-\alpha^2\left\{\frac{\rho_1\rho_2\rho_3}{(1+n)^3}+\frac{\rho_1\rho_2+\rho_1\rho_3-\rho_2\rho_3}{(1+n)^2}\right\}+\frac{\rho_1\rho_2\rho_3\alpha^3}{(1+n)^3},\right.$$

$$\left.1+\frac{\sigma_1}{1+n}+\alpha\left\{\frac{\sigma_2+\sigma_3-\sigma_1}{(1+n)}+\frac{\sigma_1(\sigma_2+\sigma_3)}{(1+n)^2}\right\}+\alpha^2\left\{\frac{\sigma_1\sigma_2\sigma_3}{(1+n)^3}+\frac{\sigma_2\sigma_3-\sigma_1\sigma_2-\sigma_1\sigma_3}{(1+n)^2}\right\}-\frac{\sigma_1\sigma_2\sigma_3\alpha^3}{(1+n)^3}\right]$$

Thus, $I(T_A\,I_A\,F_A)$

$$= \frac{1}{2N}\sum_{n=0}^{N} a_2b_2c_2\left[2+\frac{\sigma_1-\rho_1}{1+n}+\frac{\rho_1+\sigma_2+\sigma_3-\sigma_1-\rho_2-\rho_3}{2(1+n)}+\frac{\rho_1(\rho_2+\rho_3)+\sigma_1(\sigma_2+\sigma_3)}{2(1+n)^2}-\frac{\rho_1\rho_2+\rho_1\rho_3-\rho_2\rho_3+\sigma_1\sigma_2+\sigma_1\sigma_3-\sigma_2\sigma_3}{3\,(1+n)^2}+\frac{\sigma_1\sigma_2\sigma_3-\rho_1\rho_2\rho_3}{(1+n)^3}-\frac{\sigma_1\sigma_2\sigma_3+\rho_1\rho_2\rho_3}{4(1+n)^3}\right] \quad \text{(A.4)}$$

Appendix B

Here we shall study the $\alpha-cuts$ of $(T_A,\ I_A\ and\ F_A)$ when all the components occur proper dense property of fuzzy sets[stated in (2), (6) and (8)]. In this case the $\alpha-cut$ of T_A that is $(T_A)_\alpha$ will remains the same.

But the $\alpha-cuts$ of $(F_A)_\alpha$ are given by $\left\{\frac{b_2e^{-n}-x}{\rho_2b_2e^{-n}}\right\}\geq\alpha\rightarrow x\leq b_2e^{-n}(1-\alpha\rho_2)$ and $\left\{\frac{x-b_2e^{-n}}{\sigma_2b_2e^{-n}}\right\}\geq\alpha\rightarrow x\geq b_2e^{-n}(1+\alpha\sigma_2)$

Thus $(F_A)_\alpha=[b_2e^{-n}(1-\alpha\rho_2),b_2e^{-n}(1+\alpha\sigma_2)]=b_2e^{-n}[(1-\alpha\rho_2),(1+\alpha\sigma_2)]$ (B.1)

Similarly, $(I_A)_\alpha=c_2e^{-n}[(1-\alpha\rho_3),(1+\alpha\sigma_3)]$ (B.2)

Therefore, $I(T_A)=\frac{1}{2N}\sum_{n=0}^{N}a_2\left\{2+\frac{(\sigma_1-\rho_1)}{2(1+n)}\right\}$ (B.3)

$I(I_A)=\frac{c_2}{2N}\left\{2+\frac{1}{2}(\sigma_3-\rho_3)\right\}\sum_{n=0}^{N}e^{-n}$ (B.4)

$I(F_A)=\frac{b_2}{2N}\left\{2+\frac{1}{2}(\sigma_2-\rho_2)\right\}\sum_{n=0}^{N}e^{-n}$ (B.5)

Again, $(T_A\,F_A)_\alpha=(T_A)_\alpha(F_A)_\alpha=a_2\left[\left(1-\frac{\rho_1}{1+n}+\frac{\rho_1\alpha}{1+n}\right),\left(1+\frac{\sigma_1}{1+n}-\frac{\sigma_1\alpha}{1+n}\right)\right]\{b_2e^{-n}[(1-\alpha\rho_2),(1+\alpha\sigma_2)]\}$

$$= a_2b_2e^{-n}\left[\left(1-\frac{\rho_1}{1+n}+\frac{\rho_1\alpha}{1+n}\right)(1-\alpha\rho_2),\left(1+\frac{\sigma_1}{1+n}-\frac{\sigma_1\alpha}{1+n}\right)(1+\alpha\sigma_2)\right]$$

$$= a_2b_2e^{-n}\begin{bmatrix}1-\frac{\rho_1}{1+n}+\frac{\rho_1\alpha}{1+n}-\alpha\rho_2+\frac{\alpha\rho_1\rho_2}{1+n}-\frac{\alpha^2\rho_1\rho_2}{1+n},\\ 1+\frac{\sigma_1}{1+n}-\frac{\sigma_1\alpha}{1+n}+\alpha\sigma_2+\frac{\alpha\sigma_1\sigma_2}{1+n}-\frac{\alpha^2\sigma_1\sigma_2}{1+n}\end{bmatrix}$$

$$= a_2b_2e^{-n}\begin{bmatrix}1-\frac{\rho_1}{1+n}+\alpha\left(\frac{\rho_1+\rho_1\rho_2}{1+n}-\rho_2\right)-\frac{\alpha^2\rho_1\rho_2}{1+n},\\ 1+\frac{\sigma_1}{1+n}+\alpha\left(\frac{\sigma_1\sigma_2-\sigma_1}{1+n}+\sigma_2\right)-\frac{\alpha^2\sigma_1\sigma_2}{1+n}\end{bmatrix}$$

(B.6)

Thus, $I(T_A\,F_A)=\frac{a_2b_2}{2N}\sum_{n=0}^{N}e^{-n}\left[2+\frac{\sigma_1-\rho_1}{1+n}+\frac{1}{2}\left(\frac{\rho_1-\sigma_1+\sigma_1\sigma_2+\rho_1\rho_2}{1+n}+\sigma_2-\rho_2\right)-\frac{\rho_1\rho_2+\sigma_1\sigma_2}{3(1+n)}\right]$ (B.7)

Similarly, $I(T_A\,I_A)=$

$\frac{a_2c_2}{2N}\sum_{n=0}^{N}e^{-n}\left[2+\frac{\sigma_1-\rho_1}{1+n}+\frac{1}{2}\left(\frac{\rho_1-\sigma_1+\sigma_1\sigma_3+\rho_1\rho_3}{1+n}+\sigma_3-\rho_3\right)-\frac{\rho_1\rho_3+\sigma_1\sigma_3}{3(1+n)}\right]$ (B.8)

For, $I(F_A\,I_A)$

We write, $(I_A\,F_A)_\alpha=(I_A)_\alpha(F_A)_\alpha$

$$= b_2c_2e^{-2n}[(1-\alpha\rho_2),(1+\alpha\sigma_2)][(1-\alpha\rho_3),(1+\alpha\sigma_3)]$$

$$= b_2c_2e^{-2n}[1-\alpha\rho_2-\alpha\rho_3+\alpha^2\rho_2\rho_3,\ 1+\alpha\sigma_2+\alpha\sigma_3+\alpha^2\sigma_2\sigma_3]$$

Therefore,

$I(F_A\,I_A)=$

$\frac{b_2c_2}{2N}\sum_{n=0}^{N}e^{-2n}\left[2+\frac{1}{2}(\sigma_2+\sigma_3-\rho_2-\rho_3)+\frac{\rho_2\rho_3+\sigma_2\sigma_3}{3}\right]$

$$=\frac{b_2c_2}{2N}\left[2+\frac{1}{2}(\sigma_2+\sigma_3-\rho_2-\rho_3)+\frac{\rho_2\rho_3+\sigma_2\sigma_3}{3}\right]\sum_{n=0}^{N}e^{-2n}$$

(B.9)

For, $I(T_A\,I_A\,F_A)$ we take,

$$(T_A I_A F_A)_\alpha = (T_A)_\alpha (I_A)_\alpha (F_A)_\alpha = a_2 b_2 c_2 e^{-2n}[1-\alpha\rho_2-\alpha\rho_3+\alpha^2\rho_2\rho_3, 1+\alpha\sigma_2+\alpha\sigma_3+\alpha^2\sigma_2\sigma_3]\left[\left(1-\frac{\rho_1}{1+n}+\frac{\rho_1\alpha}{1+n}\right),\left(1+\frac{\sigma_1}{1+n}-\frac{\sigma_1\alpha}{1+n}\right)\right]$$

$$= a_2 b_2 c_2 e^{-2n}\left[\left(1-\frac{\rho_1}{1+n}\right)(1-\alpha\rho_2-\alpha\rho_3+\alpha^2\rho_2\rho_3)+\frac{\rho_1}{1+n}(\alpha-\alpha^2\rho_2-\alpha^2\rho_3+\alpha^3\rho_2\rho_3), \left(1+\frac{\sigma_1}{1+n}\right)(1+\alpha\sigma_2+\alpha\sigma_3+\alpha^2\sigma_2\sigma_3)-\frac{\sigma_1}{1+n}(\alpha+\alpha^2\sigma_2+\alpha^2\sigma_3+\alpha^3\sigma_2\sigma_3)\right]$$

Therefore,

$$I(T_A I_A F_A) = \frac{a_2 b_2 c_2}{2N}\sum_{n=0}^{N} e^{-2n}\left[\left(1-\frac{\rho_1}{1+n}\right)\left(1-\frac{1}{2}\rho_2-\frac{1}{2}\rho_3+\frac{1}{3}\rho_2\rho_3\right)+\frac{\rho_1}{1+n}\left(\frac{1}{2}-\frac{1}{3}\rho_2-\frac{1}{3}\rho_3+\frac{1}{4}\rho_2\rho_3\right)+\left(1+\frac{\sigma_1}{1+n}\right)\left(1+\frac{1}{2}\sigma_2+\frac{1}{2}\sigma_3+\frac{1}{3}\sigma_2\sigma_3\right)-\frac{\sigma_1}{1+n}\left(\frac{1}{2}+\frac{1}{3}\sigma_2+\frac{1}{3}\sigma_3+\frac{1}{4}\sigma_2\sigma_3\right)\right]$$

(B.10)

Hence, using (B.3)-(B.10) the expected values of the given NS can be obtained as

$I(S) = I(T_A) + I(I_A) + I(F_A) - I(T_A I_A) - I(I_A F_A) - I(T_A F_A) + I(T_A I_A F_A)$ where

$$I(S) = \frac{1}{2N}\sum_{n=0}^{N} a_2\left\{2+\frac{(\sigma_1-\rho_1)}{2(1+n)}\right\} + \frac{b_2}{2N}\left\{2+\frac{1}{2}(\sigma_2-\rho_2)\right\}\sum_{n=0}^{N} e^{-n} + \frac{c_2}{2N}\left\{2+\frac{1}{2}(\sigma_3-\rho_3)\right\}\sum_{n=0}^{N} e^{-n}$$

$$-\frac{a_2 b_2}{2N}\sum_{n=0}^{N} e^{-n}\left[2+\frac{\sigma_1-\rho_1}{1+n}+\frac{1}{2}\left(\frac{\rho_1-\sigma_1+\sigma_1\sigma_2+\rho_1\rho_2}{1+n}+\sigma_2-\rho_2\right)-\frac{\rho_1\rho_2+\sigma_1\sigma_2}{3(1+n)}\right]$$

$$-\frac{b_2 c_2}{2N}\sum_{n=0}^{N} e^{-2n}\left[2+\frac{1}{2}(\sigma_2+\sigma_3-\rho_2-\rho_3)+\frac{\rho_2\rho_3+\sigma_2\sigma_3}{3}\right]$$

$$-\frac{a_2 c_2}{2N}\sum_{n=0}^{N} e^{-n}\left[2+\frac{\sigma_1-\rho_1}{1+n}+\frac{1}{2}\left(\frac{\rho_1-\sigma_1+\sigma_1\sigma_3+\rho_1\rho_3}{1+n}+\sigma_3-\rho_3\right)-\frac{\rho_1\rho_3+\sigma_1\sigma_3}{3(1+n)}\right]$$

$$+\frac{a_2 b_2 c_2}{2N}\sum_{n=0}^{N} e^{-2n}\left[\left(1-\frac{\rho_1}{1+n}\right)\left(1-\frac{1}{2}\rho_2-\frac{1}{2}\rho_3+\frac{1}{3}\rho_2\rho_3\right)+\frac{\rho_1}{1+n}\left(\frac{1}{2}-\frac{1}{3}\rho_2-\frac{1}{3}\rho_3+\frac{1}{4}\rho_2\rho_3\right)+\left(1+\frac{\sigma_1}{1+n}\right)\left(1+\frac{1}{2}\sigma_2+\frac{1}{2}\sigma_3+\frac{1}{3}\sigma_2\sigma_3\right)-\frac{\sigma_1}{1+n}\left(\frac{1}{2}+\frac{1}{3}\sigma_2+\frac{1}{3}\sigma_3+\frac{1}{4}\sigma_2\sigma_3\right)\right] \quad \text{(B.11)}$$

Note that, if the fuzzy components are experienced with different interactions then we shall calculate the expected score values as follows:

$$I(S) = \frac{1}{2M}\sum_{m=0}^{M} a_2\left\{2+\frac{(\sigma_1-\rho_1)}{2(1+m)}\right\} + \frac{b_2}{2N}\left\{2+\frac{1}{2}(\sigma_2-\rho_2)\right\}\sum_{n=0}^{N} e^{-n} + \frac{c_2}{2P}\left\{2+\frac{1}{2}(\sigma_3-\rho_3)\right\}\sum_{p=0}^{P} e^{-p}$$

$$-\frac{a_2b_2}{2N_1}\sum_{n=0}^{N_1} e^{-n}\Big[2+\frac{\sigma_1-\rho_1}{1+n} +\frac{1}{2}\Big(\frac{\rho_1-\sigma_1+\sigma_1\sigma_2+\rho_1\rho_2}{1+n}+\sigma_2 -\rho_2\Big)-\frac{\rho_1\rho_2+\sigma_1\sigma_2}{3(1+n)}\Big]$$

$$-\frac{b_2c_2}{2N_2}\sum_{n=0}^{N_2} e^{-2n}\Big[2 +\frac{1}{2}(\sigma_2+\sigma_3-\rho_2-\rho_3) +\frac{\rho_2\rho_3+\sigma_2\sigma_3}{3}\Big]$$

$$-\frac{a_2c_2}{2N_3}\sum_{n=0}^{N_3} e^{-n}\Big[2+\frac{\sigma_1-\rho_1}{1+n} +\frac{1}{2}\Big(\frac{\rho_1-\sigma_1+\sigma_1\sigma_3+\rho_1\rho_3}{1+n}+\sigma_3 -\rho_3\Big)-\frac{\rho_1\rho_3+\sigma_1\sigma_3}{3(1+n)}\Big]$$

$$+\frac{a_2b_2c_2}{2N_4}\sum_{n=0}^{N_4} e^{-2n}\Big[\Big(1-\frac{\rho_1}{1+n}\Big)\Big(1-\frac{1}{2}\rho_2-\frac{1}{2}\rho_3+\frac{1}{3}\rho_2\rho_3\Big)+\frac{\rho_1}{1+n}\Big(\frac{1}{2}-\frac{1}{3}\rho_2-\frac{1}{3}\rho_3+\frac{1}{4}\rho_2\rho_3\Big)+\Big(1+\frac{\sigma_1}{1+n}\Big)\Big(1+\frac{1}{2}\sigma_2+\frac{1}{2}\sigma_3+\frac{1}{3}\sigma_2\sigma_3\Big)-\frac{\sigma_1}{1+n}\Big(\frac{1}{2}+\frac{1}{3}\sigma_2+\frac{1}{3}\sigma_3+\frac{1}{4}\sigma_2\sigma_3\Big)\Big] \quad (B.12)$$

Where $N_1 = min(M,N)$, $N_2 = min(N,P)$, $N_3 = min(M,P)$ and $N_4 = min(M,N,P)$

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Applications of Fuzzy and Neutrosophic Logic in Solving Multi-criteria Decision Making Problems

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Abstract. In daily life, decision makers around the world are seeking for the appropriate decisions while facing many challenges due to conflicting criteria and the presence of many alternatives. In the way of pursuit a powerful decision making process, many researches act in multi-criteria decision making (MCDM) field and many methods were developed. This paper sheds some lights on the applicability of fuzzy set theory and neutrosophic logic in solving multi-criteria decision making problems. Also, it presents the possible applications of each method in MCDM different fields.

Keywords: Fuzzy Set Theory, Neutrosophic logic, Multi-criteria Decision Making.

1 Introduction

The multi-criteria decision making (MCDM) can be defined as the process of ranking a set of alternatives and selecting the most suitable one based on decision criteria [1]. During the second half of the 20th century, MCDM research area has undergone remarkable and fast development, and many MCDM methods have been developed to introduce better solution for multi-criteria decision making problems [1]. MCDM process components are a set of decision criteria (at least two), decision makers, and a set of alternatives which sorted and ranked based on the decision criteria [2]. With a goal of helping decision makers to rank different alternatives and choose the best one that satisfies organization's needs, MCDM has been used to support a wide range of decisions in many areas such as: portfolio optimization, benefit-risk assessment, technology assessment, and software selection [3–4].

This paper analyses two multi-criteria decision making methods and determines their applicability to different situations by evaluating their relative advantages and disadvantages. A comprehensive literature review is conducted to allow a summary of the two methods. A review of the use of these methods and an examination of the evolution of their use is then performed.

This paper is organized as follows: Section 2 introduces a brief background of fuzzy set theory. Fuzzy applications in different MCDM fields are discussed in Section 3. Section 4 introduces a brief background of neutrosophic logic. Section 5 presents the role of neutrosophic logic in solving multi-criteria decision making problems. Finally, conclusions and potential future scope of research are described in Conclusion section.

2 Fuzzy Set Theory

Fuzzy set theory was first introduced in 1965 by Zadeh [5]. It is an extension of classical set theory that helps solving problems with uncertain data and handling information expressed in vague and imprecise terms [6]. Its great strength appears in handling imprecise input and problems with great complexity; however, fuzzy systems are considered difficult and complex to develop, and, in many cases, they may require numerous simulations before being used in the real world [7]. Fuzzy set theory is established and has been used in many applications such as engineering, economics, environmental and social sciences, medicine, and management [7].

Zadeh [5] introduced many definitions of fuzzy sets such as:

Let X be a space of points with a generic element of X denoted by x. Thus X = {x}.

A fuzzy set A in X is characterized by a membership function fA(x) which associates with each point in X a real number in the interval [0,1], with the values of fA(x) at x representing the "grade of membership" of x in A. Thus, the nearer the value of fA(x) to unity, the higher the grade of membership of x in A.

A fuzzy number $\tilde{n}$ is a fuzzy subset in the universe of discourse X whose membership function is both Convex and normal [8]. A fuzzy set is defined by a membership function used to map an item onto an interval [0, 1] that can be associated with linguistic terms [9]. A triangular fuzzy number (TFN) is a special case of a trapezoidal fuzzy number and it is a very popular and common tool in fuzzy applications [10].

Figure 1: shows a fuzzy number [5]

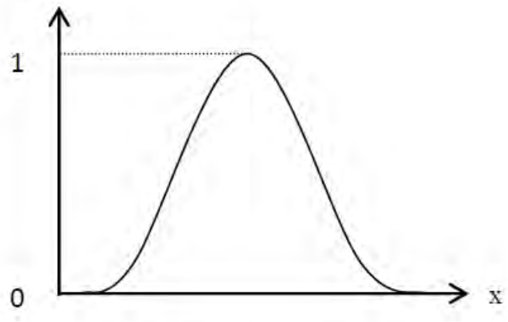

Figure1. A fuzzy number

3 Applications of Fuzzy set in MCDM

3.1 Software Selection Field

Sen et al. [11] proposed a multi criteria decision making approach for Enterprise Resource Planning (ERP) software selection using a heuristic algorithm, a fuzzy multi-criteria, and a multi objective programming model. The proposed approach aimed to evaluate the functional and non-functional ERP software characteristics. To validate the approach, the researchers applied it on an electronic company in Turkey and the results were satisfying for the company's decision makers. The researchers recommended combining their method with expert system for future work.

Lin et al. [12] first developed some aggregation operators for aggregating hesitant fuzzy linguistic information: hesitant fuzzy linguistic weighted average (HFLWA) operator, hesitant fuzzy linguistic ordered weighted average (HFLOWA) operator, and hesitant fuzzy linguistic hybrid average (HFLHA) operator, then the researchers used these operators in fuzzy approaches for solving ERP software selection problem. The proposed method was applied on a real world case study and it ensured its capability in selecting the best ERP software that suited the organization needs.

Ozturkoglu and Esendemir [13] combined the power of grey relational analysis (GRA) with an intuitionistic fuzzy set (IFS) multi-criteria method for developing a hybrid ERP software selection model. After making a survey of all criteria affecting the ERP software selection process and the software packages alternatives, the researchers used the IFS method for obtaining the weight of each criteria, then the GRA method was used for ranking the alternatives and selecting the best one. A service provider firm which offered transportation, warehousing, and packaging services was used as a case study, and the model helped the firm to select the most suitable ERP package.

Vahidi et al. [14] used the fuzzy logic for developing a model for ERP software selection. A triangular fuzzy membership function was used for processing each criterion to measure the efficiency level of each ERP system alternative. For future work, the researchers suggested using a method based on Adaptive-Network-based Fuzzy Inference Systems (ANFIS) as ANFIS method used a learning algorithm that simulate a given training data set.

Lien and Chan [15] developed a Fuzzy-Analytic Hierarchy Process (F-AHP) ERP software selection model. The proposed model was used in two case studies: a company and a college for selecting the best ERP software that mate their needs.

Cebeci [16] presented an approach for selecting the best ERP system in textile industry by using the balanced scorecard and Fuzzy-AHP method. The aims of this research were using balanced scorecard for defining the business objectives and matching them with ERP packages capabilities, and using Fuzzy-AHP model for ranking and selecting the most suitable ERP software package.

Onut and Efendigil [17] introduced a Fuzzy-AHP model for helping organizations in selecting ERP software in the presence of vagueness and with consideration to cost and quality criteria. The researchers combined Fuzzy method to the AHP model to solve the problems of ambiguities and vagueness accompanied by software selection problem. At the end of the research, a real world case study was solved using the proposed model and a comparison between AHP and Fuzzy-AHP solutions was conducted, and the results included that Fuzzy-AHP method showed more accurate results and flexibility in adding new ERP software selection criteria.

Demirtas et al. [18] presented a two stage decision making model for ERP software selection process and applied the model on an urban transportation company. At the first stage, by using Fuzzy-AHP model, the model helped the company to first take the decision whether it would develop a new software package or it would use a vendor software package. If the decision was using a vendor software package, then moving to the second stage, by using Fuzzy-Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) model, the model helped the company to select the most suitable software package fitting its needs and expectation.

Kara and Cheikhrouhou [19] proposed a four steps decision making methodology for selecting business management system to Small and Medium sized Enterprises. First the selection criteria were collected and determined by experts, then criteria weights were calculated using Fuzzy-AHP combined to TOPSIS, finally the best alternative was selected. For ensuring the methodology effectiveness, a sensitivity analysis was conducted and the results demonstrated that uncertainty was reduced.

Kilic et al. [20] used the strength of Fuzzy-AHP and TOPSIS multi-criteria decision making methods for developing a three stage hybrid model for ERP system selection and applied the model for the Airline industry. The first model stage was the determination of all ERP selection process factors and criteria and identifying ERP software packages as alternatives, the second stage was using the Fuzzy-AHP method for obtaining weights for each decision criteria, the final model stage was using the TOPSIS method for ranking the alternatives and selecting the best one. The researchers used the proposed model for helping the Turkish Airlines in selecting ERP software package for its maintenance center and the model proved its effectiveness and efficiency.

Volaric et al. [21] proposed a Fuzzy AHP-TOPSIS model for selecting the best multimedia software for learning and teaching purposes. The Fuzzy AHP method was used for assigning the weight of each criterion and demonstrating the benefit of each criterion to another, finally the TOPSIS method was used for ranking the multimedia software systems and selecting the best one.

Efe [22] developed a hybrid model by integrating Fuzzy-AHP and Fuzzy-TOPSIS for ERP software selection. First the selection criteria were determined, then the weight of each criterion was determined using Fuzzy-AHP, after that Fuzzy-TOPSIS was used for choosing the most appropriate ERP software alternative. For ensuring the model effectiveness, it was applied on an electronic firm and the results demonstrated that the model decreased the uncertainty and the information loss in group decision making. For future work, the researcher recommended using type 2 fuzzy MCDM methods in the ERP selection process.

Karsak and Ozogul [23] developed a multi-criteria decision framework using on quality function deployment (QFD), fuzzy linear regression, and zero–one goal programming for ERP software selection. The QFD method was used for determining and establishing the relationships between user demands and software characteristics, while the fuzzy linear regression method was used for assigning values to the ERP software characteristics, and finally the zero–one goal programming was used for determining the ERP software alternative that achieve the maximum values of company needs. The proposed model was applied on a Turkish automotive parts manufacturer to ensure its effectiveness.

3.2 Risk Assessment and Success Factors Evaluation

Je et al. [24] introduced an integrated fuzzy entropy-weight MCDM method and applied it to evaluate and assess risk of hydropower stations in the Xiangxi River.

Shafiee [25] used Fuzzy Analytic Network Process (F-ANP) approach, based on Chang's extent analysis for selecting the most appropriate risk mitigation strategy for offshore wind farms.

Kong and Liu [26] combined Fuzzy sets with AHP for developing a MCDA model to evaluate success factors in E-commerce projects in order to help the decision makers to determine new opportunities for their organizations.

3.3 Site Selection Field

Rezaeiniya et al. [27] used Fuzzy-ANP for selecting the appropriate location of greenhouses in Mazandaran province, Iran. The application of the model ensured its efficiency in the selection process and ranking of alternatives.

Vahidnia et al. [28] used Fuzzy-AHP in hospital site selection and determining the optimum site for a new hospital in the Tehran urban area.

Chou et al [29] developed a MCDM model by combining Fuzzy set theory and simple additive weighting (SAW) to evaluate facility locations alternatives and selecting the best one.

3.4 Supplier Selection Field

Kahraman et al. [30] proposed a Fuzzy-AHP model for supplier selection, the researchers determined the selection criteria, and then the model was used to select the most suitable supplier that mate the company needs.

Ayhan [31] presented a Fuzzy-AHP model for helping the firms to select the best supplier according to the firm selection criteria, and for ensuring the model effectiveness, it was applies on a gear motor company for assessing its suppliers and selecting the best one.

Junior et al. [32] proposed a comparative analysis of Fuzzy-AHP and Fuzzy-TOPSIS in solving the problem of supplier selection. Both methods were applied on a transmission cables for motorcycles manufacturer which needed to select the suitable supplier among five alternatives and based on five selection criteria, and the results showed that both methods were helpful, however the Fuzzy-TOPSIS method was more effective in the supplier selection problem.

Dargia et al. [33] developed a multi-criteria decision making framework for helping the Iranian automotive industry in supplier selection process. First, the researchers made a huge survey for determining the most critical factor in the supplier selection process by using the Nominated Group Technique (NGT) and the result was seven critical factors, a Fuzzy Analytical Network Process (F-ANP) was then used for determining weights of each selection factor and selecting the most appropriate supplier, the model was applied on an automotive company and it ensured its effectiveness.

Gupta et al. [34] developed an integrated Fuzzy AHP - Fuzzy Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE) model for service provider selection under conflicting criteria and uncertainty environment. First, the selection criteria were determined, then Fuzzy AHP method was used for calculating the weight of each criterion, after that Fuzzy PROMETHEE method was used for selecting the best alternative that suited the organization needs, Geometrical Analysis for Interactive Aid (GAIA) software was then used for demonstrating the model results and providing better understanding, a sensitivity analysis was conducted to ensure the model validity and model results ensured high sensitivity to change in criteria weights, finally the proposed model was applied on a real world case study, a cermet company, to select the most appropriate service provider and the model ensured its effectiveness.

Haleh and Hamidi [35] used fuzzy sets to assess and rank the suppliers and selecting the best one.

3.5 Outsourcing Selection Field

Kahraman et al. [36] tried to solve the selection problem of the right ERP outsourcing alternatives under uncertainty conditions using Fuzzy-AHP multi-criteria decision making method, the researchers applied the proposed model on an automotive firm to help it select the best ERP outsourcing alternative and the model proved its effectiveness.

Chen et al [37] integrated the triangular fuzzy method with PROMETHEE method for selecting the most appropriate outsourcing partner for organizations based on seven selection criteria and the proposed model was applied on a real world case study and helped the organization to select the most suitable outsourcing partner among four alternatives.

3.6 Other MCDM Fields

Yilmaz and Dagdeviren [38] integrated Fuzzy-PROMETHEE method with zero-one goal programming to develop a MCDA approach for equipment selection among conflicting criteria.

For handling the uncertainty problem within the quality management consultant selection process, Kabir and Sumi [39] used fuzzy set theory as it is a powerful tool for handling uncertainty, therefore Fuzzy method was integrated with the AHP method for determining the selection criteria weights, then the PROMETHEE method was used for associating a preference function to each criterion and ranking the alternatives.

For extending the power of Data envelopment analysis (DEA) MCDM method, Wen and Li [40] introduced a Fuzzy-DEA method for ranking all the decision-making units (DMUs), for solving the fuzzy model, a hybrid algorithm combined with fuzzy simulation and genetic algorithm was used, finally a numerical example was used for illustrating how the model worked.

Yuen and Ting [41] integrated the triangular fuzzy number and ranking method with PROMETHEE II method for developing a hybrid model used in text book selection and the model was applied on a case study to ensure its validity and effectiveness.

4 Neutrosophic Logic

In realistic decision making situations, information cannot always be described by unique crisp numbers, they may imply indeterminacy, and therefore Neutrosophy was originally introduced by Smarandache [42]. Neutrosophy is a branch of philosophy which studies the origin, nature and scope of neutralities and their interactions with different ideational spectra [42]. Neutrosophy studies the ideas and notions that are neutral, indeterminate, vague, unclear, ambiguous, and incomplete [43]. Neutrosophic sets are capable of dealing with uncertainty, indeterminate and inconsistent information, therefore Smarandache seek to publish the concept of neutrosophic set in all sciences branches, social sciences, and humanities [43]. Smarandache refined the neutrosophic set to n components: $t1$, $t2$, … ; $i1$, $i2$, …,ik ; $f1$, $f2$, …, fl , with j+k+l = n > 3 [43]. The basic concept of neutrosophic set is a generalization of classical set or crisp set [44, 45], fuzzy set [5], intuitionistic fuzzy set [46].

After Smarandache's introducing the concept of neutrosophic set, different sets were quickly proposed in the literature. Wang et al. [47] extended the concept of neutrosophic set to single valued neutrosophic sets (SVNSs) and they also studied the set theoretic operators and various properties of SVNSs; many other sets were introduced, such as neutrosophic soft set [48], weighted neutrosophic soft sets [49], generalized neutrosophic soft set [50], neutrosophic parametrized soft set [51], neutrosophic soft expert sets [52, 53], neutrosophic soft multi-set [54], neutrosophic bipolar set [55], neutrosophic cubic set [56, 57], rough neutrosophic set [58, 59], interval rough neutrosophic set [60], interval-valued neutrosophic soft rough sets [61, 62], etc.

5 Applications of Neutrosophic Logic in MCDM

Yang and Li [63] proposed new aggregation operators under single-valued neutrosophic environment, The researchers used single-valued neutrosophic set (SVNS) which is an extension of traditional fuzzy set, as SVNS can handle incomplete and inconsistent information, then, a MCDM method was introduced according to the proposed operators and cosine similarity measures, finally the proposed method was applied on an illustrative example of helping an investment company to select the best investment option and the results demonstrated that the proposed method was practical and effective. For future work, the researchers recommended studying new aggregation operators under neutrosophic environment.

Jency and Arockiarani [64] proposed a model based on adjustable and mean potentiality approach by means of single valued neutrosophic level soft sets, also the notion of weighted single valued neutrosophic soft set was introduced with an investigation to its applicability in decision making in an imprecise environment.

Biswas et al. [65] proposed a method with the aim of dealing with impreciseness and incompleteness information of decision maker's assessments to achieve better solution to multi-criteria decision making problems. The researcher introduced triangular fuzzy number neutrosophic sets by integrating triangular fuzzy numbers with single valued neutrosophic set. For ensuring the proposed method effectiveness, it was used to help a medical firm in selecting a medical representative.

Chi and Liu [66] introduced a MCDM model by integrating TOPSIS method with interval neutrosophic set for solving multi-criteria decision making problems in uncertainty environment. The proposed method was used in helping an investment company to select the best investment option and the results demonstrated its simplicity and ease of use.

Biswas et al. [67] presented a model for solving MCDM problems with missing or unknown information about criteria weights. They used Grey Relational Analysis (GRA) with single-value neautrosophic for developing the model, finally an illustrative example was used to ensure model practicality and effectiveness.

Dey et al. [68] extended the grey relational analysis (GRA) problems with interval neutrosophic for solving MCDM problems with incomplete or unknown weights of criteria. The researchers first developed two optimization models for recognizing criteria weights, then extended GRA was used for ranking the alternatives, finally a numerical example was used to ensure the applicability of the method.

Broumi et al. [69] proposed an extended TOPSIS model for solving MCDM problems, TOPSIS was integrated with interval neutrosophic for its great ability in handling inconsistent information. The extended TOPSIS model used interval neutrosophic for representing the values of the criteria, then alternatives were ranked using TOPSIS method. Finally an example was solved to illustrate the model effectiveness.

For solving uncertain, imprecise, incomplete, and inconsistent information in MCDM problems, Zhang and Wu [70] developed a two-stage method for single-valued neutrosophic or interval neutrosophic multi-criteria decision making. First a maximizing deviation method was introduced for assigning criteria weights under interval neutrosophic environments, then TOPSIS was used for ranking the alternatives and selecting the optimum choice. Finally the method was applied in a real world case study and proved its effectiveness.

Chen and Ye [71] introduced a projection model of neutrosophic numbers and its application for solving the MCDM problem of clay-bricks selection, an actual case was used for applying the model and the results demonstrated model's applicability and ease of use.

Ye [72] developed a single valued neutrosophic cross-entropy measure and its MCDM method was proposed based on the proposed cross entropy under single valued neutrosophic environment. Finally, an illustrative example was solved to illustrate the application of the proposed method.

Pramanik and Mondal [73] introduced a MCDM method based on interval neutrosophic sets where the rating of altenatives was expressed with interval neutrosophic values characterized by interval truth-membership degree, interval indeterminacy-membership degree, and interval falsity-membership degree. The single valued neutrosophic grey relational analysis method was extended to interval neutrosophic environment and applied MCDM problems. Finally, an illustrative example was solved to illustrate the application of the proposed method.

Mandal and Basu [74] developed a new similarity measures in neutrosophic environment based on hypercompex number system for ranking alternatives and selecting the best one while solving MCDM problems. Finally a numerical example was introduced to ensure the method effectiveness.

Mondal and Pramanik [75] introduced a MCDM method based on Dice and Jaccard similarity measures of interval rough neutrosophic set and interval neutrosophic mean operator and finally they applied the method on a laptop selection case.

Biswas et al. [76] introduced cosine similarity measure between two trapezoidal fuzzy neutrosophic numbers for solving MCDM problems under neutrosophic environment and a numerical example was solved to illustrate the method work.

Ma et al. [77] introduced a time series analysis approach integrated with interval neutrosophic sets for selecting trustworthy cloud service. Three numerical examples were used to illustrate the approach applicability and efficiency in selecting risk-sensitive service.

Mondal and Pramanik [78] developed a neutrosophic MCDM model based on hybrid score-accuracy functions of single valued neutrosophic numbers for teacher selection in recruitment process in higher education, an illustrative example was introduced for demonstrating the model work.

Mondal and Pramanik [79] also proposed a single valued neutrosophic MCDM model for selecting the best school for children. A numerical example was used to prove the model efficiency.

Ye and Smarandache [80] introduced a refined single-valued neutrosophic set (RSVNS) and a similarity measure of RSVNSs, then a MCDM method using RSVNS information was presented based on the similarity measure of RSVNSs. Finally a real case study was used for applying the metod to help a construction firm selecting the best project and the results demonstrated the method effectiveness.

Mondal and Pramanik [81] introduced a rough neutrosophic multi-attribute decision making method based on grey relational analysis by extending the neutrosophic grey relational analysis method to rough neutrosophic grey relational analysis method and applying it to multi-attribute decision making problem. In this method, the rating of all alternatives was expressed with upper and lower approximation operator and the pair of neutrosophic sets which were characterized by truth-membership degree, indeterminacy-membership degree, and falsity-membership degree. Finally a numerical example was used to demonstrate the method applicability.

Mondal and Pramanik [82] also proposed a rough neutrosophic multi-attribute decision making method based on rough accuracy score function. The rating of all alternatives was expressed with upper and lower approximation operator and the pair of neutrosophic sets which were characterized by truth-membership degree, indeterminacy-membership degree, and falsity-membership degree. Finally a numerical example was used to ensure the method effectiveness.

Peng et al. [83] introduced a new outranking approach for solving MCDM problems under neutrosophic environment by integrating simplified neutrosophic sets with ELECTRE method. Two practical examples were provided to ensure the practicality and effectiveness of the proposed approach.

Ye [84] introduced a new MCDM method using the weighted correlation coefficient or the weighted cosine similarity measure of single-valued neutrosophic sets where the alternatives evaluation was made by truth-membership degree, indeterminacy-membership degree, and falsity-membership degree under single-valued neutrosophic environment. Finally, an example was solved for proving the applicability of the proposed method.

Biswas et al. [85] proposed a ranking method for solving MCDM problems using single-valued trapezoidal neutrosophic numbers (SVTrNNs), which was a special case of single-valued neutrosophic numbers. Finally, an example was used for demonstrating the model efficiency.

Conclusion

This study demonstrated the role of fuzzy set theory and neutrosophic logic in the field of multi-criteria decision making; applications and researches of the two methods were presented to illustrate the improvements and developments made in MCDM field using those two methods. It is concluded that there is a weakness point in neutrosophic sets applications in MCDM real world case studies. Although there are many researchers that use numerical examples for applying the neutrosophic model, there is a shortage in real case studies usage. Also, neutrosophic logic should be applied more in MCDM fields like supplier selection, software selection, risk assessment and other fields, where fuzzy set theory made a noticeable development, to investigate its strength and weakness points. Therefore, there are many future works that can be done, such as:

1. Apply Neutrosophic logic on different decision support problems.
2. Apply Neutrosophic logic on software engineering.
3. Propose new adaptive mechanism to update Neutrosophic logic.
4. Solve time series forecasting.
5. Analyze the effect of hybridizing Neutrosophic logic with meta-heuristics algorithms.
6. Apply Neutrosophic logic with neural networks.
7. Design Neutrosophic logic Controller by Particle Swarm Optimization.

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Irregular Neutrosophic Graphs

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Abstract. The concepts of neighbourly irregular neutrosophic graphs, neighbourly totally irregular neutrosophic graphs, highly irregular neutrosophic graphs and highly totally irregular neutrosophic graphs are introduced. A criteria for neighbourly irregular and highly irregular neutrosophic graphs to be equivalent is discussed.

Keywords: Neutrosophic graphs, Irregular neutrosophic graphs, Neighbourly irregular neutrosophic graphs, Highly irregular neutrosophic graphs.

1 Introduction

Azriel Rosenfeld [16] introduced the notion of fuzzy graphs in 1975 which have many applications in modeling, Environmental sciences, social sciences, Geography and Linguistics. Some remarks on fuzzy graphs are given by P. Bhattacharya [4]. J. N. Mordeson and C. S. Peng defined different operations on fuzzy graphs in his paper [10]. The concept of bipolar fuzzy sets was initiated by Zhang [24]. A bipolar fuzzy set is an extension of the fuzzy set which has a pair of positive and negative membership values ranging in $[-1,1]$. In usual fuzzy sets, the membership degrees of elements range over the interval $[0,1]$. The membership degree expresses the degree of belongingness of elements to a fuzzy set. The membership degree 1 indicates that an element completely belongs to its corresponding fuzzy set, and the membership degree 0 indicates that an element does not belong to the fuzzy set. The membership degrees on the interval $(0,1)$ indicate the partial membership to the fuzzy set. Sometimes, the membership degree means the satisfaction degree of elements to some property or constraint corresponding to a fuzzy set. In Bipolar fuzzy sets membership degree range is enlarged from the interval $[0,1]$ to $[-1,1]$. In a bipolar valued fuzzy set, the membership degree 0 indicate that elements are irrelevant to the corresponding property, the membership degrees on $(0,1]$ shows that elements some what satisfy the property, and the membership degrees on $[-1,0)$ shows that elements somewhat satisfy the implicit counter-property. In many domains, it is important to be able to deal with bipolar information. It is noted that positive information represents what is granted to be possible, while negative information represents what is considered to be impossible. The first definition of bipolar fuzzy graphs was introduced by Akram [1]which are the extensions of fuzzy graphs. He defined different operations of union, intersection, complement, isomorphisms in his paper. Smarandache [21] introduced notion of neutrosophic set which is useful for dealing real life problems having imprecise, indeterminacy and inconsistent data. The theory is generalization of classical sets and fuzzy sets and is applied in decision making problems, control theory, medicines, topology and in many more real life problems. N. Shah and A. Hussain introduced the notion of soft neutrosophic graphs [17]. N. Shah introduced the notion of neutrosophic graphs and different operations like union, intersection, complement in his work [18]. Furthermore he defined different morphisms on neutrosophic graphs and proved related theorems. In the present paper the concepts of neighbourly irregular neutrosophic graphs, neighbourly totally irregular neutrosophic graphs, highly irregular neutrosophic graphs, highly totally irregular neutrosophic graphs and neutrosophic digraphs are introduced. Some results on irregularity of neutrosophic graphs are also proven.

In section 2, some basic concepts about graphs and neutrosophic sets are given. Section 3 is about neutrosophic graphs, their different operations and irregularity of neutrosophic graphs. Examples along with figures are also given to make the ideas clear.

2 PRILIMINARIES

In this section, we have given some definitions about graphs and neutrosophic sets. These will be helpful in later sections.

2.1 Definition *[22] A graph G^* consists of set of finite objects $V=\{v_1,v_2,v_3\ldots\ldots,v_n\}$ called vertices (also called points or nodes) and other set $E=\{e_1,e_2,e_3\ldots\ldots e_n\}$ whose element are called edges (also called lines or arcs).* Usually a graph is denoted as $G^*=(V,E)$. Let G^* be a graph and $\{u,v\}$ an edge of G^* . Since $\{u,v\}$ is 2-element set, we may write $\{v,u\}$ instead of $\{u,v\}$. It is often more convenient to represent this edge by uv or vu .

2.2 Definition *[15] The cardinality of V, i.e., the no.*

of vertices, is called the order of graph G^* *and denoted by* $|V|$. *The cardinality of* E *, i.e., the number of edges, is called the size of the graph and denoted by* $|E|$ *. Let* $V(G^*)=\{v_1,v_2,....v_n\}$ *and* $E(G^*)=\{e_1,e_2,...e_n\}$ *be the set of vertices and edges of a graph* G^*. *Each edge* $e_k \in E(G^*)$ *is identified with an unordered pair* (v_i,v_j) *of vertices. The vertices* v_i *and* v_j *are called the end vertices of* e_k *.*

2.3 Definition *[15] Two vertices joined with an edge are called adjacent vertices.*

2.4 Definition *[15] [20 An edge* e *of a graph* G^* *is said to be incident with a vertex* v *and vice versa if* v *is the end vertex of* e *. Any two non-parallel edges say* e_i *and* e_j *are said to be adjacent if* e_i *and* e_j *are incident with a vertex* v.

2.5 Definition *[15] The degree of any vertex* v *of* G^* *is the number of edges incident with vertex* v *. Each self-loop is counted twice. Degree of a vertex is always a positive number and is denoted as* $deg(v)$ *. The minimum degree and maximum degree of vertices in* $V(G^*)$ *are denoted by* $\delta(G^*)$ *and* $\Delta(G^*)$, *respectively* H^* *.*

2.6 Definition *[15] A vertex which is not incident with any edge is called an isolated vertex. In other words a vertex with degree zero is called an isolated vertex.*

2.7 Definition *[15] A graph without self-loops and parallel edges is called a simple graph.*

2.8 Definition *[15]A simple graph is said to be regular if all vertices of graph G are of equal degree. In other words if in a graph* G, $\delta(G^*)=\Delta(G^*)=r$ *i.e., each vertex having degree* r *then* G^* *is said to be regular of degree* r *, or simply* $r-regular$ *.*

2.9 Definition *[15] A graph* $G_1^*=(V_1,E_1)$ *is called a subgraph of* $G^*=(V,E)$ *if* $V_1(G_1^*)\subseteq V(G^*)$ *and* $E_1(G_1^*)\subseteq E(G^*)$ *and each edge of* G_1^* *has the same end vertices in* G_1^* *as in* G^*.

2.10 Definition *[15] In a graph* G^*, *a finite alternating sequence of vertices and edges,* $v_1,e_1,v_2,e_2,......e_m,v_k$ *starting and ending with vertices such that each edge in the sequence is incident with the vertices following and preceding it, is called a walk. In a walk no edge appears more than once however a vertex may appear more than once.*

2.11 Definition *[22] In a multigraph no loop are allowed but more than one edge can join two vertices, these edges are called multiple edges or parallel edges and a graph is called multigraph*

2.12 Definition *[22] Let* $G_1^*=(V_1,E_1)$ *and* $G_2^*=(V_2,E_2)$ *be two graphs. A function* $f : V_1 \to V_2$ *is called Isomorphism if i)* f *is one to one and onto.*
ii) for all $a,b\in V_1, \{a,b\}\in E_1$ *if and only if* $\{f(a),f(b)\}\in E_2$ *when such a function exists,* G_1^* *and* G_2^* *are called isomorphic graphs and is written as* $G_1^*\cong G_2^*$. *In other words, two graph* G_1^* *and* G_2^* *are said to be isomorphic to each other if there is a one to one correspondence between their vertices and between edges such that incidence relationship is preserved.*

2.13 Definition *[21] A neutrosophic set* Λ *on the universe of discourse* X *is defined as* $\Lambda=\{<x,T_\Lambda(x),I_\Lambda(x),F_\Lambda(x)>,\ x\in X\}$, *where* $T,I,F : X \to]\bar{0},1^+[$ $\bar{0} \le$

$T_\Lambda(x)+I_\Lambda(x)+F_\Lambda(x)\le 3^+$. *Hence we consider the neutrosophic set which takes the values from the subset of* $]\bar{0},1^+[$.

2.14 Definition *Let* $\Lambda=\{<x,T_\Lambda(x),I_\Lambda(x),F_\Lambda(x)>,\ x\in X\}$ *and* $\Theta=\{<x,T_\Theta(x),I_\Theta(x),F_\Theta(x)>,\ x\in X\}$ *be two neutrosophic sets on universe of discourse* X *. Then* Θ *is called neutrosophic relation on* Λ *if*

$$T_\Theta(x,y)\le \min\{T_\Lambda(x),T_\Lambda(y)\}$$
$$I_\Theta(x,y)\le \min\{I_\Lambda(x),I_\Lambda(y)\}$$
$$F_\Theta(x,y)\ge \max\{F_\Lambda(x),F_\Lambda(y)\}$$

for all $x,y\in X$. *A neutrosophic relation* Θ *on* X *is called symmetric if* $T_\Theta(x,y)=T_\Theta(y,x)$, $I_\Theta(x,y)=I_\Theta(y,x)$, $F_\Theta(x,y)=F_\Theta(y,x)$ *for all* $x,y\in X$.

3 NEUTROSOPHIC GRAPHS AND IRREGULARITY

In this section we will study some basic definitions about neutrosophic graphs and different types of degrees of vertices will be discussed. Irregularity of neutrosophic graphs and related results are also proven in this section

3.1 Definition *[18] Let* $G^* = (V, E)$ *be a simple graph and* $E \subseteq V \times V$ *. Let* T_Λ, I_Λ, F_Λ : $V \to [0,1]$ *denote the truth-membership, indeterminacy-membership and falsity- membership of an element* $x \in V$ *and* $T_\Theta, I_\Theta, F_\Theta$: $E \to [0,1]$ *denote the truth-membership, indeterminacy-membership and falsity-membership of an element* $(x,y) \in E$. *By a neutrosophic graph, we mean a* 3 *-tuple* $G = (G^*, \Lambda, \Theta)$ *such that*

$T_\Theta(x,y) \le \min\{T_\Lambda(x), T_\Lambda(y)\}$

$I_\Theta(x,y) \le \min\{I_\Lambda(x), I_\Lambda(y)\}$

$F_\Theta(x,y) \ge \max\{F_\Lambda(x), F_\Lambda(y)\}$

for all $(x,y) \in E$.

