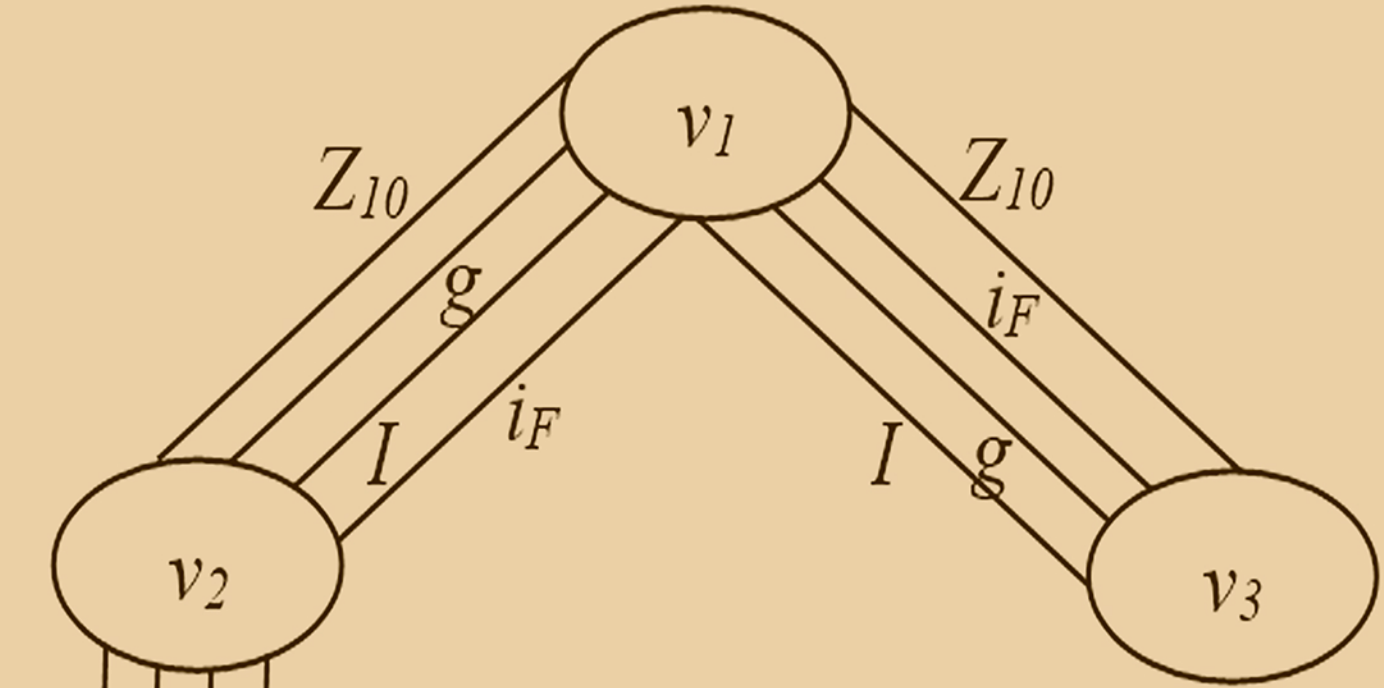
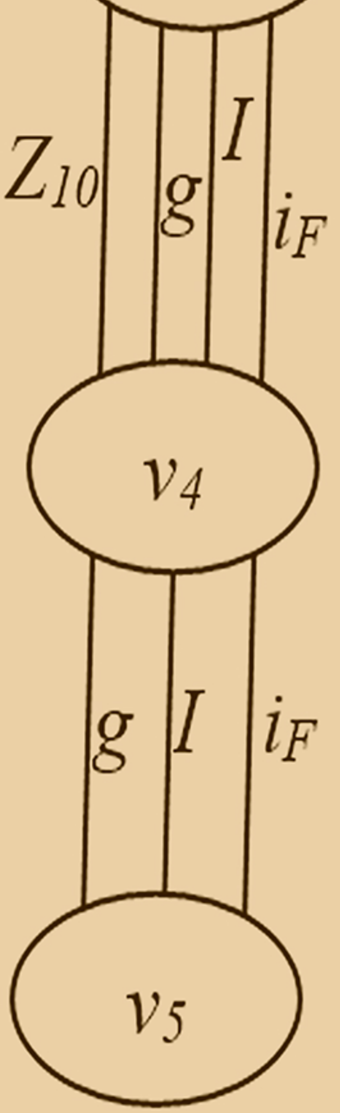


W.B.VASANTHA KANDASAMY
K. ILANTHENRAL
FLORENTIN SMARANDACHE

SUBSET VERTEX MULTIGRAPHS AND NEUTROSOPHIC MULTIGRAPHS FOR SOCIAL MULTI NETWORKS



Subset Vertex Multigraphs and Neutrosophic Multigraphs for Social Multi Networks

W. B. Vasantha Kandasamy
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Florentin Smarandache

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CONTENTS

PREFACE

In this book authors introduce the notion of subset vertex multigraphs for the first time. The study of subset vertex graphs was introduced in 2018, however they are not multiedged, further they were unique once the vertex subsets are given. These subset vertex multigraphs are also unique once the vertex subsets are given and the added advantage is that the number of common elements between two vertex subsets accounts for the number of edges between them, when there is no common element there is no edge between them. In case the two vertex subsets have only one common element then only one edge exists between them. When we do not associate any direction, we call them as subset vertex multigraphs of type I, when we associate direction, that is when they are directed graphs, we define them as subset vertex multigraphs of type II. If the subsets of these multigraphs are taken from the subsets of a neutrosophic set, we call them as subset vertex neutrosophic multigraphs.

We define multi network as a network whose underlying graph is a multigraph which is also known as multirelational structure or multivariate network or multiple networks or multi line networks. We have defined and only use the terminology multi network when they use multigraphs. Recently researchers have used multigraphs in the study of social networks. It is important to record that multigraphs can be used in transportation networks, computer networks apart from social networks.

Several interesting properties are derived about these subset vertex multigraphs and over 250 figures of multigraphs of various types are provided in this book. At the end of each chapter we have provided several problems for the reader. Some open problems are also suggested.

We wish to acknowledge Dr. K Kandasamy for his sustained support and encouragement in the writing of this book.

W.B.VASANTHA KANDASAMY
ILANTHENRAL K
FLORENTIN SMARANDACHE

Chapter One

BASIC CONCEPTS

In this chapter authors give a brief introduction to basic concepts as well some insight into the contents of this book. Basically, the elements of the vertices are taken as the subsets from the power set P(S) of the set S. Thus the vertices of these special type of graphs are subsets, hence called as subset vertex graphs [53-4].

There are two types of subset vertex graphs viz. type I and type II. Type I subset vertex graphs are not directed whereas subset vertex graphs of type II are directed. For more about these notions refer [53-4]. For completeness of the chapter, we recall the definition of type I and type II subset vertex graphs.

Definition 1.1: *Let P(S) be the power set of the set S. Let G be the graph with vertices from P(S) where $v_1, \ldots, v_n \in P(S)$. We say an edge from v_i to v_j exists if $v_i \cap v_j \neq \phi$; $i \neq j$. A graph G with $v_1, \ldots, v_n$ as vertex subsets and edges defined in this way is defined as the subset vertex graph of type I.*

We will give an example of subset vertex graph.

Example 1.1. Let S = {1, 2, …, 9} be a set and P(S) the power set of S. Let G be the subset vertex graph of type I with vertex subsets from P(S) given in the following

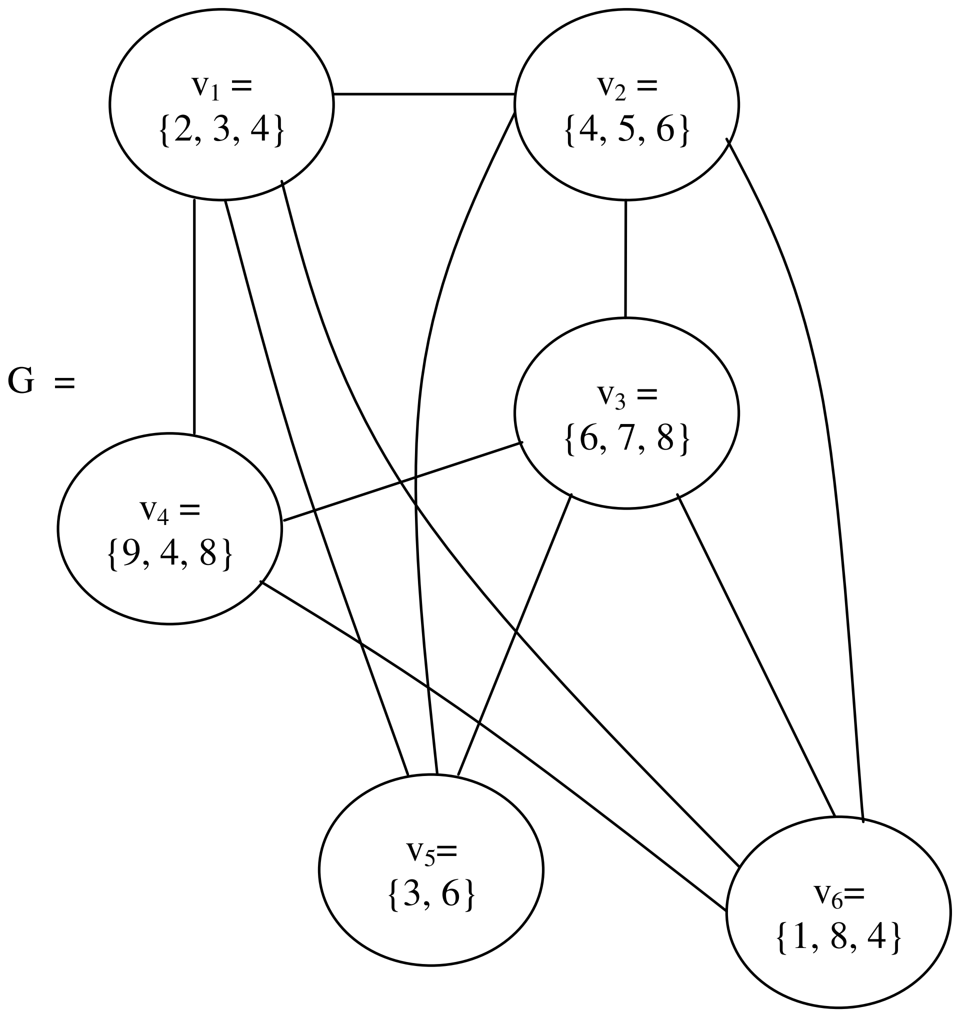


Figure 1.1

We see $v_1 \cap v_2 = \{4\}$, $v_1 \cap v_4 = \{4\}$ $v_1 \cap v_3 = \phi$, $v_1 \cap v_6 = 4$ and $v_5 \cap v_1 = \{3\}$. Similarly, we can find $v_i \cap v_j$; $i \neq j$ and find the edges. Once the vertex subset is known the edges are uniquely fixed by the very definition of the subset vertex graph of type I.

We can find subset vertex subgraphs of type I.

Definition 1.2. *The subset vertex subgraphs of the vertex subset graph G are those subset vertex graphs whose vertex subset is the proper subset from the collection of the vertex subsets of G.*

We provide an example of this.

Example 1.2. Let S = {1, 2, 3, …, 12} and P(S) the power set of S. Consider the subset vertex graph G of type I is given by the following figure.

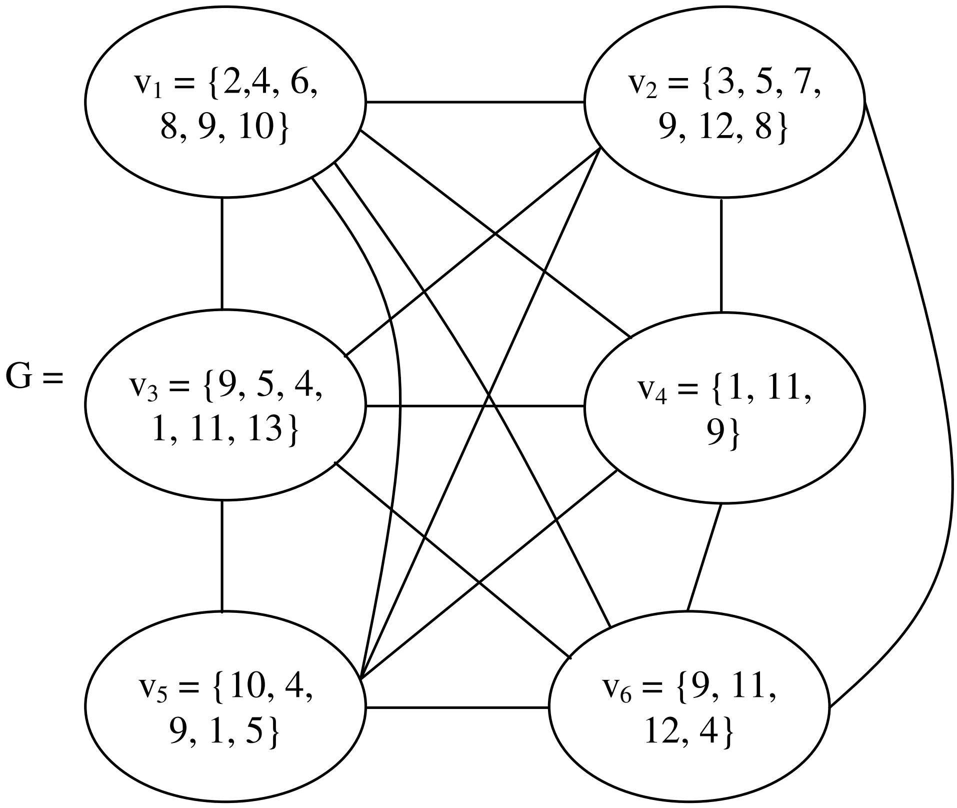


Figure 1.2

Consider H_1 the subset vertex subgraph with vertex subset $\{v_1, v_2, v_3, v_5\}$ given by the following figure.

H_1 =

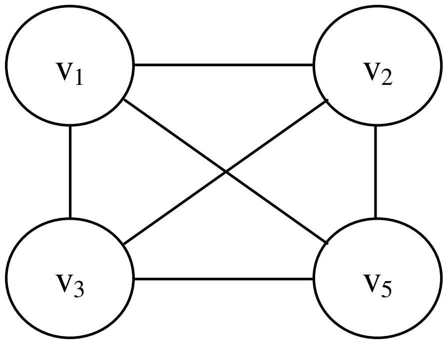


Figure 1.3

Clearly both G and H_1 are complete subset vertex graph and subgraph respectively.

It is left as an exercise for the reader to prove every subset vertex subgraph of G is a complete subset vertex subgraph of G.

Other related properties of subset vertex graphs of type I can be had from [53-4].

Now we give definition of the other type of subset vertex subgraph namely the special subset vertex subgraph in the following.

Definition 1.3. *Let P(S) be the power set of S. G be a subset vertex graph of type I with $v_1, \ldots, v_n$ as the vertex subsets. We define H to be a special subset vertex subgraph of type I if H has n subset vertices got as $u_i \subseteq v_i$; i = 1, 2, …, n where at least one of these subsets u_i is a proper subset of v_i. Thus, all special subset vertex subgraphs of G have the same number of vertex subsets as that of G.*

We provide an example of this.

Example 1.3. Let S = {1, 2, …, 20} and P(S) be the power set of S. Let G be the subset vertex graph given by the following figure.

G =

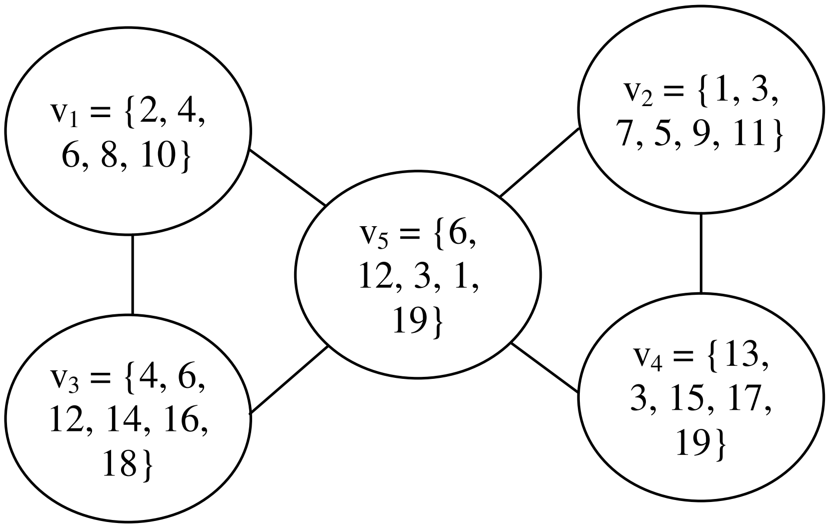


Figure 1.4

Let K_1 be a special subset vertex subgraph given by the following figure.

K_1 =

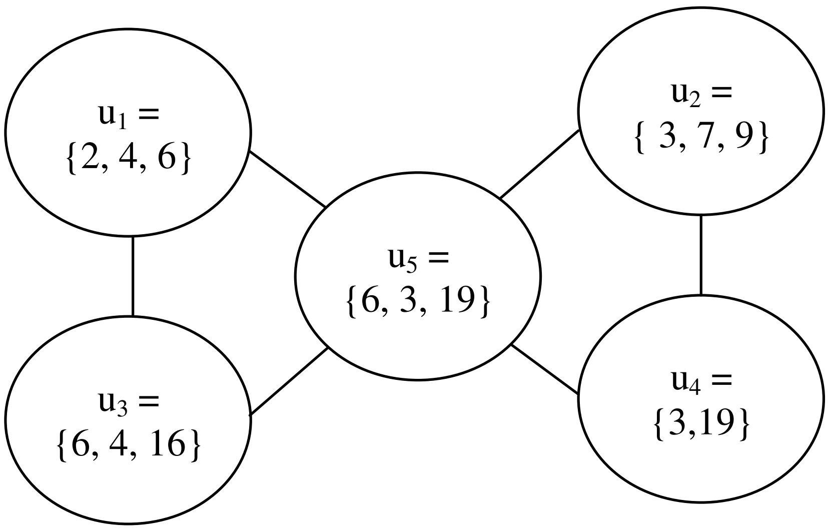


Figure 1.5

We see both K_1 and G enjoy the same structure so K_1 is a special type of hyper subset vertex subgraph of G.

We find the universal complement of G and the local complement of K_1 relative to G in the following. The universal complement G of G is as follows: $a_i = S \setminus v_i$, i = 1 to 5.

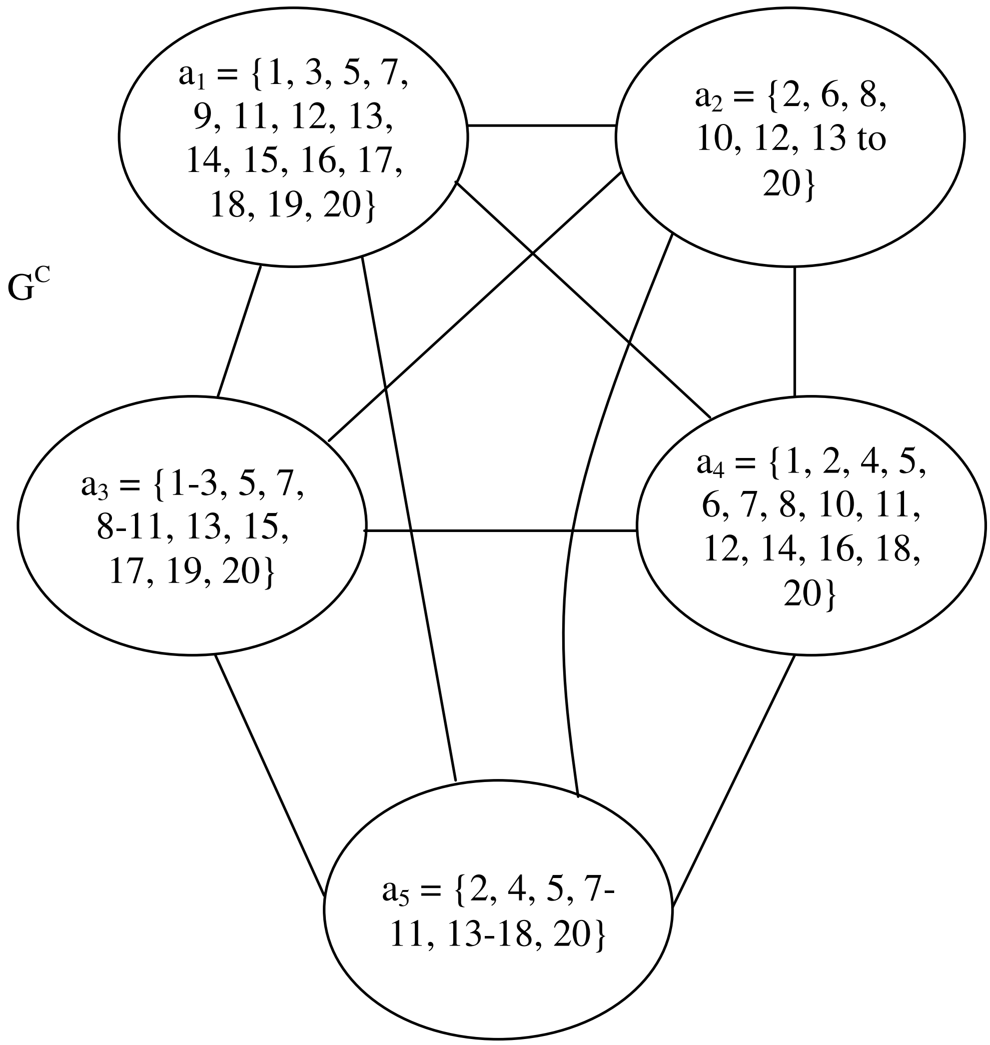


Figure 1.6

Now we give the local complement K_1^C of K_1 relative to G in the following where $d_i = v_i \setminus u_i$; i = 1, 2, 3, 4, 5.

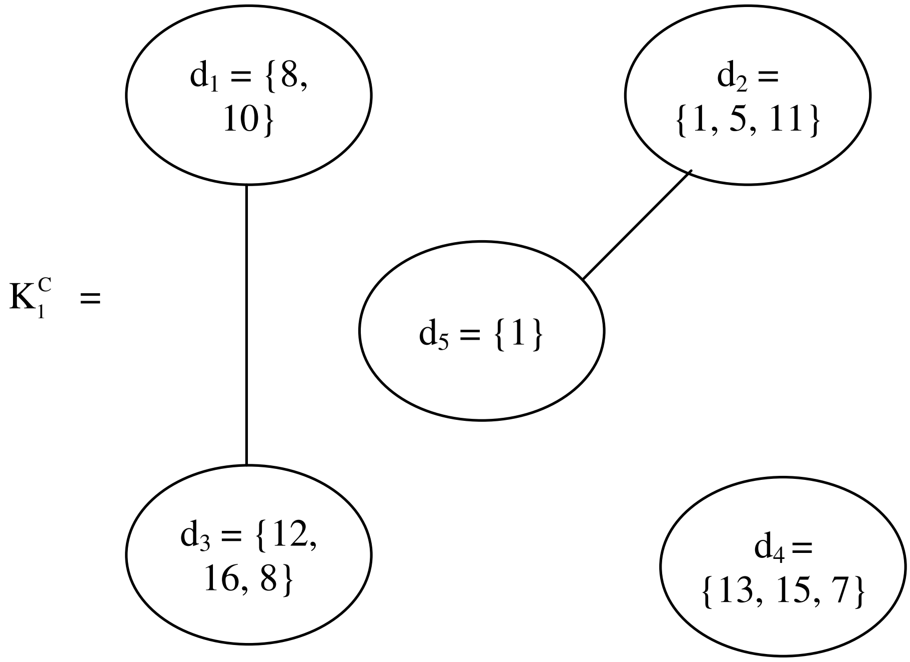
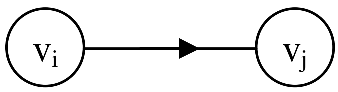


Figure 1.7

Clearly the local complement K_1^C of K_1 relative to G is only a disconnected special subset vertex subgraph of G. K_1^C is not structurally the same as K_1 of G.

Now we proceed to describe subset vertex graphs of type II in the following.

Definition 1.4. *Let P(S) be the power set of S. G be a graph with vertices from P(S). Let V = {v_1, v_2, …, v_n} be the set of vertex subsets of G from P(S). The edge from v_i to v_j ($i \neq j$) exists if and only if $v_i \subset v_j$; v_i is properly contained as a subset of v_j then it is denoted as*



Thus, G endowed with V as the vertex subsets and the edges defined as mentioned is defined as the injective subset vertex graph of type II.

Clearly G is a directed graph and the edges are unique once the vertex subset V is given.

We will describe this by an example.

Example 1.4: Let S = {1, 2, …, 15} be a set and P(S) the power set of S. Let G be the injective subset vertex graph of type II given by the following figure.

G =

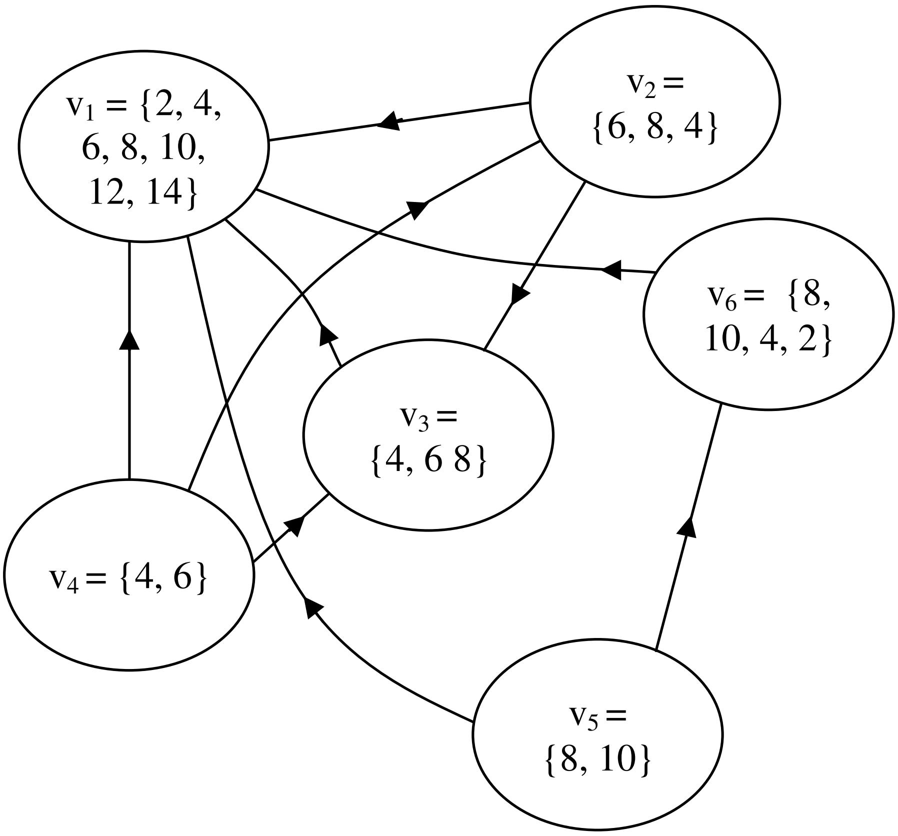


Figure 1.8

We do not use the term injective for from the very graph one can understand this.

We describe the notion of subset vertex subgraph and special subset vertex subgraphs from this example.

Let H be the subset vertex subgraph of G given by the following figure.

H =

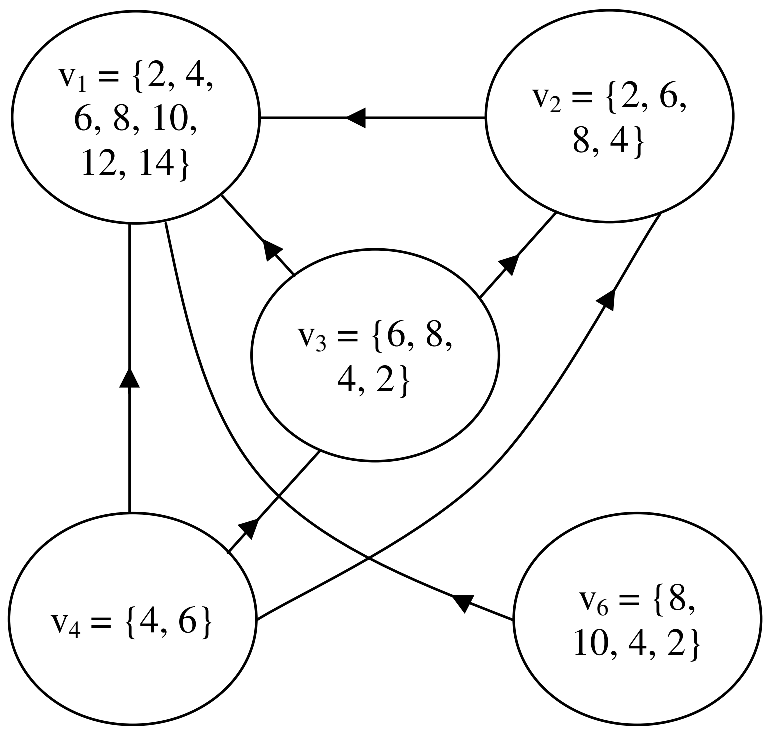


Figure 1.9

Let K be the special subset vertex subgraph of G given by the following figure:

K =

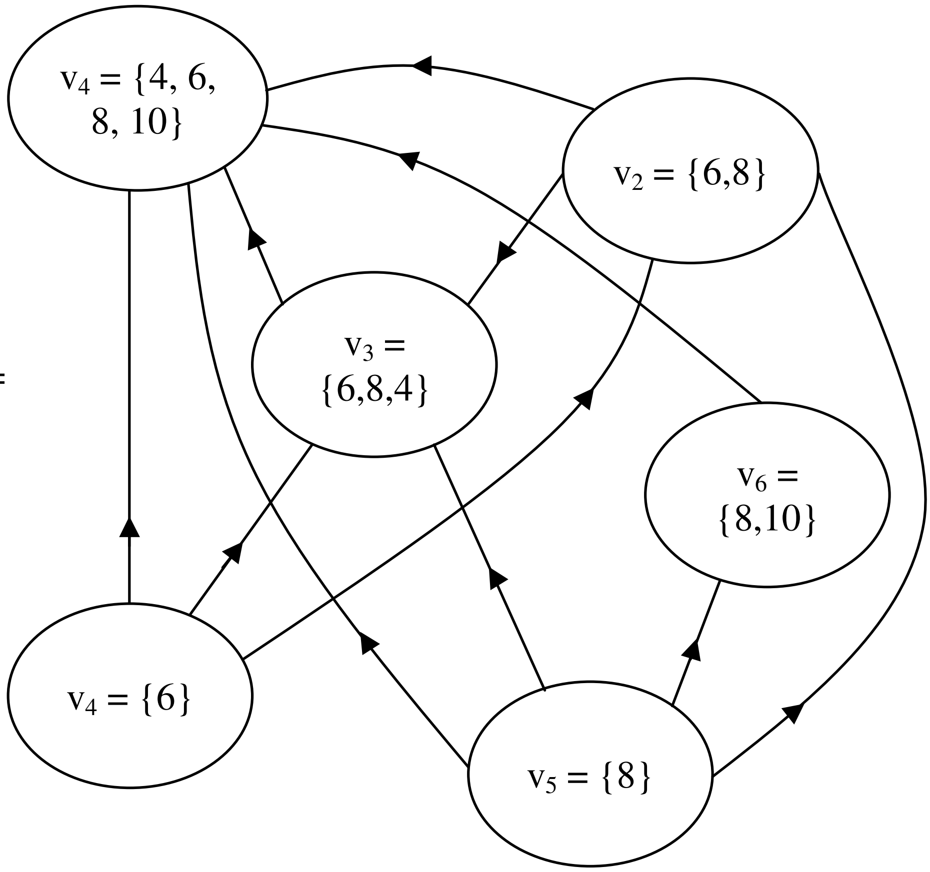
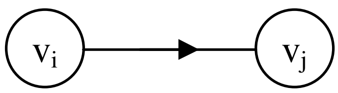


Figure 1.10

Clearly K the special subset vertex subgraph is pseudo complete though G is not a pseudo complete subset vertex multigraph.

Now in case of projective subset vertex multigraphs of type II the arrows are reversed for the injective that is if $v_i \subset v_j$ the we denote it graphically as



That is v_j is projected onto v_i; $i \neq j$.

We give one example of this situation.

Example 1.5: Let S = {1, 2, …, 20} and P(S) be the power set of S. Let G be the projective subset vertex graph of type II given by the following figure

G =

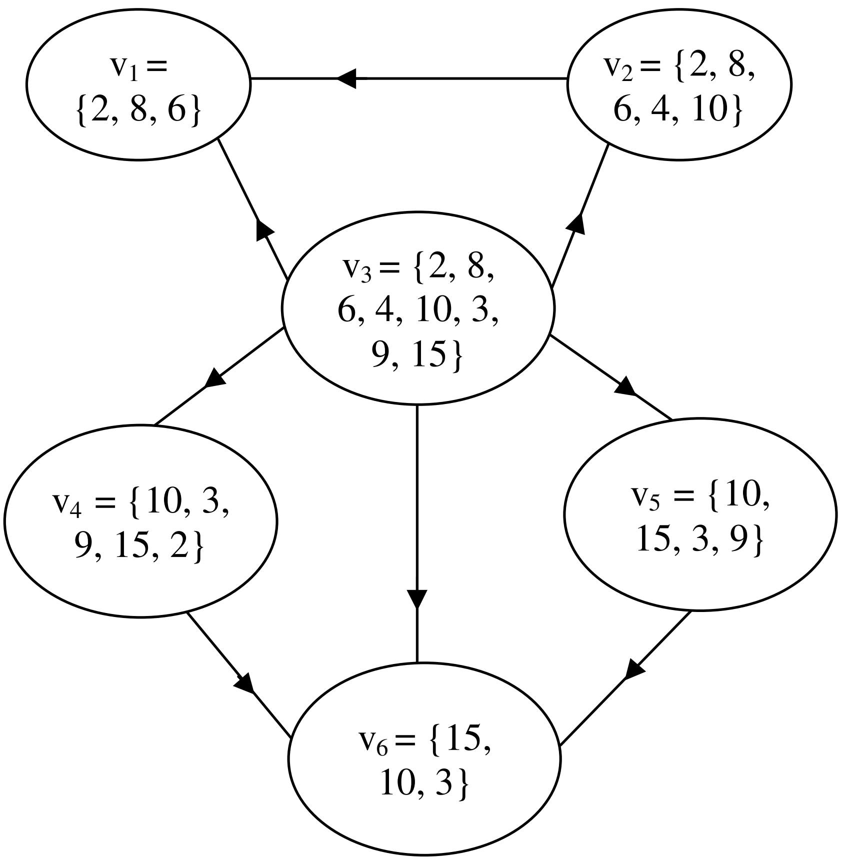


Figure 1.11

It is left as an exercise for the reader to find the subset vertex subgraph of type II of G and special subset vertex subgraph of G of type II.

For more about these concepts refer [53-4].

In this book we do not indulge in the study of multigraphs we only define, develop and describe the notion of subset vertex multigraphs of type I and type II. As in case of subject vertex graph the subset vertex multigraphs are also unique once the vertex subset from P(S) is provided.

We call the usual subset vertex multigraphs like subset vertex graphs as ordinary subset vertex multigraphs which forms chapter II of this book.

However, multigraphs with labeled edge weights depending on the elements of S, so basically on P(S) are defined as subset vertex multigraphs where the edges connecting any two relevant subsets may be imaginary, real, neutrosophic (or indeterminate) or dual number and so on.

For these subset vertex multigraphs also subset vertex multisubgraphs and special subset vertex multisubgraphs are defined. In case of subset vertex multigraphs also we have two types viz. type I and type II defined in a similar way. This is done in chapter III of this book.

Finally, these special subset vertex multisubgraphs can be used as fault tolerant multigraphs. Use of these special vertex subset multisubgraphs as fault tolerant multisubgraphs can save both time and money.

Further these subset vertex multigraphs can be efficiently used in the social networks when the researcher wants each and every particular property or custom or practice or feature to be represented exactly and properly.

However, the hyper multigraphs of these subset vertex multigraphs can play the role of fault tolerant multigraphs of fault tolerant multi-networks (Here the term hypermulti-subgraphs is used differently from the classical hypergraphs).

The feasibility of the use of these multigraphs depends on the researcher to apply them appropriately.

Further some of the edges connecting the vertex subsets v_i and v_j ($i \neq j$) can be imaginary ($a + ib$), purely imaginary (bi) ($a, b \in$ Reals), pure neutrosophic (bI), just neutrosophic ($a + bI$), complex neutrosophic ($a + bi + cI + diI$), dual ($a + bg$ / $g^2 = 0$ is a dual number) and so on and so forth.

Thus, adopting these subset vertex multigraphs with various multi-edges can make the result better, accurate as well as better in dealing real and commercial networks. Finally, we wish to state ambiguity in finding the edges does not exists as in case of subset vertex multigraphs the edges are uniquely labeled their by avoiding the bias. These new structures are explained elaborately with substantiating examples so that the reader can understand them easily.

Several types of subset vertex multigraphs like star subset vertex multigraphs, complete subset vertex multigraphs, circle/ring subset vertex multigraphs and pseudo complete subset vertex multigraphs are defined and described as they form a basis for the study and analysis of social networks.

Also, the new notion of universal complement of a subset vertex multigraph is defined in this book. It is proved that in general the subset vertex multigraph and its universal complement do not enjoy the same structure.

Another new concept viz local complements of subset - subset vertex multisubgraphs of a subset vertex multigraph is defined and described.

This study lead of the notion of fault tolerant multigraphs. We see the subset vertex multisubgraph H of G and its local complement H^C of H relative to G happen to be subset - subset vertex multisubgraph of G. However, we have given the condition for the local complement to exist, for in general given a subset - subset vertex multisubgraph its local complement may or may not exist.

There is a lot of scope for any researcher to work on this new notion of subset vertex multigraphs of type I and type II. None of these multigraphs are Boolean algebras.

Chapter Two

ORDINARY SUBSET VERTEX MULTIGRAPHS

In this chapter we for the first time introduce the notion of subset multigraphs and derive some interesting results related with them. The authors have in [53-4] introduced the notion of subset vertex graphs of type I and II. Multigraphs have been introduced by [10].

However, the notion of ordinary subset vertex multigraphs is new and in these new types of graphs, the number edges automatically become fixed from one vertex subset to the other.

We will illustrate this situation by some examples before the formal definition is made.

Example 2.1. Let P(S) be the power set of the set S = {1, 2, …, 27}.

Let G = {V, E} where V = {{2, 3, 4, 5, 8}, {20, 21, 26,2, 5, 3}, {20, 12, 3, 4, 16, 2}, {5, 8, 16, 26, 20}, {3, 5, 16, 2, 20}} is the vertex set of G.

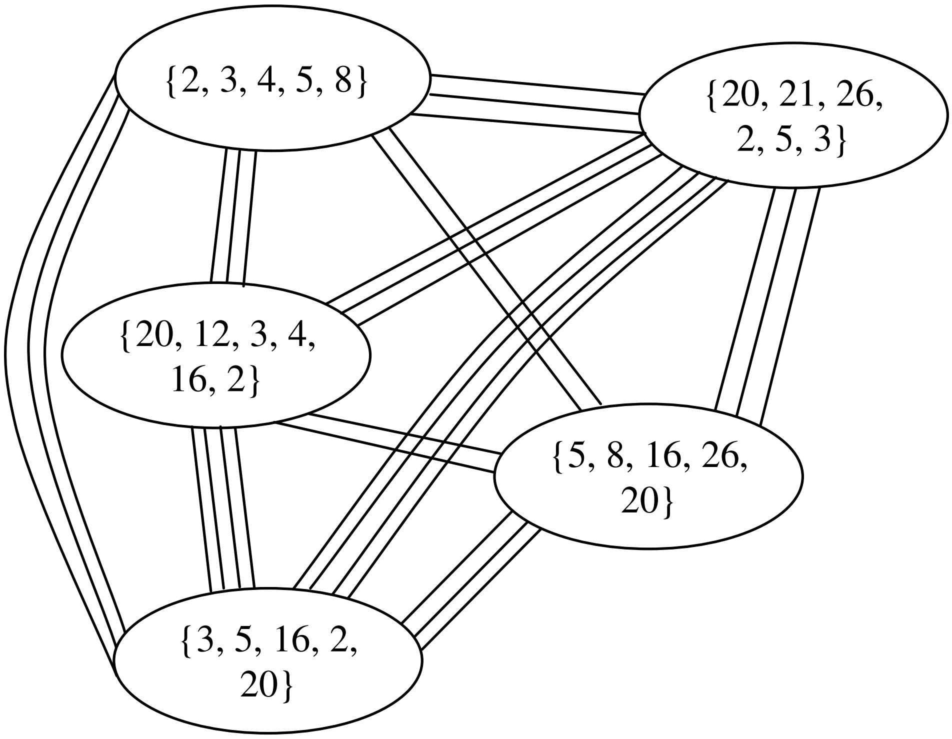
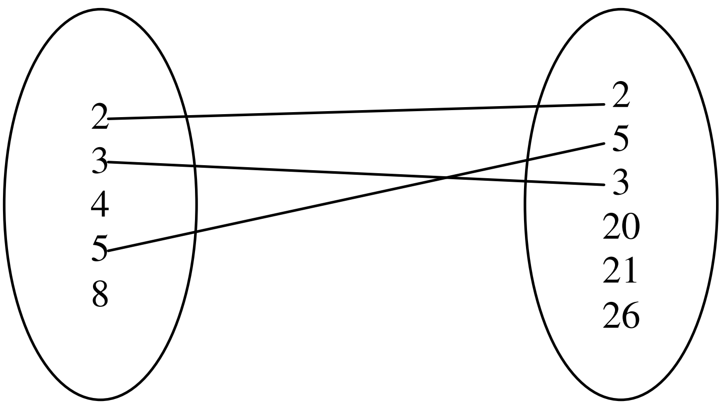
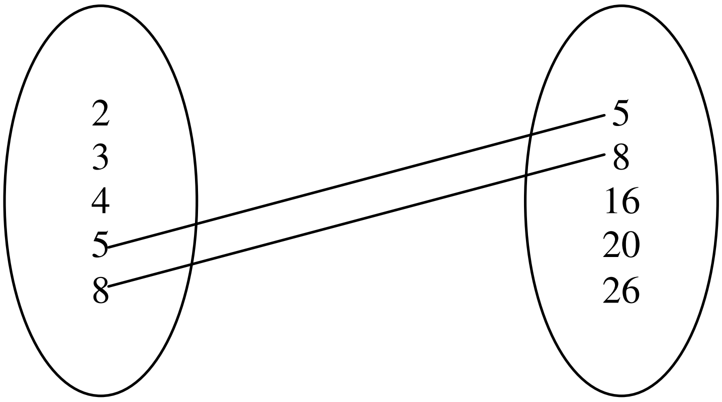


Figure 2.1

The mappings are

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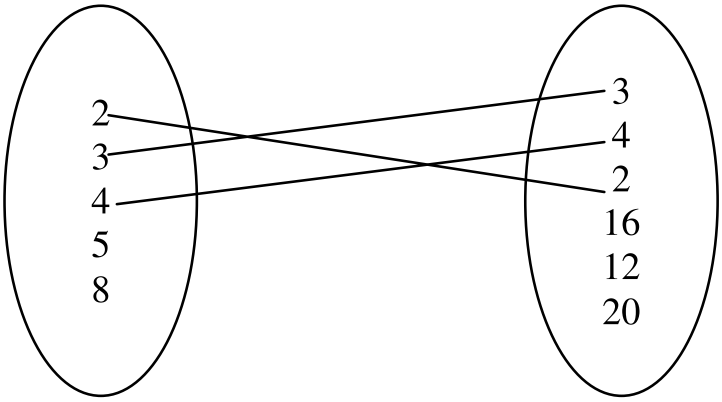
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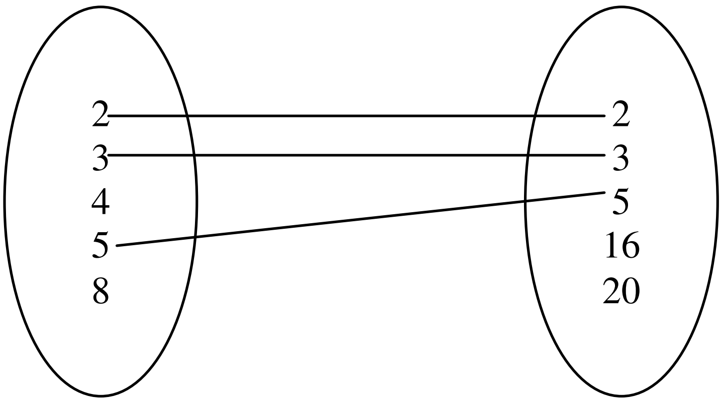
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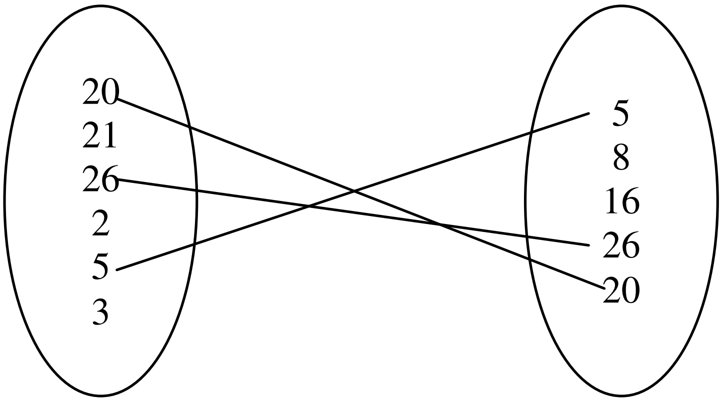
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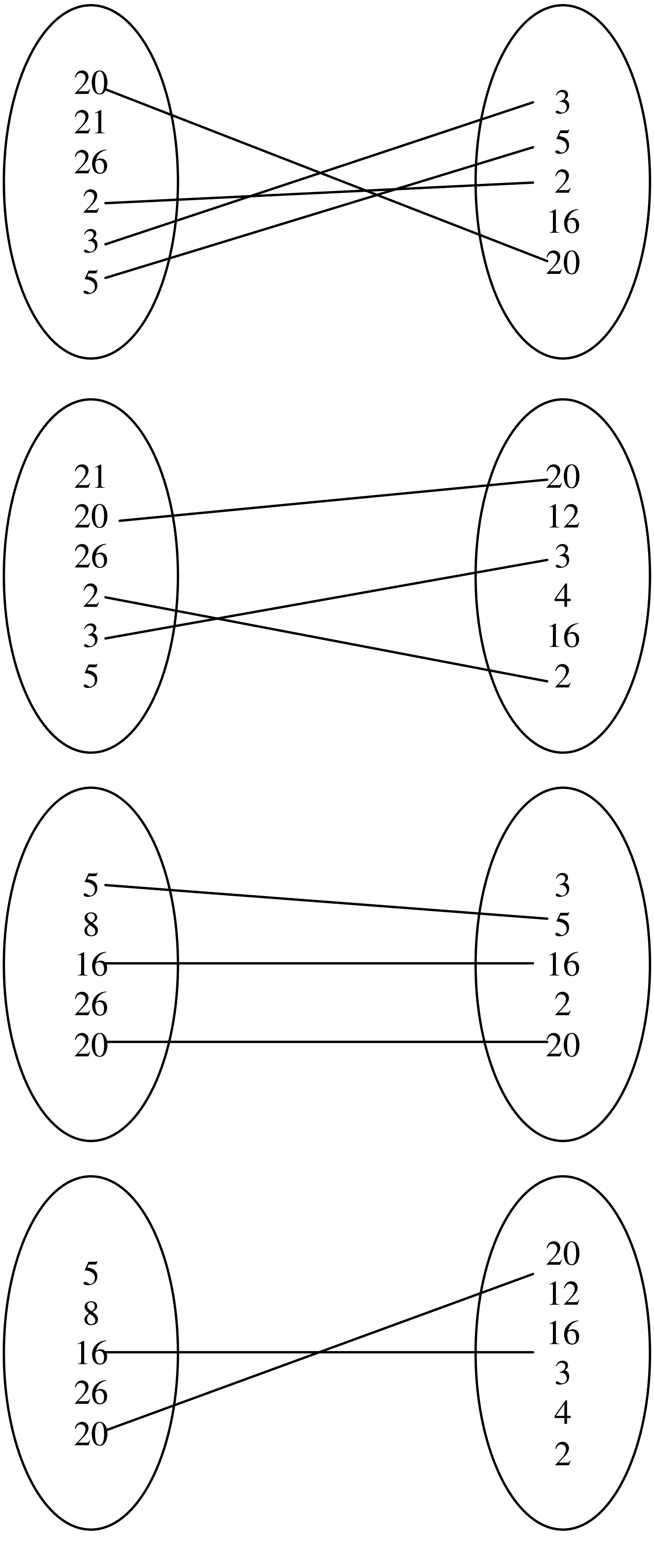
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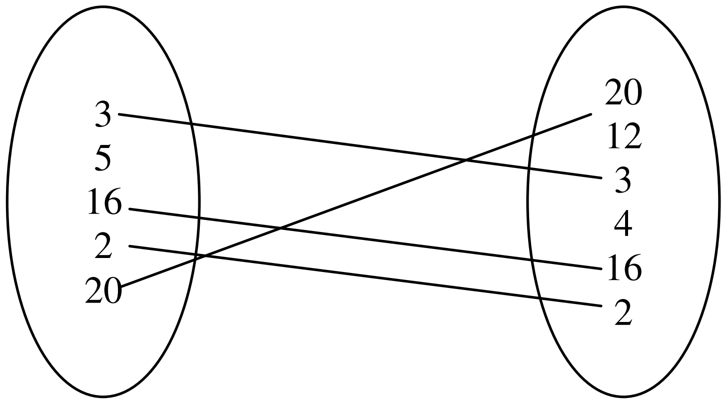


Figures 2.2

This is the multigraph defined as the ordinary subset vertex multigraph.

Example 2.2. Let S = {1, 2, 3, …, 28} be the finite set and P(S) be the power set of S.

Consider the ordinary subset vertex multigraph G_1 given by the following figure.

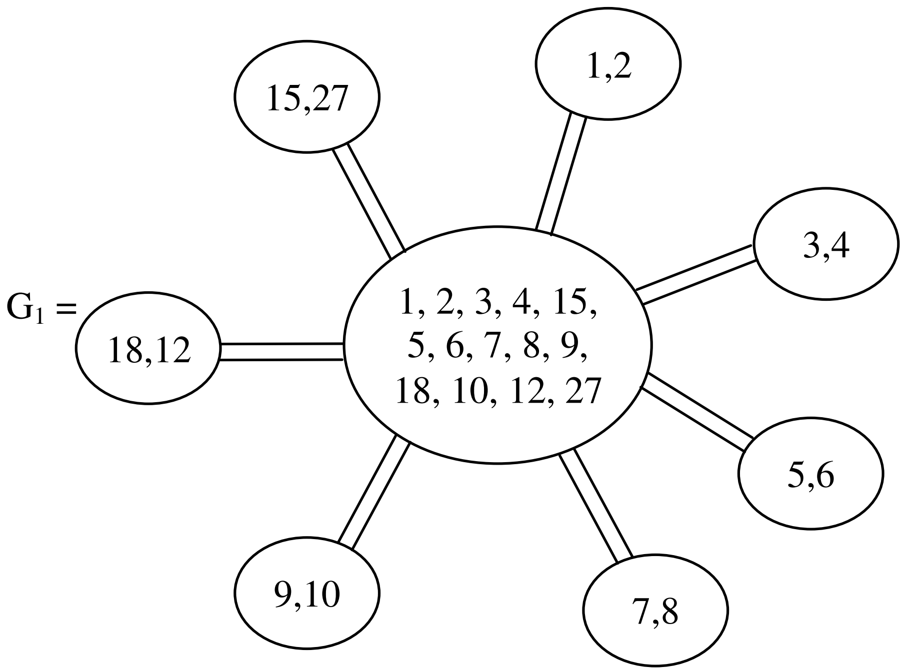


Figure 2.3

Clearly G is a star subset vertex multigraph.

Consider the ordinary subset vertex multigraph G_2 given by the following figure.

G_2 =

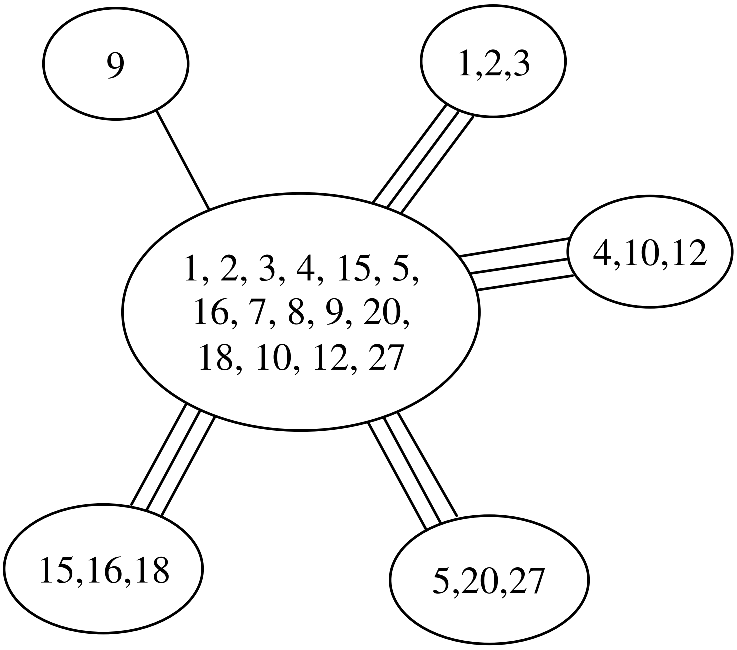


Figure 2.4

We see there is difference between the two-subset vertex multi-star graphs (star multigraphs).

Now we proceed onto give more examples of these ordinary subset vertex multigraphs.

Example 2.3. Let S = {1, 2, …, 24} be a set of order 24. P(S) the power set of S. Let V = {{1, 3, 4, 5}, {4, 5, 9, 2} {9, 2, 6, 8}, {6, 8, 10, 12}, {10, 12, 16, 19}, {16, 19, 20, 22}, {22, 20, 1, 3}} be the vertex set of the ordinary subset vertex multigraph G given by the following figure.

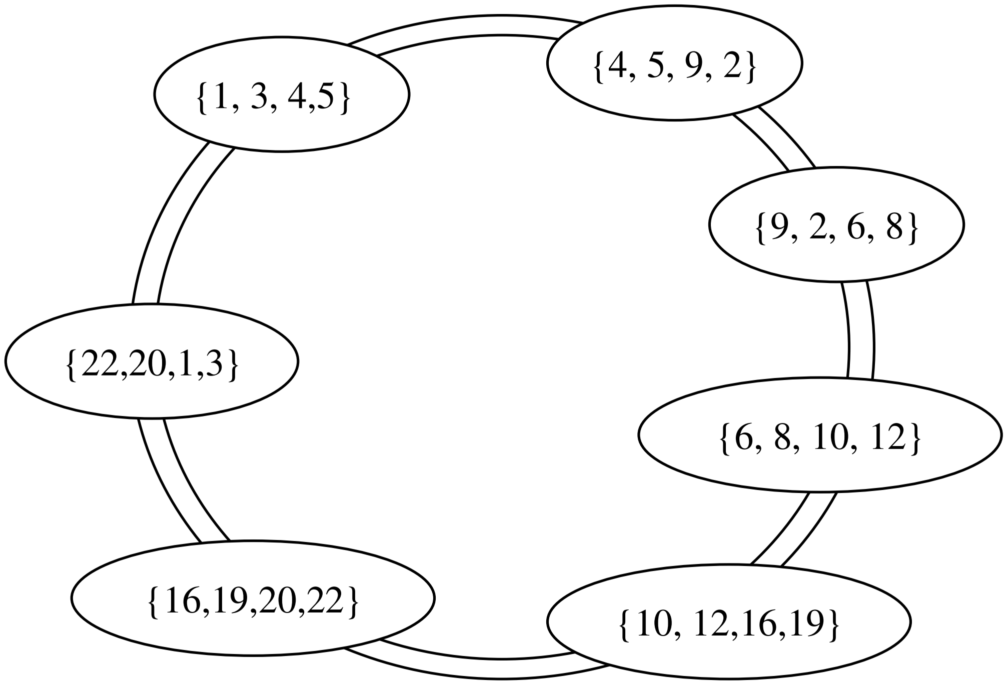


Figure 2.5

It is an ordinary subset vertex multigraph which is a ring or a circle multigraph. Before we make the formal definition, we must express or describe some of vital properties enjoyed by them or the restrictions imposed on them.

The property which is enjoyed by these ordinary subset vertex multigraphs is that, these graphs are not directed. Further these graphs get the edges and the number of edges get automatically fixed, thereby avoiding all bias of the investigator/researcher.

We have just now provided in the example the subset vertex circle multigraph.

Now we proceed onto provide another example of the subset vertex circle multigraph which is clearly different from the existing one in the following.

Example 2.4. Let P(S) be the power set of the set S = {1, 2, …, 36}.

Let G = {V, E} be the ordinary subset vertex multigraph given by the following figure.

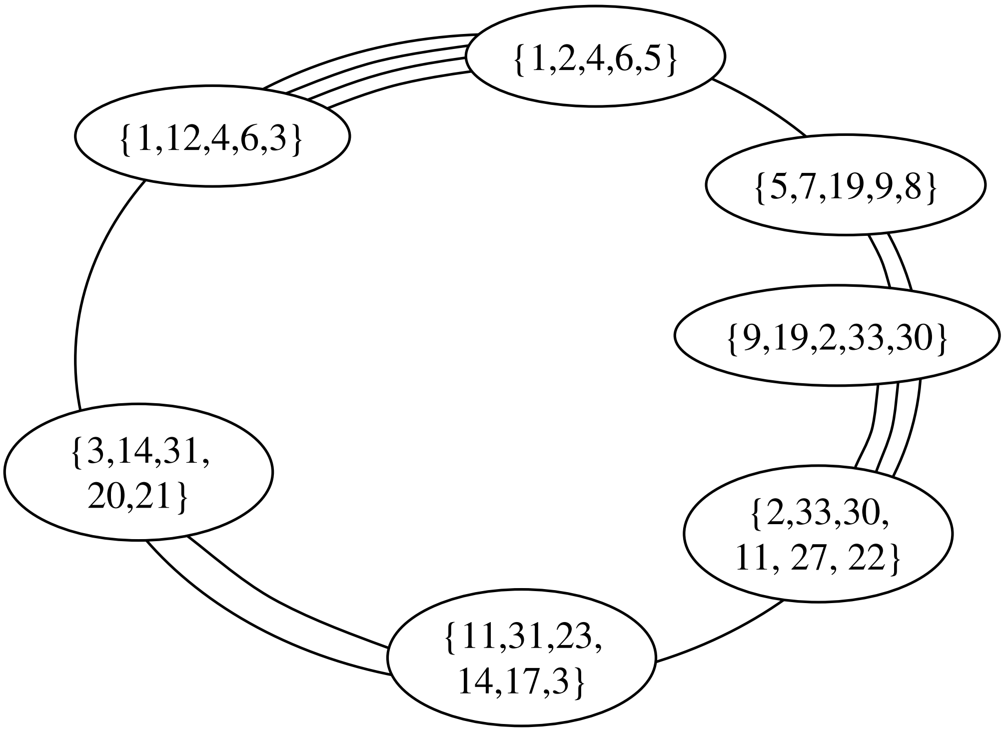


Figure 2.6

We see this multigraph is an ordinary subset vertex circle multigraph. When we compare this one with the subset vertex multigraph given in Figure 2.5 we see the number of multiedges in Figure 2.5 is two whereas in this multigraph the edges vary in a very different and distinct way.

We now provide more examples of them.

Example 2.5. Let S = {1, 2, …., 23} be the set, P(S) the power set of S.

Let K = {V, E} be the subset vertex multigraph the figure of which is given in the following.

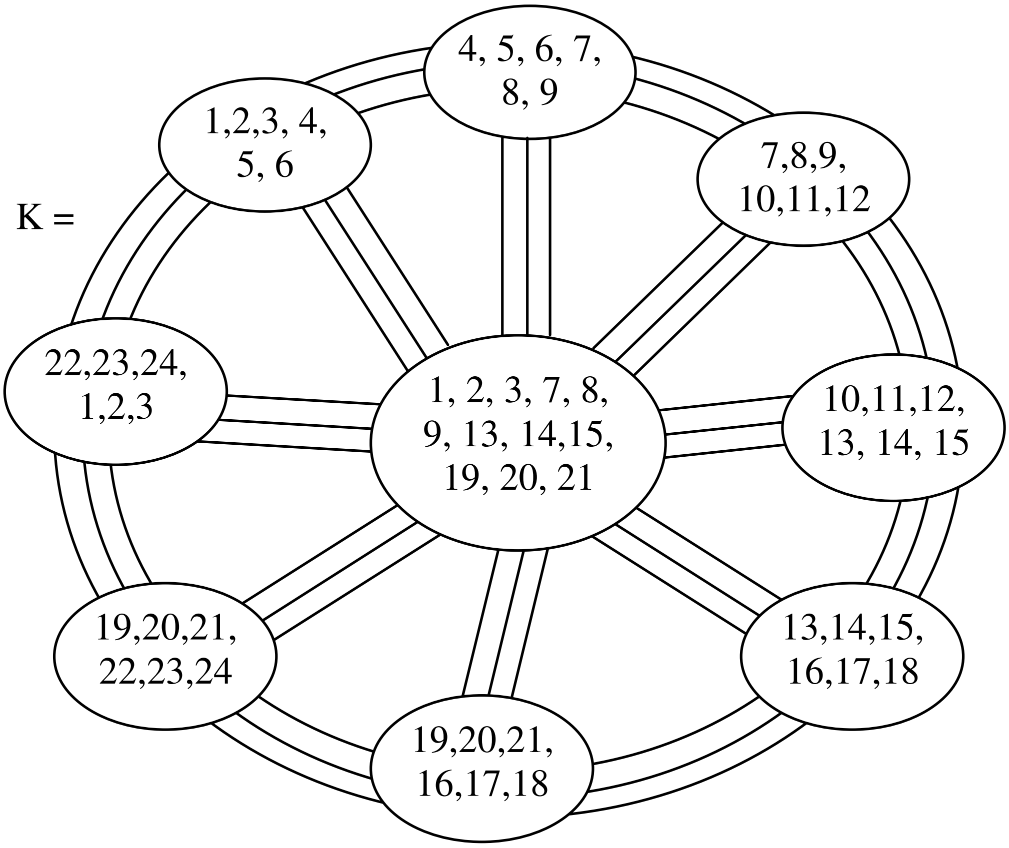


Figure 2.7

We see K is a ordinary subset vertex multigraph which is a wheel. The number of edges in each of the multigraphs are only three which is unique and clearly K is an ordinary subset vertex wheel multigraph.

We proceed onto give yet another example of it little different from the K mentioned above.

Example 2.6. Let S = {1, 2, …., 36} be the set and P(S) the power set of S. W = {V, E} be ordinary subset vertex multigraph given by the following figure.

W =

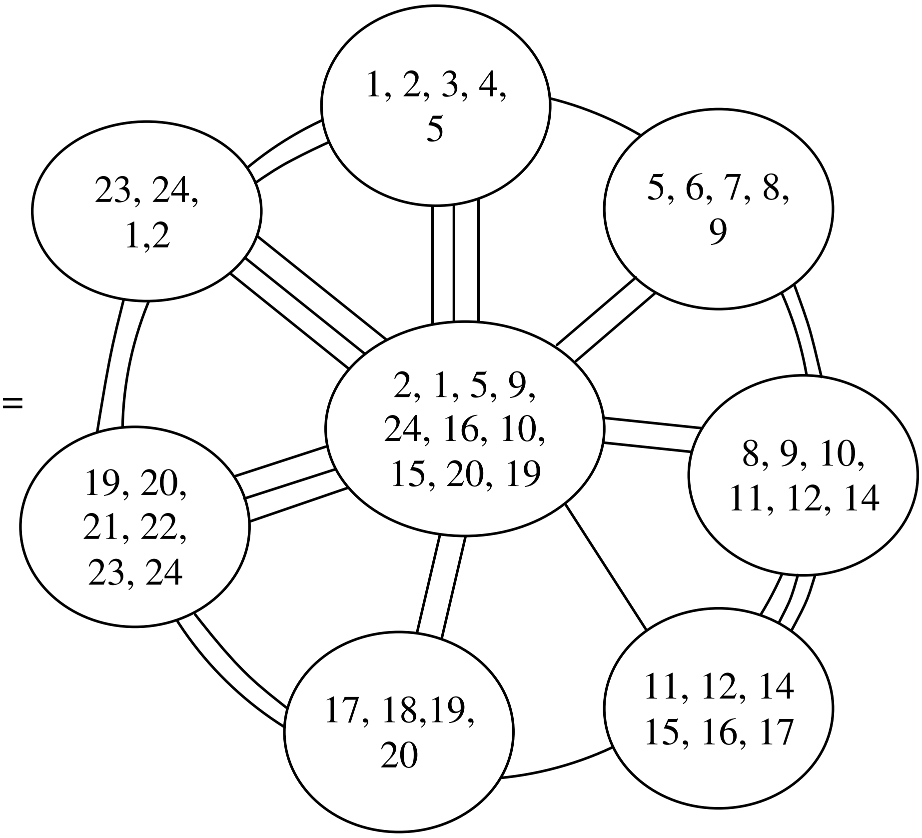


Figure 2.8

W is again a subset vertex wheel multigraph with a difference from K mentioned in the example 2.7.

Here the edges are different whereas in K all the edges have the same number viz. 3. Study in this direction is interesting and innovative.

We provide yet another example of the situation.

Example 2.7. Let S = {1, 2, …, 27} be the set and P(S) the power set of S. Let M be the subset vertex multigraph given by the following figure.

M =

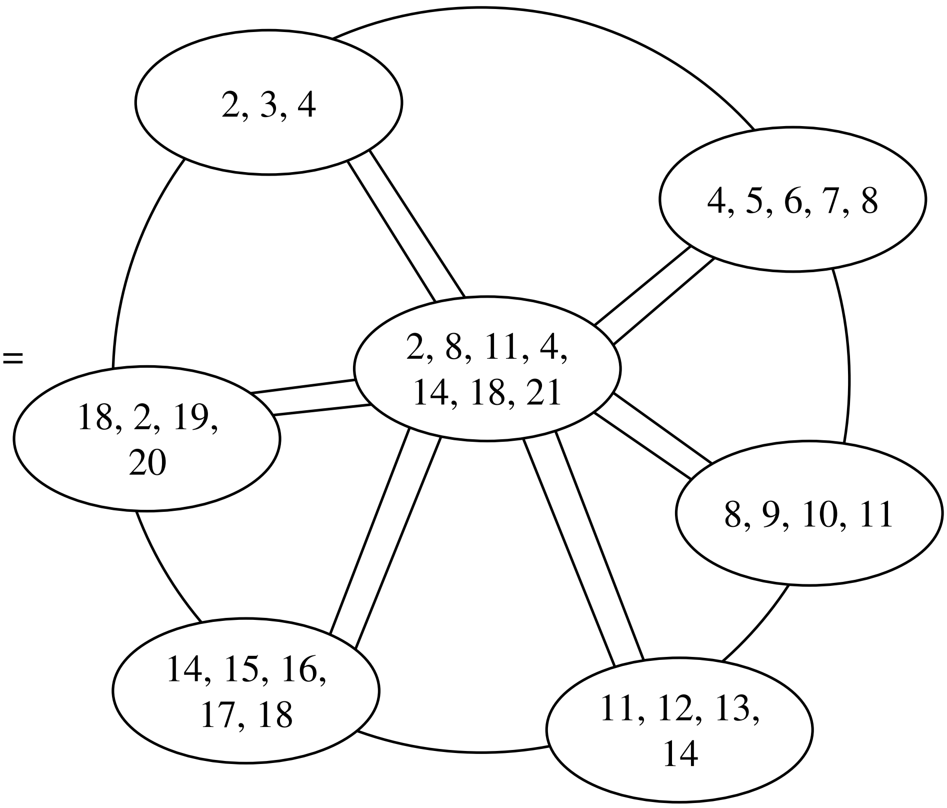


Figure 2.9

We see M is a subset vertex wheel multigraph. Further the star component of M has uniformly two edges and the wheel component as one edges.

This is also different from subset vertex multigraphs W and K described earlier.

Now we proceed onto describe yet another example for the same powerset P(S).

Consider B the ordinary subset vertex multigraph given by the following figure.

B =

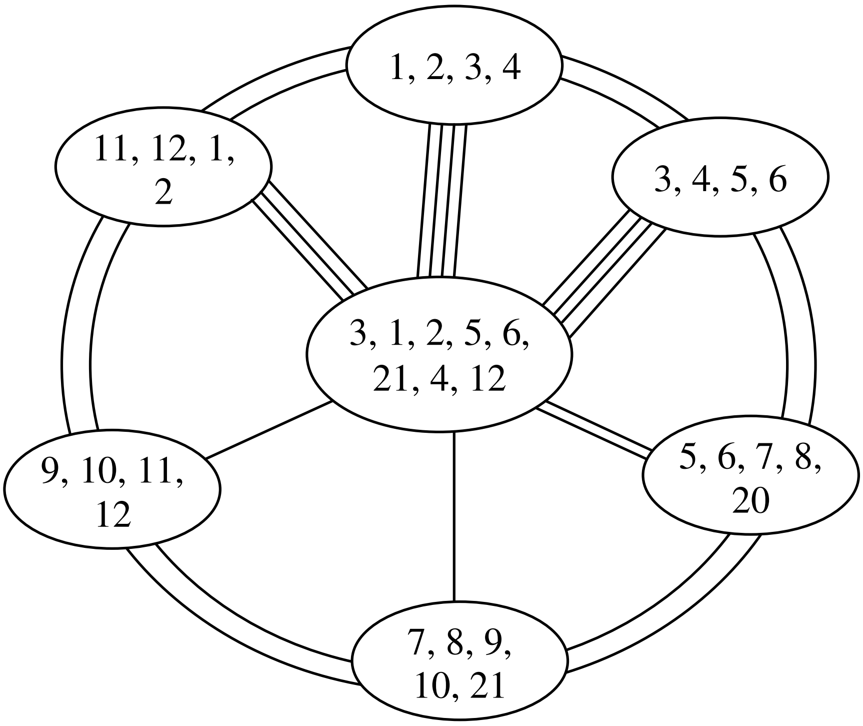


Figure 2.10

We see B is an ordinary subset vertex multigraph which is a wheel graph and the circle component of B has two edges whereas the star component has varying edges.

Next for the same power set we give yet another example.

Let D be the ordinary subset vertex multigraph given by the following figure.

D =

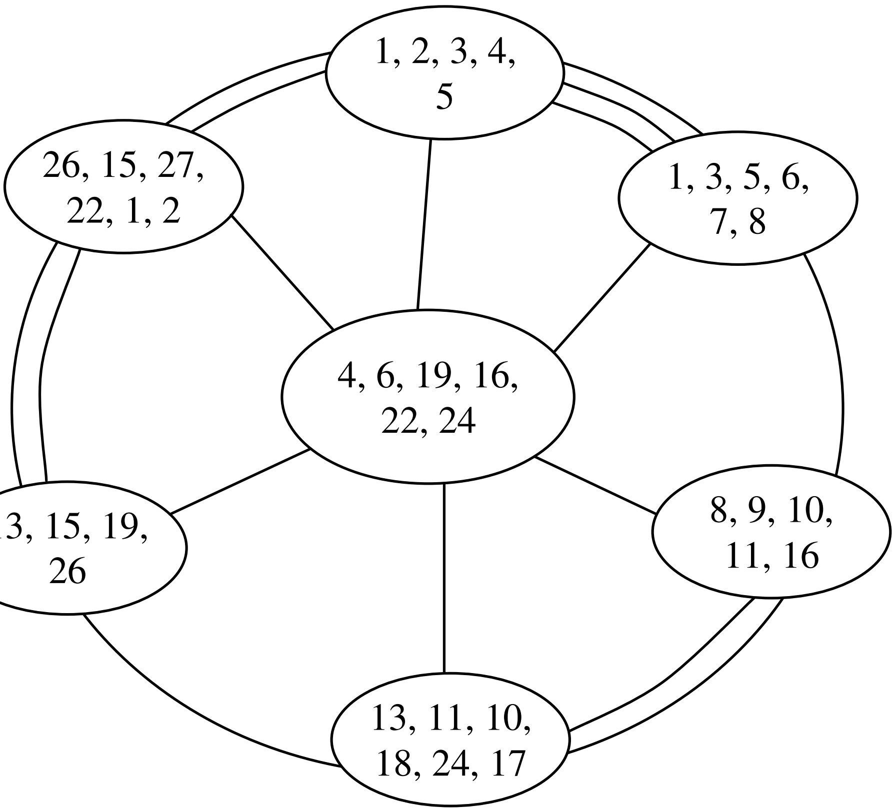


Figure 2.11

We see D is a subset vertex multigraph which is a wheel the star part of the wheel D has only one edges whereas the wheel part has different edges.

Thus, we have illustrated 5 types of subset vertex wheel multigraphs all of which are different in the edges.

Based on these we define more properties in due course of time.

Now we illustrate by examples the notion of subset vertex line multigraph.

Example 2.8. Let S = {1, 2, …, 28} be a set of order 28 and P(S) the power set of S.

Let A be the ordinary subset vertex multigraph given by the following figure.

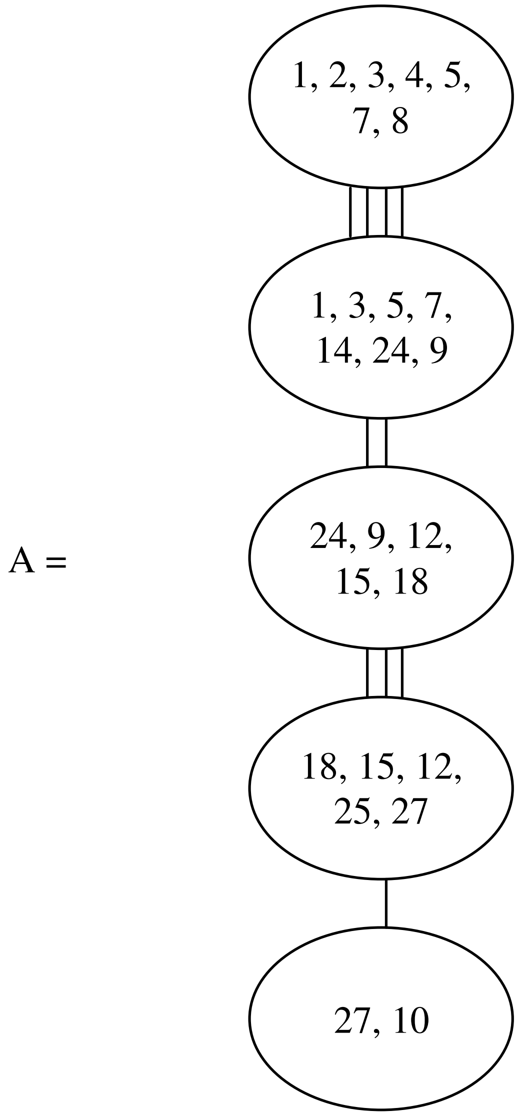


Figure 2.12

Clearly A is a subset vertex line multigraph of length 5.

We now give yet another example of a ordinary subset vertex multigraph B given by the following figure.

B =

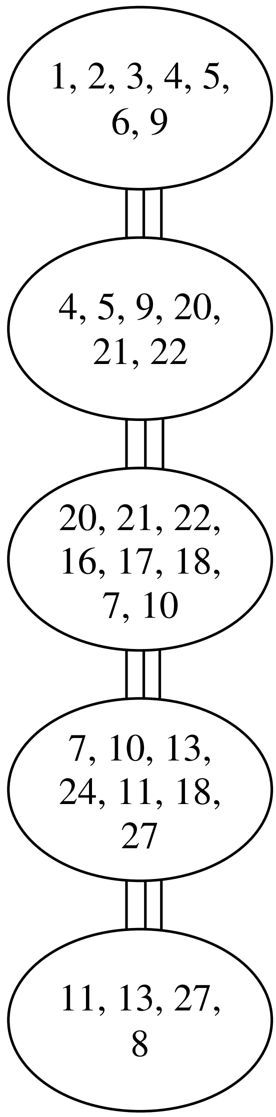


Figure 2.13

When we compare A and B, we see edges in A are varying whereas in case of B the edges are the same between any two nodes.

Next, we proceed onto develop the notion of ordinary subset vertex tree multigraphs.

First, we supply examples of the same.

Example 2.9. Let S = {1, 2, …, 36} be a set of order 36. P(S) be the power set of S.

Consider the ordinary subset vertex multigraph H given by the following figure.

H =

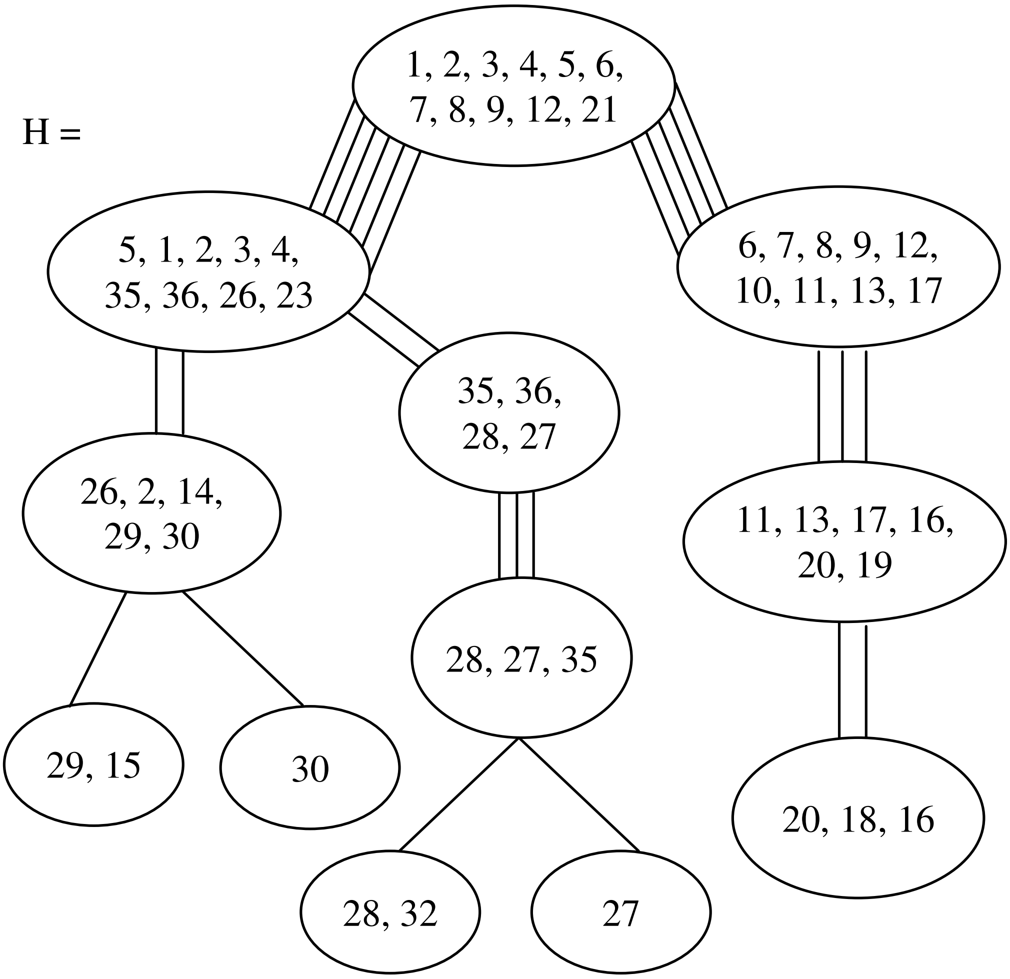


Figure 2.14

We see H is a subset vertex multigraph which is an ordinary multitree.

However, the number of edges connecting the nodes are not the same.

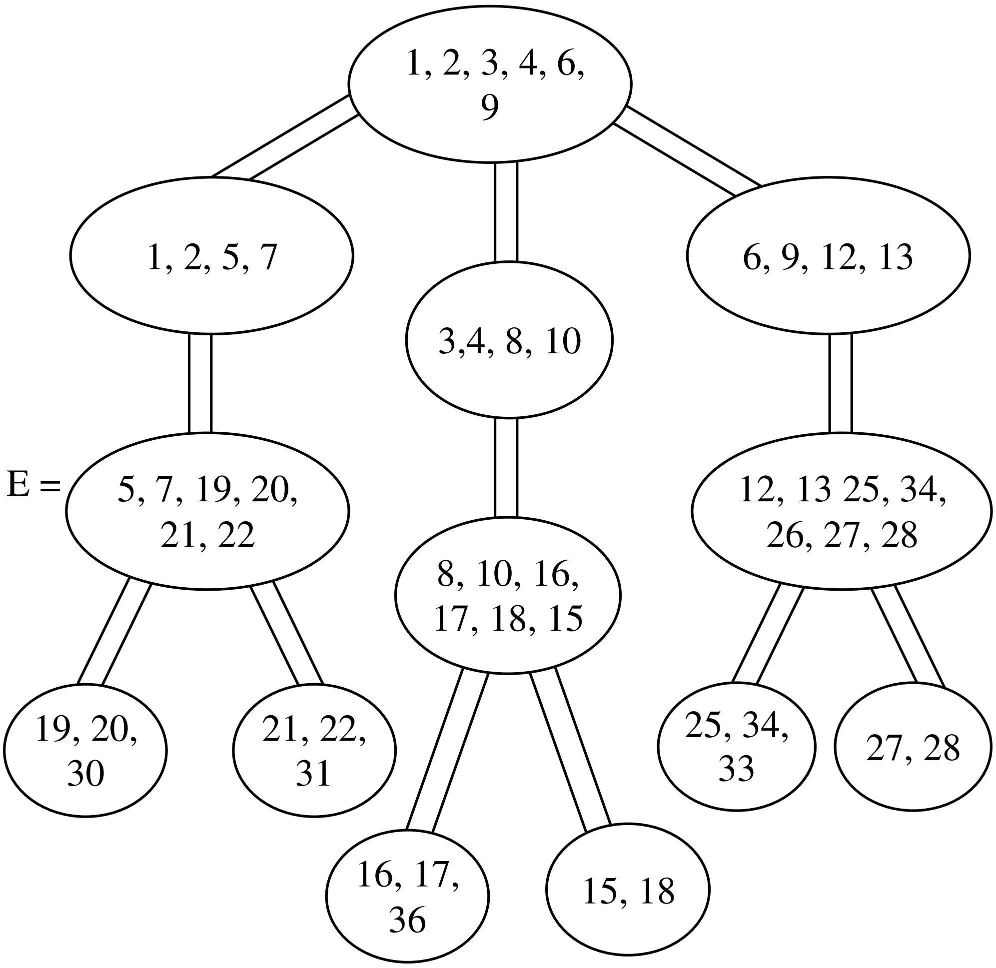


Figure 2.15

We see the above ordinary subset vertex multigraph E is also a multitree with a distinct difference from H, that the edges between nodes in E are the same viz. 2, however in case of H it varies arbitrarily from 1 to 4.

In the next examples we proceed onto describe the ordinary subset vertex complete graph.

Example 2.10. Let S = {1,2, …, 45} be a set of order 45, P(S) the power set of S. Consider the subset vertex multigraph given by the following figure.

