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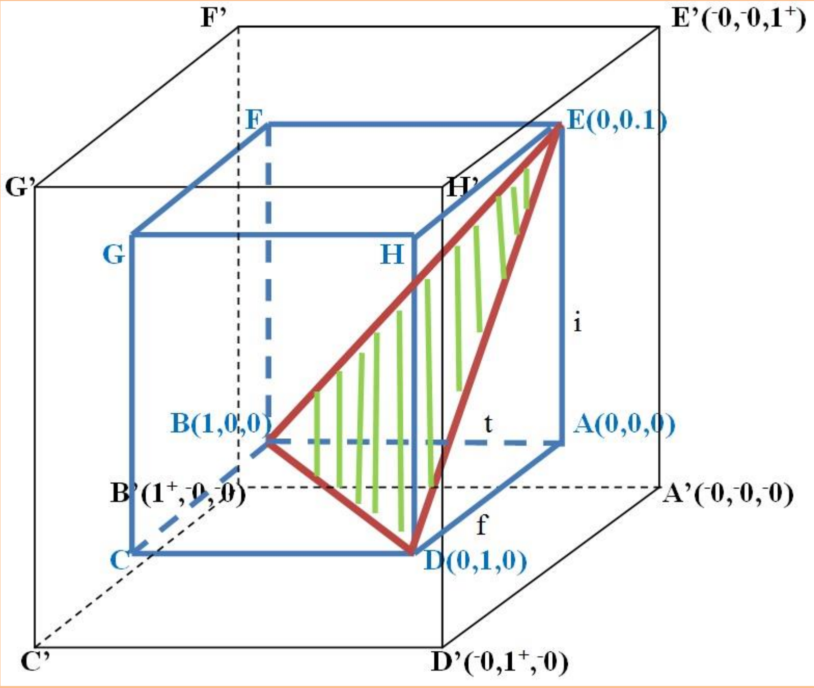

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Contents

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"Neutrosophic Sets and Systems" has been created for publications on advanced studies in neutrosophy, neutrosophic set, neutrosophic logic, neutrosophic probability, neutrosophic statistics that started in 1995 and their applications in any field, such as the neutrosophic structures developed in algebra, geometry, topology, etc.

The submitted papers should be professional, in good English, containing a brief review of a problem and obtained results.

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e. notions or ideas supporting neither <A> nor <antiA>). The <neutA> and <antiA> ideas together are referred to as <nonA>.

Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only).

According to this theory every idea <A> tends to be neutralized and balanced by <antiA> and <nonA> ideas - as a state of equilibrium.

In a classical way <A>, <neutA>, <antiA> are disjoint two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well.

Neutrosophic Set and *Neutrosophic Logic* are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and respectively intuitionistic fuzzy logic). In neutrosophic logic a proposition has a degree of truth (T), a degree of indeter
minacy (I), and a degree of falsity (F), where T, I, F are standard or non-standard subsets of $]^-0, 1^+[$.

Neutrosophic Probability is a generalization of the classical probability and imprecise probability.

Neutrosophic Statistics is a generalization of the classical statistics.

What distinguishes the neutrosophics from other fields is the <neutA>, which means neither <A> nor <antiA>.

<neutA>, which of course depends on <A>, can be indeterminacy, neutrality, tie game, unknown, contradiction, ignorance, imprecision, etc.

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Multi-valued Neutrosophic Sets and its Application in Multi-criteria Decision-making Problems

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Abstract. In recent years, hesitant fuzzy sets and neutrosophic sets have aroused the interest of researchers and have been widely applied to multi-criteria decision-making problems. The operations of multi-valued neutrosophic sets are introduced and a comparison method is developed based on related research of hesitant fuzzy sets and intuitionistic fuzzy sets in this paper. Furthermore, some multi-valued neutrosophic number aggregation operators are proposed and the desirable properties are discussed as well. Finally, an approach for multi-criteria decision-making problems was explored applying the aggregation operators. In addition, an example was provided to illustrate the concrete application of the proposed method.

Keywords: Multi-valued neutrosophic sets; multi-criteria decision-making; aggregation operators

1. Introduction

Atanassov introduced intuitionistic fuzzy sets (AIFSs) [1-4], which an extension of Zadeh's fuzzy sets(FSs) [5]. As for the present, AIFS has been widely applied in solving multi-criteria decision-making (MCDM) problems [6-10], neural networks [11, 12], medical diagnosis [13], color region extraction [14, 15], market prediction [16]. Then, AIFS was extended to the interval-valued intuitionistic fuzzy sets (AIVIFSs) [17]. AIFS took into account membership degree, non-membership degree and degree of hesitation simultaneously. So it is more flexible and practical in addressing the fuzziness and uncertainty than the traditional FSs. Moreover, in some actual cases, the membership degree, non-membership degree and hesitation degree of an element in AIFS may not be only one specific number. To handle the situations that people are hesitant in expressing their preference over objects in a decision-making process, hesitant fuzzy sets (HFSs) were introduced by Torra [18] and Narukawa [19]. Then generalized HFSs and dual hesitant fuzzy sets (DHFSs) were developed by Qian and Wang [20] and Zhu et al. [21] respectively.

Although the FS theory has been developed and generalized, it can not deal with all sorts of uncertainties in different real physical problems. Some types of uncertainties such as the indeterminate information and inconsistent information can not be handled. For example, when we ask the opinion of an expert about certain statement, he or she may say that the possibility that the statement is true is 0.6, the statement is false is 0.3 and the degree that he or she is not sure is 0.2 [22]. This issue is beyond the field of the FSs and AIFSs. Therefore, some new theories are required.

Florentin Smarandache coined neutrosophic logic and neutrosophic sets (NSs) in 1995 [23, 24]. A NS is a set where each element of the universe has a degree of truth, indeterminacy and falsity respectively and which lies in $]0^{-}, 1^{+}[$, the non-standard unit interval [25]. Obviously, it is the extension to the standard interval [0, 1] as in the AIFS. And the uncertainty present here, i.e. indeterminacy factor, is independent of truth and falsity values while the incorporated uncertainty is dependent of the degree of belongingness and degree of non belongingness in AIFSs [26]. So for the aforementioned example, it can be expressed as $x(0.6, 0.3, 0.2)$ in the form of NS.

However, without being specified, it is difficult to apply in the real applications. Hence, a single-valued neutrosophic sets (SVNSs) was proposed, which is an instance of the NSs [22, 26]. Furthermore, the information energy of SVNSs, correlation and correlation coefficient of SVNSs as well as a decision-making method based on SVNSs were presented [27]. In addition, Ye also introduced the concept of simplified neutrosophic sets (SNSs), which can be described by three real numbers in the real unit interval [0,1], and proposed a MCDM using aggregation operators for SNSs [28]. Majumdar et al. introduced a measure of entropy of a SNS [26]. Wang et al. and Lupiáñez proposed the concept of interval-valued neutrosophic sets (IVNS) and gave the set-theoretic operators of IVNS [29, 30]. Furthermore, Ye proposed the similarity measures between SVNS and INSs based on the relationship between similarity measures and distances [31, 32].

However, in some cases, the operations of SNSs in Ref. [28] might be irrational. For instance, the sum of any element and the maximum value should be equal to the maximum one, while it does not hold with the operations in Ref. [28]. Furthermore, decision-makers also hesitant to express their evaluation values for each membership in SNS. For instance, in the example given above, if decision-maker think that the possibility that statement is true is 0.6 or 0.7, the statement is false is 0.2 or 0.3 and the degree that he or she is not sure is 0.1 or 0.2. Then how to handle these circumstances with SVNS is also a problem. At the same time, if the operations and comparison method of SVNSs are extended to multi-valued in SVNS, then there exist shortcomings else as we discussed earlier. Therefore, the definition of multi-valued neutrosophic sets (MVNSs) and its operations along with comparison approach between multi-valued neutrosophic numbers (MVNNs), and aggregation operators for MVNS are defined in this paper. Thus, a MCDM method is established based on the proposed operators, an illustrative example is given to demonstrate the application of the proposed method.

The rest of paper is organized as follows. Section 2 briefly introduces the concepts and operations of NSs and SNSs. The definition of MVNS along with its operations and comparison approach for MVNSs is defined on the basis of AIFS and HFSs in Section 3. Aggregation operators MVNNs are given and a MCDM method is developed in Section 4. In Section 5, an illustrative example is presented to illustrate the proposed method and the comparative analysis and discussion were given. Finally, Section 6 concludes the paper.

2. Preliminaries

In this section, definitions and operations of NSs and SNSs are introduced, which will be utilized in the rest of the paper.

Definition 1 [25]. Let X be a space of points (objects), with a generic element in X denoted by x. A NS A in X is characterized by a truth-membership function $T_A(x)$, a indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]0^-, 1^+[$, that is, $T_A(x): X \to]0^-, 1^+[$, $I_A(x): X \to]0^-, 1^+[$, and $F_A(x): X \to]0^-, 1^+[$. There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0^- \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$.

Definition 2 [25]. A NS A is contained in the other NS B, denoted as $A \subseteq B$, if and only if $\inf T_A(x) \le \inf T_B(x)$, $\sup T_A(x) \le \sup T_B(x)$, $\inf I_A(x) \le \inf I_B(x)$, $\sup I_A(x) \le \sup I_B(x)$, $\inf F_A(x) \le \inf F_B(x)$ and $\sup F_A(x) \le \sup F_B(x)$ for $x \in X$.

Since it is difficult to apply NSs to practical problems, Ye reduced NSs of nonstandard intervals into a kind of SNSs of standard intervals that will preserve the operations of the NSs [26].

Definition 3 [28]. Let X be a space of points (objects), with a generic element in X denoted by x. A NS A in X is characterized by $T_A(x)$, $I_A(x)$ and $F_A(x)$, which are singleton subintervals/subsets in the real standard [0, 1], that is $T_A(x): X \to [0,1]$, $I_A(x): X \to [0,1]$, and $F_A(x): X \to [0,1]$. Then, a simplification of A is denoted by

$$A = \{< x, T_A(x), I_A(x), F_A(x) > \mid x \in X\},$$

which is called a SNS. It is a subclass of NSs.

The operational relations of SNSs are also defined in Ref. [28].

Definition 4 [31]. Let A and B are two SNSs. For any $x \in X$,

$$(1) A + B = \left\langle \begin{array}{l} T_A(x) + T_B(x) - T_A(x) \cdot T_B(x), \\ I_A(x) + I_B(x) - I_A(x) \cdot I_B(x), \\ F_A(x) + F_B(x) - F_A(x) \cdot F_B(x) \end{array} \right\rangle,$$

$$(2) A \cdot B = \left\langle T_A(x) \cdot T_B(x), I_A(x) \cdot I_B(x), F_A(x) \cdot F_B(x) \right\rangle,$$

$$(3) \lambda \cdot A = \left\langle 1-(1-T_A(x))^\lambda, 1-(1-I_A(x))^\lambda, 1-(1-F_A(x))^\lambda \right\rangle, \lambda > 0,$$

$$(4) A^\lambda = \left\langle T_A^\lambda(x), I_A^\lambda(x), F_A^\lambda(x) \right\rangle, \lambda > 0.$$

It has some limitations in Definition 9.

(1) In some situations, the operations, such as $A+B$ and $A \cdot B$, as given in Definition 9, might be irrational. This is shown in the example below.

Let $a =< 0.5, 0.5, 0.5 >$, $a^* =< 1, 0, 0 >$ be two simplified neutrosophic numbers (SNNs). Obviously, $a^* =< 1, 0, 0 >$ is the maximum of the SNS. It is notorious that the sum of any number and the maximum number should be equal to the maximum one. However, according to the equation (1) in Definition 9, $a+b =< 1, 0.5, 0.5 > \neq b$. Hence, the equation (1) does not hold. So does the other equations in Definition 9. It shows that the operations above are incorrect.

(2) The correlation coefficient for SNSs in Ref. [27] on basis of the operations does not satisfy in some special cases.

Let $a_1 =< 0.8, 0, 0 >$ and $a_2 =< 0.7, 0, 0 >$ be two SNSs, and $a^* =< 1, 0, 0 >$ be the maximum of the SNS. According to the MCDM based on the correlation coefficient for SNSs under the simplified neutrosophic environment in Ref. [29], we can obtain the result $W_1(a_1, a^*) = W_2(a_2, a^*) = 1$. We cannot distinguish the best one. However, it is clear that the alternative a_1 is superior to alternative a_2.

(3) In addition, the similarity measure for SNSs in Ref. [32] on basis of the operations does not satisfy in special cases.

Let $a_1 =< 0.1,0,0 >$, $a_2 =< 0.9,0,0 >$ be two SNSs, and $a^* =< 1,0,0 >$ be the maximum of the SNS. According to the decision making method based on the cosine similarity measure for SNSs under the simplified neutrosophic environment in Ref. [28], we can obtain the result $S_1(a_1, a^*) = S_2(a_2, a^*) = 1$, that is, the alternative a_1 is equal to alternative a_2. We cannot distinguish the best one else. However, $T_{a_2}(x) > T_{a_1}(x)$, $I_{a_2}(x) > I_{a_1}(x)$ and $F_{a_2}(x) > F_{a_1}(x)$, it is clear that the alternative a_2 is superior to alternative a_1.

(4) If $I_A = I_B$, then A and B are reduced to two AIFNs. However, above operations are not in accordance with the laws for two AIFSs in [4,6-10,30].

3. Multi-valued neutrosophic sets and theirs operations

In this section, MVNSs is defined, and its operations based on AIFSs [4,6-10,30] are developed as well.

Definition 5. Let X be a space of points (objects), with a generic element in X denoted by x. A *MVNS* A in X is characterized by three functions $\tilde{T}_A(x)$, $\tilde{I}_A(x)$ and $\tilde{F}_A(x)$ in the form of subset of [0, 1], which can be denoted as follows:

$$A=\{<x,\tilde{T}_A(x),\tilde{I}_A(x),\tilde{F}_A(x)>|\,x\in X\}$$

Where $\tilde{T}_A(x)$, $\tilde{I}_A(x)$, and $\tilde{F}_A(x)$ are three sets of some values in [0,1], denoting the truth-membership degree, indeterminacy-membership function and falsity-membership degree respectively, with the conditions: $0\le\gamma,\eta,\xi\le 1, 0\le\gamma^+ +\eta^+ +\xi^+\le 3$, where $\gamma\in\tilde{T}_A(x),\eta\in\tilde{I}_A(x),\xi\in\tilde{F}_A(x)$, and $\gamma^+=\sup\tilde{T}_A(x)$, $\eta^+=\sup\tilde{I}_A(x)$ and $\xi^+=\sup\tilde{F}_A(x)$. $\tilde{T}_A(x)$, $\tilde{I}_A(x)$, and $\tilde{F}_A(x)$ is set of crisp values between zero and one. For convenience, we call $A=\{<\tilde{T}_A,\tilde{I}_A,\tilde{F}_A>\}$ the multi-valued neutrosophic number (MVNN). Apparently, MVNSs are an extension of NSs.

Especially, if $\tilde{T}_A,\tilde{I}_A$ and $\tilde{F}_A$ have only one value γ,η and ξ , respectively, and $0\le\gamma+\eta+\xi\le 3$, then the MVNSs are reduced to SNS; If $\tilde{I}_A=\varnothing$, then the MVNSs are reduced to DHFSs ;If $\tilde{I}_A=\tilde{F}_A=\varnothing$, then the MVNSs are reduced to HFSs. Thus the MVNSs are an extension of these sets above.

The operational relations of MVNSs are also defined as follows.

Definition 6. The complement of a MVNS $A=\{<\tilde{T}_A,\tilde{I}_A,\tilde{F}_A>\}$ is denoted by A^C and is defined by $A^C=<\bigcup_{\gamma\in\tilde{T}_A}\{1-\gamma\},\bigcup_{\eta\in\tilde{I}_A}\{1-\eta\},\bigcup_{\xi\in\tilde{F}_A}\{1-\xi\}>$.

Definition 7.The MVNS $A=\{<\tilde{T}_A,\tilde{I}_A,\tilde{F}_A>\}$ is contained in the other MVNS $B=\{<\tilde{T}_B,\tilde{I}_B,\tilde{F}_B>\}$, $A\subseteq B$ if and only if $\gamma_A^-\le\gamma_B^+,\eta_A^-\ge\eta_B^+$ and $\xi_A^-\ge\xi_B^+$.

Where $\gamma_A^-=\inf\tilde{T}_A$, $\gamma_A^+=\sup\tilde{T}_A$, $\eta_A^-=\inf\tilde{I}_A$, $\eta_A^+=\sup\tilde{I}_A$ and $\xi_A^-=\inf\tilde{F}_A$, $\xi_A^+=\sup\tilde{F}_A$.

Definition 8. Let $A=<\tilde{T}_A,\tilde{I}_A,\tilde{F}_A>$, $B=<\tilde{T}_B,\tilde{I}_B,\tilde{F}_B>$ be two MVNNs, and $\lambda>0$. The operations for MVNNs are defined as follows.

(1) $$\lambda A=\left\langle\begin{array}{l}\bigcup_{\gamma_A\in\tilde{T}_A}\{1-(1-\gamma_A)^\lambda\},\\ \bigcup_{\eta_A\in\tilde{I}_A}\{(\eta_A)^\lambda\},\\ \bigcup_{\xi_A\in\tilde{F}_A}\{(\xi_A)^\lambda\}\end{array}\right\rangle;$$

(2) $$A^\lambda=\left\langle\begin{array}{l}\bigcup_{\gamma_A\in\tilde{T}_A}\{(\gamma_A)^\lambda\},\\ \bigcup_{\eta_A\in\tilde{I}_A}\{1-(1-\eta_A)^\lambda\},\\ \bigcup_{\xi_A\in\tilde{F}_A}\{1-(1-\xi_A)^\lambda\}\end{array}\right\rangle;$$

(3) $$A+B=\left\langle\begin{array}{l}\bigcup_{\gamma_A\in\tilde{T}_A,\gamma_B\in\tilde{T}_B}\{\gamma_A+\gamma_B-\gamma_A\cdot\gamma_B\},\\ \bigcup_{\eta_A\in\tilde{I}_A,\eta_B\in\tilde{I}_B}\{\eta_A\cdot\eta_B\},\\ \bigcup_{\xi_A\in\tilde{F}_A,\xi_B\in\tilde{F}_B}\{\xi_A\cdot\xi_B\}\end{array}\right\rangle;$$

(4) $$A\cdot B=\left\langle\begin{array}{l}\bigcup_{\gamma_A\in\tilde{T}_A,\gamma_B\in\tilde{T}_B}\{\gamma_A\cdot\gamma_B\},\\ \bigcup_{\eta_A\in\tilde{I}_A,\eta_B\in\tilde{I}_B}\{\eta_A+\eta_B-\eta_A\cdot\eta_B\},\\ \bigcup_{\xi_A\in\tilde{F}_A,\xi_B\in\tilde{F}_B}\{\xi_A+\xi_B-\xi_A\cdot\xi_B\}\end{array}\right\rangle.$$

Apparently, if there is only one specified number in $\tilde{T}_A,\tilde{I}_A$ and $\tilde{F}_A$, then the operations in Definiton 8 are reduced to the operations for MVNNs as follows:

(5) $\lambda A=<1-(1-T_A)^\lambda,(I_A)^\lambda,(F_A)^\lambda>$;

(6) $A^\lambda=<(T_A)^\lambda,1-(1-I_A)^\lambda,1-(1-F_A)^\lambda>$;

(7) $A+B=<T_A+T_B-T_A\cdot T_B,I_A\cdot I_B,F_A\cdot F_B>\}$;

(8) $A\cdot B=<T_A\cdot T_B,I_A+I_B-I_A\cdot I_B,F_A+F_B-F_A\cdot F_B>\}$.

Note that the operations for MVNNs are coincides with operations of AIFSs in Ref. [7,34].

Theorem 1. Let $A = <\tilde{T}_A, \tilde{I}_A, \tilde{F}_A>$, $B = <\tilde{T}_B, \tilde{I}_B, \tilde{F}_B>$, $C = <\tilde{T}_C, \tilde{I}_C, \tilde{F}_C>$ be three MVNNs, then the following equations are true.

(1) $A + B = B + A,$

(2) $A \cdot B = B \cdot A,$

(3) $\lambda(A + B) = \lambda A + \lambda B, \lambda > 0,$

(4) $(A \cdot B)^{\lambda} = A^{\lambda} + B^{\lambda}, \lambda > 0,$

(5) $\lambda_1 A + \lambda_2 A = (\lambda_1 + \lambda_2) A, \lambda_1 > 0, \lambda_2 > 0,$

(6) $A^{\lambda_1} \cdot A^{\lambda_2} = A^{(\lambda_1 + \lambda_2)}, \lambda_1 > 0, \lambda_2 > 0,$

(7) $(A + B) + C = A + (B + C),$

(8) $(A \cdot B) \cdot C = A \cdot (B \cdot C).$

3.2 Comparison rules

Based on the score function and accuracy function of AIFS [35-38], the score function, accuracy function and certainty function of a MVNN are defined in the following.

Definition 9. Let $A = <\tilde{T}_A, \tilde{I}_A, \tilde{F}_A>$ be a MVNN, and then score function $s(A)$, accuracy function $a(A)$ and certainty function $c(A)$ of an MVNN are defined as follows:

(1) $$s(A) = \frac{1}{l_{\tilde{T}_A} \cdot l_{\tilde{I}_A} \cdot l_{\tilde{F}_A}} \times \sum_{\gamma_i \in \tilde{T}_A, \eta_j \in \tilde{I}_A, \xi_k \in \tilde{F}_A} (\gamma_i + 1 - \eta_j + 1 - \xi_k)/3;$$

(2) $$a(A) = \frac{1}{l_{\tilde{T}_A} \cdot l_{\tilde{F}_A}} \sum_{\gamma_i \in \tilde{T}_A, \xi_k \in \tilde{F}_A} (\gamma_i - \xi_k);$$

(3) $$c(A) = \frac{1}{l_{T_A}} \sum_{\gamma_i \in T_A} \gamma_i .$$

Where $\gamma_i \in \tilde{T}_A, \eta_j \in \tilde{I}_A, \xi_k \in \tilde{F}_A$, $l_{\tilde{T}_A}, l_{\tilde{I}_A}$ and $l_{\tilde{F}_A}$ denotes the element numbers in $\tilde{T}_A, \tilde{I}_A$ and $\tilde{F}_A$, respectively.

The score function is an important index in ranking the MVNNs. For a MVNN *A*, the truth-membership $\tilde{T}_A$ is bigger, the MVNN is greater. And the indeterminacy-membership $\tilde{I}_A$ is less, the MVNN is greater. Similarly, the false-membership $\tilde{F}_A$ is smaller, the MVNN is greater. For the accuracy function, if the difference between truth and falsity is bigger, then the statement is more affirmative. That is, the larger the values of $\tilde{T}_A$, $\tilde{I}_A$ and $\tilde{F}_A$, the more the accuracy of the MVNN. As to the certainty function, the value of truth- membership $\tilde{T}_A$ is bigger, it means more certainty of the MVNSN.

On the basis of Definition 9, the method to compare MVNNs can be defined as follows.

Definition 10. Let A and B be two MVNNs. The comparision methods can be defined as follows:

(1) If $s(A) > s(B)$, then A is greater than B, that is, A is superior to B, denoted by $A \succ B$.

(2) If $s(A) = s(B)$ and $a(A) > a(B)$, then A is greater than B, that is, A is superior to B, denoted by $A \succ B$.

(3) If $s(A) = s(B)$, $a(A) = a(B)$ and $c(A) > c(B)$, then A is greater than B, that is, A is superior to B, denoted by $A \succ B$.

(4) If $s(A)=s(B)$, $a(A)=a(B)$ and $c(A)=c(B)$, then A is equal to B, that is, A is indifferent to B, denoted by $A \sim B$.

4. Aggregation operators of MVNNs and their application to multi-criteria decision-making problems

In this section, applying the MVNSs operations, we present aggregation operators for MVNNs and propose a method for MCDM by utilizing the aggregation operators.

4.1 MVNN aggregation operators

Definition 11. Let $A_j = \langle \tilde{T}_{A_j}, \tilde{I}_{A_j}, \tilde{F}_{A_j} \rangle$ ($j=1,2,\cdots,n$) be a collection of MVNNs, and let

$\mathrm{MVNNWA}: \mathrm{MVNN}^n \to \mathrm{MVNN}$,

$$MVNNWA\ (A_1, A\ ,\cdots, A_n) = w_1A_1 + w_2A_2 + \cdots + w_nA_n = \sum_{j=1}^{n} w_jA_j, \quad (1)$$

then $MVNNWA$ is called the multi-valued neutrosophic number weighted averaging operator of dimension n, where $W=(w_1, w\ ,\cdots, w_n)$ is the weight vector of A_j $(j=1,2,\cdots,n)$, with $w_j \geq 0$ $(j=1,2,\cdots,n)$ and $\sum_{j=1}^{n} w_j = 1$.

Theorem 2. Let $A_j = \langle \tilde{T}_{A_j}, \tilde{I}_{A_j}, \tilde{F}_{A_j} \rangle$ ($j=1,2,\cdots,n$) be a collection of MVNNs, $W=(w_1, w\ ,\cdots, w_n)$ be the weight vector of A_j $(j=1,2,\cdots,n)$, with $w_j \geq 0$ $(j=1,2,\cdots,n)$ and $\sum_{j=1}^{n} w_j = 1$, then their aggregated result using the MVNNWA operator is also an MVNN, and

$$MVNNWA\ (A_1, A_2, \cdots, A_n) = \left\langle \begin{array}{l} \bigcup_{\gamma_j \in \tilde{T}_{A_j}} \left\{ 1-\prod_{j=1}^{n}(1-\gamma_j)^{w_j} \right\}, \\ \bigcup_{\eta_j \in \tilde{I}_j} \left\{ \prod_{j=1}^{n} \eta_j^{w_j} \right\}, \bigcup_{\xi_j \in \tilde{F}_j} \left\{ \prod_{j=1}^{n} \xi_j^{w_j} \right\} \end{array} \right\rangle \quad (2)$$

Where $W=(w_1, w\ ,..., w_n)$ is the vector of $A_j\,(j=1,2,\cdots,n)$, $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$.

It is obvious that the MVNNWA operator has the following properties.

(1) (Idempotency): Let $A_j\,(j=1,2,...,n)$ be a collection of MVNNs. If all $A_j\,(j=1,2,...,n)$ are equal, i.e., $A_j = A$, for all $j \in \{1,2,\cdots,n\}$, then

$$MVNNWA\ (A_1, A\ ,\cdots, A_n) = A.$$

(2) (Boundedness): If $A_j = \langle \tilde{T}_{A_j}, \tilde{I}_{A_j}, \tilde{F}_{A_j} \rangle$ $(j=1,2,\cdots,n)$ is a collection of MVNNs and

$A^- = \left\langle \min_j T_{A_j}, \max_j I_{A_j}, \max F_{A_j} \right\rangle$,

$A^+ = \left\langle \max_j T_{A_j}, \min_j I_{A_j}, \min F_{A_j} \right\rangle$, for all $j \in \{1,2,\cdots,n\}$, then

$$A^- \subseteq MVNNWA\ (A_1, A\ ,\cdots, A_n) \quad A\ .$$

(3) (Monotonity): Let $A_j\,(j=1,2,...,n)$ a collection of MVNNs. If $A_j \subseteq A_j^*$, for $j \in \{1,2,\cdots,n\}$, then

$SNNWA_w(A_1, A_2, \cdots, A_n) \subseteq SNNWA_w(A_1^*, A_2^*, \cdots, A_n^*)$.

Definition 12. Let $A_j = \langle \tilde{T}_{A_j}, \tilde{I}_{A_j}, \tilde{F}_{A_j} \rangle$ ($j = 1, 2, \cdots, n$) be a collection of MVNNs, and let

$\mathrm{MVNNWG}: \mathrm{MVNN}^n \to \mathrm{MVNN}$:

$$MVNNWG_w(A_1, A\ , \cdots, A_n) = \prod_{j=1}^{n} A_j^{w_j} \quad , \qquad (3)$$

then MVNNWG is called an multi-valued neutrosophic number weighted geometric operator of dimension n, where $W = (w_1, w\ , \cdots, w_n)$ is the weight vector of A_j $(j = 1, 2, \cdots, n)$, with $w_j \geq 0$ $(j = 1, 2, \cdots, n)$ and $\sum_{j=1}^{n} w_j = 1$.

Theorem 3. Let $A_j = \langle \tilde{T}_{A_j}, \tilde{I}_{A_j}, \tilde{F}_{A_j} \rangle$ ($j = 1, 2, \cdots, n$) be a collection of MVNNs, we have the following result:

$$MVNNWG_w(A_1, A\ , \cdots, A_n) \quad \left\langle \begin{array}{l} \bigcup_{\gamma_j \in \tilde{T}_{A_j}} \left\{ \prod_{j=1}^{n} (\gamma_j)^{w_j} \right\}, \\ \bigcup_{\eta \in \tilde{I}_{A_j}} \left\{ \quad - \prod_{j=1}^{n} (1 - \eta_j)^{w_j} \right\}, \\ \bigcup_{\xi_j \in \tilde{F}_j} \left\{ 1 - \prod_{j=1}^{n} (1 - \xi_j)^{w_j} \right\} \end{array} \right\rangle, \quad (4)$$

where $W = (w_1, w\ , ..., w_n)$ is the vector of $A_j (j = 1, 2, \cdots, n)$, $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$.

Definition 13. Let $A_j = \langle \tilde{T}_{A_j}, \tilde{I}_{A_j}, \tilde{F}_{A_j} \rangle$ ($j = 1, 2, \cdots, n$) be a collection of MVNNs, and let

$\mathrm{MVNNOWA}: \mathrm{MVNN}^n \to \mathrm{MVNN}$:

$$MVNNOWA\ (A_1, A\ , \cdots, A_n) = w_1 A_{\sigma(1)} + w_2 A_{\sigma(2)} + \cdots + w_n A_{\sigma(n)} = \sum_{j=1}^{n} w_j A_{\sigma(j)} \qquad (5)$$

then *MVNNOWA* is called the multi-valued neutrosophic number ordered weighted averaging operator of dimension n, where $A_{\sigma(j)}$ is the *j*-th largest value.

$W = (w_1, w\ , \cdots, w_n)$ is the weight vector of A_j $(j = 1, 2, \cdots, n)$, with $w_j \geq 0$ $(j = 1, 2, \cdots, n)$ and $\sum_{j=1}^{n} w_j = 1$.

Theorem 4. Let $A_j = \langle \tilde{T}_{A_j}, \tilde{I}_{A_j}, \tilde{F}_{A_j} \rangle$ ($j = 1, 2, \cdots, n$) be a collection of MVNNs, $W = (w_1, w\ , \cdots, w_n)$ be the weight vector of A_j $(j = 1, 2, \cdots, n)$, with $w_j \geq 0$ $(j = 1, 2, \cdots, n)$ and $\sum_{j=1}^{n} w_j = 1$, then their aggregated result using the MVNNOWA operator is also an MVNN, and

$$MVNNOWA_w(A_1, A_2, \cdots, A_n) = \left\langle \begin{array}{l} \bigcup_{\gamma_j \in \tilde{T}_{A_{\sigma(j)}}} \left\{ 1 - \prod_{j=1}^{n} (1 - \gamma_j)^{w_j} \right\}, \\ \bigcup_{\eta_j \in \tilde{I}_{A_{\sigma(j)}}} \quad \prod_{j=1}^{n} \eta_j^{w_j} \Big\}, \\ \bigcup_{\xi_j \in \tilde{F}_{A_{\sigma(j)}}} \quad \prod_{j=1}^{n} \xi_j^{w_j} \Big\} \end{array} \right\rangle, \quad (6)$$

where $A_{\sigma(j)}$ is the *j*-th largest value according to the total order: $A_{\sigma(1)} \geq A_{\sigma(2)} \geq \cdots \geq A_{\sigma(n)}$.

Definition 14. Let $A_j = \langle T_{A_j}, I_{A_j}, F_{A_j} \rangle$ ($j = 1, 2, \cdots, n$) be a collection of MVNNs, and let

$\mathrm{MVNNOWG}: MV\mathrm{NN}^n \to MV\mathrm{NN}$:

$$MVNNOWG\ (A_1, A_2, \cdots, A_n) = \prod_{j=1}^{n} A_{\sigma(j)}^{w_j} \qquad (7)$$

then MVNNOWG is called an multi-valued neutrosophic number ordered weighted geometric operator of dimension n, where $A_{\sigma(j)}$ is the *j*-th largest value and $W = (w_1, w\ , \cdots, w_n)$ is the weight vector of $A_j\ (j = 1, 2, \cdots, n)$, with $w_j \geq 0\ \ (j = 1, 2, \cdots, n)$ and $\sum_{j=1}^{n} w_j = 1$.

Theorem 5. Let $A_j = \langle \tilde{T}_{A_j}, \tilde{I}_{A_j}, \tilde{F}_{A_j} \rangle$ ($j = 1, 2, \cdots, n$) be a collection of MVNNs, we have the following result:

$$MVNNOWG_w(A_1, A_2, \cdots, A_n)$$

$$= \left\langle \begin{array}{l} \bigcup_{\gamma_j \in \tilde{T}_{A_{\sigma(j}}} \left\{ \prod_{j=1}^{n} (\gamma_j)^{w_j} \right\}, \\ \bigcup_{\eta_j \in \tilde{I}_{\sigma(j)}} \left\{ 1 - \prod_{j=1}^{n} (1\ \ \eta_j\ ^{w_j} \right., \\ \bigcup_{\xi_j \in \tilde{F}_{A_{\sigma(j)}}} \left\{ 1 - \prod_{j=1}^{n} (1 - \xi_j)^{w_j} \right. \end{array} \right\rangle \qquad (8)$$

where $W = (w_1, w\ , ..., w_n)$ is the vector of $A_j (j = 1, 2, \cdots, n)$, $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, and $A_{\sigma(j)}$ is the *j*-th largest value according to the total order: $A_{\sigma(1)} \geq A_{\sigma(2)} \geq \cdots \geq A_{\sigma(n)}$.

Definition 15. Let $A_j = \langle \tilde{T}_{A_j}, \tilde{I}_{A_j}, \tilde{F}_{A_j} \rangle$ ($j = 1, 2, \cdots, n$) be a collection of MVNNs, and let

$$\mathrm{MVNNHOWA} : \mathrm{MVNN}^n \to \mathrm{MVNN} :$$

$$MVNNHOWA\ (A_1, A\ , \cdots, A_n)$$

$$= w_1 \dot{A}_{\sigma(1)} + w_2 \dot{A}_{\sigma(2)} + \cdots + w_n \dot{A}_{\sigma(n)} = \sum_{j=1}^{n} w_j \dot{A}_{\sigma(j)} \qquad (9)$$

then *MVNNHOWA* is called the multi-valued neutrosophic number hybrid ordered weighted averaging operator of dimension n, where $\dot{A}_{\sigma(j)}$ is the *j*-th largest of the weighted value $\dot{A}_j (\dot{A}_j = n w_j A_j, j = 1, 2, \cdots, n)$, $W = (w_1, w\ , \cdots, w_n)$ is the weight vector of $A_j\ \ (j = 1, 2, \cdots, n)$, with $w_j \geq 0\ (j = 1, 2, \cdots, n)$ and $\sum_{j=1}^{n} w_j = 1$, and *n* is the balancing coefficient.

Theorem 6. Let $A_j = \langle \tilde{T}_{A_j}, \tilde{I}_{A_j}, \tilde{F}_{A_j} \rangle$ ($j = 1, 2, \cdots, n$) be a collection of MVNNs, $W = (w_1, w\ , \cdots, w_n)$ be the weight vector of $A_j\ (j = 1, 2, \cdots, n)$, with $w_j \geq 0$ $(j = 1, 2, \cdots, n)$ and $\sum_{j=1}^{n} w_j = 1$, then their aggregated result using the SNNHOWA operator is also a MVNN, and

$$MVNNHOWA_w(A_1, A_2, \cdots, A_n)$$

$$= \left\langle \begin{array}{l} \bigcup_{\gamma_j \in \tilde{T}_{A_{\sigma(j}}} \left\{ 1 - \prod_{j=1}^{n} (1 - \gamma_j)^{w_j} \right\}, \\ \bigcup_{\eta_j \in \tilde{I}_{A_{\sigma(j)}}} \left\{ \prod_{j=1}^{n} \eta_j^{w_j} \right., \\ \bigcup_{\xi_j \in \tilde{F}_{A_{\sigma(j)}}} \left\{ \prod_{j=1}^{n} \xi_j^{w_j} \right\} \end{array} \right\rangle \qquad (10)$$

where $\dot{A}_{\sigma(j)}$ is the *j*-th largest of the weighted value $\dot{A}_j (\dot{A}_j\ \ A_j^{n w_j}, j = 1, 2, \cdots, n)$.

Definition 16. Let $A_j = \langle \tilde{T}_{A_j}, \tilde{I}_{A_j}, \tilde{F}_{A_j} \rangle$ ($j = 1, 2, \cdots, n$) be a collection of MVNNs, and let

$$\text{MVNNHOWG}:\text{MVNN}^n \rightarrow \text{MVNN}$$

$$MVNNHOWG\ (A_1, A_2, \cdots, A_n) = \prod_{j=1}^{n} \dot{A}_{\sigma(j)}^{w_j} \qquad (11)$$

then MVNNHOWG is called the multi-valued neutrosophic number hybrid ordered weighted geometric operator of dimension n, where $\dot{A}_{\sigma(j)}$ is the j-th largest of the weighted value $\dot{A}_j(\dot{A}_j = nw_j A_j, j=1,2,\cdots,n)$, $W=(w_1, w\ ,\cdots, w_n)$ is the weight vector of A_j $(j=1,2,\cdots,n)$, with $w_j \geq 0$ $(j=1,2,\cdots,n)$ and $\sum_{j=1}^{n} w_j = 1$, and n is the balancing coefficient.

Theorem 7. Let $A_j = \langle \tilde{T}_{A_j}, \tilde{I}_{A_j}, \tilde{F}_{A_j} \rangle$ ($j=1,2,\cdots,n$) be a collection of MVNNs, $W=(w_1, w\ ,\cdots, w_n)$ be the weight vector of A_j $(j=1,2,\cdots,n)$, with $w_j \geq 0$ $(j=1,2,\cdots,n)$ and $\sum_{j=1}^{n} w_j = 1$,

$$MVNNHOWG\ (A_1, A\ , \cdots, A_n) = \left\langle \begin{array}{l} \bigcup_{\gamma_j \in \tilde{T}_{A_{\sigma(j)}}} \left\{ \prod_{j=1}^{n} (\gamma_j)^{w_j} \right. , \\ \bigcup_{\eta_j \in \tilde{I}_{A_{\sigma(j)}}} \left\{ 1 - \prod_{j=1}^{n} (1-\eta_j)^{w_j} \right. \\ \bigcup_{\xi_j \in \tilde{F}_{A_{\sigma(j)}}} \left\{ 1 - \prod_{j=1}^{n} (1-\xi_j \quad {}^{w_j} \right\} \end{array} \right\rangle \qquad (12)$$

Where $\dot{A}_{\sigma(j)}$ is the j-th largest of the weighted value $\dot{A}_j(\dot{A}_j \quad A_j^{nw_j}, j=1,2,\cdots,n)$.

Similarly, it can be proved that the mentioned operators have the same properties as the MVNNWA operator.

4.2 Multi-criteria decision-making method based on the MVNN aggregation operators

Assume there are n alternatives $A=\{a_1, a_2, \cdots, a_n\}$ and m criteria $C=\{c_1, c_2, \cdots, c_m\}$, whose criterion weight vector is $w=(w_1, w\ ,\cdots, w_m)$, where $w_j \geq 0$ ($j=1,2,\cdots,m$), $\sum_{j=1}^{m} w_j = 1$. Let $R=(a_{ij})_{n\times m}$ be the simplified neutrosophic decision matrix, where $a_{ij} = \langle T_{a_{ij}}, I_{a_{ij}}, F_{a_{ij}} \rangle$ is a criterion value, denoted by MVNN, where $T_{a_{ij}}$ indicates the truth-membership function that the alternative a_i satisfies the criterion c_j, $I_{a_{ij}}$ indicates the indeterminacy-membership function that the alternative a_i satisfies the criterion c_j and $F_{a_{ij}}$ indicates the falsity-membership function that the alternative a_i satisfies the criterion c_j.

In the following, a procedure to rank and select the most desirable alternative(s) is given.

Step 1: Aggregate the MVNNs.

Utilize the MVNNWA operator or the MVNNWG operator or MVNNHOWA operator or the MVNNHOWG to aggregate MVNNs and we can get the individual value of the alternative a_i ($i=1,2,\cdots,n$, $j=1,2,\cdots,m$).

$$x_i = MVNNWA_w(a_{i1}, a_i \ ,\cdots, a_{im}) \text{ , or}$$

$$x_i = MVNNWG_w(a_{i1}, a_i \ ,\cdots, a_{im}) \text{ , or}$$

$$x_i = MVNNOWA_w(a_{i1}, a_i \ ,\cdots, a_{im}) \text{ , or}$$

$$x_i = MVNNOWA_w(a_{i1}, a_i \ ,\cdots, a_{im}) \text{ , or}$$

$$x_i = MVNNHOWA_w(a_{i1}, a_i \ ,\cdots, a_{im}) \text{ , or}$$

$$x_i = MVNNHOWG_w(a_{i1}, a_i \ ,\cdots, a_{im}) \text{ .}$$

Step 2: Calculate the score function value $s(y_i)$, accuracy function value $a(y_i)$ and certainty function value $c(y_i)$ of y_i ($i=1,2,\cdots,m$) by Definition 9.

Step 3: Rank the alternatives. According to Definition 10, we could get the priority of the alternatives a_i ($i=1,2,\cdots,m$) and choose the best one.

5. Illustrative example

In this section, an example for the multi-criteria decision making problem of alternatives is used as the demonstration of the application of the proposed decision making method, as well as the effectiveness of the proposed method.

Let us consider the decision making problem adapted from Ref. [28]. There is an investment company, which wants to invest a sum of money in the best option. There is a panel with four possible alternatives to invest the money:

(1) A_1 is a car company;

(2) A_2 is a food company;

(3) A_3 is a computer company;

(4) A_4 is an arms company.

The investment company must take a decision according to the following three criteria:

(1) C_1 is the risk analysis;

(2) C_2 is the growth analysis;

(3) C_3 is the environmental impact analysis, where C_1 and C_2 are benefit criteria, and C_3 is a cost criterion. The weight vector of the criteria is given by $W=(0.35, 0.25, 0.4)$. The four possible alternatives are to be evaluated under the above three criteria by the form of MVNNs, as shown in the following simplified neutrosophic decision matrix *D*:

$$D = \begin{bmatrix} \langle\{0.4,0.5\},\{0.2\},\{0.3\}\rangle & \langle\{0.4\},\{0.2,0.3\},\{0.3\}\rangle & \langle\{0.2\},\{0.2\},\{0.5\}\rangle \\ \langle\{0.6\},\{0.1,0.2\},\{0.2\}\rangle & \langle\{0.6\},\{0.1\},\{0.2\}\rangle & \langle\{0.5\},\{0.2\},\{0.1,0.2\}\rangle \\ \langle\{0.3,0.4\},\{0.2\},\{0.3\}\rangle & \langle\{0.5\},\{0.2\},\{0.3\}\rangle & \langle\{0.5\},\{0.2,0.3\},\{0.2\}\rangle \\ \langle\{0.7\},\{0.1,0.2\},\{0.1\}\rangle & \langle\{0.6\},\{0.1\},\{0.2\}\rangle & \langle\{0.4\},\{0.3\},\{0.2\}\rangle \end{bmatrix}$$

The procedures of decision making based on MVNS are shown as following.

Step 1: Aggregate the MVNNs.

Utilize the MVNNWA operator or the MVNNWG operator to aggregate MVNNs of each decision maker, and we can get the individual value of the alternative a_i ($i=1,2,\cdots,n$, $j=1,2,\cdots,m$).

By using MVNNWA operator, the alternatives matrix A_{WA} can be obtained:

$$A_{WA}=\begin{bmatrix} \langle\{0.327,0.368\},\{0.200,0.221\},\{0.368\}\rangle \\ \langle\{0.563\},\{0.132,0.168\},\{0.152,0.200\}\rangle \\ \langle\{0.438,0.467\},\{0.200,0.235\},\{0.255\}\rangle \\ \langle\{0.574\},\{0.155,0.198\},\{0.157\}\rangle \end{bmatrix}.$$

With MVNNWG operator, the alternatives matrix A_{WG} is as follows:

$$A_{WG}=\begin{bmatrix} \langle\{0.303,0.328\},\{0.200,0.226\},\{0.388\}\rangle \\ \langle\{0.558\},\{0.141,0.176\},\{0.161,0.200\}\rangle \\ \langle\{0.418,0.462\},\{0.200,0.242\},\{0.262\}\rangle \\ \langle\{0.538\},\{0.186,0.219\},\{0.166\}\rangle \end{bmatrix}.$$

Step 2: Calculate the score function value, accuracy function value and certainty function value.

To the alternatives matrix A_{WA}, by using Definition 9, then we have:

$$s_{A_{WA}}=(0.590,0.746,0.660,0.747).$$

Apparently, there is no need to compute accuracy function value and certainty function value.

To the alternatives matrix A_{WG}, by using Definition 10, the function matrix of A_{WG} is as follows:

$$s_{A_{WG}}=(0.571,0.739,0.653,0.723).$$

Apparently, there is no need to compute accuracy function value and certainty function value else.

Step 3: Get the priority of the alternatives and choose the best one.

According to Definition 10 and results in step 2, for A_{WA}, we have $a_4 \succ a_2 \succ a_3 \succ a_1$. Obviously, the best alternative is a_4. for A_{WG}, we have $a_2 \succ a_4 \succ a_3 \succ a_1$. Obviously, the best alternative is a_2.

Similarly, if the other two aggregation operators are utilized, then the results can be founded in Table 1.

From the results in Table 1, we can see that if the *MVNNOWA* and *MVNNHOWA are utilized in Step 1, then* we can obtain the results: $a_4 \succ a_2 \succ a_3 \succ a_1$. The best one is a_4 while the worst is a_1. If the *MVNNOWG* and *MVNNHOWG* operators are used, then the final ranking is $a_2 \succ a_4 \succ a_3 \succ a_1$, the best one is a_2 while the worst one is a_1.

In most cases, the different aggregation operator may lead to different rankings. However, all weighted average operators and all geometry operators also lead to the same rankings respectively. So we have two ranks of four alternatives and the best one is always the A_4 or A_2, the worst one is always the A_1.At the same time, decision-makers can choose different aggregation operator according to their preference.

