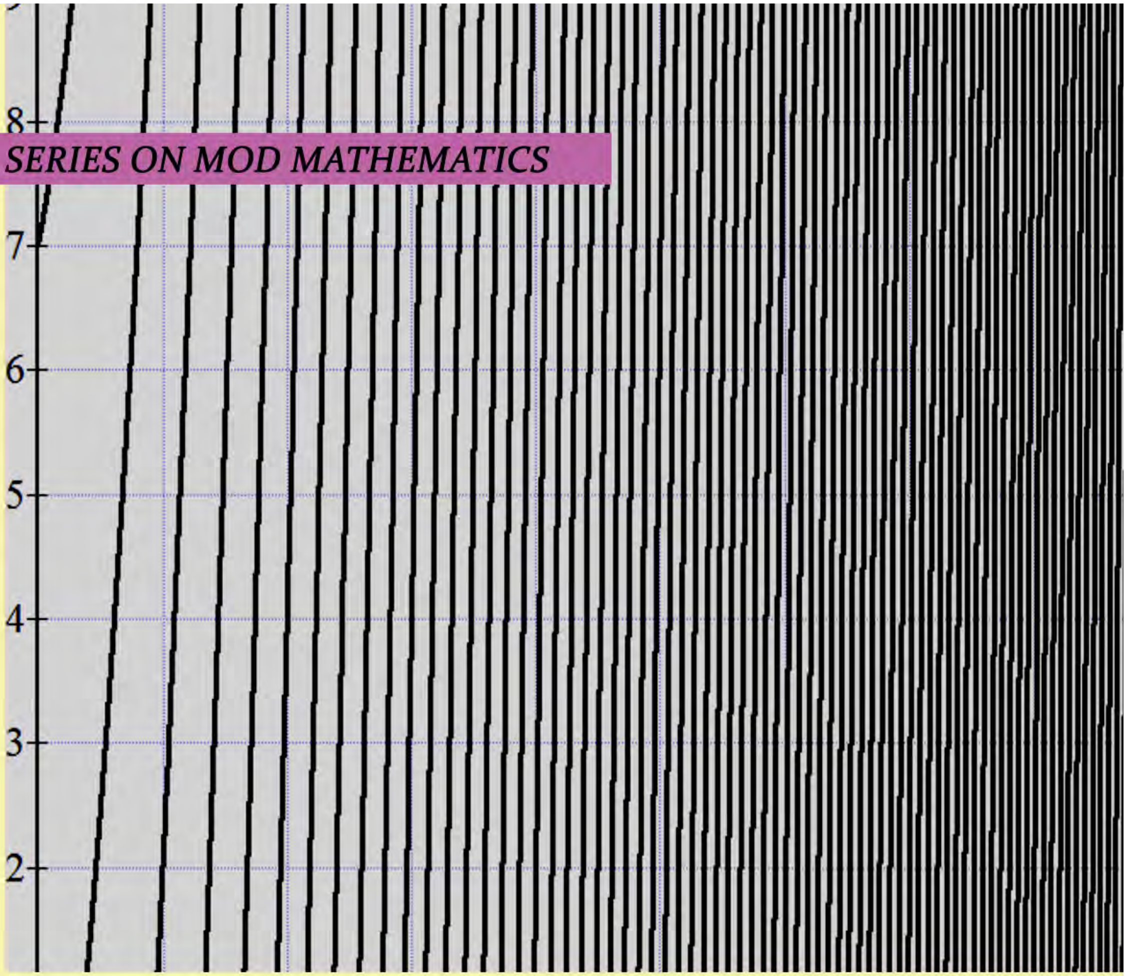
SERIES ON MOD MATHEMATICS



MOD GRAPHS

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MOD Graphs

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PREFACE History of Neutrosophic Theory and its Applications

Zadeh introduced the *degree of membership/truth* (t) in 1965 and defined the fuzzy set.

Atanassov introduced the *degree of nonmembership/ falsehood* (f) in 1986 and defined the intuitionistic fuzzy set.

Smarandache introduced the *degree of indeterminacy/ neutrality* (i) as independent component in 1995 (published in 1998) and defined the neutrosophic set on three components (t, i, f) = (truth, indeterminacy, falsehood):
http://fs.gallup.unm.edu/FlorentinSmarandache.htm

Etymology.

The words “neutrosophy” and “neutrosophic” were coined/ invented by F. Smarandache in his 1998 book.

Neutrosophy: A branch of philosophy, introduced by F. Smarandache in 1980, which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

Neutrosophy considers a proposition, theory, event, concept, or entity, "A" in relation to its opposite, "Anti-A" and that which is not A, "Non-A", and that which is neither "A" nor "Anti-A", denoted by "Neut-A".

Neutrosophy is the basis of neutrosophic logic, neutrosophic probability, neutrosophic set, and neutrosophic statistics.
{From: *The Free Online Dictionary of Computing*, edited by Denis Howe from England. Neutrosophy is an extension of the Dialectics.}

Neutrosophic Logic is a general framework for unification of many existing logics, such as fuzzy logic (especially intuitionistic fuzzy logic), paraconsistent logic, intuitionistic logic, etc.

The main idea of NL is to characterize each logical statement in a 3D-Neutrosophic Space, where each dimension of the space represents respectively the truth (T), the falsehood (F), and the indeterminacy (I) of the statement under consideration, where T, I, F are standard or non-standard real subsets of $]^{-}0, 1^{+}[$ with not necessarily any connection between them.

For software engineering proposals the classical unit interval [0, 1] may be used.

T, I, F are *independent components*, leaving room for incomplete information (when their superior sum < 1), paraconsistent and contradictory information (when the superior sum > 1), or complete information (sum of components = 1).

For software engineering proposals the classical unit interval [0, 1] is used.

For single valued neutrosophic logic, the sum of the components is:

- $0 \leq t+i+f \leq 3$ when all three components are independent;
- $0 \leq t+i+f \leq 2$ when two components are dependent, while the third one is independent from them;
- $0 \leq t+i+f \leq 1$ when all three components are dependent.

When three or two of the components T, I, F are independent, one leaves room for incomplete information (sum < 1), paraconsistent and contradictory information (sum > 1), or complete information (sum = 1).

If all three components T, I, F are dependent, then similarly one leaves room for incomplete information (sum < 1), or complete information (sum = 1).

In general, the sum of two components x and y that vary in the unitary interval [0, 1] is: $0 \leq x + y \leq 2 - d°(x, y)$, where d°(x, y) is the degree of dependence between x and y, while d°(x, y) is the degree of independence between x and y.

In 2013 Smarandache *refined the neutrosophic set to n components*: (T_1, T_2, ...; I_1, I_2, ...; F_1, F_2, ...);
see http://fs.gallup.unm.edu/n-ValuedNeutrosophicLogic-PiP.pdf .

The Most Important Books and Papers in the Development of Neutrosophics

1995-1998 - *introduction of neutrosophic set/logic/probability/statistics;*
generalization of dialectics to neutrosophy;
http://fs.gallup.unm.edu/ebook-neutrosophics6.pdf (last edition)

2003 – *introduction of neutrosophic numbers (a+bI, where I = indeterminacy)*
2003 – *introduction of I-neutrosophic algebraic structures*
2003 – *introduction to neutrosophic cognitive maps*
http://fs.gallup.unm.edu/NCMs.pdf

2005 - *introduction of interval neutrosophic set/logic*
http://fs.gallup.unm.edu/INSL.pdf

2006 – introduction of degree of dependence and degree of independence
between the neutrosophic components T, I, F
http://fs.gallup.unm.edu/ebook-neutrosophics6.pdf (p. 92)
http://fs.gallup.unm.edu/NSS/DegreeOfDependenceAndIndependence.pdf

2007 – The Neutrosophic Set was extended [Smarandache, 2007] to Neutrosophic Overset (when some neutrosophic component is > 1), since he observed that, for example, an employee working overtime deserves a degree of membership > 1, with respect to an employee that only works regular full-time and whose degree of membership = 1;
and to Neutrosophic Underset (when some neutrosophic component is < 0), since, for example, an employee making more damage than benefit to his company deserves a degree of membership < 0, with respect to an employee that produces benefit to the company and has the degree of membership > 0;

and to and to Neutrosophic Offset (when some neutrosophic components are off the interval [0, 1], i.e. some neutrosophic component > 1 and some neutrosophic component < 0).

Then, similarly, the Neutrosophic Logic/Measure/Probability/Statistics etc. were extended to respectively Neutrosophic Over-/Under-/Off- Logic, Measure, Probability, Statistics etc.

http://fs.gallup.unm.edu/SVNeutrosophicOverset-JMI.pdf
http://fs.gallup.unm.edu/IV-Neutrosophic-Overset-Underset-Offset.pdf
https://arxiv.org/ftp/arxiv/papers/1607/1607.00234.pdf

2007 – Smarandache *introduced the Neutrosophic Tripolar Set* and *Neutrosophic Multipolar Set*
and consequently
– *the Neutrosophic Tripolar Graph* and *Neutrosophic Multipolar Graph*
http://fs.gallup.unm.edu/ebook-neutrosophics6.pdf (p. 93)
http://fs.gallup.unm.edu/IFS-generalized.pdf

2009 – *introduction of N-norm and N-conorm*
http://fs.gallup.unm.edu/N-normN-conorm.pdf

2013 - *development of neutrosophic probability*
(chance that an event occurs, indeterminate chance of occurrence,
chance that the event does not occur)
http://fs.gallup.unm.edu/NeutrosophicMeasureIntegralProbability.pdf

2013 - *refinement of components* $(T_1, T_2, \ldots; I_1, I_2, \ldots; F_1, F_2, \ldots)$
http://fs.gallup.unm.edu/n-ValuedNeutrosophicLogic.pdf

2014 – *introduction of the law of included multiple middle (<A>; <neut1A>, <neut2A>, ...; <antiA>)*
http://fs.gallup.unm.edu/LawIncludedMultiple-Middle.pdf

2014 - *development of neutrosophic statistics (indeterminacy is introduced into classical statistics with respect to the sample/population, or with respect to the individuals that only partially belong to a sample/population)*
http://fs.gallup.unm.edu/NeutrosophicStatistics.pdf

2015 - *introduction of neutrosophic precalculus and neutrosophic calculus*
http://fs.gallup.unm.edu/NeutrosophicPrecalculusCalculus.pdf

2015 – *refined neutrosophic numbers* ($a+ b_1I_1 + b_2I_2 + \ldots + b_nI_n$)*, where* $I_1, I_2, \ldots, I_n$ *are subindeterminacies of indeterminacy I;*
2015 – *(t,i,f)-neutrosophic graphs;*
2015 - *Thesis-Antithesis-Neutrothesis, and Neutrosynthesis, Neutrosophic Axiomatic System, neutrosophic dynamic systems, symbolic neutrosophic logic, (t, i, f)-Neutrosophic Structures, I-Neutrosophic Structures, Refined Literal Indeterminacy, Multiplication Law of Subindeterminacies*:
http://fs.gallup.unm.edu/SymbolicNeutrosophicTheory.pdf
2015 – *Introduction of the subindeterminacies of the form* $I_0^n = \frac{k}{0}$*, for* $k \in \{0, 1, 2, \ldots, n-1\}$*, into the ring of modulo integers* Z_n - called *natural neutrosophic indeterminacies* [Vasantha-Smarandache]

http://fs.gallup.unm.edu/MODNeutrosophicNumbers.pdf

2015 – Introduction of *neutrosophic triplet structures* and *m-valued refined neutrosophic triplet structures* [Smarandache - Ali]

Submit papers on neutrosophic set/logic/probability/statistics to the international journal "Neutrosophic Sets and Systems", to the editor-in-chief: smarand@unm.edu

(see http://fs.gallup.unm.edu/NSS)

ABOUT THE BOOK

In this book authors for the first time introduce study and develop the notion of MOD graphs, MOD directed graphs, MOD finite complex number graphs, MOD neutrosophic graphs, MOD dual number graphs and so on using edge weights from Z_n, $C(Z_n)$ $\langle Z_n \cup I\rangle$, $\langle Z_n \cup g\rangle$ and so on.

Likewise MOD directed natural neutrosophic graphs are defined. Further type I, type II and type III. MOD directed graphs and MOD natural neutrosophic graphs are defined and developed.

This book has over 185 examples and over 250 figures.

The notion of MOD bipartite graphs and MOD natural neutrosophic bipartite graphs using Z_n^I, $C^I(Z_n)$ $\langle Z_n \cup I\rangle_I$ and so on are described.

This book gives the probable applications of these concepts to MOD mathematical models like MOD Cognitive Maps model and MOD Relational Maps model which have been introduced by the authors.

There are open conjectures which can help the researchers in graph theory. Several innovative results are obtained.

We wish to acknowledge Dr. K Kandasamy for his sustained support and encouragement in the writing of this book.

W.B.VASANTHA KANDASAMY
ILANTHENRAL K
FLORENTIN SMARANDACHE

Chapter One

BASIC CONCEPTS

In this book for the first time authors venture to study MOD graphs using Z_n, Z_n^I, $C(Z_n)$, $\langle Z_n \cup I\rangle$, $\langle Z_n \cup g\rangle$, $C^I(Z_n)$, $\langle Z_n \cup h\rangle_I$ and so on.

MOD graphs take vertex set and (or) edge sets from any of the sets Z_n, $C(Z_n)$, $\langle Z_n \cup I\rangle$, $\langle Z_n \cup g\rangle$, $\langle Z_n \cup h\rangle$ and $\langle Z_n \cup k\rangle$.

These MOD graphs are special for these lead to MOD Cognitive Maps model [68]. So an exhaustive study of MOD graphs is carried out in this book.

Next we study MOD natural neutrosophic graphs and directed graphs with edge weights / vertex sets from Z_n^I or $\langle Z_n \cup h\rangle_I$ or $\langle Z_n \cup I\rangle_I$ or $C^I(Z_n)$ or $\langle Z_n \cup g\rangle_I$ or $\langle Z_n \cup k\rangle_I$. Such study is thoroughly carried out in this book. These graphs find applications in the study of MOD natural neutrosophic Cognitive Maps model [68].

So a systematic study is made in this book. For the first time we visualize edges and vertex sets to be natural neutrosophic, natural neutrosophic dual numbers, natural neutrosophic-neutrosophic edges / vertices and so on. Such study is only new and innovative but can find application in MOD Cognitive Maps models. [17-25, 68].

Next we proceed onto introduce and newly describe the new notion of MOD bipartite graphs and MOD n-partite graph with edge / vertex set from any one of the sets Z_n, $\langle Z_n \cup g\rangle$, $C(Z_n)$, $\langle Z_n \cup h\rangle$, $\langle Z_n \cup I\rangle$,$\langle Z_n \cup k\rangle$.

These structures find applications in MOD Relational Maps model with edge weights from Z_n or $\langle Z_n \cup g\rangle$ or $\langle Z_n \cup I\rangle$ or $C(Z_n)$ or $\langle Z_n \cup h\rangle$ or $\langle Z_n \cup k\rangle$ [69].

These models will be new for edge weights / vertex sets can be complex or dual numbers or neutrosophic or special dual like numbers or special quasi dual numbers. So such study is not only new and innovative but is very useful.

Next we study of MOD n-partite graphs with vertex sets / edge sets from Z_n or $C(Z_n)$ or $\langle Z_n \cup I\rangle$ or $\langle Z_n \cup h\rangle$ or $\langle Z_n \cup g\rangle$ or $\langle Z_n \cup k\rangle$.

We now proceed onto describe MOD natural neutrosophic bipartite graph with edge weights / vertex sets from $\langle Z_n \cup I\rangle_I$ or Z_n^I or $C^I(Z_n)$ or $\langle Z_n \cup h\rangle_I$ or $\langle Z_n \cup g\rangle_I$ or $\langle Z_n \cup k\rangle_I$.

These MOD natural neutrosophic bipartite graphs can find applications in MOD natural neutrosophic Relational Maps model [69].

The edge weights can be from Z_n^I or $C^I(Z_n)$ or $\langle Z_n \cup I\rangle_I$ or $\langle Z_n \cup h\rangle_I$ or $\langle Z_n \cup k\rangle_I$ or $\langle Z_n \cup g\rangle_I$.

Such study is new and innovative for we can have the nodes to be natural neutrosophic or complex or dual number or special quasi dual number or special dual like numbers. For more about these concepts refer [47-50, 55].

Chapter Two

MOD GRAPHS

In this chapter we for the first time introduce the notion of MOD graphs and MOD directed graphs.

A MOD graph is a graph where the vertex sets is either a subset of Z_n or whole of Z_n ($2 \leq n < \infty$).

MOD directed graphs are of three types.

In type I MOD directed graphs the vertex set can be any thing but the edge weights are from Z_n; $2 \leq n < \infty$.

In type II MOD directed graphs both vertices as well as edge weights are from Z_n.

In type III MOD directed graphs vertices are from Z_n but edge weights from the set $\{0, 1\}$.

The general MOD graphs are graphs whose vertex sets are from Z_n or subsets of Z_n may not be directed.

We will first provide some examples of each of the situations and also suggest problems.

Example 2.1: Let {G} be the MOD graphs with vertex set from $Z_2 = \{0, 1\}$.

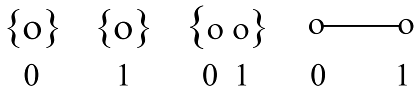


Figure 2.1

are the only four MOD graphs using the vertex set Z_2.

Example 2.2: Let {G} be the MOD graphs with vertex set for $Z_3 = \{0, 1, 2\}$.

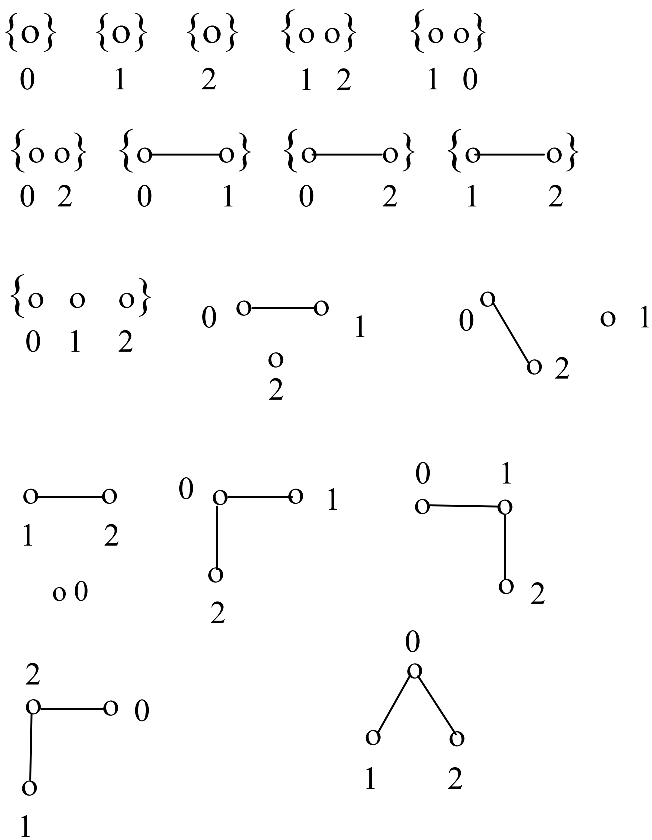
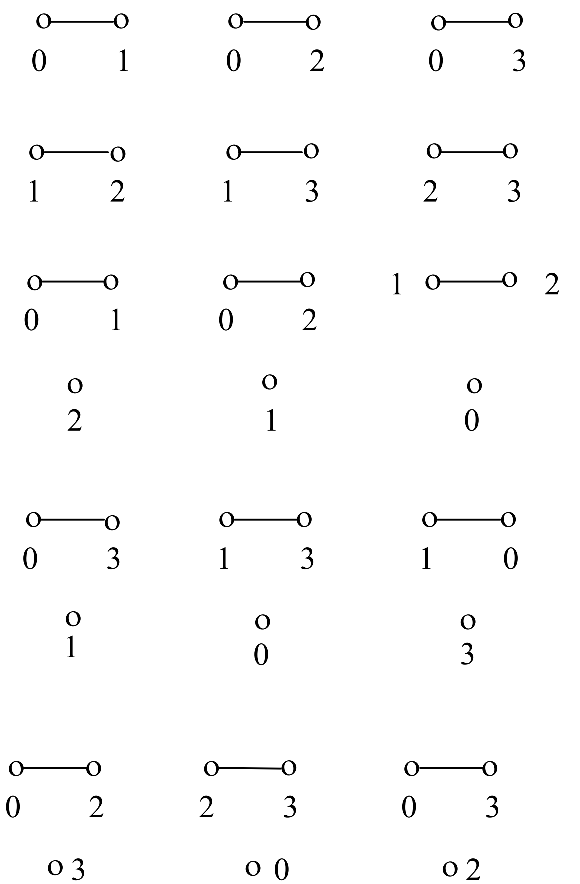


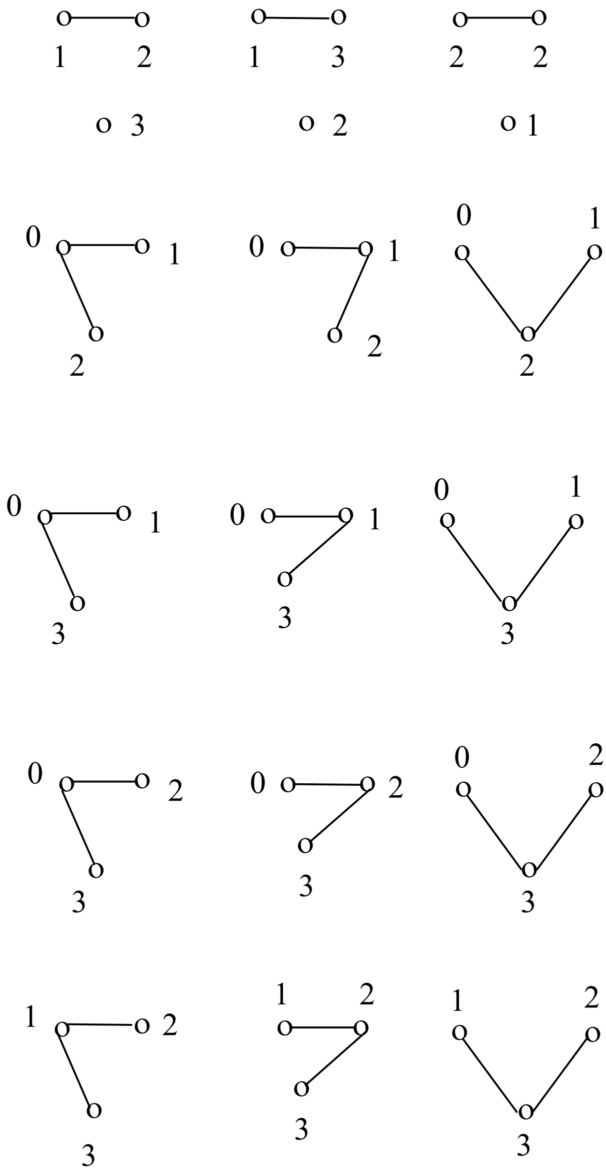
Figure 2.2

There are 17 MOD graphs using the vertex set
$Z_3 = \{0, 1, 2\}$.

Example 2.3: Let $\{G_i\}$ be the collection of all MOD graphs using vertex set from $Z_4 = \{0, 1, 2, 3\}$.

$\{0\}$, $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 2\}$ $\{0, 2\}$, $\{0, 1\}$, $\{0, 3\}$, $\{1, 3\}$, $\{2, 3\}$, $\{0, 1, 2\}$, $\{0, 1, 3\}$ $\{0, 2, 3\}$, $\{1, 2, 3\}$, $\{0, 1, 2, 3\}$,




1 2
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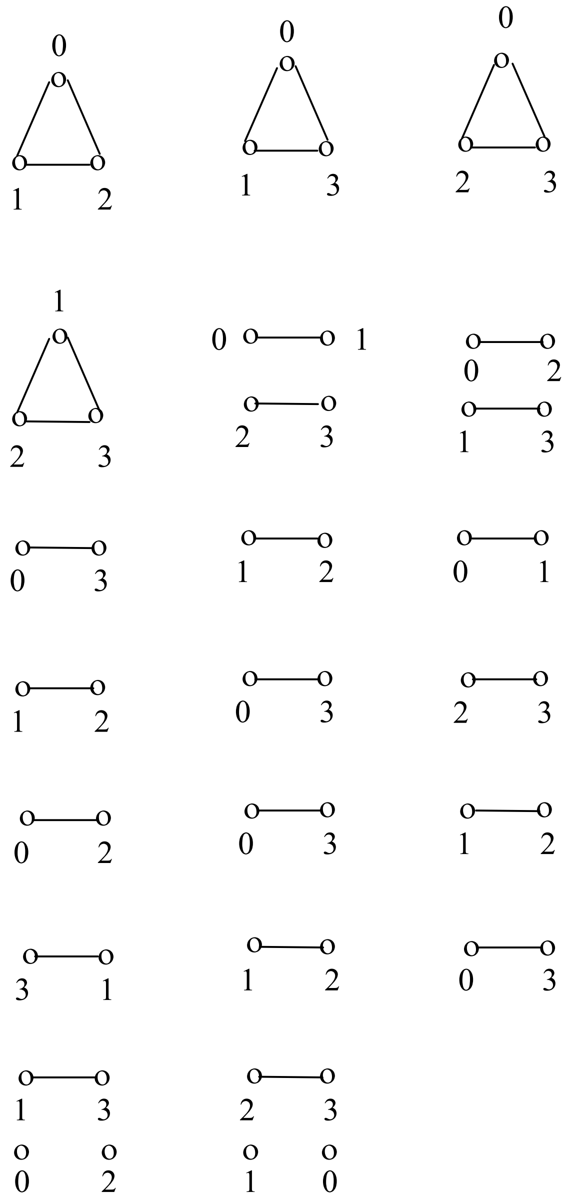


and so on.

Figure 2.3

So even in case of Z_4 we see getting all the MOD graphs happens to be a challenging problem.

We take $\{0, 1, 2, 3\} = Z_4$ as the vertices $\{v_0, v_1, v_2, v_3\}$ for one can easily work with vertices for it is non abstract.

We see when we work with MOD integers Z_n they can be applied to semi automaton, automaton or in networking.

We leave open the following conjecture.

Conjecture 2.1: Let $\{G_i\}$ be the collection of all MOD graphs with vertex set from Z_n; $2 \leq n < \infty$.

Find the number of such MOD graphs which take its vertices from subsets of Z_n or Z_n; $2 \leq n < \infty$.

We have provided examples of them.

This is introduced mainly for appropriate applications.

In case of MOD graphs the elements of Z_n can be given face value ordering or the vertices can be given face values which would be useful in case of networking or semi automaton or automation.

For the associated face values for their vertices can predict the importance or otherwise of these vertices from Z_n; $2 \leq n < \infty$.

Next we proceed onto describe type I MOD directed graphs by examples.

Example 2.4: Let G be the MOD directed graph with edge weights from Z_5 and $v_1, v_2, \ldots, v_7$ are the vertices of G.

G =

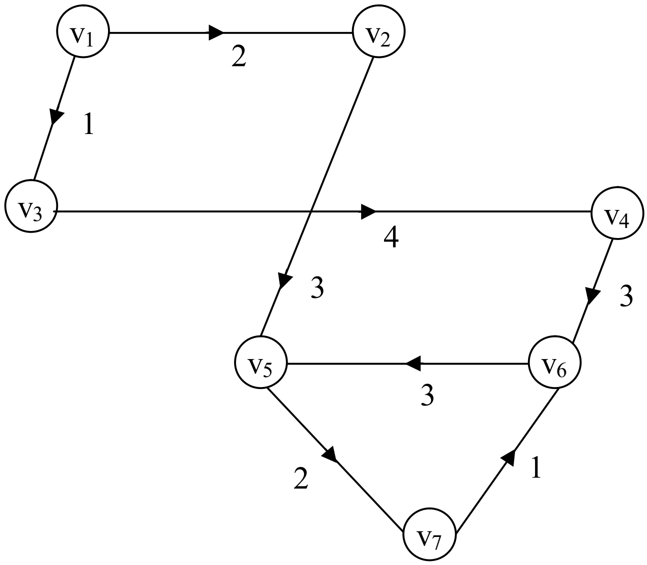


Figure 2.4

The MOD type I matrix M associated with G is as follows:

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}.$$

Example 2.5: Let V be a MOD directed graph with edge weights from the subset of Z_{12}. $v_1, v_2, \ldots, v_5$ are the vertices associated with V.