T =

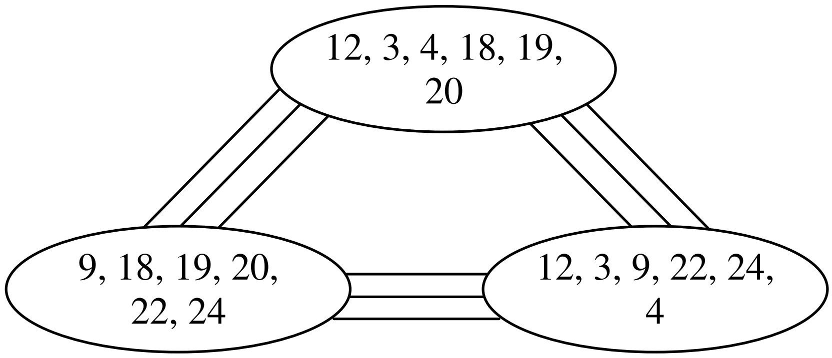


Figure 2.16

Clearly T is a subset vertex multigraph which is complete, and degree of each node is 6.

Consider the following subset vertex multigraph P given by the following figure.

P =

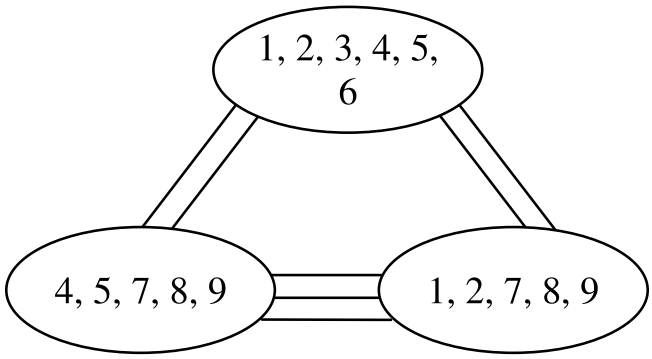


Figure 2.17

Clearly the subset vertex multigraph P is only pseudo complete. Compare this P with T and see the degree of every node of T is 6 whereas that of P is 4 and 5.

Let B be the subset vertex multigraph given by the following figure.

B =

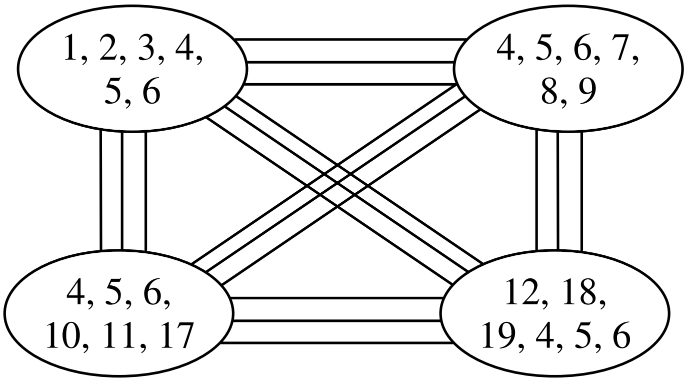


Figure 2.18

Clearly B is a subset vertex complete multigraph the degree of each node is 9.

Now consider the following subset vertex multigraph D given by the following figure.

D =

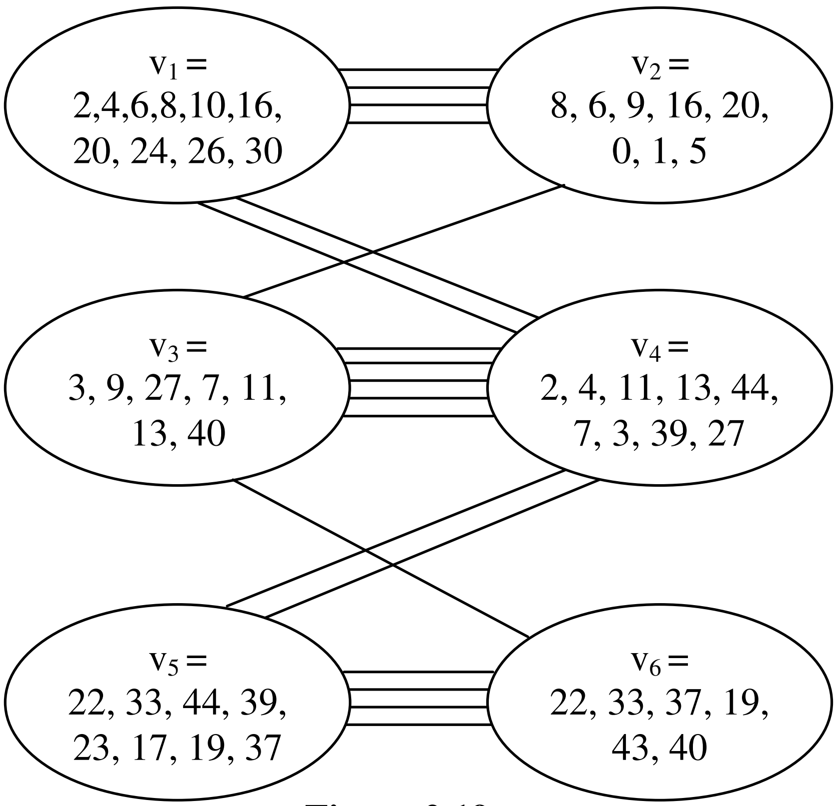


Figure 2.19

Where $v_1 = \{2, 4, 6, 8, 10, 16, 20, 26, 24, 30\}$,

$v_2 = \{8, 6, 9, 16, 20, 0, 1, 5\}$,

$v_3 = \{3, 9, 27, 7, 11, 13, 40\}$,

$v_4 = \{2, 4, 11, 13, 44, 7, 3, 39, 27\}$,

$v_5 = \{22, 33, 44, 39, 23, 17, 19, 37\}$ and

$v_6 = \{22, 33, 37, 19, 40, 43\}$; here $S = Z_{45}$.

Clearly the ordinary subset vertex multigraph D is a bigraph or to be more precise multibigraph or multibipartite graph.

It is pertinent to keep on record that these ordinary subset vertex multibigraphs can be used in the fuzzy relational maps models and in also soft computing (ANN).

We consider yet another ordinary subset vertex multitripartite graph K given by the following figure taking subset vertex elements from P(S) where $S = \{Z_{55}\}$.

Clearly these subset vertex multigraphs can be used in ANN with hidden layers or more specifically in DNN (Deep Neural Networks).

K =

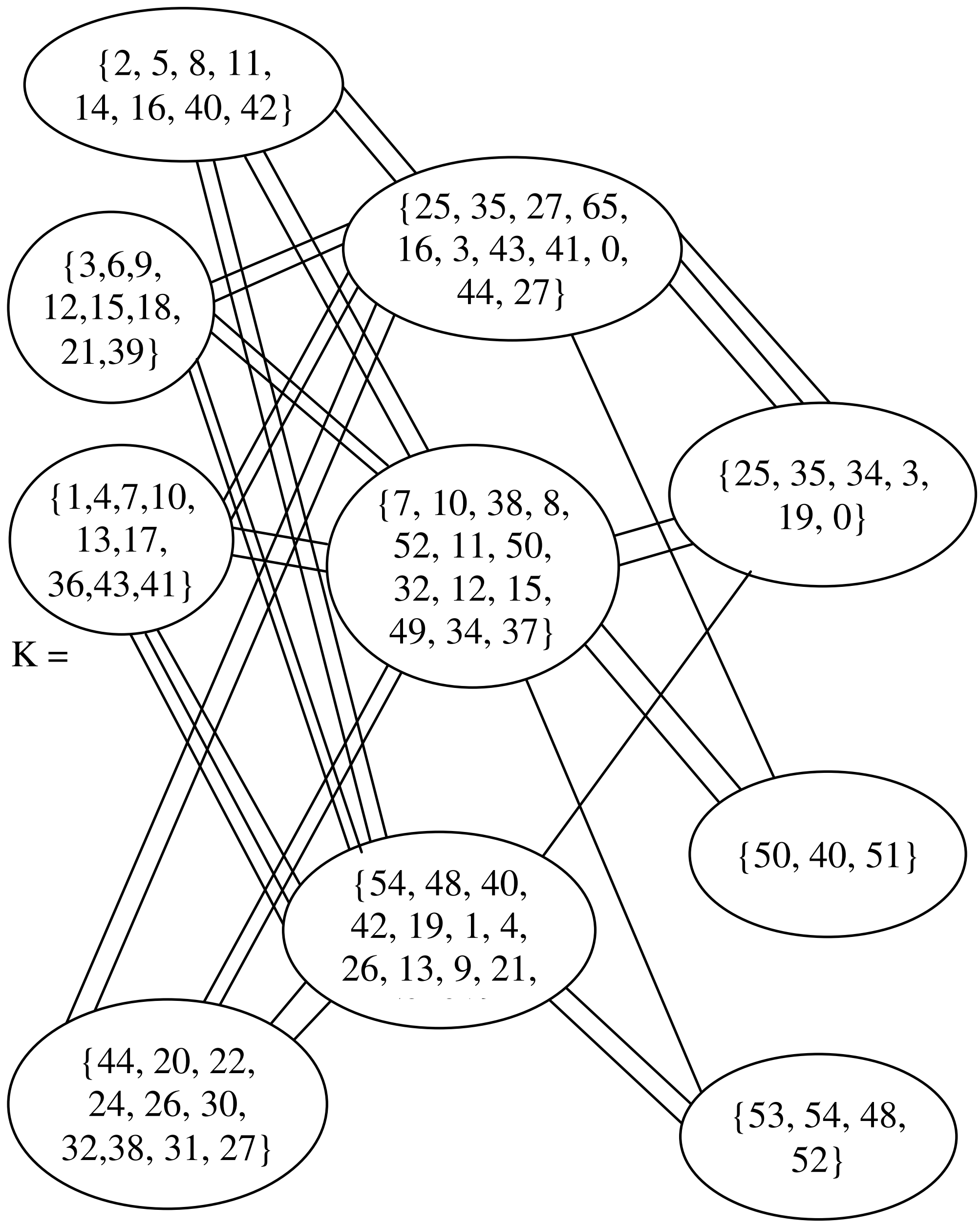


Figure 2.20

We see K is a subset vertex multigraph. These graphs can also be used in deep neural network (DNN). We can have several layers of multi n-graphs. K is a tripartite multigraph.

Further we wish to state that we need not always put the term "ordinary" subset vertex multigraph, for it can be easily understood from the very structure for the subsets are from one set whereas in the next chapter we mark the edges according to their attributes.

Now we just keep on record that this type of subset vertex n-partite multigraphs can be used in DNN, several layered *S be a set of finite* neural network and so on.

Now as these ordinary subset vertex multigraphs are not directed, we define them as type I multigraphs.

We now give the definition of the same.

Definition 2.1. *Let or infinite order, P(S) the powerset of S. Let G be a subset vertex graph of type I. If in G we draw the number of edges between any two subset vertices as the number of common elements between the two subset vertices, then we define the subset vertex graph of type I as ordinary subset vertex multigraph of type I.*

Clearly these subset vertex multigraphs of type I are not directed. Further we see the edge between two subset vertex pair exists if and only if $v_i \cap v_j \neq \phi$ that is if $v_i \cap v_j = A$ then there are $|A|$ edges between v_i and v_j (we assume $v_i \neq v_j$).

Next, we proceed onto describe two types of multisubgraphs of a subset vertex multigraph of type I by some examples.

We define a classical subset vertex multigraph with n vertex subsets $v_1, v_2, \ldots, v_n$ as all subset vertex multisubgraphs built using 2 vertex subsets v_i, v_j or 3 vertex subsets v_i, v_j, v_k and so on. However, if the subset vertex multisubgraph is to be proper we take a maximum of only (n – 1) vertex subsets from $v_1, v_2, \ldots, v_n$.

Example 2.11. Let S = {Z_{63}} be the set of order 63. P(S) be the power set of S. Let G be the ordinary vertex multigraph of type I with vertex subset; v_1, v_2, v_3, v_4, v_5 and v_6 given by the following figure:

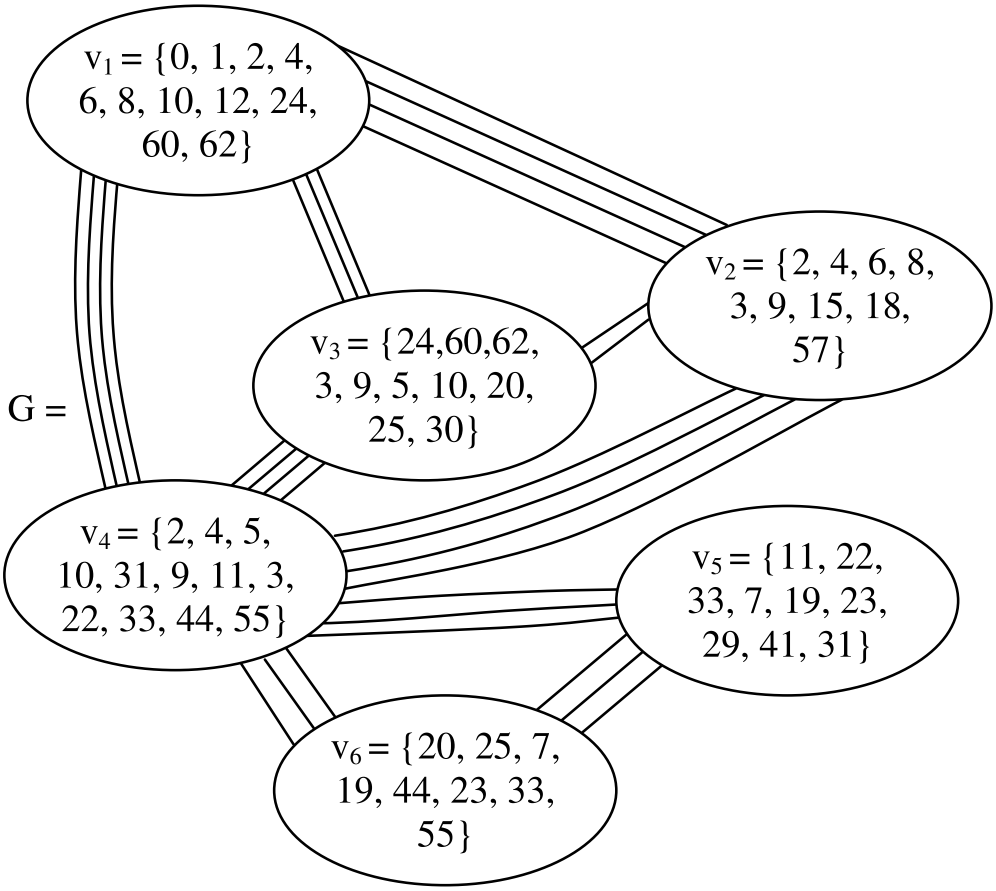


Figure 2.21

In the first place we wish to keep on record that there are $6C_2 + 6C_3 + 6C_4 + 6C_5$ number classical subset vertex multisubgraphs barring the 6-vertex subset vertex multisubgraphs.

There are 15 subset vertex multisubgraphs with two vertex subsets.

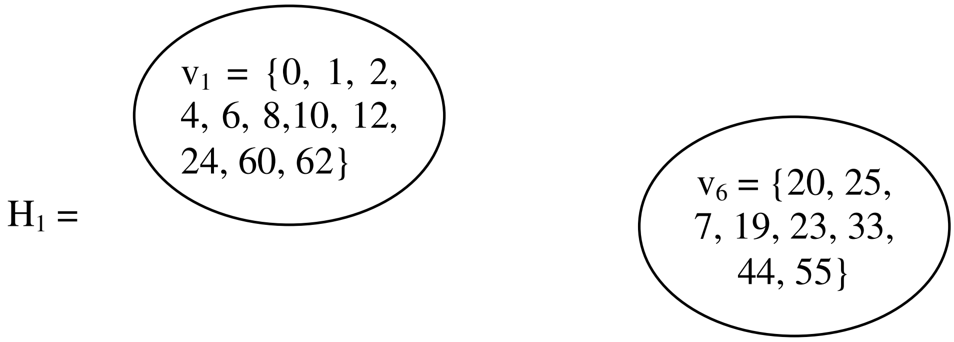


Figure 2.22

Clearly H_1 is just an empty subset vertex multisubgraph. Consider H_2 the subset vertex multisubgraph with two vertex subsets.

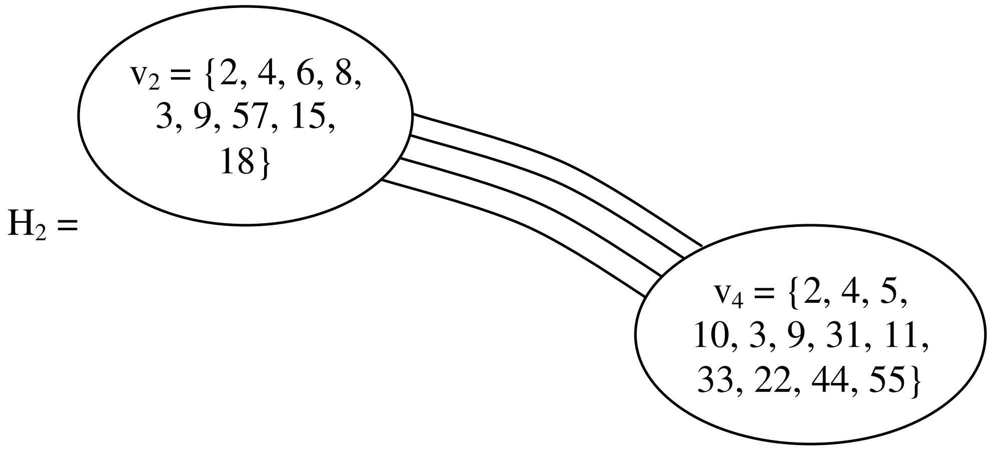


Figure 2.23

H_2 is a subset vertex multisubgraph with 4 edges. Let H_3 be the subset vertex multisubgraph with vertex subsets v_4 and v_5 is as follows.

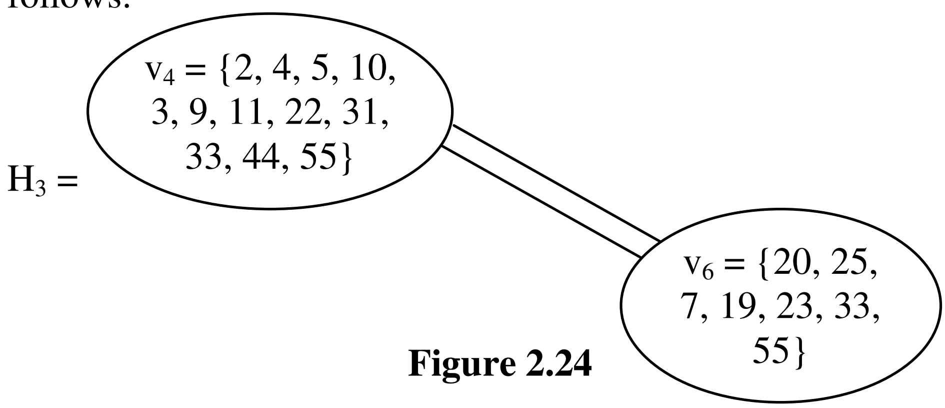


Figure 2.24

This is a subset vertex multisubgraph with two edges. We have four subset vertex multisubgraph with two subset vertices which has only two edges and only one subset vertex multisubgraph with 5 edges.

Next, we proceed onto provide a few subset vertex multisubgraphs with three vertex subsets.

Let K_1 be the subset vertex multisubgraph given by the following figure:

K_1 =

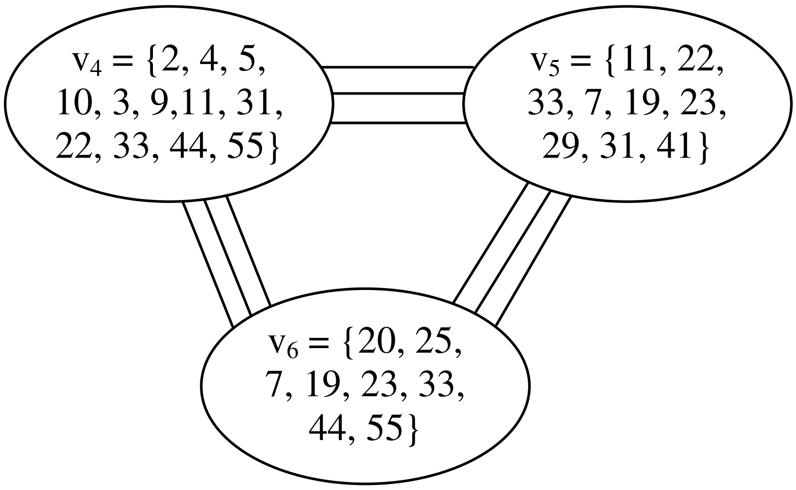


Figure 2.25

Clearly K_1 is a uniform complete triad.

We see the edges connecting any two subset vertices is three. Infact there are 20 subset vertex multisubgraphs with three subset vertices. None of the collection is an empty multisubgraph.

Consider K_2 the subset vertex multisubgraph given by the following figure;

K_2 =

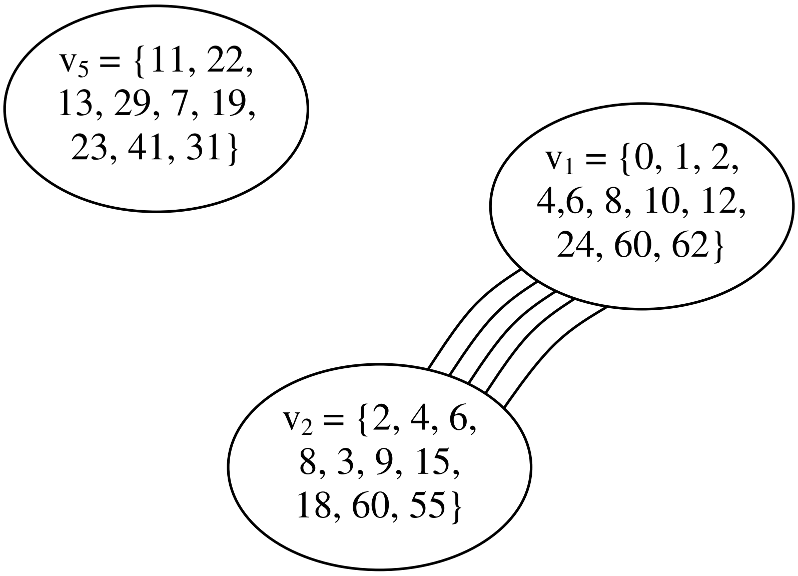


Figure 2.26

Clearly K_2 is a disconnected subset vertex multisubgraph. Let K_3 be the subset vertex multisubgraph given by the following figure.

K_3 =

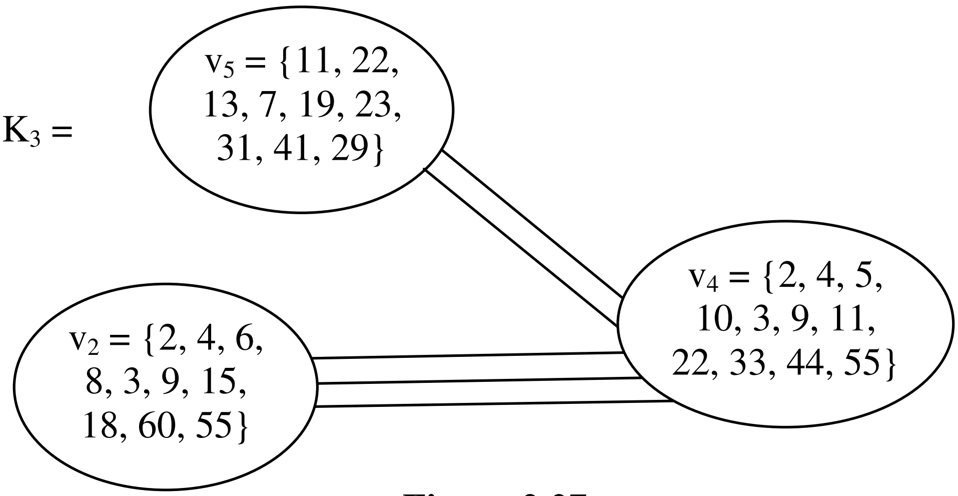


Figure 2.27

The subset vertex multisubgraph with vertex subsets v_2, v_5 and v_4 is a forbidden triad which is not uniform as the number of multi edges are only 2 and three connecting v_5 and v_4 and v_2 and v_4 respectively. Now we see the structures enjoyed by subset vertex multisubgraph M_1 with four subset vertices.

M_1 =

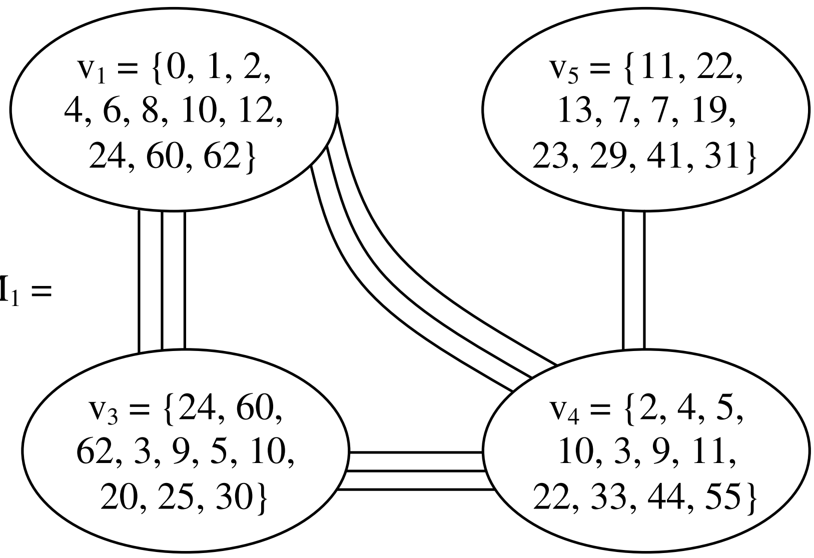


Figure 2.28

M_1 is not a complete subset vertex multisubgraph. Consider the subset vertex multisubgraph M_2 given by the following figure.

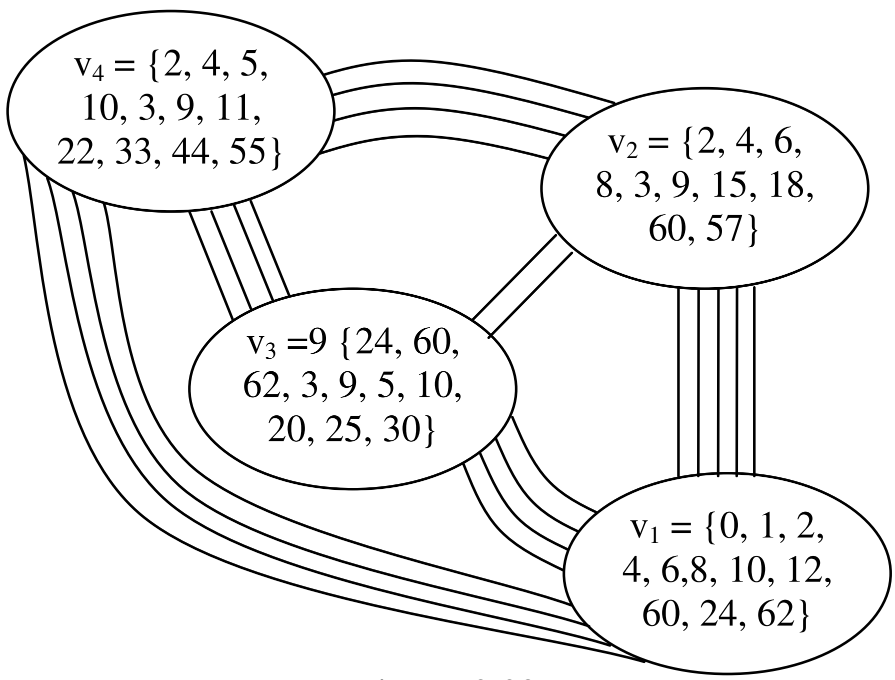


Figure 2.29

M_2 is a pseudo complete non uniform multisubgraph of order 4. There are 15 subset vertex multisubgraphs with four vertex subsets. None of them is an empty subset vertex multisubgraph.

Now we enlist some of the subset vertex multisubgraphs with five vertex subsets. Let P_1 be a subset vertex multisubgraph with v_1, v_2, v_3, v_4 and v_5 as vertex subsets given by the following figure.

P_1 =

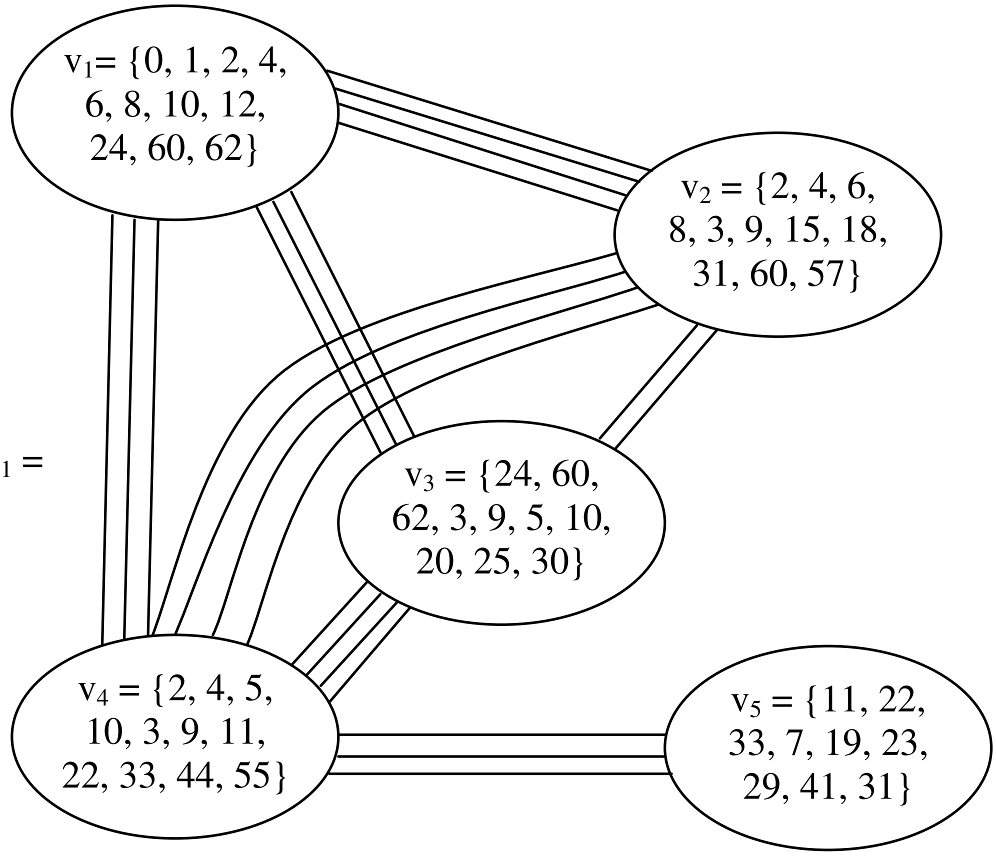


Figure 2.30

Further P_1 is not a complete subset vertex multisubgraph. Let P_2 be the subset vertex multisubgraph given by the following figure with v_1, v_2, v_3, v_4 and v_6 as vertex subsets.

P₂ =

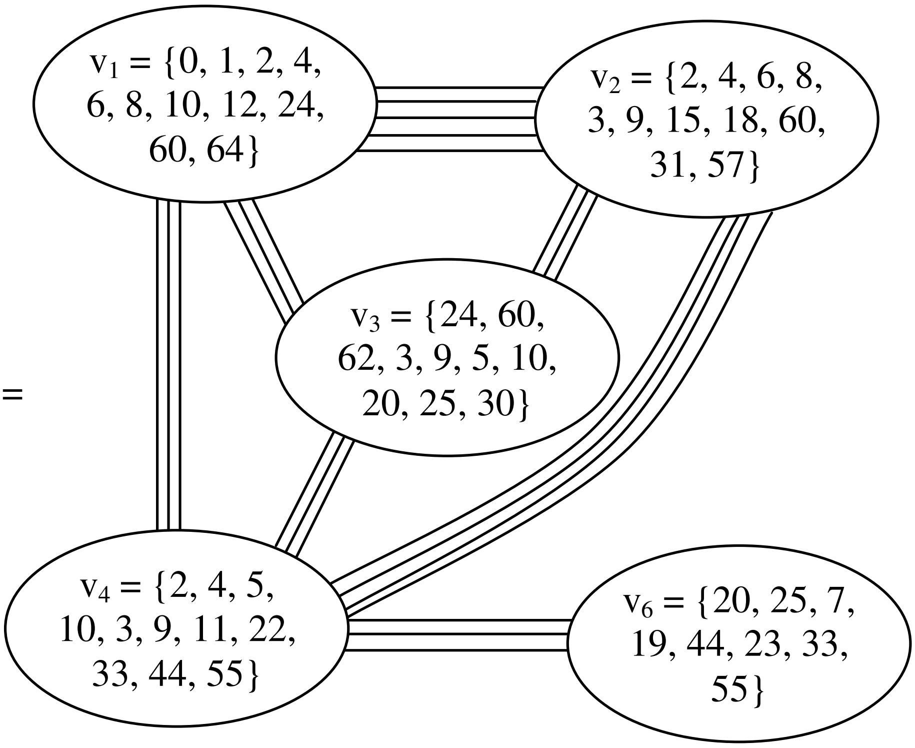


Figure 2.31

P_2 is not a complete subset vertex multisubgraph. We see G cannot contain a subset vertex multisubgraph with 5 subset vertices which can be complete.

In view of all these we have the following theorem.

Theorem 2.1: *Let S = {any set of elements} be a finite or infinite set. P(S) be the powerset of S. Let G be a subset vertex multi graph with n number of vertex subsets $v_1, v_2, \ldots, v_n$.*

The number of proper classical subset vertex multisubgraphs of G including the single subset vertex is $nC_1 + \ldots + nC_{n-1}$.

Proof: Direct from the very definition of classical subset vertex multisubgraphs of G.

We describe the notion of subset-subset vertex multigraphs of type I.

Example 2.12. Let S = $\{Z_{108}\}$ be the given set and P(S) the power set of S. G the subset vertex star multigraph of type I given below;

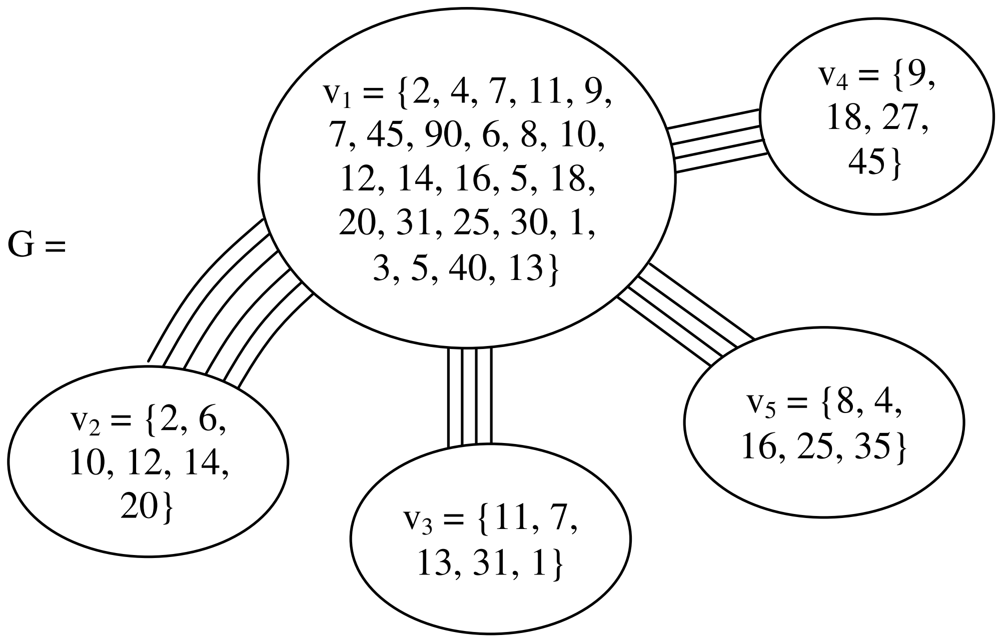


Figure 2.32

G is a subset vertex multistar graph of type I. H_1 be the subset-subset vertex multisubgraph of G given by the following figure.

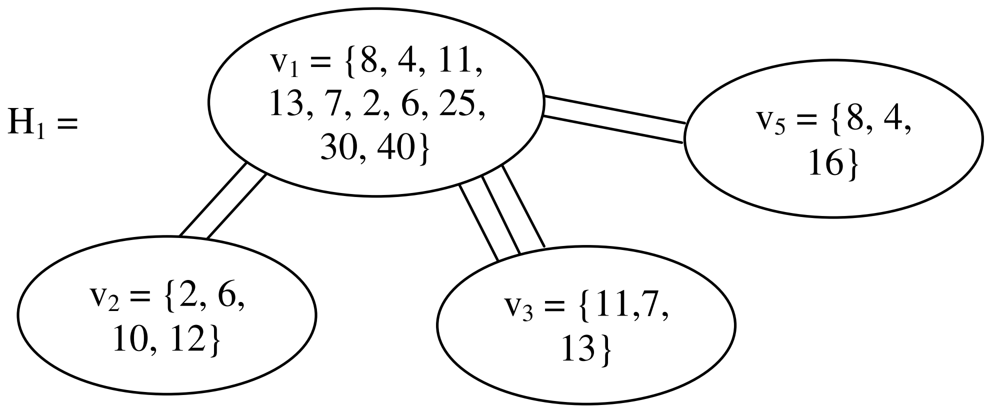


Figure 2.33

We see H_1 is again a subset-subset vertex multisubgraph which is again a star multisubgraph.

However, it is important to note that if the subset-subset vertex multisubgraph does not include a subset from the subset vertex v_1 then all those subset-subset vertex multisubgraphs will only be an empty subset-subset vertex multisubgraphs. We give another example of subset-subset vertex multisubgraph H_2 is as follows.

H_2 =

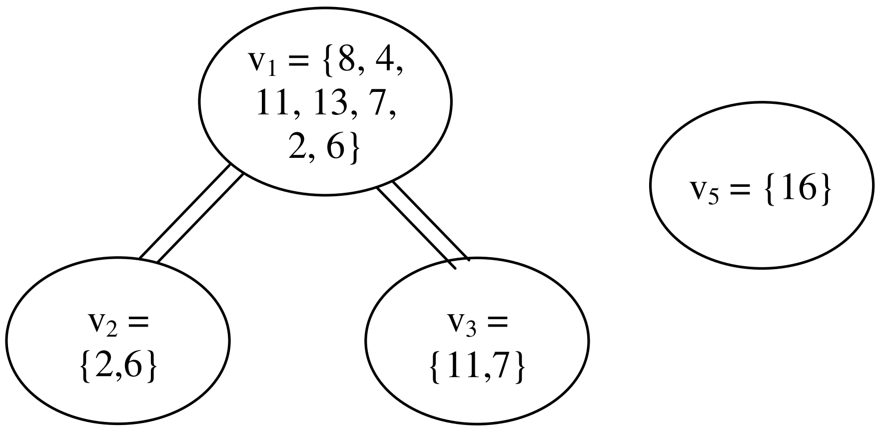


Figure 2.34

H_2 is not a connected subset-subset vertex multisubgraph whereas H_1 is a connected subset-subset vertex multisubgraph.

Let H_3 be the subset-subset vertex multisubgraph given by the following figure.

s

H_3 =

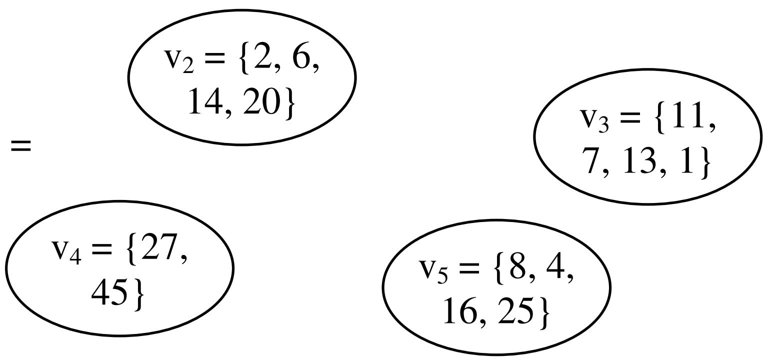


Figure 2.35

Clearly H_5 is a subset-subset vertex multisubgraph which is an empty multisubgraph of G.

It is interesting to note that the structure in general need not be preserved by subset-subset vertex multisubgraphs, as we take only subset of the vertex subsets $v_1, v_2, \ldots, v_n$.

We will give yet another example of the subset-subset vertex multisubgraphs.

Example 2.13. Let S = {Z_{45}} be the set. P(S) be the power set of S.

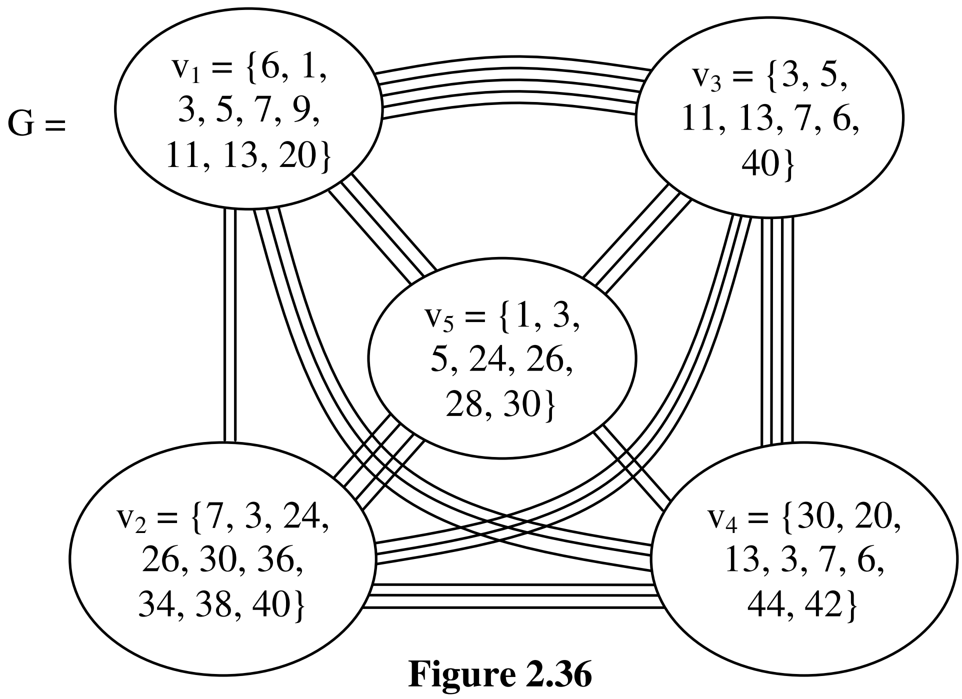


Figure 2.36

Clearly G in figure is a subset vertex multigraph with five vertex subsets v_1, v_2, v_3, v_4 and v_5. G is a pseudo complete non uniform subset vertex multigraph.

Now we find the universal complement of this subset vertex multigraph G by taking the vertex subsets as $S\backslash v_1$, $S\backslash v_2$, $S\backslash v_3$, $S\backslash v_4$ and $S\backslash v_5$ in the following

$G^c =$

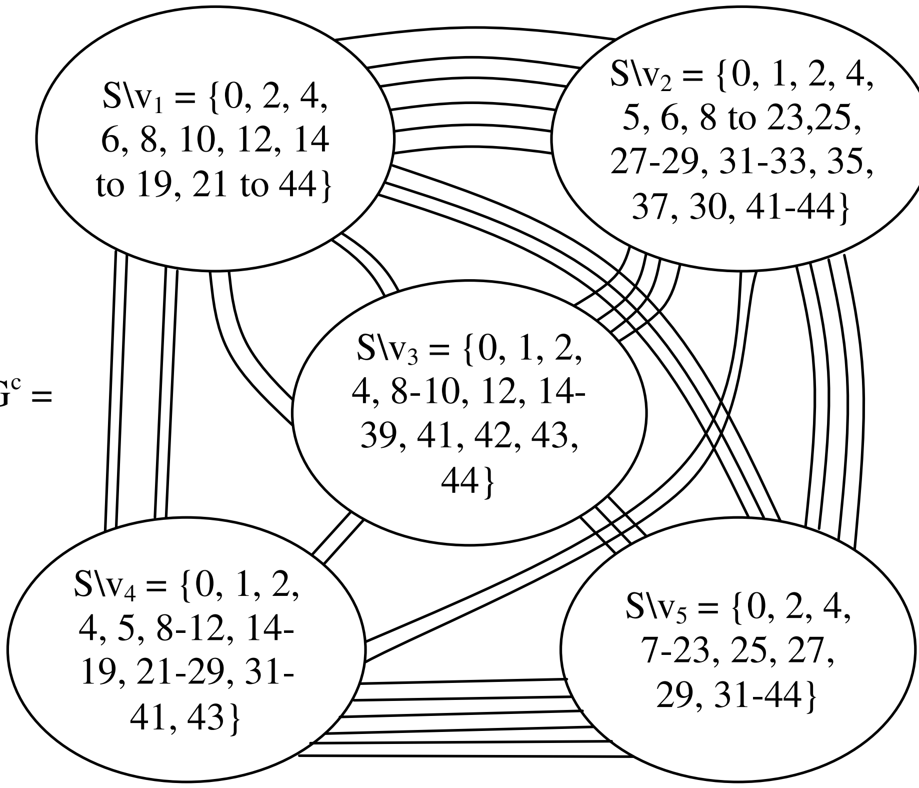


Figure 2.37

We see the universal complement G is a complete pseudo or otherwise subset vertex multigraph of G.

The reader is left with task of checking whether G^c is uniform complete or pseudo complete.

Now we find for the following subset vertex multi graph H the universal complement.

The vertex subsets are from P(S) where S = {0, 1, 2, …, 17}.

H =

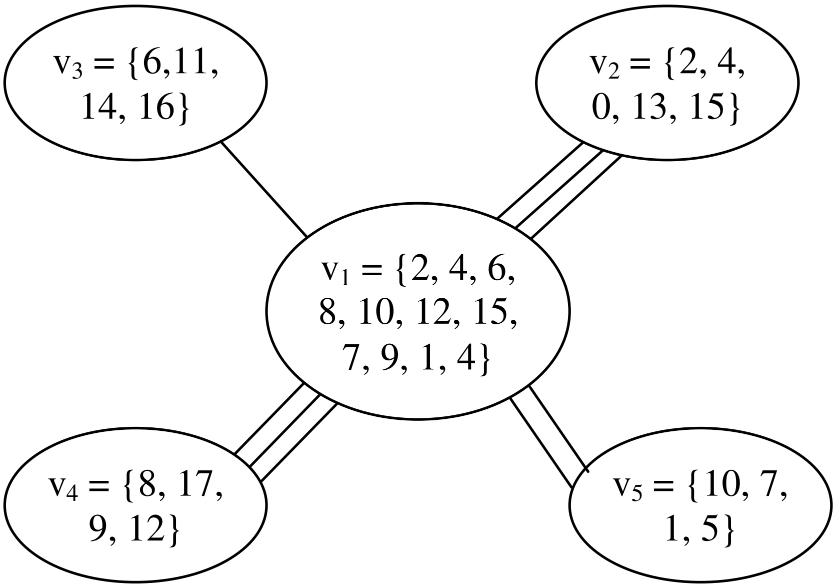


Figure 2.38

Now we find the universal complement H^c of H relative to the subset in P(S). H^c the subset vertex multigraph is as follows.

H^c =

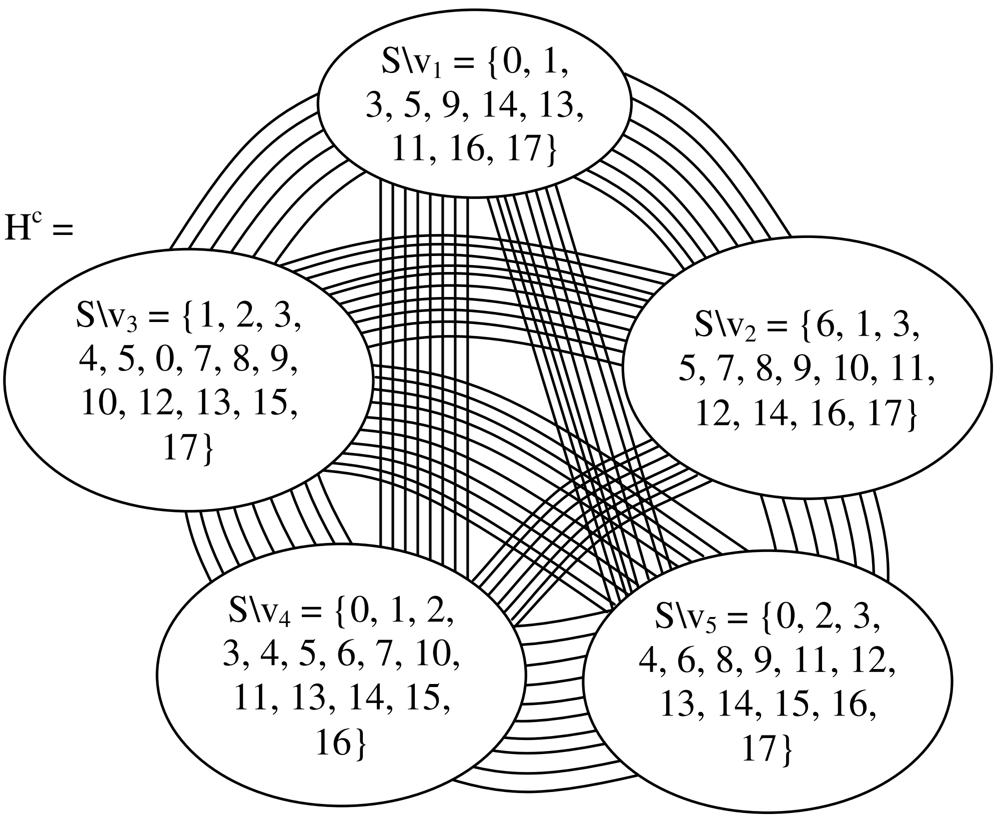


Figure 2.39

Clearly the universal complement of the subset vertex multistargraph happens to be a non uniform complete subset vertex multigraph.

Thus, it is interesting to keep on record that in general the universal complements G^c of a subset vertex multigraph G does not preserve the structure. In view of this we have the following theorem.

Theorem 2.2: *Let S be a set and P(S) the power set of S. Let G be an ordinary subset vertex multigraph using subset vertex set from P(S). In general, the universal complement G^c of G and G do not enjoy the same structure.*

Proof. Can be proved by counter examples.

We give examples of ordinary subset vertex multigraphs and their universal complements.

Example 2.14. Let S = $\{Z_{12}\}$ be a finite set and P(S) the power set of S and G the subset vertex multigraph given by the following figure.

G =

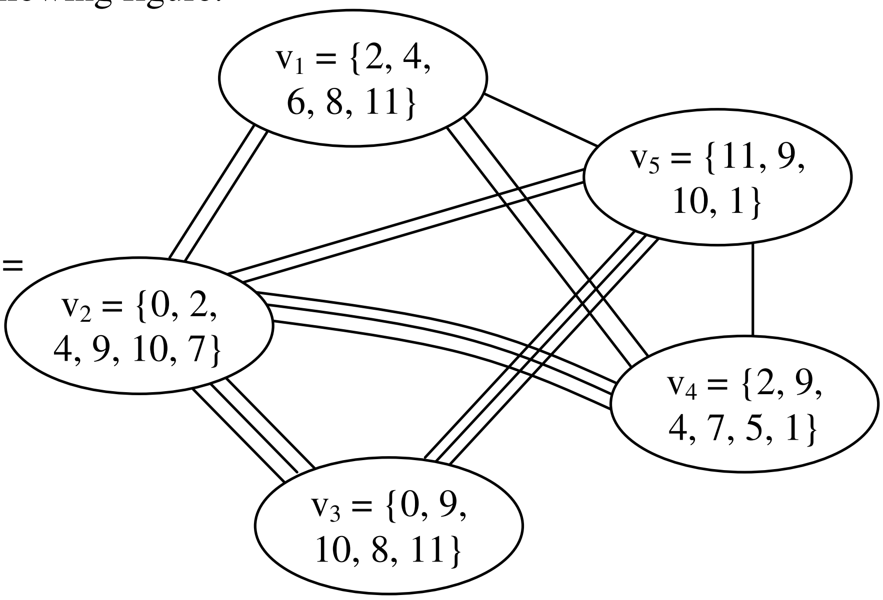


Figure 2.40

Now we find the universal complement G^c of G in the following.

$a_1 = S \backslash v_1 = \{0, 1, 3, 5, 7, 9, 10\}$,

$a_2 = S \backslash v_2 = \{1, 3, 5, 6, 8, 11\}$,

$a_3 = S \backslash v_3 = \{1, 2, 3, 4, 5, 6, 7\}$,

$a_4 = S \backslash v_4 = \{0, 9, 3, 6, 8, 10, 11\}$ and

$a_5 = S \backslash v_5 = \{0, 2, 3, 4, 5, 6, 7, 8\}$.

Let G^c be the universal complement of G with complement vertex subsets a_1, a_2, a_3, a_4 and a_5 given in the following.

G^c =

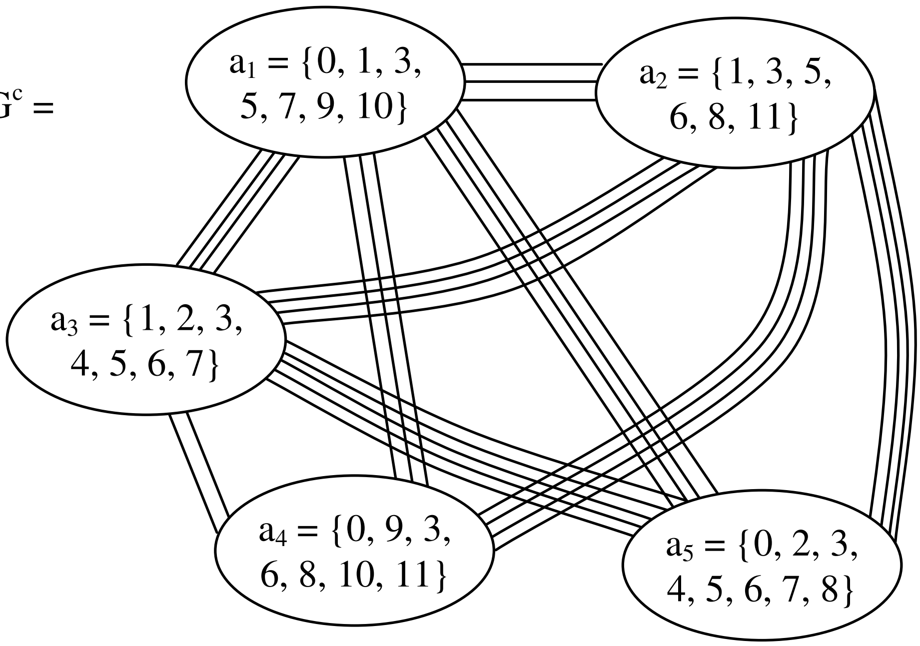


Figure 2.41

Clearly G is not complete or pseudo complete subset vertex multigraph; however, the universal complement of G is a non uniform complete subset vertex multigraph of G^c of G.

Now we give a subset-subset vertex multisubgraph H of G in the following.

H =

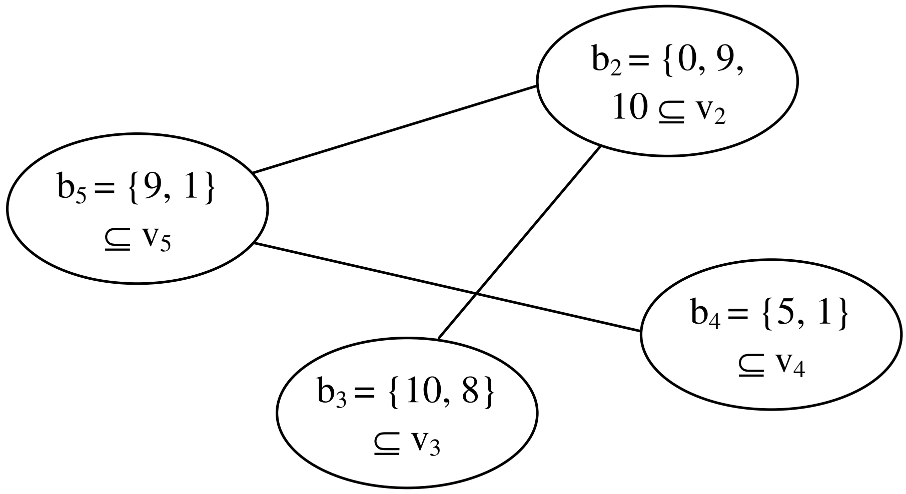


Figure 2.42

We see the structure of G and H are different. Now we find the local complement K of H relative to G.

K =

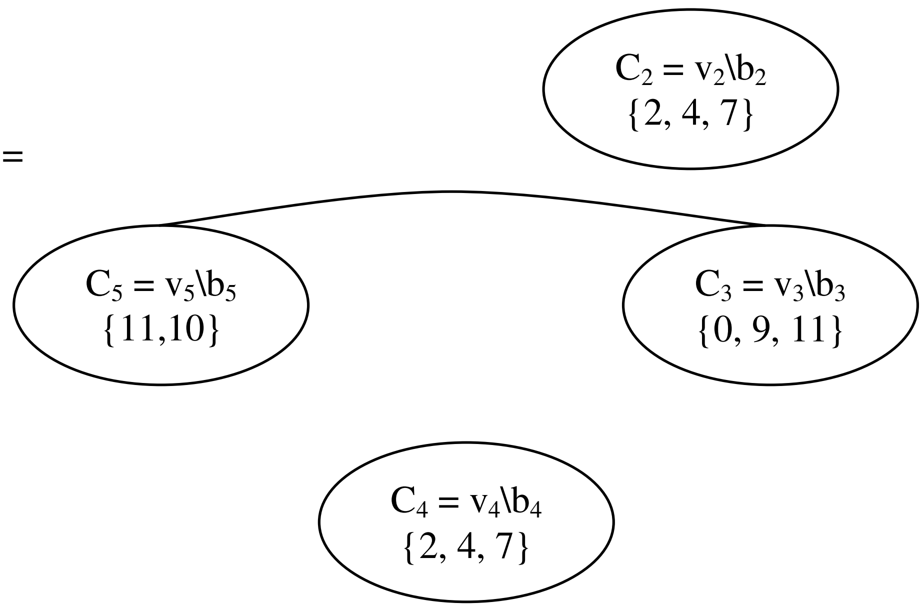


Figure 2.43

Since $C_2 = C_4$ we see the local complement is not defined. Thus for this subset-subset vertex multigraph the local complement is not defined.

Let M be the subset-subset vertex multisubgraph of G given in the following.

M =

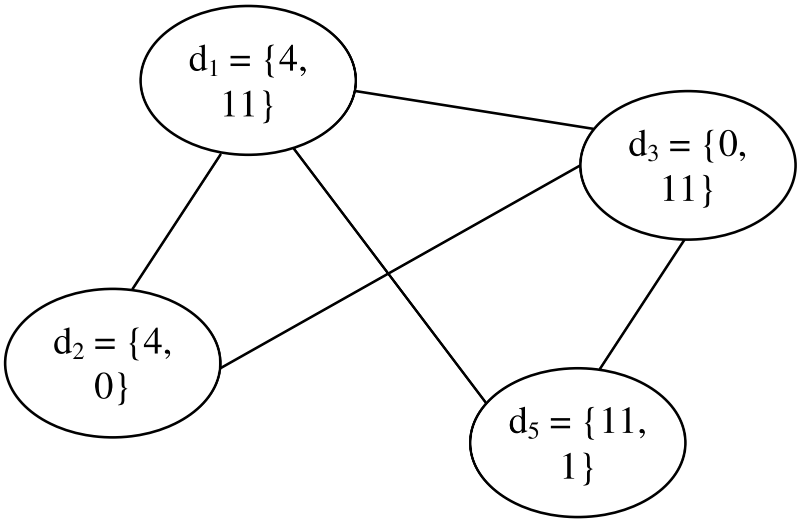


Figure 2.44

Now we define subset-subset vertex multisubgraph and its local complement.

Definition 2.2. *Let G be a vertex subset multigraph with V = $\{v_1, v_2, \ldots, v_n\}$ as vertex set from P(S), the power set of a set S. Let H be a subgraph of G with t number of vertices taken from V such that $u_i \leq v_i$; i = 1, 2, …, t, $u_i \leq v_i$. $2 < t < n$. We call H the subset-subset vertex multisubgraph of G.*

The subset-subset vertex multisubgraph exists if and only if $u_i \neq u_j$ if $i \neq j$; $1 \leq i, j \leq t$. The local complement H^C of H relative to G has its vertex set $w_1, w_2, \ldots, w_t$ where $w_i = v_i \setminus u_i$ $1 \leq i \leq t$.

The local complement H^C of H exists if and only if $w_i \neq w_j$ if $i \neq j$, $1 \leq i, j \leq t$.

Thus, one of the main criteria for the existence of a local complement of a subset-subset vertex multisubgraph relative to the subset vertex multigraph is that none of the vertices of the local complement should be coincident. If coincident the local complement cannot be defined.

In view of this we have the following result.

Theorem 2.3. *Let S be any set. P(S) the power set of S. Let G be a subset vertex multigraph of type I. Suppose H be subset-subset vertex multisubgraph of G. The local complement of H relative to G exists if and only if no two of the local complement vertex subsets are equal.*

Proof. Let G be a subset vertex multigraph of type I with vertex subsets $v_1, v_2, \ldots, v_n$; ($v_i \neq v_j$; $i \neq j$; $1 \leq i, j \leq n$). Let H be a subset-subset vertex multisubgraph with subset vertex subsets $u_1, u_2, \ldots, u_{i-1}, u_{i+1}, \ldots, u_t$; where $u_j \leq v_j$; $j = 1, 2, \ldots, i-1, i+1, \ldots, t$, let $s_j = v_j \setminus u_j$; $j = 1, 2, \ldots, i-1, i+1, \ldots, t$ with $s_j \neq s_k$ if $j \neq k$. Suppose K be the subset-subset vertex multisubgraph with vertex subsets $s_1, s_2, \ldots, s_t$, then K exists. Suppose $s_j = s_k$ for $j \neq k$ then K does not exist. Hence the result.

We have also given some examples of them.

Next, we proceed onto describe the notion of ordinary subset vertex multigraphs of type II. Before we proceed onto define this new concept, we will give some examples of the same.

Example 2.15. Let S = $\{Z_{27}\}$ be a set of order 27. P(S) be the power set of S. Let $v_1 = \{2, 0, 4, 6, 9, 11, 15, 18\}$, $v_2 = \{2, 0, 13, 7, 5, 19, 23\}$, $v_3 = \{2, 0, 4, 6, 15\}$, $v_4 = \{2, 0, 5, 19\}$, $v_5 = \{11, 15, 18\}$, $v_6 = \{13, 19, 23\}$ and $v_7 = \{2, 0, 4\}$ be the subset vertices of the ordinary subset vertex multigraph G of type II given by the following figure.

We know type II graphs are directed.

G =

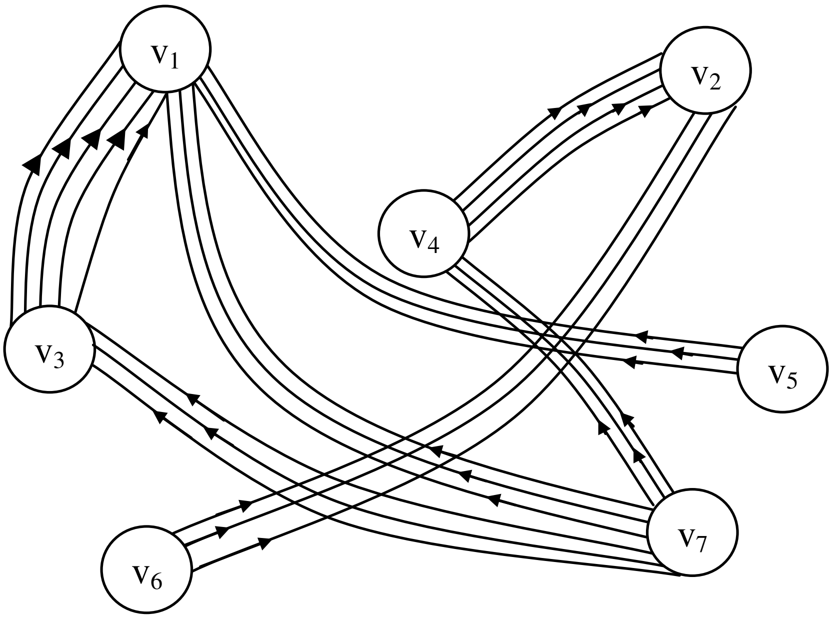


Figure 2.45

Clearly G is an ordinary injective subset vertex multigraph of type II.

We in a casual way indicate these multigraphs in this book mostly as subset vertex multigraphs of type II; from the very context it is easily understood that these subset vertex multigraphs are ordinary or injective or projective.

Thus, we give the definition in the following.

Definition 2.3. Let S be any set. P(S) the power set of S. Let v_1, v_2, …, v_n be n subset vertex sets of s. we can have t-multiedges connecting v_i and v_j if and only if $v_i \subset v_j$ ($v_i \neq v_j$; $i \neq j$; 1, 2, …,

n) where $o(v_i) = |v_i| = t$ and the map from v_i to v_j is injective; that is t such edges.

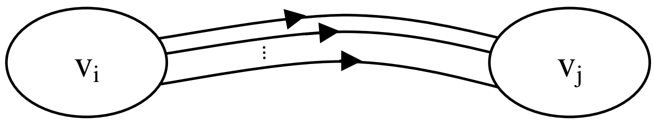


Figure 2.46

The resulting graph G is defined as the ordinary injective subset vertex multigraph of type II.

Clearly G is a directed multigraph we provide yet another example of the same.

Example 2.16. Let S = $\{Z_{45}\}$ be the set and P(S) the power set of S. Let G be the injective subset vertex multigraph given by the following figure.

G =

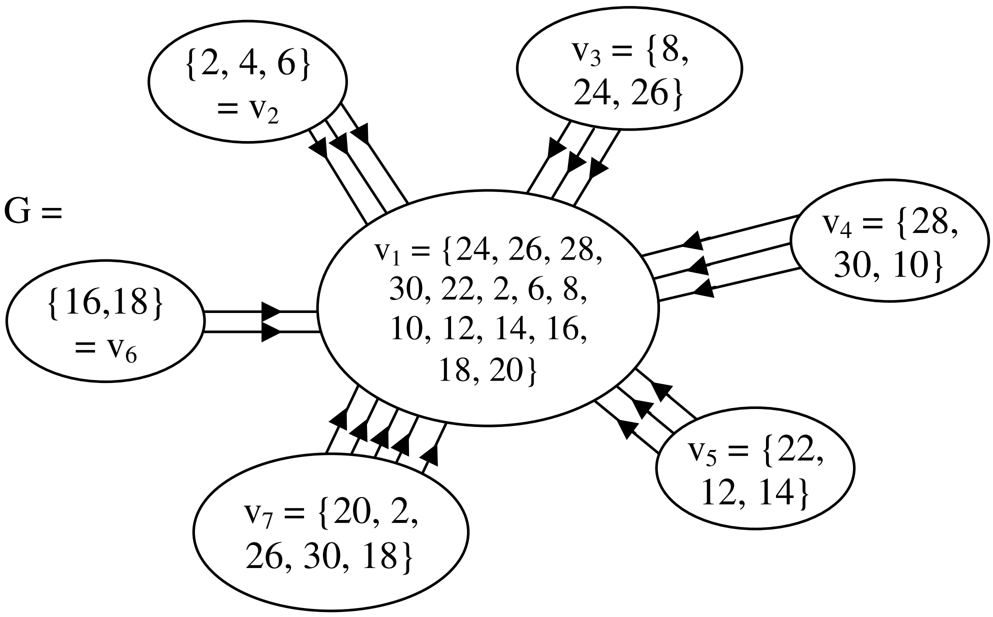


Figure 2.47

Clearly G is an ordinary injective subset vertex multigraph which is a star multigraph.

Consider H the injective ordinary subset vertex multigraph given by the following figure.

H =

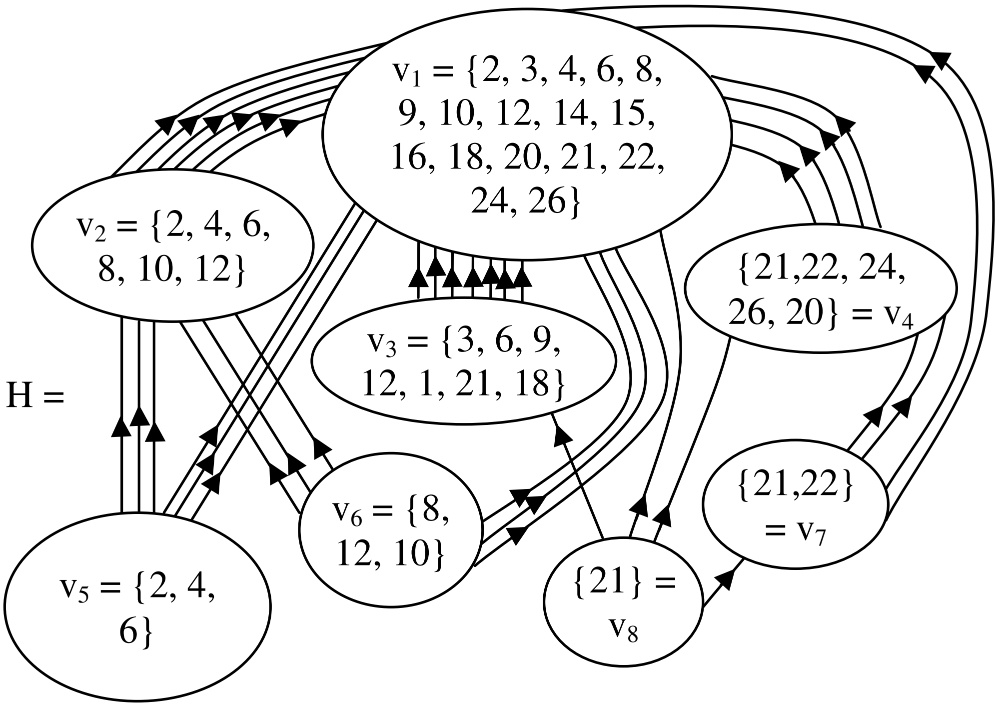


Figure 2.48

Clearly H is a subset vertex multigraph of type II. Clearly H is directed.

Thus we can have subset vertex multigraph of type II which is not complete.

Consider the subset vertex multigraph K given by the following figure.

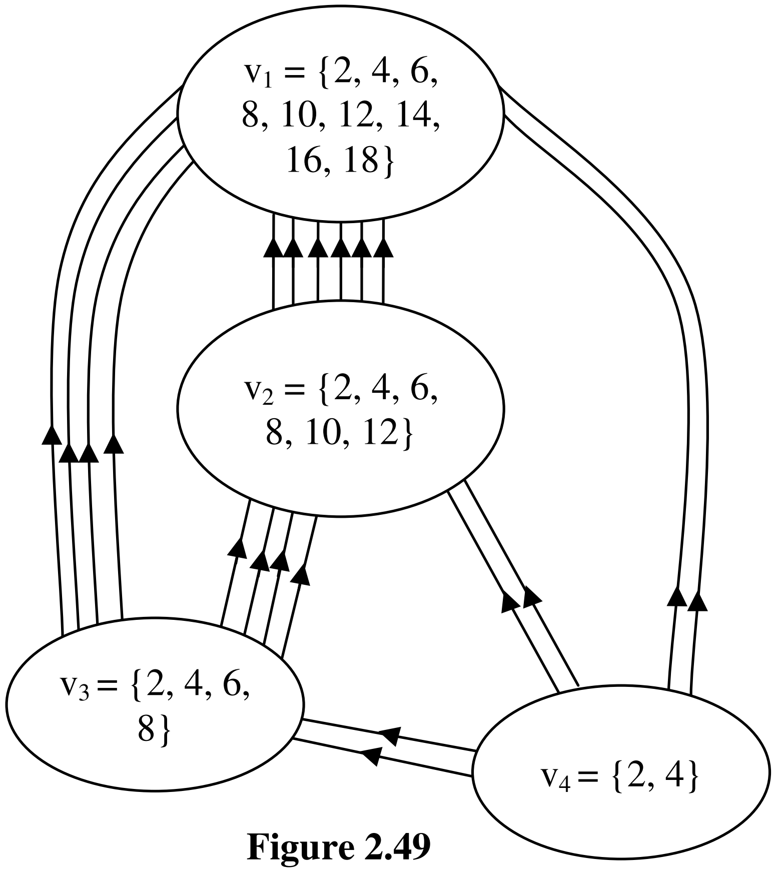
K = 

Figure 2.49

We see K is a pseudo complete subset vertex multigraph of type II. Now we have the following interesting property enjoyed by subset vertex multigraphs of type II. If we have a chain of subsets related by inclusion (say for instance).

$\{2\} \subseteq \{2, 4\} \subseteq \{2, 4, 6\} \subseteq \{2, 4, 6, 8\} \subseteq \{2, 4, 6, 8, 10\} \subseteq \{2, 4, 6, 8, 10, 12\}$;

where $\subseteq$ denotes that the subset {2} is contained "$\subseteq$" in {2, 4] and likewise for other subsets or chain of subsets. Then if T is a subset vertex multigraph with vertex subsets v_1 = {2, 4, 6, 8, 10, 12}, v_2 = {2, 4, 6, 8, 10}, v_3 = {2, 4, 6, 8}, v_4 = {2, 4, 6}, v_5 = {2, 4} and v_6 = {2}; is given in the following;

T =

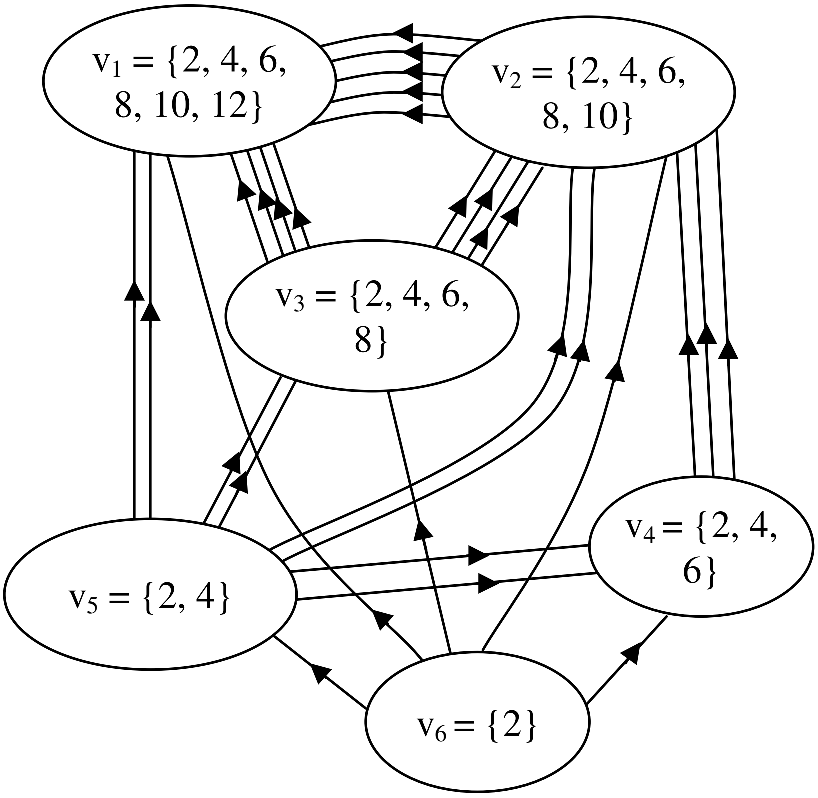


Figure 2.50

We make the following observations from the subset vertex multigraph T.

This subset vertex multigraph is complete or pseudo complete. All the edges from the vertex subset v_6 emerge out that is v_6 is an into map to other subset vertex sets v_j. None of the edges. In case of v_5 there are three multiedge and only one single edge between v_5 and v_6. There are 3 multiedges which emerge out to other subset vertices. Consider v_4; of the four edges connecting v_4 to other v_i's two of the edges are multiedges of order three both of which emerge out the rest of the two edges get in and so on.

Thus we can say a chain of subsets which is totally ordered by inclusion can only yield a pseudo complete subset vertex multigraph of type II which is directed.

Thus if we have set S = $\{Z_5\}$ with say 5 elements then the larges totally ordered set using subsets of S is

$$\{\phi\} \subseteq \{0\} \subseteq \{0, 1\} \subseteq \{0, 2, 1\} \subseteq \{0, 1, 2, 3\}$$

$$\subseteq \{0, 1, 2, 3, 4\} \qquad \ldots I$$

The subset vertex multigraph P using this chain I is as follows:

P =

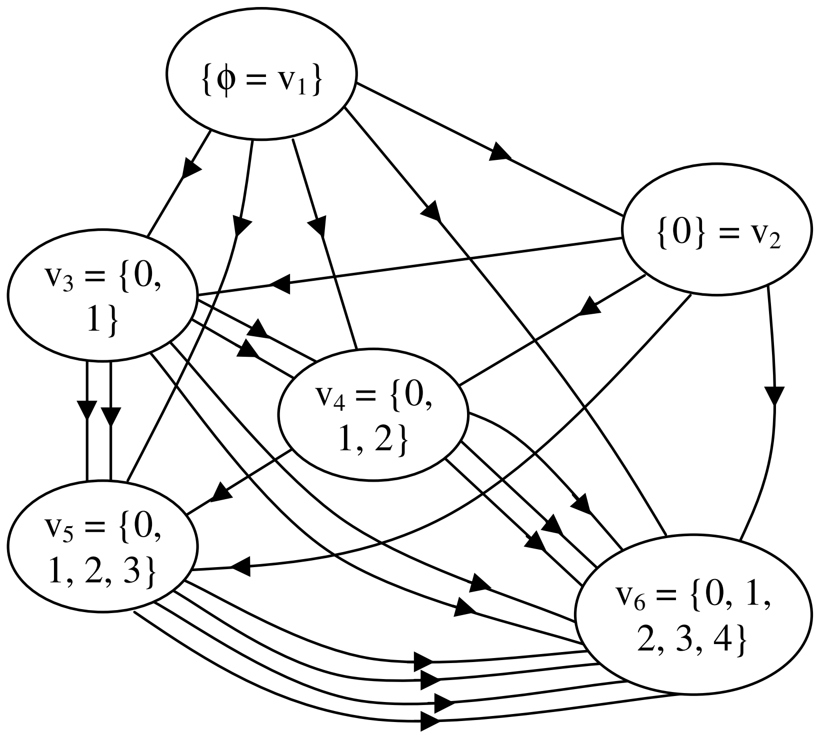


Figure 2.51

Clearly P is an injective pseudo complete subset vertex multigraph. In view of these we can have the following result.

Theorem 2.4: *Let S = $\{Z_n\}$ = $\{a_0, \ldots, a_{n-1}\}$ be a set of order n. P(S) be the power set of S. All totally ordered chains of maximal* *length is only {n + 1} (and related to these chain),these subsets as vertex subsets of multigraph results in an injective pseudo complete multigraph with (n + 1) vertices which are subsets from P(S).*

Proof is direct and hence left as an exercise to the reader.

However, it is pertinent to keep on record that we can have subset vertex multigraph which are complete associated with chains of lesser length.

Thus with every chain lattice of length n we have a pseudo complete subset vertex multigraph. The converse question is left as an exercise for the reader.

Example 2.17. Let S = {a, b} be a set of order two. P(S) be the power set of S. P(S) = {ϕ, {a}, {b}, {a, b}}. Now the subset vertex multigraph M associated with v_1 = {ϕ}, v_2 = {a}, v_3 = {b} and v_4 = {a, b} is as follows:

M =

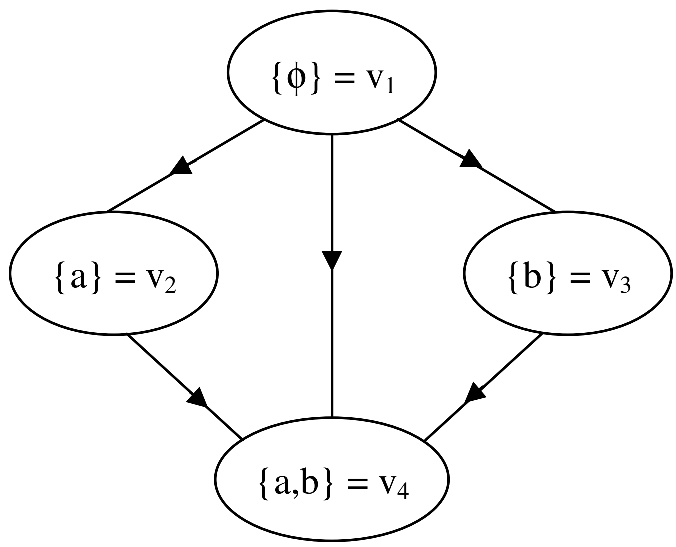


Figure 2.52

The lattice with P(S) which is a Boolean algebra B is as follows:

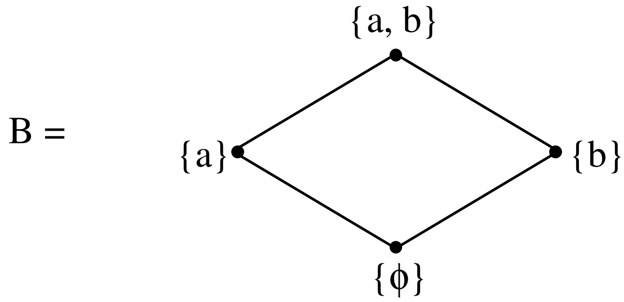


Figure 2.53

Clearly B and M are different. Infact we do not have any subset vertex multigraph of type I or type II to be a Boolean algebra but type I subset vertex multigraphs can be chain lattices of a special type.

Consider the subset vertex multigraph of type II with vertex subset as whole of P(S) where S = {a, b, c}.

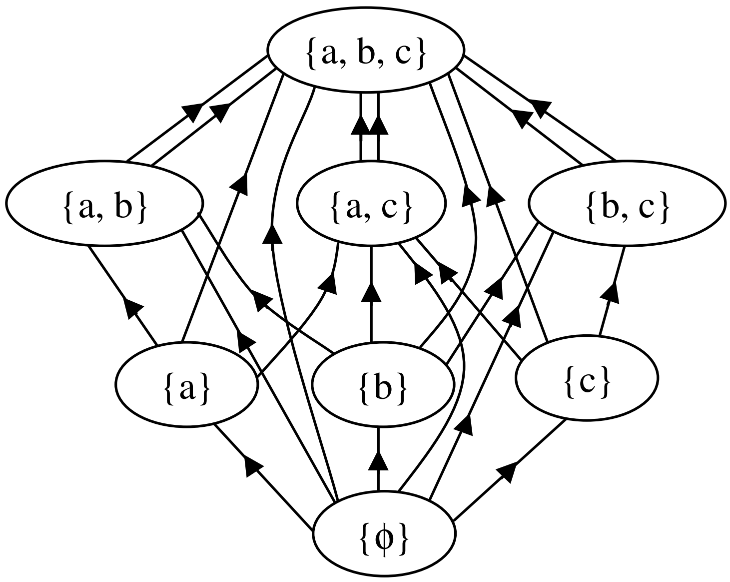
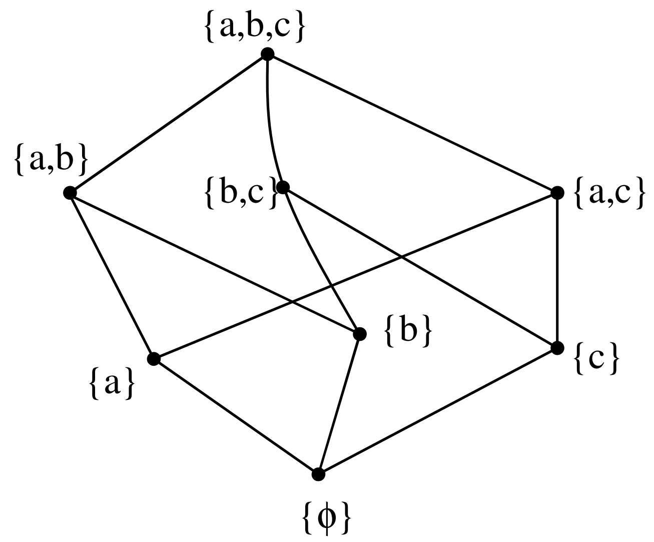


Figure 2.54

However the Boolean algebra of order 8 is as follows:

B =



B =

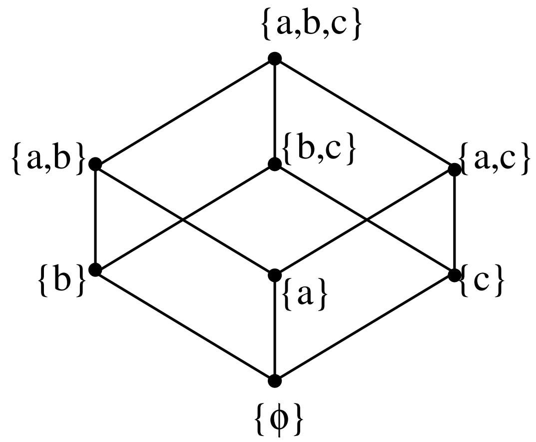


Figure 2.55

We give yet another example of chain lattice and its corresponding subset vertex multigraph of type I and injective subset vertex multigraph of type II where P(S) = {subsets from Z_7}.