Table 1: The rankings as aggregation operator changes

Operators	The final ranking	The best alternative(s)	The worst alternative(s)
MVNNWA	$a_4 \succ a_2 \succ a_3 \succ a_1$	a_4	a_1
MVNNWG	$a_2 \succ a_4 \succ a_3 \succ a_1$	a_2	a_1
MVNHOWA	$a_4 \succ a_2 \succ a_3 \succ a_1$	a_4	a_1
MVNNOWG	$a_2 \succ a_4 \succ a_3 \succ a_1$	a_2	a_1
MVNNHOWA	$a_4 \succ a_2 \succ a_3 \succ a_1$	a_4	a_1
MVNNHOWG	$a_2 \succ a_4 \succ a_3 \succ a_1$	a_2	a_1

6. Conclusion

MVNSs can be applied in addressing problems with uncertain, imprecise, incomplete and inconsistent information existing in real scientific and engineering applications. However, as a new branch of NSs, there is no enough research about MVNSs. Especially, the existing literature does not put forward the aggregation operators and MCDM method for MVNSs. Based on the related research achievements in AIFSs, the operations of MVNSs were defined. And the approach to solve MCDM problem with MVNNs was proposed. In addition, the aggregation operators of *MVNNWA*, *MVNNWG*, *MVNNOWA*, *MVNNOWG*, *MVNNHOWA* and *MVNNHOWG* were given. Thus, a MCDM method is established based on the proposed operators. Utilizing the comparison approach, the ranking order of all alternatives can be determined and the best one can be easily identified as well. An illustrative example demonstrates the application of the proposed decision making method, and the calculation is simple. In the further study, we will continue to investigate the related comparison method for MVNSs.

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More on neutrosophic soft rough sets and its modification

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Abstract. This paper aims to introduce and discuss anew mathematical tool for dealing with uncertainties, which is a combination of neutrosophic sets, soft sets and rough sets, namely neutrosophic soft rough set model. Also, its modification is introduced. Some of their properties are studied and supported with proved propositions and many counter examples. Some of rough relations are redefined as a neutrosophic soft rough relations. Comparisons among traditional rough model, suggested neutrosophic soft rough model and its modification, by using their properties and accuracy measures are introduced. Finally, we illustrate that, classical rough set model can be viewed as a special case of suggested models in this paper.

Keywords: Neutrosophic set, soft set, rough set approximations, neutrosophic soft set, neutrosophic soft rough set approximations.

1 Introduction

In recent years, many theories based on uncertainty have been proposed, such as fuzzy set theory [36], intuitionistic fuzzy set theory [5], vague set theory [10] and interval-valued fuzzy set theory [11].

In 1982, Pawlak [22] initiated his rough set model, based on equivalence relations, as a new approach towards soft computing finding a wide application. Rough set model has been developed, in many papers, as a generalization models. These models based on reflexive relation, symmetric relation, preference relation, tolerance relation, any relation, coverings, different neighborhood operators, using uncertain function, etc. [12, 15, 16, 24, 25, 29, 32-34, 37]. Also, many papers, recently, have been appeared to apply it in many real life applications such as [2, 3, 7, 17, 27, 28, 30, 35].

In 1995, Smarandache, started his study of the theory of neutrosophic set as a new mathematical tool for handling problems involving imprecise data. Neutrosophic logic is a generalization of intuitionistic fuzzy logic. In neutrosophic logic a proposition is $t\%$ true, $i\%$ indeterminate, and $f\%$ false. For example, let's analyze the following proposition: Let $x(0.6, 0.4, 0.3)$ belongs to A means, with probability of 60% (x in A), with probability of 30% (x not in A)and with probability of 40% (undecidable).

Soft set theory [21], proposed by Molodtsov in 1999, is also a mathematical tool for dealing with uncertainties. Recently, traditional soft model has been developed and applied in some decision making problems in many papers such as [1, 4, 6, 8, 13, 14, 18, 19, 31].

In 2011, Feng et al. [9] introduced the soft rough set model and proved its properties. In 2013, Maji [20] introduced neutrosophic soft set.

In this paper, we introduce a combination of neutrosophic sets, soft sets and rough sets, called neutrosophic soft rough set model. Also, a modification of it is introduced. Basic properties and concepts of suggested models are deduced. We compare between traditional rough model and proposed models to illustrate that traditional rough model is a special case of these proposed models.

2 Preliminaries

In this section we recall some definitions and properties regarding rough set, neutrosophic set, soft set and neutrosophic soft set theories required in this paper.

The following definitions and proposition are given in [22], as follows

Definition 2.1 An equivalence class of an element $x \in U$, determined by the equivalence relation E is

$$[x]_E = \{x' \in U : E(x) = E(x')\}.$$

Definition 2.2 Lower, upper and boundary approximations of a subset $X \subseteq U$ are defined as

$$\underline{E}(X) = \cup\{[x]_E : [x]_E \subseteq X\},$$

$$\overline{E}(X) = \cup\{[x]_E : [x]_E \cap X \neq \varphi\},$$

$$BND_E(X) = \overline{E}(X) - \underline{E}(X).$$

Definition 2.3 Pawlak determined the degree of crispness of any subset $X \subseteq U$ by a mathematical tool, named the accuracy measure of it, which is defined as

$$\alpha_E(X) = \underline{E}(X) / \overline{E}(X), \overline{E}(X) \neq \varnothing.$$

Properties of Pawlak's approximations are listed in the following proposition.

Proposition 2.1 Let (U, E) be a Pawlak approximation space and let $X, Y \subseteq U$. Then,

(a) $\underline{E}(X) \subseteq X \subseteq \overline{E}(X)$.

(b) $\underline{E}(\phi) = \phi = \overline{E}(\phi)$ and $\underline{E}(U) = U = \overline{E}(U)$.

(c) $\overline{E}(X \cup Y) = \overline{E}(X) \cup \overline{E}(Y)$.

(d) $\underline{E}(X \cap Y) = \underline{E}(X) \cap \underline{E}(Y)$.

(e) $X \subseteq Y$, then $\underline{E}(X) \subseteq \underline{E}(Y)$ and $\overline{E}(X) \subseteq \overline{E}(Y)$.

(f) $\underline{E}(X \cup Y) \supseteq \underline{E}(X) \cup \underline{E}(Y)$.

(g) $\overline{E}(X \cap Y) \subseteq \overline{E}(X) \cap \overline{E}(Y)$.

(h) $\underline{E}(X^c) = [\overline{E}(X)]^c$, X^c is the complement of X.

(i) $\overline{E}(X^c) = [\underline{E}(X)]^c$.

(j) $\underline{E}(\underline{E}(X)) = \overline{E}(\underline{E}(X)) = \underline{E}(X)$.

(k) $\overline{E}(\overline{E}(X)) = \underline{E}(\overline{E}(X)) = \overline{E}(X)$.

Definition 2.4 [23] An information system is a quadruple $IS = (U, A, V, f)$, where U is a non-empty finite set of objects, A is a non-empty finite set of attributes, $V = \cup\{V_e, e \in A\}$, V_e is the value set of attribute e, an $f : U \times A \to V$, is called an information (knowledge) function.

Definition 2.5 [21] Let U be an initial universe set, E be a set of parameters, $A \subseteq E$ and let $P(U)$ denotes the power set of U. Then, a pair $S = (F, A)$ is called a soft set over U, where F is a mapping given by $F : A \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of U. For $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of S.

Definition 2.6 [26] A neutrosophic set A on the universe of discourse U is defined as

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle : x \in U\}, where$$

$$^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+, where$$

$$T, I, F : U \to]^-0, 1^+[.$$

Definition 2.7 [20] Let U be an initial universe set and E be a set of parameters. Consider $A \subset E$, and let $P(U)$ denotes the set of all neutrosophic sets of U. The collection (F, A) is termed to be the neutrosophic soft set over U, where F is a mapping given by

$$F : A \to P(U).$$

3 Neutrosophic soft lower and upper concepts and their properties

In this section, neutrosophic soft rough lower and upper approximations are introduced and their properties are deduced and proved. Moreover, many counter examples are obtained.

For more illustration the meaning of neutrosophic soft set, we consider the following example

Example 3.1 Let U be a set of cars under consideration and E is the set of parameters (or qualities). Each parameter is a generalized neutrosophic word or sentence involving generalized neutrosophic words. Consider E = {beautiful, cheap, expensive, wide, modern,in good repair, costly, comfortable}. In this case, to define a neutrosophic soft set means to point out beautiful cars, cheap cars and so on. Suppose that, there are five cars in the universe U, given by, $U = \{h_1, h_2, h_3, h_4, h_5\}$ and the set of parameters $A = \{e_1, e_2, e_3, e_4\}$, where each e_i is a specific criterion for cars: e_1 stands for (beautiful), e_2 stands for (cheap), e_3 stands for (modern), e_4 stands for (comfortable). Suppose that,

F(beautiful) =
$\{\langle h_1, 0.6, 0.6, 0.2\rangle, \langle h_2, 0.4, 0.6, 0.6\rangle, \langle h_3, 0.6, 0.4, 0.2\rangle, \langle h_4, 0.6, 0.3, 0.3\rangle, \langle h_5, 0.8, 0.2, 0.3\rangle\}$

F(cheap) =
$\{\langle h_1, 0.8, 0.4, 0.3\rangle, \langle h_2, 0.6, 0.2, 0.4\rangle, \langle h_3, 0.8, 0.1, 0.3\rangle, \langle h_4, 0.8, 0.2, 0.2\rangle, \langle h_5, 0.8, 0.3, 0.2\rangle\}$,

F(modern) =

$\{\langle h_1, 0.7, 0.4, 0.3\rangle, \langle h_2, 0.6, 0.4, 0.3\rangle, \langle h_3, 0.7, 0.2, 0.5\rangle, \langle h_4, 0.5, 0.2, 0.6\rangle, \langle h_5, 0.7, 0.3, 0.4\rangle\}$,

F(comfortable)=

$\{\langle h_1, 0.8, 0.6, 0.4\rangle, \langle h_2, 0.7, 0.6, 0.6\rangle, \langle h_3, 0.7, 0.6, 0.4\rangle, \langle h_4, 0.7, 0.5, 0.6\rangle, \langle h_5, 0.9, 0.5, 0.7\rangle\}$.

In order to store a neutrosophic soft set in a computer, we could represent it in the form of a table as shown in Table 1 (corresponding to the neutrosophic soft set in Example 3.1). In this table, the entries are c_{ij} corresponding to the car h_i and the parameter e_j, where c_{ij} = (true membership value of h_i, indeterminacy-membership value of h_i, falsity membership value of h_i) in $F(e_j)$. Table 1, represents the neutrosophic soft set (F, A) as follows

U	e_1	e_2	e_3	e_4
h_1	(0.6, 0.6, 0.2)	(0.8, 0.4, 0.3)	(0.7, 0.4, 0.3)	(0.8, 0.6, 0.4)
h_2	(0.4, 0.6, 0.6)	(0.6, 0.2, 0.4)	(0.6, 0.4, 0.3)	(0.7, 0.6, 0.6)
h_3	(0.6, 0.4, 0.2)	(0.8, 0.1, 0.3)	(0.7, 0.2, 0.5)	(0.7, 0.6, 0.4)
h_4	(0.6, 0.3, 0.3)	(0.8, 0.2, 0.2)	(0.5, 0.2, 0.6)	(0.7, 0.5, 0.6)
h_5	(0.8, 0.2, 0.3)	(0.8, 0.3, 0.2)	(0.7, 0.3, 0.4)	(0.9, 0.5, 0.7)

Table1: Tabular representation of (F, A) of Example 3.1.

Definition 3.1 Let (G, A) be a neutrosophic soft set on a universe U. For any element $h \in U$, a neutrosophic right neighborhood, with respect to $e \in A$ is defined as follows

$$h_e = \{h_i \in U : T_e(h_i) \geq T_e(h), I_e(h_i) \geq I_e(h), F_e(h_i) \leq F_e(h)\}.$$

Definition 3.2 Let (G, A) be a neutrosophic soft set on a universe U. For any element $h \in U$, a neutrosophic right neighborhood, with respect to all parameters A is defined as follows

$$h]_A = \cap\{h_{e_i} : e_i \in A\}.$$

For more illustration of Definitions 3.1 and 3.2, the following example is introduced.

Example 3.2 According Example 3.1, we can deduce the following results:

$h_{1e_1} = h_{1e_2} = h_{1e_3} = h_{1e_4} = \{h_1\}$, $h_{2e_1} = h_{2e_3} = \{h_1, h_2\}$, $h_{2e_2} = \{h_1, h_2, h_4, h_5\}$, $h_{2e_4} = \{h_1, h_2, h_3\}$, $h_{3e_1} = h_{3e_4} = \{h_1, h_3\}$, $h_{3e_2} = \{h_1, h_3, h_4, h_5\}$, $h_{3e_3} = \{h_1, h_3, h_5\}$, $h_{4e_1} = \{h_1, h_3, h_4\}$, $h_{4e_2} = \{h_4, h_5\}$, $h_{4e_3} = U$, $h_{4e_4} = \{h_1, h_2, h_3, h_4\}$, $h_{5e_1} = h_{5e_2} = h_{5e_4} = \{h_5\}$, $h_{5e_3} = \{h_1, h_5\}$. It follows that, $h_1]_A = \{h_1\}$, $h_2]_A = \{h_1, h_2\}$, $h_3]_A = \{h_1, h_3\}$, $h_4]_A = \{h_4\}$ and $h_5]_A = \{h_5\}$.

Proposition 3.1 Let (G, A) be a neutrosophic soft set on a universe U, ξ is the family of all neutrosophic right neighborhoods on it, and let

$R_e : U \to \xi, R_e(h) = h_e$. Then,

(a) R_e is reflexive relation.

(b) R_e is transitive relation.

(c) R_e may be not symmetric relation.

Proof Let

$\langle h_1, T_e(h_1), I_e(h_1), F_e(h_1)\rangle$, $\langle h_2, T_e(h_2), I_e(h_2), F_e(h_2)\rangle$ and $\langle h_3, T_e(h_3), I_e(h_3), F_e(h_3)\rangle \in G(A)$. Then,

(a) Obviously,

$T_e(h_1) = T_e(h_1)$, $I_e(h_1) = I_e(h_1)$ and $F_e(h_1) = F_e(h_1)$. Hence, for every $e \in A$, $h_1 \in h_{1e}$. Then $h_1 \, R_e \, h_1$ and then R_e is reflexive relation.

(b) Let $h_1 \, R_e \, h_2$ and $h_2 \, R_e \, h_3$. Then, $h_2 \in h_{1e}$ and $h_3 \in h_{2e}$. Hence, $T_e(h_2) \geq T_e(h_1)$, $I_e(h_2) \geq I_e(h_1)$, $F_e(h_2) \leq F_e(h_1)$, $T_e(h_3) \geq T_e(h_2)$, $I_e(h_3) \geq I_e(h_2)$ and $F_e(h_3) \leq F_e(h_2)$. Consequently, we have $T_e(h_3) \geq T_e(h_1)$, $I_e(h_3) \geq I_e(h_1)$ and $F_e(h_3) \leq F_e(h_1)$. It follows that, $h_3 \in$

h_{1e}. Then $h_1 \; R_e \; h_3$ and then R_e is transitive relation.

The following example proves (c), of Proposition 3.1.

Example 3.3 From Example 3.2, we have,

$h_{1e_1} = \{h_1\}$ and $h_{3e_1} = \{h_1, h_3\}$. Hence, $(h_3, h_1) \in R_{e_1}$ but $(h_1, h_3) \notin R_{e_1}$. Then, R_e isn't symmetric relation.

Neutrosophic soft lower and upper approximations are defined as follows

Definition 3.3. Let (G, A) be a neutrosophic soft set on U. Then, neutrosophic soft lower, upper and boundary approximations of $X \subseteq U$, respectively, are

$$\underline{NR}X = \cup\{h]_A : h \in U, h]_A \subseteq X\},$$

$$\overline{NR}X = \cup\{h]_A : h \in U, h]_A \cap X \neq \varnothing\},$$

$$b_{NR}X = \overline{NR}X - \underline{NR}X.$$

Properties of neutrosophic soft rough set approximations are introduced in the following proposition.

Proposition 3.2 Let (G, A) be a neutrosophic soft set on U, and let $X, Z \subseteq U$. Then the following properties hold

(a) $\underline{NR}X \subseteq X \subseteq \overline{NR}X$.

(b) $\underline{NR}\varnothing = \overline{NR}\varnothing = \varnothing$.

(c) $\underline{NR}U = \overline{NR}U = U$.

(d) $X \subseteq Z \Rightarrow \underline{NR}X \subseteq \underline{NR}Z$.

(e) $X \subseteq Z \Rightarrow \overline{NR}X \subseteq \overline{NR}Z$.

(f) $\underline{NR}(X \cap Z) = \underline{NR}X \cap \underline{NR}Z$.

(g) $\underline{NR}(X \cup Z) \supseteq \underline{NR}X \cup \underline{NR}Z$.

(h) $\overline{NR}(X \cap Z) \subseteq \overline{NR}X \cap \overline{NR}Z$.

(i) $\overline{NR}(X \cup Z) = \overline{NR}X \cup \overline{NR}Z$.

Proof

(a) From Definition 3.3, obviously, we can deduce that, $\underline{NR}X \subseteq X$. Also, let $h \in X$, but R_e, defined in Proposition 3.1, is reflexive relation. Then, for all $e \in A$, there exists h_e such that, $h \in h_e$, and then $h \in h]_A$. So $h]_A \cap X \neq \varnothing$. Hence, $h \in \overline{NR}X$. Therefor $\underline{NR}X \subseteq X \subseteq \overline{NR}X$.

(b) Proof of (b), follows directly, from Definition 3.3 and Property (a).

(c) From Property (a), we have $U \subseteq \overline{NR}U$, but U is the universe set, then $\overline{NR}U = U$. Also, from Definition 3.3, we have $\underline{NR}U = \cup\{h]_A : h]_A \subseteq U\}$, but for all $h \in U$, we have $h \in h]_A \subseteq U$. Hence, $\underline{NR}U = U$. Therefor $\underline{NR}U = \overline{NR}U = U$.

(d) Let $X \subseteq Z$ and $p \in \underline{NR}X$. Then, there exists $h]_A$ such that, $p \in h]_A \subseteq X$. But $X \subseteq Z$, then $p \in h]_A \subseteq Z$. Hence, $p \in \underline{NR}Z$. Therefor $\underline{NR}X \subseteq \underline{NR}Z$.

(e) Let $X \subseteq Z$ and $p \in \overline{NR}X$. Then, there exists $h]_A$ such that, $p \in h]_A$, $h]_A \cap X \neq \varnothing$. But $X \subseteq Z$, then $h]_A \cap Z \neq \varnothing$. Hence, $p \in \overline{NR}Z$. Therefor $\overline{NR}X \subseteq \overline{NR}Z$.

(f) Let $p \in \underline{NR}(X \cap Z) = \cup\{h]_A : h]_A \subseteq (X \cap Z)\}$. So, there exists $h]_A$ such that, $p \in h]_A \subseteq (X \cap Z)$, then $p \in h]_A \subseteq X$ and $p \in h]_A \subseteq Z$. Consequently, $p \in \underline{NR}X$ and $p \in \underline{NR}Z$, then $p \in \underline{NR}X \cap \underline{NR}Z$. Thus, $\underline{NR}(X \cap Z) \subseteq \underline{NR}X \cap \underline{NR}Z$. Conversely, let $p \in \underline{NR}X \cap \underline{NR}Z$. Hence $p \in \underline{NR}X$ and $p \in \underline{NR}Z$. Then there exists $h]_A$ such that, $p \in h]_A \subseteq X$ and $p \in h]_A \subseteq Z$, then $p \in h]_A \subseteq (X \cap Z)$. Consequently, $p \in \underline{NR}(X \cap Z)$, it follows that $\underline{NR}X$

$\cap$ $\underline{NR}Z \subseteq \underline{NR}(X\cap Z)$. Therefor $\underline{NR}(X\cap Z) = \underline{NR}X \cap \underline{NR}Z$.

(g) Let $p \notin \underline{NR}(X\cup Z) = \cup\{h]_A : h]_A \subseteq X\cup Z\}$. So, for all $h]_A$, such that $p\in h]_A$, we have $h]_A \nsubseteq X\cup Z$, then for all $h]_A$ containing p, we have $h]_A \nsubseteq X$ and $h]_A \nsubseteq Z$. Consequently, $p\notin \underline{NR}X$ and $p\notin \underline{NR}Z$, then $p\notin \underline{NR}X \cup \underline{NR}Z$. Therefor $\underline{NR}(X\cup Z) \supseteq \underline{NR}X \cup \underline{NR}Z$.

(h) Let $p\in \overline{NR}(X\cap Z) = \cup\{h]_A : h]_A \cap (X\cap Z)\neq\emptyset\}$. So, there exists $h]_A$ such that, $p\in h]_A$ and $h]_A \cap (X\cap Z) \neq \emptyset$, then $h]_A \cap X \neq \emptyset$ and $h]_A \cap Z \neq \emptyset$. Consequently, $p\in \overline{NR}X$ and $p\in \overline{NR}Z$, then $p\in \overline{NR}X \cap \overline{NR}Z$. Therefor $\overline{NR}(X\cap Z) \subseteq \overline{NR}X \cap \overline{NR}Z$.

(i) Let $p\notin \overline{NR}(X\cup Z) = \cup\{h]_A : h]_A \cap (X\cup Z)\neq\emptyset\}$. So, for all $h]_A$ containing p, we have $h]_A \cap (X\cup Z) = \emptyset$, then for all $h]_A$ containing p, we have $h]_A \cap X = \emptyset$ and $h]_A \cap Z = \emptyset$. Consequently, $p\notin \overline{NR}X$ and $p\notin \overline{NR}Z$, then $p\notin \overline{NR}X \cup \overline{NR}Z$. Then, $\overline{NR}(X\cup Z) \supseteq \overline{NR}X \cup \overline{NR}Z$. Conversely, let $p\in \overline{NR}(X\cup Z)$. Then, there exists $h]_A$ such that, $p\in h]_A$ and $h]_A \cap (X\cup Z) \neq \emptyset$, it follows that, $h]_A \cap X \neq \emptyset$ or $h]_A \cap Z \neq \emptyset$. Consequently, $p\in \overline{NR}X$ or $p\in \overline{NR}Z$, hence, $p\in \overline{NR}X \cup \overline{NR}Z$, then $\overline{NR}X \cup \overline{NR}Z \supseteq \overline{NR}(X\cup Z)$. Therefor $\overline{NR}X \cup \overline{NR}Z = \overline{NR}(X\cup Z)$.

The following example illustrates that, containments of Property (a), may be proper.

Example 3.4 From Example 3.1, If $X=\{h_1\}$, then $\underline{NR}X = \{h_1\}$ and $\overline{NR}X = \{h_1,h_2,h_3\}$. Hence, $\underline{NR}X \neq X$ and $X \neq \overline{NR}X$.

The following example illustrates that, containments of Properties (d) and (e), may be proper.

Example 3.5 From Example 3.1, If $X = \{h_2\}$ and $Z=\{h_2,h_4\}$, then $\underline{NR}X = \emptyset$, $\underline{NR}Z = \{h_4\}$, $\overline{NR}X = \{h_1,h_2\}$ and $\overline{NR}Z = \{h_1,h_2,h_4\}$. Hence, $\underline{NR}X \neq \underline{NR}Z$ and $\overline{NR}X \neq \overline{NR}Z$.

The following example illustrates that, a containment of Property (g), may be proper.

Example 3.6 From Example 3.1, If $X = \{h_1\}$ and $Z=\{h_2\}$, then $\underline{NR}X = \{h_1\}$, $\underline{NR}Z = \emptyset$ and $\underline{NR}(X\cup Z) = \{h_1,h_2\}$. Therefor $\underline{NR}(X\cup Z) \neq \underline{NR}X \cup \underline{NR}Z$.

The following example illustrates that, a containment of Property (h), may be proper.

Example 3.7 From Example 3.1, If $X = \{h_1,h_4\}$ and $Z=\{h_2,h_4\}$, then $\overline{NR}X = \{h_1,h_2,h_3,h_4\}$, $\overline{NR}Z = \{h_1,h_2,h_4\}$ and $\overline{NR}(X\cap Z) = \{h_4\}$. Therefor $\overline{NR}(X\cap Z) \neq \overline{NR}X \cap \overline{NR}Z$.

Proposition 3.3 Let (G,A) be a neutrosophic soft set on a unverse U, and let $X,Z\subseteq U$. Then the following properties hold.

(a) $\underline{NR}\;\underline{NR}X = \underline{NR}X$.

(b) $\underline{NR}\;\overline{NR}X = \overline{NR}X$.

Proof

(a) Let $W=\underline{NR}X$ and $p\in W = \cup\{h]_A : h]_A \subseteq X\}$. Then, there exists some $h]_A$ containing p, such that $h]_A \subseteq W$. So, $p\in \underline{NR}W$. Hence, $W \subseteq \underline{NR}W$.

Thus, $\underline{NR}X \subseteq \underline{NR}\ \underline{NR}X$. Also, from Property (a), of Proposition 3.2, we have $\underline{NR}X \subseteq X$ and by using Property (d), of Proposition 3.2, we get $\underline{NR}\ \underline{NR}X \subseteq \underline{NR}X$. Therefor $\underline{NR}\ \underline{NR}X = \underline{NR}X$.

(b) Let $W = \overline{NR}X$, by using Property (a), of Proposition 3.2, we have $\underline{NR}W \subseteq W$. Conversely, let $p \in W = \cup\{h]_A : h]_A \cap X \neq \emptyset\}$, hence there exists $h]_A$ containing p such that, $p \in h]_A \subseteq W$, it follows that, $p \in \underline{NR}W$. Consequently, $W \subseteq \underline{NR}W$, then $W = \underline{NR}W$, but $W = \overline{NR}X$. Thus, $\underline{NR}\overline{NR}X = \overline{NR}X$.

Proposition 3.4 Let (G, A) be a neutrosophic soft set on U, and let $X, Z \subseteq U$. Then, the following properties don't hold

(a) $\overline{NR}\ \overline{NR}X = \overline{NR}X$.

(b) $\overline{NR}\ \underline{NR}X = \underline{NR}X$.

(c) $\underline{NR}X^c = [\overline{NR}X]^c$.

(d) $\overline{NR}X^c = [\underline{NR}X]^c$.

(e) $\underline{NR}(X-Z) = \underline{NR}X - \underline{NR}Z.$

(f) $\overline{NR}(X-Z) = \overline{NR}X - \overline{NR}Z.$

The following example proves (a) of Proposition 3.4.

Example 3.8 From Example 3.1, If $X = \{h_2\}$, then $\overline{NR}X = \{h_1, h_2\}$ and $\overline{NR}\ \overline{NR}X = \{h_1, h_2, h_3\}$. Hence, $\overline{NR}\ \overline{NR}X \neq \overline{NR}X$.

The following example proves (b) of Proposition 3.4.

Example 3.9 From Example 3.1, If $X = \{h_1\}$, then $\underline{NR}X = \{h_1\}$ and $\overline{NR}\ \underline{NR}X = \{h_1, h_2, h_3\}$. Hence, $\overline{NR}\ \underline{NR}X \neq \overline{NR}X$.

The following example proves (c) of Proposition 3.4.

Example 3.10 From Example 3.1, If $X = \{h_2\}$, then $\underline{NR}X^c = \{h_1, h_3, h_4, h_5\}$ and $[\overline{NR}X]^c = \{h_3, h_4, h_5\}$. Therefor $\underline{NR}X^c \neq [\overline{NR}X]^c$.

The following example proves (d) of Proposition 3.4.

Example 3.11 From Example 3.1, If $X = \{h_1, h_3, h_4, h_5\}$, then $\overline{NR}X^c = \{h_1, h_2\}$ and $[\underline{NR}X]^c = \{h_2\}$. Therefor $\overline{NR}X^c \neq [\underline{NR}X]^c$.

The following example proves (e), (f) of Proposition 3.4.

Example 3.12 From Example 3.1, If $X = \{h_1, h_2\}$ and $Z = \{h_1, h_3\}$, then $\underline{NR}X = \{h_1, h_2\}$, $\underline{NR}Z = \{h_1, h_3\}$, $\underline{NR}(X-Z) = \emptyset$, $\overline{NR}X = \{h_1, h_2, h_3\}$, $\overline{NR}Z = \{h_1, h_2, h_3\}$, $\overline{NR}(X-Z) = \{h_1, h_2\}$. Therefor $\underline{NR}(X-Z) \neq \underline{NR}X - \underline{NR}Z$ and $\overline{NR}(X-Z) \neq \overline{NR}X - \overline{NR}Z$.

4 Modification of suggested neutrosophic soft rough set approximations

In this section, we introduce a modification of suggested neutrosophic soft rough set approximations, introduced in Section 3. Some basic properties are introduced and proved. Finally, a comparison among traditional rough set model, suggested neutrosophic soft rough set model and its modification, by using their properties.

Modified neutrosophic soft lower and upper approximations are defined as follows

Definition 4.1 Let (G, A) be a neutrosophic soft set on U. Then, modified neutrosophic soft lower, upper and boundary approximations of $X \subseteq U$, respectively, are

$$N_R X = \cup\{h]_A : h \in U, h]_A \subseteq X\},$$

$$N^R X = [N_R X^c]^c,$$

$$b_{NR} X = N^R X - N_R X.$$

Modified neutrosophic soft lower and upper approximations properties are introduced in the following proposition.

Proposition 4.1 Let (G, A) be a neutrosophic soft set on

U, and let $X, Z \subseteq U$. Then the following properties hold

(a) $N_R X \subseteq X \subseteq N^R X$.

(b) $N_R \varnothing = N^R \varnothing = \varnothing$.

(c) $N_R U = N^R U = U$.

(d) $X \subseteq Z \Rightarrow N_R X \subseteq N_R Z$.

(e) $X \subseteq Z \Rightarrow N^R X \subseteq N^R Z$.

(f) $N_R (X \cap Z) = N_R X \cap N_R Z$.

(g) $N_R (X \cup Z) \supseteq N_R X \cup N_R Z$.

(h) $N^R (X \cap Z) \subseteq N^R X \cap N^R Z$.

(i) $N^R (X \cup Z) = N^R X \cup N^R Z$.

(j) $N_R \; N_R X = N_R X$.

(k) $N^R \; N^R X = N^R X$.

(l) $N_R X^c = [N^R X]^c$.

(m) $N^R X^c = [N_R X]^c$.

Proof

Properties (a)-(i) are proved at the same way as Proposition 3.2.

(j) Let

$W = N_R X$ and $p \in W = \cup\{h]_A : h]_A \subseteq X\}$.

Then, there exists some $h]_A$ containing p, such that $h]_A \subseteq W$. So, $p \in N_R W$. Hence, $W \subseteq N_R W$. Thus, $N_R X \subseteq N_R \; N_R X$. Also, from Property (a), of Proposition 3.2, we have $N_R X \subseteq X$ and by using Property (d), of Proposition 3.2, we can deduce that $N_R \; N_R X \subseteq N_R X$. Therefor $N_R \; N_R X = N_R X$.

(k) $N^R \; N^R X = N^R [N_R X^c]^c = [N_R ([N_R X^c]^c)^c]^c = [N_R \; N_R X^c]^c$, from Property (j) of Proposition 4.1, we can deduce that $N_R \; N_R X^c = N_R X$. Then $[N_R \; N_R X^c]^c = [N_R X^c]^c$, from Definition 4.1, we have $[N_R X^c]^c = N^R X$. Therefor $N^R \; N^R X = N^R X$.

Properties (l), (m) can be proved, directly, by using Definition 4.1.

The following example illustrates that, containments of Property (a), may be proper.

Example 4.1 From Example 3.1, If $X = \{h_1\}$, then $N_R X = \{h_1\}$ and $N^R X = \{h_1, h_2, h_3\}$. Hence, $N_R X \neq X$ and $X \neq N^R X$.

The following example illustrates that, containments of Properties (d) and (e), may be proper.

Example 4.2 From Example 3.1, If $X = \{h_2\}$ and $Z = \{h_2, h_4\}$, then $N_R X = \varnothing$, $N_R Z = \{h_4\}$, $N^R X = \{h_2\}$ and $N^R Z = \{h_2, h_4\}$. Hence, $N_R X \neq N_R Z$ and $N^R X \neq N^R Z$.

The following example illustrates that, a containment of Property (g), may be proper.

Example 4.3 From Example 3.1, If $X = \{h_1\}$ and $Z = \{h_2\}$, then $N_R Z = \varnothing$, $N_R X = \{h_1\}$ and $N_R (X \cup Z) = \{h_1, h_2\}$. Therefor $N_R (X \cup Z) \neq N_R X \cup N_R Z$.

The following example illustrates that, a containment of Property (h), may be proper.

Example 4.4 From Example 3.1, If $X = \{h_1, h_4\}$ and $Z = \{h_2, h_4\}$, then $N^R X = \{h_1, h_2, h_3, h_4\}$, $N^R Z = \{h_2, h_4\}$ and $N^R (X \cap Z) = \{h_4\}$. Therefor

$N^R(X \cap Z) \neq N^R X \cap N^R Z$.

Proposition 4.2 Let (G, A) be a neutrosophic soft set on a unverse U, and let $X, Z \subseteq U$. Then, the following properties don't hold

(a) $N_R \; N^R X = N^R X$.

(b) $N^R \; N_R X = N_R X$.

(c) $N_R(X - Z) = N_R X - N_R Z$.

(d) $N^R(X - Z) = N^R X - N^R Z$.

The following example proves (a) of Proposition 4.2.

Example 4.5 From Example 3.1, If $X = \{h_2\}$, then $N^R X = \{h_2\}$ and $N_R \; N^R X = \varnothing$. Hence, $N_R \; N^R X \neq N^R X$.

The following example proves (b) of Proposition 4.2.

Example 4.6 From Example 3.1, If $X = \{h_1\}$, then $N_R X = \{h_1\}$ and $N^R \; N_R X = \{h_1, h_2, h_3\}$. Hence, $N^R \; N_R X \neq N^R X$.

The following example proves (c), (d) of Proposition 4.2.

Example 4.7 From Example 3.1, If $X = \{h_1, h_2\}$ and $Z = \{h_1, h_3\}$, then $N_R X = \{h_1, h_2\}$, $N_R Z = \{h_1, h_3\}$, $N_R(X - Z) = \varnothing$, $N^R X = \{h_1, h_2, h_3\}$, $N^R Z = \{h_1, h_2, h_3\}$, $N^R(X - Z) = \{h_2\}$. Therefor $N_R(X - Z) \neq N_R X - N_R Z$ and $N^R(X - Z) \neq N^R X - N^R Z$.

Remark 4.1 A comparison among traditional rough model, suggested neutrosophic soft rough model and its modification, by using their properties, is concluded in Table 2, where traditional rough are symboled by (T), neutrosophic soft rough by(N), its modification by (M) and (*) means that, this property is satisfied, as follows

Rough properties	T	N	M
$\underline{E}(\varnothing) = \overline{E}(\varnothing) = \varnothing$	*	*	*
$\underline{E}(U) = \overline{E}(U) = U$	*	*	*
$\underline{E}(X) \subseteq X \subseteq \overline{E}(X)$	*	*	*
$\overline{E}(X \cup Y) = \overline{E}(X) \cup \overline{E}(Y)$	*	*	*
$\underline{E}(X \cap Y) = \underline{E}(X) \cap \underline{E}(Y)$	*	*	*
$\overline{E}(X \cap Y) \subseteq \overline{E}(X) \cap \overline{E}(Y)$	*	*	*
$\underline{E}(X \cup Y) \supseteq \underline{E}(X) \cup \underline{E}(Y)$	*	*	*
$\underline{E}(X^c) = [\overline{E}(X)]^c$	*		*
$\overline{E}(X^c) = [\underline{E}(X)]^c$	*		*
$X \subseteq Y \to \underline{E}(X) \subseteq \underline{E}(Y)$	*	*	*
$X \subseteq Y \to \overline{E}(X) \subseteq \overline{E}(Y)$	*	*	*
$\underline{E}(\underline{E}(X)) = \underline{E}(X)$	*	*	*
$\underline{E}(\overline{E}(X)) = \overline{E}(X)$	*	*	
$\overline{E}(\underline{E}(X)) = \underline{E}(X)$	*		
$\overline{E}(\overline{E}(X)) = \overline{E}(X)$	*		*

Table 2: Comparison among traditional rough and suggested models, by using their properties.

To compare between suggested neutrosophic soft upper approximation and its modification, the following proposition is introduced.

Proposition 4.3 Let (G, A) be a neutrosophic soft set on a unverse U. For any considered set $X \subseteq U$, the following property holds

$$N^R X \subseteq \overline{NRX}$$

Proof Obvious.

The following example illustrates that a containment relationship between suggested neutrosophic soft upper and its modification, may be proper.

Example 4.7 According to Example 3.1, Table 3 can be created as follows

X	$N^R X$	$\overline{NR}X$
$\{h_2\}$	$\{h_2\}$	$\{h_1, h_2\}$
$\{h_3\}$	$\{h_3\}$	$\{h_1, h_3\}$
$\{h_2, h_3\}$	$\{h_2, h_3\}$	$\{h_1, h_2, h_3\}$
$\{h_2, h_4\}$	$\{h_2, h_4\}$	$\{h_1, h_2, h_4\}$
$\{h_2, h_5\}$	$\{h_2, h_5\}$	$\{h_1, h_2, h_5\}$
$\{h_3, h_4\}$	$\{h_3, h_4\}$	$\{h_1, h_3, h_4\}$
$\{h_3, h_5\}$	$\{h_3, h_5\}$	$\{h_1, h_3, h_5\}$
$\{h_2, h_3, h_4\}$	$\{h_2, h_3, h_4\}$	$\{h_1, h_2, h_3, h_4\}$
$\{h_2, h_3, h_5\}$	$\{h_2, h_3, h_5\}$	$\{h_1, h_2, h_3, h_5\}$
$\{h_2, h_4, h_5\}$	$\{h_2, h_4, h_5\}$	$\{h_1, h_2, h_4, h_5\}$
$\{h_3, h_4, h_5\}$	$\{h_3, h_4, h_5\}$	$\{h_1, h_3, h_4, h_5\}$
$\{h_2, h_3, h_4, h_5\}$	$\{h_2, h_3, h_4, h_5\}$	U

Table 3: Comparison between suggested upper approximation and its modification.

From Table 3, we can deduce that, for any considered set X, the modified upper approximation is decreased. It follows that its boundary region is decreased.

5 Neutrosophic soft rough concepts and their modification

In this section, some of neutrosophic soft rough concepts are defined as a generalization of rough concepts. Their modification are introduced and compare with them.

Neutrosophic soft rough NR-definability and N_R-definability of any subset $X \subseteq U$, is defined as follows

Definition 5.1. Let (G, A) be a neutrosophic soft set on U, and let $X \subseteq U$. A subset $X \subseteq U$, is called

(a) NR-definable, if $\underline{NR}X = \overline{NR}X = X$.

(b) N_R-definable, if $N_R X = N^R X = X$.

(c) Internally NR-definable, if $\underline{NR}X = X$ and $\overline{NR}X \neq X$.

(d) Internally N_R-definable, if $N_R X = X$ and $N^R X \neq X$.

(e) Externally NR-definable, if $\underline{NR}X \neq X$ and $\overline{NR}X = X$.

(f) Externally N_R-definable, if $N_R X \neq X$ and $N^R X = X$.

(g) NR-rough, if $\underline{NR}X \neq X$ and $\overline{NR}X \neq X$.

(h) N_R-rough, if $N_R X \neq X$ and $N^R X \neq X$.

Proposition 5.1 Let (G, A) be a neutrosophic soft set on U. For any considered set $X \subseteq U$, the following properties hold

(a) X is NR-definable set $\rightarrow$ X is N_R-definable set.

(b) X is externally NR-definable set $\rightarrow$ X is externally N_R-definable set.

(c) X is N_R-rough set $\rightarrow$ X is NR-rough set.

Proof Obvious.

The following example proves that the inverse of Proposition 5.1, does not hold.

Example 5.1 According to Example 3.1, Table 4 can be created, where (Ex) means externally definable and (R) means rough as follows

Ex- NR	Ex- N_R	N_R -R	NR -R
	$\{h_2\}$		$\{h_2\}$
	$\{h_3\}$		$\{h_3\}$
	$\{h_2,h_3\}$		$\{h_2,h_3\}$
	$\{h_2,h_4\}$		$\{h_2,h_4\}$
	$\{h_2,h_5\}$		$\{h_2,h_5\}$
	$\{h_3,h_4\}$		$\{h_3,h_4\}$
	$\{h_3,h_5\}$		$\{h_3,h_5\}$
	$\{h_2,h_3,h_4\}$		$\{h_2,h_3,h_4\}$
	$\{h_2,h_3,h_5\}$		$\{h_2,h_3,h_5\}$
	$\{h_2,h_4,h_5\}$		$\{h_2,h_4,h_5\}$
	$\{h_3,h_4,h_5\}$		$\{h_3,h_4,h_5\}$
	$\{h_2,h_3,h_4,h_5\}$		$\{h_2,h_3,h_4,h_5\}$

Table 4: Comparison between NR -definabilityand its modification.

From Table 4, it is clear that, by using a modified suggested upper approximation, any considered set has a big chance to change from NR -rough set to externally N_R -definable set. The reason of this is that its suggested modified upper approximation is decreased in some degrees.

In the following definition neutrosophic soft rough membership relations and their modifications are defined.

Definition 5.2 Let (G,A) be a neutrosophic soft set on U , and let $x\in U$, $X\subseteq U$. Then

$$x \underline{\in}_{NR} X, if \quad x\in \underline{NR}X,$$

$$x \overline{\in}_{NR} X, if \quad x\in \overline{NR}X,$$

$$x \overline{\in}_{N_R} X, if \quad x\in N^R X.$$

Proposition 5.2 Let (G,A) be a neutrosophic soft set on a unverse U , and let $x\in U$, $X\subseteq U$. Then,

$$x \underline{\in}_{NR} X \rightarrow x\in X \rightarrow x \overline{\in}_{N_R} X \rightarrow x \overline{\in}_{NR} X.$$

Proof From Propositions 3.2, 4.1 and 4.3, we can deduce that

$\underline{NR}X \subseteq X \subseteq N^R X \subseteq \overline{NR}X$. Then, by using Definition 5.2, we get the proof, directly.

The following example illustrates that, the inverse of Proposition 5.2, doesn't hold.

Example 5.2 In Example 3.1, if

$X=\{h_1\}$ and $Z=\{h_2\}$, then $\underline{NR}Z = \varnothing$, $\overline{NR}Z = \{h_1,h_2\}$, $N^R Z = \{h_2\}$ and $N^R X = \{h_1,h_2,h_3\}$. Hence, $h_2 \underline{\notin}_{NR} Z$, although, $h_2\in Z$, $h_1 \overline{\notin}_{N_R} Z$, although, $h_1 \overline{\in}_{NR} Z$ and $h_3 \notin X$, although, $h_3 \overline{\in}_{N_R} X$.

In the following definition neutrosophic soft rough inclusion relations and their modifications are defined.

Definition 5.3 Let (G,A) be a neutrosophic soft set on U , and let $X,Z\subseteq U$. Then

$$X \underset{\rightarrow}{\subset}_{NR} Z, if\ \underline{NR}X \subseteq \underline{NR}Z,$$

$$X \overset{\rightarrow}{\subset}_{NR} Z, if \quad \overline{NR}X \subseteq \overline{NR}Z,$$

$$X \overset{\rightarrow}{\subset}_{N_R} Z, if \quad N^R X \subseteq N^R Z.$$

In the following definition neutrosophic soft rough equality relations and their modifications are defined.

Definition 5.4 Let (G, A) be a neutrosophic soft set on a unverse U, and let $X, Z \subseteq U$. Then

$$X \underset{\to NR}{=} Z, if \quad \underline{NR}X = \underline{NR}Z,$$

$$X \overset{\to}{=}_{NR} Z, if \quad \overline{NR}X = \overline{NR}Z,$$

$$X \overset{\to}{=}_{N_R} Z, if \quad N^R X = N^R Z,$$

$$X \approx_{NR} Z, if \quad X \underset{\to NR}{=} Z \quad and \quad X \overset{\to}{=}_{NR} Z,$$

$$X \approx_{N_R} Z, if \quad X \underset{\to N_R}{=} Z \quad and \quad X \overset{\to}{=}_{N_R} Z.$$

The following examples illustrate Definition 5.4.

Example 5.3 In Example 3.1, if $X_1 = \{h_2\}$, $X_2 = \{h_3\}$, $X_3 = \{h_1, h_2\}$ and $X_4 = \{h_1, h_3\}$. Then, $\underline{NR}X_1 = \underline{NR}X_2 = \varnothing$ and $\overline{NR}X_3 = \overline{NR}X_4 = N^R X_3 = N^R X_4 = \{h_1, h_2, h_3\}$. Consequently, $X_1 \underset{\to NR}{=} X_2$, $X_3 \overset{\to}{=}_{NR} X_4$ and $X_3 \overset{\to}{=}_{N_R} X_4$.

Example 5.4 According to Example 3.1, if $A' = \{e_1, e_3\}$. Tabular representation of Neutrosophic soft set (G, A') can be seen in Table 5, as follows

A'	h_1	h_2	h_3	h_4	h_5
e_1	(.6, .6, .2)	(.4, .6, .6)	(.6, .4, .2)	(.6, .3, .3)	(.8, .2, .3)
e_3	(.7, .4, .3)	(.6, .4, .3)	(.7, .2, .5)	(.5, .2, .6)	(.7, .3, .4)

Table 5: Tabular representation of neutrosophic soft set in Example 5.4.

It follows that,

$h_1]_{A'} = \{h_1\}$, $h_2]_{A'} = \{h_1, h_2\}$, $h_3]_{A'} = \{h_1, h_3\}$, $h_4]_{A'} = \{h_1, h_3, h_4\}$ and $h_5]_{A'} = \{h_5\}$.

If we take $X_1 = \{h_3\}$ and $X_2 = \{h_3, h_4\}$, then $\underline{NR}X_1 = \underline{NR}X_2 = \varnothing$ and $\overline{NR}X_1 = \overline{NR}X_2 = \{h_1, h_3, h_4\}$ and $N^R X_1 = N^R X_2 = \{h_3, h_4\}$. Therefor $X_1 \approx_{NR} X_2$ and $X_5 \approx_{N_R} X_2$.

Proposition 5.3 Let (G, A) be a neutrosophic soft set on a unverse U, $X, Z \subseteq U$ and let $I \in \{NR, N_R\}$. Then,

(a) $X \underset{\to NR}{=} \underline{NR}X$.

(b) $X \underset{\to N_R}{=} N_R X$.

(c) $X \overset{\to}{=}_{N_R} N^R X$.

(d) $X = Y \to X \approx_I Z$.

(e) $X \subseteq Z \to X \underset{\to I}{\subset} Z$ and $X \overset{\to}{\subset}_I Z$.

(f) $X \subseteq Z$, $Z \underset{\to I}{=} \varnothing \to X \underset{\to I}{=} \varnothing$.