3.2 Definition *[18] Let* $G_1^* = (V_1, E_1)$ *and* $G_2^* = (V_2, E_2)$ *be two simple graphs. The union of two neutrosophic graphs* $G_1 = (G_1^*, \Lambda_1, \Theta_1)$ *and* $G_2 = (G_2^*, \Lambda_2, \Theta_2)$ *is denoted by* $G = (G^*, \Lambda, \Theta), G^* = G_1^* \cup G_2^*$, $\Lambda = \Lambda_1 \cup \Lambda_2, \Theta = \Theta_1 \cup \Theta_2$, *where the truth-membership, indeterminacy-membership and falsity-membership of union are as follows*

$$T_\Lambda(x) = \begin{cases} T_{\Lambda_1}(x) & \text{if } x \in V_1 - V_2 \\ T_{\Lambda_2}(x) & \text{if } x \in V_2 - V_1 \\ \max\{T_{\Lambda_1}(x), T_{\Lambda_2}(x)\} & \text{if } x \in V_1 \cap V_2 \end{cases},$$

$$I_\Lambda(x) = \begin{cases} I_{\Lambda_1}(x) & \text{if } x \in V_1 - V_2 \\ I_{\Lambda_2}(x) & \text{if } x \in V_2 - V_1 \\ \max\{I_{\Lambda_1}(x), I_{\Lambda_2}(x)\} & \text{if } x \in V_1 \cap V_2 \end{cases}$$

$$F_\Lambda(x) = \begin{cases} F_{\Lambda_1}(x) & \text{if } x \in V_1 - V_2 \\ F_{\Lambda_2}(x) & \text{if } x \in V_2 - V_1 \\ \min\{F_{\Lambda_1}(x), F_{\Lambda_2}(x)\} & \text{if } x \in V_1 \cap V_2 \end{cases},$$

Also

$$T_\Theta(x,y) = \begin{cases} T_{\Theta_1}(x,y) & \text{if } (x,y) \in E_1 - E_2 \\ T_{\Theta_2}(x,y) & \text{if } (x,y) \in E_2 - E_1 \\ \max\{T_{\Theta_1}(x,y), T_{\Theta_2}(x,y)\} & \text{if } (x,y) \in E_2 \end{cases}$$

$$I_\Theta(x,y) = \begin{cases} I_{\Theta_1}(x,y) & \text{if } (x,y) \in E_1 - E_2 \\ I_{\Theta_2}(x,y) & \text{if } (x,y) \in E_2 - E_1 \\ \max\{I_{\Theta_1}(x,y), I_{\Theta_2}(x,y)\} & \text{if } (x,y) \in E_1 \cap E_2 \end{cases}$$

$$F_\Theta(x,y) = \begin{cases} F_{\Theta_1}(x,y) & \text{if } (x,y) \in E_1 - E_2 \\ F_{\Theta_1}(x,y) & \text{if } (x,y) \in E_2 - E_1 \\ \min\{F_{\Theta_1}(x,y), F_{\Theta_1}(x,y))\} & \text{if } (x,y) \in E_1 \cap E_2 \end{cases}$$

3.3 Definition *[18] The intersection of two neutrosophic graphs* $G_1 = (G_1^*, \Lambda_1, \Theta_1)$ *and* $G_2 = (G_2^*, \Lambda_2, \Theta_2)$ *is denoted by* $G = (G^*, \Lambda, \Theta)$ *where* $G^* = G_1^* \cap G_2^*$, $\Lambda = \Lambda_1 \cap \Lambda_2, \Theta = \Theta_1 \cap \Theta_2, V = V_1 \cap V_2$ *and the truth-membership, indeterminacy-membership and falsity-membership of intersection are as follows*

$T_\Lambda(x) = \{\min\{T_{\Lambda_1}(x), T_{\Lambda_2}(x)\}$,

$I_\Lambda(x) = \{\min\{\{I_{\Lambda_1}(x), I_{\Lambda_2}(x)\},$

$F_\Lambda(x) = \{\max\{\{F_{\Lambda_1}(x), F_{\Lambda_2}(x)\}$

also

$T_\Theta(x,y) = \{\min\{T_{\Theta_1}(x,y), T_{\Theta_2}(x,y)\}$

$I_\Theta(x,y) = \{\min\{I_{\Theta_1}(x,y), I_{\Theta_2}(x,y)\},$

$F_\Theta(x,y) = \{\max\{F_{\Theta_1}(x,y), F_{\Theta_2}(x,y)\}$

3.4 Definition *Let* $G = (G^*, \Lambda, \Theta)$ *be a neutrosophic graph. The nbhd of a vertex in is defined as* $N(x) = (N_T(x), N_I(x), N_F(x))$ *where* $N_T(x) = \{y \in V : T_\Theta(x,y) \le \min\{T_\Lambda(x), T_\Lambda(y)\}\}$, $N_I(x) = \{y \in V : I_\Theta(x,y) \le \min\{I_\Lambda(x), I_\Lambda(y)\}\}, N_F(x) = \{y \in V : F_\Theta(x,y) \ge \max\{F_\Lambda(x), F_\Lambda(y)\}\}$.

3.5 Definition *Let* $G = (G^*, \Lambda, \Theta)$ *be a* neutrosophic graph. The nbhd degree of a vertex *x* in *G* defined by

$$\deg_T(x) = \sum_{y \in N_{Tg}(x)} T_\Lambda(y),$$

$$\deg_I(x) = \sum_{y \in N_{g_I}(x)} I_\Lambda(y), \deg_F(x) = \sum_{y \in N_{F_g}(x)} F_\Lambda(y).$$

3.6 Definition *Let* $G = (G^*, \Lambda, \Theta)$ *be a neutrosophic graph. The closed nbhd degree of a vertex in is defined as* $\deg[x] = (\deg_T[x], \deg_I[x], \deg_F[x])$ *, where*

$$\deg_T[x] = \sum_{y \in N_T(x)} T_\Lambda(y) + T_\Lambda(x),$$

$$, \deg_I[x] = \sum_{y \in N_I(x)} I_\Lambda(y) + I_\Lambda(x),$$

$$\deg_F[x] = \sum_{y \in N_F(x)} F_\Lambda(y) + F_\Lambda(x).$$

3.7 Definition *Let* $G = (G^*, \Lambda, \Theta)$ *be a neutrosophic graph. The order of neutrosophic graph denoted by* $O(G)$ *is defined as*

$$O(G) = (O_T(G), O_I(G), O_F(G)), \text{ Where}$$

$$O_T(G) = \sum_{x \in V} T_\Lambda(x),$$

$$O_I(G) = \sum_{x \in V} I_\Lambda(x), O_F(G) = \sum_{x \in V} F_\Lambda(x).$$

The size of a neutrosophic graph $G = (G^*, \Lambda, \Theta)$ *is denoted by* $S(G)$ *and is defined as*

The size of a neutrosophic graph $G = (G^*, \Lambda, \Theta)$ *is denoted by* $S(G)$ *and is defined as*

$$S(G) = (S_T(G), S_I(G), S_F(G)), \text{ where}$$

$$S_T(G) = \sum_{(x.y) \in E} T_\Theta(x, y),$$

$$S_I(G) = \sum_{(x.y) \in E} I_\Theta(x, y),$$

$$S_F(G) = \sum_{(x.y) \in E} F_\Theta(x, y)$$

3.8 Definition *A neutrosophic graph* $G = (G^*, \Lambda, \Theta)$ *is called regular if all the vertices have the same open nbhd degree.*

3.9 Definition *Let* $G = (G^*, \Lambda, \Theta)$ *be a neutrosophic graph. If there is a vertex which is adjacent to vertices with distinct neighborhood degrees then is called a irregular neutrosophic graph.*

3.10 Example *Let* $G^* = (V, E)$ *be a simple graph with* $V = \{x_1, x_2, x_3\}$ *and* $E = \{(x_1, x_2)(x_{2,}, x_3), (x_1, x_3)\}$. *A neutrosophic graph G is given in table* 1 *below and* $T_\Theta(x_i, x_j) = 0, \quad I_\Theta(x_i, x_j) = 0$ *and* $F_\Theta(x_i, x_j) = 1$ *for all* $(x_i, x_j) \in E \setminus \{(x_1, x_2), (x_2, x_3), (x_1, x_3)\}$.

Table 1

Λ	x_1	x_2	x_3
T_Λ	0.1	0.1	0.2
I_Λ	0.3	0.3	0.4
F_Λ	0.4	0.3	0.6
Θ	(x_1, x_2)	$(x_{2,}, x_3)$	(x_1, x_3)
T_Θ	0.1	0.1	0.1
I_Θ	0.2	0.3	0.3
F_Θ	0.8	0.7	0.8

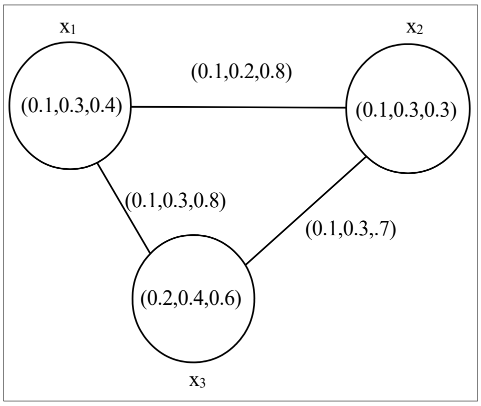

Figure 1

Here $\deg(x_1) = (0.3, 0.7, 0.9)$. *Similarly,*

$\deg(x_2) = (0.3, 0.7, 0.1), \deg(x_3) = (0.2, 0.6, 0.7).$

Clearly，G *is an irregular neutrosophic graph.*

3.11 Definition *Let* $G = (G^*, \Lambda, \Theta)$ *be a neutrosophic graph. If there is a vertex which is adjacent to vertices with distinct closed neighborhood degrees, then is called a totally irregular neutrosophic graph.*

3.12 Example *Consider a neutrosophic graph below with* $V = \{x_1, x_2, x_3, x_4, x_5\}$,

$E = \{(x_1, x_2), (x_{2,}, x_3), (x_{3,}, x_4), (x_4, x_2), (x_1, x_3), (x_4, x_5), (x_4, x_1)\}$

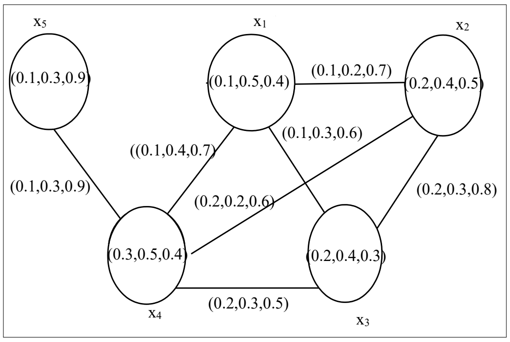

Figure 2

$\deg[x_1] = (0.8, 1.8, 1.6).$

$\deg[x_2] = (0.8, 1.8, 1.6), \deg[x_3] = (0.8, 1.8, 1.6),$

$\deg[x_4] = (0.9, 2.1, 2.5), \deg[x_5] = (0.4, 0.8, 1.3).$

Clearly G *is totally irregular neutrosophic graph.*

3.13 Definition *Let* $G = (G^*, \Lambda, \Theta)$ *be a connected neutrosophic graph. If every two adjacent vertices of have distinct open neighborhood degrees, then is called neighbourly irregular neutrosophic graph*

3.14 Example *Consider a neutrosophic graph* G *below* $with V = \{x_1, x_2, x_3, x_4\}$,

$E = \{(x_1, x_2), (x_{2,}, x_3), (x_{3,}, x_4), (x_4, x_1)\}.$

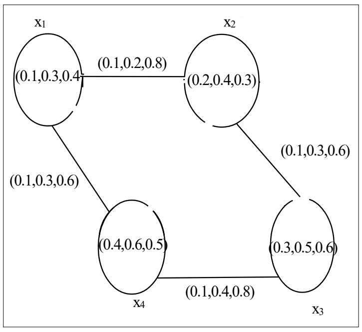

Figure 3

Here $\deg(x_1) = (0.6, 01, 0.8)$. *Similarly,*

$\deg(x_2) = (0.4, 0.8, 01), \deg(x_3) = (0.6, 01, 0.8).$

$\deg(x_4) = (0.4, 0.8, 01,),$ *Clearly*, G *is neighbourly irregular neutrosophic graph..*

3.15 Definition *A connected neutrosophic graph* $G = (G^*, \Lambda, \Theta)$ *is called neighbourly totally irregular neutrosophic graph if every two adjacent vertices of* G *have distinct closed neighborhood degrees.*

3.16 Example *An example of neighbourly totally irregular neutrosophic graph* G *is given below with*

$V = \{x_1, x_2, x_3, x_4\}$,

$E = \{(x_1, x_2), (x_{2,}, x_3), (x_{3,}, x_4), (x_4, x_1)\}.$

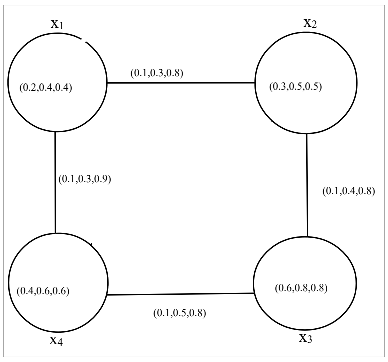

Figure 4

$\deg[x_1(=]0.9, 1.5, 1.5),$

$\deg[x_2(=]1.1, 1.7, 1.7),$

$\deg[x_3] = (1.3, 1.9, 1.9)$,

$\deg[x_4] = (1.2, 1.8, 1.8)$

3.17 Definition *A connected neutrosophic graph* $G = (G^*, \Lambda, \Theta)$ *is called highly irregular neutrosophic graph if every vertex of* G *is adjacent to vertices with distinct neighborhood degrees.*

Note (i) A highly irregular neutrosophic graph may not be neighbourly irregular neutrosophic graph.

3.18 Example *From figure* 5 *below ,it can be seen that a highly irregular neutrosophic graph may not be neighbourly irregular neutrosophic graph.*

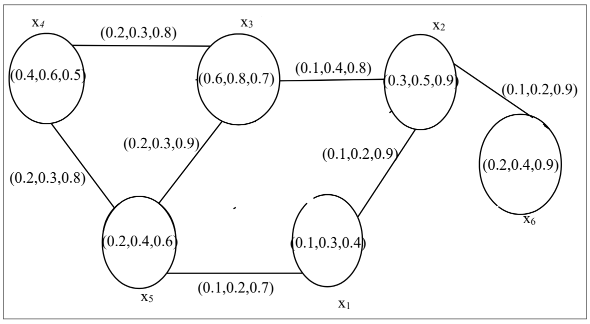

Figure 5

Here $x_2 \in V$, which is adjacent to the vertices x_1, x_3, x_6 with distinct nbhd degrees. But $\deg(x_2) = \deg(x_3)$.

So G is highly irregular neutrosophic graph but it is not a neighbourly irregular.

ii) A neighbourly irregular neutrosophic graph may not be highly irregular neutrosophic graph.

3.19 Example Consider the graph below

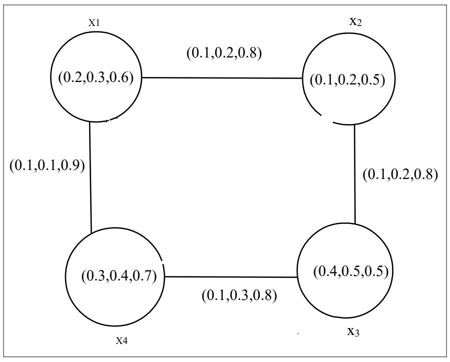

Figure 6

Here $\deg(x_1) = (0.4, 0.6, 1.2)$,

$\deg(x_2) = (0.6, 0.8, 1.1)$, $\deg(x_3) = (0.4, 0.6, 1.2)$,

$\deg(x_4) = (0.6, 0.8, 1.1)$

Clearly every two adjacent vertices have distinct nbhd degree, but x_2 is adjacent to x_1 and x_3 having same degree. Hence G is neighbourly irregular neutrosophic graph but not highly irregular neutrosophic graph.

(iii) A neighbourly irregular neutrosophic graph may not be a neighbourly totally irregular neutrosophic graph.

3.20 Example *Consider a neutrosophic graph such that* $V = \{x_1, x_2, x_3, x_4\}$,

$$E = \{(x_1, x_2), (x_2, x_3), (x_3, x_4), (x_4, x_1)\}.$$

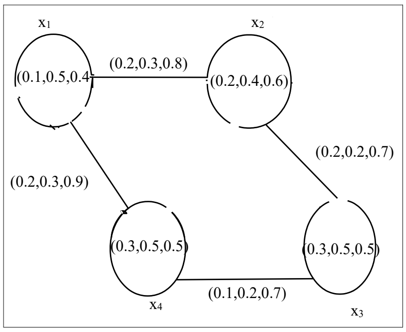

Figure 7

$\deg(x_1) = (0.5, 0.9, 1.1), \deg(x_2) = (0.6, 1, 0.9)$,

$\deg(x_3) = (0.5, 0.9, 1.1), \deg(x_4) = (0.6, 1, 0.9)$

And

$\deg[x_1] = (0.6, 1.4, 1.5), \deg[x_2] = (0.6, 1.4, 1.5)$,

$\deg[x_3] = (0.8, 1.4, 1.6), \deg[x_4] = (0.7, 1.5, 1.4)$.

We see that $\deg[x_1] = \deg[x_2]$ *. Hence* G *is neighbourly irregular neutrosophic graph but not a neighbourly totally irregular neutrosophic graph.*

(iv) A neighbourly totally irregular neutrosophic graph may not be a neighbourly irregular neutrosophic graph.

3.21 Example *Consider a neutrosophic graph* G *such that*

$V = \{x_1, x_2, x_3, x_4\}$,

$E = \{x_1x_2, x_2x_3, x_3x_4, x_4x_1\}$.

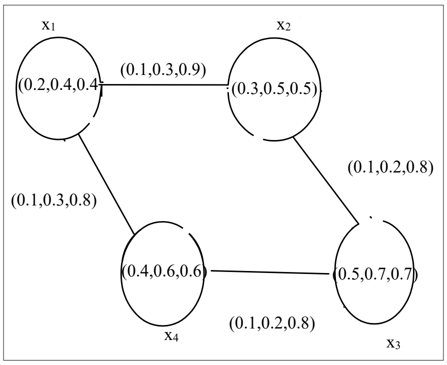

Figure 8

$\deg[x_1] = (0.9,1.5,1.5), \deg[x_2] = (1.0,1.6,1.6),$
$\deg[x_3] = (1.2,1.8,1.8), \deg[x_4] = (1.1,1.7,1.7).$

But

$\deg(x_1) = (0.7,1.1,1.1), \deg(x_2) = (0.7,1.1,1.1),$
$\deg(x_3) = (0.7,1.1,1.1), \deg(x_4) = (0.7,1.1,1.1).$

Hence $\deg(x_1) = \deg(x_2) = \deg(x_3) = \deg(x_4)$. *So* G *is neighbourly totally irregular neutrosophic graph but not a neighbourly irregular neutrosophic graph.*

3.22 Proposition

Let be a Neutrosophic graph. Then G *is highly irregular neutrosophic graph and neighbourly irregular Neutrosophic graph iff the neighborhood degrees of all the vertices of* G *are distinct.*

Proof Let G be a neutrosophic graph with n-vertices $x_1, x_2, x_3, ..., x_n$. Suppose G is both highly irregular and neighbourly irregular neutrosophic graph. We want to show the neighborhood degrees of all vertices of are distinct. Let $\deg(x_i) = (p_i, q_i, r_i)$, $i = 1,2,\ldots,n.$

Let the adjacent vertices of x_1 are $x_2, x_3, \ldots, x_n$ with nbhd degrees (p_2, q_2, r_2) , $(p_3, q_3, r_3), ..., (p_n, q_n, r_n)$ respectively. Since G is highly irregular so $p_2 \neq p, \ldots, \neq p_n$, $q_2 \neq q_3 \neq, ..., \neq q_n$, $r_2 \neq r_3 \neq, ..., \neq r_n$. Also $p_1 \neq p_2 \neq p, ..., \neq p_n$, $q_1 \neq q_2 \neq q_3 \neq, ..., \neq q_n$, $r_1 \neq \quad r_2 \neq r_3 \neq, ..., \neq r_n$ because G is neighbourly irregular. So

$$(p_1, q_1, r_1) \neq (p_2, q_2, r_2) \neq (p_3, q_3, r_3) \neq, ..., \neq (p_n, q_n, r_n).$$

Hence the neighborhood degrees of all the vertices of are distinct.

Conversely, suppose that the neighborhood degrees of all the vertices are distinct. Now we want to show that is highly irregular and neighbourly irregular neutrosophic graph. Let $deg(x_i) = (p_i, q_i, r_i)$, $i = 1,2,\ldots,n$ given that $p_1 \neq p_2 \neq p, ..., \neq p_n$, $q_1 \neq q_2 \neq q_3 \neq, \ldots, \neq q_n$, and $r_1 \neq r_2 \neq r_3 \neq, ..., \neq r_n \quad \Rightarrow$ Every two adjacent vertices have distinct neighborhood degrees and to every vertex, the adjacent vertices have distinct neighborhood degrees, which completes the proof.

3.23 Proposition

Let G *be a neutrosophic graph. If* G *is neighbourly irregular neutrosophic graph and* $(T_\Lambda, I_\Lambda, F_\Lambda)$ *is a constant function, then* G *is a neighbourly totally irregular neutrosophic graph.*

Proof Let G be neighbourly irregular neutrosophic graph. Let $x_i, x_j \in V$, where x_i and x_j are adjacent vertices with distinct neighborhood degrees (p_1, q_1, r_1) and (p_2, q_2, r_2) respectively. Let us assume that $(T_\Lambda(x_i), I_\Lambda(x_i), F_\Lambda(x_i)) = (T_\Lambda(x_j),$, where $I_\Lambda(x_j), F_\Lambda(x_j)) = (k_1, k_2, k_3)$
k_1, k_2, k_3 are constants and $k_1, k_2, k_3 \in [0,1]$. Therefore, $\deg_T[x_i] = \deg_T(x_i) + T_\Lambda(x_i) = p_1 + k_1,$

$deg_I[x_i] = deg_I(x_i) + I_\Lambda(x_i) = q_1 + k_2,$
$deg_F[x_i] = deg_F(x_i) + F_\Lambda(x_i) = r_1 + k_3,$
$deg_T[x_j] = deg_T(x_j) + T_\Lambda(x_j) = p_2 + k_1,$
$deg_I[x_j] = deg_I(x_j) + I_\Lambda(x_j) = q_2 + k_2,$
$deg_F[x_j] = deg_F(x_j) + F_\Lambda(x_j) = r_2 + k_3.$ We want to show

$degT[xi \neq]degT[xj], degI[xi \neq]degI[xj], degF[xi \neq]degF[xj$

Suppose that on contrary,

$\deg_T[x_i] = \deg_T[x_j] \Rightarrow p_1 + k_1 = p_2 + k_1$ Which $\Rightarrow p_1 - p_2 = k_1 - k_1 = 0 \Rightarrow p_1 = p_2$ is a contradiction because $p_1 \neq p_2$. Similarly

$deg_I[x_i] = \deg_I[x_j] \Rightarrow q_1 - q_2 = k_2 - k_2 = 0 \Rightarrow q_1 = q_2.$

Which is a contradiction since $q_1 \neq q_2$. Consider

$degF[xi =]degF[xj \Rightarrow]r1 \ +k3 \ =r2 \ +k3$

Which is a contradiction since $r_1 \neq r_2$. Therefore G is neighbourly totally irregular neutrosophic graph.

3.24 Proposition

A neutrosophic graph G of G^*, where G^* is a cycle with 3 vertices is neighbourly irregular and highly irregular iff the truth- membership, indeterminacy- membership and falsity- membership values of vertices between every pair of vertices are all distinct.

Proof Suppose that truth membership, indeterminacy and falsity membership between every pair of vertices are all distinct. Let $x_i, x_j, x_k \in V$ and

$T_\Lambda(x_i) \neq T_\Lambda(x_j) \neq T_\Lambda(x_k)$,

$I_\Lambda(x_i) \neq I_\Lambda(x_j) \neq I_\Lambda(x_k)$,

$F_\Lambda(x_i) \neq F_\Lambda(x_j) \neq F_\Lambda(x_k)$.

Which implies that

$$\sum_{x \in N(x_i)} F_\Lambda(x_i) \neq \sum_{x \in N(x_j)} F_\Lambda(x_j) \neq \sum_{x \in N(x_k)} F_\Lambda(x_k),$$

$$\sum_{x \in N(x_i)} I_\Lambda(x_i) \neq \sum_{x \in N(x_j)} I_\Lambda(x_j) \neq \sum_{x \in N(x_k)} I_\Lambda(x_k),$$

$$\sum_{x \in N(x_i)} T_\Lambda(x_i) \neq \sum_{x \in N(x_j)} T_\Lambda(x_j) \neq \sum_{x \in N(x_k)} T_\Lambda(x_k).$$

That is, $\deg_T(x_i) \neq \deg_T(x_j) \neq \deg_T(x_k)$.

Similarly we can show

$\deg_I(x_i) \neq \deg_I(x_j) \neq \deg_I(x_k)$,

$\deg_F(x_i) \neq \deg_F(x_j) \neq \deg_F(x_k)$ showing that $\deg(x_i) \neq \deg(x_j) \neq \deg(x_k)$. Hence is neighbourly irregular and highly irregular neutrosophic graph.

Conversely, suppose that is neighbourly irregular and highly irregular. Let $deg(x_i) = (p_i, q_i, r_i)$, $i = 1, 2, 3, \ldots, n$. Suppose that, truthfulness, falsity and indeterminacy of two vertices are same. . Let $x_1, x_2 \in V$ with $T_\Lambda(x_1) = T_\Lambda(x_2)$, $I_\Lambda(x_1) = I_\Lambda(x_2), F_\Lambda(x_1) = F_\Lambda(x_2)$. Then $\deg_T(x_1) = \deg_T(x_2)$, $\deg_I(x_1) = \deg_I(x_2)$, $\deg_F(x_1) = \deg_F(x_2)$. Which implies $\deg(x_1) = \deg(x_2)$. Since G^* is a cycle, so we have a contradiction to the fact that G is neighbourly irregular and highly irregular neutrosophic graph. Hence the truth-membership, indeterminacy-membership and falsity-membership values of vertices between every pair of vertices are all distinct.

3.25 Proposition

Let G be a neutrosophic graph. If G is neighbourly totally irregular neutrosophic graph and $(T_\Lambda, I_\Lambda, F_\Lambda)$ is a constant function, then G is a neighbourly irregular neutrosophic graph.

Proof We suppose G is neighbourly totally irregular neutrosophic graph. Then by definition, the closed neighborhood degree of every two adjacent are distinct.

Let $x_i, x_j \in V$, where x_i and x_j are adjacent vertices with distinct degrees (p_1, q_1, r_1) and (p_2, q_2, r_2) respectively. Let us assume that

$(T_\Lambda(x_i), I_\Lambda(x_i), F_\Lambda(x_i)) = (T_\Lambda(x_j), I_\Lambda(x_j), F_\Lambda(x_j)) = (k_1, k_2, k_3)$,

where k_1, k_2, k_3 are constants and $k_1, k_2, k_3 \in [0,1]$ and $\deg[x_i] \neq \deg[x_j]$. We want to show $\deg(x_i) \neq \deg(x_j)$. Since $\deg[x_i] \neq \deg[x_j]$, so $\deg_T[x_i] \neq \deg_T[x_j]$, $\deg_I[x_i] \neq \deg_I[x_j]$, $\deg_F[x_i] \neq \deg_F[x_j]$. Now

$\deg_T[x_i] \neq \deg_T[x_j] \Rightarrow p_1 + k_1 \neq p_2 + k_1$

$\Rightarrow p_1 - p_2 \neq k_1 - k_1 = 0 \Rightarrow p_1 \neq p_2$

Similarly

$\deg_I[x_i] \neq \deg_I[x_j] \Rightarrow q_1 - q_2 \neq k_2 - k_2 = 0$

$\Rightarrow q_1 \neq q.$

$\deg_F[x_i] \neq \deg_F[x_j] \Rightarrow r_1 + k_3 \neq r_2 + k_3$

$\Rightarrow r_1 - r_2 \neq k_3 - k_3 = 0 \Rightarrow r_1 \neq r_2$.

Hence the degrees of $x_i, x_j \in V$ are distinct. This is true for every pair of adjacent vertices in G . Therefore G is neighbourly irregular neutrosophic graph.

Conclusion

Neutrosophic sets are the generalization of the classical sets and of the fuzzy sets, and have many applications in real world problems when the data is imprecise, indeterminant or inconsistent. In this paper, we initiated the idea of the irregular neutrosophic graphs, and discussed different properties of such graphs. We have seen how neighbourly irregular and highly irregular neutrosophic graphs are equivalent. In future, we will extend our work to other graph theory areas by using neutrosophic graphs.

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Neutrosophic Features for Image Retrieval

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Abstract The goal of an Image Retrieval System is to retrieve images that are relevant to the user's request from a large image collection. In this paper, we present texture features for images embedded in the neutrosophic domain. The aim is to extract a set of features to represent the content of each image in the training database to be used for the purpose of retrieving images from the database similar to the image under consideration.

Keywords: Content-Based Image Retrieval (CBIR), Text-based Image Retrieval (TBIR), Neutrosophic Domain, Neutrosophic Entropy, Neutrosophic Contrast, Neutrosophic Energy, Neutrosophic Homogeneity.

1 Introduction

An Image Retrieval System is a computer system for searching and retrieving images from a large database of digital images. The traditional way to image retrieval is the text-based image retrieval (TBIR) which proposed late 1970s [17, 43]. Such techniques commence by annotating the images by text and then use text-based database management systems to retrieve images [8].
Although there are several progresses have been made to TBIR techniques. Such as data modeling. Multidimensional indexing, and query evaluation, there are some limitations when using such techniques. For instance, the problem of annotating images in large volumes of databases and that only one language is valid for image retrieval. Furthermore, the problems due to the subjectivity of human perception arising from the responsibility of the end-user; as well as the queries that cannot be described at all, but tap into the visual features of the image [2, 3, 4, 5].
Later on during the 1990's, another way was invented to retrieve images, which is Content-based Image Retrieval (CBIR) technique. The new techniques came up to deal with the rapid increase of using digital images databases on the internet. Used for retrieving, managing and navigating large digital images databases, the CBIR techniques index the images by their own visual content, such as color and texture, instead of annotated the image manually by text-based key words [11, 16, 22, 23, 36].
The Neutrosophic logic which proposed by Samarandache in [40] is a generalization of fuzzy sets which introduced by Zada at 1965 [45], The fundamental concepts of neutrosophic set, introduced by Samarandache in [41, 42] and Salama etl in [1, 14, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35].

2 Image Retrieval Technique

2.1 Content-Based Image Retrieval (CBIR)

The Content-Based Image Retrieval was used to depict the experiments of automatic retrieval images from a database, that depended on colors and shapes. After that it used to retrieve images from a large collection of database based on syntactical image features. CBIR used some techniques, tools and algorithms which taken from some fields such as statistics, pattern recognition, signal processing and computer vision. In CBIR, the images indexed by the description of the visual content of the images. Most of the CBIR systems are concerned with the approximate queries, because it is aim to find the images visually similar to the target image. The target of CBIR system is to duplicate the human perception of image similarity as possible as it can.
Feature Extraction is the basic of CBIR. Features may contain both text-based features (key, words, annotations) and visual features (color, texture, shape, faces). The goal of feature extraction is to create high-level data (pixel

values). The visual features ordered in three levels: low level features (primitive), middle level features (logical) and high level features (abstract). All recently system were depended on low level features (color, shape). But now both mid-level and high-level image representations are in demand. The efficiency of a CBIR system depends on extracted features [6].

The stages of the CBIR process are:

1- Image acquisition: to acquire a digital image [9]. Image database: it consists of the collection of n number of images which depends on the user range and choice [9].
2- Image processing: it used to improve the image by increased the chances for success of the other processes. At first, the image processed to extract the features that depict its contents. This process contains filtering, normalization, segmentation, and object identification. For example, the process of image segmentation is used to divide an image into multiple parts and its output is a set of significant regions and objects [9].
3- Feature extraction: the features such as shape, texture, color are used to depict the content of the image. The features can be characterized as low-level and high-level features. The visual information in this step extracted from the image and saved as feature vector in a features database. The image description for each pixel is found in the form of feature vector by using the feature extraction. These feature vectors are used to make a compare between query with other images and retrieval [9].
4- Similarity matching: for each image, its information stored in its feature vectors for computation process and these feature vectors are matched with the feature vectors of query image to helps in similarity measure. This step contains the matching of the above stated features to have that is visually similar with the use of similarity measure method called as Distance method. There are another distance methods such as Euclidean distance, City block distance, Canberra distance [9].
5- Resultant retrieval images: this process searched for the prior maintained information to find the matched images from database. Its output will be the similar images with the same or very closest features as that of the query image [18].
6- User interface and feedback which controls the display of the outcomes, their ranking and the type of user interaction with possibility of refining the search by some automatic or manual preferences scheme [24].

2.1.1 Color Features for Image Retrieval

Color is widely used low-level visual features and it is invariant to image size and orientation [9].

- Color Histogram: In CBIR, one of the most popular features is the color histogram in HSV color space, which used in MPEG-7 descriptor. At first, the images converted to the HSV color space, and uniformly quantizing H, S, and V components into 16, 2, and 2 regions respectively generates the 64-bit color histogram [44].
- Color moments: To form a 9-dimensional feature vector, the mean μ, standard deviation σ, and skew g are extracted from the R, G, B color spaces. The best known space color and commonly used for visualization is the RGB space color. It can be depict as a cube where the horizontal x-axis as red values increasing to the left, y-axis as blue increasing to the lower right and the vertical z-axis as green increasing towards the top [21].

2.1.2 Texture Feature for Image Retrieval

In the texture feature extraction, using the gray level co-occurrence matrix for the query image and the first image in the database to extract the texture feature vector [19]. The co-occurrence matrix representation is a technique used to give the intensity values and the distribution of the intensities. The features which selected for retrieving texture properties are Energy, Entropy, Inverse difference, Moment of inertia, Mean, Variance, Skewness, Distribution uniformity, Local stationary and Homogeneity [15].

2.1.3 Shape Features for Image Retrieval

The shape defined as the characteristic surface configuration of an object: an outline or contour. The object can be distinguished from its surroundings by its outline [9].

We can divide the shape representations into two categories:

1- Boundary-based shape representation: it uses only the outer boundary of the shape. It works by describing the considered region by using its external characteristics. For example, the pixels along the object boundary [39].
2- Region-based shape representation: it uses the entire shape region. It works by describing the considered region using its internal characteristics. For example, the pixels which the region contained [39].

3 Images in the Neutrosophic Domain with Hesitancy degree

The image in the neutrosophic domain is considered as an array of neutrosophic singletons [25]. Let U be a universe of discourse and W is a set in U which composed of bright pixels. A neutrosophic images

P_{NS} is characterized by three sub sets T, I, and F. which can be defined as T is the degree of membership, I is the degree of indeterminacy, and F is the degree of non-membership. In the image, a pixel P is described as P(T,I,F) which belongs to W by it is t% is true in the bright pixel, i% is the indeterminate and f% is false where t varies in T, i varies in I, and f varies in F. In the image domain, the pixel p(i,j) is transformed to $NDP_{NS}(i,j) = \{T(i,j),\ I(i,j),\ F(i,j)\}$. Where $T(i,j)$ belongs to white set, $I(i,j)$ belongs to indeterminate set and $F(i,j)$ belongs to non-white set.
Which can be defined as [7]:

$$p_{NS}(i,j) = \{T(i,j), I(i,j), F(i,j)$$

$$T(i,j) = \frac{\overline{g(i,j)} - \bar{g}_{\min}}{\bar{g}_{\max} - \bar{g}_{\min}}$$

$$I(i,j) = 1 - \frac{H_0(i,j) - H_0}{H_{0\max} - H_{0\min}}$$

$$F(i,j) = 1 - T(i,j)$$

$$H_0(i,j) = abs(g(i,j) - \overline{g(i,j)}$$

Where $\overline{g(i,j)}$ can be defined as the local mean value of the pixels of window size, and $H_0(i,j)$ can be defined as the homogeneity value of T at (i,j), which described by the absolute value of difference between intensity $\overline{g(i,j)}$ and its local mean value $g(i,j)$.
The second transformation for $NDP_{NS}(i,j) = \{T(i,j),\ I(i,j),\ F(i,j), \pi(i,j)\}$
Where $\pi(i,j) = 3 - T(i,j) - I(i,j) - F(i,j)$ in [25].

4 Texture features in neutrosophic domain

4.1 Neutrosophic Entropy

Shannons Entropy provides an absolute limit on the best possible average length of lossless encoding or compression of an information source.
Generally, you need $log_2(n)$ bits to represent a variable that can take one of n values if n is a power of 2. If these values are equally probable, the entropy is equal to the number of bits equality between number of bits and shannons holds only while all outcomes are equally probable. If one of the events is more probable than others, observations of that event is less informative.
Conversely, rare events provide more information when observed. Since observation of less probable events occurs more rarely, the net effect is that the entropy received from non-uniformly distributed data is than $log_2(n)$. Entropy is zero when one outcome is certain. Shannon entropy quantifies all these considerations exactly quantifies all these considerations exactly when a probability distribution of the source is known. Entropy only takes into account the probability of observing a specific event, so the information which encapsulates is information about the underlying probability distribution, not the meaning of the events themselves [37].
Entropy is defined as [12]:

$$Entropy = \sum_i \sum_j P(i,j)\log(i,j)$$

Although the Neutrosophic Set Entropy was defined in one dimension, presented in [10], we will define it in two dimension to be as follows:

$$EntNS = EnT + EnI + EnF$$

$$EntT = \sum_i \sum_j P_T(i,j) \log P_T(i,j)$$

$$EntI = \sum_i \sum_j P_I(i,j) \log P_I(i,j)$$

$$EntF = \sum_i \sum_j P_F(i,j) \log P_F(i,j)$$

where P contains the histogram counts.
Because, we used the interval between 0 and 1, $logP(i,j)$ may have negative values.
So, we use the absolute of $En_T, En_I, and\ En_F$

4.2 Neutrosophic Contrast

Contrast is the difference in luminance or color that makes an object distinguishable. In visual perception of the real world, contrast is determined by the difference in the color and brightness of the object and other objects within the same field of view. The human visual system is more sensitive to contrast than absolute luminance. The maximum contrast of an image is the contrast ratio or dynamic range. It is the measure of the intensity contrast between a pixel and its neighbor over the whole image, it can be defined as [38]:

$$Contrast = \sum_i \sum_j (i-j)^2 P(i,j)$$

We will define the Neutrosophic set Contrast to be as follows:

$$ContNS = ContT + Cont_I + Cont_F$$

$$Cont_T = \sum_i \sum_j (i-j)^2 P_T(i,j)$$

$$Cont_I = \sum_i \sum_j (i-j)^2 P_I(i,j)$$

$$Cont_F = \sum_i \sum_j (i,j)^2 P_F(i,j)$$

4.3 Neutrosophic Energy

It is the sum of squared elements. Which defined as [13]:

$$Energy = \sum_i \sum_j P^2(i, j)$$

We will define the Neutrosophic set Energy to be as follows:

$$EnNS = EnT + En_I + En_F$$

$$En_T = \sum_i \sum_j P_T^2(i, j)$$

$$En_I = \sum_i \sum_j P_I^2(i, j)$$

$$En_F = \sum_i \sum_j P_F^2 9i, j)$$

4.4 Neutrosophic Homogeneity

Homogeneity describe the properties of a data set, or several datasets. Homogeneity can be studied to several degrees of complexity. For example, considerations of homoscedasticity examine how much the variability of data-values changes throughout a dataset. However, questions of homogeneity apply to all aspects of the statistical distributions, including the location parameter. Homogeneity relates to the validity of the often convenient assumption that the statistical properties of any one part of an overall dataset are the same as any other part. In meta-analysis, which combines the data from several studies, homogeneity measures the difference or similarities between the several studies.
That is a value which measures the closeness of the distribution of elements. Which defined as [20]:

$$Homogeneity = \sum_i \sum_j \frac{p(i, j)}{1 + |i - j|}$$

We will define the Neutrosophic set Homogeneity to be as follows:

$$HomoNS = HomoT + HomoI + Homo_F$$

$$HomoT = \sum_i \sum_j \frac{PT(i, j)}{1 + |i - j|}$$

$$HomoI = \sum_i \sum_j \frac{PI(i, j)}{1 + |i - j|}$$

$$HomoF = \sum_i \sum_j \frac{PF(i, j)}{1 + |i - j|}$$

Recently, the Euclidean distance is calculated between the query image and the first image in the database and stored in an array. This process is repeated for the remaining images in the database followed by storing their values respectively. The array is stored now in ascending order and displayed the first 8 closest matches.

5. Conclusion and Future Work

In this paper, we introduced a survey of the Text-Based Image Retrieval (TBIR) and the Content-Based Image Retrieval (CBIR). We also introduced the image in neutrosophic domain and the texture feature in neutrosophic domain. In the future, we plan to introduce some similarity measurement which may be used to determine the distance between the image under consideration and each image in the database, using the features we introduced in this paper. Hence, the images similar to the image under consideration can be retrieved.

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Truss Design Optimization using Neutrosophic Optimization Technique

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Abstract: In this paper, we develop a neutrosophic optimization (NSO) approach for optimizing the design of plane truss structure with single objective subject to a specified set of constraints. In this optimum design formulation, the objective functions are the weight of the truss and the deflection of loaded joint; the design variables are the cross-sections of the truss members; the constraints are the stresses in members. A classical truss optimization example is presented to demonstrate the efficiency of the neutrosophic optimization approach. The test problem includes a two-bar planar truss subjected to a single load condition. This single-objective structural optimization model is solved by fuzzy and intuitionistic fuzzy optimization approach as well as neutrosophic optimization approach. A numerical example is given to illustrate our NSO approach. The result shows that the NSO approach is very efficient in finding the best optimal solutions.

Keywords: Neutrosophic Set, Single Valued Neutrosophic Set, Neutrosophic Optimization, Non-linear Membership Function, Structural Optimization.

1 Introduction

In the field of civil engineering nonlinear structural design optimizations are of great importance. So the description of structural geometry and mechanical properties like stiffness are required for a structural system. However the system description and system inputs may not be exact due to human errors or some unexpected situations. At this juncture fuzzy set theory provides a method which deals with ambiguous situations like vague parameters, non-exact objective and constraint. In structural engineering design problems, the input data and parameters are often fuzzy/imprecise with nonlinear characteristics that necessitate the development of fuzzy optimum structural design method. Fuzzy set (FS) theory has long been introduced to handle inexact and imprecise data by Zadeh [2], Later on Bellman and Zadeh [4] used the fuzzy set theory to the decision making problem. The fuzzy set theory also found application in structural design. Several researchers like Wang et al. [8] first applied α-cut method to structural designs where the non-linear problems were solved with various design levels α, and then a sequence of solutions were obtained by setting different level-cut value of α. Rao [3] applied the same α-cut method to design a four–bar mechanism for function generating problem. Structural optimization with fuzzy parameters was developed by Yeh et al. [9]. Xu [10] used two-phase method for fuzzy optimization of structures. Shih et al. [5] used level-cut approach of the first and second kind for structural design optimization problems with fuzzy resources. Shih et al. [6] developed an alternative α-level-cuts methods for optimum structural design with fuzzy resources. Dey et al. [11] used generalized fuzzy number in context of a structural design. Dey et al used basic t-norm based fuzzy optimization technique for optimization of structure. Dey et al. [13] developed parameterized t-norm based fuzzy optimization method for optimum structural design. Also, Dey et.al [14] Optimized shape design of structural model with imprecise coefficient by parametric geometric programming.

In such extension, Atanassov [1] introduced Intuitionistic fuzzy set (IFS) which is one of the generalizations of fuzzy set theory and is characterized by a membership function, a non- membership function and a hesitancy function. In fuzzy sets the degree of acceptance is only considered but IFS is characterized by a membership function and a non-membership function so that the sum of both values is less than one. A transportation model was solved by Jana et al.[15]using multi-objective intuitionistic fuzzy linear programming. Dey et al. [12] solved two bar truss non-linear problem by using intuitionistic fuzzy optimization problem. Dey et al. [16] used intuitionistic fuzzy optimization technique for multi objective optimum structural design. Intuitionistic fuzzy sets consider both truth membership and falsity membership. Intuitionistic fuzzy sets can only handle incomplete information not the indeterminate information and inconsistent information.

In neutrosophic sets indeterminacy is quantified explicitly and truth membership, indeterminacy membership and falsity membership are independent. Neutrosophic theory was introduced by Smarandache [7]. The motivation of the present study is to give computational algorithm for solving multi-objective structural problem by single valued neutrosophic optimization approach. Neutrosophic optimization technique is very rare in application to structural optimization. We also aim to study the impact of truth membership, indeterminacy membership and falsity membership function in such optimization process. The results are compared numerically both in fuzzy optimization technique, intuitionistic fuzzy optimization technique and neutrosophic optimization technique. From our numerical result, it is clear that neutrosophic optimization technique provides better results than fuzzy optimization and intuitionistic fuzzy optimization.

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2 Single-objective structural model

In sizing optimization problems,the aim is to minimize single objective function,usually the weight of the structure under certain behavioural constraints on constraint and displacement. The design variables are most frequently chosen to be dimensions of the cross sectional areas of the members of the structures. Due to fabrications limitations the design variables are not continuous but discrete for belongingness of cross-sections to a certain set. A discrete structural optimization problem can be formulated in the following form

$$Minimize\ WT(A)$$

$$subject\ to\ \sigma_i(A) \le 0, i = 1,2,........,m$$

$$A_j \in R^d,\ \ j = 1,2,..........,n$$

where $WT(A)$ represents objective function, $\sigma_i(A)$ is the behavioural constraints, m and n are the number of constraints and design variables respectively. A given set of discrete value is expressed by R^d and in this paper objective function is taken as $WT(A) = \sum_{i=1}^{m} \rho_i l_i A_i$ and constraint are chosen to be stress of structures as follows $\sigma_i(A) \lesssim \sigma_i$ with allowable tolerance σ_i^0 for $i = 1,2,........,m$ where ρ_i and l_i are weight of unit volume and length of i^{th} element respectively, m is the number of structural element, σ_i and σ_i^0 are the i^{th} stress ,allowable stress respectively.

3 Mathematical preliminaries

3.1 Fuzzy set

Let X be a fixed set. A fuzzy set A set of X is an object having the form $\tilde{A} = \{(x, T_A(x)) : x \in X\}$ where the function $T_A : X \to [0,1]$ defined the truth membership of the element $x \in X$ to the set A .

3.2 Intuitionistic fuzzy set

Let a set X be fixed. An intuitionistic fuzzy set or IFS $\tilde{A}^i$ in X is an object of the form

$$\tilde{A}^i = \{< X, T_A(x), F_A(x) > | x \in X\} \text{ where}$$

$$T_A : X \to [0,1] \text{ and } F_A : X \to [0,1]$$

define the truth membership and falsity membership respectively, for every element of $x \in X$ $0 \le T_A + F_A \le 1$.

3.3 Neutrosophic set

Let a set X be a space of points (objects) and $x \in X$ .A neutrosophic set $\tilde{A}^n$ in X is defined by a truth membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity membership function $F_A(x)$,and denoted by $\tilde{A}^n = \{< x, T_A(x), I_A(x), F_A(x) > | x \in X\}$.

$T_A(x)$ $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$ .That is $T_A(x) : X \to]0^-, 1^+[$, $I_A(x) : X \to]0^-, 1^+[$, and $F_A(x) : X \to]0^-, 1^+[$, .There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ so $0^- \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$.

3.4 Single valued neutrosophic set

Let a set X be the universe of discourse. A single valued neutrosophic set $\tilde{A}^n$ over X is an object having the form $\tilde{A}^n = \{< x, T_A(x), I_A(x), F_A(x) > | x \in X\}$ where $T_A : X \to [0,1]$, $I_A : X \to [0,1]$, and $F_A : X \to [0,1]$ with $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$ for all $x \in X$.

3.5 Complement of neutrosophic Set

Complement of a single valued neutrosophic set A is denoted by $c(A)$ and is defined by $T_{c(A)}(x) = F_A(x)$, $I_{c(A)}(x) = 1 - F_A(x)$, $F_{c(A)}(x) = T_A(x)$.

3.6 Union of neutrosophic sets

The union of two single valued neutrosophic sets A and B is a single valued neutrosophic set C, written as $C = A \cup B$,whose truth membership, indeterminacy-membership and falsity-membership functions are given by

$T_{c(A)}(x) = \max(T_A(x), T_B(x)),$

$I_{c(A)}(x) = \max(I_A(x), I_B(x)),$

$F_{c(A)}(x) = \min(F_A(x), F_B(x))$ for all $x \in X$.

3.7 Intersection of neutrosophic sets

The intersection of two single valued neutrosophic sets A and B is a single valued neutrosophic set C , written as $C = A \cap B$,whose truth membership, indeterminacy-membership and falsity-membership functions are given by

$T_{c(A)}(x) = \min(T_A(x), T_B(x)),$

$I_{c(A)}(x) = \min(I_A(x), I_B(x)),$

$F_{c(A)}(x) = \max(F_A(x), F_B(x))$ for all $x \in X$.