$C_6 =$

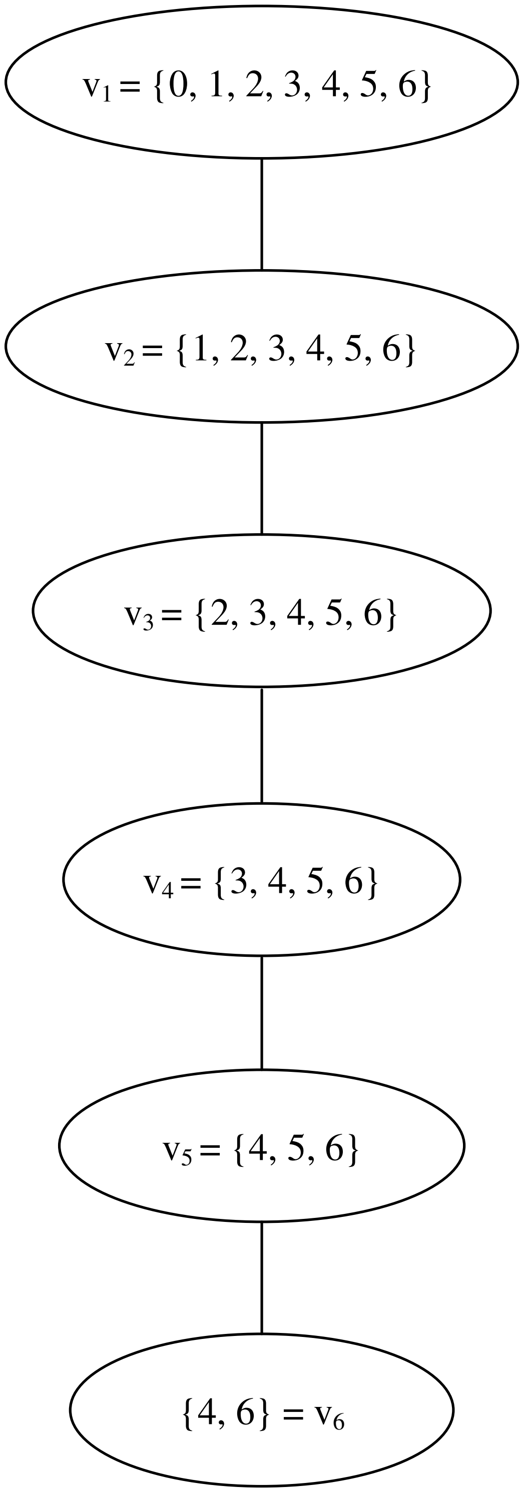


Figure 2.56

$v_6 \subset v_5 \subset v_4 \subset v_3 \subset v_2 \subset v_1$ is a totally ordered chain of order six.

B =

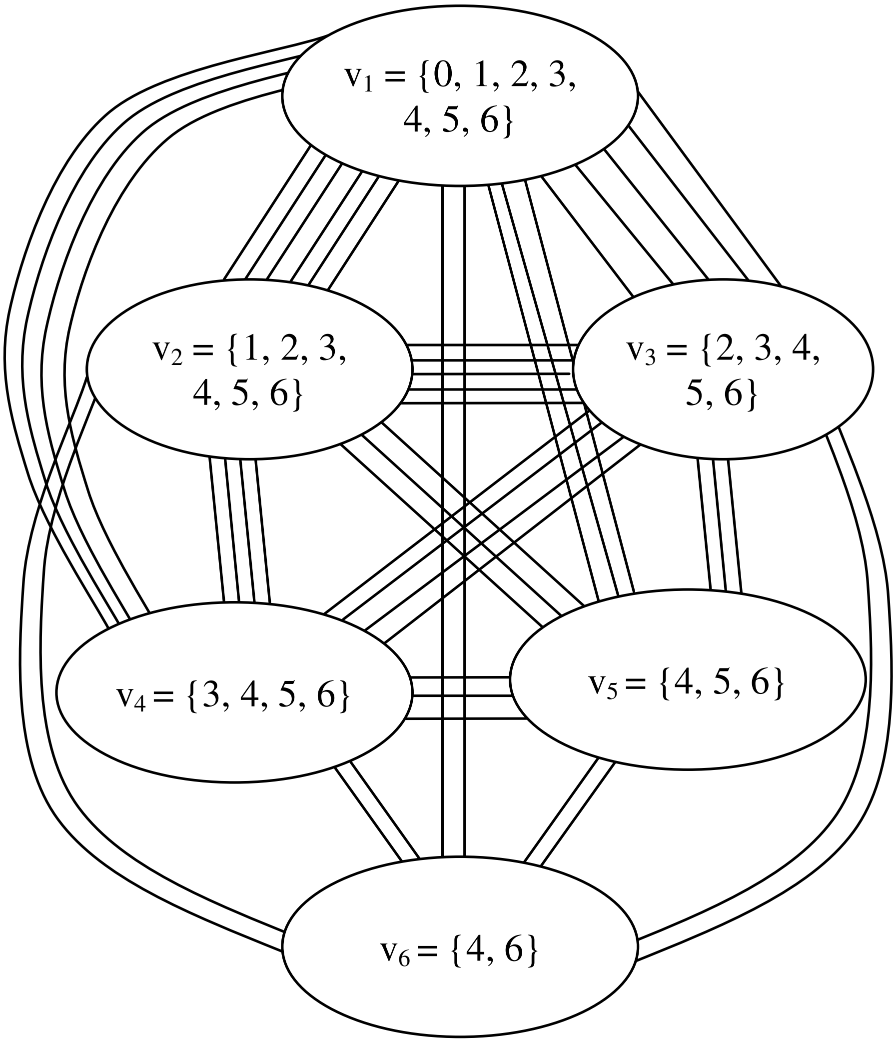


Figure 2.57

B is the subset vertex multigraph of type I. Clearly B is a pseudo complete multigraph.

Let C be the injective subset vertex multigraph of type II given by the following figure.

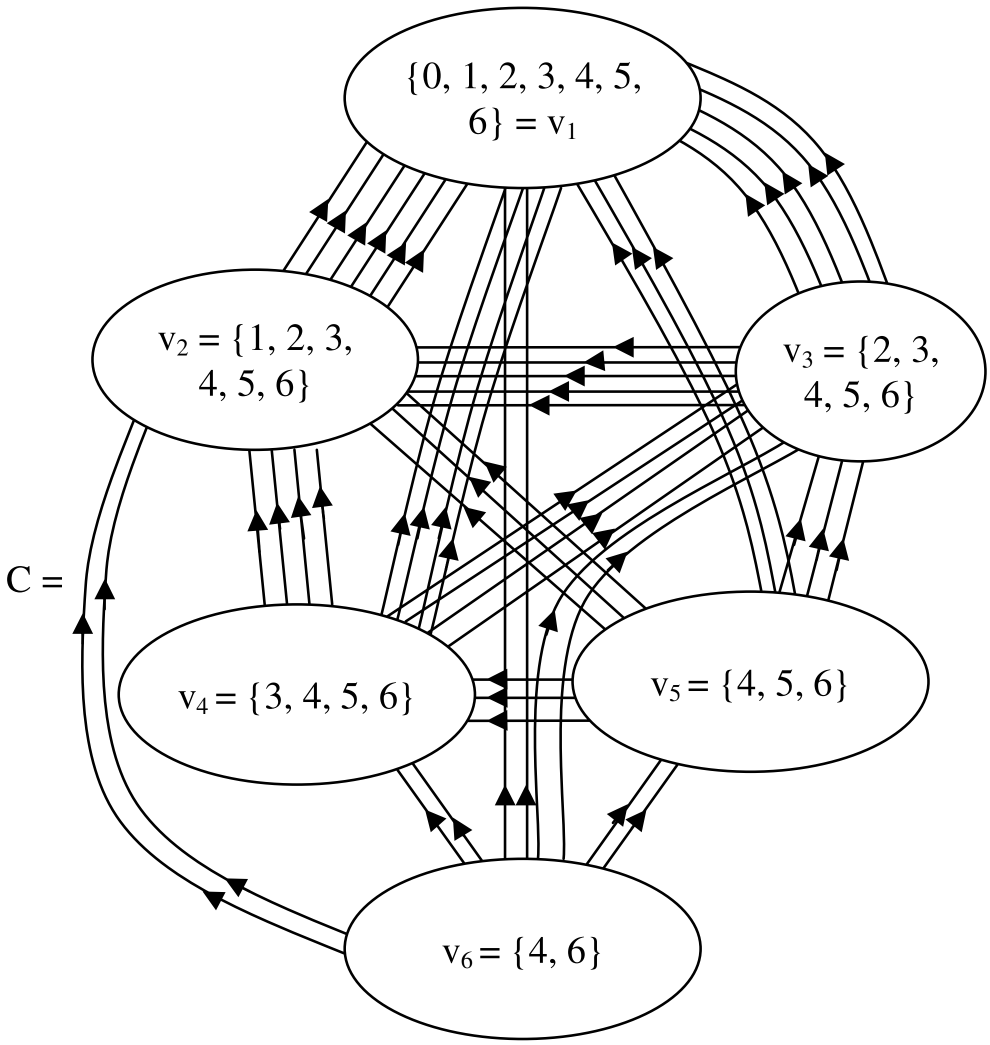


Figure 2.58

Clearly C is a pseudo complete subset vertex multigraph of type II. The only difference between C and D is that D is not directed but C is a directed one.

Now we will just study the subset vertex multisubgraphs of type II using examples.

Example 2.18. Let S = {Z_{18}} be the set of order 18. P(S) the power set of S. Let G be the subset vertex multigraph of type II given by the following figure.

G =

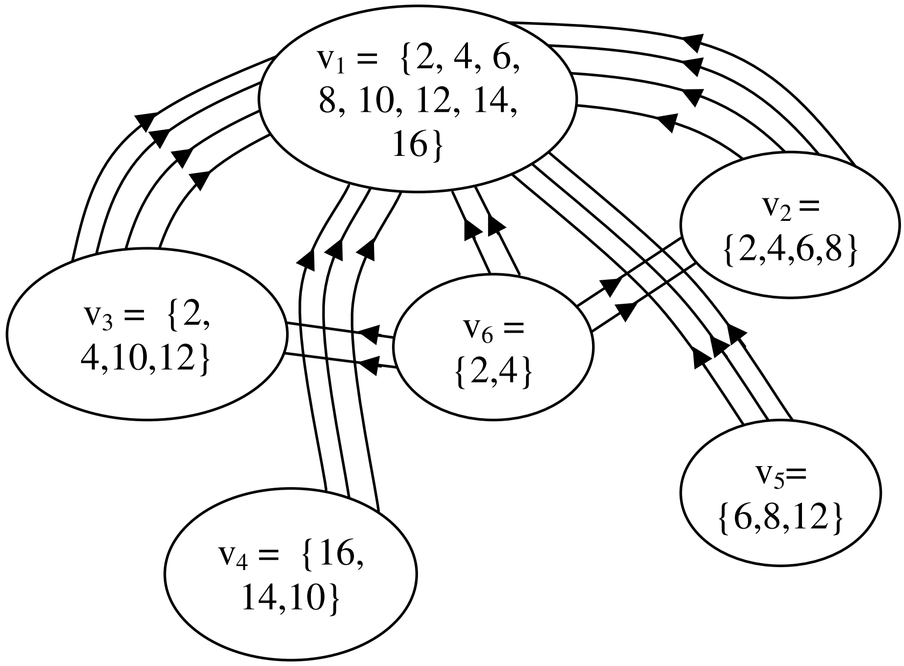


Figure 2.59

We see there exists 6 + $6C_2$ + $6C_3$ + $6C_4$ + $6C_5$ number of proper subset vertex multisubgraphs of type II which are directed, of course ordinary injective subset vertex multisubgraphs of G. Consider the vertex subset v_1, v_2, v_3, v_5 and v_6; let H_1 be the subset vertex multisubgraph of type II given in the following:

H_1 =

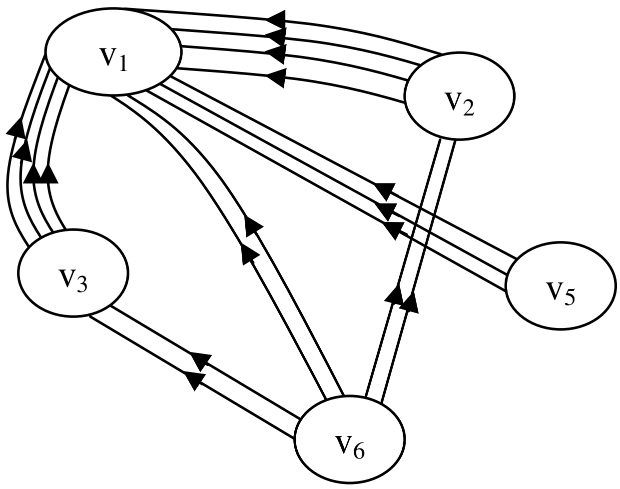


Figure 2.60

Now we describe H_2 the subset vertex multisubgraph of G in the following:

H_2 =

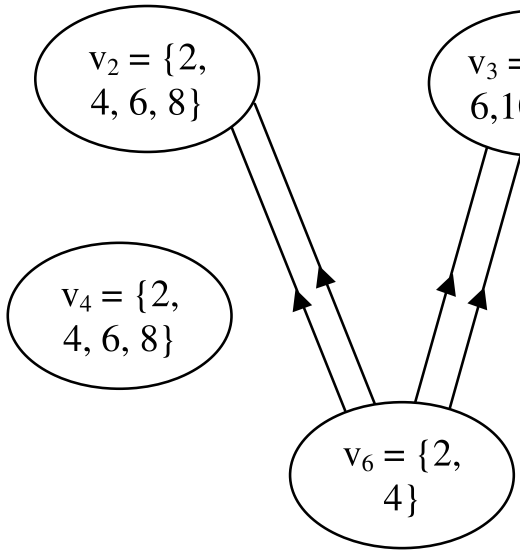


Figure 2.61

We see H_1 is a connected subset vertex multisubgraph and H_2 is a disconnected subset vertex multisubgraph of type II; which is the major difference between these two multisubgraphs H_1 and H_2. Now consider H_3 the subset vertex multisubgraph of G is given by the following figure:

H_3 =

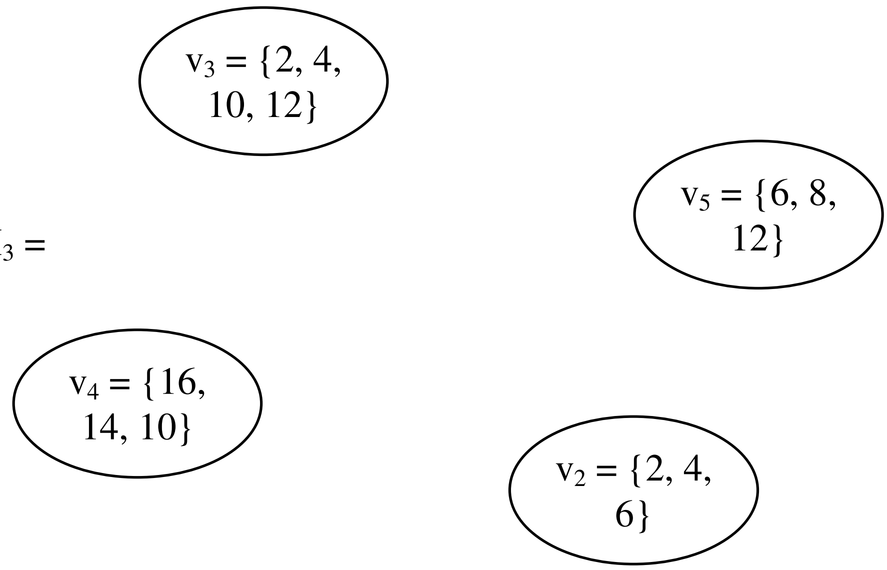


Figure 2.62

We see H_3 is only an empty subset vertex multisubgraph and in fact the biggest empty multisubgraph of G. Several related properties can be studied by any interested reader.

Next, we proceed onto describe by examples the notion of subset-subset vertex multisubgraphs. Let us consider the same subset vertex multigraph G.

Consider the subsets of vertex subsets u_1, u_2, u_3, u_4 and u_6 contained in v_1, v_2, v_3, v_4 and v_6 respectively. Let K_1 be the subset-subset vertex multisubgraph of G associated with the vertex subsets u_1, u_2, u_3, u_4 and u_6 given by the following figure:

K_1 =

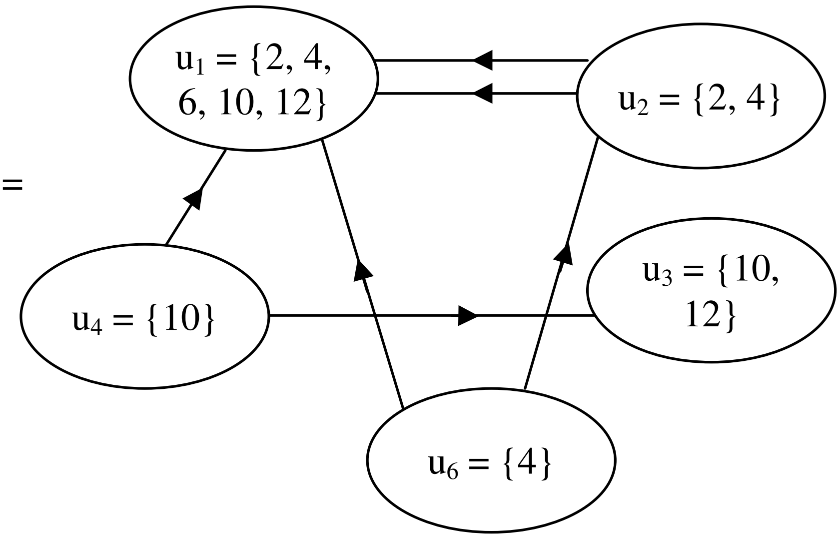


Figure 2.63

Let K_2 be a subset-subset vertex multisubgraph of G with subsets of vertex subsets s_1, s_3, s_4, s_5 contain in v_1, v_3, v_4 and v_5 respectively given by the following figure:

K_2 =

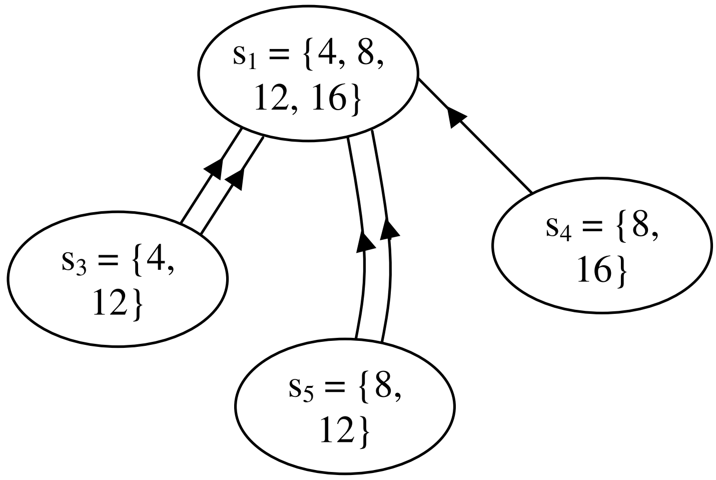


Figure 2.64

We see K_2 is a non uniform subset-subset vertex multisubgraph which a star multisubgraph of G.

We can have several such subset-subset vertex multisubgraphs of type II.

Infact finding the number of subset-subset vertex multisubgraphs of a given subset vertex multigraph happens to be a difficult problem.

We now give the definition of the same.

Definition 2.4. *Let S be any set. P(S) the power set of S. Let v_1, v_2, …, v_n be the vertex subset of a subset vertex multigraph G of type II. K is defined to be a subset-subset vertex multisubgraph of G if and only if*

i) *K has less than n vertices.*
ii) *The vertex subset of K is a collection of subsets of v_j for some $j \in \{1, 2, …, n\}$.*

We have subset-subset vertex multisubgraphs of G to be star graphs even if G is not one such.

We will provide some more examples of the same.

Example 2.19. Let S = $\{Z_{36}\}$ be the given set. P(S) the power set of S. Let G be the subset vertex multigraph given by the following figure:

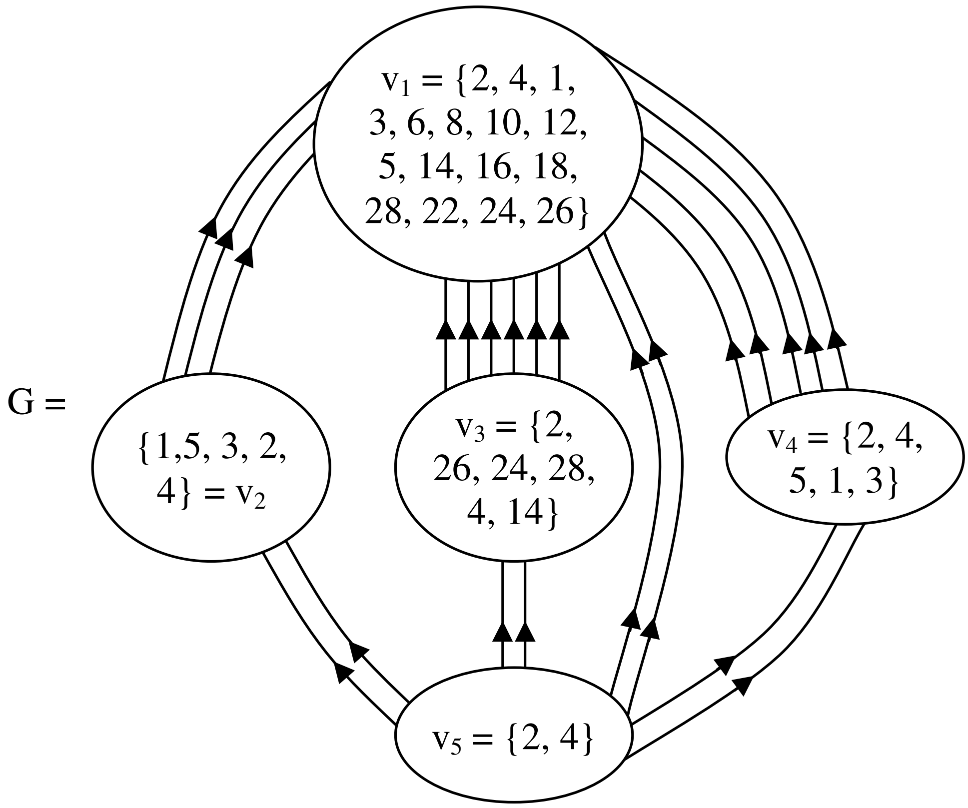


Figure 2.65

Let K_1 be the subset-subset vertex multisubgraph given by the following figure:

K_1 =

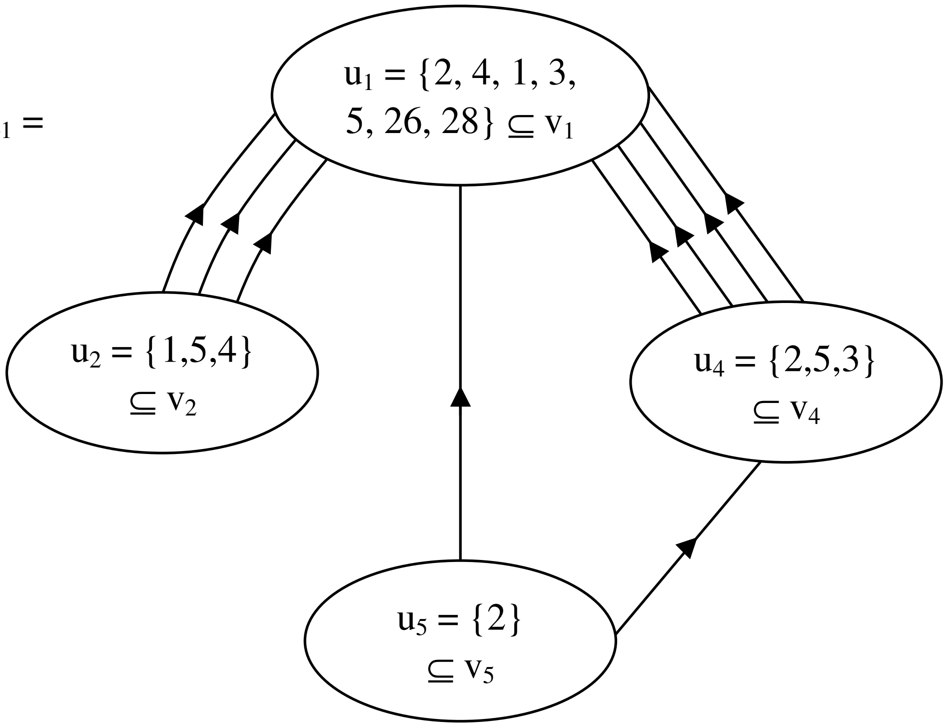


Figure 2.66

K_1 is not complete only a subset-subset vertex multisubgraph of G of type II.

Let K_2 be the subset-subset vertex multisubgraph of G given by the following figure:

K_2 =

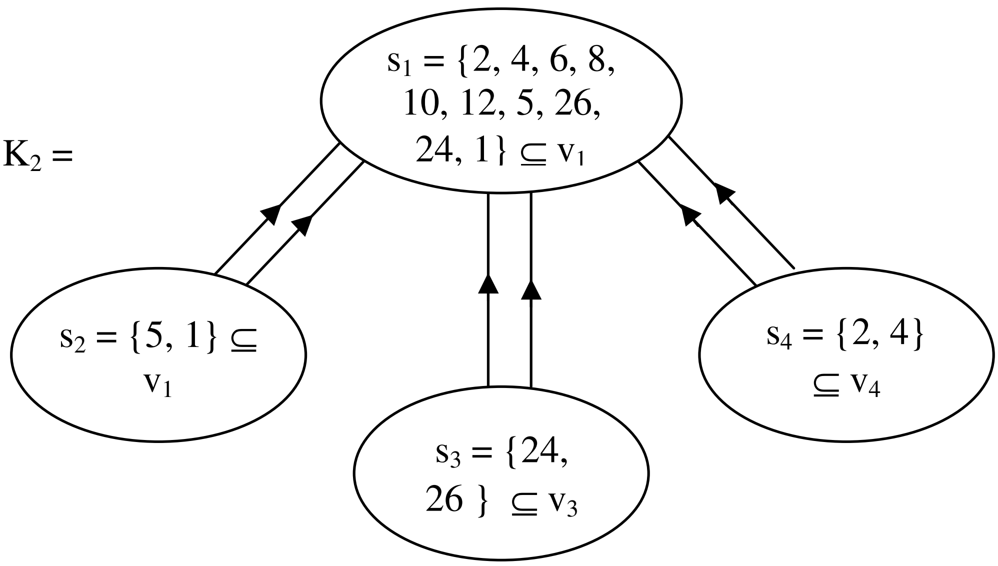


Figure 2.67

Clearly K_2 is a subset-subset vertex uniform star multisubgraph of G of type II.

Let K_3 be the subset-subset vertex multisubgraph of G given by the following figure:

K_3 =

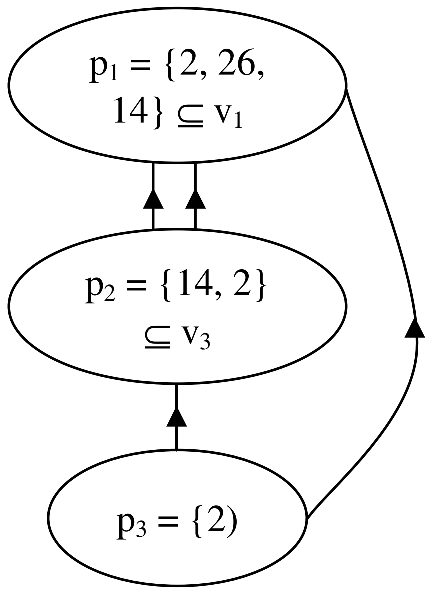


Figure 2.68

K_3 is a subset-subset vertex multisubgraph which is a pseudo triad of type II.

There are several subset-subset vertex multisubgraphs which are pseudo triads of type II.

Let K_4 be the subset-subset vertex multisubgraph given by the following figure:

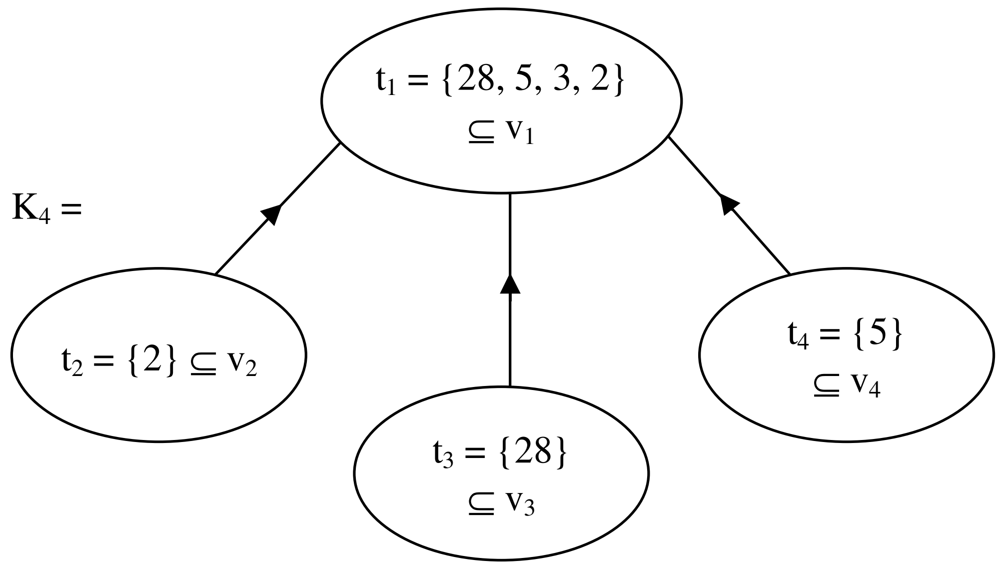


Figure 2.69

K_4 is again a subset-subset vertex multisubgraph which is a tree or a star graph.

Next we proceed onto describe for subset vertex multigraph G the universal complement of G by some examples.

Example 2.20. Let S = {Z_{18}} be a set of order 18. P(S) be the power set of S.

Suppose G be the subset vertex multigraph of type II with vertex set $v_1, v_2, \ldots, v_7$ given by the following figure:

G =

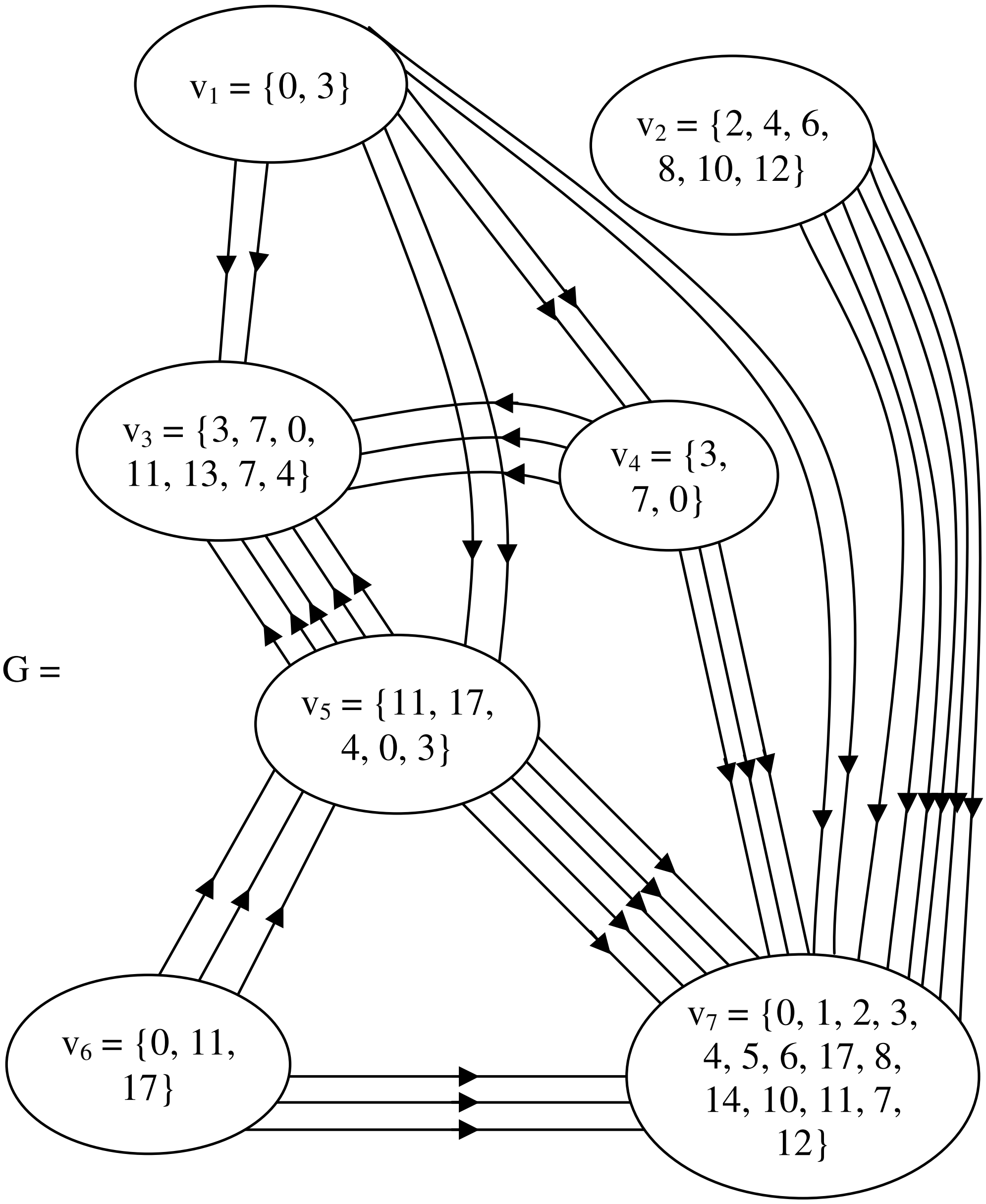


Figure 2.70

We now find the universal complement of G.

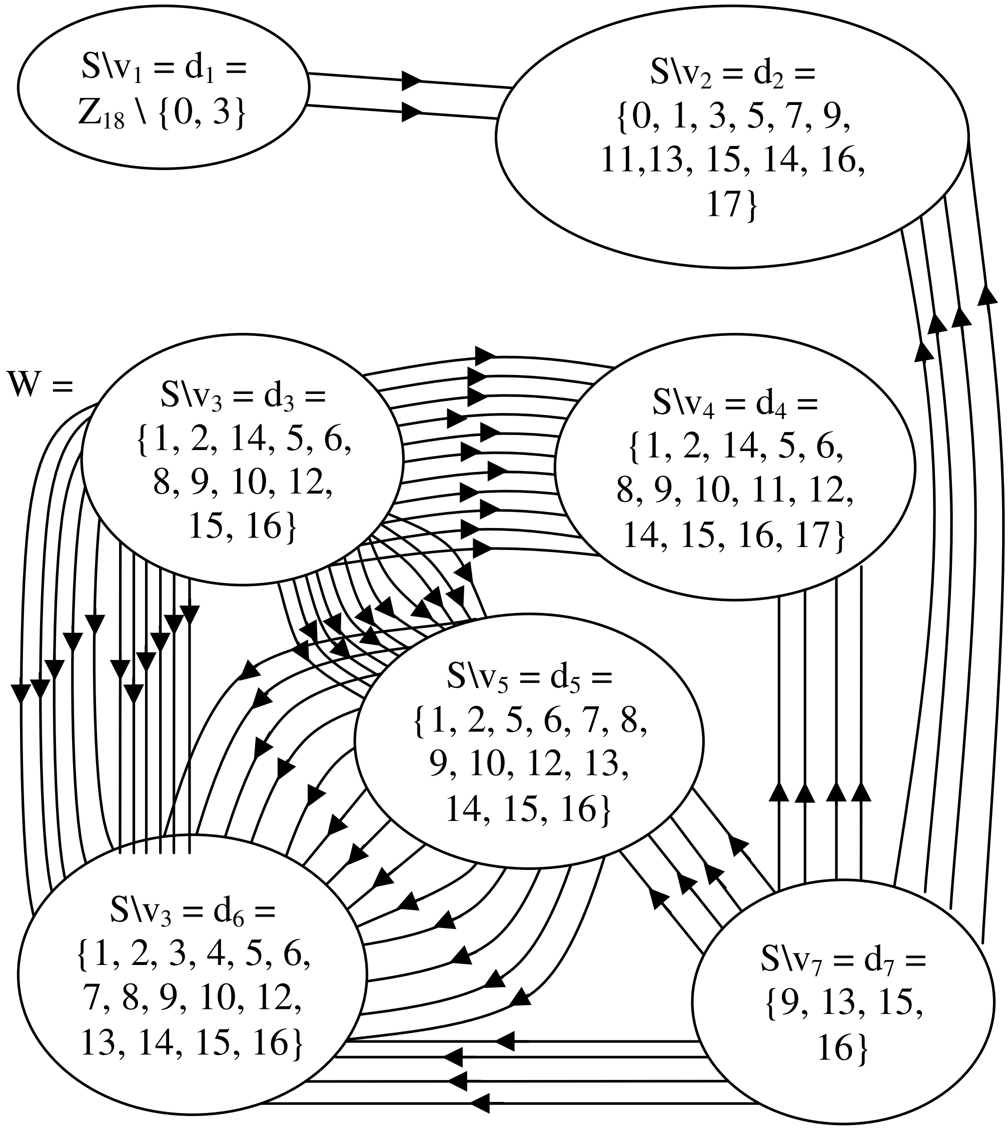


Figure 2.71

W is the universal complement of G. However the universal complement subset vertex multigraph is G and G are of different structures.

Example 2.21. Let S = {Z_{12}} be the set of order 12. P(S) be the power set of S. Let G be the subset vertex multigraph given by the following figure:

G =

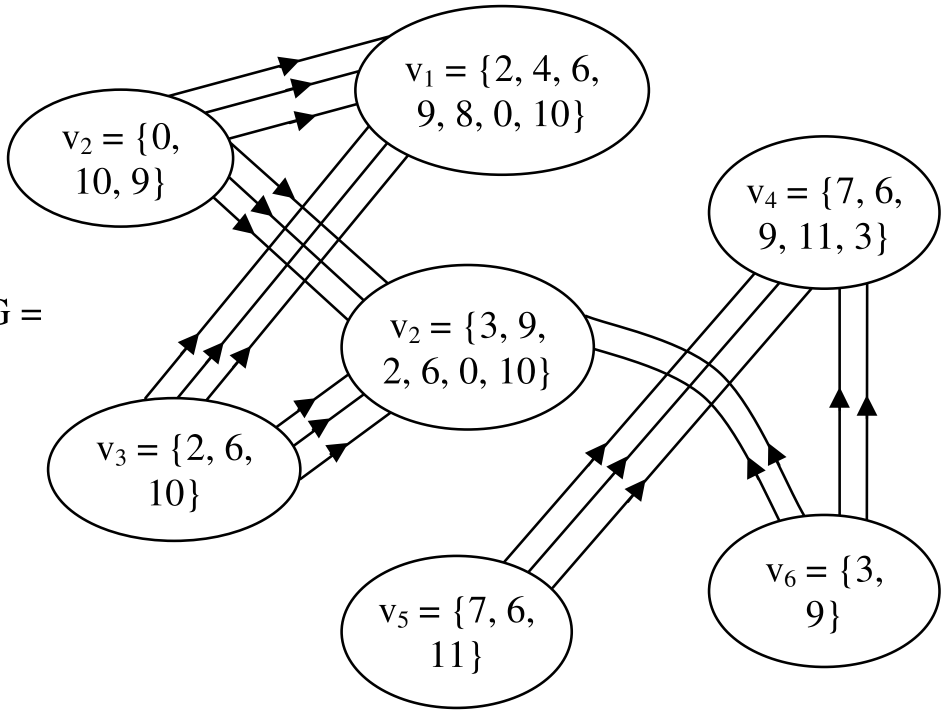


Figure 2.72

Let H be the universal complement of G given by the following figure.

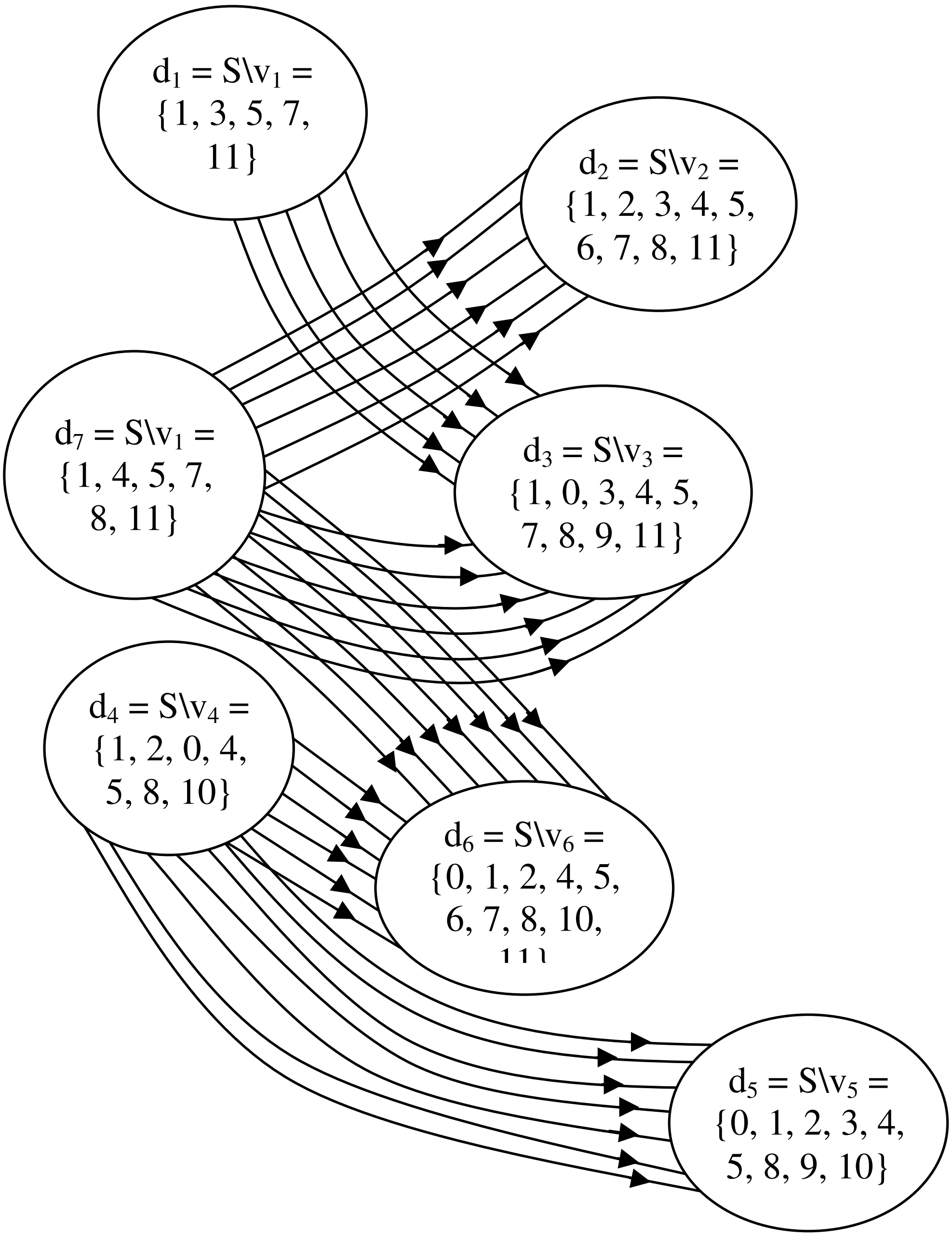


Figure 2.73

The universal complement of G has more edges connecting any two relevant subset vertices.

H the subset vertex multigraph is also connected. However the universal complement does not preserve structure.

We give one more example before we proceed onto describe the local complements of the subset-subset vertex multigraphs of the subset vertex multigraph G of type II.

Example 2.22. Le S = $\langle Z_4 \cup I\rangle$ be the set of order 16. P(S) the power set of S. Let G be any subset vertex multigraph with vertex set v_1, v_2, v_3, v_4, v_5 and v_6 whose figure is given in the following.

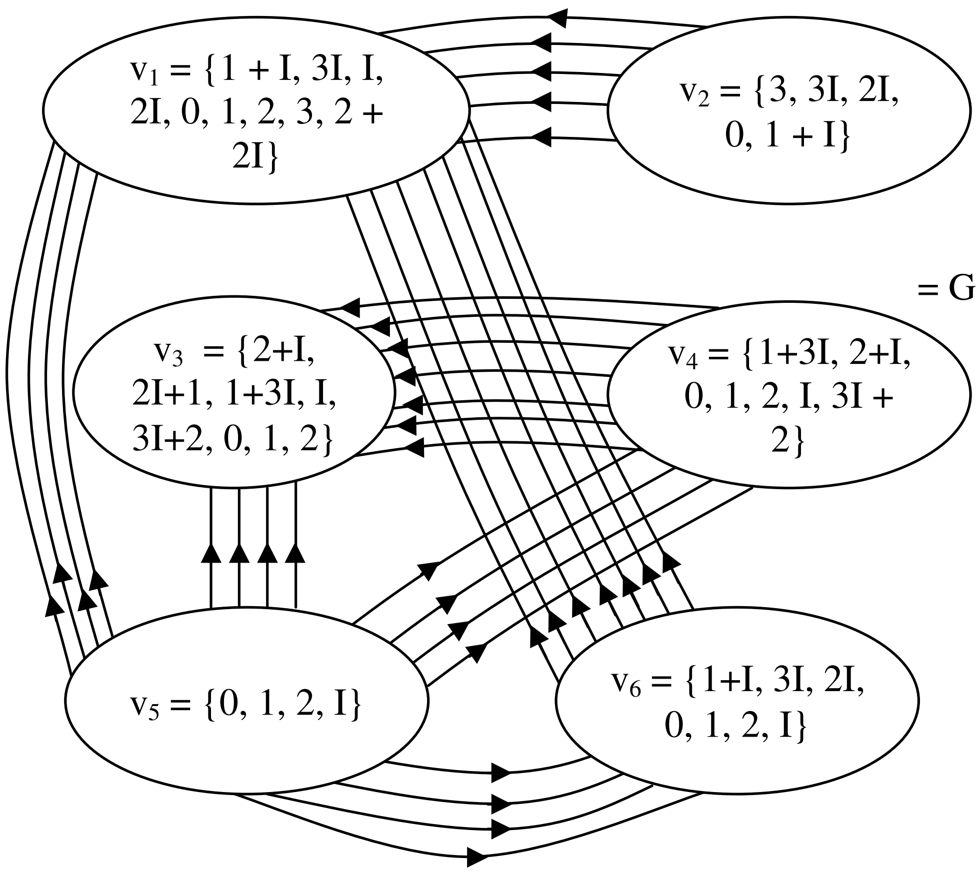


Figure 2.74

Now we find the universal complement of G. First we find the universal complement of the six vertex subsets v_1, v_2, v_3, v_4, v_5 and v_6,

$s_1 = S \setminus v_1 = \{2 + I, 3 + I, 1 + 2I, 3 + 2I, 1 + 3I, 2 + 3I, 3 + 3I\}$,

$s_2 = S\backslash v_2 = \{1, 2, I, 2 + I, 3 + I, 2 + 2I, 1 + 2I, 2 + 3I, 3 + 3I, 3I + 1, 3 + 2I\}$,

$s_3 = S\backslash v_3 = \{2I, 3, 3I, 1 + I, 3 + I, 3 + 3I, 2 + 2I, 3 + 2I\}$

$s_4 = S\backslash v_4 = \{3, 2I, 1 + I, 3 + I, 3I, 1 + 2I, 2 + 2I, 3 + 2I, 3 + 3I\}$,

$s_5 = S\backslash v_5 = \{2I, 3I, 3 + 1 + I, 2 + I, 3 + I, 2I + 1, 2I + , 2I + 3, 3I + 1, 3I + 2, 3I + 3\}$ and $s_4 = S\backslash v_6 = \{3, 2 + I, 3 + I, 1 + 2I, 3 + 2I, 1 + 3I, 2 + 2I, 2 + 3I, 3 + 3I\}$.

Clearly $s_1 \subseteq s_6$, $s_1 \subseteq s_2$, $s_4 \subseteq s_5$, $s_3 \subseteq s_5$, $s_1 \subseteq s_5$, $s_3 \subseteq s_4$, $s_6 \subseteq s_5$.

Let H be the universal complement of G given by the following figure.

H =

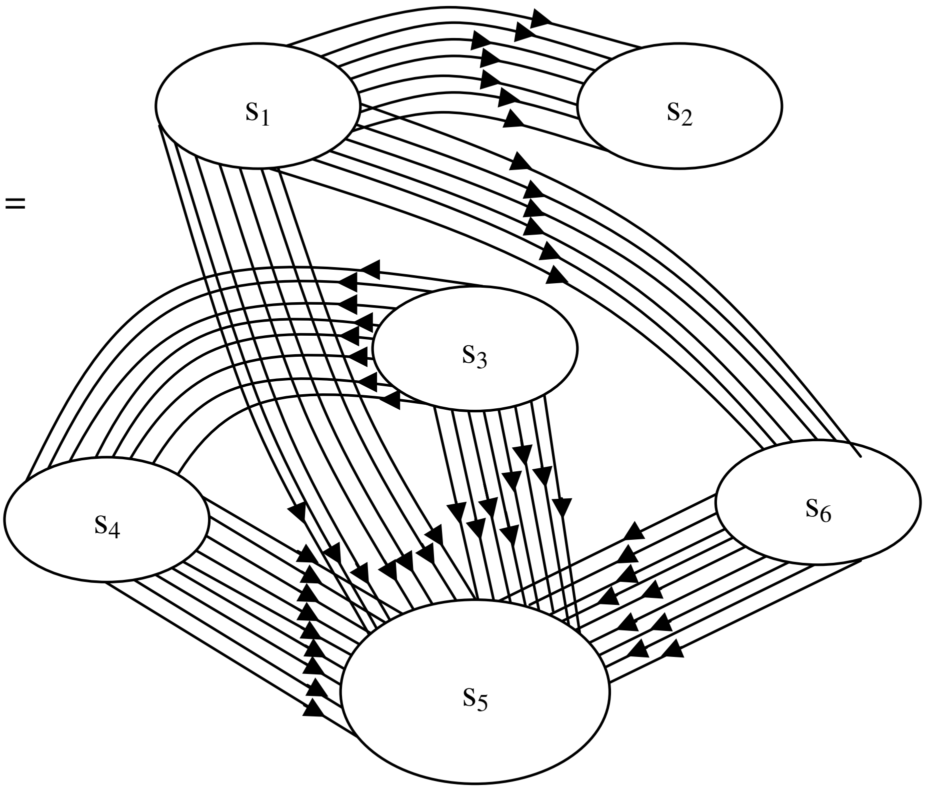


Figure 2.75

The universal complement subset vertex multigraph of G is not structure preserving.

We give yet another simple example.

Example 2.23. Let S = {a_1, a_2, …, a_9} be a set of order 9. P(S) be the powerset of S. Let G be the subset vertex multigraph given by the following figure.

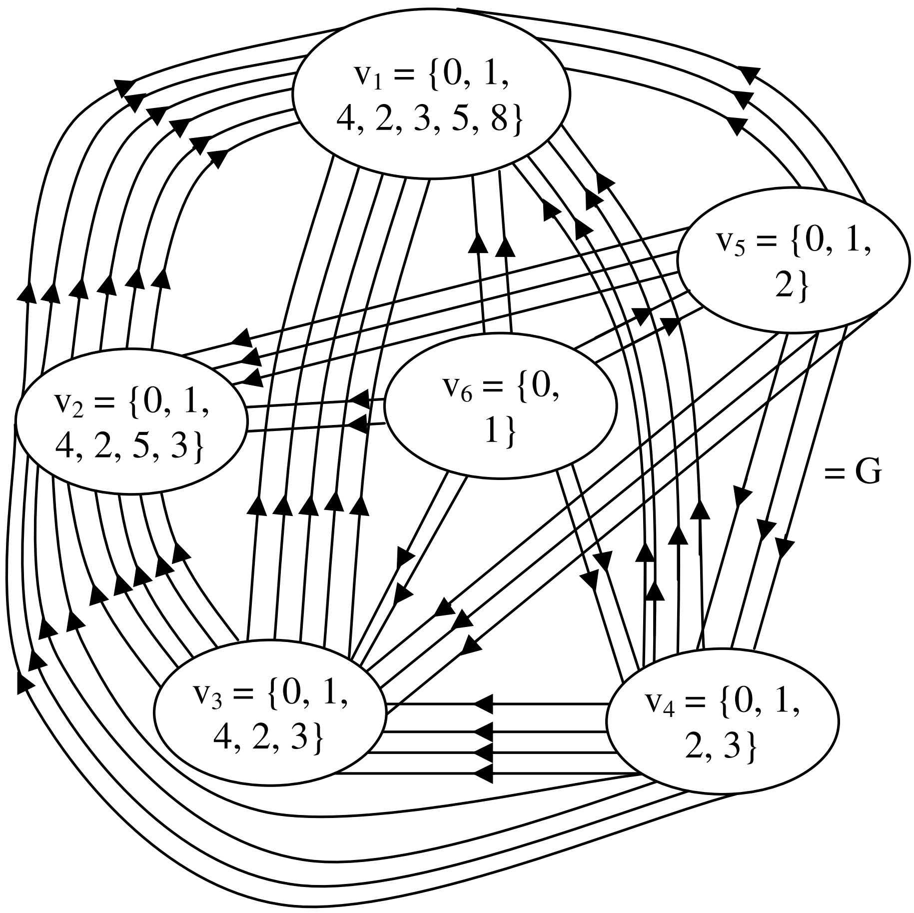


Figure 2.76

Clearly G is a pseudo complete subset vertex multigraph.

Now we find the local complement H of G for the multigraph H given below

H =

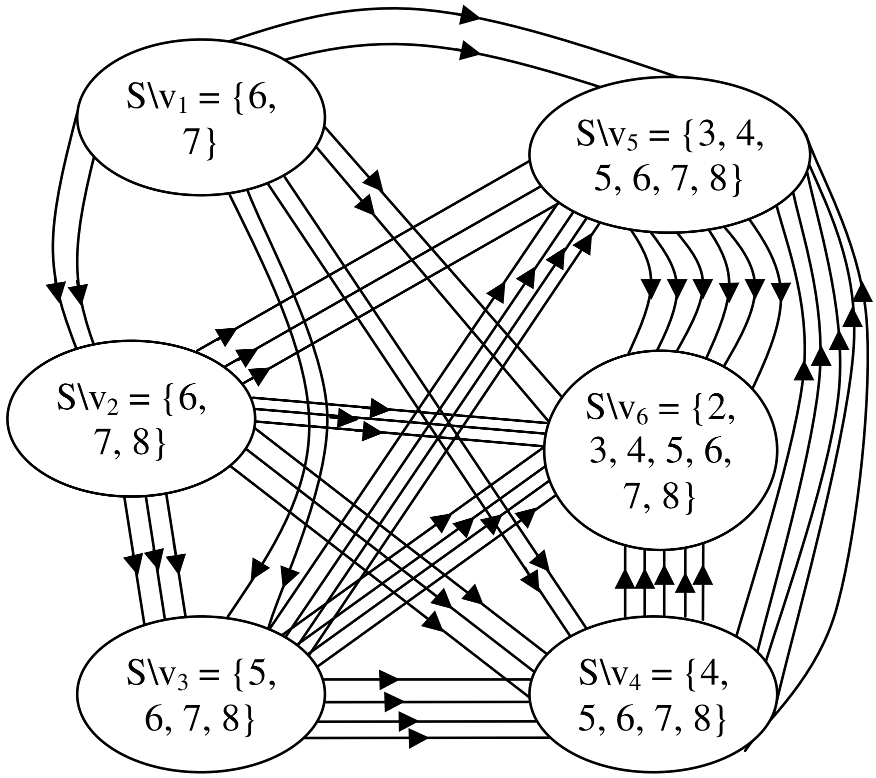


Figure 2.77

Clearly H is also a subset-subset vertex multigraph which is pseudo complete.

How we proceed onto describe by examples the notion of local complements of subset-subset vertex multigraphs of a subset-subset vertex multigraph H of the subset vertex multigraph.

Example 2.24. Let S = $\{Z_{12}\}$ be the set with 12 elements P(S) be the power set of S. Let G be the subset vertex multigraph given by the following figure.

G =

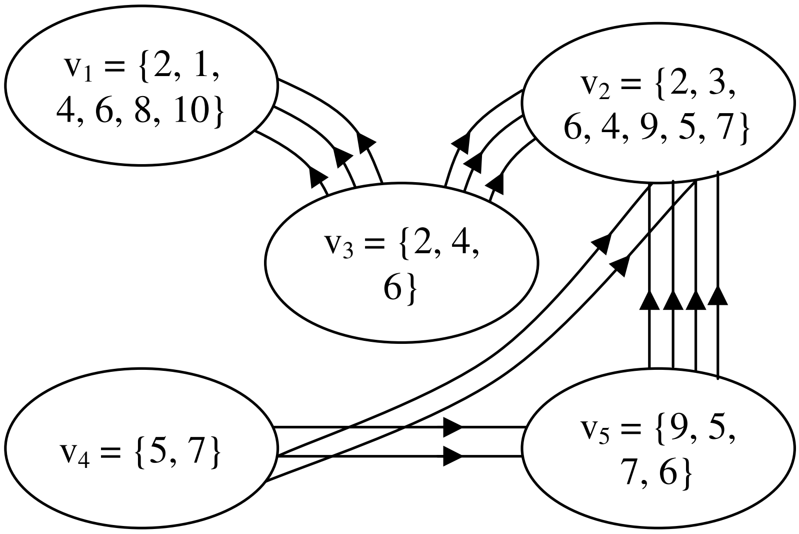


Figure 2.78

Let H be the subset-subset vertex multisubgraph of G given by the following figure.

H =

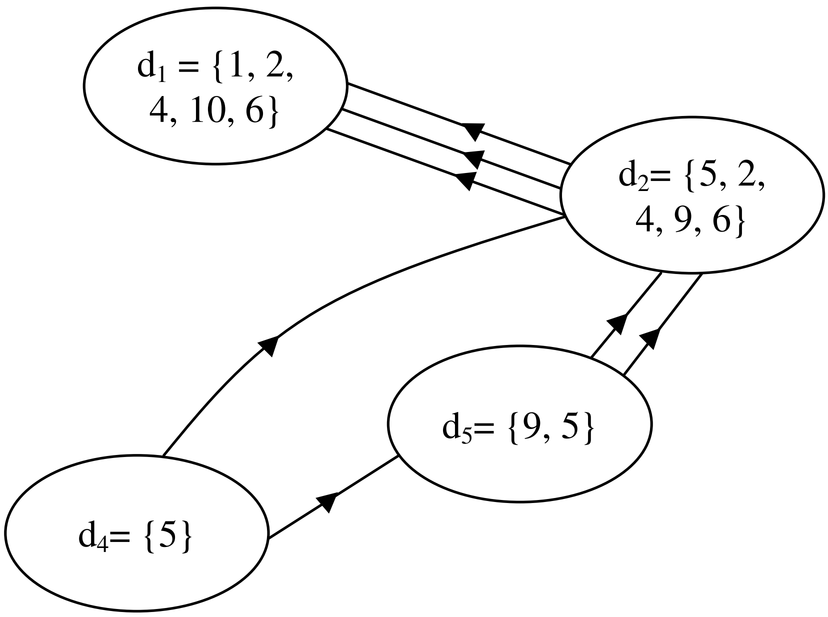


Figure 2.79

The local vertex subset complements of d_1, d_2, d_4 and d_5 relative to v_1, v_2, v_4 and v_5 respectively be s_1, s_2, s_4 and s_5 where $s_1 = v_1 \setminus d_1 = \{8\}$, $s_2 = v_2 \setminus d_2 = \{7, 3\}$, $s_4 = v_4 \setminus d_4 = \{7\}$ and $s_5 = v_5 \setminus d_5 = \{6, 7\}$.

Let K be the local complement subset-subset vertex multisubgraph of H relative to G given by the following figure.

K =

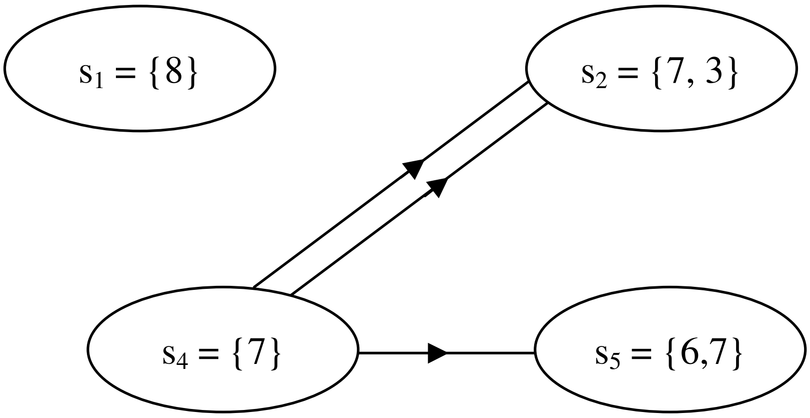


Figure 2.80

Clearly K and H have different structures.

Let H_2 be another subset-subset vertex multisubgraph of G given by the following figure.

H_2 =

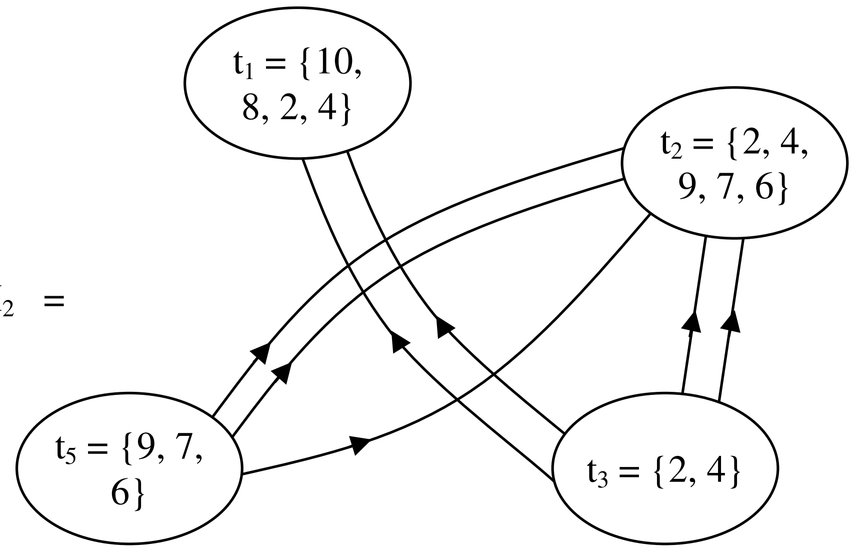


Figure 2.81

The local complement subset-subset vertex multisubgraph of H_2 be T given by the following figure :

T =

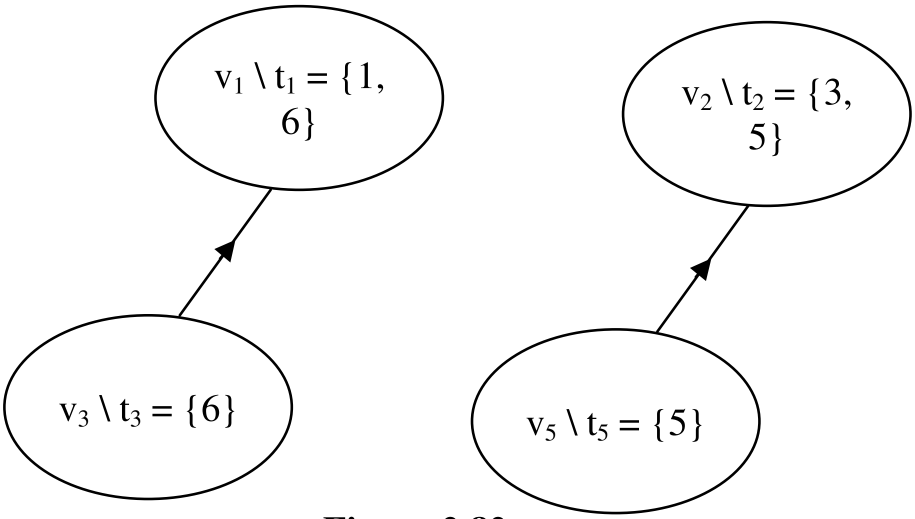


Figure 2.82

We see H_2 is a connected subset-subset vertex multisubgraph however its local complement T is disconnected.

We now give some more examples of local complements of subset-subset vertex multisubgraphs.

Example 2.25. Let S = {a_1, a_2, a_3, a_4, a_5, a_6, a_7, …, a_{15}} = Z_{15} be a set of order 15. P(S) be the power set of S. Let G be the subset vertex multigraph given by the following figure.

G =

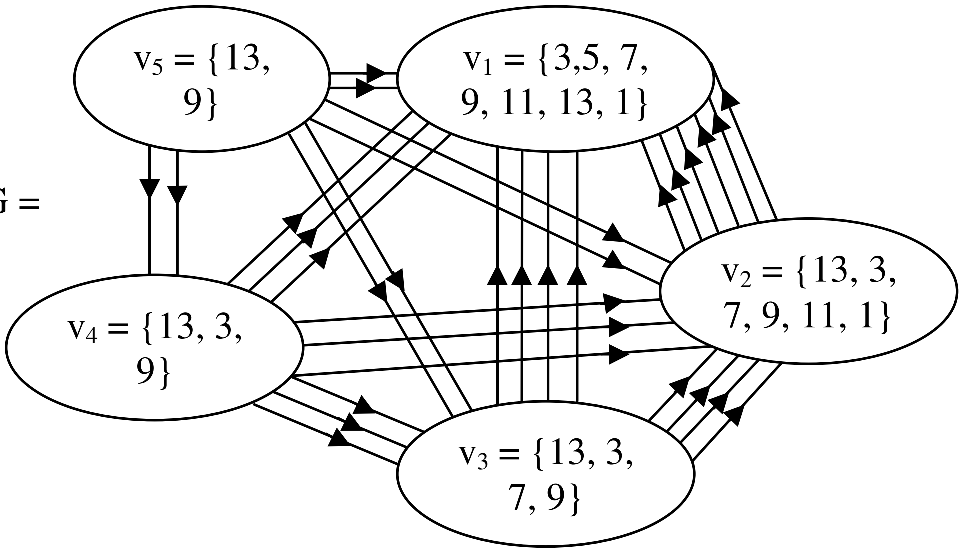


Figure 2.83

Clearly G is a pseudo complete subset vertex multigraph of type II.

Let H be the universal complement of G given by the following figure.

H =

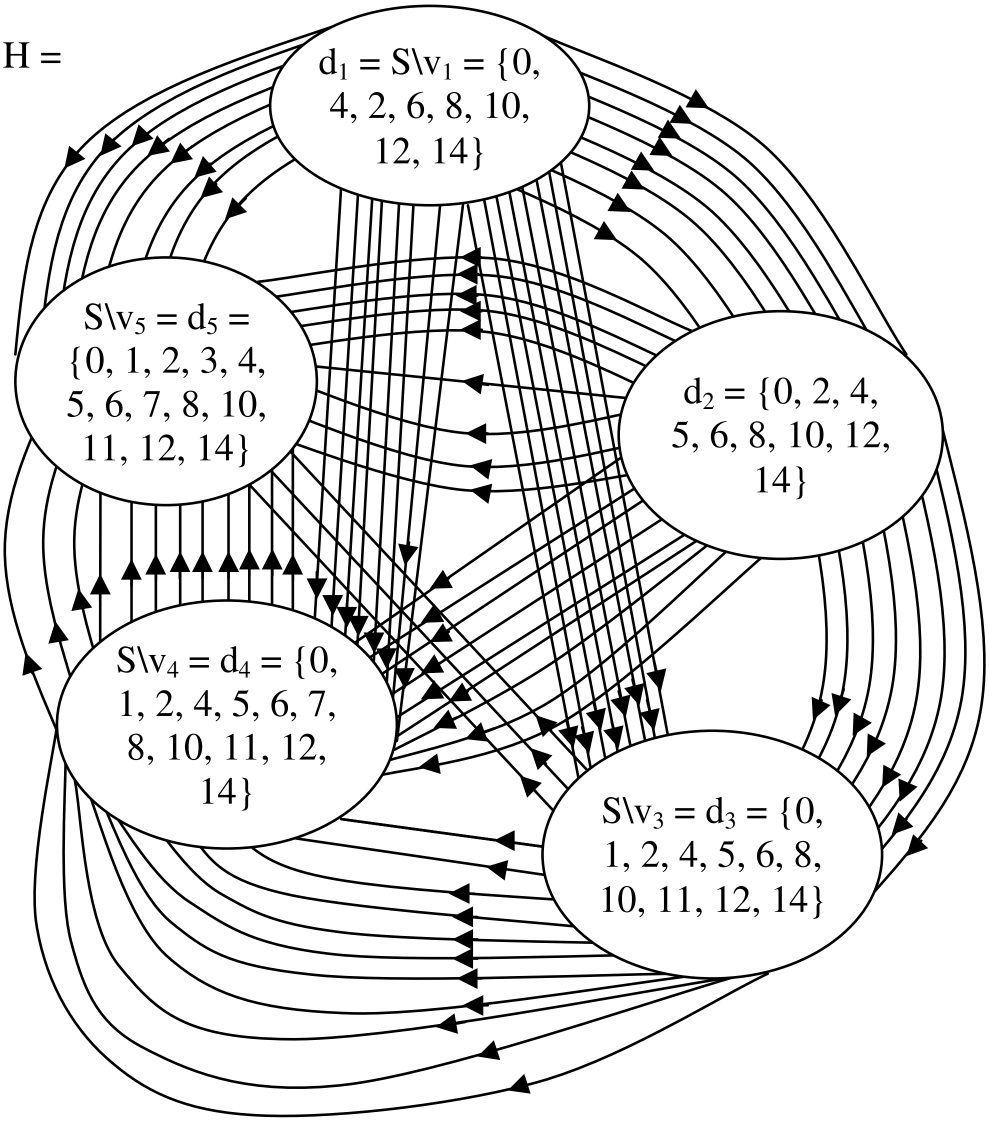


Figure 2.84

H is a pseudo complete subset vertex multigraph. Clearly H has more multiedges than G.

Now we find a few subset-subset vertex multisubgraphs of G.

Let K_1 be a subset-subset vertex multigraph of G given by the following figure.

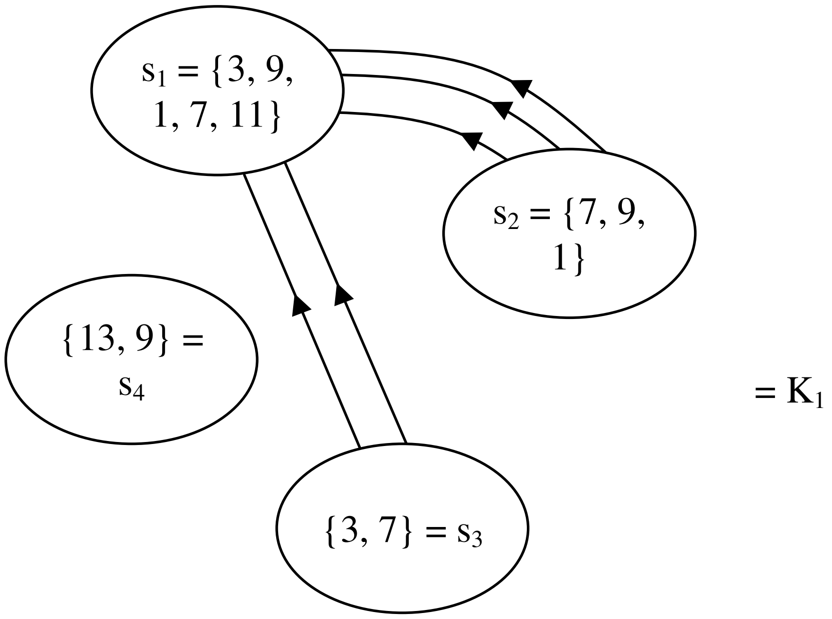


Figure 2.85

Clearly K_1 is a disjoint subset-subset vertex multisubgraph of G.

Now we find the local complement of K_1 relative to G.

Let P_1 be the local complement of K_1 relative to G given by the following figure.

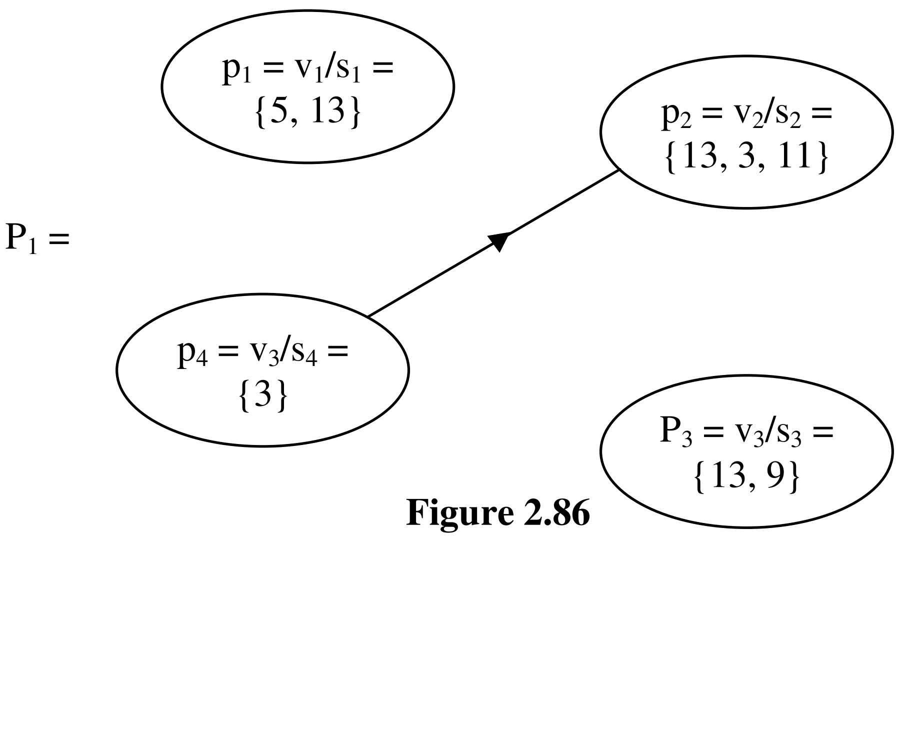


Figure 2.86

P_1 is disconnected also K_1 is the disconnected one. We see P_1 is the local complement of K_1 and vice versa relative to G.

Let K_2 be a subset-subset vertex multisubgraph of G given by the following figure;

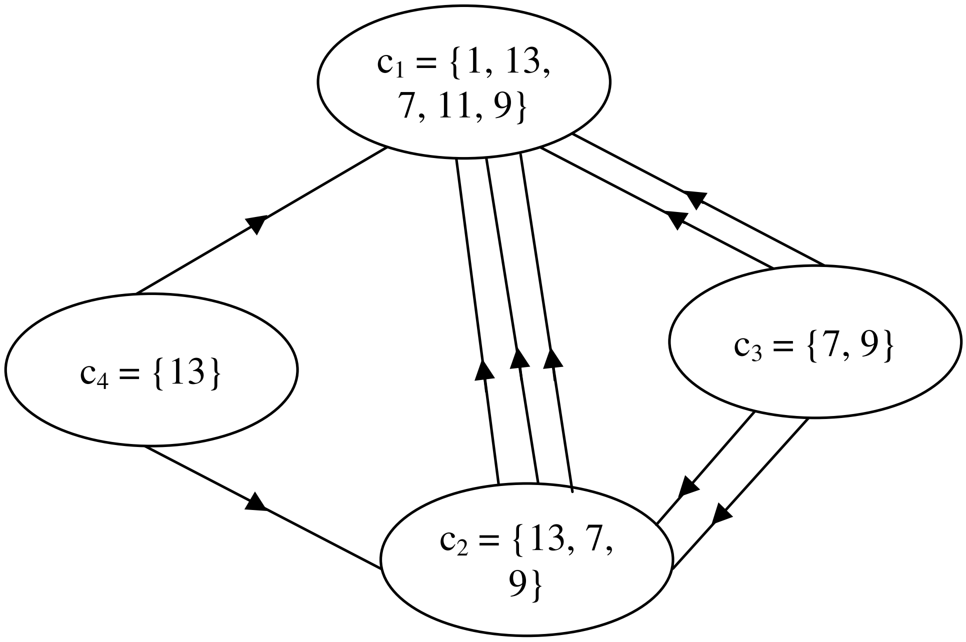


Figure 2.87

K_2 is a connected subset-subset vertex multisubgraph of G. Now we find the local complement P_2 of K_2 relative to G.

P_2 =

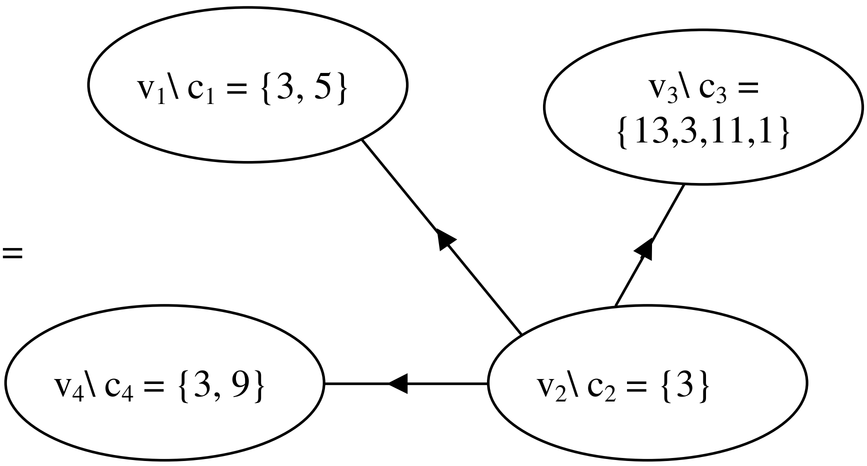


Figure 2.88

We see P_2 the local complement of K_2 relative to G is star graph which is not a multistar graph.

Next, we proceed onto briefly describe the ordinary projective subset vertex multigraph of type II by some examples.

Example 2.26. Let S = $\{Z_{27}\}$ be a set of order 27. P(S) the powerset of S.

Let G be the projective ordinary subset vertex multigraph of type II given by the following figure with vertex set from P(S).

G =

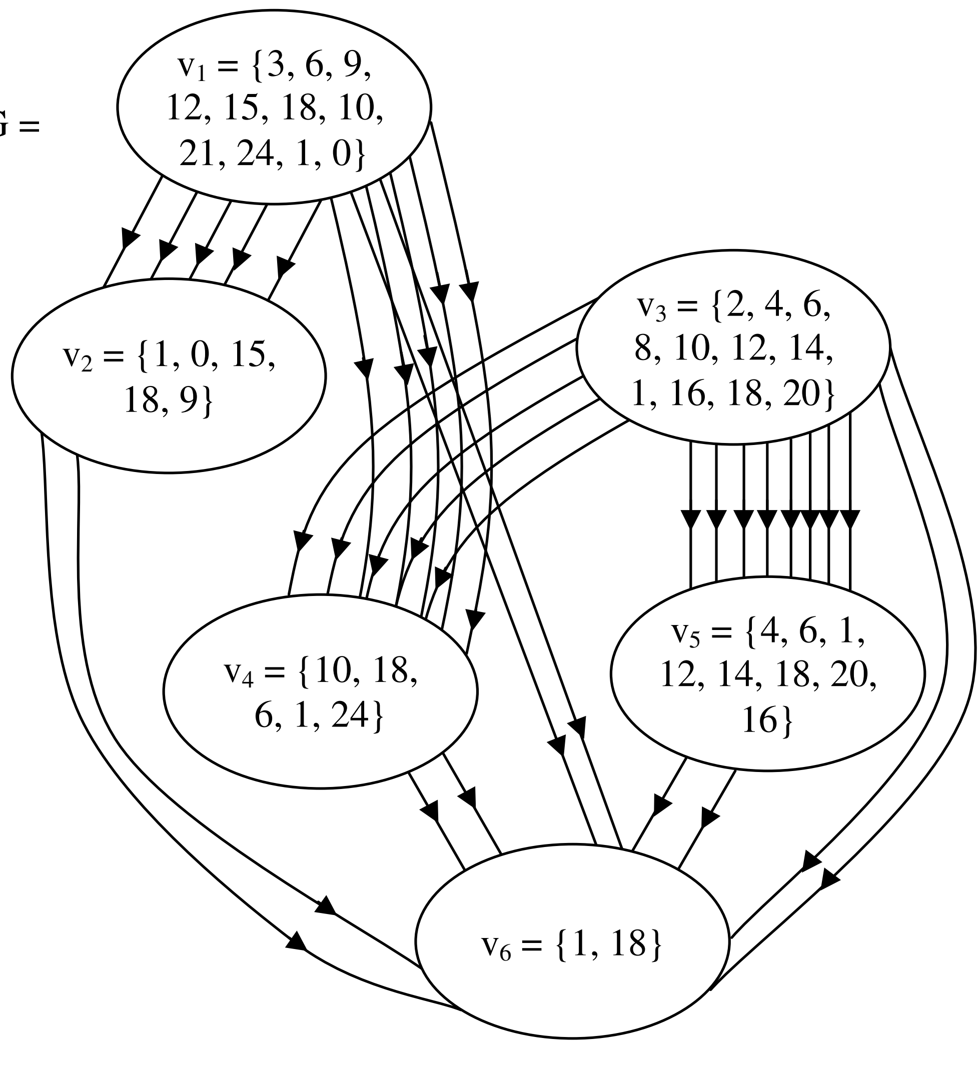


Figure 2.89

Now it is pertinent to keep on record that G is projective however if we consider for the same set of vertex subsets v_1, v_2, …, v_6 and get the ordinary subset vertex injective multigraph H we see G and H are similar except for the direction of the multiedges. Even the number of edges is the same only direction is just opposite.

Thus, we do not indulge in elaborately discussing the properties of ordinary projective subset vertex multigraphs of type II as they are like injective subset vertex multigraph only the direction is changed.