(g) $X \subseteq Z$, $X \underset{\to I}{=} U \to Z \underset{\to I}{=} U$.

(h) $X \subseteq Z$, $Z \overset{\to}{=}_I \varnothing \to X \overset{\to}{=}_I \varnothing$.

(i) $X \subseteq Z$, $X \overset{\to}{=}_I U \to Z \overset{\to}{=}_I U$.

Proof From Propositions 3.2, 3.3 and 4.1, we get the proof, directly.

We can determine the degree of neutrosophic soft NR-definability and N_R-definability of $X \subseteq U$, by using their accuracy measures denoted by $C_{NR} X$ and $C_{N_R} X$, respectively, which are defined as follows

Definition 5.5 Let (G, A) be a neutrosophic soft set on U and let $X \subseteq U$. Then,

$$C_{NR} X = \frac{\underline{\underline{NR}}X}{\overline{\overline{NR}}X}, \quad X \neq \phi,$$

$$C_{N_R} X = \frac{N_R X}{N^R X}, \quad X \neq \phi.$$

Proposition 5.4 Let (G, A) be a neutrosophic soft set on U and let $X \subseteq U$, the following statements are satisfied

(a) $0 \leq C_{NR}(X) \leq C_{N_R}(X) \leq 1.$

(b) X is NR-definable, if and only if, $C_{NR}(X) = 1.$

(c) X is N_R -definable, if and only if, $C_{N_R} = 1.$

Proof From Definitions 3.3, 4.1, 5.1 and 5.5, we get the proof, directly.

A comparison between suggested neutrosophic soft rough model and its modification, by using their accuracy measures, is concluded in Table 6.

Example 5.5 From Example 3.1, we can create Table 6, as follows

Accuracy measures / Sets	C_{NR}	C_{N_R}
$\{h_3, h_4\}$	0.33	0.50
$\{h_2, h_4\}$	0.33	0.50
$\{h_2, h_5\}$	0.33	0.50
$\{h_2, h_4, h_5\}$	0.50	0.67
$\{h_3, h_4, h_5\}$	0.50	0.67
$\{h_2, h_3, h_4, h_5\}$	0.40	0.50

Table 6: Comparison between suggested neutrosophic soft rough model and its modification, by using their accuracy measures.

From Table 6, by using suggested modified approximations, the degree of definability of all these subsets is increased. It means that, when we use suggested modified approximations, we notice that, for any considered neutrosophic soft rough set, its boundary region is decreased. It leads to more accurate results of any real life application.

Remark 5.1 Let (G, A) be a neutrosophic soft set on a unverse U , and let $h \in U$, $X \subseteq U$. If we consider the following case : If

$T_e(h_i) > 0.5$, then $e(h) = 1$, otherwise, $e(h) = 0$.

Hence, neutrosophic right neighborhood of an element h is replaced by the following equivalence class $[h] = \{h_i \in U : e(h_i) = e(h), \ e \in A\}$. It follows that, neutrosophic soft rough set approximations will be returned to Pawlak's rough set approximations. Consequently, all properties of traditional rough set approximations will be satisfied. Hence, Pawlak's approach to rough sets is a special case of the proposed approaches in this paper.

Conclusion

The difference in neutrosophic logic is that there is a component of indeterminate I , which means, for example in decision making and control theory, that we have ($I\%$) hesitating to take a decision. It follows that proposed models, in this paper, are more realistic than Pawlak's model. Pawlak's approach to rough sets can be viewed as a special case of neutrosophic soft approach to rough sets. Our future work, aims to apply them in solving many practical problems in medical science.

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Solution of Multi-Criteria Assignment Problem using Neutrosophic Set Theory

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Abstract: Assignment Problem (AP) is a very well-known and also useful decision making problem in real life situations. It becomes more effective when different criteria are added. To solve Multi-Criteria Assignment Problem (MCAP), the different criteria have been considered as neutrosophic elements because Neutrosophic Set Theory (NST) is a generalization of the classical sets, conventional fuzzy sets, Intuitionistic Fuzzy Sets (IFS) and Interval Valued Fuzzy Sets (IVFS). In this paper two different methods have been proposed for solving MCAP. In the first method, we have calculated evaluation matrix, score function matrix, accuracy matrix and ranking matrix of the MCAP. The rows represent the alternatives and columns represent the projects of the MCAP. From the ranking matrix, the ranking order of the alternatives and the projects are determined separately. From the above two matrices, composite matrix is formed and it is solved by Hungarian Method to get the optimal assignment.

In the second one, Cosine formula for Vector Similarity Measure [1] on neutrosophic set is used to calculate the degree of similarity between each alternative and the ideal alternative. From the similarity matrix, the ranking order of the alternatives and the projects are determined in the same way as above. Finally the problem is solved by Hungarian Method to obtain the optimal solution.

Keywords: Assignment, Neutrosophic Set, Similarity Measures.

1. Introduction:-

NST is a powerful formal framework which generalizes the concepts of classical set, fuzzy set, IFS, IVFS etc. In the year 1965 Zadeh [2] first introduced the concept of fuzzy set which is a very effective tool to measure uncertainty in real life situation. After two decades, Turksen [3] proposed the concept of IVFS. Atanassov [4] introduced IFS which not only describes the degree of membership, but also the degree of non-membership function. Wang et. al [5] proposed a different concept of imprecise data which gives indeterminate information. F. Smarandache introduced the degree of indeterminacy/neutrality [6] as independent concept in 1995 (published in 1998) and defined the neutrosophic set. He coined the words 'neutrosophy' and 'neutrosophic'. In 2013, he refined the neutrosophic set to 'n' components: t1, t2,…… ; i1, i1,….; f1, f2,….. .

Different authors have solved Multi-Criteria Decision Making (MCDM) problems in different ways. But in neutrosophy, MCAP has not been solved earlier. In real life situation, truth value and falsity (membership and non-membership function) are not sufficient; indeterminacy is also a very important part for decision making problem. NST is a different and more practical concept of fuzzy set where degree of truth value, falsity and indeterminacy are all considered and so it is more relevant to solve MCDM problems.

Several mathematicians have worked on the concept of similarity measures of fuzzy sets. Xu. Z. S [7] used similarity measures of IFS and their applications to multiple attribute DM problems. Li et. al [8] also worked on IFS using similarity measures. Zhizhen Liang, Pengfei Shi (2003) [9] also worked on similarity measures on IFS. Smeg-Hyuk Cha [10] worked on distance similarity measures between probability density functions. Santini S. et. al [11] developed a similarity measure which is based on fuzzy logic. The model is dubbed Fuzzy Feature Contrast (FFC) and they used it to model similarity assessment from fuzzy judgment of properties. Wen-Liang Hung et. al [12] worked on similarity measures between two IFSs. Said Broumi, F. Samarandache [13] calculated the degree of similarity between neutrosophic sets.

In this paper we have developed two methods to solve MCAP. One is based on score function and another one is on vector similarity measure for neutrosophic set. The methods have been demonstrated by a numerical example. The paper is organized as follows- In section 2 preliminaries have been given; section 3 describes the MCAP method and its solution procedures along with the two algorithms. Section 4 illustrates the numerical example and finally section 5 concludes the paper.

2. Preliminaries:-

2.1 Neutrosophic Set:-

Let U be the space of points (or objects) with generic element 'x'. A neutrosophic set A in U is characterized by a truth membership function T_A, and indeterminacy function I_A and a falsity membership function F_A, where T_A, I_A and F_A are real standard or non-standard subsets of $]^-0, 1^+[$, i.e $T_A: x \rightarrow]^-0, 1^+[$

$$F_A: x \rightarrow]^-0, 1^+[$$

$$I_A: x \rightarrow]^-0, 1^+[$$

A neutrosophic set A upon U as an object is defined as -

$$\frac{x}{T_A(x), I_A(x), F_A(x)} = \left\{ \frac{x}{T_A, I_A, F_A} : x \in U \right\}$$

where $T_A(x)$, $I_A(x)$ and $F_A(x)$ are subintervals or union of subintervals of [0, 1].

2.2 Algebraic Operations with Neutrosophic Set:-

For two neutrosophic sets A and B where and

a> Complement of A
$A^{/} = \{ \frac{x}{T,I,F} \mid T = 1 - T_A, I = 1 - I_A, F = 1 - F_A \}$

b> Intersection of A and B
$A \cap B = \{ \frac{x}{T,I,F} \mid T = T_A T_B, I = I_A I_B, F = F_A F_B \}$

c> Union of A and B
$A \cup B = \{ \frac{x}{T,I,F} \mid T = T_A + T_B - T_A T_B, I = I_A + I_B - I_A I_B, F = F_A + F_B - F_A F_B \}$

d> Cartesian Product of A and B

$$A \times B = \{ (\frac{x}{T_A, I_A, F_A}, \frac{y}{T_B, I_B, F_B}) \mid \frac{x}{T_A, I_A, F_A} \in A, \frac{y}{T_B, I_B, F_B} \in B \}$$

e> A is a subset of B

$$A \subseteq B \ \forall \ \frac{x}{T_A, I_A, F_A} \in A \text{ and } \frac{y}{T_B, I_B, F_B} \in B, T_A \leq T_B \text{ and } F_A \geq F_B$$

f> Difference of A and B
$A \backslash B = \{ \frac{x}{T,I,F} \mid T = T_A - T_A T_B, I = I_A - I_A I_B, F = F_A - F_A F_B \}$

NST can be used in assignment problem (AP) and Generalized Assignment Problem (GAP).

2.3 Cosine formula for vector similarity measure:-

Cosine formula for vector similarity measure is

$WS_c(A_i, A^*) =$

$$\frac{\sum_{j=1}^{n} w_j [a_{ij} a_j^* + b_{ij} b_j^* + c_{ij} c_j^*]}{\sqrt{\sum (a_{ij}^2 + b_{ij}^2 + c_{ij}^2)}\sqrt{\sum (a_j^{*2} + b_j^{*2} + c_j^{*2})}}$$

…………………………..[1]

Where A_i is the alternative, A^* is the ideal alternative, w_j represents the weight of the alternatives s.t. $\sum_{j=1}^{n} w_j = 1$.

The criteria are divided into two types – one is cost criterion and the other is benefit criterion (profit, efficiency, quality etc). For these two types ideal alternatives have been defined as –

a> Ideal alternative for cost criterion, A^* is $\alpha_j^* = <(a_j^*, b_j^*, c_j^*)> =$

$$\left\langle \left[\min_i (a_{ij}), \max_i (b_{ij}), \max_i (c_{ij}) \right] \right\rangle$$

……………………..[2]

b> Ideal alternative for benefit criterion, A^* is $\alpha_j^* = <(a_j^*, b_j^*, c_j^*)> =$

$$\left\langle \left[\max_i (a_{ij}), \min_i (b_{ij}), \min_i (c_{ij}) \right] \right\rangle$$

……………………..[3]

3. MCAP using NST:-

In this section we have formulated the MCAP using NST. The AP has been solved by different mathematicians in various ways [14], [15]. Here we have proposed two methods – i> In the first method, to compute the best final result, the evaluation of the alternatives with regard to each criteria are must. So from the decision matrix, evaluation matrix has been calculated. Then score function of each alternative has been computed. To find the degree of accuracy ($H(A_i)$) of neutrosophic elements, accuracy matrix has been evaluated. The larger value of $H(A_i)$, the more is the degree of accuracy of an alternative A_i. To evaluate all the above matrices weights must be considered because the larger the value of $W(E(A_i))$, the more is the suitability to which the alternative A_i satisfies the decision maker's requirement. Using the above, ranking matrix has been computed. Then the alternatives (Teams) are ranked with respect to the criteria (Projects) row-wise and the opposite is done column-wise. From the above two matrices, composite matrix has been formed. Finally assignment is done using Hungarian method. ii> By using the cosine formula for vector similarity measure on neutrosophic elements. Similarity matrix is computed and the Hungarian method, as mentioned earlier, is again used to get the optimal assignment.

3.1 Solution procedure for MCAP:-

Method 1:

To solve MCAP we have considered the elements of the criteria as neutrosophic elements (T, I, F), where T is the truth membership degree, I is indeterminacy and F represents falsity degree. From the input data, evaluation matrix E(A), score function matrix S(A) and accuracy matrix H(A) of the alternatives are determined. Algorithm 1 is applied to find the ranking matrix R(A) using the above three matrices and weights of the criteria.

Algorithm 1:

<u>Step 1</u>: Construct the matrix of neutrosophic MCAP.

<u>Step 2</u>: Determine the evaluation matrix $E(A) = (E(A_{ij}))_{mxn}$ of the alternatives as $E(A) = \left[T_{A_i}^l, T_{A_i}^u\right]$ where

$[T_{A_i}^l, T_{A_i}^u]=$

$$\begin{bmatrix} \min\left(\left(\frac{T_{A_{ij}} + I_{A_{ij}}}{2}\right), \left(\frac{1 - F_{A_{ij}} + I_{A_{ij}}}{2}\right)\right), \\ \max\left(\left(\frac{T_{A_{ij}} + I_{A_{ij}}}{2}\right), \left(\frac{1 - F_{A_{ij}} + I_{A_{ij}}}{2}\right)\right) \end{bmatrix}$$

……[4]

<u>Step 3</u>: Compute the score function matrix $S(A) = (S(A_{ij}))_{mxn}$ of an alternative using the formula

$S(A) = 2\left[T_{A_i}^{\ U} - T_{A_i}^{\ L}\right]$ where $0 \leq S(A_i) \leq 1$……[5]

Step 4: Compute Accuracy matrix H(A) = $(H(A_{ij}))_{mxn}$ to evaluate degree of accuracy of the neutrosophic elements as –

$$H(A) = 0.5\left[T_{A_i}{}^{U} + T_{A_i}{}^{L}\right] \ldots\ldots\ldots\ldots\ldots\ldots[6]$$

Step 5: Using E(A), H(A), S(A) and w_j, ranking matrix R(A) = $(R(A_{ij}))_{mxn}$ of the alternatives is determined by the formula

$$R(A)=\sum_{j=1}^{n}\left[(S(A_{ij}))^2 - \frac{1-H(A_{ij})}{2}\right]w_j \ldots\ldots\ldots[7]$$

Step 6: Form R_1 matrix by considering the rank of the teams and R_2 matrix for the projects.

Step 7: Form the composite matrix by taking the product of R_1 and R_2.

Step 8: Solve the composite matrix by Hungarian method to get the optimal assignment.

Step 9: End.

Method 2:

This method is based on the concept of similarity measures. Here the cosine formula, previously mentioned, has been used. The ideal alternatives for the two types of criteria (cost and benefit) have been defined in the equations [2] and [3]. The cosine formula for similarity measures which is defined in equation [1] has been used to find the degree of similarity and the ranking matrix has been evaluated. The alternative which has the maximum value of the degree of similarity is more similar to the ideal alternative A^* and can be considered as the best choice.

Algorithm 2:

Step 1: Categorize the criteria in two ways – cost criterion and benefit criterion.

Step 2: Determine the ideal alternative for both of the types of criteria defined as in equation [2] and [3].

Step 3: Consider the weights of the criteria w_j and use cosine formula (equation [1]) for vector similarity measures on NS to find the similarity matrix.

Step 4: Follow steps 6 to 8 of Method 1 Algorithm 1.

Step 5: End.

Numerical Example:-

Let us consider an AP consisting of three projects and four teams with three criteria. The three criteria are – cost, profit and efficiency of the team which are considered as neutrosophic elements and the data are as follows.

Table – 1

Input Data Table

Teams	Projects								
	I			II			III		
	c_1	c_2	c_3	c_1	c_2	c_3	c_1	c_2	c_3
A	[(0.75,0.39,0.1)	(0.6,0.5,0.25)	(0.8,0.4,0.2)]	[(0.3,0.2,0.5)	(0.8,0.6,0.1)	(0.2,0.3,0.5)]	[(0.1,0.2,0.4)	(0.2,0.55,0.6)	(0.3,0.5,0.7)]
B	[(0.8,0.6,0.15)	(0.68,0.46,0.2)	(0.45,0.1,0.05)]	[(0.1,0.3,0.4)	(0.5,0.6,0.8)	(0.4,0.5,0.6)]	[(0.2,0.3,0.5)	(0.4,0.5,0.7)	(0.3,0.2,1)]
C	[(0.4,0.8,0.45)	(0.75,0.9,0.05)	(1,0.5,1)]	[(0.25,0.2,0.4)	(0.3,0.5,0.6)	(0.4,0.7,0.8)]	[(0.6,0.5,0.1)	(0.3,0.5,0.6)	(0.6,0.7,0.8)]
D	[(0.4,0.6,0.3)	(0.5,0.4,0.8)	(0.5,0.6,0.9)]	[(0.15,0.3,0.5)	(0.4,0.5,0.6)	(0.7,0.8,1)]	[(0.3,0.4,0.5)	(0.2,0.3,0.4)	(0.6,0.7,1)]

The weights of the criteria are $w_1 = 0.35$, $w_2 = 0.40$ and $w_3 = 0.25$ such that $\sum_{j=1}^{3} w_j = 1$.

The problem is to find the optimal assignment.

Solution:

Method 1:

First we calculate the evaluation matrix $E(A_i)$ of the alternatives by applying formula [4] –

Table 2:

Evaluation Matrix ($E(A_i)$)

Projects \ Teams	I	II	III
A	(0.57,0.645),(0.55,0.625),(0.6,0.6)	(0.25,0.35),(0.7,0.75),(0.25,0.4)	(0.15,0.4),(0.375,0.475),(0.4,0.4)
B	(0.7,0.725),(0.57,0.63),(0.275,0.525)	(0.2,0.45),(0.4,0.55),(0.45,0.45)	(0.25,0.4),(0.4,0.45),(0.1,0.25)
C	(0.6,0.675),(0.825,0.925),(0.25,0.75)	(0.225,0.4),(0.4,0.45),(0.45,0.55)	(0.55,0.7),(0.4,0.45),(0.45,0.65)
D	(0.5,0.65),(0.3,0.45),(0.35,0.55)	(0.225,0.4),(0.45,0.45),(0.4,0.75)	(0.35,0.45),(0.25,0.45),(0.35,0.65)

The score function matrix is calculated by the formula [5].

Table 3:

Score Function Matrix ($S(A_i)$)

Projects \ Teams	I	II	III
A	(0.15,0.15,0)	(0.2,0.1,0.3)	(0.5,0.2,0)
B	(0.05,0.12,0.5)	(0.5,0.3,0)	(0.3,0.1,0.3)
C	(0.15,0.2,1)	(0.35,1,0.2)	(0.3,0.1,0.4)
D	(0.3,0.3,0.4)	(0.35,0,0.7)	(0.2,0.4,0.6)

Accuracy matrix $H(A_i)$ has been calculated by formula [6].

Table 4:

Accuracy Matrix ($H(A_i)$)

Projects \ Teams	I	II	III
A	(0.6075,0.5875,0.6)	(0.3,0.725,0.325)	(0.275,0.425,0.4)
B	(0.7125,0.6,0.4)	(0.325,0.475,0.45)	(0.325,0.425,0.175)
C	(0.6375,0.875,0.5)	(0.3125,0.475,0.5)	(0.625,0.425,0.55)
D	(0.575,0.375,0.45)	(0.3125,0.45,0.575)	(0.4,0.35,0.5)

Now we calculate the ranking matrix $R(A_i)$ using formula [7].

Table 5:

Ranking Matrix ($R(A_i)$)

Projects \ Teams	I	II	III
A	- 0.184	- 0.222	- 0.075
B	- 0.136	- 0.169	- 0.279
C	0.124	0.165	- 0.161
D	- 0.161	- 0.118	- 0.129

Table 6:

Ranking Indices (R_1) of the project w.r.t the team

Projects / Teams	I	II	III
A	2	3	1
B	1	2	3
C	2	1	3
D	3	1	2

Table 7:

Ranking Indices (R_2) of the team w.r.t the project

Projects / Teams	I	II	III
A	4	4	1
B	2	3	4
C	1	1	3
D	3	2	2

Table 8:

Composite matrix R_1R_2

Projects / Teams	I	II	III
A	8	12	1
B	2	6	12
C	2	1	9
D	9	2	4

To solve the above matrix, a dummy column has been added and the AP is solved by Hungarian method to get the optimal assignment.

Table 9:

Solution Matrix (a)

Projects / Teams	I	II	III	IV
A	6	11	[0]	0
B	[0]	5	11	0
C	0	[0]	8	0
D	7	1	3	[0]

Therefore optimal assignment is A→III, B→I, C→II, and D→IV.

Method 2:

Here the three criteria are –

c_1 → Cost, c_2→ Profit, c_3 → Efficiency

For cost criterion, ideal alternative A^* is –

$$\alpha_j^* = < (a_j^*, b_j^*, c_j^*)> = \left\langle \left[\min_i (a_{ij}), \max_i (b_{ij}), \max_i (c_{ij}) \right] \right\rangle$$

For benefit criteria (profit and efficiency), ideal alternative A^* is –

$$\alpha_j^* = < (a_j^*, b_j^*, c_j^*)> = \left\langle \left[\max_i (a_{ij}), \min_i (b_{ij}), \min_i (c_{ij}) \right] \right\rangle$$

Therefore cosine formula for similarity measure is –

$$WS_c\ (A_i, A^*) = \frac{\sum_{j=1}^{n} w_j [a_{ij} a_j^* + b_{ij} b_j^* + c_{ij} c_j^*]}{\sqrt{\sum (a_{ij}^2 + b_{ij}^2 + c_{ij}^2)} \sqrt{\sum (a_j^{*2} + b_j^{*2} + c_j^{*2})}}$$

where weights of the criteria c_1, c_2 and c_3 are w_1 = 0.35, w_2 = 0.40 and w_3 = 0.25 such that $\sum_{j=1}^{3} w_j = 1$.

Table 10:

Degree of similarity matrix (WS_c (A_i, A^*))

Projects / Teams	I	II	III
A	0.289	0.323	0.441
B	0.303	0.275	0.293
C	0.280	0.251	0.264
D	0.244	0.263	0.274

Table 11:

Ranking Indices (R_3) of the project w.r.t the team

Projects / Teams	I	II	III
A	3	2	1
B	1	3	2
C	1	3	2
D	3	2	1

Table 12:

Ranking Indices (R_4) of the team w.r.t the project

Projects / Teams	I	II	III
A	2	1	1
B	1	2	2
C	3	4	4
D	4	3	3

Table 13:

Composite matrix R_3R_4

Projects / Teams	I	II	III
A	6	2	1
B	1	6	4
C	3	12	8
D	12	6	3

Solving composite matrix by Hungarian method-

Table 14:

Projects / Teams	I	II	III	IV
A	5	0	0	0
B	0	4	3	0
C	2	10	7	0
D	11	4	2	0

Table 15:

Solution Matrix (b)

Projects / Teams	I	II	III	IV
A	7	[0]	0	2
B	[0]	2	1	0
C	2	8	5	[0]
D	11	2	[0]	0

Therefore optimal assignment is A→II, B→I, C→IV, and D→III.

4. Conclusion:-

This paper proposes two different approaches to solve MCAP. Both of them are simple but very efficient and have not been used earlier. The numerical example demonstrates the application and effectiveness of the methods with the incomplete and indeterminate information which exist commonly in real life situations.

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Taylor Series Approximation to Solve Neutrosophic Multiobjective Programming Problem

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Abstract. In this paper, Taylor series is used to solve neutrosophic multi-objective programming problem (NMOPP). In the proposed approach, the truth membership, Indeterminacy membership, falsity membership functions associated with each objective of multi-objective programming problems are transformed into a single objective linear programming problem by using a first order Taylor polynomial series. Finally, to illustrate the efficiency of the proposed method, a numerical experiment for supplier selection is given as an application of Taylor series method for solving neutrosophic multi-objective programming problem at end of this paper.

Keywords: Taylor series; Neutrosophic optimization; Multiobjective programming problem.

1 Introduction

In 1995,Smarandache [1] starting from philosophy (when he fretted to distinguish between *absolute truth* and *relative truth* or between *absolute falsehood* and *relative falsehood* in logics, and respectively between *absolute membership* and *relative membership* or *absolute non-membership* and *relative non-membership* in set theory) [1] began to use the non-standard analysis. Also, inspired from the sport games (winning, defeating, or tie scores), from votes (pro, contra, null/black votes), from positive/negative/zero numbers, from yes/no/NA, from decision making and control theory (making a decision, not making, or hesitating), from accepted/rejected/pending, etc. and guided by the fact that the law of excluded middle did not work any longer in the modern logics. [1] combined the non-standard analysis with a tri-component logic/set/probability theory and with philosophy .How to deal with all of them at once, is it possible to unity them? [1].

The words “neutrosophy” and “neutrosophic” were invented by F. Smarandache in his 1998 book [1]. Etymologically, “neutro-sophy” (noun) [French *neutre* < Latin *neuter*, neutral, and Greek *sophia*, skill / wisdom] means knowledge of neutral thought. While “neutrosophic” (adjective), means having the nature of, or having the characteristic of Neutrosophy.

Netrosophic theory means Neutrosophy applied in many fields in order to solve problems related to indeterminacy. Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. This theory considers every entity <A> together with its opposite or negation <antiA> and with their spectrum of neutralities <neutA> in between them (i.e. entities supporting neither <A> nor<antiA>). The <neutA> and <antiA> ideas together are referred to as <nonA>.

Neutrosophy is a generalization of Hegel's dialectics (the last one is based on <A> and <antiA> only). According to this theory every entity <A> tends to be neutralized and balanced by <antiA> and <nonA> entities - as a state of equilibrium. In a classical way <A>, <neutA>, <antiA> are disjoint two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that <A>, <neutA>, <antiA> (and <nonA> of course) have common parts two by two, or even all three of them as well. Hence, in one hand, the Neutrosophic Theory is based on the triad <A>, <neutA>, and <antiA>. In the other hand, Neutrosophic Theory studies the indeterminacy, labeled as I, with $I^n = I$ for $n \geq 1$, and $mI + nI = (m+n)I$, in neutrosophic structures developed in algebra, geometry, topology etc.

The most developed fields of Neutrosophic theory are Neutrosophic Set, Neutrosophic Logic, Neutrosophic Probability, and Neutrosophic Statistics - that started in 1995, and recently Neutrosophic Precalculus and

Neutrosophic Calculus, together with their applications in practice. Neutrosophic Set and Neutrosophic Logic are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and respectively intuitionistic fuzzy logic). In neutrosophic logic a proposition has a degree of truth (*T*), a degree of indeterminacy (*I*), and a degree of falsity (*F*), where *T,I,F* are standard or non-standard subsets of $]^-0, 1^+[$.
Multi-objective linear programming problem (MOLPP) a prominent tool for solving many real decision making problems like game theory, inventory problems, agriculture based management systems, financial and corporate planning, production planning, marketing and media selection, university planning and student admission, health care and hospital planning, air force maintenance units, bank branches etc.

Our objective in this paper is to propose an algorithm to the solution of neutrosophic multi-objective programming problem (NMOPP) with the help of the first order Taylor's theorem. Thus, neutrosophic multi-objective linear programming problem is reduced to an equivalent multi-objective linear programming problem. An algorithm is proposed to determine a global optimum to the problem in a finite number of steps. The feasible region is a bounded set. In the proposed approach, we have attempted to reduce computational complexity in the solution of (NMOPP). The proposed algorithm is applied to supplier selection problem .

The rest of this article is organized as follows. Section 2 gives brief some preliminaries. Section 3 describes the formation of the problem. Section 4 presents the implementation and validation of the algorithm with practical application. Finally, Section 5 presents the conclusion and proposals for future work.

2 Some preliminaries

Definition 1. [1] A triangular fuzzy number $\tilde{J}$ is a continuous fuzzy subset from the real line R whose triangular membership function $\mu_{\tilde{J}}(J)$ is defined by a continuous mapping from R to the closed interval [0,1], where

(1) $\mu_{\tilde{J}}(J)=0$ for all $J \in (-\infty, a_1]$,

(2) $\mu_{\tilde{J}}(J)$ is strictly increasing on $J \in [a_1, m]$,

(3) $\mu_{\tilde{J}}(J)=1$ for $J = m$,

(4) $\mu_{\tilde{J}}(J)$ is strictly decreasing on $J \in [m, a_2]$,

(5) $\mu_{\tilde{J}}(J)=0$ for all $J \in [a_2, +\infty)$.

This will be elicited by:

$$\mu_{\tilde{J}}(J)=\begin{cases} \dfrac{J-a_1}{m-a_1}, & a_1 \le J \le m, \\ \dfrac{a_2-J}{a_2-m}, & m \le J \le a_2, \\ 0, & otherwise. \end{cases} \quad (1)$$

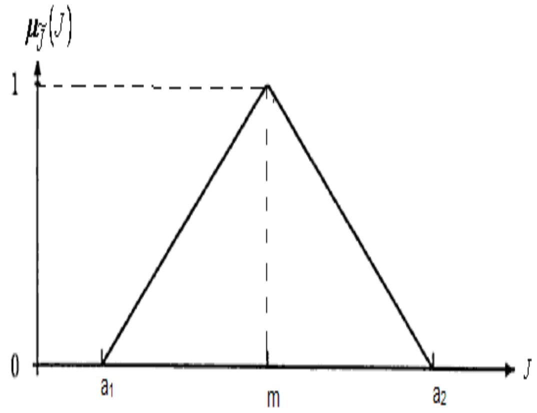

Figure 1: Membership Function of Fuzzy Number *J*.

where m is a given value and a_1 , a_2 denote the lower and upper bounds. Sometimes, it is more convenient to use the notation explicitly highlighting the membership function parameters. In this case, we obtain

$$\mu(J;a_1,m,a_2)=\text{Max}\left\{\text{Min}\left[\frac{J-a_1}{m-a_1},\frac{a_2-J}{a_2-m}\right],0\right\} \quad (2)$$

In what follows, the definition of the α-level set or α-cut of the fuzzy number $\tilde{J}$ is introduced.

Definition 2. [1] Let $X = \{x_1, x_2, ..., x_n\}$ be a fixed non-empty universe. An intuitionistic fuzzy set IFS *A* in *X* is defined as

$$A=\{\langle x,\mu_A(x),\upsilon_A(x)\rangle | x\in X\} \quad (3)$$

which is characterized by a membership function $\mu_A : X \to [0,1]$ and a non-membership function $\upsilon_A : X \to [0,1]$ with the condition $0 \le \mu_A(x)+\upsilon_A(x) \le 1$ for all $x \in X$ where μ_A and υ_A represent ,respectively, the degree of membership and non-membership of the element *x* to the set *A*. In addition, for each IFS *A* in X, $\pi_A(x)=1-\mu_A(x)-\upsilon_A(x)$ for all $x \in X$ is called the degree of hesitation of the element x to the set A . Especially, if $\pi_A(x) = 0,$, then the IFS A is degraded to a fuzzy set.

Definition 3. [4] The α-level set of the fuzzy parameters $\tilde{J}$ in problem (1) is defined as the ordinary set $L_\alpha(\tilde{J})$ for which the degree of membership function exceeds the level, α, $\alpha \in [0,1]$, where:

$$L_\alpha(\tilde{J}) = \left\{ J \in R \,\middle|\, \mu_{\tilde{J}}(J) \geq \alpha \right\} \tag{4}$$

For certain values α_j^* to be in the unit interval,

Definition 4. [1] Let X be a space of points (objects) and $x \in X$. A neutrosophic set A in X is defined by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. It has been shown in figure 2. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or real nonstandard subsets of $]0^-,1^+[$. That is $T_A(x):X \to]0^-,1^+[$, $I_A(x):X \to]0^-,1^+[$ and $F_A(x):X \to]0^-,1^+[$. There is not restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0^- \leq \sup T_A(x) \leq \sup I_A(x) \leq F_A(x) \leq 3^+$.
In the following, we adopt the notations $\mu_A(x)$, $\sigma_A(x)$ and $\nu_A(x)$ instead of $T_A(x)$, $I_A(x)$ and $F_A(x)$, respectively. Also we write SVN numbers instead of single valued neutrosophic numbers.

Definition 5. [10] Let X be a universe of discourse. A single valued neutrosophic set A over X is an object having the form
$A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle : x \in X\}$
where $\mu_A(x):X \to [0,1]$, $\sigma_A(x):X \to [0,1]$ and $\nu_A(x):X \to [0,1]$ with $0 \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 3$ for all $x \in X$. The intervals $\mu_A(x)$, $\sigma_A(x)$ and $\nu_A(x)$ denote the truth- membership degree, the indeterminacy-membership degree and the falsity membership degree of x to A, respectively.
For convenience, a SVN number is denoted by $A = (a,b,c)$, where $a,b,c \in [0,1]$ and $a+b+c \leq 3$.

Definition 6

Let $\tilde{J}$ be a neutrosophic triangular number in the set of real numbers R, then its truth-membership function is defined as

$$T_{\tilde{J}}(J) = \begin{cases} \dfrac{J - a_1}{a_2 - a} & a_1 \leq J \leq a_2, \\ \dfrac{a_2 -}{a_3 - a} & a_2 \leq J \leq a_3, \\ 0, & otherwise. \end{cases} \tag{5}$$

its indeterminacy-membership function is defined as

$$I_{\tilde{J}}(J) = \begin{cases} \dfrac{J - b_1}{b_2 - b} & b_1 \leq J \leq b_2, \\ \dfrac{b_2 -}{b_3 - b} & b_2 \leq J \leq b_3, \\ 0, & otherwise. \end{cases} \tag{6}$$

and its falsity-membership function is defined as

$$F_{\tilde{J}}(J) = \begin{cases} \dfrac{J - \ _1}{c_2 - c}, & c_1 \leq J \leq c_2, \\ \dfrac{c_2 -}{c_3 - c} & c_2 \leq J \leq c_3, \\ 1, & otherwise. \end{cases} \tag{7}$$

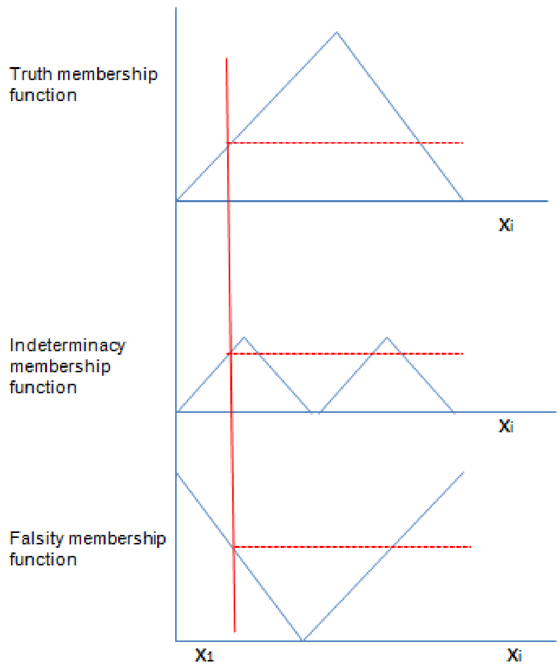

Figure 2: Neutrosophication process [11]

3 Formation of The Problem

The multi-objective linear programming problem and the multi- objective neutrosophic linear programming problem are described in this section.

A. Multi-objective Programming Problem (MOPP)

In this paper, the general mathematical model of the MOPP is as follows[6]:

$$\min/\max\ \left[z_1(x_1,\ldots,x_n), z_2(x_1,\ldots,x_n),\ldots,z_p(x_1,\ldots,x_n)\right] \tag{8}$$

$$subject\ to\ \ x \in S, x$$

$$S = \left\{ x \in R^n \left| AX \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} b,\ \ X \ge 0. \right. \right\} \tag{9}$$

B. Neutrosophic Multi-objective Programming Problem (NMOPP)

If an imprecise aspiration level is introduced to each of the objectives of MOPP, then these neutrosophic objectives are termed as neutrosophic goals.

Let $z_i \in \left\lfloor z_i, z_i^U \right\rceil$ denote the imprecise lower and upper bounds respectively for the i^{th} neutrosophic objective function.

For maximizing objective function, the truth membership, indeterminacy membership, falsity membership functions can be expressed as follows:

$$\mu_i^I(z_i) = \begin{cases} 1, & if\ \ z_i \ge z_i^U, \\ \dfrac{z_i - z^L}{z_i^U - z_i}, & if\ \ z_i^L \le z_i \le z_i^U, \\ 0, & if\ \ z_i \le z^L \end{cases} \tag{10}$$

$$\sigma_i^I(z_i) = \begin{cases} 1, & if\ \ z_i \ge z_i^U, \\ \dfrac{z_i - z^L}{z_i^U - z_i}, & if\ \ z_i^L \le z_i \le z_i^U, \\ 0, & if\ \ z_i \le z^L \end{cases} \tag{11}$$

$$\upsilon_i^I(z_i) = \begin{cases} 0, & if\ \ z_i \ge z_i^U, \\ \dfrac{z_i - z^L}{z_i^U - z_i}, & if\ \ z_i^L \le z_i \le z_i^U, \\ 1, & if\ \ z_i \le z^L \end{cases} \tag{12}$$

for minimizing objective function, the truth membership, Indeterminacy membership, falsity membership functions can be expressed as follows:

$$\mu_i^I(z_i) = \begin{cases} 1, & if\ \ z_i \le z_i^L, \\ \dfrac{z_i^U - z_i}{z_i^U - z_i}, & if\ \ z_i^L \le z_i \le z_i^U, \\ 0, & if\ \ z_i \ge z_i^U \end{cases} \tag{13}$$

$$\sigma_i^I(z_i) = \begin{cases} 1, & if\ \ z_i \le z_i^L, \\ \dfrac{z_i^U - z_i}{z_i^U - z_i}, & if\ \ z_i^L \le z_i \le z_i^U, \\ 0, & if\ \ z_i \ge z_i^U \end{cases} \tag{14}$$

$$\upsilon_i^I(z_i) = \begin{cases} 0, & if\ \ z_i \le z_i^L, \\ \dfrac{z_i^U - z_i}{z_i^U - z_i}, & if\ \ z_i^L \le z_i \le z_i^U, \\ 1, & if\ \ z_i \ge z_i^U \end{cases} \tag{15}$$

4 Algorithm for Neutrosophic Multi-Objective Programming Problem

The computational procedure and proposed algorithm of presented model is given as follows:

Step 1. Determine $x_i^* = \left(x_{i1}^*, x_{i2}^*, \ldots, x_{in}^*\right)$ that is used to maximize or minimize the i^{th} truth membership function $\mu_i^I(X)$, the indeterminacy membership $\sigma_i^I(X)$, and the falsity membership functions $\upsilon_i^I(X)$. $i=1,2,..,p$ and n is the number of variables.

Step 2. Transform the truth membership, indeterminacy membership, falsity membership functions by using first-order Taylor polynomial series

$$\mu_i^I(x) \cong \mu_i^I(x_i^*) + \sum_{j=1}^{n}(x_j - x_{ij})\frac{\partial \mu^I(x_i^*)}{\partial x_j} \quad (16)$$

$$\sigma_i^I(x) \cong \sigma_i^I(x_i^*) + \sum_{j=1}^{n}(x_j - x_{ij})\frac{\partial \sigma^I(x_i^*)}{\partial x_j} \quad (17)$$

$$\upsilon_i^I(x) \cong \upsilon_i^I(x_i^*) + \sum_{j=1}^{n}(x_j - x_{ij})\frac{\partial \upsilon^I(x_i^*)}{\partial x_j} \quad (18)$$

Step 3. Find satisfactory $x_i^* = (x_{i1}^*, x_{i2}^*, ..., x_{in}^*)$ by solving the reduced problem to a single objective for the truth membership, indeterminacy membership, falsity membership functions respectively.

$$p(x) = \sum_{i=1}^{p}\left[\mu_i^I(x_i^*) + \sum_{j=1}^{n}(x_j - x_{ij}^*)\frac{\partial \mu_i^I(x_i^*)}{\partial x_j}\right]$$

$$q(x) = \sum_{i=1}^{p}\left[\sigma_i^I(x_i) + \sum_{j=1}^{n}(x_j - x_{ij}^*)\frac{\partial \sigma_i^I(x_i^*)}{\partial x_j}\right] \quad (19)$$

$$h(x) = \sum_{i=1}^{p}\left[\upsilon_i^I(x_i) + \sum_{j=1}^{n}(x_j - x_{ij}^*)\frac{\partial \upsilon_i^I(x_i^*)}{\partial x_j}\right]$$

Thus neutrosophic multiobjective linear programming problem is converted into a new mathematical model and is given below:

Maximize or Minimize *p(x)*
Maximize or Minimize *q(x)*
Maximize or Minimize *h(x)*

Where $\mu_i^I(X)$, $\sigma_i^I(X)$ and $\upsilon_i^I(X)$ calculate using equations (10), (11), and (12) or equations (13), (14), and (15) according to type functions maximum or minimum respectively.

4.1 Illustrative Example

A multi-criteria supplier selection is selected from [2]. For supplying a new product to a market assume that three suppliers should be managed. The purchasing criteria are net price, quality and service. The capacity constraints of suppliers are also considered.

It is assumed that the input data from suppliers' performance on these criteria are not known precisely. The neutrosophic values of their cost, quality and service level are presented in Table 1.

The multi-objective linear formulation of numerical example is presented as :

$$\begin{aligned}
&\min\ z_1 = 5x_1 + 7x_2 + 4x_3,\\
&\max\ z_2 = 0.80x_1 + 0.90x_2 + 0.85x_3,\\
&\max\ z_3 = 0.90x_1 + 0.80x_2 + 0.85x_3,\\
&s.t.:\\
&x_1 + x_2 + x_3 = 800,\\
&x_1 \le 400,\\
&x_2 \le 450,\\
&x_3 \le 450,\\
&x_i \ge 0, \quad = 1,2,3.
\end{aligned}$$

Table 1: Suppliers quantitative information

	Z1 Cost	Z2Quality (%)	Z3 Service (%)	Capacity
Supplier 1	5	0.80	0.90	400
Supplier 2	7	0.90	0.80	450
Supplier 3	4	0.85	0.85	450

The truth membership, Indeterminacy membership, falsity membership functions were considered to be neutrosophic triangular. When they depend on three scalar parameters (a1,m,a2). z_1 depends on neutrosophic aspiration levels (3550,4225,4900), when z_2 depends on neutrosophic aspiration levels (660,681.5,702.5), and z3 depends on neutrosophic aspiration levels (657.5,678.75,700).

The truth membership functions of the goals are obtained as follows:

$$\mu_1^I(z_1) = \begin{cases} 0, & if \quad z_1 \le 3550, \\ \dfrac{4225 - z_1}{4225 - 3550}, & if \quad 3550 \le z_1 \le 4225, \\ \dfrac{4900 - z_1}{4900 - 4225}, & if \quad 4225 \le z_1 \le 4900, \\ 0, & if \quad z_1 \ge 4900 \end{cases}$$

$$\mu_2^I(z_2) = \begin{cases} 0, & if \quad z_2 \ge 702.5, \\ \dfrac{z_2 - 681.5}{702.5 - 681.5}, & if \quad 681.5 \le z_2 \le 702.5, \\ \dfrac{z_2 - 660}{681.5 - 660}, & if \quad 660 \le z_2 \le 681.5, \\ 0, & if \quad z_2 \le 660. \end{cases}$$

$$\mu_3^I(z_3)=\begin{cases}0, & if \quad z_3 \geq 700,\\ \dfrac{z_3-678.75}{700-678.75}, & if \quad 678.75 \leq z_3 \leq 700,\\ \dfrac{z_3-657.5}{678.75-657.5}, & if \quad 657.5 \leq z_3 \leq 678.75,\\ 0, & if \quad z_3 \leq 657.5.\end{cases}$$

If

$$\mu_1^I(z_1)=\max\left(\min\left(\frac{4225-(5x_1+7x_2+4x_3)}{675},\frac{4900-(5x_1+7x_2+4x_3)}{675},0\right)\right)$$

$$\mu_2^I(z_2)=\min(\max(\frac{(0.8x_1+0.9x_2+0.85x_3)-681.5}{21},\frac{(0.8x_1+0.9x_2+0.85x_3)-660}{21},1))$$

$$\mu_3^I(z_3)=\min(\max(\frac{(0.9x_1+0.8x_2+0.85x_3)-678.75}{21.25},\frac{(0.9x_1+0.8x_2+0.85x_3)-657.5}{21.25},1))$$

Then

$\mu_1^{I*}(350,0,450)$, $\mu_2^{I*}(0,450,350)$, $\mu_3^{I*}(400,0,400)$

The truth membership functions are transformed by using first-order Taylor polynomial series

$$\widehat{\mu}_1^I(x)=\mu_1^I(350,0,450)+\left[(x_1-350)\frac{\partial\mu_1^I(350,0,450)}{\partial x_1}\right]+\left[(x_2-0)\frac{\partial\mu_1^I(350,0,450)}{\partial x_2}\right]+\left[(x_3-450)\frac{\partial\mu_1^I(350,0,450)}{\partial x_3}\right]$$

$$\widehat{\mu}_1^I(x) \approx -0.00741x_1-0.0104x_2-0.00593x_3+5.2611$$

In the similar way, we get

$$\widehat{\mu}_2^I(x) \approx 0.0381x_1+0.0429x_2+0.0405x_3-33.405$$

$$\widehat{\mu}_3^I(x) \approx 0.042x_1+0.037x_2+0.0395x_3-32.512$$

The the p(x) is

$$p(x)=\widehat{\mu}_1^I(x)+\widehat{\mu}_2^I(x)+\widehat{\mu}_3^I(x)$$

$$p(x) \approx 0.07259x_1+0.0695x_2+0.0741x_3-60.6559$$

$s.t.:$

$x_1+x_2+x_3=800,$

$x_1 \leq 400,$

$x_2 \leq 450,$

$x_3 \leq 450,$

$x_i \geq 0, \ i=1,2,3.$

The linear programming software LINGO 15.0 is used to solve this problem. The problem is solved and the optimal solution for the truth membership model is obtained is as follows: (x_1, x_2, x_3) = (350,0,450) z_1=3550, z_2=662. 5, z_3=697. 5.

The truth membership values are $\mu_1=1$, $\mu_2=0.1163$, $\mu_3=0.894$. The truth membership function values show that both goals z_1 , z_3 and z_2 are satisfied with 100% , 11.63% and 89.4% respectively for the obtained solution which is x1=350; x2= 0, x3=450.

In the similar way, we get $\sigma_i^I(X)$, *q(x)* Consequently we get the optimal solution for the Indeterminacy membership model is obtained is as follows: (x_1,x_2,x_3)=(350,0,450) z_1=3550, z_2=662.5, z_3=697.5

and the Indeterminacy membership values are $\mu_1=1$, $\mu_2=0.1163$, $\mu_3=0.894$. The Indeterminacy membership function values show that both goals z_1 , z_3 and z_2 are satisfied with 100% , 11.63% and 89.4% respectively for the obtained solution which is x1=350; x2= 0, x3=450.