4 Mathematical analyses

4.1 Neutrosophic optimization technique to solve minimization type Single-Objective

Let a nonlinear single-objective optimization problem be

$$Minimize \ \ f(x) \tag{2}$$

Such that

$$g_j(x) \le b_j \qquad j = 1, 2, ..., m$$

$$x \ge 0$$

Usually constraints goals are considered as fixed quantity .But in real life problem ,the constraint goal cannot be always exact. So we can consider the constraint goal for less than type constraints at least b_j and it may possible to extend to $b_j + b_j^0$ .This fact seems to take the constraint goal as a neutrosophic fuzzy set and which will be more realistic descriptions than others. Then the NLP becomes NSO problem with neutrosophic resources, which can be described as follows

$$Minimize \ \ f(x) \tag{3}$$

Such that

$$g_j(x) \le \tilde{b}_j^n \qquad j = 1, 2, ..., m$$

$$x \ge 0$$

To solve the NSO (3), we are presenting a solution procedure for single-objective NSO problem (3) as follows

Step-1: Following warner's approach solve the single objective non-linear programming problem without tolerance in constraints $(i.e\ g_j(x) \le b_j)$, with tolerance of acceptance in constraints (i.e $g_j(x) \le b_j + b_j^0$) by appropriate non-linear programming technique

Here they are
Sub-problem-1

$$Minimize \ \ f(x) \tag{4}$$

Such that

$$g_j(x) \le b_j \qquad j = 1, 2, ..., m$$

$$x \ge 0$$

Sub-problem-2

$$Minimize \ \ f(x) \tag{5}$$

Such that

$$g_j(x) \le b_j + b_j^0, \quad j = 1, 2, ..., m$$

$$x \ge 0$$

We may get optimal solution $x^* = x^1, f(x^*) = f(x^1)$ and $x^* = x^1, f(x^*) = f(x^1)$

Step-2: From the result of step 1 we now find the lower bound and upper bound of objective functions. If $U^T_{f(x)}, U^I_{f(x)}, U^F_{f(x)}$ be the upper bounds of truth, indeterminacy , falsity function for the objective respectively and $L^T_{f(x)}, L^I_{f(x)}, L^F_{f(x)}$ be the lower bound of truth, indeterminacy, falsity membership functions of objective respectively. then

$$U^F_{f(x)} = U^T_{f(x)}, L^F_{f(x)} = L^T_{f(x)} + \varepsilon_{f(x)} \ \ where\ 0 < \varepsilon_{f(x)} < \left(U^T_{f(x)} - L^T_{f(x)}\right)$$

$$U^F_{f(x)} = U^T_{f(x)}, L^F_{f(x)} = L^T_{f(x)} + \varepsilon_{f(x)} \ \ where\ 0 < \varepsilon_{f(x)} < \left(U^T_{f(x)} - L^T_{f(x)}\right)$$

$$L^I_{f(x)} = L^T_{f(x)}, U^I_{f(x)} = L^T_{f(x)} + \xi_{f(x)} \ \ where\ 0 < \xi_{f(x)} < \left(U^T_{f(x)} - L^T_{f(x)}\right)$$

Step-3: In this step we calculate membership for truth, indeterminacy and falsity membership function of objective as follows

$$T_{f(x)}(f(x)) = \begin{cases} 1 & if\ \ f(x) \le L^T_{f(x)} \\ 1 - \exp\left\{-\psi\left(\dfrac{U^T_{f(x)} - f(x)}{U^T_{f(x)} - L^T_{f(x)}}\right)\right\} & if\ \ L^T_{f(x)} \le f(x) \le U^T_{f(x)} \\ 0 & if\ \ f(x) \ge U^T_{f(x)} \end{cases}$$

$$I_{f(x)}\left(f(x)\right)=\begin{cases}1 & if\ \ f(x)\le L^{I}_{f(x)}\\ \exp\left\{\dfrac{U^{I}_{f(x)}-f(x)}{U^{I}_{f(x)}-L^{I}_{f(x)}}\right\} & if\ \ L^{I}_{f(x)}\le f(x)\le U^{I}_{f(x)}\\ 0 & if\ \ f(x)\ge U^{I}_{f(x)}\end{cases}$$

$$F_{f(x)}\left(f(x)\right)=\begin{cases}0 & if\ f(x)\le L^{F}_{f(x)}\\ \dfrac{1}{2}+\dfrac{1}{2}\tanh\left\{\left(f(x)-\dfrac{U^{F}_{f(x)}+L^{F}_{f(x)}}{2}\right)\tau_{f(x)}\right\} & if\ \ L^{F}_{f(x)}\le f(x)\le U^{F}_{f(x)}\\ 1 & if\ \ f(x)\ge U^{F}_{f(x)}\end{cases}$$

where ψ,τ are non-zero parameters prescribed by the decision maker.

Step-4: In this step using exponential and hyperbolic membership function we calculate truth , indeterminacy and falsity membership function for constraints as follows

$$T_{g_j(x)}\left(g_j(x)\right)=\begin{cases}1 & if\ \ g_j(x)\le b_j\\ 1-\exp\left\{-\psi\left(\dfrac{U^{T}_{g_j(x)}-g_j(x)}{U^{T}_{g_j(x)}-L^{T}_{g_j(x)}}\right)\right\} & if\ \ b_j\le g_j(x)\le b_j+b^0_j\\ 0 & if\ \ g_j(x)\ge b_j+b^0_j\end{cases}$$

$$I_{g_j(x)}\left(g_j(x)\right)=\begin{cases}1 & if\ \ g_j(x)\le b_j\\ \exp\left\{\dfrac{\left(b_j+\xi_{g_j(x)}\right)-g_j(x)}{\xi_{g_j(x)}}\right\} & if\ \ b_j\le g_j(x)\le b_j+\xi_{g_j(x)}\\ 0 & if\ \ g_j(x)\ge b_j+\xi_{g_j(x)}\end{cases}$$

$$F_{g_j(x)}\left(g_j(x)\right)=\begin{cases}0 & if\ g_j(x)\le b_j+\varepsilon_{g_j(x)}\\ \dfrac{1}{2}+\dfrac{1}{2}\tanh\left\{\left(g_j(x)-\dfrac{2b_j+b^0_j+\varepsilon_{g_j(x)}}{2}\right)\tau_{g_j(x)}\right\} & if\ \ b_j+\varepsilon_{g_j(x)}\le g_j(x)\le b_j+b^0_j\\ 1 & if\ \ g_j(x)\ge b_j+b^0_j\end{cases}$$

where ψ,τ are non-zero parameters prescribed by the decision maker and for $j=1,2,.....,m$ $0<\varepsilon_{g_j(x)},\xi_{g_j(x)}<b^0_j$.

Step-5: Now using NSO for single objective optimization technique the optimization problem (2) can be formulated as

$$Maximize\ \left(\alpha+\gamma-\beta\right) \tag{6}$$

Such that

$T_{f(x)}(x)\ge\alpha;\ T_{g_j}(x)\ge\alpha;$

$I_{f(x)}(x)\ge\gamma;\ I_{g_j}(x)\ge\gamma;$

$F_{f(x)}(x)\le\beta;\ F_{g_j}(x)\le\beta;$

$\alpha+\beta+\gamma\le 3;\ \alpha\ge\beta;\alpha\ge\gamma;$

$\alpha,\beta,\gamma\in[0,1]$

where

$\alpha=T_{\tilde{D}^n}(x)=\min\left\{T_{f(x)}\left(f(x)\right),T_{g_j(x)}\left(g_j(x)\right)\right\}$

for $j=1,2,...,m$

$\gamma=I_{\tilde{D}^n}(x)=\min\left\{I_{f(x)}\left(f(x)\right),I_{g_j(x)}\left(g_j(x)\right)\right\}$

for $j=1,2,...,m$ and

$\beta=F_{\tilde{D}^n}(x)=\min\left\{F_{f(x)}\left(f(x)\right),F_{g_j(x)}\left(g_j(x)\right)\right\}$ for $j=1,2,...,m$

are the truth ,indeterminacy and falsity membership function of decision set $\tilde{D}^n=f^n(x)\bigcap_{j=1}^{m}g^n{}_j(x)$. Now if non-linear membership be considered the above problem (6) can be reduced to following crisp linear programming problem

$$Maximize\ \left(\theta+\kappa-\eta\right) \tag{7}$$

Such that

$f(x)+\theta\dfrac{\left(U^{T}_{f(x)}-L^{T}_{f(x)}\right)}{\psi}\le U^{T}_{f(x)};$

$f(x)+\kappa\xi_{f(x)}\le U^{T}_{f(x)};$ b

$f(x)+\dfrac{\eta}{\tau_{f(x)}}\le\dfrac{U^{T}_{f(x)}+L^{T}_{f(x)}+\varepsilon_{f(x)}}{2};$

$g_j(x)+\theta\dfrac{b^0_j}{\psi}\le b_j+b^0_j;$

$g_j(x)+\kappa\xi_{g_j(x)}\le b^0_j+\xi_{g_j(x)};$

$g_j(x)+\dfrac{\eta}{\tau_{g(x)}}\le\dfrac{2b_j+b^0_j+\varepsilon_{g_j(x)}}{2};$

$\theta+\kappa+\eta\le 3;$

$\theta\ge\kappa;\theta\ge\eta;$

$$\theta,\kappa,\eta\in[0,1]$$

where $\theta=-\ln(1-\alpha)$; $\psi=4$; $\tau_{f(x)}=\dfrac{6}{\left(U^F_{f(x)}-L^F_{f(x)}\right)}$;

$\tau_{g_j(x)}=\dfrac{6}{\left(b_j^0-\varepsilon_j\right)}$, $for\ j=1,2,........,m$ $\kappa=\ln\gamma$;

$\eta=-\tanh^{-1}(2\beta-1)$.

This crisp nonlinear programming problem can be solved by appropriate mathematical algorithm.

5. Solution of Single-objective Structural Optimization Problem (SOSOP) by Neutrosophic Optimization Technique

To solve the SOSOP (1), step 1 of 4 is used and we will get optimum solutions of two sub problem as A^1 and A^2 .After that according to step 2 we find upper and lower bound of membership function of objective function as $U^T_{WT(A)},U^I_{WT(A)},U^F_{WT(A)}$ and $U^T_{WT(A)},U^I_{WT(A)},U^F_{WT(A)}$ where

$$U^T_{WT(A)}=\max\left\{WT\left(A^1\right),WT\left(A^2\right)\right\},L^T_{WT(A)}=\min\left\{WT\left(A^1\right),WT\left(A^2\right)\right\},$$

$$U^F_{WT(A)}=U^T_{WT(A)},L^F_{WT(A)}=L^T_{WT(A)}+\varepsilon_{WT(A)}\ \ where\ 0<\varepsilon_{WT(A)}<\left(U^T_{WT(A)}-L^T_{WT(A)}\right)$$

$$L^I_{WT(A)}=L^T_{WT(A)},U^I_{WT(A)}=L^T_{WT(A)}+\xi_{WT(A)}\ \ where\ 0<\xi_{WT(A)}<\left(U^T_{WT(A)}-L^T_{WT(A)}\right)$$

Let the non-linear membership function for objective function $WT(A)$ be

$$T_{WT(A)}\left(WT(A)\right)=\begin{cases}1 & if\ \ WT(A)\le L^T_{WT(A)}\\ 1-\exp\left\{-\psi\left(\dfrac{U^T_{WT(A)}-WT(A)}{U^T_{WT(A)}-L^T_{WT(A)}}\right)\right\} & if\ \ L^T_{WT(A)}\le WT(A)\le U^T_{WT(A)}\\ 0 & if\ \ WT(A)\ge U^T_{WT(A)}\end{cases}$$

$$I_{WT(A)}\left(WT(A)\right)=\begin{cases}1 & if\ \ WT(A)\le L^I_{WT(A)}\\ \exp\left\{\dfrac{U^I_{WT(A)}-WT(A)}{U^I_{WT(A)}-L^I_{WT(A)}}\right\} & if\ \ L^I_{WT(A)}\le WT(A)\le U^I_{WT(A)}\\ 0 & if\ \ WT(A)\ge U^I_{WT(A)}\end{cases}$$

$$I_{\sigma_i(A)}\left(\sigma_i(A)\right)=\begin{cases}1 & if\ \ \sigma_i(A)\le\sigma_i\\ \exp\left\{\dfrac{\left(\sigma_i+\xi_{g_j(x)}\right)-\sigma_i(A)}{\xi_{\sigma_i(x)}}\right\} & if\ \ \sigma_i\le\sigma_i(A)\le\sigma_i+\xi_{\sigma_i(x)}\\ 0 & if\ \ \sigma_i(A)\ge\sigma_i+\xi_{\sigma_i(x)}\end{cases}$$

$$F_{\sigma_i(A)}\left(\sigma_i(A)\right)=\begin{cases}0 & if\ \ \sigma_i(A)\le\sigma_i+\varepsilon_{\sigma_i(A)}\\ \dfrac{1}{2}+\dfrac{1}{2}\tanh\left\{\left(\sigma_i(A)-\dfrac{2b_j+b_j^0+\varepsilon_{\sigma_i}}{2}\right)\tau_{\sigma_i}\right\} & if\ \ \sigma_i+\varepsilon_{\sigma_i(A)}\le\sigma_i(A)\le\sigma_i+\sigma_i^0\\ 1 & if\ \ \sigma_i(A)\ge\sigma_i+\sigma_i^0\end{cases}$$

where ψ,τ are non-zero parameters prescribed by the decision maker and for $j=1,2,.....,m$ $0<\varepsilon_{\sigma_i(A)},\xi_{\sigma_i(A)}<\sigma_i^0$

then neutrosophic optimization problem can be formulated as

$$Max\ (\alpha+\beta-\gamma) \tag{8}$$

such that

$$T_{WT(A)}\left(WT(A)\right)\ge\alpha;\ T_{\sigma_i(A)}\left(\sigma_i(A)\right)\ge\alpha;$$

$$I_{WT(A)}\left(WT(A)\right)\ge\gamma;\ I_{\sigma_i(A)}\left(\sigma_i(A)\right)\ge\gamma;$$

$$F_{WT(A)}\left(WT(A)\right)\le\beta;\ F_{\sigma_i(A)}\left(\sigma_i(A)\right)\le\beta$$

$$\alpha+\beta+\gamma\le 3;\alpha\ge\beta,\alpha\ge\gamma;$$

$$\alpha,\beta,\gamma\in[0,1]$$

The above problem can be reduced to following crisp linear programming problem, for non-linear membership as

$$Maximize\ (\theta+\kappa-\eta) \tag{9}$$

such that

$$WT(A)+\theta\frac{\left(U^T_{WT(A)}-L^T_{WT(A)}\right)}{\psi}\le U^T_{WT(A)};$$

$$WT(A)+\frac{\eta}{\tau_{WT(A)}}\le\frac{U^T_{WT(A)}+L^T_{WT(A)}+\varepsilon_{WT(A)}}{2};$$

$$WT(A)+\kappa\xi_{WT(A)}\le U^T_{WT(A)};$$

$$\sigma_i(A)+\theta\frac{\sigma_i^0}{\psi}\le\sigma_i+\sigma_i^0;$$

$\sigma_i\left(A\right)+\kappa\xi_{\sigma_i(A)}\leq\sigma_i^0+\xi_{\sigma_i(A)};$

$\sigma_i\left(A\right)+\dfrac{\eta}{\tau_{\sigma_i(A)}}\leq\dfrac{2\sigma_i+\sigma_i^0+\varepsilon_{\sigma_i(A)}}{2};$

$\theta+\kappa-\eta\leq 3;\ \theta\geq\kappa;\theta\geq\eta;$

$\theta,\kappa,\eta\in\left[0,1\right]$

where

$\theta=-\ln\left(1-\alpha\right);\ \psi=4;\ \tau_{WT(A)}=\dfrac{6}{\left(U_{WT(A)}^F-L_{WT(A)}^F\right)};$

$\kappa=\ln\gamma;\ \eta=-\tanh^{-1}\left(2\beta-1\right).\ and\ \tau_{\delta(A)}=\dfrac{6}{\left(U_{\delta(A)}^F-L_{\delta(A)}^F\right)};$

This crisp nonlinear programming problem can be solved by appropriate mathematical algorithm.

6 Numerical illustration

A well-known two-bar [17] planar truss structure is considered. The design objective is to minimize weight of the structural $WT\left(A_1,A_2,y_B\right)$ of a statistically loaded two-bar planar truss subjected to stress $\sigma_i\left(A_1,A_2,y_B\right)$ constraints on each of the truss members $i=1,2$.

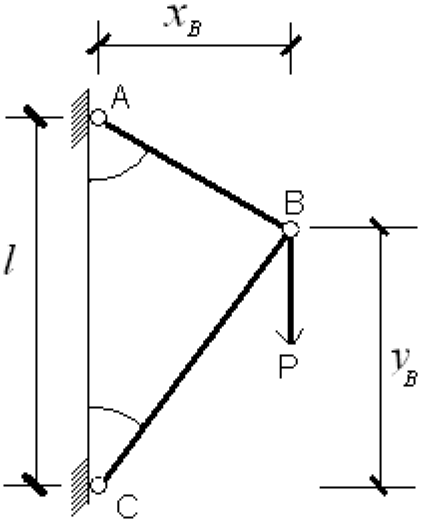

Figure 1. Design of the two-bar planar truss

The multi-objective optimization problem can be stated as follows

$$Minimize\ WT\left(A_1,A_2,y_B\right)=\rho\left(A_1\sqrt{x_B^2+\left(l-y_B\right)^2}+A_2\sqrt{x_B^2+{y_B}^2}\right)\quad(10)$$

Such that

$$\sigma_{\mathrm{AB}}\left(A_1,A_2,y_B\right)\equiv\frac{P\sqrt{x_B^2+\left(l-y_B\right)^2}}{lA_1}\leq\left[\sigma_{AB}^T\right];$$

$$\sigma_{\mathrm{BC}}\left(A_1,A_2,y_B\right)\equiv\frac{P\sqrt{x_B^2+{y_B}^2}}{lA_2}\leq\left[\sigma_{BC}^C\right];$$

$$0.5\leq y_B\leq 1.5$$

$$A_1>0,A_2>0;$$

where $P=$ nodal load ; $\rho=$ volume density ; $l=$ length of AC ; $x_B=$ perpendicular distance from AC to point B . $A_1=$ Cross section of bar- AB ; $A_2=$ Cross section of bar- BC . $\left[\sigma_T\right]=$ maximum allowable tensile stress , $\left[\sigma_C\right]=$ maximum allowable compressive stress and $y_B=y$ -co-ordinate of node B .

Input data for crisp model (10) is in table 1.

Solution : According to step 2 of 4,we find upper and lower bound of membership function of objective function as

$U_{WT(A)}^T,U_{WT(A)}^I,U_{WT(A)}^F$

and $L_{WT(A)}^T,L_{WT(A)}^I,L_{WT(A)}^F$ where

$U_{WT(A)}^T=14.23932=U_{WT(A)}^F$

$L_{WT(A)}^T=12.57667=L_{WT(A)}^I$

$L_{WT(A)}^F=12.57667+\varepsilon_{WT(A)},\ 0<\varepsilon_{WT(A)}<1.66265$

$U_{WT(A)}^I=L_{WT(A)}^T+\xi_{WT(A)},\ 0<\xi_{WT(A)}<1.66265$

Now using the bounds we calculate the membership functions for objective as follows

$$T_{WT\left(A_1,A_2,y_B\right)}\left(WT\left(A_1,A_2,y_B\right)\right)=\begin{cases}1 & if\ WT\left(A_1,A_2,y_B\right)\leq 12.57667\\ 1-\exp\left\{-4\left(\dfrac{14.23932-WT\left(A_1,A_2,y_B\right)}{1.66265}\right)\right\} & if\ 12.57667\leq WT\left(A_1,A_2,y_B\right)\leq 14.23932\\ 0 & if\ WT\left(A_1,A_2,y_B\right)\geq 14.23932\end{cases}$$

$$I_{WT\left(A_1,A_2,y_B\right)}\left(WT\left(A_1,A_2,y_B\right)\right)=\begin{cases}1 & if\ WT\left(A_1,A_2,y_B\right)\leq 12.57667\\ \exp\left\{\dfrac{\left(12.57667+\xi_{WT}\right)-WT\left(A_1,A_2,y_B\right)}{\xi_{WT}}\right\} & if\ 12.57667\leq WT\left(A_1,A_2,y_B\right)\leq 12.57667+\xi_{WT}\\ 0 & if\ WT\left(A_1,A_2,y_B\right)\geq 12.57667+\xi_{WT}\end{cases}$$

$$F_{WT(A_1,A_2,y_B)}\left(WT(A_1,A_2,y_B)\right)=\begin{cases}0 & if\ WT(A_1,A_2,y_B)\le 12.57667+\varepsilon_{WT}\\ \frac{1}{2}+\frac{1}{2}\tanh\left\{\left(WT(A_1,A_2,y_B)-\frac{(26.81599+\varepsilon_{WT})}{2}\right)\frac{6}{1.66265-\varepsilon_{WT}}\right\} & if\ 12.57667+\varepsilon_{WT}\le WT(A_1,A_2,y_B)\le 14.23932\\ 1 & if\ WT(A_1,A_2,y_B)\ge 14.23932\end{cases}$$

Similarly the membership functions for tensile stress are

$$T_{\sigma_T(A_1,A_2,y_B)}\left(\sigma_T(A_1,A_2,y_B)\right)=\begin{cases}1 & if\ \sigma_T(A_1,A_2,y_B)\le 130\\ 1-\exp\left\{-4\left(\frac{150-\sigma_T(A_1,A_2,y_B)}{20}\right)\right\} & if\ 130\le\sigma_T(A_1,A_2,y_B)\le 150\\ 0 & if\ \sigma_T(A_1,A_2,y_B)\ge 150\end{cases}$$

$$I_{\sigma_T(A_1,A_2,y_B)}\left(\sigma_T(A_1,A_2,y_B)\right)=\begin{cases}1 & if\ \sigma_T(A_1,A_2,y_B)\le 130\\ \exp\left\{\frac{(130+\xi_{\sigma_T})-\sigma_T(A_1,A_2y_B)}{\xi_{\sigma_T}}\right\} & if\ 130\le\sigma_T(A_1,A_2,y_B)\le 130+\xi_{\sigma_T}\\ 0 & if\ \sigma_T(A_1,A_2,y_B)\ge 130+\xi_{\sigma_T}\end{cases}$$

$$F_{\sigma_T(A_1,A_2,y_B)}\left(\sigma_T(A_1,A_2,y_B)\right)=\begin{cases}0 & if\ \sigma_T(A_1,A_2,y_B)\le 130+\varepsilon_{\sigma_T}\\ \frac{1}{2}+\frac{1}{2}\tanh\left\{\left(\sigma_T(A_1,A_2,y_B)-\left(\frac{280+\varepsilon_{\sigma_T}}{2}\right)\right)\frac{6}{20-\varepsilon_{\sigma_T}}\right\} & if\ 130+\varepsilon_{\sigma_T}\le\sigma_T(A_1,A_2,y_B)\le 150\\ 1 & if\ \sigma_T(A_1,A_2,y_B)\ge 150\end{cases}$$

where $0<\varepsilon_{\sigma_T},\xi_{\sigma_T}<20$

and the membership functions for compressive stress constraint are

$$T_{\sigma_C(A_1,A_2,y_B)}\left(\sigma_C(A_1,A_2,y_B)\right)=\begin{cases}1 & if\ \sigma_C(A_1,A_2,y_B)\le 90\\ 1-\exp\left\{-4\left(\frac{100-\sigma_C(A_1,A_2,y_B)}{10}\right)\right\} & if\ 90\le\sigma_C(A_1,A_2,y_B)\le 100\\ 0 & if\ \sigma_C(A_1,A_2,y_B)\ge 100\end{cases}$$

$$I_{\sigma_C(A_1,A_2,y_B)}\left(\sigma_C(A_1,A_2,y_B)\right)=\begin{cases}1 & if\ \sigma_C(A_1,A_2,y_B)\le 90\\ \exp\left\{\frac{(90+\xi_{\sigma_C})-\sigma_C(A_1,A_2,y_B)}{\xi_{\sigma_C}}\right\} & if\ 90\le\sigma_C(A_1,A_2,y_B)\le 90+\xi_{\sigma_C}\\ 0 & if\ \sigma_C(A_1,A_2,y_B)\ge 90+\xi_{\sigma_C}\end{cases}$$

$$F_{\sigma_C(A_1,A_2,y_B)}\left(\sigma_C(A_1,A_2,y_B)\right)=\begin{cases}0 & if\ \sigma_C(A_1,A_2,y_B)\le 90+\varepsilon_{\sigma_C}\\ \frac{1}{2}+\frac{1}{2}\tanh\left\{\left(\sigma_C(A_1,A_2,y_B)-\left(\frac{190+\varepsilon_{\sigma_C}}{2}\right)\right)\frac{6}{10-\varepsilon_{\sigma_C}}\right\} & if\ 90+\varepsilon_{\sigma_C}\le\sigma_C(A_1,A_2,y_B)\le 100\\ 1 & if\ \sigma_C(A_1,A_2,y_B)\ge 100\end{cases}$$

where $0<\varepsilon_{\sigma_C},\xi_{\sigma_C}<10$.

Now , using above mentioned truth, indeterminacy and falsity membership function NLP (7) can be solved by NSO technique for different values of $\varepsilon_{WT},\varepsilon_{\sigma_T},\varepsilon_{\sigma_C}$ and $\xi_{WT},\xi_{\sigma_T},\xi_{\sigma_C}$. The optimum solution of SOSOP(10) is given in table (2) and the solution is compared with fuzzy and intuitionistic fuzzy optimization technique.

Table 1: Input data for crisp model (10)

Applied load $P(KN)$	Volume density $\rho\left(KN/m^3\right)$	Length $l(m)$	Maximum allowable tensile stress $\lfloor\sigma_T\rfloor(Mpa)$	Maximum allowable compressive stress $\lfloor\sigma_C\rfloor(Mpa)$	Distance of x_B from AC (m)
100	7.7	2	130 with tolerance 20	90 with tolerance 10	1

Table 2: Comparison of Optimal solution of SOSOP (10) based on different methods

Methods	$A_1(m^2)$	$A_2(m^2)$	$WT(A_1,A_2)(KN)$	$y_B(m)$
Fuzzy single-objective non-linear programming (FSONLP) with non-linear membership functions	.5883491	.7183381	**14.23932**	1.013955
Intuitionistic fuzzy single-objective nonlinear programming (IFSONLP) with non-linear membership functions $\varepsilon_1=0.8,\varepsilon_2=16,\varepsilon_3=8$	.6064095	.6053373	**13.59182**	.5211994
Neutosophic optimization(NSO) with non-linear membership functions $\varepsilon_1=0.8,\varepsilon_2=16,\varepsilon_3=8$ $\xi_1=0.66506,\xi_2=8,\xi_3=4$	.5451860	.677883	**13.24173**	.7900455

Here we get best solutions for the different tolerance ξ_1, ξ_2 and ξ_3 for indeterminacy exponential membership function of objective functions for this structural optimization problem. From the table 2, it shows that NSO technique gives better Pareto optimal result in the perspective of Structural Optimization.

7 Conclusions

The main objective of this work is to illustrate how neutrosophic optimization technique using non-linear membership function can be utilized to solve a nonlinear structural problem. The concept of neutrosophic optimization technique allows one to define a degree of truth membership, which is not a complement of degree of falsity; but rather they are independent with degree of indeterminacy. The numerical illustration shows the superiority of neutrosophic optimization over fuzzy optimization as well as intuitionistic fuzzy optimization. The results of this study may lead to the development of effective neutrosophic technique for solving other model of nonlinear programming problem in other engineering field.

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Las habilidades del marketing como determinantes que sustentaran la competitividad de la Industria del arroz en el cantón Yaguachi. Aplicación de los números SVN a la priorización de estrategias.

Marketing skills as determinants that underpin the competitiveness of the rice industry in Yaguachi canton. Application of SVN numbers to the prioritization of strategies

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Abstract:
In Ecuador, specifically in the Yaguachi Canton, there is an enormous potential in the rice production, which unfortunately is not being well used and driven by marketing strategies. In this work, marketing strategies were developed that help to sustain the commercial activity of rice in Yaguachi Canton and its surroundings. The proposed strategies were analyzed and prioritized using SVN and Euclidean distance for the treatment of neutralities. The paper ends with conclusion and future work proposal for the application of neutrosophy to new areas of marketing.

Keywords: rice, marketing, neutrosophy, SVN numbers.

1. INTRODUCCION

1.1 Antecedentes:

Cuando escuchamos la palabra Marketing, nuestro posicionamiento cognoscitivo nos enrumba a detallar imágenes que en el mundo actual están presentes en toda gestión de oferta y demanda, nos dejamos llevar por la fantasía de la publicidad y nos dejamos envolver por los curiosos y entretenidos videos que recrean nuestra decisión de compra, nuestra mente se ocupa casi en su totalidad en productos de áreas específicas de la suntuosidad, diversión, mercado del entretenimiento entre otros; pero muy poco relacionamos productos de primera necesidad (tales como arroz, azúcar, harina etc.) con estrategias de Marketing. En Ecuador, específicamente en el Cantón Yaguachi existe un enorme potencial en la Industria de la Produccion-Comercializacion del arroz, que desafortunadamente no está siendo bien aprovechada y enrumbada por estrategias de la Mezcla del Marketing que le den sitial necesario, para que esta Industria siga creciendo y contribuyendo a las economías sostenibles y al impacto que ejercen en lo laboral ofreciendo empleo en todas las áreas que engloban las actividades del agro-comercio en el país.

Es por esto que en este presente estudio se desarrollaran estrategias de marketing que ayuden a sostener la actividad comercial del arroz proveniente de la zona geográfica de Yaguachi y sus alrededores, planteándonos objetivos alcanzables y justificando el accionar de los modelos conceptuales que sostengan las estrategias a desarrollarse, concluyendo con una propuesta enriquecedora en torno a la nueva forma de manejar la competitividad, diferenciándonos en las habilidades sin descuidar las debilidades que nos ayudara a enriquecer la estrategia de crecimiento y competitividad en el mercado de consumo, recomendando situaciones encontradas no alcanzables ni medibles en el presente estudio.

1.2 Definición del Problema:

El arroz es el cultivo que más extensión abarca en el Ecuador ocupando 399.535 Has. según la Encuesta de Superficie y Producción Agropecuaria Continua (ESPAC) 2015 [1].
La mayor área sembrada de arroz en el país se encuentra en la región Costa, pero también se siembra en las estribaciones andinas y en la Amazonía pero en cantidades poco significantes (Tabla 1).

REGIÓN	Superficie (Has.)		Produccion (Tm.)	Ventas (Tm.)
	Sembrio	Cosecha		
Total Nacional	399.535	375.117	1.652.793	1.534.4
Region Sierra	1.701	1.564	11.472	9.100
Region Costa	397.231	372.953	1.639.978	1.524.2
Region Oriente	528	526	1.245	1.005
Zona No Delimitad	75	75	98	97

Tabla 1. Superficie, produccion y ventas segun region.
Fuente: INEC 2015

Dos provincias, Guayas y Los Ríos, representan el 94% de la superficie sembrada de la gramínea en el Ecuador. En cuanto a la producción, de forma correspondiente, Guayas y Los Ríos tienen el 71,83% y % y 23.81% respectivamente (Tabla 2).

PROVINCIA	SUPERFICIE (Has.)		Producc (Tm.)	Venta (Tm.)
	Sembrio	Cosecha		
Cañar	128	118	829	829
Loja	1.541	1.414	10.575	8.252
Santo Domingo	32	32	69	18
El Oro	3.999	3.896	12.390	10.362
Esmeralda	100	100	179	.
Guayas	274.992	258.620	1.187.15	1.120
Los Rios	100.961	94.278	383.106	345.96
Manabi	17.180	16.060	57.169	47.88
Morona	3	3	3	3
Orellana	350	349	914	743
Sucumbio	174	174	328	259

Tabla # 2 Segun Provincias Fuente: INEC 2015

La Industria del arroz tiene su concentración máxima entre tres provincias casi exclusivamente, Guayas, Los Ríos y Manabí que son quienes marcan las pautas de la siembra, cosecha, pilada, y comercialización. [2]. En la Industria alimentaria, la gramínea del arroz constituye una de las fuentes principales de la canasta familiar.

Por su parte el cantón Yaguachi (fig. 1) cuya cabecera cantonal se encuentra a 29 km. de Guayaquil presenta una población de 47,600 habitantes en un área de 512 km². Está asentada a 15 m.s.n.m. y su temperatura promedio es de 25°C, su precipitación promedio anual está entre 500 y 1000 mm [3]. Su área cultivada de arroz corresponde a 15,521 has [4].

En otras instancias el problema se concentra mucho mas en la comercializacion, porque las culturas y costumbres que los obligan a usar metodos caducos y pocos enrumbados hacia las tecnologias no ayudan a mejorar las ofertas en un mercado en donde hay tradicion de padres a hijos.

Las piladoras que es el intermediario que se desenvuelve en el negocio del pilado, algunos aun siguen haciendolo de manera muy artesanal regar el arroz en un tendedero de secado al sol, son muy poco los mas proactivos que ya han empezado a tecnificar sus procesos en una cadena de instalaciones industrializados desde el tamizado hasta el secado en hornos.

Los arrozeros se quejan de que el gobierno no apoya a las asociaciones inyectando dinero en las infraestructuras, no tienen agua, no tienen semillas de buena calidad, el kit de insecticidas que ofertan estan siendo mal distribuidos pues solo se aprovechan los agricultores cercanos. El programa de alto rendimiento estaba constituido por un kit que traia 20 insumos: sacos de urea, semillas, insecticidas entre otros.

Figura 1. Cantón Yaguachi

1.3 Justificación del Problema:

Debido a este fenómeno, el Cantón Yaguachi enfrenta condiciones desfavorables inclusive en la provincia del Guayas, para mantenerse activa comercialmente siguiendo patrones ancestrales de procesos en la producción y específicamente por el desconocimiento de estrategias de marketing que motiven al consumo de la gramínea, pues con el crecimiento desmedido que presenta la Industria del Fitness en las sociedades modernas en donde el patrón de la belleza conllevan dejar de consumir productos de altos porcentajes de carbohidratos, el arroz es el primer producto que se evita consumir en las dietas alimenticias y considerando aún más que el arroz peruano está siendo concebido, como una gramínea con mejor calidad y menor precio en el mercado de consumo, ahondando más la situación de los productores del sector de Yaguachi y sus alrededores. El problema de este estudio se concentra entonces en la falta de estrategias de mercadotecnia que ayudarían a crecer a las empresas que conforman la Industria de producción y comercialización del arroz en el cantón Yaguachi

1.4 Objetivos:

Objetivo General

El objetivo fundamental de este proyecto es Proponer estrategias de comercialización sustentadas en el mix del marketing, que promuevan y ayuden al crecimiento sostenible de las empresas de la Industria del arroz en el cantón Yaguachi.

Objetivos Específicos

- Describir las estrategias de Marketing, haciendo un análisis macro y micro ambiental
- Plantear estrategias de recolección de Datos de tipo experimental y documental (caso empresarial) para obtener resultados proyectados.
- Proponer las estrategias de marketing para el crecimiento sostenible de la Industria del arroz en el cantón Yaguachi.
- Priorizar las distintas estrategias utilizando lógica Neutrosofica

1.5 Justificación de la Investigación:

Si consideramos que el problema esta circunscrito en un producto de consumo masivo, y que la graminea es un sustento social en las localidades mas olvidadas en el avance tecnologico y de crecimiento estructural, el canton Yaguachi viene a contribuir al sostenimiento alimenticio del Ecuador el mismo que debe ser enrumbado en una politica social de inclusion en los programas gubernamentales.

Es por esto que este estudio concentro la metodologia del analisis de la voz directa del agricultor – pilador; para este proposito elegimos una familia muy representativa en la siembra y pilado del arroz, quienes ademas han iniciado con procesos artesanales llegando a los actuales momentos a poseer procesos industriales bien estructurados y con planes de crecimiento sostenible. Es el caso de la empresa Timecorpoc S.A. la misma que fue fundada el año 2009, despues de haberse separado del nucleo familiar inicial, con quienes tambien llego a semi indutrializar el proceso no concluyendolo. En la actualidad esta empresa se dedica a la siembra, pilada, compra , venta y comercializacion del arroz, esta ubicada en el km 2,5 de la via Yaguachi-Jujan. La misma que nos dara los parametros indispensables para el analisis estrategico.

1.6 Marco Teórico de la Industria del arroz en el Ecuador

El eje de las estadísticas referenciales nos han permitido crear el contexto del problema, y si nos enfocamos hacia al aspecto nutricional el arroz es el alimento más consumido en el mundo y una quinta parte de la población del planeta depende de su cultivo, recientes estudios asocian un mayor consumo del arroz con el aumento de la obesidad y de factores de riesgo de padecer enfermedades cardiovasculares y diabetes tipo 2 [5, 6]. Sin embargo existen prácticas de cultivos, variedades y formas de consumirlos que mitigan o eliminan tanto los impactos medioambientales como los nutricionales. Esta visión nos hace considerar una amenaza la falta de estrategias de marketing en las acciones comerciales de la gramínea.

Por otro lado existen varios factores que están afectando el cultivo y producción de arroz a nivel mundial. En cuanto al aspecto medioambiental, su producción esta asociada de entre el 7-17% de todas las emisiones de metano de origen humano y, siendo el metano un potente gas de efecto invernadero representa un aporte significativo al calentamiento global [7].

Otro aspecto negativo es el empleo de grandes volumen de agua para anegar los terrenos[5]. Con estos datos, solo referenciamos algunas de las acciones del marco teórico, que nos harán recorrer los conceptos de grandes investigadores y pensadores estratégicos.[8]. Comprometidos con el medio ambiente y la economia sostenible, el marketing bien enrubado y sustentado en la herramienta mas eficaz de analisis de datos FODA nos ayuda a visualizar y delinear mejor las estrategias para la comercializacion de la graminea de los agricultores de Yaguachi.
En el siguiente cuadro resumimos lo expresado:

Fortalezas	Oportunidades	Debilidades	Amenazas
Producto consumo masivo	Inclusion social en la Matriz Productiva	cultivos sin control del medio ambiente	Desconocimiento de estrategias de mercadeo

las realidades del entorno nos ayuda a enfocar nuestro analisis basandonos en las estrategias de PENETRACION en el mercado, sabiendo que la competitividad dependera de la fuerza competitiva del sector y que la determinara la rentabilidad del mismo.

2. METODOLOGIA DE LA INVESTIGACION

Se realiza bajo un enfoque cualitativo, que utiliza recolección de datos e investigación preliminar [9, 10]. La misma se realizo en fuentes primarias con entrevistas a las empresas de los hermanos Menendez.

2.1 Diseño de la Investigación

La construcción del contexto cualitativo son armadas desde la logica del analisis de caso y modelo y niveles epistemológicos en la génesis e historia de la investigación social.[9].

2.2 Tipo de Estudio

Tiene un alcance descriptivo y no experimental porque se consideran las características de la empresa con éxito en el mercado del arroz y que tiene posicionada la marca de ARROZ MARAVILLA muy reconocida entre los compradores por su calidad. El horizonte espacial esta definido por la actividad del cultivo que actualmente están en esta industria en el cantón Yaguachi y que son especificamente agricultores que terminan su ciclo con el pilado del arroz, accion que ya no depende de ellos sino de una operacion aislada, siendo muy pocos los agricultores que terminan su proceso.

Despues de las reuniones establecidas en el sitio se escogio una empresa tipo para hacer el estudio, de la misma se resume lo siguiente:

La empresa que nos sirve de analisis de caso es ecuatoriana denominada Timecorpoc S.A. esta ubicada en el km 2,5 de la via Yaguachi-Jujan. fue fundada el año 2009 por Jose Avilio Menendez Mendoza en un acto de inmaculación, los fondos utilizados para la creación de Timecorpoc S.A. fueron recaudados mediante apalancamiento bancario y ahorros producidos del trabajo eficaz y exhaustivo de Avilio Menéndez cuando era la cabeza principal de la piladora Hermanos Menéndez, la cual está ubicada a 6 cuadras del centro del cantón Yaguachi. Cuando siendo su administrador logro dejarla semi indutrializada, la misma que no ha tenido ninguna evolucion ni crecimiento en los ultimos 7 años corooborando asi que sin administracion ordenada y sistematica no se consiguen resultados optimos.

Timecorpoc pertenece al sector Agropecuario y su principal actividad es la producción y comercialización del arroz. En la produccion esta empresa se dedica a la siembra y pilada, cuenta con una hacienda con un área de 168 Hectareas. Con una producción de 70 sacas de 200 lbs por hectárea, tiene 2 ciclos de siembra la de invierno en el mes de enero y se cosecha en el mes de abril, (4 meses de cultivo) y la de verano se siembra en julio y se cosecha en noviembre.
El costo de producción por hectárea es de $1.400, el costo de un saco de las 200 lbs de arroz es de $42. En los meses de mayo y junio se prepara la tierra para su respectiva siembra al igual que el mes de diciembre. La variedad de arroz que se siembra es el 09, f 25. Que son arroces grano largo de 4 meses, cabe recalcar que hay arroces de ciclo corto y a su vez se lo conoce como iniap 14 o grano corto, con ese tipo de arroz no trabajan por cuanto solo tienen mercado para arroces grano largo.
E[arroz una vez cosechado se lo lleva a la piladora para su debido pílado, el costo de cosecha y traslado a la planta es de $3.30.
En la planta se lo pesa por báscula manejada por sistemas, y se lo deposita en los silos de almacenamiento (existen 3 silos de 3000 quintales cada uno), para que después vayan a las dos secadoras de flujo continuo para su respectivo secado en càscara, la secadora puede secar 1600 quintales cada 12 horas. Luego de este proceso de secado se lo almacena por 7 días para pasar a su respectivo pílado. La píladora es de marca SATAKE (Japonesa), tiene una capacidad de pilado de 120qq por hora y una

capacidad diaria por 10 horas arrojando una produccion de 28800 quintales mensuales.

En la Comercializacion se dedica a la Compra-venta, compra arroz natural a los campesinos del medio y lo almacena para el envejecido natural lo que conlleva a mejorar el precio cuando hay sobreproduccion; o les da servicio de pilado y secado.

Entre los productos que maneja son: pilado natural, envejecido natural segun el tiempo y tiene derivados tales como: ½ arrocillo, ¾ arrocillo, tiza, yelen y polvillo. Dentro del arroz pilado natural la empresa comercializa diferentes variedades como 09, f-11, f-25 y corriente.
Posee dos marcas comerciales siendo estas:
GRAN ARROZ MARAVILLA y
ARROZ CONEJO.
La presentación de sus sacos predominan los colores: naranja, café claro y negro, en el saco Maravilla se envasa el arroz envejecido naturalmente, mientras que en el saco conejo se envasa el arroz pilado natural, los demás derivados del arroz son envasados en sacos blancos sin diseños.

Uno de los principales problemas para nuestro arroz nacional es el ingreso del arroz peruano por contrabando, ya que este tipo de arroz tiene mejores características que el arroz Ecuatoriano, esto se le atribuye al tipo de clima que tiene Perú, un clima de mayor luminosidad óptimo para el proceso de fotosíntesis para la planta, en comparación a nuestro clima.
Por las características del arroz Peruano, un arroz mas brilloso de menos almidón, de menor costo y de mejor presencia los clientes prefieren irse por ese tipo de arroz lo cual a mermado las ventas.

2.3 Instrumento de la Investigación

Usamos el Método Delphi que nos permitió elaborar escenarios futuros, haciendo converger la opinión de un grupo de expertos que nos ayudaron hacer estudios de tendencias y verificaciones con la correlación de los consumidores frente a diversidad de temas que tienen que ver con la industria del consumo de la gramínea.

2.4 Análisis e interpretación de resultados

Una vez realizados el análisis del caso empresarial se interpretan los resultados desde el punto de vista de las valoraciones según los éxitos y fracasos.

3. MARCO TEORICO

Existen diferentes tipos de estrategias, en este trabajo presentaremos una variedad agrupada de la siguiente manera:
Estrategias Intensivas (Crecimiento sostenible).
Estrategias de Diversificacion (abrir hacia nuevos productos).
Estrategias Defensivas (ventaja competitiva en el sector)

Fig 1 : Tipos de Estrategias[11].

3.1 Estrategias de Marketing adoptadas en el Crecimiento Sostenible.

Las teorías que se analizan en el presente proyecto están sustentadas en " la lógica de la mercadotecnia" con la cual la unidad de negocios espera alcanzar sus objetivos, y consiste en definir estrategias específicas para mercados meta, considerando hacer un rediseño profundo del negocio para determinar los planes estrategicos [12].
"El marketing puede producirse en cualquier momento en que una persona o una organización se afane por intercambiar algo de valor con otra persona u organización, en este contexto el marketing consta de actividades ideadas para generar y facilitar intercambios con la intención de facilitar necesidades o deseos de las personas o las organizaciones"[13].

Segun el modelo del analisis de las 5 fuerzas de Michel Porter, las fuerzas competitivas que formulan la estructura basica de un sector y define su rentabilidad son:

Competidores actuales y su intensa rivalidad
Entrada de nuevos competidores
Presion de productos sustitutivos
Experiencia negociadora de los clientes
Experiencia negociadora de los proveedores

- Al no constituirse en un sector de fuerte crecimiento no se deben desarrollar estrategias muy agresivas ya que no se ganara cuota de mercado pero si se permitira corregir errores. Los errores a corregirse saldran del analisis de sus practicas de procedimientos en la oferta y demanda.
- Debe implementarse cambio tecnologico para rejuvenecer el sector y aprovechar el apoyo gubernamental en politicas instauradas en la Matriz Productiva del plan Nacional, con un acercamiento de propuestas regularizadas por planes de negocios formales que sustenten el ROI.
- La experiencia conquistada por el conocimiento del manejo del producto debe ser rescatada en un

plan de mercadeo exaustivo del manejo de la promocion y publicidad, tomando en consideracion los diversos segmentos asi se tendra en cuenta al consumidor minorista tanto como al consumidor mayorista.
- Debe priorizarse el trabajo de los proveedores, ya que de esto depende en gran medida el crecimiento sostenible pues sin las cantidades adecuadas de la produccion de lagraminea se muere la linea del crecimiento y mucho mas el sostenimiento de la industria en la region.

3.2 Estrategias de Marketing adoptada en la Diversificacion.

El ciclo de vida del producto determina la madurez del sector, y en nuestro caso especifico del arroz afecta profundamente a la Gestion.

Tomando el analisis del ciclo de vida del producto en todas sus etapas tenemos:

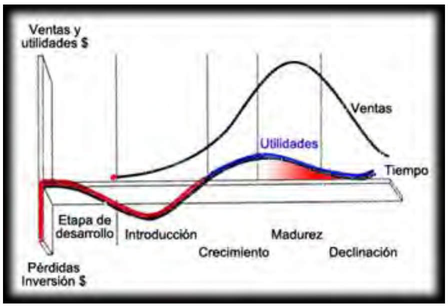

Fig 2 ciclo de vida del sector [14].

Fases ciclo	competencia	Ventas	Estrategia Producto
Introduccion	Entran pocos	Poco Beneficios negativos	Unico
crecimiento	Entran muchos	Aumentan Beneficios positivos	Mejorarlo Ampliar la marca Crear marca
madurez	Gran competencia	Ventas maximas Beneficios altos	Diferenciarlo Nuevos usos Segmentos nuevos
declive	Disminuyen	Vents y beneficios disminuyen	Modificarlo Eliminarlo sustituirlo

Fig 3 Fases de analisis del ciclo de vida

Desde la aparición del marketing, muchas corrientes ideológicas han surgido dando lugar a “Modelos” que pueden ser usados o adaptados a diferentes entornos o circunstancias, dependiendo del producto o servicio que se va a proponer[15].

Hay tres tipos de estrategias de Diversificacion:

- Concentrica: adicionar productos nuevos pero relacionados para nuevos clientes.
- Horizontal: adicionar productos nuevos sin relacionarse y para clientes actuales.
- Conglomerada: es la suma de productos nuevos no relacionados aqui juega papel importante la innovacion.

Nuestra decision es la Diversificacion Concentrica, ya que nos determina aprovechar los subproductos tales como la cascarilla que se usa como combustible y bio combustible y el desperdicio tales como arrozillo grueso y fino los mismos que se convierten en polvillo convirtiendolos en productos de nuevos clientes que se concentran en elaborar balanceados.

Deben ser fabricados bajo una nueva marca que permita expandir el mercado y afianzar la estrategia competitiva.

3.3 Estrategias de Marketing adoptada en la Ventaja Competitiva.

Un mercado de competencia perfecta es aquel que carece de barreras de entrada y salida y cuyos productos están estandarizados, es decir cualquiera puede entrar y competir en el negocio y los compradores adquieren los productos exclusivamente en función del precio[16].

En un mercado con esas caracteristicas no seria posible obtener beneficios a mediano y a largo plazo,sin embargo debido a sus caracteristicas estructurales los mercados presentan algunas imperfecciones (marcas, patentes,regulaciones gubernamentales)

1. Debera trabajarse un plan de Marca, el mismo que debe relacionarse con los colores y el nombre que ya esta posicionado como es el de ARROZ MARAVILLA.

2. Enfocar los desarrollos de nuevos productos a patentarlos para determinar la rentabilidad del crecimiento de marca.

3. Crear un plan exhaustivo de publicidad y promocion el mismo que debe incluir redes sociales y parametros tecnologicos de ultima generacion.

Los parámetros de análisis serán concentradas en el costo – beneficio – factibilidad.

3.4 Priorizacion de las estrategias de Marketing mediante SVN.

La neutrosofía fue propuesta por y Smarandache [17] para el tratamiento de la neutralidades. Esta ha sido la base para una serie de teorías matemáticas que generalizan las teorías clásicas y difusas.