We just present a few more examples of finding universal complement and local complements of ordinary subset vertex multigraph of type II which are projective.

Example 2.27: Let S = {Z_{12}} be the set of order 12. P(S) the power set of S. Let G be the ordinary projective subset vertex multigraph given by the following figure.

G =

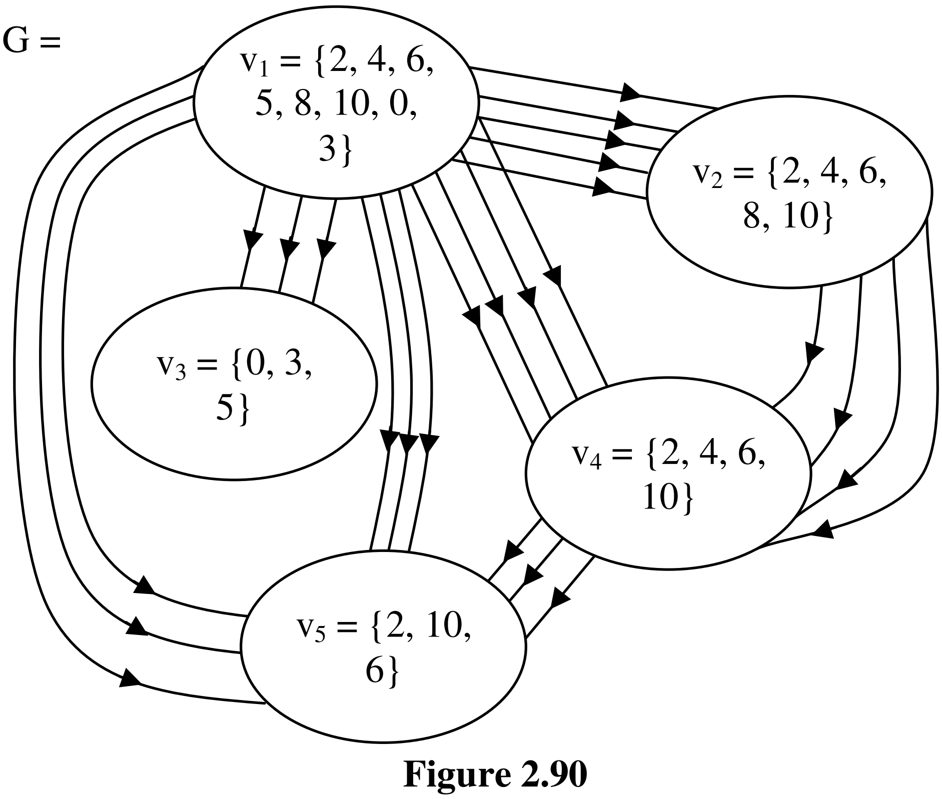


Figure 2.90

Now we find the universal complement of G. Let H be the projective subset vertex multigraph of type II which is the universal complement of G given by the following figure.

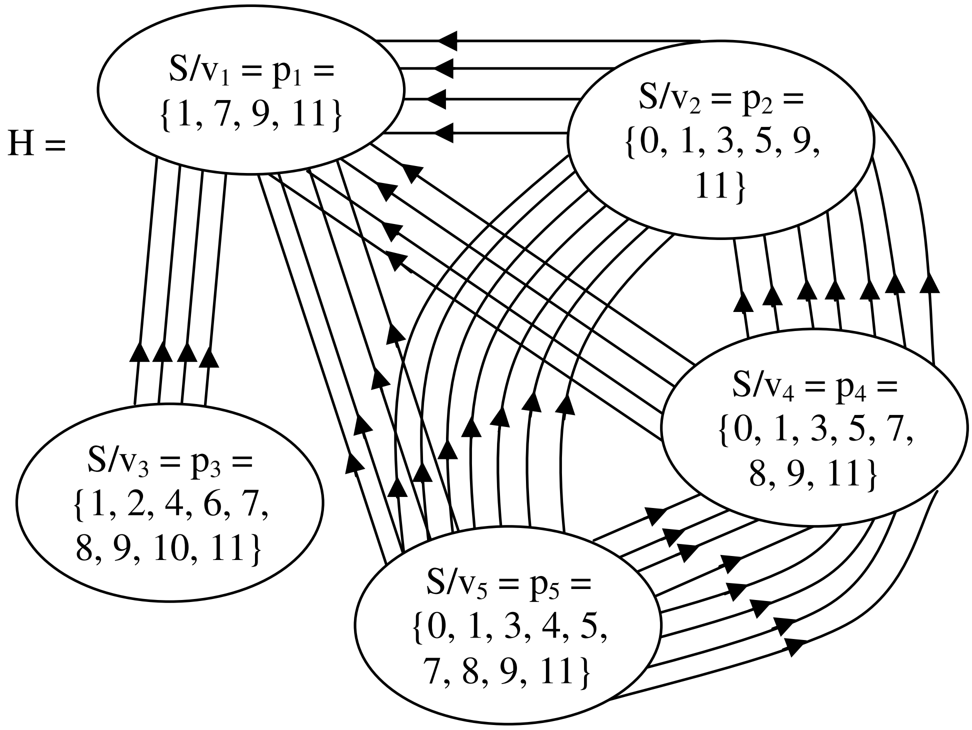


Figure 2.91

Clearly in this case the universal complement is structure preserving. However, H has more multiedges. Let P be the subset-subset vertex multigraph of G given by the following figure.

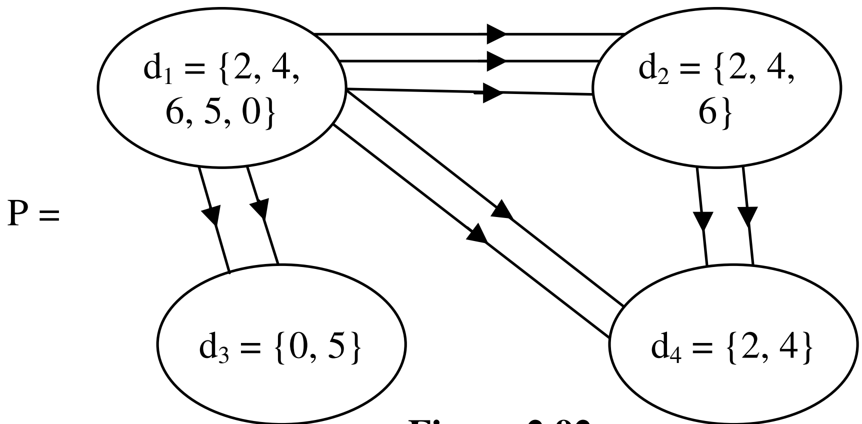


Figure 2.92

Now we find the local complement L of P relative to G which is given by the following figure.

L =

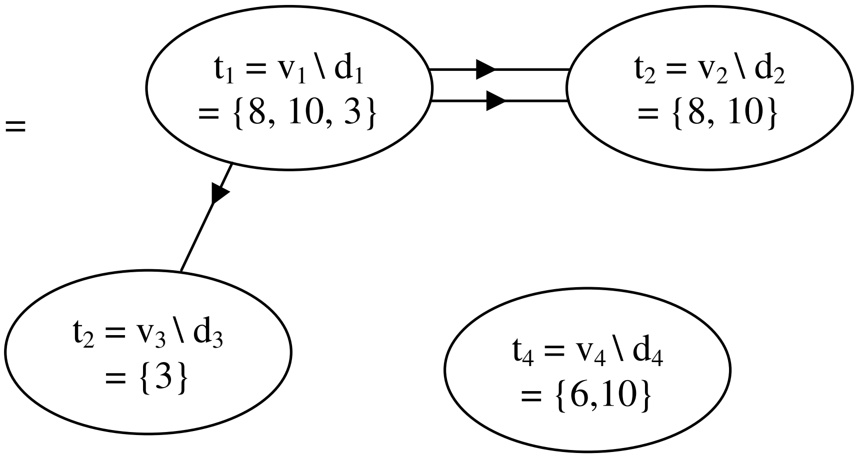


Figure 2.93

We see P is a connected subset-subset vertex multisubgraph whereas L is not a connected subset-subset vertex multisubgraph which is the local complement of P.

Thus, in this case, they do not in general inherit the structure.

This property is also true in case of injective subset-subset vertex multisubgraphs also.

However, when we just use the vertex subset of P and L and built the corresponding subset-subset vertex multisubgraph of type I we have the following structure.

Let D denote the subset-subset vertex multisubgraph of type I using the vertex subsets d_1, d_2, d_3 and d_4 which is given by the following figure.

D =

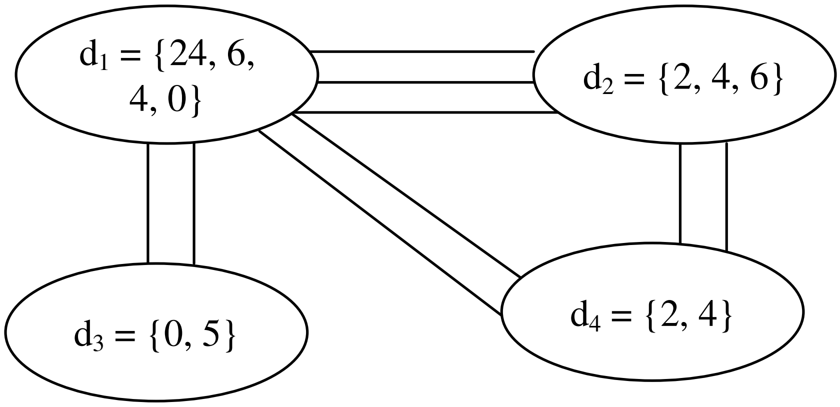


Figure 2.94

We see except for the direction of edges D and P have the same structure.

Now the subset-subset vertex multisubgraph of type I M using t_1, t_2, t_3 and t_4 is described below.

M =

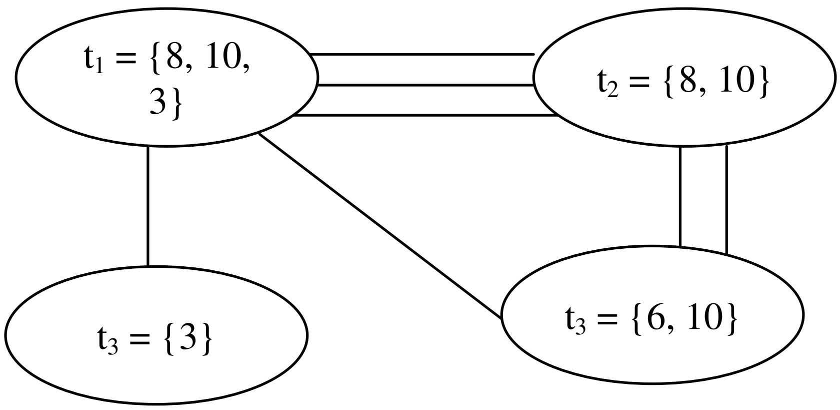


Figure 2.95

Clearly L is disconnected however M is connected so the local complements of type I and type II multigraphs in general do not behave alike or to be precise may not preserve structure.

Let W be the subset-subset vertex multisubgraph of type II given by the following figure.

W =

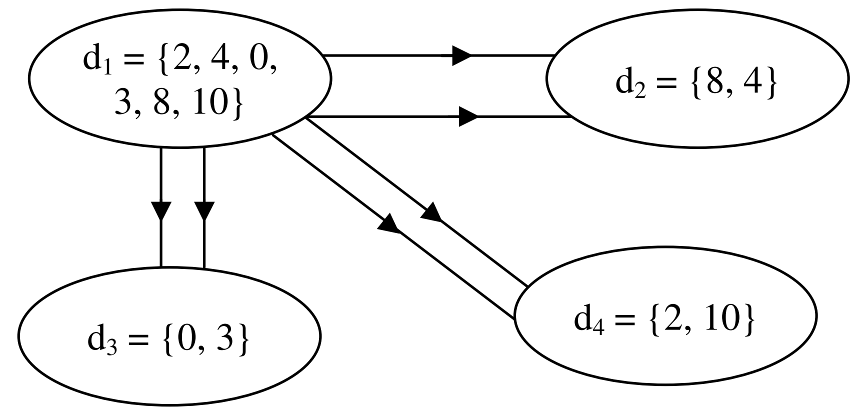


Figure 2.96

Clearly W is a subset-subset vertex multistar subgraph of G. Now we find the local complement of W relative to G. Let M denote the subset-subset vertex multisubgraph of type II which is the local complement of W given by the following figure.

M =

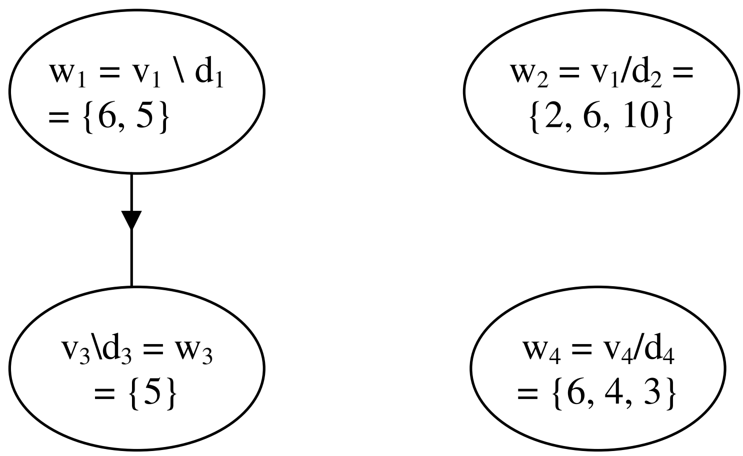


Figure 2.97

We see W is a star subset-subset vertex multisubgraph of type II where as M the local complement is a disconnected one which is not a star subset-subset vertex multisubgraph of type II.

Now one of the main roles played by these subset-subset vertex multisubgraphs of type I or type II is in social information networks. We see if we have some n social groups and they be represented by subsets vertices of the multi graph and suppose it so happens a few of the group perish and also some of the characteristic or features with these social groups die away with the societal development then in that case to describe them as network the subset-subset vertex multisubgraphs of type I (or type II) will be handy. So these subset vertex multigraphs can be best suited to depict a social network.

We will illustrate this situation by some examples.

Example 2.28. Suppose there are some n social groups which enjoyed some / several common features from the set of characteristics / properties features represented by M = {a_1, a_2, …, a_f} where f is a large number much bigger than n. Let the n social groups be denoted by S_1, S_2, …, S_n.

These S_i take fixed subsets from the power set P(M). Suppose there is a subset vertex multigraph of type II associated with these n socials groups given by the following multigraph S.

S=

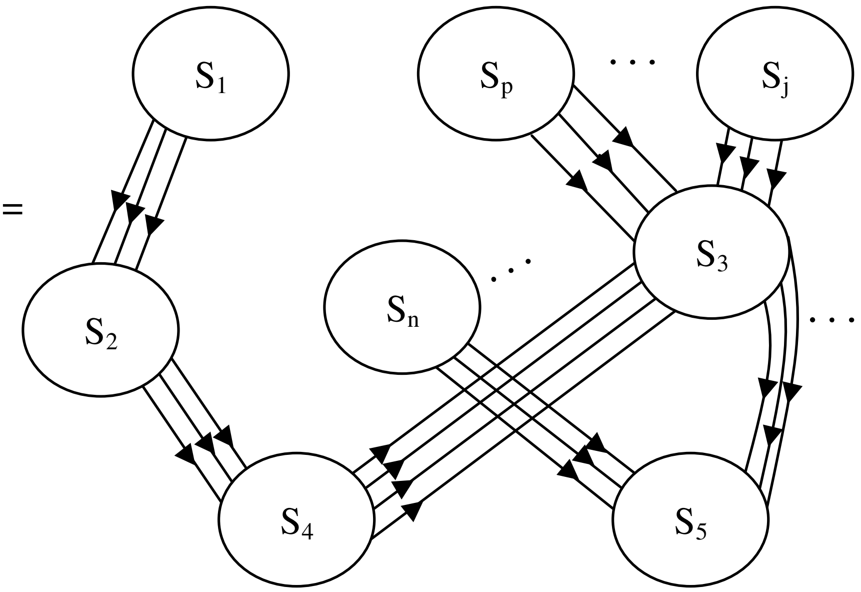


Figure 2.98

Suppose in due course of many years say some r – number of social groups have persisted and the some of the features or characteristics associated with these social groups have died. Then we get a subset-subset vertex multisubgraph R with (n – r) subset vertices and these n – r subset vertices are only subsets of these n – r subset vertices.

Now to depict the existing present social information network we can use the subset-subset vertex multisubgraph R of type II and study the relations among them.

To be more specific we will give a non abstract example of the same.

Example 2.29. Let there be some 9 social groups denoted by S_1, S_2, …, S_9. Let the features or properties or characteristics enjoyed by these 9 social groups be denoted by M = {a_1, a_2, …, a_{45}}. Let P(M) be the power set of M. These 9 social groups can be and are described by subsets of P(M).

So let $S_1, \ldots, S_9$ denote the subsets of P(M). The subset vertex multigraph S with $S_1, S_2, \ldots, S_9$ as vertex subsets is given by the following figure.

S =

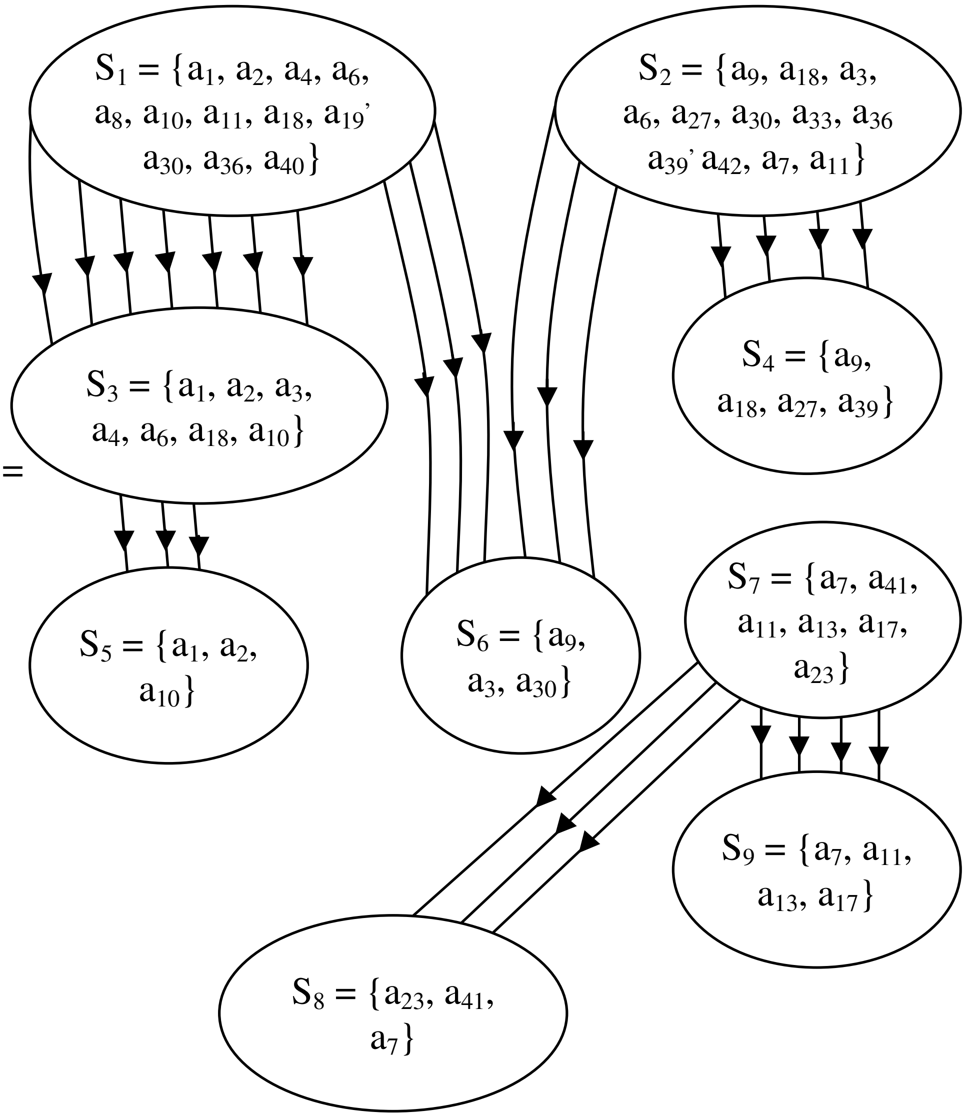


Figure 2.99

S is a disconnected subset - vertex multigraph of type II. However, the following observations among the 9 social groups are mandatory. Though these 9 social groups form a disconnected multigraph of type II they have some interconnecting features for S_7 feature is present in S_9 and S_8, thereby forcing the researcher to suggest whether type I graphs would be useful or better than type II. Secondly the feature a_3 is present in S_1, S_2, S_6 and S_3. Some researchers may feel type I subset vertex multigraph would be better suited than type II.

Now suppose some 34 of the clans or social groups have perished due to war or migration or any other reason and further few of the features or characteristics enjoyed by the existing social groups once and not in vogue now then we can conveniently represent the resulting present social group by the subset-subset vertex multisubgraph of type II.

Let us assume P_1, P_2, P_7, P_9 and P_3 are the social groups which exist after a period of time with a few features lost which is depicted by the following subset-subset vertex multisubgraph P given by the following figure.

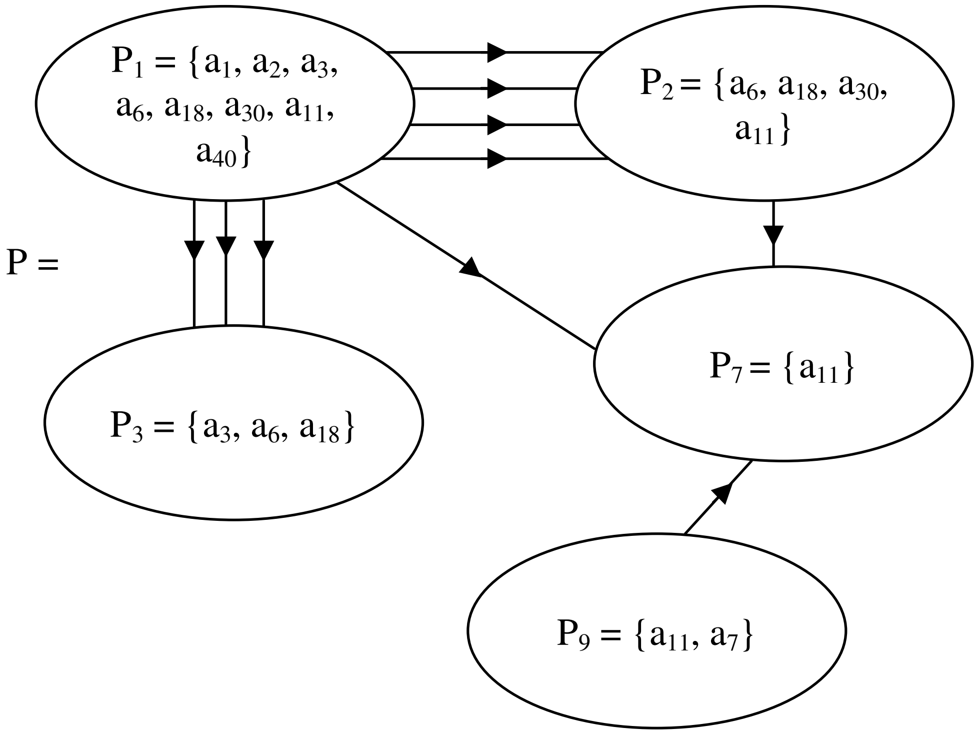


Figure 2.100

Now the disconnected subset vertex multigraphs S has a subset-subset vertex multisubgraph P which is connected. Those social group which were unrelated in S has become related or connected in P. The researcher can decide to use the appropriate subset vertex multigraph of type I or type II. However, these subset -subset vertex multisubgraphs would be a boon to study a social group in which some social groups have died and where some of the characteristics or features have faded and does not find a place in these social groups.

We suggest a few problems for the researcher.

Problems

1. Let S = {Z_{18}} be the set P(S) the power set of S.

   a. Find all ordinary subset vertex multigraphs using the vertex subset from P(S).
   b. How many of these ordinary subset vertex multigraphs are star graphs?
   c. Can these ordinary subset vertex multigraphs be a tree with more than two layers? Justify.
   d. Can these ordinary subset vertex multigraphs be line multigraphs?
   e. Find all uniform complete subset vertex multigraphs using this P(S).
   f. Find all pseudo complete subset vertex multigraphs using this P(S).

2. What are the special features associated with the ordinary injective subset vertex multigraphs of type II?

3. Compare the ordinary injective subset vertex multigraphs of type II with ordinary projective subset vertex multigraphs of type II.

4. Construct a real-world social network and use these models to analyse the problem.

5. Give any other appropriate innovative applications of these new type of subset vertex multigraphs.

6. Compare ordinary subset vertex multigraphs of type I with ordinary subset vertex injective (projective) multigraphs of type II.

7. Give an example of a social network in which type I multigraphs are better than type II multigraphs.

8. Prove these subset vertex multigraphs which are n-partite can be used in ANN and DNN.

9. For a given set S and the corresponding power set P(S).

   Prove we have a greater number of subset vertex multigraphs of type I than that of subset vertex multigraphs of type II.

10. Show if G is any subset vertex multigraph of type II (injective or projective) than G with the same vertex set of G we have a subset vertex multigraph of type I. Prove the converse in general is not true.

11. Let S = $\{Z_{18}\}$ be a set and P(S) the power set of S. Using the vertex subsets v_1 = {2, 3, 4, 5, 8, 9, 10, 16, 17}, v_2 = {1, 0, 4, 8, 9, 16, 17}, v_3 = {2, 3, 4, 5, 16, 17},

v_4 = {1, 0, 16, 17}, v_5 = {7, 11, 13, 5, 3, 16, 14}, v_6 = {16, 17}, v_7 = {7, 11, 13, 16} and v_8 = {4, 8, 16} find the following.

a. Subset vertex multigraph G_1 of type I.

b. Subset vertex multigraph G_2 of type II.

c. Compare G_1 with G_2.

d. Prove G_1 has more multiedges than G_2.

e. Find the universal complement D_1 and D_2 of G_1 and G_2 respectively.

f. Which of them is structure preserving?

g. Using the following subsets of the vertex subsets u_1, u_2, u_3, u_6 and u_7 where $u_1 = \{2, 3, 4, 8, 9, 16, 17\} \subseteq v_1$, $u_2 = \{16, 17, 9, 8\} \subseteq v_6$ and $u_7 = \{16\} \subseteq v_8$, find the subset-subset vertex multisubgraphs P_1 and P_2 of type I and type II and compare them.

h. Find the local complements of P_1 and P_2 relative to G_1 and G_2 respectively and compare them.

i. Which of the subset vertex multigraphs G_1 or G_2 has a greater number of subset-subset vertex multisubgraphs?

j. Which of the subset vertex multigraphs G_1 or G_2 has a greater number of empty multisubgraphs?

k. Obtain any other special feature enjoyed by these subset vertex multigraphs of type I and type II.

12. Let S = {Z_{27}} be the set and P(S) the power set of S.

Let G be the subset vertex multigraph of type II given by the following figure.

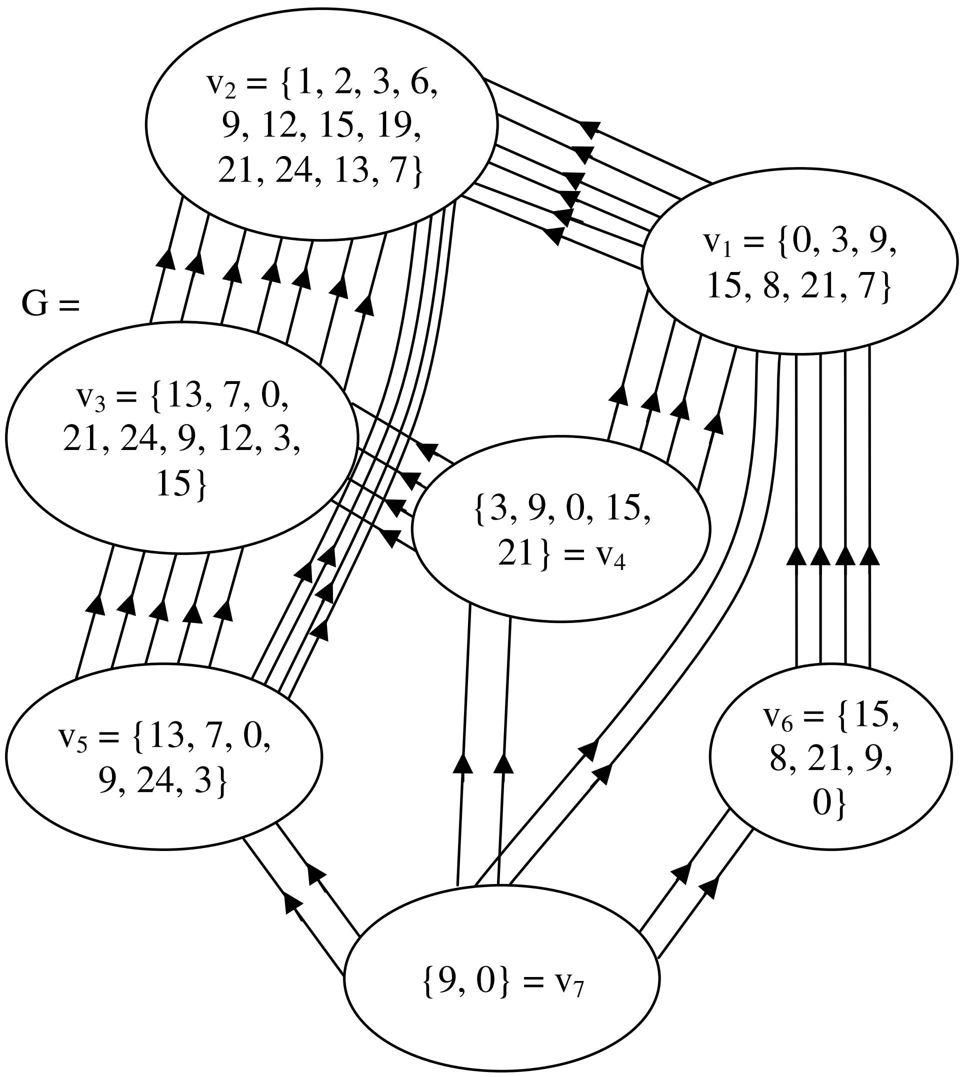


Figure 2.101

a. Complete the multigraph and find the universal complement of G.

b. Find all subset vertex multisubgraphs of G.

c. Find all subset-subset vertex multisubgraphs H_i of G.

d. Find all local complements of H_i in (iii).

e. How many of these H_i's are empty subset-subset vertex multisubgraphs?

f. Can there be subset-subset vertex multisubgraphs of G which are multistar graphs?

g. How many of these subset-subset vertex multisubgraphs of G are complete?

h. How many of these subset-subset vertex multisubgraphs of G are triads?

i. How many of these subset-subset vertex multisubgraphs are forbidden triads?

j. Obtain any other special property enjoyed by these subset vertex multigraphs of type I and type II.

13. Let G be a subset vertex complete multigraph of type II which is uniform with 6 vertex subset from P(S) where $S = \{Z_{45}\}$.

a. Can G have subset-subset vertex multisubgraph which is a tree?

b. Can G have subset-subset vertex multisubgraph which is a star graph?

c. Find all subset-subset vertex multigraphs which are (a) triads (b) forbidden triads.

d. How many subset-subset vertex multisubgraphs can be obtained using G (which ever G you have considered with 6 vertex subsets but being uniform)?

e. Obtain / discuss any other special feature associated with that G.

14. Let S = {Z_{153}} and P(S) the power set of S

    a. How many subset vertex multigraphs of type I can be built using the vertex set S?

    b. What will be the largest number of vertex subsets that can yield the biggest uniform subset vertex multigraph of type I (and type II)?

    c. Which one type I or type II will have a greater number of vertex subsets?

    d. Study the above three questions (a), (b) and (c) using S = 15.

    e. Prove / disprove the largest vertex subset multigraph of type II need not be the same largest vertex subset multigraph of type I using same set of vertex subsets.

15. Prove that larger the cardinality of S we get larger subset vertex multigraphs with many multiedges.

16. Apply a 5 partite subset vertex multigraph using subsets from as appropriate subsets of P(S) to ANN or DNN. Discuss the merits and uses of study of such application.

17. Let S = {Z_{22}} and P(S) be the power set of S. Let G be the subset vertex multigraph of type I given by the following figure.

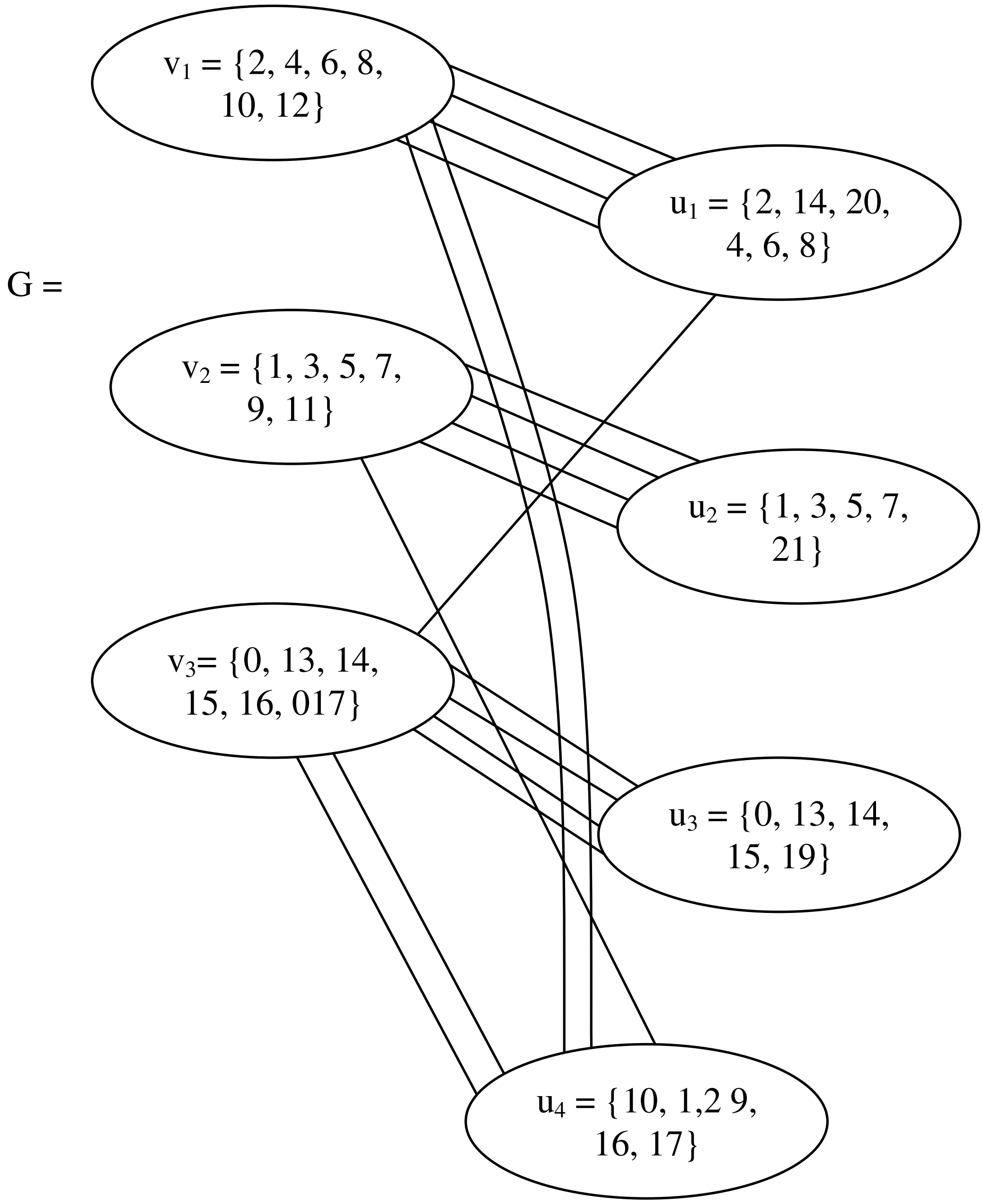


Figure 2.102

18. a) Prove these types of bipartite subset vertex multigraphs of type I can be used for the study of Fuzzy Relational Multimaps models and Neutrosophic Relational multimaps models.

    b) Prove this type of subsets vertex multigraphs can be used in soft computing.

19. Let S = {a_1, a_2, a_3, a_4, a_5, a_6, …, a_{15}} = Z_{15} be a set of cardinality 15. P(S) be the power set of S. Let G be the subset vertex multigraph of type I given by the following figure.

G =

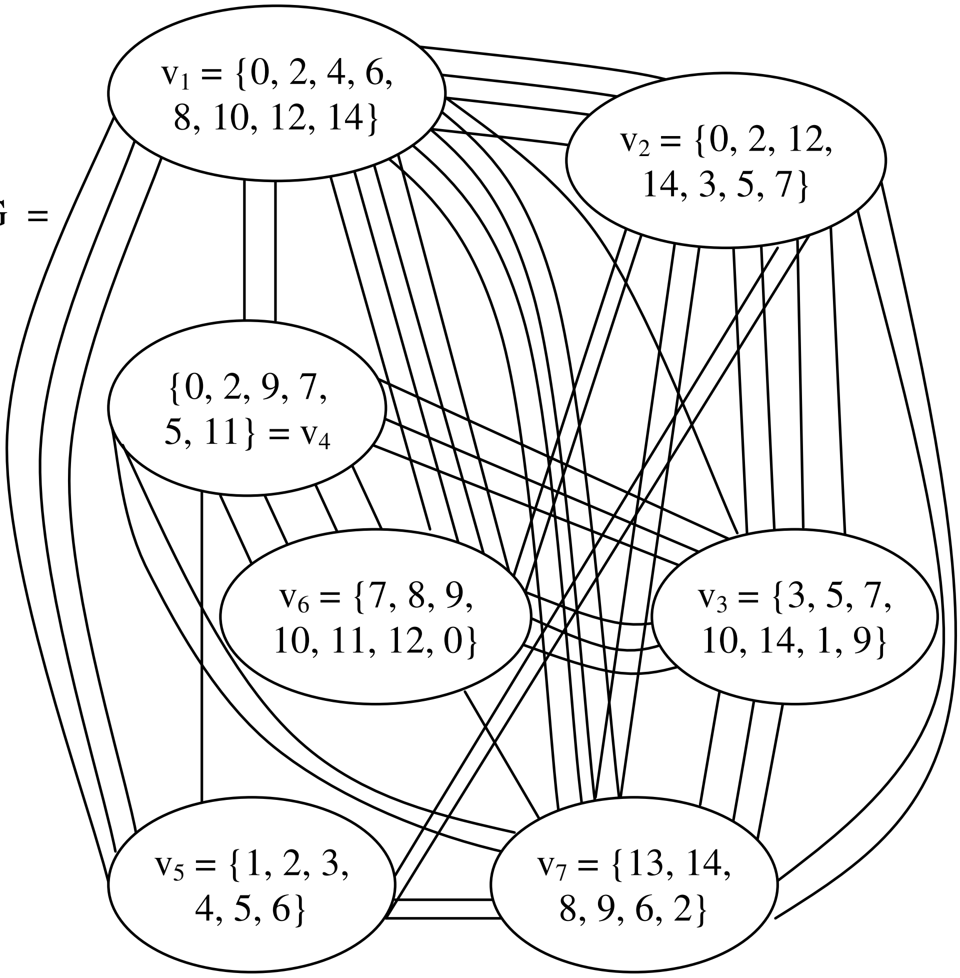


Figure 2.103

i. Prove these types of subset vertex multigraphs can be used in the study of Fuzzy cognitive multimaps model.
ii. Obtain using this type of model get a multi-structured opinion.

iii. Use this type of multimaps models to real world problems.

iv. Give any other viable applications of these subset vertex multigraphs.

20. Study subset vertex multigraphs in the context of multilayered neural networks.

Chapter Three

SUBSET VERTEX MULTIGRAPHS AND NEUTROSOPHIC MULTIGRAPHS OF TYPE I

In the earlier chapters we have introduced the notion of subset vertex graphs of type I and type II which are multigraphs. Further the edges were defined in such a way that given a set of subset vertices there is one and only graph associated with it. Thus, the problem of Freeman index or any form of difficulty or ambiguity or arbitrariness in forming these subset vertex graphs of type I and type II does not exist.

The only drawback by that definition was we cannot go for multigraphs so we in this chapter alter the definition and thereby these subset vertex graphs can be made into a multisubset vertex graphs or multivertex subset graphs or multigraphs with subset vertices.

We will first illustrate this situation by some examples.

Example 3.1. Suppose S = $\{Z_5, Z_{12}\}$ = $\{a_0, a_1, a_2, a_3, a_4, b_0, b_1, \ldots, b_{11} / a_i \in Z_5, b_j \in Z_{12}, a_0 = 0, a_1 = 1, a_2 = 2, a_3 = 3, a_4 = 4, b_0 = 0, b_1 = 1, \ldots, b_{11} = 11, 0 \leq i \leq 4$ and $0 \leq j \leq 11\}$ be a finite set. We now find the power set of S. P(S) the power set of S is of finite order. In fact $|P(S)| = 2^{17}$; which includes ϕ and S the empty set and the whole set. Consider the vertex subset V = $\{v_1 = \{a_1, a_2, b_1, b_2, b_{10}\}, v_2 = \{a_1, a_4, a_5, b_{10}, b_2, b_5, b_7, b_8\}, v_3 = \{b_{10}, b_1, b_6, b_9\}, \{a_1, a_2, a_0, a_4\} = v_4, \{a_5, b_{10}\} = v_5\}$ we give the multigraph with subset vertex by the following figure.

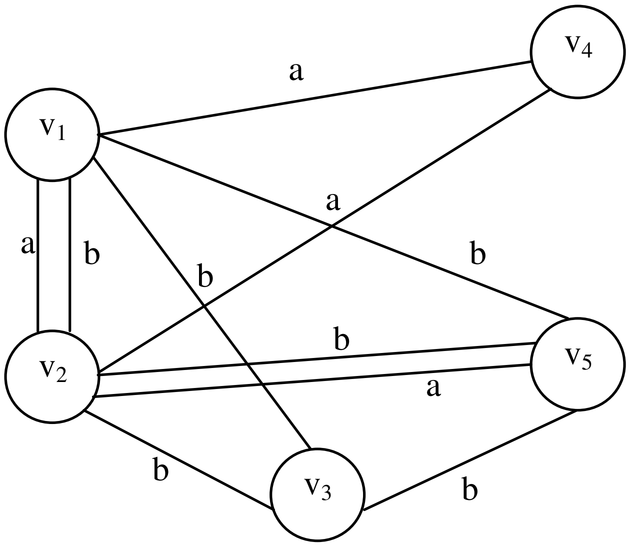


Figure 3.1

Now we proceed onto describe the graph in figure 3.1.

The vertex v_1 has two elements $a_1, a_2 \in Z_5$ and $b_1, b_2, b_{10} \in Z_{12}$; so this vertex $v_1 = (a_1, a_2, b_1, b_2, b_{10}) = (\{a_1, a_2\}, \{b_1, b_2, b_{10}\})$, that is it has two different types (or properties or concepts put together). These two concepts cannot be mixed for basically they convey some distinct results.

Further if algebraic operations like + or × is to be used on them; for $a_1 + b_2$ or / and $a_1 \times b_2$ has no meaning as $a_1 \in Z_5$

and $b_2 \in Z_{12}$. As they mean two different concepts it would be appropriate, we use the multigraph for we cannot have any meaningful function defined relating Z_5 and Z_{12}. Under these circumstances it is vital we use multigraphs to represent the very model.

Thus, the vertex set in the figure 3.1 can be restructured as follows. This is done only for the easy finding of the edges V = $\{v_1 = \{a_1, a_2, b_1, b_2, b_{10}\} = \{\{a_1, a_2\}, \{b_1, b_2, b_{10}\}\}, v_2 = \{a_1, a_4, a_5, b_{10}, b_2, b_5, b_7, b_8\} = \{\{a_1, a_4, a_5\}, \{b_2, b_5, b_7, b_8, b_{10}\}\}, v_3 = \{b_1, b_{10}, b_6, b_9\} = \{b_1, b_6, b_9, b_{10}\}, v_4 = \{a_1, a_0, a_2, a_4\} = \{a_1, a_0, a_2, a_4\}, v_5 = \{a_5, b_{10}\} = \{\{a_5\}, \{b_{10}\}\}\}$.

With these representing (or restructuring the vertex set) we see marking edges as in case of subset vertex graph except we have to do to two subsets for each node in this case in contrary to only one subset in case of subset vertex graphs of type I. Thus the subset vertex restructured graph is given below.

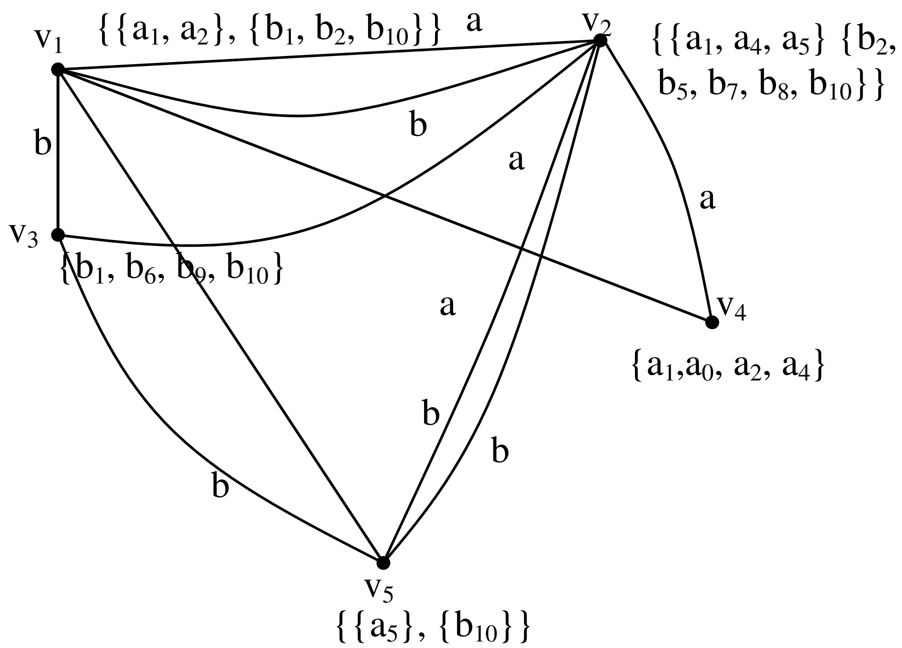


Figure 3.2

Further the edges are distinguished by showing them as a and b. The two edges are drawn if for v_i and v_j. $v_i \cap v_j$ = ({a's}, {b's}) $\neq \phi$; $i \neq j$.

However, loops are not drawn in this graph. Further these graphs are not directed.

We have no edges connecting v_i and v_j if $v_j \cap v_j = \{\{\phi\}, \{\phi\}\}$.

We have one edge connecting them if $v_i \cap v_j$ = {{ϕ}, {b's}} or $v_i \cap v_j$ = {{a's}, ϕ}. ($i \neq j$).

The term 'or' is used in the mutually exclusive sense.

Only following these definitions this multigraph is drawn.

Further this multigraph can have a maximum of only two edges between any relevant vertices. Clearly, they are not directed.

Also, these multigraphs can be considered as an extension of the usual subset vertex graph of type I. Other properties related with them will be explained.

We however provide a few more examples of them.

Example 3.2. Let S = $\{Z_9, Z_{15}, Z_7, Z_{19}\}$ be a set. We give the notational description of S = $\{a_i \in Z_9; 0 \leq i \leq 8, b_j \in Z_{15}; 0 \leq j \leq 14, c_k \in Z_7; 0 \leq k \leq 6; d_t \in Z_{19}; 0 \leq t \leq 18\}$; P(S) be the powerset of S. Any subset vertex multigraph takes its vertex set from P(S). Thus we can also represent P(S) as {{a_j's collection}, {b_j's collection}, {c_k's collection}, {d_t's collection} where $a_i \in Z_9$, $b_j \in Z_{15}$, $c_k \in Z_7$ and $d_k \in Z_{19}$}. It is pertinent to keep on record |P(S)| = 2n, where n = 9x15x7x19 in either way.

Now we give a simple representation of the subset vertex multigraph of type I. However, we will give the definition of the same after we illustrate them by examples.

Let V = {v_1, v_2, v_3, v_4, v_5, v_6, v_7} be the vertex set contained in P(S) the powerset of S where S = {Z_{19}, Z_{15}, Z_7, Z_9}. The elements in v_i's are described in the following $1 \leq i \leq 7$. v_1 = {a_0, a_2, a_4, a_6}, v_2 = {b_3, b_6, b_9, b_{12}, b_{11}, b_{13}}, v_3 = {c_0, c_2, c_5}, v_4 = {d_0, d_2, d_4, d_6, d_8, d_{10}, d_{12}}, v_5 = {{a_2, a_4}, {b_6, b_{12}, b_9}, {c_2, c_0}, {d_2, d_4, d_8}}, v_6 = {{a_0, a_6}, {b_3, b_{11}, b_{13}}, {c_0}, {d_0, d_{10}, d_{12}}} and v_7 = {{a_2, a_6}, {c_0, c_6}, {d_2, d_8, d_{10}}, {b_{11}, b_6, b_9}}.

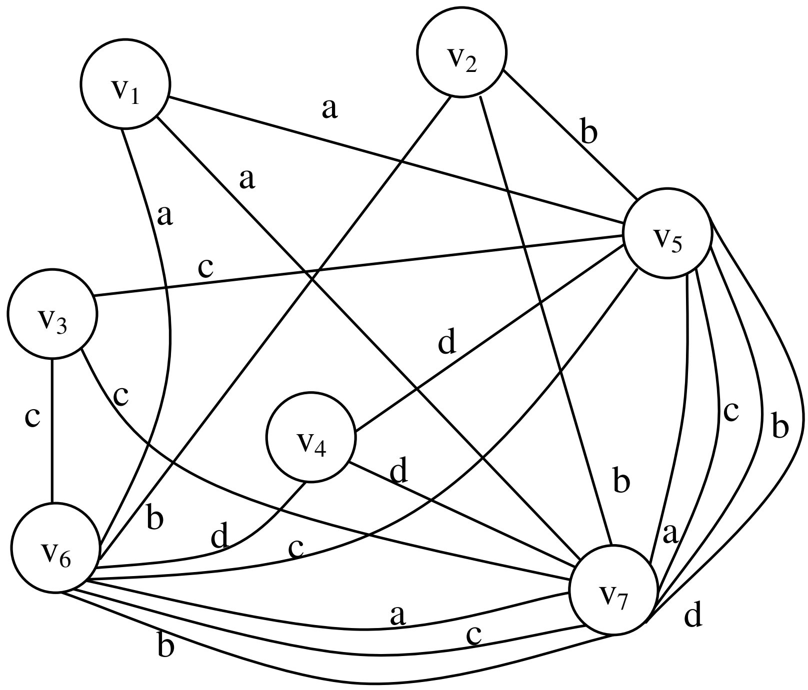


Figure 3.3

Clearly the figure 3.3 is the subset vertex multigraph M using the vertices $v_1, v_2, v_3, \ldots, v_7$.

We see the maximum number of multiedges M can have is only 4. This occurs in the subset vertex multigraph M only with the vertices v_5 and v_7.

This is illustrated in the following.

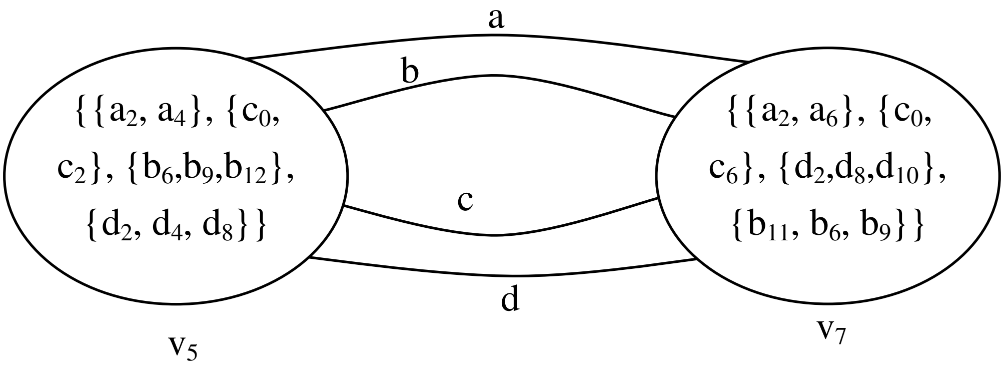


Figure 3.4

We see $v_5 \cap v_7 = \{\{a_2\}, \{c_0\}, \{b_6, b_9\}, \{d_2, d_8\}\}$.

Now we see $v_1 \cap v_7 = \{\{a_2, a_6\}\}$.

$v_2 \cap v_7 = \{b_{11}, b_6, b_9\}$

$v_3 \cap v_7 = \{c_0\}$, $v_4 \cap v_7 = \{d_2, d_8, d_{10}\}$.

So with v_7 these vertices v_1, v_2, v_3 and v_4 have only one edge so they do not form multigraph.

Thus if $W = \{v_1, v_2, v_3, v_4, v_7\}$ is the vertex subset of V; the subset vertex multigraph K associated with W is K

K =

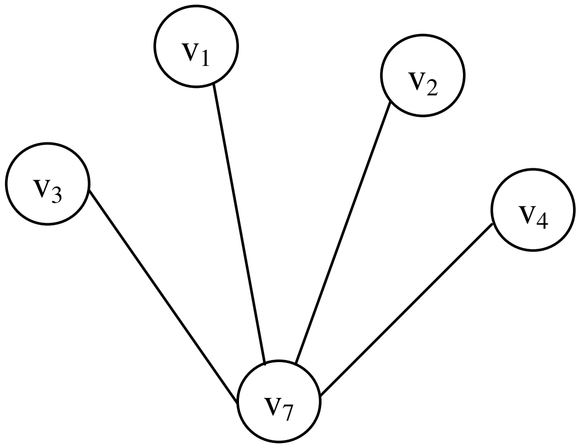


Figure 3.5

The subset multigraph is a star graph and has no multiedges. Thus we see a subset vertex multigraph can have simple subset vertex graph as a subgraph, figure 2.5.

Consider the set R = {v_7, v_6, v_5, v_3} $\subseteq$ V; we find the subset vertex multigraph H associated with R.

H =

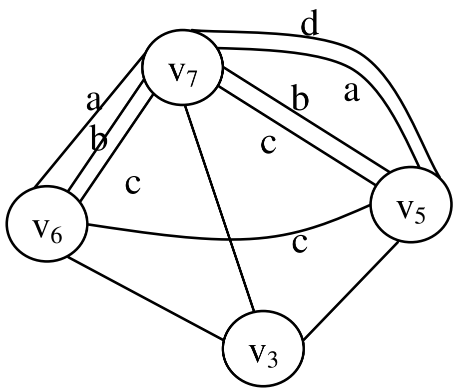


Figure 3.6

We see the subset vertex multigraph H associated with the vertex set R is a subset vertex multiset subgraph in contrast with the subset vertex multisubgraph K which is only a vertex subset subgraph as it has no multiedges.

It is important to keep on record that once given a set of subset vertices there exists a unique subset vertex multigraph.

Now we make the formal definition of them.

Definition 3.1. *Let S = {A_1, A_2, A_3, …, A_n} where each of the A_i's enjoy a special type of property which is distinctly different from other A_i's so that intermingling or mixing them cannot be entertained.*

Let P(S) denote the power set of S. We take the vertex set from the subsets of P(S) for the graph. Let A_1 = { a_1^i/i is finite positive integer, with denotes cardinality of A_1}; A_2 = { a_2^j/j is a finite positive integer which denotes the cardinality of A_2} and so on A_n = { a_n^k/ k is a finite positive integer which denotes the cardinality of A_n}. Any $v_i \in P(S)$ will be a collection of the form v_i = {{ a_1^i }, { a_2^j }, …, { a_n^k }}. If $v_j \in P(S)$ and v_j = {{ a_1^t }, { a_2^r }, …, { a_n^s }} then we define $v_i \cap v_j$ = {{ a_1^i } $\cap$ { a_1^t }, { a_2^j } $\cap$ { a_2^r }, …,{ a_n^k } $\cap$ { a_n^s }}; if all of them are nonempty we put n edges connecting the vertex subset v_i with the vertex subset v_j. If r of them are nonempty then we have only r number of edges connecting v_i with v_j r < n. If all of them are empty, there is no edge connecting v_i with v_j. Using this principle, we draw the subset vertex graph. So if we have say some m subset vertices {v_1, …, v_m} $\subseteq$ P(S) then we draw the graph with v_1, …, v_m as vertices and define this graph as the subset vertex multigraphs M of type I.

Clearly M is not an ordinary multigraph mentioned I chapter II of this book and for a given set of vertices the subset vertex multigraph of type I is unique.

We will illustrate this situation by some examples.

Example 3.3. Let S= $\{Z_{10}, \langle Z_5 \cup I\rangle, \langle Z_3 \cup g\rangle, Z_{11}, C(Z_4)\}$ be the given set. P(S) be the power set of S.

Let V = $\{v_1, v_2, v_3, v_4, v_5\} \subseteq$ P(S) be the vertex subset associated with the subset vertex multigraph M where each of the v_i's are described in the following.

$\{v_1 = \{\{2, 4, 6, 8\}, \{0, I, 1 + I, 4I, 2 + 3I\}, \{g + 1, 2g, 2g + 2\}, \{0, 1, 3, 4, 9, 10\}, \{2i_F, 1 + i_F, 3 + i_F, 4 + 2i_F, 3\}\}$,

$v_2 = \{\{2, 4, 3, 9, 6\}, \{I + 1 + I, 2I, 0, 2\}, \{g, 2g, 1 + g, 2 + g\}, \{0, 3, 6, 9, 4, 5\}, \{i_F, 3, 1 + i_F, 4, 2i_F + 1\}\}$

$v_3 = \{\{8, 0, 5, 1\}, \{0, 1 + I, 4I + 1, 3I\}, \{0, 1, 2, 2g\}, \{5, 6, 7, 8, 10\} \{0, i_F, 2 + i_F, 3 + 3i_F\}$,

$v_4 = \{\{5, 7, 0\}, \{0, 1 + I, I, 2I\}, \{g, g + 1, 0, 2g\}, \{1, 2, 3, 4\}, \{i_F, 2i_F, 3i_F, 0\}\}$ and

$v_5 = \{\{2, 3, 7, 8\}, \{I, 2I, 3I, 4I, 0\}, \{g, 0\}, \{10, 9, 8, 7\}, \{i_F, 2 + i_F 0\}\}\} = V$.

To get the subset vertex multigraph we find the values of $v_i \cap v_j$; $1 \neq j$, $1 \leq i, j \leq 5$, $v_1 \cap v_2 = \{\{2, 4, 6\}, \{0, I, 1 + I\}, \{1 + g, 2g\}, \{0, 3, 4, 9\}, \{3, 1 + i_F\}\}$,

$v_1 \cap v_3 = \{\{8\}, \{0, 1 + I\}, \{2g\}, \{10\}, \phi\}$,

$v_1 \cap v_4 = \{\{\phi\}, \{0, 1 + I\}, \{g + 1, 2g\}, \{1, 3, 4\}, \{i_F\}\}$,

$v_1 \cap v_5 = \{\{2,8\}, \{I, 0\}, \{\phi\}, \{9, 10\}, \{\phi\}\}$,

$v_2 \cap v_3 = \{\{\phi\}, \{0, 1 + I\}, \{2g\}, \{6, 5\}, \{i_F\}$.

$v_2 \cap v_4 = \{\{\phi\}, \{0, 1 + I, I, 2I\}, \{g + 1 + g, 2g\}, \{3, 4\}, [2i_F\}\}$,

$v_2 \cap v_5 = \{\{2, 3\}, \{I, 2I, 0\}, \{g\}, \{9\}, \{i_F\}\}$,

$v_3 \cap v_4 = \{\{0\}, \{0, 1 + I\}, \{0, 2g\}, \{\phi\}, \{0, i_F\}$,

$v_3 \cap v_5 = \{\{\phi\}, \{3I, 0\}, \{0\}, \{10, 8\}< \{0, i_F\}\}$ and

$v_4 \cap v_5 = \{\{7\}, \{0, I\}, \{g, 0\}, \{\phi\}, \{i_F, 0\}\}$.

We now give the subset vertex multigraph M related with $v_1, \ldots, v_5$ which is given by the following figure;

M =

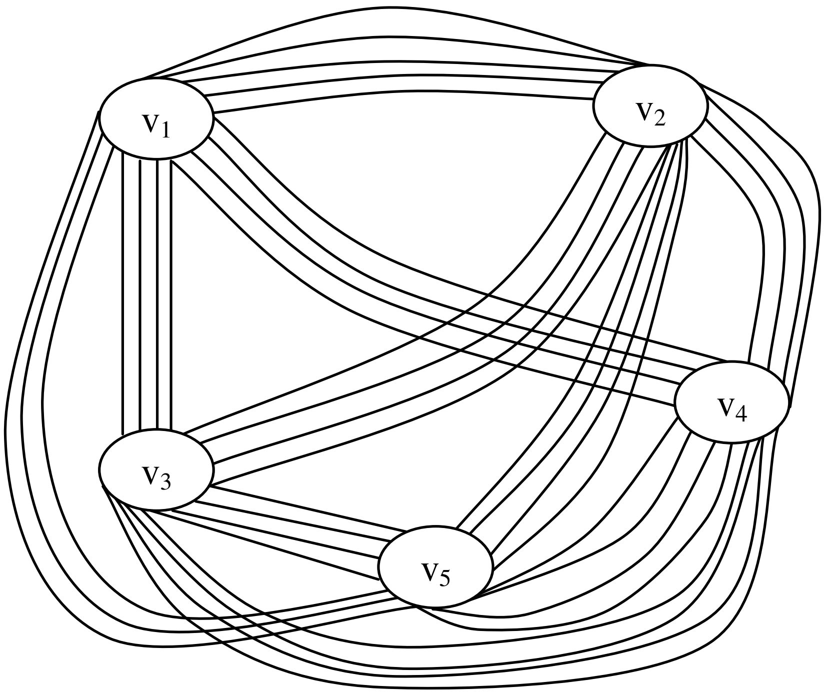


Figure 3.7

We see this subset vertex multigraph M can have a maximum of 5 edges connection with any of the other relevant vertices. In case of this M, the number of edges connecting any of the two vertices are either 5 or 4 or 3 only. In view of all these facts we put forth the following definition.

Definition 3.2. *Let G be a subset vertex multigraph with M vertex subsets from a power P(S) of a set S with M set of distinct attributes. If G has for every vertices M edges connecting them then we define G to be a subset vertex complete multigraph.*

We give a few examples to this effect.

Example 3.4: Let S = {collection of all subsets from the set B = $\{C(Z_4), \langle Z_3 \cup g\rangle, \langle Z_{10} \cup I\rangle\}\}$ = P(B) the power set of B. o(P(B)) = 2^m where m = $|C(Z_4)| \times |\langle Z_3 \cup g\rangle| \times | Z_{10} \cup I\rangle|$.

All subset vertex multigraphs built using P(B) can at most have only 3 edges connecting any pair of vertices.

We now give an example of a subset vertex complex, dual number, indeterminate edged multigraph which is complete.

Consider V = $\{v_1, v_2, v_3, v_4\} \subseteq$ P(B), where $v_1 = \{\{1 + i_F, 2, 0, 3i_F, 3 + i_F\}, \{1 + g, 2, 2g, 0\}, \{I, 5I, 2, 3 + 7I, 0\}\}$, $v_2 = \{\{3i_F, 0, 2, 2 + 2i_F, 2i_F\}, \{g, 2g, 0, 2, 1\}, \{I, 5I, 2, 0, 6I + 6, 6, 6I\}\}$, $v_3 = \{\{2i_F, 2 + 2i_F, 2, 0, 3i_F\}, \{ I, 2I, 2I +3 \}, \{2, 2g, 0, 1\}$ and $v_4 = \{\{0, 2, 2i_F\}, \{1 + g, 2, I, g\}, \{I, 5I, 0, 6I, 2\}\}$.

We see the subset vertex multigraph associated with the vertices v_1, v_2, v_3 and v_4 is given by the following figure.

T =

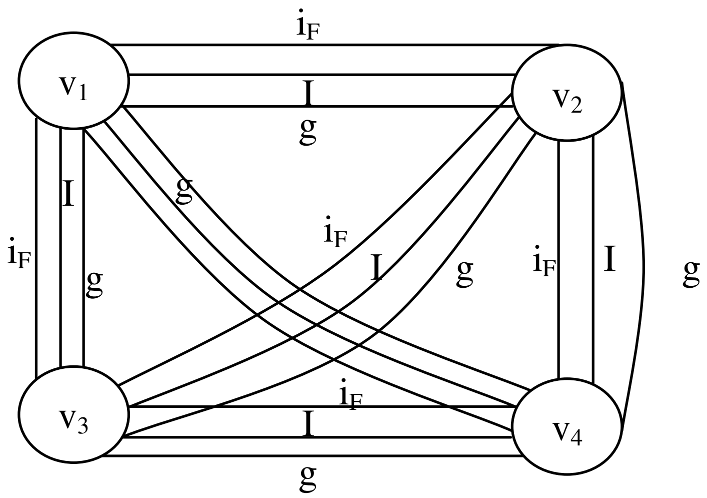


Figure 3.8

Clearly T is a complete subset vertex multigraph of type I.

We cannot say all subset vertex multigraphs are complete. Infact we see when we define the concept of subset vertex multigraphs; a subset vertex multigraph is complete if the following conditions are true.

i) It should be a complete graph that is the number of edges incident at every vertex is the same and it is the maximum possible edges.
ii) Every edge must have the maximal number of edges.
iii) If G is a multisubset graph with n-vertices where (m is the maximal number of edges two vertices can enjoy) each vertex v_i has $(n - 1)$ maximum multiedges, that is v_i has $m \times (n - 1)$ edges.

Only under these three conditions we define the subset multigraph as a complete graph of type I.

We will illustrate this situation by some more examples.

Example 3.5: Let S = {$C(Z_7)$, Z_{10}, {⟨$Z_5 \cup I$⟩}} be the given set P(S) be the power set of S. Clearly |P(S)| = $2^{|Z_{10}|\times|\langle Z_5 \cup I\rangle|\times|C(Z_7)|}$.

Clearly the maximum number of edges any subset vertex graph can have with vertex set from P(S) is three. There can be some vertices from P(S) which can be joined by two edges or one edge or no edge.

We now give examples of each of them using the P(S).

Let v_1 = {{5 + $2i_F$, i_F, 0, $6i_F$, 2}, {1, 2, 0, 5, 7}, {I, 2I + 1, 3I + 3, 0}} and v_2 = {{6, 5, $4i_F$, 1}, {3, 4, 8}, {1, 0, 3I, I, 2I + 4}} be two subsets of P(S).

The line graph connecting v_1 and v_2 is as follows.

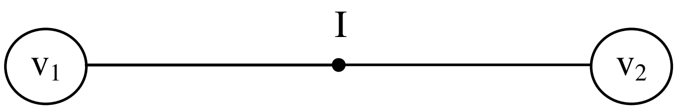


Figure 3.9

We see $v_1 \cap v_2$ = {{ϕ}, {ϕ}, {0, I}}.

Thus the subset vertex multiline neutrosophic graph is only the usual subset vertex line graph.

Now consider the vertex set V = {v_1, v_2, v_3, v_4} where v_1 = {{$2i_F$ + 3, i_F + 1, 1, 0, 5, 3}, {I, 2I, 3I, 0}, {2, 4, 6, 8, 0}}, v_2 = {{5, 3, 7, 1}, {0, I, 2, 3, 2 + 2I}, {$3i_F$, $5i_F$, 1, 0, $3i_F$ + $5i_F$}}, v_3 = {1, 0}, {I_2}, {4 + 4i, 5 + $5i_F$}} and v_4 = {{5, 4}, {I, 3, 0}, {$3i_F$, 0, 1 + i_F}} be the vertex set.

We first find out how many edges can there between the vertices v_1 and v_2.

$v_1 \cap v_2 = \{\{1, 0\}, \{0, I\}, \{\phi\}\}$

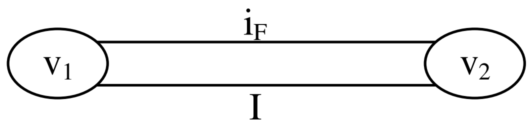


Figure 3.10

The number of edges between v_1 and v_3 is as follows;

$v_1 \cap v_3 = \{\{0\}, \{I\}, \{\phi\}\}$.

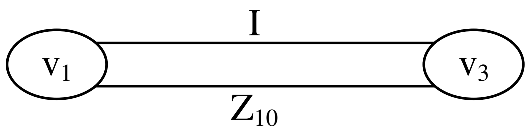


Figure 3.11

The number of edges between v_1 and v_4 is three which is as follows

$v_1 \cap v_4 = \{\{4\}, \{I, 0\}, \{0, 1 + i_F\}\}$

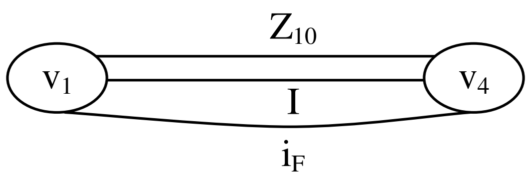


Figure 3.12

The number of edges between v_2 and v_3 is only two

$v_2 \cap v_3 = \{\{1\}, \{2, I\}, \{\phi\}\}$.

The line graph between v_2 and v_3 is

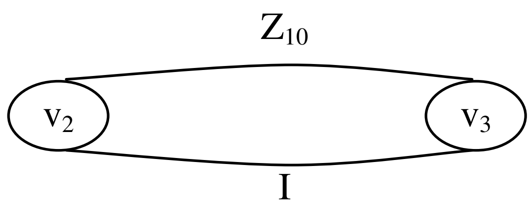


Figure 3.13

The number of edges between v_2 and v_4 is three

$v_2 \cap v_4 = \{\{5\}, \{I, 3, 0\}, \{3i_F, 0\}\}$

The line graph between v_2 and v_4 is

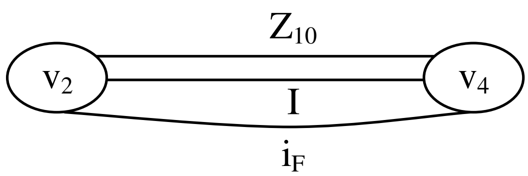


Figure 3.14

The number of edges between v_3 and v_4 is I the line graph between v_3 and v_4 is as follows;

$v_3 \cap v_4 = \{\{\phi\}, \{I\}, \{\phi\}\}$.

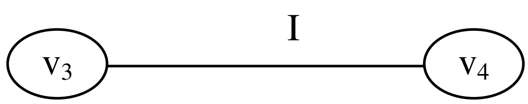


Figure 3.15

Now we give the subset vertex multigraph G using v_1, v_2, v_3 and v_4 which is given in the following.

G =

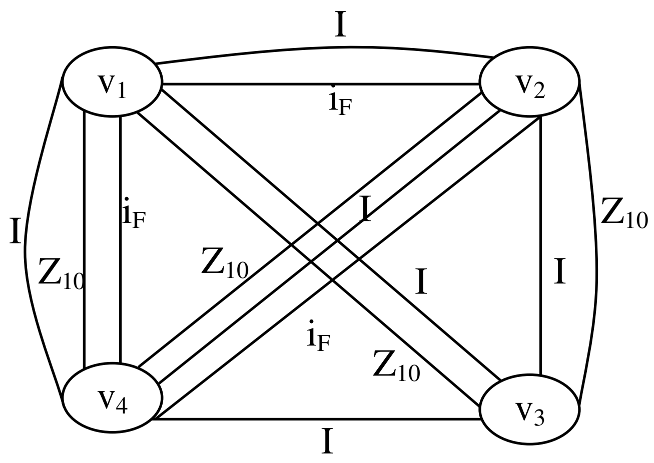


Figure 3.16

Clearly G is not a complete subset vertex multigraph. This has only two-line graphs as subgraphs which has the maximum number of edges. In this subset vertex multigraph the maximum number of edges that can exist between two vertices (vertex set from this P(S)) is only three.

Further G cannot have any subset vertex multisubgraph which is complete.

However, we call the subset vertex multigraphs in which every vertex set is connected with every other vertex set as a subset vertex pseudo multigraph. Clearly G is a subset pseudo multigraph of order four.

The number of edges in this case should be greater than or equal to 6.

Thus, the number of edges varies from

$$6 \leq \left\{ \begin{array}{c} \text{number of edges of a subset vertex} \\ \text{pseudo complete multigraph} \end{array} \right\} \leq 6 \times 3.$$

In view of all these we give subset vertex multigraphs with vertex set from P(S) with 3 vertices. Let $\{v_1, v_2, v_3\} \subseteq$ P(S).

The subset vertex complete multigraph is as follows.

G_1 =

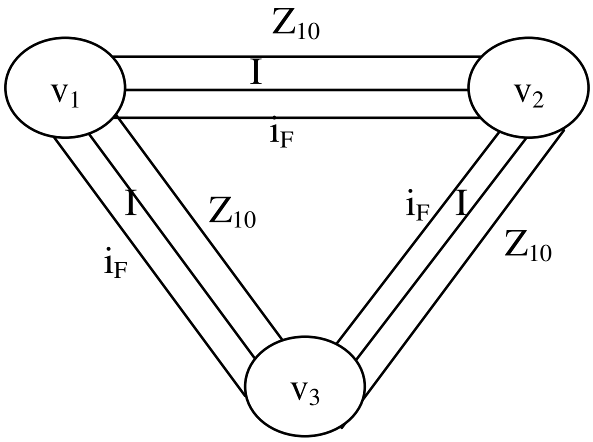
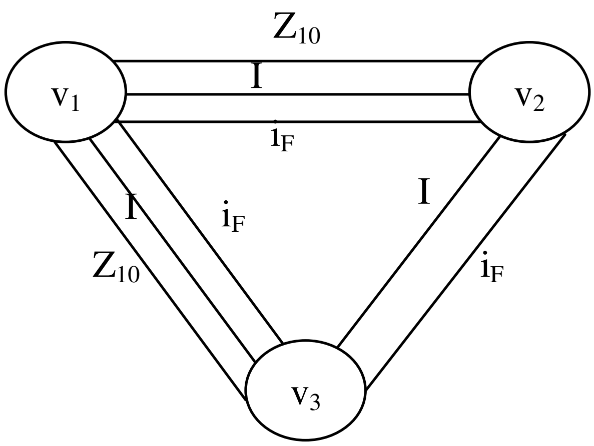


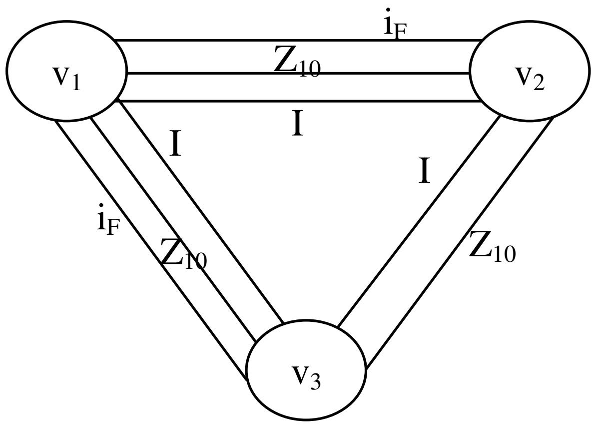
Figure 3.17

The number of edges in G given in figure 3.17 is 9. We can have the following types of subset vertex multigraphs with three vertices.

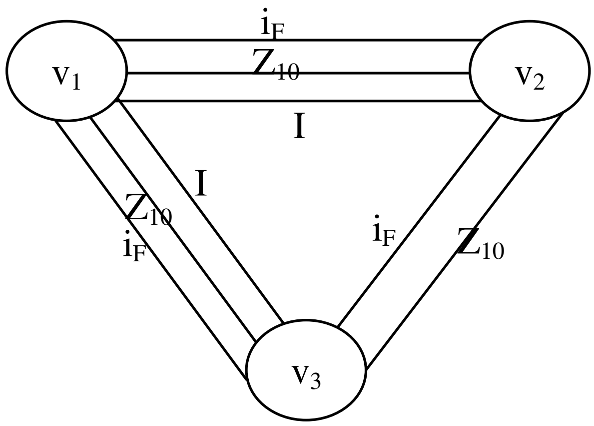
G_2 =



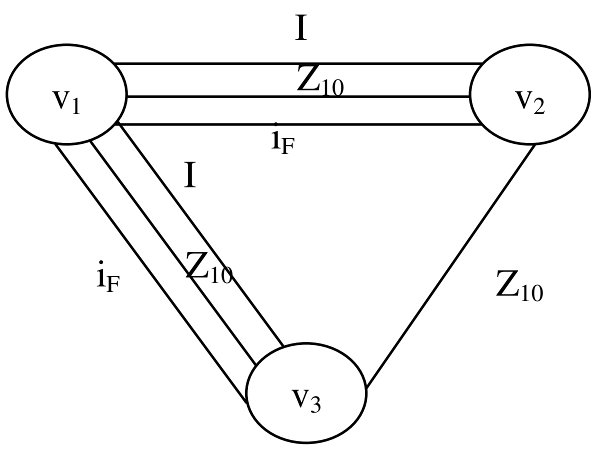
$G_3 =$


v1
v2
v3
iF
Z10
I
I
I
iF
Z10
Z10

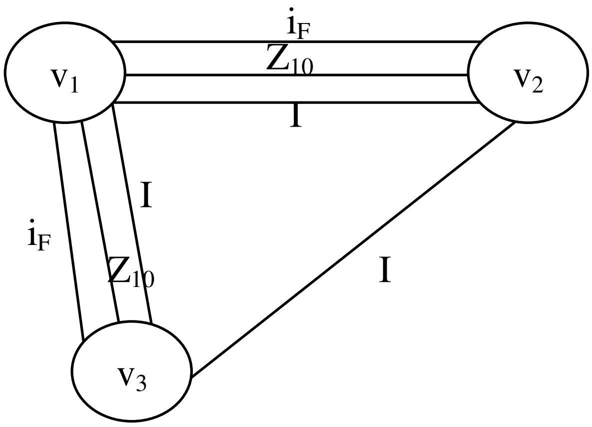
$G_4 =$


v1
v2
v3
iF
Z10
I
I
Z10
iF
iF
Z10

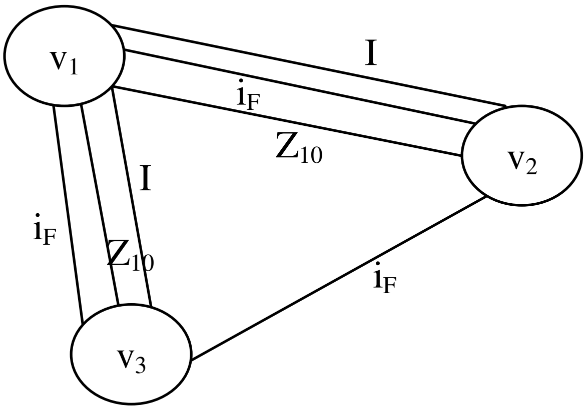
$G_5 =$


v1
v2
v3
I
Z10
iF
I
iF
Z10
Z10

$G_6 =$



$G_7 =$



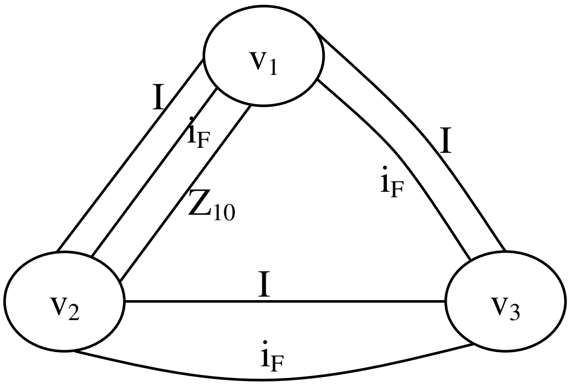
Figures 3.18

The number edges in G_4, G_2 and G_3 is 8 so G_1, G_2 and G_3 can only be subset vertex pseudo complete multigraphs.

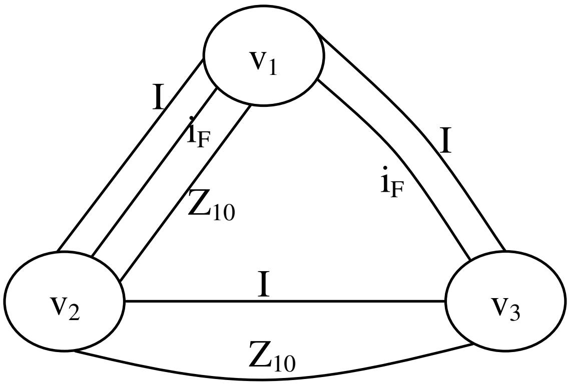
The number of edges of G_7, G_5 and G_6 is 7.

Consider

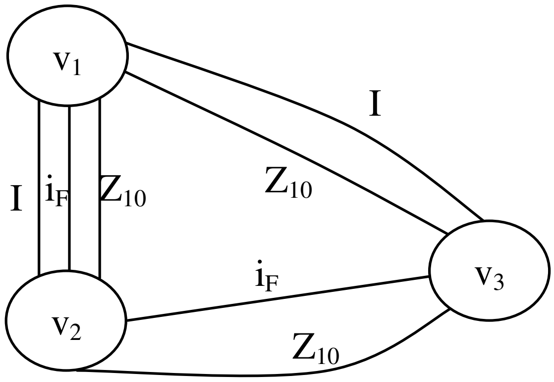
G_8 =



G_9 =



G_{10} =

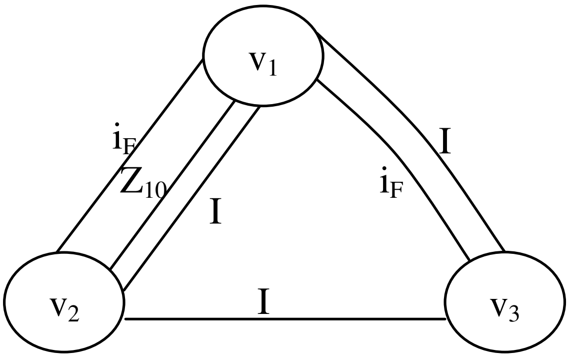


Figures 3.19

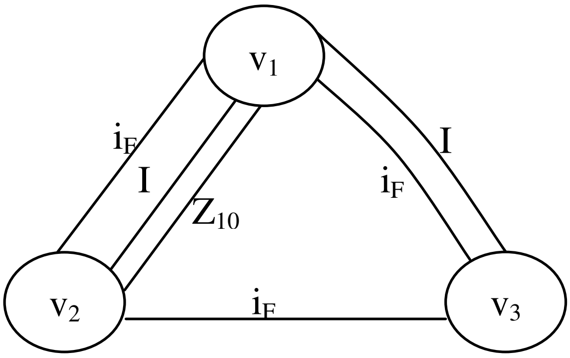
We see the subset vertex pseudo multigraph G_7, G_8, G_9 and G_{10} has only 7 edges, these are different from G_4, G_5 and G_6.