In the similar way, we get $\upsilon_i^I(X)$ *and h(x)* Consequently we get the optimal solution for the falsity membership model is obtained is as follows: (x_1,x_2,x_3)=(350,0,450) z_1=3550, z_2=662.5, z_3=697.5

and the falsity membership values are $\mu_1=0$, $\mu_2=0.8837$, $\mu_3=0.106$. The falsity membership function values show that both goals z_1 , z_3 and z_2 are satisfied with 0% , 88.37% and 10.6% respectively for the obtained solution which is x1=350; x2= 0, x3=450.

5 Conclusions and Future Work

In this paper, we have proposed a solution to Neutrosophic Multiobjective programming problem (NMOPP). The truth membership, Indeterminacy membership, falsity membership functions associated with each objective of the problem are transformed by using the first order Taylor polynomial series. The neutrosophic multi-objective programming problem is reduced to an equivalent multiobjective programming problem by the proposed method. The solution obtained from this method is very near to the solution of MOPP. Hence this method gives a more accurate solution as compared with other methods. Therefore the complexity in solving NMOPP, has reduced to easy computation. In the future studies, the proposed algorithm can be solved by metaheuristic algorithms.

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Decision Making Based on Some similarity Measures under Interval Rough Neutrosophic Environment

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Abstract: This paper is devoted to propose cosine, Dice and Jaccard similarity measures of interval rough neutrosophic set and interval neutrosophic mean operator. Some of the properties of the proposed similarity measures have been established. We have proposed multi attribute decision making approaches based on proposed simlarity measures. To demonstrate the applicability and efficiency of the proposed approaches, a numerical example is solved and comparision has been done among the proposed the apprqaches.

Keywords: Tangent similarity measure, Single valued neutrosophic set, Cosine similarity measure, Medical diagnosis

1 Introduction

The concept of neutrosophic set was grounded by one of the greatest mathematician and philosopher Smarandache [1, 2, 3, 4, 5]. The root of neutrosophic set is the neutrosophy, a new branch of philosophy initiated by Smarandache [1]. Neutrosophy studies the ideas and notions that are neutral, indeterminate, unclear, vague, ambiguous, incomplete, contradictory, etc. Inherently, neutrosophic set is capable of dealing with uncertainty, indeterminate and inconsistent information. Smarandache endeavored to propagate the concept of neutrosophic set in all branches of sciences, social sciences and humanities. To use neutrosophic sets in practical fields such as real scientific and engineering applications, Wang et al.[6] extended the concept of neutrosophic set to single valued neutrosophic sets (SVNSs) and studied the set theoretic operators and various properties of SVNSs. Recently, single valued neutrosophic set has caught much attention to the researcher on various topics such as artificial intelligence [7], conflict resolution [8], education [9, 10], decision making [11-27] medical diagnosis [28], social problems [29, 30], etc. Smarandache's original ideas blossomed into a comprehensive corpus of methods and tools for dealing with membership degrees of truth, falsity, indeterminacy and non-probabilistic uncertainty. In essence, the basic concept of neutrosophic set is a generalization of classical set or crisp set [31, 32], fuzzy set [33], intuitionistic fuzzy set [34]. The field has experienced an enormous development, and Smarandache's seminal concept of neutrosophic set [1] has naturally evolved in different directions. Different sets were quickly proposed in the literature such as neutrosophic soft set [35], weighted neutrosophic soft sets [36], generalized neutrosophic soft set [37], Neutrosophic parametrized soft set [38], Neutrosophic soft expert sets [39, 40], neutrosophic refined sets [41, 42]. Neutrosophic soft multi-set [43], neutrosophic bipolar set (44), neutrosophic cubic set (45, 46), neutrosophic complex set (47), rough neutrosophic set (48, 49), interval rough neutrosophic set [50], Interval-valued neutrosophic soft rough sets [51, 52], etc.

Broumi et al. [48, 49] recently proposed new hybrid intelligent structure namely, rough neutrosophic set combing the concept of rough set theory [53] and the concept of neutrosophic set theory to deal with uncertainty and incomplete information. Rough neutrosophic set [48, 49] is the generalization of rough fuzzy sets [54], [55] and rough intuitionistic fuzzy sets [56]. Several studies of rough neutrosophic sets have been reported in the literature. Mondal and Pramanik [57] applied the concept of rough neutrosophic set in multi-attribute decision making based on grey relational analysis. Pramanik and Mondal [58] presented cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis. Pramanik and Mondal [59] also proposed some rough neutrosophic similarity measures namely Dice and Jaccard similarity measures of rough neutrosophic environment. Mondal and Pramanik [60] proposed rough neutrosophic multi attribute decision making based on rough score accuracy function. Pramanik and Mondal [61] presented cotangent similarity measure of rough neutrosophic sets and its application to medical diagnosis.

In 2015, Broumi and Smarandache [50] combined the concept of rough set theory [53] and interval neutrosophic set theory [62] and defined interval rough neutrosophic set.

In this paper, we develop some similarity measures namely, cCosine, Dice, Jaccard similarity measures based on interval rough neutrosophic sets [50].
Rest of the paper is organized in the following way. Section 2 describes preliminaries of neutrosophic sets and rough neutrosophic sets and interval rough neutrosophic sets. Section 3 presents definitions and propositions of the proposed functions. Section 4 is devoted to present multi attribute decision-making method based on similarity functions. In Section 5, we provide a numerical example of the proposed approaches. Section 6 presents the comparision of results of the three proposed approaches. Finally section 7 presents concluding remarks and future scopes of research.

2 Mathematical preliminaries

2.1 Neutrosophic set

Definition 2.1[1]
Let U be an universe of discourse. Then the neutrosophic set A can be presented of the form:
$A = \{< x{:}T_A(x), I_A(x), F_A(x)>, x \in U\}$, where the functions T, I, F: U→ $]^-0,1^+[$ define respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element $x \in U$ to the set A satisfying the following the condition.

$$^-0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+ \quad (1)$$

For two netrosophic sets (NSs), $A_{NS} = \{<x: T_A(x), I_A(x), F_A(x) > | x \in X\}$ and $B_{NS} = \{< x, T_B(x), I_B(x), F_B(x) > | x \in X\}$ the two relations are defined as follows:

(1) $A_{NS} \subseteq B_{NS}$ if and only if $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, $F_A(x) \ge F_B(x)$

(2) $A_{NS} = B_{NS}$ if and only if $T_A(x) = T_B(x)$, $I_A(x) = I_B(x)$, $F_A(x) = F_B(x)$

2.2 Single valued neutrosophic set (SVNS)

Definition 2.2 [6]

From philosophical point of view, the neutrosophic set assumes the value from real standard or non-standard subsets of $]^-0, 1^+[$. So instead of $]^-0, 1^+[$ one needs to take the interval [0, 1] for technical applications, because $]^-0, 1^+[$ will be difficult to apply in the real applications such as scientific and engineering problems. Wang et. al [6] introduced single valued neutrosophic set (SVNS).

Let X be a space of points with generic element $x \in X$. A SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity membership function $F_A(x)$, for each point $x \in X$, $T_A(x), I_A(x), F_A(x) \in [0, 1]$. When X is continuous, a SVNS A can be written as follows:

$$A = \int_X \frac{< T_A(x), I_A(x), F_A(x) >}{x} : x \in X$$

When X is discrete, a SVNS A can be written as follows:

$$A = \sum_{i=1}^{n} \frac{< T_A(x_i), I_A(x_i), F_A(x_i) >}{x_i} : x_i \in X$$

For two SVNSs , $A_{SVNS} = \{<x: T_A(x), I_A(x), F_A(x)> | x \in X\}$ and $B_{SVNS} = \{<x, T_B(x), I_B(x), F_B(x)> | x \in X\}$ the two relations are defined as follows:

1. $A_{SVNS} \subseteq B_{SVNS}$ if and only if $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, $F_A(x) \ge F_B(x)$
2. $A_{SVNS} = B_{SVNS}$ if and only if $T_A(x) = T_B(x)$, $I_A(x) = I_B(x)$, $F_A(x) = F_B(x)$ for any $x \in X$

2.3 Interval neutrosophic sets

Definition 2.3.1 [62]

Let X be a space of points (bjects) with generic element $x \in X$. An interval neutrosophic set (INS) A in X is characterized by truth-membership function $T_A(x)$, indeterminacy-membership function $I_A(x)$, and falsity-membership function $F_A(x)$. For each point $x \in X$., we have, $T_A(x), I_A(x), F_A(x) \in [0, 1]$.

For two IVNS,
$A_{INS} =$
$\{< x, ([T_A^L(x), T_A^U(x)], [I_A^L(x), I_A^U(x)], [F_A^L(x), F_A^U(x)]) > | x \in X\}$ and
$B_{INS} =$
$\{<x, ([T_B^L(x), T_B^U(x)], [I_B^L(x), I_B^U(x)], [F_B^L(x), F_B^U(x)]) > | x \in X\}$ the two relations are defined as follows:

1. $A_{INS} \subseteq B_{INS}$ if and only if $T_A^L \le T_B^L$, $T_A^U \le T_B^U$; $I_A^L \ge I_B^L$, $I_A^U \ge I_B^U$; $F_A^L \ge F_B^L$, $F_A^U \ge F_B^U$
2. $A_{INS} = B_{INS}$ if and only if $T_A^L = T_B^L$, $T_A^U = T_B^U$; $I_A^L = I_B^L$, $I_A^U = I_B^U$; $F_A^L = F_B^L$, $F_A^U = F_B^U$
for all $x \in X$

2.4 Rough neutrosophic set

Definition 2.4.1 [48, 49]: Let Z be a non-zero set and R be an equivalence relation on Z. Let P be neutrosophic set in Z with the membership function T_P, indeterminacy function I_P and non-membership function F_P. The lower and the upper approximations of *P* in the approximation (Z, R) denoted by $\underline{N}(P)$ and $\overline{N}(P)$ are respectively defined as follows:

$$\underline{N}(P) = \left\langle \begin{matrix} < x, T_{\underline{N}(P)}(x), I_{\underline{N}(P)}(x), F_{\underline{N}(P)}(x) > / \\ z \in [x]_R, x \in Z \end{matrix} \right\rangle \quad (2)$$

$$\overline{N}(P) = \left\langle \begin{array}{l} < x, T_{\overline{N}(P)}(x), I_{\overline{N}(P)}(x), F_{\overline{N}(P)}(x) > / \\ z \in [x]_R, x \in Z \end{array} \right\rangle \quad (3)$$

Where, $T_{\underline{N}(P)}(x) = \wedge_z \in [x]_R\, T_P(z)$,

$I_{\underline{N}(P)}(x) = \wedge_z \in [x]_R\, I_P(z)$, $F_{\underline{N}(P)}(x) = \wedge_z \in [x]_R\, F_P(z)$,

$T_{\overline{N}(P)}(x) = \vee_z \in [x]_R\, T_P(z)$, $I_{\overline{N}(P)}(x) = \vee_z \in [x]_R\, T_P(z)$,

$F_{\overline{N}(P)}(x) = \vee_z \in [x]_R\, I_P(z)$

So, $0 \le T_{\underline{N}(P)}(x) + I_{\underline{N}(P)}(x) + F_{\underline{N}(P)}(x) \le 3$

$0 \le T_{\overline{N}(P)}(x) + I_{\overline{N}(P)}(x) + F_{\overline{N}(P)}(x) \le 3$

The symbols $\vee$ and $\wedge$ denote "max" and "min'' operators respectively. $T_P(z)$, $I_P(z)$ and $F_P(z)$ are the membership, indeterminacy and non-membership of z with respect to P. It is easy to see that $\underline{N}(P)$ and $\overline{N}(P)$ are two neutrosophic sets in Z.

Thus NS mapping $\underline{N}$, $\overline{N}$: N(Z) $\rightarrow$ N(Z) are, respectively, referred to as the lower and upper rough NS approximation operators, and the pair $(\underline{N}(P), \overline{N}(P))$ is called the rough neutrosophic set in (Z, R).

From the above definition, it is seen that $\underline{N}(P)$ and $\overline{N}(P)$ have constant membership on the equivalence clases of R if $\underline{N}(P) = \overline{N}(P)$; .e. $T_{\underline{N}(P)}(x) = T_{\overline{N}(P)}(x)$,

$I_{\underline{N}(P)}(x) = I_{\overline{N}(P)}(x)$, $F_{\underline{N}(P)}(x) = F_{\overline{N}(P)}(x)$

for any x belongs to Z.

P is said to be a definable neutrosophic set in the approximation (Z, R). It can be easily proved that zero neutrosophic set (0_N) and unit neutrosophic sets (1_N) are definable neutrosophic sets.

2.5 Interval neutrosophic rough sets [50]

Interval neutrosophic rough set [50] is the hybrid structure of rough sets and interval neutrosophic sets. According to Broumi and Smarandache [50] interval neutrosophic rough set is the generalizations of interval valued intuitionistic fuzzy rough set [63].

Definition 2.5.1 [53]

Let R be an equivalence relation on the universal set U. Then the pair (U, R) is called a Pawlak approximation space [5, 6]. An equivalence class of R containing x will be denoted by $[x]_R$ for X ⊆ U, the lower and upper approximation of X with respect to (U, R) are denoted by respectively R^*X and R_*X and are defined by

$R^*X = \{x \in U: [x]_R \subseteq X\}$,

$R_*X = \{x \in U: [x]_R \cap X \neq \emptyset\}$.

Now if $R^*X = R_*X$, then X is called definable; otherwise X is called a rough set.

Definition 2.5.2 [50]

Let U be a universe and X, a rough set in U. An intuitionistic fuzzy rough set A in U is characterized by a membership function μ_A:U→ [0, 1] and non-membership function ν_A: U→ [0, 1] such that $\mu_A(\underline{R}X) = 1$ and $\nu_A(\underline{R}X) = 0$

ie, $[\mu_A(x), \nu_A(x)] = [1, 0]$ if $x \in (\underline{R}X)$ and $\mu_A(U - \overline{R}X) = 0$

$\nu_A(U - \overline{R}X) = 1$

ie, $0 \le [\mu_A(\overline{R}X - \underline{R}X) + \nu_A(\overline{R}X - \underline{R}X)] \le 1$

2.5.1 Basic concept of rough approximations of an interval valued neutrosophic set and their properties

Definition 2.5.3 [50]

Assume that, (U, R) be a Pawlak approximation space, for an interval neutrosophic set

$$A = \{< x, [T_A^L(x), T_A^U(x)], [I_A^L(x), I_A^U(x)], [F_A^L(x), F_A^U(x)] > \mid x \in U\}.$$

The lower approximation $\underline{A}_R$ and the upper approximation $\overline{A}_R$ of A in the Pawlak approximation space (U, R) are expressed as follows:

$$\begin{aligned}\underline{A}_R = \{< x, &[\wedge_{y\in[x]_R}\{T_A^L(y)\}, \wedge_{y\in[x]_R}\{T_A^U(y)\}],\\ &[\vee_{y\in[x]_R}\{I_A^L(y)\}, \vee_{y\in[x]_R}\{I_A^U(y)\}],\\ &[\vee_{y\in[x]_R}\{F_A^L(y)\}, \vee_{y\in[x]_R}\{F_A^U(y)\}] > \mid x \in U\}\end{aligned}$$

$$\begin{aligned}\overline{A}_R = \{< x, &[\vee_{y\in[x]_R}\{T_A^L(y)\}, \vee_{y\in[x]_R}\{T_A^U(y)\}],\\ &[\wedge_{y\in[x]_R}\{I_A^L(y)\}, \wedge_{y\in[x]_R}\{I_A^U(y)\}],\\ &[\wedge_{y\in[x]_R}\{F_A^L(y)\}, \wedge_{y\in[x]_R}\{F_A^U(y)\}] > \mid x \in U\}\end{aligned}$$

The symbols $\wedge$ and $\vee$ indicate "min" and "max" operators respectively. R denotes an equivalence relation for interval neutrosophic set A. Here $[x]_R$ is the equivalence class of the element x. It is obvious that

$[\wedge_{y\in[x]_R}\{T_A^L(y)\}, \wedge_{y\in[x]_R}\{T_A^U(y)\}] \subset [0,1]$

$[\vee_{y\in[x]_R}\{I_A^L(y)\}, \vee_{y\in[x]_R}\{I_A^U(y)\}] \subset [0,1]$

$[\vee_{y\in[x]_R}\{F_A^L(y)\}, \vee_{y\in[x]_R}\{F_A^U(y)\}] \subset [0,1]$

and

$$0 \le \wedge_{y\in[x]_R}\{T_A^U(y)\}] + \vee_{y\in[x]_R}\{I_A^U(y)\}] + \vee_{y\in[x]_R}\{F_A^U(y)\}] \le 3$$

Then $\underline{A}_R$ is an interval neutrosophic set (INS)

Similarly, we have

$[\vee_{y\in[x]_R}\{T_A^L(y)\}, \vee_{y\in[x]_R}\{T_A^U(y)\}] \subset [0,1]$

$[\wedge_{y\in[x]_R}\{I_A^L(y)\}, \wedge_{y\in[x]_R}\{I_A^U(y)\}] \subset [0,1]$

$[\wedge_{y\in[x]_R}\{F_A^L(y)\}, \wedge_{y\in[x]_R}\{F_A^U(y)\}] \subset [0,1]$

and

$0 \le \vee_{y\in[x]_R}\{T_A^U(y)\}] + \wedge_{y\in[x]_R}\{I_A^U(y)\}] +$
$\wedge_{y\in[x]_R}\{F_A^U(y)\}] \le 3$

Then $\overline{A}_R$ is an interval neutrosophic set.

If $\underline{A}_R = \overline{A}_R$ then A is a definable set, otherwise A is an interval valued neutrosophic rough set. Here, $\underline{A}_R$ and $\overline{A}_R$ are called the lower and upper approximations of interval neutrosophic set with respect to approximation space (U, R) respectively. $\underline{A}_R$ and $\overline{A}_R$ are simply denoted by $\underline{A}$ and $\overline{A}$ respectively.

Proposition1 [50]: Let A and B be two interval neutrosophic sets and $\underline{A}$ and $\overline{A}$ the lower and upper approximation of interval neutrosophic set A with respect to approximation space (U, R) respectively. $\underline{B}$ and $\overline{B}$ are the lower and upper approximation of interval neutrosophic set B with respect to approximation space (U, R), respectively. Then the following relations hold good.

1. $\underline{A} \subseteq A \subseteq \overline{A}$
2. $\overline{A \cup B} = \overline{A} \cup \overline{B}$ and $\underline{A \cap B} = \underline{A} \cap \underline{B}$
3. $\overline{A \cap B} = \overline{A} \cap \overline{B}$ and $\underline{A \cup B} = \underline{A} \cup \underline{B}$
4. $\overline{\overline{A}} = \underline{\overline{A}} = \overline{A}$ and $\underline{\underline{A}} = \overline{\underline{A}} = \underline{A}$
5. $\underline{U} = U$ and $\overline{\phi} = \phi$
6. If $A \subseteq B$ then, $\underline{A} \subseteq \underline{B}$ and $\overline{A} \subseteq \overline{B}$
7. $\underline{A^c} = \overline{A}^c$ and $\overline{A^c} = \underline{A}^c$

Definition2.5.4 [50]

Assume that, (U, R) be a Pawlak approximation space and A and B are two interval neutrosophic sets over U.

If $\underline{A} = \underline{B}$ then A and B are called interval neutrosophic lower rough equal. If $\overline{A} = \overline{B}$, then A and B are called interval neutrosophic upper rough equal.

If $\underline{A} = \underline{B}$, $\overline{A} = \overline{B}$, then A and B are called interval neutrosophic rough equal.

Proposition2 [50]

Assume that (U, R) be a Pawlak approximation space and A and B two interval neutrosophic sets over U. then

1. $\underline{A} = \underline{B} \Rightarrow \underline{A \cap B} = \underline{A}$ and $\underline{A \cap B} = \underline{B}$
2. $\overline{A} = \overline{B} \Rightarrow \overline{A \cup B} = \overline{A}$ and $\overline{A \cup B} = \overline{B}$
3. $\overline{A} = \overline{A^c}$ and $\overline{B} = \overline{B^c} \Rightarrow \overline{A \cup B} = \overline{A^c \cup B^c}$
4. $\overline{A} = \overline{A^c}$ and $\overline{B} = \overline{B^c} \Rightarrow \underline{A \cap B} = \underline{A^c \cap B^c}$
5. $A \subseteq B$ and $\underline{B} = \underline{\phi}$ then $\underline{A} = \underline{\phi}$
6. $A \subseteq B$ and $\underline{B} = \underline{U}$ then $\underline{A} = \underline{U}$
7. $\underline{B} = \underline{\phi}$ and $\underline{A} = \underline{\phi}$ then $\underline{A \cap B} = \underline{\phi}$
8. $\overline{A} = \overline{U}$ and $\overline{B} = \overline{U} \Rightarrow \overline{A \cup B} = \overline{U}$
9. $\overline{A} = \overline{U} \Rightarrow A = B$
10. $\overline{A} = \overline{\phi} \Rightarrow A = \phi$

3. Cosine, Dice, Jaccard similarity measures of interval rough neutrosophic environment

Cosine, Dice and Jaccard similarity measure are proposed in interval rough neutrosophic environment in the following subsections.

3.1 Cosine similarity measure of interval rough neutrosophic environment

Definition 3.1.1: Assume that there are two interval rough neutrosophic sets

$$A = \left\langle \begin{pmatrix} \{[\underline{T}_A(x_i)]^L, [\underline{T}_A(x_i)]^U\}, \\ \{[\underline{I}_A(x_i)]^L, [\underline{I}_A(x_i)]^U\}, \\ \{[\underline{F}_A(x_i)]^L, [\underline{F}_A(x_i)]^U\} \end{pmatrix}, \begin{pmatrix} \{[\overline{T}_A(x_i)]^L, [\overline{T}_A(x_i)]^U\}, \\ \{[\overline{I}_A(x_i)]^L, [\overline{I}_A(x_i)]^U\}, \\ \{[\overline{F}_A(x_i)]^L, [\overline{F}_A(x_i)]^U\} \end{pmatrix} \right\rangle \text{ and}$$

$$B = \left\langle \begin{pmatrix} \{[\underline{T}_B(x_i)]^L, [\underline{T}_B(x_i)]^U\}, \\ \{[\underline{I}_B(x_i)]^L, [\underline{I}_B(x_i)]^U\}, \\ \{[\underline{F}_B(x_i)]^L, [\underline{F}_B(x_i)]^U\} \end{pmatrix}, \begin{pmatrix} \{[\overline{T}_B(x_i)]^L, [\overline{T}_B(x_i)]^U\}, \\ \{[\overline{I}_B(x_i)]^L, [\overline{I}_B(x_i)]^U\}, \\ \{[\overline{F}_B(x_i)]^L, [\overline{F}_B(x_i)]^U\} \end{pmatrix} \right\rangle$$

in $X = \{x_1, x_2, \ldots, x_n\}$.

A cosine similarity measure between interval rough neutrosophic sets A and B is defined as follows:

$$C_{IRNS}(A,B) = \frac{1}{n}\sum_{i=1}^{n} \frac{(\Delta T_A(x_i)\Delta T_B(x_i) + \Delta I_A(x_i)\Delta I_B(x_i) + \Delta F_A(x_i)\Delta F_B(x_i))}{\left(\sqrt{(\Delta T_A(x_i))^2 + (\Delta I_A(x_i))^2 + (\Delta F_A(x_i))^2}\right)\left(\sqrt{(\Delta T_B(x_i))^2 + (\Delta I_B(x_i))^2 + (\Delta F_B(x_i))^2}\right)} \quad (4)$$

Where $\Delta T_A(x_i) =$

$$\left(\frac{[\underline{T}_A(x_i)]^L + [\underline{T}_A(x_i)]^U + [\overline{T}_A(x_i)]^L + [\overline{T}_A(x_i)]^U}{4}\right),$$

$\Delta T_B(x_i) =$

$$\left(\frac{[\underline{T}_B(x_i)]^L+[\underline{T}_B(x_i)]^U+[\overline{T}_B(x_i)]^L+[\overline{T}_B(x_i)]^U}{4}\right),$$

$$\Delta I_A(x_i) = \left(\frac{[\underline{I}_A(x_i)]^L+[\underline{I}_A(x_i)]^U+[\overline{I}_A(x_i)]^L+[\overline{I}_A(x_i)]^U}{4}\right),$$

$\Delta I_B(x_i) =$

$$\left(\frac{[\underline{I}_B(x_i)]^L+[\underline{I}_B(x_i)]^U+[\overline{I}_B(x_i)]^L+[\overline{I}_B(x_i)]^U}{4}\right),$$

$\Delta F_A(x_i) =$

$$\left(\frac{[\underline{F}_A(x_i)]^L+[\underline{F}_A(x_i)]^U+[\overline{F}_A(x_i)]^L+[\overline{F}_A(x_i)]^U}{4}\right),$$

$\Delta F_B(x_i) =$

$$\left(\frac{[\underline{F}_B(x_i)]^L+[\underline{F}_B(x_i)]^U+[\overline{F}_B(x_i)]^L+[\overline{F}_B(x_i)]^U}{4}\right).$$

Proposition 3

Let *A* and *B* be interval rough neutrosophic sets then

1. $0 \le C_{IRNS}(A,B) \le 1$
2. $C_{IRNS}(A,B) = C_{IRNS}(B,A)$
3. $C_{IRNS}(A, B) = 1$, iff A = B

Proofs :

1. It is obvious because all positive values of cosine function are within 0 and 1.
2. It is obvious that the proposition is true.
3. When A = B, then obviously $C_{IRNS}(A, B) = 1$. On the other hand if $C_{IRNS}(A, B) =1$ then,

$\Delta T_A(x_i) = \Delta T_B(x_i)$,

$\Delta I_A(x_i) = \Delta I_B(x_i)$,

$\Delta F_A(x_i) = \Delta F_B(x_i)$ ie,

$[\underline{T}_A(x_i)]^L = [\underline{T}_B(x_i)]^L$,

$[\underline{T}_A(x_i)]^U = [\underline{T}_B(x_i)]^U$,

$[\overline{T}_A(x_i)]^L = [\overline{T}_B(x_i)]^L$,

$[\overline{T}_A(x_i)]^U = [\overline{T}_B(x_i)]^U$,

$[\underline{I}_A(x_i)]^L = [\underline{I}_B(x_i)]^L$,

$[\underline{I}_A(x_i)]^U = [\underline{I}_B(x_i)]^U$,

$[\overline{I}_A(x_i)]^L = [\overline{I}_B(x_i)]^L$,

$[\overline{I}_A(x_i)]^U = [\overline{I}_B(x_i)]^U$,

$[\underline{F}_A(x_i)]^L = [\underline{F}_B(x_i)]^L$,

$[\underline{F}_A(x_i)]^U = [\underline{F}_B(x_i)]^U$,

$[\overline{F}_A(x_i)]^L = [\overline{F}_B(x_i)]^L$,

$[\overline{F}_A(x_i)]^U = [\overline{F}_B(x_i)]^U$

This implies that A = B.

If we consider the weight w_i of each element x_i, a weighted interval rough cosine similarity measure between interval rough neutrosophic sets A and B can be defined as follows:

$C_{WIRNS}(A,B) =$

$$\sum_{i=1}^{n} w_i \frac{(\Delta T_A(x_i)\Delta T_B(x_i) + \Delta I_A(x_i)\Delta I_B(x_i) + \Delta F_A(x_i)\Delta F_B(x_i))}{\left(\begin{array}{l}\sqrt{(\Delta T_A(x_i))^2+(\Delta I_A(x_i))^2+(\Delta F_A(x_i))^2}\,. \\ \sqrt{(\Delta T_B(x_i))^2+(\Delta I_B(x_i))^2+(\Delta F_B(x_i))^2}\end{array}\right)} \quad (5)$$

$w_i \in [0,1]$, i = 1, 2,…, n and $\sum_{i=1}^{n} w_i = 1$. If we take $w_i = \frac{1}{n}$, i = 1, 2,…, n, then $C_{WIRNS}(A, B) = C_{IRNS}(A, B)$.

The weighted interval rough cosine similarity measure between two interval rough neutrosophic sets A and B also satisfies the following properties:

Proposition4

1. $0 \le C_{WIRNS}(A,B) \le 1$
2. $C_{WIRNS}(A,B) = C_{WIRNS}(B,A)$
3. $C_{WIRNS}(A, B) = 1$, iff A = B

Proof :

The proofs of above properties are similar to the profs of the propertyies of the proposition (3).

3.2 Dice similarity measure of interval rough neutrosophic environment

Definition 3.2.2

A Dice similarity measure between interval rough neutrosophic sets A and B (defined in 3.1.1) is defined as follows:

$DIC_{IRNS}(A,B) =$

$$\frac{1}{n}\sum_{i=1}^{n} \frac{2.[\Delta T_A(x_i)\Delta T_B(x_i) + \Delta I_A(x_i)\Delta I_B(x_i) + \Delta F_A(x_i)\Delta F_B(x_i)]}{\left[\begin{array}{l}(\Delta T_A(x_i))^2+(\Delta I_A(x_i))^2+(\Delta F_A(x_i))^2 \\ +(\Delta T_B(x_i))^2+(\Delta I_B(x_i))^2+(\Delta F_B(x_i))^2\end{array}\right]} \quad (6)$$

Where, $\Delta T_A(x_i) =$

$$\left(\frac{[\underline{T}_A(x_i)]^L+[\underline{T}_A(x_i)]^U+[\overline{T}_A(x_i)]^L+[\overline{T}_A(x_i)]^U}{4}\right),$$

$\Delta T_B(x_i) =$

$$\left(\frac{[\underline{T}_B(x_i)]^L+[\underline{T}_B(x_i)]^U+[\overline{T}_B(x_i)]^L+[\overline{T}_B(x_i)]^U}{4}\right),$$

$$\Delta I_A(x_i) = \left(\frac{[\underline{I}_A(x_i)]^L + [\underline{I}_A(x_i)]^U + [\bar{I}_A(x_i)]^L + [\bar{I}_A(x_i)]^U}{4}\right),$$

$$\Delta I_B(x_i) = \left(\frac{[\underline{I}_B(x_i)]^L + [\underline{I}_B(x_i)]^U + [\bar{I}_B(x_i)]^L + [\bar{I}_B(x_i)]^U}{4}\right),$$

$$\Delta F_A(x_i) = \left(\frac{[\underline{F}_A(x_i)]^L + [\underline{F}_A(x_i)]^U + [\bar{F}_A(x_i)]^L + [\bar{F}_A(x_i)]^U}{4}\right),$$

$$\Delta F_B(x_i) = \left(\frac{[\underline{F}_B(x_i)]^L + [\underline{F}_B(x_i)]^U + [\bar{F}_B(x_i)]^L + [\bar{F}_B(x_i)]^U}{4}\right).$$

Proposition 5

Let *A* and *B* be interval rough neutrosophic sets then

1. $0 \le DIC_{IRNS}(A, B) \le 1$
2. $DIC_{IRNS}(A, B) = DIC_{IRNS}(B, A)$
3. $DIC_{IRNS}(A, B) = 1$, iff A = B

Proofs :

1. It is obvious because all positive values of Dice function are within 0 and 1.
2. It is obvious that the proposition is true.
3. When A = B, then obviously $DIC_{IRNS}(A, B) = 1$. On the other hand if $DIC_{IRNS}(A, B) = 1$ then,

$\Delta T_A(x_i) = \Delta T_B(x_i)$,

$\Delta I_A(x_i) = \Delta I_B(x_i)$,

$\Delta F_A(x_i) = \Delta F_B(x_i)$ ie,

$[\underline{T}_A(x_i)]^L = [\underline{T}_B(x_i)]^L$,

$[\underline{T}_A(x_i)]^U = [\underline{T}_B(x_i)]^U$,

$[\bar{T}_A(x_i)]^L = [\bar{T}_B(x_i)]^L$,

$[\bar{T}_A(x_i)]^U = [\bar{T}_B(x_i)]^U$,

$[\underline{I}_A(x_i)]^L = [\underline{I}_B(x_i)]^L$,

$[\underline{I}_A(x_i)]^U = [\underline{I}_B(x_i)]^U$,

$[\bar{I}_A(x_i)]^L = [\bar{I}_B(x_i)]^L$,

$[\bar{I}_A(x_i)]^U = [\bar{I}_B(x_i)]^U$,

$[\underline{F}_A(x_i)]^L = [\underline{F}_B(x_i)]^L$,

$[\underline{F}_A(x_i)]^U = [\underline{F}_B(x_i)]^U$,

$[\bar{F}_A(x_i)]^L = [\bar{F}_B(x_i)]^L$,

$[\bar{F}_A(x_i)]^U = [\bar{F}_B(x_i)]^U$

This implies that A = B.

If we consider the weight w_i of each element x_i, a weighted interval rough Dice similarity measure between interval rough neutrosophic sets A and B is defined as follows:

$$DIC_{WIRNS}(A, B) = \sum_{i=1}^{n} w_i \frac{2.[\Delta T_A(x_i)\Delta T_B(x_i) + \Delta I_A(x_i)\Delta I_B(x_i) + \Delta F_A(x_i)\Delta F_B(x_i)]}{\left[\begin{array}{l}(\Delta T_A(x_i))^2 + (\Delta I_A(x_i))^2 + (\Delta F_A(x_i))^2 \\ + (\Delta T_B(x_i))^2 + (\Delta I_B(x_i))^2 + (\Delta F_B(x_i))^2\end{array}\right]} \quad (7)$$

$w_i \in [0,1]$, i = 1, 2,…, n and $\sum_{i=1}^{n} w_i = 1$. If we take $w_i = \frac{1}{n}$, i = 1, 2,…, n, then $DIC_{WIRNS}(A, B) = DIC_{IRNS}(A, B)$.

The weighted interval rough Dice similarity measure between two interval rough neutrosophic sets A and B also satisfies the following properties:

Proposition6

1. $0 \le DIC_{WIRNS}(A, B) \le 1$
2. $DIC_{WIRNS}(A, B) = DIC_{WIRNS}(B, A)$
3. $DIC_{WIRNS}(A, B) = 1$, iff A = B

Proof :

The proofs of above properties are similar to the proofs of the properties of the proposition (5).

3.3 Jaccard similarity measure of interval rough neutrosophic environment

Definition 3.3.1 A Jaccard similarity measure between interval rough neutrosophic sets A and B (defined in 3.1.1) is defined as follows:

$$JAC_{IRNS}(A, B) = \frac{1}{n}\sum_{i=1}^{n} \frac{[\Delta T_A(x_i)\Delta T_B(x_i) + \Delta I_A(x_i)\Delta I_B(x_i) + \Delta F_A(x_i)\Delta F_B(x_i)]}{\left[\begin{array}{l}(\Delta T_A(x_i))^2 + (\Delta I_A(x_i))^2 + (\Delta F_A(x_i))^2 + \\ (\Delta T_B(x_i))^2 + (\Delta I_B(x_i))^2 + (\Delta F_B(x_i))^2 \\ + \Delta T_A(x_i)\Delta T_B(x_i) + \Delta I_A(x_i)\Delta I_B(x_i) \\ + \Delta F_A(x_i)\Delta F_B(x_i)\end{array}\right]} \quad (8)$$

where

$$\Delta T_A(x_i) = \left(\frac{[\underline{T}_A(x_i)]^L + [\underline{T}_A(x_i)]^U + [\bar{T}_A(x_i)]^L + [\bar{T}_A(x_i)]^U}{4}\right),$$

$$\Delta T_B(x_i) = \left(\frac{[\underline{T}_B(x_i)]^L + [\underline{T}_B(x_i)]^U + [\overline{T}_B(x_i)]^L + [\overline{T}_B(x_i)]^U}{4}\right),$$

$$\Delta I_A(x_i) = \left(\frac{[\underline{I}_A(x_i)]^L + [\underline{I}_A(x_i)]^U + [\overline{I}_A(x_i)]^L + [\overline{I}_A(x_i)]^U}{4}\right),$$

$$\Delta I_B(x_i) = \left(\frac{[\underline{I}_B(x_i)]^L + [\underline{I}_B(x_i)]^U + [\overline{I}_B(x_i)]^L + [\overline{I}_B(x_i)]^U}{4}\right),$$

$$\Delta F_A(x_i) = \left(\frac{[\underline{F}_A(x_i)]^L + [\underline{F}_A(x_i)]^U + [\overline{F}_A(x_i)]^L + [\overline{F}_A(x_i)]^U}{4}\right),$$

$$\Delta F_B(x_i) = \left(\frac{[\underline{F}_B(x_i)]^L + [\underline{F}_B(x_i)]^U + [\overline{F}_B(x_i)]^L + [\overline{F}_B(x_i)]^U}{4}\right).$$

Proposition 7

Let *A* and *B* be interval rough neutrosophic sets then

1. $0 \le JAC_{IRNS}(A,B) \le 1$
2. $JAC_{IRNS}(A,B) = JAC_{IRNS}(B,A)$
3. $JAC_{IRNS}(A, B) = 1$, iff A = B

Proofs :

1. It is obvious because all positive values of Jaccard function are within 0 and 1.
2. It is obvious that the proposition is true.
3. When A = B, then obviously $JAC_{IRNS}(A, B) = 1$. On the other hand if $JAC_{IRNS}(A, B) = 1$ then,

$\Delta T_A(x_i) = \Delta T_B(x_i)$,
$\Delta I_A(x_i) = \Delta I_B(x_i)$,
$\Delta F_A(x_i) = \Delta F_B(x_i)$ ie,
$[\underline{T}_A(x_i)]^L = [\underline{T}_B(x_i)]^L$,
$[\underline{T}_A(x_i)]^U = [\underline{T}_B(x_i)]^U$,
$[\overline{T}_A(x_i)]^L = [\overline{T}_B(x_i)]^L$,
$[\overline{T}_A(x_i)]^U = [\overline{T}_B(x_i)]^U$,
$[\underline{I}_A(x_i)]^L = [\underline{I}_B(x_i)]^L$,
$[\underline{I}_A(x_i)]^U = [\underline{I}_B(x_i)]^U$,
$[\overline{I}_A(x_i)]^L = [\overline{I}_B(x_i)]^L$,
$[\overline{I}_A(x_i)]^U = [\overline{I}_B(x_i)]^U$,
$[\underline{F}_A(x_i)]^L = [\underline{F}_B(x_i)]^L$,
$[\underline{F}_A(x_i)]^U = [\underline{F}_B(x_i)]^U$,
$[\overline{F}_A(x_i)]^L = [\overline{F}_B(x_i)]^L$,
$[\overline{F}_A(x_i)]^U = [\overline{F}_B(x_i)]^U$

This implies that A = B.

If we consider the weight w_i of each element x_i, a weighted interval rough Jaccard similarity measure between interval rough neutrosophic sets A and B can be defined as follows:

$$JAC_{WIRNS}(A,B) = \sum_{i=1}^{n} w_i \frac{[\Delta T_A(x_i)\Delta T_B(x_i) + \Delta I_A(x_i)\Delta I_B(x_i) + \Delta F_A(x_i)\Delta F_B(x_i)]}{\begin{bmatrix} (\Delta T_A(x_i))^2 + (\Delta I_A(x_i))^2 + (\Delta F_A(x_i))^2 + \\ (\Delta T_B(x_i))^2 + (\Delta I_B(x_i))^2 + (\Delta F_B(x_i))^2 \\ + \Delta T_A(x_i)\Delta T_B(x_i) + \Delta I_A(x_i)\Delta I_B(x_i) \\ + \Delta F_A(x_i)\Delta F_B(x_i) \end{bmatrix}} \quad (9)$$

$w_i \in [0,1]$, i = 1, 2,…, n and $\sum_{i=1}^{n} w_i = 1$. If we take $w_i = \frac{1}{n}$, i = 1, 2,…, n, then $JAC_{WIRNS}(A, B) = JAC_{IRNS}(A, B)$

The weighted interval rough Jaccard similarity measure between two interval rough neutrosophic sets A and B also satisfies the following properties:

Proposition 8

1. $0 \le JAC_{WIRNS}(A,B) \le 1$
2. $JAC_{WIRNS}(A,B) = JAC_{WIRNS}(B,A)$
3. $JAC_{WIRNS}(A, B) = 1$, iff A = B

Proof :

The proofs of above properties are similar to the proofs of the properties of proposition (7).

4. Decision making based on cosine, Dice and Jaccard hamming similarity operator under interval rough neutrosophic environment

In this section, we apply interval rough similarity measures between IRNSs to the multi-criteria decision making problem. Assume that, A ={ A_1, A_2,..., A_m }be a set of alternatives and C = {C_1, C_2, ..., C_n} be the set of attributes.

The proposed decision making approach is described using the following steps..

Step 1: **Construct of the decision matrix with interval rough neutrosophic number**

The decision maker fomrs a decision matrix with respect to m alternatives and n attributes in terms of interval rough neutrosophic numbers (see the Table 1).

Table1: Interval rough neutrosophic decision matrix

$$D = \left\langle \underline{d}_{ij}^{L}, \bar{d}_{ij}^{U} \right\rangle_{m\times n} =$$

$$\begin{array}{c|cccc} & C_1 & C_2 & \cdots & C_n \\ \hline A_1 & \left\langle \underline{d}_{11}^{L}, \bar{d}_{11}^{U} \right\rangle & \left\langle \underline{d}_{12}^{L}, \bar{d}_{12}^{U} \right\rangle & \cdots & \left\langle \underline{d}_{1n}^{L}, \bar{d}_{1n}^{U} \right\rangle \\ A_2 & \left\langle \underline{d}_{21}^{L}, \bar{d}_{21}^{U} \right\rangle & \left\langle \underline{d}_{22}^{L}, \bar{d}_{22}^{U} \right\rangle & \cdots & \left\langle \underline{d}_{2n}^{L}, \bar{d}_{2n}^{U} \right\rangle \\ . & \cdots & \cdots & \cdots & \cdots \\ . & \cdots & \cdots & \cdots & \cdots \\ A_m & \left\langle \underline{d}_{m1}^{L}, \bar{d}_{m1}^{U} \right\rangle & \left\langle \underline{d}_{m2}^{L}, \bar{d}_{m2}^{U} \right\rangle & \cdots & \left\langle \underline{d}_{mn}^{L}, \bar{d}_{mn}^{U} \right\rangle \end{array} \quad (10)$$

Here $\left\langle \underline{d}_{ij}^{L}, \bar{d}_{ij}^{U} \right\rangle$ is the interval rough neutrosophic number according to the i-th alternative and the j-th attribute.

Step 2: **Determine interval rough neutrosophic mean operator (IRNMO)**

$$\left\langle \Delta T(x_i), \Delta I(x_i), \Delta F(x_i) \right\rangle =$$

$$\left(\begin{array}{l} \dfrac{[\underline{T}(x_i)]^L + [\underline{T}(x_i)]^U + [\bar{T}(x_i)]^L + [\bar{T}(x_i)]^U}{4}, \\ \dfrac{[\underline{I}(x_i)]^L + [\underline{I}(x_i)]^U + [\bar{I}(x_i)]^L + [\bar{I}(x_i)]^U}{4}, \\ \dfrac{[\underline{F}(x_i)]^L + [\underline{F}(x_i)]^U + [\bar{F}(x_i)]^L + [\bar{F}(x_i)]^U}{4} \end{array} \right) \quad (11)$$

i= 1, 2, ..., n.

Step 3: **Determine the weights of the attributes**

Assume that the weight of the attributes C_j (j= 1, 2, ..., n) considered by the decision-maker is w_j (j = 1, 2, … , n). Where, all w_j $\in$ belongs to [0, 1]

And $\sum_{j=1}^{n} w_j = 1$.

Step 4: **Determine the benefit type attributes and cost type attributes**

The evaluation attribute can be categorized into two types: benefit attribute and cost attribute. In the proposed decision-making method, an ideal alternative can be identified by using a maximum operator for the benefit attribute and a minimum operator for the cost attribute to determine the best value of each criterion among all the alternatives. Therefore, we define an ideal alternative as follows.

$A^* = \{C_1^*, C_2^*, \dots, C_m^*\}$.

Where benefit attribute

$$C_j^* = \left[\max_i T_{C_j}^{(A_i)}, \min_i I_{C_j}^{(A_i)}, \min_i F_{C_j}^{(A_i)} \right] \quad (12)$$

The cost attribute

$$C_j^* = \left[\min_i T_{C_j}^{(A_i)}, \max_i I_{C_j}^{(A_i)}, \max_i F_{C_j}^{(A_i)} \right] \quad (13)$$

Step 5: **Determine the weighted interval rough neutrosophic similarity measure of the alternatives**

Using the equations (5), (7), and (9), the weighted interval rough neutrosophic similarity functions can be written as follows.

$$C_{WIRNS}(A,B) = \sum_{j=1}^{n} w_j C_{IRNS}(A,B) \quad (14)$$

$$DIC_{WIRNS}(A,B) = \sum_{j=1}^{n} w_j DIC_{IRNS}(A,B) \quad (15)$$

$$JAC_{WIRNS}(A,B) = \sum_{j=1}^{n} w_j JAC_{IRNS}(A,B) \quad (16)$$

Step 6: **Rank the alternatives**

Through the weighted interval rough neutrosophic similarity measure between each alternative and the ideal alternative, the ranking order of all alternatives can be determined based on the descending order of similarity measures.

Step 7: End

5. Numerical Example

Assume that a decision maker intends to select the most suitable laptop for random use from the four initially chosen laptops (S_1, S_2, S_3) by considering four attributes namely: features C_1, reasonable Price C_2, Customer care C_3, risk factor C_4. Based on the proposed approach discussed in section 4, the considered problem is solved by the following steps:

Step 1: **Construct the decision matrix with interval rough neutrosophic number**

The decision maker forms a decision matrix with respect to three alternatives and four attributes in terms of interval rough neutrosophic numbers as follows.