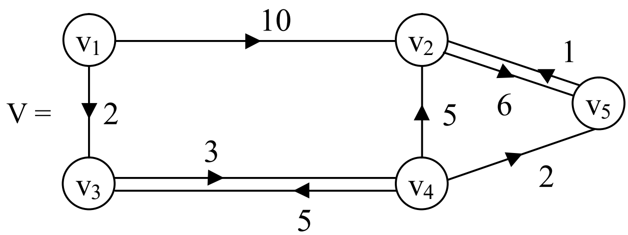


Figure 2.5

The MOD type I matrix S associated with V is as follows:

$$S = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 10 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 5 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

The edge weights of graph V are only from a subset of Z_{12}.

These types of MOD directed graphs have been already used in MOD Cognitive Maps model [68].

We have some advantages of using these type I MOD graphs.

For we see we can find S^2, S^3 and so on.

$$\text{Now } S^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}\!\!\begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 10 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 5 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array} \times \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}\!\!\begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 10 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 5 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$= \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}\!\!\begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 6 & 0 \\ 0 & 6 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 6 \\ 0 & 2 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 6 \end{bmatrix} \end{array}.$$

S^2 is associated with a MOD directed graph with edge weights from Z_{12} but has loops.

The graph associated with S^2 is as follows.

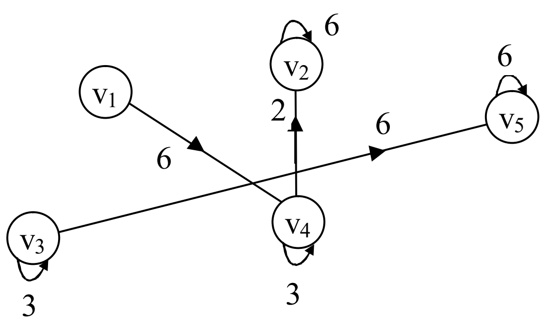


Figure 2.6

Now we find

$$S^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}\!\!\begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 6 & 0 \\ 0 & 6 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 6 \\ 0 & 2 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 6 \end{bmatrix} \end{array} \times \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}\!\!\begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 10 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 5 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$= S^2 \times S = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}\!\!\begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 6 & 0 & 9 & 0 \\ 0 & 0 & 3 & 0 & 6 \\ 0 & 6 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

The type I MOD directed graph associated with S^3 is as follows:

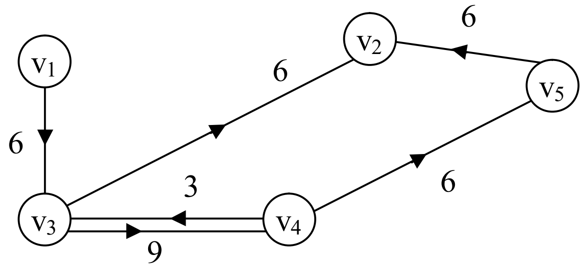


Figure 2.7

The type I MOD directed graph associated with S^3 has no loops.

Consider $S^4 = S^2 \times S^2$

$$= \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 6 & 0 \\ 0 & 6 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 6 \\ 0 & 2 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 6 \end{bmatrix} \end{array} \times \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 6 & 0 \\ 0 & 6 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 6 \\ 0 & 2 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 6 \end{bmatrix} \end{array}$$

$$= \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 9 & 0 & 6 \\ 0 & 6 & 0 & 9 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

The graph related to S^4 has two loops. Thus we can conjecture only the following:

Conjecture 2.2: Let G be type I MOD directed graph with related adjacency matrix M.

Edge weights of G are from Z_n.

Characterize those type I MOD graph G so that.

i) The type I MOD directed graph H related with M^2 have always loops (specify under what conditions it will have no loops).

ii) Can type I MOD directed graph P related with M^3 be free of loops?

iii) Characterize those MOD directed graph related to say odd powers of matrix M say M^{2t+1} will have no loops and that of even powers M^{2t} will have loops.

Next we proceed onto describe one more type I MOD directed graph G and the related MOD adjacency matrix by an example.

Example 2.6: Let G be the type I MOD directed graph with edge weights from Z_7 given by the following figure with vertex set v_1, v_2, v_3, v_4, v_5 and v_6 with no loops.

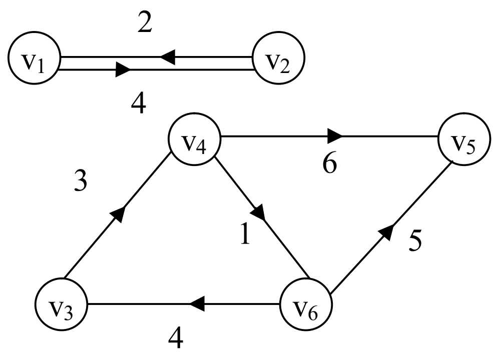
G = 

Figure 2.8

The related type I MOD adjacency matrix M related with G is as follows:

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 4 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 5 & 0 \end{bmatrix} \end{array}.$$

$$M^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 3 \\ 0 & 0 & 4 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 \end{bmatrix} \end{array}.$$

The type I MOD graph related with M^2 has two loops.

Now we find M^3 in the following

$$M^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 4 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 1 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 5 \end{bmatrix} \end{array}.$$

This the type I MOD directed graph associated with the adjacency matrix M^3 has three loops.

We see the type I MOD directed graph matrix behaves in such a way so that the following conjecture is made.

Conjecture 2.3: Characterize those type I MOD directed graphs G so that their squares, cubes, etc. represented by their MOD type I matrices i) has no loops, ii) always has loops.

It is pertinent to keep on record that these type I MOD directed graphs with edge weight from Z_n, have already been applied to MOD Cognitive Maps model [68].

Thus they will find applications in mathematical modelling.

Next we proceed onto describe type II MOD directed graphs. These type II MOD directed graphs take both edge values as vertex sets from subsets of Z_n.

We will describe this situation by some examples.

Example 2.7: Let G be a MOD directed graph with vertices v_1, v_2, …, v_7 from Z_{10} and edge weights from the set Z_{10} given by the following figure.

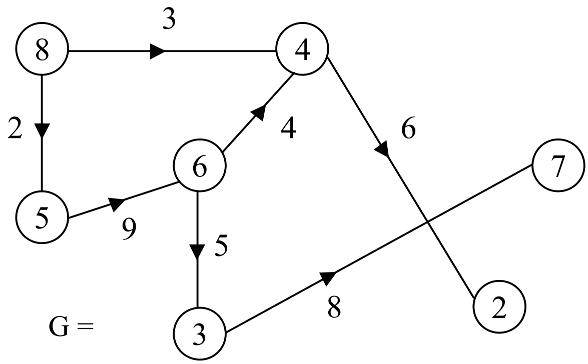


Figure 2.9

This G is a type II MOD directed graph.
The type II MOD adjacency matrix M associated with G is as follows:

$$M = \begin{array}{c} \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \begin{array}{c} \begin{array}{ccccccc} 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 9 & 0 & 0 \\ 0 & 5 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 2 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

The following rules is to be compulsorily followed to avoid confusion.

We know there is a face value ordering in Z_{10} also 0 is the least and 9 is a greatest so the vertex with $v_1 = 2$, $v_2 = 3$, $v_3 = 4$, $v_4 = 5$, $v_5 = 6$, $v_6 = 7$ and $v_7 = 8$.

Thus we have the vertices arrange according to the face value ordering in Z_n.

We will give one more example of type II MOD directed graph in the following.

Example 2.8: Let H be the type II MOD directed graph with vertices and edge weights from Z_{15} given by the following figure.

H =

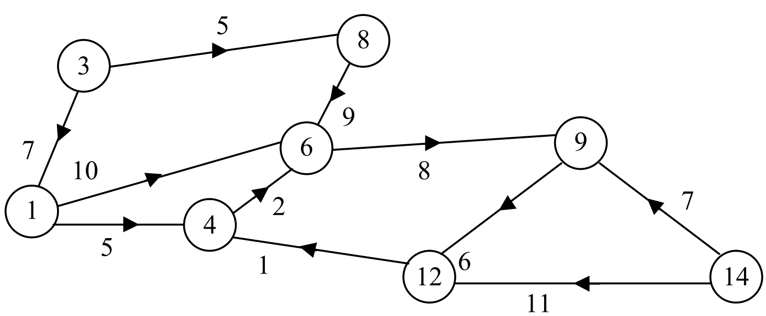


Figure 2.10

The type II MOD adjacency matrix N of the graph H is as follows:

$$N = \begin{array}{c} \\ 1 \\ 3 \\ 4 \\ 6 \\ 8 \\ 9 \\ 12 \\ 14 \end{array} \begin{array}{c} \begin{array}{cccccccc} 1 & 3 & 4 & 6 & 8 & 9 & 12 & 14 \end{array} \\ \begin{bmatrix} 0 & 0 & 5 & 10 & 0 & 0 & 0 & 0 \\ 7 & 0 & 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 7 & 11 & 0 \end{bmatrix} \end{array}.$$

Now we can adopt this MOD directed graph of type II for automaton, semi automaton and networking apart from mathematical modeling.

We proceed onto enumerate the properties enjoyed by the MOD type II matrices and their related graphs.

Example 2.9: Let G be the type II MOD directed graph given by the following figure with edge weights and vertex set from Z_{18}.

G =

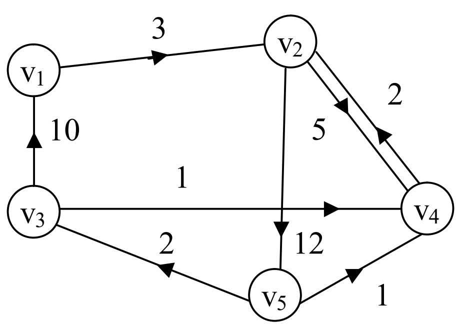


Figure 2.11

($v_1 = 3$, $v_2 = 7$, $v_3 = 10$, $v_4 = 15$ and $v_5 = 16$).

Let M be the type II MOD adjacency matrix associated with G.

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}\begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 12 \\ 10 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 \end{bmatrix} \end{array}.$$

We find $M \times M = M^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}\begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 15 & 0 \\ 0 & 10 & 6 & 12 & 0 \\ 0 & 14 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 & 6 \\ 2 & 2 & 0 & 2 & 0 \end{bmatrix} \end{array}.$

In the type II MOD directed graph associated with MOD matrix M^2 we see the MOD graph G_2 has two loops and has more edges connected; G_2 is as follows:

G_2 =

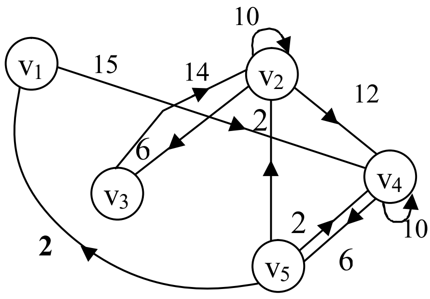


Figure 2.12

Next we find the value of M^3 and the corresponding type II MOD directed graph G_3.

$$M^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 12 & 0 & 0 & 0 \\ 6 & 6 & 0 & 2 & 12 \\ 0 & 0 & 0 & 16 & 6 \\ 0 & 2 & 12 & 6 & 0 \\ 0 & 10 & 0 & 10 & 6 \end{bmatrix} \end{array}.$$

Clearly the type II MOD directed graph G_3 is as follows:

G_3 =

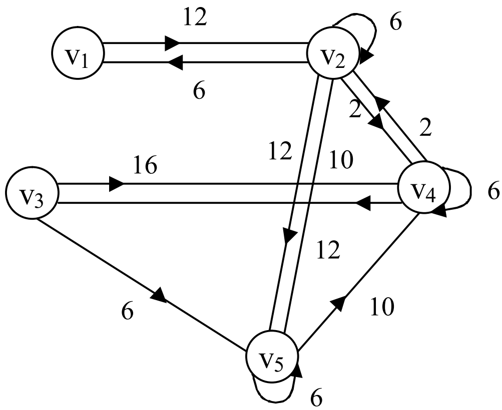


Figure 2.13

The number of loops have increased. The number of edges has increased. Some of the weights of the directed edges has also increased.

Next we find M^6 and the related type II MOD directed graph G_6.

$$M^6 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 6 & 0 \\ 0 & 16 & 6 & 0 & 0 \\ 0 & 2 & 12 & 12 & 0 \\ 12 & 6 & 0 & 16 & 6 \\ 6 & 14 & 12 & 14 & 12 \end{bmatrix} \end{array}.$$

The type II MOD directed graph has 4 loops and 15 directed weighted edges. The MOD directed type II graph is as follows.

$G_6 =$

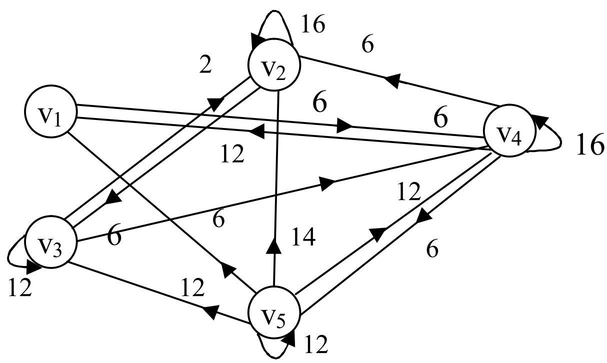


Figure 2.14

Consider

$$M^9 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 12 & 0 & 0 & 0 \\ 6 & 6 & 0 & 2 & 12 \\ 12 & 0 & 0 & 16 & 6 \\ 0 & 2 & 12 & 6 & 0 \\ 12 & 16 & 6 & 12 & 6 \end{bmatrix} \end{array}.$$

There are only three loops.

Let G_9 be the type II MOD directed graph given by the following figure.

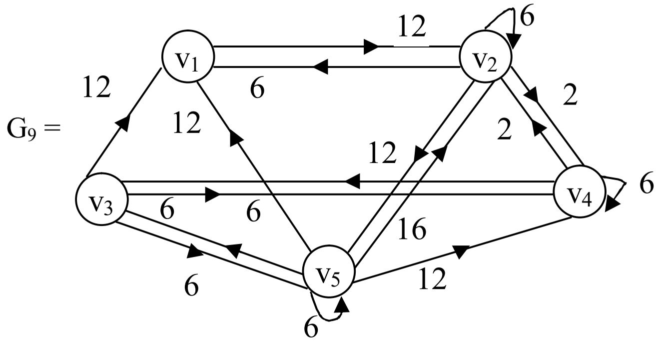
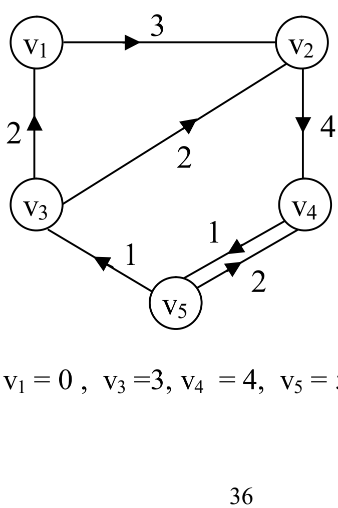


Figure 2.15

Thus we cannot say anything about this type II MOD directed graph.

We see as we product it if we choose to call so then it is clearly seen there is increase in directed edges. We conclude this notion with one more example by taking a small value of n for $Z_{n.}$

Example 2.10: Let G be the type II MOD directed graph which is as follows with edge weights from Z_6.



$v_1 = 0$, $v_3 = 3$, $v_4 = 4$, $v_5 = 5$, $v_2 = 2$

Figure 2.16

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 2 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 2 & 0 \end{bmatrix} \end{array}.$$

The type II MOD adjacency matrix. We find M^2, M^3, M^4, M^6 and their related type II MOD graphs.

$$M^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 2 & 2 & 0 & 0 & 2 \end{bmatrix} \end{array}.$$

The type II MOD directed graph associated with M^2 be G_2 which is as follows.

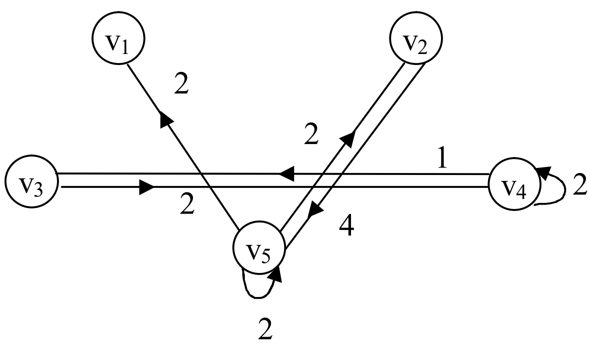


Figure 2.17

This MOD directed graph of type II has seven edges of which two are just loops.

We now find M^3 in the following.

$$M^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}\begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 2 & 2 & 0 & 0 & 2 \\ 0 & 0 & 2 & 0 & 0 \end{bmatrix} \end{array}.$$

The type II MOD directed graph G_3 related with M^3 is as follows :

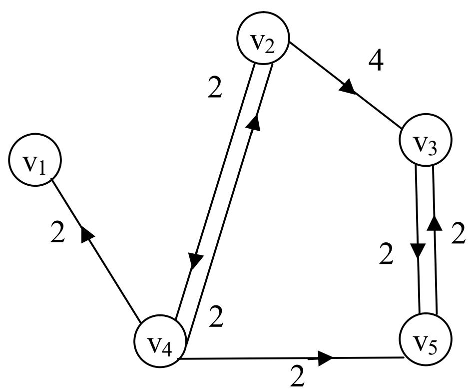
$G_3 =$ 

Figure 2.18

Clearly G_3 has no loops only six edges.

The edges has reduced for seven to six.

Now we find M^4;

$$M^4 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}\begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 & 2 \\ 0 & 0 & 2 & 4 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 4 & 4 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let G_4 be the type II MOD directed graph given by M^4. This has two loops and 8 weighted edges given by the following figure.

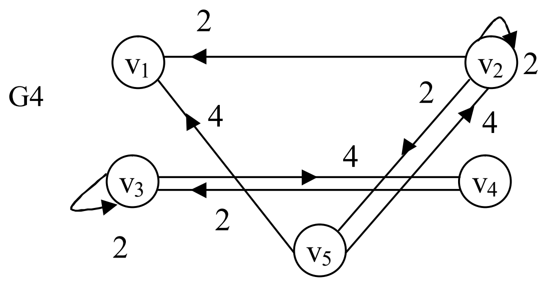


Figure 2.19

We know find M^5,

$$M^5 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}\begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 4 & 4 & 0 & 0 & 4 \\ 4 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \end{bmatrix} \end{array}.$$

Let G_5 be the type II MOD directed graph of M^5 given by the following figure.

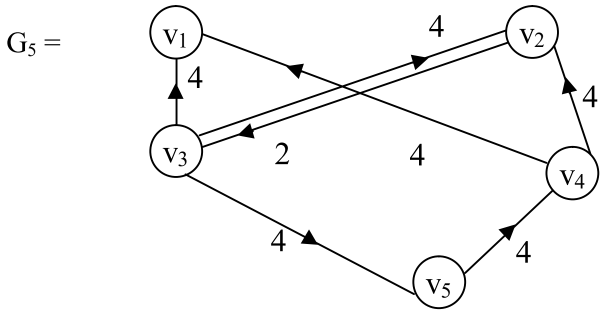


Figure 2.20

Let M^6 be the MOD matrix of type II.

$$M^6 = \begin{array}{c c} & \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 4 & 4 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix} \end{array}.$$

The type II MOD directed graph G_6 associated with M^6 is as follows:

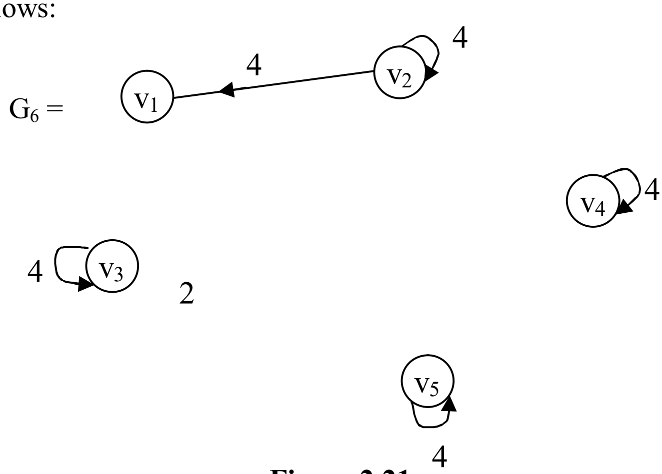


Figure 2.21

The graph has only 3 edges and four loops.

Finally before we comment on this graph G find M^7.

$$M^7 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 2 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 4 & 2 & 0 \end{bmatrix} \end{array}.$$

Clearly the type I MOD directed graph G_7 associated with M^7 has 9 edges and no loops is given below.

G_7 =

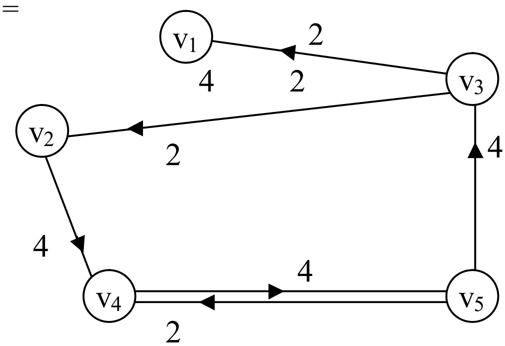


Figure 2.22

$$M^8 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 4 & 2 & 0 \\ 2 & 4 & 0 & 0 & 2 \end{bmatrix} \end{array}.$$

This type II MOD directed graph G_8 has two loops and six edges given by the following figure.

G_8 =

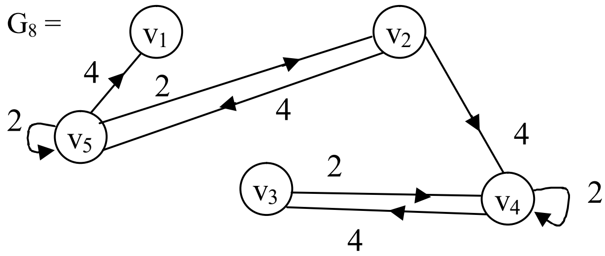


Figure 2.23

Now we find M^9,

$$M^9 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 2 & 2 & 0 & 0 & 2 \\ 0 & 0 & 2 & 0 & 0 \end{bmatrix} \end{array}.$$

Thus the type II MOD directed graph associated with M^9 has only 6 edges and no loops. The graph G_9 is as follows.

G_9 =

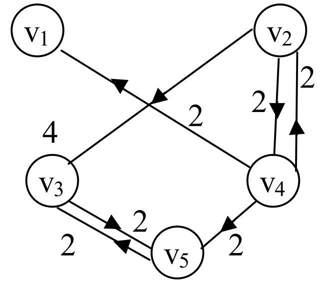


Figure 2.24

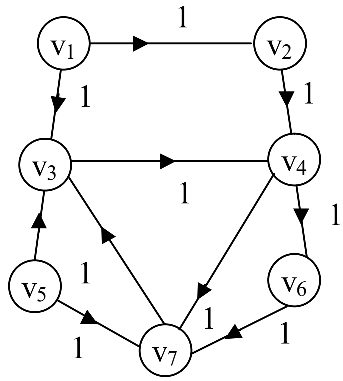
From this graph it is easily seen all graphs G_{2n+1} have no loops whereas all G_{2n} has loops.

Study in this direction is left as an exercise to the reader.

Next we proceed onto describe type III MOD directed graphs by examples.

Example 2.11: Let G be the type III MOD directed graph given by the following figure. The vertices take their values from Z_7.

G =



where $v_1 = 0$, $v_2 = 1$, $v_3 = 2$, $v_4 = 3$, $v_5 = 4$, $v_6 = 5$ and $v_7 = 6$.

Figure 2.25

Let M be the type III MOD adjacency matrix related to G

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

We find M^2

$$M^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$\to \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

The type III MOD directed graph associated with M^2 be G_2 which is as follows:

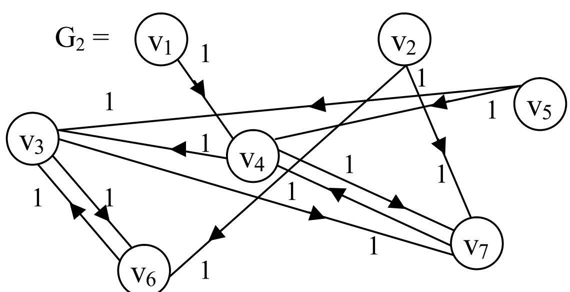
$G_2 =$ 

Figure 2.26

This is the way special product operation is performed.

In the usual product operation M^2 is kept as it is

$$M^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Here there are two methods apart from the special one which thresholds all values greater than one to one.

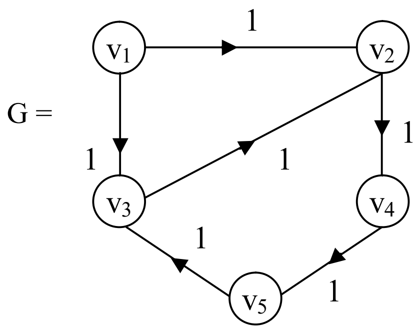
Other one keeps the value as it is as long as the values are in mod 7 as vertex set is from Z_7.

So if 8 occurs in M^t then it will be 1 and so on ($t \geq 2$).

Yet another type of operation is the expert wishes to take weight from any Z_n; $2 \leq n < \infty$ and the product is performed.

We will describe each by an example.

Example 2.12: Let G be the type II MOD directed graph with edge weights from {0, 1} and vertex set from Z_6 given by the following figure:



$v_1 = 0$, $v_2 = 1$, $v_3 = 2$, $v_4 = 3$ and $v_5 = 4$.

Figure 2.27

The type II MOD matrix B of G is as follows:

$$B = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}\begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}.$$

Now we find B^2

$$B^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}\begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

The MOD type III directed graph B_2 is as follows:

B_2 =

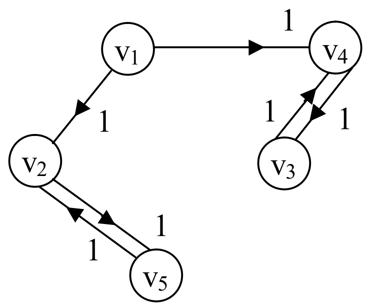


Figure 2.28

We find B^3

$$B^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}.$$

The MOD type III directed graph B_3 is as follows.

B_3 =

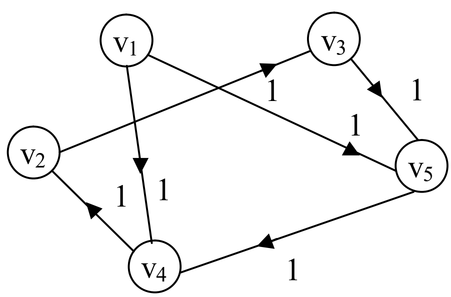


Figure 2.29

$$\text{Let } B^4 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

The type III MOD directed graph B_4 represented by M^4 is as follows.

$B_4 =$

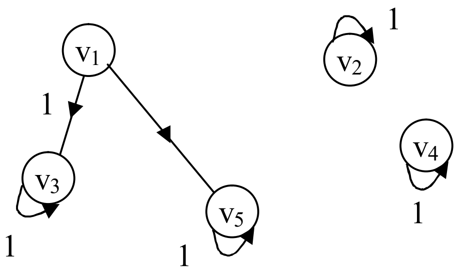


Figure 2.30

This has only one edge and four loops.