Consider

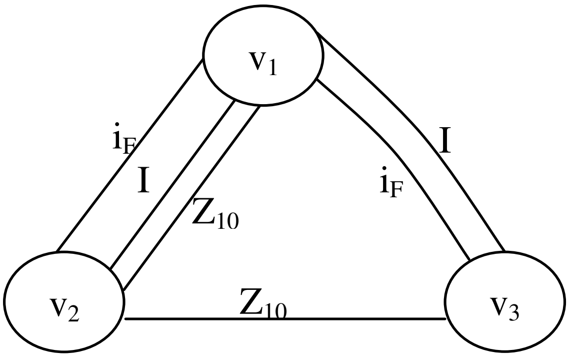
$G_{11} =$



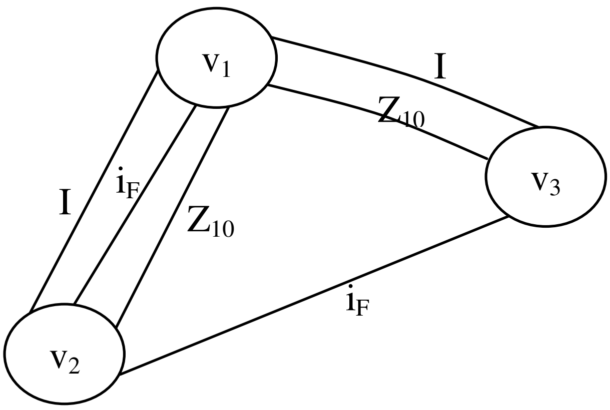
$G_{12} =$



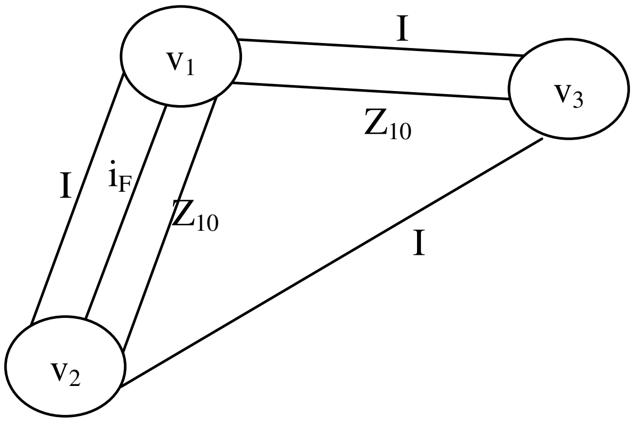
$G_{13} =$



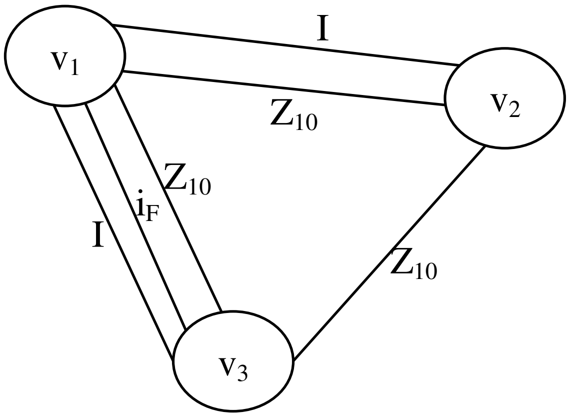
$G_{14} =$



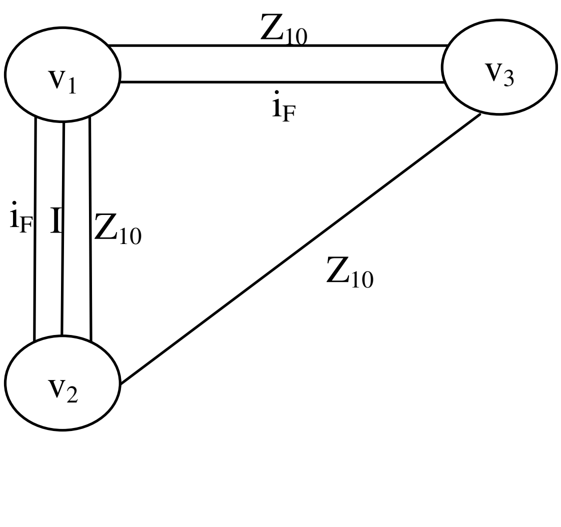
$G_{15} =$


v1
v2
v3
I
Z10
I
iF
Z10
I

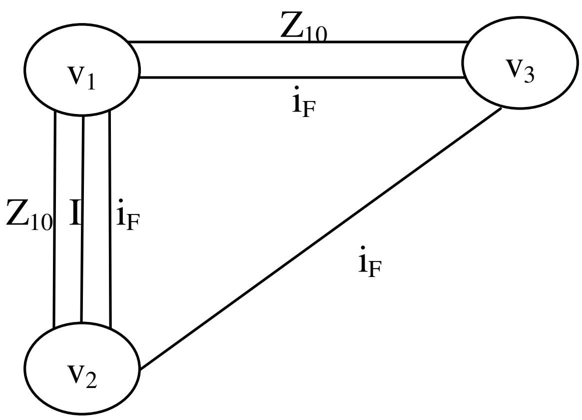
$G_{16} =$


v1
v2
v3
I
Z10
Z10
iF
I
Z10

$G_{17} =$


v1
v2
v3
Z10
iF
iF
I
Z10
Z10

G_{18} =



G_{19} =

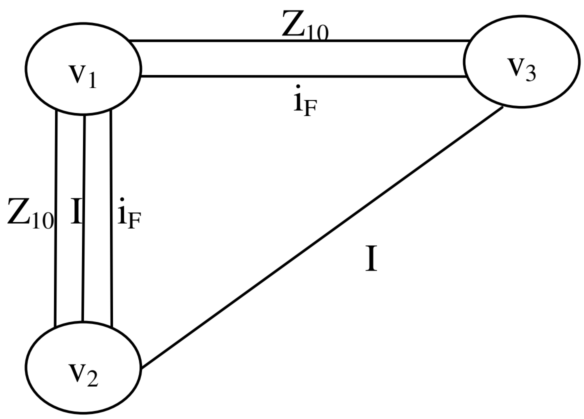
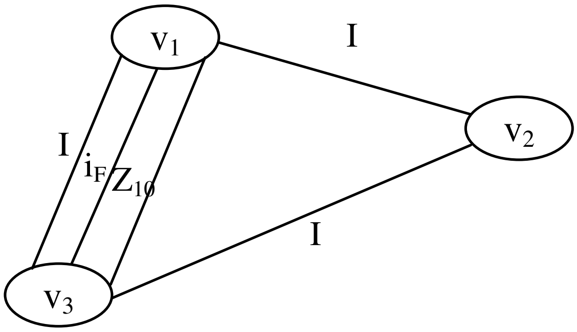


Figure 3.20

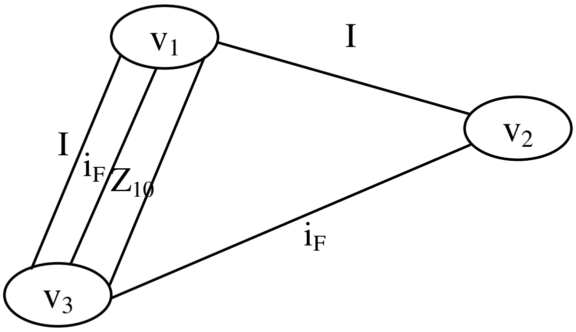
The subset vertex multigraphs G_{11} to G_{19} are pseudo complete Neutrosophic complex subset vertex multigraphs which only six edges and all of them are distinct.

Consider the subset vertex multigraphs

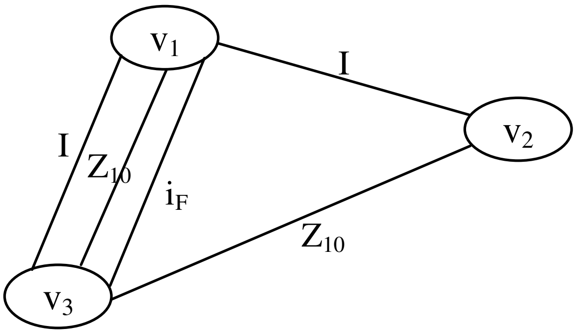
G_{20} =



$G_{21} =$

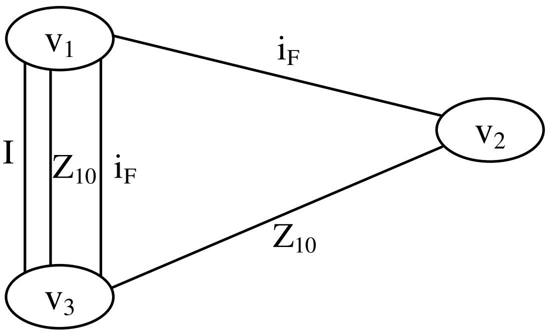


$G_{22} =$

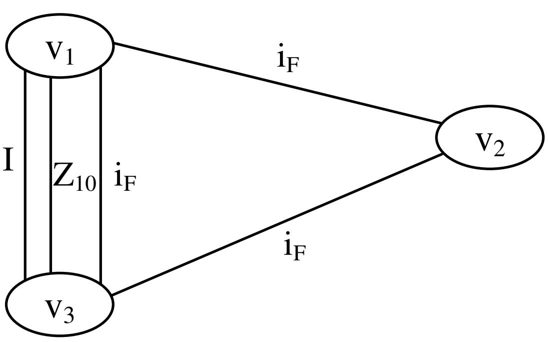


$= G$

$G_{23} =$



$G_{24} =$



$G_{25} =$

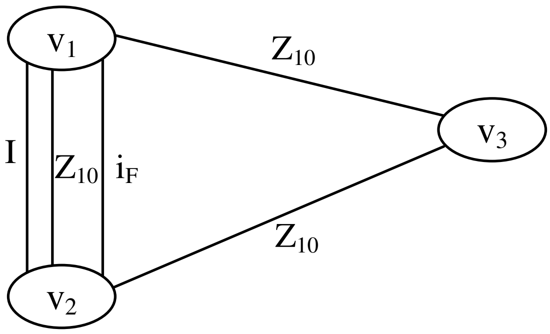
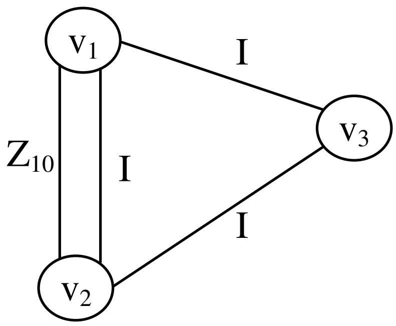


Figure 3.21

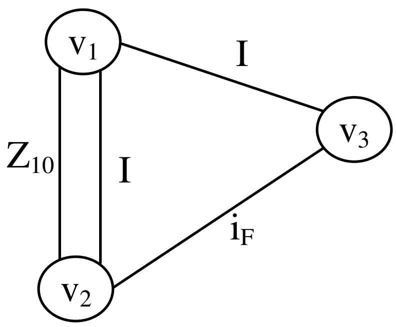
We see the subset vertex multigraph G_{20} to G_{25} are subset vertex pseudo complete multigraphs with 5 edges each. There are six graphs of this type.

Consider the following graphs which are subset vertex multigraphs with the vertex set from P(S).

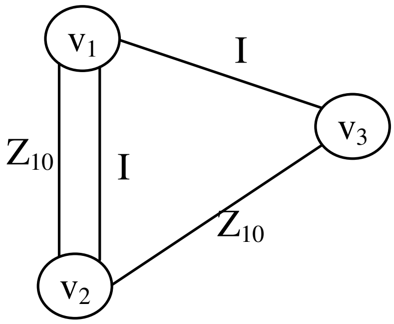
$G_{26} =$



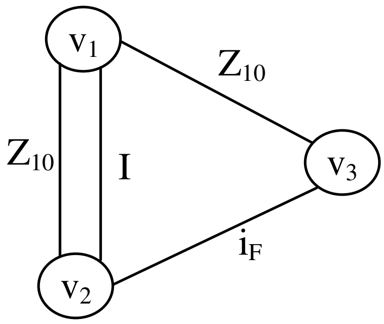
$G_{27} =$



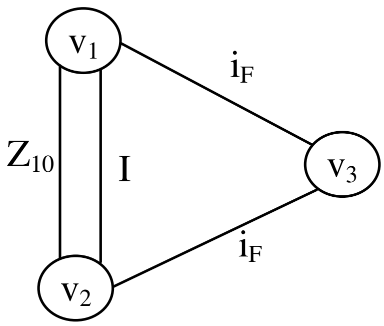
G_{28} =



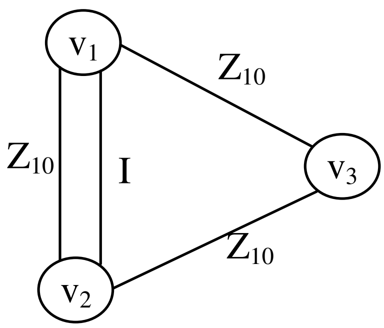
G_{29} =



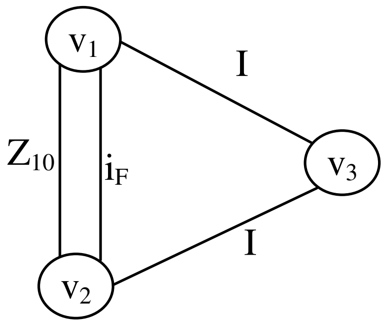
G_{30} =



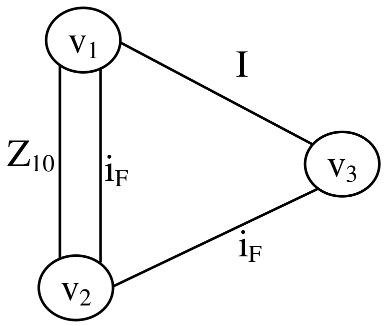
G_{31} =



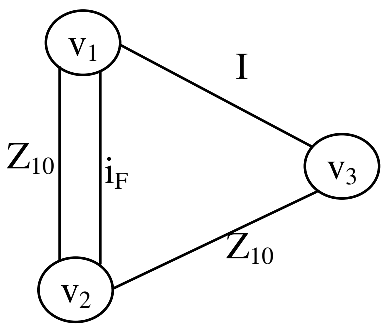
$G_{32} =$



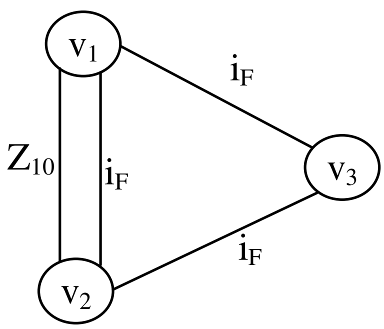
$G_{33} =$



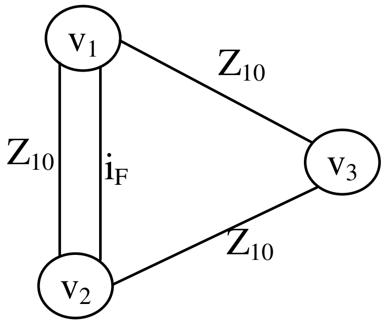
$G_{34} =$



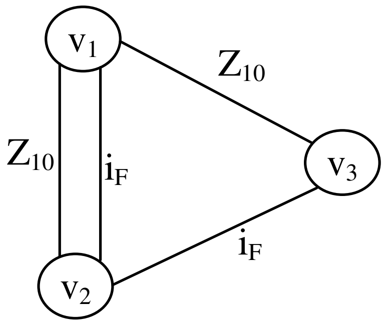
$G_{35} =$



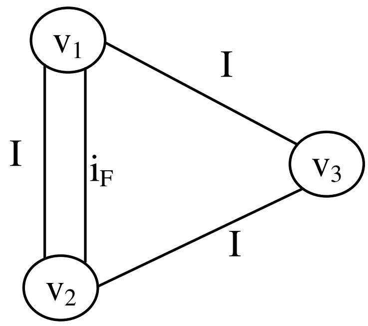
$G_{36} =$



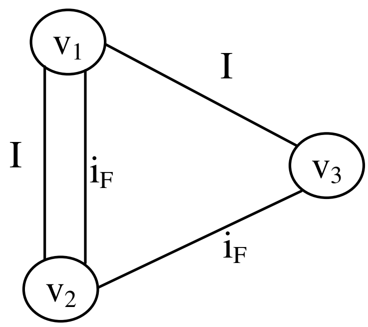
$G_{37} =$



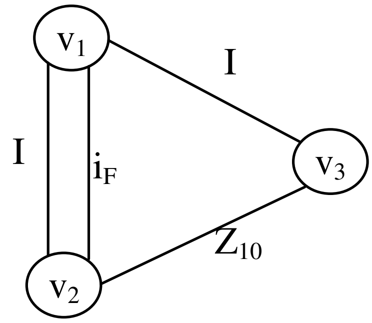
$G_{38} =$



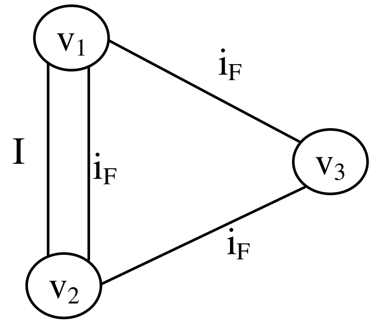
$G_{39} =$



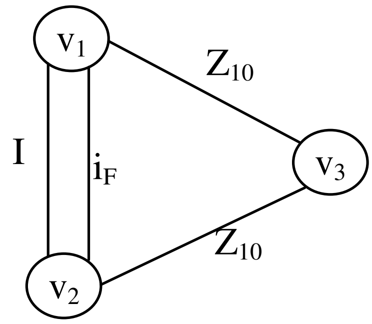
$G_{40} =$



$G_{41} =$



$G_{42} =$



$G_{43} =$

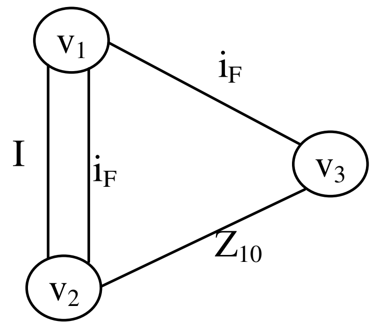
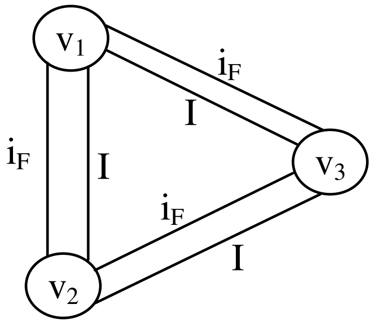


Figure 3.22

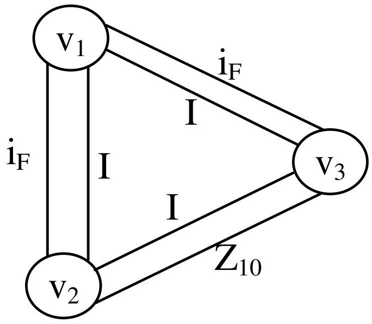
Thus, we see G_{26} to G_{43} are subset vertex multigraphs which are pseudo complete and have only four edges.

Next, we describe the following graphs.

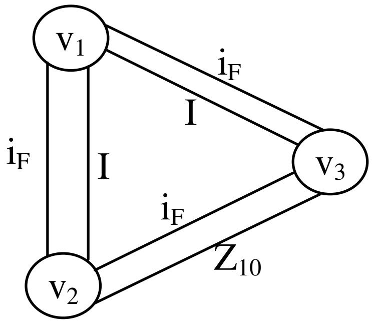
G_{44} =



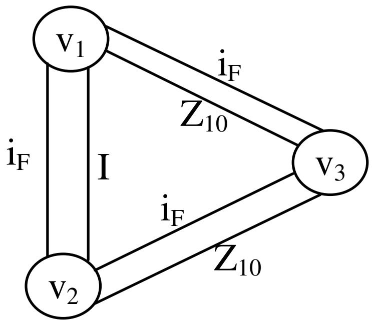
G_{45} =



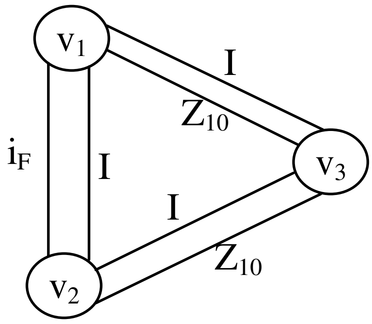
G_{46} =



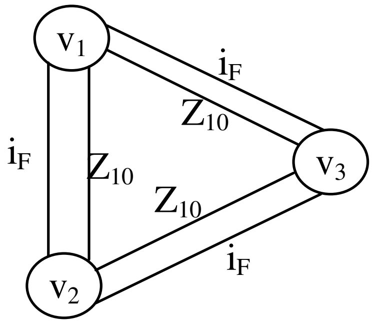
G_{47} =



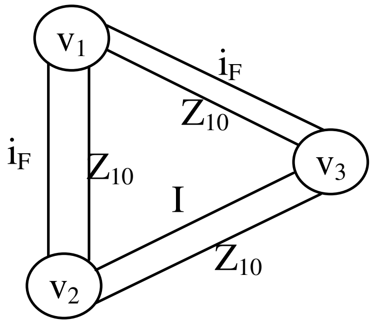
G_{48} =



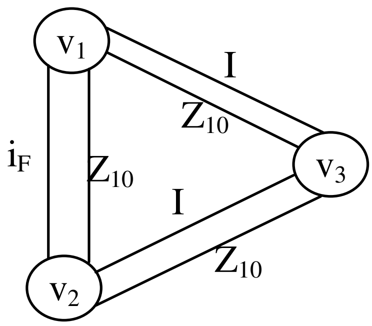
G_{49} =



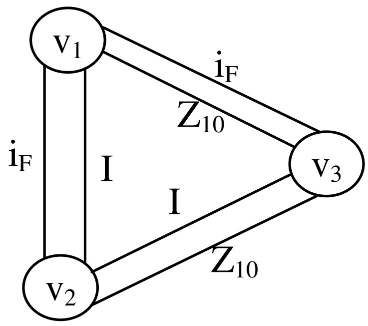
G_{50} =



G_{51} =



G_{52} =



G_{53} =

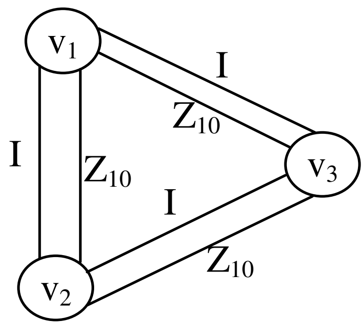
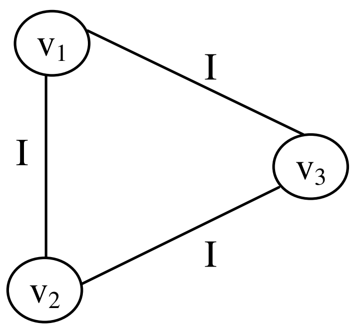


Figure 3.23

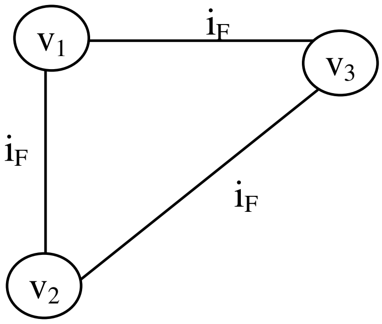
We see there are 10 subset vertex pseudo complete multigraphs with six edges. These multigraphs we also define as subset vertex pseudo complete uniform multigraphs.

Next we give the subset vertex multigraphs which are pseudo complete with 3 edges in the following

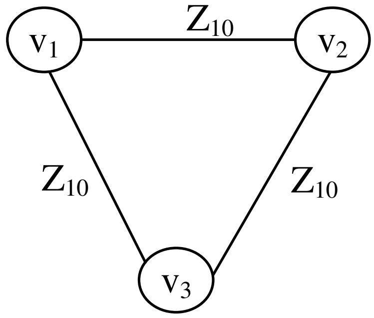
G_{54} =



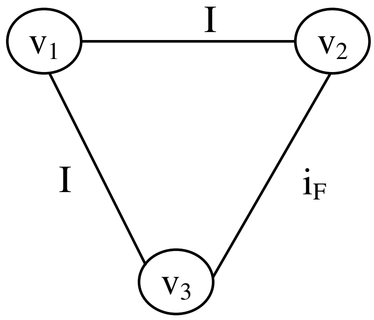
G_{55} =



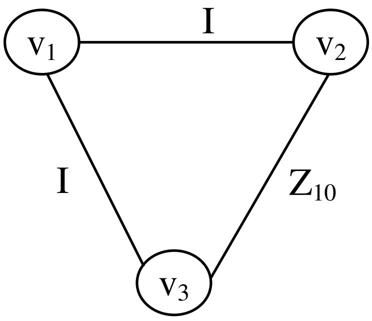
G_{56} =



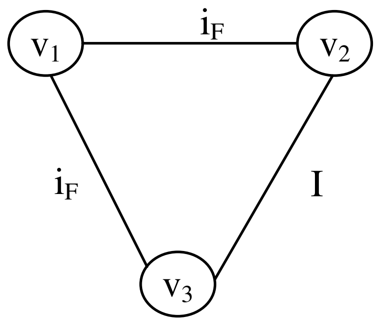
G_{57} =



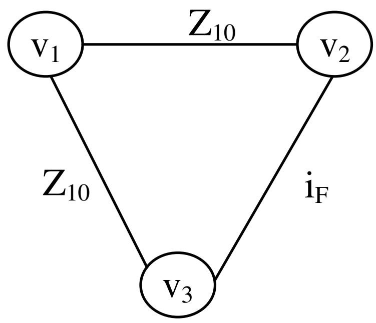
G_{58} =



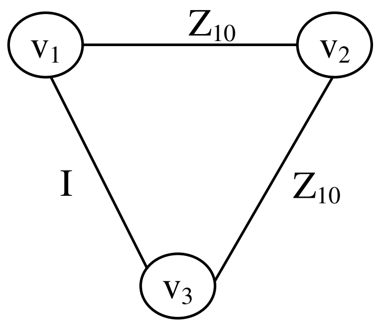
G_{59} =



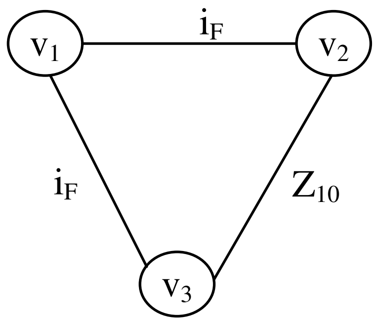
G_{60} =



G_{61} =



G_{62} =



G_{63} =

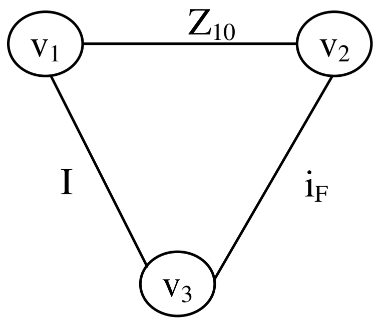


Figure 3.24

There are 10 subset vertex multigraphs which are pseudo complete as well as they are infact subset vertex pseudo complete uniform multigraphs.

Now we proceed to define the notion of subset vertex pseudo complete uniform multigraphs.

Definition 3.3. *Let S be a finite set which has n distinct different sets of objects / events / attributes. P(S) be the power set of S.*

Let v_1, …, v_m be n vertex subsets from P(S). The maximum number of edges that can exists between vertex subsets v_i and v_j is n. Let G be the subset vertex multigraph with vertex set v_1, v_2, …, v_n. If every vertex is connected with all other vertices and the number of multiedges between any two vertices is m, $1 \leq m <$; then G is subset vertex pseudo complete multigraph which we define as a subset vertex pseudo complete uniform multigraph of uniformity of m edges.

We see all pseudo uniform complete subset vertex multigraph is a subset vertex pseudo complete multigraph and not conversely.

To this effect we give the following example.

Example 3.6. Let G = {v_1, v_2, v_3, v_4; $1 \leq i, j \leq 4$} be the subset vertex multigraph given by the following figure.

G =

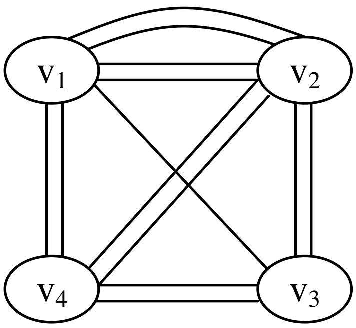


Figure 3.25

Here $v_1, v_2, v_3, v_4 \in P(S)$ where S = {$\langle Z_{10} \cup g\rangle$, $C(Z_{11})$, Z_4, $\langle Z_5 \cup I\rangle$}.

We see G is subset vertex pseudo complete multigraph. This has multisubgraphs which are subset vertex pseudo complete uniform multigraph.

However G is not a subset vertex pseudo complete uniform edge multigraph.

H_1 =

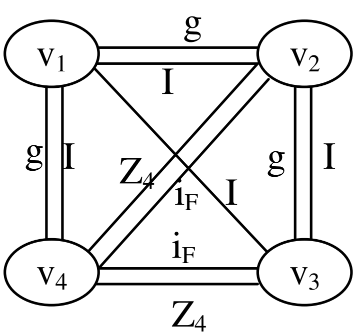


Figure 3.26

is a subset vertex pseudo complete multisubgraph which is not a subset vertex pseudo complete uniform multisubgraph.

$H_2 =$

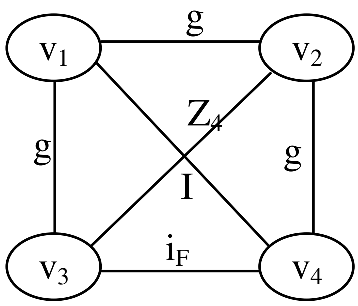


Figure 3.27

is a subset vertex pseudo complete uniform multisubgraph.

$H_3 =$

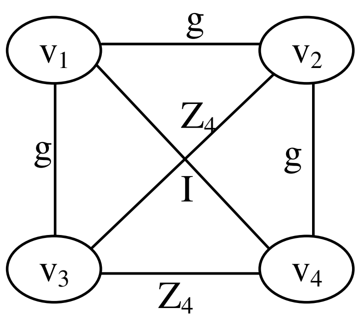


Figure 3.28

is again a subset vertex pseudo complete uniform multisubgraph.

$H_4 =$

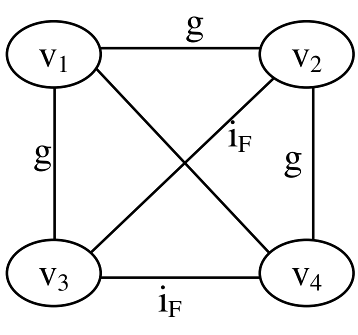


Figure 3.29

H_5 =

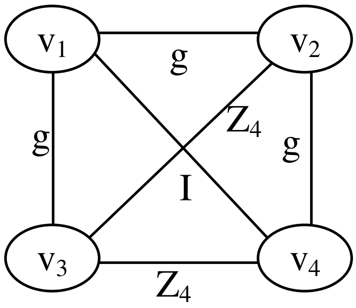


Figure 3.30

and so on. This we have several subset vertex pseudo complete uniform multigraph.

Infact all multigraphs with all the vertices from G that is vertex set is a subset or otherwise of each v_i; $1 \leq i \leq 4$ gives forth to subset vertex pseudo complete subgraphs.

We will find whether other types of subgraphs can be had for this we have to first define the vertex set previously or give values from the elements of S; that is specify the subsets of P(S) to make this verification.

Let us assume the values of v_1, v_2, v_3, v_4, v_5 defined to be as follows from P(S) (P(S)given earlier).

$v_1 = \{\{5, 5g, 3g + 2, 9, g, g + 9\}, \{3 + i_F, 0, 1, i_F, 2 + 3i_F, 5i_F\}, \{0, 2, 3\}, \{I, I + 3, 4 + 2I, 3I, 4I\}\}$,

$v_2 = \{\{9 + 9g, g, g + 9, 5g, 2g, 2g + 1, 4g\}, \{3 + i_F, 2 + 3i_F, 5i_F, 5, 0, 1\}, \{0, 1, 3\}, \{I, I + 3, 3I, 2I, 0,1, 3\}\}$,

$v_3 = \{\{5, 5g, g, 9\}, \{0, 1, i_F, 5i_F\}, \{0, 2, 1\}, \{4 + 2I, 2 + 4I, 4I\}\}$,

$v_4 = \{\{g, 4g+4, 8g + 9\}, \{i_F, 5, 5i_F + 5, 5i_F, 5\}, \{2, 1\}, \{2 + 2I, 4 + 4I, 2, 4, 0\}\}$ and

$v_5 = \{\{3 + 3g, 3, 3g, 1 + g, 1, g, 0\}, \{2, 0\} (9 + 9i_F, 9, 9i_F, 6 + 3i_F, 6 + 3i_F, 6, 3i_F\}, \{4 + 4I, 3 + 2I, 2 + 3I, I, 3I, 2I, 0\}\}$.

Now we give the subset vertex multigraph using the vertex subsets $v_1, v_2, \ldots, v_5$ by the following figure.

K =

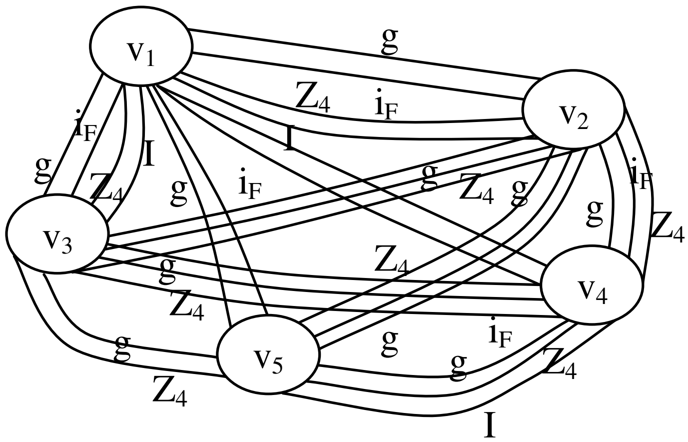


Figure 3.31

Clearly K is a subset vertex pseudo complete multigraph.

Now we consider a subset vertex multisubgraph H_1 of K taking the only vertex set v_1, v_2 and v_4. The figure (graph) of H_1 is as follows.

H_1 =

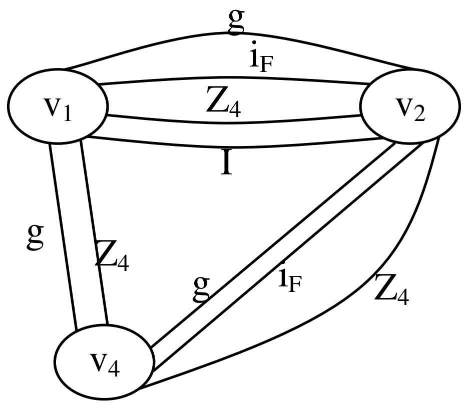


Figure 3.32

Clearly H_1 is also a subset vertex pseudo complete multi subgraph of K.

Let H_2 be the subset vertex multigraph given by the vertex subset v_1, v_3, v_4 and v_5.

$H_2 =$

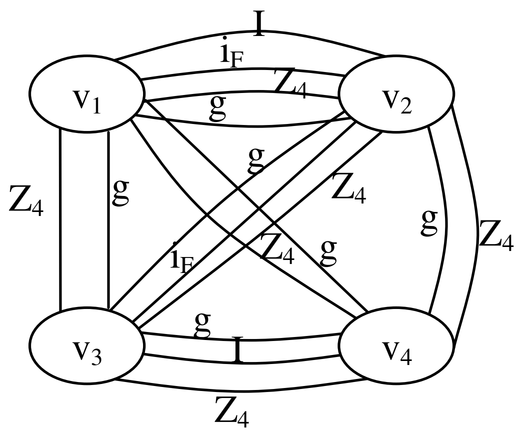


Figure 3.33

We see H_2 is also a subset vertex pseudo complete multisubgraph of K.

Infact K has all subset vertex multisubgraphs to be only subset vertex pseudo complete multiset subgraphs they have either three vertices or four vertices. However, two vertex subset multigraphs are not taken in this collection.

These subgraphs will be taken as the subset vertex classical multisubgraphs where only vertex subset sub-collection are taken to construct the multisubgraphs with vertex subsets.

If G is a subset vertex multigraph with vertices v_1, v_2, …, v_n then we can have $nC_2 + nC_3 + nC_4 + \ldots + nC_{n-1}$ number of nontrivial vertex subset multisubgraphs. All these multisubgraphs are classical that is they are a part of the subset vertex multigraph G.

Example 3.7. Let G = {V, E_m} be a subset vertex multiset multigraph given by the following figure. V = {v_1, v_2, v_3, v_4, v_5, v_6} $\subseteq$ P(S) here S = {$\langle Z_7 \cup I\rangle$, Z_9, $\langle Z_{10} \cup g\rangle$};

G =

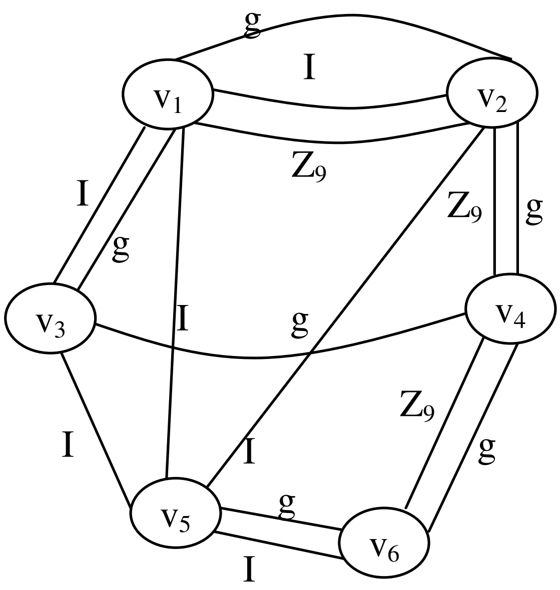


Figure 3.34

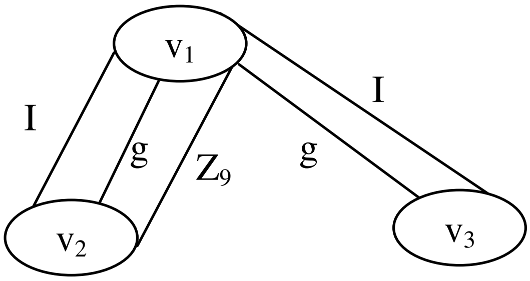
Clearly G is neither a subset vertex complete multigraph or a subset vertex pseudo complete multigraph.

The subset vertex multisubgraphs of G are as follows.

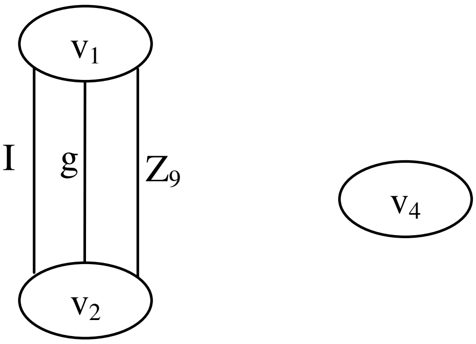
We just do not give the subset vertex multisubgraphs with two vertices from (v_1, v_2, …, v_6 or single point set subset vertex multisubgraphs).

The number of subset vertex multisubgraphs with 3 vertices is $6C_3 = 20$ given by the following figures.

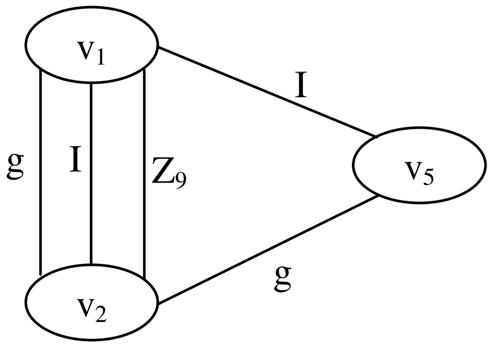
H_1 =



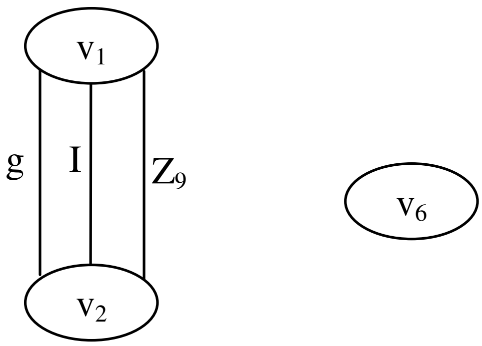
H_2 =



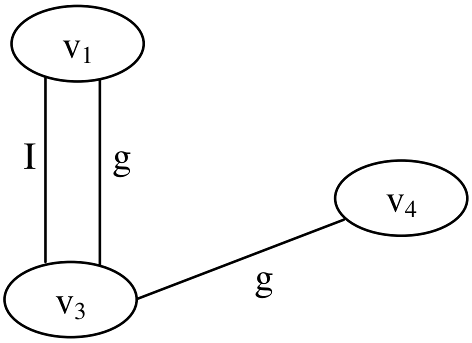
H_3 =



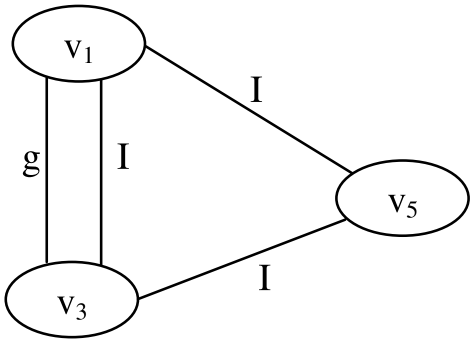
H_4 =



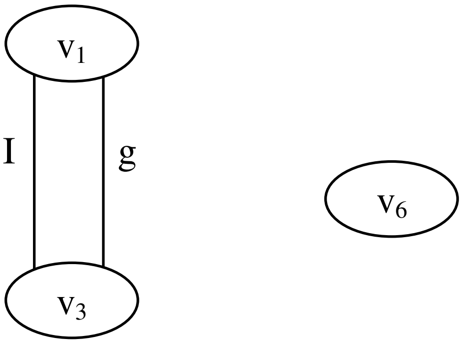
H_5 =



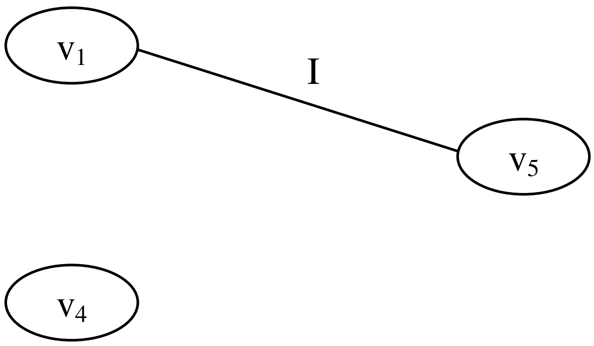
H_6 =



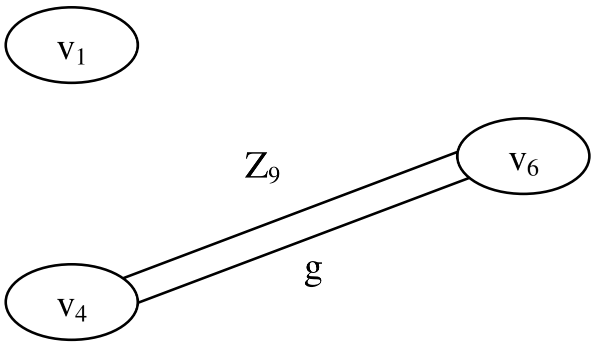
H_7 =



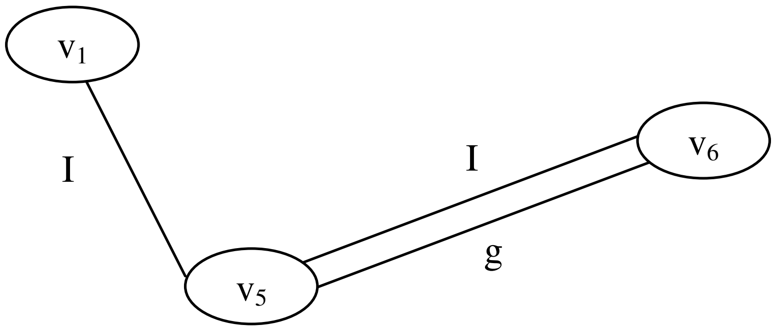
H_8 =



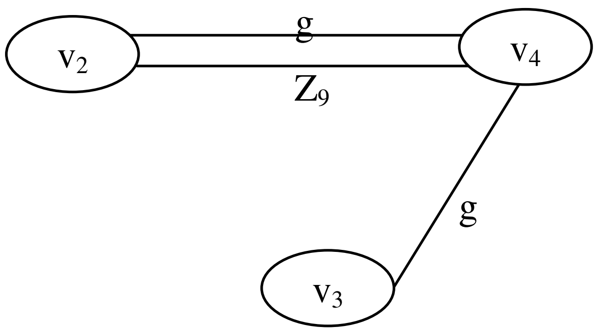
H_9 =



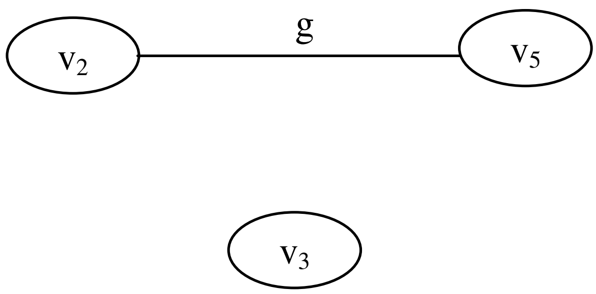
H_{10} =



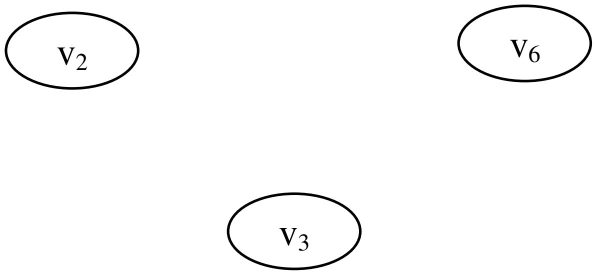
H_{11} =



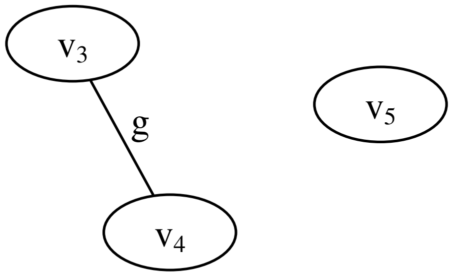
H_{12} =



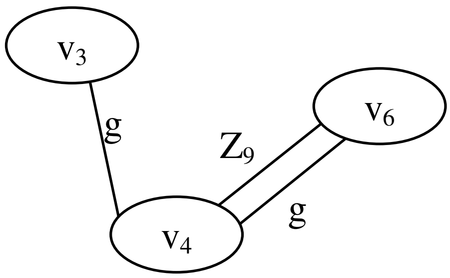
H_{13} =



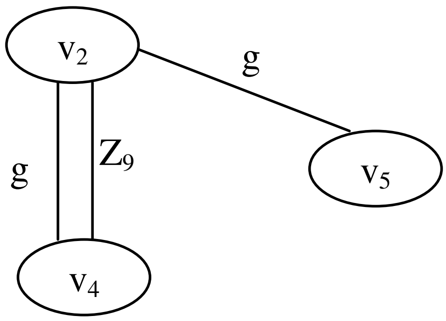
H_{14} =



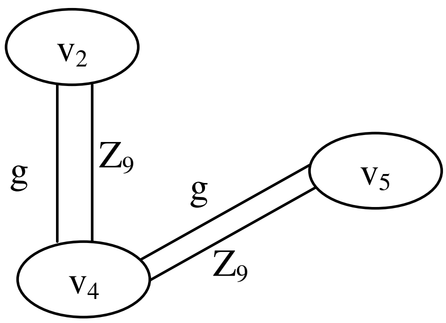
H_{15} =



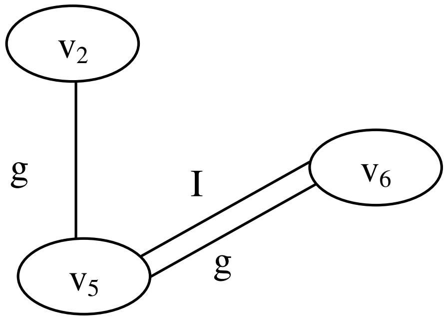
H_{16} =



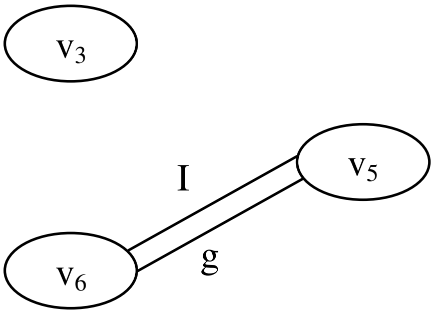
H_{17} =



H_{18} =



H_{19} =



H_{20} =

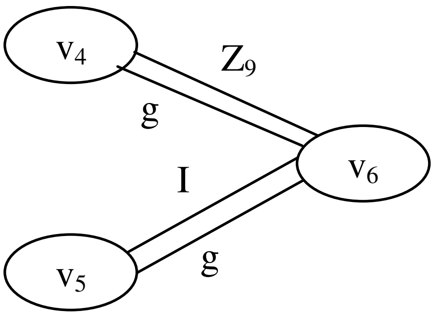
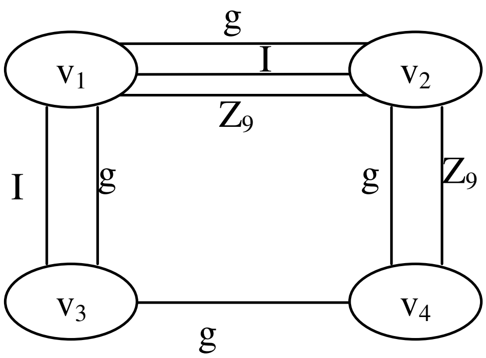


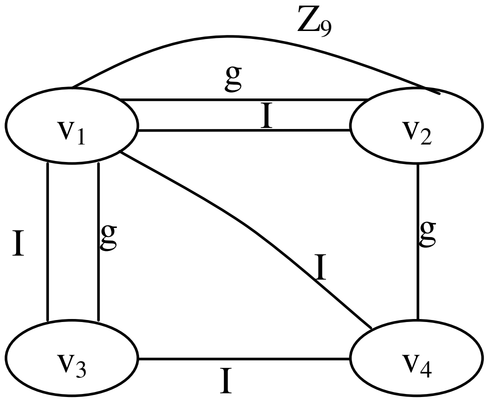
Figure 3.35

Of the 20 subset vertex multisubgraphs H_3 and H_6 are subset vertex pseudo complete multi subgraphs or subset vertex multi triads. H_1, H_5, H_{10}, H_{11}, H_{15}, H_{16}, H_{17} and H_{18} are subset vertex multisubgraphs which are subset vertex multi forbidden triads. Only H_{13} is a subset vertex empty multiset subgraph of G. Next, we consider all subset vertex multi subgraphs with four vertices from the vertex set $\{v_1, v_2, \ldots, v_6\}$ which are given by the following figures.

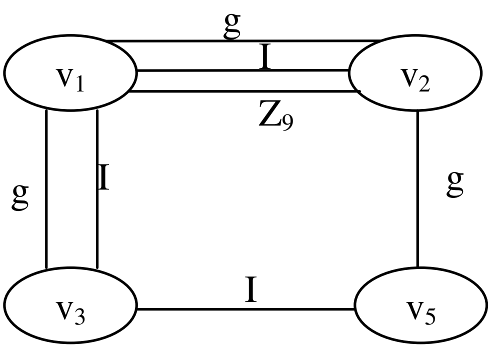
$K_1 =$


g
I
Z9
v1
v2
I
g
g
Z9
v3
v4
g

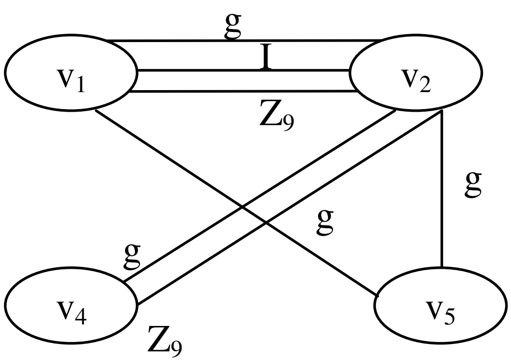
$K_2 =$


Z9
g
I
v1
v2
I
g
g
I
v3
v4
I

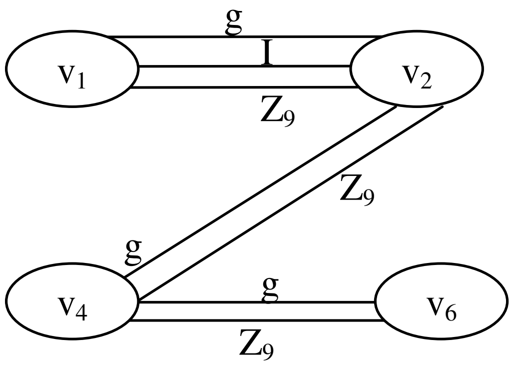
$K_3 =$


g
I
Z9
v1
v2
g
I
g
I
v3
v5

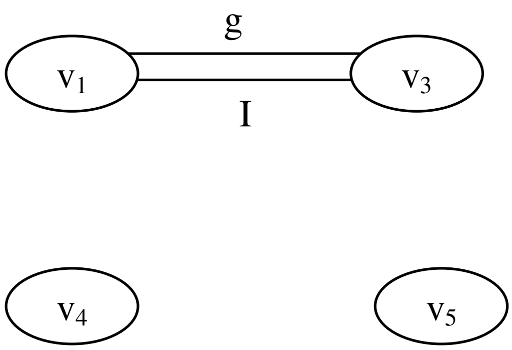
$K_4 =$


g
I
Z9
v1
v2
g
g
g
v4
v5
Z9

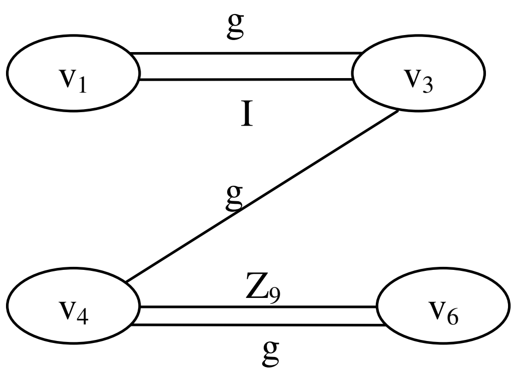
K_5 =


g
I
v_1
v_2
Z_9
Z_9
g
g
v_4
v_6
Z_9

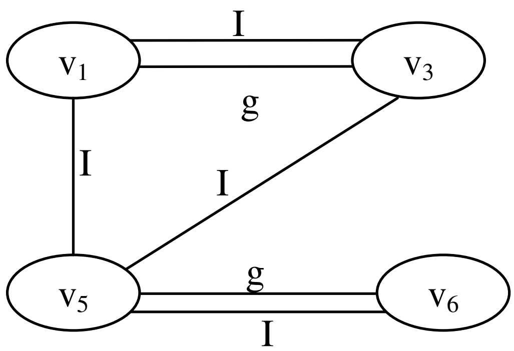
K_6 =


g
v_1
v_3
I
v_4
v_5

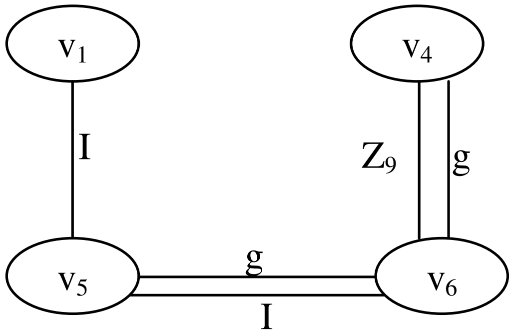
K_7 =


g
v_1
v_3
I
g
Z_9
v_4
v_6
g

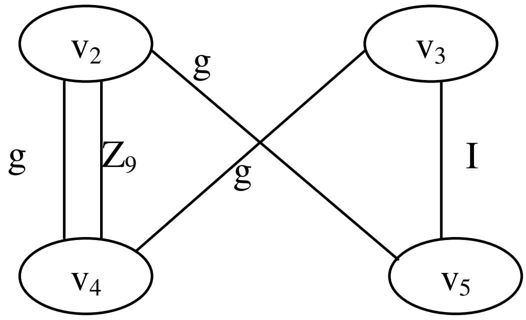
K_8 =


I
v_1
v_3
g
I
I
g
v_5
v_6
I

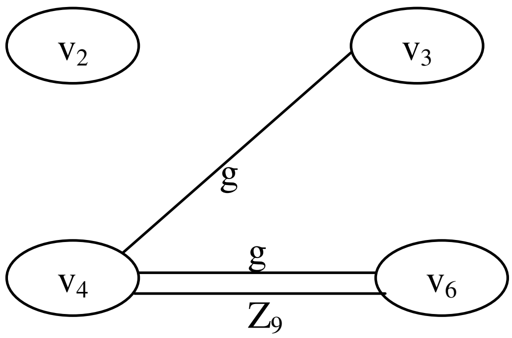
K_9 =



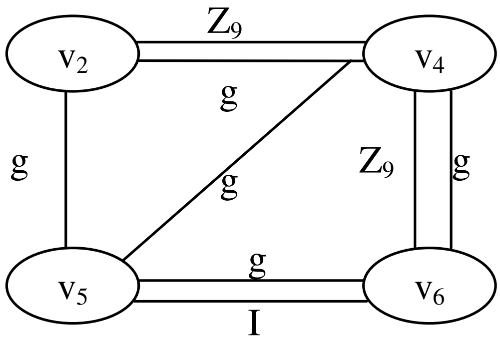
K_{10} =



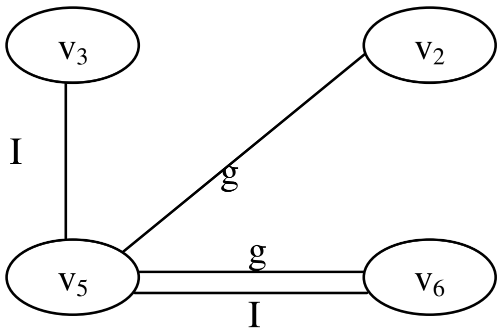
K_{11} =



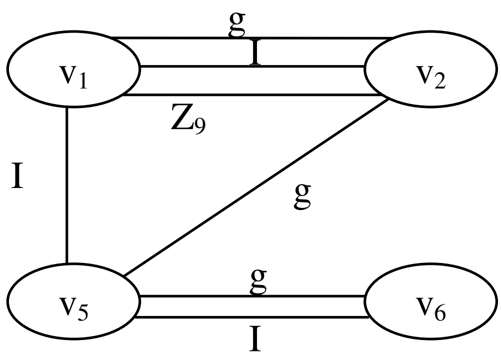
K_{12} =



K_{13} =



K_{14} =



K_{15} =

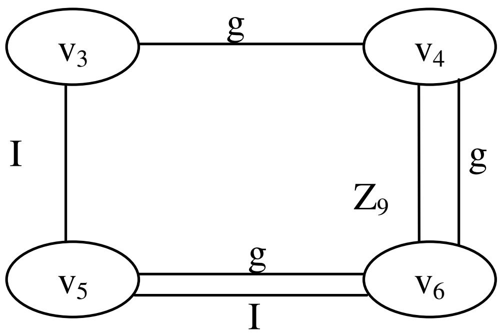
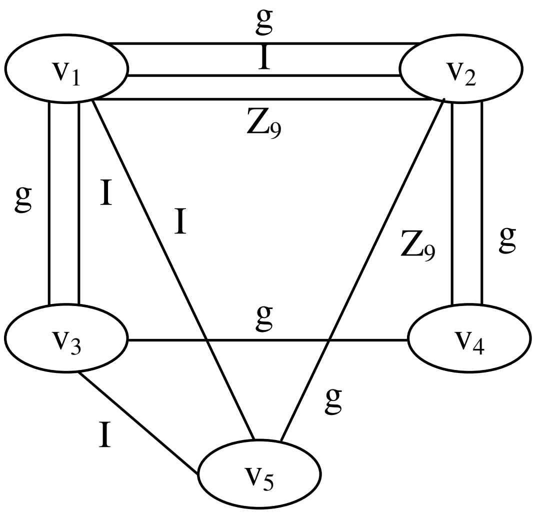


Figure 3.36

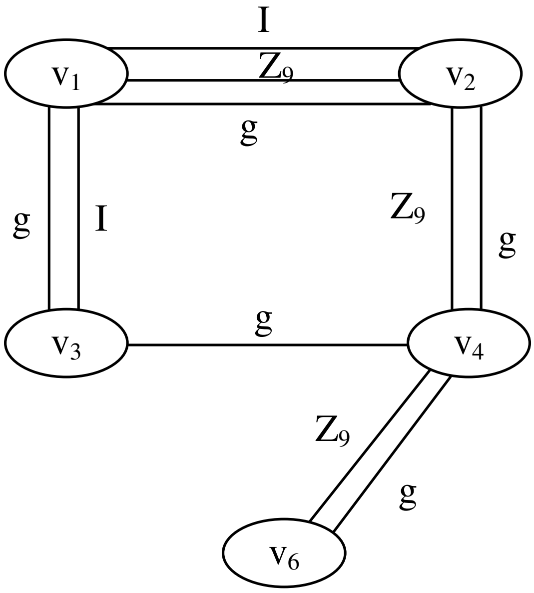
We see none of the 15 subset vertex multisubgraphs are complete or pseudo complete.

None of them is a subset vertex empty multisubgraph. Consider the collection of all subset vertex multisubgraphs with 5 vertices.

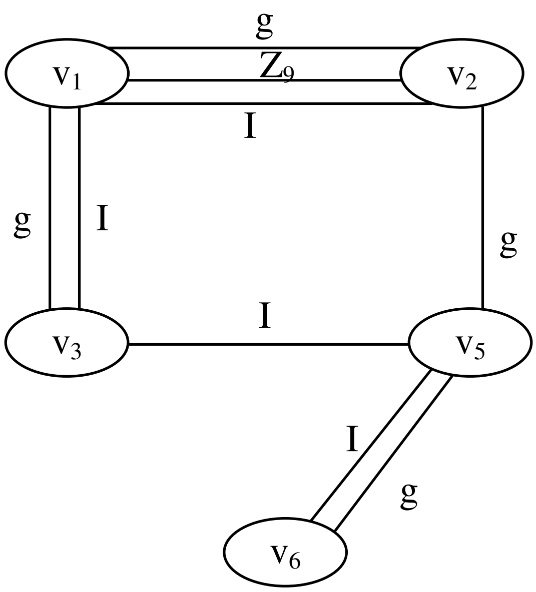
$R_1 =$


g
I
Z9
v1
v2
v3
v4
v5
g
I
I
Z9
g
g
g
I

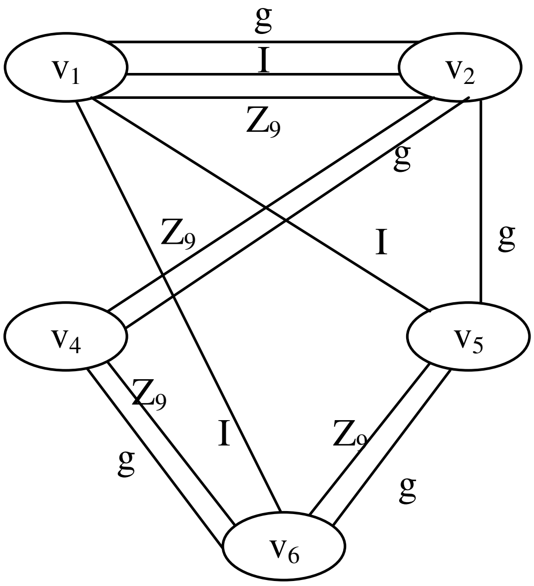
$R_2 =$


I
Z9
g
v1
v2
v3
v4
v6
g
I
Z9
g
g
Z9
g

$R_3 =$


g
Z9
I
v1
v2
v3
v5
v6
g
I
g
I
I
g

R_4 =



R_5 =

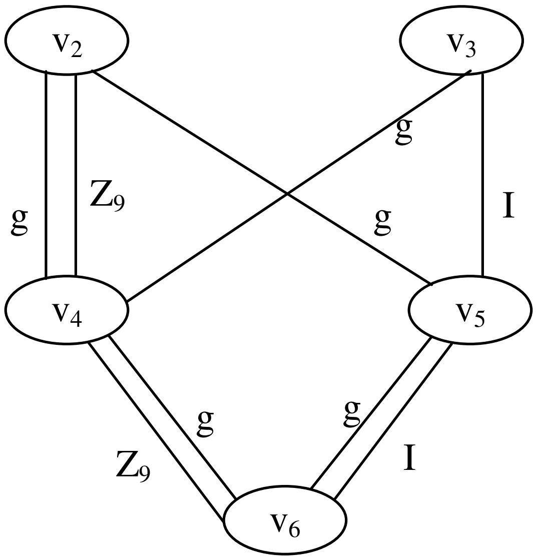


Figure 3.37

We see none of the subset vertex multisubgraphs with 5 vertices is a subset vertex multisubgraph which is complete or pseudo complete.

We see finding subset vertex multisubgraphs using a sub collection from the vertex sets forms a subgraph as in case of classical graphs. We call these subset vertex multisubgraphs as classical subset vertex multisubgraphs.

Definition 3.4. *Let G = {V, E_m} be a subset vertex multigraph with n distinct vertices $v_1, v_2, \ldots, v_n \in P(S)$; the power set of S. The classical subset vertex multisubgraph of G is got by taking some m vertex sets from V and getting the multisubgraphs of G. There are infact n point subset vertex multisubgraphs.*

nC_2 number just denote two vertices subset vertex multisubgraph which have multiedges or just empty nC_3 number of 3 vertex subset multisubgraphs which may be empty or forbidden triads or triads

nC_4 number of 4 vertex subset multigraphs and so on.

Thus we have $nC_1 + nC_2 + \ldots + nC_{n-1}$ number of classical subset vertex multisubgraphs.

Now we first illustrate the subset-subset vertex multi subgraphs which are not classical by some examples.

Example 3.8. Let S = {Z_{10}, Z_8, $\langle Z_{12} \cup I\rangle$, $\langle Z_6 \cup g\rangle$} be the set with four different entities. Let P(S) be the power set of S. Let G be a subset vertex multigraph with vertex set V = {v_1, v_2, v_3, v_4, v_5} $\subseteq$ P(S); where v_1 = {{0, 1, 2, 5, 6, 8, 9}, {0, 2, 4,6, 5}, {9, 9I, 9 + 9I, 6 + 9I, 9 + 6I, 3, 2, 7I}, {2 + 2g, 3, 4g, 5g, g, 1, 0}}, v_2 = {{0, 1, 3, 4, 6, 7}, {0, 1, 2, 5, 3, 7}, {9, 9I, 3, 3I + 3, 6I + 3, 9 + 6I, 0, 1}, {2, 2g, 2 + 4g, 4g + 2, 5g + 4, 1, 0, 3g}}, v_3 = {{0, 5, 6, 2, 4, 7}, {0, 2, 4, 6, 5}, {9, 9I, 9 + 9I, 3, 2}, {3g, 2g + 2, 4g, 0, 1, 5g + 5}}, v_4 = {{4, 2, 9}, {0, 1, 2}, {2g + 2, 3g, 4g, 0, 1} and v_5 = {{I, 9, 9I, 9 + 9I, 6 + 3I, 3I + 6, 0, 1}, {2, 4, 6, 8, 9, 3}, {0, 1, 2, 4}, {9, 0, 1, 2 + 5g, 5 + 2g, 2 + 4g, 4 + 2g}}.

Let G be the subset vertex multigraph given by the following figure with v_1, v_2, v_3, v_4 and v_5 as the vertex subsets.

G =

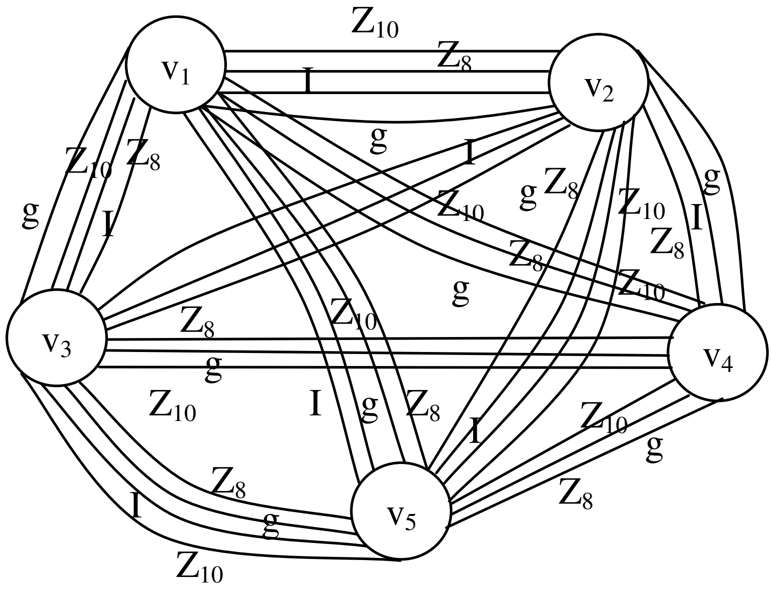


Figure 3.38

Clearly G is a vertex subset pseudo complete multigraph.

Consider the subset-subset vertex multisubgraph W given by the following vertices.

$y_1 \subseteq v_1$, $y_2 \subseteq v_2$, $y_3 \subseteq v_3$ and $y_4 \subseteq v_4$ where

y_1 = {0, 8, 9}, y_2 = {0, 2, 3}, y_3 = {9, 9I, 9 + 9 + 9I} and

y_4 = {3g, 4g}.

W the subset vertex multisubgraph is as follows.

W =

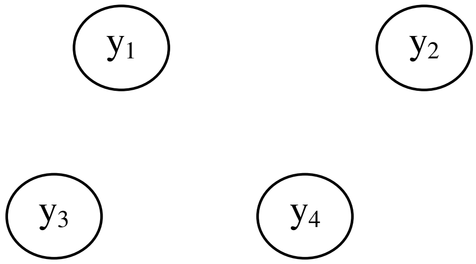


Figure 3.39

Clearly W is a subset vertex empty multisubgraph of G.

Thus, G can have subset vertex multisubgraphs which can be subset vertex complete multisubgraphs, subset vertex pseudo complete multisubgraphs or subset vertex pseudo uniform complete multisubgraphs and subset vertex empty multisubgraphs.

Now our next analysis would be to give examples of subset vertex multigraphs which are trees.

Let us consider the following example.

Example 3.9. Let S = $\{Z_6, \langle Z_7 \cup g\rangle, \langle Z_{11} \cap I\rangle, C(Z_{15})\}$ be the set with four different attributes or concepts. Let P(S) be the power set of S.

Consider the following subset vertex multigraph T given by the following figure which has the following vertex subsets.

$v_1 = \{\{2, 3, 4, 0, 1\}$ $\{g, 2g, 3g, 4g, 2g + 2, 4g + 4\}$, $\{I, 3I, 5I, 7I, 9I, 2 + 2I, 4 + 4I, 0, 6 + 6I, 8 + 8I, 10 + 10I\}$, $\{i_f, 3i_F,$

$5i_F$, $7i_F$, $9i_F$, $11i_F$, $13i_F$, $1 + 2i_F$, $3 + 4i_F$, $5 + 6i_F$, 1, $7 + 8i_F$, $9 + 10i_F$, $11 + 12i_F$, $13 + 14i_F$, 0}}

v_2 = {{2, 5, 3}, {6g, 3g, 2g + 2}, {10I, 3I, 5I, 0}, {$6i_F$, $3i_F$, $5i_F$, $7i_F$}},

v_3 = {{0, 4}, {2g, 4g, 4g + 4}, {7I, 9I, 2 + 2I, 4 + 4I}, {$9i_F$, $11i_F$, $13i_F$, $1 + 2i_F$, $5 + 6i_F$}}

v_4 = {{5}, {6g, 0}, {10, 10I}, {$6i_F$, $8i_F$, $4i_F$}} and

v_5 = {{6g}, {10I}, {$6i_F$}}.

T =

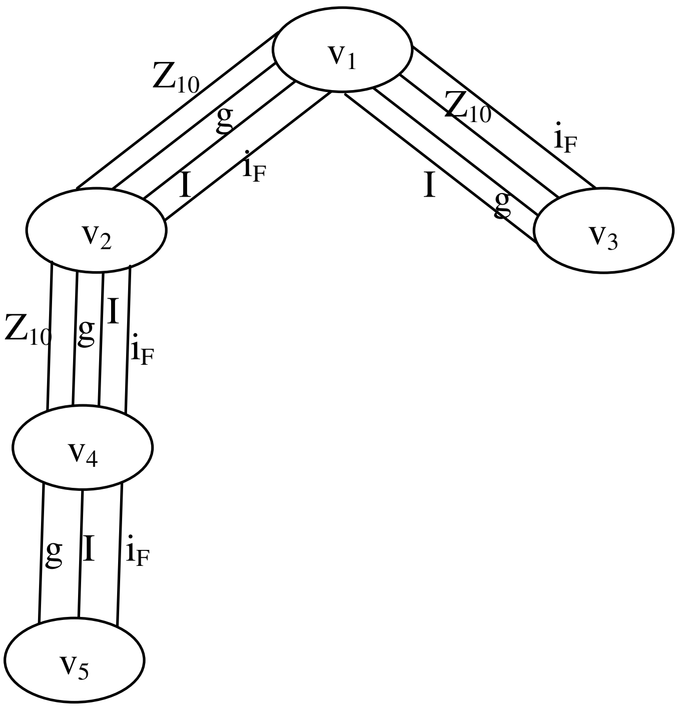


Figure 3.40

This is a subset vertex tree multigraph with four layers and v_1 is the root of the multitree. The multitree is not uniform. We call a subset vertex tree multigraph to be a subset vertex uniform tree multigraph if the number of edges connecting any two nodes is the same.

Consider Z the subset vertex tree multigraph whose vertices are given in the following.

a_1 = {{1, 2, 3, 4}, {g, 2g, 4g, 3, 5, 2 + 3g}, {I, 0, 5I, 2I, 6, 7, 10}, {i_F, 3i_F, 2 + 3i_F, 5, 7, 8}},

a_2 = {{2, 4, 0} {g + 2g, 6, 2, 4}, {i_F, 3i_F, 2, 4, 6} {5I, 0, 2I, 2, 4, 8, 6}}

a_3 = {1, 3, 5}, {5, 3, 4g, 1 + g}, {5, 7, 8, 1 + i_F, 3 + 3i_F}, {5 + 2I, 7I,10},

a_4 = {{0}, {2, 4, 5g}, {2, 4, 6i_F + 6}, {4 + 4I, 2, 4, 10I}} and

a_5 = {{5}, {1 + g}, {1 + i_F}, {7I, 3 + 8I}}.

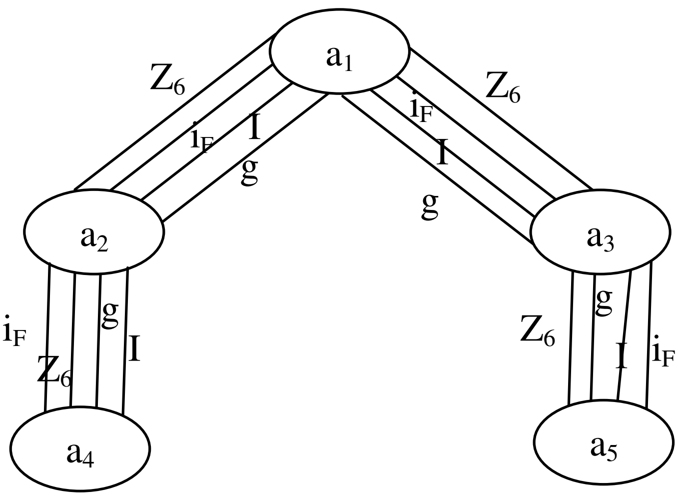


Figure 3.41

Now we see if G is a subset vertex multi tree then it has all its subset vertex multi subgraphs to be only subset vertex multi subtrees or it can be an empty subgraph. However, it is important to keep on record that the number of edges connecting any two of the nodes of the tree may vary.

Here we just give the most generalized form of the subset vertex multigraphs in particular multitrees.

Suppose we have a set of say n attributes; $a_1, \ldots, a_n$ on which the power set P(S) is built. In our assumption $a_1, a_2, \ldots, a_n$ are sets which has more than one element and each of the a_i enjoys a distinct property. That is $a_i \subseteq a_j$ or $a_j \subseteq a_i$ if $i \neq j$; $1 \leq i, j \leq n$. If $X \subseteq P(S)$ the subset X may contain only one attribute subset from a_i or it may contain subsets from both a_i and a_j ($i \neq j$) or it may contain subsets from three sets a_i, a_k and a_j ($i \neq j$, $i \neq k$ and $j \neq k$) ($1 \leq i, j, k \leq n$) and so on or it may contain subsets from every a_j, $1 \leq j \leq n$. These subsets of P(S) are taken as vertices of the subset vertex multigraph. Further if X and Y are two subsets from P(S); $X \neq Y$ and if X has t distinct attribute subsets and Y has say s distinct attribute subsets. Let $X = \{x_1, \ldots, x_t\}$ and $Y = \{y_1, y_2, \ldots, y_s\}$; $\{s \neq t\}$ with $x_i \cap y_i \neq \phi$ for some r values of i. $1 \leq i < t$ and s.

Then we say the vertex subset X and the vertex subset Y are connected by r edges. This is the way a subset vertex multigraph is constructed (we for sake of better understanding and working use for the different attributes the value Z_n, $\langle Z_n \cup I\rangle$, $\langle Z_t \cup g\rangle$ $C(Z_p)$ and so on).

The edges from X to Y are denoted by $a_{i1}, a_{i2}, \ldots, a_{ir}$ where $1 \leq i_1, i_2, \ldots, i_r \leq n$. So in general any subset vertex multigraph can have a maximum of n edges connecting any two vertices or one or two or $n - 1$ or no edges connecting them.

We call these multigraphs which has maximum possible n edges as subset vertex n-multigraphs or subset vertex multigraphs with n-maximum edges between any two possible vertex subsets.

Vertex subset multigraphs can be binary trees or n-ary trees or star graphs or complete or pseudo complete or uniform pseudo complete multigraphs.

We just provide examples of subset vertex multigraphs which are trees. Already we have provided illustrative examples of subset vertex multigraphs which are complete pseudo complete and uniform pseudo complete.

Example 3.10. Let S = $\{Z_{12}, Z_7, \langle Z_{10} \cup g\rangle, C(Z_{15})\}$ be a set and P(S) be the power set of S. Let K be the subset vertex multigraph with v_1, v_2, v_3, v_4 and v_5 as subsets of P(S).

Here $v_1 = \{\{0, 2, 4, 8\}, \{3, 5, 1\}, \{g, 5g, 5 + 5g, 9g, 0\}, \{i_F, 9 + 9i_F, 9, 9i_F, 2 + 3i_F, i_F + 2, 0\}\}$,

$v_2 = \{\{0, 1, 3, 5, 2\}, \{1, 2, 3, 4\}, \{2g, 9g, 0, 3 + 3g, 3g\}, \{9i_F, 9, 2 + 3i_F, i_F, 0\}\}$,

$v_3 = \{\{9, 7, 6\}, \{6, 0\}, \{9, 9i_F, 9 + 9i_F, i_F, 0, 1\}, \{9g, 0, 9 + 9g, 2, 4, 8g\}\}$,

$v_4 = \{\{9, 7, 2, 0\}, \{3, 6, 0\}, \{9g, 5g, 2g, g, 1\}, \{9, 9i_F, 0\}\}$ and $v_5 = \{\{Z_{12}\}, Z_7, Z_{10}g, Z_{15}i_F\}$ are the vertices of K the graph of which is as follows. Clearly once given the vertex set the multigraph with subset vertices the multigraph is unique illustrated by the following figure;

K =

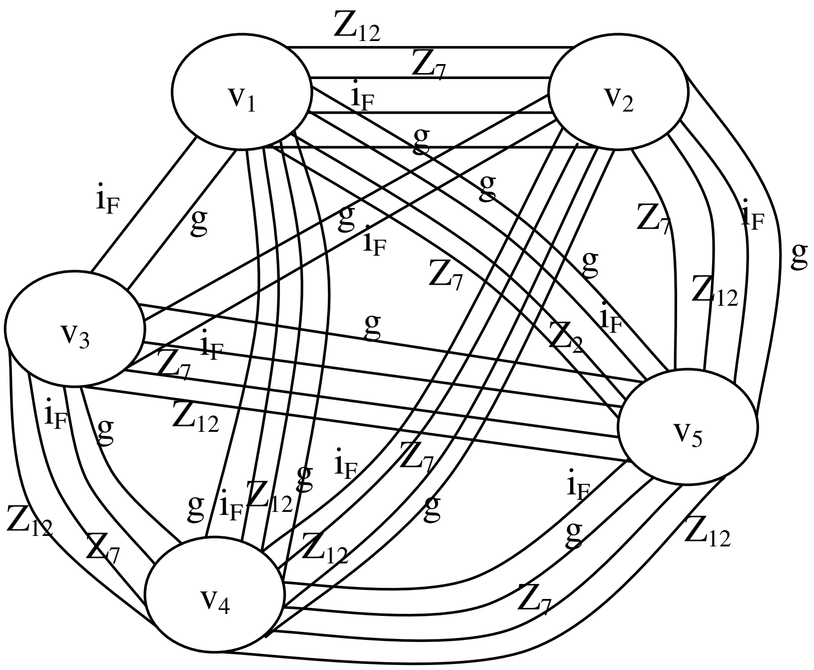


Figure 3.42

Let G_1 be the subset vertex multi subgraph with vertex set v_1, v_2, v_3 and v_4 given by the following figure.

G_1 =

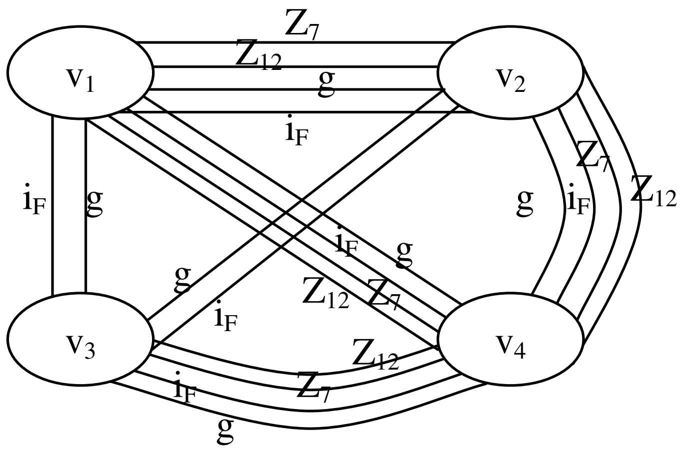


Figure 3.43

Clearly G_1 is a pseudo complete subset vertex multi subgraph of K which is also a hyper multigraph.

Let G_2 be the subset vertex multisubgraph of K with vertex set v_1, v_2, v_3 and v_5 given by the following figure.

G_2 =

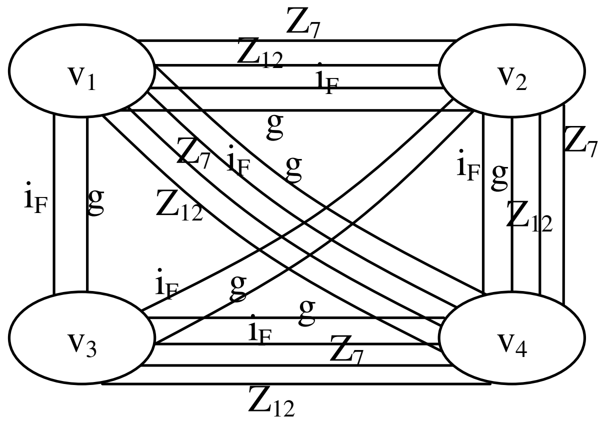


Figure 3.44

Clearly G_2 is also a pseudo complete subset vertex multi subgraph which is also a special hyper multisubgraph of K.