Table2. Decision matrix with interval rough neutrosophic number

$$d_S = \left\langle \underline{N}(P)^L, \overline{N}(P)^U \right\rangle_{3\times 4} =$$

	C_1	C_2	C_3	C_4
A_1	$\left\langle \begin{pmatrix}[0.6, 0.7],[0.3, 0.5],\\ [0.3, 0.4]\end{pmatrix}, \begin{pmatrix}[0.8, 0.9],[0.1, 0.3],\\ [0.1, 0.2]\end{pmatrix} \right\rangle$	$\left\langle \begin{pmatrix}[0.5, 0.7],[0.3, 0.4],\\ [0.1, 0.2]\end{pmatrix}, \begin{pmatrix}[0.7, 0.9],[0.3, 0.5],\\ [0.3, 0.4]\end{pmatrix} \right\rangle$	$\left\langle \begin{pmatrix}[0.5, 0.6],[0.4, 0.5],\\ [0.4, 0.6]\end{pmatrix}, \begin{pmatrix}[0.7, 0.8],[0.2, 0.4],\\ [0.3, 0.4]\end{pmatrix} \right\rangle$	$\left\langle \begin{pmatrix}[0.8, 0.9],[0.3, 0.4],\\ [0.5, 0.6]\end{pmatrix}, \begin{pmatrix}[0.7, 0.8],[0.3, 0.5],\\ [0.3, 0.5]\end{pmatrix} \right\rangle$
A_2	$\left\langle \begin{pmatrix}[0.7, 0.8],[0.2, 0.3],\\ [0.0, 0.2]\end{pmatrix}, \begin{pmatrix}[0.7, 0.9],[0.1, 0.2],\\ [0.1, 0.2]\end{pmatrix} \right\rangle$	$\left\langle \begin{pmatrix}[0.6, 0.7],[0.1, 0.2],\\ [0.0, 0.2]\end{pmatrix}, \begin{pmatrix}[0.6, 0.7],[0.1, 0.3],\\ [0.1, 0.3]\end{pmatrix} \right\rangle$	$\left\langle \begin{pmatrix}[0.5, 0.7],[0.2, 0.3],\\ [0.1, 0.2]\end{pmatrix}, \begin{pmatrix}[0.6, 0.9],[0.3, 0.5],\\ [0.2, 0.4]\end{pmatrix} \right\rangle$	$\left\langle \begin{pmatrix}[0.7, 0.8],[0.3, 0.5],\\ [0.1, 0.3]\end{pmatrix}, \begin{pmatrix}[0.5, 0.7],[0.5, 0.6],\\ [0.2, 0.3]\end{pmatrix} \right\rangle$
A_3	$\left\langle \begin{pmatrix}[0.6, 0.7],[0.3, 0.4],\\ [0.0, 0.3]\end{pmatrix}, \begin{pmatrix}[0.6, 0.9],[0.1, 0.2],\\ [0.1, 0.2]\end{pmatrix} \right\rangle$	$\left\langle \begin{pmatrix}[0.5, 0.7],[0.2, 0.4],\\ [0.2, 0.4]\end{pmatrix}, \begin{pmatrix}[0.6, 0.8],[0.1, 0.3],\\ [0.1, 0.2]\end{pmatrix} \right\rangle$	$\left\langle \begin{pmatrix}[0.6, 0.8],[0.2, 0.4],\\ [0.3, 0.4]\end{pmatrix}, \begin{pmatrix}[0.6, 0.8],[0.2, 0.5],\\ [0.3, 0.5]\end{pmatrix} \right\rangle$	$\left\langle \begin{pmatrix}[0.4, 0.7],[0.2, 0.4],\\ [0.4, 0.5]\end{pmatrix}, \begin{pmatrix}[0.5, 0.8],[0.2, 0.5],\\ [0.0, 0.2]\end{pmatrix} \right\rangle$

(17)

Step 2: **Determine the interval rough neutrosophic mean operator (IRNMO)**

Using IRNMO, the transferred decision matrix is as follows.

Table 3: Transformed decision matrix

	C_1	C_2	C_3	C_4
A_1	$\langle 0.750, 0.300, 0.250\rangle$	$\langle 0.700, 0.375, 0.250\rangle$	$\langle 0.650, 0.375, 0.425\rangle$	$\langle 0.800, 0.375, 0.475\rangle$
A_2	$\langle 0.775, 0.200, 0.125\rangle$	$\langle 0.650, 0.175, 0.150\rangle$	$\langle 0.675, 0.350, 0.225\rangle$	$\langle 0.675, 0.475, 0.225\rangle$
A_3	$\langle 0.700, 0.250, 0.150\rangle$	$\langle 0.650, 0.250, 0.225\rangle$	$\langle 0.700, 0.325, 0.375\rangle$	$\langle 0.600, 0.325, 0.275\rangle$

(18)

Step 3: Determine the weights of attributes

The weight vectors considered by the decision maker are 0.35, 0.25, 0.25 and 0.15 respectively.

Step 4: Determine the benefit type attribute and cost type attribute

Here three benefit type attributes C_1, C_2, C_3 and one cost type attribute C_4. Using equation (12), (13) and (18) we calculate the ideal alternative as follows.

A^* = [(0.775, 0.200, 0.125), (0.700, 0.175, 0.150), (0.700, 0.325, 0.225),(0.600, 0.475, 0.475)]

Step 5: Calculate the weighted interval rough neutrosophic similarity scores of the alternatives

Calculated values of weighted interval rough neutrosophic similarity values presented as follows.

$$C_{WIRNS}(A^*, A_1) = 0.9754$$

$$C_{WIRNS}(A^*, A_2) = 0.9979$$

$$C_{WIRNS}(A^*, A_3) = 0.9878$$

$$DIC_{WIRNS}(A^*, A_1) = 0.9716$$

$$DIC_{WIRNS}(A^*, A_2) = 0.9971$$

$$DIC_{WIRNS}(A^*, A_3) = 0.9835$$

$$JAC_{WIRNS}(A^*, A_1) = 0.9448$$

$$JAC_{WIRNS}(A^*, A_2) = 0.9943$$

$$JAC_{WIRNS}(A^*, A_3) = 0.9678$$

Step 6: Rank the alternatives

Ranking the alternatives is prepared based on the descending order of similarity measures (see the table 6). Highest value reflects the best alternative.

Hence, the laptop A_2 is the best alternative for random use.

6. Comparision between three proposed approaches

Weighted interval rough similarity measures	**Measured value**	**Ranking order**
Weighted interval rough cosine similarity	$C_{WIRNS}(A_1, A^*) = 0.9754$ $C_{WIRNS}(A_2, A^*) = 0.9979$ $C_{WIRNS}(A_3, A^*) = 0.9878$	$A_2 \succ A_3 \succ A_1$

measure		
Weighted interval rough Dice similarity measure	$D_{WIRNS}(A_1, A^*) = 0.9716$ $D_{WIRNS}(A_2, A^*) = 0.9971$ $D_{WIRNS}(A_3, A^*) = 0.9835$	$A_2 \succ A_3 \succ A_1$
Weighted interval rough Jaccard similarity measure	$J_{WIRNS}(A_1, A^*) = 0.9448$ $J_{WIRNS}(A_2, A^*) = 0.9943$ $J_{WIRNS}(A_3, A^*) = 0.9678$	$A_2 \succ A_3 \succ A_1$

Conclusion

In this paper, we have proposed cosine, Dice and Jaccard similarity measures of interval rough neutrosophic set and proved some of their basic properties. We have presented an application, namely selection of best laptop for random use. The thrust of the concept presented in the paper will be in pattern recognition, medical diagnosis, personnel selection, etc. in interval neutrosophic environment..

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Neutralité neutrosophique et expressivité dans le style journalistique

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Abstract. This study is inspired by Neutrosophy theory (Smarandache 1995, 1998), a new concept of states treatment with a generous applicability to logic, communication theory and applied linguistics, among other sciences. Neutrosophy considers a proposition, theory, concept, event A in relation to its opposite Anti-A which is not A, Non-A on that which is neither A nor Anti-A, denoted by "Neut A". Together, A, Anti-A and Neut-A combined two by two and also all three of them form the Neutro-Synthesis. The classical reasoning development about evidences –the triad thesis-anti-thesis-synthesis- known as dialectics is extended in the Neutrosophy by the tetrad thesis-anti-thesis-neutro-thesis-neutro-synthesis, which carries on the unification on synthesis regarding the opposites and their neutrals. Neutrosophic logic also makes a distinction between a 'relative truth' and an 'absolute truth', while fuzzy logic (Zadeh 1965) does not.

Our aim is to analyze a series of Romanian printed press chronicles reflecting the same event of the political stage but each in a different view and positioning (from neutrality to polemic attitude). Methods for text examination are speech acts and modality analysis, exploring how the author is discursively positioned in the sample text material. The study tries to argue that the paradox of journalistic communication lies in the double constraint the authors of news articles have to face: to be convincing (i.e. argumentative) while keeping their credibility. They have to be neutral about the facts presented and the political agents implied, unless they are accused of taking sides. There is no credibility without neutrality, but, on the other hand, without a definite position on the part of journalists, they will not succeed in passing their messages along to the public.

Keywords: argumentation, subjectivity, neutrality, neutrosophy, press articles.

1 Préliminaires

L'échange de messages et le partage des sens sont aussi anciens que la société elle-même. Ce processus de transmission d'informations d'un émetteur à un destinataire, par différents canaux et supports est appelé communication. Au cours de l'histoire, le taux de l'information et le niveau de développement du réseau informationnel ont connu une évolution sensible dans le domaine de la communication. On peut affirmer aujourd'hui sans nous tromper qu'une société est d'autant plus avancée qu'elle est développée du point de vue informationnel. Il faut tenir compte également du fait que, à coté des voies, des moyens et des méthodes de communication, le contenu et la forme que revêtent les messages échangés sont aussi importants. Ainsi, la qualité de l'information transmise est un facteur important de la société informationnelle. Quant à la forme/au style, si on peut parler d'une configuration communicationnelle de celle-ci/celui-ci, on pourrait dire qu'elle/il est une image fidèle de l'époque du point de vue de la demande informationnelle. La transmission rapide de l'information est fondamentale pour les diverses activités socio-économiques et ce processus s'est réalisé, massivement, pendant les derniers deux siècles, par l'intermédiaire des journaux et des périodiques, les seuls supports capables de circuler vite et d'atteindre simultanément un grand nombre de lecteurs. Ainsi, pour atteindre son but (l'information de masse), la presse écrite est devenue et restée pendant plus d'un siècle le premier moyen d'information en masse. Le terme d'origine anglaise *mass media* récupère le terme latin *medium*, qui signifie tant le moyen ou l'intermédiaire que la voie (le canal). Tard, dans les années '20 du siècle passé, la radio, premier moyen de transmission auditive des messages, a fait son apparition et, quelques décennies plus tard, la télévision a fait irruption dans l'espace médiatique. On a commencé ensuite à faire la distinction entre la presse écrite et la presse audio-visuelle, dont la mission commune, primordiale, est la transmission massive de l'information. L'information, conditionnée et organisée par les medias, constitue aujourd'hui le principal produit médiatique. Elle est transmise continuellement, massivement, sur tous les canaux médiatiques, à l'échelle planétaire, de sorte qu'on parle de plus en plus souvent de la société informationnelle, à savoir une société dont l'existence et le fonctionnement ne seraient possibles sans l'accès immédiat et massif à l'information. Avec l'extension à l'échelle planétaire des systèmes de communication par voie électronique, des réseaux de communications tels que l'Internet ou l'Xnet, on parle de société informatisée, qui permet la circulation instantanée de l'information de tous les domaines, d'un fournisseur situé à n'importe quel point du globe terrestre jusqu'au bénéficiaire situé à n'importe quel autre point. Ce qui plus est, les réseaux électroniques de communication englobent maintenant tous les autres formes et moyens de communication- la presse écrite, la radio, la télévision- étant capables de retransmettre des éditions de journaux, des émissions de radio et de télévision, etc. En outre, Internet peut contenir des sites d'informations spécialisés dans divers domaines, ainsi que des bibliothèques virtuelles. Ainsi, de la diffusion de l'information au niveau régional et national on en est arrivé, en quelques décennies seulement, à une diffusion globale. Ce phénomène de globalisation requiert une forme plus simple et plus directe de transmission de l'information.

2 La transmission de l'information: de la neutralité à la polémique

2.1 Le style neutre- l'idéal de la presse écrite

Pour amorcer, dans ce contexte, une discussion sur le style, nous avouerons notre attente que celui-ci soit également le plus simple, direct et efficace possible, autrement dit qu'on communique le maximum d'information en un minimum de mots. Mais en réalité, sur la majorité des sites Internet, la préoccupation pour le style est tout à fait marginale et ceux qui transmettent l'information le font d'une manière plutôt négligente et même familière. Le style de la presse écrite conserve, en général, les propriétés linguistiques et communicatives nécessaires à une transmission efficace de l'information, étant dans la plupart des cas simple, clair et direct. Sa propriété primordiale est la concision, dont le premier effet est l'efficacité. Le style neutre, concis et direct est spécifique des agences de presse professionnelles. En dehors de la concision, l'efficacité provient aussi de la clarté de l'expression et de la formulation. La clarté et la concision, l'objectivité entendue comme absence de l'immixtion émotionnelle sont doublées par la propriété et la précision des termes utilisés. La propriété des termes est définie comme leur qualité d'exprimer avec exactitude le concept ou l'idée visés. La précision, quant à elle, concerne l'adéquation des termes à l'information qui doit être transmise. Enfin, le style neutre est en relation directe avec l'objectivité, la transmission de l'information étant ainsi mise à l'abri des risques et des complications. Le style neutre caractérise non seulement la langue de la presse, mais aussi celle de l'administration, vouée à un public de niveau culturel moyen, qui valorise l'utilité de l'information. Ayant le plus haut degré d'adressabilité, le style neutre manifeste une tendance à l'universalité. Il s'oppose au style imagé, qui présente, comme on verra plus tard, des tendances de personnalisation.

À vrai dire, le journaliste est un « parent pauvre» de l'écrivain, n'étant plus guère qu'un capteur de l'information et un facteur de (re)structuration et de transmission de celle-ci, à travers le processus de rédaction des nouvelles, des articles, surtout dans le cas des agences de presse, des matériels écrits « à chaud » et sous la pression de la diffusion rapide. Dans ce cas-là, la rédaction prévaut sur l'élaboration, sur le commentaire et sur l'expression des opinions. Le style neutre suppose une économie de moyens et de matériel linguistique et conséquemment une rigueur de la rédaction et un aspect soutenu, clair et concis. Ce type de message comporte une dimension dénotative accentuée et une capacité accrue de pénétration de tous les milieux sociaux, car il est le résultat d'un effort de médiation. On peut conclure que le style neutre est subordonné à la communication référentielle.

2.2 Attitude, prise en charge et argumentativité

Le caractère informatif d'un texte de presse n'exclut pas pourtant son caractère argumentatif. En fait, le caractère argumentatif est intriqué au réseau informatif du texte, d'une manière naturelle. Tout énoncé est pourvu d'une orientation argumentative. Comme Anscombre et Ducrot l'ont montré dès les années '80, l'argumentativité est un trait inhérent de tout discours. Mais lorsque l'attitude du journaliste devient visiblement subjective et il perd sa neutralité, le texte passe de l'information à l'opinion, manifestation d'une prise en charge argumentative du contenu du message. Par ce type de texte, son auteur essaye d'influencer, délibérément, la conscience de ses lecteurs et de les convaincre d'adopter son opinion ou d'admettre sa thèse. On peut constater une gradualité de la prise en charge du message par le journaliste, une échelle qui va de l'attitude objective, caractérisée sur le plan de l'expression par le style neutre, à l'attitude argumentative et critique, voire combative, produite et entretenue par l'existence d'un conflit d'opinions. Ainsi, en dehors de la force argumentative découlant de l'organisation discursive et dépendant du statut sémantique et pragmatique des arguments proprement dits (éléments appartenant au plan idéatico-logique qui s'adressent à l'intellect), il s'y glisse une intention de persuader, d'emporter l'adhésion des lecteurs par des moyens qui appartiennent moins à la logique et à la raison qu'à l'émotion et à l'irrationnel. L'action de persuader tient au désir de convaincre quelqu'un, de le faire croire et agir de la manière dont nous souhaitons qu'il le fasse. Dans le cas du journaliste, cela signifie qu'il voudrait déterminer le lecteur à adopter ses convictions, ses attitudes par rapport aux faits qu'il présente, sans lui donner l'occasion, la chance de se forger une opinion personnelle sur lesdits faits, après la lecture de l'article. La persuasion relève donc de la capacité ou, si l'on peut dire, le talent d'influencer l'auditeur/ le lecteur pour qu'il adopte notre point de vue/ notre thèse. L'auteur d'un article d'opinion doit, bien sûr, apporter des arguments en faveur de sa thèse, mais comme en politique les arguments factuels ou les preuves sont parfois difficiles à procurer, il recourt aux hypothèses, aux suppositions et aux insinuations. On voit donc comment, de l'argumentation rationnelle, qui reste en grande partie objective (se rapprochant de ce point de vue de la démonstration) on peut vite glisser vers la prise de position polémique, qui, elle, témoigne d'un degré plus haut de subjectivité. Le terme de polémique provient d'ailleurs du grec ancien *polemikos* « relatif à la guerre ». Il s'agit d'un conflit ouvert et déclaré, où l'attitude critique se transforme en parti pris violent et parfois agressif. La polémique se caractérise par l'action du principe de contradiction argumentative, par l'emploi d'objections, de contre-arguments et par la mise en œuvre des stratégies de réfutation. C'est une stratégie argumentative réactive, par laquelle le sujet exprime son désaccord et apporte des objections contre un acte ou un contenu exprimé au préalable par l'interlocuteur. L'étiquette de 'polémique' s'applique à une interaction verbale (discursive ou textuelle), de nature argumentative, qui se définit par un conflit ou un désaccord par rapport à un contenu, une situation, etc. La polémique vise à la disqualification de l'adversaire et, dans ce but, tend à manipuler les contenus par la déviation des sens. Dans la presse, les articles polémiques peuvent être plus tempérés ou plus agressifs, leur violence étant en étroite relation avec la force contestataire, la virulence du style, l'abondance des actes de langage offensifs (la négation polémique, contestation, la réfutation, le démenti) etc. Parfois le discours polémique revêt la forme du pamphlet, aspect outrageux, violent, offenseur, illustré dans la presse écrite roumaine après '90 par le journal *România Mare* de Corneliu Vadim Tudor.

3 Bref aperçu de la vie politique et des partis roumains après 1989

La scène politique roumaine après '90 est partagée entre plusieurs partis se réclamant de la gauche sociale-démocrate, du centre et de la droite modérée (du libéralisme) qui se succèdent l'un à l'autre au gouvernement, ou qui forment des alliances plus ou moins opportunistes afin de s'assurer la majorité dans le parlement. Le Parti social-démocrate (PSD) est un parti politique fondé en 1992, héritier du Parti de la Démocratie Sociale de Roumanie (PDSR), parti issu du Front du Salut National (Frontul Salvării Naţionale) première formation politique au pouvoir en Roumanie après 1989. Ses opposants l'accusent d'abriter des anciens du Parti Communiste Roumain, le parti unique entre 1948 et 1989, même s'il n'y a pas de lien organique entre les deux partis, et de perpétuer certaines mentalités et coutumes spécifiques à l'époque communiste.

Depuis février 2011, il est allié au Parti National libéral et au Parti Conservateur au sein de l'Union sociale libérale. Les sociaux-démocrates, en coalition avec le Parti National Libéral et avec l'Union Démocratique des Magyares de Roumanie, ont retrouvé le chemin du gouvernement au début de l'année 2012, après quatre années passées dans l'opposition. Victor Ponta, leur ancien président, est devenu le Premier ministre de la Roumanie. Les dernières élections au sein du PSD ont marqué, par l'arrivée du nouveau leader Liviu Dragnea, un changement que les commentateurs de la vie politique ont interprété de manières différentes.

4 Hypothèse et corpus de la recherche

L'hypothèse de notre recherche est que l'usage de l'argumentation dans les articles de presse détermine les lecteurs à faire certaines inférences et associations pour arriver aux conclusions poursuivies par l'auteur, d'une manière implicite. La présentation tendancieuse des faits de la réalité sociopolitique conduit à une interprétation (pré)déterminée, apte à susciter chez le lecteur les attitudes et les sentiments que l'auteur de l'article désire éveiller. Par contre, dans le cas de l'attitude et du style neutres, la façon de présenter l'information (les événements et les déclarations des acteurs politiques) fait appel à la raison et au discernement des lecteurs, répondant aux besoins d'un public qui se considère comme étant constitué de citoyens réflexifs. Comme Habermas le montre, la fonction de la communication dans la sphère publique est la construction des identités sociales et des relations viables au sein d'une société démocratique. Ainsi, la communication est constituée d'éléments linguistiques capables de servir les positions des participants, fonction essentielle dans la construction des rôles de citoyens actifs (Fairclough, 1992). Nous avons choisi pour l'illustration de notre hypothèse une série d'articles de presse qui reflètent tous les mêmes événements (évolutions) sur l'échiquier de la vie politique roumaine, mais d'une manière très différente. Tandis que les agences de presse Mediafax et Agerpres se contentent de raconter les événements d'une manière neutre et de reprendre les déclarations des acteurs politiques impliqués sans les commenter, les journaux d'opinion tels que *Gândul* transmettent des échos variés des événements en question, illustrant des positions qui vont du scepticisme à l'optimisme, soutenues par des argumentations plus ou moins subjectives.

5 Analyse du corpus de presse

L'analyse des actes de langage accomplis dans le discours journalistique montre la manière de laquelle les lecteurs sont invités à participer à l'acte de communication. Les actes de langage sont conventionnels et étroitement liés au système social. Leur analyse, corroborée avec l'étude de la communication politique et sociale peut contribuer à la description des normes communicatives en vigueur dans la société et à élucider certaines pratiques qui constituent la manifestation des normes et des valeurs de la société en question. (Fowler, 1991: 88; C. Kerbrat-Orecchioni 2002). L'analyse des modalités logiques et des marques de modalisation discursive (R. Vion 2012) mettra en évidence le degré de prise en charge des contenus informationnels véhiculés par le discours de l'article de presse et la colorature affective imprimée aux énoncés. La modalité a affaire au rapport que le locuteur entretient avec le contenu propositionnel et contribue à la construction des identités discursives et sociales. Par la manière dont il communique les faits et par l'expression de son attitude envers ceux-ci, les évaluations que le journaliste donne de la réalité sont projetées dans l'univers du récepteur. Ainsi, la liberté d'interprétation du lecteur peut être sérieusement affectée par la présence d'une attitude explicite du journaliste dans le message. Les éléments linguistiques qui peuvent véhiculer l'expression de l'attitude du journaliste sont : les auxi-verbes modaux (M. Tuţescu 2005) *savoir, croire, pouvoir, devoir, falloir, sembler*, etc. ; les modes et les temps verbaux (à comparer par exemple l'emploi de l'indicatif présent par rapport au conditionnel journalistique) ; les adverbes modalisateurs d'énoncé et d'énonciation ; les adjectifs évaluatifs ou appréciatifs (C. Kerbrat-Orecchioni 1980, 1999) et la modalisation autonymique (italique, guillemets, incises, etc.).

5.1. Le premier extrait que nous avons soumis à l'analyse a été publié sur le site de l'agence de presse Mediafax à la veille des élections au sein du PSD. Son titre a la forme d'une citation d'un candidat à la présidence du parti, Liviu Dragnea. La neutralité ressort de l'absence de commentaires sur les déclarations reprises et de l'emploi de verbes de citation et d'autres marqueurs évidentiels spécifiques du discours rapporté neutre : *Dragnea, despre candidatura sa la şefia PSD :* […], *afirmă, a spus, a explicat, a adăugat, a precizat*. Il y a quand même des marqueurs (verbes de déclaration) qui expriment un commentaire critique des déclarations respectives : *a evitat, a recunoscut*. À la fin de l'article, une précision faite sur un ton sec rappelle que le recours du candidat en question est en train d'être jugé et qu'il avait été condamné en première instance à une année de prison avec suspension dans un dossier de fraude au referendum national. Ce commentaire a le rôle d'informer le lecteur sur le statut judiciaire du candidat, mais aussi de mettre une distance entre le journaliste-énonciateur et les propos qu'il vient de rapporter.

5.2 L'extrait suivant, intitulé *Dragnea, stafia lui Ceauşescu* (*Dragnea, le fantôme de Ceausescu*) est un article d'opinion publié après les élections dans le journal en ligne www.gândul.info. L'auteur donne une évaluation positive de la situation, argumentant que l'élection de Dragnea constitue un vrai changement du paradigme des chefs du parti social-démocrate. La sympathie de l'auteur pour le personnage transparaît, bien qu'elle ne soit pas avouée explicitement, à travers les dénominations qu'il emploie, les actions et les qualités attribuées à Dragnea qu'il choisit de mettre en relief, les adjectifs évaluatifs et axiologiques, etc. La stratégie argumentative indirecte qu'il adopte est, à notre avis, d'une grande efficacité et possède un pouvoir persuasif nettement supérieur aux stratégies directes ou à la démonstration. L'auteur commence par citer les détracteurs de Dragnea, pour qui celui-ci est un « étranger » (roum. « venetic ») et un « transfuge » d'un autre parti. Mais, dans la bouche des conservateurs du PSD, représentés par la personnalité controversée de l'ancien président Ion Iliescu, connu pour son attachement à la gauche communiste, cet appellatif devient un argument favorable, un atout de Dragnea. Le journaliste a du mal à cacher son enthousiasme pour l'élection du premier président du PSD qui « ne porte pas dans son ADN politique le gène modifié de l'activiste du PCR ». Afin d'emporter l'adhésion des lecteurs à sa thèse, il recourt à plusieurs stratégies de persuasion : l'emploi de la première personne du pluriel, qui inclut l'interlocuteur, la métaphore (*ADN-ul său politic, gena modificată a activistului pecerist*), l'ironie amicale (*copilul „din trandafiri" al lui Ion Iliescu, „Titulescu lui Năstase", „copilul răzvrătit al vechilor emanicipări pediste"*) et la suggestion d'une connivence entre le public lecteur et le personnage du nouveau chef du PSD :

***Ne** vine să **credem** sau nu, asta e situaţia. După 25 de ani de la materializarea primei emanaţii revoluţionare, partidul lui Ion Iliescu va fi condus de un cetăţean care nu poartă în ADN-ul său politic gena modificată a activistului pecerist, campat în Kiseleff, ci, mai degrabă, pe cea cu parfumul vag al cozeriei casei de oaspeţi a Lupeascăi, din Modrogan.*

*Nu se poate să nu **vi**-l amintiţi, de exemplu, în cadrul acela, remarcabil, din „Noaptea preşedintelui Geoană", când le ţinea spatele liderului-blitz şi Mihaelei, dragostea lui! Nu se poate să-i fi uitat privirea-lamă, „Gillette Stainless Steel"!*

Il compare l'aspect de l'homme politique à celui d'un agent secret ou des acteurs ayant incarné des gardes du corps et des super héros dans les films d'aventure produits à Hollywood, pour insinuer en ce qui suit qu'il a la taille d'un vrai homme d'Etat, qui a joué jusqu'alors le rôle de l'homme de main (en roum. „omul din umbră"), du lieutenant, en attendant que sa chance arrive. L'argumentation, très bien conduite, opère tout à la fois par dichotomisation, polarisation et procédés rhétoriques variés : dérision (ex : *Încet dar sigur, PSD a trecut în nefiinţă. Fireşte că nu se află întins pe năsălie, ca să mergem să-l aplaudăm cu cozi de trandafiri în palme*), ironie, appels au pathos mais aussi à la raison. Le „coup de grâce" de cette argumentation (en faveur de Dragnea et contre la vieille garde communiste du PSD) est représenté par le paragraphe final qui cite la réplique de Dragnea à Iliescu, réplique qui bénéficie d'une mise en scène théâtrale (la métaphore du rideau y est d'ailleurs convoquée) :

Atac căruia Dragnea i-a răspuns, sec, ca o ***cădere de cortină*** *peste trecutul comunist al PSD. Sau ca o „dezîmpingere", pur şi simplu: "Nu ştiu ce-şi dorea Ceauşescu, pentru că nu l-am cunoscut foarte bine şi nu pot să mă pronunţ".*

5.3. Le troisième extrait soumis à l'analyse est un article du même journal qui se situe sur une position antagonique à celle de l'article précédent. L'auteur soutient la thèse que l'élection de Dragnea à la tête du PSD n'a rien changé aux mœurs des membres du parti et que même son nouveau leader affiche un masque dont on n'est pas dupe. La polémique est entamée depuis le titre de l'article : *Liviu Dragnea s-a rupt (în figuri) de comunism* (*L. Dragnea a rompu avec le communisme, mon oeil!).* Ce jeu de mots contient une contradiction entre les sens du verbe *a se rupe (de)* « rompre avec » et celui de l'expression figée appartenant au registre familier *a se rupe în figuri* « jouer du théâtre, poser ». Ce titre, d'un grand effet rhétorique, véhicule d'une manière expressive la thèse soutenue par l'article : que les déclarations du nouveau chef du PSD ne sont que de la poudre aux yeux du public et qu'en réalité ce parti ne sera jamais reformé. Le discours polémique est amorcé par une négation polémique explicite :

Ce am înţeles noi astăzi, din Congresul PSD? Că niciodată acest partid al nemuririi comuniste nu se va schimba; nu se va reforma, nu va cunoaşte beneficiul exorcizării, nu se va rupe de trecut – minciuna, prefăcătoria şi agapa poltronilor care îl conduc alungând, la infinit, politicienii oneşti şi electoratul cu „fibră" de stânga.

L'argumentation fait usage de stratégies directes : la réfutation, l'interrogation, l'interpellation de l'adversaire et l'exemple. Pour éviter la situation ingrate du discours monogéré où le polémiste est seul maître à bord, l'auteur simule un échange avec Dragnea, en reprenant quelques déclarations de celui-ci auxquelles il répond par des contre-arguments. Il ne s'agit pas, en ce cas, de persuader l'adversaire, mais de s'adresser au lecteur, qui assiste à l'échange polémique et dont les vues, susceptibles de vaciller, sont en attente d'être confirmées et nourries. Le journaliste termine son argumentation par une interpellation directe de Liviu Dragnea contenant un dernier argument, destiné à renforcer la réfutation de la thèse adverse :

Nu, domnu' Dragnea, asta nu e despărţirea de comunism, ci doar o încercare rizibilă de a desprinde partidul de imaginea lui Ion Iliescu. Desprinderea de comunism ar fi fost aia remarcată de Ţuţea: "A te opune comunismului înseamnă a apăra puritatea Codului Penal".
Mai mult n-are ce să fie.

5.4. Le quatrième article sur les élections au sein du PSD, publié toujours dans le journal en ligne *Gândul*, porte le titre ironique *O exorcizare ratată* (*Une exorcisation ratée*), se référant aux efforts de Dragnea pour cosmétiser l'image du parti sans rompre véritablement avec le passé communiste de celui-ci. La position soutenue est la même que celle de l'article de sous 5.3 mais, si le ton de l'extrait précédent est sérieux et indigné, le ton du texte signé par Clarice Dinu est sarcastique et sa rhétorique est basée sur de nombreuses allusions au passé: emploi des termes traditionnellement associés aux leaders communistes comme *tătuc, stalinism, baron* ; évocation des anciens présidents communistes Nicolae Ceauşescu et Ion Iliescu et des anciens leaders du parti, Adrian Năstase, Mircea Geoană et Victor Ponta, qui ont perdu aux élections présidentielles à cause de l'image du parti, compromise par la corruption de ses membres notoires.

L'allégorie de l'exorcisation, d'une grande force argumentative, est soutenue par une isotopie dont nous signalons les éléments les plus saillants : *preot, drac, a păcătui, a scoate dracii, a dezgropa, nefăcută*. Nous considérons que la stratégie argumentative choisie par l'auteur relève plutôt de la persuasion que de l'argumentation logique, bien que les preuves n'en manquent pas. La conclusion se distingue du reste de l'article par le ton amer et par l'absence d'intention ironique :

Întreg congresul social-democraţilor de duminică a fost o comédie pusă în scenă pentru Liviu Dragnea. Ca o ironie a sorţii, validarea alegerii unicului candidat a avut loc exact în aceeaşi sală în care Nicolae Ceauşescu era reales la Congresele PCR. În final, PSD a rămas acelaşi partid, la fel de tarat, dar cu un nou tătuc.

5.5. Le dernier article que nous avons sélectionné pour notre analyse, intitulé *PSD, next gen* constitue une synthèse des prises de position précédentes (5.3.et 5.4), auxquelles il fait d'ailleurs allusion :

Presa a apreciat corect momentul de stand-up comedy, interpretat de Liviu Dragnea, în congres, cu privire la „ruperea partidului de comunism".

La polémique atteint ici son climax, touchant au pamphlet : les termes qui caractérisent les « enfants du parti » sont très durs et les métaphores du domaine des sciences naturelles ont une grande force argumentative :

Cu siguranţă, PSD nu-şi poate tăia cordonul ombilical - crescut din „fibra oţelului de tanc" -, care îl leagă de trecut. Gena e prea puternică şi, după cum se vede, tot mai pregnantă, inclusiv în cazul social-democraţiei next gen, copiii partidului începând să se mănânce între ei, ca lupii. Emfaza, trufia şi fandoseala dusă până la narcisism te lasă fără cuvinte.

Le titre fait allusion au geste de Mihai Sturzu, leader du TSD (Organisation de la jeunesse social-démocrate) qui a dénoncé l'existence d'une seule candidature pour la fonction de président aux élections au sein du PSD comme coutume communiste, fait qui risquerait d'invalider le résultat de celles-ci. Ce geste, longuement commenté par la presse, a été sanctionné par la direction du parti avec l'exclusion de Sturzu, décision elle-aussi très commentée. L'auteur fait le point des opinions véhiculées dans la presse :

Iar aici, părerile jurnaliştilor au fost împărţite: „PSD poate avea viitor, prin curajul tinerilor săi", au spus unii; „Sturzu a fost naiv crezând în democraţia de partid, clamată de noul lider", au considerat alţii; „Liderul TSD a fost pus cu botul pe labe după atacul declanşat împotriva lui Liviu Dragnea", a conchis ai treilea val; „Sturzu a spus ceea ce i s-a dictat să spună, încercând să schimbe impresia instaurării dictaturii în partid", au mai adăugat vreo câţiva. Ei bine, dacă nu l-ai fi cunoscut pe Sturzu, introdus în lume de o anume aroganţă, ai fi putut crede că omul a prins o mână grandioasă şi plusează cinstit, convins că va sălta potul. […]

Ensuite il met en scène la voix d'un autre jeune social-démocrate, qui exprime un point de vue opposé sur Sturzu:

Astfel, Gabriel Petrea scrie într-un comunicat bine simţit că Mihai Sturzu, impus de Victor Ponta, în urmă cu doi ani, la şefia TSD, nu îi reprezintă pe tinerii din partid şi că, la Congresul PSD, acesta a avut cel mai ipocrit discurs din istoria partidului, vorbind fără jenă în numele propriului interes. "Duminică, 18 octombrie, Mihai Sturzu, uitând de toate aceste valori, a ţinut, de la tribuna Congresului PSD, cel mai ipocrit discurs din istoria partidului. Impus de Victor Ponta la şefia TSD, acum exact doi ani de zile, în urma unui congres în care a fost singurul candidat, Mihai Sturzu acuză, fără jenă, congresele fără competiţie. După ce doi ani la rând nu a fost vocea tinerilor în PSD, nici duminică el nu a reprezentat vocea organizaţiei. Să nu se spună că Sturzu a vorbit în numele Tineretului Social Democrat! A vorbit în numele propriului interes, pentru obiective pe care nu le cunoaştem şi de care nici nu ne pasă", susţine tânărul secretar general.

avant de donner son verdict :

Uluitor! Ăştia, de mici, se mănâncă între ei, ca o haită de lupi flămânzi. Ce-mi era Sturzu, pentru care „dictatura" e bună numai şi numai dacă dictează el, şi ce-mi este Petrea, care a acceptat dictatura, ca statuia lui Lenin, până când Dragnea, „dictatorul-dictatorilor", i-a cerut să revină, şi el, la demnitatea şi la aroganţa lui, şi să-l înfiereze, cu mânie proletară, pe cântăreţ. „'Nţeles! Trăiţi!!"

Bun tineret, distins partid.

Le sarcasme de la dernière phrase -ou plutôt antiphrase- est évident. La conclusion que l'auteur de l'article veut partager aux lecteurs est pessimiste : l'avenir du parti de gauche le plus important de la Roumanie post-communiste est compromis par l'opportunisme et le manque de scrupules de ses plus jeunes membres.

6 Conclusion

Le choix des articles de presse reflétant les dernières élections au sein du PSD met en évidence la multitude des points de vue sur le même contenu et la variété des interprétations que les journalistes donnent des faits et des évolutions respectives. Nous avons sélectionné un article informatif, qui présente les événements d'une manière objective, rédigé dans un style neutre et quatre articles d'opinion dont le degré de prise en charge du contenu transmis est variable.

On pourrait représenter le degré d'adhésion du locuteur au contenu propositionnel des énoncés par un axe allant de la neutralité totale à la prise de prise en charge :

Figure 1:

Neutralité ⟷
absolue Prise en charge maximale

Sur cet axe, les extraits choisis pourraient se succéder de gauche à droite, le premier tendant à la neutralité et les autres se situant plus du côté de la prise en charge maximale. L'axe ci-dessus pourrait se rapporter aussi à une échelle des traitements des contenus informatifs allant de l'objectif (attitude zéro) au subjectif (prise en charge totale). L'interprétation donnée aux faits commentés varie elle aussi : ainsi, l'évaluation que les journalistes en font va du négatif au positif et leur attitude oscille entre l'optimisme modéré et le pessimisme extrême. Les mêmes faits, événements, gestes et déclarations donnent naissance à une multitude d'interprétations, et chacune se réclame « vraie », « juste » et « bien fondée ». À chacun sa vérité, car toute interprétation est singulière. Cette variété des points de vue met en exergue un autre phénomène scalaire spécifique aux langues naturelles : la vérité en langue est une vérité subjective, car prise en charge par un locuteur. À notre avis, dans la communication journalistique il y a toujours une attitude, plus ou moins manifeste, à l'égard l'information présentée. L'information dans la presse ne joue plus sur les oppositions classiques du vrai et du faux, mais sur le vraisemblable. L'objectif de la presse n'est pas seulement d'informer, mais aussi d'influencer l'opinion publique et de divertir le lecteur. La dramatisation, procédé consacré par le fait divers et la presse tabloïde, est devenue la façon habituelle de pimenter l'information dans tous les journaux d'information et d'opinion. Le paradoxe de la communication journalistique consiste, selon nous, dans la double contrainte à laquelle l'auteur d'un article de presse est soumis : être persuasif (lire argumentatif) tout en gardant sa crédibilité. Il doit rester neutre par rapport aux faits présentes et aux acteurs politiques impliques, sinon il risque de se faire accuser de parti pris. Sans neutralité, il n'a pas de crédibilité, mais dans l'absence d'une prise de position, il ne réussit pas à faire passer son message auprès du public.

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Neutrosophic Semilattices and Their Properties

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Abstract: In this paper authors study neutrosophic semi-lattices and their properties. These neutrosophic semi-lattices are built using either $\cup$ or $\cap$ operation only. This application of these concepts is also discussed in this paper.

Keywords: Neutrosophic semi-lattices, pure neutrosophic semi-lattice.

1 Introduction

In this paper the new notion of neutrosophic semilattices is introduced for the first time. However the study of neutrosophic lattices started in 2004 [5,6]. But those neutrosophic lattices are of a special type as they were mainly defined to cater to the needs of applications in fuzzy models. This paper has three sections. Section one is introductory in nature. In section two different types of neutrosophic semilattices are defined and their properties developed.

Section three gives the probable applications of these concepts in data mining, sorting etc. Finally we give the conclusions based on this work.

2 Neutrosophic semilattices of different types and their properties

In this section neutrosophic semilattices of various types are defined and described. This study is new and innovative and certainly can provide lots of applications to various fields where semilattices are applied.

Example 2.1: Let S(P) = {1, a, aI, I, 0} be the neutrosophic set, {S(P), $\cap$} is a neutrosophic semilattice whose diagram is as follows.

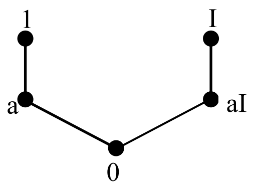


Figure 2.1

Example 2.2. Let {S(P), $\cap$} be the neutrosophic semilattice given by the following example.

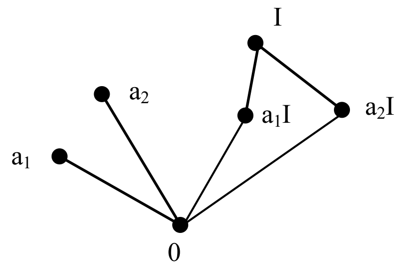


Figure 2.2

Example 2.3. Let {S(P), $\cap$} be the neutrosophic semilattice given by the following example.

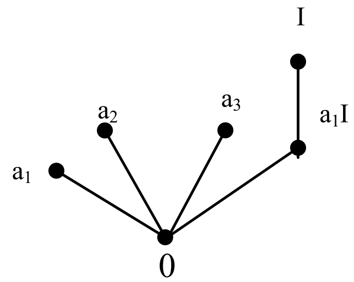


Figure 2.3

In view of this the following neutrosophic semilattice is defined.

Definition 2.1: *Let {S(P)} $\cap$} be the partially ordered neutrosophic set with 0. {S(P), $\cap$} is defined as the neutrosophic semilattice if*

$$\min\{x, y\} = x \cap y \in S(P)$$

for all x, y $\in$ S(P).

The examples given above are neutrosophic semilattices of finite order.

Similarly one can define {S(P), ∪} the neutrosophic semilattice.

Now examples of neutrosophic semilattice {S(P), ∪} are as follows.

Example 2.4: Let {S(P), ∪} be the neutrosophic semilattice which has the following figure.

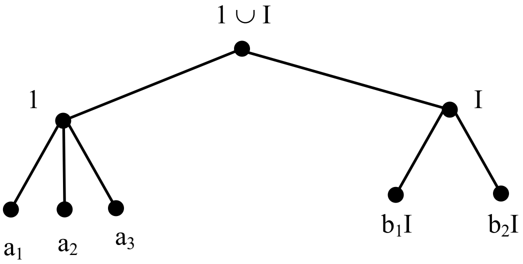


Figure 2.4

Example 2.5: Let {S(P), ∪} be the neutrosophic semilattice which has the following figure.

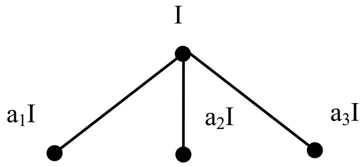


Figure 2.5

Example 2.6. Let {S(P), ∪} be the neutrosophic semilattice given by the following figure.

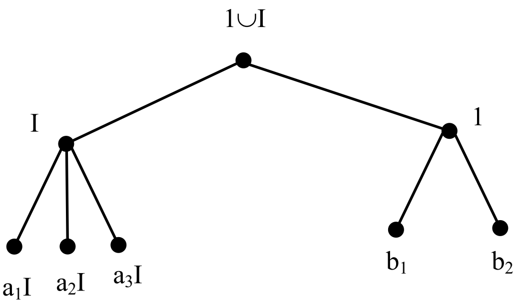


Figure 2.6

The above neutrosophic semilattice is not pure for it contains 1, b_1 and b_2 as elements of S(P).

Thus the notion of neutrosophic pure semilattice is one in which all elements of S(P) are only neutrosophic elements.

Example 2.7. Let {S(P), ∪} be the pure vertex neutrosophic semilattice whose figure is given in the following.

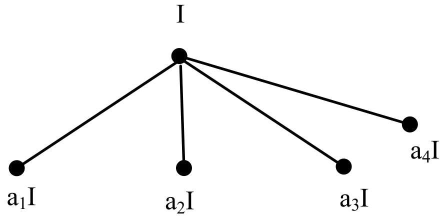


Figure 2.7

Next the notion of edge neutrosophic semilattice under {S, ∪} and {S, ∩} are defined and described in the following.

Definition 2.2: *{S, ∪} or {S, ∩} is defined to be the edge neutrosophic semilattice if all elements in S are real and is a partial ordered set. There are some edges which are neutrosophic are indeterminate.*

Examples of them are given in the following.

Example 2.8. Let {S, ∪} be the edge neutrosophic semilattice which is given by the following figure.

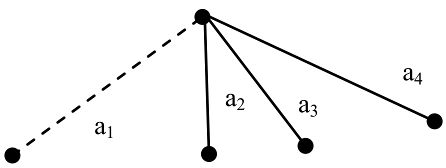


Figure 2.8

Example 2.9. The following figure gives the edge neutrosophic semilattice {S, ∩}.

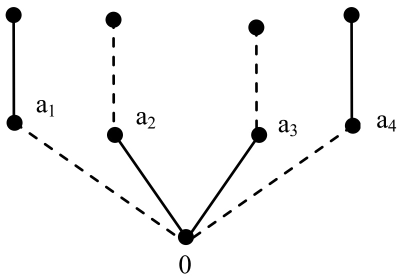


Figure 2.9

Example 2.10. Let {S, ∩} be the edge neutrosophic semilattice given by the following figure.

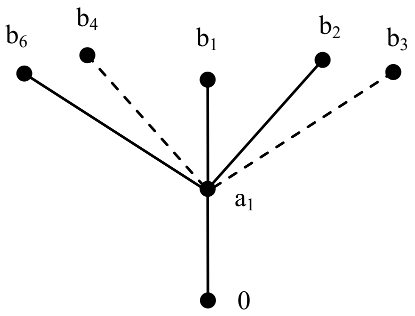


Figure 2.10

Next the notion of pure neutrosophic semilattice is defined in the following.

Definition 2.3: *Let {S, ∪} (or {S, ∩}) be the partial ordered set all of its vertes elements are neutrosophic and if every edge is also neutrosophic or an indeterminacy then {S, ∪} (or {S, ∩}) is defined as the pure neutrosophic semilattice.*

Examples of pure neutrosophic semilattices is given below.

Example 2.11: Let {S(P), ∪} be the pure neutrosophic semilattice the figure of which is as follows;

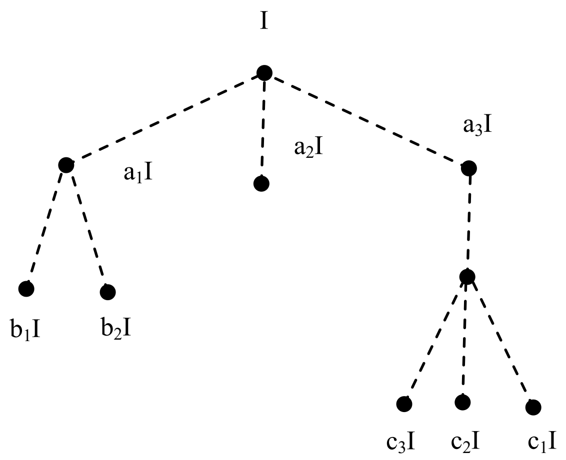


Figure 2.11

The above semilattice is a pure neutrosophic semilattice whose cardinality is 10.

Example 2.12. {S(P), ∩} be the pure neutrosophic semilattice whose Hasse diagram is as follows.

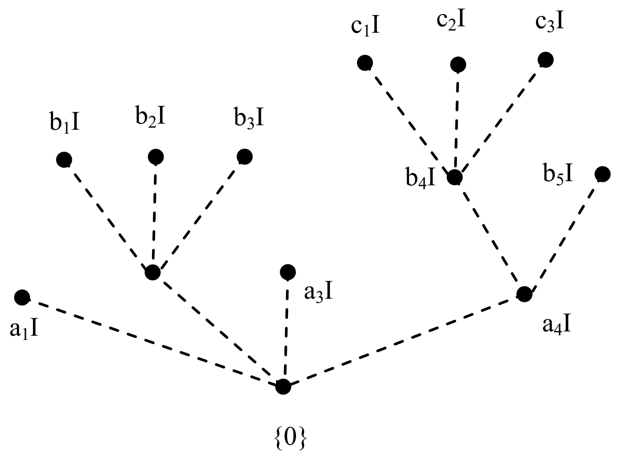


Figure 2.12

This is again a pure neutrosophic semilattice of order 13.