Consider B^5

$$B^5 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}.$$

Let B_5 be the MOD directed type III graph given by the following figure:

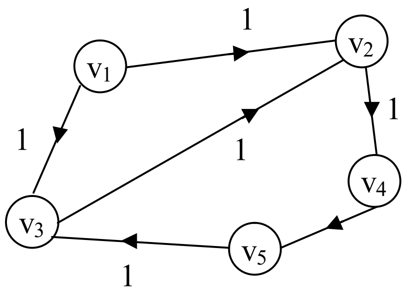


Figure 2.31

Clearly the type III MOD directed graph has no loops.

We find B^6

$$B^6 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Clearly the type III MOD directed graph has no loops.

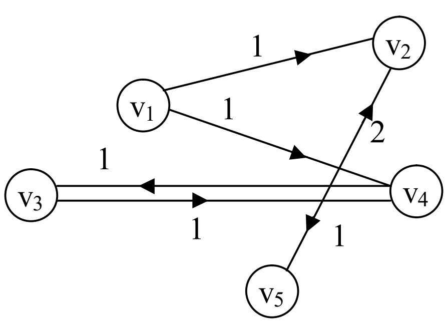


Figure 2.32

This we see after a finite number of iterations say some k iterations we will get $B^k = B$.

This type III MOD directed graph behaves in a very different way.

Next we proceed onto describe MOD graphs with vertex sets from subsets of $\langle Z_n \cup h\rangle$ or $C(Z_n)$ or $\langle Z_n \cup g\rangle$ or $\langle Z_n \cup h\rangle$ or $\langle Z_n \cup k\rangle$.

This study is not only new but also relevant for at times the vertex set can be imaginary or indeterminate or a dual number or a special dual like number or a special quasi dual number.

So to cater to these needs these new types of MOD graphs are most important.

We call MOD graph to be a MOD neutrosophic graph if the vertex sets are subsets of $\langle Z_n \cup I\rangle = \{a + bI / a, b \in Z_n, I^2 = I\}$.

We will provide some examples of such graphs.

Example 2.13: Let G be the MOD neutrosophic graph with vertex set from

$$\langle Z_{10} \cup I\rangle = \{a + bI / a, b \in Z_{10}, I^2 = I\}$$

given by the following figure:

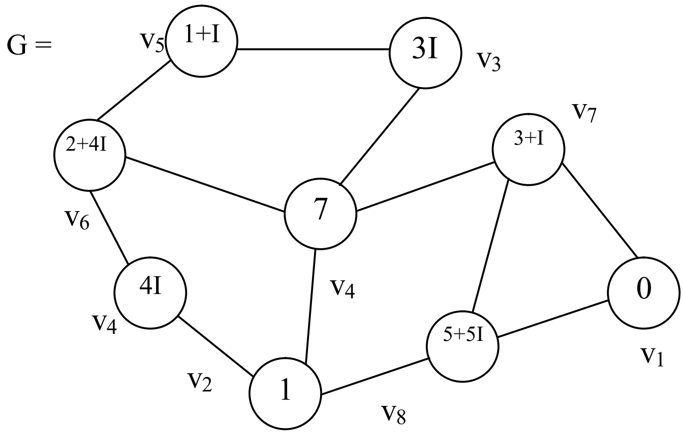


Figure 2.33

Now there are situations in machines as well as in networking where the nodes can be indeterminate at one stage (repair or over used or heated or low power) in case of machines and (in mathematical modeling where nodes can be indeterminate) respectively.

Example 2.14: Let G be the MOD neutrosophic graph with vertex weights from the set $\langle Z_3 \cup I \rangle$ given by the following figure :

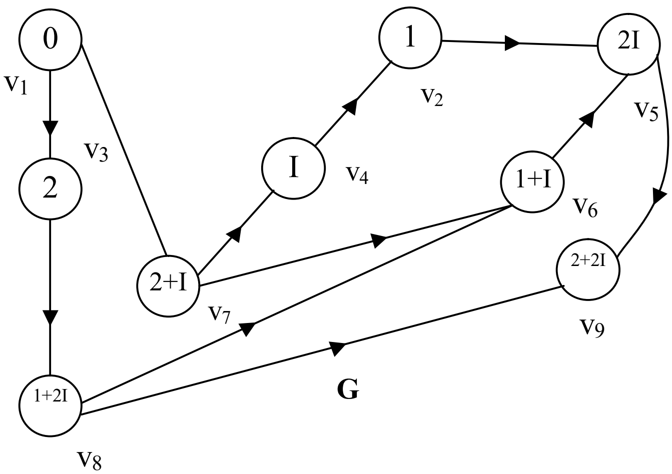


Figure 2.34

Next we proceed onto describe MOD finite complex number graphs by some examples.

We call a MOD graph which takes the vertex set values from

$$C(Z_n) = \{a + bi_F / a, b \in Z_n,\ i_F^2 = (n-1)\}$$

are defined as MOD complex graphs or MOD finite complex number graphs.

We will illustrate this situation by some examples.

Example 2.15: Let G be the MOD complex graph with vertex set from $C(Z_4)$ given by the following figure:

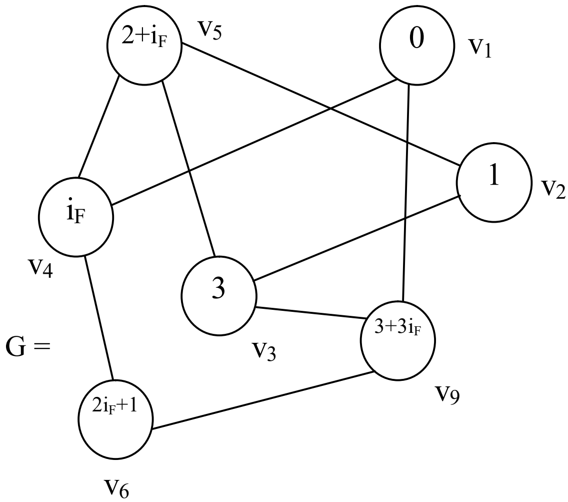


Figure 2.35

Example 2.16: Let H be the MOD finite number complex graph with edge weights from $C(Z_6)$ which is given by the following figure:

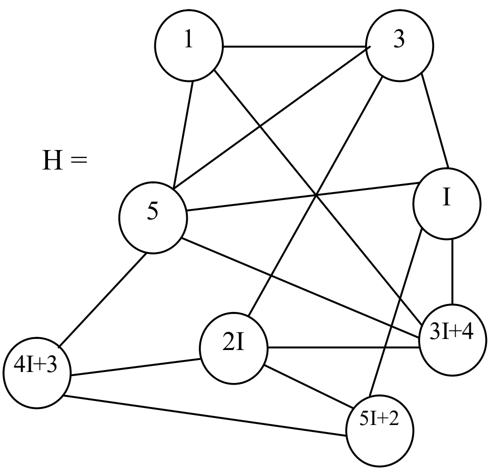


Figure 2.36

These MOD graphs will find its applications in mathematical modeling when the nodes are imaginary or mixed imaginary or real.

Next we proceed onto describe MOD dual number graphs. If a MOD graphs takes its vertex set values from the set $\langle Z_n \cup g\rangle = \{a + bg / a, b \in Z_n, g^2 = 0\}$ then we define the MOD graph as MOD dual number graph.

We will describe this situation by some examples.

Example 2.17: Let H be the MOD dual number graph given by the following figure with vertex set from $\langle Z_9 \cup g\rangle$.

H =

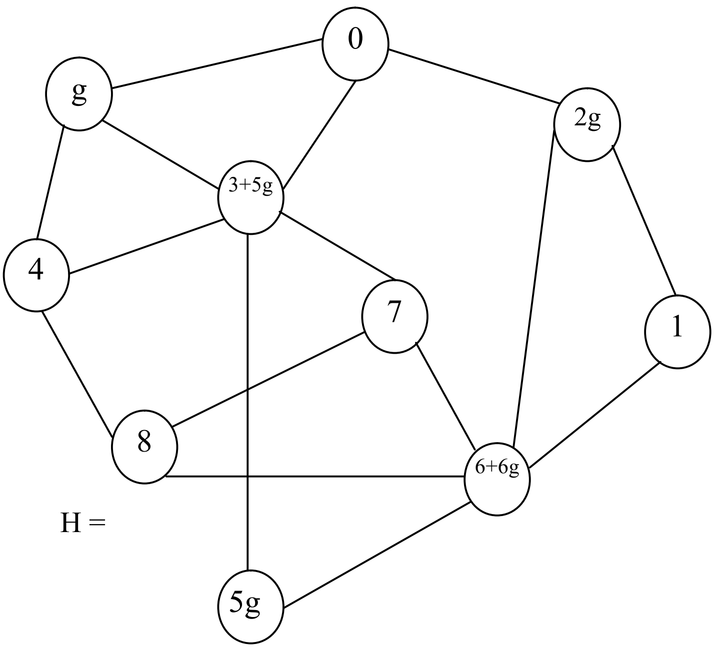


Figure 2.37

Example 2.18: Let V be the MOD dual number graph with edge weights from $\langle Z_4 \cup g\rangle$ given by the following figure:

V =

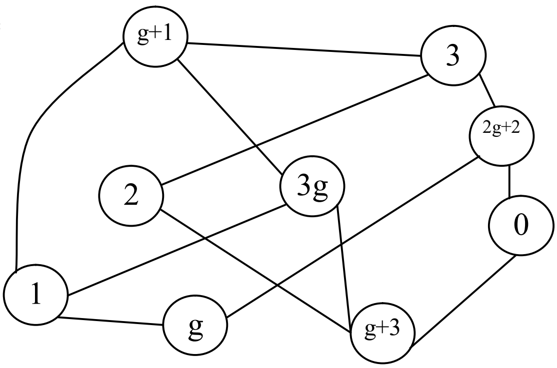


Figure 2.38

These newly constructed MOD graphs can find lots of applications in various fields.

All the more MOD dual number graphs can be very helpful when the nodes are mixed dual numbers or dual numbers or real values.

Next we describe MOD special dual like number graphs.

Let G be a MOD graphs if the vertex set is from $\langle Z_n \cup h\rangle = \{a + bh / a, b \in Z_n, h^2 = h\}$ then we define G to be a MOD special dual like number graph.

We will describe this by some examples.

Example 2.19: Let B be the MOD special dual like number graph with vertex elements from $\langle Z_7 \cup h\rangle$ given by the following figure:

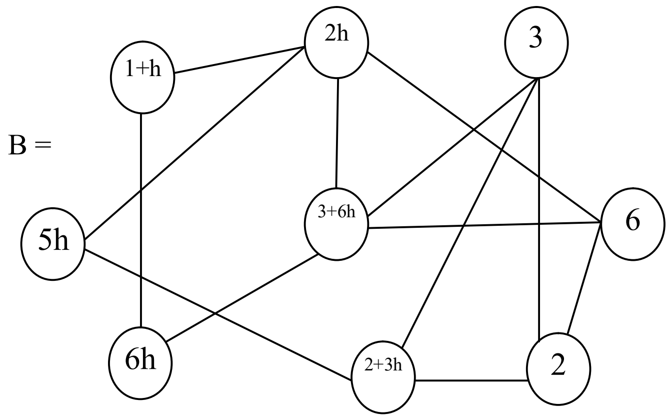


Figure 2.39

Example 2.20: Let V be the MOD special dual like number graph with vertex set from $\langle Z_{12} \cup h \rangle$ given by the following figure:

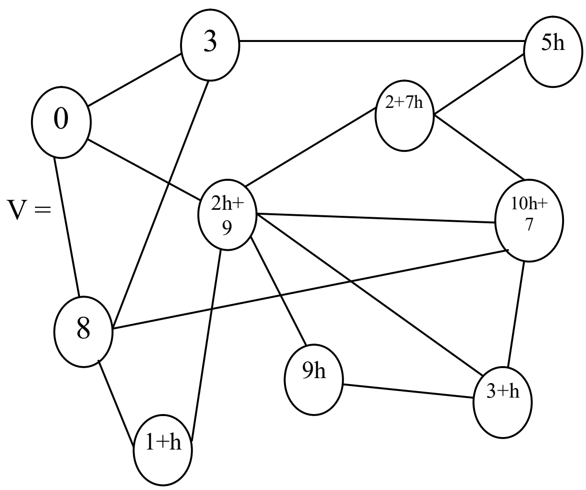


Figure 2.40

Example 2.21: Let G be a MOD graph with vertex set from $\langle Z_{15} \cup k\rangle = \{a + bk / a, b \in Z_{15}$ and $k^2 = 14k\}$ given by the following figure :

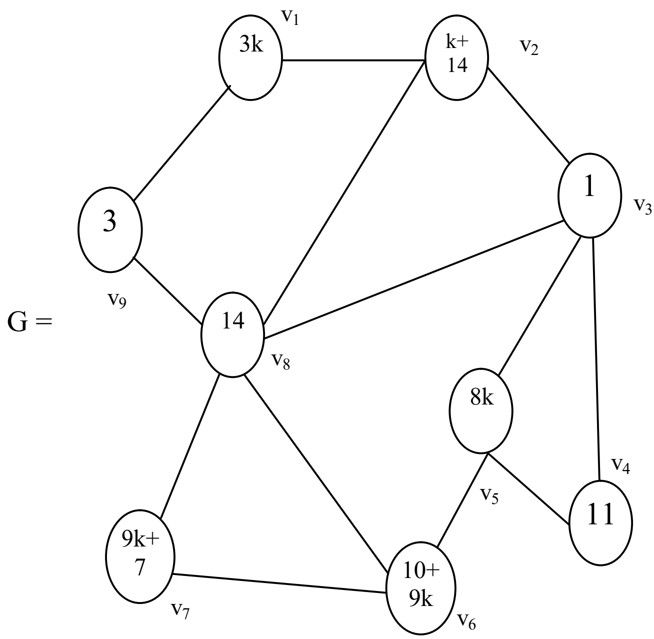


Figure 2.41

G will be known as the MOD special quasi dual number graph.

Thus if G is a MOD graph which takes vertex sets from;

$\langle Z_n \cup k\rangle = \{a + bk / a, b \in Z_n, k^2 = (n - 1) k\}$ then we define G to be a MOD special quasi dual number graph.

We will give one more example of this situation.

Example 2.22: Let H be the MOD special quasi dual number graph with vertex elements from $\langle Z_{11} \cup k\rangle$ given by the following figure:

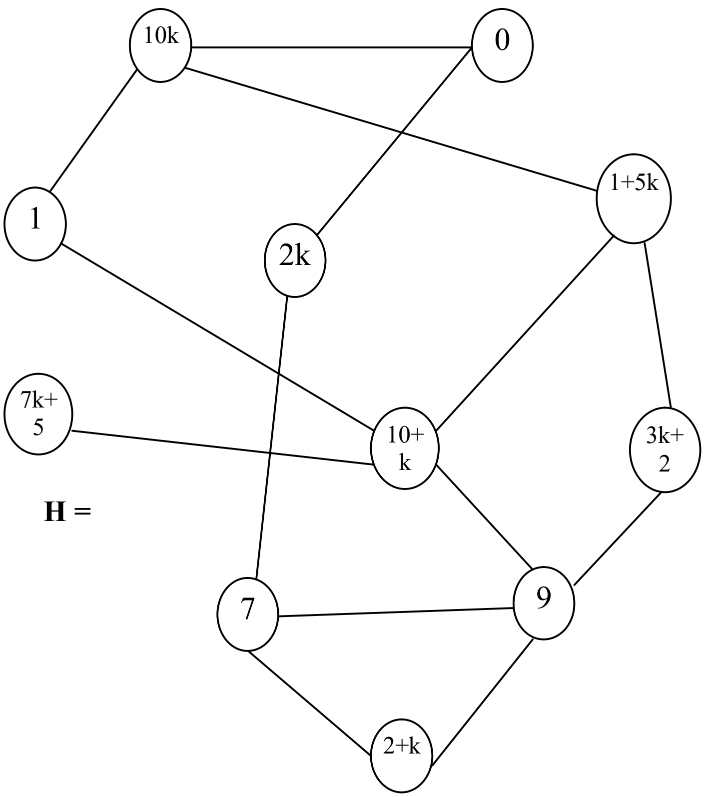


Figure 2.42

These MOD graphs will also find appropriate applications in mathematical modeling and so on.

Next we proceed onto describe type I MOD neutrosophic graphs, type I MOD dual number graphs, type I MOD complex number graphs and so on only by examples.

A MOD directed graphs which has any vertex set but whose edge weights are from $\langle Z_n \cup I\rangle$ are defined as type I MOD neutrosophic directed graph.

Example 2.23: Let V be the type I MOD neutrosophic directed graph with edge weights from $\langle Z_6 \cup I\rangle$ given by the following figure:

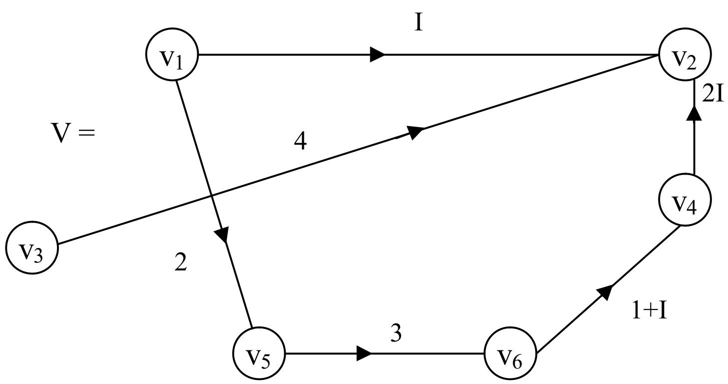


Figure 2.43

The adjacency matrix M associated with V is as follows:

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & I & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 2I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1+I & 0 & 0 \end{bmatrix} \end{array}.$$

We find

$$M^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array}\begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3+3I & 0 & 0 \\ 0 & 4I & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

The type I MOD neutrosophic directed graph associated with M^2 be V_2 which is as follows:

$V_2 =$

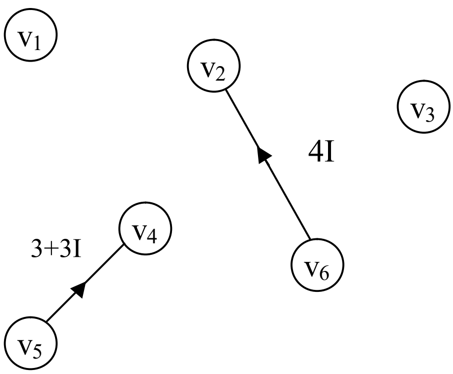


Figure 2.44

$$\text{Now we find } M^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array}\begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

The type I MOD directed neutrosophic graph is given by

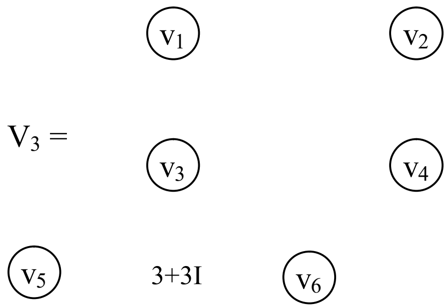


Figure 2.45

This has no edge and no loops.

Example 2.24: Let S be the type I MOD neutrosophic directed graph with edge weights from $\langle Z_5 \cup I\rangle$ given by the following figure:

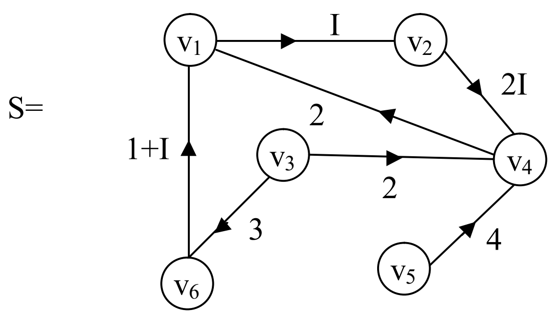


Figure 2.46

The type I MOD neutrosophic matrix P associated with S is as follows:

$$P = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2I & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 3 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 \\ 1+I & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

We now find the square of P in the following:

$$P^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 2I & 0 & 0 \\ 4I & 0 & 0 & 0 & 0 & 0 \\ 2+3I & 0 & 0 & 0 & 0 & 0 \\ 0 & 2I & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2I & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

The type I MOD neutrosophic directed graph S_2 associated with P^2 is as follows:

S_2 =

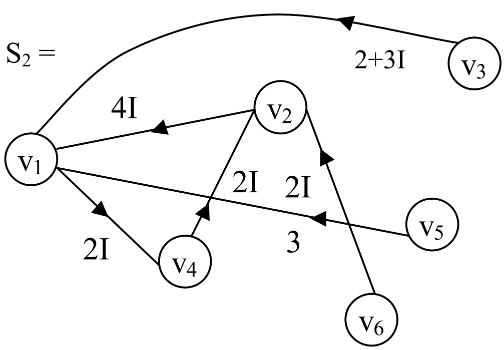


Figure 2.47

Next we find P^3.

$$P^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 4I & 0 & 0 & 0 & 0 & 0 \\ 0 & 4I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4I & 0 & 0 \\ 0 & 3I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4I & 0 & 0 \end{bmatrix} \end{array}.$$

The type I MOD neutrosophic directed graph S_3 is as follows:

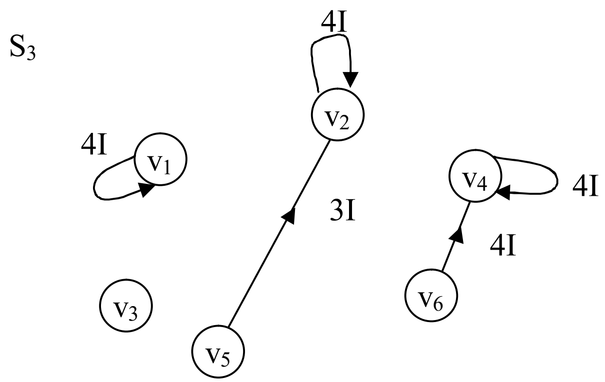


Figure 2.48

S_3 has three loops all of them are pure neutrosophic.

Edge weights of S_3 are also pure neutrosophic.

Next we find P^4 in the following:

$$P^4 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 4I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 3I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 \\ 3I & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

The type I MOD directed neutrosophic graph S_4 related with P^4 is as follows.

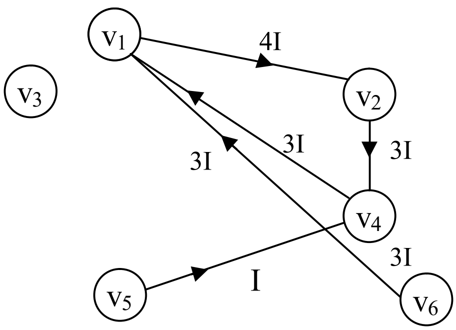
$S_4 =$ 

Figure 2.49

$$\text{Now we find } P^5 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 3I & 0 & 0 \\ I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3I & 0 & 0 & 0 & 0 \\ 2I & 0 & 0 & 0 & 0 & 0 \\ 0 & 3I & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let S_5 be the type I MOD directed neutrosophic graph associated with P^5 which is as follows:

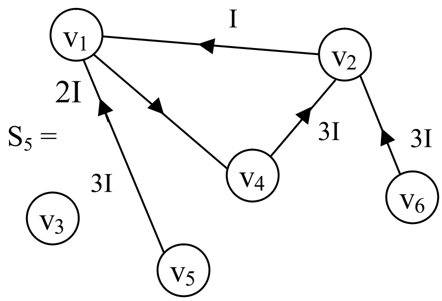


Figure 2.50

Thus we can find any number such MOD directed neutrosophic graphs of type I for a given MOD directed graph.

This will have certainly some implications in mathematical modeling as well as it would also can suggest a model given by one expert say M is related to another experts model on the same problem as $M = M^t$, $(t > 0)$.

Such study can also relate the experts opinion in a distinct and innovative way.

Another problem in this direction is can we say if S is the MOD type I neutrosophic matrix related with the MOD type I directed graph, then

$M^n = (0)$ for some n or $M^n = M$?

Study in this direction is new and left as an exercise to the reader.

Next we proceed onto define and describe MOD directed finite complex number graphs of type I.

Let G be a type I MOD directed graph if the edge weights are from $C(Z_n)$ then we define G to be a type I MOD finite complex number directed graph.

We will illustrate this situation by some examples.

Example 2.25: Let G be the type I MOD directed finite complex number graph with edge weights from $C(Z_6)$ given by the following figure.

G =

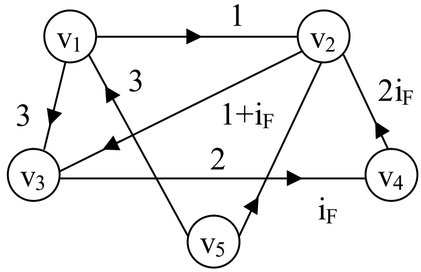


Figure 2.51

Let M be the type I MOD finite complex matrix associated with G.

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}\begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 1+i_F & 0 & 0 \\ 0 & 0 & 0 & i_F & 0 \\ 0 & 2i_F & 0 & 0 & 0 \\ 3 & 2 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

We find M^2

$$M^2 = \begin{array}{c|ccccc} & v_1 & v_2 & v_3 & v_4 & v_5 \\ \hline v_1 & 0 & 0 & 1+i_F & 3i_F & 0 \\ v_2 & 0 & 0 & 0 & i_F+5 & 0 \\ v_3 & 0 & 4 & 0 & 0 & 0 \\ v_4 & 0 & 0 & 2i_F+4 & 0 & 0 \\ v_5 & 0 & 3 & 5+2i_F & 0 & 0 \end{array}.$$

Let G_2 be the type I directed MOD finite complex number graph associated with M^2 given by the following figure:

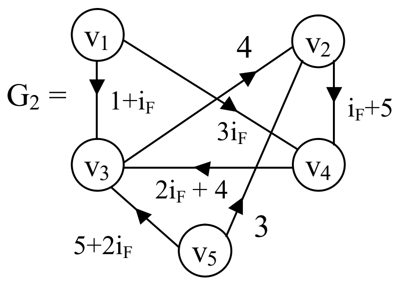


Figure 2.52

Now we find M^3

$$M^3 = \begin{array}{c|ccccc} & v_1 & v_2 & v_3 & v_4 & v_5 \\ \hline v_1 & 0 & 0 & 0 & i_F+5 & 0 \\ v_2 & 0 & 4+4i_F & 0 & 0 & 0 \\ v_3 & 0 & 0 & 4i_F+4 & 0 & 0 \\ v_4 & 0 & 0 & 0 & 4i_F+4 & 0 \\ v_5 & 0 & 0 & 3+3i_F & 5i_F+4 & 0 \end{array}.$$

The MOD directed type I graph G_3 associated with M^3 is as follows.

G_3 =

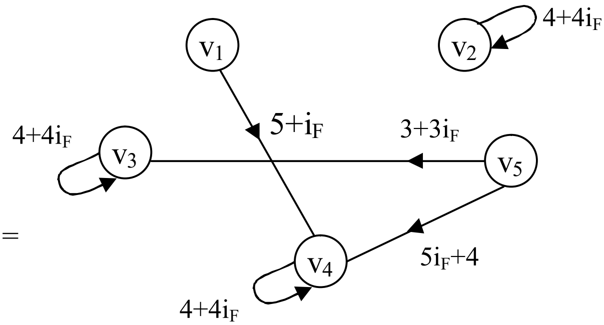


Figure 2.53

This type I MOD finite complex directed graph has three loops and 3 edges.