Consider the vertex set $\{v_1, v_2, v_4, v_5\} \subseteq \{v_1, v_2, v_3, v_4, v_5\}$; let G_3 be the subset vertex multi subgraph of K given by the following figure.

G_3 =

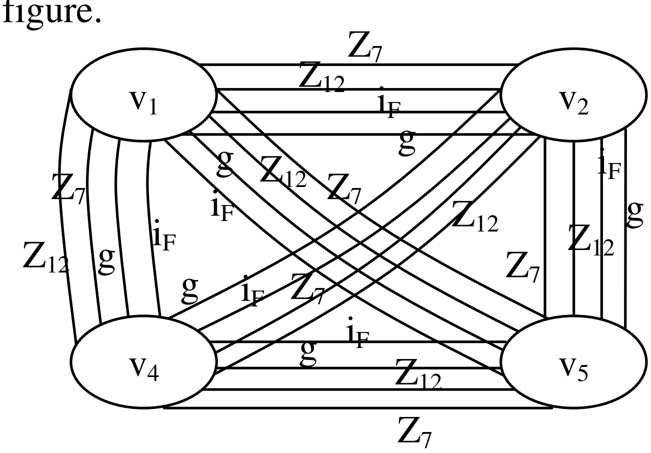


Figure 3.45

Clearly G_3 is a subset vertex multi subgraph of K which is complete, though K is not a complete subset vertex multigraph it is only a pseudo complete subset vertex multigraph.

G_3 is the complete special hyper subset vertex multi subgraph of K. Consider the subset vertex multi subgraph G_4 with vertex set $\{v_1, v_3, v_4, v_5\}$ given by the following figure.

G_4 =

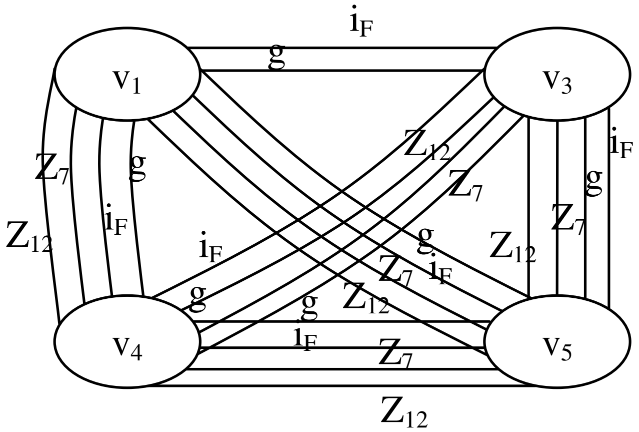


Figure 3.46

G_4 is only a pseudo complete vertex subset multi subgraph of K which is pseudo complete.

Finally consider the vertex subset multi subgraph G_5 with vertex set as $\{v_2, v_3, v_4$ and $v_5\}$ given by the following figure.

G_5 =

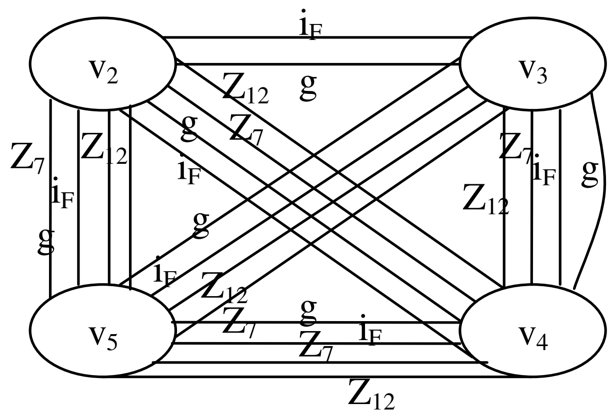


Figure 3.47

Clearly G_5 is only a pseudo complete subset vertex multisubgraph of K which is a hyper multi subgraph of K.

Thus K has 5 subset vertex multi subgraphs which are special hyper multi subgraphs of which only one subset vertex multi hyper subgraph is complete and all the others are pseudo complete.

Further it is easily verified there are four subset vertex multi subgraphs with 3 vertex set which are complete but they are not special subset vertex hyper multi subgraphs of K.

The definition of subset-subset vertex multisubgraphs which are special hyper are defined differently. In case of subset-subset vertex multisubgraphs of G if we take for all the vertex subsets of G their proper subsets then those subset-subset vertex multisubgraphs will be the special subset vertex multi subgraph.

However, subset-subset vertex multisubgraphs according to our conversation will certainly have the number of vertices (or vertex subsets) to be less than that of the subset vertex multigraph G which basically considered as the parent subset vertex multigraph. The difference between subset vertex multisubgraphs and subset-subset vertex multi subgraphs is that in case of subset vertex multisubgraphs we take vertex set from the parent graph as a sub collection of $\{v_1, v_2, \ldots, v_n\}$ say.

However, in case of subset-subset vertex multisubgraphs we take vertex subsets as subsets of vi; that is $u_i \subseteq v_i$ for some i; $1 \leq i \leq n$ with the restriction the number of u_i are strictly less than n. That is only atmost $n - 1$ subsets of the vertex subsets is chosen from $\{v_1, \ldots, v_n\}$. We will first illustrate this situation by some examples.

Example 3.11. Let S = {$\langle Z_9 \cup g\rangle$, Z_{18}, $C(Z_{12})$} be a set. P(S) be the power set of S. Let G be the subset vertex multigraph given by the following set of vertices v_1 = {{g + 1, g + 2, 3g, 5g + 5, 0, 1}, {3, 6, 8, 7, 0, 5, 10}, {10 + i_F, 2i_F, 5i_F + 4, 3i_F + 7, 0, 1 + i_F, i_F}}, v_2 = {{g + 1, 2g + 2, 0, 5g + 5, 4 + 4g, 6 + 6g}, {2, 4, 6, 8, 10, 12, 14, 16, 0}, {i_F, 10 + i_F, 2i_F, 4i_F, 7i_F + 7, 3, 2}}, v_3 = {{g + 1, 0, 2, 4, 6, 8}, {3, 5, 7, 9, 11, 13}, {i_F, 5, 4, 6, 8, 8i_F}}, v_4 = {{g + 5, 3g, 3, 5, 7}, {11, 13, 0, 1}, {i_F, 3 + 4i_F, 5 + 6i_F, 2i_F, 0}} and v_5 = {{2g, 4g, 6g, 8g, 2 + 7g, 7 + 2g}, {1, 3, 5, 7, 9, 11, 0}, {i_F + 9, 9i_F + 1, 9i_F, 9, 9 + 9i_F, 2 + 9i_F, 0}}.

Let M be the subset vertex multigraph using the vertex set {v_1, v_2, …, v_5}.

M =

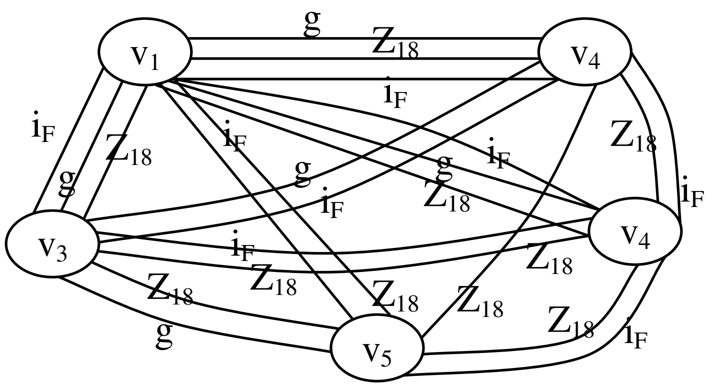


Figure 3.48

Now we give only a few example of subset-subset vertex multi subgraphs of the subset vertex multigraph M.

Let u_1 = {{g + 1, 3g}, {3, 6, 0, 8}, {2i_F, 0, i_F, 1 + i_F} ⊆ v_1, u_2 = {{g + 1, 0, 2g + 2}, {2, 4, 8, 16, 0}, {i_F, 1 + i_F, 10 + i_F} ⊆ v_2, u_3 = {{g + 1, 0, 8}, {3, 5, 11}, {i_F, 8i_F, 6}} ⊆ v_3 and u_4 = {{3g, 3, 5}, {0, 1, 11}, {i_F, 0, 2i_F}} ⊆ v_4 be the vertex subsets of the vertex subsets {v_1, v_2, …, } of M.

Now let H_1 be the subset-subset vertex multi subgraph of M given by the following figure.

H_1 =

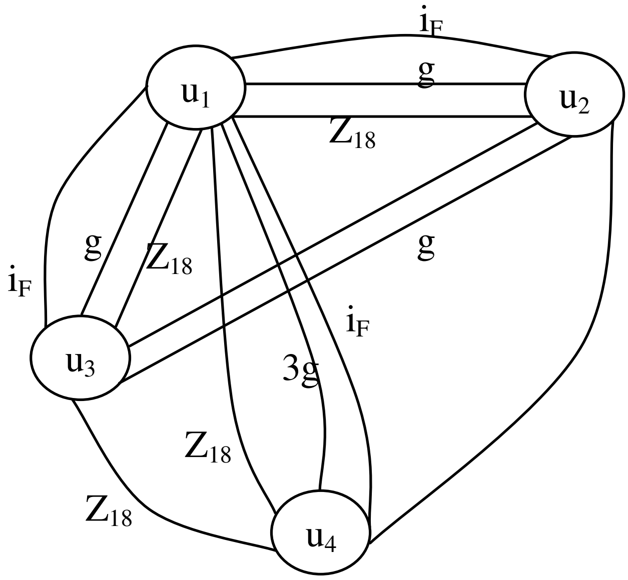


Figure 3.49

Both M and H_1 are pseudo complete subset vertex multigraphs. Infact H_1 is a subset-subset vertex multisubgraph which is also pseudo complete. We define this H_1 as a special pseudo hyper multi subgraph of G. We call it pseudo for in the first place the number of edges connecting any u_i with u_j is not the same as the number of edges connecting any relevant v_i with v_j. However, the number of vertices with H_1 has is the maximum for it to sustain as a special proper hyper multisubgraph.

Consider the subset-subset vertex multi subgraph H_2 given by the following vertex subsets $\{s_1, s_2, s_3, s_5\}$ from $\{v_1, v_2, v_3, v_5\}$.

$s_1 = \{\{5g + 5, 0, 1\}, \{5, 10\}, \{10 + i_F, 3i_F + 7\}\} \subseteq v_1$, $s_2 = \{\{5g + 5, 2g + 2\}, \{2, 4, 6, 8\}, \{10 + i_F, 4i_F, 3\}\} \subseteq v_2$, $s_3 = \{\{g + 1, 8\}, \{3, 5\}, \{i_F, 8i_F\} \subseteq v_3$ and $s_5 = \{\{2g, 8g, 7 + 2g\}, \{3,$

5, 0}, {$9i_F$, 9, 0}} ⊆ v_5. The subset-subset vertex multi subgraph H_2 is as follows:

H_2 =

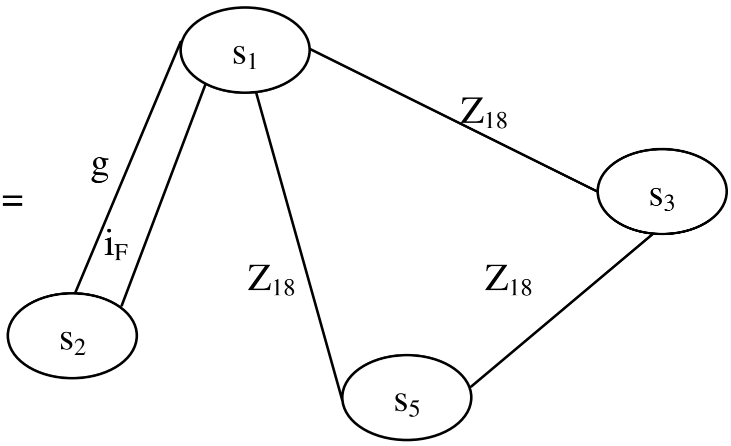


Figure 3.50

H_2 is a subset-subset vertex multi subgraph which is not pseudo complete so H_2 is not a special hyper subset vertex multi subgraph of M though H_2 has 4 vertices.

Thus even if the multisubgraph has (n – 1) of the subset vertices from the subset of n vertices we do not call them as hyper subset-subset vertex multisubgraph the demand being the structure should be preserved though not the number of edges connecting any pair of vertices be preserved.

So H_2 fails to be a hyper multisubgraph of M. It is only a subset-subset vertex multisubgraph of M.

Now consider the vertex subset {u_1, u_2, u_4, u_5} ⊆ {v_1, v_2, v_4, v_5} given in the following:

u_1 = {{3g, g + 2}, {3, 7, 10}, {10 + i_F, $5i_F$ + 4, $3i_F$ + 7}} ⊆ v_1; u_2 = {{4 + 4g, 6 + 6g}, {16, 14, 12}, {3, 2, $4i_F$, 7 + $7i_F$}}

$\subseteq v_2$; $u_4 = \{\{3, 5, 7\}, \{0, 1, 13\}, \{5 + 6i_F, 2i_F, 0\}\} \subseteq v_4$ and $u_5 = \{\{2g, 6g, 2 + 7g\}, \{1, 3, 7, 0\}, \{i_F + 9, 0, 2 + 9i_F, 9\}\} \subseteq v_5$. Let H_3 be the subset-subset vertex multi-subgraph of M given by the following figure.

H_3 =

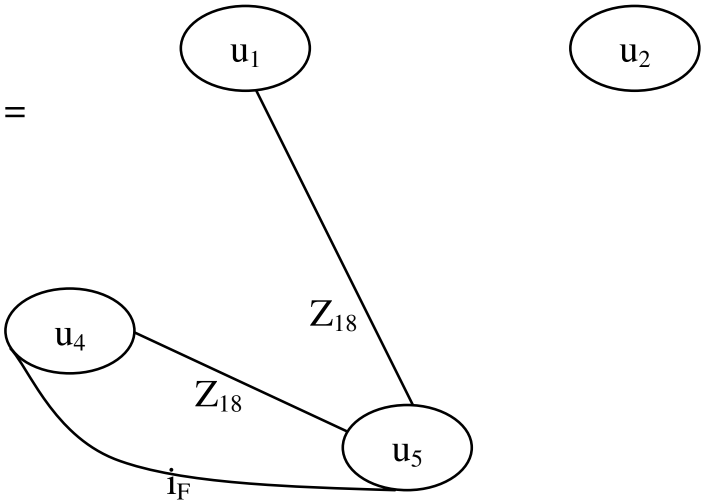


Figure 3.51

H_3 is a subset-subset vertex multi subgraph of M which is not pseudo complete so H_3 is not a special hyper multi subgraph of M.

Thus it is left as an exercise for the reader to find all subset-subset vertex multisubgraph of M. It is also an interesting task to find the number of subset-subset vertex hyper multi subgraphs of M.

Further it is pertinent to keep on record that the number a subset vertex multi subgraphs of M with 4 vertices is only 5, however the number of subset-subset vertex multi subgraphs with four vertex subset is very large. For even in this case it is a very large number. Only some of them will be hyper multi subgraphs of M. Finding the number of such hyper multi subgraphs of M also happens to be a difficult problem. We will give an example of a subset-subset vertex multi subgraphs of a subset vertex multigraph and give some examples of them.

Example 3.12. Let S = {Z_{10}, C(Z_{11})} be the finite set. P(S) be the power set of S. Let v_1, v_2, v_3 and v_4 be the vertices of the subset vertex multigraph G.

u_1 = {{2, 4, 6, 0}, {i_F, $3i_F$, $5i_F + 5$, $4i_F + 2$}}, u_2 = {3, 7, 5, 6, 2}, {$3i_F$, $2 + 2i_F$, $4 + 2i_F$, $4i_F$, i_F, $9i_F + 2$}}, u_3 = {{0, 3, 6, 1}, {$2 + 2i_F$, $4i_F$, $2i_F$, 2, i_F}} and u_4 = {{2, 8, 6, 3, 5}, {$1 + i_F$, $8i_F$, $3i_F$, i_F}} be the vertices of the subset vertex multigraph. Let H be the subset vertex multigraph given by the following figure.

H =

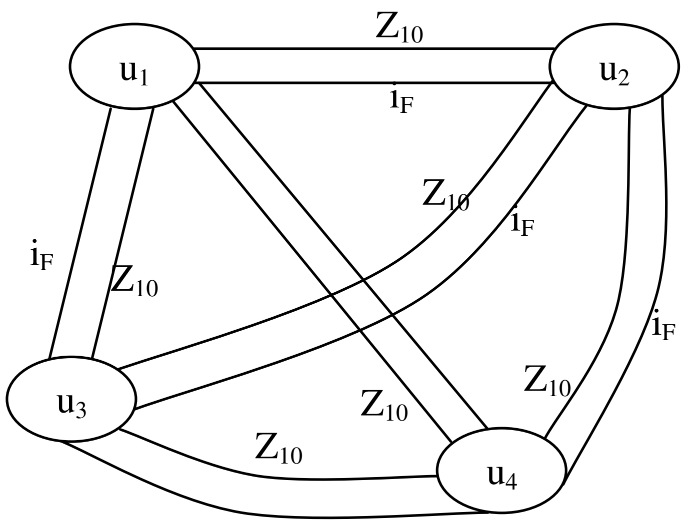


Figure 3.52

Clearly H is a complete subset vertex multigraph. We now find all the subset vertex multisubgraphs and the number of all subset-subset vertex multisubgraphs.

Clearly the number of subset vertex multi subgraphs are ${}_4C_1 + {}_4C_2 + {}_4C_3 = 4 + 6 + 4 = 14$.

However the number of subset vertex multisubgraphs given by ${}_4C_1$ are just point set multisubgraphs as we do not encourage loops we just ignore them. This has only 10 such subset vertex multi subgraphs. We find all subset-subset vertex multi subgraphs of H. The vertex subset u_1 = {{2, 4, 6, 0}, {i_F, $3i_F$, $5i_F + 5$, $4i_F + 2$}} has its power set $P(u_1) \setminus \{\phi\}$ to be $2^8 \setminus \{1\}$ the power set of $P(u_2) \setminus \{\phi\}$ is $2^{10} \setminus \{1\}$ and the power set of $P(u_4) \setminus \{\phi\}$ is $2^8 \setminus \{1\}$. Hence we have $(2^8 \setminus \{1\}) \times (2^{10} \setminus \{1\}) \times (2^9 \setminus \{1\})$ and so on number of subset-subset vertex multigraphs some of which may be defined and some may not be defined. So according to need a researcher or a technician can choose appropriately the vertex subset multisubgraphs or subset-subset vertex multisubgraph.

We will give a few lines of caution. In the first place we do not entertain the notion of empty set in the power sets created by $P(u_4)$, $P(u_2)$, $P(u_3)$ and $P(u_4)$ that is why we denote explicitly $P(u_1) \setminus \{\phi\}$, $P(u_2) \setminus \{\phi\}$ and so on.

The main reason for this is we cannot induct empty set as we have already the empty set ϕ with the universal collection. Now we see there are $2^8 \setminus \{2\}$ multigraphs taking the subsets from $P(u_4) \setminus \{\{\phi\}, u_1\}$. Just we only for the sake of understanding of the reader mention a few of these subset vertex multi subgraphs. Let K_1 take its vertex subset as {{{2}, {i_F}}, {u_2 and u_4} given by the following figure:

$K_1 =$

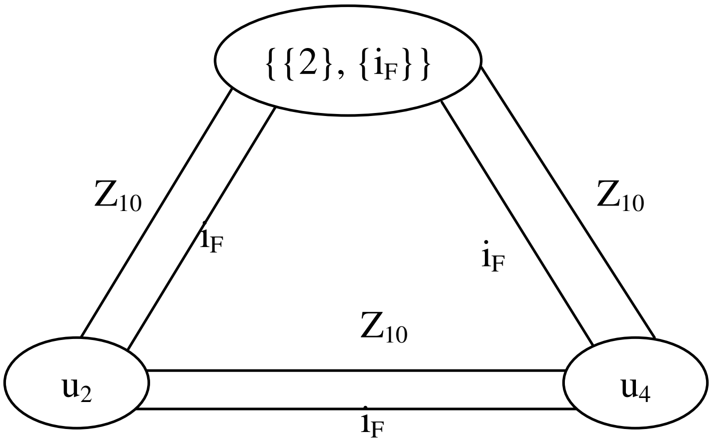


Figure 3.53

Clearly K_1 is again a complete subset-subset multi subgraph of H. It is infact a strong multigraph triad; we will give one example of a weak multigraph triad.

Let K_2 be the subset-subset vertex multi subgraph of H given by the following figure.

$K_2 =$

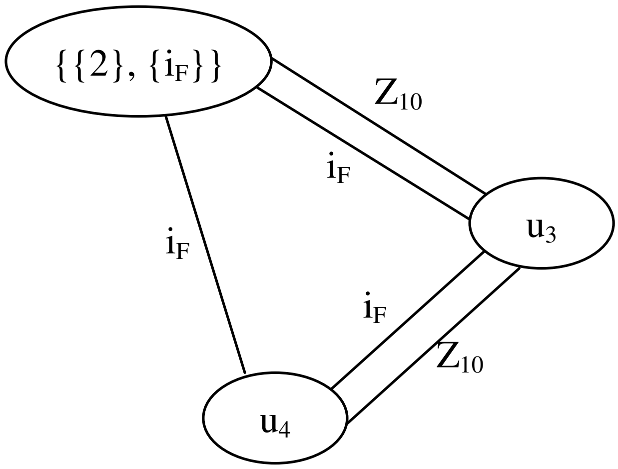


Figure 3.54

Clearly K_2 is only a pseudo complete subset-subset vertex multisubgraph of H which is an illustration of the weak multisubgraph triad.

Let K_3 be the subset-subset vertex multi subgraph of H given by the following figure.

$K_3 =$

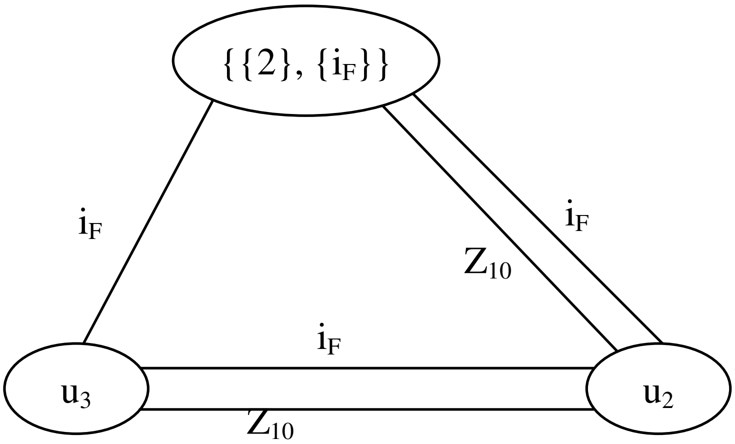


Figure 3.55

Infact K_3 is also a pseudo complete subset-subset vertex multisubgraph of H which is only a subset-subset vertex multisubgraph weak triad.

We cannot consider the multisubgraph with u_2, u_3 and u_4 as the vertex subset as it will fail to be a subset-subset vertex multisubgraph as it is only a subset vertex multigraph.

Here it is pertinent to keep on record that atleast one of the vertex subset of the subset-subset vertex multi subgraph must be a proper subset of the vertex subset of the original subset vertex multigraph. This is forbidden in the case of subset - vertex multi subgraphs.

We see $\{u_2, u_3, u_4\} \subseteq \{u_1, u_2, u_3, u_4\}$ forms only a subset vertex multi subgraph which is a triad as it is a complete subset - vertex multi subgraph L of H given by the following figure.

L =

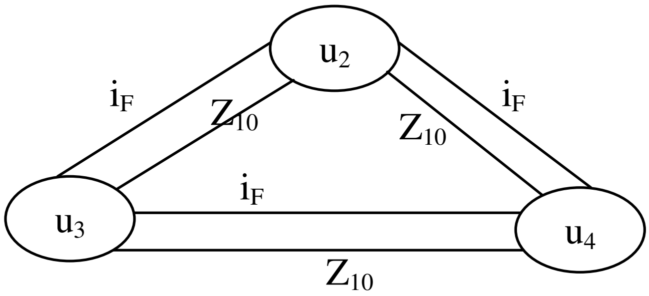


Figure 3.56

Infact all subset vertex multi subgraphs of H with three subset vertices are complete and are triads. However, all three vertex subset contribute by subset-subset vertex multi subgraphs are not triads in general. They may be strong triads, weak triads or forbidden triads or just subset-subset vertex multi subgraphs with only one edge or just an empty multigraph.

For instance consider K_4 the subset-subset vertex multi subgraph given by the vertex subset $\{\{\{i_F\}, \{2\}\}, \{\{7, 5, 2\}, \{9i_F + 2\}\}, \{\{1 + i_F\}, \{8\}\}\}$ given by the following figure:

K_4 =

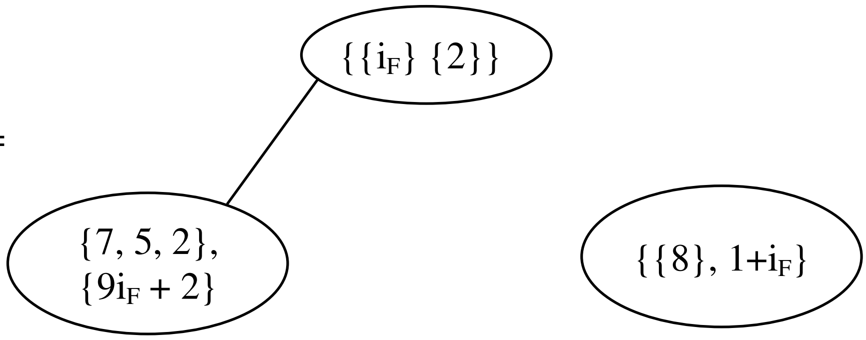


Figure 3.57

Clearly K_4 is not a triad or a forbidden triad it is just a subset-subset vertex multi subgraph which is not complete or pseudo complete.

Let K_5 be the subset-subset vertex multisubgraph given by the following figure:

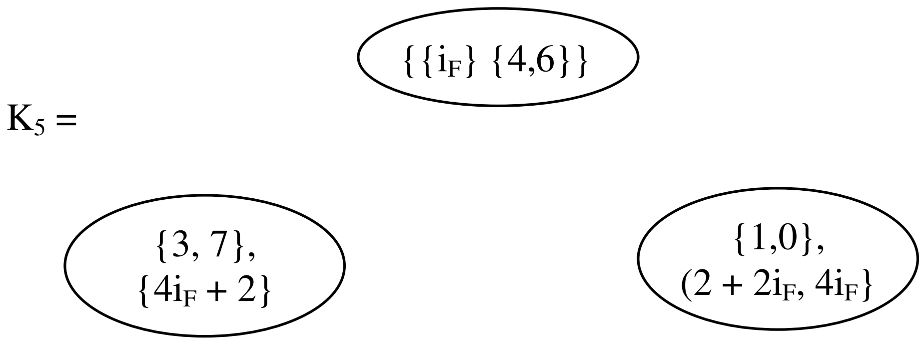


Figure 3.58

Clearly K_5 is just an empty multisubgraph of H.

Consider K_6 the subset-subset vertex multisubgraph given by the following figure.

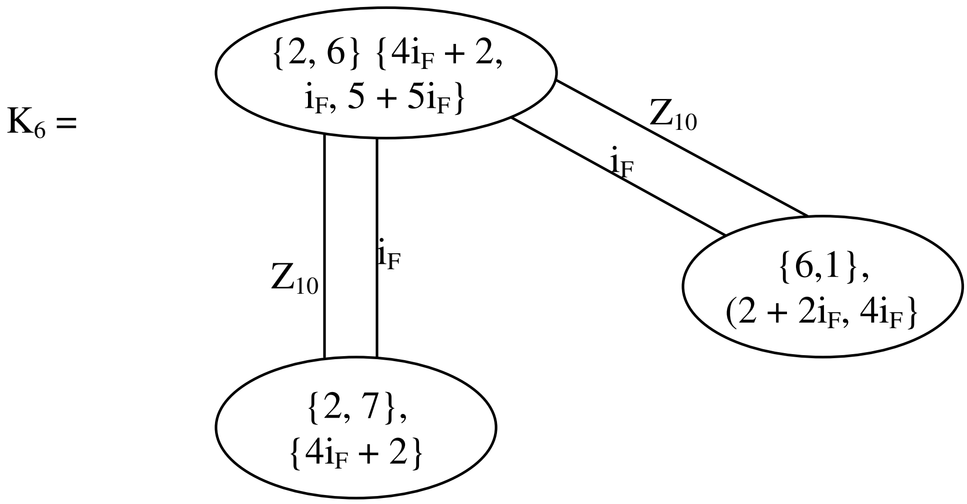


Figure 3.59

Clearly K_6 is only a subset-subset vertex multi subgraph which is not complete but is only a forbidden triad.

We consider K_7 a subset-subset vertex multi subgraph of H given by the following figure.

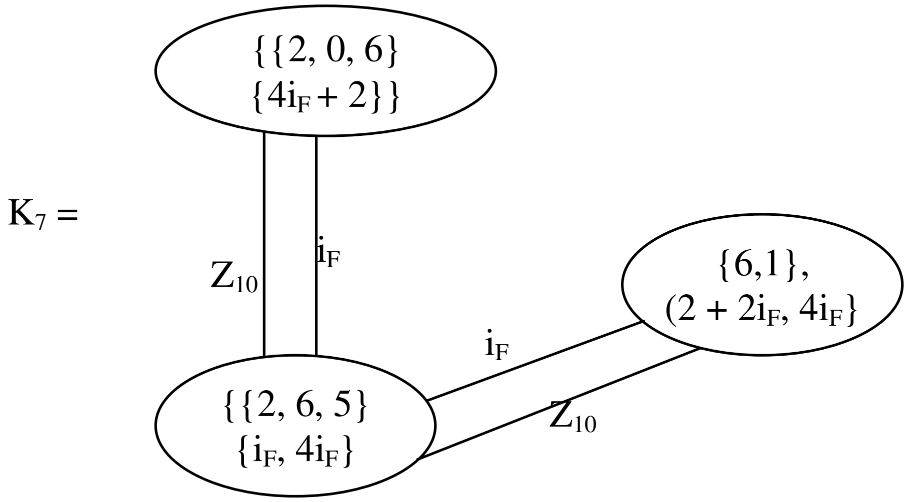


Figure 3.60

K_7 is only a forbidden triad which is a subset-subset vertex multi subgraph of H.

We call K_6 and K_7 are full forbidden triad. We will just give a sample illustration of finding how many subset-subset vertex multi subgraphs has for a given subset vertex multigraph.

Example 3.13. Let S = $\{\{Z_7\}, \langle Z_9 \cup I\rangle\}$ be the set. P(S) be the power set of S. Let G be the subset vertex multigraph given by the following figure.

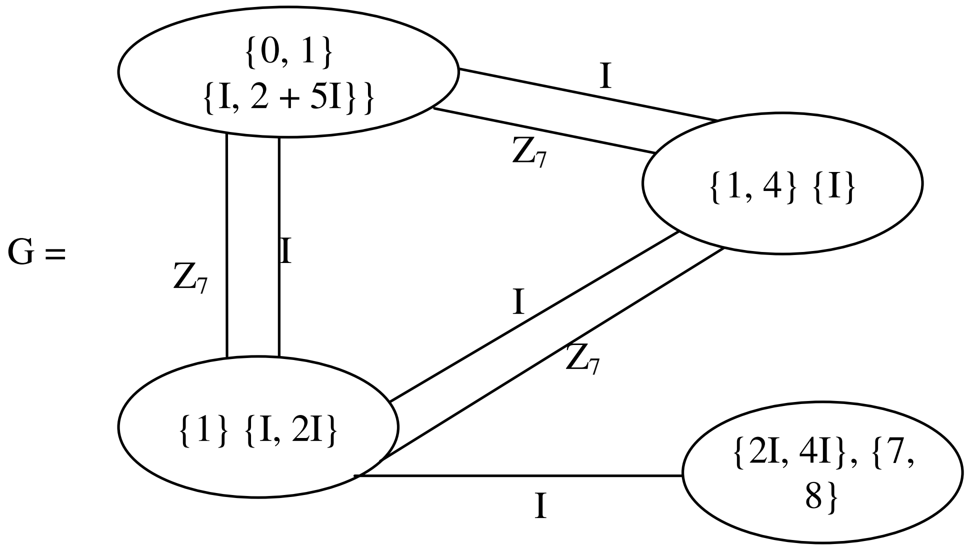


Figure 3.61

Clearly G is a subset - vertex multigraph with four vertex subsets. P({0, 1}, {I, 2 + 5I}) = {{0}, {1}, {1, 0}, {I}, {2 + 5I}, {I, 2 + 5I}, {0, 1}, {I, 2 + 5I} and other combination taking elements from subsets of {0, 1} with subsets of {I, 5I + 2}} which clearly leads of 15 possibilities. From this we can remove the subset {{0, 1}, {I, 2 + 5I}} as we need only proper subsets. Also we do not take the proper subsets from all the four vertex subsets of G; we take only from any three of them.

Subsets of {{1, 4}, {I}} = {{1}, {4}, {I}, {1, 4}}, of course if we take the usual power set it is {{1}, {1, 4}, {4}, {I}, {1, I}, {4, I}} it is only 6 as we do not take ϕ and {1, 4, I} = {{I}, {1, 4}}. The power set of {{1}, {I, 2I}} is {{1}, {I}, {2I}, {1, I}, {1, 2I}, {1, I, 2I}} is only taken barring the subsets ϕ and {1, 2I, I}.

Similarly for {{7, 8}, {2I, 4I}} the subsets are {{7}, {8}, {2I}, {4I}, {7, 2I}, {8, 2I}, {7, 4I}, {7, 8}, {8, 4I}, {7, 2I, 4I}, {8, 2I, 4I}, {7, 8, 2I}, {7, 8, 4I}} which has not included ϕ and {7, 8, 2I, 4I}.

We do not take the whole set; however if a researcher is in need of such notion that they need some vertex to be taken as that of the original graph then it is mandatory we choose at least one of the vertex subset to be proper subset of the vertex subset.

Thus, we have several subset-subset vertex multi subgraphs of G.

Let M be the subset-subset vertex multi subgraph given by the vertex subset {{{1}, {I}}, {4}, {I}}, {I, 1}} given by the following figure.

H =

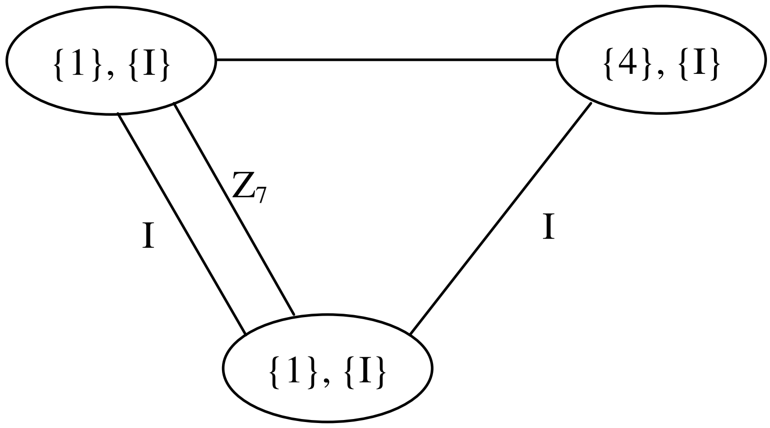


Figure 3.62

H is a subset-subset vertex multi subgraph of G. Clearly H is a pseudo complete subset-subset vertex multi triad of G. The difficult task is finding the number of subset-subset vertex multi subgraphs of a given subset vertex multigraph G.

Next we proceed on to give a subset-subset vertex multi subgraph which is just a tree.

Take the vertex subset as $\{\{I\}, \{0, 1\}\} = w_1$, $w_2 = \{1\}$ and $\{2I, I\} = w_3$. Let W be the subset-subset vertex multi subgraph given by the following figure.

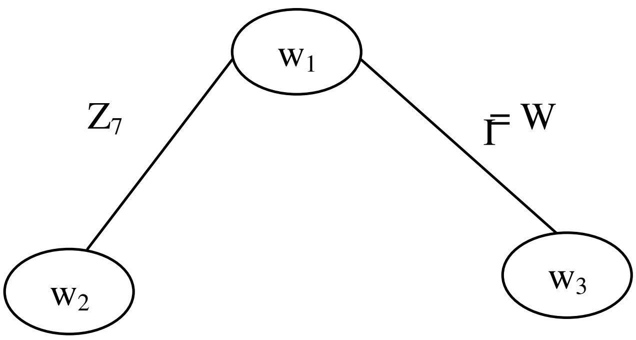


Figure 3.63

W is a subset-subset vertex multisubgraph is a tree which is also a forbidden triad. We can find several such subset-

subset vertex multi subgraphs given a subset vertex multigraph G.

Now we proceed onto describe the notion of universal complement of a subset vertex multigraph G by an example.

Example 3.14. Let S = {Z_9, $\langle Z_3 \cup I\rangle$} be the given set. P(S) be the power set of S. Let G be the subset vertex multigraph with the vertex subset v_1, v_2, v_3, v_4 and v_5 given in the following.

v_1 = {{0, 3, 6, 1}, {I, 2I + 1, 2I, 0}}, v_2 = {{0, 2, 4, 8, 6}, {2I, 1 + I, 2 + I, 0}}, v_3 = {{1, 6, 8, 5, 7}, {2I, I, 0, 1, 2}}, v_4 = {{6, 2, 8, 1, 0}, {2I, 2 + 2I, I, 1}} and v_5 = {{3, 7}, {2I + 1, 0, 2}}. The subset vertex multigraph G is given by the following figure.

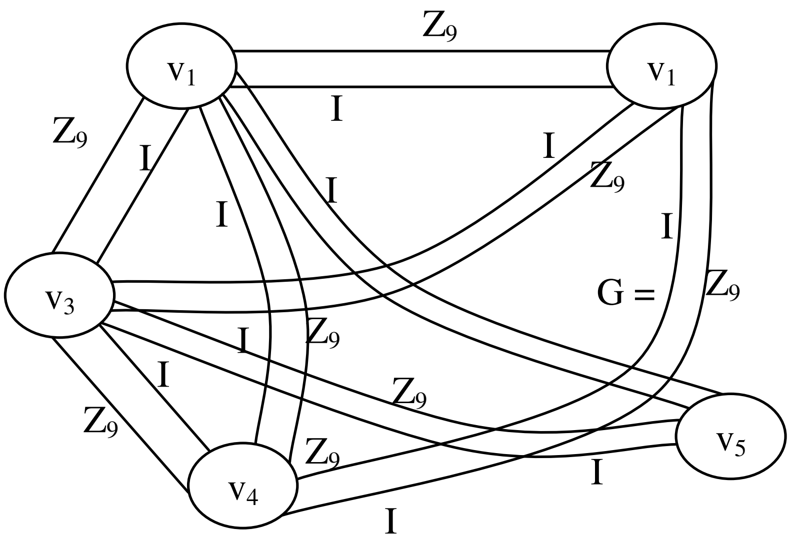


Figure 3.64

We find the universal complements of each of the subset vertices v_1, v_2, v_3, v_4 and v_5 relative to the set S.

w_1 = S \ v_1 = {{2, 4, 5, 7, 8}, {1, 1 + I, 2 + 2I, 2 + I, 2}},
w_2 = S \ v_2 = {{1, 3, 5, 7}, {1, I, 2 + 2I, 2I + 1, 2}},

$w_3 = S \setminus v_3 = \{\{0, 2, 3, 4\}, \{1 + I, 2 + I, 2 + 2I, 2I + 1\}\}$,
$w_4 = S \setminus v_4 = \{\{3, 4, 5, 7\}, \{0, 2, 1 + I, 1 + 2I, 2 + I\}\}$ and
$w_5 = S \setminus v_5 = \{\{1, 2, 0, 4, 5, 6, 8\}, \{1, I, 2I, 1 + I, 2 + 2I, 2 + I\}\}$.

Now using the vertex subsets w_1, w_2, w_3, w_4 and w_5 which are universal complements of v_1, v_2, v_3, v_4 and v_5 respectively; we get the following subset vertex multigraph H.

H =

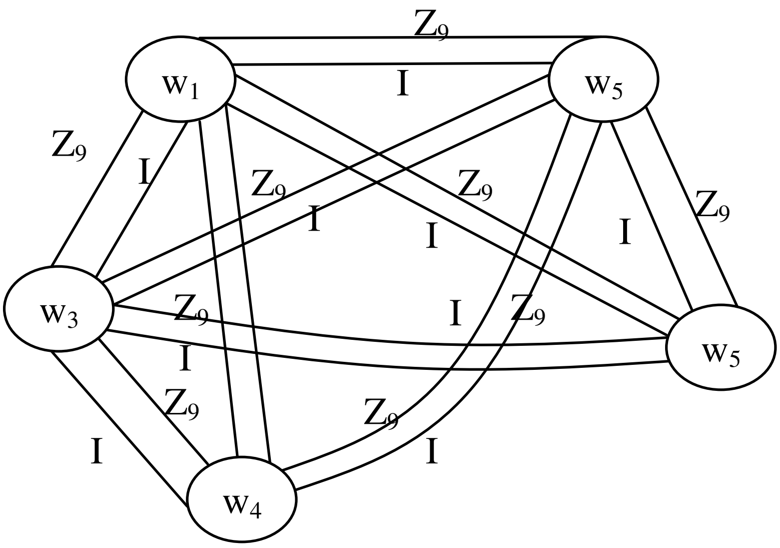


Figure 3.65

Clearly H is the universal complement of the subset vertex multigraph G. We see G is not a complete or a pseudo complete subset vertex multigraph. However, H is a complete subset vertex multigraph.

Thus the first pertinent observation is that in general the universal complement of any subset vertex multigraph need not preserve the structure of the graph for which it is a complement and vice versa. When we say structure preserving, we mean both the graphs are identical in appearance only the vertex subsets are complements of each other.

***Theorem 3.1**. Let S be a set with distinct entities say (S = {Z_n, ⟨Z_m ∪ g⟩, ⟨Z_t ∪ k⟩, C(Z_p), ⟨Z_e ∪ I⟩} (It can be any distinct attributes like) S = {{a_1, …, a_{m_t} }, {b_1, b_2, …, b_{n_r} }, …, {m_1, m_2, …, m_p}} and P(S) be the power set of S. Suppose G is any subset vertex multigraph with vertex subsets v_1, v_2, …, v_n ∈ P(S) and if H be the universal complement of G with vertex subsets {w_1 = S \ v_1, w_2 = S \ v_2, …, w_n = S \ v_n} then the universal complement subset vertex multigraph H of G need not in general be structure preserving.*

Proof. Follows from the example.

We now give some more examples of universal complement of subset vertex multigraphs G.

Example 3.15. Let S = {Z_6, ⟨Z_3 ∪ g⟩, ⟨Z_4 ∪ I⟩} be the set of some 3 attributes. P(S) be the power set of S. Let K be the subset vertex multigraph given by the following figure with v_1, v_2, v_3, v_4, v_5 and v_6 as the vertices.

v_1 = {{0, 1, 2, 3}, {0, 1, 2g, g + 1, 2}, {I + 3, 2, 3, 1, 2I, 3I, I + 2}},

v_2 = {{0, 4, 2, 5}, {0, g, 2g, 1 + 2g}, {I, 2I, 3I, 0, I + 2, 2 + 2I}},

v_3 = {{3, 1}, {2g + 2, 2, 1}, {I, 2I, 0, 2, 1}},

v_4 = {{4, 5}, {2g, g}, {3I + 3, I + 3, 3 + I}},

v_5 = {{0, 1, 2}, {g, g + 2, 2g + 1}, {0, I, 2I, I + 3}} and

v_6 = {{3, 0, 1}, {g, 2g, 2 + 2g}, {I, 2I, 3I + 3}} are the subsets of P(S).

Let G be the subset vertex multigraph with vertex set {v_1, v_2, v_3, v_4, v_5, v_6}; given by the following figure:

G =

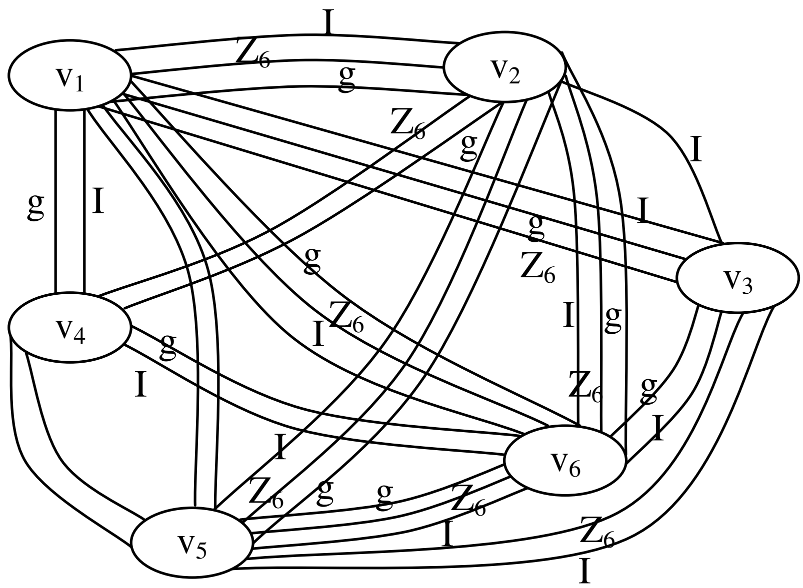


Figure 3.66

Clearly the subset vertex multigraph G is only pseudo complete. Let K denote the universal complement subset vertex multigraph of G. The vertex subset of K is as follows:

s_1 = {S \ v_1} = {{4, 5}, {g, g + 2, 2g + 2, 2g + 1}, {0, I + 1, 2I + 1, 3I + 1, 3, 2I + 2, 2I + 3, 3I + 2, 3I + 3}},

s_2 = {S \ v_2} = {{1, 3}, {1, 2, 1 + g, 2 + g, 2g + 2}, {1, 2, 3, 1 + I, 3 + I, 2I + 1, 3I + 2, 2I + 3, 3I + 1, 3I + 3}},

s_3 = {S \ v_3} = {{0, 2, 4, 5}, {0, g, 2g, 1 + g, 2 + g, 2g + 1}, {3, 3I, 1 + I, 2 + I, 3 + I, 2I + 1, 2I + 2, 2I + 3, 3I + 1, 3I + 2, 3I + 3}},

s_4 = {S \ v_4} = {{0, 1, 2, 3}, {0, 1, 2, 1 + g, 2 + g, 2g + 2, 2g + 1}, {0, 1, 2, 3, I, 2I, 3I, 1 + I, 2 + I, 2I + 1, 2I + 2, 2I + 3, 3I + 2}},

s_5 = {S \ v_5} = {{3, 4, 5}, {0, 1, 2, 2g, 1 + g, 2g + 2}, {2, 1, 3, 3I, 1 + I, 2 + I, 2I + 1, 2I + 2, 2I + 3, 3I + 1, 3I + 2, 3I + 3}} and

$s_6 = \{S \setminus v_6\} = \{\{2, 4, 5\}, \{0, 1, 2, 1 + g, 2 + g, 2g + 1\}, \{0, 1, 2, 3, 3I, 1 + I, 1 + 2I, 1 + 3I, 2 + I, 2 + 3I, 2 + 2I, 3 + I, 3 + 2I\}\}$ are the universal complements of v_1, v_2, v_3, v_4, v_5 and v_6 respectively.

In the following we give the universal complement of the subset vertex multigraph K.

K =

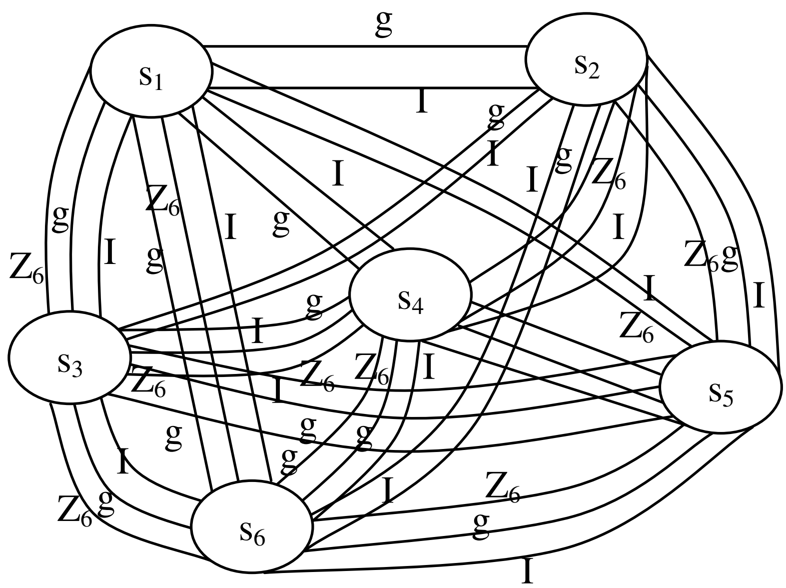


Figure 3.67

We see in this case K is also a pseudo complete subset vertex multigraph which is the universal complement of the subset vertex multigraph G.

It is to be noted that both G and K are not the same structurally though both of them happen to be pseudo complete subset vertex multigraphs.

Now let v_1, v_2, v_3, v_4 and v_5 be the vertex subset of the star graph W; where

v_1 = {{0, 1, 2, 4, 5}, {0, 1, g, 1 + g, 2 + g, 2g + 2, 2g + 1}, {0, 1, I, 2I, 3, 1 + I, 2 + I, 2 + 2I, 3 + 3I, 3 + I, 2 + 3I}},

v_2 = {{0, 3}, {g, 2g, 2}, {I, 3I, 2, 3 + 2I}},

v_3 = {{4}, {1 + g}, {3 + 3I, 2I}},

v_4 = {{2}, {2g + 2, 2 + g}, {3 + I}} and

v_5 = {{5}, {0, 1}, {2 + 3I}} are subsets from P(S). The subset vertex multigraph W is given by the following figure:

W =

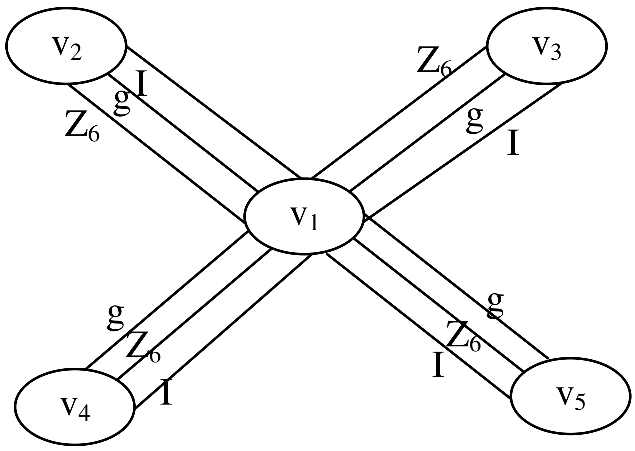


Figure 3.68

Clearly W is a uniform subset vertex multi star graph. Let Y be the universal complement subset vertex multigraph of W.

Let Y_1, Y_2, Y_3, Y_4 and Y_5 be the universal complements of v_1, v_2, v_3, v_4 and v_5 respectively, given in the following.

$Y_1 = S \setminus v_1$ = {{3}, {2, 2g}, {2, 3I, 2I + 1, 2I + 3, 2I + 1}},

$Y_2 = S \setminus v_2$ = {{1, 2, 4, 5}, {0, 1, 1 + g, 2 + g, 2g + 2, 2g + 1}, {0, 1, 3, 2I, 1 + I, 2 + I, 3 + I, 2I + 1, 2I + 2, 3I + 1, 3I + 2, 3I + 3}},

$Y_3 = S \setminus v_3 = \{\{0, 1, 2, 3, 5\}, \{0, 1, 2, 2 + g, g, 2g, 1 + 2g, 2 + 2g\}, \{0, 1, 2, 3, I, 3I, 1 + I, 1 + 2I, 1 + 3I, 2 + 2I, 2 + I, 3 + I, 3 + 2I, 2 + 3I\}\}$,

$Y_4 = S \setminus v_4 = \{\{0, 1, 3, 4, 5\}, \{0, 1, 2, g, 2g, 1 + g, 2g + 1\}, \{0, 1, 2, 3, I, 2I, 3I, 1 + I, 2 + I, 1 + 2I, 2 + 2I, 3 + 2I, 3 + 3I, 2 + 3I, 3I + 1\}\}$ and

$Y_5 = S \setminus v_5 = \{\{0, 1, 2, 3, 4\}, \{2, g, 2g, 1 + g, 2 + g, 2g + 1, 2g + 2\}, \{0, 1, 2, 3, I, 2I, 3I, 1 + I, 2 + I, 3 + I, 2 + 2I, 2I + 1, 2I + 3, 3I + 1, 3I + 3\}\}$. Now we give the subset vertex multigraph with Y_1, Y_2, Y_3, Y_4 and Y_5 as vertex subsets in the following.

Y =

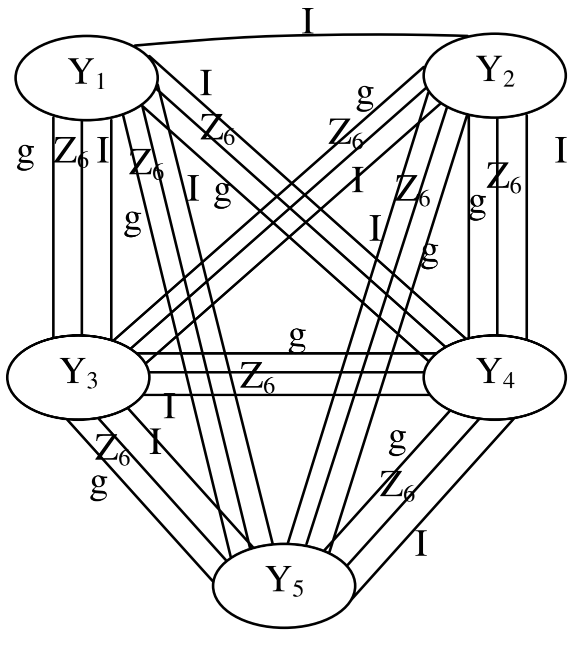


Figure 3.69

Clearly Y is a pseudo complete subset vertex multigraph. Thus for a subset vertex multi star graph its universal complement is a pseudo complete subset vertex multigraph. This is yet another interesting feature about the universal complements of the multigraphs built on subsets as vertices.

We give yet another interesting examples about the universal complement of a subset vertex multigraphs. Let x_1, x_2, x_3, x_4 and x_5 be the subsets of P(S) given in the following:

$x_1 = \{\{0, 1\}, \{2g, g\}, \{I, 2I, 3 + 3I, 2 + 2I, 1 + I\}\}$,

$x_2 = \{\{1, 4\}, \{2g, 0\}, \{2I, 1, 2\}\}$,

$x_3 = \{\{4, 3\}, \{0, 2 + g\}, \{2, 3 + I\}\}$,

$x_4 = \{\{3, 2\}, \{2 + g, 2\}, \{3 + I, 1 + 2I\}\}$ and

$x_5 = \{\{2, 0\}, \{2, g\}, \{1 + 2I, 1 + I\}\}$.

Let C be the subset vertex multigraph with x_1, x_2, x_3, x_4 and x_5 as vertex subsets given by the following figure.

C =

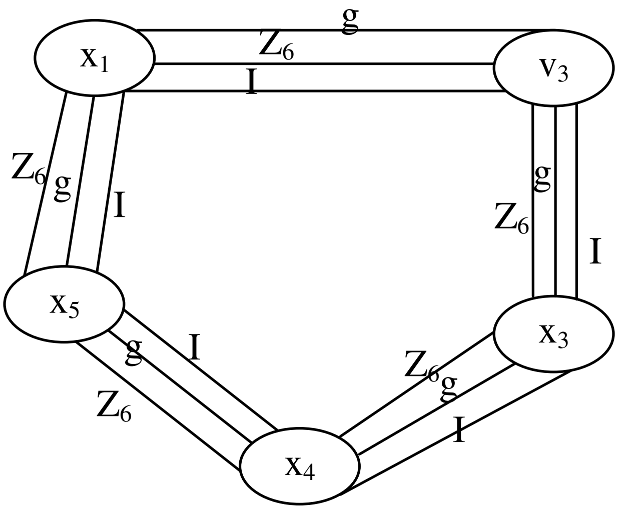


Figure 3.70

Clearly C the subset vertex multigraph is a circle multigraph.

Now we find the universal complement of C. The universal complement of x_1, x_2, x_3, x_4 and x_5 is given below. Let a_1, a_2, a_3, a_4 and a_5 denote the universal complements of the vertex subsets x_1, x_2, x_3, x_4 and x_5 respectively.

$a_1 = S \setminus x_1 = \{\{2, 3, 4, 5\}, \{0, 1, 2, 1 + g, 2 + g, 2g + 1, 2 + 2g\}, \{0, 1, 2, 3, 3I, 2I + 1, 2I + 3, 2 + I, 3 + I, 3I + 1, 3I + 2\}\}$,

$a_2 = S \setminus x_2 = \{\{0, 2, 3, 5\}, \{1, 2, g, 1 + g, 2 + g, 2g + 2, 2g + 1\}, \{0, 3, I, 3I, 1 + I, 2 + I, 3 + I, 2I + 1, 2I + 2, 2I + 3, 3I + 1, 3I + 2, 3I + 3\}\}$,

$a_3 = S \setminus x_3 = \{\{0, 1, 2, 5\}, \{1, 2, g, 2g, 1 + g, 2 + 2g, 2g + 1\}, \{0, 1, 3, I, 2I, 3I, 1 + I, 2 + I, 2 + 2I, 3 + 3I, 3I + 2, 3I + 1, 2I + 1, 2I + 3\}\}$,

$a_4 = S \setminus x_4 = \{\{0, 1, 4, 5\}, \{0, 1, g, 2g, 1 + g, 2g + 1, 2g + 2\}, \{0, 1, 2, 3, I, 2I, 3I, 1 + I, 2 + I, 2I + 2, 2I + 3, 3I + 1, 3I + 2, 3I + 3\}\}$ and

$a_5 = S \setminus x_5 = \{\{1, 3, 4, 5\}, \{0, 1, 2g, 1 + g, 2 + g, 2g + 1, 2g + 2\}, \{0, 1, 2, 3, I, 2I, 3I, 2 + I, 3 + I, 2 + 2I, 3 + 2I, 3 + 3I, 3I + 1, 3I + 2\}\}$.

D be the subset vertex multigraph with vertex subsets a_1, a_2, a_3, a_4 and a_5 respectively. Given by the following figure;

D =

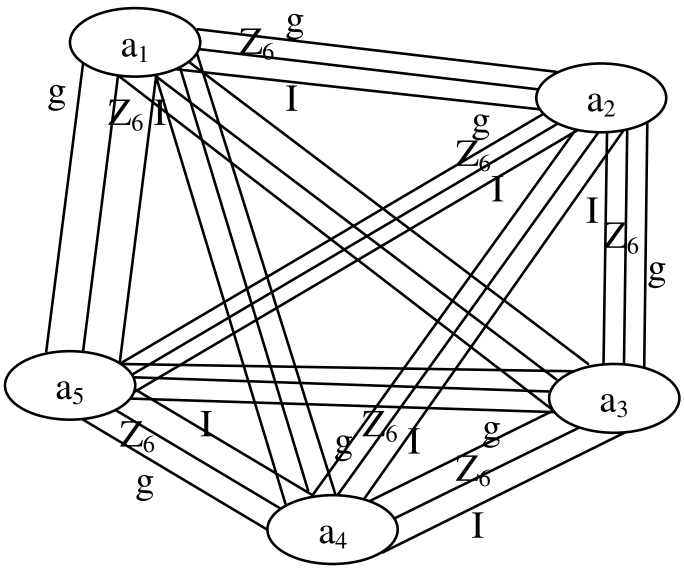


Figure 3.71

We see the subset vertex multigraph with vertex subset a_1, a_2, a_3, a_4 and a_5 which is the universal complement of C is a circle multigraph is a complete subset vertex multigraph.

Thus it is yet another special feature enjoyed by the universal complements of a subset vertex multigraph.

We just give yet another important example of this situation. Let $p_1 = \{\{0, 2\}, \{1 + g, g\}, \{I, 2, 2I + 1\}\}$, $p_2 = \{\{0, 3\}, \{g, 2 + 2g\}, \{I, 2I + 2, 3I + 3\}\}$, $p_3 = \{\{3, 4\}, \{2 + 2g, 1 + 2g\}, \{3I + 3, 3I\}\}$, $p_4 = \{\{4, 5\}, \{1 + 2g, 2 + g\}, \{3I, 3, 1\}\}$ and $p_5 = \{\{5\}, \{2 + g, 1\}, \{3, 2 + 3I\}\}$ be the vertex subsets of the subset vertex multigraph L given by the following figure.

Clearly $p_1 \cap p_2 = \{\{0\}, \{g\}, \{I\}\}$, $p_3 \cap p_1 = \{\phi\}$, $p_4 \cap p_1 = \{\phi\}$, $p_1 \cap p_5 = \phi$, $p_2 \cap p_3 = \{\{3\}, \{2g + 2\}, \{3I + 3\}\}$, $p_2 \cap p_4 = \phi$, $p_2 \cap p_5 = \phi$, $p_3 \cap p_4 = \{\{4\}, \{1 + 2g\}, \{3I\}\}$, $p_5 \cap p_3 = \phi$ and $p_4 \cap p_5 = \{\{5\}, \{2 + g\}, \{3\}\}$.

L =

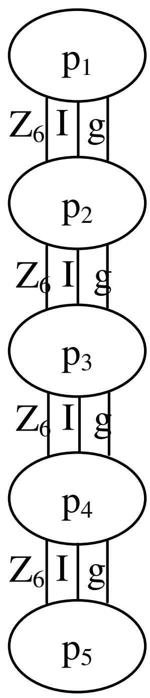


Figure 3.72

Now we find the universal complement of L. The universal complement of the vertex subsets p_1, p_2, p_3, p_4 and p_5 with respect to S are as follows: Let q_1, q_2, q_3, q_4 and q_5 be the universal complements of p_1, p_2, p_3, p_4 and p_5 respectively.

$q_1 = S \setminus p_1$ = {{1, 3, 4, 5}, {0, 1, 2, 2g, 2 + g, 2 + 2g, 2g + 1}, {0, 1, 3, 3I, 2I, 1 + I, 2 + I, 3 + I, 2 + 2I, 3 + 2I, 3I + 1, 3I + 2, 3I + 3}},

$q_2 = S \setminus p_2$ = {{1, 2, 4, 5}, {0, 1, 2, 2g, 1 + g, 2 + g, 2g + 1}, {0, 1, 2, 3, 2I, 3I, 1 + I, 2 + I, 3 + I, 2I + 1, 2I + 3, 3I + 1, 3I + 2}},

$q_3 = S \setminus p_3$ = {{0, 1, 2, 5}, {0, 1, 2, g, 2g, 1 + g, 2 + g}, {0, 1, 2, 3, I, 2I, 1 + I, 2 + I, 3 + I, 2I + 1, 2I + 2, 2I + 3, 3I + 1, 3I + 2}},

$q_4 = S \setminus p_4$ = {{0, 1, 2, 3}, {0, 1, 2, g, 2g, 1 + g, 2 + 2g}, {0, 2, I, 2I, 1 + I, 2 + I, 3 + I, 2I + 1, 2I + 2, 2I + 3, 3I + 1, 3I + 2, 3I + 3}} and

$q_5 = S \setminus p_5$ = {{0, 1, 2, 3, 4}, {0, 2, g, 2g, 1 + g, 2 + 2g, 2g + 1}, {0, 1, 2, I, 2I, 3I, 1 + I, 2 + I, 3 + I, 2I + 1, 2I + 2, 2I + 3, 3I + 1, 3I + 3}}.

Now let M be the subset vertex multigraph using the vertex subset as q_1, q_2, q_3, q_4 and q_5 which is given by the following figure.

M =

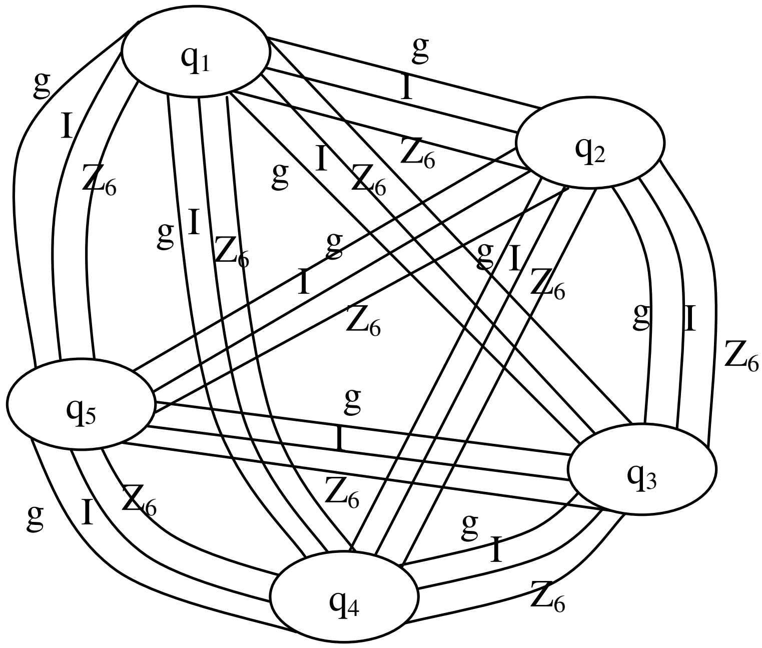


Figure 3.73

We see the multiset subset vertex multigraph M is complete. That is the universal complement of a subset vertex multiline graph is a complete subset vertex multigraph.

This is yet another interesting feature of these universal complements of subset vertex multigraphs.

We see the subset vertex multiline graph L is called as maximal uniform edged subset vertex multiline graph as the number of edges connecting any two relevant vertex subsets is the maximum in this case it is three. Thus we call this subset vertex multiline graph as maximum uniform multiline graph. The term uniform is used as every relevant subset vertices are connected with 3 edges.

We see for such line subset vertex multigraphs their universal complement also happens to be maximal uniform but not structure preserving for they happen to be complete subset vertex multigraphs which are maximal uniform.

In view of the all these we propose the following conjecture.

Conjecture 3.1. Let S be a set with n sets of distinct attributes; $S = \{\{a_{11}, a_{12}, \ldots, a_{1t_1}\}, \{a_{21}, a_{22}, a_{23}, \ldots, a_{2t_2}\}, \ldots, \{a_{n1}, a_{n2}, \ldots, a_{nt_n}\}\}$. P(S) be the power set of S. Any maximal uniform subset vertex multigraph will have n edges connecting any two relevant subset vertices.

Let L be a maximal uniform subset vertex multiline graph with m vertices. Then the universal complement of L is a maximal uniform subset vertex complete multigraph with m vertices - prove!

Interested researcher can study several properties associated with these subset vertex multigraphs and their universal complement.

Next we proceed onto study local complements of a subset - subsets vertex multi subgraphs of a subset vertex multigraph G. We call the complement as local complement relative to the subset vertex multigraph G.

Now we proceed onto describe for subset vertex multigraph G the local complement of subset-subset vertex multi subgraphs by examples.

Example 3.16. Let $S = \{\langle Z_5 \cup g\rangle, \langle Z_3 \cup I\rangle, Z_{12}\}$ be a set with three distinct attributes. Let P(S) be the power set of S. Consider the subset vertex multigraph G given by the following figure where the subset vertex subset of G are

v_1 = {{g, 2g, g + 2, 1}, {I, 2I, 0, 1, 2}, {2, 3, 4, 5, 6, 10}}, v_2 = {{g, 1, 2g + 1, 2g + 2, 4g}, {2I, I, 1 + I, 2 + I, 2 + 2I}, {2, 4, 6, 11, 7}}, v_3 = {{2g + 2, g, 1, 3g}, {I, 2 + I, 0, 2}, {2, 7, 11, 5, 4}}, v_4 = {{g, 2, 0, 3g, 1, 1 + g}, {I, 2 + 2I, 0, 1, 2}, {2, 4, 7, 9, 11, 0}} and v_5 = {{2g + 2, 1, g, 0, 4g}, {I, 2I + 1, 0, 2, 1}, {2, 0, 4, 8, 3, 9}}. The subset vertex multigraph G with vertex sets v_1, v_2, …, v_5 is given below:

G =

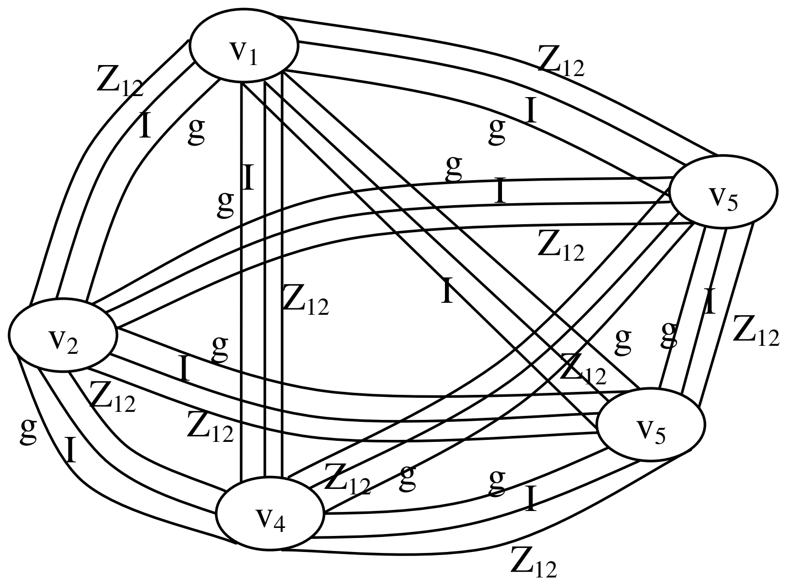


Figure 3.74

The subset vertex multigraph G is complete and maximal uniform. Let us consider a set of subsets of the vertex subsets $u_1 \subseteq v_1$, $u_2 \subseteq v_2$, $u_3 \subseteq v_3$ and $u_5 \subseteq v_5$ where u_1, u_2, u_3 and u_5 are as follows:

u_1 = {{g + 2, g}, {0, 1, 2I}, {3, 6, 10}} $\subseteq v_1$,

u_2 = {{g, 4g}, {1 + I, 2 + I}, {2, 4, 6}} $\subseteq v_2$,

u_3 = {{g, 1}, {I, 0, 2}, {7, 11}} $\subseteq v_3$ and

$u_5 = \{\{g, 2 + 2g\}, \{I, 1 + 2I\}, \{0, 9, 2\}\} \subseteq v_5$

Let H be the subset-subset vertex multi subgraph of G with u_1, u_2, u_3 and u_5 are vertex subsets given by the following figure:

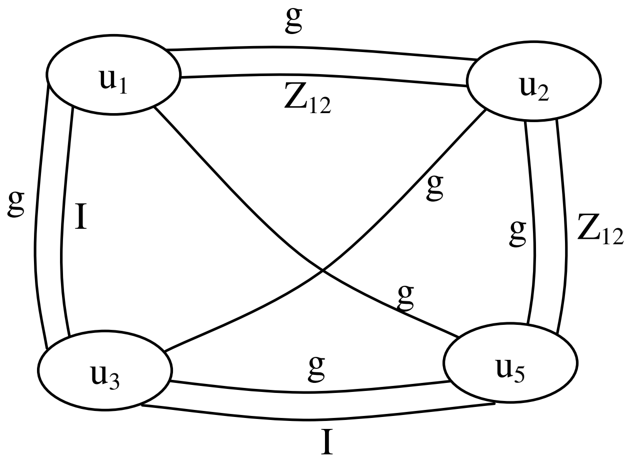
H = 

Figure 3.75

Clearly the subset-subset vertex multisubgraph H of G is only pseudo complete that is it is not maximal uniform.

Now we find the local complement of H relative to G.

The local complement of the vertex subset u_1 relative to v_1 is as follows;

Let a_1 denote the local complement of u_1 relative to v_1.

$a_1 = v_1 \setminus u_1 = \{\{1, 2g\}, \{2, I\}, \{2, 4, 5\}\}$

$a_2 = v_2 \setminus u_2 = \{\{1, 2g + 1, 2g + 2\}, \{I, 2I, 2 + 2I\}, \{11, 7\}\}$ is the local complement of u_2 relative to v_2.

Let a_3 be the local complement of u_3 relative to v_3.

$a_3 = v_3 \setminus u_3 = \{\{2g + 2, 3g\}, \{2 + I\}, \{2, 4, 5\}\}$ and

$a_5 = v_5 \setminus u_5 = \{\{1, 0, 4g\}, \{0, 1, 2\}, \{4, 8, 3\}\}$ is the local complement of u_5 relative to v_5.

Let the local complement of H relative to G given by the following figure.

A =

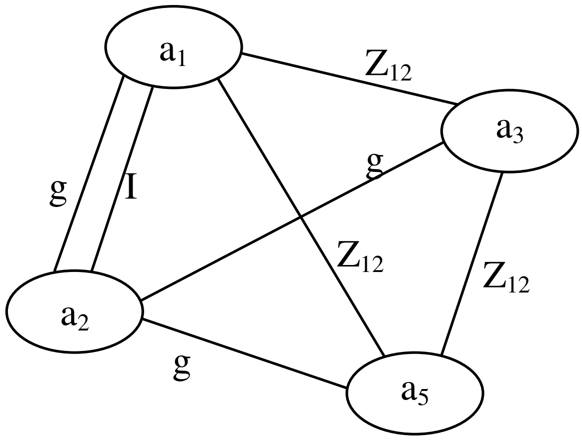


Figure 3.76

The subset-subset vertex multisubgraph A is the local complement of H in G. Clearly A is also a pseudo complete subset-subset vertex multisubgraph, but A and H enjoy different structures.

Thus for a given subset-subset vertex multi subgraphs of a subset vertex multigraph we have a unique local complement which may not in general preserve structure. Let K be the subset-subset vertex multisubgraph of G given with the following vertex subsets.

$s_1 = \{\{1, g + 2\}, \{0, 2\}, \{6, 10\}\} \subseteq v_1$

$s_3 = \{\{1, 3g\}, \{I, 2\}, \{7, 11\}\} \subseteq v_3$ and

$s_4 = \{\{g, 1 + g, 1\}, \{I, 2, 0\}, \{7, 0\}\} \subseteq v_4$ be the subsets of the vertex subsets v_1, v_3 and v_4 respectively.

The subset-subset vertex multisubgraph K of G is given in the following.

K =

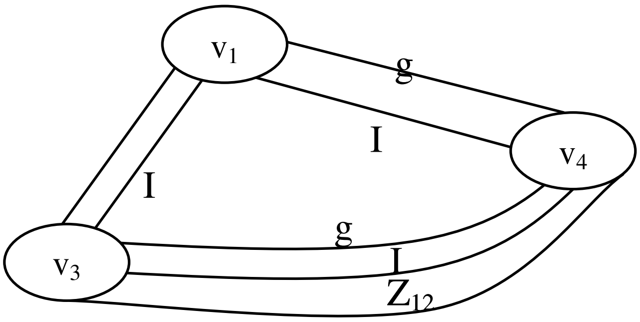


Figure 3.77

Clearly K is a pseudo complete multisubgraph of G. Now we find the local complement of K relative to G. The local complements of the vertex subsets s_1, s_3 and s_4 be denoted respectively by b_1, b_3 and b_4.

$b_1 = v_1 \setminus s_1 = \{\{g, 2g\}, \{I, 2I, 1\}, \{2, 3, 4, 5\}\}$

$b_3 = v_3 \setminus s_3 = \{\{g, 2g + 2\}, \{2 + I, 0\}, \{2, 5, 4\}\}$ and

$b_4 = v_4 \setminus s_4 = \{\{0, 2, 3g\}, \{1, 2 + 2I\}, \{2, 4, 9, 11\}\}$.

Let D denote the local complement K relative to G. The subset-subset vertex multisubgraph with vertex subset b_1, b_4 and b_3 is as follows:

D =

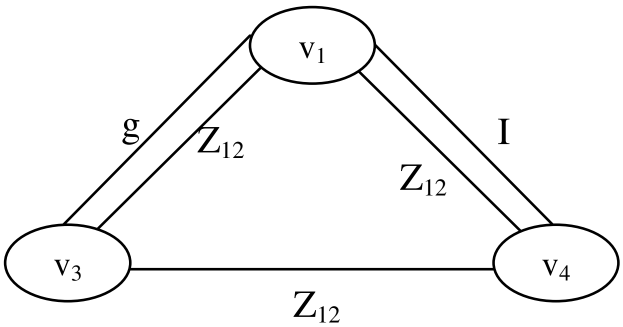


Figure 3.78

Clearly the local complement subset-subset vertex multisubgraph D of K relative to G is also a pseudo complete subset-subset vertex multisubgraph but it is not structure preserving.

In view of all these we put forth the following theorem.

Theorem 3.2. *Let S = {{a_{11}, …, a_{1r_1} }, {a_{21}, a_{22}, …, a_{1r_2} }, …, {a_{n1}, a_{n2}, …, a_{1n_1} }} be the set with n different (or distinct) attributes. Let P(S) be the power set of S. If G be a subset vertex multigraph with n subset vertices and H be a subset-subset vertex multisubgraph of G with p vertices (1 < p < n) then the local complement P of H relative to G may not in general be a structure preserving subset-subset vertex multisubgraph with H; that P and H in general may not enjoy the same structure.*

Proof. By example.

Now we provide more examples of them which may throw more light on the structure of these local complements of subset-subset vertex multigraphs.

Example 3.17. Let S = {Z_{10}, $\langle Z_4 \cup g\rangle$, $\langle Z_5 \cup I\rangle$, $C(Z_3)$} be a set with four distinct attributes. Let P(S) be the power set of S. Let G be a subset vertex multigraph with six vertex subsets given in the following:

v_1 = {{2, 5, 6, 7, 0}, {g, 2g, 1 + 2g, 1}, {1, 2, I, I + 3}, {i_F, 2, 1 + 2i_F, 1}},

v_2 = {{2, 4, 6, 8, 3, 5}, {g, 2g, 0, 1, 2, 2 + 2g}, {3I, 3 + 2I, I, 1, 2, 3}, {i_F, 2i_F, 1, 2 + 2i_F}},

v_3 = {{2, 5, 6, 9, 11}, {g, 0, 1, 2, 1 + g}, {0, 1, 2, 3, 3I + 2, 3I + 1, 3 + 2I}, {$2i_F$, 1, 2, i_F, 0}},

v_4 = {{1, 2, 3, 4, 5, 6, 7}, {1, g, 2 + 2g, g + 1, 0}, {1, I, 2, 2I, 3, 3I}, {0, 1, i_F, $2i_F$, 2, 1 + i_F}},

v_5 = {{3, 4, 6, 7, 8, 10, 9}, {1 + i_F, 1, 0, 2 + i_F, 2 + $2i_F$}, {g, 2g, 0, 1, 1 + g}, {I, 3I, 2I, 0, 1, 2, 3, 1 + 3I, 3I + 2}} and

v_6 = {{0, 2, 6, 7, 8, 9}, {g, 2g, 1 + g, 2 + 2g, 1, 2, 0}, {I, 2I, 0, 1, 2, 1 + 2I, I + 1}, {i_F, 0, 1, 2, 2 + i_F}}.

G the subset vertex multigraph is given below with vertex subsets v_1, v_2, …, v_6.

G =

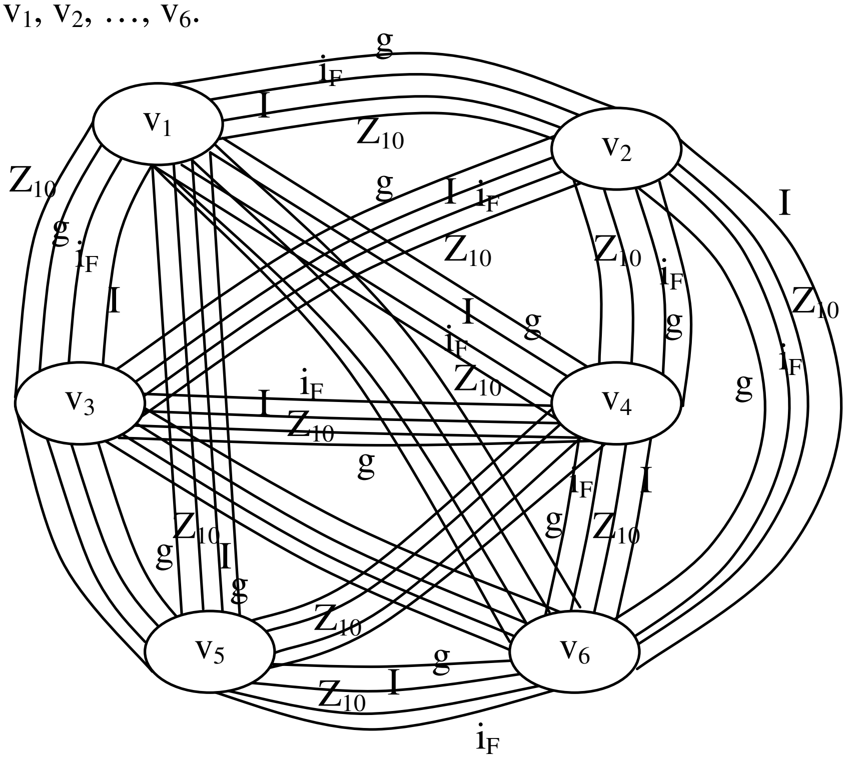


Figure 3.79

Clearly G the subset vertex multigraph is complete and is maximal uniform.

Let H be the subset-subset vertex multisubgraph of G with t_1, t_2, t_3 and t_6 as subset vertex subsets of v_1, v_2, v_3 and v_6 respectively.

$t_2 = \{\{2, 4, 6, 8, 5\}, \{g, 2g, 0, 1, 2 + 2g\}, \{I, 1, 2, 3I, 3 + 2I\}, \{i_F, 2i_F, 1\}\} \subseteq v_2$,

$t_1 = \{\{2, 5, 7\}, \{2g, 1 + 2g\}, \{I, I + 3\}, \{i_F, 2\}\} \subseteq v_1$,

$t_3 = \{\{9, 11, 2\}, \{g, 1 + g, 0\}, \{0, 3, 3 + 2I\}, \{i_F, 2i_F, 0\}\} \subseteq v_3$ and

$t_6 = \{\{0, 2, 7, 9\}, \{g, 2 + 2g, 0, 1, 2\}, \{I, 2I, 1+ 2I, 1 + I, 0\}, \{i_F, 1, 0\}\} \subseteq v_6$.

The subset-subset vertex multisubgraph M with vertex subset t_1, t_2, t_3 and t_6 is as follows.

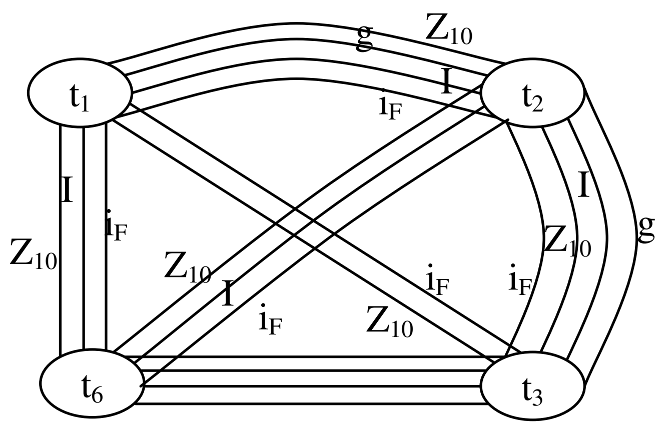
M = 

Figure 3.80

Clearly the subset-subset vertex multisubgraph M of the subset vertex multigraph G is only a pseudo complete and not maximal uniform.

Now we find the local complement P of M relative to G. The set of local complements of t_1, t_2, t_3 and t_6 the subset-subset vertex multisubgraph H relative to G is as follows.