Now having seen examples of pure vertex neutrosophic semilattices, edge neutrosophic semilattices and pure neutrosphic semilattices, examples of neutrosophic subsemilattices are provided.

Example 2.13. Let {S(P), ∪} be the vertex neutrosophic semilattice whose Hasse diagram is as follows.

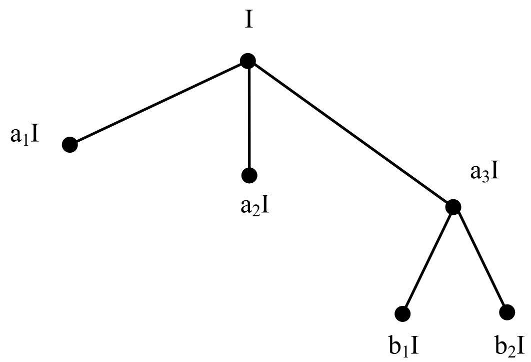
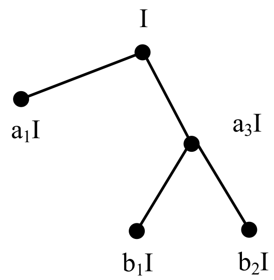
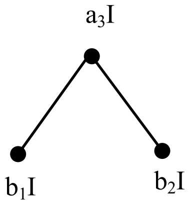


Figure 2.13

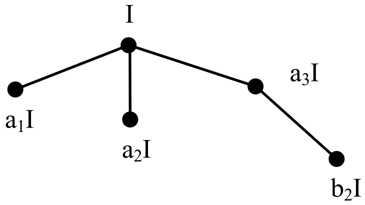
Clearly the following figures gives the vertex neutrosophic subsemilattice.



(a)



(b)



(c)

Figure 2.14 (a), (b) (c)

Next examples of edge neutrosophic semilattice and their subsemilattices are obtained.

Example 2.14. Let {S(P), $\cap$} be the edge neutrosophic semilattice whose figure is given below.

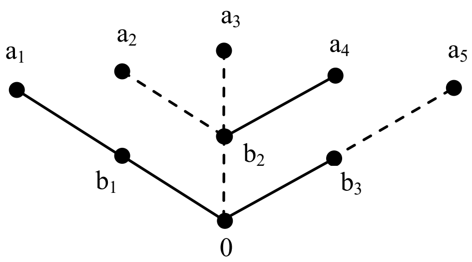
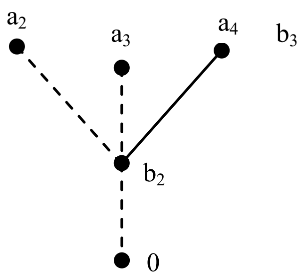


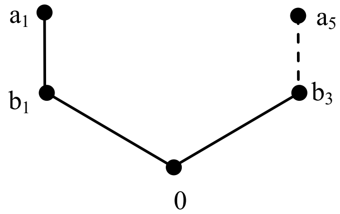
Figure 2.15

The subsemilattices of {S(P), $\cap$} need not in general be edge neutrosophic subsemilattices. They can be usual subsemilattices as well as edge neutrosophic subsemilattices.

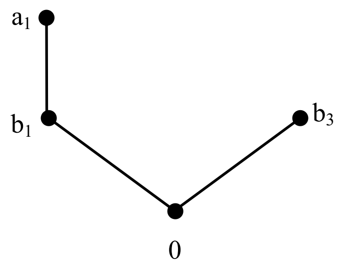
The figures associated with subsemilattices of {S(P), $\cap$} is as follows.



(a)



(b)



(c)

Figure 2.16 (a) (b) (c)

Fig 2.16(a) and (b) are edge neutrosophic subsemilattices whereas 2.16(c) is a usual subsemilattice.

Next the subsemilattices of a pure neutrosophic semilattices is described by the following example.

Example 2.15: Let *{S(P),* $\cap$*}* be the pure neutrosophic semilattice whose figure is given below.

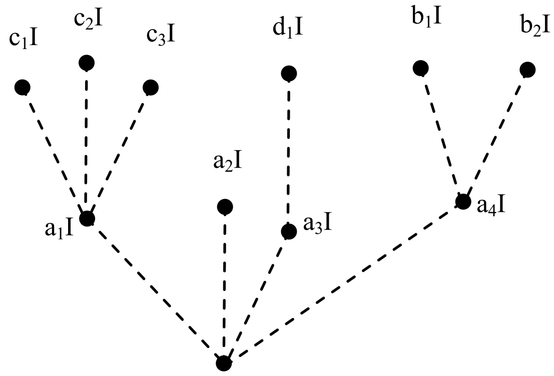


Figure 2.17

All subsemilattices of S(P) are pure neutrosophic subsemilattices only.

In view of this the following theorem is proved.

Theorem 2.1: *Let {S(P),* $\cup$*} (or {S(P),* $\cap$*} be the pure neutrosophic semilattice. Then every subsemilattice of {S(P),* $\cup$*} or {(S(P),* $\cap$*}) are also pure neutrosophic.*

Proof: Follows from the fact in a pure neutrosophic semilattice all vertices and edges are neutrosophic. Hence the claim.

Proposition 2.1: *Let {S(P), $\cup$} (or {S(P), $\cap$}) be a edge neutrosophic semilattice. Every subsemilattice need not be a edge neutrosophic subsemilattice.*

Proof: Follows from the fact a edge neutrosophic semilattice can have subsemilattices which are not in general neutrosophic edges subsemilattice.

Proposition 2.2: *Let {S(P), $\cup$} (or{S(P),$\cap$}) be a neutrosophic semilattice. Every subsemilattice of a neutrosophic semilattice need not be a neutrosophic subsemilattice.*

Proof: Evident from the examples given.

3 Application of Neutrosophic Semilattices

In this section applications of neutrosophic semilattices is briefly given.

Infact all neutrosophic semilattices are neutrosophic trees. So these will find application in all places where neutrosophic trees find their applications.

So one can apply these neutrosophic semilattices when the research or the investigator feels that indeterminancy is present in that analysis.

Conclusion

For the first time the new notion of neutrosophic semilattices is introduced and their properties are discussed. These neutrosophic lattices are also neutrosophic tress and they find their application in data mining and sorting.

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Liminality and Neutrosophy

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Abstract. This study is an application of Neutrosophy in the sphere of liminality. First, the aim of this study is to underline the importance of the concept of Neutrosophy that was introduced by the professor Florentin Smarandache correlated with the concept of the liminality. According to Arnold Van Gennep and Victor Turner, in the liminality, the rituals are conducted to put the people in an ambiguous state where everything there is not true or neither false and meaning that the threshold state is neutral. Rituals, myths or rites are representing indeed a form of communication, but on an unclear level, determined by the uncertainty. Liminality has a part which is working under the uncertainty's rules of Neutrosophy: when a person is participating in the rituals, he is searching a truth and risk a false. This means that the threshold state is improving the perception of the people from the moment when starts a ritual. But the threshold state can be generated also by the media. Rituals of the mass media are created in order to change the society's perception, persuading the idea of what is true and false.

Keywords: Neutrosophy, liminality, rituals, uncertainty, media.

1 Introduction

Professor Florentin Smarandache introduced the concept of Neutrosophy as part of thiking discourse which studies what is the nature of neutralities (Smarandache, 2002; Smarandache, 2010; Smarandache, 2015). The uncertainty between the two rituals is correlated with the people's participation. Our society is conducted by a series of opinions and belief, however, they are not only true and false, but also they have a numerous series of neutral variables. The most important element in the society is represented by the man. He is the entropy inductor (Smarandache & Vladutescu, 2014; Smarandache, Vladutescu, Dima & Voinea, 2015). In the relation with other persons, the man becomes more aware about their opinion and he respects them in order to receive the same. In our society, the probabilities of the neutral variables are determined by the conditions of what is true and false. In every aspect of life's emergence, we find communication starting with our actions and ending with our thoughts. Neutrosophy is a part of dialectics which reveals paradoxes and logics. From the moment when a person is born, he enters into reality and generates communication from every action. A communication act is created by a ritual. This religious act can be divided through a threshold state where everything is ambiguous or neutral. At the moment when a person is entering a liminal space, he will not know the actual situation. He will be put under a series of neutral variables dedicated to destabilizing the person's entire world. Describing the necessity to attend a daily communication is represented in fact the manifestation of a threshold state in order to be a part of the rituals. Based on these things, a person is condemned to become aware of the uncertainty that will come. In the threshold state, all the neutralities are transformed in order to create an exit from the liminality; the ritual is not finished until the knowledge conquers the ambiguity. The consequence of not knowing what is happening in the ambiguous state –the liminality- determines the transparency between false and truth. There are some philosophers who wrote about the thesis and antithesis as: Georg Wilhelm Friedrich Hegel (1770-1831), Karl Marx (1818-1883), Friedrich Engels (1820-1895), Immanuel Kant (1724-1804), Johann Gottlieb Fichte (1762- 1814), and Thomas Schelling (born 1921). If we think about the question of Aristotle "what is the nature of things?", we may find different responses based on science or religion. Our society creates every day new form of communication and improves the technology step by step. Regarding on the communication, through the actions, a man is generating emotions that can help supporting the basis of human kind. Therefore, in the daily rituals, there is a neutral connection that sustains the ambiguity. Liminality represents more than a simple concept. It's the important factor that can transform the perception of people. In the liminal stage or threshold state, the people's absence of knowledge embraces the need for information and communication, even everything there can be true or false. Neither of the activities that are taking place there are particularly true. On the threshold stage, the information is neutral. The communication in the rituals is consisting in a set of neutrosophic meanings. In a ritual, the communication between the person and the others creates a bond based on neutral manifestation. Everything here doesn't have any particularly elements of sensations or feelings, except the uncertainty. A person caught in the threshold stage will receive a feedback from the initiator of the rituals after the steps will be finished. Society is creating rituals in order to involve people in solid action with the purpose to persuade the population's mind. People are participating in a ritual without comprehend what has happened to them (Beech, 2010). So, the rituals are representing more than a choice, some of them are instead a cultural obligation. Rituals involve the participation of every person. But, in the ritual, when a person doesn't understand the meaning of threshold state regarding his transformation into a new person, the situation becomes more incomprehensible. His ambition to achieve the final result of the liminal space, determines the person to act properly, even he is in an ambiguous state. However, in this case, the people who participate into rituals are allocating a very large surplus of energy in order to understand the meaning of it. Based on this, liminality defines the actual situation as sacral event where the knowledge is persisting as a secondary act. The first act is all about the power to dispose the ambiguous state continue with the second act that insists on developing the knowledge after the ritual is over. Arnold Van Gennep introduced the concept of liminality to mark the importance of people's metamorphosis.

Liminality is a threshold state or a bond between two worlds where everything we see is just a vapid perception of ours. Nothing that we see in the threshold state is true or false. Victor Turner (1969, 1977) claimed that the liminality doesn't have a limited period of time, it depends on major factors, for example: when we are taking an exam, we participate in a common ritual for the students, however, the time here is something we all know, 1 or 2 hours. This means that the liminal space lasts 1 or 2 hours. At this time, we are caught under some rules that can have the power to subordinate us. If we don't act like we are supposed to, we may lose to possibility to take the exam and go further with our lives. And we may be caught again in the liminality, but this time without the possibility to know exactly how long it will last the threshold state. We act properly; we get out of the threshold state faster. It's simple. But in this period, we don't have the chance to know exactly if we chose the correct answers (Ślusarczyk & Broniszewska, 2014).

The threshold state has numerous neutral values of exam's answer, determined by the uncertainty. Here, we are condemned to a series of manifestation in order to make us to be seen as pawns in a strange round of chess. If we are just pawns, it means that the rest of the characters are representing the leaders (Voinea, 2013; Stanescu, 2015; Voinea, 2015). However, in this case, we have a series of moves limited. They determine our idea that correlate with the strict rules that game has.

2 Neutrosophy versus Liminality

The concept of liminality can be determined by neutrosophy, because the uncertainty that is maintained on an unknown level. When a ritual startthe person who participate in it, must relinquish his past life and pass through the threshold state in order to start a new life. An important factor about the threshold state is that here, the person can be seen as equal by the members of the community, but with one differentiated conditions, it doesn't have any rights. In the liminality, a person is facing three stages ; the first one is separated from his life and common things where he is induced in a new world, apart from what he knew. Here, in the same thing he is introduced in an ambiguous state, but he remained watchful with what it has happened. Nothing about what was the meaning of his life is now true. In fact, the uncertainty remains a long period. In the liminal space, the individual starts to ask himself question about what is the difference between true and false or how his life maybe was a lie until this moment. Depending of the ritual that determines the individual to conquer a new step in his life, the threshold state becomes his new home. For example: the enter in a political party represents a ritual. The determination of the person to become a member of a political party has to be much clarified in order to obtain this statute. Or another example, we can find Van Gennep's traditional society in the tribes from Africa. There, people literally renounce their values and were put under some rigorous rules with the purpose to metamorphosis their life (Cerban & Panea, 2011). Therefore, in that limited or unlimited period of time, the liminal space inducted the future members to an unknown world where they didn't do know what is true or false. The series of neutral values are the one responsible for the people's hunger in finding their self or finding the truth.

The second stage of the liminality is determined by the possibility of the future members to adapt to their new conduct of life. He becomes aware of the new truth and can see the numerous possibilities that he has in the threshold state. In between true and false are a series of values that are not overlapped with each other. In fact, the true and false can't be a presence in the liminality. A person caught in the threshold state will approach to what he finds unclear in order to achieve the knowledge. It has resulted, that the uncertainty prevails for the liminal space. He accumulates the necessary information from the group and improve our values, norms and rules in the form of their. The person in the threshold state is there to understand the reality better. He recreates his own life in function of the new set of other's values (Budica, Busu, Dumitru & Purcaru, 2015).

The final stage of the ritual is the pre-integration where the person can be seen as more than prepared to go out from the liminality. But how we can say that he is prepared? The change must come from him. This time he is leaving the uncertainty and knows exactly what he wants without the possibility to be put again questions his choices (Grabara, Kolcun & Kot, 2014).

According to Victor Turner's idea of liminality, even our common things like going to school or taking an exam are in facts rituals (Turner, 1977). So, everything we do is an on and on ritual. The incomprehensible becomes understood at every final destination of the liminality. However, at the final stage of the rituals, it appears another. We can say that the life is a circle composed of rituals: when one is finished, the other starts.

Liminality is a part of neutrosohpy; it is constructed with different forms, but at the end all the rituals have the same path. After every ritual, a person is gaining knowledge, he understands the way of life and for the most of the time, he is the one who enters another ritual. Every ritual which a person is passing, it means a gain for the human kind (Negrea, 2013 ; Dima, Grabara & Vladutescu, 2014 ; Negrea, 2015).

In our modern society, the time goes faster and faster and the people are changing unwillingly. Even if the concept of liminality introduced by Van Gennep was for the traditional society and Victor Turner named liminoid for our modern times, the idea remained the same. The Turner's term "liminoid" (Turner, 1977) didn't have much success, many scholars named the modern rituals as liminality or liminal space. In fact, both represent the path that every ritual has, starting with the peoples' wishes to change and entering in the liminality and finished with the perception of the participations changed. Everything is changing, even our life.

The determination of our perception is based on the mass media. Media is creating the society and has the power to influence it how it wants. Mass media are developing rituals through television, radio and internet (social media). The last is seen as a giant source of information, but the real truth about what is behind the scene is unknown by the media's audience. The daily media rituals are not only put us to liminality, but also to the neutrosophic theory. When people are watching the daily news, they are entering into a liminal space where everything they see may seem true, but if we analyze the situation carefully, we can discover that everything that the media generates is composed of neutral sets. Nothing we see on television is true or false (Coman, 1994; Coman, 2008 ; Thomassen, 2009). The story that news tell are more particularly between true and false, for example: if the news is about a terrible accident where 2 or 3 people were wounded, but they are out of danger, the audience will receive an information that these people are seriously hurt and they are in danger. Media system has the power to improve its information depending on the audience's impact (Ionescu, 2013).

3 Conclusion

Liminality in the Neutrosophy generates the idea that the uncertainty can be exceeded by knowledge only when a ritual is finished. The threshold state is metamorphosis the perception of the people through rituals, determined unclear moments at that time. Every ritual is ambiguous and it means that in the first moment when a person is entering in the liminality, their knowledge becomes uncertainty. After the ritual is finished and the exit of threshold state comes, the uncertainty becomes knowledge. Our society is conducted by rituals every day: starting with going to work or having an exam to entering in a political party and so on. We can say that our society is conducted by a cycle of rituals. Through mass media's rituals, society is changing every day.

The true and false state cannot be sustained by liminality, because the threshold state generated only neutral values and underlines the power of uncertainty in the people's mind through rituals.

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Application of Extended Fuzzy Programming Technique to a real life Transportation Problem in Neutrosophic environment

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Abstract. Here This paper focuses on solving the transportation problems with neutrosophic data for the first time. The indeterminacy factor has been considered in Transportation Problems (TP). The two methods of linear programming – Fuzzy Linear Programming (FLP) and Crisp Linear Programming (CLP) are discussed with reference to neutrosophic transportation problems. The first method uses the membership, non-membership and indeterminacy degrees separately to find the crisp solution using the Fuzzy Programming Technique and then the optimal solution is calculated in terms of neutrosophic data with the help of defined cost membership functions. The satisfaction degree is then calculated to check the better solution. The second method directly solves the TP to find crisp solution considering a single objective function. The cost objective function is taken as neutrosophic data and the methods have been used as such for the first time. Both the methods have been illustrated with the help of a numerical example and these are then applied to solve a real life multi - objective and multi-index transportation problem. Finally the results are compared.

Keywords: Neutrosophic Transportation Problem; Fuzzy Linear Programming; Crisp Linear Programming; Fuzzy Programming Technique; indeterminacy degree

1 Introduction

The basic transportation problem was originally developed by Hitchcock [1]. There are several classical methods to solve such transportation problems where data is given in a precise way. But in real life transportation problems, data may not be known with certainty. In such cases, the imprecise data can be considered as interval valued or fuzzy data. Fuzzy set theory was introduced by Zadeh [2]. Zimmermann [3] introduced fuzzy linear programming (LP) problems. Zimmermann [4] considered LP with fuzzy goal and fuzzy constraints and used linear membership function and min operator as an aggregator of these functions. Thus Fuzzy Linear Programming (FLP) problem was formulated. Further, Fuzzy set theory was applied to solve LPP with several objectives functions. The fuzzified constraint and objective functions were used to solve the multi-objective linear programming problems. Chanas [5] focused on Fuzzy Linear Programming model for solving transportation problems with crisp cost coefficients and fuzzy supply and demand values. Chanas and Kuchta [6] developed an algorithm for the optimal solution of TP with fuzzy coefficients which are expressed as L-R fuzzy numbers. Chanas and Kuchta [7] developed an algorithm for solving integer fuzzy transportation problem with fuzzy supply and demand. Bit and Biswal [8] applied the fuzzy programming technique with linear membership function to solve Multi-objective transportation problem (MOTP). Bit and Biswal [9] proposed an additive fuzzy programming model that considered weights and priorities for all non equivalent objectives for the transportation planning problems. Li and Lai [10] developed a fuzzy compromise programming method to obtain a non-dominated compromise solution to the MOTP in which various objectives were synthetically considered with marginal evaluation for individual objectives and the global evaluation for all objective functions. A real life multi-index multi-objective transportation problem was solved by Kour, Mukherjee and Basu in [11],[12],[13],[14] and [15] using different approaches. Intuitionistic fuzzy sets (IFS) were introduced as generalization of fuzzy set (FS). Here membership and non-membership degree were used instead of exact numbers. Intuitionistic fuzzy sets (IFSs) were introduced by Atanassov [16]. Atanassov & Gargov [17] introduced the concept of interval-valued intuitionistic fuzzy sets (IVIFSs) as a further generalization of that of IFSs. Atanassov [18] also defined some operational laws of IVIFSs. Angelov [19] reformulated the optimization problems in an intuitionistic fuzzy environment. Several works have been done taking the triangular and trapezoidal intuitionistic fuzzy number. Gani and Abbas [20] proposed a new method for intuitionistic fuzzy transportation problem using triangular intuitionistic fuzzy number. Hussain and Kumar [21] applied the fuzzy zero point method to find optimal solution of intuitionistic fuzzy transportation problems Antony [22] also developed the VAMs method for TP for triangular intuitionistic fuzzy number. Aggarwal and Gupta [23] solved the TP for generalized trapezoidal intuitionistic fuzzy number by ranking method. P. P. Angelov first introduced the Intuitionistic fuzzy optimization (IFO) in his paper [19] and solved the transportation problem with crisp data by this method. The concept of Neutrosophic set was introduced as a generalization of crisp, fuzzy, intuitionistic, interval valued Intuitionistic Fuzzy number by Smarandache[24]. The Indeterminacy function (I) was added to the two available parameters: Truth (T) and Falsity (F) membership functions. In Neutrosophic Set, the indeterminacy is quantified explicitly and truth - membership, indeterminacy membership and false - membership are completely independent. In intuitionistic fuzzy sets, the indeterminacy is 1- T (x) - F (x) (i.e. hesitancy or unknown degree) by default. In Neutrosophy, the indeterminacy membership (I(x)) is introduced as a new subcomponent so as to include the degree to which the decision maker is not sure. This type of treatment of the problem was out of scope of intuitionistic fuzzy sets. Wang et al. [25] introduced the concept of single valued neutrosophic set (SVNS).

The present paper presents the solution of transportation problems with neutrosophic data using linear programming methods. It deals with cost objective function as neutrosophic data and the Neutrosophic TP has been solved using two methods. In the first method, fuzzy linear programming (FLP) has been extended for the neutrosophic data and the second method uses the crisp linear programming method (CLP).

The formulations and solutions are illustrated with the help of solved example and then the results are compared. The uncertainties of the real life problems are considered in the form of neutrosophic data. In transportation problems, the cost of transportation, the demand and the supply may not be known exactly as crisp numbers. Thus the uncertainties can be considered in terms of their degrees of acceptance, degrees of indeterminacy and degrees of rejection. That is, neutrosophic fuzzy numbers can be used for representing the imprecise data of cost of transportation or demand or supply or all in a transportation problem. This can be explained with the help of an example. If the transportation cost is taken in terms of the neutrosophic fuzzy number (0.8,0.1,0.2), that means the degree of acceptance of the available cost is 0.8 , degrees of indeterminacy is 0.1 while the degree of rejection of the available cost is 0.2.

Finally the methods are applied to solve a real life multi-objective and multi-index neutrosophic transportation problem for the first time. The problem is solved to optimize the three objectives simultaneously namely, transportation cost, deterioration rate and underused capacity with neutrosophic data. The paper presents a better application of the method for multi-objective transportation problems.

2 Preliminaries

2.1 Single Valued Neutrosophic Set (SVNS)

An SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$ for each point x in X, $T_A(x)$, $I_A(x)$, $F_A(x)$ ϵ [0, 1]. (Wang et al.[25]

When X is continuous, an SVNS A can be written as

$$A = \int_X \frac{\langle T_A(x), I_A(x), F_A(x)\rangle}{x}, x \in X$$

When X is discrete, an SVNS A can be written as

$$A = \sum_{i=1}^{n} \frac{\langle T_A(x_i), I_A(x_i), F_A(x_i)\rangle}{x_i}, x_i \in X$$

3 Problem description and methodology

3.1 Problem Description

The transportation problem is meant for minimization of transportation cost from different sources to different destinations.

- **Classical transportation problems:**
  In the classical transportation problem cost objective function and the constraints are considered as crisp values. Therefore it is required to calculate the optimal solution which minimizes the cost objective functions and satisfies all the constraints.

Minimize $f\ (x)$

Subject to $\quad g_j(x) \le 0, j = 1,2,.....q$

(1)

- **Fuzzy transportation problem:** Later on the fuzzy transportation problem was introduced [5,6,7] and used in further works[11-15,22,26]. The degree of satisfaction of the objective function and the constraints is maximized to find the optimal solution.
- **Intuitionistic fuzzy transportation problem:** Then the intuitionistic fuzzy transportation problem (IFTP) was considered [19, 20, 21, 23]. In such case the degree of rejection $\upsilon_i(x)$ is also considered along with the degree of acceptance $\mu_i(x)$ of the cost objective function and the constraints. The degree of acceptance is maximized and the degree of rejection is minimized to find the optimal solution in such problems.
- **Neutrosophic transportation problem:** In a transportation problems with neutrosophic data , the indeterminacy factor has been considered for the first time. The degree of indeterminacy $r_i(x)$ was also considered along wtith the two available parameters, degree of acceptance $\mu_i(x)$ and degree of rejection $\upsilon_i(x)$ of the cost objective function and the constraints. The problem is to maximize degree of acceptance and minimize the degree of rejection and indeterminacy.

Model 1. Fuzzy Linear Programming Model (for Neutrosophic data):

For single-objective TP

Maximize $Z_1 = \sum\sum \mu_{ij} x_{ij}$

Minimize $Z_2 = \sum\sum r_{ij} x_{ij}$

Minimize $Z_3 = \sum\sum \upsilon_{ij} x_{ij}$

subject to $0 \le \mu(x), r(x), \upsilon(x) \le 1$,

$\sum_j x_{ij} = S_i$ where S_i denotes the supply of source i,

$\sum_i x_{ij} = D_j$ where D_j denotes the demand of destination j,

$x_{ij} \ge 0$

(2)

For Multi-objective TP, we obtain a set of similar three equations for each of the objective functions

Model 2. Crisp Linear Programming Model (for Neutrosophic data)

For Single objective transportation problems, the model is

Maximize $Z = \sum\sum(\mu_{ij} - \upsilon_{ij} - r_{ij})x_{ij}$

subject to $0 \le \mu(x), r(x), \upsilon(x) \le 1$, and other constraints mentioned in Equation(2) (3).

For Multi-objective transportation problems, we obtain a set of similar equations for each objective function.

3.2 Methodology

3.2.1 Fuzzy Linear Pogramming

The Transportation problem with neutrosophic data has been formulated as a multi-objective transportation problem as in Model 1 and has been solved by Fuzzy Linear Programming Technique (Das[26], Zimmermann [3]).

Extended Fuzzy Programming Technique

Step 1: Solve the multi-objective transportation problem as a single objective transportation problem using each time only one objective and ignoring others.

Step 2: From the results of Step 1, determine the corresponding values for every objective at each solution derived.

Then find the lower and upper bounds , Z_k^L, and Z_k^U (k=1,2,3,….,K).

Step 3 **: Linear Membership Function**

A Linear membership function $\mu_{k(x)}$ corresponding to k^{th} objective for the minimization problem is defined as

$$\mu_k(x) = \begin{cases} 1 & if \quad Z_k \le Z_k^L, \\ 1 - \dfrac{Z_k - Z_k^L}{Z_k^U - Z_k^L} & if Z_k^L < Z_k < Z_k^U \\ 0 & if \quad Z_k \ge Z_k^U \end{cases}$$

(4)

Similarly, a linear membership function can be defined for maximization problem as

$$\mu_k(x) = \begin{cases} 0 & if \quad Z_k \le Z_k^L, \\ 1 - \dfrac{Z_k - Z_k^U}{Z_k^U - Z_k^L} & if Z_k^L < Z_k < Z_k^U \\ 1 & if \quad Z_k \ge Z_k^U \end{cases}$$

(5)

The linear programming problem can further be simplified as in Model 3:

Model 3:

Maximize λ

subject to $Z_k + \lambda(Z_k^U - Z_k^L) \le Z_k^U$ (6)

for minimization problem and

$$Z_k + \lambda(Z_k^U - Z_k^L) \ge Z_k^L,$$

for maximization problem (7)

with the given constraints and non-negativity restriction as in Model 1 and $\lambda \ge 0$ (8)

Thus the Step 3 gives the values of the three objective functions, Z_1, Z_2 and Z_3 as in Model 1.

Step4:

$$Z = Z_1 - Z_2 - Z_3 \tag{9}$$

This provides the crisp optimal value for the objective functions. Then using the definition of cost membership function, the satisfaction, indeterminacy and rejection degrees of membership function of the solution are obtained as Single Valued Neutrosophic Set (SVNS)

3.2.2 Crisp Linear Pogramming

The Model 1 can further be formulated as a single-objective linear programming problem as in Model 2 and is solved as usual by standard software. The solution gives the optimal value of cost objective function Z as a crisp value. For multi-objective transportation problems, it forms a set of objective functions in equations which can be solved by fuzzy programming technique and the optimal solution can be obtained as crisp value for each objective function.

4 Numerical Illustration

4.2.1 Example 1

The problem is taken as a neutrosophic transportation problem (NTP) in which each transportation cost is taken as neutrosophic data representing the degree of acceptance, degree of indeterminacy and degree of rejection of the cost as in Table 1. The demand and the capacity are considered as crisp values.

	Market 1	Market 2	Market 3	Market 4	Capacity (S_i)
Port 1	(0.6,0.1,0.2)	(0.7,0.2,0.1)	(0.3,0.3,0.1)	(0.8,0.1,0.1)	400
Port 2	(0.5,0.2,0.3)	(0.4,0.1,0.1)	(0.5,0.3,0.1)	(0.3,0.3.0.2)	150
Port 3	(0.4,0.3,0.2)	(0.3,0.2,0.2)	(0.6,0.3,0.1)	(0.7,0.3,0.2)	300
Demand (D_j)	200	200	100	350	

(6)

Table 1: Data for NFTP .

The objective for this problem can be determined by degree of acceptance $\mu_o(x)$, degree of indeterminacy $r_o(x)$ and degree of rejection $\upsilon_o(x)$ of the cost function defined as follows:

$$\mu_o(x) = \begin{cases} 1, & \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} < 200 \\ \dfrac{(350 - \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij})}{150}, & 200 \le \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} \le 350 \\ 0, & \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} > 350 \end{cases} \tag{10}$$

$$
r_o(x)=\begin{cases}0, & \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij}<250\\ \dfrac{(\sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij}-250)^2}{10000}, & 250\le \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij}\le 350\\ 1, & \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij}>350\end{cases} \tag{11}
$$

$$
\upsilon_o(x)=\begin{cases}0, & \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij}<200\\ \dfrac{(\sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij}-200)^2}{22500}, & 200\le \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij}\le 350\\ 1, & \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij}>350\end{cases} \tag{12}
$$

where costs are considered in terms of thousand dollars.

4.2 Solution

The given problem is a neutrosophic transportation problem (NTP) and is solved by the above mentioned methods and the results are obtained.

Solution with data from Table 1 by the method based on FLP

Substituting the neutrosophic data from Table 1 in the Model 1, we get three different objective functions as

Maximize

$$Z_1 = 0.6x_{11} + 0.7x_{12} + 0.3x_{13} + 0.8x_{14} + 0.5x_{21} + 0.4x_{22} + 0.5x_{23} + 0.3x_{24} + 0.4x_{31} + 0.3x_{32} + 0.6x_{33} + 0.7x_{34}$$

Minimize

$$Z_2 = 0.1x_{11} + 0.2x_{12} + 0.3x_{13} + 0.1x_{14} + 0.2x_{21} + 0.1x_{22} + 0.3x_{23} + 0.3x_{24} + 0.3x_{31} + 0.2x_{32} + 0.3x_{33} + 0.3x_{34}$$

Minimize

$$Z_3 = 0.2x_{11} + 0.1x_{12} + 0.1x_{13} + 0.1x_{14} + 0.3x_{21} + 0.1x_{22} + 0.1x_{23} + 0.2x_{24} + 0.2x_{31} + 0.2x_{32} + 0.1x_{33} + 0.2x_{34}$$

subject to

$$\sum_j x_{ij} = S_i$$

where S_i denotes the supply of source i given in Table 1

$$\sum_i x_{ij} = D_j$$

where D_j denotes the demand of destination j given in Table 1

$$x_{ij} \ge 0 \tag{13}$$

Step 1 : The problem is solved considering as single objective taking only one objective function and neglecting others. The solution sets are obtained as:

$Z_1 = 565$, $Z_2 = 180$, $Z_3 = 140$,

Step 2 : For each solution set , the values for the other two objective functions are obtained as:

$Z_1 = 565$ $Z_1 = 505$ (for Z_2 solution set), $Z_1 = 515$ (for Z_3 solution set)

$Z_2 = 140$, $Z_2 = 180$ (for Z_1 solution set), $Z_2 = 150$ (for Z_3 solution set)

$Z_3 = 105$, $Z_3 = 140$ (for Z_1 solution set), $Z_3 = 110$ (for Z_2 solution set)

For each objective, the best and worst values are given as

$Z_1^U = 565$, $Z_1^L = 505$, $Z_2^U = 180$, $Z_2^L = 140$, $Z_3^U = 140$, $Z_3^L = 105$,

Step 3 : Using the values obtained in Step 2 in the Equations (6) and (7) obtained from Model 3 of Section 4.2, the final solution is obtained as

$\lambda = 0.9090909$, $Z_1 = 508.64$, $Z_2 = 143.64$, $Z_3 = 108.18$

Step 4: Using the values obtained in Equation (9), $Z = 256.82$

Also the degree of acceptance, indeterminacy and rejection of cost objective functions are obtained using Equations (10),(11) and (12) as

$$\mu_o = 0.62, r_o = 0.0047, \upsilon_o = 0.14$$

i.e. $(\mu_o, r_o, \upsilon_o) = (0.62, 0.0047, 0.14)$

Solution with data from Table 1 by the method based on CLP

Substituting the neutrosophic data from Table 1in the Model 2, we get

Minimize

$$Z = 0.3x_{11} + 0.4x_{12} - 0.9x_{13} + 0.6x_{14}$$
$$+ 0x_{21} + 0.2x_{22} + 0.1x_{23} - 0.8x_{24}$$
$$- 0.1x_{31} - 0.1x_{32} + 0.2x_{33} + 0.2x_{34}$$

subject to all the constraints in Equation (13)

The transportation problem is solved as single objective TP by crisp linear programming as in Model 2 and the crisp optimal solution is obtained as $Z = 260$.

The degree of acceptance, indeterminacy and rejection of cost objective functions are obtained as

$$\mu_o = 0.6, r_o = 0.01, \upsilon_o = 0.16$$

i.e. $(\mu_o, r_o, \upsilon_o) = (0.6, 0.01, 0.16)$

5 Real life Illustration

5.1 Real life multi-objective multi-index transportation problem

To illustrate the application of the proposed approach for a real life multi-objective multi-index transportation problem, following numerical example from Kour, Mukherjee and Basu [11] is considered, previously taken as approximate past records from DSP Plant, Durgapur,West Bengal, INDIA.

The problem deals with the solution of the multi-objective multi-index real life transportation problem focusing on the minimization of the transportation cost, deterioration rate and underused capacity of the transported raw materials like coal, iron ore, etc from different sources to different destination sites at Durgapur Steel Plant (DSP) by different transportation modes like train, trucks, etc. The problem is formulated by taking different parameters in the objective function as neutrosophic data and supply and demand as crisp numbers.

Consider a problem in which we have three raw materials (m=3) i.e. q=1(Coal), 2(Iron -ore), 3(Limestone).These raw materials are transported from different i^{th} sources to j^{th} destination sites by different transportation modes 'h' where h=1(train), 2(truck) as per Table 2.

Raw materials	Sources	Destinations	Mode of transportation
q=1	i=1,2	j=1,2	h=1
q=2	i=1,2,3	j=1,2	h=1,2
q=3	i=1,2,3,4,5	j=1	h=1

Table 2 Number of raw materials, sources, destinations and mode of transportation

Then we are considering the problem as a neutrosophic transportation problem (NTP) in which each transportation cost is taken as neutrosophic data representing the degree of acceptance, indeterminacy and rejection of the cost. The demand and the supply are considered as crisp numbers. i.e. Transportation Cost functions as C_{ijh}^q (C_{ij1}^1 , C_{ij1}^2 C_{ij2}^2, C_{ij1}^3), and other objective functions for Deterioration rate and underused capacity in matrix form. The data are given below.

$$C_{ij1}^1 = \begin{bmatrix} [0.8,0.01,0.15] & [0.6,0.1,0.3] \\ [0.5,0.02,0.3] & [0.9,0.01,0.01] \end{bmatrix}_{h=1}$$

$$C_{ij1}^2 = \begin{bmatrix} [0.5,0.01,0.4] & [0.75,0.01,0.02] \\ [0.55,0.02,0.3] & [0.4,0.1,0.2] \\ [0.8,0.01,0.1] & [0.9,0.02,0.03] \end{bmatrix}_{h=1}$$

$$C_{ij2}^{2}=\begin{bmatrix}[0.4,0.1,0.3] & [0.5,0.2,0.3]\\ [0.6,0.02,0.25] & [0.6,0.1,0.3]\\ [0.8,0.01,0.12] & [0.9,0.01,0.01]\end{bmatrix}_{h=2}$$

$$C_{ij1}^{3}=\begin{bmatrix}[0.85,0.01,0.02]\\ [0.78,0.02,0.2]\\ [\ 0.8,0.01,0.1]\\ [0.7,0.01,0.15]\\ [0.5,0.02,0.3]\end{bmatrix}_{h=1}$$

$$R_{ij1}^{1}=\begin{bmatrix}[0.6,0.1,0.2] & [0.5,0.1,0.3]\\ [0.8,0.01,0.1] & [0.55,0.02,0.2\end{bmatrix}_{h=1}$$

$$R_{ij1}^{2}=\begin{bmatrix}[0.6,0.1,0.3] & [0.9,0.01,0.1]\\ [0.8,0.1,0.2] & [0.7,0.01,0.2]\\ [0.4,0.3,0.1] & [0.6,0.2,0.1]\end{bmatrix}_{h=1}$$

$$R_{ij2}^{2}=\begin{bmatrix}[0.6,0.01,0.2] & [0.95,0.01,0.02]\\ [0.9,0.02,0.1] & [0.8,0.01,0.2]\\ [0.62,0.02,0.2] & [0.7,0.1,0.2]\end{bmatrix}_{h=2}$$

$$R_{ij1}^{3}=\begin{bmatrix}[0.9,0.02,0.1]\\ [0.4,0.1,0.3]\\ [0.6,0.02,0.2]\\ [0.75,0.1,0.1]\\ [0.8,0.2,0.1]\end{bmatrix}_{h=1}$$

$$U_{ij1}^{1}=\begin{bmatrix}[0.7,0.1,0.2] & [0.6,0.1,0.2]\\ [0.4,0.2,0.2] & [0.8,0.02,0.1\end{bmatrix}_{h=1}$$

$$U_{ij1}^{2}=\begin{bmatrix}[0.3,0.1,0.2] & [0.5,0.02,0.2]\\ [0.45,0.1,0.2] & [0.4,0.02,0.3]\\ [0.6,0.1,0.3] & [0.7,0.03,0.2]\end{bmatrix}_{h=1}$$

$$U_{ij2}^{2}=\begin{bmatrix}[0.4,0.02,0.1] & [0.5,0.01,0.2]\\ [0.7,0.01,0.3] & [0.8,0.02,0.2]\\ [0.6,0.02,0.3] & [0.7,0.1,0.2]\end{bmatrix}_{h=2}$$

$$U_{ij1}^{3}=\begin{bmatrix}[0.5,0.03,0.3]\\ [0.8,0.02,0.1]\\ [0.7,0.01,0.2]\\ [0.4,0.02,0.3]\\ [0.6,0.01,0.2]\end{bmatrix}_{h=1}$$

The data in the form of crisp numbers for supply and demand are as follows:

q	h	I	S_{iq}
1	1	1	182.5
1	1	2	107.5
2	1	1	59
2	1	2	40
2	1	3	30.5
2	2	1	77
2	2	2	89.5
2	2	3	51.25
3	1	1	78.05
3	1	2	47.75
3	1	3	122.5
3	1	4	147.5
3	1	5	120

Table 4 Supply data

Q	h	j	D_{jq}
1	1	1	90
1	1	2	195
2	1	1	50
2	1	2	81
2	2	1	88
2	2	2	129
3	1	1	49

Table 4 Demand data

The data for supply S_{iq}, $\forall \quad i,q$ are given in the Table 3. The data for demand D_{jq}, $\forall \quad j,q$ are given in the Table 4. The neutrosophic objective for this problem can be determined by degree of acceptance $\mu_o(x)$, degree of indeterminacy $r_o(x)$ and degree of rejection $\upsilon_o(x)$ of the three objective functions defined as follows:

For Transportation Cost:

$$\mu_o(x)=\begin{cases}1, & \sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}<200\\ \dfrac{(350-\sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij})}{150}, & 200\le\sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}\le 350\\ 0, & \sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}>350\end{cases} \tag{14}$$

$$r_o(x)=\begin{cases}0, & \sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}<210\\ \dfrac{(\sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}-200)^2}{19600}, & 210\le\sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}\le 350\\ 1, & \sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}>350\end{cases} \tag{15}$$

$$\upsilon_o(x)=\begin{cases}0, & \sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}<200\\ \dfrac{(\sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}-200)^2}{22500}, & 200\le\sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}\le 350\\ 1, & \sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}>350\end{cases} \tag{16}$$

For Deterioration Rate:

$$\mu_o(x)=\begin{cases}1, & \sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}<150\\ \dfrac{(350-\sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij})}{200}, & 200\le\sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}\le 350\\ 0, & \sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}>350\end{cases} \tag{17}$$

$$r_o(x)=\begin{cases}0, & \sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}<155\\ \dfrac{(\sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}-155)^2}{38025}, & 155\le\sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}\le 350\\ 1, & \sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}>350\end{cases} \tag{18}$$

$$\upsilon_o(x)=\begin{cases}0, & \sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}<150\\ \dfrac{(\sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}-155)^2}{40000}, & 150\le\sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}\le 350\\ 1, & \sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}>350\end{cases} \tag{19}$$

ForUnderusedcapacity:

$$\mu_o(x)=\begin{cases}1, & \sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}<100\\ \dfrac{(350-\sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij})}{200}, & 100\le\sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}\le 400\\ 0, & \sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}>400\end{cases} \tag{20}$$

$$r_o(x)=\begin{cases}0, & \sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}<150\\ \dfrac{(\sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}-155)^2}{90000}, & 150\le\sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}\le 400\\ 1, & \sum_{i=1}^{3}\sum_{j=1}^{4}C_{ij}x_{ij}>400\end{cases} \tag{21}$$

$$\upsilon_O(x) = \begin{cases} 0, & \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} < 100 \\ \dfrac{(\sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} - 100)^2}{40000}, & 100 \le \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} \le 400 \\ 1, & \sum_{i=1}^{3}\sum_{j=1}^{4} C_{ij}x_{ij} > 400 \end{cases} \quad (22)$$

The given problem is first written in the form of the formulated model, Model 4 as:

$$\text{Minimize } Z_1 = \sum_{q=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{o}\sum_{h=1}^{p} C_{ijh}^{q} X_{ijh}^{q} \quad (23)$$

$$\text{Minimize } Z_2 = \sum_{q=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{o}\sum_{h=1}^{p} R_{ijh}^{q} X_{ijh}^{q} \quad (24)$$

$$\text{Minimize } Z_3 = \sum_{q=1}^{m}\sum_{i=1}^{n}\sum_{j=1}^{o}\sum_{h=1}^{p} U_{ijh}^{q} X_{ijh}^{q} \quad (25)$$

$$\text{Subject to } \sum_{j}\sum_{h} X_{ijh}^{q} \ge S_{iq}, \quad \forall \quad i, q \quad (26)$$

$$\sum_{i}\sum_{h} X_{ijh}^{q} \le D_{jq}, \quad \forall \quad j, q \quad (27)$$

$$X_{ijh}^{q} \ge 0. \quad (28)$$

where q = type of raw material; m= number of raw materials;

n = number of sources;

o = number of destination sites;

h = transportation modes; p = number of transportation modes.

X_{ijh}^{q} = Quantity to be transported of q^{th} raw material from i^{th} source to j^{th} destination by transportation mode 'h';

C_{ijh}^{q} =Transportation cost (in billion rupees per metric tonne) of transportation of q^{th} raw material from i^{th} source to j^{th} destination by transportation mode 'h' as neutrosophic set

R_{ijh}^{q} = Deterioration rate (in tonnes per million metric tonne) while transporting q^{th} raw material from i^{th} source to j^{th} destination by transportation mode 'h'; as neutrosophic set

U_{ijh}^{q} = Underused capacity (in tonnes per thousand metric tonne) while transporting q^{th} raw material from i^{th} source to j^{th} destination by transportation (mode 'h'; as neutrosophic set
2
5
S_{iq} = Supplied quantity of q^{th}) raw material from i^{th} source (Availability) (in million metric tonnes)

D_{jq} = Demand of q^{th} raw material at j^{th} destination (Requirement) (in million metric tonnes)

Z_1, Z_2, Z_3 are the minimal values of the neutrosophic Transportation Cost, Detrioration rate and Underused capacity.

5.2 Solution

The given problem is a neutrosophic transportation problem (NTP) and is solved by the above mentioned methods.

Solution by the method based on FLP:

Substituting the above neutrosophic data in the Model 1, three different objective functions for each of the Equations (23), (24) and (25) are obtained. Then the problem can be solved using Extended neutrosophic fuzzy programming technique.