La definición original de valor de verdad en la lógica neutrosófica se muestra a continuación:

sean $N = \{(T, I, F) : T, I, F \subseteq [0, 1]\}$, una valuación neutrosófica es un mapeo de un grupo de fórmulas proposicionales a N , esto es que por cada sentencia p tenemos $v(p) = (T, I, F)$ [18].

Para facilitar la aplicación práctica a problema de la toma de decisiones y de la ingeniería los conjuntos neutrosóficos de valor único fueron propuestos[19] (SVNS por sus siglas en inglés).

Sea X un universo de discurso.

Un SVNS A sobre X es un objeto de la forma.

$A = \{\langle x, u_A(x), r_A(x), v_A(x)\rangle : x \in X\}$ (1)

donde $u_A(x): X \to [0,1]$, $r_A(x), : X \to [0,1]$ y $v_A(x): X \to [0,1]$ con $0 \leq u_A(x) + r_A(x) + v_A(x): \leq 3$ para todo $x \in X$. El intervalo $u_A(x)$, $r_A(x)$ y $v_A(x)$ denotan las mebrecia a verdadero, indeterminado y falso de x en A, respectivamente. Por cuestiones de conveniencia un número SVN será expresado como $A = (a, b, c)$, donde a, b, $c \in [0,1]$, y $+ b + c \leq 3$.

Para evaluar la alternativas proponemos construir la opción ideal[20] y ordenar las alternativas empleando las distancia euclidiana entre números neutrosficos de valor unico (SVN por sus siglas en ingles)[20, 21].

Sea $A^* = (A_1^*, A_2^*, \ldots, A_n^*)$ sea un vector de números SVN tal que $A_j^* = (a_j^*, b_j^*, c_j^*)$ j=(1,2, … , n) y $B_i = (B_{i1}, B_{i2}, \ldots, B_{im})$ (i = 1,2, … , m) sea m vectores de n SVN números tal que y $B_{ij} = (a_{ij}, b_{ij}, c_{ij})$ (i = 1,2, … , m), (j = 1,2, … , n) entonces la distancia euclidiana es definida como. Las B_i y A^* resulta[20]:

$$s_i = \left(\frac{1}{3}\sum_{j=1}^{n}\left\{\left(\left|a_{ij}-a_j^*\right|\right)^2+\left(\left|b_{ij}-b_j^*\right|\right)^2+\left(\left|c_{ij}-c_j^*\right|\right)^2\right\}\right)^{\frac{1}{2}} \quad (2)$$

(i = 1,2, … , m)

En la medida en que la alternativa de $A_i$se encuentra más próximo al punto ideal (s_i menor) mejor será esta, permitiendo establecer un orden entre alternativas [22].

Se emplean los siguientes términos lingüísticos.

Término lingüístico	Números SVN
Extremadamente buena(EB)	(1,0,0)
Muy muy buena (MMB)	(0.9, 0.1, 0.1)
Muy buena (MB)	(0.8,0,15,0.20)
Buena(B)	(0.70,0.25,0.30)
Medianamente buena (MDB)	(0.60,0.35,0.40)
Media(M)	(0.50,0.50,0.50)
Medianamente mala (MDM)	(0.40,0.65,0.60)
Mala (MA)	(0.30,0.75,0.70)
Muy mala (MM)	(0.20,0.85,0.80)
Muy muy mala (MMM)	(0.10,0.90,0.90)
Extremadamente mala (EM)	(0,1,1)

Tabla # 2 Términos lingüísticos empleados. Fuente: [20]

A continuación demuestras los resultados para 3 estrategias

E_1: Crecimiento sostenible.

E_2: Diversificación.

E_3: Ventaja competitiva en el sector.

Los criterios empleados fueron

C_1: Beneficios

C_2: Factibilidad

C_2: Costo

Posteriormente se realiza la valoración para cada estrategia con respecto a los criterios seleccionados (Tabla 4).

Estrategia	Beneficios	Factibilidad	Costo
E1	MB	B	MB
E2	EB	MB	MDB
E3	MDM	M	B

Tabla # 4 Valoración de las estrategias.

La alternativa ideal resulta :

E^+ =(EB, MB, MB)

Los resultados del calculo de las distancia nos pemiten ordenar las estrategias . En este caso el orden de prioridad es el siguiente $E_2 \succ E_1 \succ E_3$, siendo la estrategia de diversificación la mas priorizada.

Estrategia	s_i
E_1	0.21

E_2	0.2
E_3	0.7

Tabla 5. Cálculo de la distancia

Entre las ventajas planteadas por los especialistas se encuentran la relativa facilidad de la técnica. Los resultados muestran además la aplicabilidad que presentan los modelos de ayuda a la toma de decisión basados en SVN al marketing.

4. Conclusiones

En presente trabajo se desarrollaron estrategias de marketing que ayuden a sostener la actividad comercial del arroz proveniente de la zona geográfica de Yaguachi y sus alrededores, planteándonos objetivos alcanzables y justificando el accionar de los modelos conceptuales que sostengan las estrategias a desarrollarse, concluyendo con una propuesta enriquecedora en torno a la nueva forma de manejar la competitividad, diferenciándonos en las habilidades sin descuidar las debilidades que nos ayudara a enriquecer la estrategia de crecimiento y competitividad en el mercado de consumo, recomendando situaciones encontradas no alcanzables ni medibles en el presente estudio.
Las estrategias propuestas fueron analizadas y priorizadas mediante los números SVN y la distancia euclidiana para el tratamiento de la neutralidades. Como trabajos futuros se plantea la incorporacion al metodo de preorización de operadores de agregacion que pemritan expresar importancia y compensación al método. Otras areas de trabajo futuro están en el empleo de la neutrosofía a nuevas áres del marketing.

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Classical Logic and Neutrosophic Logic. Answers to K. Georgiev

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Abstract. In this paper, we make distinctions between Classical Logic (where the propositions are 100% true, or 100 false) and the Neutrosophic Logic (where one deals with partially true, partially indeterminate and partially false propositions) in order to respond to K. Georgiev's criticism [1]. We recall that if an axiom is true in a classical logic system, it is not necessarily that the axiom be valid in a modern (fuzzy, intuitionistic fuzzy, neutrosophic etc.) logic system.

Keywords: Neutrosophic Logic, Neutrosophic Logical Systems, Single Valued Neutrosophic Set, Neutrosophic Logic Negations, Degree of Dependence and Independence, Degrees of Membership, Standard and Non-Standard Real Subsets.

1 Single Valued Neutrosophic Set

We read with interest the paper [1] by K. Georgiev. The author asserts that he proposes "a general simplification of the Neutrosophic Sets a subclass of theirs, comprising of elements of R3", but this was actually done before, since the first world publication on neutrosophics [2]. The simplification that Georgiev considers is called single valued neutrosophc set.

The single valued neutrosophic set was introduced for the first time by us [Smarandache, [2], 1998].

Let

$n = t + i + f$ (1)

In Section 3.7, "Generalizations and Comments", [pp. 129, last edition online], from this book [2], we wrote:

"Hence, the neutrosophic set generalizes:

- the intuitionistic set, which supports incomplete set theories (for $0 < n < 1$*;* $0 \le t, i, f \le 1$*) and incomplete known elements belonging to a set;*

- the fuzzy set (for $n = 1$ *and* $i = 0$*, and* $0 \le t, i, f \le 1$*);*

- the classical set (for $n = 1$ *and* $i = 0$*, with t, f either 0 or 1);*

- the paraconsistent set (for $n > 1$*, with all* $t, i, f < 1$*);*

- the faillibilist set ($i > 0$*);*

- the dialetheist set, a set M whose at least one of its elements also belongs to its complement C(M); thus, the intersection of some disjoint sets is not empty;

- the paradoxist set ($t = f = 1$*);*

- the pseudoparadoxist set ($0 < i < 1$*;* $t = 1$ *and* $f > 0$ *or* $t > 0$ *and* $f = 1$*);*

- the tautological set ($i, f < 0$*)."*

It is clear that we have worked with single-valued neutrosophic sets, we mean that t, i, f were explicitly real numbers from [0, 1].

See also (Smarandache, [3], 2002, p. 426).

More generally, we have considered that: t varies in the set T, i varies in the set I, and f varies in the set F, but in the same way taking crisp numbers n = t + i + f, where all t, i, f are single (crisp) real numbers in the interval [0, 1]. See [2] pp. 123-124, and [4] pp. 418-419.

Similarly, in *The Free Online Dictionary of Computing* [FOLDOC], 1998, updated in 1999, ed. by Denis Howe [3].

Unfortunately, Dr. Georgiev in 2005 took into consideration only the neutrosophic publication [6] from year 2003, and he was not aware of previous publications [2, 3, 4] on the neutrosophics from the years 1998 - 2002.

The misunderstanding was propagated to other authors on neutrosophic set and logic, which have considered that Haibin Wang, Florentin Smarandache, Yanqing Zhang, Rajshekhar Sunderraman (2010, [5]) have defined the single valued neutrosophic set.

2 Standard and Non-Standard Real Subsets

Section 3 of paper [1] by Georgiev is called "Reducing Neutrosophic Sets to Subsets of R3". But this was done already since 1998. In our Section 0.2, [2], p. 12, we wrote:

"Let T, I, F be standard or non-standard real subsets…".

"Standard real subsets", which we talked about above, mean just the classical real subsets.

We have taken into consideration the non-standard analysis in our attempt to be able to describe the absolute truth as well [i.e. truth in all possible worlds, according to Leibniz's denomination, whose neutrosophic value is equal to 1+], and relative truth [i.e. truth in at least one world, whose truth value is equal to 1]. Similarly, for absolute indeterminacy and absolute falsehood.

We tried to get a definition as general as possible for the neutrosophic logic (and neutrosophic set respectively), including the propositions from a philosophical point of [absolute or relative] view.

Of course, in technical and scientific applications we do not consider non-standard things, we take the classical unit interval [0, 1] only, while T, I, F are classical real subsets of it.

In Section 0.2, Definition of Neutrosophic Components [2], 1998, p. 12, we wrote:

"The sets T, I, F are not necessarily intervals, but may be any real sub-unitary subsets: discrete or continuous; single-element, finite, or (countable or uncountable) infinite; union or intersection of various subsets; etc.

They may also overlap. The real subsets could represent the relative errors in determining t, i, f (in the case when the subsets T, I, F are reduced to points)."

So, we have mentioned many possible real values for T, I, F. Such as: each of T, I, F can be "single-element" {as Georgiev proposes in paper [1]}, "interval" {developed later in [7], 2005, and called interval-neutrosophic set and interval-neutrosophic logic respectively}, "discrete" [called hesitant neutrosophic set and hesitant neutrosophic logic respectively] etc.

3 Degrees of Membership > 1 or < 0 of the Elements

In Section 4 of paper [1], Georgiev says that: "Smarandache has adopted Leibniz's 'worlds' in his work, but it seems to be more like a game of words."

As we have explained above, "Leibniz's worlds" are not simply a game of words, but they help making a distinction *in philosophy* between absolute and relative truth / indeterminacy / falsehood respectively. {In technics and science yes they are not needed.}

Besides absolute and relative, the non-standard values or hyper monads (-0 and 1+) have permitted us to introduce, study and show applications of the neutrosophic overset (when there are elements into a set whose real (standard) degree of membership is > 1), neutrosophic underset (when there are elements into a set whose real degree of membership is < 0), and neutrosophic offset (when there are both elements whose real degree of membership is > 1 and other elements whose real degree of membership is < 0). Check the references [8-11].

4 Neutrosophic Logic Negations

In Section 4 of the same paper [1], Georgiev asserts that "according to the neutrosophic operations we have

$$\neg\neg A = A \quad (2)$$

and since

$$\neg\neg A \neq A \quad (3)$$

is just the assumption that has brought intuitionism to life, the neutrosophic logic could not be a generalization of any Intuitionistic logic."

First of all, Georgiev's above assertation is partially true, partially false, and partially indeterminate (as in the neutrosophic logic).

In neutrosophic logic, there is a class of neutrosophic negation operators, not only one. For some neutrosophic negations the equality (2) holds, for others it is invalid, or indeterminate.

Let *A(t, i, f)* be a neutrosophic proposition A whose neutrosophic truth value is *(t, i, f)*, where *t, i, f* are single real numbers of *[0, 1]*. We consider the easiest case.

a) For examples, if the neutrosophic truth value of $\neg A$, the negation of A, is defined as:

(1-t, 1-i, 1-f) or (f, i, t) or (f, 1-i, t) (4)

then the equality (2) is valid.

b) Other examples, if the neutrosophic truth value of $\neg A$, the negation of A, is defined as:

(f, (t+i+f)/3, t) or (1-t, (t+i+f)/3, 1-f) (5)

then the equality (2) is invalid, as in intuitionistic fuzzy logic, and as a consequence the inequality (3) holds.

c) For the future new to be designed/invented neutrosophic negations (needed/adjusted for new applications) we do not know {so (2) has also a percentage of indeterminacy.

5 Degree of Dependence and Independence between (Sub)Components

In Section 4 of [1], Georgiev also asserts that "The neutrosophic logic is not capable of maintaining modal operators, since there is no normalization rule for the components T, I, F". This is also partially true, and partially false.

In our paper [12] about the dependence / independence between components, we wrote that:

> "For single valued neutrosophic logic, the sum of the components t+i+f is:
>
> $0 \le t+i+f \le 3$ when all three components are 100% independent;
>
> $0 \le t+i+f \le 2$ when two components are 100% dependent, while the third one is 100% independent from them;
>
> $0 \le t+i+f \le 1$ when all three components are 100% dependent.
>
> When three or two of the components t, i, f are 100% independent, one leaves room for incomplete information (therefore the sum t+i +f < 1), paraconsistent and contradictory information (t+i+f > 1), or complete information (t+i+f = 1).
>
> If all three components t, i, f are 100% dependent, then similarly one leaves room for incomplete information (t+i+f < 1), or complete information (t+i+f = 1)."

Therefore, for complete information the normalization to 1, 2, 3 or so respectively {see our paper [12] for the case when one has degrees of dependence between components or between subcomponents (for refined neutrosophic set respectively) which are different from 100% or 0%} is done.

But, for incomplete information and paraconsistent information, in general, the normalization is not done.

Neutrosophic logic is capable of maintaining modal operators. The connection between Neutrosophic Logic and Modal Logic will be shown in a separate paper, since it is much longer, called Neutrosophic Modal Logic (under press).

6 Definition of Neutrosophic Logic

In Section 5, paper [1], it is said: "Apparently there isn't a clear definition of truth value of the neutrosophic formulas." The author is right that "apparently", but in reality the definition of neutrosophic logic is very simple and common sense:

In neutrosophic logic a proposition P has a degree of truth (T); a degree of indeterminacy (I) that means neither true nor false, or both true and false, or unknown, indeterminate; and a degree of falsehood (F); where T, I, F are subsets (either real numbers, or intervals, or any subsets) of the interval [0, 1].

What is unclear herein?

In a soccer game, as an easy example, between two teams, Bulgaria and Romania, there is a degree of truth about Bulgaria winning, degree of indeterminacy (or neutrality) of tie game, and degree of falsehood about Bulgaria being defeated.

7 Neutrosophic Logical Systems

a) Next sentence of Georgiev is

"in every meaningful logical system if A and B are sets (formulas) such that A ⊆ B then B ‘ A, i.e. when B is true then A is true." (6)

In other words, when B → A (B implies A), and B is true, then A is true.

This is true for the Boolean logic where one deals with 100% truths, but in modern logics we work with partial truths.

If an axiom is true in the classical logic, it does not mean that that axiom has to be true in the modern logical system. Such counter-example has been provided by Georgiev himself, who pointed out that the law of double negation {equation (2)}, which is valid in the classical logic, is not valid any longer in intuitionistic fuzzy logic.

A similar response we have with respect to his above statement on the logical system axiom (6): it is partially true, partially false, and partially indeterminate. All depend on the types of chosen neutrosophic implication operators.

In neutrosophic logic, let's consider the neutrosophic propositions $A(t_A, i_A, f_A)$ and $B(t_B, i_B, f_B)$,

and the neutrosophic implication:

$B(t_B, i_B, f_B) \rightarrow A(t_A, i_A, f_A)$, (7)

that has the neutrosophic truth value

$(B \rightarrow A)(t_{B\rightarrow A}, i_{B\rightarrow A}, f_{B\rightarrow A})$. (8)

Again, we have a class of many neutrosophic implication operators, not only one; see our publication [13], 2015, pp. 79-81.

Let's consider one such neutrosophic implication for single valued neutrosophic logic:

$(B \rightarrow A)(t_{B\rightarrow A}, i_{B\rightarrow A}, f_{B\rightarrow A})$ is equivalent to $B(t_B, i_B, f_B) \rightarrow A(t_A, i_A, f_A)$

which is equivalent to $\neg B(f_B, 1-i_B, t_B) \vee A(t_A, i_A, f_A)$

which is equivalent to $(\neg B \vee A)(max\{f_B, t_A\}, min\{1-i_B, i_A\}, min\{t_B, f_A\})$. (9)

Or:

$(t_{B\rightarrow A}, i_{B\rightarrow A}, f_{B\rightarrow A}) = (max\{f_B, t_A\}, min\{1-i_B, i_A\}, min\{t_B, f_A\})$. (10)

Now, a question arises: what does "(B →) A is true" mean in fuzzy logic, intuitionistic fuzzy logic, and respectively in neutrosophic logic?

Similarly for the "B is true", what does it mean in these modern logics? Since in these logics we have infinitely many truth values t(B) ∈ (0, 1); {we made abstraction of the truth values 0 and 1, which represent the classical logic}.

b) Theorem 1, by Georgiev, "Either A H k(A) [i.e. A is true if and only if k(A) is true] or the neutrosophic logic is contradictory."

We prove that his theorem is a nonsense.

First at all, the author forgets that when he talks about neutrosophic logic he is referring to a modern logic, not to the classical (Boolean) logic. The logical propositions in neutrosophic logic are partially true, in the form of (t, i, f), not totally 100% true or (1, 0, 0). Similarly for the implications and equivalences, they are not classical (i.e. 100% true), but partially true {i.e. their neutrosophic truth values are in the form of (t, i, f) too}.

- The author starts using the previous classical logical system axiom (6), i.e.

"since *k(A)* ⊆ A we have A ‘ *k(A)* " meaning that *A* → k(A) and when A is true, then k(A) is true.

- Next Georgiev's sentence: "Let assume k(A) be true and assume that A is not true".

The same comments as above:

What does it mean in fuzzy logic, intuitionistic fuzzy logic, and neutrosophic logic that a proposition is true? Since in these modern logics we have infinitely many values for the truth value of a given proposition. Does, for example, t(*k(A))* = 0.8 {i.e. the truth value of *k(A)* is equal to 0.8}, mean that *k(A)* is true?

If one takes t(*k(A))* = 1, then one falls in the classical logic.

Similarly, what does it mean that proposition A is not true? Does it mean that its truth value

t(A) = 0.1 or in general t(A) < 1 ? Since, if one takes t(A) = 0, then again we fall into the classical logic.

The author confuses the classical logic with modern logics.

- In his "proof" he states that "since the Neutrosophic logic is not an intuitionistic one, $\neg A$ should be true leading to the conclusion that $k(\neg A) = \neg k(A)$ is true".

For the author an "intuitionistic logic" means a logic that invalidates the double negation law {equation (3)}. But we have proved before in Section 4, of this paper, that depending on the type of neutrosophic negation operator used, one has cases when neutrosophic logic invalidates the double negation law [hence it is "intuitionistic" in his words], cases when the neutrosophic logic does not invalidate the double negation law {formula (2)}, and indeterminate cases {depending on the new possible neutrosophic negation operators to be design in the future}.

- The author continues with "We found that $k(A) \wedge \neg k(A)$ is true which means that the simplified neutrosophic logic is contradictory."

Georgiev messes up the classical logic with modern logic. In classical logic, indeed

$k(A) \wedge \neg k(A)$ is false, being a contradiction.

But we are surprised that Georgiev does not know that in modern logic we may have

$k(A) \wedge \neg k(A)$ that is not contradictory, but partially true and partially false.

For example, in fuzzy logic, let's say that the truth value (t) of k(A) is

$t(k(A)) = 0.4$, then the truth value of its negation, $\neg k(A)$, is $t(\neg k(A)) = 1 - 0.4 = 0.6$.

Now, we apply the t-norm "min" in order to do the fuzzy conjunction, and we obtain:

$t(k(A) \wedge \neg k(A)) = \min\{0.4, 0.6\} = 0.4 \neq 0$.

Hence, $k(A) \wedge \neg k(A)$ is not a contradiction, since its truth value is 0.4, not 0. Similarly in intuitionistic fuzzy logic. The same in neutrosophic logic, for example:

Let the neutrosophic truth value of k(A) be (0.5, 0.4, 0.2), that we denote as:

$k(A)(0.5, 0.4, 0.2)$, then its negation $\neg k(A)$ will have the neutrosophic truth value:

$\neg k(A)(0.2, 1\text{-}0.4, 0.5) = \neg k(A)(0.2, 0.6, 0.5)$.

Let's do now the neutrosophic conjunction:

$k(A)(0.5, 0.4, 0.2) \wedge \neg k(A)(0.2, 0.6, 0.5) = (k(A) \wedge \neg k(A))(\min\{0.5, 0.2\}, \max\{0.4, 0.6\}, \max\{0.2, 0.5\}) = (k(A) \wedge \neg k(A))(0.2, 0.6, 0.5)$.

In the same way, $k(A) \wedge \neg k(A)$ is not a contradiction in neutrosophic logic, since its neutrosophic truth value is (0.2, 0.6, 0.5), which is different from (0, 0, 1) or from (0, 1, 1). Therefore, Georgiev's "proof" that the simplified neutrosophic logic [= single valued neutrosophic logic] is a contradiction has been disproved!

His following sentence, "But since the simplified neutrosophic logic is only a subclass of the neutrosophic logic, then the neutrosophic logic is a contradiction" is false. Simplified neutrosophic logic is indeed a subclass of the neutrosophic logic, but he did not prove that the so-called simplified neutrosophic logic is contradictory (we have showed above that his "proof" was wrong).

Conclusion

We have shown in this paper that Georgiev's critics on the neutrosophic logic are not founded. We made distinctions between the Boolean logic systems and the neutrosophic logic systems.

Neutrosophic logic is developing as a separate entity with its specific neutrosophic logical systems, neutrosophic proof theory and their applications.

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Regular Bipolar Single Valued Neutrosophic Hypergraphs

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Abstract. In this paper, we define the regular and totally regular bipolar single valued neutrosophic hypergraphs, and discuss the order and size along with properties of regular and totally regular bipolar single valued neutrosophic hypergraphs. We extend work on completeness of bipolar single valued neutrosophic hypergraphs.

Keywords: bipolar single valued neutrosophic hypergraphs, regular bipolar single valued neutrosophic hypergraphs and totally regular bipolar single valued neutrosophic hyper graphs.

1 Introduction

The notion of neutrosophic sets (NSs) was proposed by Smarandache [8] as a generalization of the fuzzy sets [14], intuitionistic fuzzy sets [12], interval valued fuzzy set [11] and interval-valued intuitionistic fuzzy sets [13] theories. The neutrosophic set is a powerful mathematical tool for dealing with incomplete, indeterminate and inconsistent information in real world. The neutrosophic sets are characterized by a truth-membership function (*t*), an indeterminacy-membership function (*i*) and a falsity membership function (*f*) independently, which are within the real standard or nonstandard unit interval $]^-0, 1^+[$. In order to conveniently use NS in real life applications, Wang et al. [9] introduced the concept of the single-valued neutrosophic set (SVNS), a subclass of the neutrosophic sets. The same authors [10] introduced the concept of the interval valued neutrosophic set (IVNS), which is more precise and flexible than the single valued neutrosophic set. The IVNS is a generalization of the single valued neutrosophic set, in which the three membership functions are independent and their value belong to the unit interval [0, 1]. More works on single valued neutrosophic sets, interval valued neutrosophic sets and their applications can be found on *http://fs.gallup.unm.edu/NSS/*.

Hypergraph is a graph in which an edge can connect more than two vertices, hypergraphs can be applied to analyse architecture structures and to represent system partitions, Mordesen J.N and P.S Nasir gave the definitions for fuzzy hypergraphs. Parvathy. R and M. G. Karunambigai's paper introduced the concepts of Intuitionistic fuzzy hypergraphs and analyse its components, Nagoor Gani. A and Sajith Begum. S defined degree, order and size in intuitionistic fuzzy graphs and extend the properties. Nagoor Gani. A and Latha. R introduced irregular fuzzy graphs and discussed some of its properties.

Regular intuitionistic fuzzy hypergraphs and totally regular intuitionistic fuzzy hypergraphs are introduced by Pradeepa. I and Vimala. S in [0]. In this paper we extend regularity and totally regularity on bipolar single valued neutrosophic hypergraphs.

2 Preliminaries

In this section we discuss the basic concept on neutrosophic set and neutrosophic hyper graphs.

Definition 2.1 Let X be the space of points (objects) with generic elements in X denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by truth membership function $T_A(x)$, indeterminacy membership function $I_A(x)$ and a falsity membership function $F_A(x)$. For each point x $\in$X; $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$.

Definition 2.2 Let X be a space of points (objects) with generic elements in X denoted by x. A bipolar single valued neutrosophic set A (BSVNS A) is characterized by positive truth membership function $PT_A(x)$, positive indeterminacy membership function $PI_A(x)$ and a positive falsity membership function $PF_A(x)$ and negative truth membership function $NT_A(x)$, negative indeterminacy membership function $NI_A(x)$ and a negative falsity membership function $NF_A(x)$.

For each point $x \in$ X; $PT_A(x)$, $PI_A(x)$, $PF_A(x) \in [0, 1]$ and $NT_A(x)$, $NI_A(x)$, $NF_A(x) \in [-1, 0]$.

Definition 2.3 Let A be a BSVNS on X then support of A is denoted and defined by

$Supp(A) = \{x : x \in X, PT_A(x) > 0, PI_A(x) > 0, PF_A(x) > 0, NT_A(x) < 0, NI_A(x) < 0, NF_A(x) < 0\}$.

Definition 2.4 A hyper graph is an ordered pair $H = (X, E)$, where

(1) $X = \{x_1, x_2, \dots, x_n\}$ be a finite set of vertices.

(2) $E = \{E_1, E_2, \dots, E_m\}$ *be* a family of subsets of

X. (3) E_j for $j= 1,2,3,...,m$ and $\bigcup_j (E_j)= X$.

The set X is called set of vertices and E is the set of edges (or hyper edges).

Definition 2.5 A bipolar single valued neutrosophic hypergraph is an ordered pair $H = (X, E)$, where

(1) $X = \{x_1, x_2, \dots, x_m\}$ be a finite set of vertices.

(2) $E = \{E_1, E_2, \dots, E_m\}$ be a family of BSVNSs of X.

(3) $E_j \neq O = (0, 0, 0)$ for $j= 1,2,3,...,m$ and $\bigcup_j Supp(E_j)= X$.

The set X is called set of vertices and E is the set of BSVN-edges (or BSVN-hyper edges).

Proposition 2.6 The bipolar single valued neutrosophic hyper graph is the generalization of fuzzy hyper graphs, intuitionistic fuzzy hyper graphs, bipolar fuzzy hyper graphs and single valued neutrosophic hypergraphs.

3 Regular and totally regular BSVNHGs

Definition 3.1 The open neighbourhood of a vertex x in bipolar single valued neutrosophic hypergraphs (BSVNHGs) is the set of adjacent vertices of x, excluding that vertex and is denoted by $N(x)$.

Definition 3.2 The closed neighbourhood of a vertex x in bipolar single valued neutrosophic hypergraphs (BSVNHGs) is the set of adjacent vertices of x, including that vertex and is denoted by $N[x]$.

Example 3.3 Consider a bipolar single valued neutrosophic hypergraphs $H = (X, E)$ where, $X = \{a, b, c, d, e\}$ and $E = \{P, Q, R, S\}$, which is defined by

P = {(a, 0.1, 0.2, 0.3, -0.4, -0.6 -0.8), (b, 0.4, 0.5, 0.6, -0.4, -0.6 -0.8)}

Q = {(c, 0.1, 0.2, 0.3, -0.4, -0.4 -0.9), (d, 0.4, .5, 0.6, -0.3, -0.5 -0.6), (e, 0.7, 0.8, 0.9, -0.7, -0.9, -0.2)}

R = {(b, 0.1, 0.2, 0.3, -0.2, -0.5, -0.8), (c, 0.4, 0.5, 0.6, -0.9, -0.7 -0.4)}

S = {(a, 0.1, 0.2, 0.3, -0.7, -0.6, -0.9), (d, 0.9, 0.7, 0.6, -0.4, -0.7, -0.9)}

Then the open neighbourhood of a vertex a is the b and d, and closed neighbourhood of a vertex b is b, a and c.

Definition 3.4 Let $H = (X, E)$ be a BSVNHG, the open neighbourhood degree of a vertex x, which is denoted and defined by

$$deg(x) = (deg_{PT}(x), deg_{PI}(x), deg_{PF}(x), deg_{NT}(x), deg_{NI}(x), deg_{NF}(x))$$

where

$$deg_{PT}(x) = \sum_{x \in N(x)} PT_E(x)$$

$$deg_{PI}(x) = \sum_{x \in N(x)} PI_E(x)$$

$$deg_{PF}(x) = \sum_{x \in N(x)} PF_E(x)$$

$$deg_{NT}(x) = \sum_{x \in N(x)} NT_E(x)$$

$$deg_{NI}(x) = \sum_{x \in N(x)} NI_E(x)$$

$$deg_{NF}(x) = \sum_{x \in N(x)} NF_E(x)$$

Example 3.5 Consider a bipolar single valued neutrosophic hypergraphs $H = (X, E)$ where, $X = \{a, b, c, d, e\}$ and $E = \{P, Q, R, S\}$, which are defined by

P = {(a, .1, .2, .3, -0.1, -0.2, -0.3), (b, .4, .5, .6, -0.1, -0.2, -0.3)}

Q = {(c, .1, .2, .3, -0.1, -0.2, -0.3), (d, .4, .5, .6, -0.1, -0.2, -0.3), (e, .7, .8, .9, -0.1, -0.2, -0.3)}

R = {(b, .1, .2, .3, -0.1, -0.2, -0.3), (c, .4, .5, .6, -0.1, -0.2, -0.3)}

S = {(a, .1, .2, .3, -0.1, -0.2, -0.3), (d, .4, .5, .6, -0.1, -0.2, -0.3)}

Then the open neighbourhood of a vertex a contain b and d and therefore open neighbourhood degree of a vertex a is (.8, 1, 1.2, -0.2, -0.4, -0.6).

Definition 3.6 Let $H = (X, E)$ be a BSVNHG, the closed neighbourhood degree of a vertex x is denoted and defined by

$deg[x] = (deg_{PT}[x], deg_{PI}[x], deg_{PF}[x], deg_{NT}[x], deg_{NI}[x], deg_{NF}[x])$

which are defined by

$$deg_{PT}[x] = deg_{PT}(x) + PT_E(x)$$
$$deg_{PI}[x] = deg_{PI}(x) + PI_E(x)$$
$$deg_{PF}[x] = deg_{PF}(x) + PF_E(x)$$
$$deg_{NT}[x] = deg_{NT}(x) + NT_E(x)$$
$$deg_{NI}[x] = deg_{NI}(x) + NI_E(x)$$
$$deg_{NF}[x] = deg_{NF}(x) + NF_E(x)$$

Example 3.7 Consider a bipolar single valued neutrosophic hypergraphs *H = (X, E)* where, *X = {a, b, c, d, e}* and *E = {P, Q, R, S}*, which is defined by

P = {(a, 0.1, 0.2, 0.3, -0.1, -0.2, -0.3), (b, 0.4, 0.5, 0.6, -0.1, -0.2, -0.3)}

Q = {(c, 0.1, 0.2, 0.3, -0.1, -0.2, -0.3), (d, 0.4, 0.5, 0.6, -0.1, -0.2, -0.3), (e, 0.7, 0.8, 0.9, -0.1, -0.2, -0.3)}

R = {(b, 0.1, 0.2, 0.3, -0.1, -0.2, -0.3), (c, 0.4, 0.5, 0.6, -0.1, -0.2, -0.3)}

S = {(a, 0.1, 0.2, 0.3, -0.1, -0.2, -0.3), (d, 0.4, 0.5, 0.6, -0.1, -0.2, -0.3)}

The closed neighbourhood of a vertex *a* contain *a*, *b* and *d*, hence the closed neighbourhood degree of a vertex *a* is (0.9, .1.2, 1.5, -0.3, -0.6, -0.9).

Definition 3.8 Let *H = (X, E)* be a BSVNHG, then *H* is said to be an *n*-regular BSVNHG if all the vertices have the same open neighbourhood degree $n = (n_1, n_2, n_3, n_4, n_5, n_6)$

Definition 3.9 Let *H = (X, E)* be a BSVNHG, then *H* is said to be *m*-totally regular BSVNHG if all the vertices have the same closed neighbourhood degree $m = (m_1, m_2, m_3, m_4, m_5, m_6)$.

Proposition 3.10 A regular BSVNHG is the generalization of regular fuzzy hypergraphs, regular intuitionistic fuzzy hypergraphs, regular bipolar fuzzy hypergraphs and regular single valued neutrosophic hypergraphs.

Proposition 3.11 A totally regular BSVNHG is the generali-zation of totally regular fuzzy hypergraphs, totally regular intuitionistic fuzzy hypergraphs, totally regular bipolar fuzzy hypergraphs and totally regular single valued neu-trosophic hypergraphs.

Example 3.12 Consider a bipolar single valued neutrosophic hypergraphs *H = (X, E)* where, *X = {a, b, c, d}* and *E = {P, Q, R, S}* which is defined by

P = {(a, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3), (b, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3)}

Q = {(b, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3), (c, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3)}

R = {(c, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3), (d, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3)}

S = {(d, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3), (a, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3)}

Here the open neighbourhood degree of every vertex is *(1.6, 0.4, 0.6, -0.2, -0.4, -0.6)* hence *H* is regular BSVNHG and closed neighbourhood degree of every vertex is *(2.4, 0.6, 0.9, -0.3, -0.6, -0.9)*, Hence *H* is both regular and totally regular BSVNHG.

Theorem 3.13 Let *H = (X, E)* be a BSVNHG which is both regular and totally regular BSVNHG then *E* is constant.

Proof: Suppose *H* is an *n*-regular and *m*-totally regular BSVNHG. Then *deg(x) = n* = $(n_1, n_2, n_3, n_4, n_5, n_6)$ and *deg[x]* = *m* = $(m_1, m_2, m_3, m_4, m_5, m_6)$ $\forall x \in E_i$. Consider *deg[x] = m*. Hence by definition, *deg(x)* + $E_i(x)$ = m this implies $E_i(x) = m - n$ for all $x \in E_i$. Hence *E* is constant.

Remark 3.14 The converse of above theorem need not to be true in general.

Example 3.15 Consider a bipolar single valued neutrosophic hypergraphs *H = (X, E)* where, *X = {a, b, c, d}* and *E = {P, Q, R, S}*, which is defined by

P = {(a, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3), (b, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3)}

Q = {(b, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3), (d, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3)}

R = {(c, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3), (d, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3)}

S = {(d, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3), (a, 0.8, 0.2, 0.3, -0.1, -0.2, -0.3)}

Here *E* is constant but *deg(a) = (1.6, 0.4, 0.6, -0.2, -0.4, -0.6)* and *deg(d) = (2.4, 0.6, 0.9, -0.3, -0.6, -0.9)* i.e deg(a) and deg(d) are not equals hence *H* is not regular BSVNHG. Next *deg[a] = (2.4, 0.6, 0.9, -0.3, -0.6, -0.9)* and *deg[d]= (3.2, 0.8, 1.2, -.4, -0.8, -1.2)*, hence deg[a] and deg[d] are not equals hence *H* is not totally regular BSVNHG, Thus that *H* is neither regular and nor totally regular BSVNHG.

Theorem 3.16 Let *H = (X, E)* be a BSVNHG then *E* is constant on *X* if and only if following are equivalent,

(1) *H* is regular BSVNHG.

(2) *H* is totally regular BSVNHG.

Proof: Suppose *H = (X, E)* be a BSVNHG and *E* is constant in *H*, that is $E_i(x) = c = (c_1, c_2, c_3, c_4, c_5, c_6)\ \forall x \in E_i$.

Suppose H is n-regular BSVNHG, then $deg(x) = n = (n_1, n_2, n_3, n_4, n_5, n_6)$ $\forall x \in E_i$, consider $deg[x] = deg(x) + E_i(x) = n + c$ $\forall x \in E_i$, hence H is totally regular BSVNHG.

Next suppose that H is m-totally regular BSVNHG, then $deg[x] = m = (m_1, m_2, m_3, m_4, m_5, m_6)$ for all $x \in E_i$, that is $deg(x) + E_i(x) = m$ $\forall x \in E_i$, this implies that $deg(x) = m - c$ $\forall x \in E_i$. Thus H is regular BSVNHG, thus *(1)* and *(2)* are equivalent.

Conversely: Assume that *(1)* and *(2)* are equivalent. That is H is regular BSVNHG if and only if H is totally regular BSVNHG. Suppose contrary E is not constant, that is E_i(x) and E_i(y) not equals for some x and y in X. Let $H = (X, E)$ be n-regular BSVNHG, then $deg(x) = n = (n_1, n_2, n_3, n_4, n_5, n_6)$ for all x $\in E_i$. Consider

$$deg[x] = deg(x) + E_i(x) = n + E_i(x)$$

$$deg[y] = deg(y) + E_i(y) = n + E_i(y)$$

Since E_i(x) and E_i(y) are not equals for some x and y in X. Hence $deg[x]$ and $deg[y]$ are not equals, thus H is not totally regular BSVNHG, which contradict to our assumption.

Next let H be totally regular BSVNHG, then $deg[x] = deg[y]$, that is $deg(x) + E_i(x) = deg(y) + E_i(y)$ and $deg(x) - deg(y) = E_i(y) - E_i(x)$, since RHS of last equation is nonzero, hence LHS of above equation is also nonzero, thus $deg(x)$ and $deg(y)$ are not equals, so H is not regular BSVNHG, which is again contradict to our assumption, thus our supposition was wrong, hence E must be constant, this completes the proof.

Definition 3.17 Let $H = (X, E)$ be a regular BSVNHG, then the order of BSVNHG H is denoted and defined by

$O(H) = (p, q, r, s, t, u)$, where $p = \sum_{x \in X} PT_{E_i}(x)$, $q = \sum_{x \in X} PI_{E_i}(x)$, $r = \sum_{x \in X} PF_{E_i}(x)$, $s = \sum_{x \in X} NT_{E_i}(x)$, $t = \sum_{x \in X} NI_{E_i}(x)$, $u = \sum_{x \in X} NF_{E_i}(x)$. For every $x \in X$ and size of regular BSVNHG is denoted and defined by $S(H) = \sum_{i=1}^{n}(S_{E_i})$, where $S(E_i) = (a, b, c, d, e, f)$ which is defined by

$$a = \sum_{x \in E_i} PT_{E_i}(x)$$

$$b = \sum_{x \in E_i} PI_{E_i}(x)$$

$$c = \sum_{x \in E_i} PF_{E_i}(x)$$

$$d = \sum_{x \in E_i} NT_{E_i}(x)$$

$$e = \sum_{x \in E_i} NI_{E_i}(x)$$

$$f = \sum_{x \in E_i} NF_{E_i}(x)$$

Example 3.18 Consider a bipolar single valued neutrosophic hypergraphs $H = (X, E)$ where, $X = \{a, b, c, d\}$ and

$E = \{P, Q, R, S\}$, which is defined by

P = {(a, .8, .2, .3, -.1, -.2, -.3), (b, .8, .2, .3, -.1, -.2, -.3)}

Q = {(b, .8, .2, .3, -.1, -.2, -.3), (c, .8, .2, .3, -.1, -.2, -.3)}

R = {(c, .8, .2, .3, -.1, -.2, -.3), (d, .8, .2, .3, -.1, -.2, -.3)}

S = {(d, .8, .2, .3, -.1, -.2, -.3), (a, .8, .2, .3, -.1, -.2, -.3)}

Here order and size of H are given (3.2, .8, 1.2, -.4, -.8, -1.2) and (6.4, 1.6, 2.4, -.8, -1.6, -2.4) respectively.

Proposition 3.19 The size of an n-regular BSVNHG $H = (H, E)$ is $nk/2$, where $|X| = k$.

Proposition 3.20 If $H = (X, E)$ be m-totally regular BSVNHG then $2S(H) + O(H) = mk$, where $|X| = k$.

Corollary 3.21 Let $H = (X, E)$ be a n-regular and m-totally regular BSVNHG then $O(H) = k(m - n)$, where $|X| = k$.

Proposition 3.22 The dual of n-regular and m-totally regular BSVNHG $H = (X, E)$ is again an n-regular and m-totally regular BSVNHG.

Definition 3.23 A bipolar single valued neutrosophic hypergraph (BSVNHG) is said to be complete BSVNHG if for every x in X, $N(x) = \{x: x \text{ in } X-\{x\}\}$, that is $N(x)$ contains all remaining vertices of X except x.

Example 3.24 Consider a bipolar single valued neutrosophic hypergraphs $H = (X, E)$, where $X = \{a, b, c, d\}$ and $E = \{P, Q, R\}$, which is defined by

P = {(a, 0.4, 0.6, 0.3, -0.5, -0.2, -0.3), (c, 0.8, 0.2, 0.3, -0.1, -0.8, -0.3)}

Q = {(a, 0.8, 0.8, 0.3, -0.1, -0.6, -0.3), (b, 0.8, 0.2, 0.1, -0.1, -0.2, -0.3), (d, 0.8, 0.2, 0.1, -0.1, -0.9, -0.3)}

R = {(c, 0.4, 0.9, 0.9, -0.1, -0.2, -0.3), (d, 0.7, 0.2, 0.1, -0.5, -0.9, -0.3), (b, *0.4, 0.2, 0.1, -0.8, -0.4, -0.2)}*. Here *N(a) = {b, c, d}* , *N(b) = {a, c, d}*, *N(c) = {a, b, d}*, *N(d) = {a, b, c}* hence *H* is complete BSVNHG.

Remark 3.25 In a complete BSVNHG *H = (X, E),* the cardi-nality of *N(x)* is same for every vertex.

Theorem 3.26 Every complete BSVNHG *H = (X, E)* is both regular and totally regular if *E* is constant in *H*.

Proof: Let *H = (X, E)* be complete BSVNHG, suppose *E* is constant in *H*, so that $E_i(x) = c = (c_1, c_2, c_3, c_4, c_5, c_6)$ $\forall x \in E_i$, since BSVNHG is complete, then by definition for every vertex *x* in *X*, *N(x) = {x: x in X-{x}}*, the open neighbourhood degree of every vertex is same. That is *deg(x) = n* = $(n_1, n_2, n_3, n_4, n_5, n_6)$ $\forall x \in E_i$. Hence complete BSVNHG is regular BSVNHG. Also, *deg[x] = deg(x) +* $E_i(x)$ *= n + c* $\forall x \in E_i$. Hence *H* is totally regular BSVNHG.

Remark 3.27 Every complete BSVNHG is totally regular even if *E* is not constant.

Definition 3.28 A BSVNHG is said to be *k*-uniform if all the hyper edges have same cardinality.

Example 3.29 Consider a bipolar single valued neutrosophic hypergraphs *H = (X, E),* where *X = {a, b, c, d}* and *E = {P, Q, R},* which is defined by

P = {(a, 0.8, 0.4, 0.2,-0.4, -0.6, -0.2), (b, 0.7, 0.5, 0.3, -0.7, -0.1, -0.2)}

Q = {(b, 0.9, 0.4, 0.8, -0.3, -0.2, -0.9), (c, 0.8, 0.4, 0.2, -0.4, -0.3, -0.7)}

R = {(c, 0.8, 0.6, 0.4, -0.3, -0.7, -0.2), (d, 0.8, 0.9, 0.5, -0.4, -0.8, -0.9)}

4 Conclusion

Theoretical concepts of graphs and hypergraphs are utilized by computer science applications. Single valued neutrosophic hypergraphs are more flexible than fuzzy hypergraphs and intuitionistic fuzzy hypergraphs. The concepts of single valued neutrosophic hypergraphs can be applied in various areas of engineering and computer science. In this paper, we defined the regular and totally regular bipolar single valued neutrosophic hyper graphs. We plan to extend our research work to irregular and totally irregular on bipolar single valued neutrosophic hyper graphs.

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Neutrosophic Topology

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Abstract: In this paper, we redefine the neutrosophic set operations and, by using them, we introduce neutrosophic topology and investigate some related properties such as neutrosophic closure, neutrosophic interior, neutrosophic exterior, neutrosophic boundary and neutrosophic subspace.

Keywords: Neutrosophic set, neutrosophic topological space, neutrosophic open set, neutrosophic closed set, neutrosophic interior, neutrosophic exterior, neutrosophic boundary and neutrosophic subspace.

1 Introduction

The concept of neutrosophic sets was first introduced by Smarandache [13, 14] as a generalization of intuitionistic fuzzy sets [1] where we have the degree of membership, the degree of indeterminacy and the degree of non-membership of each element in X. After the introduction of the neutrosophic sets, neutrosophic set operations have been investigated. Many researchers have studied topology on neutrosophic sets, such as Smarandache [14] Lupianez [7–10] and Salama [12]. Various topologies have been defined on the neutrosophic sets. For some of them the De Morgan's Laws were not valid.

Thus, in this study, we redefine the neutrosophic set operations and investigate some properties related to these definitions. Also, we introduce for the first time the neutrosophic interior, neutrosophic closure, neutrosophic exterior, neutrosophic boundary and neutrosophic subspace. In this paper, we propose to define basic topological structures on neutrosophic sets, such that interior, closure, exterior, boundary and subspace.

2 Preliminaries

In this section, we will recall the notions of neutrosophic sets [13]. Moreover, we will give a new approach to neutrosophic set operations.

Definition 1 *[13] A neutrosophic set A on the universe of discourse X is defined as*

$$A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$$

where $\mu_A, \sigma_A, \gamma_A : X \to]^-0, 1+[$ and $^-0 \leq \mu_A(x) + \sigma_A(x) + \gamma_A(x) \leq 3^+$ From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0, 1^+[$. But in real life application in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]^-0, 1^+[$. Hence we consider the neutrosophic set which takes the value from the subset of $[0, 1]$. Set of all neutrosophic set over X is denoted by $\mathcal{N}(X)$.

Definition 2 *Let $A, B \in \mathcal{N}(X)$. Then,*

i. *(Inclusion) If $\mu_A(x) \leq \mu_B(x)$, $\sigma_A(x) \geq \sigma_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$, then A is neutrosophic subset of B and denoted by $A \sqsubseteq B$. (Or we can say that B is a neutrosophic super set of A.)*

ii. *(Equality) If $A \sqsubseteq B$ and $B \sqsubseteq A$, then $A = B$.*

iii. *(Intersection) Neutrosophic intersection of A and B, denoted by $A \sqcap B$, and defined by*

$$A \sqcap B = \Big\{\langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \nu_A(x) \vee \nu_B(x)\rangle : x \in X\Big\}.$$

iv. *(Union) Neutrosophic union of A and B, denoted by $A \sqcup B$, and defined by*

$$A \sqcup B = \Big\{\langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \nu_A(x) \wedge \nu_B(x)\rangle : x \in X\Big\}.$$

v. *(Complement) Neutrosophic complement of A is denoted by A^c and defined by*

$$A^c = \Big\{\langle x, \nu_A(x), 1 - \sigma_A(x), \mu_A(x)\rangle : \ x \in X\Big\}.$$

vi. *(Universal Set) If $\mu_A(x) = 1$, $\sigma_A(x) = 0$ and $\nu_A(x) = 0$ for all $x \in X$, A is said to be neutrosophic universal set, denoted by $\tilde{X}$.*

vii. *(Empty Set) If $\mu_A(x) = 0$, $\sigma_A(x) = 1$ and $\nu_A(x) = 1$ for all $x \in X$, A is said to be neutrosophic empty set, denoted by $\tilde{\emptyset}$.*

Remark 3 *According to Definition 2, $\tilde{X}$ should contain complete knowledge. Hence, its indeterminacy degree and non-membership degree are 0 and its membership degree is 1. Similarly, $\tilde{\emptyset}$ should contain complete uncertainty. So, its indeterminacy degree and non-membership degree are 1 and its membership degree is 0.*

Example 4 *Let $X = \{x, y\}$ and $A, B, C \in \mathcal{N}(X)$ such that*

$$\begin{aligned} A &= \{\langle x, 0.1, 0.4, 0.3\rangle, \langle y, 0.5, 0.7, 0.6\rangle\} \\ B &= \{\langle x, 0.9, 0.2, 0.3\rangle, \langle y, 0.6, 0.4, 0.5\rangle\} \\ C &= \{\langle x, 0.5, 0.1, 0.4\rangle, \langle y, 0.4, 0.3, 0.8\rangle\}. \end{aligned}$$

Then,

i. *We have that $A \sqsubseteq B$.*

ii. *Neurosophic union of B and C is*

$$\begin{aligned} B \sqcup C &= \Big\{\Big\langle x, (0.9 \vee 0.5), (0.2 \wedge 0.1), (0.3 \wedge 0.4)\Big\rangle, \\ &\quad \Big\langle y, (0.6 \vee 0.4), (0.4 \wedge 0.3), (0.5 \wedge 0.8)\Big\rangle\Big\} \\ &= \Big\{\langle x, 0.9, 0.1, 0.3\rangle, \langle y, 0.6, 0.3, 0.5\rangle\Big\}. \end{aligned}$$

iii. *Neurosophic intersection of A and C is*

$$\begin{aligned} A \sqcap C &= \Big\{\Big\langle x, (0.1 \wedge 0.5), (0.4 \vee 0.1), (0.3 \vee 0.4)\Big\rangle, \\ &\quad \Big\langle y, (0.5 \wedge 0.4), (0.7 \vee 0.3), (0.6 \vee 0.8)\Big\rangle\Big\} \\ &= \Big\{\langle x, 0.1, 0.4, 0.3, \rangle, \langle y, 0.5, 0.7, 0.6\rangle\Big\}. \end{aligned}$$

iv. *Neutrosophic complement of C is*

$$\begin{aligned} C^c &= \{\langle x, 0.5, 0.1, 0.4\rangle, \langle y, 0.4, 0.3, 0.8\rangle\}^c \\ &= \{\langle x, 0.4, 1 - 0.1, 0.5\rangle, \langle y, 0.8, 1 - 0.3, 0.4\rangle\} \\ &= \{\langle x, 0.4, 0.9, 0.5\rangle, \langle y, 0.8, 0.7, 0.4\rangle\}. \end{aligned}$$

Theorem 5 *Let $A, B \in \mathcal{N}(X)$. Then, followings hold.*

i. *$A \sqcap A = A$ and $A \sqcup A = A$*

ii. *$A \sqcap B = B \sqcap A$ and $A \sqcup B = B \sqcup A$*

iii. *$A \sqcap \tilde{\emptyset} = \tilde{\emptyset}$ and $A \sqcap \tilde{X} = A$*

iv. *$A \sqcup \tilde{\emptyset} = A$ and $A \sqcup \tilde{X} = \tilde{X}$*

v. *$A \sqcap (B \sqcap C) = (A \sqcap B) \sqcap C$ and $A \sqcup (B \sqcup C) = (A \sqcup B) \sqcup C$*

vi. *$(A^c)^c = A$*

Proof. It is clear.