We now find M^4,

$$M^4 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 4+4i_F & 0 & 0 & 0 \\ 0 & 0 & 2i_F & 0 & 0 \\ 0 & 0 & 0 & 2+4i_F & 0 \\ 0 & 2i_F+4 & 0 & 0 & 0 \\ 0 & 2+2i_F & 0 & 3+3i_F & 0 \end{bmatrix} \end{array}.$$

We give the type I MOD finite complex number directed graph G_4 associated with M^4 in the following:

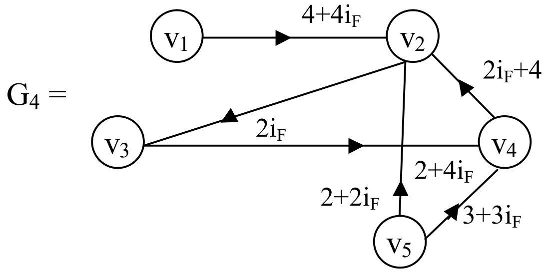


Figure 2.54

We now find M^5

$$M^5 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}\begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 2i_F & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 4+4i_F & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 4i_F & 0 & 0 \end{bmatrix} \end{array}.$$

The type I MOD directed graph of finite complex numbers G_5 associated with M^5 is as follows:

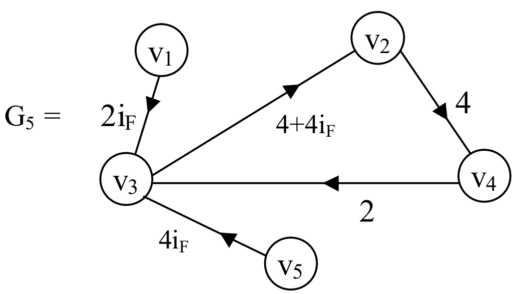


Figure 2.55

Thus we can find any M^n and its associated G_n type I MOD finite number directed graph.

Next we proceed onto describe type I MOD directed dual number graphs.

Let G be a type I MOD directed graph withi edge weights from $\langle Z_n \cup g\rangle = \{a + bg / a, b \in Z_n, g^2 = 0\}$, we call G to be the type I MOD directed dual number graph.

We will illustrate this situation by some examples.

Example 2.26: Let G be the type I MOD directed dual number graph with edge weights from $\langle Z_8 \cup g\rangle$ given by the following figure.

G =

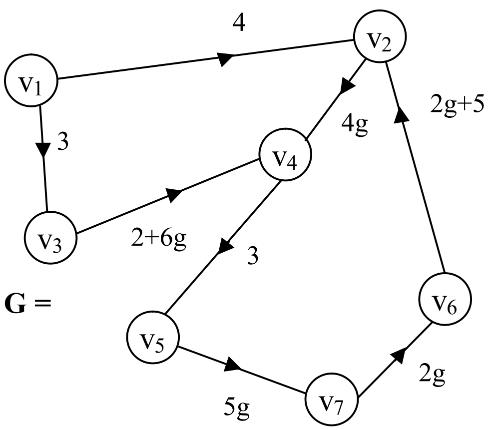


Figure 2.56

Let M be the type I MOD dual number matrix associated with G;

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 4 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4g & 0 & 0 & 0 \\ 0 & 0 & 0 & 2+6g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 5g \\ 0 & 2g+5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2g & 0 \end{bmatrix} \end{array}.$$

We find M^2

$$M^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 6+2g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4g & 0 & 0 \\ 0 & 0 & 0 & 0 & 6+2g & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 7g \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4g & 0 & 0 & 0 \\ 0 & 2g & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

The type I MOD directed dual number graph G_2 associated with M^2 is as follows.

$G_2 =$

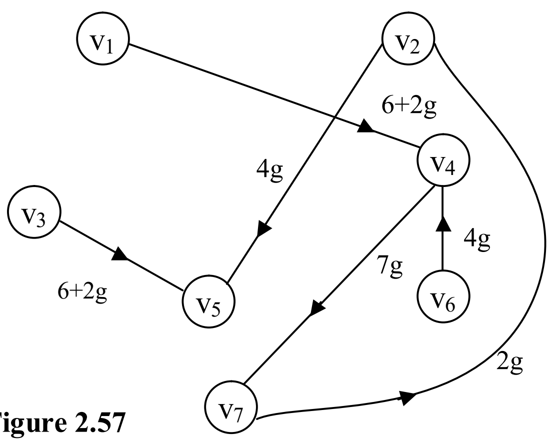


Figure 2.57

Next we find M^3 in the following

$$M^3 = \begin{array}{c c} & \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 2+6g & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4g \\ 0 & 0 & 0 & 0 & 0 & 0 & 6g \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2g & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

The type I MOD directed dual number graph G_3 is as follows.

G_3 =

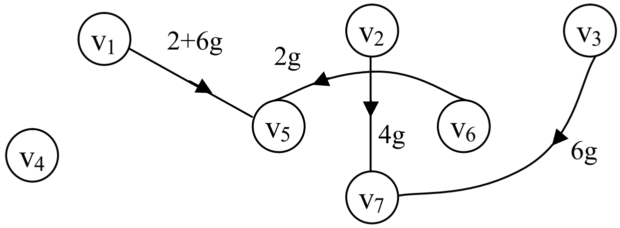


Figure 2.58

This type I MOD dual number directed graph G_3 has only four edges no loops.

$$M^4 = \begin{array}{c c} & \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 2g \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

The type I MOD dual number directed graph G_4 is as follows.

G_4 =

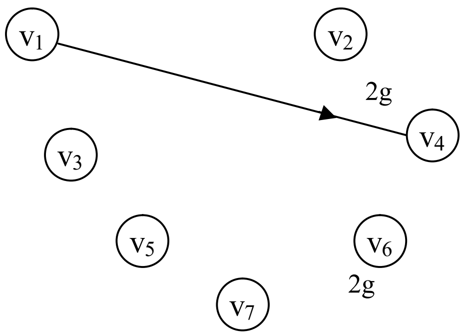


Figure 2.59

So at one stage we will have $M^n = (0)$ for some finite n, $(n > 0)$.

Interested reader can work more such type I MOD dual number directed graphs.

Now we proceed onto define type I MOD special dual like number directed graphs.

A type I MOD directed graph if it takes its edge weights from $\langle Z_n \cup h\rangle = \{a + bh / a, b \in Z_n, h^2 = h\}$ is defined as the type I MOD directed special dual like number graph.

We will illustrate this situation by some examples.

Example 2.27: Let G be the MOD type I special dual like number directed graph with edge weights from $\langle Z_{10} \cup h\rangle$.

The following figure for G is given below:

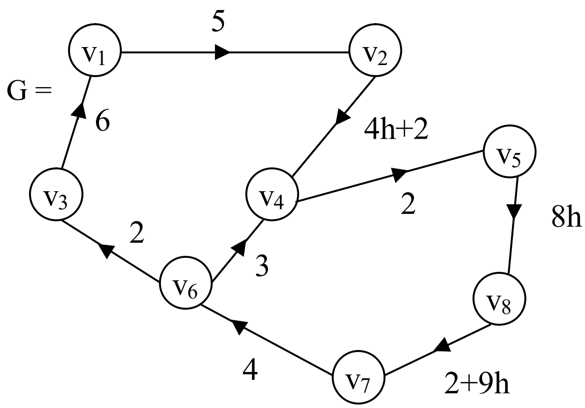


Figure 2.60

The associated type I MOD matrix of G is as follows:

$$N = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 \end{array} \\ \begin{bmatrix} 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4h+2 & 0 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8h \\ 0 & 0 & 2 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2+9h & 0 \end{bmatrix} \end{array}.$$

Now we proceed onto find $N^{2.}$ Let the corresponding type I MOD graph associated with N^2 be G_2.

$$N^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4+8h & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6h \\ 0 & 0 & 0 & 0 & 0 & 0 & 8h & 0 \\ 2 & 0 & 0 & 0 & 6 & 0 & 0 & 0 \\ 0 & 0 & 8 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8+6h & 0 & 0 \end{bmatrix} \end{array}.$$

The type I MOD special dual like number graph associated with N^2 is as follows.

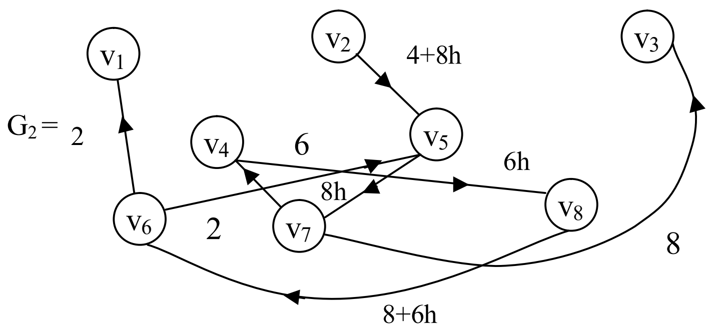


Figure 2.61

Now we find N^3 in the following:

$$N^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6h \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 6h & 0 \\ 0 & 0 & 0 & 0 & 0 & 2h & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8h \\ 8 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 6+2h & 4+8h & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

The type I MOD special dual like number directed graph G_3 associated with N^3 is as follows:

$G_3 =$

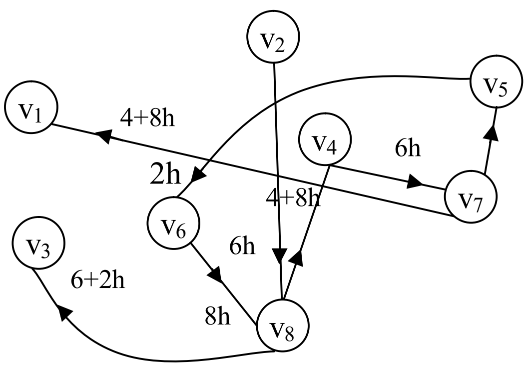


Figure 2.62

There is no loops only weighted edges.

Likewise we can find the type I MOD directed graph with edge weights from $\langle Z_{10} \cup h \rangle$.

We now give one example of the type I MOD directed graph G with edge weights from

$$\langle Z_n \cup k\rangle = \{a + bk \,/\, a, b \in Z_n, k^2 = (n-1)k\},$$
the graph G will also be known as the type I MOD special quasi dual number graph.

Example 2.28: Let G be the type I MOD directed special quasi dual number graph with edge weights from

$$\langle Z_9 \cup k\rangle = \{a + bk \,/\, a, b \in Z_9, k^2 = 8k\}.$$

The figure of G is as follows:

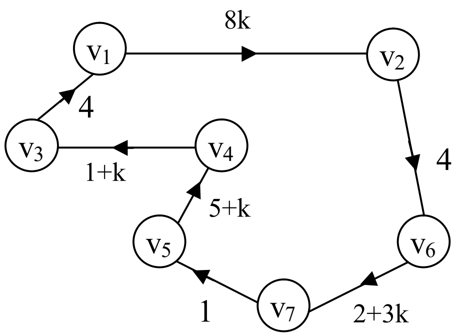


Figure 2.63

The type I MOD matrix of G is as follows:

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 8k & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1+k & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5+k & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2+3k \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}.$$

We give M^2 in the following:

$$M^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 5k & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 8+3k \\ 0 & 5k & 0 & 0 & 0 & 0 & 0 \\ 4+4k & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5+5k & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2+3k & 0 & 0 \\ 0 & 0 & 0 & 5+k & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

The type I MOD directed graph G_2 is as follows.

$G_2 =$

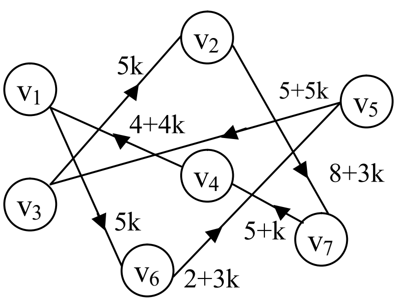


Figure 2.64

Now we find M^3 in the following:

$$M^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array}\begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 4k \\ 0 & 0 & 0 & 0 & 8+3k & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2k & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2+2k & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1+5k & 0 & 0 & 0 \\ 0 & 0 & 5+5k & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

The MOD type I directed graph G_3 is as follows:

$G_3 =$

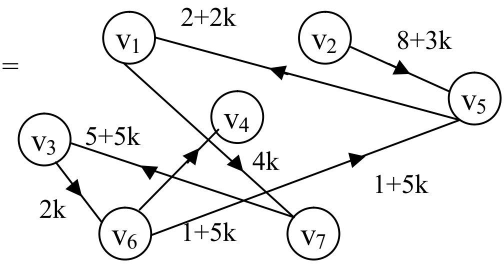


Figure 2.65

Now we find

$$M^4 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array}\begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 4k & 0 & 0 \\ 0 & 0 & 0 & 4+2k & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 7k \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1+k & 0 & 0 & 0 & 0 \\ 2+2k & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

The type I MOD special quasi dual number directed graph G_4 associated with M^4 is as follows.

G_4 =

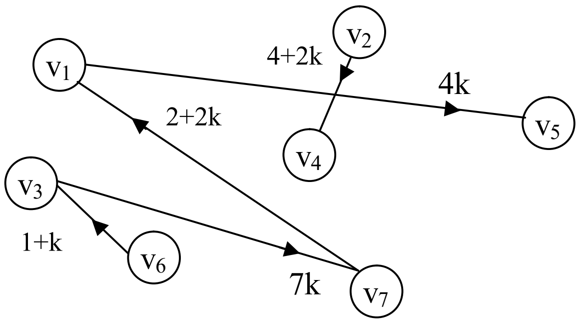


Figure 2.66

This is the way the product operation is performed using type I MOD special quasi dual number directed graphs with edge weights from

$$\langle Z_n \cup k\rangle = \{a + bk / k^2 = (n-1)k;\ a, b \in Z_n\}.$$

We now leave it for the reader to develop the properties of MOD directed graphs built using various sets like $C(Z_n)$ or $\langle Z_n \cup I\rangle$ or $\langle Z_n \cup g\rangle$ or $\langle Z_n \cup h\rangle$ or $\langle Z_n \cup k\rangle$ and analyse the special feature associated with them.

We suggest the following problems for the interested reader.

Problems

1. Let G be the MOD graph with entries from Z_7.

    i) How many such MOD graphs can be got using Z_7?
    ii) Find the number of MOD graphs using Z_n, $(2 \leq n < \infty)$.
    iii) What are the special features enjoyed by these MOD graphs?

2. Let G be the MOD directed graph given by the following figure with vertex the following figure with vertex set from Z_7.

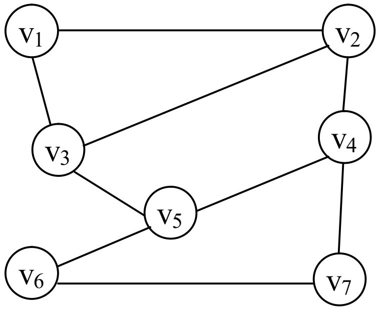
G = 

Figure 2.67

i) Find all MOD graphs isomorphic with G.
ii) All MOD graphs with seven vertices not isomorphic with G.
iii) Find all MOD graphs (distinct) with seven vertices.
iv) How many MOD graphs with six vertices from Z_7 can be constructed?
v) Study question (iv) for 5 and 4 vertices.
vi) Find the number of MOD graphs with three vertices from Z_7.

3. Study any other distinct feature associated with MOD graphs.

4. Let G be the MOD neutrosophic graph with edge set from $\langle Z_n \cup I\rangle$.

i) Show all MOD graphs are included in the MOD neutrosophic graphs.
ii) Find the number of distinct MOD neutrosophic graphs with $|\langle Z_n \cup I\rangle|$ number of vertices.

iii) Find the number of MOD neutrosophic graphs with 5 vertices.
iv) Enumerate all special features associated with MOD neutrosophic graphs.

5. What are the special and distinct features enjoyed by MOD finite complex number graphs with vertex set from $C(Z_n)$?

6. Show certainly these can find many application as we tred over finite number of vertices.

7. Study MOD dual number graphs with vertex set from $\langle Z_n \cup g \rangle$.

   Show this will have lot of application when one works with dual number as vertices.

8. Let {G} be the collection of all MOD special dual like number graph with vertex set from $\langle Z_6 \cup h \rangle$ or subsets of $\langle Z_6 \cup h \rangle$.

   i) How many graphs exist in {G}?
   ii) Does these graphs enjoy any special property?
   iii) How many of these MOD special dual like number graphs with vertex set from $\langle Z_6 \cup h \rangle$ are complete graphs?

9. Let B = {collection of all MOD special quasi dual number vertex set graphs with vertex sety from $\langle Z_{10} \cup k \rangle$ or subset of $\{\langle Z_{10} \cup k \rangle\}$.

   i) Find o(B).
   ii) How many are complete MOD graphs?
   iii) Compare the collection when $\langle Z_{10} \cup k \rangle$ is replaced by

      a) Z_{10},
      b) $\langle Z_{10} \cup g \rangle$,

c) $\langle Z_{10} \cup I\rangle$ and
d) $C(Z_{10})$.

10. What are the special features associated with type I MOD directed graph?

11. Let G = {collection of all type I MOD directed graphs with edge weights from subsets of Z_9 or Z_n}.

    i) Find o(G).
    ii) Hence find o(G) if Z_9 is replaced by Z_n, $2 \leq n < \infty$.
    iii) If G is given by the following figure:

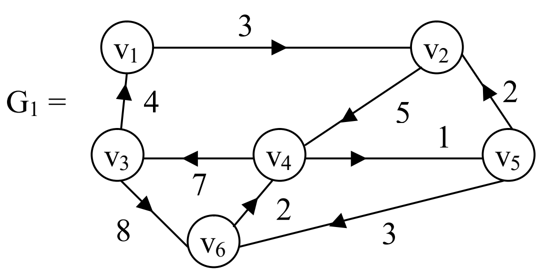


Figure 2.68

    a) Find the type I MOD connection matrix M_1 associated with G_1.
    b) Find M_1^2, M_1^3, M_1^4, …., M_1^t; $2 \leq t < \infty$ and the corresponding type I MOD graphs.
    c) Can we say $M^n = (0)$ or $M^n = M^t$ after a finite number of products t = 1 or 2 or 3?

12. Distinguish between type I MOD directed graphs and MOD graphs.

13. Let {G} be the collection of all type I MOD directed neutrosophic graphs with edge weights from $\langle Z_8 \cup I\rangle$.

a) Find o({G}).
b) Let H ∈ {G} given by the following figure:

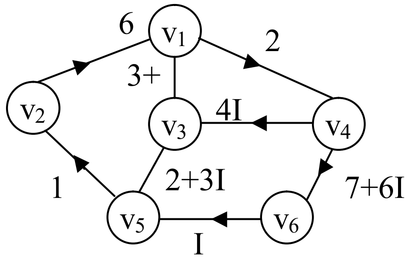


Figure 2.69

If M is the type I MOD matrix find M^2, M^3 and so on and the related H_2, H_3 and so on.

c) What type of M^t; $2 \leq t < \infty$ have loops?
d) When will $M^s = (0)$; $2 \leq s < \infty$?
e) Find $G_1 \in \{G\}$ which has 10 vertices taking edge weights from $\langle Z_8 \cup I\rangle$.

14. Let {G} be the collection of all type I MOD finite complex number directed graph with edge weights from $C(Z_{12})$.

a) Find o({G}).
b) If H ∈ {G} is given by the following figure:

G =

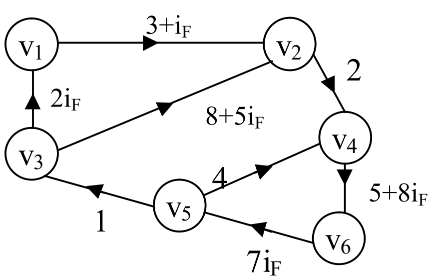


Figure 2.70

If M is the MOD associated matrix find the type I MOD graphs associated with M^2, M^3 and so on.

c) Can we say $M^t = M$ or $M^s = (0)$?

15. Let {H} be the collection of all type I MOD finite dual number directed graph with edge weights from $\langle Z_{11} \cup g\rangle$.
    i) Find o({H}).
    ii) If H_1 be a MOD finite dual number directed graph of type I by the following figure:

H_1 =

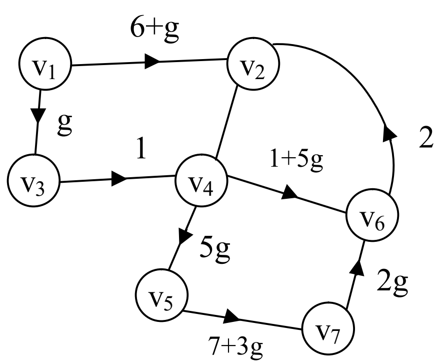


Figure 2.71

If M is the MOD matrix dual numbers associated with H_1 find M^2, M^3 and so on and obtain the corresponding MOD type I graphs.

iii) Which of the MOD type I graphs are free from loop?

iv) Enumerate all type I MOD directed graphs which has loops.

16. Let {P} be the collection of all MOD type I directed special dual like number graph with edge weights from

$$\langle Z_{21} \cup h\rangle = \{a + bh \,/\, a, b \in Z_{21}, h^2 = h\}.$$

a) Find o({P}).

b) If G_1 be a graph in {P} with 7 vertices how many type I MOD directed special dual like number graphs can be obtained.

c) How many G_i's are there?

d) Let H =

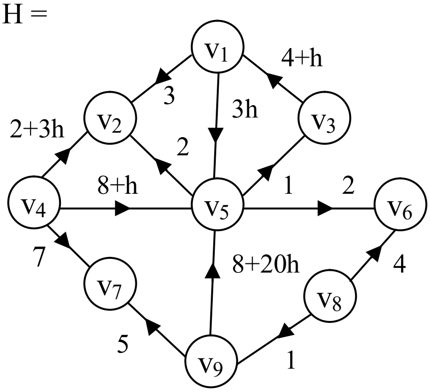


Figure 2.72

be the type I MOD directed special dual like number graph. M the associated type I MOD matrix.

Find M^2, M^3, … the corresponding MOD directed special dual like number graphs, which of them have loop?

17. Let {G} be the collection of type I MOD special quasi dual number directed graphs with edge weights from

$$\langle Z_{14} \cup k\rangle = \{a + bk / k^2 = 13k, a, b \in Z_{14}\}.$$

i) Find o({G}).
ii) Let V be the type I special quasi dual number MOD directed graph given by the following figure:

$V =$

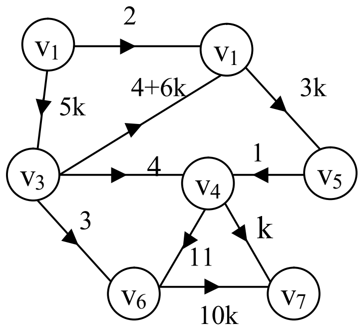


Figure 2.73

Find the type I MOD directed special quasi dual number MOD matrix M of V.

iii) Find M^2, M^3 and M^7 and their respective MOD type I directed special quasi dual number graphs V_2, V_3 and V_7.

iv) Which of these graphs have loops?

v) Describe any other special feature associated with these type I MOD directed special quasi dual number graphs.

vi) Compare this with type I MOD directed graphs, type I MOD directed dual number graph and type I finite complex number graph.

18. Describe and develop type II MOD directed graphs.

19. Let {G} be the collection of all MOD dual number directed graphs of type II with edge weights from $\langle Z_{18} \cup g\rangle$.

i) Find o({G}).

ii) How many of the type II MOD directed dual number graphs will have loops?

iii) Enumerate all special features enjoyed by type II MOD directed dual number graphs.

iv) Compare type I MOD directed dual number graph with type II MOD directed dual number graphs.

20. Study questions (i) to (iv) of problem 19 in case of type II MOD directed neutrosophic graphs with edge weights from $\langle Z_{29} \cup I \rangle$.

21. Study questions (i) to (iv) of problem 19 in case of type II MOD directed finite complex number graphs with edge weights from $C(Z_{23})$.

22. Describe all special features associated with type III MOD directed graphs using Z_n or $\langle Z_n \cup I \rangle$ or $\langle Z_n \cup g \rangle$ or $\langle Z_n \cup h \rangle$ or $\langle Z_n \cup k \rangle$ or $C(Z_n)$.

23. Distinguish MOD type III directed graphs from MOD type II directed graphs and MOD type I graphs.

24. Let G be the type III MOD directed graph G with vertex set from Z_6 and edge sets from {0, 1} given by the following figure.

G =

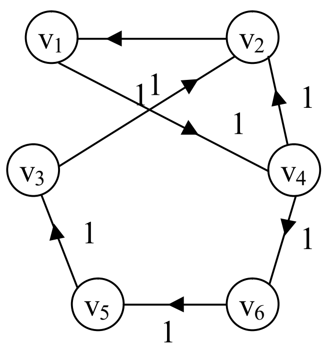


Figure 2.74

i) Find the type III MOD matrix M associated with G.
ii) Find M^2 and the related graph G_2. Does G_2 have loops?
iii) Can we say there exist a n such that $M^n = M$?
iv) Is it possible $M^t = (0)$ for $2 \leq t < \infty$?
v) Which is true in this case (iii) or (iv)?

25. Let G be the type II MOD dual number directed graph given by the following figure with edge weights from $\langle Z_{16} \cup g \rangle$.

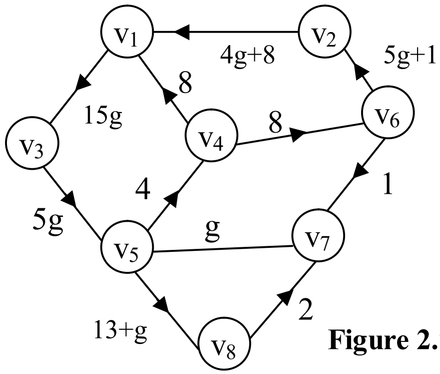


Figure 2.75

i) Find M related with G.
ii) Find M^3, M^4, M^7 and M^9 and the related graphs G_3, G_4, G_7 and G_9 respectively.
iii) Can M^t $(2 \leq t < \infty)$ have loops?
iv) What is the smallest t so that M^t has loops?
v) Can $M^t = (0)$ for some t, $2 \leq t < \infty$?
vi) Can $M^t = M$ for some t, $2 \leq t < \infty$?
vii) Enumerate any other special and interesting feature enjoyed by this type II MOD directed dual number graph G.

26. Let G be the type II MOD directed finite complex number graph given by the following figure with edge weights from $C(Z_6)$.

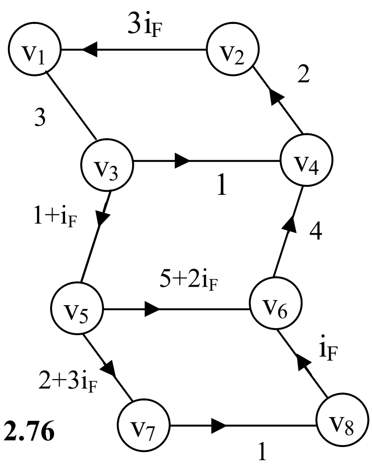


Figure 2.76

i) Find the MOD type II finite complex number matrix M associated with G.
ii) Find M^3, M^6, M^9 and M^{12} and the related MOD type II directed finite complex number graphs.
iii) Which M^t has loops?
iv) Find the smallest t so that G_t has loops.
v) Will odd order M^{2n+1} or even order M^{2n} contribute to MOD type II graphs with loops?
vi) Will $M^t = (0)$ or $M^t = M$?

27. Let V be the type II MOD directed graph with edge weights from Z_{12} given by the following figure:

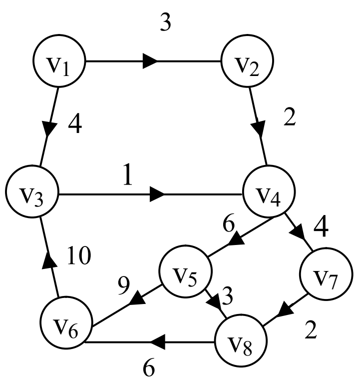


Figure 2.77

i) Find M the type II MOD matrix associated with V.
ii) Find M^2, M^4, M^8, M^{16} and M^{32} and the corresponding V_2, V_4, V_8, V_{16} and V_{32} respectively.
iii) For what power of M the relation type II MOD directed graph has loops?
iv) Can $M^t = (0)$?
v) Can $M^t = M$ or $M^s = (0)$, $(2 \leq t, s < \infty)$?