$s_1 = v_1 \setminus t_1 = \{\{6, 0\}, \{1, g\}, \{1, 2\}, \{1, 1 + 2i_F\}\}$

$s_2 = v_2 \setminus t_2 = \{\{3\}, \{2\}, \{3\}, \{2 + 2i_F\}\}$

$s_3 = v_3 \setminus t_3 = \{\{5, 6\}, \{1, 2\}, \{3I + 2, 3I + 1, 1, 2\}, \{1, 2\}\}$ and

$s_6 = v_6 \setminus t_6 = \{\{6, 8\}, \{2g, 1 + g\}, \{2\}, \{2, 2 + i_F\}\}$ are local complements of t_1, t_2, t_3 and t_6 relative to v_1, v_2, v_3 and v_6 respectively.

The subset-subset vertex multisubgraph with vertex subsets s_1, s_2, s_3 and s_6.

P =

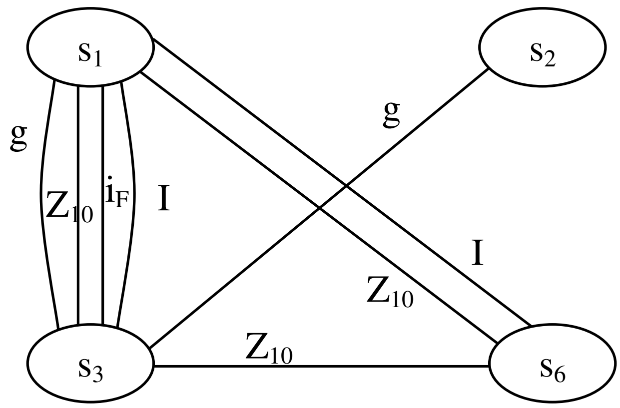


Figure 3.81

We see the local complement P of M is not even pseudo complete. Further there is no edge connecting t_1 and t_2. However, s_1 and s_3 has the maximum number of edges whereas t_1 to t_3 in the subset-subset vertex multisubgraph there is only two edges but the parent subset vertex multigraph G has (four or) the maximum number of edges.

We see the local complement of the subset-subset vertex multi subgraphs do not in general preserve structure either with the parent subset vertex multigraph G or the subset-subset vertex multisubgraph H of G.

It would be an interesting problem to study those local complements of those subset-subset vertex multi subgraphs which enjoy the same structure as that of the subset-subset vertex multi subgraph.

Now we give an example of a subset vertex star multigraph G, its subset-subset vertex multi subgraphs and its local complement.

Example 3.18. Let S = {Z_{19}, $\langle Z_{10} \cup g\rangle$, $\langle Z_{12} \cup I\rangle$} be the set with three distinct features. P(S) be the power set of S. Let G be the subset vertex multigraph given by the following figure.

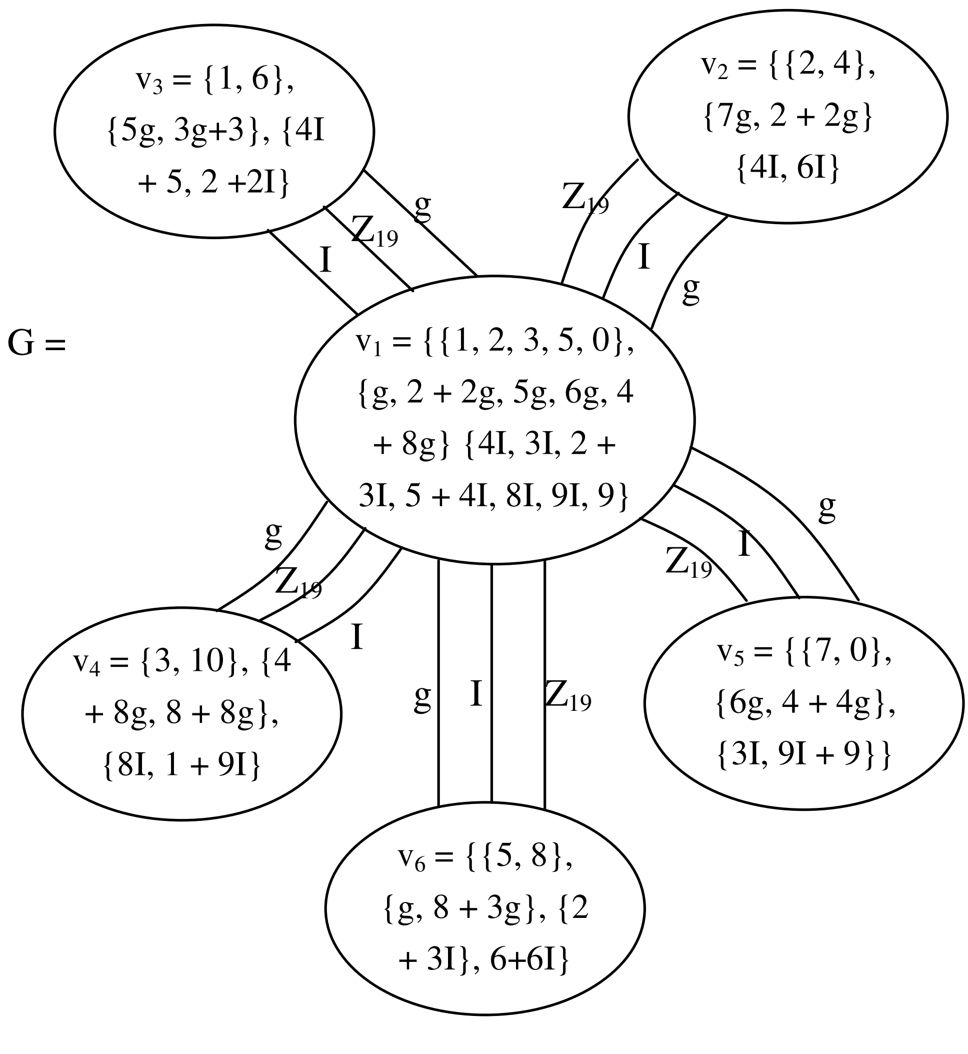


Figure 3.82

Let H be the subset-subset vertex multisubgraph of G given by the following figure.

H =

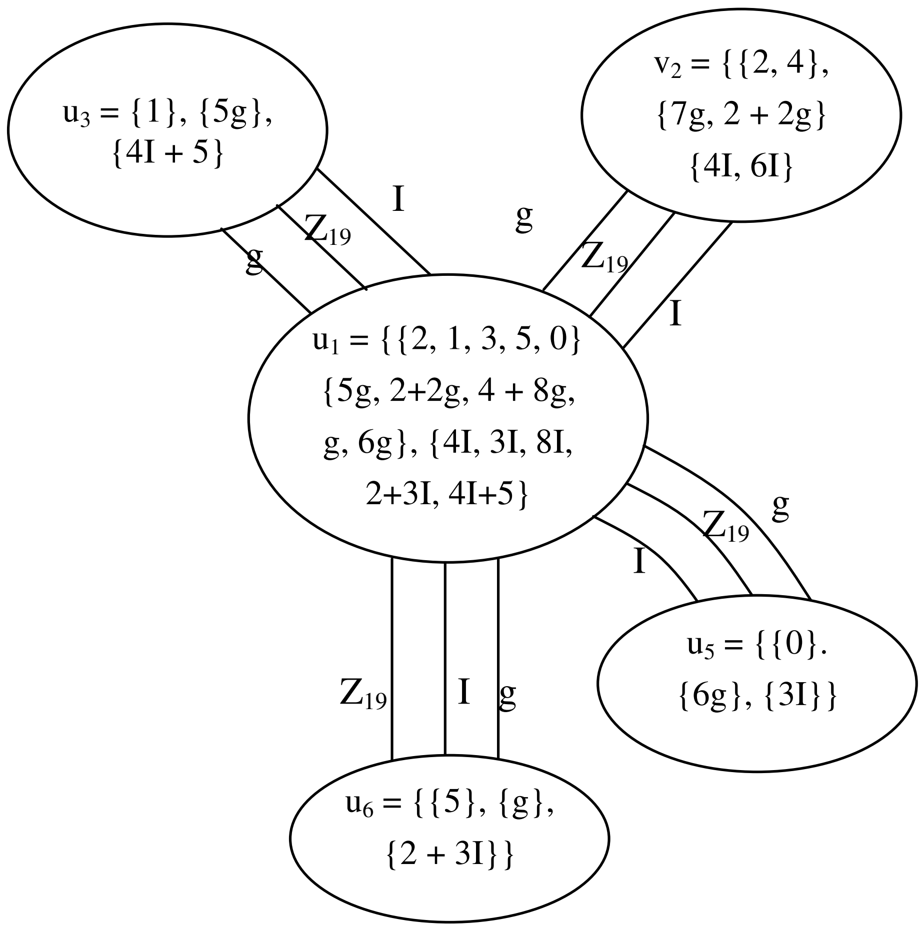


Figure 3.83

H is a subset-subset vertex multisubgraph of G. Now we find the local complement of H relative to G. The local complements of u_1, u_2, u_3, u_5 and u_6 are as follows:

$s_1 = v_1 \setminus u_1$ = {{3}, {4 + 8g}, {8I, 9I, 9}},

$s_3 = v_3 \setminus u_3$ = {{6}, {3g + 3}, {2 + 2I}},

$s_2 = v_2 \setminus u_2$ = {{4}, {7g}, {6I}},

$s_5 = v_5 \setminus u_5$ = {{7}, {4 + 4g}, {9 + 9I}} and

$s_6 = v_6 \setminus u_6$ = {{8}, {3 + 8g}, {6 + 6I}}.

We see the vertex subsets s_1, s_2, s_3, s_5 and s_6 which are local complements of u_1, u_2, u_3, u_5 and u_6 relative to v_1, v_2, v_3, v_5 and v_6.

T =

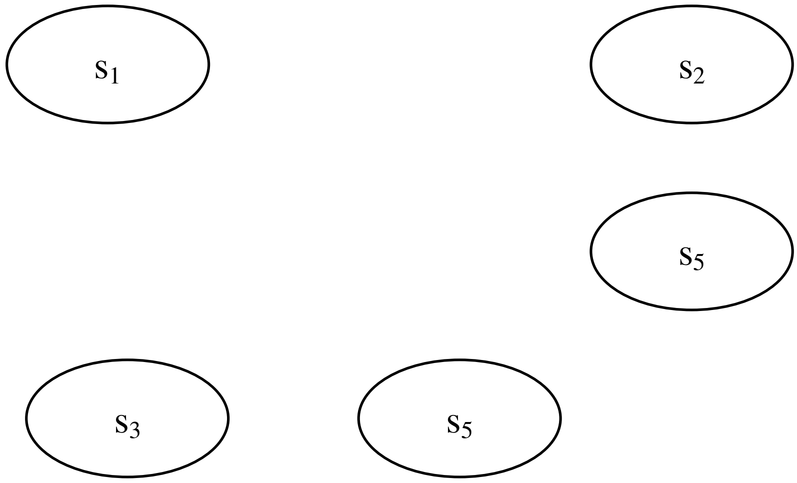


Figure 3.84

Thus, the local complement of H relative to G is only a point subset-subset vertex multi subgraph.

This is yet another special feature of this new notion of local complements of subset-subset vertex multi subgraphs. We see even a complex (maximal) subset-subset star multisubgraph may have a local complement which is just a point multi subgraph. We use the term point subset vertex multisubgraph as the subset vertex sets have more than one element.

Finally we see if we have two vertex subsets u_i and v_i, where u_i is one of the subset-subset vertex multisubgraph H of G another v_i just a vertex subset of G such that even for one set of the attributes say {a_i's 's}; i^{th} attribute; of u_i and v_i (u_i contained in v_i) coincides then we define the local complement for all other attributes we cannot assign any edge for the i^{th} attributes. So we are always guaranteed of the local complement of a subset-subset vertex multi subgraphs of a subset vertex multigraph as we basically assume they are proper subsets of the parent graph. Similarly, in case of universal complement to exist we must have subset vertex to be proper subsets of each of the distinct properties set. For instance if $S = \{Z_7, \langle Z_9 \cup I\rangle, \langle Z_{12} \cup g\rangle, C(Z_{18})\}$ and if v_1 is a vertex subset of a subset vertex multigraph of G with $v_1 = \{Z_7, X_2, X_3, X_4\}$ where X_2, X_3 and X_4 are proper subsets of $\langle Z_9 \cup I\rangle$, $\langle Z_{12} \cup g\rangle$ and $C(Z_{18})$ respectively then we define the universal complement of v_1 as $\{\langle Z_9 \cup I\rangle \setminus X_2, \langle Z_{12} \cup g\rangle \setminus X_3, C(Z_{18}) \setminus X_4\}$; for now the universal complement of v_1 in S is $S \setminus v_1$. Thus, the number of elements contributed by Z_7 is nothing and do not put the empty symbol. The main reason for this is we take only subsets of the set S only for working we group them together according to their attributes.

We will illustrate this by one more example taking the same set S.

If $v_1, v_2, \ldots, v_5$ be some vertex subsets of S for a subset vertex graph G and if u_1, u_2, u_3 and u_4 are the subsets of the vertex subsets of the subset-subset multi subgraph.

If $v_1 = \{\{2, 3, 0\}, \{8I, 3I, 4I, 2 + 2I, 1, 2, 5\}, \{8 + 9g, 9g, g, 9 + 9g, 0, 1, 2, 5, 6, 8, 9\}, \{i_F, 9 + 9i_F, 9i_F, 9, 9 + 6i_F, i_F + 1\}\}$ and $u_1 = \{\{2, 3, 0\}, \{8I, 3I\}, \{9 + 9g, 0, 1, 2, 9, 9g\}, \{i_F, 9 + 9i_F\}\}$ the local complement of u_1 relative to v_1 is

$v_1 \setminus u_1 = \{\{4I, 2 + 2I, 1, 2, 5\}, \{8 + 9g, g, 6, 8\}, \{9i_F, 9, 9 + 6i_F, i_F + 1\}\}$, that is $v_1 \setminus u_1$ has no Z_7 edge with $v_2 \setminus u_2$, $v_3 \setminus u_3$ and $v_4 \setminus u_4$ respectively. This is the way local complements are taken.

We will give an illustration of the same. Let S be as before. Let v_1, v_2, v_3 and v_4 be the subset vertex sets of the subset vertex multigraph G given in the following:

$v_1 = \{\{1, 2, 3\}, \{8I, 3 + 2I, 2 + 2I, 5I\}, \{4g, 8g, 9g, 9g + 1, 9, 6\}, \{2i_F, 4, 6i_F, 8, 10i_F, 1, 2, 14, 16, 0\}\}$,

$v_2 = \{\{1, 2, 4, 6, 0\}, \{8I, 2 + 2I, 1, 2, 3, 4\}, \{9, 6, 4g, 4g + 2\}, \{i_F, 2i_F, 4, 10i_F, 14\}\}$,

$v_3 = \{\{6, 0\}, \{2I, 2, 4, 4I, 0, 6\}, \{9, 6, 8g, g, 2g\}, \{2, 4, 6, 8i_F, 16i_F, 1\}\}$ and

$v_4 = \{\{4, 6\}, \{2I, 2, I\}, \{g, 3g, 5g\}, \{3i_F, 5i_F, 0, 5, 3\}\} \in S$.

The subset vertex multigraph G is as follows:

G =

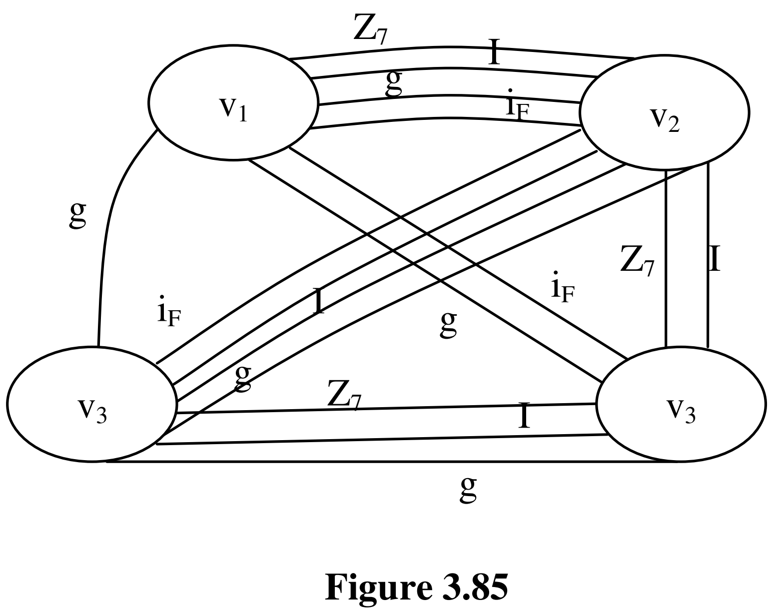


Figure 3.85

Let H be the subset-subset vertex multisubgraph given by the following set of vertex subset u_1, u_2 and u_4 given by the following.

$u_1 = \{\{1, 2, 3\}, \{8I, 2 + 2I, 5I\}, \{8g, 4g, 9, 6\}, \{2i_F, 4, 6i_F, 8\}\} \subseteq v_1$

$u_2 = \{\{1, 2, 6\}, \{8I, 2 + 2I, 1, 2, 3, 4\}, \{9, 6, 4g\}, \{i_F, 4, 14\}\} \subseteq v_2$ and

$u_4 = \{\{4, 6\}, \{2I, I\}, \{g, 3g, 5g\}, \{3i_F, 5i_F, 0\}\} \subseteq v_4$. The subset-subset vertex multisubgraph H of G is as follows.

H =

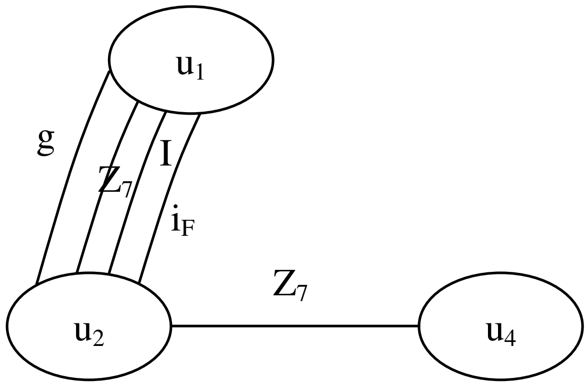


Figure 3.86

Clearly the subset-subset vertex multisubgraph H is a forbidden triad which is not maximal uniform. Now we find the local complement of H in G.

Let s_1, s_2 and s_4 be the local complements of u_1, u_2 and u_4 respectively relative to v_1, v_2 and v_4.

$s_1 = v_1 \setminus u_1 = \{\{3 + 2I\}, \{9g, 9g + 1\}, \{10i_F, 12, 14, 16, 0\}\} \subseteq v_1$

$s_2 = v_2 \setminus u_2 = \{\{4, 6\}, \{4g + 2\}\} \subseteq v_2$

$s_4 = v_4 \setminus u_4 = \{\{2\} \subseteq \{Z_9 \cup I\}, \{5, 3\} \subseteq C(Z_{18})\} \subseteq v_4$.

The local complement subset-subset vertex multisubgraph K is as follows.

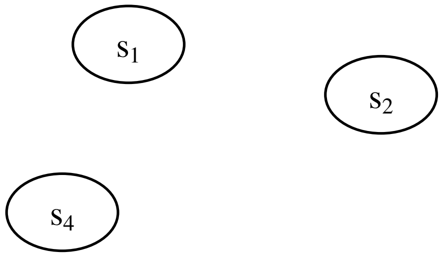
K = 

Figure 3.87

K is just a point multi subgraph. This is the way complements are taken. The reason being if X is a subset of P(S) they can have elements from all four sets $\{Z_7, \langle Z_9 \cup I\rangle, \langle Z_{12} \cup g\rangle$ and $C(Z_{18})\}$ and they can be only from 3 sets or from 2 sets or from 1 set but we take only S = {0, 1, 2, …, 6, 0, 1, 2, …, 8, 1 + I, 2 + I, …, I, 2I, …, 8I, 8 + 8I, g, 2g, …, 11g, 0, 1, …, 11, 1 + g, …, 11g + 11, 0, 1, 2, …, 17, i_F, $2i_F$, …, $17i_F$, 1 + i_F, 2 + i_F, …, 17 + $17i_F$}. Clearly taking into account the elements are picked and marked as four distinct attributes or nodes from these subset vertices or nodes.

Now we proceed onto suggest a few problems.

Problems

1. Describe type I subset vertex multigraph using P(S) the power set of set S.

2. Give an example of a subset vertex multigraph of type II which is complete.

3. Give an example of a subset vertex multigraph which is a uniform binary subset vertex multi tree.

4. Let $S = \{Z_5, \langle Z_3 \cup I\rangle\}$ be a set. P(S) be the power set of S.

a. How many subset vertex multigraphs can be constructed using the vertex subsets from P(S)?

b. How many of these subset vertex multigraphs are complete?

c. What is the maximum number of layers a subset vertex multigraph binary tree can have?

d. Study the situation of problem iii) in case of any tree.

e. What is the maximum number of vertices a subset vertex star multigraph can have using this P(S)?

f. What is the maximum number of vertex subset multigraphs we can have with 5 vertex subsets using this P(S)?

g. Let G be a subset vertex multigraph given by the following figure.

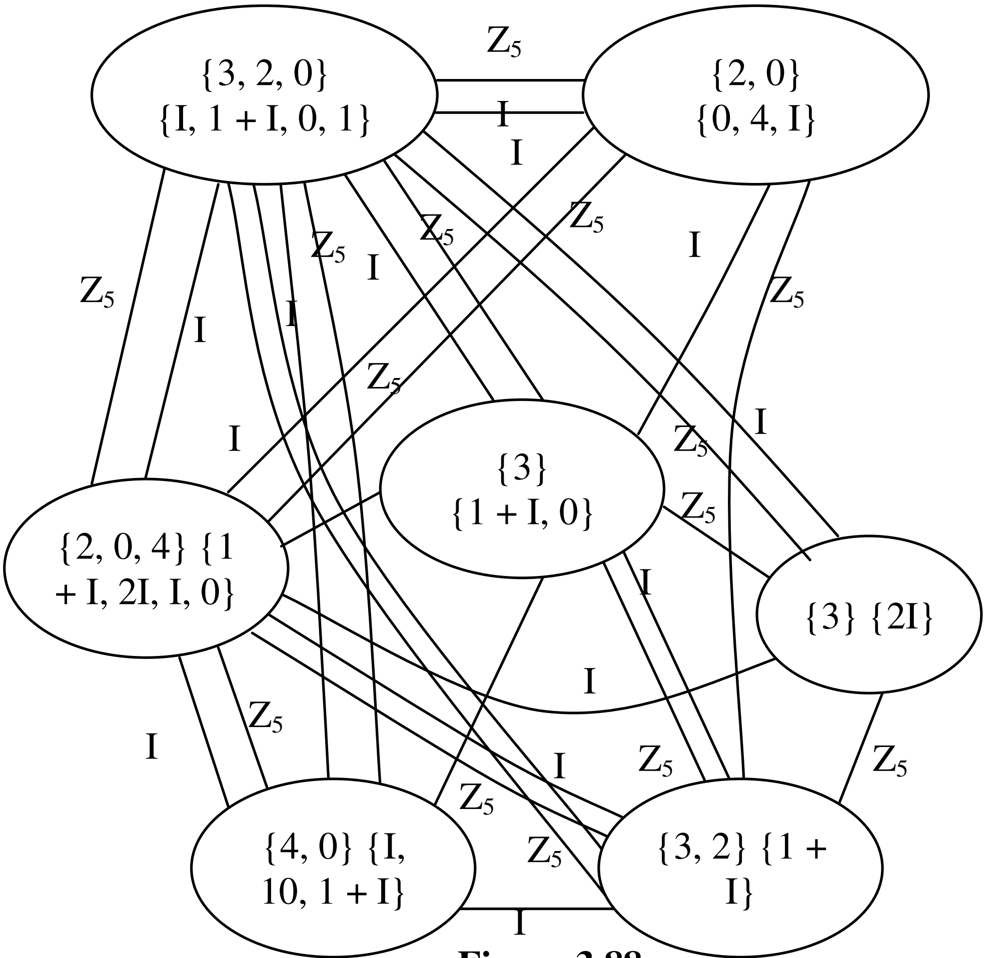


Figure 3.88

a. Find all subset vertex multi subgraphs of G.

b. Is G a subset vertex complete multigraph?

c. Find all subset vertex pseudo complete multi subgraphs of G.

d. Can G have subset vertex multi subgraphs which are pseudo uniform complete?

e. Find any other interesting property associated with this G.

5. Let S = $\{Z_{11}, Z_{16}, \langle Z_9 \cup I\rangle, \langle Z_{18} \cup g\rangle, C(Z_{10})\}$ be a set with 5 distinct attributes. P(S) be the power set of S.

   a. Study questions i) to iii) of problem for this P(S).

   b. Find a 3 - ary tree T (subset vertex multigraph which is a tree) with 8 layers.

   c. Prove or disprove T in ii) can have subset vertex multi subgraphs which can only be trees.

   d. Find a subset vertex multi star graph with 10 vertices where each of the multi edges connecting them is five.

   e. Let B be the uniform maximal subset vertex star multigraph given by the following figure.

B =

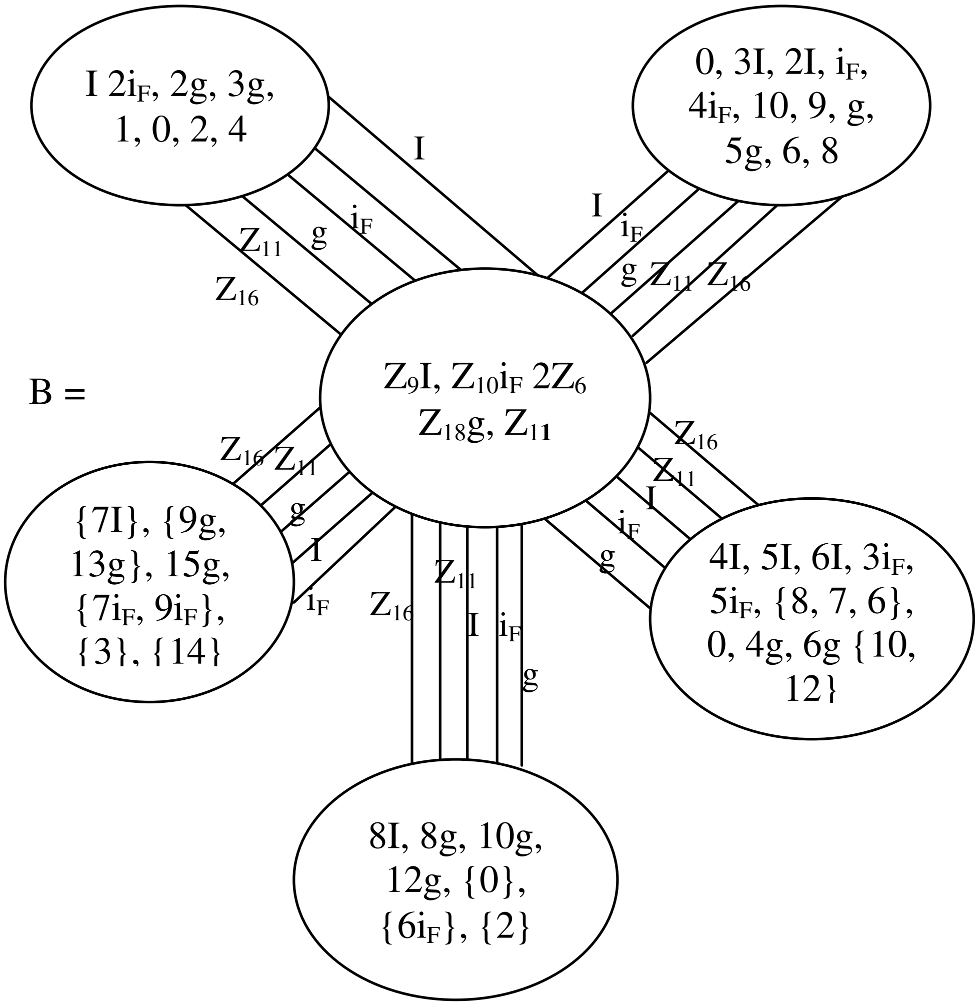


Figure 3.89

a) Show all subset vertex multi subgraphs of B are all either star graphs or empty subgraphs.

b) How many subset vertex multi subgraphs of B are there?

c) How many of them in b) are subset vertex hyper multi subgraphs?

(Recall a subset vertex multisubgraph H of B is hyper if that H is the largest possible proper subgraph of B. However, there can be many such subset vertex multi subgraphs which are hyper).

6. Let S = {Z_{12}, ⟨Z_9 ∪ g⟩, ⟨Z_8 ∪ k⟩, C(Z_{10})} be the set with four different kinds of sets. P(S) the power set of S. Let v_1 = {{2, 4, 6, 0, 9}, {g, 8g, 6g, 3 + 3g, 4g, 5g, 4, 2}, {k, k + 2, k + 5, 3 + 3k, 2k, 0}, {i_F, 9i_F + 9, 9i_F, 5i_F + 5, 6i_F + 3}}, v_2 = {{2, 4, 5, 7, 11}, {g, 6g, 2 + 2g, 3 + g, 3 + 8g, 8g}, {i_F, 3i_F + 3, 2 + 5i_F, 9i_F, 9}}, v_3 = {{2, 4, 9, 0}, {6g, 2 + 4g, 4 + 8g, 8 + 4g, 3 + g}, {5k, 3k + 2, 0, 1, 2, 4, 3 + 3k}}, v_4 = {{2 + 3g, 6g, 8g, 3 + g, 6g, 8g, 3 + g, 6g + 4}, {i_F, 3i_F + 3, 9i_F, 9, 2, 5, 7, 6}} and v_5 = {{1, 2, 3, 4, 5}, {1, 2, 2g, 3g, 3g + 3, 6g, 8g}, {k, 6k + 3, 5k + 5, 4k, 2k + 5, 3k}, {6 + 3i_F, i_F, 0, 4, 4i_F + 4, 8i_F + 8, 9i_F, 9}} be the vertex subset of the vertex subset multigraph G.

i) Draw the subset vertex multigraph G.

ii) Is G a uniform maximal complete subset vertex multigraph?

iii) Is G a pseudo complete subset vertex multigraph?

iv) Find at least two subset vertex multi subgraphs of G.

v) Find for atleast any two subset-subset vertex multi subgraphs their local complement.

vi) Find the universal complement of G.

vii) How many subset vertex point multi subgraphs of G exist?

viii) Find all subset vertex multi subgraphs of G which are pseudo complete.

ix) Can G have subset vertex multi star subgraphs?

x) Can G have subset vertex multi circle subgraphs?

xi) Can G contain subset vertex multi subgraphs which are pseudo complete triads?

xii) Find all subset vertex multi subgraphs of G which are forbidden triads.

xiii) Find a subset-subset vertex multisubgraph H of G.

xiv) Find the local complement of H relative to G.

xv) Can H have the local complement to be a pseudo complete subset-subset vertex multi subgraph?

7. Obtain any special and interesting feature enjoyed by universal complements in general of a subset vertex multigraph.

8. Characterize those subset vertex multigraphs which have their universal complement to be structure preserving.

9. Can a general formula be derived to find the number of subset-subset vertex multi subgraphs of a given subset vertex multigraph given the power set P(S) ofS?

10. Find some interesting applications of these subset vertex multigraphs in general.

11. Prove subset vertex multigraphs can find applications in social networks.

12. Given the power set P(S) of S and a fixed subset vertex multigraph G can one find the number of subset-subset vertex multi subgraphs of G.

13. Let S = $\{\{a_{11}, a_{12}, a_{13}, a_{14}, a_{15}\}, \{a_{21}, a_{22}, a_{23}, a_{24}, a_{25}, a_{26}, a_{27}\}, \{a_{31}, a_{32}, \ldots, a_{39}\}\}$ be the set having 3 distinct sets as nodes. P(S) be the power set of S.

i) Find the total number of subset vertex multigraphs that can be constructed using P(S).

ii) Given G a subset vertex multigraph given by the following figure. Study questions ii) to xv) of problem 6 for this G.

G =

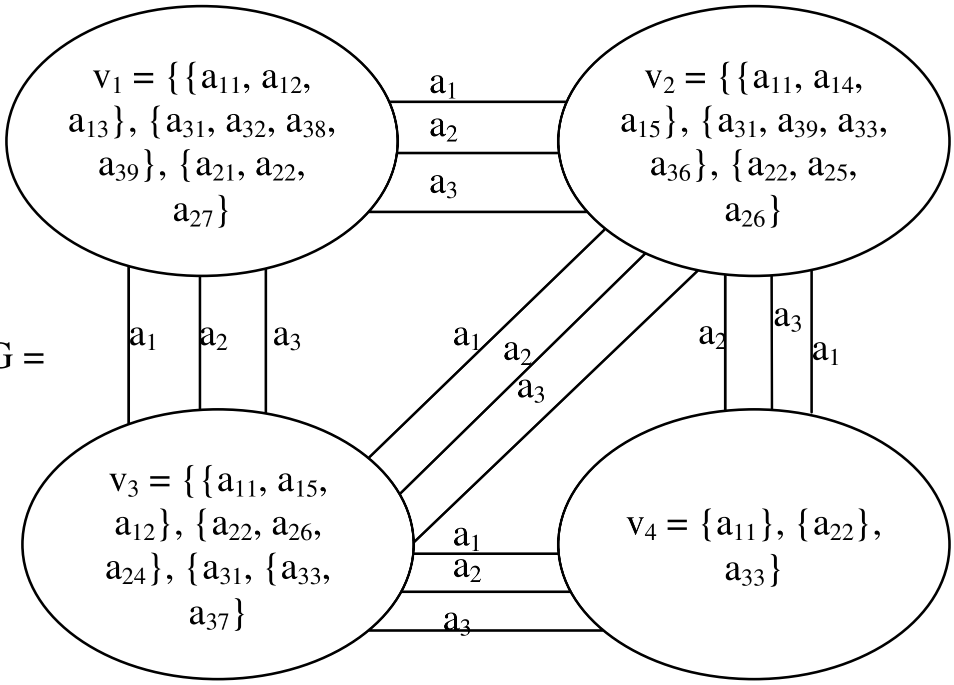


Figure 3.90

iii) Find all subset-subset vertex multi subgraphs of G.

iv) Find the local complement of H the subset-subset vertex multi subgraphs of G.

H =

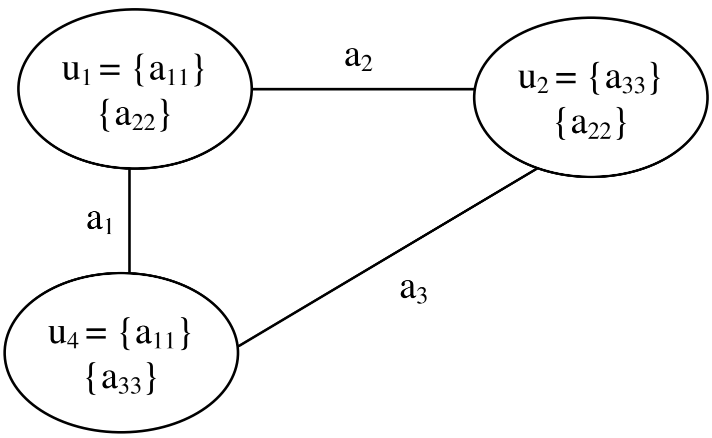


Figure 3.91

v) Is the local complement H of G pseudo complete?

14. Find all subset vertex multi subgraphs of G which are hyper. (G given in problem 13.).

15. Let S = {Z_{12}, ⟨$Z_{13} \cup I$⟩, C(Z_{18}), ⟨$Z_{15} \cup g$⟩} be the set. P(S) the power set of S.

i) Find all subset vertex multigraphs using P(S) as vertex subset.

ii) Find all uniform complete subset vertex multigraphs using P(S) as vertex subset.

iii) Find all uniform maximal subset vertex multiline graphs using P(S) as the vertex subset.

iv) Give the subset vertex multiline graph with maximum number of vertex subsets (nodes).

v) Find all subset vertex multi star graphs with maximum number of vertex subsets (nodes).

vi) Does there exist subset vertex multigraphs which has no hyper multi subgraphs?

16. Prove/Disprove that there does not exist a subset vertex multigraph which has subset-subset vertex multi subgraphs which has no local complement.

17. Let S = {Z_{45}, C(Z_{10}) ⟨$Z_9 \cup I$⟩} be a set. P(S) the power set of S.

i) Draw the subset vertex multigraph G using the vertex set v_1 = {{3, 5, 0, 7, 40, 43, 14}, {i_F, 8, $9i_F$ + 1, 9 + $9i_F$, 3, 4, $5i_F$, 5 + $5i_F$}, {0, 1, 2, 3, 4I, 6I, 2 + 3I, 5I + 4}}, v_2 = {{3, 5, 2, 4, 8, 6, 10, 42, 24, 44}, {2, 6, 4, $5i_F$, 5, 5 +

$5i_F\}, \{0, 3, 4I, 5I, 6 + 3I, 7I, 8 + 8I\}\}$, $v_3 = \{\{3, 5, 0, 6, 4, 30, 32, 36, 34, 38\}, \{i_F, 8, 5 + 5i_F, 5i_F\}, \{0, 2, 4, 4I, 3I, 4I + 2, 2I + 4\}\}$ and $v_4 = \{\{2, 3, 0, 6, 4, 30, 32, 36\}, \{i_F, 8, 9i_F + 1, 5 + 5i_F, 5\}, \{0, 8, 9 + 9I, 9I, 2I + 4, 4I + 2, 6 + 3I, 5I + 4\}\}$.

ii) Study questions i) to xv) of problem 6 for this G.

iii) Let $u_1 = \{\{3, 0, 40, 14\}, \{8, 9 + 9i_F, 4, 5 + 5i_F\}, \{1, 3, 4, I, 2 + 3I, 5I + 4\}\} \subseteq v_1$, $u_2 = \{\{5, 4, 6, 10, 44, 24\}, \{6, 5i_F, 5 + 5i_F\}, \{0, 4I, 6 + 3I, 8 + 8I\}\} \subseteq v_2$ and $u_3 = \{\{5, 0, 4, 30, 36, 34\}, \{i_F, 5 + 5i_F\}, \{0, 4, 3I, 4I + 2\}\} \subseteq v_3$ be the subset vertex s of the subset-subset vertex multisubgraph H of G. Draw H.

iv) Find the local complement of H in G.

v) Take another subset-subset vertex multisubgraph K of G and compare it with H.

vi) Can G have a subset-subset vertex multisubgraph whose local complement is complete?

vii) Can G have a subset-subset vertex multisubgraph whose local complement is just a point multi subgraph?

Chapter Four

DIRECTED SUBSET VERTEX MULTIGRAPHS AND NEUTROSOPHIC MULTIGRAPHS

In this chapter we define the notion of directed subset vertex multigraphs where the arrows may be projective or injective. From the very graph one can understand the multigraph is projective or injective. Consider two subset vertices v_1 and v_2 where

v_1 = {{3, 4, 5, 9, 12}, {I, 7I + 4, 10I, 12I + 7, 9 + 9I, 9I, 9} {g + 3, 5g + 2, 9g, 12g + 9, 9g + 3, 9 + 5g, g, 10g}, {i_F, 2 + 3i_F, 7, 4 + 5i_F}} and v_2 = {{3, 4, 9}, {I, 9 + 9I, 9I, 9}, {g + 3, 5g + 2, 12g + 9, 9g + 3}, {3i_F, i_F, 2i_F + 3, 9, 2 + 3i_F, 4 + 5i_F}}.

We see v_1 and v_2 are in no way related by containment relation as total sets, however {3, 4, 5, 9, 12} $\subseteq$ v_1 contains {3, 4, 9} $\subseteq$ v_2, {I, 9 + 9I, 9I, 9} $\subseteq$ v_2 is contained in {I, 7I + 4, 10I, 12I + 7, 9 + 9I, 9I, 9} $\subseteq$ v_1, {g + 3, 5g + 2, 9g, 12g + 9, 9g + 3, 9 + 5g, g, 10g} $\subseteq$ v_1 contains {g + 3, 5g + 2, 12g + 9, 9g + 3} $\subseteq$

v_2 but $\{i_F, 2 + 3i_F, 7, 4 + 5i_F\} \subseteq v_1$ is not contained in $\{3i_F, i_F, 2i_F + 3, 9, 2 + 3i_F,, 5i_F + 4\} \subseteq v_2$.

So the subset vertex directed multigraph is as follows.

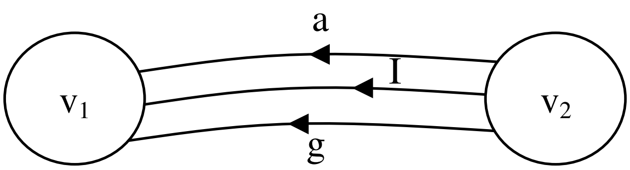


Figure 4.1

Here a denotes the real set edge, I denotes the indeterminate edge and g denotes the dual number edge. We observe the edges of the vertices connecting v_1 and v_2 the relation is injective in case of projective subset vertex multigraphs we will have relation as follows.

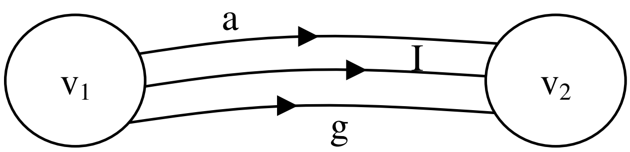


Figure 4.2

Now we give yet another example when the arrows or not in one way alone.

Let $v_1 = \{\{3, 7, 4\}, \{g, 5g, 2g, 7g, g + 1, 9g + 9\}, \{2 + k, 5k, 9k, 12k + 9\}, \{I, 2I, 9I, 8I + 9, 9I + 8\}\}$ and $v_2 = \{\{3, 7, 4\}, \{g, 5g, 1 + g, 9g + 9\}, \{5k, 9k, 12k + 9, 2 + k, 8k, 10k\} \{I, 8I, 9I\}\}$; since $\{3, 7, 4\}$ set is present in both v_1 and v_2 so we say no relation exists for such v_1 and v_2.

So it is mandatory that we can have edges if and only if none of subsets in v_1 and v_2 are identical. So when we make the abstract definition we make this mandatory.

Now consider two subset vertices v_1 = {{2, 4, 6, 8}, {I, 2I, 4I, 6 + 3I, 9I, 9I + 1}, {g, 5g, 10g, g + 9, 9, 9g, 9g + 1}, {3 + i_F, 2 + $4i_F$, i_F, $9i_F$, 2 + $9i_F$}} and v_2 = {{1, 3, 2, 4, 6, 8}, {5I, 8I, 4I}, {g, 5g, 9, 9 + g}|, {i_F, $9i_F$, 3 + i_F}}.

The injective subset vertex multigraph G with v_1 and v_2 as vertex subsets is as follows.

G =

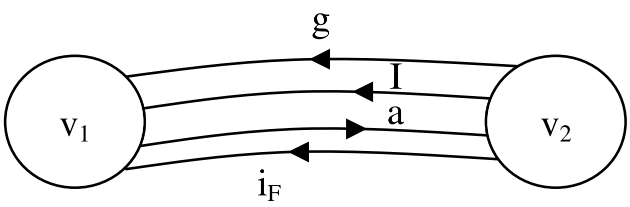


Figure 4.3

We see this is a injective subset vertex multigraph. The projective subset vertex multigraph G using the same set of vertices v_1 and v_2 is as follows.

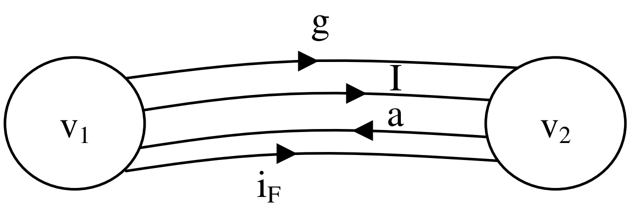


Figure 4.4

All these multigraphs with vertex sets v_1 and v_2 can be termed as multidyads.

Further we cannot say v_1 reciprocates with v_2 or v_2 reciprocates with v_1. In subsets of vertex subsets the indeterminate concept imaginary concept and the dual number concepts are sent by v_1 to v_2 however v_2 does not send these concepts but v_2 sends the real concept to v_1.

Thus in the opinion of the authors the notion of directed subset vertex multigraphs will play a significant and a special role in the social information networks. As these can work and make changes in the very unit (dyads) of the social network.

Next we proceed onto describe a few more examples.

Example 4.1: Let S = Z_{15}, $\langle Z_7 \cup g\rangle$, $\langle Z_{10} \cup I\rangle$, $C(Z_{12})\}$ be a set with four types of attributes real, (the edges connecting the real part of the subset vertices will be denoted by a), dual, (the edge associated with it will be denoted by g), indeterminate (the edge associated with will be denoted by I) and finally finite complex which will be denoted by i_F. Let P(S) denote the power set of S.

$P(S) = \langle\{P(Z_{15}), P(\langle Z_7 \cup g\rangle), P(C(Z_{12})), P(\langle Z_{10} \cup I\rangle\}\rangle$. Let K be the directed injective subset vertex multigraph with vertex subsets v_1, v_2 and v_3 given by the following figure where $v_1 = \{\{0, 1, 3, 5, 7, 9, 11\}; \{2i_F, 3 + 9i_F, 9 + i_F, 9, 9i_F, 8, 7, 3, 2, i_F\}, \{2g + 2, g + 5, 3g, g + 1\}, \{9I, 9, 9 + 9I, 2 + 2I, 3 + 3I\}\}$, $v_2 = \{\{0, 1, 3, 5, 7, 9, 11, 13, 2, 8\}, \{8, 7, 9, 9i_F\}, \{g, 3g, 5g, g + 1, g + g, 2 + 2g\}, \{9I, 9, 9 + 9I\}\}$ and $v_3 = \{\{2, 3, 5, 8, 11\}, \{8, 7, 9, 9i_F, 2i_F, 3\}, \{3g, g + 1, 5g\}, \{2 + 2I, 9I, 9, 9 + 9I\}\}$.

K =

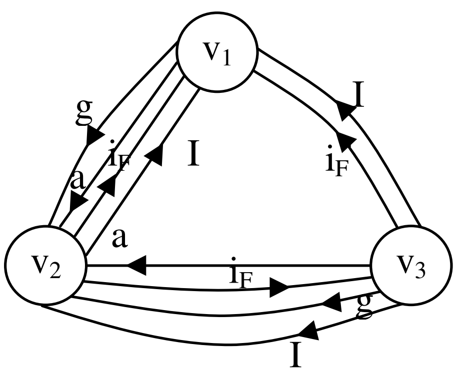


Figure 4.5

Clearly I_k is an injective directed subset vertex multigraph which is a triad. However, none of the relational tie real or complex or dual or indeterminate is transitive. Further as far as the notions real and dual are concerned the triad is only a forbidden triad.

Thus these injective directed subset vertex triads (multigraphs) can by all means some relations associated with certain concepts to be transitive and some relations can result in forbidden triads as those attributes may not result in a complete multigraph or a triad.

All the subset vertex multisubgraph of K are only dyads given in the following.

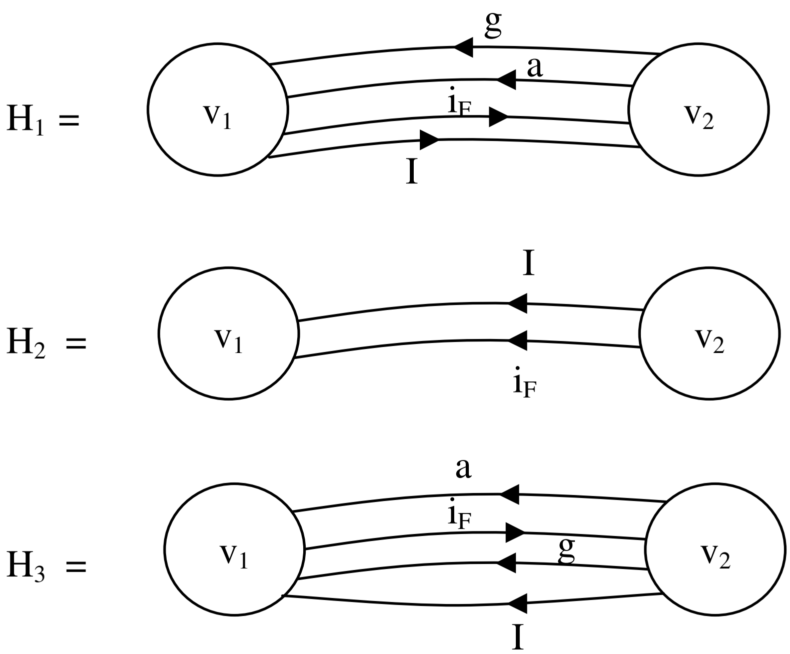


Figure 4.6

All the three dyads are non reciprocating dyads.

Now we give yet another example of the injective subset vertex multigraphs.

Example 4.2: Let S = {Z_{16}, Z_{24}, $\langle Z_{10} \cup I\rangle$, $\langle Z_{15} \cup g\rangle$, C($Z_{11}$)} be a set with 5 attributes. Edges associated with Z_{16} will be denoted by 'a' that of Z_{24} will be denoted by 'b'. Those edges relating to the attribute $\langle Z_{10} \cup I\rangle$ will be denoted by I and those with attribute $\langle Z_{15} \cup g\rangle$ by g.

Finally, i_F will denote the edge with attribute associated C(Z_{11}). Let G be the subset vertex multigraph given by the following figure.

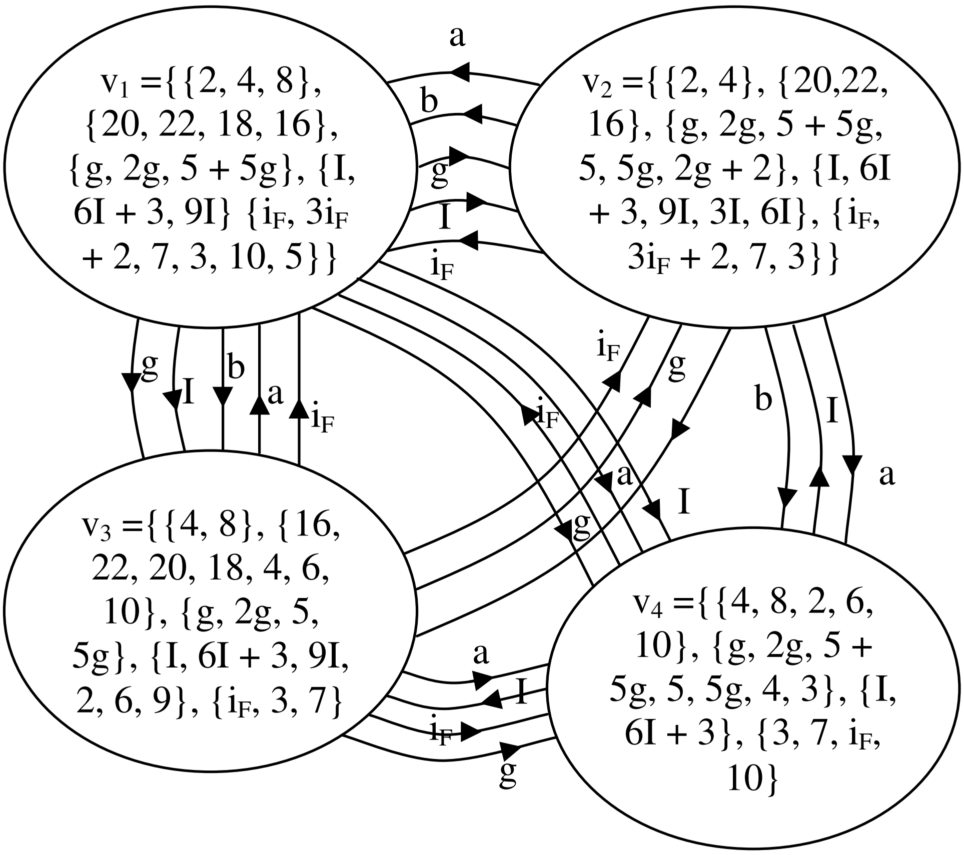
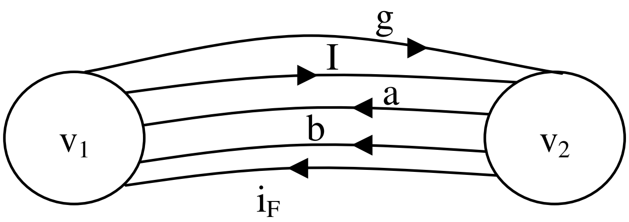


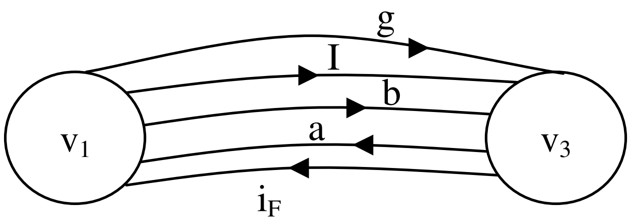
Figure 4.7

Now we have 6 subset vertex multisubgraphs of order two and 4 subset vertex multisubgraphs of order 3 which are as follows.

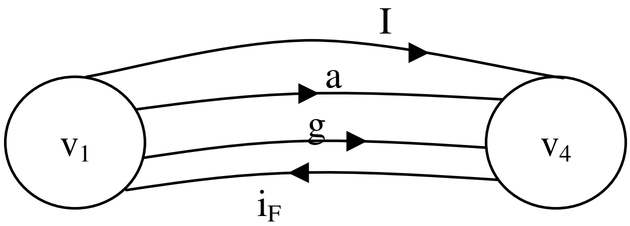
H_1 =



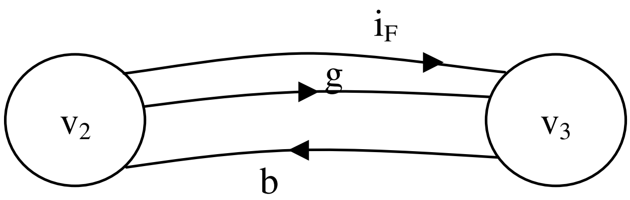
H_2 =



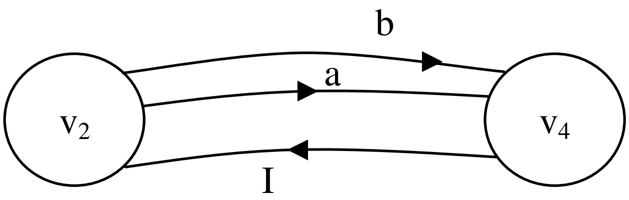
H_3 =



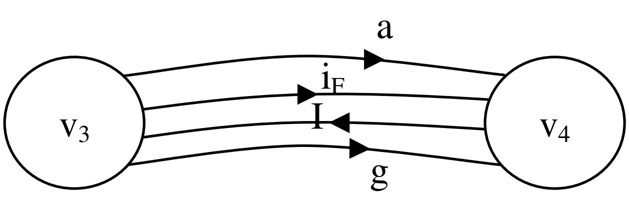
H_4 =



H_5 =

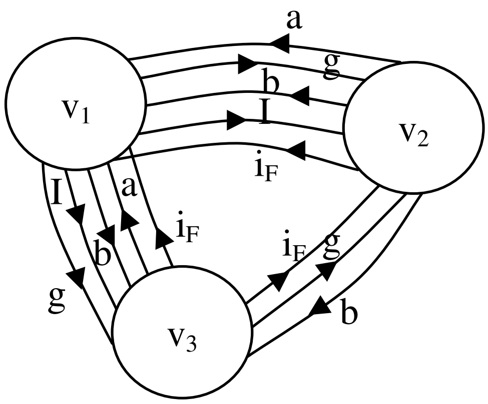


H_6 =

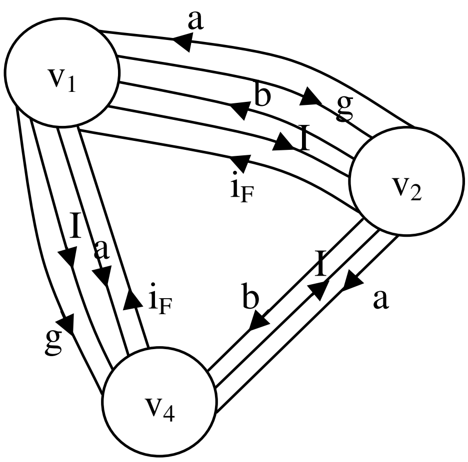


Figures 4.8

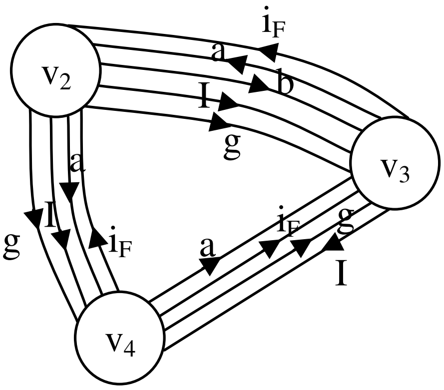
K_1 =



K_2 =



K_3 =



Figures 4.9

Now we proceed onto give the abstract definition of the injective subset vertex multigraphs.

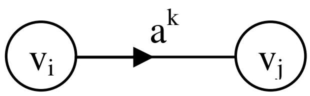
Definition 4.1. *Let* $S = \{\{ a_1^1 a_2^1,...,a_{n_1}^1 \} \{ a_1^2 a_2^2,...,a_{n_2}^2 \}, \ldots, \{ a_1^m\ a_2^n, \ldots, a_{n_m}^m \}\}$ *be a set which has m distinct attributes. P(S) be the power set of S.*

The injective subset vertex multigraph G with vertex subsets $v_1, \ldots, v_n \in P(S)$ *is defined if* $v_1, \ldots, v_n$ *satisfies the following conditions.*

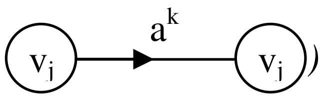
i) $v_1, \ldots, v_n$ *are attribute wise distinct if and only if the subsets in* v_i *and are such that none of the attribute subsets in* v_i *and* v_j *are identical.*

That is if $v_i = \{ a_{i_1}^1 \}\ \{ a_{i_2}^2 \}, \ldots, \{ a_{i_m}^m \}\}$ *and* $v_j = \{\{ a_{j_1}^1 \}, \{ a_{j_2}^2 \}, \ldots, \{ a_{j_m}^m \}\}$ *where* $\{ a_{i_t}^t \}$ *and* $\{ a_{j_t}^t \}$ *are subsets of* $\{ a_1^t, \ldots, a_{n_t}^t \}$ *then* $\{ a_{j_t}^t \} \neq \{ a_{i_t}^t \}$; $i \neq j$; $1 \leq t \leq m$.

We say an (injective) edge a^k *exists between* $\{ a_i^k \}$ *and* $\{ a_j^k \}$ *if* $\{ a_i^k \} \subset \{ a_j^k \}$ *(or* $\{ a_j^k \} \subset \{ a_i^k \}$*) then*



(or



) where $1 \leq k \leq m$*. Thus we have several edges connecting* v_i *and* v_j *in both directions* $1 \leq i, j \leq n$.

We will now present some more examples.

Example 4.3. Let $S = \{Z_{10}, \langle Z_{12} \cup g\rangle, \langle Z_{11} \cup I\rangle\}$ be a set with three sets of attributes the edges relating subsets of Z_{10} is denoted by 'a', that of $\langle Z_{12} \cup g\rangle$ by g and that of $\langle Z_{11} \cup I\rangle$ by I.

Let G be the injective subset vertex multigraph given by the following figure.

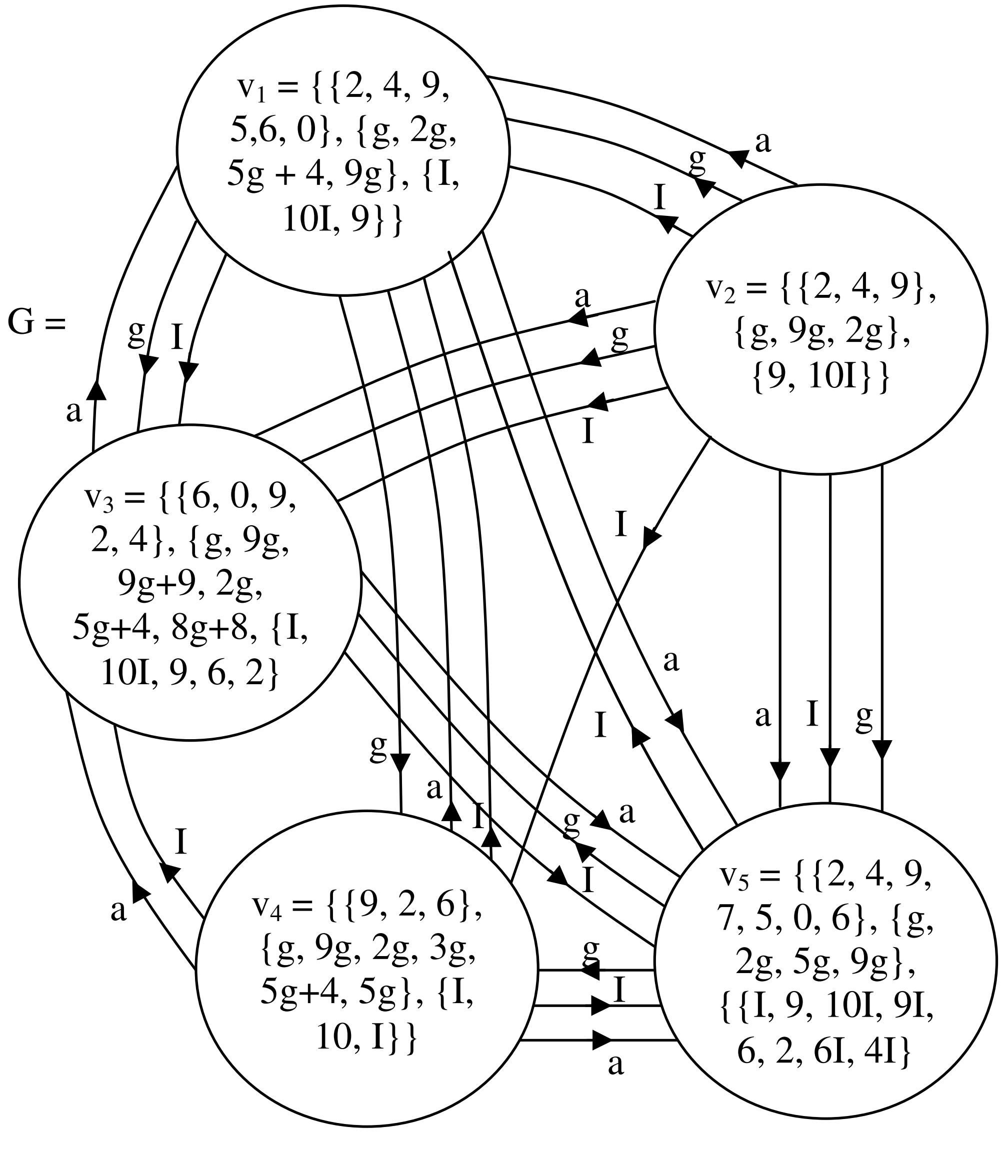


Figure 4.10

There are $5C_4 + 5C_3 + 5C_2$ number of subset vertex multisubgraphs for G.

10 of these subset vertex multisubgraphs are dyads.

5 of these subset vertex multisubgraphs are one-way dyads that is strongly non reciprocating dyads.

These are given below by the following figure.

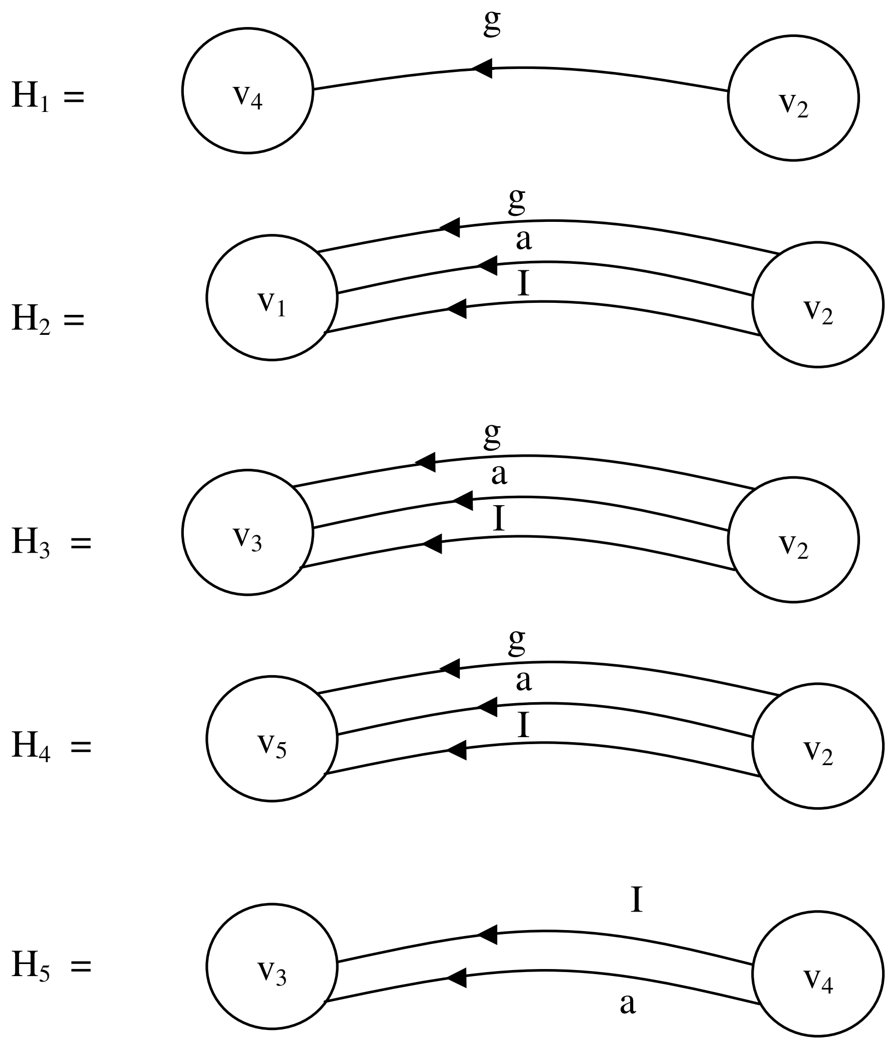


Figure 4.11

We call these as strongly non reciprocating dyads as none of these have mixed edges.

Next we proceed onto list out the mixed non reciprocating dyads in the following.

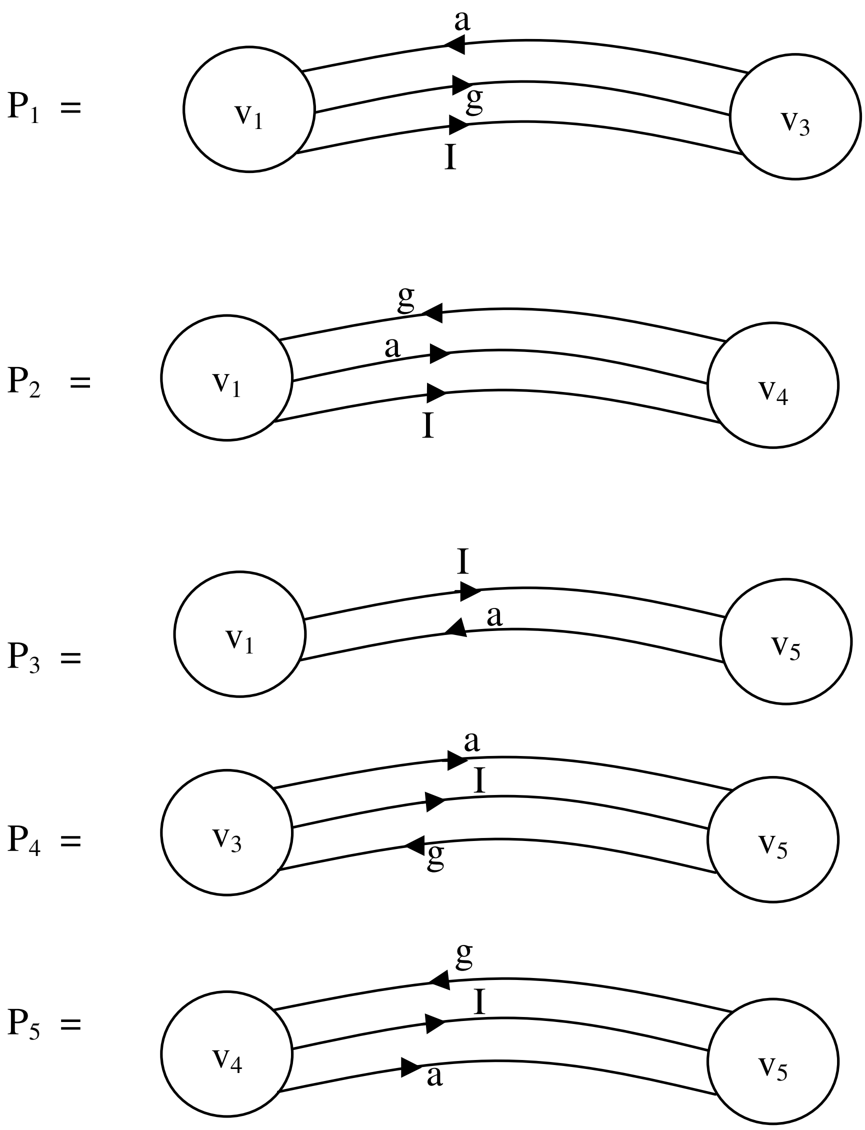


Figure 4.12

We call these dyads as mixed reciprocating subset vertex multi dyads (subgraphs).

However it is pertinent keep on record that there is no injective (or projective) subset vertex multisubgraphs which are dyads to be totally reciprocating. They can only be either non reciprocating or mixed reciprocating. This concept is very vital for in reality even in friendship they cannot be totally reciprocating.

Next we proceed onto analyze the injective subset vertex multisubgraphs which are multitriads in the following. There are 10 such injective subset vertex multisubgraphs which are multitriads by the following figures.

L_1 =

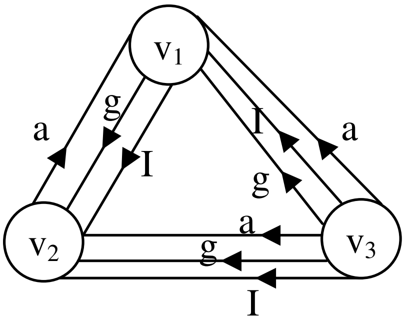
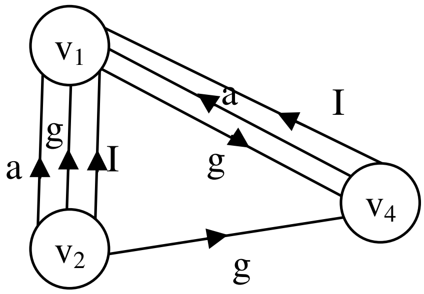


Figure 4.13

We call these multitriads as uniform neutrosophic multitriads, however they are not balanced multitriads. As the multiedges are three in all cases and only that is the possible one we call them as uniform multitriads.

L_2 =



L_3 =

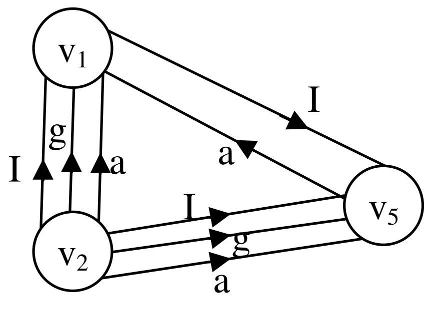
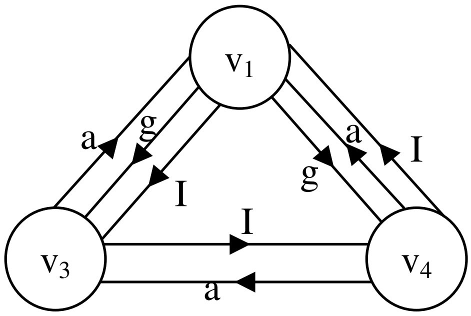


Figure 4.14

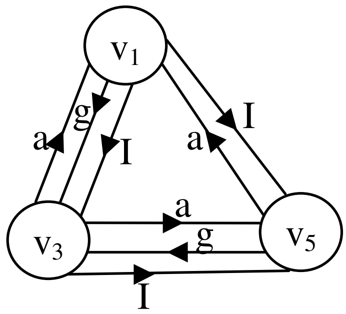
These two injective subset vertex multitriads are not uniform.

L_4 =

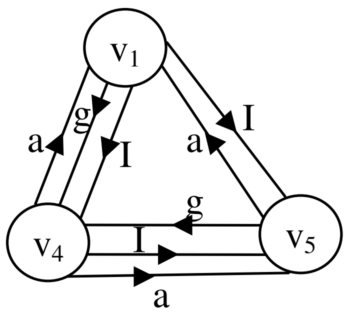


L_4 is not a uniform multitriad. However the edge I alone is balanced.

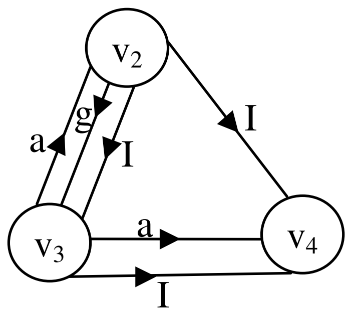
L_5 =



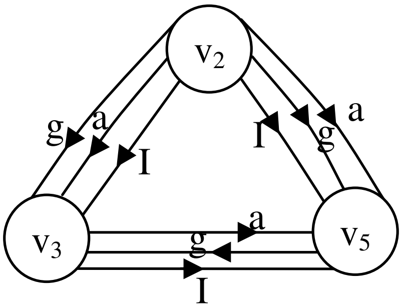
L_6 =



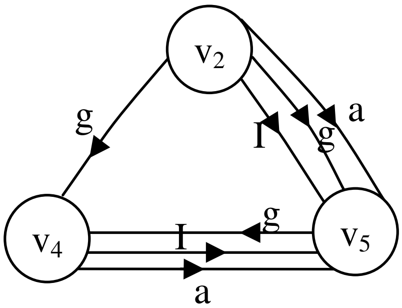
L_7 =



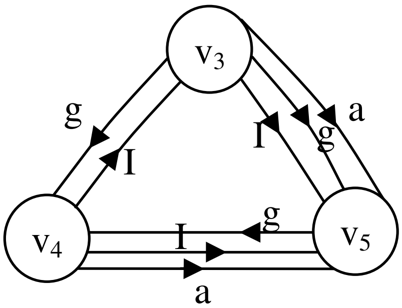
L_8 =



L_9 =



L_{10} =



Figures 4.15

Example 4.4. Let S = {Z_{15} = {a_0 = 0, a_1 = 1, a_{13} = 13, 14 = a_{14}}, $\langle Z_{12} \cup I\rangle$ = {a + bI / a, b ∈ Z_{12}}, I^2 = I $\langle Z_6 \cup g\rangle$ = {a + bg / a, b ∈ Z_6, g^2 = 0}, C(Z_7) = {a + bi_F / a, b ∈ Z_7, i_F^2 = 6}} be the set of 4 different types of attributes.

If elements are taken from Z_{15} we denote it by a as its edge if elements are taken from $\langle Z_{12} \cup I\rangle$ we will denote the edge by I, if elements are taken from $\langle Z_6 \cup g\rangle$ the edge will be denoted by g and for elements from C(Z_7) the edge connecting them will be denoted by i_F.

With this notational convenience we give the subset vertex multigraph G which is directed.

G =

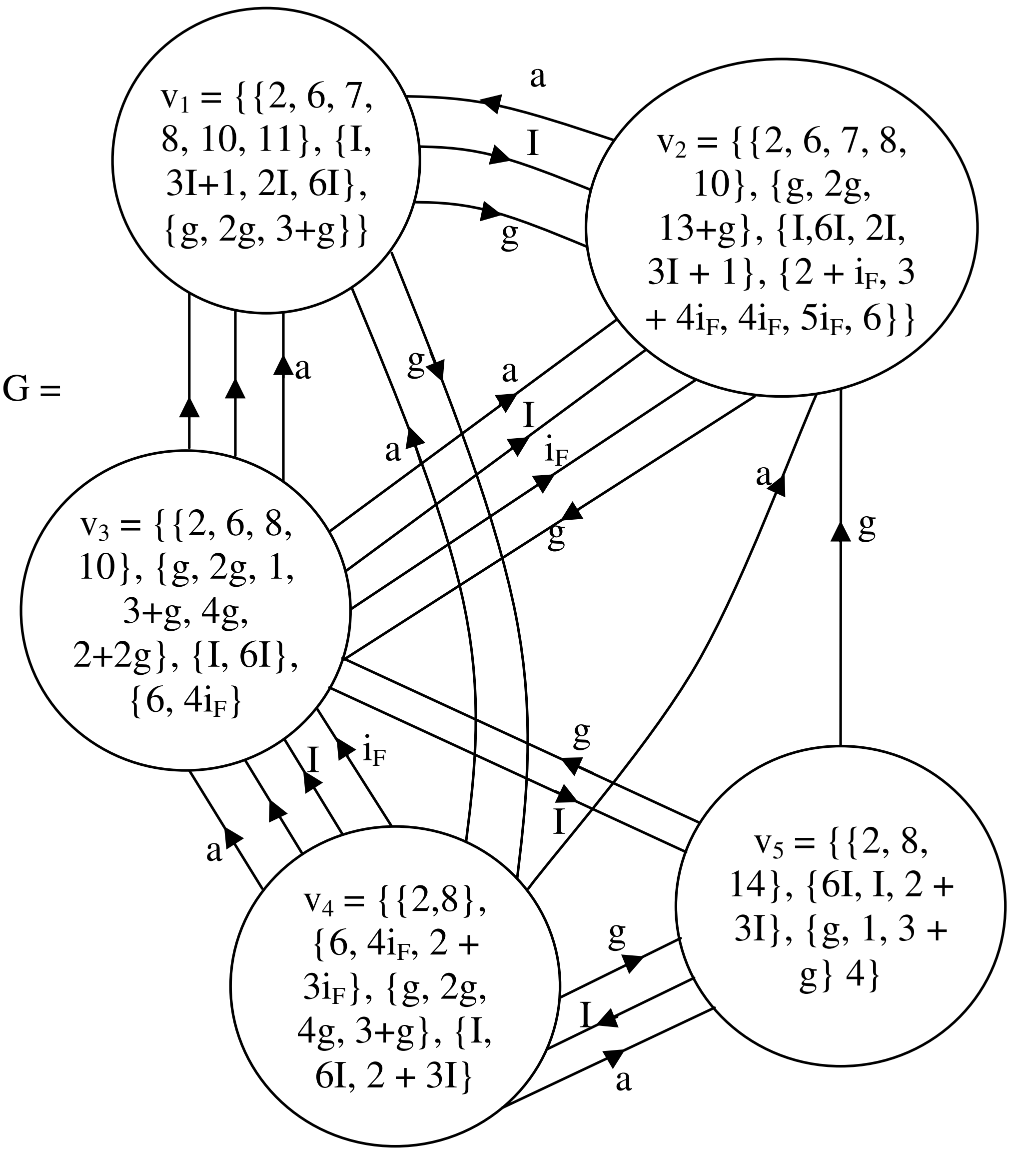


Figure 4.16

We make a few observations about the structure of G.

We see this subset vertex multigraph is neither projective nor injective it is mixed directed subset vertex multigraph. The edges from v_3 to v_1 are all injective so we see $\{v_3\} \not\subset \{v_1\}$ that is

$\{v_3\}$ is not properly contained in $\{v_1\}$ yet we get an injective relation.

Similarly $\{v_5\} \not\subset \{v_2\}$ we have only one edge connecting them, that is $\{g, 1, 3 + g\} \subseteq \{1, 3 + g, g, 2g\} \subseteq \{v_2\}$ and that there is no other relation. So it is observed that if we want to study the edges connecting v_3 and v_4 we see v_3 is in no way contained completely in v_4 or vice versa however $\{2, 8\} \subseteq v_4$ and $\{2, 6, 8, 10\} \subseteq v_3$ so $\{2, 8\} \subseteq \{2, 6, 8, 10\}$ which accounts for the edge 'a' from v_4 to v_3.

$\{g, 2g, 1, 4g, 2 + 2g, g + 3\} \subseteq v_3$ and $\{g, 2g, 4, 3 + g\} \subseteq v_4$ so this accounts for the edge from v_4 to v_3. Now $\{I, 6I\} \subseteq v_3$ and $\{I, 6I, 2 + 3I\} \subseteq v_4$ so this gives the edge I from v_3 to v_4 and so on.

There are $5C_2 + 5C_3 + 5C_4$ number of subset vertex multisubgraphs. But how many subset-subset vertex multigraphs can be defined using the subset vertex multigraph G.

Let B_1 be the subset-subset vertex multisubgraph given by the following figure.

B_1 =

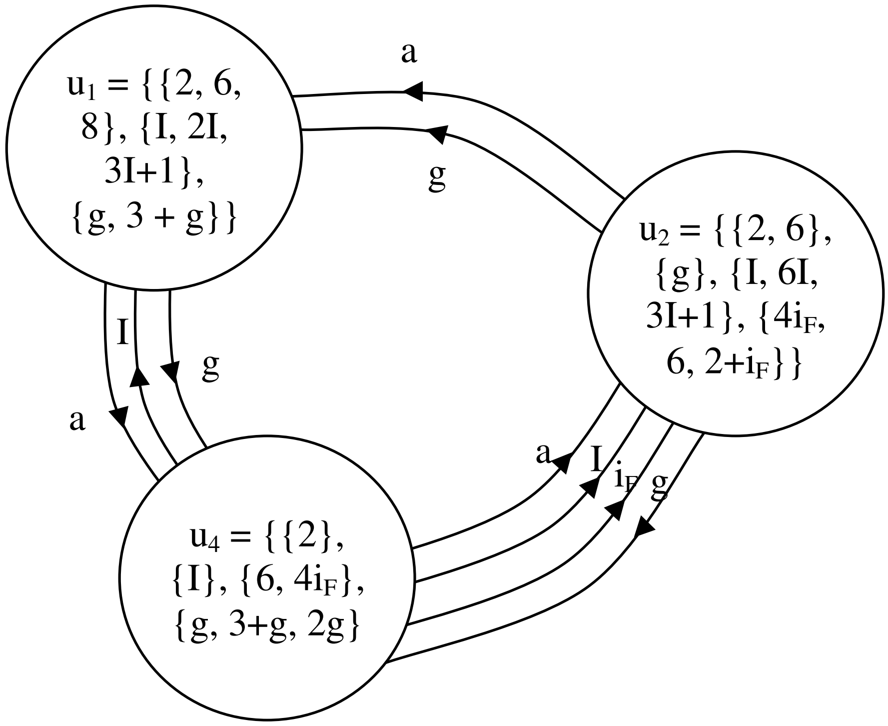


Figure 4.17

Clearly B_1 is a subset-subset vertex multisubgraph which is not a uniform multitriad and it is also not a balanced multitriad.

We can compare this with the subset vertex multisubgraph which is a subset vertex multitriad given by the following figure T_1

T_1 =

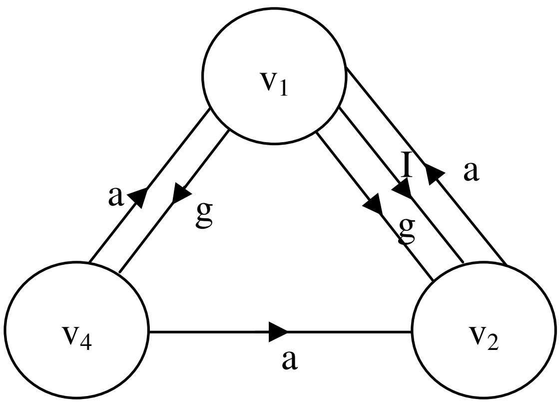


Figure 4.18

We see T_1 is a subset vertex multisubgraph which is not a uniform triad. We see B_1 can also be realized as a subset-subset vertex multisubgraph of T_1.

Now we compare the multi edge from u_1 to u_2 and v_1 to v_2 respectively as $u_1 \subset v_1$ and $u_2 \subset v_2$.