Step 1: The problem is solved considering as single objective taking only one objective function and neglecting others. The solution sets are obtained as:

1. $Z_{11} = 419.66$ 2. $Z_{12} = 21.43$ 3. $Z_{13} = 166.02$
4. $Z_{21} = 411.94$ 5. $Z_{22} = 76.64$ 6. $Z_{23} = 158.32$
7. $Z_{31} = 326.51$ 8. $Z_{32} = 34.66$ 9. $Z_{33} = 229.94$

Step 2 : For each solution set , the values for the other objective functions can be obtained. The best and worst values for each objective are obtained as:

$Z_{11}^L = 364.58$ $Z_{11}^U = 828.49$ $Z_{11}^U - Z_{11}^L = 463.91$

$Z_{12}^L = 21.43$ $Z_{12}^U = 42.13$ $Z_{12}^U - Z_{12}^L = 20.7$

$Z_{13}^L = 103.13$ $Z_{13}^U = 216.79$ $Z_{13}^U - Z_{13}^L = 113.66$

$Z_{21}^L = 376.66$ $Z_{21}^U = 843.75$ $Z_{21}^U - Z_{21}^L = 467.09$

$Z_{22}^L = 35.43$ $Z_{22}^U = 85.42$ $Z_{22}^U - Z_{22}^L = 49.99$

$Z_{23}^L = 64.12$ $Z_{23}^U = 191.88$ $Z_{23}^U - Z_{23}^L = 127.76$

$Z_{31}^L = 326.51$ $Z_{31}^U = 714.85$ $Z_{31}^U - Z_{31}^L = 388.28$

$Z_{32}^L = 31.22$ $Z_{32}^U = 56.14$ $Z_{32}^U - Z_{32}^L = 24.92$

$Z_{33}^L = 91.65$ $Z_{33}^U = 255.72$ $Z_{33}^U - Z_{33}^L = 164.07$

Step 3 : Corresponding to the three objective functions, a linear membership function can be defined. Then the problem can be solved using Equations(6)and(7) from Model 4 of Section 4.2 and the final solution is obtained as

$\lambda = 0.8365686,$

$Z_{11} = 328.75$, $Z_{12} = 0$,

$Z_{13} = 75.63$ $Z_{21} = 284.38$, $Z_{22} = 35.45$,

$Z_{23} = 85$ $Z_{31} = 263.25$, $Z_{32} = 0$, $Z_{33} = 105.25$

Step 4 : Using the values obtained in Equation (12), $Z_1 = 253.12, Z_2 = 163.93, Z_3 = 158$

The degree of acceptance, indeterminacy and rejection of different objective functions are obtained as

- Transportation cost : $\mu_o = 0.65, r_o = 0.095, \upsilon_o = 0.13$

  i.e. $(\mu_o, r_o, \upsilon_o) = (0.65.0.095,0.13)$

- Deterioration rate : $\mu_o = 0.93, r_o = 0.0021, \upsilon_o = 0.005$

  i.e. $(\mu_o, r_o, \upsilon_o) = (0.93,0.0021,0.005)$

- Underused capacity : $\mu_o = 0.8, r_o = 0.001, \upsilon_o = 0.037$

  i.e. $(\mu_o, r_o, \upsilon_o) = (0.8,0.001,0.037)$

Solution by the method based on CLP:

Substituting the neutrsophic data in the Equation (3) in the Model 2, a set of three similar equations is obtained which form a multi-objective transportation problem and thus can be solved by fuzzy programming technique. This gives the optimal solution for each objective function. The final crisp optimal solution is obtained as

$\lambda = 0.4075449$, $Z_1 = 217.665$, $Z_2 = 329.12$ $Z_3 = 169.355$

. The degree of acceptance, indeterminacy and rejection of different objective functions are obtained as

- Transportation cost : $\mu_o = 0.88, r_o = 0.003, \upsilon_o = 0.014$

  i.e. $(\mu_o, r_o, \upsilon_o) = (0.88.0.003, 0.014)$

- Deterioration rate : $\mu_o = 0.104, r_o = 0.7, \upsilon_o = 0.8$

  i.e. $(\mu_o, r_o, \upsilon_o) = (0.104, 0.7, 0.8)$

- Underused capacity : $\mu_o = 0.77, r_o = 0.0059, \upsilon_o = 0.053$

  i.e. $(\mu_o, r_o, \upsilon_o) = (0.77, 0.0059, 0.053)$

6. Results and Discussions

- The two methods are introduced for neutrosophic transportation problems and illustrated by an example in Section 4. The method is then applied for a real life multi-objective and multi-index neutrosophic transportation problem in Section 5 for the first time.
- The optimal solution for the neutrosophic transportation problems in Section 4 is obtained by the above two methods, i.e. by FLP and the other by CLP. The crisp optimal solution for the cost objective function of the given neutrosophic fuzzy transportation problem in Section 4 is obtained by FLP method using linear membership function as 256.82 (thousand dollars) as in Table 5. The degree of acceptance, indeterminacy and rejection of the obtained solution is calculated as (0.62, 0.0047, 0.14). Thus the satisfaction degree of the solution is 0.62 which means the solution is 62% acceptable 0.4% indeterminant (not known) and 76% rejectable.

λ	Z_1	Z_2	Z_3	Z	(μ_o, r_o, υ_o)
0.909	508.64	143.61	08.2	256.8	(0.6,0.0047,0.14)

Table5: Solution of Example in Section 4 using Linear membership function.

The degree of satisfaction of the optimal solution depends upon the respective defined membership, indeterminacy and non-membership function in the given problems. The degree of satisfaction and the degree of rejection need not be complement to each other. The crisp optimal solution for the cost objective transportation problem of the given neutrosophic transportation problem is obtained by crisp linear programming method as 260 (thousand dollars).The satisfaction degree of this solution is 0.6 which means the solution is 60% acceptable 0.01% indeterminant (not known) and 0.16% rejectable.

	Using Fuzzy linear programming	Using Crisp linear programming
Z	(0.62,0.0047,0.14	(0.6,0.01,0.1 6)

Table 6 : Comparison of the obtained neutrosophic solutions using FLP and CLP methods.

The crisp optimal solution for the different objective functions – transportation cost, deterioration rate and underused capacity of the given real life neutrosophic transportation problem in Section 5 is obtained by FLP method using linear membership function as 253.12, 163.93 and 158 as in Table 8. The degree of acceptance, indeterminacy and rejection of the obtained solution for transportation cost, deterioration rate and underused capacity is calculated as (0.65,0.095,0.13), (0.93,0.0021,0.005) and (0.8,0.001,0.037). Thus the satisfaction degree of the three solutions are 0.65,0.93 and 0.859 which means the first transportation cost solution is 65% acceptable, 9% indeterminant and 13% rejectable. The second deterioration rate solution is 93% acceptable, 0.2% indeterminant and 0.5% rejectable and the third underused capacity solution is 80% acceptable, 0.1% indeterminant and 3.7% rejectable.

λ	Z_1	Z_2	Z_3	Z	(μ_o, r_o, υ_o)
0.909	328.8	0	75.63	253.1	(0.65,0.09 ,0.13)
	284.38	35.45	85	163.9	(0.93,0.00 2,0.005)
	263.25	0	105.2	158	(0.8,0.001 ,0.04)

Table7: Solution of real life example in Section 5 using Linear membership function.

The crisp optimal solution for the transportation cost, deterioration rate and underused capacity is calculated as 217.67, 329.12 and 169.355 by CLP method. The satisfaction degree of this solution for transportation cost, deterioration rate and underused capacity is calculated as 0.88, 0.104 and 0.77.

	Using Fuzzy linear programming	Using Crisp linear programming
Transportation cost	(0.65.0.09 5,0.13)	(0.88,0.003,0.014)
Deterioration rate	(0.93,0.00 21,0.005)	(0.104,0.7,0.8)
Underused capacity	(0.8,0.001 ,0.037)	(0.77,0.0059,0.053)

Table 8 : Comparison of the obtained neutrosophic solutions using FLP and CLPmethods.

Thus the FLP method appears to be better method as it gives more optimal solution as compared to the crisp linear programming method.

7. Conclusions

- In this paper, the Neutrosophic Transportation Problem (NTP) is solved by two methods- FLP method and CLP method.
- The first method, FLP method gives the solution as crisp and then as SVNS which represent the degree of acceptance, indeterminacy and rejection of the solution obtained from the defined membership function for a particular problem.
- The second method, i.e., CLP method gives the solution as crisp number only. Then the degree of the acceptance, indeterminacy and rejection is calculated
- The FLP method can be seen as a better method and it gives more optimal solution.
- The SVNS data can represent real life uncertainties and so depicts more practical solutions of the problem as it helps to determine the degree of acceptance, indeterminacy and rejection of the obtained solution. Row 1, Col 2 Row 1, Col 3
- A real life multi-objective and multi-index Neutrosophic transportation problem has also been solved in Section 5 other than the numerical example in Section 4 to illustrate the two proposed methods. The results and comparisons of the large scale problem are shown in the Table 5, Table 6, Table 7 and Table 8. The results obtained are compared and the FLP method proves to give better solution compared to the CLP method for most of the circumstances.

- The solution obtained by the proposed approaches has not been compared with any of the existing approaches for NTPs, as no work has been done for neutrosophic transportation problem. It is a new type of problem.

- The application of the methods to a real life multi-objective and multi- index neutrosophic transportation problem is also a new field itself.

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TOPSIS for Single Valued Neutrosophic Soft Expert Set Based Multi-attribute Decision Making Problems

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Abstract. In the paper, we propose Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) technique for solving single valued neutrosophic soft set expert based multi-attribute decision making problems. Single valued neutrosophic soft expert sets are combination of single valued neutrosophic sets and soft expert sets. In the decision making process, the ratings of alternatives with respect to the parameters are expressed in terms of single valued neutrosophic soft expert sets to deal with imprecise or vague information. The unknown weights of the parameters are derived from maximizing deviation method. Then, we determine the rank of the alternatives and choose the best one by using TOPSIS method. Finally, a numerical example for teacher selection is presented to demonstrate the applicability and effectiveness of the proposed approach.

Keywords: Single valued neutrosophic sets, single valued neutrosophic soft expert sets, TOPSIS, multi-attribute decision making.

1 Introduction

Hwang and Yoon [1] grounded the technique for order preferece by similarity to ideal solution (TOPSIS) method for solving conventional multi-attribute decision making (MADM) problems. The basic concept of TOPSIS is straightforward. It is developed from the concept of a displaced ideal point from which the compromise solution has the shortest distance. Hwang and Yoon [1] proposed that the ranking of alternatives would be based on the shortest distance from the positive ideal solution (PIS) and the farthest from the negative ideal solution (NIS). TOPSIS approac simultaneously considers the distances to both PIS and NIS, and a preference order is ranked based on their relative closeness, and a combination of these two distance measures.

MADM is the process of identifying the most suitable alternative from a finite set of feasible alternatives with respect to numerous usually conflicting attributes. MADM has been applied to various practical problems such as learning management system evaluation [2], project portfolio selection [3], electric utility resource planning [4], economics, military affairs, etc. However, in practical decision making situation, the information about the rating of the alternative with respect to the attributes cannot be assessed due to imprecise source of information. So, traditional MADM methods are not capable to solve these types of problems.

In 1965, Zadeh [5] proposed fuzzy set which is characterized by membership function to deal with problems with imprecise information. Atanassov [6] defined intuitionistic fuzzy set by incorporating non-membership function. However, for proper description of an object in uncertain and complex environment, we require to deal with indeterminate and inconsistent information. So, Smarandache [7, 8, 9, 10] extended the concept of Atanassov [6] by introducing indeterminacy membership function as an independent component and defined neutrosophic set for dealing with the problems with incomplete, imprecise, inconsistent information. Thereafter, Wang et al. [11] defned single valued neutrosophic set (SVNS) as an instance of neutrosophic set for dealing with real scientific and engineering problems.

Molodtsov [12] initiated the concept of soft set theory for dealing with uncertainty and vagueness in 1999. Soft set is free from the limitation of variety of theories such as probability theory, fuzzy theory, rough set theory, vague set theory and it is easy to implement in practical problems. After the pioneering work of Molodtsov [12], many researchers developed diverse mathematical hybrid models such as fuzzy soft sets [13, 14, 15], intuitionistic fuzzy soft set theory [16, 17, 18], possibility fuzzy soft set [19], generalized fuzzy soft sets [20, 21], generalized intuitionistic fuzzy soft [22], possibility intuitionistic fuzzy soft set [23], vague soft set [24], possibility vague soft set [25], neutrosophic soft set [26], weighted neutrosophic soft sets [27], generalized neutrosophic soft set [28], intuitionistic neutrosophic soft set [29, 30], etc in order to solve different practical problems. However, most of the models consider only one expert and this creates difficulties for the researchers who employ questionnaires for his/her works and studies [31]. In order to overcome the difficulties, Alkhazaleh and Salleh [31] developed soft expert sets in 2011 where the researcher can observe the opinions of all experts in one model without any operations. They defined basic operations of soft expert sets and studied some of their properties and then applied the concept in decision making problem. Alkhazaleh and Salleh [32] also defined fuzzy soft expert set which is a hybridization of soft expert set and fuzzy set. Hazaymey et al. [33] introduced generalized fuzzy soft expert set by combining soft expert set due to Alkhazaleh and Salleh [31] and generalized soft set due to Majumdar and Samanta [21]. Hazaymey et al. [34] also incorporated fuzzy parameterized fuzzy soft expert set by extending the concept of fuzzy soft expert set by providing a membership value of each parameter in a set of parameters. Later, many authors have developed soft expert sets in different environment to form different structures such as vague soft expert set [35], generalized vague soft expert set [36], fuzzy parameterized soft expert set [37], possibility fuzzy soft expert set [38], intuitionistic fuzzy soft expert set [39], etc and the concepts of soft expert sets are applied to different practical problems [40, 41, 42]. Recently, Şahin et al. [43] incorporated neutrosophic soft expert set as a combination of neutrosophic set and soft expert set to deal with indeterminate and inconsistent information. Later, Broumi and Smarandache [44] explored the concept of single valued neutrosophic soft expert set (SVNSES) which is an extension of fuzzy soft expert sets and intuitionistic fuzzy soft expert sets and they investigated some related properties with supporting proofs.

In the paper, we have developed a new method for solving SVNSES based MADM problem through TOPSIS technique.

The content of the paper is constructed as follows. Section 2 presents some basic definitions concerning neutrosophic sets, SVNSs, soft sets, soft expert. Section 3 is devoted to present TOPSIS method for SVNSESs based MADM problems. Section 4 presents an algorithm of the proposed method. A hypothetical problem regarding teacher selection is solved in Section 5 to illustrate the applicability of the proposed method. Finally, Section 6 presents conclusions and future scope research.

2 Preliminaries

We present basic definitions regarding neutrosophic sets, soft sets, soft expert sets and SVNSESs in this Section as follows:

2.1 Neutrosophic Sets [7, 8, 9, 10]

Consider X be a space of objects with a generic element of X denoted by x. Then, a neutrosophic set N on X is defined as follows:

$$N = \{x, \langle \mathrm{T}_N(x), \mathrm{I}_N(x), \mathrm{F}_N(x) \rangle \mid x \in X\}$$

where, $\mathrm{T}_N(x)$, $\mathrm{I}_N(x)$, $\mathrm{F}_N(x)$: $X \to\ ]^-0, 1^+[$ represent respectively the degrees of truth-membership, indeterminacy-membership, and falsity-membership of a point $x \in X$ to the set N with the condition $^-0 \le \mathrm{T}_N(x) + \mathrm{I}_N(x) + \mathrm{F}_N(x) \le 3^+$.

2.2 Single valued neutrosophic Sets [11]

Let X be a universal space of points with a generic element of X denoted by x, then a SVNS S is presented as follows:

$$S = \{x, \langle \mathrm{T}_S(x), \mathrm{I}_S(x), \mathrm{F}_S(x) \rangle \mid x \in X\}$$

where, $\mathrm{T}_S(x)$, $\mathrm{I}_S(x)$, $\mathrm{F}_S(x)$: $X \to [0, 1]$ and $0 \le \mathrm{T}_S(x) + \mathrm{I}_S(x) + \mathrm{F}_S(x) \le 3$ for each point $x \in X$.

Definition 1 [45] The Hamming distance between two SVNSs $S_1 ==\{x_j, \langle \mathrm{T}_{S_1}(x_j), \mathrm{I}_{S_1}(x_j), \mathrm{F}_{S_1}(x_j) \rangle \mid x_j \in X\}$ and $S_2 = \{x_j, \langle \mathrm{T}_{S_2}(x_j), \mathrm{I}_{S_2}(x_j), \mathrm{F}_{S_2}(x_j) \rangle \mid x_j \in X\}$ is defined as follows:

$\mathrm{L}_{\mathrm{Ham}}(S_1, S_2) =$

$$\sum_{j=1}^{n} \left\{ \begin{array}{l} |\mathrm{T}_{S_1}(x_j) - \mathrm{T}_{S_2}(x_j)| + |\mathrm{I}_{S_1}(x_j) - \mathrm{I}_{S_2}(x_j)| + \\ |\mathrm{F}_{S_1}(x_j) - \mathrm{F}_{S_2}(x_j)| \end{array} \right\} \quad (1)$$

Definition 2 [45] The normalized Hamming distance between two SVNSs $S_1 ==\{x_j, \langle \mathrm{T}_{S_1}(x_j), \mathrm{I}_{S_1}(x_j), \mathrm{F}_{S_1}(x_j) \rangle \mid$

$x_j \in X\}$ and $S_2 = \{x_j, \langle T_{S_2}(x_j), I_{S_2}(x_j), F_{S_2}(x_j) \rangle \mid x_j \in X\}$ is defined as follows:

$$^{N}L_{Ham}(S_1, S_2) = \frac{1}{3n}\sum_{j=1}^{n}\left\{\begin{array}{l} |T_{S_1}(x_j) - T_{S_2}(x_j)| + |I_{S_1}(x_j) - I_{S_2}(x_j)| + \\ |F_{S_1}(x_j) - F_{S_2}(x_j)| \end{array}\right\} \quad (2)$$

Definition 3 [45] The Euclidean distance between two SVNSs $S_1 == \{x_j, \langle T_{S_1}(x_j), I_{S_1}(x_j), F_{S_1}(x_j) \rangle \mid x_j \in X\}$ and $S_2 = \{x_j, \langle T_{S_2}(x_j), I_{S_2}(x_j), F_{S_2}(x_j) \rangle \mid x_j \in X\}$ is defined as follows:

$$L_{Euc}(S_1, S_2) = \sqrt{\sum_{j=1}^{n}\left\{\begin{array}{l} (T_{S_1}(x_j) - T_{S_2}(x_j))^2 + (I_{S_1}(x_j) - I_{S_2}(x_j))^2 + \\ (F_{S_1}(x_j) - F_{S_2}(x_j))^2 \end{array}\right\}} \quad (3)$$

Definition 4 [45] The normalized Euclidean distance between two SVNSs $S_1 == \{x_j, \langle T_{S_1}(x_j), I_{S_1}(x_j), F_{S_1}(x_j) \rangle \mid x_j \in X\}$ and $S_2 = \{x_j, \langle T_{S_2}(x_j), I_{S_2}(x_j), F_{S_2}(x_j) \rangle \mid x_j \in X\}$ is defined as follows:

$$^{N}L_{Euc}(S_1, S_2) = \sqrt{\frac{1}{3n}\sum_{j=1}^{n}\left\{\begin{array}{l} (T_{S_1}(x_j) - T_{S_2}(x_j))^2 + (I_{S_1}(x_j) - I_{S_2}(x_j))^2 + \\ (F_{S_1}(x_j) - F_{S_2}(x_j))^2 \end{array}\right\}} \quad (4).$$

2.3 Soft set [12]

Let X be a universal set and E be a set of parameters. Assume that P (X) represents power set of X. Also, let A be a non-empty set, where $A \subset E$. Then, a pair (M, A) is called a soft set over X, where M is a mapping given by M: $A \to P(X)$.

2.4 Neutrosophic soft set [26]

Let, X be an initial universal set. Also, let E be a set of parameters and A be a non-empty set such that $A \subset E$. NS (X) represents the set of all neutrosophic subsets of X. Then, a pair (M, A) is termed to be the neutrosophic soft set over X, where M is a mapping given by M: $A \to NS(X)$.

2.5 Soft expert set [31]

Consider X be an initial universal set, E be the set of parameters, Z be a set of experts (agents) and O = {agree = 1, disagree = 0} be a set of opinions. Let, $W = E \times Z \times O$, $A \subseteq W$. Then, a pair (M, A) is called soft expert set over X, where M is a mapping given by M: $A \to P(X)$, where P (X) represents power set of X.

Definition 5 [31] An agree-soft expert set $(M, A)_1$ over X is a soft expert subset of (M, A) is defined as follows:

$$(M, A)_1 = \{M(\beta): \beta \in E \times Z \times \{1\}\}.$$

Definition 6 [31] An disagree-soft expert set $(M, A)_0$ over X is a soft expert subset of (M, A) is defined as follows:

$(M, A)_0 = \{M(\beta): \beta \in E \times Z \times \{0\}\}$.

2.6 Single valued neutrosophic soft expert set [44]

Consider $X = \{x_1, x_2, \ldots, x_n\}$be a universal set of objects, E = {e_1, e_2, …, e_n} be the set of parameters, Z = {z_1, z_2, …, z_n} be a set of experts (agents) and O = {agree = 1, disagree = 0} be a set of opinions. Let, $W = E \times Z \times O$, and A be a non-empty set such that $A \subseteq W$. A pair (M, A) is said to be SVNSES over X, where M is a mapping given by M: $A \to$ SVNSES (X), where SVNSES (X) represents all single valued neytrosophic subsets of X.

Example: Let X be the set of objects under consideration and E be the set of parameters, where every parameter is a neutrosophic word or sentence concerning neutrosophic words. Suppose there are three objects in the universe X given by $X = \{x_1, x_2, x_3\}$, E = {costly, beautiful} = {e_1, e_2} be the set of decision parameters and Z = {z_1, z_2} be a set of experts. Suppose M: $A \to$ SVNSES (X) is defined as follows:

M (e_1, z_1, 1) =
$\{\langle x_1, 0.2, 0.5, 0.7\rangle, \langle x_2, 0.4, 0.2, 0.5\rangle, \langle x_3, 0.6, 0.3, 0.4\rangle\}$,

M (e_2, z_1, 1) =
$\{\langle x_1, 0.5, 0.1, 0.2\rangle, \langle x_2, 0.5, 0.2, 0.4\rangle, \langle x_3, 0.6, 0.2, 0.2\rangle\}$,

M (e_1, z_2, 1) =
$\{\langle x_1, 0.7, 0.1, 0.3\rangle, \langle x_2, 0.8, 0.3, 0.1\rangle, \langle x_3, 0.8, 0.2, 0.4\rangle\}$,

M (e_2, z_2, 1) =
$\{\langle x_1, 0.9, 0.1, 0.2\rangle, \langle x_2, 0.3, 0.3, 0.2\rangle, \langle x_3, 0.4, 0.3, 0.1\rangle\}$,

M (e_1, z_1, 0) =
$\{\langle x_1, 0.3, 0.5, 0.1\rangle, \langle x_2, 0.5, 0.2, 0.1\rangle, \langle x_3, 0.4, 0.3, 0.2\rangle\}$,

M (e_2, z_1, 0) =
$\{\langle x_1, 0.7, 0.1, 0.5\rangle, \langle x_2, 0.6, 0.3, 0.4\rangle, \langle x_3, 0.6, 0.5, 0.4\rangle\}$,

M (e_1, z_2, 0) =
$\{\langle x_1, 0.2, 0.1, 0.4\rangle, \langle x_2, 0.6, 0.5, 0.4\rangle, \langle x_3, 0.5, 0.6, 0.3\rangle\}$,

M (e_2, z_2, 0) =

$\{\langle x_1, 0.8, 0.4, 0.2\rangle, \langle x_2, 0.7, 0.5, 0.4\rangle, \langle x_3, 0.5, 0.3, 0.3\rangle\}$.

Then, (M, A) is a SVNSES over the soft universe.

Definition 7 [44]**:** Let (M_1, A) and (M_2, B) be two SVNSESs over a common soft universe. Then, (M_1, A) is said to be single valued neutrosophic soft expert subset of (M_2, B) if

(i). $B \subseteq A$

(ii). $M_1(\delta)$ is a single valued neutrosophic subset $M_2(\delta), \forall \delta \in A$.

Definition 8 [44]**:** A null SVNSES (φ, A) is defined as follows:

$$(\varphi, A) = M(\beta) \text{ where } \beta \in W.$$

Where M (β) = <0, 0, 1>, that is $T_{M(\beta)} = 0$, $I_{M(\beta)} = 0$, $F_{M(\beta)} = 1$, $\forall \beta \in W$.

Definition 9 [44]: The complement of a SVNSES (M, A) is defined as follows:

$$(M, A)^C = \tilde{C}(M(\beta)) \; \forall \beta \in X.$$

Where, $\tilde{C}$ represents single valued neutrosophic complement.

Definition 10 [44]: Consider (M_1, A) and (M_2, B) be two SVNSESs over a common soft universe. The union (M_1, A) and (M_2, B) is denoted by $(M_1, A) \cup (M_2, B) = (M_3, C)$, where $C = A \cup B$ and is defined as follows:

$$M_3(\beta) = M_1(\beta) \cup M_2(\beta), \forall \beta \in C.$$

$$\text{Where, } M_3(\beta) = \begin{cases} M_1(\beta), \beta \in A - B \\ M_2(\beta), \beta \in B - A \\ M_1(\beta) \cup M_2(\beta), \beta \in A \cap B \end{cases}$$

where $M_1(\beta) \cup M_2(\beta) = \{<x, \text{Max}\{T_{M_1(\beta)}, T_{M_2(\beta)}\}, \text{Min}\{I_{M_1(\beta)}, I_{M_2(\beta)}\}, \text{Min}\{F_{M_1(\beta)}, F_{M_2(\beta)}\}>: x \in X\}$.

Definition 11 [44]: Suppose (M_1, A) and (M_2, B) are two SVNSESs over a common soft universe. The intersection (M_1, A) and (M_2, B) is denoted by $(M_1, A) \cap (M_2, B) = (M_4, D)$, where $D = A \cap B$ and is defined as follows:

$$M_4(\beta) = M_1(\beta) \cap M_2(\beta), \forall \beta \in D.$$

$$\text{Here, } M_4(\beta) = \begin{cases} M_1(\beta), \beta \in A - B \\ M_2(\beta), \beta \in B - A \\ M_1(\beta) \cap M_2(\beta), \beta \in A \cap B \end{cases}$$

where $M_1(\beta) \cap M_2(\beta) = \{<x, \text{Min}\{T_{M_1(\beta)}, T_{M_2(\beta)}\}, \text{Max}\{I_{M_1(\beta)}, I_{M_2(\beta)}\}, \text{Max}\{F_{M_1(\beta)}, F_{M_2(\beta)}\}>: x \in X\}$

3 TOPSIS method for MADM with single valued neutrosophic soft expert information

Let $C = \{C_1, C_2, \ldots, C_m\}$, ($m \geq 2$) be a discrete set of m feasible alternatives, $A = \{a_1, a_2, \ldots, a_n\}$, ($n \geq 2$) be a set of parameters under consideration and $w = (w_1, w_2, \ldots, w_n)^T$ be the unknown weight vector of the attributes with $0 \leq w_j \leq 1$ and $\sum_{j=1}^{n} w_j = 1$. Let, $Z = \{z_1, z_2, \ldots, z_t\}$ be a set of t experts (agents), where we consider the weights of the experts are equal and O = {agree = 1, disagree = 0} be a set of opinions. The rating of performance value of alternative C_i, (i = 1, 2, …, m) with respect to the parameters is presented by the experts and they can be expressed in terms of SVNSs. Therefore, the proposed methodology for solving single valued neutrosophic soft expert MADM problem based on TOPSIS method is presented as follows:

Step 1. Formulation of decision matrix with SVNSs

Let, the rating of alternative C_i (i = 1, 2, …, m) with respect to the parameter provided by the experts is represented by SVNSES (M, A) and they can be presented in matrix form as follows:

$$D_{SVNSES} = \langle d_{ij} \rangle_{m \times q} = \begin{bmatrix} d_{11} & d_{12} & \ldots & d_{1q} \\ d_{21} & d_{22} & \ldots & d_{2q} \\ . & . & \ldots & . \\ . & . & \ldots & . \\ d_{m1} & d_{m2} & \ldots & d_{mq} \end{bmatrix}$$

Here, $d_{ij} = (T_{ij}, I_{ij}, F_{ij})$ where $T_{ij}, I_{ij}, F_{ij} \in [0, 1]$ and $0 \leq T_{ij} + I_{ij} + F_{ij} \leq 3$, i = 1, 2, …, m; j = 1, 2, …, q, where $q = n \times t \times 2$.

Step 2. Determination of unknown weights of the parameters

In the selection process, we assume that the importance (weight) of the attributes is not same and the weights of the attributes are completely unknown. Therefore, we employ maximizing deviation method due to Yang [46] in order to to obtain the unknown weights. The deviation values of alternative C_i (i = 1, 2, …, m) to all other alternatives under the attribute A_j (j = 1, 2, …, q) can be defined as Y_{ij} (w_j) = $\sum_{k=1}^{m} y(d_{ij}, d_{kj}) w_j$, then Y_j (w_j) = $\sum_{i=1}^{m} Y_{ij} w_j$ = $\sum_{i=1}^{m}\sum_{k=1}^{m} y(d_{ij}, d_{kj}) w_j$ denotes the total deviation values of all alternatives to the other alternatives for the attribute A_j (j = 1, 2, …, q). Now Y (w_j) = $\sum_{j=1}^{n} Y_j(w_j)$ = $\sum_{j=1}^{q}\sum_{i=1}^{m}\sum_{k=1}^{m} y(d_{ij}, d_{kj}) w_j$ presents the deviation of all attributes for all alternatives to the other alternatives. The maximizing deviation model [47] is formulated as follows:

Max Y (w_j) = $\sum_{j=1}^{q}\sum_{i=1}^{m}\sum_{k=1}^{m} y(d_{ij}, d_{kj}) w_j$

Subject to $\sum_{j=1}^{q} w_j^2 = 1$, $w_j \geq 0$, j = 1, 2, …, q.

Solving the above model, we obtain

$$w_j^* = \frac{\sum_{i=1}^{m}\sum_{k=1}^{m} y(d_{ij}, d_{kj})}{\sqrt{(\sum_{j=1}^{q}\sum_{i=1}^{m}\sum_{k=1}^{m} y^2(d_{ij}, d_{kj})}} \tag{5}$$

Finally, we get normalized attribute weight based on the above model as follows:

$$w_j^* = \frac{\sum_{i=1}^{m}\sum_{k=1}^{m} y(d_{ij}, d_{kj})}{\sum_{j=1}^{q}\sum_{i=1}^{m}\sum_{k=1}^{m} y(d_{ij}, d_{kj})} \tag{6}$$

Step 3. Construction of weighted decision matrix

We obtain aggregated weighted decision matrix by multiplying weights (w_j) [48] of the parameters and aggregated decision matrix $\left\langle d_{ij}^{w_j} \right\rangle_{m\times q}$ is presented as follows:

$$D_{\mathrm{SVNSES}}^{w} = D_{\mathrm{SVNSES}} \otimes w = \left\langle d_{ij} \right\rangle_{m\times q} \otimes w_j$$

$$= \left\langle d_{ij}^{w_j} \right\rangle_{m\times q} = \begin{bmatrix} d_{11}^{w_1} & d_{12}^{w_2} & \ldots & d_{1q}^{w_q} \\ d_{21}^{w_1} & d_{22}^{w_2} & \ldots & d_{2q}^{w_q} \\ . & . & \ldots & . \\ . & . & \ldots & . \\ d_{m1}^{w_1} & d_{m2}^{w_2} & \ldots & d_{mq}^{w_q} \end{bmatrix}$$

Here, $d_{ij}^{w_j} = \left\langle T_{ij}^{w_j}, I_{ij}^{w_j}, F_{ij}^{w_j} \right\rangle$ where $T_{ij}^{w_j}$, $I_{ij}^{w_j}$, $F_{ij}^{w_j} \in [0, 1]$ and $0 \leq T_{ij}^{w_j} + I_{ij}^{w_j} + F_{ij}^{w_j} \leq 3$, i = 1, 2, …, m; j = 1, 2, …, q.

Step 4. Determination of single valued neutrosophic relative positive ideal solution (SVNRPIS) and single valued neutrosophic relative negative ideal solution (SVNRNIS)

The parameters are generally classified into two categories namely benefit type attributes (α_1) and cost type attributes (α_2). Consider $R_{\mathrm{SVNRPIS}}^{w+}$ and $R_{\mathrm{SVNRNIS}}^{w-}$ be the single valued neutrosophic relative positive ideal solution (SVNRPIS) and single valued neutrosophic relative negative ideal solution (SVNRNIS). Then, $R_{\mathrm{SVNRPIS}}^{w+}$ and $R_{\mathrm{SVNRNIS}}^{w-}$ can be defined as follows:

$R_{\mathrm{SVNRPIS}}^{w+}$ = ($\left\langle T_1^{w_1+}, I_1^{w_1+}, F_1^{w_1+} \right\rangle$, $\left\langle T_2^{w_2+}, I_2^{w_2+}, F_2^{w_2+} \right\rangle$, …, $\left\langle T_q^{w_q+}, I_q^{w_q+}, F_q^{w_q+} \right\rangle$)

$R_{\mathrm{SVNRNIS}}^{w-}$ = ($\left\langle T_1^{w_1-}, I_1^{w_1-}, F_1^{w_1-} \right\rangle$, $\left\langle T_2^{w_2-}, I_2^{w_2-}, F_2^{w_2-} \right\rangle$, …, $\left\langle T_q^{w_q-}, I_q^{w_q-}, F_q^{w_q-} \right\rangle$) where

$\left\langle T_j^{w_j+}, I_j^{w_j+}, F_j^{w_j+} \right\rangle$ = < [{ $\underset{i}{\mathrm{Max}}(T_{ij}^{w_j})$ |j $\in \alpha_1$ }; { $\underset{i}{\mathrm{Min}}(T_{ij}^{w_j})$ |j $\in \alpha_2$ }], [{ $\underset{i}{\mathrm{Min}}(I_{ij}^{w_j})$ | j $\in \alpha_1$ }; { $\underset{i}{\mathrm{Max}}(I_{ij}^{w_j})$ | j $\in \alpha_2$ }], [{ $\underset{i}{\mathrm{Min}}(F_{ij}^{w_j})$ | j $\in \alpha_1$ }; { $\underset{i}{\mathrm{Max}}(F_{ij}^{w_j})$ | j $\in \alpha_2$ }] >, j = 1, 2, …, q,

$\left\langle T_j^{w_j-}, I_j^{w_j-}, F_j^{w_j-} \right\rangle$ = < [{ $\underset{i}{\mathrm{Min}}(T_{ij}^{w_j})$ | j $\in \alpha_1$ }; { $\underset{i}{\mathrm{Max}}(T_{ij}^{w_j})$ | j $\in \alpha_2$ }], [{ $\underset{i}{\mathrm{Max}}(I_{ij}^{w_j})$ | j $\in \alpha_1$ }; { $\underset{i}{\mathrm{Min}}(I_{ij}^{w_j})$ | j $\in \alpha_2$ }], [{ $\underset{i}{\mathrm{Max}}(F_{ij}^{w_j})$ | j $\in \alpha_1$ }; { $\underset{i}{\mathrm{Min}}(F_{ij}^{w_j})$ | j $\in \alpha_2$ }] >, j = 1, 2, …, q.

Step 5. Computation of distance measure of each alternative from RPIS and RNIS

The normalized Euclidean measure of each alternative $\left\langle T_{ij}^{w_j}, I_{ij}^{w_j}, F_{ij}^{w_j} \right\rangle$ from the SVNRPIS $\left\langle T_j^{w_j+}, I_j^{w_j+}, F_j^{w_j+} \right\rangle$ for i = 1, 2, …, m; j = 1, 2, …., q can be defined as follows:

$$L_N^{i+}(d_{ij}^{w_j}, d_j^{w_j+}) = \sqrt{\frac{1}{3q}\sum_{j=1}^{q}\left\{\begin{array}{l}(T_{ij}^{w_j}(x_j) - T_j^{w_j+}(x_j))^2 + (I_{ij}^{w_j}(x_j) - I_j^{w_j+}(x_j))^2 + \\ (F_{ij}^{w_j}(x_j) - F_j^{w_j+}(x_j))^2\end{array}\right\}} \tag{7}$$

Similarly, normalized Euclidean measure of each alternative $\left\langle T_{ij}^{w_j}, I_{ij}^{w_j}, F_{ij}^{w_j} \right\rangle$ from the SVNRNIS $\left\langle T_j^{w_j-}, I_j^{w_j-}, F_j^{w_j-} \right\rangle$ for i = 1, 2, …, m; j = 1, 2, …., q can be written as follows:

$$\mathrm{L}_{\mathrm{N}}^{\mathrm{i}-}(\mathrm{d}_{\mathrm{ij}}^{w_j}, \mathrm{d}_{\mathrm{j}}^{w_j-}) = \sqrt{\frac{1}{3q}\sum_{j=1}^{q}\left\{\begin{matrix}(\mathrm{T}_{\mathrm{ij}}^{w_j}(x_j) - \mathrm{T}_{\mathrm{j}}^{w_j-}(x_j))^2 + (\mathrm{I}_{\mathrm{ij}}^{w_j}(x_j) - \mathrm{I}_{\mathrm{j}}^{w_j-}(x_j))^2 + \\ (\mathrm{F}_{\mathrm{ij}}^{w_j}(x_j) - \mathrm{F}_{\mathrm{j}}^{w_j-}(x_j))^2\end{matrix}\right\}} \quad (8)$$

Step 6. Calculation of the relative closeness co-efficient to the neutrosophic ideal solution

The relative closeness co-efficient of each alternative C_i, (i = 1, 2, …, m) with respect to the SVNRPIS is defined as follows:

$$\tau_i^* = \frac{\mathrm{L}_{\mathrm{N}}^{\mathrm{i}-}(\mathrm{d}_{\mathrm{ij}}^{w_j}, \mathrm{d}_{\mathrm{j}}^{w_j-})}{\mathrm{L}_{\mathrm{N}}^{\mathrm{i}+}(\mathrm{d}_{\mathrm{ij}}^{w_j}, \mathrm{d}_{\mathrm{j}}^{w_j+}) + \mathrm{L}_{\mathrm{N}}^{\mathrm{i}-}(\mathrm{d}_{\mathrm{ij}}^{w_j}, \mathrm{d}_{\mathrm{j}}^{w_j-})} \quad (9)$$

where, $0 \le \tau_i^* \le 1$, i = 1, 2, …, m.

Step 7. Rank the alternatives

We rank the alternatives according to the values of τ_i^*, i = 1, 2, …, m and bigger value of τ_i^*,(i = 1, 2, …, m) reflects the best alternative.

4 Proposed algorithm for MADM problem with single valued neutrosophic soft expert information

An algorithm for MADM problem with single valued neutrosophic soft expert information through TOPSIS method is given using the following steps.

Step 1. Construct the decision matrix D_{SVNSES}.

Step 2. Determine the unknown weight (w_j) of the attributes by using Eq (6).

Step 3. Formulate the weighted aggregated decision matrix $\mathrm{D}_{\mathrm{G}}^{w} = \left\langle \mathrm{d}_{\mathrm{ij}}^{w_j} \right\rangle_{m\times q}$.

Step 4. Recognize the SVNRPIS ($\mathrm{R}_{\mathrm{SVNRPIS}}^{w+}$) and SVNRNIS ($\mathrm{R}_{\mathrm{SVNRNIS}}^{w-}$).

Step 5. Calculate the distance of each alternative from SVNRPIS ($\mathrm{R}_{\mathrm{SVNRPIS}}^{w+}$) and SVNRNIS ($\mathrm{R}_{\mathrm{SVNRNIS}}^{w-}$) using Eqs. (7) and (8) respectively.

Step 6. Determine the relative closeness co-efficient τ_i^* (i = 1, 2, …, m) using Eq. (9) of each alternative C_i.

Step 7. Rank the preference order of alternatives in accordance with the order of their relative closeness.

5 A numerical example

In this section, we solve a hypothetical problem to show the effectiveness of the proposed approach. Suppose that a school authority is going to recruit an assistant teacher in Mathematics to fill the vacancy on contractual basis for six months. After preliminary screening, three candidates (alternatives) C_1, C_2, C_3 are short-listed for further assessment. A committee consisting of two members namely 'Senior Mathematics teacher (z_1) and 'an external expert on the relevant subject' (z_2) is formed to conduct the interview in order to select the most suitable teacher and O = {1 = agree, 0 = disagree} be the set of opinions of the selection committee members. The committee considers two parameters a_i, i = 1, 2, where a_1 denotes 'pedagogical knowledge' and a_2 denotes 'personality'. After the interview of the candidates, the select committee provides the following SVNSESs.

M (a_1, z_1, 1) =
$\{\langle x_1, 0.7, 0.5, 0.2\rangle, \langle x_2, 0.6, 0.2, 0.3\rangle, \langle x_3, 0.8, 0.3, 0.3\rangle\}$,
M (a_2, z_1, 1) =
$\{\langle x_1, 0.5, 0.1, 0.4\rangle, \langle x_2, 0.9, 0.2, 0.2\rangle, \langle x_3, 0.8, 0.1, 0.2\rangle\}$,
M (a_1, z_2, 1) =
$\{\langle x_1, 0.7, 0.3, 0.5\rangle, \langle x_2, 0.9, 0.2, 0.1\rangle, \langle x_3, 0.7, 0.1, 0.4\rangle\}$,
M (a_2, z_2, 1) =
$\{\langle x_1, 0.6, 0.2, 0.3\rangle, \langle x_2, 0.9, 0.1, 0.1\rangle, \langle x_3, 0.8, 0.3, 0.2\rangle\}$,
M (a_1, z_1, 0) =
$\{\langle x_1, 0.3, 0.4, 0.3\rangle, \langle x_2, 0.5, 0.3, 0.2\rangle, \langle x_3, 0.2, 0.3, 0.5\rangle\}$,
M (a_2, z_1, 0) =
$\{\langle x_1, 0.4, 0.1, 0.3\rangle, \langle x_2, 0.3, 0.3, 0.1\rangle, \langle x_3, 0.4, 0.3, 0.4\rangle\}$,
M (a_1, z_2, 0) =
$\{\langle x_1, 0.5, 0.1, 0.2\rangle, \langle x_2, 0.4, 0.2, 0.3\rangle, \langle x_3, 0.5, 0.1, 0.4\rangle\}$,
M (a_2, z_2, 0) =
$\{\langle x_1, 0.5, 0.2, 0.3\rangle, \langle x_2, 0.3, 0.3, 0.2\rangle, \langle x_3, 0.5, 0.2, 0.5\rangle\}$.

Then, the proposed procedure for solving the problem is provided using the following steps.

Step 1: Formulation of decision matrix

We present the SVNSESs in the tabular form(see the table 1) as given below (see Table 1)

Step 2. Calculation of the weights of the attributes

We use Hamming distance and obtained the weights of the parameters using Eq. (6) as follows:
$w_1 = 0.12$, $w_2 = 0.14$, $w_3 = 0.16$, $w_4 = 0.14$, $w_5 = 0.12$, $w_6 = 0.12$, $w_7 = 0.08$, $w_8 = 0.12$, where $\sum_{j=1}^{8} w_j = 1$.

Step 3. Construction of weighted decision matrix

The tabular form of the weighted decision matrix is presented the Table 2.

Step 4. Determination of SVNRPIS and SVNRNIS

The SVNRPIS ($R^{w+}_{SVNRPIS}$) and SVNRNIS ($R^{w-}_{SVNRNIS}$) can be obtained from the weighted decision matrix (see Table 2) as follows:

$R^{w+}_{SVNRPIS}$ = < (0.176, 0.824, 0.824); (0.276, 0.724, 0.798); (0.308, 0.692, 0.692); (0.276, 0.724, 0.724); (0.08, 0.865, 0.824); (0.059, 0.758, 0.758); (0.054, 0.832, 0.879); (0.08, 0.824, 0.824) >,

$R^{w-}_{SVNRNIS}$ = < (0.104, 0.92, 0.865); (0.092, 0.798, 0.88); (0.175, 0.825, 0.895); (0.12, 0.845, 0.845); (0.026, 0.896, 0.92); (0.042, 0.865, 0.896); (0.04, 0.879, 0.929); (0.042, 0.865, 0.92) >.

Step 5. Compute the distance measure of each alternative from the SVNRPIS and SVNRNIS

The Euclidean distance measures of each alternative from the SVNRPIS are calculated by using Eq. (7) as follows:

$L^{1+}_{N} = 0.1542$, $L^{2+}_{N} = 0.0393$, $L^{3+}_{N} = 0.0753$.

Similarly, the Euclidean distance measures of each alternative from the SVNRNIS are determined by using Eq. (8) as follows:

$L^{1-}_{N} = 0.1736$, $L^{2-}_{N} = 0.1565$, $L^{3-}_{N} = 0.1542$.

Step 6. Calculation of the relative closeness coefficient

We calculate the relative closeness co-efficient τ_i^* (i = 1, 2, 3) by using Eq. (9) are shown as follows:

$\tau_1^* = 0.5296$, $\tau_2^* = 0.7993$, $\tau_3^* = 0.6719$.

Step 7. Rank the alternatives

The ranking order of alternatives according to the relative closeness coefficient is presented as follows:

$C_2 \succ C_3 \succ C_1$.

Consequently, C_2 is the best candidate.

Table 1. Tabular form of the given SVNSESs

U_1	(0.7, 0.5, 0.2)	(0.5, 0.1, 0.4)	(0.7, 0.3, 0.5)	(0.6, 0.2, 0.3)	(0.3, 0.4, 0.3)	(0.4, 0.1, 0.3)	(0.5, 0.1, 0.2)	(0.5, 0.2, 0.3)
U_2	(0.6, 0.2, 0.3)	(0.9, 0.2, 0.2)	(0.9, 0.2, 0.1)	(0.9, 0.1, 0.1)	(0.5, 0.3, 0.2)	(0.3, 0.3, 0.1)	(0.4, 0.2, 0.3)	(0.3, 0.3, 0.2)
U_3	(0.8, 0.3, 0.3)	(0.8, 0.1, 0.2)	(0.9, 0.2, 0.1)	(0.7, 0.1, 0.4)	(0.8, 0.3, 0.2)	(0.2, 0.3, 0.5)	(0.4, 0.3, 0.4)	(0.5, 0.2, 0.5)

Table 2. Formulation of weighted decision matrix of the given SVNSESs

U_1	(0.134, 0.920, 0.824)	(0.092, 0.724, 0.880)	(0.175, 0.825, 0.895)	(0.120, 0.798, 0.845)	(0.042, 0.896, 0.865)	(0.059, 0.758, 0.865)	(0.054, 0.832, 0.879)	(0.08, 0.824, 0.865)
U_2	(0.104, 0.824, 0.865)	(0.276, 0.798, 0.798)	(0.308, 0.773, 0.692)	(0.276, 0.724, 0.724)	(0.080, 0.865, 0.824)	(0.042, 0.865, 0.758)	(0.04, 0.879, 0.908)	(0.042, 0.865, 0.824)
U_3	(0.176, 0.865, 0.920)	(0.202, 0.724, 0.798)	(0.175, 0.692, 0.864)	(0.202, 0.845, 0.798)	(0.026, 0.865, 0.920)	(0.054, 0.832, 0.929)	(0.04, 0.879, 0.908)	(0.042, 0.865, 0.824)

6 Conclusion

SVNSES is an effective and useful decision making tool to describe indeterminate and inconsistent information and it is also possible for a user to view the opinions of all experts in a single model. In this study, we have investigated a TOPSIS method for solving MADM problems with single valued neutrosophic soft expert information. The rating of performance values of the alternatives with respect to the parameters are presented in terms of SVNSs. We determine the weights of the parameters by maximizing deviation method and formulate weighted decision matrix. We identify SVNRPIS and SVNRNIS from the weighted decision matrix and normalized Euclidean distance measure is used to calculate distances of each alternative from SVNRPISs as well as SVNRNISs. Relative closeness co-efficient of each alternative is then calculated to select the most desirable alternative. Finally, an application of the proposed method for teacher selection is given.