Chapter Three

MOD NATURAL NEUTROSOPHIC GRAPHS AND THEIR PROPERTIES

In this chapter for the first time we introduce the notion of MOD natural neutrosophic graphs in a systematic way.

However in [68] we have used this concept in the MOD natural neutrosophic Cognitive Maps model.

Further in this book we use zero dominant MOD natural neutrosophic product that is 0. $I_m^t = 0$; t = n or g or h or c or I or k, $m \in Z_n$ is a zero divisor or nilpotent or an idempotent.

We will proceed onto describe this notion first by examples.

Example 3.1: Let G be the MOD natural neutrosophic graph with vertex set from subsets of Z_6^I or whole of Z_6^I by the following figures.

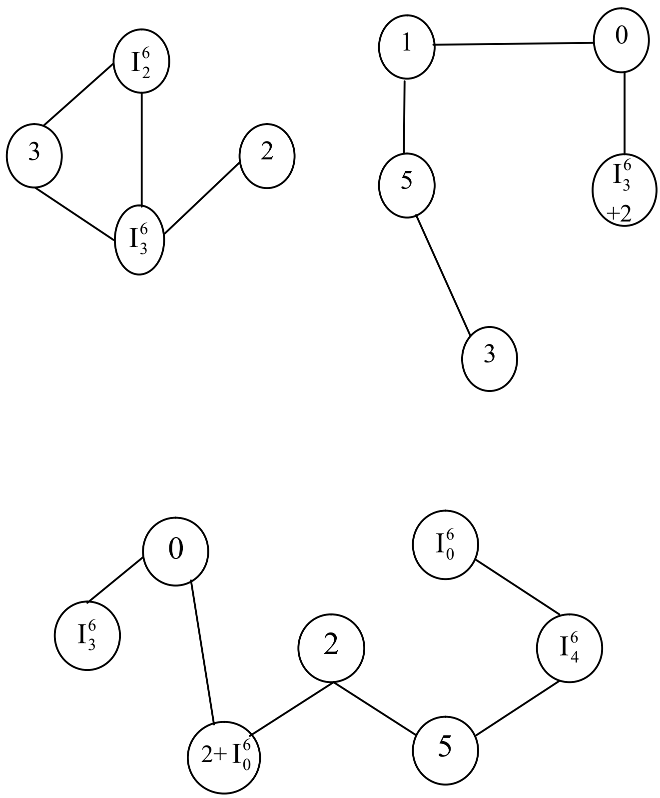


Figure 3.1

There are several MOD natural neutrosophic graphs using Z_6^I.

Infact finding the number of MOD natural neutrosophic graphs with vertex set Z_6^I happens to be challenging problem.

Conjecture 3.1: Let be the MOD natural neutrosophic set.

Finding the total number of MOD natural neutrosophic graphs happens to be a challenging one.

Example 3.2: Let Z_4^I be the MOD natural neutrosophic set.

$G_1 =$

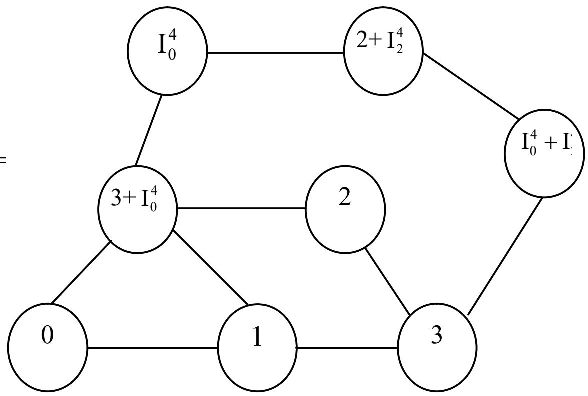


Figure 3.2

The MOD natural neutrosophic graph with vertex set from Z_4^I is given in Figure 3.2

$G_2 =$

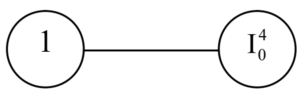


Figure 3.3

is a MOD natural neutrosophic graph with two vertices.

$G_3 =$

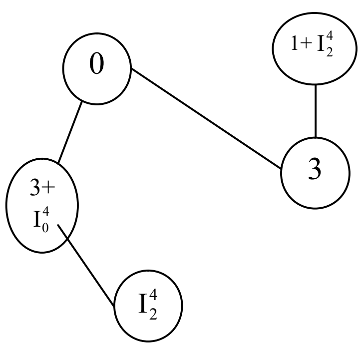


Figure 3.4

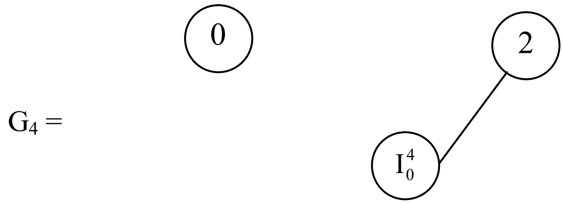


Figure 3.5

G_1, G_2, G_3 and G_4 are the MOD natural neutrosophic graphs with vertex set from Z_4^I.

Example 3.3: Let G be the MOD natural neutrosophic graph with entries from Z_{10}^I given by the following figures:

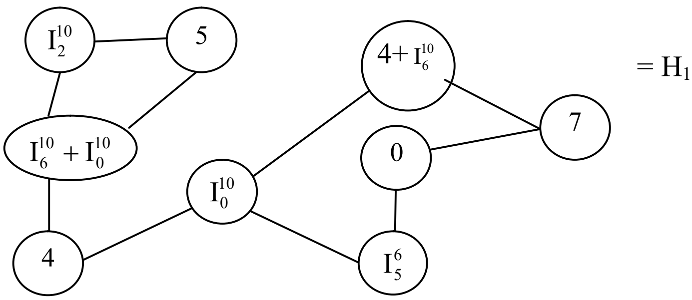


Figure 3.6

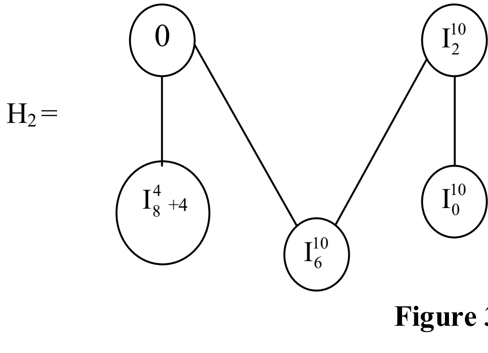


Figure 3.7

Next we can construct MOD natural neutrosophic finite complex number graph with vertex set from $C^I(Z_n)$.

This will be described by the following examples.

Example 3.4: Let G_1 be the MOD natural neutrosophic finite complex numbers with vertex set from $C^I(Z_{12})$.

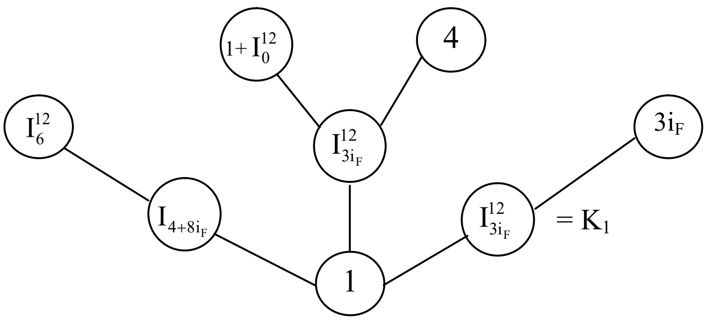


Figure 3.8

$K_2 =$

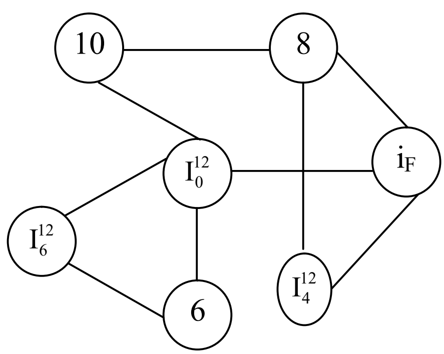


Figure 3.9

K_1 and K_2 are MOD natural neutrosophic finite complex number graphs with vertex set from $C^I(Z_n)$.

Example 3.5: Let G be the MOD natural neutrosophic finite complex number graph with vertex set from $C^I(Z_5)$.

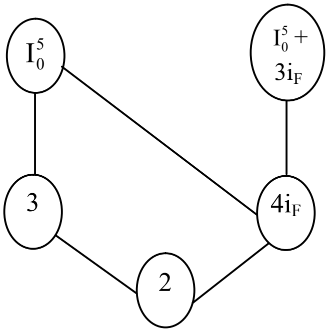


Figure 3.10

Next we just give an example or two of MOD natural neutrosophic dual number graphs in the following.

A MOD graph which takes its vertex set from the MOD natural neutrosophic dual number set
$\langle Z_n \cup g\rangle = \{a + bg / a, b \in Z_n, g^2 = 0\}$ will be known as the MOD natural neutrosophic dual number graph.

Example 3.6: Let G be the MOD natural neutrosophic dual number graph with vertex set from set $\langle Z_9 \cup g\rangle$. G is given by the following figure:

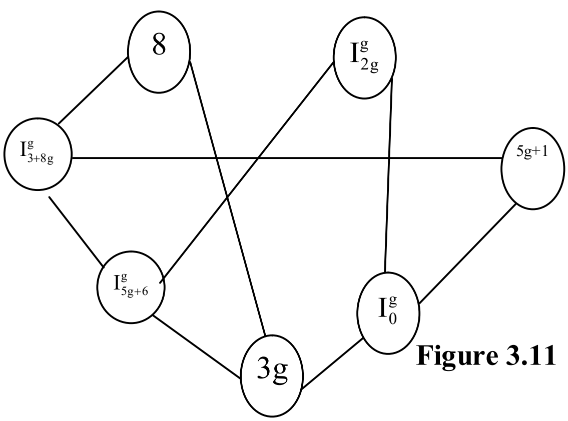


Figure 3.11

We next give one more example of MOD natural neutrosophic dual number graphs.

Example 3.7: Let G be the MOD natural neutrosophic dual number graph with vertex set from $\langle Z_3 \cup g\rangle$ given by the following figure:

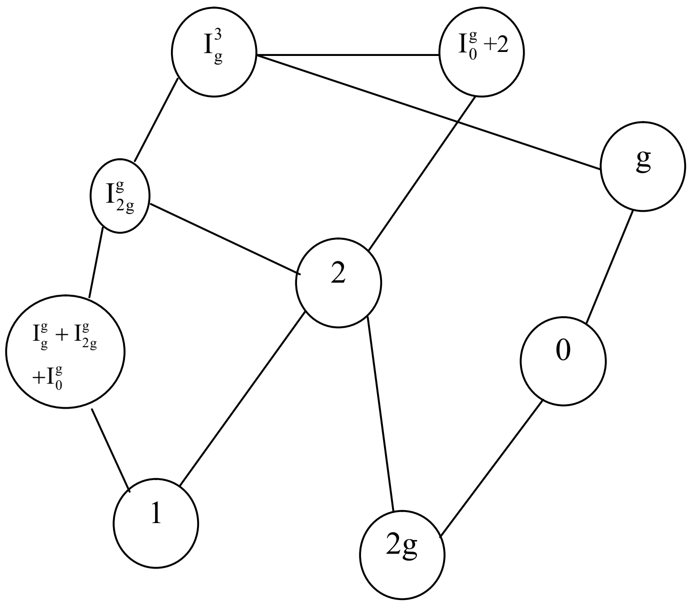


Figure 3.12

When we need labeling differently these MOD graphs will play a vital role.

For the labeled graphs can get the labeling from Z_n^I or $C^I(Z_n)$ or $\langle Z_n \cup g\rangle$ or $\langle Z_n \cup I\rangle$ or $\langle Z_3 \cup k\rangle$ or $\langle Z_3 \cup h\rangle$.

We can also obtain the adjacency matrix of a labeled graph. Thus both MOD graphs and MOD natural neutrosophic graphs can take the vertex values or distinctly labeled as per need.

Vertex labeling as usual realized as a function from vertex set to Z_n or $\langle Z_n \cup g\rangle$ or $\langle Z_n \cup h\rangle$ or $C(Z_n)$ or $\langle Z_n \cup I\rangle$ or $\langle Z_n \cup k\rangle$ or Z_n^I or $\langle Z_n \cup I\rangle_I$ or $\langle Z_n \cup g\rangle_I$ or $\langle Z_n \cup h\rangle_I$ or $C^I(Z_n)$.

So these vertex labeled graphs will also be known as MOD graphs MOD dual number graphs so on and MOD natural neutrosophic graphs or MOD natural neutrosophic dual number graphs and so on as the vertex set is from these sets.

All properties associated with vertex labeled graphs can be also developed for these all types of MOD graphs.

Now we will first given an example of a adjacency matrix and describe the MOD natural neutrosophic special dual like number graphs by a few examples.

Example 3.8: Let G be the MOD natural neutrosophic number graph with vertex set from $\langle Z_{12}^I \rangle$ given by the following figure

G =

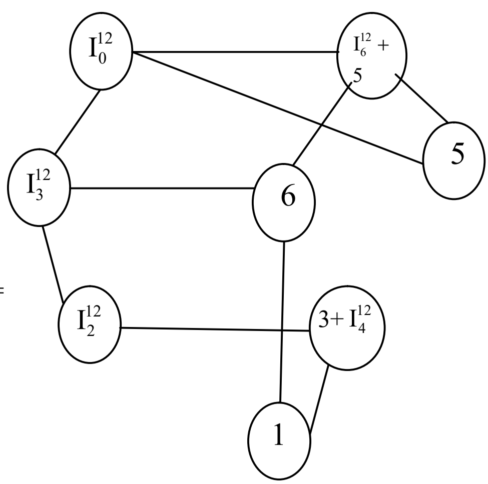


Figure 3.13

The MOD adjacency matrix associated with G is as follows:

$$\begin{array}{c} \\ 1 \\ 5 \\ 6 \\ I_0^{12} \\ I_2^{12} \\ I_3^{12} \\ 3+I_4^{12} \\ 5+I_6^{12} \end{array}\begin{array}{c} \begin{array}{cccccccc} 1 & 5 & 6 & I_0^{12} & I_2^{12} & I_3^{12} & I_4^{12}+3 & I_6^{12}+5 \end{array} \\ \left[\begin{array}{cccccccc} 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}.$$

Thus the MOD natural neutrosophic graph which takes its vertex set from $\langle Z_n \cup h\rangle_I$ will be defined as MOD natural neutrosophic special dual like number graph.

Example 3.9: Let H be the MOD natural neutrosophic special dual like number graph with vertex set from $\langle Z_{11} \cup h\rangle_I$ given by the following figure:

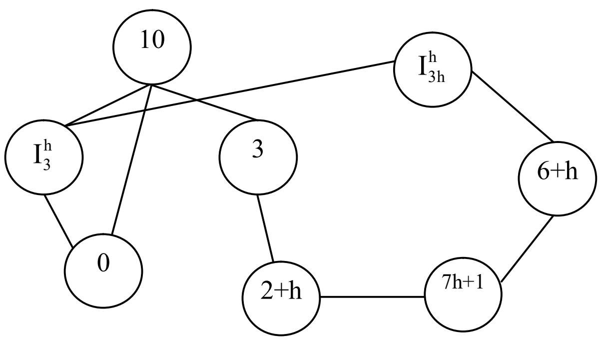


Figure 3.14

The MOD adjacency special dual like number matrix M is follows:

$$M = \begin{array}{c} \\ 0 \\ 3 \\ 10 \\ 2+h \\ 7h+1 \\ 6+h \\ I_3^h \\ I_{3h}^h \end{array} \begin{array}{c} \begin{array}{cccccccc} 0 & 3 & 10 & h+2 & 7h+1 & 6+h & I_3^h & I_{3h}^h \end{array} \\ \left[\begin{array}{cccccccc} 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{array}\right] \end{array}.$$

Interested reader can work more with such MOD natural neutrosophic special dual like number graphs which are nothing but a special type of labeled graphs with vertex set from $\langle Z_{12} \cup h\rangle_I$. That is there is a function from the vertex set to the set $\langle Z_n \cup h\rangle_I$.

The reader is expected to work with more examples and derive all properties associated with labeled graphs.

Next we proceed onto define and describe the new notion of MOD natural neutrosophic graphs of type I in the following.

Let G be the MOD natural neutrosophic directed type I graph with edge weights from Z_n^I and vertex set can be anything.

We give some examples of them.

Example 3.10: Let G be the MOD natural neutrosophic directed type I graph with edge weights from Z_{10}^I given by the following figure:

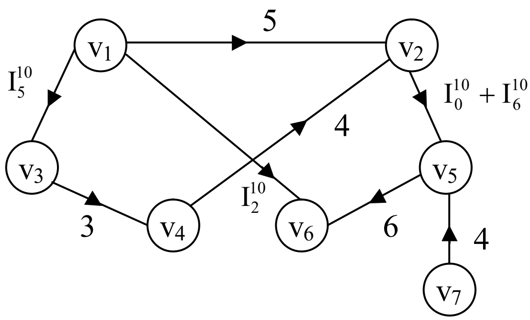


Figure 3.15

Let M be the MOD type I connection matrix of G which is given in the following.

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 5 & I_5^{10} & 0 & 0 & I_2^{10} & 0 \\ 0 & 0 & 0 & 0 & I_0^{10}+I_6^{10} & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 \end{bmatrix} \end{array}.$$

We can find M^2 and the corresponding type I MOD directed graph G_2 and so on for M^3 and M^4 in the following:

$$M^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array}\begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \left[\begin{array}{ccccccc} 0 & 0 & 0 & I_5^{10} & I_0^{10}+I_6^{10} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_0^{10}+I_6^{10} & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_0^{10}+I_6^{10} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 \end{array}\right] \end{array}$$

The MOD directed graph G_2 of natural neutrosophic type I is as follows.

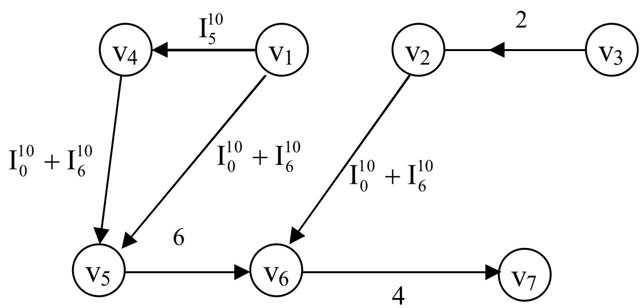


Figure 3.16

We now find M^3 in the following.

$$M^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & I_5^{10} & 0 & 0 & 0 & I_0^{10}+I_6^{10} & 0 \\ 0 & 0 & 0 & 0 & 0 & I_0^{10}+I_6^{10} & 0 \\ 0 & 0 & 0 & 0 & I_0^{10}+I_6^{10} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_0^{10}+I_6^{10} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 \end{bmatrix} \end{array}$$

Let G_3 be the type I MOD natural neutrosophic directed graph.

$G_3 =$

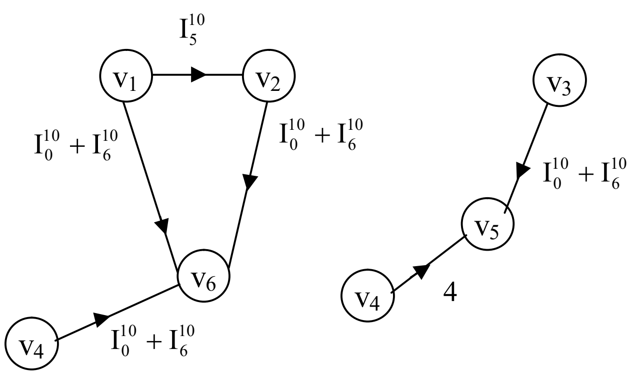


Figure 3.17

Now we find M^4 in the following.

$$M^4 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & I_0^{10} & I_0^{10}+I_6^{10} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_0^{10}+I_6^{10} & 0 \\ 0 & 0 & 0 & 0 & 0 & I_0^{10}+I_6^{10} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Let G_4 be the type I MOD natural neutrosophic directed graph.

$G_4 =$

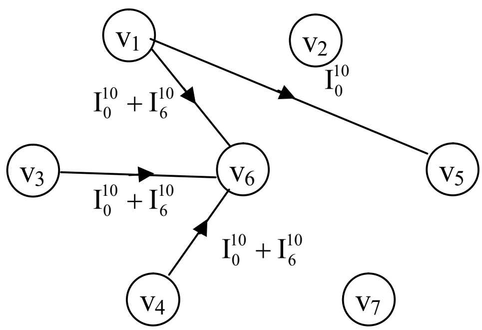


Figure 3.18

We next find M^6 in the following.

$$M^6 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & I_0^{10} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Thus the MOD natural neutrosophic directed graph of type I G_6 is as follows.

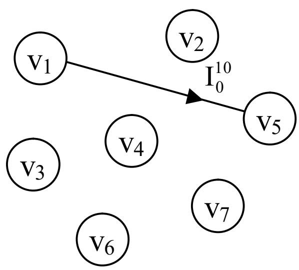
$G_6 =$ 

Figure 3.19

Now we find M^5 in the following:

$$M^5 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & I_0^{10} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_0^{10}+I_6^{10} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

The type I MOD natural neutrosophic directed graph G_5 is as follows.

$G_5 =$

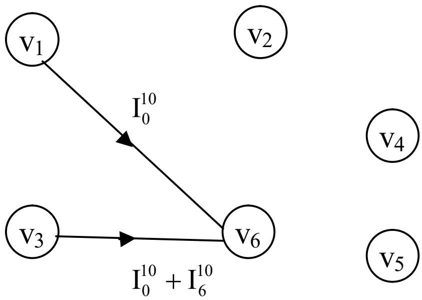


Figure 3.20

The reader is left with the task of find the n such that $M^n = (0)$.

Example 3.11: Let G be the type I MOD natural neutrosophic finite complex number directed graph with edge weights from $C(Z_6)$ in the following figure

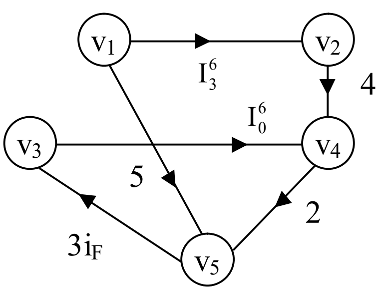


Figure 3.21

Let M be the MOD type I finite complex matrix is given in the following:

$$M = \begin{array}{c c} & \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} & \begin{bmatrix} 0 & I_3^6 & 0 & 0 & 5 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & I_0^6 & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 3i_F & 0 & 0 \end{bmatrix} \end{array}.$$

We first find M^2 in the following :

$$M^2 = \begin{array}{c c} & \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} & \begin{bmatrix} 0 & 0 & 3i_F & I_3^6 & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & I_0^6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_0^6 & 0 \end{bmatrix} \end{array}.$$

The MOD type I directed graph G_2 associated with the MOD finite complex number matrix is as follows.

G_2 =

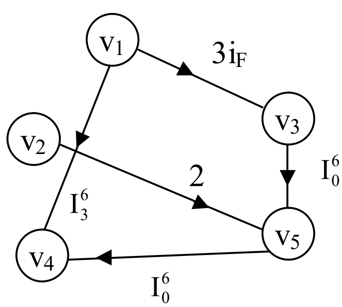


Figure 3.22

We find now M^3 in the following :

$$M^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & I_0^6 & I_3^6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_0^6 & 0 \\ 0 & 0 & I_0^6 & 0 & 0 \end{bmatrix} \end{array}.$$

The type I MOD directed graph G_3 associated with M^3 is as follows:

G_3 =

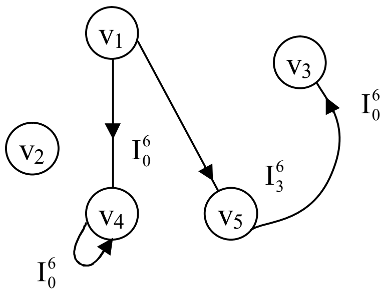


Figure 3.23

We find now M^4 in the following

$$M^4 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & I_0^6 \\ 0 & 0 & 0 & I_0^6 & 0 \\ 0 & 0 & 0 & I_0^6 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let G_4 be the MOD natural neutrosophic finite complex number directed graph is given in the following:

G_4 =

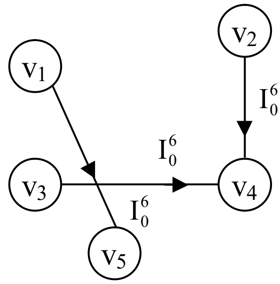


Figure 3.24

We find M^6 in the following

$$M^6 = \begin{array}{c c} & \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} & \begin{bmatrix} 0 & 0 & I_0^6 & I_0^6 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_0^6 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

This is the way MOD graphs associated with powers of M, M the adjacency matrix associated with G.

The MOD graph of natural neutrosophic elements of type I is as follows.

G_6 =

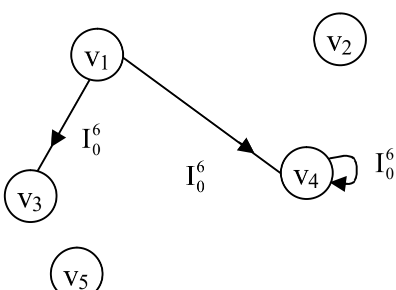


Figure 3.25

Next we proceed onto describe type I MOD natural neutrosophic-neutrosophic directed graphs.

Let G be a type I MOD natural neutrosophic directed graph with edge weights from $\langle Z_n \cup I\rangle_I$.

Then G is defined as n the type I MOD natural neutrosophic - neutrosophic directed graph.

We will give examples of them.

Example 3.12: Let G be a type I MOD natural neutrosophic-neutrosophic directed graph G with edge weights from $\langle Z_4 \cup I\rangle_I$, given by the following figure:

G =

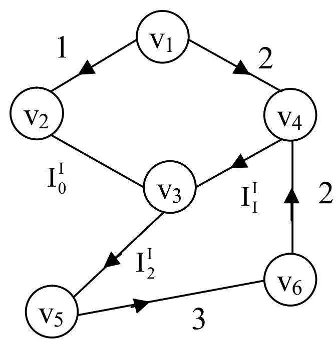


Figure 3.26

The MOD type I connection matrix M associated with is as follows:

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 2 & 0 & 0 \\ 0 & 0 & I_0^I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_2^I & 0 \\ 0 & 0 & I_I^I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 2 & 0 & 0 \end{bmatrix} \end{array}.$$

We now find M^2 in the following

$$M^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 0 & I_0^I + I_I^I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_0^I & 0 \\ 0 & 0 & 0 & 0 & 0 & I_2^I \\ 0 & 0 & 0 & 0 & I_{2I}^I & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & I_I^I & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let G_2 be the type I MOD natural neutrosophic-neutrosophic directed graph, which is given in the following:

$G_2 =$

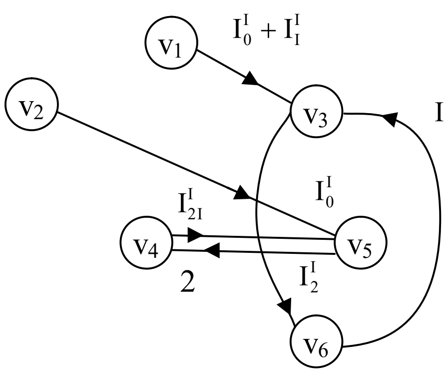


Figure 3.27

Next we find M^3 in the following.

$$M^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & I_0^I + I_{2I}^I & 0 \\ 0 & 0 & 0 & 0 & 0 & I_0^I \\ 0 & 0 & 0 & I_2^I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{2I}^I \\ 0 & 0 & I_I^I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{2I}^I & 0 \end{bmatrix} \end{array}.$$

The type I MOD natural neutrosophic-neutrosophic graph G_3 associated with M^3 is given by the following figure.