Theorem 6 *Let $A, B \in \mathcal{N}(X)$. Then, De Morgan's law is valid.*

i. $\left(\bigsqcup_{i \in I} A_i\right)^c = \prod_{i \in I} A_i^c$

ii. $\left(\prod_{i \in I} A_i\right)^c = \bigsqcup_{i \in I} A_i^c$

Proof.

i. From Definition 2 *v.*

$$\begin{aligned} \left(\bigsqcup_{i \in I} A_i\right)^c &= \Big\{\Big\langle x, \bigvee_{i \in I} \mu_{A_i}(x), \bigwedge_{i \in I} \sigma_{A_i}(x), \\ &\quad \bigwedge_{i \in I} \nu_{A_i}(x)\Big\rangle : x \in X\Big\}^c \\ &= \Big\{\Big\langle x, \bigwedge_{i \in I} \nu_{A_i}(x), 1 - \bigwedge_{i \in I} \sigma_{A_i}(x), \\ &\quad \bigvee_{i \in I} \mu_{A_i}(x)\Big\rangle : x \in X\Big\} \\ &= \prod_{i \in I} A_i^c \end{aligned}$$

ii. It can proved by similar way to *i.*

Theorem 7 *Let $B \in \mathcal{N}(X)$ and $\{A_i : i \in I\} \subseteq \mathcal{N}(X)$. Then,*

i. $B \sqcap \left(\bigsqcup_{i \in I} A_i\right) = \bigsqcup_{i \in I}(B \sqcap A_i)$

ii. $B \sqcup \left(\prod_{i \in i} A_i\right) = \prod_{i \in I}(B \sqcup A_i)$.

Proof. It can be proved easily from Definition 2.

3 Neutrosophic topological spaces

In this section, we will introduce neutrosophic topological space and give their properties.

Definition 8 *Let $\tau \subseteq \mathcal{N}(X)$, then τ is called a neutrosophic topology on X if*

i. *$\tilde{X}$ and $\tilde{\emptyset}$ belong to τ,*

ii. *The union of any number of neutrosophic sets in τ belongs to τ,*

iii. *The intersection of any two neutrosophic sets in τ belongs to τ.*

The pair (X, τ) is called a neutrosophic topological space over X. Moreover, the members of τ are said to be neutrosophic open sets in X. If $A^c \in \tau$, then $A \in \mathcal{N}(X)$ is said to be neutrosophic closed set in X

Theorem 9 *Let (X, τ) be a neutrosophic topological space over X. Then*

i. *$\tilde{\emptyset}$ and $\tilde{X}$ are neutrosophic closed sets over X.*

ii. The intersection of any number of neutrosophic closed sets is a neutrosophic closed set over X.

iii. The union of any two neutrosophic closed sets is a neutrosophic closed set over X.

Proof. Proof is clear.

Example 10 *Let $\tau = \{\tilde{\emptyset}, \tilde{X}\}$ and $\sigma = \mathcal{N}(X)$. Then, (X,τ) and (X,σ) are two neutrosophic topological spaces over X. Moreover, they are called neutrosophic discrete topological space and neutrosophic indiscrete topological space over X, respectively.*

Example 11 *Let $X = \{a, b\}$ and $A \in \mathcal{N}(X)$ such that*

$$A = \{\langle a, 0.2, 0.4, 0.6\rangle, \langle b, 0.1, 0.3, 0.5\rangle\}.$$

Then, $\tau = \{\tilde{\emptyset}, \tilde{X}, A\}$ is a neutrosophic topology on X.

Theorem 12 *Let (X,τ_1) and (X,τ_2) be two neutrosophic topological spaces over X, then $(X,\tau_1 \cap \tau_2)$ is a neutrosophic topological space over X.*

Proof. Let (X,τ_1) and (X,τ_2) be two neutrosophic topological spaces over X. It can be seen clearly that $\tilde{\emptyset}, \tilde{X} \in \tau_1 \cap \tau_2$. If $A, B \in \tau_1 \cap \tau_2$ then, $A, B \in \tau_1$ and $A, B \in \tau_2$. It is given that $A \sqcap B \in \tau_1$ and $A \sqcap B \in \tau_2$. Thus, $A \sqcap B \in \tau_1 \cap \tau_2$. Let $\{A_i : i \in I\} \subseteq \tau_1 \cap \tau_2$. Then, $A_i \in \tau_1 \cap \tau_2$ for all $i \in I$. Thus, $A_i \in \tau_1$ and $A_i \in \tau_2$ for all $i \in I$. So, we have $\bigsqcup_{i\in I} A_i \in \tau_1 \cap \tau_2$.

Corollary 13 *Let $\{(X,\tau_i) : i \in I\}$ be a family of neutrosophic topological spaces over X. Then, $(X, \bigcap_{i\in I} \tau_i)$ is a neutrosophic topological space over X.*

Proof. It can proved similar way Theorem 12.

Remark 14 *If we get the union operation instead of the intersection operation in Theorem 12, the claim may not be correct. This situation can be seen following example.*

Example 15 *Let $X = \{a, b\}$ and $A, B \in \mathcal{N}(X)$ such that*

$$A = \{\langle a, 0.2, 0.4, 0.6\rangle, \langle b, 0.1, 0.3, 0.5\rangle\}$$
$$B = \{\langle a, 0.4, 0.6, 0.8\rangle, \langle b, 0.3, 0.5, 0.7\rangle\}.$$

Then, $\tau_1 = \{\tilde{\emptyset}, \tilde{X}, A\}$ and $\tau_2 = \{\tilde{\emptyset}, \tilde{X}, B\}$ are two neutrosophic topology on X. But, $\tau_1 \cup \tau_2 = \{\tilde{\emptyset}, \tilde{X}, A, B\}$ is not neutrosophic topology on X. Because, $A \sqcap B \notin \tau_1 \cup \tau_2$. So, $\tau_1 \cup \tau_2$ is not neutrosophic topological space over X.

Definition 16 *Let (X,τ) be a neutrosophic topological space over X and $A \in \mathcal{N}(X)$. Then, the neutrosophic interior of A, denoted by $\mathrm{int}(A)$ is the union of all neutrosophic open subsets of A. Clearly $\mathrm{int}(A)$ is the biggest neutrosophic open set over X which containing A.*

Theorem 17 *Let (X,τ) be a neutrosophic topological space over X and $A, B \in \mathcal{N}(X)$. Then*

i. $\mathrm{int}(\tilde{\emptyset}) = \tilde{\emptyset}$ *and* $\mathrm{int}(\tilde{X}) = \tilde{X}$.

ii. $\mathrm{int}(A) \sqsubseteq A$.

iii. A is a neutrosophic open set if and only if $A = \mathrm{int}(A)$.

iv. $\mathrm{int}(\mathrm{int}(A)) = \mathrm{int}(A)$.

v. $A \sqsubseteq B$ *implies* $\mathrm{int}(A) \sqsubseteq \mathrm{int}(B)$.

vi. $\mathrm{int}(A) \sqcup \mathrm{int}(B) \sqsubseteq \mathrm{int}(A \sqcup B)$.

vii. $\mathrm{int}(A \sqcap B) = \mathrm{int}(A) \sqcap \mathrm{int}(B)$.

Proof. *i.* and *ii.* are obvious.

iii. If A is a neutrosophic open set over X, then A is itself a neutrosophic open set over X which contains A. So, A is the largest neutrosophic open set contained in A and $\mathrm{int}(A) = A$. Conversely, suppose that $\mathrm{int}(A) = A$. Then, $A \in \tau$.

iv. Let $\mathrm{int}(A) = B$. Then, $\mathrm{int}(B) = B$ from *iii.* and then, $\mathrm{int}(\mathrm{int}(A)) = \mathrm{int}(A)$.

v. Suppose that $A \sqsubseteq B$. As $\mathrm{int}(A) \sqsubseteq A \sqsubseteq B$. $\mathrm{int}(A)$ is a neutrosophic open subset of B, so from Definition 16, we have that $\mathrm{int}(A) \sqsubseteq \mathrm{int}(B)$.

vi. It is clear that $A \sqsubseteq A \sqcup B$ and $B \sqsubseteq A \sqcup B$. Thus, $\mathrm{int}(A) \sqsubseteq \mathrm{int}(A \sqcup B)$ and $\mathrm{int}(B) \sqsubseteq \mathrm{int}(A \sqcup B)$. So, we have that $\mathrm{int}(A) \sqcup \mathrm{int}(B) \sqsubseteq \mathrm{int}(A \sqcup B)$ by *v.*

vii. It is known that $\mathrm{int}(A \sqcap B) \sqsubseteq \mathrm{int}(A)$ and $\mathrm{int}(A \sqcap B) \sqsubseteq \mathrm{int}(B)$ by *v.* so that $\mathrm{int}(A \sqcap B) \sqsubseteq \mathrm{int}(A) \sqcap \mathrm{int}(B)$. Also, from $\mathrm{int}(A) \sqsubseteq A$ and $\mathrm{int}(B) \sqsubseteq B$, we have $\mathrm{int}(A) \sqcap \mathrm{int}(B) \sqsubseteq A \sqcap B$. These imply that $\mathrm{int}(A \sqcap B) = \mathrm{int}(A) \sqcap \mathrm{int}(B)$.

Example 18 *Let $X = \{a, b\}$ and $A, B, C \in \mathcal{N}(X)$ such that*

$$A = \{\langle a, 0.5, 0.5, 0.5\rangle, \langle b, 0.3, 0.3, 0.3\rangle\}$$
$$B = \{\langle a, 0.4, 0.4, 0.4\rangle, \langle b, 0.6, 0.6, 0.6\rangle\}$$
$$C = \{\langle a, 0.7, 0.7, 0.7\rangle, \langle b, 0.2, 0.2, 0.2\rangle\}.$$

Then, $\tau = \{\tilde{\emptyset}, \tilde{X}, A\}$ is a neutrosophic soft topological space over X. Therefore, $\mathrm{int}(B) = \tilde{\emptyset}$, $\mathrm{int}(C) = \tilde{\emptyset}$ *and* $\mathrm{int}(B \sqcup C) = A$. *So,* $\mathrm{int}(B) \sqcup \mathrm{int}(C) \neq \mathrm{int}(B \sqcup C)$.

Definition 19 *Let (X,τ) be a neutrosophic topological space over X and $A \in \mathcal{N}(X)$. Then, the neutrosophic closure of A, denoted by $\mathrm{cl}(A)$ is the intersection of all neutrosophic closed super sets of A. Clearly $\mathrm{cl}(A)$ is the smallest neutrosophic closed set over X which contains A.*

Example 20 *In the Example 10, according to the neutrosophic topological space (X,σ), neutrosophic interior and neutrosophic closure of each element of $\mathcal{N}(X)$ is equal to itself.*

Theorem 21 *Let (X,τ) be a neutrosophic topological space over X and $A, B \in \mathcal{N}(X)$. Then*

i. $\mathrm{cl}(\tilde{\emptyset}) = \tilde{\emptyset}$ *and* $\mathrm{cl}(\tilde{X}) = \tilde{X}$.

ii. $A \sqsubseteq \mathrm{cl}(A)$.

iii. *A is a neutrosophic closed set if and only if* $A = \mathrm{cl}(A)$.

iv. $\mathrm{cl}(\mathrm{cl}(A)) = \mathrm{cl}(A)$.

v. $A \sqsubseteq B$ *implies* $\mathrm{cl}(A) \sqsubseteq \mathrm{cl}(B)$.

vi. $\mathrm{cl}(A \sqcup B) = \mathrm{cl}(A) \sqcup \mathrm{cl}(B)$.

vii. $\mathrm{cl}(A \sqcap B) \sqsubseteq \mathrm{cl}(A) \sqcap \mathrm{cl}(B)$.

Proof. *i.* and *ii.* are clear. Moreover, proofs of *vi.* and *vii.* are similar to Theorem 17 *vi.* and *vii.*.

iii. If A is a neutrosophic closed set over X then A is itself a neutrosophic closed set over X which contains A. Therefore, A is the smallest neutrosophic closed set containing A and $A = \mathrm{cl}(A)$. Conversely, suppose that $A = \mathrm{cl}(A)$. As A is a neutrosophic closed set, so A is a neutrosophic closed set over X.

iv. A is a neutrosophic closed set so by *iii.*, then we have $A = \mathrm{cl}(A)$.

v. Suppose that $A \sqsubseteq B$. Then every neutrosophic closed super set of B will also contain A. This means that every neutrosophic closed super set of B is also a neutrosophic closed super set of A. Hence the neutrosophic intersection of neutrosophic closed super sets of A is contained in the neutrosophic intersection of neutrosophic closed super sets of B. Thus $\mathrm{cl}(A) \sqsubseteq \mathrm{cl}(B)$.

Example 22 *Let $X = \{a, b\}$ and $A, B \in \mathcal{N}(X)$ such that*

$$\begin{aligned} A &= \{\langle a, 0.5, 0.5, 0.5\rangle, \langle b, 0.4, 0.4, 0.4\rangle\} \\ B &= \{\langle a, 0.6, 0.6, 0.6\rangle, \langle b, 0.3, 0.3, 0.3\rangle\}. \end{aligned}$$

Then,

$$\tau = \{\tilde{\emptyset}, \tilde{X}, A, B, A \sqcap B, A \sqcup B\}$$

is a neutrosophic topology on X. Moreover, set of neutrosophic closed sets over X is

$$\{\tilde{X}, \tilde{\emptyset}, A^c, B^c, (A \sqcap B)^c, (A \sqcup B)^c\}.$$

Therefore

$$\begin{aligned} A^c &= \{\langle a, 0.5, 0.5, 0.5\rangle, \langle b, 0.4, 0.6, 0.4\rangle\} \\ B^c &= \{\langle a, 0.6, 0.4, 0.6\rangle, \langle b, 0.3, 0.7, 0.3\rangle\} \\ (A \sqcap B)^c &= \{\langle a, 0.6, 0.4, 0.5\rangle, \langle b, 0.4, 0.6, 0.4\rangle\} \\ (A \sqcup B)^c &= \{\langle a, 0.5, 0.5, 0.6\rangle, \langle b, 0.3, 0.7, 0.4\rangle\}. \end{aligned}$$

Thus, we have that

$$\begin{aligned} A \sqcap B &= \{\langle a, 0.5, 0.5, 0.6\rangle, \langle b, 0.3, 0.7, 0.4\rangle\} \\ \mathrm{cl}(A) &= \tilde{X} \\ \mathrm{cl}(B) &= \tilde{X} \\ \mathrm{cl}(A \sqcap B) &= (A \sqcup B)^c \\ \mathrm{cl}(A \sqcap B) &\sqsubseteq \mathrm{cl}(A) \sqcap \mathrm{cl}(B). \end{aligned}$$

Remark 23 *Example 18 and Example 22 show that there is not equality in Theorem 17* vi. *and Theorem 21* vii.

Theorem 24 *Let (X,τ) be a neutrosophic topological space over X and $A, B \in \mathcal{N}(X)$. Then*

i. $\mathrm{int}(A^c) = (\mathrm{cl}(A))^c$,

ii. $\mathrm{cl}(A^c) = (\mathrm{int}(A))^c$.

Proof. Let $A, B \in \mathcal{N}(X)$. Then,

i. It is known that

$$\mathrm{cl}(A) = \bigsqcap_{\substack{B^c \in \tau \\ A \sqsubseteq B}} B.$$

Therefore, we have that

$$(\mathrm{cl}(A))^c = \bigsqcup_{\substack{B^c \in \tau \\ B^c \sqsubseteq A^c}} B^c.$$

Right hand of above equality is $\mathrm{int}(A^c)$, thus $\mathrm{int}(A^c) = (\mathrm{cl}(A))^c$.

ii. If it is taken A^c instead of A in *i.*, then it can be seen clearly that $(\mathrm{cl}(A^c))^c = \mathrm{int}((A^c)^c) = \mathrm{int}(A)$. So, $\mathrm{cl}(A^c) = (\mathrm{int}(A))^c$.

Definition 25 *Let (X,τ) be a neutrosophic topological space over X then the neutrosophic exterior of a neutrosophic set A over X is denoted by* $\mathrm{ext}(A)$ *and is defined as* $\mathrm{ext}(A) = \mathrm{int}(A^c)$.

Theorem 26 *Let (X,τ) be a neutrosophic topological space over X and $A, B \in \mathcal{N}(X)$. Then*

i. $\mathrm{ext}(A \sqcup B) = \mathrm{ext}(A) \sqcap \mathrm{ext}(B)$

ii. $\mathrm{ext}(A) \sqcup \mathrm{ext}(B) \sqsubseteq \mathrm{ext}(A \sqcap B)$

Proof. Let $A, B \in \mathcal{N}(X)$. Then,

i. By Definition 25, Theorem 6 and Theorem 17 *vii.*

$$\begin{aligned} \mathrm{ext}(A \sqcup B) &= \mathrm{int}((A \sqcup B)^c) \\ &= \mathrm{int}(A^c \sqcap B^c) \\ &= \mathrm{int}(A^c) \sqcap \mathrm{int}(B^c) \\ &= \mathrm{ext}(A) \sqcap \mathrm{ext}(B) \end{aligned}$$

ii. It is similar to *i.*

Definition 27 *Let (X, τ) be a neutrosophic topological space over X and $A \in \mathcal{N}(X)$. Then, the neutrosophic boundary of a neutrosophic set A over X is denoted by $\mathrm{fr}(A)$ and is defined as $\mathrm{fr}(A) = \mathrm{cl}(A) \sqcap \mathrm{cl}(A^c)$. It must be noted that $\mathrm{fr}(A) = \mathrm{fr}(A^c)$.*

Example 28 *Let consider the neutrosophic sets A and B in the Example 22. According to the neutrosophic topology in Example 11 we have $\mathrm{fr}(A) = \tilde{\emptyset}$ and $\mathrm{fr}(C) = (A \sqcap B)^c$.*

Theorem 29 *Let (X, τ) be a neutrosophic topological space over X and $A, B \in \mathcal{N}(X)$. Then*

i. $(\mathrm{fr}(A))^c = \mathrm{ext}(A) \sqcup \mathrm{int}(A)$.

ii. $\mathrm{cl}(A) = \mathrm{int}(A) \sqcup \mathrm{fr}(A)$.

Proof. Let $A, B \in \mathcal{N}(X)$. Then,

i. By Theorem 24 *i.*, we have

$$\begin{aligned}(\mathrm{fr}(A))^c &= (\mathrm{cl}(A) \sqcap \mathrm{fr}(A^c))^c \\ &= (\mathrm{cl}(A))^c \sqcup (\mathrm{fr}(A^c))^c \\ &= (\mathrm{cl}(A))^c \sqcup ((\mathrm{int}(A))^c)^c \\ &= \mathrm{ext}(A) \sqcup \mathrm{int}(A).\end{aligned}$$

ii. By Theorem 24 *i.*, we have

$$\begin{aligned}\mathrm{int}(A) \sqcup \mathrm{fr}(A) &= \mathrm{int}(A) \sqcup (\mathrm{cl}(A) \sqcap \mathrm{fr}(A^c)) \\ &= (\mathrm{int}(A) \sqcup \mathrm{cl}(A)) \sqcap (\mathrm{int}(A) \sqcup \mathrm{fr}(A^c)) \\ &= \mathrm{cl}(A) \sqcap (\mathrm{int}(A) \sqcup (\mathrm{int}(A))^c) \\ &= \mathrm{cl}(A) \sqcap \tilde{X} \\ &= \mathrm{cl}(A).\end{aligned}$$

Theorem 30 *Let (X, τ) be a neutrosophic topological space over X and $A \in \mathcal{N}(X)$. Then*

i. A is a neutrosophic open set over X if and only if $A \sqcap \mathrm{fr}(A) = \tilde{\emptyset}$.

ii. A is a neutrosophic closed set over X if and only if $\mathrm{fr}(A) \sqsubseteq A$.

Proof. Let $A \in \mathcal{N}(X)$. Then

i. Assume that A is a neutrosophic open set over X. Thus $\mathrm{int}(A) = A$. By Theorem 24, $\mathrm{fr}(A) = \mathrm{cl}(A) \sqcap \mathrm{fr}(A^c) = \mathrm{cl}(A) \sqcap (\mathrm{int}(A))^c$. So,

$$\begin{aligned}\mathrm{fr}(A) \sqcap \mathrm{int}(A) &= \mathrm{cl}(A) \sqcap (\mathrm{int}(A))^c \sqcap \mathrm{int}(A) \\ &= \mathrm{cl}(A) \sqcap A^c \sqcap A \\ &= \tilde{\emptyset}.\end{aligned}$$

Conversely, let $A \sqcap \mathrm{fr}(A) = \tilde{\emptyset}$. Then, $A \sqcap \mathrm{cl}(A) \sqcap \mathrm{fr}(A^c) = \tilde{\emptyset}$ or $A \sqcap \mathrm{fr}(A^c) = \tilde{\emptyset}$ or $\mathrm{cl}(A) \sqsubseteq A^c$ which implies A^c is a neutrosophic set and so A is a neutrosophic open set.

ii. Let A be a neutrosophic closed set. Then, $\mathrm{cl}(A) = A$. By Definition 27, $\mathrm{fr}(A) = \mathrm{cl}(A) \sqcap \mathrm{fr}(A^c) \sqsubseteq \mathrm{cl}(A) = A$. Therefore, $\mathrm{fr}(A) \sqsubseteq A$. Conversely, $\mathrm{fr}(A) \sqsubseteq A$. Then $\mathrm{fr}(A) \sqcap A^c = \tilde{\emptyset}$. From $\mathrm{fr}(A) = \mathrm{fr}(A^c)$, $\mathrm{fr}(A^c) \sqcap A^c = \tilde{\emptyset}$. By *i.*, A^c is a neutrosophic open set and so A is a neutrosophic closed set.

Theorem 31 *Let (X, τ) be a neutrosophic topological space over X and $A \in \mathcal{N}(X)$. Then*

i. $\mathrm{fr}(A) \sqcap \mathrm{int}(A) = \tilde{\emptyset}$

ii. $\mathrm{fr}(\mathrm{int}(A)) \sqsubseteq \mathrm{fr}(A)$

Proof. Let $A \in \mathcal{N}(X)$. Then,

i. From Theorem 30 *i.*, it is clear.

ii. By Theorem 24 *ii.*,

$$\begin{aligned}\mathrm{fr}(\mathrm{int}(A)) &= \mathrm{cl}(\mathrm{int}(A)) \sqcap \mathrm{cl}(\mathrm{int}(A)) \\ &= \mathrm{cl}(\mathrm{int}(A)) \sqcap \mathrm{fr}(A^c) \\ &\sqsubseteq \mathrm{cl}(A) \sqcap \mathrm{fr}(A^c) \\ &= \mathrm{fr}(A).\end{aligned}$$

Definition 32 *Let (X, τ) be a neutrosophic topological space and Y be a non-empty subset of X. Then, a neutrosophic relative topology on Y is defined by*

$$\tau_Y = \{A \sqcap \tilde{Y} : A \in \tau\}$$

where

$$\tilde{Y}(x) = \begin{cases} \langle 1, 0, 0 \rangle, & x \in Y \\ \langle 0, 1, 1 \rangle, & \textit{otherwise}. \end{cases}$$

Thus, (Y, τ_Y) is called a neutrosophic subspace of (X, τ).

Example 33 *Let $X = \{a, b, c\}$, $Y = \{a, b\} \subseteq X$ and $A, B \in \mathcal{N}(X)$ such that*

$$\begin{aligned}A &= \{\langle a, 0.4, 0.2, 0.2 \rangle, \langle b, 0.5, 0.4, 0.6 \rangle, \langle c, 0.2, 0.5, 0.7 \rangle\} \\ B &= \{\langle a, 0.4, 0.5, 0.3 \rangle, \langle b, 0.5, 0.6, 0.5 \rangle, \langle c, 0.3, 0.7, 0.8 \rangle\}.\end{aligned}$$

Then,

$$\tau = \{\tilde{\emptyset}, \tilde{X}, A, B, A \sqcap B, A \sqcup B\}$$

is a neutrosophic topology on X. Therefore

$$\tau_Y = \{\tilde{\emptyset}, \tilde{Y}, C, M, L, K\}$$

is a neutrosophic relative topology on Y such that $C = \tilde{Y} \sqcap A$, $M = \tilde{Y} \sqcap B$, $L = \tilde{Y} \sqcap (A \sqcap B)$ and $K = \tilde{Y} \sqcap (A \sqcup B)$.

4 Conclusion

In this work, we have redefined the neutrosophic set operations in accordance with neutrosophic topological structures. Then, we have presented some properties of these operations. We have also investigated neutrosophic topological structures of neutrosophic sets. Hence, we hope that the findings in this paper will help researchers enhance and promote the further study on neutrosophic topology.

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Neutrosophic crisp Sets via Neutrosophic crisp Topological Spaces $NCTS$

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Abstract: In this paper, the structure of some classes of neutrosophic crisp nearly open sets are investigated via topology, and some applications are given. Finally, we generalize the crisp topological and neutrosophic crisp studies to the notion of neutrosophic crisp set.

Keywords: Set Theory, Topology, Neutrosophic crisp set theory, Neutrosophic crisp topology, Neutrosophic crisp α-open set, Neutrosophic crisp $semi$-open set, Neutrosophic crisp continuous function.

1 Introduction

Neutrosophy has laid the foundation for a whole family of new mathematical theories generalizing both their crisp and fuzzy counterparts, such as a neutrosophic set theory in [9, 11, 10]. It followed the introduction of the neutrosophic set concepts in [13, 12, 14, 15, 5, 7, 8, 16, 17] and the fundamental definitions of neutrosophic set operations. Smarandache [9, 11] and Salama et al. in [13, 18] provide a natural foundation for treating mathematically the neutrosophic phenomena which exist pervasively in our real world and for building new branches of neutrosophic mathematics.

In this paper, we introduce the concept of neutrosophic crisp sets. We investigate the properties of continuous, open and closed maps in the neutrosophic crisp topological spaces, also give relations between neutrosophic crisp pre-continuous mapping and neutrosophic crisp $semi$-precontinuous mapping and some other continuous mapping, and show that the category of intuitionistic fuzzy topological spaces is a bireflective full subcategory of neutrosophic crisp topological spaces.

2 Terminology

We recollect some relevant basic preliminaries, and in particular, the work of Smarandache in [9, 11, 10], and Salama et al. [13, 12, 14, 15, 5, 7, 8, 16, 17, 6]. Smarandache introduced the neutrosophic components T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where $\rfloor^{-}0, 1^{+}\lfloor$ is a non-standard unit interval. Hanafy and Salama et al.[8, 16] considered some possible definitions for basic concepts of the neutrosophic crisp set and its operations.

Definition 1 *[20] Let X be a non-empty fixed set. A neutrosophic crisp set (NCS) A is an object having the form $A = \{A_1, A_2, A_3\}$, where A_1, A_2, and A_3 are subsets of X satisfying $A_1 \cap A_2 = \phi$, $A_1 \cap A_3 = \phi$, and $A_2 \cap A_1 = \phi$.*

Remark 2 *[20] Neutrosophic crisp set $A = \{A_1, A_2, A_3\}$ can be identified as an ordered triple $\{A_1, A_2, A_3\}$ where A_1, A_2, and A_3 are subsets on X, and one can define several relations and operations between $NCSs$.*

Types of $NCSs$ ϕ_N and X_N [20] in X as follows:

1- ϕ_N may be defined in many ways as a NCS, as follows

1. $\phi_N = \langle \phi, \phi, X \rangle$ or
2. $\phi_N = \langle \phi, X, X \rangle$ or
3. $\phi_N = \langle \phi, X, \phi \rangle$ or
4. $\phi_N = \langle \phi, \phi, \phi \rangle$

2- X_N may be defined in many ways as a NCS, as follows

1. $X_N = \langle X, \phi, \phi \rangle$ or
2. $X_N = \langle X, X, \phi \rangle$ or
3. $X_N = \langle X, X, X \rangle$ or

Definition 3 *[20] Let X is a non-empty set, and the $NCSs$ A and B in the form $A = \{A_1, A_2, A_3\}$, $B = \{B_1, B_2, B_3\}$. then we may consider two possible definition for subsets $A \subseteq B$, may defined in two ways:*

1. *$A \subseteq B \Leftrightarrow A_1 \subseteq B_1$, $A_2 \subseteq B_2$, and $A_3 \supseteq B_3$ or*
2. *$A \subseteq B \Leftrightarrow A_1 \subseteq B_1$, $A_2 \supseteq B_2$, and $A_3 \supseteq B_3$*

Definition 4 *[20] Let X is a non-empty set, and the $NCSs$ A and B in the form $A = \{A_1, A_2, A_3\}$, $B = \{B_1, B_2, B_3\}$. Then*

1. $A \cap B$ may defined in two way:

   i) $A \cap B = \langle A_1 \cap B_1, A_2 \cap B_2, A_3 \cup B_3 \rangle$

   ii) $A \cap B = \langle A_1 \cap B_1, A_2 \cup B_2, A_3 \cup B_3 \rangle$

2. $A \cup B$ may defined in two way:

   i) $A \cup B = \langle A_1 \cup B_1, A_2 \cap B_2, A_3 \cap B_3 \rangle$

   ii) $A \cup B = \langle A_1 \cup B_1, A_2 \cup B_2, A_3 \cap B_3 \rangle$

3. $[\,]A = \langle A_1, A_2, A_1^c \rangle$

4. $<> A = \langle A_3^c, A_2, A_3 \rangle$

Definition 5 *[20] A neutrosophic crisp topology (NCT) on a non-empty set X is a family Γ of neutrosophic crisp subsets in X satisfying the following axioms.*

1. $\phi_N, X_N \in \Gamma$.

2. $A_1 \cap A_2 \in \Gamma$, *for any* A_1 *and* $A_2 \in \Gamma$.

3. $\cup A_j \in \Gamma, \forall \{A_j : j \in J\} \subseteq \Gamma$.

In this case the pair (X, Γ) is said to be a neutrosophic crisp topological space $(NCTS)$ in X. The elements in Γ are said to be neutrosophic crisp open sets $(NCOSs)$ in Y. A neutrosophic crisp set F is closed $(NCCS)$ if and only if its complement F^c is an open neutrosophic crisp set.

Remark 6 *[20] Neutrosophic crisp topological spaces are very natural generalizations of topological spaces and intuitionistic topological spaces, and they allow more general functions to be members of topology:*

$$TS \Rightarrow ITS \Rightarrow NCTS$$

Definition 7 *[20] Let (X, Γ) be $NCTS$ and $A = \{A_1, A_2, A_3\}$ be a NCS in X. Then the neutrosophic crisp closure of A ($NCcl(A)$ for short) and neutrosophic crisp interior ($NCint(A)$ for short) of A are defined by*
$NCcl(A) = \cap\{K : is\, a\, NCCS\, in\, X\, and\, A \subseteq K\}$
$NCint(A) = \cup\{G : G\, is\, a\, NCOS\, in\, X\, and\, G \subseteq A\}$,
where NCS is a neutrosophic crisp closed set, and $NCOS$ is a neutrosophic crisp open set. Note that for any NCS in (X, Γ), we have
(1) $NCcl(A^c) = (NCcl(A))^c$, and
(2) $NCint(A^c) = (NCint(A))^c$
It can be also shown that $NCcl(A)$ is $NCCS$ (neutrosophic crisp closed set) and $NCint(A)$ is a $CNOS$ in X.

1. *A is in X if and only if $NCcl(A) \supseteq A$.*

2. *A is a $NCOS$ in X if and only if $NCint(A) = A$.*

Definition 8 *[20] Let (X, Γ) be a $NCTS$ and A, B be a NCS in X, then the following properties hold:*

1. $NCint(A) \subseteq A$,

2. $A \subseteq NCcl(A)$.

3. $A \subseteq B \Longrightarrow NCint(A) \subseteq NCint(B)$,

4. $A \subseteq B \Longrightarrow NCcl(A) \subseteq NCcl(B)$,

5. $NCint(A \cap B) = NCint(A) \cap NCint(B)$,

6. $NCint(A \cup B) = NCint(A) \cup NCint(B)$,

7. $NCint(X_N) = X_N$, $NCcl(\phi N) = \phi_N$

Definition 9 *[21] Let (X, Γ) be a $NCTS$ and $A = \{A_1, A_2, A_3\}$ be a NCS in X, then A is said to be*

1. *Neutrosophic crisp α-open set $(NC\alpha OS)$ iff $A \subseteq NCint(NCcl(NCint(A)))$,*

2. *Neutrosophic crisp semi-open set $(NCSOS)$ iff $A \subseteq NCcl(NCint(A))$.*

3. *Neutrosophic crisp pre-open set $(NCPOS)$ iff $A \subseteq NCint(NCcl(A))$.*

The class of all neutrosophic crisp α-open sets $NC\Gamma^{\alpha}$ which is finer than $NC\Gamma$, the class of all neutrosophic crisp $semi$-open sets $NC\Gamma^{s}$, and the class of all neutrosophic crisp pre-open sets $NC\Gamma^{p}$.

Definition 10 *[20] Let (X, Γ) be $NCTS$ and $A = \{A_1, A_2, A_3\}$ be a NCS in X. Then the α-neutrosophic crisp closure of A ($\alpha NCCl(A)$ for short) and α-neutrosophic crisp interior ($\alpha NCInt(A)$ for short) of A are defined by*

1. $\alpha NCcl(A) = \cap\{K : is\, an\, NC\alpha CS\, in\, X\, and\, A \subseteq K\}$,

2. $\alpha NCint(A) = \cup\{G : G\, is\, an\, NC\alpha OS\, in\, X\, and\, G \subseteq A\}$,

Proposition 11 *[20] Let (X, Γ) be $NCTS$ and A, B be two neutrosophic crisp sets in . Then the following properties hold:*

1. $NCint(A) \subseteq A$,

2. $A \subseteq NCcl(A)$,

3. $A \subseteq B \Rightarrow NCint(A) \subseteq NCint(B)$,

4. $A \subseteq B \Rightarrow NCcl(A) \subseteq NCcl(B)$,

5. $NCint(A \cap B) = NCint(A) \cap NCint(B)$,

6. $NCcl(A \cup B) = NCcl(A) \cup NCcl(B)$

7. $NCint(X_N) = X_N$,

8. $NCcl(\phi_N) = \phi_N$

Example 12 *Let $X = \{a, b, c, d\}, \phi_N, X_N$ be any types of theuniversal and empty subsets, and A, B two neutrosophic crisp subsets on X defined by $A = \langle\{a\}, \{b, d\}, \{c\}\rangle$, $B = \langle\{a\}, \{b\}, \{c, d\}\rangle$ then the family $\Gamma = \{\phi_N, X_N, A, B\}$ is a neutrosophic crisp topology on X.*

3 Neutrosophic Crisp Open Set

In this section, we will present an equivalent definition to Neutrosophic crisp α-open set and prove many special properties of it. Moreover, we will explain the relationship between different classes of neutrosophic crisp open sets by diagram.

Definition 13 *Let (X, Γ) be a $NCTS$ and $A = \{A_1, A_2, A_3\}$ be a NCS in X, then A is said to be*

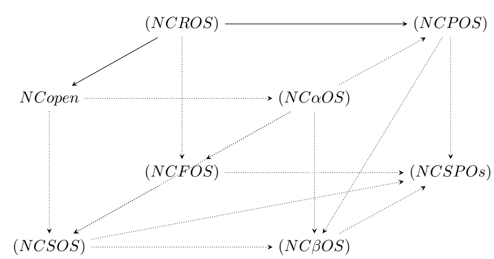
1. *Neutrosophic crisp feebly-open $(NCFOS)$if there is a Neutrosophic crisp open set U such that $U \subseteq A \subseteq sNCcl(U)$, where $sNCcl(U)$ is denote neutrosophic closure with respect to $NC\Gamma^s$, is defined by the intersection of all Neutrosophic crisp semi closed sets containing A.*
2. *Neutrosophic crisp β-open set $(NC\beta OS)$ iff $A \subseteq NCcl(NCint(NCcl(A)))$,*
3. *Neutrosophic crisp $semipre$-open set $(NCSPOs)$ iff there exists a neutrosophic crisp preopen set U such that $U \subseteq A \subseteq NCcl(U)$.*
4. *Neutrosophic crisp regular-open set $(NCROS)$ iff $A = NCint(NCcl(A))$.*
5. *Neutrosophic crisp semiα-open $(NCS\alpha OS)$ iff there exists a Neutrosophic crisp α-open set U such that $U \subseteq A \subseteq NCcl(U)$*

The class of all neutrosophic crisp feebly-open sets $NC\Gamma^{feebly}$, the calls all neutrosophic crisp β-open sets $NC\Gamma^{\beta}$, the class of all neutrosophic crisp semipre-open sets $NC\Gamma^{sp}$, the class of all neutrosophic crisp regular-open sets $NC\Gamma^{r}$, and the class of all neutrosophic crisp semiα-open sets $NC\Gamma^{s\alpha}$.

A neutrosophic crisp A is said to be a neutrosophic crisp semi-closed set, neutrosophic crisp α-closed set, neutrosophic crisp preclosed set, and neutrosophic crisp regular closed set, Neutrosophic crisp feebly-open, Neutrosophic crisp β-open set, Neutrosophic crisp $semipre$-open set, Neutrosophic crisp semiα-open respectively ($(NCSCS)$, $(NC\alpha CS)$, $(NCPCS)$,$(NCRCS)$,$(NCFCS)$,$(NC\beta CS)$,$(NCSPCs)$, $(NCS\alpha CS)$, see the following table.

Table of Abbreviations	
Abbreviations	Neutrosophic crisp open sets
$NCFOS$	Neutrosophic crisp $feebly$-open
$NC\beta OS$	Neutrosophic crisp β-open
$NCSPOs$	Neutrosophic crisp $semipre$-open
$NCROS$	Neutrosophic crisp $regular$-open
$NCS\alpha OS$	Neutrosophic crisp semiα-open
$NC\alpha OS$	Neutrosophic crisp α-open set
$NCSOS$	Neutrosophic crisp $semi$-open
$NCPOS$	Neutrosophic crisp pre-open

Remark 14 *From above the following implication and none of these implications is reversible as shown by examples given below*



Example 15 *Let $X = \{a, b, c, d\}$, ϕ_N, X_N be any types of the universal and empty subset, and $A_1 = \langle\{a\}, \{b\}, \{c\}\rangle$ $A_2 = \langle\{a\}, \{b, d\}, \{c\}\rangle$, then the family $\Gamma = \{\phi_N, X_N, A_1, A_2\}$ is a neutrosophic crisp topology on X. The NCS A_1 & and A_2 are neutrosophic crisp open $(NCOS)$, then its neutrosophic crisp α-open sets i.e ($A \subseteq NCint(NCcl(NCint(A)))$) neutrosophic crisp pre-open sets i.e ($A \subseteq NCint(NCcl(A))$), neutrosophic crisp $semi$-open sets i.e ($A \subseteq NCcl(NCint(A))$). Also A_2 is neutrosophic crisp β-open sets, hence its Neutrosophic crisp $semipre$-open set.*
If $A_3 = \langle\{a\}, \{d\}, \{c\}\rangle$, then its clear A_3 neutrosophic crisp α-open set but not neutrosophic crisp open set.
If $A_4 = \langle\{a, b\}, \{c\}, \{d\}\rangle$, then A_4 neutrosophic crisp pre-open set but not neutrosophic crisp regular-open set, and we can see also that A_4 is neutrosophic crisp β-open but not neutrosophic crisp $semi$-open set.

Theorem 16 *An neutrosophic crisp A in a $NCTS$ (X, Γ) is a $NC\alpha OS$ if and only if it is both a $(NCSOS)$ and a $(NCPOS)$.*
Proof. *Necessity follows from the diagram given above. Suppose that A is both a $(NCSOS)$ and a $(NCPOS)$. Then*

$A \subseteq cl(int(A))$, and so

$$cl(A) \subseteq cl(cl(int(A))) = cl(int(A)).$$

It follows that $A \subseteq int(cl(A)) \subseteq int(cl(int(A)))$, so that A is a $(NC\alpha OS)$. We give condition(s) for a NCS to be a $(NC\alpha OS)$.

Theorem 17 *Let A be a NCS in a $NCTS$ (X, Γ). If B is a $(NCSOS)$ such that $B \subseteq A \subseteq int(cl(B))$, then A is a $(NC\alpha OS)$.*
Proof. *Since B is a $(NCSOS)$, we have $B \subseteq cl(int(B))$. Thus, $A \subset int(cl(B)) \subseteq int(cl(cl(int(B)))) \subseteq int(cl(int(B)))$,and so A is a $(NC\alpha OS)$.*

Lemma 18 *Any union of $(NC\alpha OS)$ (resp., $(NCPOS)$) is a $(NC\alpha OS)$ (resp., $(NCPOS)$).*
Proof. *The proof is straightforward.*

Definition 19 *Let $\langle a_{i1}, a_{i2}, a_{i3}\rangle \subseteq X$. A neutrosophic crisp point (NCP for short) $p(a_1, a_2, a_3)$ of X is a NCS of X defined by $a_{i1} \cap a_{i2} = \phi$, $a_{i1} \cap a_{i3} = \phi$, $a_{i2} \cap a_{i3} = \phi$. Let $p(a_{i1}, a_{i2}, a_{i3})$ be a NCP of a $NCTS$ (X, Γ). An NCS A of X is said to be a neutrosophic crisp neighborhood (NCN) of $p(a_{i1}, a_{i2}, a_{i3})$ if there exists a $NCOS$ B in X such that $p(a_{i1}, a_{i2}, a_{i3}) \in B \subseteq A$.*

Theorem 20 *Let (X, Γ) be a $NCTS$. Then a neutrosophic crisp A of X is a neutrosophic crisp α-open (resp., neutrosophic crisp pre-open) if and only if for every (NCP) $P_{(a_{i1},a_{i2},a_{i3})} \in A$, there exists a $(NC\alpha OS)$ (resp., $(NCPOS)$ $B_{p(\alpha,\beta)}$) such that $P_{(a_{i1},a_{i2},a_{i3})} \in B_{p(a_{i1},a_{i2},a_{i3})} \subseteq A$.*
Proof. *If A is a $(NC\alpha OS)$ (resp., $(NCPOS)$), then we may take $B_{p(\alpha,\beta} = A$ for every $P_{(a_{i1},a_{i2},a_{i3})} \in A$. Conversely assume that for every $(NCPOS)$ $P_{(a_{i1},a_{i2},a_{i3})} \in A$, there exists a $(NC\alpha OS)$ (resp., $(NCPOS)$) $B_{p(a_{i1},a_{i2},a_{i3}}$ such that $P_{(a_{i1},a_{i2},a_{i3})} \in B_{p(a_{i1},a_{i2},a_{i3}} \subseteq A$. Then,*

$$A = \bigcup\{P_{(a_{i1},a_{i2},a_{i3})} | P_{(a_{i1},a_{i2},a_{i3})} \in A\} \subseteq \bigcup\{B_{p(a_{i1},a_{i2},a_{i3})} | P_{(a_{i1},a_{i2},a_{i3})} \in A\} \subseteq A$$

Theorem 21 *Let (X, Γ) be a $NCTS$,*

1. *If $V \in NCSOS(X)$ and $A \in NC\alpha OS(X)$, then $V \cap A \in NCSOS(X)$.*
2. *If $V \in NCPOS(X)$ and $A \in NC\alpha OS(X)$, then $V \cap A \in NCPOS(X)$.*

Proof. *(1) Let $V \in NCSOS(X)$ and $A \in NC\alpha OS(X)$. Then we obtain,*

$$\begin{aligned} V \cap A \subset & NCcl(NCint(V)) \cap NCnt(NCcl(NCint(A))) \\ & \subset NCcl[NCint(V) \cap NCint(NCcl(NCint(A)))] \\ & \subset NCcl[NCint(V) \cap NCcl(NCint(A))] \\ & \subset NCcl[NCcl[NCint(V) \cap NCint(A)]] \\ & \subset NCcl[NCint(V \cap A)]. \end{aligned}$$

This shows that $V \cap A \in NCSOS(X)$
(2) Let $V \in NCPOS(X)$ and $A \in NC\alpha OS(X)$. Then we obtain,

$$\begin{aligned} V \cap A \subset & NCint(NCcl(V)) \cap NCint(NCcl(NCint(A))) \\ & = NCint[NCint(V) \cap NCcl(NCint(A))] \\ & \subset NCint[NCcl(NCint(V) \cap NCint(A))] \\ & \subset NCint[NCcl[NCCL(V) \cap NCint(A)]] \\ & \subset NCint[NCcl[NCCL[V \cap NCint(A)]]] \\ & \subset NCint[NCcl[V \cap A]]. \end{aligned}$$

This shows that $V \cap A \in NCPOS(X)$.

Theorem 22 *Let A be a subset of a neutrosophic crisp topological space (X, Γ). Then the following properties hold:*

1. *A subset A of X is $NC\alpha OS$ if and only if it is $NCPOS$ and $NCSOS$,*
2. *If A is $NCSOS$, then A is $NC\beta OS$.*
3. *If A is $NCPOS$, then A is $NC\beta OS$.*

Proof. *(1) Necessity: This is obvious.*
Sufficiency: Let A be $NCSOS$ and $NCPOS$. Then we have

$$\begin{aligned} A & \subseteq NCint(NCcl(A)) \\ & \subseteq NCint(NCcl(NCcl(NCint(A)))) \\ & \subseteq NCint(NCcl(NCint(A))). \end{aligned}$$

This shows that A is $NC\alpha OS$.
(2) Since A is $NCSOS$, we have

$$\begin{aligned} A & \subseteq NCcl(NCint(A)) \\ & \subseteq NCcl(NCint(A)) \\ & \subseteq NCcl(NCint(c(A))) \end{aligned}$$

This shows that A is $NC\beta OS$.
(3) The proof is obvious.

Definition 23 *Let (X, Γ) be $NCTS$ and $A = \{A_1, A_2, A_3\}$ be a NCS in X. Then the $*$-neutrosophic crisp closure of A ($* - NCCl(A)$ for short) and $*$-neutrosophic crisp interior ($* - NCInt(A)$ for short) of A are defined by*

1. *$pNCcl(A) = \cap\{K : is\, a\, NCPCS\, in\, X\, and\, A \subseteq K\}$,*
2. *$pNCint(A) = \cup\{G : G\, is\, a\, NCPOS\, in\, X\, and\, G \subseteq A\}$,*
3. *$sNCcl(A) = \cap\{K : is\, a\, NCSCS\, in\, X\, and\, A \subseteq K\}$,*
4. *$sNCint(A) = \cup\{G : G\, is\, a\, NCSOS\, in\, X\, and\, G \subseteq A\}$,*
5. *$\beta NCcl(A) = \cap\{K : is\, a\, NC\beta CS\, in\, X\, and\, A \subseteq K\}$,*
6. *$\beta NCint(A) = \cup\{G : G\, is\, a\, NC\beta OS\, in\, X\, and\, G \subseteq A\}$,*

7. $rNCcl(A) = \cap\{K : is\, a\, NCRCS\, in\, X\, and\, A \subseteq K\}$,

8. $rNCint(A) = \cup\{G : G\, is\, a\, NCROS\, in\, X\, and\, G \subseteq A\}$,

Theorem 24 *For any neutrosophic crisp subset A of NCT S X. A is said to be neutrosophic crisp α-open set if and only if there exists a neutrosophic crisp open set G such that* $G \subseteq A \subseteq NCint(NCcl(G))$.
Proof. *Necessity : If A be a neutrosophic crisp α-open set* $\Longrightarrow A \subseteq NCint(NCcl(A))$. *Hence* $G \subseteq A \subseteq NCint(NCcl(G))$, *where* $G = NCint(A)$
Sufficiency : obvious.
This completes the proof of the theorem.