In future, the proposed method can be used for dealing with interval-valued neutrosophic soft expert based MADM problems and different practical problems such as pattern recognition, medical diagnosis, information fusion, supplier selection, etc.

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Neutrosophic Quadruple Numbers, Refined Neutrosophic Quadruple Numbers, Absorbance Law, and the Multiplication of Neutrosophic Quadruple Numbers

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Abstract. In this paper, we introduce for the first time the neutrosophic quadruple numbers (of the form $\boldsymbol{a+bT+cI+dF}$) and the refined neutrosophic quadruple numbers.
Then we define an absorbance law, based on a prevalence order, both of them in order to multiply the neutrosophic components $\boldsymbol{T, I, F}$ or their sub-components $\boldsymbol{T_j, I_k, F_l}$ and thus to construct the multiplication of neutrosophic quadruple numbers.

Keywords: neutrosophic quadruple numbers, refined neutrosophic quadruple numbers, absorbance law, multiplication of neutrosophic quadruple numbers, multiplication of refined neutrosophic quadruple numbers.

1 Neutrosophic Quadruple Numbers

Let's consider an entity (i.e. a number, an idea, an object, etc.) which is represented by a known part (a) and an unknown part $(bT+cI+dF)$.

Numbers of the form:

$$NQ = a + bT + cI + dF, \tag{1}$$

where a, b, c, d are real (or complex) numbers (or intervals or in general subsets), and

T = truth / membership / probability,
I = indeterminacy,
F = false / membership / improbability,

are called Neutrosophic Quadruple (Real respectively Complex) Numbers (or Intervals, or in general Subsets).

"a" is called the known part of NQ, while "$bT + cI + dF$" is called the unknown part of NQ.

2 Operations

Let

$$NQ_1 = a_1 + b_1T + c_1I + d_1F, \tag{2}$$

$$NQ_2 = a_2 + b_2T + c_2I + d_2F \tag{3}$$

and $\alpha \in \mathbb{R}$ (or $\alpha \in \mathbb{C}$) a real (or complex) scalar.

Then:

2.1 Addition

$$NQ_1 + NQ_2 = (a_1 + a_2) + (b_1 + b_2)T + (c_1 + c_2)I + (d_1 + d_2)F. \tag{4}$$

2.2 Substraction

$$NQ_1 - NQ_2 = (a_1 - a_2) + (b_1 - b_2)T + (c_1 - c_2)I + (d_1 - d_2)F. \tag{5}$$

2.3 Scalar Multiplication

$$\alpha \cdot NQ = NQ \cdot \alpha = \alpha a + \alpha bT + \alpha cI + \alpha dF. \tag{6}$$

One has:

$$0 \cdot T = 0 \cdot I = 0 \cdot F = 0, \tag{7}$$

and

$$mT + nT = (m+n)T, \tag{8}$$

$$mI + nI = (m+n)I, \tag{9}$$

$$mF + nF = (m+n)F. \tag{10}$$

3 Refined Neutrosophic Quadruple Numbers

Let us consider that Refined Neutrosophic Quadruple Numbers are numbers of the form:

$$RNQ = a + \sum_{i=1}^{p} b_i T_i + \sum_{j=1}^{r} c_j I_j + \sum_{k=1}^{s} d_k F_k, \tag{11}$$

where a, all b_i, all c_j, and all d_k are real (or complex) numbers, intervals, or, in general, subsets,

while $T_1, T_2, \ldots, T_p$ are refinements of T;
$I_1, I_2, \ldots, I_r$ are refinements of I;
and $F_1, F_2, \ldots, F_s$ are refinements of F.

There are cases when the known part *(a)* can be refined as well as a_1, a_2, … .

The operations are defined similarly.

Let

$$RNQ^{(u)} = a^{(u)} + \sum_{i=1}^{p} b_i^{(u)} T_i + \sum_{j=1}^{r} c_j^{(u)} I_j + \sum_{k=1}^{s} d_k^{(u)} F_k, \quad (12)$$

for $u = 1$ or 2.
Then:

3.1 Addition

$$RNQ^{(1)} + RNQ^{(2)} = \left[a^{(1)} + a^{(2)}\right] + \sum_{i=1}^{p} \left[b_i^{(1)} + b_i^{(2)}\right] T_i + \sum_{j=1}^{r} \left[c_j^{(1)} + c_j^{(2)}\right] I_j + \sum_{k=1}^{s} \left[d_k^{(1)} + d_k^{(2)}\right] F_k. \quad (13)$$

3.2 Substraction

$$RNQ^{(1)} - RNQ^{(2)} = \left[a^{(1)} - a^{(2)}\right] + \sum_{i=1}^{p} \left[b_i^{(1)} - b_i^{(2)}\right] T_i + \sum_{j=1}^{r} \left[c_j^{(1)} - c_j^{(2)}\right] I_j + \sum_{k=1}^{s} \left[d_k^{(1)} - d_k^{(2)}\right] F_k. \quad (14)$$

3.3 Scalar Multiplication

For $\alpha \in \mathbb{R}$ (or $\alpha \in \mathbb{C}$) one has:

$$\alpha \cdot RNQ^{(1)} = \alpha \cdot a^{(1)} + \alpha \cdot \sum_{i=1}^{p} b_i^{(1)} T_i + \alpha \cdot \sum_{j=1}^{r} c_j^{(1)} I_j + \alpha \cdot \sum_{k=1}^{s} d_k^{(1)} F_k. \quad (15)$$

4 Absorbance Law

Let S be a set, endowed with a total order $x \prec y$, named “x prevailed by y” or “x less stronger than y” or “x less preferred than y”. We consider $x \preccurlyeq y$ as “x prevailed by or equal to y” “x less stronger than or equal to y”, or “x less preferred than or equal to y”.

For any elements $x, y \in S$, with $x \preccurlyeq y$, one has the absorbance law:

$$\boxed{x \cdot y = y \cdot x = \text{absorb}\ (x, y) = \max\{x, y\} = y,} \quad (16)$$

which means that the bigger element absorbs the smaller element (the big fish eats the small fish!).

Clearly,

$$x \cdot x = x^2 = \text{absorb}\ (x, x) = \max\{x, x\} = x, \quad (17)$$

and

$$x_1 \cdot x_2 \cdot \ldots \cdot x_n = \text{absorb}(\ldots \text{absorb}(\text{absorb}(x_1, x_2), x_3) \ldots, x_n) = \max\{\ldots \max\{\max\{x_1, x_2\}, x_3\} \ldots, x_n\} = \max\{x_1, x_2, \ldots, x_n\}. \quad (18)$$

Analougously, we say that “$x \succ y$” and we read: “x prevails to y” or “x is stronger than y” or “x is preferred to y”.

Also, $x \succcurlyeq y$, and we read: “x prevails or is equal to y” “x is stronger than or equal to y”, or “x is preferred or equal to y”.

5 Multiplication of Neutrosophic Quadruple Numbers

It depends on the prevalence order defined on $\{T, I, F\}$.

Suppose in an optimistic way the neutrosophic expert considers the prevalence order $T \succ I \succ F$. Then:

$$\begin{aligned} NQ_1 \cdot NQ_2 &= (a_1 + b_1T + c_1I + d_1F) \cdot (a_2 + b_2T + c_2I + d_2F) \\ &= a_1a_2 + (a_1b_2 + a_2b_1 + b_1b_2 + b_1c_2 + c_1b_2 + b_1d_2 + d_1b_2)T \\ &\quad + (a_1c_2 + a_2c_1 + c_1d_2 + c_2d_1)I + (a_1d_2 + a_2d_1 + d_1d_2)F, \end{aligned} \quad (19)$$

since $TI = IT = T, TF = FT = T, IF = FI = I$, while $T^2 = T, I^2 = I, F^2 = F$.

Suppose in an pessimistic way the neutrosophic expert considers the prevalence order $F \succ I \succ T$. Then:

$$\begin{aligned} NQ_1 \cdot NQ_2 &= (a_1 + b_1T + c_1I + d_1F) \cdot (a_2 + b_2T + c_2I + d_2F) \\ &= a_1a_2 + (a_1b_2 + a_2b_1 + b_1b_2)T + (a_1c_2 + a_2c_1 + b_1c_2 + b_2c_1 + c_1c_2)I \\ &\quad + (a_1d_2 + a_2d_1 + b_1d_2 + b_2d_1 + c_1d_2 + c_2d_1 + d_1d_2)F, \end{aligned} \quad (20)$$

since

$$F \cdot I = I \cdot F = F, F \cdot T = T \cdot F = F, I \cdot T = T \cdot I = I$$

while similarly $F^2 = F, I^2 = I, T^2 = T$.

5.1 Remark

Other prevalence orders on $\{T, I, F\}$ can be proposed, depending on the application/problem to solve, and on other conditions.

6 Multiplication of Refined Neutrosophic Quadruple Numbers

Besides a neutrosophic prevalence order defined on $\{T, I, F\}$, we also need a sub-prevalence order on $\{T_1, T_2, \ldots, T_p\}$, a sub-prevalence order on $\{I_1, I_2, \ldots, I_r\}$, and another sub-prevalence order on $\{F_1, F_2, \ldots, F_s\}$.

We assume that, for example, if $T \succ I \succ F$, then $T_j \succ I_k \succ F_l$ for any $j \in \{1, 2, \ldots, p\}$, $k \in \{1, 2, \ldots, r\}$, and $l \in \{1, 2, \ldots, s\}$. Therefore, any prevalence order on $\{T, I, F\}$ imposes a prevalence suborder on their corresponding refined components.

Without loss of generality, we may assume that

$$T_1 \succ T_2 \succ \cdots \succ T_p$$

(if this was not the case, we re-number the subcomponents in a decreasing order).

Similarly, we assume without loss of generality that:

$I_1 \succ I_2 \succ \cdots \succ I_r$, and

$F_1 \succ F_2 \succ \cdots \succ F_s$.

6.1 Exercise for the Reader

Let's have the neutrosophic refined space

$$NS = \{T_1, T_2, T_3, I, F_1, F_2\},$$

with the prevalence order $T_1 \succ T_2 \succ T_3 \succ I \succ F_1 \succ F_2$.

Let's consider the refined neutrosophic quadruples

$NA = 2 - 3T_1 + 2T_2 + T_3 - I + 5F_1 - 3F_2$, and

$NB = 0 + T_1 - T_2 + 0 \cdot T_3 + 5I - 8F_1 + 5F_2$.

By multiplication of sub-components, the bigger absorbs the smaller. For example:

$T_2 \cdot T_3 = T_2$,

$T_1 \cdot F_1 = T_1$,

$I \cdot F_2 = I$,

$T_2 \cdot F_1 = T_2$, etc.

Multiply NA with NB.

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On Refined Neutrosophic Algebraic Structures

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Abstract.The objective of this paper is to develop refined neutrosophic algebraic structures. In particular, we study refined neutrosophic group and we present some of its elementary properties.

Keywords:neutrosophic logic, neutrosophic set,refined neutrosophic algebraic structures, refined neutrosophic group,refined neutrosophic numbers.

1 Introduction

In neutrosophic logic, each proposition is approximated to have the percentage of truth in a subset (T), the percentage of indeterminacy in a subset (I), and the percentage of falsity in a subset (F), where T, I, F are standard or non-standard subsets of the non-standard unit interval $]^-0, 1^+[$.

The concept of neutrosophic numbers of the form $a + bI$, where I is the indeterminacy with $I^n = I$, and, a and b are real or complex numbers, was introduced by Kandasamy and Smarandache in 2003. In the same year, Kandasamy and Smarandache introduced the concept of neutrosophic algebraic structures by combining the indeterminate element I with the elements of a given algebraicstructure $(X, *)$ to form a new algebraic structure $(X(I), *) =<X, I>$ generated by X and I and they called it a neutrosophic algebraic structure. Some of the neutrosophic algebraic structures developed and studied by Kandasamy and Smarandache include neutrosophic groupoids, neutrosophic semigroups, neutrosophic groups, neutrosophic loops, neutrosophic rings, neutrosophic fields, neutrosophic vector spaces, neutrosophic modules, neutrosophic bigroupoids, neutrosophic bisemigroups, neutrosophic bigroups, neutrosophic biloops, neutrosophic N-groups, neutrosophic N-semigroups, neutrosophic N-loops, and so on.

In [5], Smarandache introduced the refined neutrosophic logic and neutrosophic set where it was shown that it is possible to split the components $<T, I, F>$ into the form $<T_1,T_2, \dots , T_p; I_1, I_2, \dots , F_r; F_1, F_2, \dots, F_s>$. Also in [6], Smarandache extended the neutrosophic numbers $a + bI$ into refined neutrosophic numbers of the form $a + b_1I_1 + b_2I_2 + \dots + b_nI_n$, where $a, b_1, b_2, \dots , b_n$ are real or complex numbers and considered the refined neutrosophic set based on these refined neutrosophic numbers.

2 Refined Neutrosophic Algebraic Structures

Consider the split of the indeterminacy I into two indeterminacies I_1 and I_2 defined as follows:

I_1= contradiction (true (T) and false (F)), (1)

I_2 = ignorance (true (T) or false) (F). (2)

It can be shown from (1) and (2) that:

$I_1^2 = I_1$; (3)

$I_2^2 = I_2$; (4)

$I_1I_2 = I_2I_1 = I_1$. (5)

Now, let X be a nonempty set and let I_1 and I_2 be two indeterminacies. Then the set

$X(I_1, I_2) =< X, I_1,I_2> = \{(x, yI_1, zI_2):x, y, z \in X\}$(6)

is called a refined neutrosophic set generated by X, I_1 and I_2, and (x, yI_1, zI_2) is called a refined neutrosophic element of $X(I_1, I_2)$. If + and . are ordinary addition and multiplication, I_k with k = 1, 2 have the following properties:

(1) $I_k+I_k+\cdots+I_k=nI_k$.

(2) $I_k+(-I_k)=0$.

(3) $I_k.I_k.\cdots.I_k= I_k^n=I_k$ for all positive integer $n>1$.

(4) $0.I_k=0$.

(5) I_k^{-1} is undefined and therefore does not exist.

If $* : X(I_1, I_2) \times X(I_1, I_2) \to X(I_1, I_2)$ is a binary operation defined on $X(I_1, I_2)$, then the couple $(X(I_1, I_2),*)$ is called a refined neutrosophic algebraic structure and it is named according to the laws (axioms) satisfied by $*$. If $(X(I_1, I_2),*)$ and $(Y(I_1, I_2),*')$ are two refined neutrosophic algebraic structures, the mapping $\varphi: (X(I_1, I_2),*) \to(Y(I_1, I_2),*')$ is called a neutrosophic homomorphism if the following conditions hold:

(1) $\varphi((a, bI_1, cI_2) * (d, eI_1, fI_2)) = \varphi((a, bI_1, cI_2)) *' \varphi((d, eI_1, fI_2))$.

(2) $\varphi(I_k) = I_k \; \forall(a, bI_1, cI_2), (d, eI_1, fI_2) \in X(I_1, I_2)$ and $k = 1, 2$.

Definition 2.1.

Let $(X(I_1, I_2), +, .)$ be any refined neutrosophic algebraic structure where + and . are ordinary addition and multiplication respectively. For any two elements $(a, bI_1, cI_2), (d, eI_1, fI_2) \in X(I_1, I_2)$, we define

$(a, bI_1, cI_2) + (d, eI_1, fI_2)=(a + d, (b + e)I_1, (c + f)I_2)$,(7)

$(a, bI_1, cI_2).(d, eI_1, fI_2) = (ad, (ae + bd + be + bf + ce)I_1,$
$(af + cd + cf)I_2)$. (8)

Definition 2.2.

Let $(G, *)$ be any group. We call the couple $(G(I_1, I_2), *)$ a refined neutrosophic group generated by G, I_1 and I_2. $(G(I_1, I_2), *)$ is said to be commutative if for all $x, y \in G(I_1, I_2)$, we have $x * y = y * x$. Otherwise, we call $(G(I_1, I_2), *)$ a non-commutative refined neutrosophic group.

$(G(I_1, I_2), *)$ is called a finite refined neutrosophic group if the elements in $G(I_1, I_2)$ are countable. Otherwise, $G(I_1, I_2)$ is called an infinite refined neutrosophic group. If the number of elements in $G(I_1, I_2)$ is n, we call n the order of $G(I_1, I_2)$ and we write $o(G(I_1, I_2)) = n$. For an infinite refined neutrosophic group $G(I_1, I_2)$, we write $o(G(I_1, I_2)) = \infty$.

Example 1.

$(\mathbb{Z}(I_1, I_2), +)$, $(\mathbb{R}(I_1, I_2), +)$, $(\mathbb{C}(I_1, I_2), +)$, $(\mathbb{R}(I_1, I_2), .)$ and $(\mathbb{C}(I_1, I_2), +.)$ are commutative refined neutrosophic groups.

Example 2.

$\left(M^{\mathbb{R}}_{2\times 2}(I_1, I_2, .)\right)$, where

$$M^{\mathbb{R}}_{2\times 2}(I_1, I_2, .) = \left\{\begin{bmatrix} w & x \\ y & z \end{bmatrix} : w, x, y, z \in \mathbb{R}(I_1, I_2)\right\}$$

is a non-commutative refined neutrosophic group.

Example 3.

Let $\mathbb{Z}_2(I_1, I_2) = \{(0, 0, 0), (1, 0, 0), (0, I_1, 0), (0, 0, I_2), (0, I_1, I_2), (1, I_1, 0), (1, 0, I_2), (1, I_1, I_2)\}$.

Then $(\mathbb{Z}2(I_1, I_2), +)$ is a commutative refined neutrosophic group of integers modulo 2. Generally for a positive integer $n \geq 2$, $(\mathbb{Z}_n(I_1, I_2), +)$ is a finite commutative refined neutrosophic group of integers modulo n.

Theorem 2.3.

(1) Every refined neutrosophic group is a semi group but not agroup.
(2) Every refined neutrosophic group contains a group.

Corollary 2.4.

Every refined neutrosophic group $(G(I_1, I_2), +)$ is a group.

Theorem 2.5.

Let $(G(I_1, I_2), *)$ and and $(H(I_1, I_2), *)$ be two refined neutrosophic groups. Then $G(I_1, I_2) \times H(I_1, I_2) = \{(x, y) : x \in G(I_1, I_2), y \in H(I_1, I_2)\}$ is a refined neutrosophic group.

Definition 2.6.

Let $(G(I_1, I_2), *)$ be a refined neutrosophic group and let $A(I_1, I_2)$ be a nonempty subset of $G(I_1, I_2)$. $A(I_1, I_2)$ is called a refined neutrosophic sub-group of $G(I_1, I_2)$ if $(A(I_1, I_2), *)$ is a refined neutrosophic group. It is essential that $A(I_1, I_2)$ contains a proper subset which is a group.

Otherwise, $A(I_1, I_2)$ will be called a pseudo refined neutrosophic subgroup of $G(I_1, I_2)$.

Example 4.

Let $G(I_1, I_2) = (\mathbb{Z}(I_1, I_2), +)$ and let $A(I_1, I_2) = (3\,\mathbb{Z}(I_1, I_2), +)$. Then $A(I_1, I_2)$ is a refined neutrosophic subgroup of $G(I_1, I_2)$.

Example 5.

Let $G(I_1, I_2) = (\mathbb{Z}_6(I_1, I_2), +)$ and let

$A(I_1, I_2) = \{(0, 0, 0), (0, I_1, 0), (0, 0, I_2), (0, I_1, I_2),$
$(0, 2I_1, 0), (0, 0, 2I_2), (0, 2I_1, 2I_2),$
$(0, 3I_1, 0), (0, 0, 3I_2), (0, 3I_1, 3I_2),$
$(0, 4I_1, 0), (0, 0, 4I_2), (0, 4I_1, 4I_2),$
$(0, 5I_1, 0), (0, 0, 5I_2), (0, 5I_1, 5I_2)\}$.

Then $A(I_1, I_2)$ is a pseudo refined neutrosophic subgroup of $G(I_1, I_2)$.

Theorem 2.7.

Let $\{A_k(I_1, I_2)\}_1^n$ be a family of refined neutrosophic subgroups(pseudo refined neutrosophic subgroups) of a refined neutrosophic group $G(I_1, I_2)$. Then $\bigcap_1^n A_k(I_1, I_2)\}$ is a refined neutrosophic subgroup (pseudo refined neutrosophic subgroup) of $G(I_1, I_2)$.

Definition 2.9.

Let $A(I_1, I_2)$ and $B(I_1, I_2)$ be any two refined neutrosophic sub-groups (pseudo refined neutrosophic subgroups) of a refined neutrosophic group $(G(I_1, I_2), +)$. We define the sum $A(I_1, I_2) + B(I_1, I_2)$ by the set

$A(I_1, I_2) + B(I_1, I_2) = \{a + b : a \in A(I_1, I_2),$
$b \in B(I_1, I_2)\}$ (9)

which is a refined neutrosophic subgroup (pseudo refined neutrosophic subgroup) of $G(I_1, I_2)$.

Theorem 2.9.

Let $A(I_1, I_2)$ be any refined neutrosophic subgroup of a refined neutrosophic group $(G(I_1, I_2), +)$ and let $B(I_1, I_2)$ be any pseudo refined neutrosophic subgroup $(G(I_1, I_2), +)$. Then:

(1) $A(I_1, I_2) + A(I_1, I_2) = A(I_1, I_2)$.
(2) $B(I_1, I_2) + B(I_1, I_2) = B(I_1, I_2)$.
(3) $A(I_1, I_2) + B(I_1, I_2)$ is a refined neutrosophic subgroup of $G(I_1, I_2)$.

Definition 2.10.

Let $(G(I_1, I_2), *)$ and $(H(I_1, I_2), *')$ be two refined neutrosophic groups. The mapping $\varphi : (G(I_1, I_2), *) \to (H(I_1, I_2), *')$ is called a neutrosophic homomorphism if the following conditions hold:

(1) $\varphi(x * y) = \varphi(x) *' \varphi(y)$.

(2) $\varphi(I_k) = I_k \ \forall x, y \in G(I_1, I_2)$ and $k = 1, 2$.

The image of φ is defined by the set

$Im\varphi = \{y \in H(I_1, I_2) : y = \varphi(x),$ (10)

for some $x \in G(I_1, I_2)\}$.

If $G(I_1, I_2)$ and $H(I_1, I_2)$ are additive refined neutrosophic groups, then the kernel of the neutrosophic homomorphism $\varphi : (G(I_1, I_2), +) \to (H(I_1, I_2), +)$ is defined by the set

$Ker\varphi = \{x \in G(I_1, I_2) : \varphi(x) = (0, 0, 0)\}$.

Epimorphism, monomorphism, isomorphism, endomorphism and automorphism of φ have the same definitions as those of the classical cases.

Example 6.

Let $(G(I_1, I_2), *)$ and $(H(I_1, I_2), *')$ be two refined neutrosophic groups. Let $\varphi : G(I_1, I_2) \times H(I_1, I_2) \to G(I_1, I_2)$ be a mapping defined by $\varphi(x, y) = x$ and let $\psi: G(I_1, I_2) \times H(I_1, I_2) \to H(I_1, I_2)$ be a mapping defined by $\psi(x, y) = y$. Then φ and ψ are refined neutrosophic group homomorphisms.

Theorem 2.11.

Let $\varphi : (G(I_1, I_2), *) \to (H(I_1, I_2), *')$ be a refined neutro-sophic group homomorphism. Then $Im\varphi$ is a neutrosophic subgroup of $H(I_1, I_2)$.

Theorem 2.12.

Let $\varphi : (G(I_1, I_2), +) \to (H(I_1, I_2), +)$ be a refined neutro-sophic group homomorphism. Then $Ker\varphi$ is a subgroup of G and not a neutrosophic subgroup of $G(I_1, I_2)$.

Example 7.

Let $\varphi : \mathbb{Z}_2(I_1, I_2) \times \mathbb{Z}_2(I_1, I_2) \to \mathbb{Z}_2(I_1, I_2)$ be a neutrosophic group homomorphism defined by $\varphi(x, y) = x$ for all $x, y \in \mathbb{Z}_2(I_1, I_2)$. Then

$Im\varphi = \{(0, 0, 0), (1, 0, 0), (0, I_1, 0), (0, 0, I_2), (0, I_1, I_2), (1, I_1, 0), (1, 0, I_2), (1, I_1, I_2)\}$.

$Ker\varphi = \{((0, 0, 0), (0, 0, 0)), ((0, 0, 0), (1, 0, 0)), ((0, 0, 0), (0, I_1, 0)),((0, 0, 0), (0, I_1, I_2)), ((0, 0, 0), (0, 0, I_2)), ((0, 0, 0), (1, I_1, 0)),((0, 0, 0), (1, 0, I_2)), ((0, 0, 0), (1, I_1, I_2))\}$.

Conclusion

By splitting the usual indeterminacy I into two indeterminacies I_1 and I_2, we have developed a new neutrosophic set $X(I_1, I_2)$ called a refined neutrosophic set and we have generated a new neutrosophic algebraic structure $(X(I_1, I_2), *)$ from X, I_1 and I_2 which we called a refined neutrosophic algebraic structure. In particular, we have studied refined neutrosophic group and we have presented some of its elementary properties.

Using the same approach as in this paper, other refined neutrosophic algebraic structures involving rings, fields, vector spaces, modules, group rings, loops, hypergroups, hyperrings, algebras, and so on could be developed. We hope to look into these in our future papers.

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Neutrosophic Actions, Prevalence Order, Refinement of Neutrosophic Entities, and Neutrosophic Literal Logical Operators

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Abstract. In this paper, we define for the first time three *neutrosophic actions* and their properties. We then introduce the *prevalence order* on {T, I, F} with respect to a given *neutrosophic operator* "o", which may be subjective - as defined by the neutrosophic experts; and the refinement of *neutrosophic entities* <A>, <neutA>, and <antiA>. Then we extend the classical logical operators to *neutrosophic literal logical operators* and to *refined literal logical operators*, and we define the *refinement neutrosophic literal space*.

Keywords: neutrosophic logic, neutrosophic actions, prevalence order, neutrosophic operator, neutrosophic entities, neutrosophic literal logical operators, refined literal logical operators, refinement neutrosophic literal space, neutrosophic conjunction, neutrosophic disjunction, neutrosophic Sheffer's stroke, neutrosophic equivalence.

1 Introduction

In Boolean Logic, a proposition $\mathcal{P}$ is either true (T), or false (F). In Neutrosophic Logic, a proposition $\mathcal{P}$ is either true (T), false (F), or indeterminate (I).

For example, in Boolean Logic the proposition $\mathcal{P}_1$:

$"1 + 1 = 2 \text{ (in base 10)}"$

is true, while the proposition $\mathcal{P}_2$:

$"1 + 1 = 3 \text{ (in base 10)}"$

is false.

In neutrosophic logic, besides propositions $\mathcal{P}_1$ (which is true) and $\mathcal{P}_2$ (which is false), we may also have proposition $\mathcal{P}_3$:

$"1 + 1 = ? \text{ (in base 10)}"$,

which is an incomplete/indeterminate proposition (neither true, nor false).

1.1 Remark

All *conjectures in science* are indeterminate at the beginning (researchers not knowing if they are true or false), and later they are proved as being either true, or false, or indeterminate in the case they were unclearly formulated.

1.2 Notations

In order to avoid confusions regarding the operators, we note them as:

a. Boolean (classical) logic:

$\neg, \quad \wedge, \quad \vee, \quad \underline{\vee}, \quad \rightarrow, \quad \leftrightarrow$

b. Fuzzy logic:

$\underset{F}{\neg}, \quad \underset{F}{\wedge}, \quad \underset{F}{\vee}, \quad \underset{F}{\underline{\vee}}, \quad \underset{F}{\rightarrow}, \quad \underset{F}{\leftrightarrow}$

c. Neutrosophic logic:

$\underset{N}{\neg}, \quad \underset{N}{\wedge}, \quad \underset{N}{\vee}, \quad \underset{N}{\underline{\vee}}, \quad \underset{N}{\rightarrow}, \quad \underset{N}{\leftrightarrow}$

2 Three Neutrosophic Actions

In the frame of neutrosophy, we have considered [1995] for each entity $\langle A \rangle$, its opposite $\langle \text{anti}A \rangle$, and their neutrality $\langle \text{neut}A \rangle$ {i.e. neither $\langle A \rangle$, nor $\langle \text{anti}A \rangle$}.

Also, by $\langle \text{non}A \rangle$ we mean what is not $\langle A \rangle$, i.e. its opposite $\langle \text{anti}A \rangle$, together with its neutral(ity) $\langle \text{neut}A \rangle$; therefore:

$\langle \text{non}A \rangle = \langle \text{neut}A \rangle \vee \langle \text{anti}A \rangle.$

Based on these, we may straightforwardly introduce for the first time the following neutrosophic actions with respect to an entity <A>:

1. **To neutralize** (or **to neuter**, or simply **to neutize**) the entity <A>. [As a noun: neutralization, or neuter-ization, or simply neut-ization.]

   We denote it by <neutA> or neut(A).

2. **To antithetic-ize** (or **to anti-ize**) the entity <A>. [As a noun: antithetic-ization, or anti-ization.]

   We denote it by <antiA> ot anti(A).

   This action is 100% opposition to entity <A> (strong opposition, or strong negation).

3. To **non-ize** the entity <A>. [As a noun: non-ization].

   We denote it by <nonA> or non(A).

   It is an opposition in a percentage between (0, 100]% to entity <A> (weak opposition).

Of course, not all entities <A> can be neutralized, or antithetic-ized, or non-ized.

2.2 Example

Let

$\langle A \rangle$ = "Phoenix Cardinals beats Texas Cowboys".

Then,

$\langle \text{neut}A \rangle$ = "Phoenix Cardinals has a tie game with Texas Cowboys";

$\langle \text{anti}A \rangle$ = "Phoenix Cardinals is beaten by Texas Cowboys";

$\langle \text{non}A \rangle$ = "Phoenix Cardinals has a tie game with Texas Cowboys, or Phoenix Cardinals is beaten by Texas Cowboys".

3 Properties of the Three Neutrosophic Actions

$\text{neut}(\langle \text{anti}A \rangle) = \text{neut}(\langle \text{neut}A \rangle) = \text{neut}(A)$;

anti(⟨antiA⟩) = A; anti(⟨neutA⟩) = ⟨A⟩ or ⟨antiA⟩;

non(⟨antiA⟩) = ⟨A⟩ or ⟨neutA⟩; non(⟨neutA⟩) = ⟨A⟩ or ⟨antiA⟩.

4 Neutrosophic Actions' Truth-Value Tables

Let's have a logical proposition P, which may be true (T), Indeterminate (I), or false (F) as in previous example. One applies the neutrosophic actions below.

4.1 Neutralization (or Indetermination) of *P*

neut(P)	*T*	*I*	*F*
	I	*I*	*I*

4.2 Antitheticization (Neutrosophic Strong Opposition to *P*

anti(P)	*T*	*I*	*F*
	F	$T \vee F$	*T*

4.3 Non-ization (Neutrosophic Weak Opposition to *P*)

non(P)	*T*	*I*	*F*
	$I \vee F$	$T \vee F$	$T \vee I$

5 Refinement of Entities in Neutrosophy

In neutrosophy, an entity $\langle A \rangle$ has an opposite $\langle \text{anti}A \rangle$ and a neutral $\langle \text{neut}A \rangle$. But these three categories can be refined in sub-entities $\langle A \rangle_1, \langle A \rangle_2, \ldots, \langle A \rangle_m$, and respectively $\langle \text{neut}A \rangle_1, \langle \text{neut}A \rangle_2, \ldots, \langle \text{neut}A \rangle_n$, and also $\langle \text{anti}A \rangle_1, \langle \text{anti}A \rangle_2, \ldots, \langle \text{anti}A \rangle_p$, where *m, n, p* are integers ≥ 1, but $m + n + p \geq 4$ (meaning that at least one of $\langle A \rangle$, $\langle \text{anti}A \rangle$ or $\langle \text{neut}A \rangle$ is refined in two or more sub-entities).

For example, if $\langle A \rangle$ = white color,

then $\langle \text{anti}A \rangle$ = black color,

while $\langle \text{neut}A \rangle$ = colors different from white and black.

If we refine them, we get various nuances of white color: $\langle A \rangle_1, \langle A \rangle_2, \ldots$, and various nuances of black color: $\langle \text{anti}A \rangle_1, \langle \text{anti}A \rangle_2, \ldots$, and the colors in between them (red, green, yellow, blue, etc.): $\langle \text{neut}A \rangle_1, \langle \text{neut}A \rangle_2, \ldots$.

Similarly as above, we want to point out that not all entities <A> and/or their corresponding (if any) <neutA> and <antiA> can be refined.

6 The Prevalence Order

Let's consider the classical literal (symbolic) truth *(T)* and falsehood *(F)*.

In a similar way, for neutrosophic operators we may consider the literal (symbolic) truth *(T)*, the literal (symbolic) indeterminacy *(I)*, and the literal (symbolic) falsehood *(F)*.

We also introduce the *prevalence order* on $\{T, I, F\}$ with respect to a given binary and commutative neutrosophic operator "o".

The neutrosophic operators are: neutrosophic negation, neutrosophic conjunction, neutrosophic disjunction, neutrosophic exclusive disjunction, neutrosophic Sheffer's stroke, neutrosophic implication, neutrosophic equivalence, etc.

The prevalence order is *partially objective* (following the classical logic for the relationship between *T* and *F*), and *partially subjective* (when the indeterminacy *I* interferes with itself or with *T* or *F*).

For its subjective part, the prevalence order is determined by the neutrosophic logic expert in terms of the application/problem to solve, and also depending on the specific conditions of the application/problem.

For $X \neq Y$, we write $X ⓟ Y$, or $X \succ_o Y$, and we read "X" prevails to Y with respect to the neutrosophic binary commutative operator "o", which means that $XoY = X$.

Let's see the below examples. We mean by "o": conjunction, disjunction, exclusive disjunction, Sheffer's stroke, and equivalence.

7 Neutrosophic Literal Operators & Neutrosophic Numerical Operators

7.1 If we mean by *neutrosophic literal proposition*, a proposition whose truth value is a letter: either T or I or F. The operators that deal with such logical propositions are called *neutrosophic literal operators*.

7.2. And by *neutrosophic numerical proposition*, a proposition whose truth value is a triple of numbers (or in general of numerical subsets of the interval [0, 1]), for examples A(0.6, 0.1, 0.4) or B([0, 0.2], {0.3, 0.4, 0.6}, (0.7, 0.8)).

The operators that deal with such logical propositions are called *neutrosophic numerical operators*.

8 Truth-Value Tables of Neutrosophic Literal Operators

In Boolean Logic, one has the following truth-value table for negation:

8.1 Classical Negation

$\neg$	T	F
	F	T

In Neutrosophic Logic, one has the following neutrosophic truth-value table for the neutrosophic negation:

8.2 Neutrosophic Negation

$\neg_N$	T	I	F
	F	I	T

So, we have to consider that the negation of I is I, while the negations of T and F are similar as in classical logic.

In classical logic, one has:

8.3 Classical Conjunction

$\wedge$	T	F
T	T	F
F	F	F

In neutrosophic logic, one has:

8.4 Neutrosophic Conjunction (AND_N), version 1

$\wedge_N$	T	I	F
T	T	I	F
I	I	I	I
F	F	I	F

The objective part (circled literal components in the above table) remains as in classical logic, but when indeterminacy I interferes, the neutrosophic expert may choose the most fit prevalence order.

There are also cases when the expert may choose, for various reasons, to entangle the classical logic in the objective part. In this case, the prevalence order will be totally subjective.

The prevalence order works for classical logic too. As an example, for classical conjunction, one has $F \succ_c T$, which means that $F \wedge T = F$. While the prevalence order for the neutrosophic conjunction in the above tables was:

$I \succ_c F \succ_c T$,

which means that $I \wedge_N F = I$, and $I \wedge_N T = I$.

Other prevalence orders can be used herein, such as:

$F \succ_c I \succ_c T$,

and its corresponding table would be:

8.5 Neutrosophic Conjunction (AND_N), version 2

$\wedge_N$	T	I	F
T	T	I	F
I	I	I	F
F	F	F	F

which means that $F_{\wedge_N} I = F$ and $I_{\wedge_N} I = I$; or another prevalence order:

$F \succ_c T \succ_c I$,

and its correspongING table would be:

8.6 Neutrosophic Conjunction (AND_N), version 3

$\wedge_N$	T	I	F
T	T	T	F
I	T	I	F
F	F	F	F

which means that $F_{\wedge_N} I = F$ and $T_{\wedge_N} I = T$.

If one compares the three versions of the neutrosophic literal conjunction, one observes that the objective part remains the same, but the subjective part changes.

The subjective of the prevalence order can be established in an optimistic way, or pessimistic way, or according to the weights assigned to the neutrosophic literal components T, I, F by the experts.

In a similar way, we do for disjunction.

In classical logic, one has:

8.7 Classical Disjunction

$\vee$	T	F
T	T	T
F	T	F

In neutrosophic logic, one has:

8.8 Neutrosophic Disjunction (OR_N)

$\vee_N$	T	I	F
T	T	T	T
I	T	I	F
F	T	F	F

where we used the following prevalence order:

$T \succ_d F \succ_d I$,

but the reader is invited (as an exercise) to use another prevalence order, such as:

$T \succ_d I \succ_d F$,

or $I \succ_d T \succ_d F$, etc.,

for all neutrosophic logical operators presented above and below in this paper.

In classical logic, one has:

8.9 Classical Exclusive Disjunction

$\underline{\vee}$	T	F
T	F	T
F	T	F

In neutrosophic logic, one has:

8.10 Neutrosophic Exclusive Disjunction

$\underline{\vee}_N$	T	I	F
T	F	T	T
I	T	I	F
F	T	F	F

using the prevalence order

$T \succ_d F \succ_d I$.

In classical logic, one has:

8.11 Classical Sheffer's Stroke

$\mid$	T	F
T	F	T
F	T	T

In neutrosophic logic, one has:

8.12 Neutrosophic Sheffer's Stroke

$\mid_N$	T	I	F
T	F	T	T
I	T	I	I
F	T	I	T

using the prevalence order

$T \succ_d I \succ_d F$.

In classical logic, one has:

8.13 Classical Implication

$\rightarrow$	T	F
T	T	F
F	T	T

In neutrosophic logic, one has:

8.14 Neutrosophic Implication

$\rightarrow_N$	T	I	F
T	T	I	F
I	T	T	F
F	T	T	T

using the subjective preference that $I \rightarrow_N T$ is true (because in the classical implication T is implied by anything), and $I \rightarrow_N F$ is false, while $I \rightarrow_N I$ is true because is similar to the classical implications $T \rightarrow T$ and $F \rightarrow F$, which are true.

The reader is free to check different subjective preferences.

In classical logic, one has:

8.15 Classical Equivalence

$\leftrightarrow$	T	F
T	T	F
F	F	T

In neutrosophic logic, one has:

8.16 Neutrosophic Equivalence

$\leftrightarrow_N$	T	I	F
T	T	I	F
I	I	T	I
F	F	I	T

using the subjective preference that $I \leftrightarrow_N I$ is true, because it is similar to the classical equivalences that $T \rightarrow T$ and $F \rightarrow F$ are true, and also using the prevalence:

$I \succ_e F \succ_e T$.

9 Refined Neutrosophic Literal Logic

Each particular case has to be treated individually.

In this paper, we present a simple example.

Let's consider the following neutrosophic logical propositions:

T = Tomorrow it will rain or snow.

T is split into

→ Tomorrow it will rain.

→ Tomorrow it will snow.

F = Tomorrow it will neither rain nor snow.

F is split into

→ Tomorrow it will not rain.

→ Tomorrow it will not snow.

I = Do not know if tomorrow it will be raining, nor if it will be snowing.

I is split into

→ Do not know if tomorrow it will be raining or not.

→ Do not know if tomorrow it will be snowing or not.

Then:

$\neg_N$	T_1	T_2	I_1	I_2	F_1	F_2
	F_1	F_2	$T_1 \vee F_1$	$T_2 \vee F_2$	T_1	T_2

It is clear that the negation of T_1 (Tomorrow it will raining) is F_1 (Tomorrow it will not be raining). Similarly for the negation of T_2, which is F_2.

But, the negation of I_1 (Do not know if tomorrow it will be raining or not) is "Do know if tomorrow it will be raining or not", which is equivalent to "We know that tomorrow it will be raining" (T_1), or "We know that tomorrow it will not be raining" (F_1). Whence, the negation of I_1 is $T_1 \vee F_1$, and similarly, the negation of I_2 is $T_2 \vee F_2$.

9.1 Refined Neutrosophic Literal Conjunction Operator

$\wedge_N$	T_1	T_2	I_1	I_2	F_1	F_2
T_1	T_1	$T_{1\,2}$	I_1	I_2	F_1	F_2
T_2	$T_{1\,2}$	T_2	I_1	I_2	F_1	F_2
I_1	I_1	I_1	I_1	I	F_1	F_2
I_2	I_2	I_2	I	I_2	F_1	F_2
F_1	F_1	F_1	F_1	F_1	F_1	F
F_2	F_2	F_2	F_2	F_2	F	F_2

where $T_{1\,2} = T_1 \wedge T_2$ = "Tomorrow it will rain and it will snow".

Of course, other prevalence orders can be studied for this particular example.

With respect to the neutrosophic conjunction, F_l prevail in front of I_k, which prevail in front of T_j, or

$F_l \succ I_k \succ T_j$,

for all $l, k, j \in \{1, 2\}$.

9.2 Refined Neutrosophic Literal Disjunction Operator

$\vee_N$	T_1	T_2	I_1	I_2	F_1	F_2
T_1	T_1	T	T_1	T_1	T_1	T_1
T_2	T	T_2	T_2	T_2	T_2	T_2
I_1	T_1	T_2	I_1	I	F_1	F_2
I_2	T_1	T_2	I	I_2	F_1	F_2
F_1	T_1	T_2	F_1	F_1	F_1	$F_1 \vee F_2$
F_2	T_1	T_2	F_2	F_2	$F_1 \vee F_2$	F_2

With respect to the neutrosophic disjunction, T_j prevail in front of F_l, which prevail in front of I_k, or

$T_j \succ F_l \succ I_k$,

for all $j, l, k \in \{1, 2\}$.

For example, $T_1 \vee T_2 = T$, but

$F_1 \vee F_2 \notin \{T, I\ F\} \cup \{T_1, T_2, I_1, I_2, F_1, F_2\}$.

10 The Refinement Neutrosophic Literal Space

The Refinement Neutrosophic Literal Space $\{T_1, T_2, I_1, I_2, F_1, F_2\}$ is not closed under neutrosophic negation, neutrosophic conjunction, and neutrosophic disjunction.

The reader can check the closeness under other neutrosophic literal operations.

A *neutrosophic refined literal space*

$$S_N = \{T_1, T_2, \dots, T_p;\ I_1, I_2, \dots, I_r;\ F_1, F_2, \dots, F_s\},$$

where p, r, s are integers ≥ 1, is said to be **closed** under a given neutrosophic operator "θ_N", if for any elements $X, Y \in S_N$ one has $X_{\theta_N} Y \in S_N$.

Let's denote the extension of S_N with respect to a single θ_N by:

$S_{N_*}^C = (S_N, \theta_N)$.

If S_N is not closed with respect to the given neutrosophic operator θ_N, then $S_{N_*}^C \neq S_N$, and we extend S_N by adding in the new elements resulted from the operation $X \theta_N Y$, let's denote them by $A_1, A_2, \dots A_m$.

Therefore,

$S_{N_*}^C \neq S_N \cup \{A_1, A_2, \dots A_m\}$.

$S_{N_*}^C$ encloses S_N.

Similarly, we can define the *closeness* of the neutrosophic refined literal space S_N with respect to the two or more neutrosophic operators $\theta_{1_N}, \theta_{2_N}, \dots, \theta_{w_N}$, for $w \geq 2$.

S_N is closed under $\theta_{1_N}, \theta_{2_N}, \dots, \theta_{w_N}$ if for any $X, Y \in S_N$ and for any $i \in \{1, 2, \dots, w\}$ one has $X_{\theta_{i_N}} Y \in S_N$.

If S_N is not closed under these neutrosophic operators, one can extend it as previously.

Let's consider: $S_{N_{**}}^C = (S_N, \theta_{1_N}, \theta_{2_N}, \dots, \theta_{w_N})$, which is S_N closed with respect to all neutrosophic operators $\theta_{1_N}, \theta_{2_N}, \dots, \theta_{w_N}$, then $S_{N_{**}}^C$ encloses S_N.

Conclusion

We have defined for the first time three *neutrosophic actions* and their properties. We have introduced the *prevalence order* on {T, I, F} with respect to a given *neutrosophic operator* "o", the *refinement of neutrosophic entities* <A>, <neutA>, and <antiA>, and the *neutrosophic literal logical operators*, the *refined literal logical operators*, as well as the *refinement neutrosophic literal space*.

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Abstract

This volume is a collection of thirteen papers, written by different authors and co-authors (listed in the order of the papers): J. J. Peng and J. Q. Wang, E. Marei, S. Kar, K. Basu, S. Mukherjee, I. M. Hezam, M. Abdel-Baset and F. Smarandache, K. Mondal, S. Pramanik, A. Ionescu, M. R. Parveen and P. Sekar, B. Teodorescu, D. Kour and K. Basu, P. P. Dey and B. C. Giri, A. A. A. Agboola.

In first paper, the authors studied Multi-valued Neutrosophic Sets and its Application in Multi-criteria Decision-Making Problems. More on neutrosophic soft rough sets and its modification is discussed in the second paper. Solution of Multi-Criteria Assignment Problem using Neutrosophic Set Theory are studied in third paper. In fourth paper, Taylor Series Approximation to Solve Neutrosophic Multiobjective Programming Problem. Similarly in fifth paper, Decision Making Based on Some similarity Measures under Interval Rough Neutrosophic Environment is discussed. In paper six, Neutralité neutrosophique et expressivité dans le style journalistique is studied by the author. Neutrosophic Semilattices and Their Properties given in seventh paper. Liminality and Neutrosophy is proposed in the next paper. Application of Extended Fuzzy Program-ming Technique to a real life Transportation Problem in Neutrosophic environment in the next paper. Further, TOPSIS for Single Valued Neutrosophic Soft Expert Set Based Multi-attribute Decision Making Problems is discussed by the authors in the tenth paper. In eleventh paper, Neutrosophic Quadruple Numbers, Refined Neutrosophic Quadruple Numbers, Absorbance Law, and the Multiplication of Neutrosophic Quadruple Numbers have been studied by the author. In the next paper, On Refined Neutrosophic Algebraic Structures. At the end, Neutrosophic Actions, Prevalence Order, Refinement of Neutrosophic Entities, and Neutrosophic Literal Logical Operators are introduced by the authors.

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