$G_3 =$

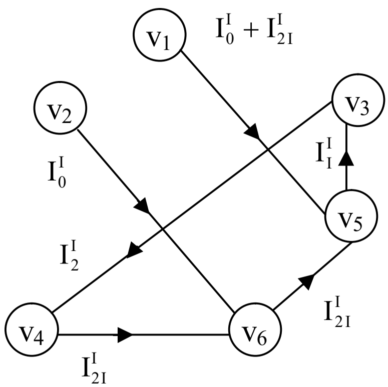


Figure 3.28

Interested reader can work with any M^t; $2 \leq t < \infty$ and get the corresponding type I MOD natural neutrosophic neutrosophic directed graph G_4.

We give get another example of this situation.

Example 3.13: Let G be the MOD natural neutrosophic neutrosophic directed graph of type I with edge weights from $\langle Z_{10} \cup I\rangle_I$ given by the following figure:

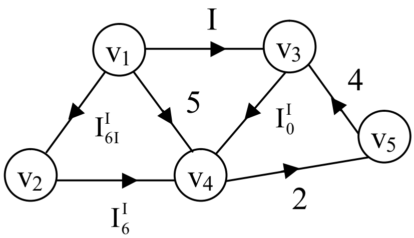


Figure 3.29

Let M be the MOD connection matrix associated with G given in the following.

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & I_{6I}^{I} & I & 5 & 0 \\ 0 & 0 & 0 & I_6^I & 0 \\ 0 & 0 & 0 & I_0^I & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 4 & 0 & 0 \end{bmatrix} \end{array}$$

Now we find M^2,

$$M^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & I_{6I}^{I} + I_0^I & 0 \\ 0 & 0 & 0 & 0 & I_6^I \\ 0 & 0 & 0 & 0 & I_0^I \\ 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & I_0^I & 0 \end{bmatrix} \end{array}.$$

Let G_2 be the MOD directed natural neutrosophic-neutrosophic type I graph associated with M^2.

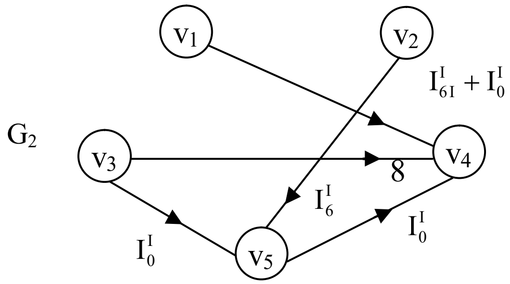


Figure 3.30

Now we find M^3 in the following

$$M^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & I_{6I}^{I} + I_0^I \\ 0 & 0 & I_6^I & 0 & 0 \\ 0 & 0 & I_0^I & 0 & 0 \\ 0 & 0 & 0 & I_0^I & 0 \\ 0 & 0 & 0 & 0 & I_0^I \end{bmatrix} \end{array}.$$

Now we give the type I MOD directed natural neutrosophic neutrosophic type I graph G_3 in the following.

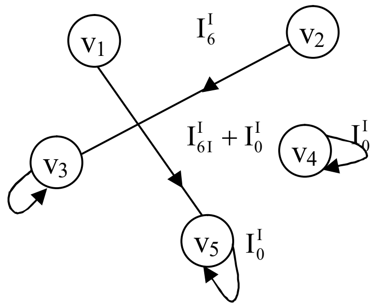
$G_3 =$ 

Figure 3.31

There three loops and two edges.

Next we find

$$M^4 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & I^I_{6I} + I^I_0 & 0 & 0 \\ 0 & 0 & 0 & I^I_0 & 0 \\ 0 & 0 & 0 & I^I_0 & 0 \\ 0 & 0 & 0 & 0 & I^I_0 \\ 0 & 0 & I^I_0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let G_4 be the type I MOD directed natural neutrosophic neutrosophic graph associated with M^4.

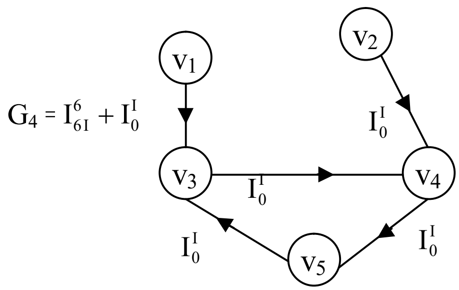


Figure 3.32

The graph G_3 has only 5 edges no loops.

We now proceed onto find M^5 and M^6 and the corresponding MOD natural neutrosophic-neutrosophic directed graphs G_5 and G_6 of type I respectively.

$$M^5 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & I^I_0 & I^I_0 \\ 0 & 0 & 0 & 0 & I^I_0 \\ 0 & 0 & 0 & 0 & I^I_0 \\ 0 & 0 & I^I_0 & 0 & 0 \\ 0 & 0 & 0 & I^I_0 & 0 \end{bmatrix} \end{array}.$$

Let G_5 be the type I MOD directed natural neutrosophic-neutrosophic graph given by the following figure:

$G_5 =$

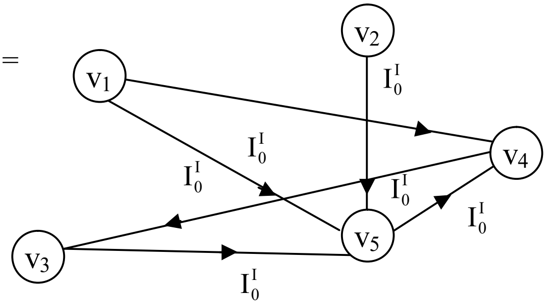


Figure 3.33

This graph has only give edges.

Now we proceed onto find M^6;

$$M^6 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & I_0^I \\ 0 & 0 & I_0^I & 0 & 0 \\ 0 & 0 & I_0^I & 0 & 0 \\ 0 & 0 & 0 & I_0^I & 0 \\ 0 & 0 & 0 & 0 & I_0^I \end{bmatrix} \end{array}.$$

Now let G_6 be the type I MOD directed natural neutrosophic neutrosophic graphs associated with M^6 given by the following figure.

$G_6 =$

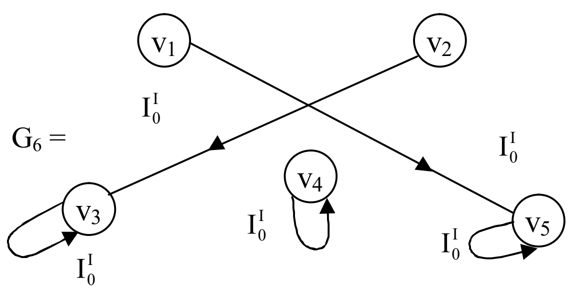


Figure 3.34

This is the same as graph G_3 with only a little variation in the edge weight v_1 to v_6.

Interested reader can work for the following result.

i) Can we find a^t and s so that $M^t = M^s$ $2 \leq t, s < \infty$?
ii) Can $M^n = (0)$; $2 \leq t, s < \infty$?
iii) Will even power of M or odd powers of M alone contribute for loops?
iv) Obtain any other special feature associated with M.

Next we proceed onto work with type I MOD natural neutrosophic directed dual number graphs?

Let G be a type I MOD natural neutrosophic directed graph with edge weights from $\langle Z_n \cup g\rangle_I$.

G will be defined as the type I MOD natural neutrosophic dual number directed graph. We will first illustrate this situation by some examples.

Example 3.14: Let H be the type I MOD directed natural neutrosophic dual number graph with edge weights from $\langle Z_{10} \cup g\rangle_I$ given by the following figure:

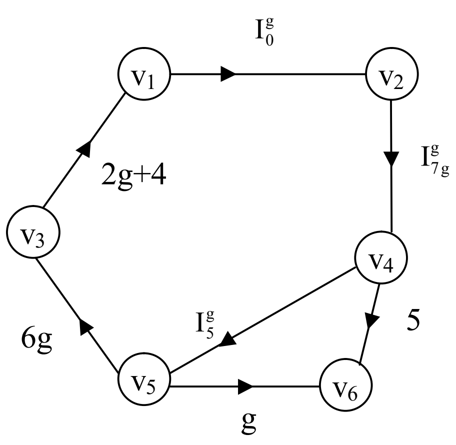


Figure 3.35

Let N be the MOD natural neutrosophic dual number matrix associated with the graph H

$$N = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & I_0^g & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{7g}^g & 0 & 0 \\ 2g+4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_5^g & 5 \\ 0 & 0 & 6g & 0 & 0 & g \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

We now find N^2;

$$N^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & I_0^g & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{5g}^g & I_{7g}^g \\ 0 & I_0^g & 0 & 0 & 0 & 0 \\ 0 & 0 & I_5^g & 0 & 0 & I_5^g \\ 4g & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Let H_2 be the type I MOD directed natural neutrosophic dual number graph associated with N^2

$H_2 =$

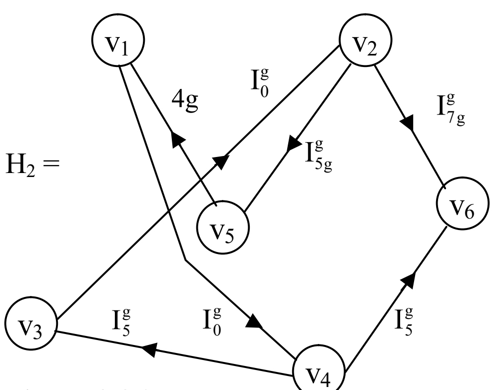


Figure 3.36

Now find N^3

$$N^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & I_0^g \\ 0 & 0 & I_{5g}^g & 0 & 0 & I_{5g}^g \\ 0 & 0 & 0 & I_0^g & 0 & 0 \\ I_5^g & 0 & 0 & 0 & 0 & 0 \\ 0 & I_0^g & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

The type I MOD directed natural neutrosophic dual number graph H_3 is as follows:

$H_3 =$

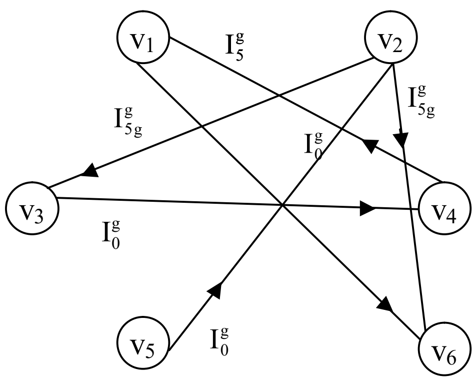


Figure 3.37

Now we find

$$N^4 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 0 & I_0^g & 0 & 0 & I_0^g \\ I_{5g}^g & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_0^g & I_0^g \\ 0 & I_0^g & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_0^g & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

The type I MOD directed natural neutrosophic dual number graph H_4 is as follows:

H_4 =

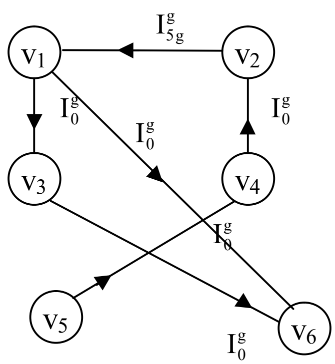


Figure 3.38

Next we proceed onto describe type I MOD directed natural neutrosophic special dual like number graphs.

Let G be a type I MOD directed natural neutrosophic graph with edge weights from $\langle Z_n \cup h\rangle_I$, then we define G to be a type I MOD natural neutrosophic special dual like number directed graph.

We will provide examples of them.

Example 3.15: Let G be the type I MOD directed special dual like number natural neutrosophic graph with edge weights from $\langle Z_7 \cup h\rangle_I$ given by the following figure:

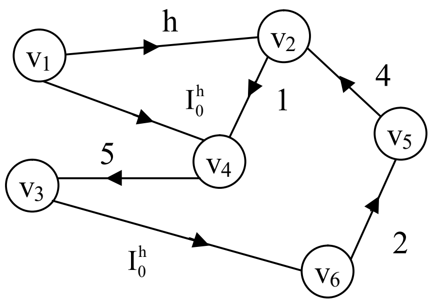


Figure 3.39

Let S be the MOD natural neutrosophic special dual like number type I matrix associated with the graph G.

$$S = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & h & 0 & I_0^h & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_0^h \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \end{bmatrix} \end{array}.$$

S^2 of the matrix S is as follows

$$S^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 0 & I_0^h & h & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_0^h & 0 \\ 0 & 0 & 0 & 0 & 0 & I_0^h \\ 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let G_2 be the type I MOD directed natural neutrosophic special dual like number graph associated with S^2.

G_2 =

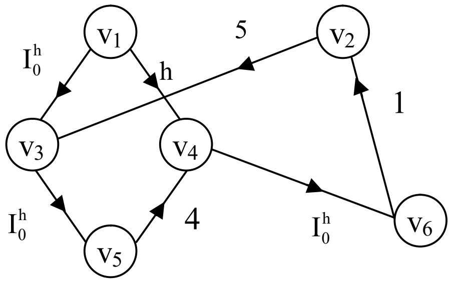


Figure 3.40

Now we find S^3 in the following

$$S^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 5h & 0 & 0 & I_0^h \\ 0 & 0 & 0 & 0 & 0 & I_0^h \\ 0 & I_0^h & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_0^h & 0 \\ 0 & 0 & 6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}.$$

The MOD type I directed natural neutrosophic dual like number graph G_3 associated with the matrix S^3 is as follows.

G_3 =

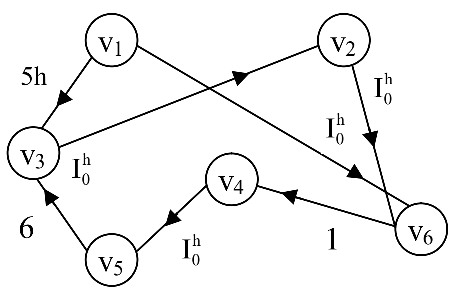


Figure 3.41

We just find S^5 in the following.

$$S^5 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & I_0^h & 0 & 0 & I_0^h & 0 \\ 0 & I_0^h & 0 & 0 & 0 & 0 \\ 0 & 0 & I_0^h & 0 & 0 & 0 \\ 0 & 0 & 0 & I_0^h & 0 & 0 \\ 0 & 0 & 0 & 0 & I_0^h & 0 \\ 0 & 0 & 0 & 0 & 0 & I_0^h \end{bmatrix} \end{array}.$$

The type I MOD graph G_5 associated with S^5 is as follows:

$G_5 =$

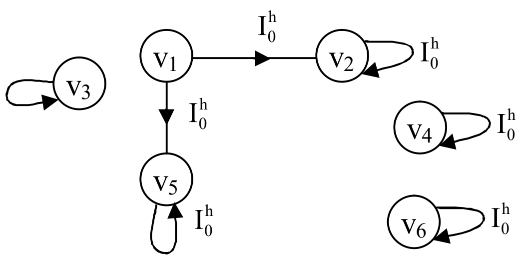


Figure 3.42

In similar ways one can find MOD type I graphs G_t using the matrix S^t; $2 \leq t < \infty$. Finding the number of loops and edges of these graphs happens to be an interesting research.

This is left as an exercise to the reader.

Problem 3.1: Can we have a MOD type I natural neutrosophic graph G which has M to be the related matrix such that G_t corresponding to $M^t = \underbrace{M \times M \times \ldots \times M}_{t-\text{times}}$ for all t; $2 \leq t < \infty$ has no loops?

Next we provide one more example.

Example 3.16: Let K be the type I MOD directed natural neutrosophic special dual like number graph with edge weights from $\langle Z_6 \cup h \rangle_I$ given by the following figure.

K =

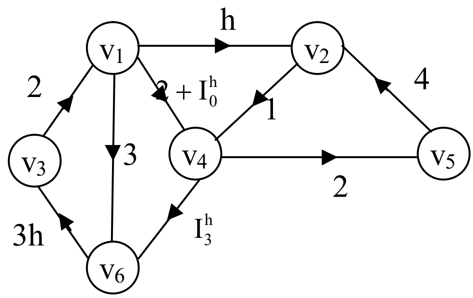


Figure 3.43

Let P be the MOD type I matrix associated with the type I MOD graph K.

$$P = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & h & 0 & 2+I_0^h & 0 & 3 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & I_3^h \\ 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3h & 0 & 0 & 0 \end{bmatrix} \end{array}$$

We find

$$P^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 3h & h & 4+I_0^h & I_3^h + I_0^h \\ 0 & 0 & 0 & 0 & 2 & I_3^h \\ 0 & 2h & 0 & 4+I_0^h & 0 & 0 \\ 0 & 2 & I_3^h & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let K_2 be the MOD directed special dual like number graph associated the matrix P^2.

$K_2 =$

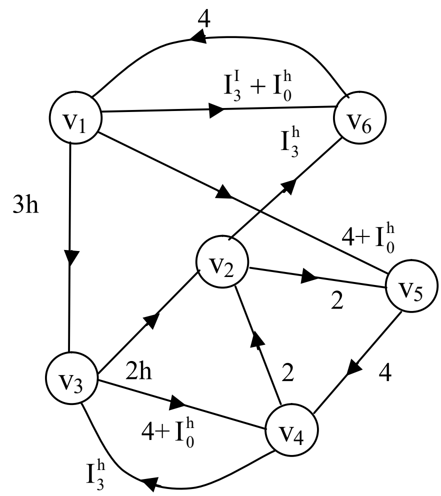


Figure 3.44

Now we see K has only 8 edges where as the type I MOD graph K_2 has 11 edges. We find P^3 in the following.

$$P^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 4+I_0^h & I_3^h+I_0^h & 0 & 2h & I_3^h \\ 0 & 2 & I_3^h & 0 & 0 & 0 \\ 0 & 0 & 0 & 2h & 4+I_0^h & I_3^h+I_0^h \\ I_3^h & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & I_3^h \\ 0 & 4h & 0 & 2+I_0^h & 0 & 0 \end{bmatrix} \end{array}.$$

The MOD type I graph K_3 associated with P^3 is as follows:

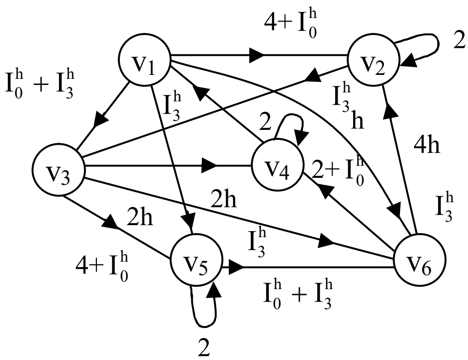
$K_3 =$ 

Figure 3.45

This MOD graph K_3 has more number of edges than K_2 and K_1. K_3 has two loops.

Just we find P^4

$$P^4 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} I_3^h + I_0^h & 2h & I_3^h & I_0^h + 4 & 0 & 0 \\ I_3^h & 0 & 0 & 2 & 0 & 0 \\ 0 & 2 + I_0^h & I_3^h + I_0^h & 0 & 4h & I_3^h \\ 0 & I_3^h & 0 & I_0^h + I_3^h & 4 & I_3^h \\ 0 & 2 & I_3^h & 0 & 0 & 0 \\ 0 & 0 & 0 & 4h & 4 + I_0^h & I_3^h + I_0^h \end{bmatrix} \end{array}.$$

The type I MOD graph K_4 associated with P^4 is as follows:

$K_4 =$

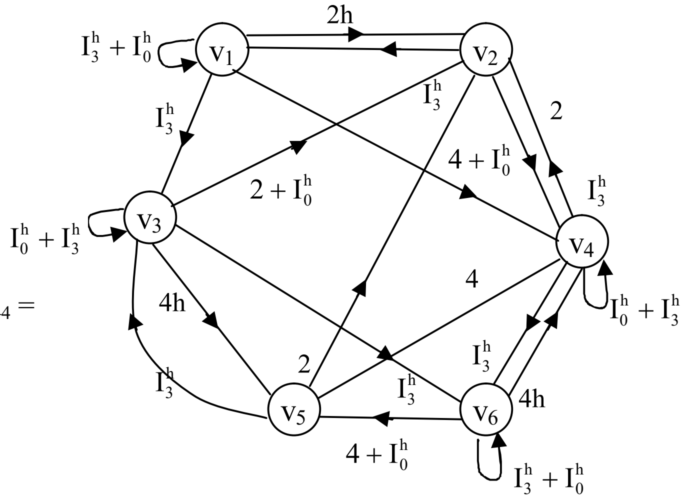


Figure 3.46

Clearly K_4 has more edges and more loops than K_3, K_2 and K.

So we leave it for the reader to find the maximum t so that K_t has maximum number of edges or loops.

We just for interest sake find.

$P^5 =$

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array}\begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} I_3^h + I_0^h & I_3^h + I_0^h & I_3^h + I_0^h & I_3^h + I_0^h & 2 + I_3^h + I_0^h & I_3^h \\ I_3^h + I_0^h & I_3^h & I_3^h & I_3^h + I_0^h & 2 & I_3^h \\ I_3^h + I_0^h & I_3^h + I_0^h + 2 & I_3^h + I_0^h & I_3^h + I_0^h & I_3^h + I_0^h & I_3^h + I_0^h \\ I_3^h & 2 + I_3^h + I_0^h & I_3^h & I_3^h + I_0^h & I_3^h + I_0^h & I_3^h + I_0^h \\ I_3^h & I_3^h + I_0^h & I_3^h + I_0^h & I_3^h + I_0^h & 2h + I_3^h & I_3^h + I_0^h \\ 0 & 2 + I_3^h + I_0^h & I_3^h + I_0^h & I_3^h + I_0^h & 2h + I_3^h + I_0^h & I_3^h + I_0^h \end{bmatrix} \end{array}$$

We see the number edges and loops have increased. Interested reader can draw the type I MOD natural neutrosophic directed special dual like number graph K_8.

Next we proceed onto describe and define type I MOD natural neutrosophic special quasi dual number graphs.

Let G be the type I MOD natural neutrosophic number directed graph with edge weights from $\langle Z_n \cup k\rangle_I$.

Then we define G to be the type I MOD natural neutrosophic special quasi dual number directed graph with edge weights from $\langle Z_n \cup k\rangle_I$.

We will illustrate this by some examples.

Example 3.17: Let G be the type I MOD natural neutrosophic special quasi dual number directed graph with edge weights from $\langle Z_{12} \cup k\rangle_I$ given by the following figure:

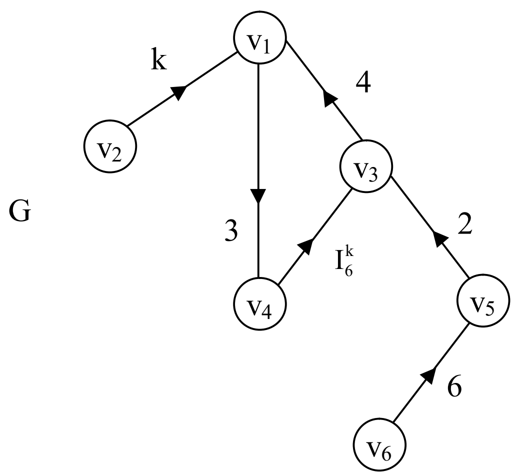


Figure 3.47

Let L be the type I MOD matrix associated with G.

$$L = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 3 & 0 & 0 \\ k & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_6^k & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 & 0 \end{bmatrix} \end{array}.$$

We find L,

$$L^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 0 & I_6^k & 0 & 0 & 0 \\ 0 & 0 & 0 & 3k & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ I_6^k & 0 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

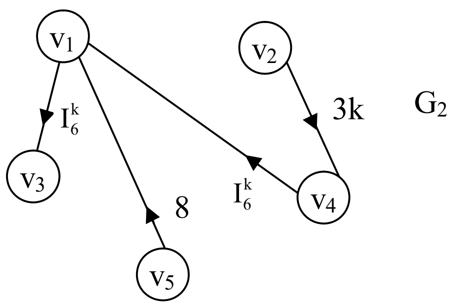


Figure 3.48

The MOD type I natural neutrosophic special quasi dual number directed graph associated with L^2.

The edges have reduced from 6 to four.

We now find L^3 and the corresponding MOD graph $G_{2.}$

$$L^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} I_6^k & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_6^k & 0 & 0 & 0 \\ 0 & 0 & I_6^k & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

The type I MOD graph G_3 associated with L^3 is as follows.

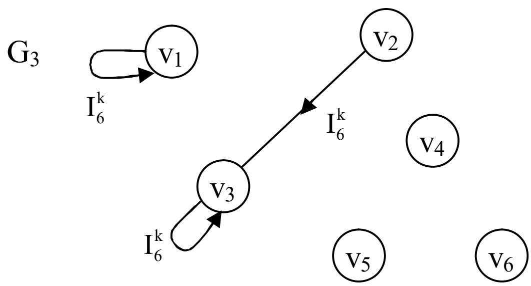


Figure 3.49

Next we find matrix L^4 in the following.

$$L^4 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ I_6^k & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_0^k & 0 & 0 & 0 \\ 0 & 0 & I_6^k & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let G_4 be the type I MOD natural neutrosophic directed graph G_4 of special quasi dual numbers.

G_4 =

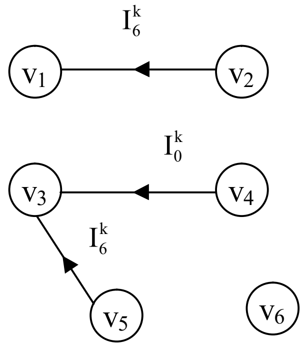


Figure 3.50

Next we find L^5 in the following.

$$L^5 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 0 & I_0^k & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ I_0^k & 0 & 0 & 0 & 0 & 0 \\ I_6^k & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

The type I MOD directed natural neutrosophic special quasi dual number graph G_5 associated with L^5.

G_5 =

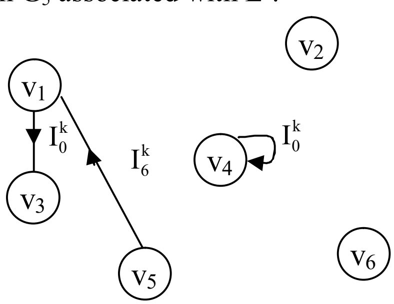


Figure 3.51

We give L^{10} in the following.

$$L^{10} = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array}\begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_0^k & 0 & 0 & 0 \\ 0 & 0 & I_0^k & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

The graph G_{10} associated with L^{10} is

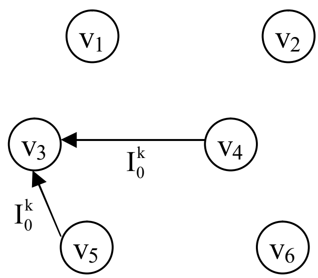
$G_{10} =$ 

Figure 3.52

Thus we see this graph G is such that G_t gives only a vertex set here for some t.

Next we proceed onto describe type II MOD directed natural neutrosophic graph with takes its edge weights and vertex set from Z_n^I or $\langle Z_n \cup I\rangle_I$ or $\langle Z_n \cup g\rangle_I$ or $\langle Z_n \cup h\rangle_I$ or $\langle Z_n \cup k\rangle_I$ or $C^I(Z_n)$.

We will illustrate all these situations by some examples.

Example 3.18: Let G be the type II directed MOD natural neutrosophic graph with edge weights from Z_9^I given by the following figure:

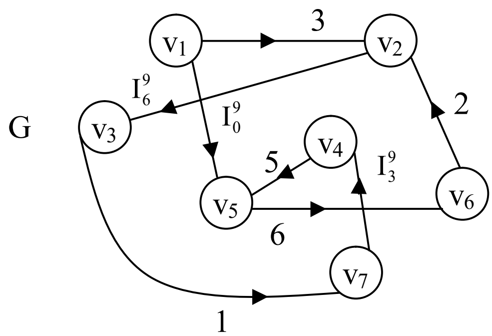


Figure 3.53

Let N be the MOD type II matrix associated with the type II MOD graph G.

$$N = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_6^9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_3^9 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

We find the N^2 in the following.

$$N^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 0 & I_6^9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I_6^9 \\ 0 & 0 & 0 & I_3^9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_6^9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_3^9 & 0 & 0 \end{bmatrix} \end{array}.$$

The related MOD directed graph associated with N^2 is as follows.