We see there are 3 edges from v_1 to v_2

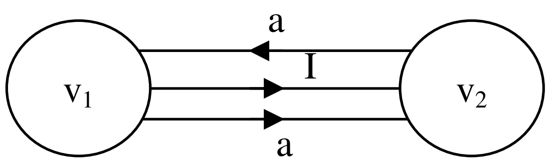


Figure 4.19

However from u_1 to u_2 there are only two edges

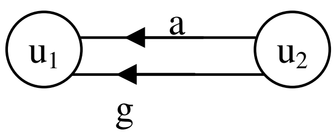


Figure 4.20

Clearly the multidyad with vertex subset u_1 to u_2 is only totally non reciprocating however the multidyad using the vertex subsets v_1 and v_2 where the same weight g in case of u_1 to v_2 is such that it is from u_2 (v_2) to u_1(v_1) so the direction is changed or reversed. On same lines we find the subset vertex multisubgraph (multidyad of u_1 to u_4 and v_1 to v_4 is given in the following for comparison, $u_1 \subset v_1$ and $u_4 \subset v_4$).

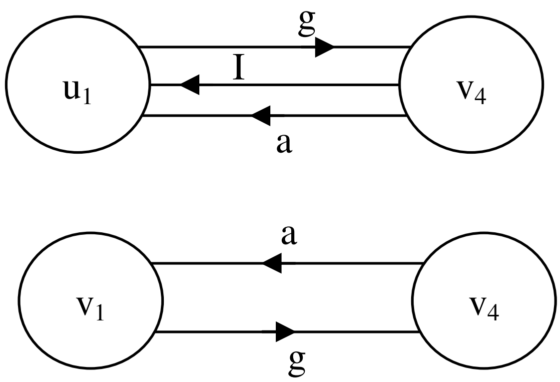


Figure 4.21

We see the multidyad connecting u_1 to u_4 has 3 edges whereas the multidyad connecting v_1 to v_4 has only two edges; however the direction happens to be the same.

Consider the multidyad connecting u_2 to u_v and v_2 to v_4 $u_2 \subseteq v_2$ and $u_4 \subseteq v_4$ given by the following figures.

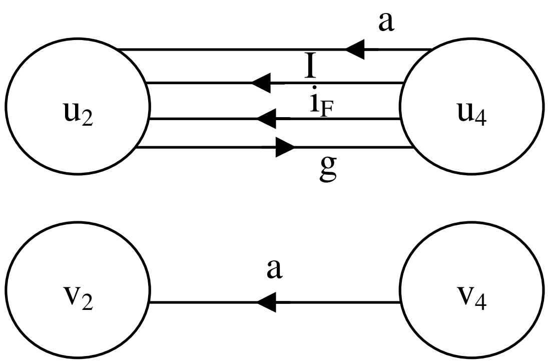


Figure 4.22

Observe the two multidyads we know the multidyad, with vertex subsets u_4 and u_2 is a injective subset vertex multisubgraph of the multidyad with vertex subsets v_4 and v_2. This multidyad has only one edge whereas the subset vertex multisubgraph of this which is a multidyad has four edges.

Thus, it is very interesting to note that in case of injective subset vertex multigraphs their injective subset vertex multisubgraphs may have more number of edges than the original. This is a very striking property enjoyed by injective subset vertex multisubgraphs of an injective subset vertex multigraph.

So in studying the social networks one may be very surprised to know by taking the subcollection of the existing collection the bonds or the relations may increase. This sort of property nodes cannot be got from the common or usual multigraphs.

Consider the subset vertex multisubgraph K of G given by the following figure.

K =

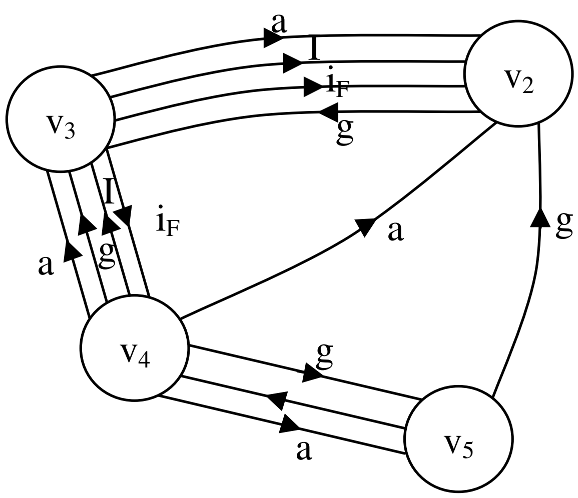


Figure 4.23

Consider the subset vertex subset multisubgraph H of G given by the following figure with subset vertex subsets $w_2 \subseteq v_2$, $w_3 \subseteq v_3$, $w_4 \subseteq v_4$ and $w_5 \subseteq v_5$.

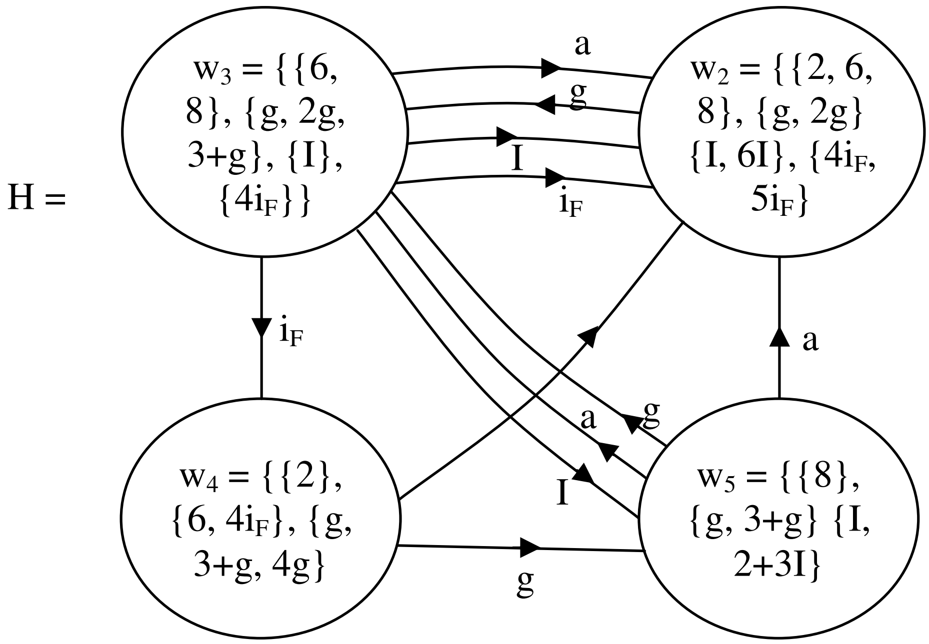


Figure 4.24

When we compare the subset vertex subset multisubgraph with the original vertex subset multigraph G we see the edges in H are reduced from that of G.

We have provided examples of them. We also define the subset - vertex multigraph H as the special subset-subset vertex multisubgraph of K.

As in case of subset vertex graph we can for any injective (or projective) subset vertex multisubgraph of G find the universal complement. Such study is considered as a matter of routine so we leave it as an exercise to the reader.

However, for given subset vertex multigraph G we can find the subset vertex multisubgraph H of G. For this H we can find special subset-subset vertex multisubgraph K of H. For this K of H we can find the local complement.

We see these subset-subset vertex multisubgraphs act very differently in many cases from the original graphs. At times these have more multiedges this special feature can be applied to social network problem whereby reducing the subset vertex one has more edges.

We first provide some more examples of them.

Example 4.5. Let S = $\{Z_{10}, Z_4, \langle Z_9 \cup I\rangle\}$ = $\{\{a_0, a_1, \ldots, a_9\}, \{b_1, b_0, b_2, b_3\}$ $\{a + bI / a, b \in$ a, $b \in Z_9, I^2 = I\}$ be a finite set. P(S) the power set of S. Let $v_1 = \{\{a_0, a_4, a_6, a_8\}, \{b_1, b_0\},$ $\{3 + 4I, 6 + 8I, 4I, 8I, 6, 3\}\}$, $v_2 = \{\{a_4, a_6, a_8\}, \{b_1, b_0, b_2\}, \{3 + 4I, 4I, 8I, 6, 3\}\}$, $v_3 = \{a_6, a_8\}, \{b_2, b_1\}, \{3 + 4I, 8I, 6\}\}$ and $v_4 = \{\{a_6, a_8, a_0\}, \{b_1, b_0, b_3, b_4\}, \{3 + 4I, 6\}\}$

G =

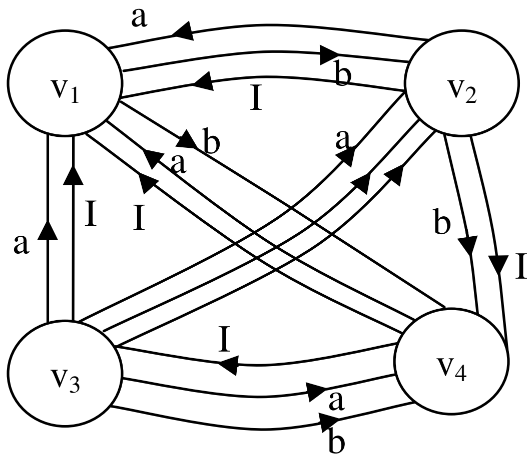


Figure 4.25

Clearly G is a subset vertex mixed multigraph of type II. We see some edges connecting the node v_1 to v_2 is in one direction and some in the opposite direction. How do we define them? It is interesting to note that if v_i and v_j are two subset vertices. v_i need not in general be contained in v_j or vice versa $(i \neq j)$ what is demanded is if we have the edge a subset of Z_{10} in

v_i must be contained in v_j or vice versa. Similar demands in case of Z_4 and $\langle Z_9 \cup I\rangle$.

Now we proceed onto give a subset vertex multisubgraph H of G given by the following figure.

H =

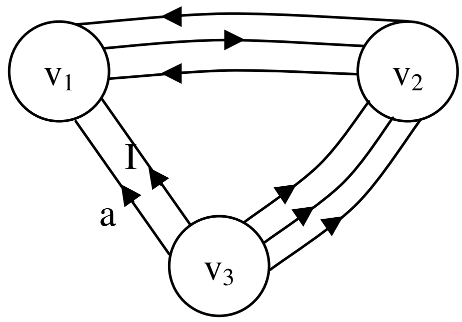


Figure 4.26

Let K be the subset-subset vertex multisubgraph of the subset vertex multisubgraph H given by the following figure.

K =

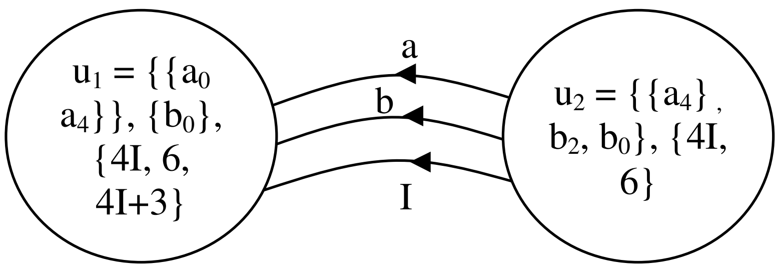


Figure 4.27

We see K is a subset vertex multisubgraph of subset vertex multisubgraph H of K and is multidyad. However H is not a reciprocating triad. So in some cases it may so happen by taking subsets of the subset vertex the links or tie or connections may be lost sometimes even disconnecting the very nodes their by disabling one to study them as a social network.

Now we find the local complement of K relative to H. This is given by the following figure.

K^C =

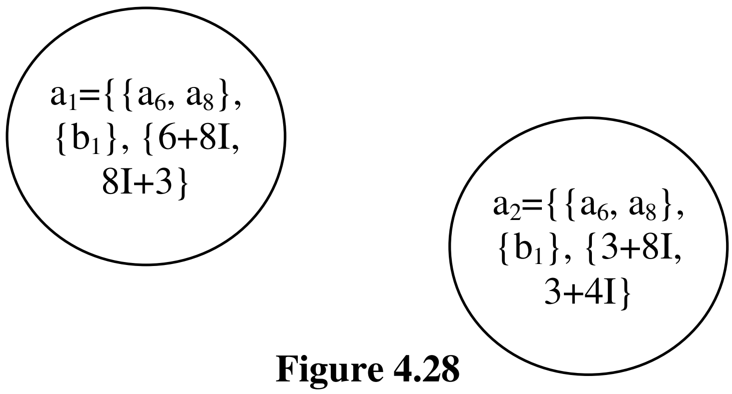


Figure 4.28

We see K^C is not defined as ($a_1 = v_1 \setminus u_1$, $a_2 = v_2 \setminus u_2$) both the nodes related to edge a and edge 4 yield same subset vertices in a_1 and a_2.

We now study the universal complement of G. Clearly G is a pseudo complete subset - vertex multigraph.

The universal complement G of G is given by the following figure.

G^C =

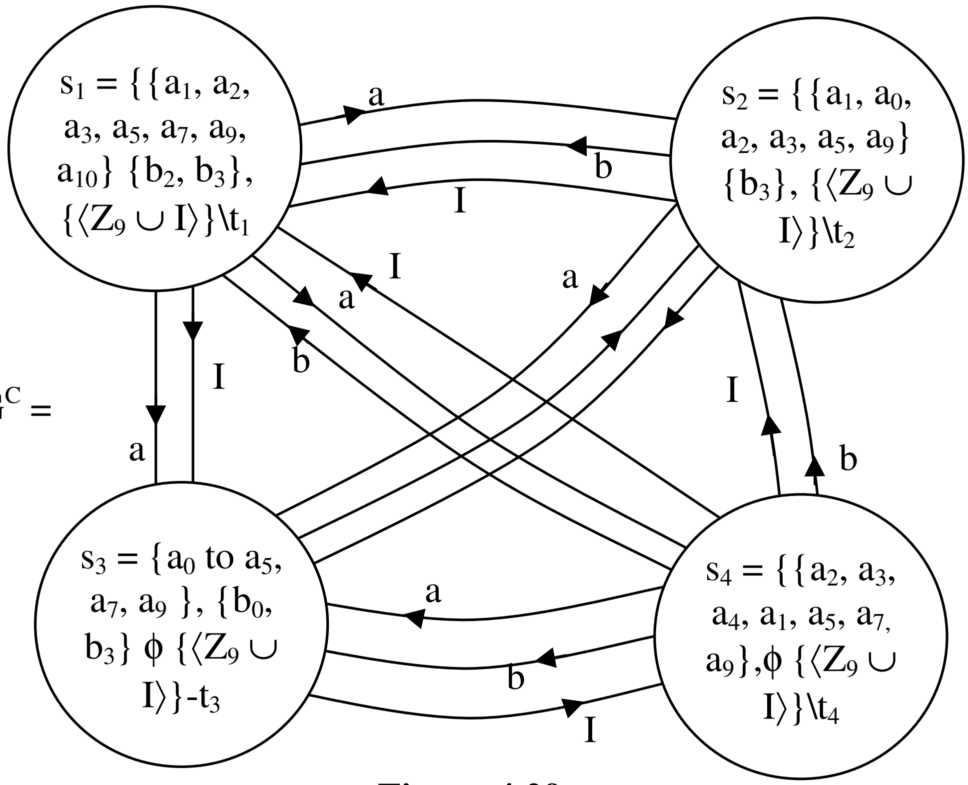


Figure 4.29

We see G^C is also a pseudo complete subset vertex multigraph. However, structure is not preserved as directions of the multiedges in G and G^C are different.

We provide some more examples.

Consider the subset vertex multigraph where the vertex subsets are totally ordered by inclusion given by the following examples.

Example 4.6. Let S = {⟨Z_{12} ∪ I⟩, Z_{18}, ⟨Z_{15} ∪ g⟩, C(Z_{16})} be a set with 4 attributes the edges of which are described by I, a, g and i_F associated with indeterminate I; reals a and dual number g and complex values i_F.

Here we give the subset vertex multigraph G for which the vertex subset forms a chain in the following.

v_1 = {{2, 4}, {6I + 6, 3 + 3I, 2 + 2I}, {g, 2g, 5g + 4}, {2i_F, 4i_F, 6i_F, 8i_F}} ⊆ v_2 = {{2, 4, 6, 8}, {6 + 6I, 3 + 3I, 2 + 2I, 6, 6I, 3I}, {g, 5g + 4, 2g, 2g + 4, 8g}, {2i_F, 4i_F, 6i_F, 8i_F, 2 + 6i_F, 4 + 4i_F}} ⊆ v_3 = {{2, 4, 6, 8, 10, 12}, {6 + 6I, 3 + 3I, 2+ 2I, 6, 6I, 3I, 3I + 6, 6 + 2I}, {g, 5g + 4, 2g, 2g + 4, 8g, 10g, 5g + 5}, {2i_F, 4i_F, 6i_F, 8i_F, 2 + 6i_F, 4 + 4i_F, 6i_F + 8, 8 + 8i_F} ⊆ v_4 = {{2, 4, 6, 8, 10, 12, 16}, {6 + 6I, 3 + 3I, 2 + 2I, 6, 6 I, 3I, 3I + 6, 6 + 2I, 10I, 10 + 5I, 10 + 10I}, {g, 5g + 4, 2g, 2g + 4, 8g, 10g, 5 + 5g}, {2i_F, 4i_F, 6i_F, 8i_F, 2 + 6i_F, 4 + 4i_F, 6i_F + 8, 8 + 8i_F, 10 + 10i_F, 10i_F, 10} ⊆ v_5 = {{2, 4, 6, 8, 10, 12, 16, 5, 7}, {6 + 6I, 3 + 3I, 2 + 2I, 3I,3I + 6, 6 + 2I, 10I, 10 + 5I, 10 + 10I, 3, 7I, 7 + 7I}, {2i_F, 4i_F, 6i_F, 8i_F, 2 + 6i_F, 4 + 4i_F, 6i_F + 8, 8 + 8i_F, 10 + 10i_F, 10i_F, 10, 11, 11i_F, 11 + i_F}, {g, 5g + 4, 2g, 2g + 4, 8g, 10g, 5 + 5g, 6g, 6 + 6g, 6 + 8g}}.

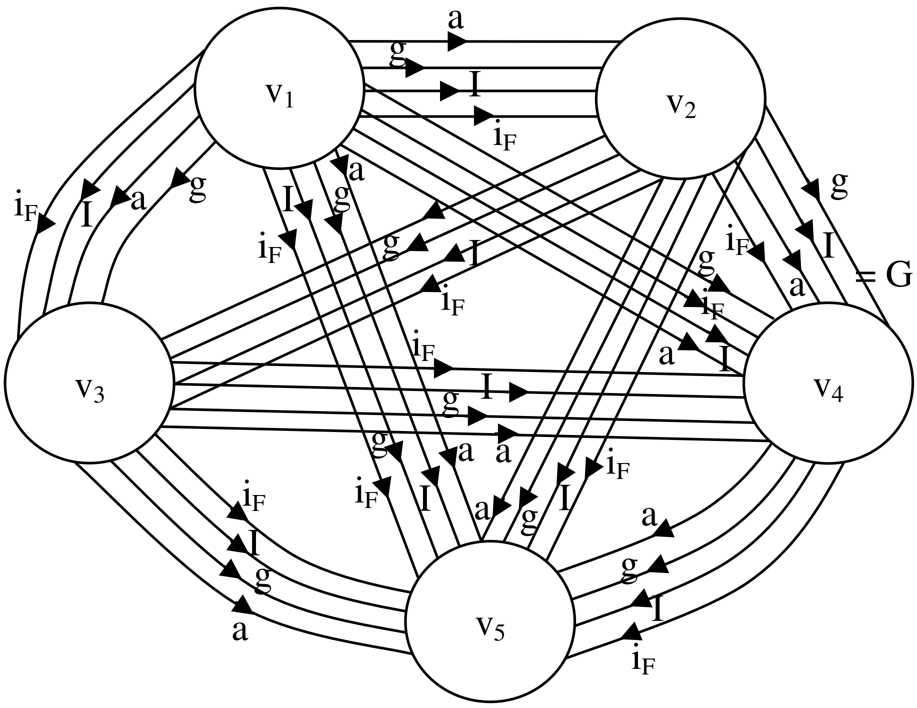


Figure 4.30

We call G a subset vertex complete full uniform multigraph. The term full means that all the 4 attributes are used, uniform implies every pair of nodes have same number of edges.

The main observation is that the vertex subsets of G form a chain or a totally ordered collection.

We give subset vertex multisubgraphs of G. All subset vertex multisubgraphs of G are also full uniform complete subset vertex multigraphs only.

We just consider the subset vertex multigraph H given by the following figure.

H =

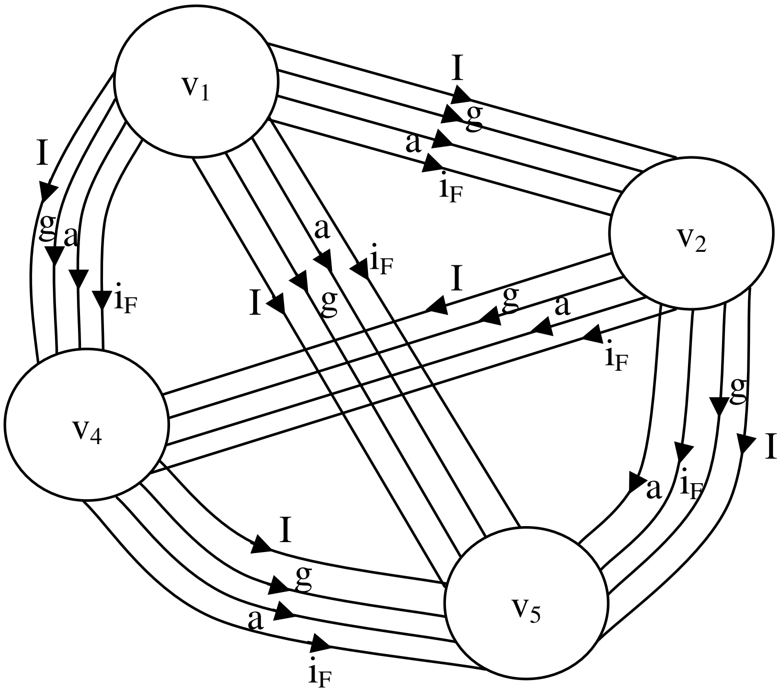


Figure 4.31

We now see H is also a complete subset vertex full uniform multisubgraph of G.

Now we give a few subset-subset vertex multisubgraphs of H in the following.

Let K_1 be a subset-subset vertex multisubgraphs of H given by the following figure.

K_1 =

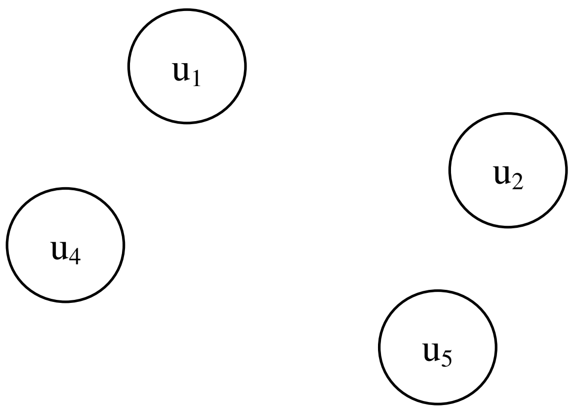


Figure 4.32

u_1 = {{2}, {6 + 6I, 2 + 2I}, {g, 5g + 4}, {$2i_F$, $4i_F$}} ⊆ u_2 = {{4, 6}, {3 + 3I, 3I}, {8g, 2g}, {$2i_F$, $6i_F$, 2 + $6i_F$}} ⊆ u_4 = {{10, 12}, {6 + 2I, 10I, 10 + 5I, 10 +10I}, {10g, 5 + 5g, 2g}, {$8i_F$, 2 + $6i_F$, 4 + $4i_F$} ⊆ u_5 = {{ 7, 5 16}, {7I, 6, 3I, 3I + 6}, {$10i_F$, 10, 11, $11i_F$}, {6g, 8g, 10g}}.

Clearly K_1 is a null subset-subset vertex neutrosophic multisubgraph of G of order 4. However, G is a uniform full complete subset vertex multigraph.

Now we proceed onto find the local complement of H relative to G. The vertex subsets of K are $s_1 = v_1 \setminus u_1$, $v_2 \setminus u_2 = s_2$, $s_4 = v_4 \setminus u_4$ and $s_5 = v_5 \setminus u_5$ where s_1 = {{4}, {3 + 3I}, {2g}, {$6i_F$, $8i_F$}}, s_2 = {{2, 4}, {6 + 6I, 6I, 6, 2 + 2I}, {g + 4, 2g + 4}, {$4i_F$, $8i_F$, 4 + $4i_F$}}, s_4 = {{2, 4, 6, 8, 16}, {6 + 6I, 3 + 3I, 2 + 2I, 6, 6I, 3I, 3I + 6}, {10, $10i_F$, $2i_F$, $4i_F$, $6i_F$, $6i_F$ + 8, 10 + $10i_F$, {g, 5g + 4, 8 + $8i_F$, 2g + 4, 8g}} and s_5 = {{2, 4, 6, 8, 10, 12}, {6 + 6I, 3 + 3I, 2 + 2I, 6I, 3, 6 + 2I, 10I, 10 + 5I, 10 + 10I, 7 + 7I}, {$2i_F$, $4i_F$, 6_{iF}, 8_{iF}, 2 + $6i_F$, 4 + $4i_F$, $6i_F$ + 8, 8 + $8i_F$, 10 + $10i_F$}, {g, 5g + 4, 2g, 2g + 4, 5 + 5, 6 + 6g, 6 + 8g}}.

The local complement K are given in the following.

H^C = K =

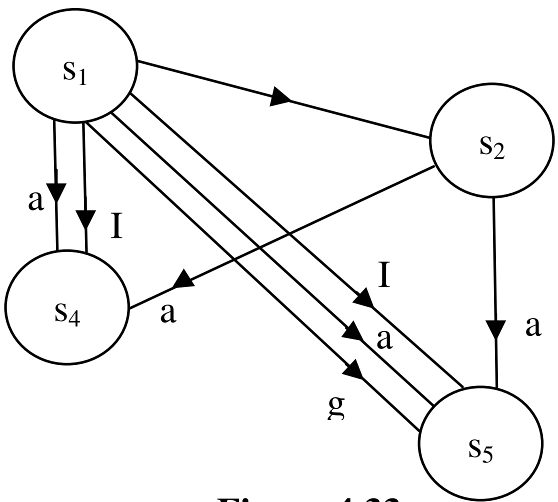


Figure 4.33

Clearly K is also not a pseudo complete subset - vertex multisubgraph of G.

We see neither the subset-subset vertex neutrosophic multisubgraph of G nor its local complement related to G enjoy any form of common features.

Next we proceed onto describe by examples the subset vertex circle multigraphs and subset vertex line multigraphs.

Example 4.7. Let S = $\{Z_{22}, \langle Z_{20} \cup g\rangle, C(Z_{15}), \langle Z_{16} \cup I\rangle\}$ be a set and P(S) the power set of S. Let G be the line multigraph with subset vertices given by the following figure whose vertex sets are described below.

$v_1 = \{\{2, 4\}, \{10, g\}\}$

$v_2 = \{\{2, 4, 6, 9\}, \{10, g, 7\}, \{i_F, 12i_F, 3i_F + 3, 6i_F\}, \{I, 2I, 6I\}\}$,

$v_3 = \{\{6, 9\}, \{2g, 8g, 1, 5 + 8g, 0, 6 + 10g\}, \{i_F, 12i_F, 3 + 3i_F, 6i_F, 9i_F, 9 + 9i_F\}, \{I, 2I, 6I, 6 + 8I, 9I, 9 + 9I\}$,

$v_4 = \{\{8, 10, 16, 18, 20\}, \{5 + 8g, 0, 1\}, \{9 + 9i_F, 9i_F, 6i_F\}, \{9I + 9, 9I, 2I, I\}\}$

$v_5 = \{\{18, 20\}, \{9 + 9g, 10 + 10g, 12 + 12g, 15 + 15g, 5 + 5g\}, \{9 + 9I, 15 + 15I, 10 + 10I, 0, 1\}\}$ and $v_6 = \{\{12 + 12g, 15 + 15g\}, \{15 + 15I, 10 + 10I, 0, 1\}, \{7i, 11, 13\}, \{7i_F, 11i_F, 13i_F, 3i_F, 5i_F\}\}$

G =

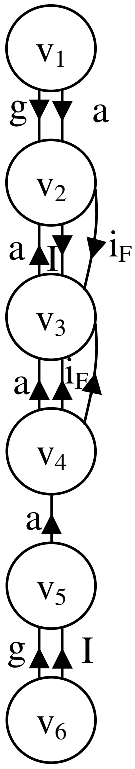


Figure 4.34

We see this line subset vertex multigraph is pseudo connected as through none of the attribute we can transverse in one direction from v_1 to v_6 or v_6 to v_1.

Further no attribute links it fully for v_6 to v_5 g and I links but v_5 to v_4 a links and v_4 to v_3 g, i_F and I links and v_2 to v_3 a, I and i_f links. Infact from v_2 to v_4 v_3 is the meeting node of I and i_F and not the intermediate or linking node from v_1 to v_2 the attributes g and a link.

Further the attribute a is the meeting node serving as the meeting edge between v_1 and v_3.

We define such type of directed subset vertex multiline graphs as pseudo linked line multigraphs.

We proceed to give more examples of them.

Example 4.8. Let S = $\{Z_{16}, \langle Z_{18} \cup g\rangle \langle Z_{20} \cup I\rangle\}$ be a set with three distinct attributes. P(S) be the power set of S.

Let $v_1 = \{\{2, 4\}, \{g, 3g, 5g\}, \{3I, 7I, 5I\},$

$v_2 = \{\{1, 2, 4, 7, 6\}, \{g, 3g, 5g, 7g, 8g\}, \{10I, 12I, 16I, 2 + I\}\},$

$v_3 = \{\{7, 6\}, \{5g, 7g\}, \{10I, 12I, 2 + I, 14I, 16I\}\},$

$v_4 = \{\{14I, 16I\}, \{0, 1, 3, 5, 7\}, \{2 + 2g, 3 + 3g, 4 + 4g\}\}$ and $v_5 = \{14I, 16I, 17I, 19I\}, \{0, 1, 3\}, \{2 + 2g, 3 + 3g, 4 + 4g, 4g, 5g\}\}.$

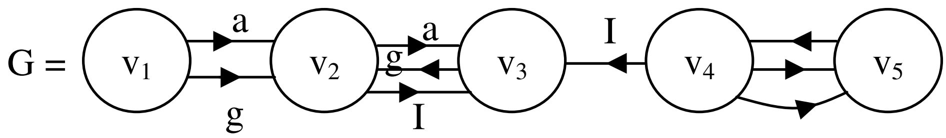


Figure 4.35

We see the line multigraph is pseudo linked.

We give some of its subset vertex multisubgraphs of G.

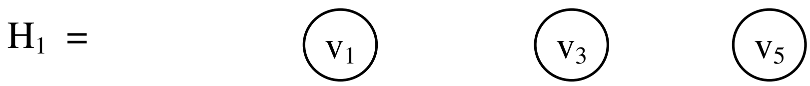


Figure 4.36

Clearly H is a empty vertex subset multisubgraph of G.

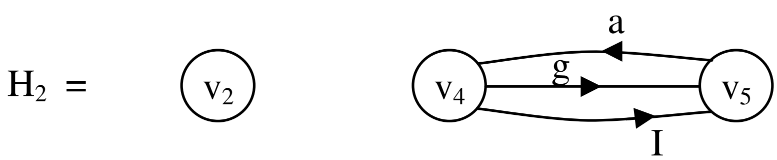


Figure 4.37

H_3 is a disconnected subset vertex multisubgraph of G.

H_3 =

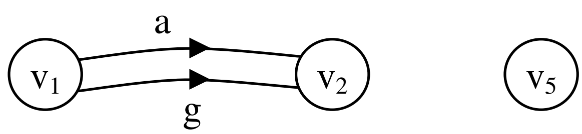


Figure 4.38

We see only if the vertex subsets are consecutive edges exists otherwise, they are disconnected or sometimes only empty subset vertex multisubgraphs.

Our main study is, can the subset-subset vertex multisubgraphs of H_i, i = 1, 2, 3 be a chain?

Let K_1 be the subset-subset vertex multisubgraph of K_1 given by the following figure.

$u_1 = \{\{4\}, \{5g\}, \{3I, 7I\}\} \subseteq v_1$

$u_3 = \{\{6\}, \{5g\} \{14I, 16I\} \subseteq v_3$

$u_5 = \{\{0, 1\}, \{14I, 16I, 17I\}, \{5g, 3 + 3g, 4g\}\} \subseteq v_5$ given by the following figure.

K_1 =

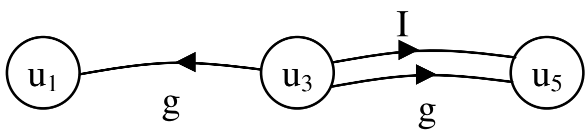


Figure 4.39

We see H_1 is a empty subset vertex multisubgraph of G whereas the subset-subset vertex multisubgraph K_1 of H is a pseudo linked subset-subset vertex multisubgraph of H_1. Now we find the local complement L_1 of K_1 relative to H_1.

Let $a_1 = v_1 \setminus u_1$, $a_3 = v_3 \setminus u_3$ and $a_5 = v_5 \setminus u_5$.

a_1 = {{2}, {3g, g}, {5I}}, a_3 = {{7}, {7g}, {10I, 2 + I, 12I}} and

a_5 = {{19I}, {3}, {2 + 2g, 4 + 4g}}.

We give the local complement L_1 of K_1 by the following figure.

L_1 =

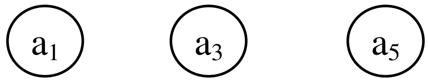


Figure 4.40

Clearly L_1 is a subset-subset vertex multisubgraph of H_1 which is empty. So H_1 and L_1 enjoy the same structure.

Now we find a subset-subset vertex multisubgraph K_2 of H_2. The subset vertex subset of K_2 is as follows.

b_2 = {{1, 2, 6}, {g, 5g, 7g}, {10I, 16I}} $\subseteq v_2$,

b_4 = {{1}, {2 + 2g}, {16I}} $\subseteq v_4$

and b_5 = {{0, 1}, {16I, 19I}, {2 + 2g, 5g}} $\subseteq v_5$.

The graph K_2 is given by the following figure.

K_2 =

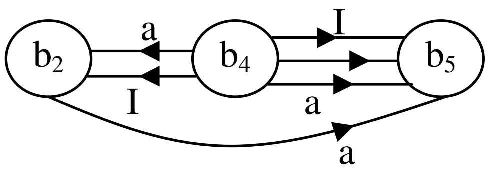


Figure 4.41

K_2 the subset-subset vertex neutrosophic multigraph is not a chain in fact a triad which is nonuniform.

Now we find the local complement L_2 of K_2 relative to H_2.

L_2 has vertex subsets as $v_2 \setminus b_2 = C_2$, $C_4 = v_4 \setminus b_4$ and $C_5 = v_5 \setminus b_5$.

$C_2 = \{\{4, 7\}, \{3g, 8g\}, \{12I, 2 + I\}\} \subseteq v_2$,

$C_4 = \{\{0, 3, 5, 7\}, \{3 + 3g, 4 + 4g, 4g, 5g\}, \{14I\}\} \subseteq v_4$ and

$C_5 = \{\{3\}, \{14I, 17I\}, \{3 + 3g, 4 + 4g, 4g\}\} \subseteq v_5$

L_2 =

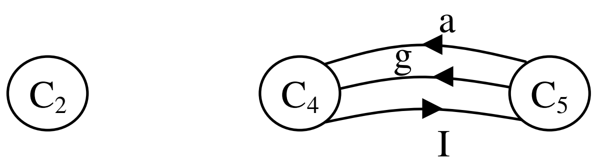


Figure 4.42

We see L_2 is a disconnected subset-subset vertex neutrosophic multisubgraph of H_2. However only H_2 and L_2 enjoy same structure but the directions of the edges are changed in some attributes.

Further K_2 happens to yield a triad a pseudo complete multigraph whereas both H_2 and L_2 are disconnected triads.

Next we work with H_3 by finding a subset-subset vertex multisubgraph K_3 of H_3.

Let $d_1 \subseteq v_1$, $d_2 \subseteq v_2$ and $d_5 \subseteq v_5$ be the subset-subset vertex set of K_3, where

$d_1 = \{\{4,\}, \{g, 3g\}, \{3I\}\} \subseteq v_1$,

$d_2 = \{\{6, 4, 7\}, \{g, 3g, 7g\}, \{2 + I, 10I\}\} \subseteq v_2$ and

$d_5 = \{\{3\}, \{2 + 2g, 3 + 3g\}, \{19I, 17\}\}$.

The subset-subset vertex multisubgraph K_3 is given by the following figure.

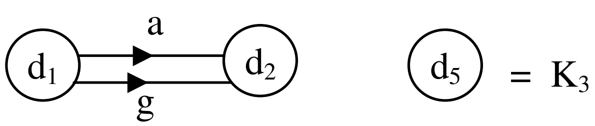


Figure 4.43

Now we find L_3 the local complement of K_3 relative to H_3. Let $n_1 = v_1 \setminus d_1$, $n_2 = v_2 \setminus d_2$ and $n_5 = v_5 \setminus d_5$ where $n_1 = \{\{2\}, \{5g\}, \{7I, 5I\}\} \subseteq v_1$, $n_2 = \{\{1, 2\}, \{5g, 8g\}, \{12I, 16I\}\} \subseteq v_2$ and $n_5 = \{\{0, 1\}, \{4 + 4g, 4g, 5g\}, \{14I, 16I\}\}$.

The local complement is a subset-subset vertex multisubgraph L_3 of K_3 which is as follows.

$L_3 =$

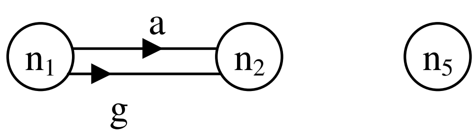


Figure 4.44

We see H_3, L_3 and K_3 all the 3-subset vertex multisubgraphs enjoy the same or identical structure.

Now we proceed onto construct circle subset vertex multigraph which are described by some examples.

Example 4.9. Let $S = \{Z_{12}, \langle Z_{10} \cup I\rangle, C(Z_8)\}$ be the set with three attributes. P(S) be the powerset of S.

Let G be the subset vertex multigraph given by the following vertex subsets.

$v_1 = \{\{6, 10, 2, 0\}, \{5I, 4I, I\}, \{2i_F, 4i_F, 6i_F\}\}$,

$v_2 = \{\{6, 10, 2, 0, 9, 7, 1\}, \{5I, 4I, I, 2I, 3 + 4I\}, \{2i_F, 4i_F, 6i_F, 8i_F, 3i_F\}\}$,

$v_3 = \{\{9, 7, 1, 0\}, \{4I, 3 + 4I\}, \{3i_F, 8i_F, 2i_F\}$ and

$v_4 = \{6, 10, 2, 0, 10, 11\}, \{5I, 4I, I, 9I, 9 + 9I\}, \{2i_F, 4i_F, 8i_F, 3i_F, 2 + 2i_F\}$

The subset vertex multigraph G with v_1, v_2, v_3 and v_4 are as follows.

G =

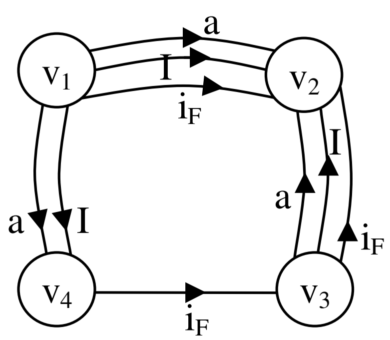


Figure 4.45

We see G is a circle subset vertex multigraph.

There are $4C_2 + 4C_3 = 9$ subset vertex multisubgraphs of G.

Consider the subset vertex neutrosophic multisubgraph H of G with vertex subsets v_1, v_4 and v_3 given by the following figure.

H =

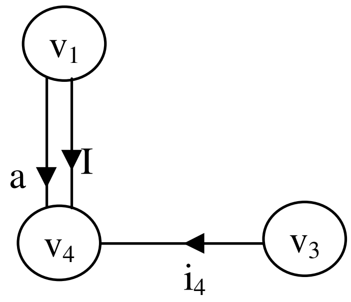


Figure 4.46

We given some of subset-subset vertex neutrosophic multisubgraphs of H given by the following sets of vertices.

$u_1 = \{\{6, 10\}, \{I\} \{2i_F, 4i_F\}\} \subseteq v_1$,

$u_3 = \{\{9, 7\}, \{3 +4I\}, \{2i_F\}\} \subseteq v_3$ and

$u_4 = \{\{6, 10, 0\}, \{I, 4I\}, \{4i_F, 2i_F, 8i_F\}\} \subseteq v_4$

K_1 =

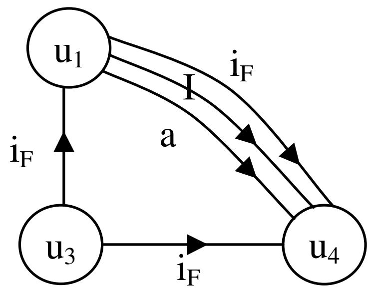


Figure 4.47

K_1 is a triad which is a subset-subset vertex neutrosophic multisubgraph of the subset vertex neutrosophic multisubgraph H which is only a forbidden triad.

Consider K_2 a subset-subset vertex neutrosophic multisubgraph of H with the following set of subset vertices.

$a_1 = \{\{0, 2\}, \{4I, I\}, \{2i_F\}\} \subseteq v_1$,

$a_3 = \{\{0\}, \{4I\}, \{2i_F, 8i_F\}\} \subseteq v_3$

and $a_4 = \{\{0, 2, 10, 11\}, \{4I, I, 9I\}, \{2i_F, 8i_F, 2 + 2i_F\}\} \subseteq v_4$

The subset-subset vertex neutrosophic multisubgraph K_2 of H is given by the following figure.

$K_2 =$

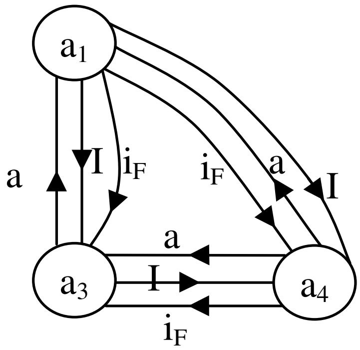


Figure 4.48

We see K_2 is a uniform full triad subset-subset vertex multisubgraph of H where H is only a forbidden triad which is not even full. So structure is not preserved by K_2. Now we proceed onto find the local complement L_2 of K_2 relative to H.

The vertex subsets of L_2 are $m_1 = v_1 \setminus a_1 = \{\{6, 10\}, \{5I\}, \{6i_F, 4i_F\} \subseteq v_1$; $m_3 = v_3 \setminus a_3 = \{\{7, 9, 1\}, \{3 + 4I\}, \{3i_F\}\} \subseteq v_3$ and $m_4 = v_4 \setminus a_4 = \{\{6, 10\}, \{5I, 9 + 9I\}, \{4i_F, 3i_F\}\}$.

Clearly the subset-subset vertex multisubgraph L_2 the complement of K_2 does not exist as m_1 and m_4 have {6, 10} to be a subset in the subset vertex.

Thus the local complement of K_2 is not defined.

So K_2 of H a full complete subset-subset vertex multisubgraph has its local complement to be undefined relative to H.

Hence the subset-subset vertex multisubgraphs of a subset vertex multigraph enjoys a very different structure and more so its local complement may exist or may not exist.

Next we proceed onto describe the concept of subset vertex multigraphs which are trees by some examples.

Example 4.10. Let S = {Z_{18}, $\langle Z_1 \cup I\rangle$ C(Z_{12})} be a set and P(S) the powerset of S. Let G be a subset vertex neutrosophic multigraph given by the following figure.

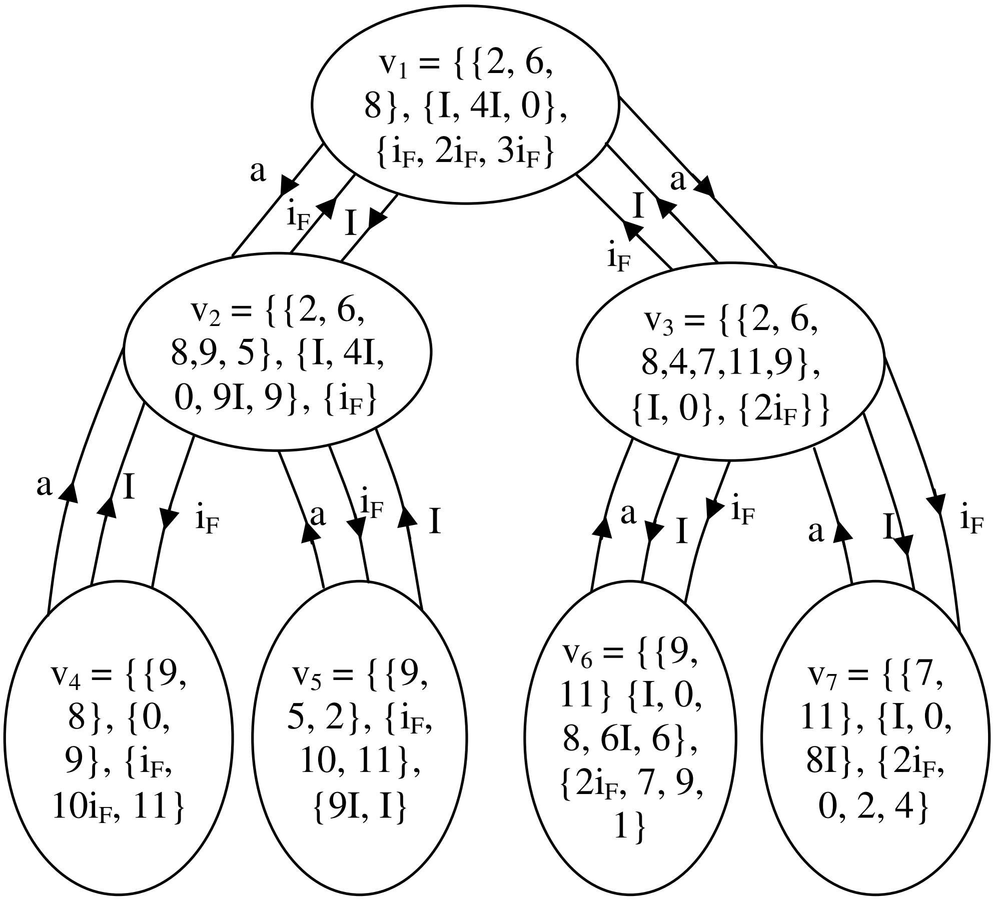


Figure 4.49

We define this type of subset vertex multigraphs which are trees as full subset vertex multigraph binary tree.

We prove some subset vertex multisubgraphs of G.

Let K_1 be a subset vertex neutrosophic multisubgraph of G given by the following figure.

K_1 =

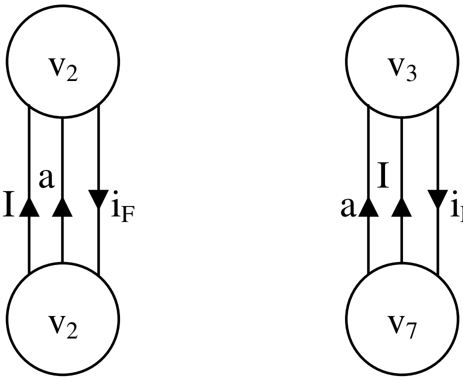


Figure 4.50

Clearly K_1 which is a subset vertex multisubgraph of G is not a tree. It is two full dyads but they are neither mutual nor one way reciprocative

Now we find subset-subset vertex multisubgraph L_1 of K_1. L_2 is given by the following figure: $u_i \subseteq v_i$; i = 2, 4, 3 and 7.

L_1 =

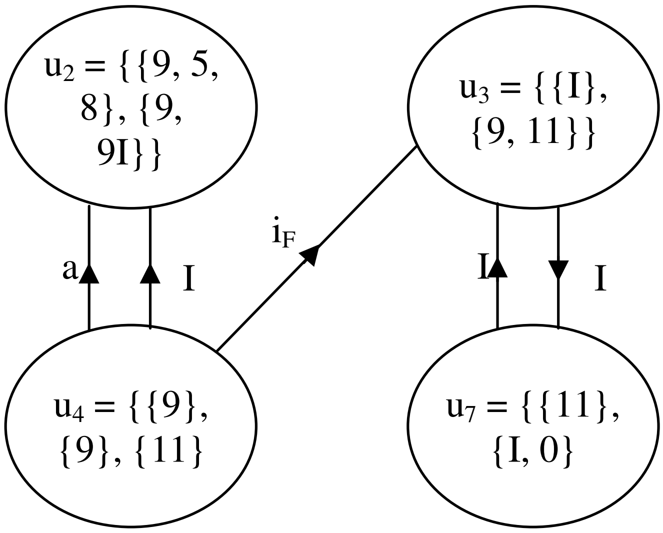


Figure 4.51

We see the subset-subset vertex multisubgraph L_1 of K_1 is not a tree; infact it does not preserve the structure of K_1 or G.

Clearly the local complement of L_1 does not exist as the subsets of $\{i_F\}$ of v_2 and $\{2i_F\}$ of v_3 cannot be defined or should be taken as it is we take the convention of here that if singleton sets are taken the notion of subset-subset vertex multisubgraphs do not exist so L_1 is undefined at the subsets of vertex subsets.

Hence L_1 given though is a wrong concept, this is mainly done to make the reader understand that by looking at the very vertex subsets of a subset vertex multisubgraph dispose of it as the non existence of the subset-subset vertex multisubgraphs.

Thus we have the following remark.

Remark 4.1: Let S be a set with different sets of attributes. P(S) the powerset of S. Let G be a subset vertex multigraph. Let H be a subset vertex multisubgraph of G, if even one of the vertex subsets of H (some of which are that of G also) has singleton subsets then the subset-subset vertex multisubgraph of G is not defined, so by very observation of the subset vertices one can make this conclusion.

We provide some more examples of subset vertex multigraphs which are multitrees.

Example 4.11. Let S = $\{Z_{15}, \langle Z_{21} \cup I\rangle, \langle Z_{18} \cup I\rangle\}$ be a set P(S) the power set of S. Let G be the subset vertex multigraph which is a multitree given by the following figure.

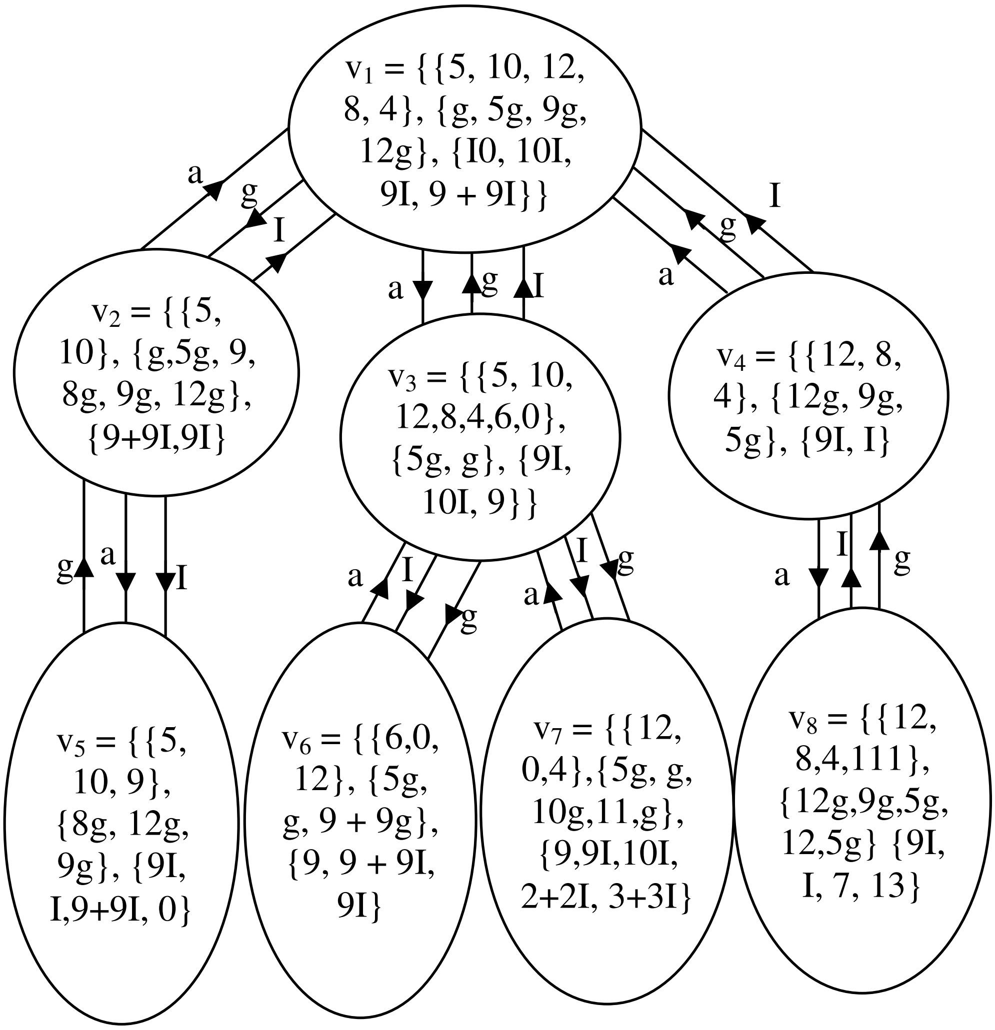


Figure 4.52

Now we are guaranteed that this has both subset vertex multisubgraphs as well as subset-subset vertex multisubgraphs.

Let B be the subset vertex multisubgraph with vertex subsets $\{v_2, v_8, v_6$ and $v_7\}$. Clearly B is an empty subset vertex multisubgraph of G.

We now find the subset-subset vertex multisubgraph C of B given by the following figure. $u_i \subseteq v_i$; i = 2, 6, 7 and 8.

C =

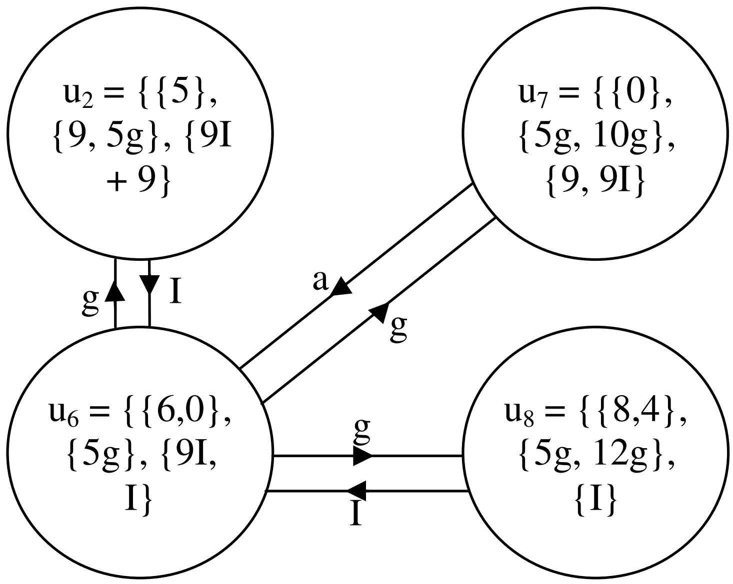


Figure 4.53

Clearly C is a neutrosophic multitree with uniform edges, however the child node u_6 has become a parent node for their subset vertex neutrosophic multisubgraph tree with one layer.

Now we find the local complement D of C relative to subset vertex empty multisubgraph B of G. The vertex subset of D are $a_2 = v_2 \setminus u_2$, $a_6 = v_6 \setminus u_6$, $a_7 = v_7 \setminus u_7$ and $a_8 = v_8 \setminus u_8$ given by the following figure.

D =

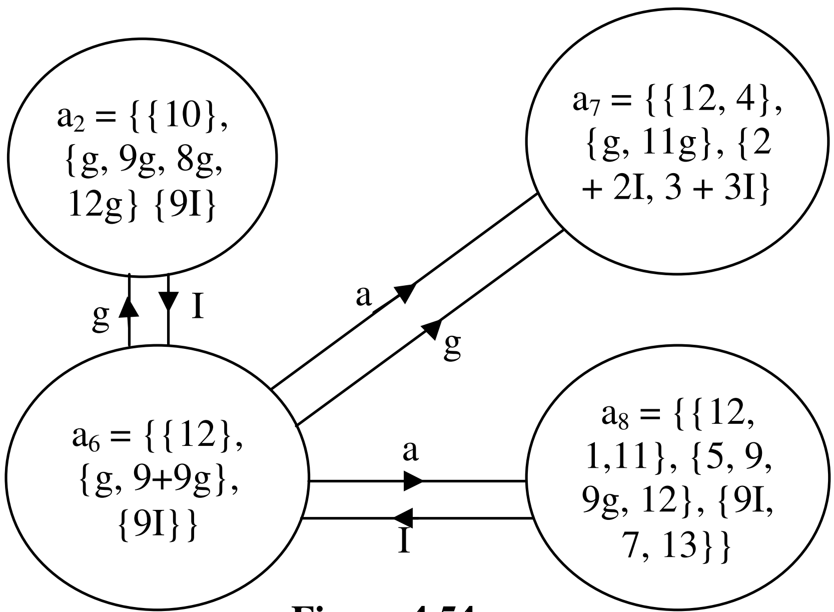


Figure 4.54

Clearly D is not a tree and has a very different structure from that of B and G.

Empty subset vertex multisubgraph has subset-subset vertex multisubgraphs which is a tree and a multisubgraph of very different structure.

Now we proceed onto give one more example of a empty subset vertex multigraphs and its subgraphs subset-subset vertex multigraphs.

Example 4.12. Let S = $\{Z_9, \langle Z_9 \cup I\rangle, \langle Z_6 \cup g\rangle, C(Z_5)\}$ be a set and P(S) the powerset of S. Let G be the empty subset vertex neutrosophic multigraph where the vertex subsets are described in the following.

$v_1 = \{\{3, 6, 8\}, \{2I + 2, 3I + 3, 2I\}, \{4g, 2g, 3g, g + 1\}, \{i_F, 2 + 2i_F, 2i_F, 4i_F + 1\}$, $v_2 = \{\{3, 0, 1, 4\}, \{2I, I, 2I + 2, 3, 3I, 1\}, \{2g, 3g, 2g + 2, 3g + 3\}, \{i_F, 2i_F, 3 + 3i_F\}\}$, $v_3 = \{\{6, 1, 5\}, \{2I, 2+2I, 1 + I, 3I\}, \{4 + 4i_F, 4, 4i_F + 1, 2i_F, 4i_F\}, \{2g, 3g, 3g + 2, 2g + 1\}\}$ and $v_4 = \{\{3, 0, 6, 1\}, \{2I + 2, 2I, 3I, 2, 1 + I\}, \{2g, 3g, 2g + 2, 2g + 1\}, \{i_F, 2i_F, 4, 3 + i_F\}\}$.

G = $\{v_1, v_2, v_3, v_4\}$ is a subset vertex multigraph which is empty. All subset vertex multisubgraphs of G are also empty.

Now consider the empty subset vertex neutrosophic multisubgraph $H_1 = \{v_1, v_4, v_2\}$.

Clearly H_1 is empty we find some of subset-subset vertex multisubgraphs of H_1 in the following.

Let P_1 be the subset-subset vertex multisubgraph of H_1 given in the following figure; $u_i \subseteq v_i$, i = 1, 3 and 4.

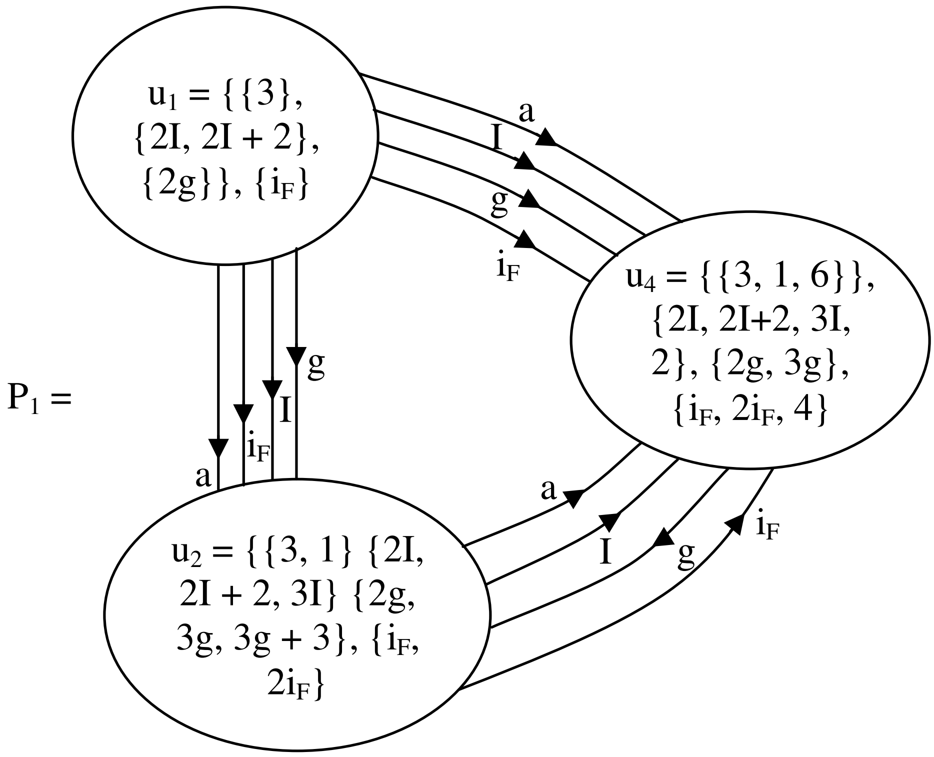


Figure 4.55

Clearly P_1 the subset-subset vertex multisubgraph of H_1 is a uniform full complete subset-subset vertex multisubgraph of the empty subset vertex multisubgraph.

This is the vital point we wish to discuss in social networks and in political scenario. We may have a empty of subset vertex multisubgraph but its subset of subset vertices can yield a full uniform complete subset-subset vertex multisubgraph (a complete neutrosophic multi network). Thus these structures are endowed with very many special features which when applied in any social networking for studying their social structure or political structure will yield such sensitive accurate solutions to problems.

Now we find the local complement M_1 of P_1 relative to H. M_1 is described by the following figure.

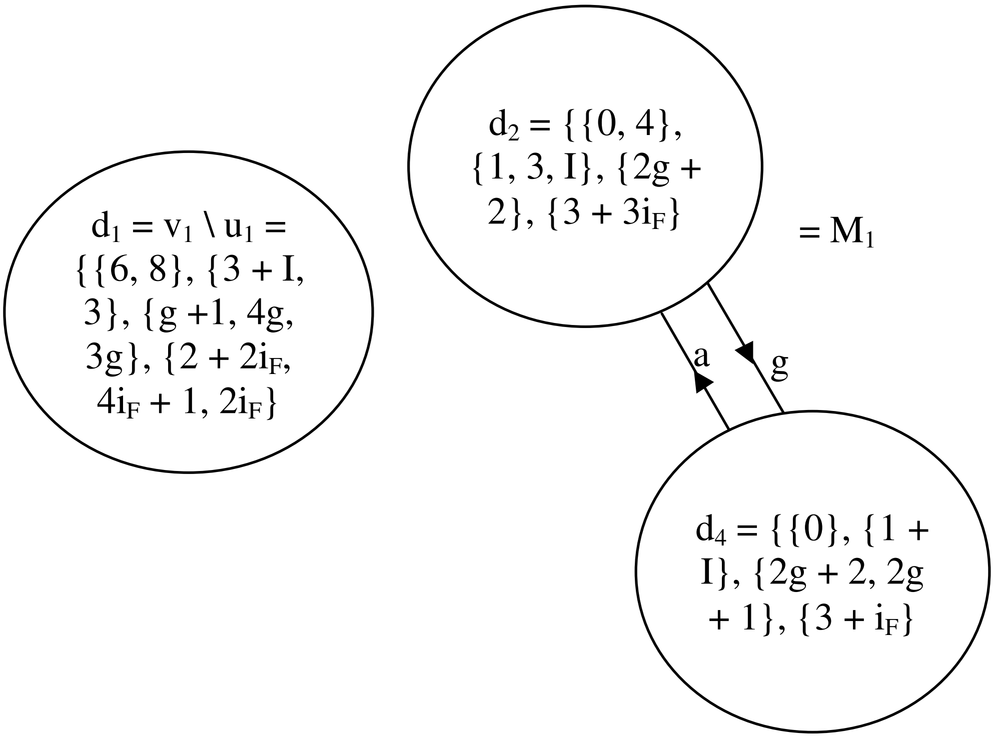


Figure 4.56

We see the local complement of P_1 is a triadic state with one multiedge and is a d is connected subset-subset vertex neutrosophic multisubgraph of the empty multigraph.

Study in this direction will certainly yield innovative results and can give a better solution to social problems when adopted on social network analysis.

The very distinct behavior of these subset vertex multigraphs are that some empty subset vertex multigraphs can have subset vertex multisubgraphs which can yield uniform full complete subset-subset vertex multisubgraphs, which in turn implies that is a society of a social setup which is totally disconnected or is an empty subset vertex multigraph can have subsets of subsets which can yield a strong bondage in the same society when a sub collection is taken. So a socio scientist can build a dismantled society into a complete full strongly knit community by picking up some vital attributes sub collection from the total collection.

This property can also be exploited in political situations where the purity looks like a dismantled one but can built it back with strong multi ties or multi relations by dropping out the appropriate (manifesto) or persons from the specific attribute. Interested researcher can work in this direction.

Finally, we wish to mention that subset vertex multigraphs will play a major role in the social net on one side and in soft computing in other side, when the attributes of these concepts are different from ON and OFF states.

We just give the following theorem.

Theorem 4.1. *Let S be any set with m distinct attributes. P(S) the powerset of S. G be a empty subset vertex multigraph.*

i) All subset vertex multisubgraphs H of G are empty

ii) H can have subset-subset vertex multisubgraphs which can be complete depending on the vertex subsets choosen from G.

Proof (i) is obvious from the very definition of subset vertex multisubgraphs H of G will always be empty if G is empty

ii) This can be proved depending on the subset vertices of the multisubgraph H and the subsets choosen from the vertex subsets of H.

We propose a open problem.

Problem 4.1 Let S be a set with m attribute. P(S) the power set of S.

Characterize those empty subset vertex multigraphs G and the subset vertex multisubgraphs H of G which has subset - subset vertex multisubgraphs which are complete or equivalently characterize all empty subset vertex multigraphs G whose subset vertex multisubgraphs has all of its subset-subset vertex multisubgraphs to be empty.

We propose a few problems for the interested reader some of which are at research level.

Problems

1. Find all subset vertex neutrosophic multigraphs of order four using the directed projective vertex subset from P(S) where S = {$\langle Z_{14} \cup I\rangle$, Z_{12}, $C(Z_{10})$, $\langle Z_{15} \cup g\rangle$}.

2. Enumerate all special features enjoyed by injective subset vertex multigraphs.

3. Differentiate between subset vertex multigraphs and injective subset vertex multigraphs.

4. How can one relate projective subset vertex multigraphs with injective subset vertex multigraphs?

5. Find all projective subset vertex neutrosophic multigraphs using vertex subsets from P(S) where S = {Z_{15}, $\langle Z_9 \cup I\rangle$, $\langle Z_{18} \cup g\rangle$, $C(Z_{27})$} with four distinct attributes.

6. Find all subset vertex neutrosophic multigraphs that can be obtained using injective subsets vertex from P(S) where S = {Z_9, C(Z_{18}), ⟨Z_{15} ∪ I⟩} of order 5.

7. How many pseudo complete injective subset vertex complex multigraph of order 5 exist using P(S) given in problem 6?

8. Let G be a subset vertex multigraph with entries from P(S) where S = {Z_{20}, Z_{45}, C(Z_{25}), ⟨Z_{30} ∪ I⟩, ⟨Z_{35} ∪ g⟩}. If the vertex subsets of G form a totally ordered set, will G be a complete subset vertex multigraph? Justify your claim.

9. Let S = {Z_{40}, ⟨Z_{12} ∪ I⟩m ⟨Z_{35} ∪ g⟩} be a set and P(S) the power set of S. Let G be a subset vertex multigraph with vertex subset which forms the following chain:

   v_1 = {2, 4, 8, 16} ⊆ {{2, 17, 4, 18, 16}, {3I, 6I}} = v_2 ⊆ v_3 = {{2, 7, 4, 1, 8, 16, 36}, {3I, 6I, 2 + 3I, 4 + 6I}} ⊆ v_4 = {{2, 7, 4, 1, 8, 16, 36, 32, 34, 30}, {3I, 6I, 2 + 3I, 4 + 6I, 4, 2, 8, 10}, {3g, 5g, 8g, 4 + 4g, 5 + 5g}} ⊆ v_5 = {{2, 1, 7, 4, 8, 16, 36, 32, 34, 30, 28, 20, 18}, {3I, 6I, 2 + 3I, 4 + 6I, 4, 2, 8, 10, 4I, 2I, 8I}, {3g, 5g, 8g, 4 + 4g, 5 + 5g, 5 + 9g, 9g, 9}}.

   i) Prove G is not a complete subset vertex multigraph

   ii) Is G a pseudo complete non uniform subset vertex multigraph?

iii) Prove there are only $5C_2 + 5C_3 + 5C_4$ number of subset vertex multigraphs.

iv) Find all subset-subset vertex multisubgraphs of H where H are, subset vertex multisubgraphs of G.

v) How many subset-subset vertex multisubgraph of H with vertex subsets v_1, v_3, v_4 and v_5 are possible?

10. Find any of the interesting features associated with subset vertex complete multigraphs in general.

11. Let G be a subset vertex multistar graph given by the following figure. Vertex subsets are taken from P(S) where $S = \{C(Z_{20}), Z_{40}, \langle Z_{30} \cup I\rangle\}$

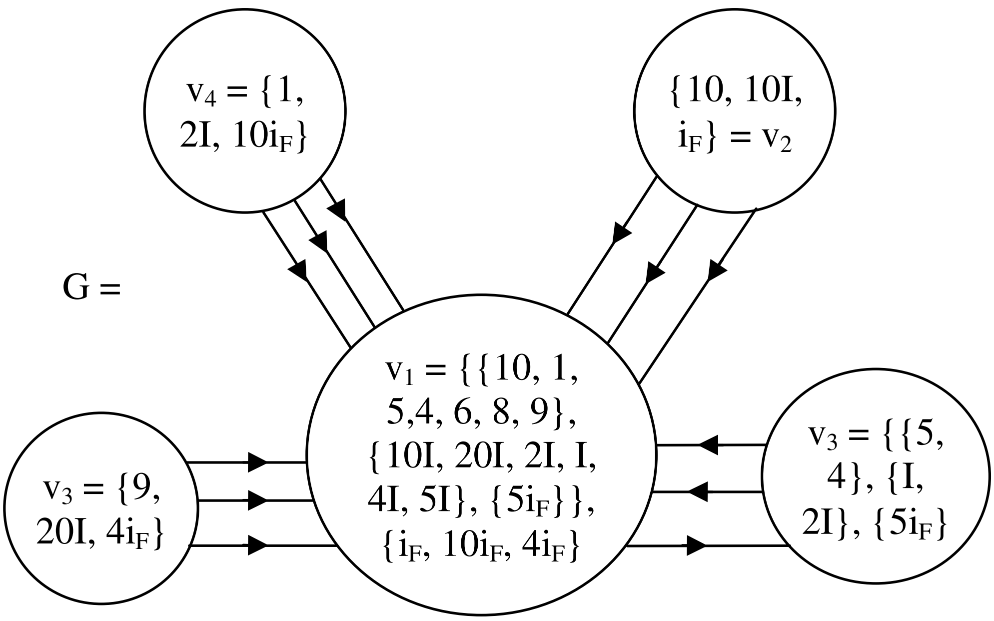


Figure 4.57

i) Find all subset vertex multisubgraphs of G.

ii) Can we say all subset vertex multisubgraphs of G will be star graphs?

iii) Is the subset vertex multisubgraph K using the vertex subsets $\{v_1, v_3, v_4\}$ a subset vertex multistar subgraph?

iv) Find all subset-subset vertex multisubgraph of K.

v) Find the global complement of G.

vi) Find the local complement of atleast two subset-subset vertex multisubgraphs of G.

vii) Is the global complement of G a subset vertex multigraph which is a star graph?

12. Can there be subset vertex multigraph which are cycle graphs? (Prove or disprove)

13. Can we have subset vertex multigraphs which are line graphs?

14. Let $S = \{C(Z_{20}), \langle Z_{19} \cup I\rangle \langle Z_{12} \cup I\rangle, Z_{24}\}$ be a set and P(S) the power set of S.

i) How many empty subset vertex multigraphs can be constructed using the vertex subsets from P(S)?

ii) Let $v_1 = \{\{i_F, 3i_F, 5i_F, 7i_F, 10i_F, 12i_F, 14i_F, 16i_F, 2 + 2i_F, 4 +4i_F\}, \{I, 8I, 3 + 3I, 4 + 4I, 5I, 6I\}, \{2, 4, 6, 8, 10, 12, 7, 9, 11, 13\}, \{g, 7g + 4, 2g, 4g, 6g, 8g, 3g, 5g, 5 + 5g\}\}$, $v_2 = \{\{i_F, 5i_F, 10i_F,$

$14i_F$, $2 + 2i_F$, $4 + 4i_F$, $16i_F$, $9i_F$, $9 + 9i_F$, $18i_F$, $18i_F + 9$, $9 + i_F$}, {I, 8I, 3 + 3I, 8 + I, 6 + I, 6I, 6 + 6I, 5I, 5I + 5}, {2, 4, 6, 8, 14, 11, 9, 1, 13}, {g, 2g, 4g, 6g, 9g, 5 + 5g, 8g, 3g}} and v_3 = {{2, 4, 6, 8, 9, 1, 11, 17, 15, 16, 13, 19}, {I, 8I, 3 + 3I, 8 + I, 5I, 6I, 6 + I, 6 + 6I, 18I, 16I + 1}, {i_F, $5i_F$, $3i_F$, $10i_F$, $2 + 2i_F$, $16i_F$, $9i_F$, 9, $9 + 9i_F$}, {g, 2g, 4g, 7g + 4, 3g, 5g, 2 + 2g, 3 + 4g, 5 + g}} be the vertex subsets of P(S). Find the structure of the multigraph G with v_1, v_2 and v_3 as vertices.

iii) Find all vertex subset - subsets multisubgraphs which are uniform and full of subset vertex empty multidyads.

iv) How many empty subset-subset vertex multidyads exists?

v) Find all subset-subset vertex dyads with single edge.

15. Let S = {Z_{12}, $\langle Z_{15} \cup g\rangle$, $C(Z_{18})$} be a set and P(S) the power set of S.

i) Find all trees using vertex subsets from P(S).

ii) Prove or disprove subset vertex multisubgraphs H of G are trees?

iii) Find those subset-subset vertex multisubgraphs whose child node vertex subset becomes the parent node.

iv) Can there be subset-subset vertex multisubgraphs of H be complete or pseudo complete? (H is a subset vertex multisubgraph of a multitree G).

v) Can there be subset vertex multisubgraphs of G which are empty but their subset-subset vertex multisubgraphs are connected?

vi) Can a subset-subset vertex multisubgraph of an empty subset vertex multisubgraph be a tree?

16. Obtain all special features related with subset vertex multigraph?

17. Compare multigraphs with subset vertex multigraphs.

18. Compare subset-subset vertex multisubgraphs with subset vertex multisubgraphs of a subset vertex multigraph.

19. Find those subset-subset vertex multisubgraphs H of K a subset vertex multigraph K whose local complement enjoys the same structure as that of H.

20. Characterize all those subset vertex multisubgraphs H of a subset vertex multigraph G, and K be the subset-subset vertex multisubgraphs of H so that K and H enjoy the same structure.

21. Prove or disprove that a subset-subset vertex multisubgraphs of a complete subset vertex multisubgraph can also be empty.

22. Can a subset-subset vertex multisubgraph of a subset vertex multisubgraph H which is a tree be always a tree? Justify your claim.

23. Let S= {⟨$Z_{10} \cup g$⟩, ⟨$Z_{15} \cup I$⟩, C(Z_{18})} be the set with 3 attributes. P(S) the powerset of S.

    Let G be the directed subset vertex multigraph given by the following vertex subsets v_1 = {{g, g + 2, g + 3, 2g + 1, 5}, {6I, 8I, 5 + I, 5I + 1}, {3i_F + 4, 5i_F + 2, 3i_F, 9, 9 + 9i_F, 9, 9i_F}}, v_2 = {{g, g + 2, g + 3}, {6I, 8I, 9I, 9 + 9I}, {3i_F + 4, 9, 9i_F}}, v_3 = {{g + 3, g + 2, g, 5, g + 7, 9g + 9, 9g, 9}, {9, 9i_F, 3i_F + 4, 2, 7}, {5 + 7, 5I + 1}, v_4 = {g + 3, g + 2, 5}, {9 + 9i_F, 9, 9i_F}, {6I, 8I}}, v_5 = {{5 + I, 5I + 1, 6I}, {g + 2, g + 3, g, 8g, 8 + 8g}, {9, 9i_F}} and v_6 = {{6I, 8I}, 9, 9 + 9i_F, 3i_F + 9}, {6I, 8I, 5I + 1, 9I, 9I + 9}}.

    i) Draw the multigraph G with vertex subsets {v_1, v_2, …, v_6}.

    ii) Is G a star directed subset vertex multigraph?

    iii) Is G a pseudo completed directed subset vertex multigraph?

    iv) Draw using the vertex subsets {v_1, …, v_6} a non directed subset vertex multigraph H of type I and compare it with G.

    v) Using vertex subsets {v_1, v_3, v_5 and v_6} draw the projective (injective) subset vertex multisubgraphs K of G.

vi) Find all subset-subset vertex multisubgraphs L_i of K.

vii) Find those L_i's of K which has same structure as that of K.

viii) Find those local complements of L_i relative to K which has the same structure as that of both L_i and K.

ix) Can K contain subset-subset vertex multisubgraphs P_i which are empty and their local complements are complete?

x) Can G contain subset vertex multisubgraphs which are trees?

xi) Can subset-subset vertex multisubgraphs of any L_i be a tree?

xii) Can subset-subset vertex multisubgraphs of any L_i be a circle graph?

xiii) Obtain any other special feature enjoyed by subset-subset vertex multisubgraphs of L_i?

xiv) Using vertex subsets from P(S) construct subset - subset multisubgraphs which are full uniform trees with 6 layers?

xv) What is the maximum number of layers a full uniform subset vertex multitree can be constructed using this P(S)?

xvi) Prove or disprove in general one can have more layers if the subset vertex multitree is not full and uniform.

Let S = {C(Z_{16}), ⟨Z_{12}∪I⟩, ⟨Z_{30} ∪ g⟩, Z_{18}} be a set. P(S) the powerset of S. Let G be the subset vertex multigraph given by the following figure.

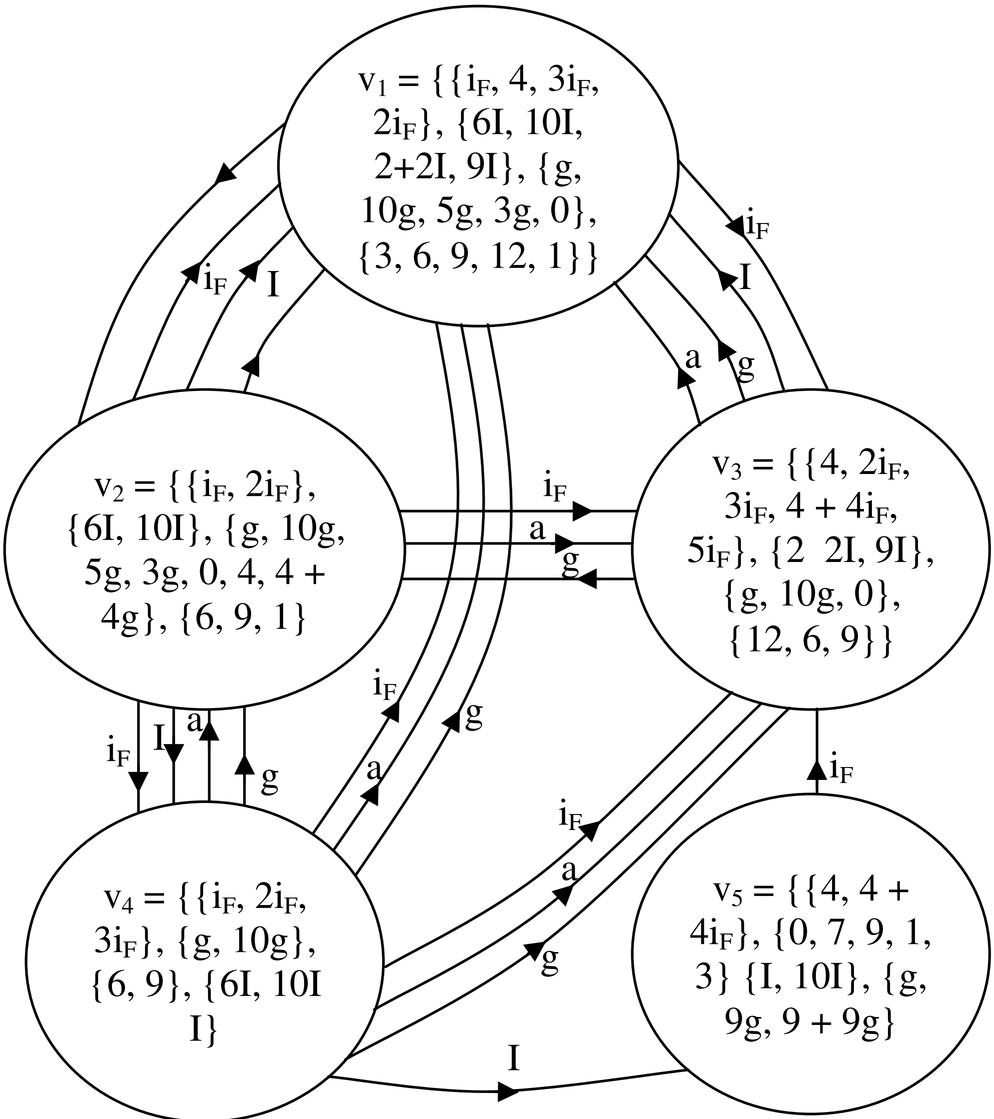


Figure 4.58

i) Is G a directed complete subset vertex multigraph?

ii) Draw H be the subset vertex multisubgraph with vertex subset {v_1, v_2, v_3 and v_4}

iii) Can subset-subset vertex multisubgraphs of H be full uniform complete?

iv) Can subset-subset vertex multisubgraphs of H be empty?

v) Can subset-subset vertex multisubgraphs of H be a tree?

vi) Is it possible to have a subset-subset vertex multisubgraph of H to be star multigraph?

vii) Describe all subset-subset vertex multisubgraphs of H which are empty but their local complements are pseudo complete subset-subset vertex multisubgraphs?

viii) Give any of the special features associated with this H.

25. Show by examples these subset-subset vertex neutrosophic multisubgraphs concept is a best-known tool to study disintegrated society or nation.

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INDEX

M

N

O

P

R

S

T

U

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On India's 60th Independence Day, Dr.Vasantha was conferred the Kalpana Chawla Award for Courage and Daring Enterprise by the State Government of Tamil Nadu in recognition of her sustained fight for social justice in the Indian Institute of Technology (IIT) Madras and for her contribution to mathematics. The award, instituted in the memory of Indian-American astronaut Kalpana Chawla who died aboard Space Shuttle Columbia, carried a cash prize of five lakh rupees (the highest prize-money for any Indian award) and a gold medal.

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In this book authors have designed, developed and described subset vertex multigraphs and neutrosophic multigraphs. The notion of neutrosophic subset vertex multigraphs will play a vital role in social multi networks. The special feature associated with these multigraphs are that they are unique once a set of vertex subsets are given.

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