Theorem 25 *For any neutrosophic crisp subset of* $NC\Gamma\ X$, *the following properties are equivalent:*

1. $A \in NC\alpha OS(X)$.

2. *There exists a neutrosophic crisp open set say G such that* $G \subseteq A \subseteq NCcl(NCint(NCcl(G)))$.

3. $A \subseteq NCcl(NCint(NCcl(A)))$

4. $NCcl(A) = NCcl(NCint(NCcl(A)))$

Proof. $(1) \Rightarrow (2)$. *Let* $A \in NC\alpha OS(X)$, *there exists a neutrosophic crisp α-open set U in X such that* $U \subseteq A \subseteq NCcl(U)$. *Hence there exists G neutrosophic crisp open set such that* $G \subseteq U \subseteq NCint(NCcl(G))$ *(by Theorem 24). Therefore* $NCcl(G) \subseteq NCcl(U) \subseteq NCcl(NCint(NCcl(G)))$. *Then* $G \subseteq U \subseteq A \subseteq NCcl(U) \subseteq NCcl(NCint(NCcl(G)))$. *Therefore* $G \subseteq A \subseteq NCcl(NCint(NCcl(G)))$ *for some G neutrosophic crisp open sets.*
$(2) \Rightarrow (3)$. *Let there exists a neutrosophic crisp open set say G such that* $G \subseteq A \subseteq NCcl(NCint(NCcl(G)))$. *Hence* $NCcl(G) \subseteq NCcl(NCint(A))$, *then* $NCint(NCcl(G)) \subseteq NCint(NCcl(NCint(A)))$. *Therefore,* $NCcl(NCint(NCcl(G))) \subseteq NCcl(NCint(NCcl(NCint(A))))$. *Then (by hypothesis)* $A \subseteq NCcl(NCint(NCcl(NCint(A))))$.
$(3) \Rightarrow (4)$. *Obvious.*
$(4) \Rightarrow (1)$. *Let* $NCcl(A) = NCcl(NCint(NCcl(NCint(A))))$. *Then* $A \subseteq NCcl(NCint(NCcl(NCint(A))))$. *To prove* $A \in NC\alpha OS(X)$. *Since NCint* $NCcl(NCint(A)) \subseteq NCint(NCcl(A))$, *therefore* $NCcl(NCint(NCcl(NCint(A)))) \subseteq NCcl(NCint(A)) \Rightarrow A \subseteq NCcl(NCint(A))$. *let* $U = NCint(A)$ *Hence there exists a neutrosophic crisp open set U such that* $U \subseteq A \subseteq NCcl(U)$. *On othere hand, U is neutrosophic crisp α-open set. Hence* $A \in NC\alpha OS(X)$.

Proposition 26 *Let* (X, Γ) *be a* $NCTS$, *then arbitrary union of neutrosophic crisp α-open set is a neutrosophic crisp α-open set and arbitrary intersection neutrosophic crisp α-closed set is neutrosophic crisp α-closed set.*

Proof. *Let* $A = \{A_i, A_i, A_i \mid i \in \Lambda\}$ *be a collection of neutrosophic crisp α-open sets. Then, for each* $i \in \Lambda$, $A_i \subseteq NCint(NCcl(NCint(A_i)))$. *It follows that*

$$\begin{aligned}\bigcup A_i &\subseteq \bigcup NCint(NCcl(NCint(A_i))) \\ &\subseteq NCint(\bigcup NCcl(NCint(A_i))) \\ &= NCint(NCcl(\bigcup NCint(A_i))) \\ &\subseteq NCint(NCcl(NCint(\bigcup A_i)))\end{aligned}$$

Hence $\cup A_i$ *is a neutrosophic crisp α-open set. The second part follows immediately from the first part by taking complements.*

Having shown that arbitrary union of neutrosophic crisp α-open sets is a neutrosophic crisp α-open set, it is natural to consider whether or not the intersection of neutrosophic crisp α-open sets is a neutrosophic crisp α-open set, and the following example shows that the intersection of neutrosophic crisp α-open sets is not a neutrosophic crisp α-open set.

Example 27 *Let* $X = \{a, b, c, d\}$, ϕ_N, X_N *be any types of the universal and empty subset, and* $A_1 = \langle\{a\}, \{b\}, \{c\}\rangle$ $A_2 = \langle\{a\}, \{b, d\}, \{c\}\rangle$, *then the family* $\Gamma = \{\phi_N, X_N, A_1, A_2\}$ *is a neutrosophic crisp topology on X. Let* $A_3 = \langle\{b\}, \{c\}, \{d\}\rangle$ *The* NCS A_1 *& and* A_3 *are neutrosophic crisp open (NCOS), then its sets neutrosophic crisp α-open sets i.e* $(A \subseteq NCint(NCcl(NCint(A))))$. *In fact,* $A_1 \cap A_3$ *is a* NCS *on X given by* $A_1 \cap A_3 = \langle\phi, \phi, \{d, c\}\rangle$ *or* $A_2 \cap A_3 = \langle\phi, \{d, b, c\}, \{d, c\}\rangle$ *and so* $A_2 \cap A_3 \nsubseteq NCint(NCcl(NCint(A_2 \cap A_3)))$ *and hence the intersection is not neutrosophic crisp α-open set.*

Proposition 28 *In a* $NCTS$ (X, Γ), *a* NCS *A is neutrosophic crisp α-closed if and only if* $A = \alpha NCcI(A)$.
Proof. *Assume that A is a neutrosophic crisp α-closed set. Obviously,*
$A \in \{B_i | B_i$ *is a neutrosophic crisp α-closed set and* $A \subseteq B_i\}$.
Also, $A \in \bigcap\{B_i | B_i$ *is a neutrosophic crisp α-closed set and* $A \subseteq B_i\} = NCcl(A)$.
Conversely, suppose that $A = \alpha NCcI(A)$, *which shows that.*
$A \in \{B_i | B_i$ *is a neutrosophic crisp α-closed set and* $A \subseteq B_i\}$
Hence A is a neutrosophic crisp α-closed set.

4 Neutrosophic Crisp Continuity

Definition 29 *Let* (X, Γ_1) *and* (Y, Γ_2) *be two* $NCTS$ *and let* $f : X \to Y$ *be a function then f is said to be*
(1) Continuous [20] iff the preimage of each NCS *in* Γ_2 *is a* NCS *in* Γ_1. *i.e* $f^{-1}(B)$ *is neutrosophic crisp open set in X for each neutrosophic crisp open set B in Y where* $B = \{B_1, B_2, B_3\}$, *then the preimage of B under f, denoted by* $f^{-1}(B)$, *is neutrosophic crisp open in X defined by* $f^{-1}(B) = \langle f^{-1}(B_1), ^{-1}(B_2), f^{-1}(B_3)\rangle$.

(2) Open [20], iff the image of each NCS in Γ_1 is a NCS in Γ_2. i.e, if $A = \{A_1, A_2, A_3\}$ is a NCS in X, then the image of A under f denoted by $f(A)$ is NCS in Y defined by $f(A) = \{f(A_1), f(A_2), f(A_3)^c\}$.

Corollary 30 *[20] Let $A = \{A_i, i \in J\}$, be neutrosophic crisp sets in X, and $B = \{B_j, j \in K\}$ neutrosophic crisp sets in Y, and $: X \to Y$ be a function. Then*

1. *$A_1 \subseteq A_2 \Leftrightarrow f(A_1) \subseteq f(A_2)$, and $B_1 \subseteq B_2 \Leftrightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$,*
2. *$f^{-1}(\bigcup B_i) = \bigcup f^{-1}(B_i)$, $f^{-1}(\bigcap B_i) = \bigcap f^{-1}(B_i)$,*
3. *$f^{-1}(Y_N) = X_N$, $f^{-1}(\phi_N) = \phi_N$,*
4. *$A \subseteq B \Rightarrow NCcl(A) \subseteq NCcl(B)$,*
5. *$A \subseteq f^{-1}(f(A))$, and if f is surjective, then $A = f^{-1}(f(A))$.*

Definition 31 *Let $f : X \to Y$ be a function from a $NCTS$ (X, Γ_1) into a $NCTS$ (Y, Γ_2) is said to be*

1. *neutrosophic crisp α-continuous if $f^{-1}(B)$ is α-neutrosophic crisp open set in X for each neutrosophic crisp open set B in Y.*
2. *neutrosophic crisp pre-continuous if $f^{-1}(B)$ is neutrosophic crisp pre-open set in X for each neutrosophic crisp open set B in Y.*
3. *neutrosophic crisp $semi$-continuous if $f^{-1}(B)$ is neutrosophic crisp $semi$-open set in X for each neutrosophic crisp open set B in Y.*
4. *neutrosophic crisp $semipre$-continuous if $f^{-1}(B)$ is neutrosophic crisp $semi$-open set in X for each neutrosophic crisp open set B in Y.*
5. *neutrosophic crisp β-continuous if $f^{-1}(B)$ isneutrosophic crisp $semi$-open set in X for each neutrosophic crisp open set B in Y.*

Theorem 32 *For a mapping f from a $NCTS$ (X, Γ_1) to a $NCTS$ (X, Γ_2), the following are equivalent.*

1. *f is neutrosophic crisp pre-continuouss.*
2. *$f^{-1}(B)$ is a $NCPCS$ in X for every $NCCS$ B in Y.*
3. *$NCcl(NCint(f^{-1}(A))) \subseteq f^{-1}(NCcl(A))$ for every NCS A in Y.*

Proof. $(1) \Rightarrow (1)$. *The proof is straightforward.*
$(2) \Rightarrow (3)$. *Let A be a NCS in Y. Then $cl(A)$ is neutrosophic crisp closed. It follows from 2 that $f^{-1}(NCcl(A))$ is a $NCPCS$ in X so that*

$$NCcl(NCint(f^{-1}(A))) \subseteq NCcl(NCint(f^{-1}(NCcl(A)))) \subseteq f^{-1}(NCcl(A))$$

$(3) \Rightarrow (1)$. *Let A be a $NCOS$ in Y. Then A is a $NCCS$ in Y, and so*

$$NCcl(NCint(f^{-1}(\overline{A}))) \subseteq f^{-1}(NCcl(\overline{A})) = f^{-1}(\overline{A}).$$

This implies

$$\begin{aligned}\overline{NCint(NCcl(f^{-1}(A)))} &= NCcl(\overline{NCcl(f^{-1}(A))}) \\ &= NCcl(NCint(\overline{f^{-1}(A)})) \\ &= NCcl(NCint(f^{-1}(\overline{A}))) \\ &\subseteq f^{-1}(\overline{A}) = f^{-1}(\overline{A}) = \overline{f^{-1}(A)},\end{aligned}$$

and thus $f^{-1}(A) \subseteq NCint(NCcl(f^{-1}(A)))$. Hence $f^{-1}(A)$ is a $NCPOS$ in X, and f is neutrosophic crisp pre-continuous.

Theorem 33 *Let f be a mapping from a $NCTS$ (X, Γ_1) to a $NCTS$ (Y, Γ_2). Then the following assertions are equivalent.*

1. *f is neutrosophic crisp pre-continuous.*
2. *For each NCP $p(a_{i1}, a_{i2}, a_{i3}) \in X$ and every (NCN) A of $f(p(a_{i1}, a_{i2}, a_{i3}))$, there exists a $NCPOS$ B in X such that $p(a_{i1}, a_{i2}, a_{i3}) \in B \subseteq f^{-1}(A)$.*
3. *For each NCP $p(a_{i1}, a_{i2}, a_{i3}) \in X$ and every (NCN) A of $f(p(a_{i1}, a_{i2}, a_{i3}))$, there exists a $NCPOS$ B in X such that $p(a_{i1}, a_{i2}, a_{i3}) \in B \subseteq A$.*

Proof. $(1) \Rightarrow (2)$. *Let $p(a_{i1}, a_{i2}, a_{i3})$ be a NCP in X and let A be a NCN of $f(p(a_{i1}, a_{i2}, a_{i3}))$. Then there exists a $NCOS$ B in Y such that $f(p(a_{i1}, a_{i2}, a_{i3})) \in B \subseteq A$. Since f is neutrosophic crisp pre-continuous,*
we know that $f^{-1}(B)$ is a $NCPOS$ in X and

$$p(a_{i1}, a_{i2}, a_{i3}) \in f^{-1}(f(p(a_{i1}, a_{i2}, a_{i3}))) \subseteq f^{-1}(B) \subseteq f^{-1}(A).$$

Thus (2) is valid.
$(2) \Rightarrow (3)$. *Let $p(a_{i1}, a_{i2}, a_{i3})$ be a NCP in X and let A be a NCN of $f(p(a_{i1}, a_{i2}, a_{i3}))$. The condition (2) implies that there exists a $NCPOS$ B in X such that $p(a_{i1}, a_{i2}, a_{i3}) \in B \subseteq f^{-1}(A)$ so that $p(a_{i1}, a_{i2}, a_{i3}) \in B$ and $f(B) \subseteq f(f^{-1}(A)) \subseteq A$. Hence (3) is true.*

$(3) \Rightarrow (1)$ *Let B be a $NCOS$ in Y and let $p(a_{i1}, a_{i2}, a_{i3}) \in f^{-1}(B)$. Then $f(p(a_{i1}, a_{i2}, a_{i3})) \in B$, and so B is a NCN of $f(p(a_{i1}, a_{i2}, a_{i3}))$ since B is a $NCOS$. It follows from (3) that there exists a $NCPOS$ A in X such that $p(a_{i1}, a_{i2}, a_{i3}) \in A$ and $f(A) \subseteq B$ so that.*

$$p(a_{i1}, a_{i2}, a_{i3}) \in A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(B).$$

Applying Theorem 20 induces that $f^{-1}(B)$ is a $NCPOS$ in X. Therefore, f is neutrosophic crisp pre-continuous.

Theorem 34 *Let f be a mapping from $NCTS$ (X, Γ_1) to $NCTS$ (Y, Γ_2) that satisfies*

$$NCcl(NCint(f^{-1}(NCcl(B)))) \subseteq f^{-1}(NCcl(B))$$

for every NCS B in Y. Then f is neutrosophic crisp α-continuous.

Proof. *Let B be a $NCOS$ in Y. Then B is a $NCCS$ in Y, which implies from hypothesis that*

$$NCcl(NCint(f^{-1}(NCcl(\overline{B})))) \subseteq f^{-1}(NCcl(\overline{B})) = f^{-1}(\overline{B}).$$

Its follows

$$\overline{NCint(NCcl(NCint(f^{-1}(B))))}$$
$$= NCcl(\overline{NCcl(NCint(f^{-1}(B)))})$$
$$= NCcl(NCint(\overline{NCint(f^{-1}(B))}))$$
$$= NCcl(NCint(NCcl(\overline{f^{-1}(B)})))$$
$$= NCcl(NCint(NCcl(f^{-1}\overline{(B)})))$$
$$\subseteq f^{-1}\overline{(B)}$$
$$= \overline{f^{-1}(B)}$$

so that $f^{-1}(B) \subseteq NCint(NCcl(NCint(f^{-1}(B))))$. This shows that $f^{-1}(B)$ is a $NC\alpha OS$ in X. Hence, f is neutrosophic crisp α-continuous.

Theorem 35 *Let f be a mapping from a $NCTS$ (X, Γ_1) to a $NCTS$ (Y, Γ_2). Then the following assertions are equivalent.*

1. *f is neutrosophic crisp α-continuous.*
2. *For each NCP $p(a_{i1}, a_{i2}, a_{i3}) \in X$ and every (NCN) A of $f(p(a_{i1}, a_{i2}, a_{i3}))$, there exists a $NC\alpha OS$ B in X such that $p(a_{i1}, a_{i2}, a_{i3}) \in B \subseteq f^{-1}(A)$.*
3. *For each NCP $p(a_{i1}, a_{i2}, a_{i3}) \in X$ and every (NCN) A of $f(p(a_{i1}, a_{i2}, a_{i3}))$, there exists a $NC\alpha OS$ B in X such that $p(a_{i1}, a_{i2}, a_{i3}) \in B$ and $f(B) \subseteq A$.*

Proof. (1) $\Rightarrow$ (2). *Let $p(a_{i1}, a_{i2}, a_{i3})$ be a NCP in X and let A be a NCN of $f(p(a_{i1}, a_{i2}, a_{i3}))$. Then there exists a $NCOS$ B in Y such that $f(p(a_{i1}, a_{i2}, a_{i3})) \in C \subseteq A$. Since f is neutrosophic crisp α-continuous,*
we know that $f^{-1}(B)$ is a $NC\alpha OS$ in X and

$$p(a_{i1}, a_{i2}, a_{i3}) \in {}^{-1}(f(p(a_{i1}, a_{i2}, a_{i3}))) \subseteq f^{-1}(C) = B \subseteq f^{-1}(A).$$

Thus (2) is valid.
(2) $\Rightarrow$ (3). *Let $p(a_{i1}, a_{i2}, a_{i3})$ be a NCP in X and let A be a NCN of $f(p(a_{i1}, a_{i2}, a_{i3}))$. The condition (2) implies that there exists a $NC\alpha OS$ B in X such that $p(a_{i1}, a_{i2}, a_{i3}) \in B \subseteq f^{-1}(A)$, by (2). Thus, we have $p(a_{i1}, a_{i2}, a_{i3}) \in B$ and $f(B) \subseteq f(f^{-1}(A)) \subseteq A$. Hence (3) is true.*

(3) $\Rightarrow$ (1) *Let B be a $NCOS$ in Y and let $p(a_{i1}, a_{i2}, a_{i3}) \in f^{-1}(B)$. Then $f(p(a_{i1}, a_{i2}, a_{i3})) \inf(f^{-1}(B)) \subseteq B$ and so B is a NCN of $f(p(a_{i1}, a_{i2}, a_{i3}))$ since B is a $NCOS$. It follows from (3) that there exists a $NC\alpha OS$ A in X such that $p(a_{i1}, a_{i2}, a_{i3}) \in A$ and $f(A) \subseteq B$ so that.*

$$p(a_{i1}, a_{i2}, a_{i3}) \in A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(B).$$

Using Theorem 20 induces that $f^{-1}(B)$ is a $NC\alpha OS$ in X. and hence f is neutrosophic crisp α-continuous.

Combining Theorems 35, 34, we have the following characterization of a neutrosophic crisp α-continuous mapping.

Theorem 36 *Let f be a mapping from $NCTS$ (X, Γ_1) to $NCTS$ (Y, Γ_2). Then the following assertions are equivalent.*

1. *f is neutrosophic crisp α-continuous.*
2. *If C is a $NCCS$ in Y, then $f^{-1}(C)$ is a $NC\alpha CS$ in X.*
3. *$NCcl(NCint(f^{-1}(NCcl(B)))) \subseteq f^{-1}(NCcl(B))$ for every NCS B in Y.*
4. *For each NCP $p(a_{i1}, a_{i2}, a_{i3}) \in X$ and every (NCN) A of $f(p(a_{i1}, a_{i2}, a_{i3}))$, there exists a $NC\alpha OS$ B in X such that $p(a_{i1}, a_{i2}, a_{i3}) \in B \subseteq f^{-1}(A)$.*
5. *For each NCP $p(a_{i1}, a_{i2}, a_{i3}) \in X$ and every (NCN) A of $f(p(a_{i1}, a_{i2}, a_{i3}))$, there exists a $NC\alpha OS$ B in X such that $p(a_{i1}, a_{i2}, a_{i3}) \in B$ and $f(B) \subseteq A$.*

Some aspects of neutrosophic crisp continuity, neutrosophic crisp α-continuity, neutrosophic crisp pre-continuity, neutrosophic crisp $semi$-continuity, and neutrosophic crisp β-continuity are studied in this paper and as well as in several papers, see [20]. The relation among these types of neutrosophic crisp continuity is given as follows, where NC means neutrosophic crisp.

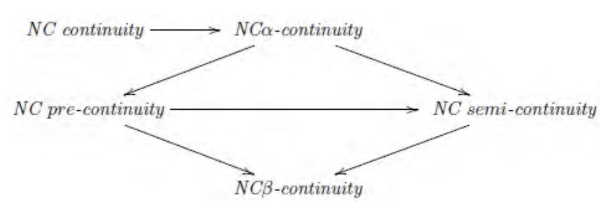


Figure 1: Diagram 2

Remark 37 *The reverse implications are not true in the above diagram in general.*

Example 38 *Let (X, Γ_0) and (Y, Ψ_0) be two $NCTS$. If $: X \to Y$ is continuous in the usual sense, then in this case, f is continuous in the sense of $f(A) = \{f(A_1), f(A_2), f(A_3)^c\}$. Here we* *consider the $NCTS$ on X and Y, respectively, as follows: $\Gamma_1 = \{\langle G, \phi, G^c : G \in \Gamma_0\rangle\}$ and $\Gamma_2 = \{\langle H, \phi, H^c : H \in \Psi_0\rangle\}$, in this case we have $\langle H, \phi, H^c\rangle \in \Gamma_2$, $H \in \Psi_0$, $f^{-1}\langle H, \phi, H^c\rangle = \langle f^{-1}(H), f^{-1}(\phi), f^{-1}(H^c)\rangle = \langle f^{-1}(H), f(\phi), (f(H))^c\rangle \in \Gamma_1$.*

Example 39 *Let f be a mapping from a $NCTS$ (X, Γ_1) to a $NCTS$ (Y, Γ_2), and let $X \doteq Y \doteq \{a, b, c, d\}$, ϕ_N, X_N be any types of the universal and empty subset, and $A_1 \doteq \langle\{a\}, \{b\}, \{c\}\rangle$ $A_2 \doteq \langle\{a\}, \{b, d\}, \{c\}\rangle$, then the family $\Gamma_1 \doteq \Gamma_2 \doteq \{\phi_N, X_N, A_1, A_2\}$ is a neutrosophic crisp topology on X and Y. Then f is neutrosophic crisp continuous function, since $f^{-1}(A_1) \doteq A_1$ & and $f^{-1}(A_2) \doteq A_2$ are neutrosophic crisp open in X ($NCOS$), and hence its neutrosophic crisp α-continuous, since $f^{-1}(A_1), f^{-1}(A_2)$ is α-neutrosophic crisp open set in X.*

Example 40 *Let $X = \{a, b, c, d\}$, $Y = \{u, v, w\}$ and $A_1 = \langle\{a\}, \{b\}, \{c\}\rangle$, $A_2 = \langle\{a\}, \{b, d\}, \{c\}\rangle$, $A_3 = \langle\{u\}, \{v\}, \{w\}\rangle$. Then $\Gamma_1 = \{\phi_N, X_N, A_1, A_2\}$, $\Gamma_2 = \{\phi_N, X_N, A_3\}$ are neutrosophic crisp topology on X and Y respectively. Defined a mapping $f : (X, \Gamma_1) \longrightarrow (Y, \Gamma_2)$ by $f(\{a\}) = \{u\}$, $f(\{d\}) = \{v\}$ and $f(\{c\}) = \{w\}$. Then f is neutrosophic crisp α-continuous function but not neutrosophic crisp continuous function.*

Example 41 *Let $X = \{a, b, c, d\}$, $Y = \{u, v, w\}$ and $A_1 = \langle\{a\}, \{b\}, \{c\}\rangle$, $A_2 = \langle\{a\}, \{b, d\}, \{c\}\rangle$, $A_3 = \langle\{u\}, \{v\}, \{w\}\rangle$. Then $\Gamma_1 = \{\phi_N, X_N, A_1, A_2\}$, $\Gamma_2 = \{\phi_N, X_N, A_3\}$ are neutrosophic crisp topology on X and Y respectively. Defined a mapping $f : (X, \Gamma_1) \longrightarrow (Y, \Gamma_2)$ by $f(\{a\}) = f(\{b\}) = \{u\}$, $f(\{c\}) = \{v\}$ and $f(\{d\}) = \{w\}$. Then f is neutrosophic crisp pre-continuous function but not neutrosophic crisp regular-continuous function. Also f is neutrosophic crisp β-continuous function but not neutrosophic crisp $semi$-continuous function, since $f^{-1}(A_3) = \langle\{a, b\}, \{c\}, \{d\}\rangle$ is neutrosophic crisp pre-open set but not neutrosophic crisp regular-open set, also A_4 is neutrosophic crisp β-open but not neutrosophic crisp $semi$-open set.*

Theorem 42 *Let f be a mapping from $NCTS$ (X, Γ_1) to $NCTS$ (Y, Γ_2). If f is both neutrosophic crisp pre-continuous and neutrosophic crisp $semi$-continuous, then it is neutrosophic crisp α-continuous.*

Proof. *Let B be a $NCOS$ in Y. Since f is both neutrosophic crisp pre-continuous and neutrosophic crisp $semi$-continuous, $f^{-1}(B)$ is both a $NCPOS$ and a $NCSOS$ in X. It follows from Theorem 17 that $f^{-1}(B)$ is a $NC\alpha OS$ in X so that f is ineutrosophic crisp α-continuous.*

5 Conclusions and Discussions

In this paper, we have introduced neutrosophic crisp β-open, Neutrosophic crisp semipre-open, Neutrosophic crisp regular-open, Neutrosophic crisp semiα-open sets and studied some of their basic properties. Also we study the relationship between the newly introduced sets and some of the Neutrosophic crisp open sets that already existed. In this paper, we also introduced Neutrosophic crisp closed sets and studied some of their basic properties. Finally, we introduced the definition of neutrosophic crisp continuous function, and studied some of its basic properties.

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Rough Neutrosophic TOPSIS for Multi-Attribute Group Decision Making

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Abstract: This paper is devoted to present Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method for multi-attribute group decision making under rough neutrosophic environment. The concept of rough neutrosophic set is a powerful mathematical tool to deal with uncertainty, indeterminacy and inconsistency. In this paper, a new approach for multi-attribute group decision making problems is proposed by extending the TOPSIS method under rough neutrosophic environment. Rough neutrosophic set is characterized by the upper and lower approximation operators and the pair of neutrosophic sets that are characterized by truth-membership degree, indeterminacy membership degree, and falsity membership degree. In the decision situation, ratings of alternatives with respect to each attribute are characterized by rough neutrosophic sets that reflect the decision makers' opinion. Rough neutrosophic weighted averaging operator has been used to aggregate the individual decision maker's opinion into group opinion for rating the importance of attributes and alternatives. Finally, a numerical example has been provided to demonstrate the applicability and effectiveness of the proposed approach.

Keywords: Multi-attribute group decision making; Neutrosophic set; Rough set; Rough neutrosophic set; TOPSIS

1 Introduction

Hwang and Yoon [1] put forward the concept of Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) in 1981 to help select the best alternative with a finite number of criteria. Among numerous multi criteria decision making (MCDM) methods developed to solve real-world decision problems, (TOPSIS) continues to work satisfactorily in diverse application areas such as supply chain management and logistics [2, 3, 4, 5], design, engineering and manufacturing systems [6, 7], business and marketing management [8, 9], health, safety and environment management[10, 11], human resources management [12, 13, 14], energy management [15], chemical engineering [16], water resources management [17, 18], bi-level programming problem [19, 20], multi-level programming problem [21], medical diagnosis [22], military [23], education [24], others topics [25, 26, 27, 28, 29, 30], etc. Behzadian et al. [31] provided a state-of the-art literature survey on TOPSIS applications and methodologies. According to C. T. Chen [32], crisp data are inadequate to model real-life situations because human judgments including preferences are often vague. Preference information of alternatives provided by the decision makers may be poorly defined, partially known and incomplete. The concept of fuzzy set theory grounded by L. A. Zadeh [33] is capable of dealing with impreciseness in a mathematical form. Interval valued fuzzy set [34, 35, 36, 37] was proposed by several authors independently in 1975 as a generalization of fuzzy set. In 1986, K. T. Atanassov [38] introduced the concept of intuitionistic fuzzy set (IFS) by incorporating non-membership degree as independent entity to deal non-statistical impreciseness. In 2003, mathematical equivalence of intuitionistic fuzzy set (IFS) with interval-valued fuzzy sets was proved by Deschrijver and Kerre [39]. C. T. Chen [32] studied the TOPSIS method in fuzzy environment for solving multi-attribute decision making problems. Boran et al. [12] studied TOPSIS method in intuitionistic fuzzy environment and provided an illustrative example of personnel selection in a manufacturing company for a sales manager position. However, fuzzy sets and interval fuzzy sets are not capable of all types of uncertainties in different real physical problems involving indeterminate information.

In order to deal with indeterminate and inconsistent information, the concept of neutrosophic set [40, 41, 42, 43] is useful. In neutrosophic set each element of the universe is characterized by the truth membership degree, indeterminacy membership degree and falsity membership degree lying in the non-standard unit interval $]^-0, 1^+[$. However, it is difficult to apply directly the neutrosophic set in real engineering and scientific applications. Wang et al. [44] introduced single-valued neutrosophic set (SVNS) to face real scientific and engineering fields involving imprecise, incomplete, and inconsistent information. However, the idea was envisioned some years earlier by Smarandache [43] SVNS, a subclass of NS, can also represent each element of universe with the truth membership values, indeterminacy membership values and falsity membership values lying in the real unit interval [0, 1]. SVNS has caught much attention to the researchers on various topics such as, medical diagnosis [45], similarity measure [46, 47, 48, 49, 50], decision making [51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70], educational problems [71, 72], conflict resolution [73], social problem [74, 75], optimization [76, 77, 78, 79, 80, 81], etc.

Pawlak [82] proposed the notion of rough set theory for the study of intelligent systems characterized by inexact, uncertain or insufficient information. It is a useful mathematical tool for dealing with uncertainty or incomplete information. Broumi et al. [83, 84] proposed new hybrid intelligent structure called rough neutrosophic set by combining the concepts of single valued neutrosophic set and rough set. The theory of rough neutrosophic set [83, 84] is also a powerful mathematical tool to deal with incompleteness. Rough neutrosophic set can be applied in addressing problems with uncertain, imprecise, incomplete and inconsistent information existing in real scientific and engineering applications. In rough neutrosophic environment, Mondal and Pramanik [85] proposed rough neutrosophic multi-attribute decision-making based on grey relational analysis. Mondal and Pramanik [86] also proposed rough neutrosophic multi-attribute decision-making based on rough accuracy score function. Pramanik and Mondal [87] proposed cotangent similarity measure of rough neutrosophic sets and its application to medical diagnosis. Pramanik and Mondal [88] also proposed cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis. Pramanik and Mondal [88] also proposed some similarity measures namely, Dice and Jaccard similarity measures in rough neutrosophic environment and applied them for multi attribute decision making problem. Pramanik and Mondal [90] studied decision making in rough interval neutrosophic environment in 2015. Mondal and Pramanik [91] studied cosine, Dice and Jaccard similarity measures for interval rough neutrosophic sets and presented their applications in decision making problem. So decision making in rough neutrosophic environment appears to be a developing area of study. Mondal et al. [92] proposed rough trigonommetric Hamming similarity measures such as cosine, sine and cotangent rough similarity meaures and proved their basic properties. In the same study Mondal et al. [92] also provided a numerical example of selection of a smart phone for rough use based on the proposed methods.

The objective of the study is to extend the concept of TOPSIS method for multi-attribute group decision making (MAGDM) problems under single valued neutrosophic rough neutrosophic environment. All information provided by different domain experts in MAGDM problems about alternative and attribute values take the form of rough neutrosophic set. In a group decision making process, rough neutrosophic weighted averaging operator is used to aggregate all the decision makers' opinions into a single opinion to select best alternative.

The remaining part of the paper is organized as follows: section 2 presents some preliminaries relating to neutrosophic set, section 3 presents the concept of rough neutrosophic set. In section 4, basics of TOPSIS method are discussed. Section 5 is devoted to present TOPSIS method for MAGDM under rough neutrosophic environment. In section 6, a numerical example is provided to show the effectiveness of the proposed approach. Finally, section 7 presents the concluding remarks and scope of future research.

2 Neutrosophic sets and single valued neutrosophic set [43, 44]

2.1 Definition of Neutrosophic sets [40, 41, 42, 43]

Definition 2.1.1. [43]:

Assume that V be a space of points and v be a generic element in V. Then a neutrosophic set G in V is characterized by a truth membership function T_G, an indeterminacy membership function I_G and a falsity membership function F_G. The functions T_G, I_G and F_G are real standard or non-standard subsets of $]^-0, 1^+[$ i.e. $T_G: V \to]^-0, 1^+[$, $I_G: V \to]^-0, 1^+[$, $F_G: V \to]^-0, 1^+[$, and $^-0 \leq T_G(v) + I_G(v) + F_G(v) \leq 3^+$.

2.1.2.[43]:

The complement of a neutrosophic set G is denoted by G^c and is defined by

$T_{G^c}(v) = \{1^+\} - T_G(v)$; $I_{G^c}(v) = \{1^+\} - I_G(v)$;

$F_{G^c}(v) = \{1^+\} - I_G(v)$

Definition 2.1.3. [43]:

A neutrosophic set G is contained in another neutrosophic set H, $G \subseteq H$ iff the following conditions holds.

$\inf T_G(v) \leq \inf T_H(v)$ $\sup T_G(v) \leq \sup T_H(v)$

$\inf I_G(v) \geq \inf I_H(v)$, $\sup I_G(v) \geq \sup I_H(v)$

$\inf F_G(v) \geq \inf F_H(v)$, $\sup F_G(v) \geq \sup F_H(v)$

for all v in V.

Definition 2.1.4. [44]:

Assume that V be a universal space of points, and v be a generic element of V. A single-valued neutrosophic set P is characterized by a truth membership function $T_P(v)$, a falsity membership function $I_P(v)$, and an indeterminacy membership function $F_P(v)$. Here, $T_P(v)$, $I_P(v)$, $F_P(v) \in [0, 1]$. When V is continuous, a SVNS P can be written as

$$P = \int_V \langle\langle T_P(v), F_P(v), I_P(v)\rangle / v, v \in V.$$

When V is discrete, a SVNS P can be written as

$$P = \sum \langle T_P(v), F_P(v), I_P(v)\rangle / v, \ \forall v \in V$$

It is obvious that for a SVNS P,

$$0 \le \sup T_P(v) + \sup F_P(v) + \sup I_P(v) \le 3, \ \forall v \in V$$

Definition 2.1.5. [44]:
The complement of a SVNS set P is denoted by P^C and is defined as follows:

$$T_P^C(v) = F_P(v); \ I_P^C(v) = 1 - I_P(v); \ F_P^C(v) = T_P(v)$$

Definition 2.1.6. [44]:
A SVNS P_G is contained in another SVNS P_H, denoted as $P_G \subseteq P_H$ if the following conditions hold.

$$T_{P_G}(v) \le T_{P_H}(v) \ ; \ I_{P_G}(v) \ge I_{P_H}(v) \ ; \ F_{P_G}(v) \ge F_{P_H}(v) \ , \ \forall v \in V.$$

Definition 2.1.7. [44]:
Two SVNSs P_G and P_H are equal, i.e., $P_G = P_H$, iff $P_G \subseteq P_H$ and $P_G \supseteq P_H$

Definition 2.1.8. [44]:
The union of two SVNSs P_G and P_H is a SVNS P_Q, written as $P_Q = P_G \cup P_H$.
Its truth, indeterminacy and falsity membership functions are as follows:

$$T_{P_Q}(v) = \max(T_{P_G}(v), T_{P_H}(v)) ;$$

$$I_{P_Q}(v) = \min(I_{P_G}(v), I_{P_H}(v)) ;$$

$$F_{P_Q}(v) = \min(F_{P_G}(v), F_{P_H}(v)), \ \forall v \in V.$$

Definition 2.1.9. [44]:
The intersection of two SVNSs P_G and P_H is a SVNS P_C written as $P_C = P_G \cap P_H$. Its truth, indeterminacy and falsity membership functions are as follows:

$$T_{P_C}(v) = \min(T_{P_G}(v), T_{P_H}(v)) ;$$

$$I_{P_C}(v) = \max(I_{P_G}(v), I_{P_H}(v)) ;$$

$$F_{P_C}(v) = \max(F_{P_G}(v), F_{P_H}(v)), \ \forall v \in V.$$

Definition 2.1.10. [44]:
Wang et al. [44] defined the following operation for two SVNS P_G and P_H as follows:

$$P_G \otimes P_H = \left\langle \begin{matrix} T_{P_G}(v).T_{P_H}(v), \ I_{P_G}(v) + I_{P_H}(v) - I_{P_G}(v).I_{P_H}(v), \\ F_{P_G}(v) + F_{P_H}(v) - F_{P_G}(v).F_{P_H}(v) \end{matrix} \right\rangle,$$

$\forall v \in V$.

Definition 2.1.11. [93]
Assume that

$$P_G = \left\{ \begin{matrix} (v_1 / (T_{P_G}(v_1), I_{P_G}(v_1), F_{P_G}(v_1))), \cdots, \\ (v_n / (T_{P_G}(v_n), I_{P_G}(v_n), F_{P_G}(v_n))) \end{matrix} \right\}$$

$$P_H = \left\{ \begin{matrix} (v_1 / (T_{P_H}(v_1), I_{P_H}(v_1), F_{P_H}(v_1))), \cdots, \\ (v_n / (T_{P_H}(v_n), I_{P_H}(v_n), F_{P_H}(v_n))) \end{matrix} \right\}$$

be two SVNSs in $v = \{v_1, v_2, v_3, \ldots, v_n\}$

Then the Hamming distance [93] between two SVNSs P_G and P_H is defined as follows:

$$d_P(P_G, P_H) = \sum_{i=1}^{n} \left\langle \begin{matrix} |T_{P_G}(v) - T_{P_H}(v)| + |I_{P_G}(v) - I_{P_H}(v)| \\ + |F_{P_G}(v) - F_{P_H}(v)| \end{matrix} \right\rangle \quad (1)$$

and normalized Hamming distance [93] between two SVNSs P_G and P_H is defined as follow

$$^N d_P(P_G, P_H) = \frac{1}{3n} \sum_{i=1}^{n} \left\langle \begin{matrix} |T_{P_G}(v) - T_{P_H}(v)| + |I_{P_G}(v) - I_{P_H}(v)| \\ + |F_{P_G}(v) - F_{P_H}(v)| \end{matrix} \right\rangle \quad (2)$$

with the following two properties

i. $0 \le d_P(P_G, P_H) \le 3$

ii. $0 \le {}^N d_P(P_G, P_H) \le 1$

Distance between two SVNSs:
Majumder and Samanta [93] studied similarity and entropy measure by incorporating Euclidean distances of SVNSs.

Definition 2.1.12. [93]: (Euclidean distance)

$$\text{Let } P_G = \left\{ \begin{matrix} (v_1 | \langle T_{P_G}(v_1), I_{P_G}(v_1), F_{P_G}(v_1)\rangle), \cdots, \\ (v_n | \langle T_{P_G}(v_n), I_{P_G}(v_n), F_{P_G}(v_n)\rangle) \end{matrix} \right\} \text{ and}$$

$$P_H = \left\{ \begin{matrix} (v_1 | \langle T_{P_H}(v_1), I_{P_H}(v_1), F_{P_H}(v_1)\rangle), \cdots, \\ (v_n | \langle T_{P_H}(v_n), I_{P_H}(v_n), F_{P_H}(v_n)\rangle) \end{matrix} \right\} \text{ be two}$$

SVNSs for $v_i \in V$, where i = 1, 2, . . . , n. Then the Euclidean distance between two SVNSs P_G and P_H can be defined as follows:

$$D_{euclid}(P_G, P_H) = \left\langle \sum_{i=1}^{n} \left(\begin{matrix} (T_{P_G}(v_i) - T_{P_H}(v_i))^2 + (I_{P_G}(v_i) - I_{P_H}(v_i))^2 \\ + (F_{P_G}(v_i) - F_{P_H}(v_i))^2 \end{matrix} \right) \right\rangle^{0.5} \quad (3)$$

and the normalized Euclidean distance [93] between two SVNSs P_G and P_H can be defined as follows:

$$D^N_{euclid}(P_G, P_H) = \frac{1}{3n} \left\langle \sum_{i=1}^{n} \left(\begin{matrix} (T_{P_G}(v_i) - T_{P_H}(v_i))^2 + (I_{P_G}(v_i) - I_{P_H}(v_i))^2 \\ + (F_{P_G}(v_i) - F_{P_H}(v_i))^2 \end{matrix} \right) \right\rangle^{0.5} \quad (4)$$

Definition 2.1.13. (Deneutrosophication of SVNS) [53]:
Deneutrosophication of SVNS P_G can be defined as a process of mapping P_G into a single crisp output $\theta^* \in V$ i.e. $f : P_G \to \theta^*$ for $v \in V$. If P_G is discrete set then the vector $P_G = \{v \mid \langle T_{P_G}(v), I_{P_G}(v), F_{P_G}(v)\rangle \mid v \in V\}$ is reduced to a single scalar quantity $\theta^* \in V$ by deneutrosophication. The obtained scalar quantity $\theta^* \in V$ best represents the aggregate distribution of three membership degrees of neutrosophic element $\langle T_{P_G}(v), I_{P_G}(v), F_{P_G}(v)\rangle$

3 Rough neutrosophic set [83, 84]

Rough set theory [82] has been developed based on two basic components. The components are crisp set and equivalence relation. The rough set logic is based on the approximation of sets by a couple of sets. These two are known as the lower approximation and the upper approximation of a set. Here, the lower and upper approximation operators are based on equivalence relation. Rough neutrosophic sets [83, 84] are the generalization of rough fuzzy sets [94, 95, 96] and rough intuitionistic fuzzy sets [97].

Definition 3.1. **Rough neutrosophic set [83,84]**

Assume that S be a non-null set and ρ be an equivalence relation on S. Assume that E be neutrosophic set in S with the membership function T_E, indeterminacy function I_E and non-membership function F_E. The lower and the upper approximations of E in the approximation (S, ρ) denoted by $\underline{L}(E)$ and $\overline{U}(E)$ are respectively defined as follows:

$$\underline{L}(E)=\left\langle < v, T_{\underline{L}(E)}(v), I_{\underline{L}(E)}(v), F_{\underline{L}(E)}(v) > / s \in [v]_\rho, v \in S \right\rangle \quad (5)$$

$$\overline{U}(E)=\left\langle < v, T_{\overline{U}(E)}(v), I_{\overline{U}(E)}(v), F_{\overline{U}(E)}(v) > / s \in [v]_\rho, v \in S \right\rangle \quad (6)$$

Here, $T_{\underline{L}(E)}(v) = \wedge_s \in [v]_\rho \, T_E(s)$, $I_{\underline{L}(E)}(v) = \wedge_s \in [v]_\rho \, I_E(s)$, $F_{\underline{L}(E)}(v) = \wedge_s \in [v]_\rho \, F_E(s)$, $T_{\overline{U}(E)}(v) = \vee_s \in [v]_\rho \, T_E(s)$, $I_{\overline{U}(E)}(v) = \vee_s \in [v]_\rho \, T_E(s)$, $F_{\overline{U}(E)}(v) = \vee_s \in [v]_\rho \, I_E(s)$.

So, $0 \le T_{\underline{L}(E)}(v) + I_{\underline{L}(E)}(v) + F_{\underline{L}(E)}(v) \le 3$

$0 \le T_{\overline{U}(E)}(v) + I_{\overline{U}(E)}(v) + F_{\overline{U}(E)}(v) \le 3$

The symbols ∨ and ∧ indicate "max" and "min'' operators respectively. $T_E(s)$, $I_E(s)$ and $F_E(s)$ represent the membership , indeterminacy and non-membership of S with respect to E. $\underline{L}(E)$ and $\overline{U}(E)$ are two neutrosophic sets in S.

Thus the mapping $\underline{L}$, $\overline{U}$: N(S) → N(S) are, respectively, referred to as the lower and upper rough neutrosophic approximation operators, and the pair $(\underline{L}(E), \overline{U}(E))$ is called the rough neutrosophic set in (S, ρ).

$\underline{L}(E)$ and $\overline{U}(E)$ have constant membership on the equivalence classes of ρ if $\underline{L}(E) = \overline{U}(E)$; i.e. $T_{\underline{L}(E)}(v) = T_{\overline{U}(E)}(v)$, $I_{\underline{L}(E)}(v) = I_{\overline{U}(E)}(v)$, $F_{\underline{L}(E)}(v) = F_{\overline{U}(E)}(v)$ for any v belongs to S.

E is said to be definable neutrosophic set in the approximation (S, ρ). It is obvious that zero neutrosophic set (0_N) and unit neutrosophic sets (1_N) are definable neutrosophic sets.

Definition 3.2 [83, 84].

If N(E) = ($\underline{L}(E), \overline{U}(E)$) be a rough neutrosophic set in (S, ρ), the complement of N(E) is the rough neutrosophic set and is denoted as $\sim N(E) = (\underline{L}(E)^C, \overline{U}(E)^C)$,where $\underline{L}(E)^C, \overline{U}(E)^C$ are the complements of neutrosophic sets of $\underline{L}(E), \overline{U}(E)$ respectively.

$\underline{L}(E)^c = \left\langle < v, T_{\underline{L}(E)}(v), 1 - I_{\underline{L}(E)}(v), F_{\underline{L}(E)}(v) > / \, v \in S \right\rangle$ and

$\overline{U}(E)^c = \left\langle < v, T_{\overline{U}(E)}(v), 1 - I_{\overline{U}(E)}(v), F_{\overline{U}(E)}(v) > / \, v \in S \right\rangle$

Definition 3. 3 [83, 84]

If $N(E_1)$ and $N(E_2)$ be two rough neutrosophic sets in S, then the following definitions hold:

$N(E_1) = N(E_2) \Leftrightarrow \underline{L}(E_1) = \underline{L}(E_2) \wedge \overline{U}(E_1) = \overline{U}(E_2)$

$N(E_1) \subseteq N(E_2) \Leftrightarrow \underline{L}(E_1) \subseteq \underline{L}(E_2) \wedge \overline{U}(E_1) \subseteq \overline{U}(E_2)$

$N(E_1) \bigcup N(E_2) = < \underline{L}(E_1) \bigcup \underline{L}(E_2), \overline{U}(E_1) \bigcup \overline{U}(E_2) >$

$N(E_1) \bigcap N(E_2) = < \underline{L}(E_1) \bigcap \underline{L}(E_2), \overline{U}(E_1) \bigcap \overline{U}(E_2) >$

$N(E_1) + N(E_2) = < \underline{L}(E_1) + \underline{L}(E_2), \overline{U}(E_1) + \overline{U}(E_2) >$

$N(E_1) \,.\, N(E_2) = < \underline{L}(E_1) \,.\, \underline{L}(E_2), \overline{U}(E_1) \,.\, \overline{U}(E_2) >$

If α, β, γ be rough neutrosophic sets in (S, ρ), then the following properties are satisfied.

Properties I:

1. $\sim(\sim \alpha) = \alpha$
2. $\alpha \cup \beta = \beta \cup \alpha$, $\beta \cup \alpha = \alpha \cup \beta$
3. $(\gamma \cup \beta) \cup \alpha = \gamma \cup (\beta \cup \alpha)$,
   $(\gamma \cap \beta) \cap \alpha = \gamma \cap (\beta \cap \alpha)$
4. $(\gamma \cup \beta) \cap \alpha = (\gamma \cup \beta) \cap (\gamma \cup \alpha)$,
   $(\gamma \cap \beta) \cup \alpha = (\gamma \cap \beta) \cup (\gamma \cap \alpha)$

Proof. For proofs of the properies, see [83,84].

Properties II:

De Morgan's Laws are satisfied for rough neutrosophic sets

1. $\sim (N(E_1) \cup N(E_2)) = (\sim N(E_1)) \cap (\sim N(E_2))$
2. $\sim (N(E_1) \cap N(E_2)) = (\sim N(E_1)) \cup (\sim N(E_2))$

Proof. For proofs of the properies, see [83,84].

Properties III:

If E_1 and E_2 are two neutrosophic sets of universal collection (U) such that $E_1 \subseteq E_2$, *then* 1. $N(E_1) \subseteq N(E_2)$

2. $N(E_1 \bigcap E_2) \subseteq N(E_2) \bigcap N(E_2)$
3. $N(E_1 \bigcup E_2) \supseteq N(E_2) \bigcup N(E_2)$

Proof. For proofs of the properies, see [83,84].

Properties IV:

1. $\underline{L}(E) = \sim \overline{U}(\sim E)$
2. $\overline{U}(E) = \sim \underline{L}(\sim E)$
3. $\underline{L}(E) \subseteq \overline{U}(E)$

Proof. For proofs of the properies, see [83,84].

4 TOPSIS

The TOPSIS is used to determine the best alternative from the compromise solutions. The best compromise solution should have the shortest Euclidean distance from the positive ideal solution (PIS) and the farthest Euclidean distance from the negative ideal solution (NIS). The TOPSIS method can be described as follows. Assume that K = {K_1, K_2, …,K_m} be the set of alternatives, L = {L_1, L_2, …, L_n} be the set of criteria and $p_{ij}, i=1,2,\ldots,m; j=1,2,\ldots,n$ is the rating of the alternative K_i with respect to the criterion L_j, w_j is the weight of the j- th criterion L_j.

The procedure of TOPSIS method is presented using the following steps:

Step 1. Normalization the decision matrix

Calculation of the normalized value $[\vartheta]_{ij}^{N}$ is as follows:

For benefit criterion, $\vartheta_{ij}=(\vartheta_{ij}-\vartheta_j^-)/(\vartheta_j^+-\vartheta_j^-)$, where $\vartheta_j^+=\max_i (v_{ij})$ and $\vartheta_j^-=\min_i (v_{ij})$

or setting ϑ_j^+ is the desired level and ϑ_j^- is the worst level.