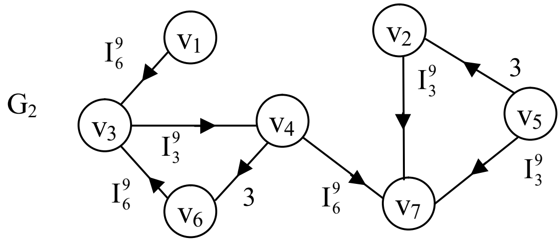


Figure 3.54

Now we found N^3 in the following

$$N^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & I_6^9 \\ 0 & 0 & 0 & I_0^9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_3^9 & 0 & 0 \\ 0 & 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_6^9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I_6^9 \\ 0 & 0 & 0 & 0 & 0 & I_3^9 & 0 \end{bmatrix} \end{array}.$$

The MOD directed graph associated with N^3 is as follows

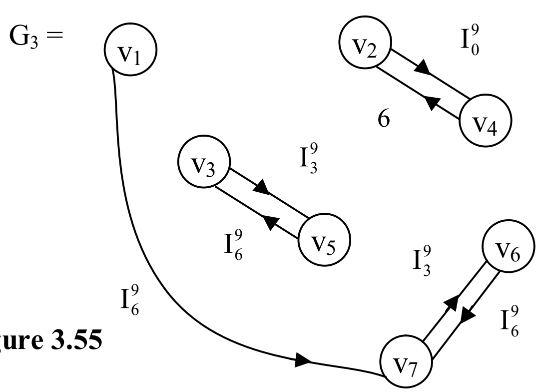


Figure 3.55

Now we find N^4 in the following.

$$N^4 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & I_0^9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_0^9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_3^9 & 0 \\ 0 & 0 & I_6^9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I_6^9 \\ 0 & 0 & 0 & I_0^9 & 0 & 0 & 0 \\ 0 & I_3^9 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let G_4 be the type II MOD directed natural neutrosophic graph in the following figure:

$G_4 =$

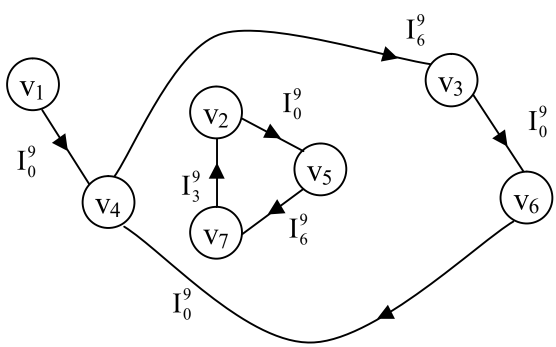


Figure 3.56

Now we find N^5 in the following :

$$N^5 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & I_0^9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_0^9 & 0 \\ 0 & I_3^9 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I_6^9 \\ 0 & 0 & 0 & I_0^9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_0^9 & 0 & 0 \\ 0 & 0 & I_0^9 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

The MOD directed type II graph G_5 in the following.

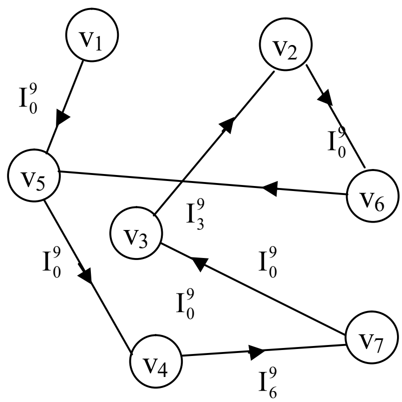
$G_5 =$ 

Figure 3.57

N^6 is given in the following.

$$N^6 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & I_0^9 & 0 \\ 0 & I_0^9 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_0^9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_0^9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_0^9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_3^9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I_0^9 \end{bmatrix} \end{array}.$$

Let G_6 be the type II MOD directed graph given by the following figure:

$G_6 =$

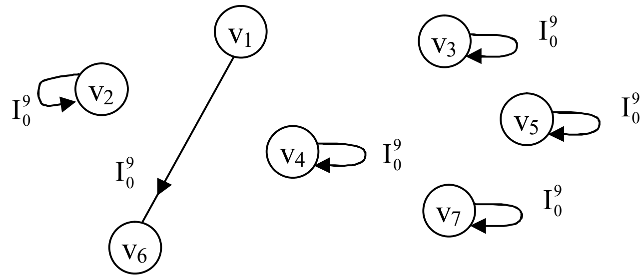


Figure 3.58

We see there are 6 loops and only one edge. Now we find N^7 in the following :

$$N^7 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \left[\begin{array}{ccccccc} 0 & I_0^9 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_0^9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I_0^9 \\ 0 & 0 & 0 & 0 & I_0^9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_0^9 & 0 \\ 0 & I_0^9 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_0^9 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}.$$

The graph G_7 associated with N^7 is as follows:

$G_7 =$

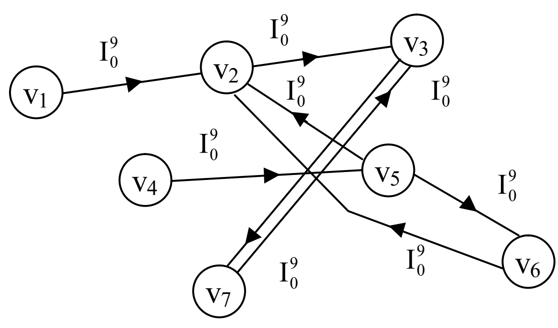


Figure 3.59

We see G_7 has no loops.
We find N^8 in the following:

$$N^8 = \begin{array}{c|ccccccc} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \\ \hline v_1 & 0 & 0 & I_0^9 & 0 & 0 & 0 & 0 \\ v_2 & 0 & 0 & 0 & 0 & 0 & 0 & I_0^9 \\ v_3 & 0 & 0 & 0 & I_0^9 & 0 & 0 & 0 \\ v_4 & 0 & 0 & 0 & 0 & 0 & I_0^9 & 0 \\ v_5 & 0 & I_0^9 & 0 & 0 & 0 & 0 & 0 \\ v_6 & 0 & 0 & I_0^9 & 0 & 0 & 0 & 0 \\ v_7 & 0 & 0 & 0 & 0 & I_0^9 & 0 & 0 \end{array}.$$

We now give the MOD graph G_8 in the following:

$G_8 =$

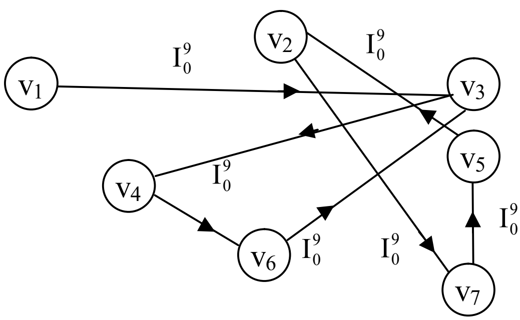


Figure 3.60

We give one more example of this situation.

Example 3.19: Let G be the type II MOD natural neutrosophic directed graph with edge weights from Z_{12}^I.

The vertex set is also from Z_{12}^I.

G is given below:

G =

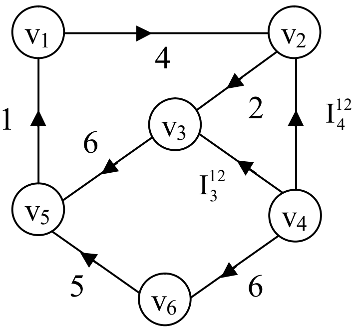


Figure 3.61

Let S be the type I MOD matrix which is as follows:

$$S = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & I_4^{12} & I_3^{12} & 0 & 0 & 6 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 \end{bmatrix} \end{array}.$$

We find S^2 in the following.

$$S^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_4^{12} & 0 & I_3^{12}+6 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

The MOD graph G_2 associated with S^2 is as follows:

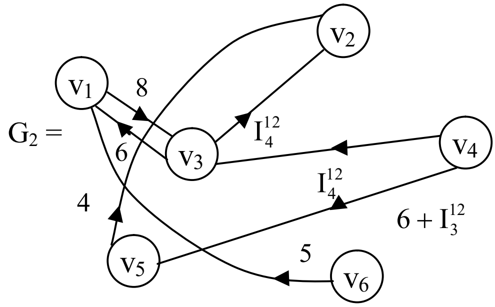


Figure 3.62

We find S^3 in the following :

$$S^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 6+I_3^{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

The MOD type II directed graph associated with S^3 is as follows.

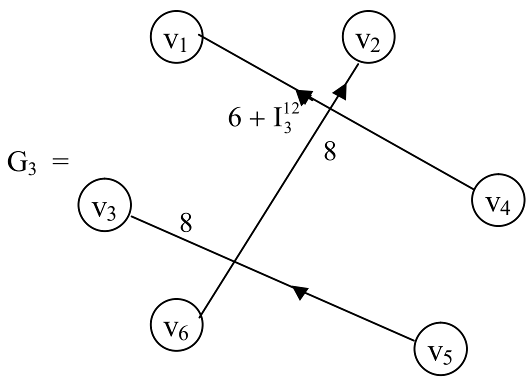


Figure 3.63

Now we find S^4 in the following:

$$S^4 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ I_4^{12} & I_3^{12}+6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let G_4 be the MOD directed graph given by the following figure:

G_4 =

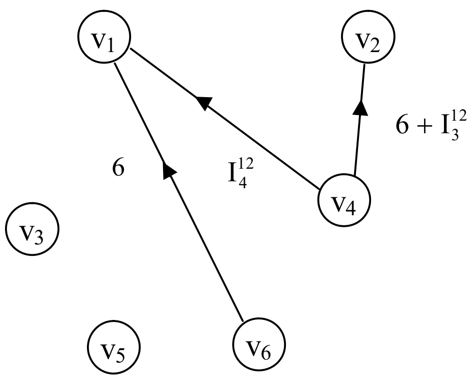


Figure 3.64

Interested reader can find more powers of S and their corresponding MOD graphs of type II of natural neutrosophic numbers with edge weights and vertex set from Z_n^I.

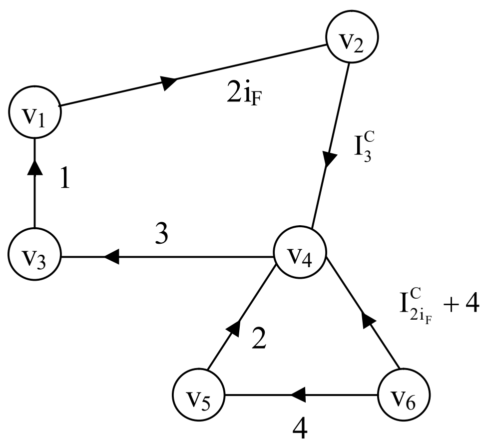
Now we proceed onto describe type II MOD natural neutrosophic finite complex number graphs.

Let G be a MOD natural neutrosophic type II graph with edge weights and vertex sets from $C^I(Z_n)$, then we define G to be a MOD natural neutrosophic finite complex number graph of type II.

We will illustrate this situation by some examples.

Example 3.20: Let G be the type II MOD natural neutrosophic finite complex number directed graph with vertex set and edge weights from $C^I(6)$ given by the following figure:

G =



$v_1 = 1 + i_F$, $v_2 = 3i_F$ $v_3 = 2$, $v_4 = 5$ and $v_6 = I^C_{3i_F+4}$.

Figure 3.65

Let B be the MOD natural neutrosophic finite complex number type II matrix associated with the graph G.

$$B = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 2i_F & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_3^C & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & I_{2i_F}^C + 4 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 \end{bmatrix} \end{array}.$$

$$\text{We find } B^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & I_3^C & 0 & 0 \\ 0 & 0 & I_3^C & 0 & 0 & I_3^C + I_0^C \\ 0 & 2i_F & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 4 + I_{2i_F}^C & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 + I_{2i_F}^C \\ 0 & 0 & 0 & 2 & 0 & 0 \end{bmatrix} \end{array}.$$

The type II MOD directed natural neutrosophic finite complex number graph G_2 is as follows:

$G_2 =$

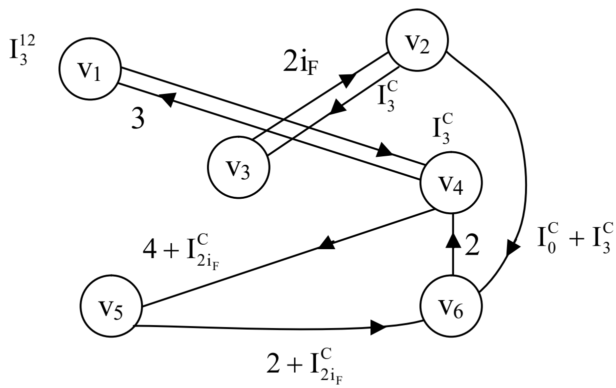


Figure 3.66

Clearly the type II MOD natural neutrosophic complex valued graph G_2 has no loops. The edges has increased by one edge.

Now we find B^3 in the following:

$$B^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 0 & I_3^C & 0 & 0 & I_3^C + I_0^C \\ I_3^C & 0 & 0 & 0 & I_3^C + I_0^C & 0 \\ 0 & 0 & 0 & I_3^C & 0 & 0 \\ 0 & 0 & 0 & 2 + I_{2i_F}^C & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 + I_{2i_F}^C & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{2i_F}^C + 2 \end{bmatrix} \end{array}.$$

The MOD type II natural neutrosophic finite complex number graph G_3 associated with B^3 is as follows.

$G_3 =$

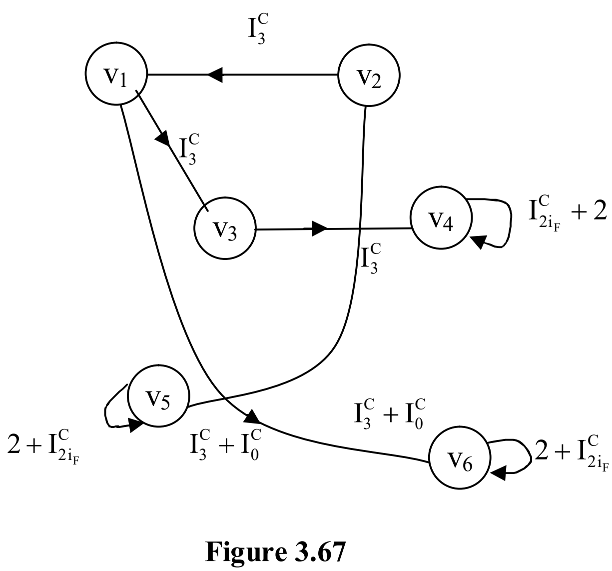


Figure 3.67

Thus the type II MOD natural neutrosophic complex number graph G_3 has three loops and five edges.

Next we find B^4 and the corresponding graph G_4 in the following.

$$B^4 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} I_3^C & 0 & 0 & 0 & I_3^C + I_0^C & 0 \\ 0 & I_3^C & 0 & I_3^C + I_0^C & 0 & 0 \\ 0 & 0 & I_3^C & 0 & 0 & I_3^C + I_0^C \\ 0 & 0 & 0 & I_3^C & 0 & 2 + I_{2i_F}^C + I_2^C \\ 0 & 0 & 0 & 4 + I_{2i_F}^C & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 + I_{2i_F}^C & 0 \end{bmatrix} \end{array}.$$

Now we give the related MOD type II directed graph G_4 of B^4 in the following.

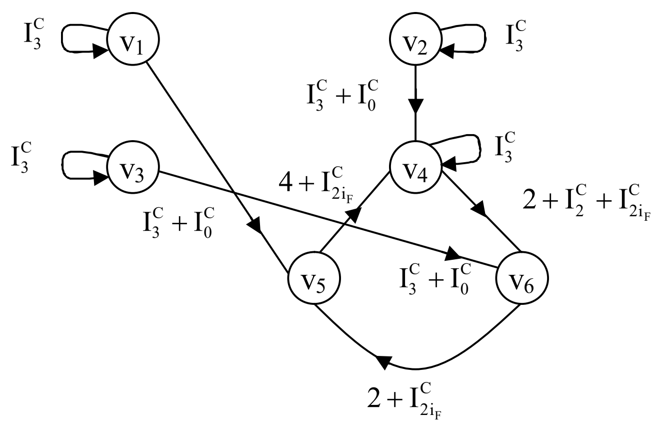


Figure 3.68

This has four loops and six edges.

Interested reader can work with graph G_t related to matrix B^t; $2 \leq t < \infty$.

Example 3.21: Let V be the type II MOD natural neutrosophic directed finite complex number graph with edge weights from $C^I(Z_{12})$ and vertices from $C^I(Z_{12})$ given by the following figure:

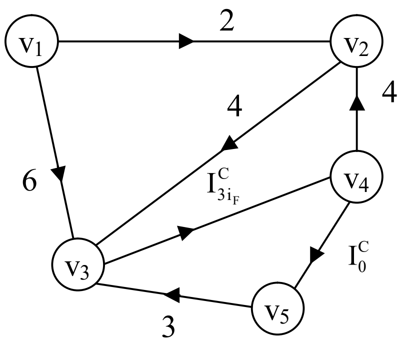
V = 

Figure 3.69

The type II MOD matrix associated with V is as follows:

$$P = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 2 & 6 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & I^C_{3i_F} & 0 \\ 0 & 4 & 0 & 0 & I^C_0 \\ 0 & 0 & 3 & 0 & 0 \end{bmatrix} \end{array}.$$

We see V has 7 edges and no loops. We now find P^2;

$$P^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 8 & I^C_{3i_F} & 0 \\ 0 & 0 & 0 & I^C_{3i_F} & 0 \\ 0 & I^C_{3i_F} & 0 & 0 & I^C_0 \\ 0 & 0 & 4+I^C_0 & 0 & 0 \\ 0 & 0 & 0 & I^C_{3i_F} & 0 \end{bmatrix} \end{array}.$$

Let V_2 be the MOD type II directed natural neutrosophic graph associated with the matrix P^2.

V_2 =

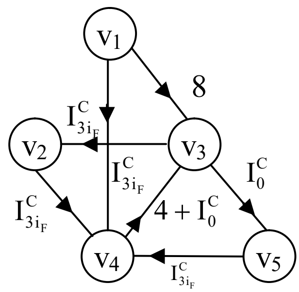


Figure 3.70

Now we find P^3 in the following:

$$P^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & I^C_{3i_F} & 0 & I^C_{3i_F} & I^C_0 \\ 0 & I^C_{3i_F} & 0 & 0 & I^C_0 \\ 0 & 0 & I^C_{3i_F} + I^C_0 & 0 & 0 \\ 0 & 0 & 0 & I^C_0 + I^C_{3i_F} & 0 \\ 0 & I^C_{3i_F} & 0 & 0 & I^C_0 \end{bmatrix} \end{array}.$$

The type II MOD directed graph V_3 associated with the MOD matrix P^3 is as follows:

$V_3 =$

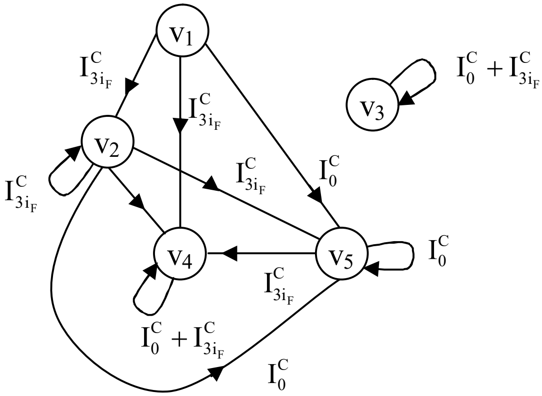


Figure 3.71

We find P^4 in the following:

$$P^4 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & I^C_{3i_F} & I^C_0 + I^C_{3i_F} & 0 & I^C_0 \\ 0 & 0 & I^C_0 + I^C_{3i_F} & 0 & 0 \\ 0 & 0 & 0 & I^C_3 + I^C_0 & 0 \\ 0 & I^C_0 + I^C_{3i_F} & 0 & 0 & I^C_0 \\ 0 & 0 & I^C_0 + I^C_{3i_F} & 0 & 0 \end{bmatrix} \end{array}.$$

Let V_4 be the MOD type II natural neutrosophic finite complex number directed graph associated with P^4.

$V_4 =$

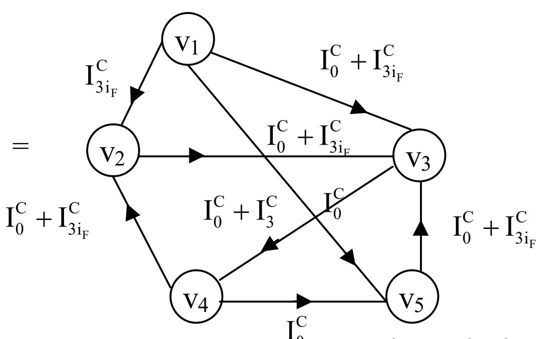


Figure 3.72

We can find the MOD natural neutrosophic directed complex finite integer graph V_t of type II for any P^t; $2 \leq t < \infty$.

Such work is left as an exercise to the reader.

Next we develop and describe MOD natural neutrosophic neutrosophic type II directed graph using edge weights and vertices from $\langle Z_n \cup I\rangle_I$.

Let G be a type II MOD natural neutrosophic directed graph with edge weights and vertex set from $\langle Z_n \cup I\rangle_I$, then G is defined as the MOD natural neutrosophic-neutrosophic directed graph of type II.

We provide a few examples of them.

Example 3.21: Let K be the type II MOD natural neutrosophic - neutrosophic directed graph with vertex set and edge weights from $\langle Z_8 \cup I\rangle_I$ given by the following figure:

K =

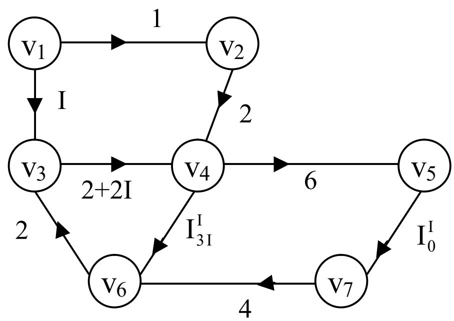


Figure 3.73

Let M be the type II MOD matrix associated with K.

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 1 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2+2I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 & 0 & I_{3I}^{I} \\ 0 & 0 & 0 & 0 & 0 & 0 & I_0^I \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 \end{bmatrix} \end{array}.$$

We find M^2 in the following:

$$M^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 2+4I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & I_{3I}^{I} \\ 0 & 0 & 0 & 0 & 4+4I & 0 & I_{3I}^{I} \\ 0 & 0 & 0 & 0 & 0 & I_{3I}^{I} & I_0^I \\ 0 & 0 & 0 & 0 & 0 & I_0^I & 0 \\ 0 & 0 & 4+4I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

The type II MOD natural neutrosophic-neutrosophic directed graph K_2 is as follows:

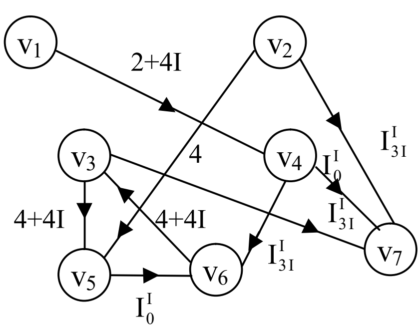
$K_2 =$ 

Figure 3.74

Now we find M^3 in the following :

$$M^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 4 & 0 & I_{3I}^{I} \\ 0 & 0 & 0 & 0 & 0 & I_{3I}^{I} & I_0^I \\ 0 & 0 & 0 & 0 & 0 & I_{3I}^{I} & I_0^I \\ 0 & 0 & 0 & 0 & 0 & I_0^I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I_{3I}^{I} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

The MOD type II natural neutrosophic - neutrosophic directed graph K_3 associated with M^3

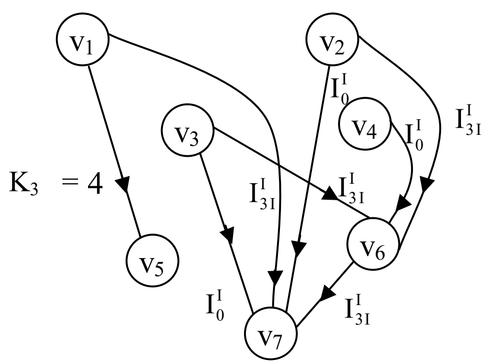


Figure 3.75

We now find the MOD type II natural neutrosophic-neutrosophic matrix M^4 is found in the following.

$$M^4 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array}\begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & I_{3I}^I & I_0^I \\ 0 & 0 & 0 & 0 & 0 & I_0^I & 0 \\ 0 & 0 & 0 & 0 & 0 & I_0^I & 0 \\ 0 & 0 & I_{3I}^I & 0 & 0 & 0 & 0 \\ 0 & 0 & I_0^I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I_{3I}^I \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

The MOD type II directed graph K_4 associated with M^4 is as follows.

$K_4 =$

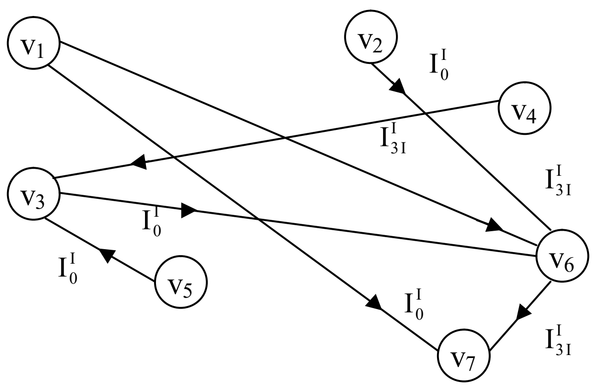


Figure 3.76

The author is left with the task of finding MOD type I graphs associated with powers of M.

Next we provide one more example of this same type of graph.

Example 3.22: Let W be the type II MOD natural neutrosophic neutrosophic directed graph with edge weights and vertex set from $\langle Z_7 \cup I\rangle_I$ given by the following figure:

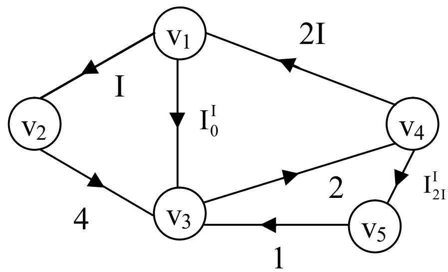
W = 

Figure 3.77

The MOD matrix L associated with W is as follows:

$$L = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & I & I_0^I & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 2I & 0 & 0 & 0 & I_{2I}^I \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}.$$

We find L^2 in the following

$$L^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 4I & I_0^I & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 4I & 0 & 0 & 0 & I_{2I}^I \\ 0 & 2I & I_0^I + I_{2I}^I & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \end{bmatrix} \end{array}.$$

Let W_2 be the type II MOD natural neutrosophic-neutrosophic directed graph associated with L^2.

$W_2 =$

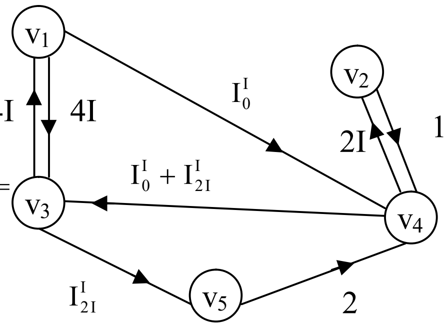


Figure 3.78

Next we find L^3 in the following.

$$L^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \left[\begin{array}{ccccc} I_0^I & 0 & 0 & I & I_0^I \\ 2I & 0 & 0 & 0 & I_{2I}^I \\ 0 & 4I & I_0^I + I_{2I}^I & 0 & 0 \\ 0 & 0 & I & I_0^I + I_{2I}^I & 0 \\ 4I & 0 & 0 & 0 & I_{2I}^I \end{array}\right] \end{array}.$$

Let W_3 be the MOD type II natural neutrosophic neutrosophic directed graph associated with L^3.