For cost criterion, $\vartheta_{ij}=(\vartheta_j^- -\vartheta_{ij})/(\vartheta_j^- -\vartheta_j^+)$

Step 2. Weighted normalized decision matrix

In the weighted normalized decision matrix, the upgraded ratings are calculated as follows:

$\eta_{ij}=w_j\times\eta_{ij}$ for i = 1, 2, . . . , m and j = 1, 2, . . . , n. Here w_j is the weight of the j-th criterion such that $w_j\geq 0$ for j = 1, 2, . . . , n and $\sum_{j=1}^{n}w_j=1$

Step 3. The positive and the negative ideal solutions

The positive ideal solution (PIS) and the negative ideal solution (NIS) are calculated as follows:

$PIS=M^+=\left\langle\eta_1^+,\eta_2^+,\cdots\eta_n^+\right\rangle$ =

$\left\langle\left(\max_j\eta_{ij}/j\in C_1\right),\left(\min_j\eta_{ij}/j\in C_2\right):j=1,2,\cdots,n\right\rangle$ and

$NIS=M^-=\left\langle\eta_1^-,\eta_2^-,\cdots\eta_n^-\right\rangle$ =

$\left\langle\left(\min_j\eta_{ij}/j\in C_1\right),\left(\max_j\eta_{ij}/j\in C_2\right):j=1,2,\cdots,n\right\rangle$

where $C_1$and C_2 are the benefit and cost type criteria respectively.

Step 4. Calculation of the separation measures for each alternative from the PIS and the NIS

The separation values for the PIS and the separation values for the NIS can be determined by using the n-dimensional Euclidean distance as follows:

$\delta_i^+=\left\langle\sum_{j=1}^{n}\left(\eta_{ij}-\eta_j^+\right)^2\right\rangle^{0.5}$ for i = 1, 2, . . . , m.

$\delta_i^-=\left\langle\sum_{j=1}^{n}\left(\eta_{ij}-\eta_j^-\right)^2\right\rangle^{0.5}$ for i = 1, 2, . . . , m.

Step 5. Calculation of the relative closeness coefficient to the PIS

The relative closeness coefficient for the alternative K_i with respect to M^+ is

$$\chi_i=\frac{\delta_i^-}{(\delta_i^+ +\delta_i^-)} \text{ for i = 1, 2, . . . m.}$$

Obviously, $0\leq\chi_i\leq 1$. According to relative closeness coefficient to the ideal alternative, larger value of χ_i indicates the better alternative K_i.

Step 6. Ranking the alternatives

Rank the alternatives according to the descending order of the relative-closeness coefficients to the PIS.

5 Topsis method for multi-attribute decision making under rough neutrosophic environment

Assume that a multi-attribute decision-making problem be characterized by m alternatives and n attributes. Assume that K = (K_1, K_2,..., K_m) be the set of alternatives, and L = (L_1, L_2, ..., L_n) be the set of attributes. The rating measured by the decision maker describes the performance of the alternative K_i against the attribute L_j. Assume that W = {w_1, w_2 . . . , w_n} be the weight vector assigned for the attributes L_1, L_2, ..., L_n by the decision makers. The values associated with the alternatives for multi-attribute decision-making problem (MADM) with respect to the attributes can be presented in rough neutrosophic decision matrix (see Table 1).

Table1: Rough neutrosophic decision matrix

$D=\left\langle\underline{d}_{ij},\overline{d}_{ij}\right\rangle_{m\times n}=$

	L_1	L_2	$\cdots$	L_n
K_1	$\langle\underline{d}_{11},\overline{d}_{11}\rangle$	$\langle\underline{d}_{12},\overline{d}_{12}\rangle$	...	$\langle\underline{d}_{1n},\overline{d}_{1n}\rangle$
K_2	$\langle\underline{d}_{21},\overline{d}_{21}\rangle$	$\langle\underline{d}_{22},\overline{d}_{22}\rangle$	...	$\langle\underline{d}_{2n},\overline{d}_{2n}\rangle$
.	...	...	...	...
.	...	...	...	...
K_m	$\langle\underline{d}_{m1},\overline{d}_{m1}\rangle$	$\langle\underline{d}_{m2},\overline{d}_{m2}\rangle$	...	$\langle\underline{d}_{mn},\overline{d}_{mn}\rangle$

(7)

Here $\left\langle\underline{d}_{ij},\overline{d}_{ij}\right\rangle$ is the rough neutrosophic number according to the i-th alternative and the j-th attribute.

In decision-making situation, there exist many attributes of alternatives. Some of them are important and others may be less important. So it is important to select proper weights of attributes for decision-making situation.

Definition 5.1. Accumulated geometric operator (AGO) [85]

Assume a rough neutrosophic number in the form: $\left\langle\underline{L}_{ij}(\underline{T}_{ij},\underline{I}_{ij},\underline{F}_{ij}),\overline{U}_{ij}(\overline{T}_{ij},\overline{I}_{ij},\overline{F}_{ij})\right\rangle$. We transform the rough neutrosophic number into SVNNs using the accumulated geometric operator (AGO). The operator is expressed as follows.

$$N_{ij}\left\langle T_{ij},I_{ij},F_{ij}\right\rangle=\left\langle\underline{L}_{ij}.\overline{U}_{ij}\right\rangle^{0.5}=$$
$$N_{ij}\left\langle(\underline{T}_{ij}\overline{T}_{ij})^{0.5},(\underline{I}_{ij}\overline{I}_{ij})^{0.5},(\underline{F}_{ij}\overline{F}_{ij})^{0.5}\right\rangle \quad (8)$$

Using AGO operator [85], the rating of each alternative with respect to each attribute is transformed into SVNN for MADM problem. The rough neutrosophic values (transformed as SVNN) associated with the alternatives for

MADM problems can be represented in decision matrix (see Table 2).

Table 2. Tranformed rough neutrosiphic decision matrix

$$D=\langle d\rangle_{m\times n}=\langle T_{ij}, I_{ij}, F_{ij}\rangle_{m\times n}=$$

$$\begin{array}{c|cccc} & L_1 & L_2 & \cdots & L_n \\ \hline K_1 & \langle T_{11}, I_{11}, F_{11}\rangle & \langle T_{12}, I_{12}, F_{12}\rangle & \cdots & \langle T_{1n}, I_{1n}, F_{1n}\rangle \\ K_2 & \langle T_{21}, I_{21}, F_{21}\rangle & \langle T_{22}, I_{22}, F_{22}\rangle & \cdots & \langle T_{2n}, I_{2n}, F_{2n}\rangle \\ . & \cdots & \cdots & \cdots & \cdots \\ . & \cdots & \cdots & \cdots & \cdots \\ K_m & \langle T_{m1}, I_{m1}, F_{m1}\rangle & \langle T_{m2}, I_{m2}, F_{m2}\rangle & \cdots & \langle T_{mn}, I_{mn}, F_{mn}\rangle \end{array} \quad (9)$$

In the matrix $\langle d\rangle_{m\times n}=\langle T_{ij}, I_{ij}, F_{ij}\rangle_{m\times n}$, T_{ij}, I_{ij} and F_{ij} (i = 1, 2,..., n and j = 1, 2,..., m) denote the degree of truth membership value, indeterminacy membership value and falsity membership value of alternative K_i with respect to attribute L_j.

The ratings of each alternative with respect to the attributes can be explained by the neutrosophic cube [98] proposed by Dezert. The vertices of neutrosophic cube are (0, 0, 0), (1, 0, 0), (1, 0, 1), (0, 0, 1), (0, 1, 0), (1, 1, 0), (1, 1, 1) and (0, 1, 1). The acceptance ratings [53, 99] in neutrosophic cube are classified in three types namely,

I. Highly acceptable neutrosophic ratings,

II. Manageable neutrosophic rating

III. Unacceptable neutrosophic ratings.

Definition 5.2. (Highly acceptable neutrosophic ratings) [99]

In decision making process, the sub cube (Θ) of a neutrosophic cube (Ω) (i.e. $\Theta\subset\Omega$) reflects the field of highly acceptable neutrosophic ratings (Ψ). Vertices of Λ are defined with the eight points (0.5, 0, 0),(1, 0, 0),(1, 0, 0.5), (0.5, 0, 0.5), (0.5, 0, 0.5), (1, 0, 0.5), (1, 0.5, 0.5) and (0.5, 0.5, 0.5). U includes all the ratings of alternative considered with the above average truth membership degree, below average indeterminacy degree and below average falsity membership degree for multi-attribute decision making. So, Ψ has a great role in decision making process and can be defined as follows:

$\Psi = \langle(\underline{T}_{ij}\overline{T}_{ij})^{0.5}, (\underline{I}_{ij}\overline{I}_{ij})^{0.5}, (\underline{F}_{ij}\overline{F}_{ij})^{0.5}\rangle$ where $0.5 < (\underline{T}_{ij}\overline{T}_{ij})^{0.5} < 1$, $0 < (\underline{I}_{ij}\overline{I}_{ij})^{0.5} < 0.5$ and $0 < (\underline{F}_{ij}\overline{F}_{ij})^{0.5} < 0.5$, for i = 1, 2, . . . , m and j = 1, 2, . . . , n.

Definition 5.3. (Unacceptable neutrosophic ratings) [99]

The field Σ of unacceptable neutrosophic ratings Λ is defined by the ratings which are characterized by 0% membership degree, 100% indeterminacy degree and 100% falsity membership degree. Hence, the set of unacceptable ratings Λ can be considered as the set of all ratings whose truth membership value is zero.

$\Lambda = \langle(\underline{T}_{ij}\overline{T}_{ij})^{0.5}, (\underline{I}_{ij}\overline{I}_{ij})^{0.5}, (\underline{F}_{ij}\overline{F}_{ij})^{0.5}\rangle$ where $(\underline{T}_{ij}\overline{T}_{ij})^{0.5} = 0$, $0 < (\underline{I}_{ij}\overline{I}_{ij})^{0.5} \le 1$ and $0 < (\underline{F}_{ij}\overline{F}_{ij})^{0.5} \le 1$, for i = 1, 2, . . . , m and j = 1, 2, . . . , n.

In decision making situation, consideration of Λ should be avoided.

Definition 5.4. (Manageable neutrosophic ratings) [99]

Excluding the field of high acceptable ratings and unacceptable ratings from a neutrosophic cube, tolerable neutrosophic rating field Φ ($=\Omega\cap\neg\Theta\cap\neg\Sigma$) is determined. The tolerable neutrosophic rating (Δ) considered membership degree is taken in decision making process. Δ can be defined by the expression as follows:

$\Delta = \langle(\underline{T}_{ij}\overline{T}_{ij})^{0.5}, (\underline{I}_{ij}\overline{I}_{ij})^{0.5}, (\underline{F}_{ij}\overline{F}_{ij})^{0.5}\rangle$ where $0 < (\underline{T}_{ij}\overline{T}_{ij})^{0.5} < 0.5$, $0.5 < (\underline{I}_{ij}\overline{I}_{ij})^{0.5} < 1$ and $0.5 < (\underline{F}_{ij}\overline{F}_{ij})^{0.5} < 1$.

for i = 1, 2, . . . , m and j = 1, 2, . . . , n.

Definition 5.5 [53].

Fuzzification of transformed rough neutrosophic set $N=\langle T_N(v), I_N(v), F_N(v)\rangle$ for any v ∈ V can be defined as a process of mapping N into fuzzy set F = $\{v/\mu_F(v)/v\in V\}$ i.e. f: N → F for v ∈ V. The representative fuzzy membership degree $\mu_F(v)\in[0,1]$ of the vector $\{v/\langle T_N(v), I_N(v), F_N(v)\rangle, v\in V\}$ is defined from the concept of neutrosophic cube. It can be obtained by determining the root mean square of $1-T_N(v)$, $I_N(v)$, and $F_N(v)$ for all v ∈ V. Therefore the equivalent fuzzy membership degree is defined as follows:

$$\mu_F(v)=\begin{cases}1-\left\langle\left\{(1-T_N(v))^2+(I_N(v))^2+(F_N(v))^2\right\}/3\right\rangle^{0.5} & \forall v\in\psi\cup\Delta \\ 0 & \forall v\in\Lambda\end{cases} \quad (10)$$

Now the steps of decision making using TOPSIS method under rough neutrosophic environment are stated as follows.

Step 1. Determination of the weights of decision makers

Assume that a group of k decision makers having their own decision weights involved in the decision making. The importance of the decision makers in a group may not be equal. Assume that the importance of each decision maker is considered with linguistic variables and expressed it by rough neutrosophic numbers.

Assume that $\langle\underline{N}_k(\underline{T}_k, \underline{I}_k, \underline{F}_k), \overline{N}_k(\overline{T}_k, \overline{I}_k, \overline{F}_k)\rangle$ be a rough neutrosophic number for the rating of k−th decision maker. Using AGO operator, we obtain $E_k = \langle T_k, I_k, F_k\rangle$ as a single valued neutrosophic number for the rating of k−th decision maker. Then, according to equation (10) the weight of the k−th decision maker can be written as:

$$\xi_k=\frac{1-\left\langle\left\{(1-T_k(v))^2+(I_k(v))^2+(F_k(v))^2\right\}/3\right\rangle^{0.5}}{\sum_{k=1}^{r}\left\langle 1-\left\{\left((1-T_k(v))^2+(I_k(v))^2+(F_k(v))^2\right)/3\right\}^{0.5}\right\rangle} \quad (11)$$

and $\sum_{k=1}^{r}\xi_k=1$

Step 2. Construction of the aggregated rough neutrosophic decision matrix based on the assessments of decision makers

Assume that $D^k=\left\langle \underline{d}^{(k)}_{ij},\overline{d}^{(k)}_{ij}\right\rangle_{m\times n}$ be the rough neutrosophic decision matrix of the k−th decision maker. According to equation (11), $D^k=\left(d^{(k)}_{ij}\right)_{m\times n}$ be the single-valued neutrosophic decision matrix corresponding to the rough neutrosophic decision matrix and $\xi=(\xi_1,\xi_2,...,\xi_k)^T$ be the weight vector of decision maker such that each $\xi_k\in[0, 1]$. In the group decision making process, all the individual assessments need to be accumulated into a group opinion to make an aggregated single valued neutrosophic decision matrix. This aggregated matrix can be obtained by using rough neutrosophic aggregation operator as follows:

$D=(d_{ij})_{m\times n}$ where,

$$(d_{ij})_{m\times n}=RNWA_{\xi}\left(d^1_{ij},d^2_{ij},\cdots,d^r_{ij}\right)=\xi_1 d^1_{ij}\oplus\xi_2 d^2_{ij}\oplus\cdots\oplus\xi_r d^r_{ij}$$

$$=\left\langle 1-\prod_{k=1}^{r}(1-T^{(r)}_{ij})^{\xi_k},\prod_{k=1}^{r}(I^{(r)}_{ij})^{\xi_k},\prod_{k=1}^{r}(F^{(r)}_{ij})^{\xi_k},\right\rangle \quad (12)$$

$$Here,\ d^r_{ij}=\left\langle \underline{d}^r_{ij}.\overline{d}^r_{ij}\right\rangle^{0.5}$$

Now the aggregated rough neutrosophic decision matrix is defined as follows:

$$(d_{ij})_{m\times n}=\left\langle (\underline{T}_{ij}.\overline{T}_{ij})^{0.5},(\underline{I}_{ij}.\overline{I}_{ij})^{0.5},(\underline{F}_{ij}.\overline{F}_{ij})^{0.5}\right\rangle_{m\times n}$$

	L_1	L_2	...	L_n
K_1	$\langle T_{11},I_{11},F_{11}\rangle$	$\langle T_{12},I_{12},F_{12}\rangle$	...	$\langle T_{1n},I_{1n},F_{1n}\rangle$
K_2	$\langle T_{21},I_{21},F_{21}\rangle$	$\langle T_{22},I_{22},F_{22}\rangle$	...	$\langle T_{2n},I_{2n},F_{2n}\rangle$
.	...	...	...	...
.	...	...	...	...
K_m	$\langle T_{m1},I_{m1},F_{m1}\rangle$	$\langle T_{m2},I_{m2},F_{m2}\rangle$	...	$\langle T_{mn},I_{mn},F_{mn}\rangle$

(13)

Here, $d_{ij}=\langle T_{ij},I_{ij},F_{ij}\rangle=\left\langle (\underline{T}_{ij}.\overline{T}_{ij})^{0.5},(\underline{I}_{ij}.\overline{I}_{ij})^{0.5},(\underline{F}_{ij}.\overline{F}_{ij})^{0.5}\right\rangle$ is the aggregated element of rough neutrosophic decision matrix D for i = 1, 2, . . . m and j = 1, 2, . .. n.

Step 3. Determination of the attribute weights

In the decision-making process, all attributes may not have equal importance. So, every decision maker may have their own opinion regarding attribute weights. To obtain the group opinion of the chosen attributes, all the decision makers' opinions need to be aggregated. Assume that $\left\langle \underline{w}^j_{(k)},\overline{w}^j_{(k)}\right\rangle$ be rough neutrosophic number (RNN) assigned to the attribute L_j by the k−th decision maker. According to equation (8) w^j_k be the neutrosophic number assigned to the attribute L_j by the k−th decision maker. Then the combined weight W = (w_1, w_2 . . . , w_n) of the attribute can be determined by using rough neutrosophic weighted aggregation (RNWA) operator

$$w_j=RNWA_{\xi}(w^{(1)}_j,w^{(2)}_j,\cdots,w^{(r)}_j)=\xi_1 w^{(1)}_j\oplus\xi_2 w^{(2)}_j\oplus\cdots\oplus\xi_r w^{(r)}_j$$

$$=\left\langle 1-\prod_{k=1}^{r}(1-T^{(r)}_j)^{\xi_k},\prod_{k=1}^{r}(I^{(r)}_j)^{\xi_k},\prod_{k=1}^{r}(F^{(r)}_j)^{\xi_k}\right\rangle \quad (14)$$

$Here,\ \xi^r_{ij}=\left\langle \underline{d}_{ij}.\overline{d}_{ij}\right\rangle;\ w_j=\left\langle T^{(r)}_j,I^{(r)}_j,F^{(r)}_j\right\rangle=$

$\left\langle (\underline{T}^{(r)}_j.\overline{T}^{(r)}_j)^{0.5},(\underline{I}^{(r)}_j.\overline{I}^{(r)}_j)^{0.5},(\underline{F}^{(r)}_j.\overline{F}^{(r)}_j)^{0.5}\right\rangle$ for j = 1, 2, . . . n.

W = (w_1, w_2 . . . , w_n) (15)

Step 4. Aggregation of the weighted rough neutrosophic decision matrix

In this section, the obtained weights of attribute and aggregated rough neutrosophic decision matrix need to be further fused to make the aggregated weighted rough neutrosophic decision matrix. Then, the aggregated weighted rough neutrosophic decision matrix can be defined by using the multiplication properties between two neutrosophic sets as follows:

$$D\otimes W=D^W=\left\langle d^{w_j}_{ij}\right\rangle_{m\times n}=\left\langle T^{w_j}_{ij},I^{w_j}_{ij},F^{w_j}_{ij}\right\rangle_{m\times n}=$$

	L_1	L_2	...	L_n
K_1	$\langle T^{w_1}_{11},I^{w_1}_{11},F^{w_1}_{11}\rangle$	$\langle T^{w_2}_{12},I^{w_2}_{12},F^{w_2}_{12}\rangle$	...	$\langle T^{w_n}_{1n},I^{w_n}_{1n},F^{w_n}_{1n}\rangle$
K_2	$\langle T^{w_1}_{21},I^{w_1}_{21},F^{w_1}_{21}\rangle$	$\langle T^{w_2}_{22},I^{w_2}_{22},F^{w_2}_{22}\rangle$	...	$\langle T^{w_n}_{2n},I^{w_n}_{2n},F^{w_n}_{2n}\rangle$
.	...	...	...	...
.	...	...	...	...
K_m	$\langle T^{w_1}_{m1},I^{w_1}_{m1},F^{w_1}_{m1}\rangle$	$\langle T^{w_2}_{m2},I^{w_2}_{m2},F^{w_2}_{m2}\rangle$	...	$\langle T^{w_n}_{mn},I^{w_n}_{mn},F^{w_n}_{mn}\rangle$

(16)

Here, $d^{w_j}_{ij}=\left\langle T^{w_j}_{ij},I^{w_j}_{ij},F^{w_j}_{ij}\right\rangle$ is an element of the aggregated weighted rough neutrosophic decision matrix D^W for i = 1, 2, . . . , m and j = 1, 2, . . . , n.

Step 5. Determination of the rough relative positive ideal solution (RRPIS) and the rough relative negative ideal solution (RRNIS)

After transferring RNS decision matrix, assume $D_N=\left\langle d^W_{ij}\right\rangle_{m\times n}=\left\langle T_{ij},I_{ij},F_{ij}\right\rangle_{m\times n}$ be a SVNS based decision matrix, where, T_{ij}, I_{ij} and F_{ij} are the membership degree, indeterminacy degree and non-membership degree of evaluation for the attribute L_j with respect to the alternative K_i. In practical siuation, two types of attributes namely, benefit type attribute and cost type attribute are considered in multi-attribute decision making problems.

Definition 5.6.

Assume that $C_1$and C_2 be the benefit type attribute and cost type attribute respectively. Suppose that G^+_N is the relative rough neutrosophic positive ideal solution (RRNPIS) and G^-_N is the relative rough neutrosophic negative ideal solution (RRNNIS).

Then G^+_N can be defined as follows:

$$G^+_N=\left\langle d^{w+}_1,d^{w+}_2,\cdots,d^{w+}_n\right\rangle \quad (17)$$

Here $d^{w+}_j=\left\langle T^{w+}_j,I^{w+}_j,F^{w+}_j\right\rangle$ for j = 1, 2, . . . , n.

$$T^{w+}_j=\{(\max_i\{T^{w_j}_{ij}\}/j\in C_1),(\min_i(T^{w_j}_{ij})/j\in C_2)\}$$

$$I^{w+}_j=\{(\min_i\{I^{w_j}_{ij}\}/j\in C_1),(\max_i(I^{w_j}_{ij})/j\in C_2)\}$$

$F_j^{w+} = \{(\min_i \{F_{ij}^{wj}\} / j \in C_1), (\max_i (F_{ij}^{wj}) / j \in C_2)\}$

Then G_N^- can be defined as follows:

$$G_N^- = \langle d_1^{w-}, d_2^{w-}, \cdots, d_n^{w-} \rangle \quad (18)$$

Here $d_j^{w-} = \langle T_j^{w-}, I_j^{w-}, F_j^{w-} \rangle$ for j = 1, 2, . . . , n.

$T_j^{w-} = \{(\min_i \{T_{ij}^{wj}\} / j \in C_1), (\max_i (T_{ij}^{wj}) / j \in C_2)\}$

$I_j^{w-} = \{(\max_i \{I_{ij}^{wj}\} / j \in C_1), (\min_i (I_{ij}^{wj}) / j \in C_2)\}$

$F_j^{w-} = \{(\max_i \{F_{ij}^{wj}\} / j \in C_1), (\min_i (F_{ij}^{wj}) / j \in C_2)\}$

Step 6. Determination of the distance measure of each alternative from the RRNPIS and the RRNNIS

The normalized Euclidean distance measure of all alternative $\langle T_{ij}^{wj}, I_{ij}^{wj}, F_{ij}^{wj} \rangle$ from the RRNPIS $\langle d_1^{w+}, d_2^{w+}, ..., d_n^{w+} \rangle$ for i = 1, 2, …, m and j = 1, 2, …, n can be written as follows:

$$\delta_{euclid}^{i+}(d_{ij}^{wj}, d_{ij}^{w+}) = \frac{1}{3n} \left\langle \sum_{j=1}^{n} \begin{pmatrix} (T_{ij}^{wj}(v_j) - T_j^{w+}(v_j))^2 + (I_{ij}^{wj}(v_j) - I_j^{w+}(v_j))^2 \\ + (F_{ij}^{wj}(v_j) - F_j^{w+}(v_j))^2 \end{pmatrix} \right\rangle^{0.5} \quad (19)$$

The normalized Euclidean distance measure of all alternative $\langle T_{ij}^{wj}, I_{ij}^{wj}, F_{ij}^{wj} \rangle$ from the RRNPIS $\langle d_1^{w-}, d_2^{w-}, ..., d_n^{w-} \rangle$ for i = 1, 2, …, m and j = 1, 2, …, n can be written as follows:

$$\delta_{euclid}^{i-}(d_{ij}^{wj}, d_{ij}^{w-}) = \frac{1}{3n} \left\langle \sum_{j=1}^{n} \begin{pmatrix} (T_{ij}^{wj}(v_j) - T_j^{w-}(v_j))^2 + (I_{ij}^{wj}(v_j) - I_j^{w-}(v_j))^2 \\ + (F_{ij}^{wj}(v_j) - F_j^{w-}(v_j))^2 \end{pmatrix} \right\rangle^{0.5} \quad (20)$$

Step 7. Determination of the relative closeness co-efficient to the rough neutrosophic ideal solution for rough neutrosophic sets

The relative closeness coefficient of each alternative K_i with respect to the neutrosophic positive ideal solution G_N^+ is defined as follows:

$$\chi_i^* = \frac{\langle \delta_{euclid}^{i-}(d_{ij}^{wj}, d_{ij}^{w-}) \rangle}{\langle \delta_{euclid}^{i-}(d_{ij}^{wj}, d_{ij}^{w-}) + \delta_{euclid}^{i+}(d_{ij}^{wj}, d_{ij}^{w+}) \rangle} \quad (21)$$

Here $0 \le \chi_i^* \le 1$. According to the relative closeness coefficient values larger the values of χ_i^* reflects the better alternative K_i for i = 1, 2, …, n.

Step 8. Ranking the alternatives

Rank the alternatives according to the descending order of the relative-closeness coefficients to the RRNPIS.

6 Numerical example

In order to demonstrate the proposed method, logistic center location selection problem is described here. Suppose that a new modern logistic center is required in a town. There are three locations K_1, K_2, K_3. A committee of three decision makers or experts D_1, D_2, D_3 has been formed to select the most appropriate location on the basis of six parameters obtained from expert opinions, namely, cost (L_1), distance to suppliers (L_2), distance to customers (L_3), conformance to government and law (L_4), quality of service (L_5), and environmental impact (L_6).

Based on the proposed approach the considered problem is solved using the following steps:

Step 1. Determination of the weights of decision makers

The importance of three decision makers in a selection committee may be different based on their own status. Their decision values are considered as linguistic terms (seeTable-3). The importance of each decision maker expressed by linguistic term with its corresponding rough neutrosophic values shown in Table-4. The weights of decision makers are determined with the help of equation (11) as:

$\xi_1 = 0.398$, $\xi_2 = 0.359$, $\xi_3 = 0.243$.

We transform rough neutrosophic number (RNN) to neutrosophic number (NN) with the help of AGO operator [85] in Table 3, Table 4 and Table 5.

Step 2. Construction of the aggregated rough neutrosophic decision matrix based on the assessments of decision makers

The linguistic terms along with RNNs are defined in Table-5 to rate each alternative with respect to each attribute. The assessment values of each alternative K_i (i = 1, 2, 3) with respect to each attribute L_j (j = 1, 2, 3, 4, 5, 6) provided by three decision makers are listed in Table-6. Then the aggregated neutrosophic decision matrix can be obtained by fusing all the decision maker opinions with the help of aggregation operator (equation 12) (see Table 7).

Step 3. Determination of the weights of attributes

The linguistic terms shown in Table-3 are used to evaluate each attribute. The importance of each attribute for every decision maker is rated with linguistic terms shown in Table-6. Three decision makers' opinions need to be aggregated to final opinion.

The fused attribute weight vector is determined by using equation (14) as follows:

$$W = \begin{Bmatrix} \langle 0.761, 0.205, 0.195 \rangle, \langle 0.800, 0.181, 0.159 \rangle, \langle 0.737, 0.241, 0.196 \rangle, \\ \langle 0.761, 0.223, 0.169 \rangle, \langle 0.774, 0.203, 0.172 \rangle, \langle 0.804, 0.184, 0.172 \rangle \end{Bmatrix} \quad (23)$$

Step 4. Construction of the aggregated weighted rough neutrosophic decision matrix

Using equation (16) and clculating the combined weights of the attributes and the ratings of the alternatives, the aggregated weighted rough neutrosophic decision matrix is obtained (see Table-8).

Step 5. Determination of the rough neutrosophic relative positive ideal solution and the rough neutrosophic relative negative ideal solution

The RNRPIS can be calculated from the aggregated weighted decision matrix on the basis of attribute types i.e. benefit type or cost type by using equation (17) as

$$G_N^+ = \begin{bmatrix} \langle 0.670, 0.289, 0.274\rangle, \langle 0.694, 0.284, 0.252\rangle, \langle 0.588, 0.388, 0.309\rangle, \\ \langle 0.607, 0.374, 0.286\rangle, \langle 0.642, 0.331, 0.303\rangle, \langle 0.708, 0.270, 0.253\rangle \end{bmatrix} \quad (25)$$

Here $d_1^{w+} = \langle T_1^{w+}, I_1^{w+}, F_1^{w+} \rangle$ is calculated as:

$T_1^{w+} =$ max [0.670, 0.485, 0.454] = 0.670, $I_1^{w+} =$ min [0.289, 0.449, 0.471] = 0.289,

$F_1^{w+} =$ min [0.274, 0.377, 0.463]= 0.274.

Similarly, other RNRPISs are calculated.

Using equation (18), the RNRNIS are calculated from aggregated weighted decision matrix based on attribute types i.e. benefit type or cost type.

$$G_N^- = \begin{bmatrix} \langle 0.454, 0.471, 0.463\rangle, \langle 0.588, 0.377, 0.353\rangle, \langle 0.469, 0.480, 0.309\rangle, \\ \langle 0.522, 0.441, 0.358\rangle, \langle 0.524, 0.429, 0.372\rangle, \langle 0.512, 0.435, 0.414\rangle \end{bmatrix} \quad (26)$$

Here, $d_1^{w-} = \langle T_1^{w-}, I_1^{w-}, F_1^{w-} \rangle$ is calculated as

T_1^{w} = min [0.670, 0.485, 0.454] = 0.454, $I_1^{w-} =$ max [0.289, 0.449, 0.471] = 0.471,

$F_1^{w-} =$ max [0.274, 0.377, 0.463] = 0.463.

Other RNRNISs are calculated in similar way.

Step 6. Determination of the distance measure of each alternative from the RRNPIS and the RRNNIS and relative closeness co-efficient

Normalized Euclidean distance measures defined in equation (19) and equation (20) are used to determine the distances of each alternative from the RRNPIS and the RNNIS.

Step 7. Determination of the relative closeness co-efficient to the rough neutrosophic ideal solution for rough neutrosophic sets

Using equation (21) and distances, relative closeness coefficient of each alternative K_1 , K_2 , K_3 with respect to the rough neutrosophic positive ideal solution G_N^+ is calculated (see Table 9).

Table 9. Distance measure and relative closeness co-efficient

$Alternatives\,(K_i)$	δ_{euclid}^{i+}	δ_{euclid}^{i-}	χ_i^*
K_1	0.0078	0.1248	0.9411
K_2	0.1192	0.0682	0.3639
K_3	0.1025	0.0534	0.3425

(27)

Step 9. Ranking the alternatives

According to the values of relative closeness coefficient of each alternative (see Table 9), the ranking order of three alternatives is obtained as follows:

$K_1 \succ K_2 \succ K_3$.

Thus K_1 is the best the logistic center.

7 Conclusion

In general, realistic MAGDM problems adhere to uncertain, imprecise, incomplete, and inconsistent data and rough neutrosophic set theory is adequate to deal with it. In this paper, we have proposed rough neutrosophic TOPSIS method for MAGDM. We have also proposed rough neutrosophic aggregate operator and rough neutrosophic weighted aggregate operator. In the decision-making situation, the ratings of each alternative with respect to each attribute are presented as linguistic variables characterized by rough neutrosophic numbers. Rough neutrosophic aggregation operator has been used to aggregate all the opinions of decision makers. Rough neutrosophic positive ideal and rough neutrosophic negative ideal solution have been defined to form aggregated weighted decision matrix. Euclidean distance measure has been used to calculate the distances of each alternative from positive as well as negative ideal solutions for relative closeness co-efficient of each alternative. The proposed rough neutrosophic TOPSIS approach can be applied in pattern recognition, artificial intelligence, and medical diagnosis in rough neutrosophic environment.

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Table 3. Linguistic terms for rating attributes

Linguistic Terms	Rough neutrosophic numbers	Neutrosophic numbers
Very good / Very important (VG/VI)	$\langle (0.85, 0.05, 0.05), (0.95, 0.15, 0.15) \rangle$	$\langle 0.899, 0.087, 0.087 \rangle$
Good / Important(G /I)	$\langle (0.75, 0.15, 0.10), (0.85, 0.25, 0.20) \rangle$	$\langle 0.798, 0.194, 0.141 \rangle$
Fair / Medium(F/M)	$\langle (0.45, 0.35, 0.35), (0.55, 0.45, 0.55) \rangle$	$\langle 0.497, 0.397, 0.439 \rangle$
Bad / Unimportant (B / UI)	$\langle (0.25, 0.55, 0.65), (0.45, 0.65, 0.75) \rangle$	$\langle 0.335, 0.598, 0.698 \rangle$
Very bad/Very Unimportant (VB/VUI)	$\langle (0.05, 0.75, 0.85), (0.15, 0.85, 0.95) \rangle$	$\langle 0.087, 0.798, 0.899 \rangle$

Table 4. Importance of decision makers expressed in terms of rough neutrosophic numbers

DM	D_1	D_2	D_3
LT	VI	I	M
RNN	$\left\langle \begin{matrix}(0.85, 0.05, 0.05),\\(0.95, 0.15, 0.15)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.75, 0.15, 0.10),\\(0.85, 0.25, 0.20)\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(0.45, 0.35, 0.35),\\(0.55, 0.45, 0.55)\end{matrix} \right\rangle$
NN	$\langle 0.899, 0.087, 0.087 \rangle$	$\langle 0.798, 0.194, 0.141 \rangle$	$\langle 0.497, 0.397, 0.439 \rangle$

Table 5. Linguistic terms for rating the candidates innterms of rough neutrosophic numbers and neutrosophic numbers

Linguistic terms	RNNs	NNs
Extremely Good/High (EG/EH)	$\langle (1.00,0.00,0.00),(1.00,0.00,0.00) \rangle$	$\langle 1.000, 0.000, 0.000 \rangle$
Very Good/High (VG/VH)	$\langle (0.85,0.05,0.05),(0.95,0.15,0.15) \rangle$	$\langle 0.899, 0.087, 0.087 \rangle$
Good/High (G/H)	$\langle (0.75,0.15,0.10),(0.85,0.25,0.20) \rangle$	$\langle 0.798, 0.194, 0.141 \rangle$
Medium Good/High (MG/MH)	$\langle (0.55,0.30,0.25),(0.65,0.40,0.35) \rangle$	$\langle 0.598, 0.346, 0.296 \rangle$
Medium/Fair (M/F)	$\langle (0.45,0.45,0.35),(0.55,0.55,0.55) \rangle$	$\langle 0.497, 0.497, 0.439 \rangle$
MediumBad/MediumLaw(MB/ML)	$\langle (0.30,0.60,0.55),(0.40,0.70,0.65) \rangle$	$\langle 0.346, 0.648, 0.598 \rangle$
Bad/Law (G/L)	$\langle (0.15,0.70,0.75),(0.25,0.80,0.85) \rangle$	$\langle 0.194, 0.748, 0.798 \rangle$
Very Bad/Low (VB/VL)	$\langle (0.05,0.80,0.85),(0.15,0.90,0.95) \rangle$	$\langle 0.087, 0.849, 0.899 \rangle$
VeryVeryBad/low(VVB/VVL)	$\langle (0.05,0.95,0.95),(0.05,0.85,0.95) \rangle$	$\langle 0.050, 0.899, 0.950 \rangle$

Table 6. Assessments of alternatives and attribute in terms of linguisterm terms given by three decision makers

Alternatives (K_i)	Decision Makers	L_1	L_2	L_3	L_4	L_5	L_6
K_1	D_1	VG	G	G	G	G	VG
	D_2	VG	VG	G	G	G	VG
	D_3	G	VG	G	G	VG	G
K_2	D_1	M	G	M	G	G	M
	D_2	G	MG	G	G	MG	G
	D_3	M	G	M	MG	M	M
K_3	D_1	M	VG	G	MG	VG	M
	D_2	M	M	G	G	M	G
	D_3	G	M	M	MG	G	VG

Table 7. Aggregated transformed rough neutrosophic decision matrix

	L_1	L_2	L_3	L_4	L_5	L_6
K_1	$\langle 0.881, 0.106, 0.098 \rangle$	$\langle 0.867, 0.126, 0.111 \rangle$	$\langle 0.798, 0.194, 0.141 \rangle$	$\langle 0.798, 0.194, 0.141 \rangle$	$\langle 0.830, 0.160, 0.125 \rangle$	$\langle 0.880, 0.106, 0.098 \rangle$
K_2	$\langle 0.637, 0.307, 0.292 \rangle$	$\langle 0.741, 0.239, 0.184 \rangle$	$\langle 0.637, 0.315, 0.292 \rangle$	$\langle 0.761, 0.223, 0.169 \rangle$	$\langle 0.677, 0.284, 0.242 \rangle$	$\langle 0.637, 0.307, 0.292 \rangle$
K_3	$\langle 0.597, 0.334, 0.333 \rangle$	$\langle 0.735, 0.217, 0.231 \rangle$	$\langle 0.748, 0.231, 0.186 \rangle$	$\langle 0.686, 0.281, 0.227 \rangle$	$\langle 0.787, 0.182, 0.175 \rangle$	$\langle 0.755, 0.212, 0.197 \rangle$

Table 8. Aggregated weighted rough neutrosophic decision matrix

	L_1	L_2	L_3	L_4	L_5	L_6
K_1	$\langle 0.670, 0.289, 0.274 \rangle$	$\langle 0.694, 0.284, 0.252 \rangle$	$\langle 0.588, 0.388, 0.309 \rangle$	$\langle 0.607, 0.374, 0.286 \rangle$	$\langle 0.642, 0.331, 0.303 \rangle$	$\langle 0.708, 0.270, 0.253 \rangle$
K_2	$\langle 0.485, 0.449, 0.377 \rangle$	$\langle 0.593, 0.377, 0.344 \rangle$	$\langle 0.469, 0.480, 0.431 \rangle$	$\langle 0.579, 0.396, 0.309 \rangle$	$\langle 0.524, 0.429, 0.372 \rangle$	$\langle 0.512, 0.435, 0.414 \rangle$
K_3	$\langle 0.454, 0.471, 0.463 \rangle$	$\langle 0.588, 0.359, 0.353 \rangle$	$\langle 0.551, 0.416, 0.346 \rangle$	$\langle 0.522, 0.441, 0.358 \rangle$	$\langle 0.609, 0.348, 0.317 \rangle$	$\langle 0.607, 0.357, 0.335 \rangle$

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NSS

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Introduction to Neutrosophic Soft Groups

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Abstract. The notion of neutrosophic soft group is introduced, together with several related properties. Its structural characteristics are investigated with suitable examples. The Cartesian product on neutrosophic soft groups and on neutrosophic soft subgroup is defined and illustrated by examples.
Related theorems are established.

Keywords: neutrosophic soft set, neutrosophic soft groups, neutrosophic soft subgroup, Cartesian product.

1 Introduction

The concept of Neutrosophic Set (NS), firstly introduced by Smarandache [1], is a generalisation of classical sets, fuzzy set [2], intuitionistic fuzzy set [3] etc. Researchers in economics, sociology, medical science and many other several fields deal daily with the complexities of modelling uncertain data. Classical methods are not always successful because the uncertainty appearing in these domains may be of various types. While probability theory, theory of fuzzy set, intuitionistic fuzzy set and other mathematical tools are well known and often useful approaches to describe uncertainty, each of these theories has its different difficulties, as pointed out by Molodtsov [4].

In 1999, Molodtsov [4] introduced a new concept of soft set theory, which is free from the parameterization inadequacy syndrome of different theories dealing with uncertainty. This makes the theory very convenient and easy to apply in practice. The classical group theory was extended over fuzzy set, intuitionistic fuzzy set and soft set by Rosenfeld [5], Sharma [6], Aktas et.al. [7], and many others. Consequently, several authors applied the theory of fuzzy soft sets, intuitionistic fuzzy soft sets to different algebraic structures, e.g. Maji et. al. [8, 9, 10], Dinda and Samanta [11], Ghosh et. al. [12], Mondal [13], Chetia and Das [14], Basu et. al. [15], Augunoglu and Aygun [16], Yaqoob et. al [17], Varol et. al. [18], Zhang [19].

Later, Maji [20] has introduced a combined concept, the Neutrosophic Soft Set (NSS). Using this concept, several mathematicians have produced their research works in different mathematical structures, e.g. Sahin et. al [21], Broumi [22], Bera and Mahapatra [23], Maji [24], Broumi and Smarandache [25]. Later, the concept has been redefined by Deli and Broumi [26].

This paper presents the notion of neutrosophic soft groups along with an investigation of some related properties and theorems. Section 2 gives some useful definitions. In Section 3, neutrosophic soft group is defined, along with some properties. Section 4 deals with the Cartesian product of neutrosophic soft groups. Finally, the concept of neutrosophic soft subgroup is studied, with suitable examples, in Section 5.

2 Preliminaries

We recall basic definitions related to fuzzy set, soft set, and neutrosophic soft.

2.1 Definition: [27]

A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is continuous t - norm if it satisfies the following conditions:

(i) $*$ is commutative and associative.

(ii) $*$ is continuous,

(iii) $a * 1 = 1 * a = a,\ \forall a \in [0,1]$,

(iv) $a * b \leq c * d$ if $a \leq c, b \leq d$,

with $a, b, c, d \in [0,1]$.

A few examples of continuous t-norm are $a * b = ab$, $a * b = min\,(a,b)$, $a * b = max\,(a + b - 1, 0)$.

2.2 Definition: [27]

A binary operation $\diamond : [0,1] \times [0,1] \to [0,1]$ is continuous t - conorm (s - norm) if it satisfies the following conditions:

(i) $\diamond$ is commutative and associative,

(ii) $\diamond$ is continuous,

(iii) $a \diamond 0 = 0 \diamond a = a,\ \forall a \in [0,1]$,

(iv) $a \diamond b \leq c \diamond d$ if $a \leq c,\ b \leq d$,

with $a, b, c, d \in [0,1]$.

A few examples of continuous s-norm are $a \diamond b = a + b - ab$, $a \diamond b = \max(a,b)$, $a \diamond b = \min(a + b, 1)$. $\forall\, a \in [0,1]$, if $a * a = a$ and $a \diamond a = a$, then $*$ is called an idempotent t-norm and $\diamond$ is called an idempotent s-norm.

2.3 Definition: [1]

A neutrosophic set (NS) on the universe of discourse U is defined as : $A = \{x, T_A(x), I_A(x), F_A(x) > : x \in U\}$,

where $T, I, F : U \longrightarrow]^-0,1^+[$ and
$^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$.

From a philosophical point of view, the neutrosophic set (NS) takes its values from real standard or nonstandard subsets of $]^-0,1^+[$. But in real life application, in scientific and engineering problems, it is difficult to use NS with values from real standard or nonstandard subset of $]^-0,1^+[$. Hence, we consider the NS which takes the values from the subset of $[0,1]$.

2.4 Definition: [4]

Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denote the power set of U. Then for $A \subseteq E$, a pair (F, A) is called a soft set over U, where $F: A \rightarrow P(U)$ is a mapping.

2.5 Definition: [20]

Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denote the set of all NSs of U. Then for $A \subseteq E$, a pair (F, A) is called an NSS over U, where $F: A \rightarrow P(U)$ is a mapping.

This concept has been modified by Deli and Broumi [26] as given below.

2.6 Definition: [26]

Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denote the set of all NSs of U. Then, a neutrosophic soft set N over U is a set defined by a set valued function f_N representing a mapping $f_N : E \rightarrow P(U)$ where f_N is called approximate function of the neutrosophic soft set N. In other words, the neutrosophic soft set is a parameterized family of some elements of the set $P(U)$ and therefore it can be written as a set of ordered pairs, $N = \{(e, \{< x, T_{f_N(e)}(x), I_{f_N(e)}(x), F_{f_N(e)}(x) >: x \in U\}) : e \in E\}$ where $T_{f_N(e)}(x), I_{f_N(e)}(x), F_{f_N(e)}(x) \in [0,1]$, respectively, called the truth-membership, indeterminacy-membership, falsity-membership function of $f_N(e)$. Since supremum of each T, I, F is 1 so the inequality $0 \leq T_{f_N(e)}(x) + I_{f_N(e)}(x) + F_{f_N(e)}(x) \leq 3$ is obvious.

2.6.1 Example

Let, $U = \{h_1, h_2, h_3\}$ be a set of houses and $E = \{e_1(beautiful), e_2(wooden), e_3(costly)\}$ be a set of parameters with respect to which the nature of houses is described. Let

$f_N(e_1) = \{<h_1, (0.5, 0.6, 0.3)>, <h_2, (0.4, 0.7, 0.6)>, <h_3, (0.6, 0.2, 0.3)>\}$;

$f_N(e_2) = \{<h_1, (0.6, 0.3, 0.5)>, <h_2, (0.7, 0.4, 0.3)>, <h_3, (0.8, 0.1, 0.2)>\}$;

$f_N(e_3) = \{<h_1, (0.7, 0.4, 0.3)>, <h_2, (0.6, 0.7, 0.2)>, <h_3, (0.7, 0.2, 0.5)>\}$;

Then, $N = \{[e_1, f_N(e_1)], [e_2, f_N(e_2)], [e_3, f_N(e_3)]\}$ is an NSS over (U, E).

The tabular representation of the NSS N is as:

	$f_N(e_1)$	$f_N(e_2)$	$f_N(e_3)$
h_1	(0.5,0.6,0.3)	(0.6,0.3,0.5)	(0.7,0.4,0.3)
h_2	(0.4,0.7,0.6)	(0.7,0.4,0.3)	(0.6,0.7,0.2)
h_3	(0.6,0.2,0.3)	(0.8,0.1,0.2)	(0.7,0.2,0.5)

Table 1: Tabular form of NSS *N*.

2.6.2 Definition: [26]

The complement of a neutrosophic soft set N is denoted by N^c and is defined as:

$$N^c = \{(e, \{< x, F_{f_N(e)}(x), 1 - I_{f_N(e)}(x), T_{f_N(e)}(x) >: x \in U\}) : e \in E\}$$

2.6.3 Definition: [26]

Let N_1 and N_2 be two NSSs over the common universe (U, E). Then N_1 is said to be the neutrosophic soft subset of N_2 if

$$T_{f_{N_1}(e)}(x) \leq T_{f_{N_2}(e)}(x),\ I_{f_{N_1}(e)}(x) \geq I_{f_{N_2}(e)}(x),$$
$$F_{f_{N_1}(e)}(x) \geq F_{f_{N_2}(e)}(x);\ \forall e \in E \text{ and } x \in U.$$

We write $N_1 \subseteq N_2$ and then N_2 is the neutrosophic soft superset of N_1.

2.6.4 Definition: [26]

1. Let N_1 and N_2 be two NSSs over the common universe (U, E). Then their union is denoted by $N_1 \cup N_2 = N_3$ and is defined as :

$$N_3 = \{(e, \{< x, T_{f_{N_3}(e)}(x), I_{f_{N_3}(e)}(x), F_{f_{N_3}(e)}(x) >: x \in U\}) : e \in E\},$$

where

$$T_{f_{N_3}(e)}(x) = T_{f_{N_1}(e)}(x) \diamond T_{f_{N_2}(e)}(x),$$
$$I_{f_{N_3}(e)}(x) = I_{f_{N_1}(e)}(x) * I_{f_{N_2}(e)}(x),$$
$$F_{f_{N_3}(e)}(x) = F_{f_{N_1}(e)}(x) * F_{f_{N_2}(e)}(x);$$

2. Let N_1 and N_2 be two NSSs over the common universe (U, E). Then their intersection is denoted by $N_1 \cap N_2 = N_4$ and it is defined as:

$$N_4 = \{(e, \{< x, T_{f_{N_4}(e)}(x), I_{f_{N_4}(e)}(x), F_{f_{N_4}(e)}(x) >: x \in U\}) : e \in E\}$$

where

$$T_{f_{N_4}(e)}(x) = T_{f_{N_1}(e)}(x) * T_{f_{N_2}(e)}(x),$$
$$I_{f_{N_4}(e)}(x) = I_{f_{N_1}(e)}(x) \diamond I_{f_{N_2}(e)}(x),$$
$$F_{f_{N_4}(e)}(x) = F_{f_{N_1}(e)}(x) \diamond F_{f_{N_2}(e)}(x);$$

2.7 Definition: [8]

Let (F, A) be a soft set over the group G. Then (F, A) is called a soft group over G if $F(a)$ is a subgroup of G, $\forall a \in A$.

3 Neutrosophic soft groups

In this section, we define the neutrosophic soft group and some basic properties related to it. Unless otherwise stated, E is treated as the parametric set throughout this paper and $e \in E$, an arbitrary parameter.

3.1 Definition:

A neutrosophic set $A = \{< x, T_A(x), I_A(x), F_A(x) >: x \in G\}$ over a group $(G,\circ)$ is called a neutrosophic subgroup of $(G,\circ)$ if

(i) $T_A(x \circ y) \geq T_A(x) * T_A(y)$,
$I_A(x \circ y) \leq I_A(x) \diamond I_A(y)$,
$F_A(x \circ y) \leq F_A(x) \diamond F_A(y)$;
for $x, y \in G$.

(ii) $T_A(x^{-1}) \geq T_A(x)$, $I_A(x^{-1}) \leq I_A(x)$, $F_A(x^{-1}) \leq F_A(x)$, for $x \in G$.

An NSS N over a group $(G,\circ)$ is called a neutrosophic soft group if $f_N(e)$ is a neutrosophic subgroup of $(G,\circ)$ for each $e \in E$.

3.1.1 Example:

1. Let us consider the Klein's -4 group $V = \{e, a, b, c\}$ and $E = \{\alpha, \beta, \gamma, \delta\}$ be the set of parameters. We define $f_N(\alpha), f_N(\beta), f_N(\gamma), f_N(\delta)$ as given by the following table:

	$f_N(\alpha)$ $f_N(\gamma)$	$f_N(\beta)$ $f_N(\delta)$
e	(0.65, 0.34, 0.14) (0.72, 0.21, 0.16)	(0.88, 0.12,0.72) (0.69, 0.31, 0.32)
a	(0.71, 0.22, 0.78) (0.84, 0.16, 0.25)	(0.71, 0.19, 0.44) (0.62, 0.32, 0.42)
b	(0.75, 0.25, 0.52) (0.69, 0.31, 0.39)	(0.83, 0.11, 0.28) (0.58, 0.41, 0.66)
c	(0.67, 0.32, 0.29) (0.79, 0.19, 0.41)	(0.75, 0.21, 0.19) (0.71, 0.27, 0.53)

Table 2: Tabular form of neutrosophic soft group *N*.

Corresponding t-norm $(*)$ and s-norm $(\diamond)$ are defined as $a * b = max\,(a + b - 1, \;\; 0)$, $a \diamond b = min\,(a + b, 1)$. Then, N forms a neutrosophic soft group over (V, E).

2. Let $E = \mathbb{N}$ (the set of natural no.), be the parametric set and $G = (\mathbb{Z}, +)$ be the group of all integers. Define a mapping $f_M : \mathbb{N} \to NS(\mathbb{Z})$ where, for any $n \in \mathbb{N}$ and $x \in \mathbb{Z}$,

$$T_{f_{M(n)}}(x) = \begin{cases} 0 & \text{if } x \text{ is odd} \\ \frac{1}{n} & \text{if } x \text{ is even} \end{cases}$$

$$I_{f_{M(n)}}(x) = \begin{cases} \frac{1}{2n} & \text{if } x \text{ is odd} \\ 0 & \text{if } x \text{ is even} \end{cases}$$

$$F_{f_{M(n)}}(x) = \begin{cases} 1 - \frac{1}{n} & \text{if } x \text{ is odd} \\ 0 & \text{if } x \text{ is even} \end{cases}$$

Corresponding t-norm $(*)$ and s-norm $(\diamond)$ are defined as $a * b = min\,(a, b), a \diamond b = max\,(a, b)$.

Then, M forms a neutrosophic soft set as well as neutrosophic soft group over $[(\mathbb{Z}, +), \mathbb{N}]$.

3.2 Proposition:

An NSS N over the group $(G,\circ)$ is called a neutrosophic soft group iff followings hold on the assumption that truth membership (T), indeterministic membership (I) and falsity membership (F) functions of an NSS obey the idempotent t-norm and idempotent s-norm disciplines.

$$T_{f_{N(e)}}(x \circ y^{-1}) \geq T_{f_{N(e)}}(x) * T_{f_{N(e)}}(y),$$

$$I_{f_{N(e)}}(x \circ y^{-1}) \leq I_{f_{N(e)}}(x) \diamond I_{f_{N(e)}}(y),$$

$$F_{f_{N(e)}}(x \circ y^{-1}) \leq F_{f_{N(e)}}(x) \diamond F_{f_{N(e)}}(y);$$
$$\forall\, x, y \in G; \quad \forall e \in E;$$

Proof:

Firstly, suppose N is an NSS group over $(G,\circ)$.
Then,

$$T_{f_{N(e)}}(x \circ y^{-1}) \geq T_{f_{N(e)}}(x) * T_{f_{N(e)}}(y^{-1}) \geq T_{f_{N(e)}}(x) * T_{f_{N(e)}}(y),$$

$$I_{f_{N(e)}}(x \circ y^{-1}) \leq I_{f_{N(e)}}(x) \diamond I_{f_{N(e)}}(y^{-1}) \leq I_{f_{N(e)}}(x) \diamond I_{f_{N(e)}}(y),$$

$$F_{f_{N(e)}}(x \circ y^{-1}) \leq F_{f_{N(e)}}(x) \diamond F_{f_{N(e)}}(y^{-1}) \leq F_{f_{N(e)}}(x) \diamond F_{f_{N(e)}}(y);$$

Conversely, for the identity element e_G in G;

$$T_{f_{N(e)}}(e_G) = T_{f_{N(e)}}(x \circ x^{-1}) \geq T_{f_{N(e)}}(x) * T_{f_{N(e)}}(x) = T_{f_{N(e)}}(x),$$

$$I_{f_{N(e)}}(e_G) = I_{f_{N(e)}}(x \circ x^{-1}) \leq I_{f_{N(e)}}(x) \diamond I_{f_{N(e)}}(x) = I_{f_{N(e)}}(x),$$

$$F_{f_{N(e)}}(e_G) = F_{f_{N(e)}}(x \circ x^{-1}) \leq F_{f_{N(e)}}(x) \diamond F_{f_{N(e)}}(x) = F_{f_{N(e)}}(x);$$

Now,

$$T_{f_{N(e)}}(x^{-1}) = T_{f_{N(e)}}(e_G \circ x^{-1}) \geq T_{f_{N(e)}}(e_G) * T_{f_{N(e)}}(x^{-1}) \geq T_{f_{N(e)}}(x) * T_{f_{N(e)}}(x) = T_{f_{N(e)}}(x),$$

$$I_{f_{N(e)}}(x^{-1}) = I_{f_{N(e)}}(e_G \circ x^{-1}) \leq I_{f_{N(e)}}(e_G) \diamond I_{f_{N(e)}}(x^{-1}) \leq I_{f_{N(e)}}(x) \diamond I_{f_{N(e)}}(x) = I_{f_{N(e)}}(x),$$

$F_{f_{N(e)}}(x^{-1}) = F_{f_{N(e)}}(e_G \circ x^{-1})$
$\leq F_{f_{N(e)}}(e_G) \diamond I_{f_{N(e)}}(x^{-1})$
$\leq F_{f_{N(e)}}(x) \diamond F_{f_{N(e)}}(x)$
$= F_{f_{N(e)}}(x);$

Next,

$T_{f_{N(e)}}(x \circ y) = T_{f_{N(e)}}(x \circ (y^{-1})^{-1})$
$\geq T_{f_{N(e)}}(x) * T_{f_{N(e)}}(y^{-1})$
$\geq T_{f_{N(e)}}(x) * T_{f_{N(e)}}(y),$

$I_{f_{N(e)}}(x \circ y) = I_{f_{N(e)}}(x \circ (y^{-1})^{-1})$
$\leq I_{f_{N(e)}}(x) \diamond I_{f_{N(e)}}(y^{-1})$
$\leq I_{f_{N(e)}}(x) \diamond I_{f_{N(e)}}(y),$

$F_{f_{N(e)}}(x \circ y) = F_{f_{N(e)}}(x \circ (y^{-1})^{-1})$
$\leq F_{f_{N(e)}}(x) \diamond F_{f_{N(e)}}(y^{-1})$
$\leq F_{f_{N(e)}}(x) \diamond F_{f_{N(e)}}(y).$

This completes the proof.

3.2.1 Proposition:

Let N be a neutrosophic soft group over the group $(G,\circ)$. Then for $x \in G$, followings hold.

(i) $T_{f_{N(e)}}(x^{-1}) = T_{f_{N(e)}}(x)$, $I_{f_{N(e)}}(x^{-1}) = I_{f_{N(e)}}(x)$, $F_{f_{N(e)}}(x^{-1}) = F_{f_{N(e)}}(x)$;

(ii) $T_{f_{N(e)}}(e_G) \geq T_{f_{N(e)}}(x)$, $I_{f_{N(e)}}(e_G) \leq I_{f_{N(e)}}(x)$, $F_{f_{N(e)}}(e_G) \leq F_{f_{N(e)}}(x)$;

if T follows the idempotent t-norm and I, F follow the idempotent s-norm disciplines, respectively. (e_G being the identity element of G.)

Proof:

(i) $T_{f_{N(e)}}(x) = T_{f_{N(e)}}(x^{-1})^{-1} \geq T_{f_{N(e)}}(x^{-1})$
$I_{f_{N(e)}}(x) = I_{f_{N(e)}}(x^{-1})^{-1} \leq I_{f_{N(e)}}(x^{-1})$
$F_{f_{N(e)}}(x) = F_{f_{N(e)}}(x^{-1})^{-1} \leq F_{f_{N(e)}}(x^{-1})$

Now, from definition of neutrosophic soft group, the result follows.

(ii) For the identity element e_G in G,

$T_{f_{N(e)}}(e_G) = T_{f_{N(e)}}(x \circ x^{-1})$
$\geq T_{f_{N(e)}}(x) * T_{f_{N(e)}}(x)$
$= T_{f_{N(e)}}(x)$,

$I_{f_{N(e)}}(e_G) = I_{f_{N(e)}}(x \circ x^{-1})$
$\leq I_{f_{N(e)}}(x) \diamond I_{f_{N(e)}}(x)$
$= I_{f_{N(e)}}(x)$,

$F_{f_{N(e)}}(e_G) = F_{f_{N(e)}}(x \circ x^{-1})$
$\leq F_{f_{N(e)}}(x) \diamond F_{f_{N(e)}}(x)$
$= F_{f_{N(e)}}(x)$;

Hence, the proposition is proved.

3.3 Theorem:

Let N_1 and N_2 be two neutrosophic soft groups over the group $(G,\circ)$. Then, $N_1 \cap N_2$ is also a neutrosophic soft group over $(G,\circ)$.

Proof:

Let $N_1 \cap N_2 = N_3$; Now for $x, y \in G$;

$T_{f_{N_3(e)}}(x \circ y)$
$= T_{f_{N_1(e)}}(x \circ y) * T_{f_{N_2(e)}}(x \circ y)$
$\geq \left[T_{f_{N_1(e)}}(x) * T_{f_{N_1(e)}}(y)\right] *$
$\left[T_{f_{N_2(e)}}(x) * T_{f_{N_2(e)}}(y)\right]$
$= \left[T_{f_{N_1(e)}}(x) * T_{f_{N_1(e)}}(y)\right] *$
$\left[T_{f_{N_2(e)}}(y) * T_{f_{N_2(e)}}(x)\right]$
(as $*$ is commutative)
$= T_{f_{N_1(e)}}(x) * \left[T_{f_{N_1(e)}}(y) * T_{f_{N_2(e)}}(y)\right]$
$* T_{f_{N_2(e)}}(x)$ (as $*$ is associative)
$= T_{f_{N_1(e)}}(x) * T_{f_{N_3(e)}}(y) * T_{f_{N_2(e)}}(x)$
$= T_{f_{N_1(e)}}(x) * T_{f_{N_2(e)}}(x) * T_{f_{N_3(e)}}(y)$
(as $*$ is commutative)
$= T_{f_{N_3(e)}}(x) * T_{f_{N_3(e)}}(y)$

Also,

$T_{f_{N_3(e)}}(x^{-1}) = T_{f_{N_1(e)}}(x^{-1}) * T_{f_{N_2(e)}}(x^{-1})$
$\geq T_{f_{N_1(e)}}(x) * T_{f_{N_2(e)}}(x)$
$= T_{f_{N_3(e)}}(x);$

Next,

$I_{f_{N_3(e)}}(x \circ y)$
$= I_{f_{N_1(e)}}(x \circ y) \diamond I_{f_{N_2(e)}}(x \circ y)$

$\leq \left[I_{f_{N_1(e)}}(x) \diamond I_{f_{N_1(e)}}(y)\right] \diamond$
$\left[I_{f_{N_2(e)}}(x) \diamond I_{f_{N_2(e)}}(y)\right]$
$= \left[I_{f_{N_1(e)}}(x) \diamond I_{f_{N_1(e)}}(y)\right] \diamond$
$\left[I_{f_{N_2(e)}}(y) \diamond I_{f_{N_2(e)}}(x)\right]$
(as $\diamond$ is commutative)
$= I_{f_{N_1(e)}}(x) \diamond \left[I_{f_{N_1(e)}}(y) \diamond I_{f_{N_2(e)}}(y)\right]$
$\diamond I_{f_{N_2(e)}}(x)$ (as $\diamond$ is associative)
$= I_{f_{N_1(e)}}(x) \diamond I_{f_{N_3(e)}}(y) \diamond I_{f_{N_2(e)}}(x)$
$= I_{f_{N_1(e)}}(x) \diamond I_{f_{N_2(e)}}(x) \diamond I_{f_{N_3(e)}}(y)$
(as $\diamond$ is commutative)
$= I_{f_{N_3(e)}}(x) \diamond I_{f_{N_3(e)}}(y)$

Also,

$I_{f_{N_3(e)}}(x^{-1}) = I_{f_{N_1(e)}}(x^{-1}) \diamond I_{f_{N_2(e)}}(x^{-1})$
$\leq I_{f_{N_1(e)}}(x) \diamond I_{f_{N_2(e)}}(x)$
$= I_{f_{N_3(e)}}(x);$

Similarly,

$F_{f_{N_3(e)}}(x \circ y) \leq F_{f_{N_3(e)}}(x) \diamond F_{f_{N_3(e)}}(y),$
$F_{f_{N_3(e)}}(x^{-1}) \leq F_{f_{N_3(e)}}(x);$

This ends the theorem. The theorem is also true for a family of neutrosophic soft groups over a group.

3.3.1 Remark:

For two neutrosophic soft groups N_1 and N_2 over the group G, $N_1 \cup N_2$ is not generally a neutrosophic soft group over G. It is possible if anyone is contained in other.

For example, let, $G = (\mathbb{Z}, +)$, $E = 2\mathbb{Z}$. Consider two neutrosophic soft groups N_1 and N_2 over G as following. For $x, n \in \mathbb{Z}$,

$$T_{f_{N_1}(2n)}(x) = \begin{cases} \frac{1}{2} & \text{if } x = 4\text{kn}, \exists \text{k} \in \mathbb{Z} \\ 0 & \text{others} \end{cases}$$

$$I_{f_{N_1}(2n)}(x) = \begin{cases} 0 & \text{if } x = 4\text{kn}, \exists \text{k} \in \mathbb{Z} \\ \frac{1}{4} & \text{others} \end{cases}$$

$$F_{f_{N_1}(2n)}(x) = \begin{cases} 0 & \text{if } x = 4\text{kn}, \exists \text{k} \in \mathbb{Z} \\ \frac{1}{10} & \text{others} \end{cases}$$

and

$$T_{f_2(2n)}(x) = \begin{cases} \frac{2}{3} & \text{if } x = 6\text{kn}, \exists \text{k} \in \mathbb{Z} \\ 0 & \text{others} \end{cases}$$

$$I_{f_{N_2}(2n)}(x) = \begin{cases} 0 & \text{if } x = 6\text{kn}, \exists \text{k} \in \mathbb{Z} \\ \frac{1}{5} & \text{others} \end{cases}$$

$$F_{f_{N_2}(2n)}(x) = \begin{cases} \frac{1}{6} & \text{if } x = 6\text{kn}, \exists \text{k} \in \mathbb{Z} \\ 1 & \text{others} \end{cases}$$

Corresponding t-norm ($*$) and s-norm ($\diamond$) are defined as $a * b = min\,(a, b)$, $a \diamond b = max\,(a, b)$. Let $N_1 \cup N_2 = N_3$; Then for $n = 3$, $x = 12$, $y = 18$ we have,

$$T_{f_{N_3}(6)}(12 - 18) = T_{f_{N_1}(6)}(-6) \diamond T_{f_{N_2}(6)}(-6) = \max\,(0, 0) = 0$$ and

$$T_{f_{N_3}(6)}(12) * T_{f_{N_3}(6)}(18) = \left\{ T_{f_{N_1}(6)}(12) \diamond T_{f_{N_2}(6)}(12) \right\} * \left\{ T_{f_{N_1}(6)}(18) \diamond T_{f_{N_2}(6)}(18) \right\}$$

$$= min\left\{ max\left(\frac{1}{2}, 0\right), max\left(0, \frac{2}{3}\right) \right\} = min\left(\frac{1}{2}, \frac{2}{3}\right) = \frac{1}{2}$$

Hence,

$$T_{f_{N_3}(6)}(12 - 18) < T_{f_{N_3}(6)}(12) * T_{f_{N_3}(6)}(18);$$

i,e $N_1 \cup N_2$ is not a neutrosophic soft group, here.

Now, if we define N_2 over G as following:

$$T_{f_{N_2}(2n)}(x) = \begin{cases} \frac{1}{8} & \text{if } x = 8\text{kn}, \exists \text{k} \in \mathbb{Z} \\ 0 & \text{others} \end{cases}$$

$$I_{f_{N_2}(2n)}(x) = \begin{cases} 0 & \text{if } x = 8\text{kn}, \exists \text{k} \in \mathbb{Z} \\ \frac{2}{5} & \text{others} \end{cases}$$

$$F_{f_{N_2}(2n)}(x) = \begin{cases} \frac{1}{6} & \text{if } x = 8\text{kn}, \exists \text{k} \in \mathbb{Z} \\ \frac{1}{2} & \text{others} \end{cases}$$

Then, it can be easily verified that $N_2 \subseteq N_1$ and $N_1 \cup N_2$ is a neutrosophic soft group over G.

3.4 Definition:

1. Let N_1 and N_2 be two NSSs over the common universe (U, E). Then their 'AND' operation is denoted by $N_1 \wedge N_2 = N_3$ and is defined as:

$$N_3 = \{[(a, b), \{< x, T_{f_{N_3}(a,b)}(x),\ I_{f_{N_3}(a,b)}(x),\ F_{f_{N_3}(a,b)}(x) >: x \in U\}]: (a, b) \in E \times E\}$$ where

$$T_{f_{N_3}(a,b)}(x) = T_{f_{N_1}(a)}(x) * T_{f_{N_2}(b)}(x),$$
$$I_{f_{N_3}(a,b)}(x) = I_{f_{N_1}(a)}(x) \diamond I_{f_{N_2}(b)}(x),$$
$$F_{f_{N_3}(a,b)}(x) = F_{f_{N_1}(a)}(x) \diamond F_{f_{N_2}(b)}(x);$$

2. Let N_1 and N_2 be two NSSs over the common universe (U, E). Then their 'OR' operation is denoted by $N_1 \vee N_2 = N_4$ and is defined as:

$$N_4 = \{[(a, b), \{< x, T_{f_{N_4}(a,b)}(x),\ I_{f_{N_4}(a,b)}(x),\ F_{f_{N_4}(a,b)}(x) >: x \in U\}]: (a, b) \in E \times E\}$$

where

$$T_{f_{N_4}(a,b)}(x) = T_{f_{N_1}(a)}(x) \diamond T_{f_{N_2}(b)}(x),$$
$$I_{f_{N_4}(a,b)}(x) = I_{f_{N_1}(a)}(x) * I_{f_{N_2}(b)}(x),$$
$$F_{f_{N_4}(a,b)}(x) = F_{f_{N_1}(a)}(x) * F_{f_{N_2}(b)}(x);$$

3.5 Theorem:

Let N_1 and N_2 be two neutrosophic soft groups over the group $(G, \circ)$. Then, $N_1 \wedge N_2$ is also a neutrosophic soft group over $(G, \circ)$.

Proof:

Let $N_1 \wedge N_2 = N_3$. Then for $x, y \in G$ and $(a, b) \in E \times E$,

$$T_{f_{N_3}(a,b)}(x \circ y)$$
$$= T_{f_{N_1}(a)}(x \circ y) * T_{f_{N_2}(b)}(x \circ y)$$
$$\geq \left[T_{f_{N_1}(a)}(x) * T_{f_{N_1}(a)}(y) \right] * \left[T_{f_{N_2}(b)}(x) * T_{f_{N_2}(b)}(y) \right]$$
$$= \left[T_{f_{N_1}(a)}(x) * T_{f_{N_1}(a)}(y) \right] * \left[T_{f_{N_2}(b)}(y) * T_{f_{N_2}(b)}(x) \right]$$
(as $*$ is commutative)
$$= T_{f_{N_1}(a)}(x) * \left[T_{f_{N_1}(a)}(y) * T_{f_{N_2}(b)}(y) \right] * T_{f_{N_2}(b)}(x)$$ (as $*$ is associative)
$$= T_{f_{N_1}(a)}(x) * T_{f_{N_3}(a,b)}(y) * T_{f_{N_2}(b)}(x)$$
$$= T_{f_{N_1}(a)}(x) * T_{f_{N_2}(b)}(x) * T_{f_{N_3}(a,b)}(y)$$
(as $*$ is commutative)
$$= T_{f_{N_3}(a,b)}(x) * T_{f_{N_3}(a,b)}(y)$$

$$T_{f_{N_3}(a,b)}(x^{-1}) = T_{f_{N_1}(a)}(x^{-1}) * T_{f_{N_2}(b)}(x^{-1})$$
$$\geq T_{f_{N_1}(a)}(x) * T_{f_{N_2}(b)}(x)$$
$$= T_{f_{N_3}(a,b)}(x)$$

Similarly,
$I_{f_{N_3(a,b)}}(x \circ y) \leq I_{f_{N_3(a,b)}}(x) \diamond I_{f_{N_3(a,b)}}(y)$,
$I_{f_{N_3(a,b)}}(x^{-1}) \leq I_{f_{N_3(a,b)}}(x)$;

$F_{f_{N_3(a,b)}}(x \circ y) \leq F_{f_{N_3(a,b)}}(x) \diamond F_{f_{N_3(a,b)}}(y)$,
$F_{f_{N_3(a,b)}}(x^{-1}) \leq F_{f_{N_3(a,b)}}(x)$;

This completes the proof.

The theorem is true for a family of neutrosophic soft groups over a group.

3.6 Definition:

Let g be a mapping from a set X to a set Y. If M and N are two neutrosophic soft sets over X and Y, respectively, then the image of M under g is defined as a neutrosophic soft set $g(M) = \{[e, f_{g(M)}(e)]: e \in E\}$ over Y, where $T_{f_{g(M)}(e)}(y) = T_{f_M(e)}[g^{-1}(y)]$, $I_{f_{g(M)}(e)}(y) = I_{f_M(e)}[g^{-1}(y)]$, $F_{f_{g(M)}(e)}(y) = F_{f_M(e)}[g^{-1}(y)]$, $\forall y \in Y$.

The pre-image of N under g is defined as a neutrosophic soft set given by:
$g^{-1}(N) = \{[e, f_{g^{-1}(N)}(e)]: e \in E\}$ over X, where
$T_{f_{g^{-1}(N)}(e)}(x) = T_{f_N(e)}[g(x)]$, $I_{f_{g^{-1}(N)}(e)}(x) = I_{f_N(e)}[g(x)]$, $F_{f_{g^{-1}(N)}(e)}(x) = F_{f_N(e)}[g(x)]$,
$\forall x \in X$.

3.7 Theorem:

Let $g : X \to Y$ be an isomorphism in classical sense. If M is a neutrosophic soft group over X then $g(M)$ is a neutrosophic soft group over Y.

Proof:

Let $x_1, x_2 \in X$; $y_1, y_2 \in Y$; such that $y_1 = g(x_1)$, $y_2 = g(x_2)$. Now,

$T_{f_{g(M)}(e)}(y_1 \circ y_2)$
$= T_{f_M(e)}[g^{-1}(y_1 \circ y_2)]$
$= T_{f_M(e)}[g^{-1}(y_1) \circ g^{-1}(y_2)]$
(as g^{-1} is homomorphism)
$= T_{f_M(e)}(x_1 \circ x_2)$
$\geq T_{f_M(e)}(x_1) * T_{f_M(e)}(x_2)$
$= T_{f_M(e)}[g^{-1}(y_1)] * T_{f_M(e)}[g^{-1}(y_2)]$
$= T_{f_{g(M)}(e)}(y_1) * T_{f_{g(M)}(e)}(y_2)$

Next, $I_{f_{g(M)}(e)}(y_1 \circ y_2)$
$= I_{f_M(e)}[g^{-1}(y_1 \circ y_2)]$
$= I_{f_M(e)}[g^{-1}(y_1) \circ g^{-1}(y_2)]$
(as g^{-1} is homomorphism)
$= I_{f_M(e)}(x_1 \circ x_2)$
$\leq I_{f_M(e)}(x_1) \diamond I_{f_M(e)}(x_2)$
$= I_{f_M(e)}[g^{-1}(y_1)] \diamond I_{f_M(e)}[g^{-1}(y_2)]$
$= I_{f_{g(M)}(e)}(y_1) \diamond I_{f_{g(M)}(e)}(y_2)$

Similarly, $F_{f_{g(M)}(e)}(y_1 \circ y_2)$
$\leq F_{f_{g(M)}(e)}(y_1) \diamond F_{f_{g(M)}(e)}(y_2)$

Next, $T_{f_{g(M)}(e)}(y_1^{-1}) = T_{f_M(e)}[g^{-1}(y_1^{-1})]$
$= T_{f_M(e)}[(g^{-1}(y_1))^{-1}] = T_{f_M(e)}(x_1^{-1})$
$\geq T_{f_M(e)}(x_1) = T_{f_M(e)}[g^{-1}(y_1)]$
$= T_{f_{g(M)}(e)}(y_1)$ i,e
$T_{f_{g(M)}(e)}(y_1^{-1}) \geq T_{f_{g(M)}(e)}(y_1)$;

$I_{f_{g(M)}(e)}(y_1^{-1}) = I_{f_M(e)}[g^{-1}(y_1^{-1})]$
$= I_{f_M(e)}[(g^{-1}(y_1))^{-1}] = I_{f_M(e)}(x_1^{-1})$
$\leq I_{f_M(e)}(x_1) = I_{f_M(e)}[g^{-1}(y_1)]$
$= I_{f_{g(M)}(e)}(y_1)$ i,e
$I_{f_{g(M)}(e)}(y_1^{-1}) \leq I_{f_{g(M)}(e)}(y_1)$;

Similarly, $F_{f_{g(M)}(e)}(y_1^{-1}) \leq F_{f_{g(M)}(e)}(y_1)$;

This proves the theorem.

3.8 Theorem:

Let $g : X \to Y$ be an homomorphism in classical sense. If N is a neutrosophic soft group over Y, then $g^{-1}(N)$ is a neutrosophic soft group over X. [Note that $g^{-1}(N)$ is the inverse image of N under the mapping g. Here g^{-1} may not be a mapping.]

Proof:

Let $x_1, x_2 \in X$; $y_1, y_2 \in Y$; such that $y_1 = g(x_1)$, $y_2 = g(x_2)$. Now,
$T_{f_{g^{-1}(N)}(e)}(x_1 \circ x_2)$
$= T_{f_N(e)}[g(x_1 \circ x_2)]$
$= T_{f_N(e)}[g(x_1) \circ g(x_2)]$
(as g is homomorphism)
$= T_{f_N(e)}(y_1 \circ y_2)$
$\geq T_{f_N(e)}(y_1) * T_{f_N(e)}(y_2)$
$= T_{f_N(e)}[g(x_1)] * T_{f_N(e)}[g(x_2)]$
$= T_{f_{g^{-1}(N)}(e)}(x_1) * T_{f_{g^{-1}(N)}(e)}(x_2)$

Next, $I_{f_{g^{-1}(N)}(e)}(x_1 \circ x_2)$
$= I_{f_N(e)}[g(x_1 \circ x_2)]$
$= I_{f_N(e)}[g(x_1) \circ g(x_2)]$
(as g is homomorphism)
$= I_{f_N(e)}(y_1 \circ y_2)$
$\leq I_{f_N(e)}(y_1) \diamond I_{f_N(e)}(y_2)$
$= I_{f_N(e)}[g(x_1)] \diamond I_{f_N(e)}[g(x_2)]$
$= I_{f_{g^{-1}(N)}(e)}(x_1) \diamond I_{f_{g^{-1}(N)}(e)}(x_2)$

Similarly, $F_{f_{g^{-1}(N)}(e)}(x_1 \circ x_2)$
$\leq F_{f_{g^{-1}(N)}(e)}(x_1) \diamond F_{f_{g^{-1}(N)}(e)}(x_2)$

Next, $T_{f_{g^{-1}(N)}(e)}(x_1^{-1}) = T_{f_N(e)}[g(x_1^{-1})]$

$= T_{f_N(e)}\left[\left(g(x_1)\right)^{-1}\right] = T_{f_N(e)}(y_1^{-1}) \geq T_{f_N(e)}(y_1) = T_{f_N(e)}[g(x_1)]$

$= T_{f_{g^{-1}(N)}(e)}(x_1)$ i,e

$T_{f_{g^{-1}(N)}(e)}(x_1^{-1}) \geq T_{f_{g^{-1}(N)}(e)}(x_1)$;

Similarly, $I_{f_{g^{-1}(N)}(e)}(x_1^{-1}) \leq I_{f_{g^{-1}(N)}(e)}(x_1)$,

$F_{f_{g^{-1}(N)}(e)}(x_1^{-1}) \leq F_{f_{g^{-1}(N)}(e)}(x_1)$;

Hence, the theorem is proved.

3.9 Definition:

Let N be a neutrosophic soft group over the group G and $\lambda, \mu, \eta \in (0,1]$ with $\lambda + \mu + \eta \leq 3$. Then,

1. N is called (λ, μ, η) -identity neutrosophic soft group over G if $\forall e \in E$,
$T_{f_N(e)}(x) = \lambda,\ I_{f_N(e)}(x) = \mu,\ F_{f_N(e)}(x) = \eta$;
for $x = e_G$, the identity element of G.
$T_{f_N(e)}(x) = 0,\ I_{f_N(e)}(x) = F_{f_N(e)}(x) = 1$;
otherwise.
2. N is called (λ, μ, η) -absolute neutrosophic soft group over G if $\forall x \in G,\ e \in E,\quad T_{f_N(e)}(x) = \lambda,\ I_{f_N(e)}(x) = \mu,\ F_{f_N(e)}(x) = \eta$.

3.10 Theorem:

Let $\phi : X \rightarrow Y$ be an isomorphism in classical sense.

1. If N is a neutrosophic soft group over X, then $\phi(N)$ is a (λ, μ, η) -identity neutrosophic soft group over Y if $T_{f_N(e)}(x) = \lambda,\ I_{f_N(e)}(x) = \mu,\ F_{f_N(e)}(x) = \eta$; when $x \in Ker\phi$.
$T_{f_N(e)}(x) = 0,\ I_{f_N(e)}(x) = F_{f_N(e)}(x) = 1$;
otherwise, $\forall x \in X,\ e \in E$.
2. If N is a (λ, μ, η) -absolute neutrosophic soft group over X, then $\phi(N)$ is also so over Y.

Proof:

1. Clearly, $\phi(N)$ is a neutrosophic soft group over Y by theorem (3.7). Now, if $x \in ker\phi$ then $\phi(x) = e_Y$, the identity element of Y. Then,

$$T_{f_{\phi(N)}(e)}(e_Y) = T_{f_N(e)}[\phi^{-1}(e_Y)] = T_{f_N(e)}(x) = \lambda$$

$$I_{f_{\phi(N)}(e)}(e_Y) = I_{f_N(e)}[\phi^{-1}(e_Y)] = I_{f_N(e)}(x) = \mu$$

$$F_{f_{\phi(N)}(e)}(e_Y) = F_{f_N(e)}[\phi^{-1}(e_Y)] = F_{f_N(e)}(x) = \eta$$

Similarly, $T_{f_N(e)}(x) = 0,\ I_{f_N(e)}(x) = 1$,
$F_{f_N(e)}(x) = 1$; if x otherwise.
This ends the 1st part.

2. Let, $y = \phi(x)$ for $x \in X,\ y \in Y$. Then $\forall e \in E$,

$T_{f_{\phi(N)}(e)}(y) = T_{f_N(e)}[\phi^{-1}(y)] = T_{f_N(e)}(x) = \lambda$,
$I_{f_{\phi(N)}(e)}(y) = I_{f_N(e)}[\phi^{-1}(y)] = I_{f_N(e)}(x) = \mu$,
$F_{f_{\phi(N)}(e)}(y) = F_{f_N(e)}[\phi^{-1}(y)] = F_{f_N(e)}(x) = \eta$;

This completes the 2nd part.

4 Cartesian product of neutrosophic soft groups

4.1 Definition:

Let N_1 and N_2 be two neutrosophic soft groups over the groups X and Y, respectively. Then their cartesian product is $N_1 \times N_2 = N_3$ where $f_{N_3}(a,b) = f_{N_1}(a) \times f_{N_2}(b)$ for $(a,b) \in E \times E$. Analytically, $f_{N_3}(a,b) =$

$$\left\{< (x,y),\ T_{f_{N_3(a,b)}}(x,y)\ I_{f_{N_3(a,b)}}(x,y),\ F_{f_{N_3(a,b)}}(x,y) >: (x,y) \in X \times Y\right\}$$

where

$T_{f_{N_3(a,b)}}(x,y) = T_{f_{N_1}(a)}(x) * T_{f_{N_2}(b)}(y)$,
$I_{f_{N_3(a,b)}}(x,y) = I_{f_{N_1}(a)}(x) \diamond I_{f_{N_2}(b)}(y)$,
$F_{f_{N_3(a,b)}}(x,y) = F_{f_{N_1}(a)}(x) \diamond F_{f_{N_2}(b)}(y)$;

This definition can be extended for more than two neutrosophic soft groups.

4.2 Theorem:

Let N_1 and N_2 be two neutrosophic soft groups over the groups X and Y, respectively. Then their cartesian product $N_1 \times N_2$ is also a neutrosophic soft group over $X \times Y$.

Proof:

Let $N_1 \times N_2 = N_3$ where $f_{N_3}(a,b) = f_{N_1}(a) \times f_{N_2}(b)$ for $(a,b) \in E \times E$. Then for $(x_1,y_1),\ (x_2,y_2) \in X \times Y$,

$T_{f_{N_3(a,b)}}[(x_1,y_1) \circ (x_2,y_2)]$
$= T_{f_{N_3(a,b)}}(x_1 \circ x_2,\ y_1 \circ y_2)$
$= T_{f_{N_1}(a)}(x_1 \circ x_2) * T_{f_{N_2}(b)}(y_1 \circ y_2)$
$\geq [T_{f_{N_1}(a)}(x_1) * T_{f_{N_1}(a)}(x_2)] * [T_{f_{N_2}(b)}(y_1) * T_{f_{N_2}(b)}(y_2)]$
$= [T_{f_{N_1}(a)}(x_1) * T_{f_{N_2}(b)}(y_1)] * [T_{f_{N_1}(a)}(x_2) * T_{f_{N_2}(b)}(y_2)]$
$= T_{f_{N_3(a,b)}}(x_1,y_1) * T_{f_{N_3(a,b)}}(x_2,y_2)$

Next,

$I_{f_{N_3(a,b)}}[(x_1,y_1) \circ (x_2,y_2)]$
$= I_{f_{N_3(a,b)}}(x_1 \circ x_2,\ y_1 \circ y_2)$
$= I_{f_{N_1}(a)}(x_1 \circ x_2) \diamond I_{f_{N_2}(b)}(y_1 \circ y_2)$
$\leq [I_{f_{N_1}(a)}(x_1) I_{f_{N_1}(a)}(x_2)] \diamond [I_{f_{N_2}(b)}(y_1) \diamond I_{f_{N_2}(b)}(y_2)]$
$= [I_{f_{N_1}(a)}(x_1) \diamond I_{f_{N_2}(b)}(y_1)] \diamond [I_{f_{N_1}(a)}(x_2) \diamond I_{f_{N_2}(b)}(y_2)]$
$= I_{f_{N_3(a,b)}}(x_1,y_1) \diamond I_{f_{N_3(a,b)}}(x_2,y_2)$

Similarly, $F_{f_{N_3(a,b)}}[(x_1,y_1) \circ (x_2,y_2)] \leq F_{f_{N_3(a,b)}}(x_1,y_1) \diamond F_{f_{N_3(a,b)}}(x_2,y_2)$.

Next,

$$T_{f_{N_3}(a,b)}[(x_1, y_1)^{-1}] = T_{f_{N_3}(a,b)}(x_1^{-1}, y_1^{-1})$$
$$= T_{f_{N_1}(a)}(x_1^{-1}) * T_{f_{N_2}(b)}(y_1^{-1})$$
$$\geq T_{f_{N_1}(a)}(x_1) * T_{f_{N_2}(b)}(y_1)$$
$$= T_{f_{N_3}(a,b)}(x_1, y_1)$$

Similarly,

$I_{f_{N_3}(a,b)}[(x_1, y_1)^{-1}] \leq I_{f_{N_3}(a,b)}(x_1, y_1)$,
$F_{f_{N_3}(a,b)}[(x_1, y_1)^{-1}] \leq F_{f_{N_3}(a,b)}(x_1, y_1)$.

Hence, the theorem is proved.

5 Neutrosophic soft subgroup

5.1 Definition:

Let N_1 and N_2 be two neutrosophic groups over the group G. Then N_1 is neutrosophic soft subgroup of N_2 if

$$T_{f_{N_1}(e)}(x) \leq T_{f_{N_2}(e)}(x),\ I_{f_{N_1}(e)}(x) \geq I_{f_{N_2}(e)}(x),$$
$$F_{f_{N_1}(e)}(x) \geq F_{f_{N_2}(e)}(x);\ \forall x \in G,\ e \in E.$$

5.1.1 Example:

We consider the Klein's -4 group $V = \{ e, a, b, c \}$ and $E = \{ \alpha, \beta, \gamma, \delta \}$ be a set of parameters. The two neutrosophic soft groups M, N defined over (V, E) are given by the following tables when corresponding t-norm and s-norm are defined as

$a * b = max\,(a + b - 1, 0)$ and $a \diamond b = min\,(a + b, 1)$.

	$f_M(\alpha)$ $f_M(\gamma)$	$f_M(\beta)$ $f_M(\delta)$
e	(0.65, 0.42, 0.54) (0.70, 0.31, 0.32)	(0.68, 0.21, 0.76) (0.59, 0.38, 0.62)
a	(0.61, 0.44, 0.78) (0.67, 0.41, 0.39)	(0.62, 0.31, 0.79) (0.41, 0.49, 0.64)
b	(0.55, 0.55, 0.59) (0.60, 0.36, 0.48)	(0.59, 0.42, 0.80) (0.56, 0.43, 0.68)
c	(0.47, 0.49, 0.69) (0.48, 0.52, 0.54)	(0.67, 0.43, 0.84) (0.49, 0.50, 0.70)

Table 3: Tabular form of neutrosophic soft group *M*.

	$f_N(\alpha)$ $f_N(\gamma)$	$f_N(\beta)$ $f_N(\delta)$
e	(0.65, 0.34, 0.14) (0.72, 0.21, 0.16)	(0.88, 0.12, 0.72) (0.69, 0.31, 0.32)
a	(0.71, 0.22, 0.78) (0.84, 0.16, 0.25)	(0.71, 0.19, 0.44) (0.62, 0.32, 0.42)
b	(0.75, 0.25, 0.52) (0.69, 0.31, 0.39)	(0.83, 0.11, 0.28) (0.58, 0.41, 0.66)
c	(0.67, 0.32, 0.29) (0.79, 0.19, 0.41)	(0.75, 0.21, 0.19) (0.71, 0.27, 0.53)

Table 4: Tabular form of neutrosophic soft group *N*.

Obviously, M is the neutrosophic soft subgroup of N over (V, E).

5.2 Theorem:

Let N be a neutrosophic soft group over the group G and N_1, N_2 be two neutrosophic soft subgroups of N. If T, I, F of neutrosophic soft group N obey the disciplines of idempotent t-norm and idempotent s-norm, then,

(i) $N_1 \cap N_2$ is a neutrosophic soft subgroup of N.
(ii) $N_1 \wedge N_2$ is a neutrosophic soft subgroup of $N \wedge N$.

Proof:

The intersection($\cap$), AND ($\wedge$) of two neutrosophic soft groups is also so by theorems (3.3) and (3.5). Now to complete this theorem, we only verify the criteria of neutrosophic soft subgroup in each case.

(i) Let $N_3 = N_1 \cap N_2$. For $x \in G$,

$$T_{f_{N_3}(e)}(x) = T_{f_{N_1}(e)}(x) * T_{f_{N_2}(e)}(x)$$
$$\leq T_{f_N(e)}(x) * T_{f_N(e)}(x) = T_{f_N(e)}(x),$$

$$I_{f_{N_3}(e)}(x) = I_{f_{N_1}(e)}(x) \diamond I_{f_{N_2}(e)}(x)$$
$$\geq I_{f_N(e)}(x) \diamond I_{f_N(e)}(x) = I_{f_N(e)}(x),$$

$$F_{f_{N_3}(e)}(x) = F_{f_{N_1}(e)}(x) \diamond F_{f_{N_2}(e)}(x)$$
$$\geq F_{f_N(e)}(x) \diamond F_{f_N(e)}(x) = F_{f_N(e)}(x);$$

(ii) Let $N_3 = N_1 \wedge N_2$ and $x \in G$; Then,

$$T_{f_{N_3}(a,b)}(x) = T_{f_{N_1}(a)}(x) * T_{f_{N_2}(b)}(x)$$
$$\leq T_{f_N(a)}(x) * T_{f_N(b)}(x) = T_{f_N(a,b)}(x),$$

$$I_{f_{N_3}(a,b)}(x) = I_{f_{N_1}(a)}(x) \diamond I_{f_{N_2}(b)}(x)$$
$$\geq I_{f_N(a)}(x) \diamond I_{f_N(b)}(x) = I_{f_N(a,b)}(x),$$

$$F_{f_{N_3}(a,b)}(x) = F_{f_{N_1}(a)}(x) \diamond F_{f_{N_2}(b)}(x)$$
$$\geq F_{f_N(a)}(x) \diamond F_{f_N(b)}(x) = F_{f_N(a,b)}(x);$$

The theorems are also true for a family of neutrosophic soft subgroups of N.

5.3 Example:

We consider the group (S,.), cube root of unity where S= $\{1, \omega, \omega^2\}$ and let E= $\{ \alpha, \beta, \gamma \}$ be a set of parameters.

The t-norm and s-norm are defined as: $a * b = ab$ and $a \diamond b = a + b - ab$. The neutrosophic soft group N and it's two subgroups N_1, N_2 defined over (S,.) are given by the following tables.

	$f_N(\alpha)$	$f_N(\beta)$	$f_N(\gamma)$
1	(0.7,0.3,0.2)		(0.6,0.3,0.5)
	(0.6,0.5,0.6)		
ω			
	(0.7,0.2,0.4)		(0.7,0.3,0.5)
ω^2	(0.5,0.5,0.7)		
	(0.6,0.3,0.3)		(0.5,0.4,0.6)
	(0.4,0.4,0.6)		

Table 5: Tabular form of neutrosophic soft group *N*.

	$f_{N_1}(\alpha)$	$f_{N_1}(\beta)$	$f_{N_1}(\gamma)$
1	(0.4,0.4,0.9)		(0.6,0.6,0.6)
	(0.5,0.6,0.6)		
ω			
	(0.6,0.4,0.7)		(0.5,0.8,0.5)
ω^2	(0.4,0.5,0.7)		
	(0.3,0.5,0.8)		(0.5,0.6,0.7)
	(0.4,0.8,0.7)		

Table 6: Tabular form of neutrosophic soft subgroup N_1.

	$f_{N_2}(\alpha)$	$f_{N_2}(\beta)$	$f_{N_2}(\gamma)$
1	(0.6,0.5,0.2)		(0.6,0.4,0.6)
	(0.5,0.5,0.7)		
ω			
	(0.7,0.3,0.4)		(0.6,0.4,0.5)
ω^2	(0.4,0.5,0.8)		
	(0.6,0.4,0.3)		(0.5,0.5,0.7)
	(0.3,0.6,0.7)		

Table 7: Tabular form of neutrosophic soft subgroup N_2.

	$f_M(\alpha)$	$f_M(\beta)$	$f_M(\gamma)$
1	(0.24,0.70,0.92)		(0.36,0.76,0.84)
	(0.25,0.80,0.88)		
ω			
	(0.42,0.58,0.82)		(0.30,0.88,0.75)
ω^2	(0.16,0.75,0.94)		
	(0.18,0.70,0.86)		(0.25,0.80,0.91)
	(0.12,0.92,0.91)		

Table 8: Tabular form of neutrosophic soft subgroup $M = N_1 \cap N_2$.

	$f_P(\alpha,\alpha)$ $f_P(\alpha,\beta)$ $f_P(\alpha,\gamma)$	$f_P(\beta,\alpha)$ $f_P(\beta,\beta)$ $f_P(\beta,\gamma)$	$f_P(\gamma,\alpha)$ $f_P(\gamma,\beta)$ $f_P(\gamma,\gamma)$
	(0.24,0.70,0.92)		(0.36,0.80,0.68)
1	(0.30,0.80,0.68)		
	(0.24,0.64,0.96)		(0.36,0.76,0.84)
	(0.30,0.76,0.84)		
ω	(0.20,0.70,0.97)		(0.30,0.80,0.88)
	(0.25,0.80,0.88)		
	(0.42,0.58,0.82)		(0.35,0.86,0.70)
ω^2	(0.28,0.65,0.82)		
	(0.36,0.64,0.85)		(0.30,0.88,0.75)
	(0.24,0.70,0.85)		
	(0.24,0.70,0.94)		(0.20,0.90,0.90)
	(0.16,0.75,0.94)		
	(0.18,0.70,0.86)		(0.30,0.76,0.79)
	(0.24,0.88,0.79)		
	(0.15,0.75,0.94)		(0.25,0.80,0.91)
	(0.20,0.90,0.91)		
	(0.09,0,80,0.94)		(0.15,0.84,0.91)
	(0.12,0.92,0.91)		

Table 9: Tabular form of neutrosophic soft subgroup $P = N_1 \wedge N_2$.

	$f_P(\alpha,\alpha)$ $f_P(\alpha,\beta)$ $f_P(\alpha,\gamma)$	$f_P(\beta,\alpha)$ $f_P(\beta,\beta)$ $f_P(\beta,\gamma)$	$f_P(\gamma,\alpha)$ $f_P(\gamma,\beta)$ $f_P(\gamma,\gamma)$
	(0.49,0.51,0.36)		(0.42,0.51,0.60)
1	(0.42,0.65,0.68)		
	(0.42,0.51,0.60)		(0.36,0.51,0.75)
	(0.36,0.65,0.80)		
ω	(0.42,0.65,0.68)		(0.36,0.65,0.80)
	(0.36,0.75,0.84)		
	(0.49,0.36,0.64)		(0.49,0.44,0.70)
ω^2	(0.35,0.60,0.82)		
	(0.49,0.44,0.70)		(0.49,0.51,0.75)
	(0.35,0.65,0.85)		
	(0.35,0.60,0.82)		(0.35,0.65,0.85)
	(0.25,0.75,0.91)		
	(0.36,0.51,0.51)		(0.30,0.58,0.72)
	(0.24,0.58,0.72)		
	(0.30,0.58,0.72)		(0.25,0.64,0.84)
	(0.20,0.64,0.84)		
	(0.24,0.58,0.72)		(0.20,0.64,0.84)
	(0.16,0.64,0.84)		

Table 10: Tabular form of neutrosophic soft subgroup $P = N \wedge N$.

Tables 5 & 8 show the 1st result and Tables 9 & 10 show the 2nd result in theorem (5.2).

5.4 Theorem:

Let N_1 and N_2 be two neutrosophic soft groups over the group X such that N_1 is the neutrosophic soft subgroup of N_2. Let $g : X \to Y$ be an isomorphism in classical sense. Then $g(N_1)$ and $g(N_2)$ are two neutrosophic soft groups over Y. Moreover $g(N_1)$ is the neutrosophic soft subgroup of $g(N_2)$.

Proof:

The 1st part is already proved in theorem (3.7).

Let $x \in X, y \in Y$ so that $y = g(x)$. Then,

$$T_{f_{N_1}(e)}(x) \leq T_{f_{N_2}(e)}(x)$$
$$\Rightarrow T_{f_{N_1}(e)}[g^{-1}(y)] \leq T_{f_{N_2}(e)}[g^{-1}(y)]$$
$$\Rightarrow T_{f_{g(N_1)}(e)}(y) \leq T_{f_{g(N_2)}(e)}(y)$$

Similarly, $I_{f_{g(N_1)}(e)}(y) \geq I_{f_{g(N_2)}(e)}(y)$, $F_{f_{g(N_1)}(e)}(y) \geq F_{f_{g(N_2)}(e)}(y)$;

Hence, the theorem is proved.

Conclusion

In the present paper, the theoretical point of view of neutrosophic soft group has been discussed with suitable examples. Here, we also have defined the Cartesian product on neutrosophic soft groups and neutrosophic soft subgroup. Some theorems have been established. We extended the concept of group in NSS theory context. This concept will bring a new opportunity in research and development of NSS theory.

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Abstract

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