$W_3 =$

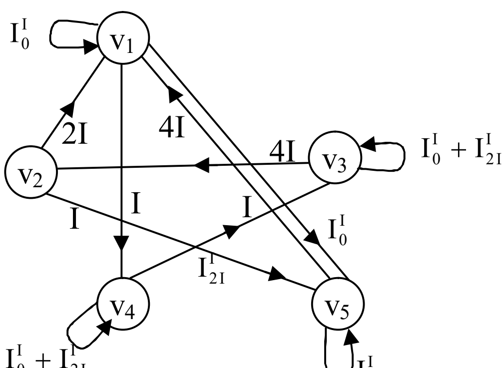


Figure 3.79

We see W_3 has four loops and 8 edges.

Now we find L^4 in the following and the corresponding type II MOD graph W_4 using L^4

$$L^4 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \left[\begin{array}{ccccc} 2I & I_0^I & I_0^I & 0 & 0 \\ 0 & 2I & I_0^I + I_{2I}^I & 0 & 0 \\ 0 & 0 & 2I & I_0^I + I_{2I}^I & 0 \\ I_0^I + I_{2I}^I & 0 & 0 & 2I & I_0^I + I_{4I}^I \\ 0 & 4I & I_0^I + I_{2I}^I & 0 & 0 \end{array}\right] \end{array}.$$

Let W_4 be the type II MOD interval neutrosophic-neutrosophic directed graph associated with L^4.

W_4 =

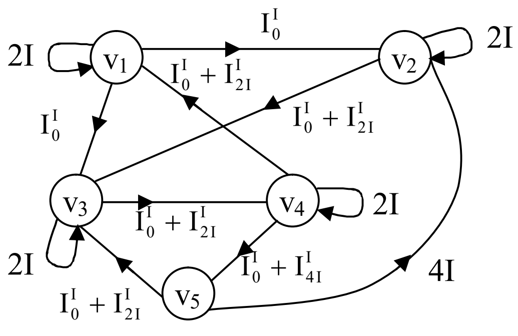


Figure 3.80

Likewise interested reader can find out the type II MOD graphs associated with any power of L.

Next we proceed onto describe and develop the notion of type II MOD natural neutrosophic dual number directed graphs.

Let A be the type II MOD natural neutrosophic directed graph with edge weights from $\langle Z_n \cup g\rangle_I$.

A is defined as the type II MOD natural neutrosophic dual number directed graph, as the edge weights are from $\langle Z_n \cup g\rangle_I$.

We will describe this situation by some examples.

Example 3.23: Let B be the type II MOD natural neutrosophic dual number directed graph with entries from $\langle Z_{10} \cup g\rangle_I$ given by the following figure:

B =

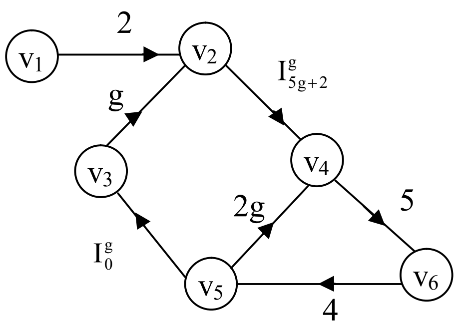


Figure 3.81

Let S be the MOD matrix associated with the graph B.

$$S = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I^g_{5g+2} & 0 & 0 \\ 0 & g & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & I^g_0 & 2g & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 \end{bmatrix} \end{array}.$$

We find S^2 in the following.

$$S^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & I^g_{5g+2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I^g_{5g+2} \\ 0 & 0 & 0 & I^g_{5g+2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I^g_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I^g_0 & 8g & 0 & 0 \end{bmatrix} \end{array}.$$

To find the MOD type II graph B_2 associated with S^2 is as follows.

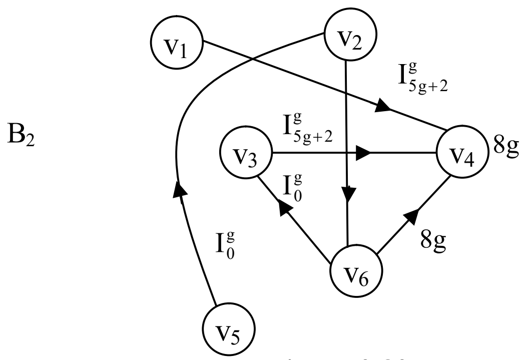


Figure 3.82

We find S^3 in the following.

$$S^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & I^g_{5g+2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I^g_{5g+2} \\ 0 & 0 & I^g_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I^g_0 & 0 & 0 \\ 0 & I^g_0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let B_3 be the type II MOD natural neutrosophic dual number directed graph associated with S^3 is given below:

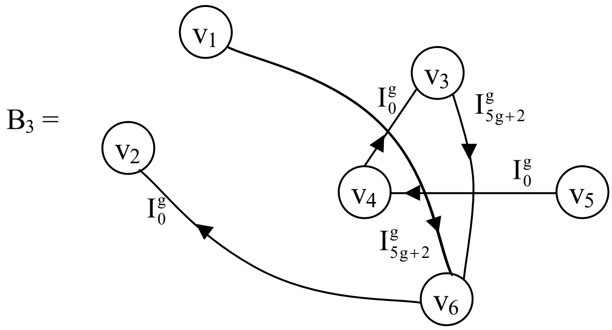


Figure 3.83

We now find S^4 in the following

$$S^4 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_0^g & I_{5g+2}^g & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_0^g \\ 0 & 0 & 0 & I_0^g & 0 & 0 \end{bmatrix} \end{array}.$$

Let B_4 be the type II MOD natural neutrosophic dual number directed graph associated with S^4

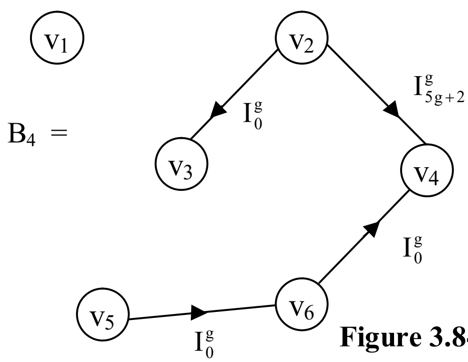


Figure 3.84

Interested reader can find the type II MOD directed graphs associated with S^t; $2 \leq t < \infty$.

Example 3.24: Let D be the type II MOD natural neutrosophic dual number directed graph given by the following figure with edge weights and vertex set from $\langle Z_5 \cup g\rangle_I$.

D =

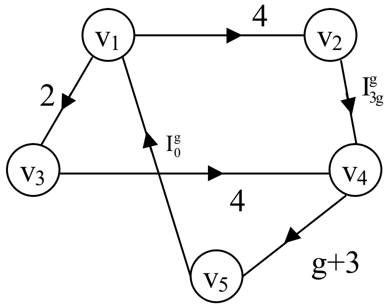


Figure 3.85

Let N be the MOD matrix associated with D

$$N = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 4 & 2 & 0 & 0 \\ 0 & 0 & 0 & I_{3g}^g & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & g+3 \\ I_0^g & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Now we find N^2 using N in the following:

$$N^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & I^g_{3g}+3 & 0 \\ 0 & 0 & 0 & 0 & I^g_{3g} \\ 0 & 0 & 0 & 0 & 4g+2 \\ I^g_0 & 0 & 0 & 0 & 0 \\ 0 & I^g_0 & I^g_0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let D_2 be the type II MOD natural neutrosophic dual number directed graph given in the following.

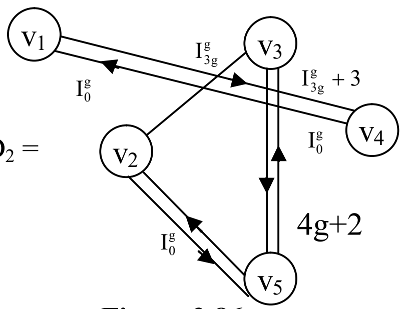
$D_2 =$ 

Figure 3.86

Now we find N^3 using N

$$N^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 3g+4+I^g_{3g} \\ I^g_0 & 0 & 0 & 0 & 0 \\ I^g_0 & 0 & 0 & 0 & 0 \\ 0 & I^g_0 & I^g_0 & 0 & 0 \\ 0 & 0 & 0 & I^g_0 & 0 \end{bmatrix} \end{array}.$$

Using N^3 we find the MOD type II natural neutrosophic dual number directed graph D_3 is as follows:

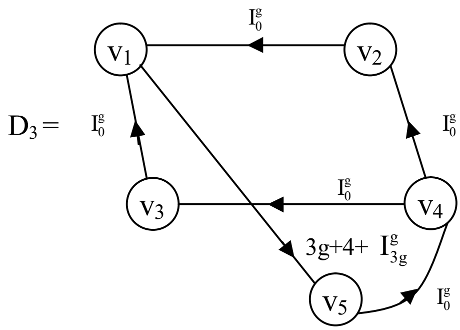


Figure 3.87

We find N^4 using N in the following:

$$N^4 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} I_0^g & 0 & 0 & 0 & 0 \\ 0 & I_0^g & I_0^g & 0 & 0 \\ 0 & I_0^g & I_0^g & 0 & 0 \\ 0 & 0 & 0 & I_0^g & 0 \\ 0 & 0 & 0 & 0 & I_0^g \end{bmatrix} \end{array}.$$

The type II MOD natural neutrosophic dual number directed graph D_4 is as follows.

$D_4 =$

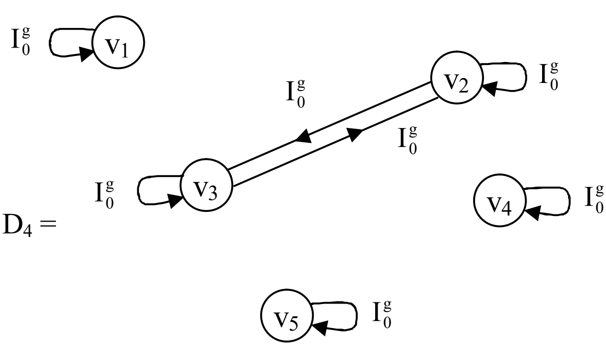


Figure 3.88

The reader is expected to find the MOD directed graph relative to N^t for $2 \leq t < \infty$.

Next we proceed onto describe the MOD natural neutrosophic special dual like number graph of type II by the following example.

Example 3.25: Let G be the MOD natural neutrosophic special dual like number directed graph of type II with edge weights and vertex set from $\langle Z_6 \cup h \rangle_I$ given by the following figure.

G =

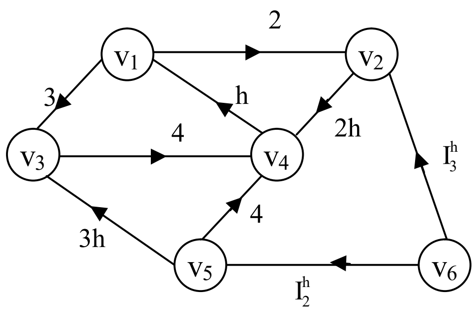


Figure 3.89

Let P be the MOD matrix associated with

$$P = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 2 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2h & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 \\ h & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3h & 4 & 0 & 0 \\ 0 & I_3^h & 0 & 0 & I_2^h & 0 \end{bmatrix} \end{array}.$$

Now we find N^2 using N in the following.

$$N^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 4h & 0 & 0 \\ 2h & 0 & 0 & 0 & 0 & 0 \\ 4h & 0 & 0 & 0 & 0 & 0 \\ 0 & 2h & 3h & 0 & 0 & 0 \\ 4h & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_2^h & I_3^h + I_2^h & 0 & 0 \end{bmatrix} \end{array}.$$

Let P_2 be the type II MOD natural neutrosophic special dual like number directed graph.

$P_2 =$

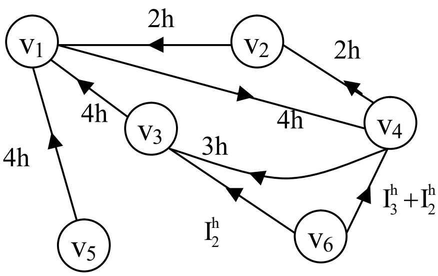


Figure 3.90

We now find N^3 in the following

$$N^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 4h & 0 & 0 & 0 & 0 & 0 \\ 0 & 4h & 0 & 0 & 0 & 0 \\ 0 & 2h & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4h & 0 & 0 \\ 0 & 2h & 0 & 0 & 0 & 0 \\ I_3^h + I_2^h & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let P^3 be the type II MOD natural neutrosophic special dual like number directed graph associated with N^3.

P_3 =

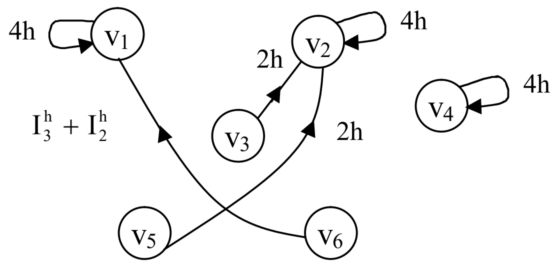


Figure 3.91

This graph has 3 loops and 3 edges.

Interested reader can find any type II MOD graphs associated with N^t; $2 \leq t < \infty$.

We give another example.

Example 3.26: Let S be the type II MOD natural neutrosophic special dual like number directed graph with edge weights and vertex set from $\langle Z_3 \cup h \rangle$.

S =

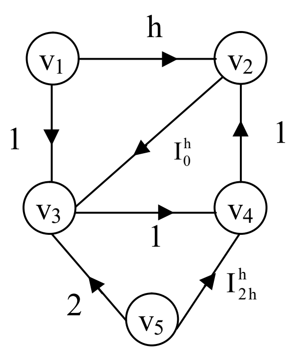


Figure 3.92

Let x be the related MOD connection matrix associated with S is as follows.

$$x = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & h & 1 & 0 & 0 \\ 0 & 0 & h & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & I_{2h}^{h} & 0 \end{bmatrix} \end{array}.$$

We find x^2 in the following.

$$x^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & h & 1 & 0 \\ 0 & 0 & 0 & h & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & h & 0 & 0 \\ 0 & I_{2h}^{h} & 0 & 2 & 0 \end{bmatrix} \end{array}.$$

The MOD type II graph S_2 associated with x^2 is as follows:

S_2 =

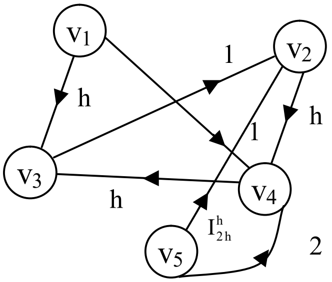


Figure 3.93

Now we find x^3 in the following

$$x^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & h & 0 \\ 0 & h & 0 & 0 & 0 \\ 0 & 0 & h & 0 & 0 \\ 0 & 0 & 0 & h & 0 \\ 0 & 2 & I_{2h}^{h} & 0 & 0 \end{bmatrix} \end{array}.$$

The type II MOD directed graph S_3 associated with matrix x^3 is as follows.

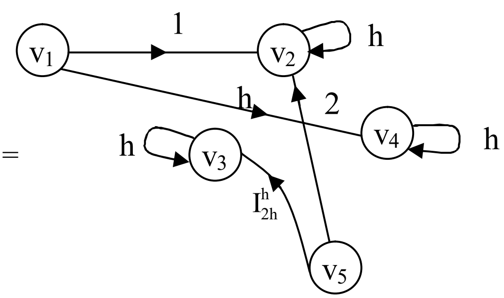
$S_3 =$ 

Figure 3.94

We find x^4 in the following

$$x^4 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & h & h & 0 & 0 \\ 0 & 0 & h & 0 & 0 \\ 0 & 0 & 0 & h & 0 \\ 0 & h & 0 & 0 & 0 \\ 0 & 0 & 2h & I_{2h}^{h} & 0 \end{bmatrix} \end{array}.$$

Let S_4 be the MOD type II graph associated with the matrix x^4.

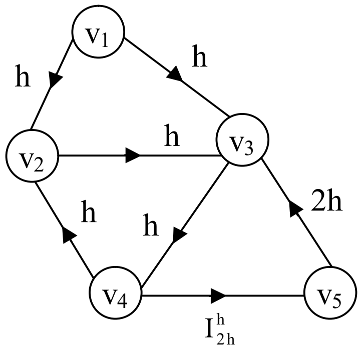
$S_4 =$ 

Figure 3.95

The task of finding type II MOD graphs associated with x^t for $2 \leq t < \infty$ is left as an exercise to the reader.

Next we proceed onto describe the type II MOD natural neutrosophic special quasi dual number directed graphs.

Let G be any type II MOD natural neutrosophic graph if the edge weights and vertex sets are taken from $\langle Z_n \cup k\rangle_I$ then we define G to be a type II MOD natural neutrosophic special quasi dual number directed graphs.

We will illustrate this situation by an example or two.

Example 3.27: Let G be the type II MOD natural neutrosophic special quasi dual number directed graphs with vertex set and edge weights from $\langle Z_6 \cup k\rangle_I$ given by the following figure.

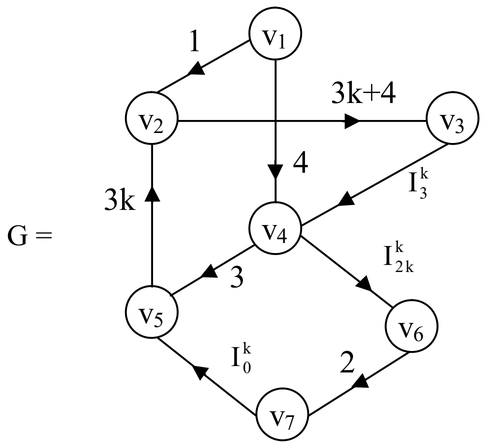


Figure 3.96

Let B be the MOD natural neutrosophic matrix associated with G.

$$B = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 3k+4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_3^k & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & I_{2k}^k & 0 \\ 0 & 3k & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & I_0^k & 0 & 0 \end{bmatrix} \end{array}.$$

We find B^2 in the following.

$$B^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 3k+4 & 0 & 0 & I_{2k}^k & 0 \\ 0 & 0 & 0 & I_3^k & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_3^k & I_0^k & 0 \\ 0 & 3k & 0 & 0 & 0 & 0 & I_{2k}^k \\ 0 & 0 & 3k & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_0^k & 0 & 0 \\ 0 & I_0^k & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let G_2 be the type II MOD natural neutrosophic graph associated with B^2.

G_2 =

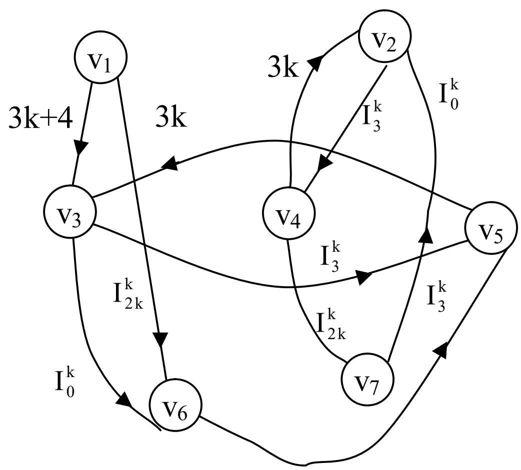


Figure 3.97

We now find B^3 in the following :

$$B^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \left[\begin{array}{ccccccc} 0 & 0 & 0 & I_3^k & 0 & 0 & I_{2k}^k \\ 0 & 0 & 0 & 0 & I_3^k & I_0^k & 0 \\ 0 & I_3^k & 0 & 0 & 0 & 0 & I_0^k \\ 0 & 0 & 3k & 0 & I_0^k & 0 & 0 \\ 0 & 0 & 0 & I_3^k & 0 & 0 & 0 \\ 0 & I_0^k & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_0^k & 0 & 0 & 0 & 0 \end{array}\right] \end{array}.$$

Let G_3 be the MOD natural neutrosophic special quasi dual number directed graph associated with the MOD matrix B^3.

G_3 =

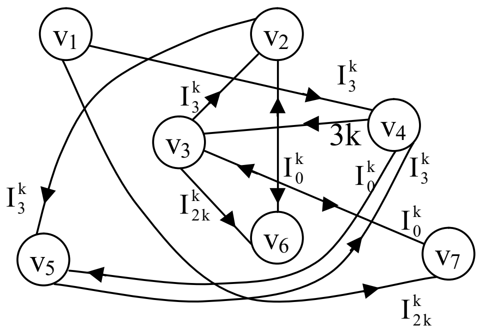


Figure 3.98

This is the way we can work with MOD type II natural neutrosophic special quasi dual number directed graphs and then associated power of matrices.

We just give one more example of this situation.

Example 3.28: Let G be the type II MOD natural number graph with vertex set and edge weights from $\langle Z_{11} \cup k\rangle_I$ given by the following figure:

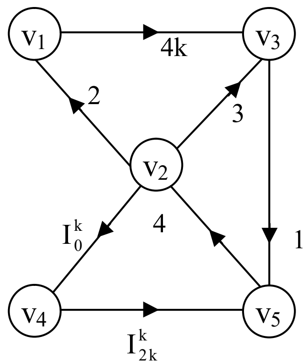


Figure 3.99

Let B be the MOD matrix associated with the type II MOD graph G.

$$B = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 4k & 0 & 0 \\ 2 & 0 & 3 & I_0^k & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & I_{2k}^k \\ 0 & 4 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

We find B^2 in the following.

$$B^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 4k \\ 0 & 0 & 8k & 0 & 3+I_0^k \\ 0 & 4 & 0 & 0 & 0 \\ 0 & I_{2k}^k & 0 & 0 & 0 \\ 8 & 0 & 1 & I_0^k & 0 \end{bmatrix} \end{array}.$$

The MOD type II natural neutrosophic special quasi dual number graph G_2 associated with B^2 is as follows:

G_2 =

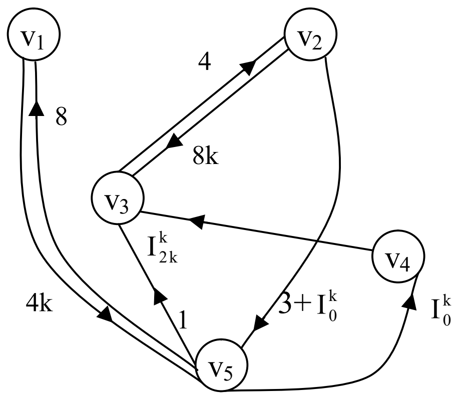


Figure 3.100

Next we just find B^4 in the following.

$$B^4 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 10k & 0 & 4k & I_0^k & 0 \\ 2+I_0^k & 10k & 3+I_0^k & I_0^k & 0 \\ 0 & 0 & 10k & 0 & 1+I_0^k \\ 0 & 0 & I_{2k}^k & 0 & I_{2k}^k+I_0^k \\ 0 & 4+I_0^k & 0 & 0 & 10k \end{bmatrix} \end{array}.$$

Let G_4 be the MOD type II directed graph associated with B^4.

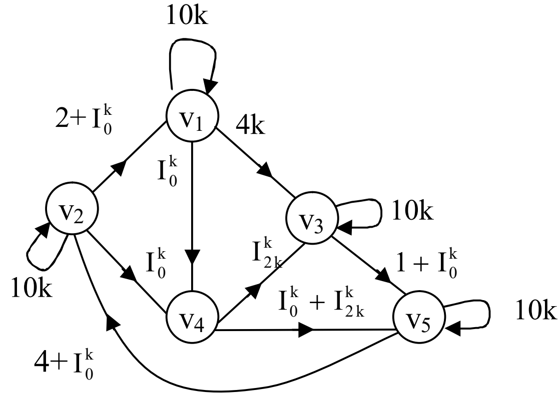
$G_4 =$ 

Figure 3.101

The reader is given the task of finding type II MOD directed graphs G_t using the power of matrices B^t; $2 \leq t < \infty$.

Now we wish to discuss about the special features associated MOD natural neutrosophic special quasi dual number graphs of type II and also all type II MOD natural neutrosophic graphs built using $\langle Z_n \cup I\rangle_I$, $\langle Z_n \cup h\rangle_h$, $C^I(Z_n)$ and Z_n^I.

In the case of type II MOD directed graphs we see they are labeled with elements of Z_n^I (or $\langle Z_n \cup g\rangle_I$ or $\langle Z_n \cup h\rangle_I$ or $\langle Z_n \cup k\rangle_I$ or $C^I(Z_n)$) as well they take edge values also from these respective sets.

Thus these MOD type II directed graphs described here have the edge weights and labels to be from the MOD natural neutrosophic sets Z_n^I, $C^I(Z_n)$ and so on.

Thus the labeling which does on face values is very unique and innovative as the values can be real or complex or neutrosophic or dual or special dual like or special quasi dual or mixed or neutrosophic idempotent or nilpotents or zero divisors.

Such diversity cannot be imagined with usual labeling. Further these graphs can be used in networking where the indeterminacy concept or dual number concept or imaginary value concept are used.

Further these MOD graphs can also be used in automation and semi automation as we have only finite number of elements in Z_n^I or $C^I(Z_n)$ or $\langle Z_n \cup I\rangle_I$ or $\langle Z_n \cup g\rangle_I$ or $\langle Z_n \cup h\rangle_I$ or $\langle Z_n \cup k\rangle_I$.

However we have not used any algebraic structure or operations on them in a systematic way.

But these graphs have been used in [ ] for MOD Cognitive Maps model and their generalizations.

These mathematical models can be exploited in medical sciences, technological research apart from taking social problems where indeterminacy of all types are involved.

Next we proceed onto describe type III MOD directed graphs using Z_n^I, $C^I(Z_n)$, $\langle Z_n \cup g\rangle_I$, $\langle Z_n \cup h\rangle_I$ or $\langle Z_n \cup I\rangle_I$ or $\langle Z_n \cup k\rangle_I$ in the following.

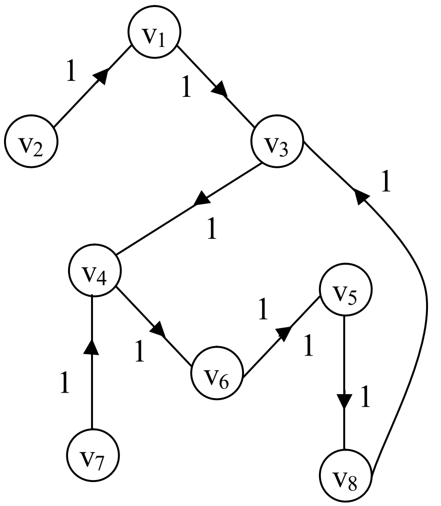
These graphs get their edge weighs from $\{0, 1\}$ but the vertex set from Z_n^I or $C^I(Z_n)$ or $\langle Z_n \cup g\rangle_I$ or so on.

We will describe them by examples.

Example 3.29: Let G be the MOD directed graph whose vertex set is from Z_{10}^I and edge weights are from the set $\{0, 1\}$.

G will be known as type III MOD natural neutrosophic directed graph.

The figure of graph G is as follows:



= G

Figure 3.102

$v_1 = 9$, $v_2 = 4$ $v_3 = I_5^{10}$, $v_4 = I_2^{10} + 3$, $v_5 = I_2^{10} + I_0^{10} + 8$, $v_6 = I_8^{10}$, v_7 6, $v_8 = 5$, $v_i \in Z_{10}^{I}$, $1 \leq i \leq 8$.

The MOD matrix P associated with G is as follows.

$$P = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 \end{array} \\ \left[\begin{array}{cccccccc} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}.$$

We can find P^2

$$P^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} .$$

The type III MOD natural neutrosophic directed graph G_2 associated with P^2 is as follows :

$G_2 =$

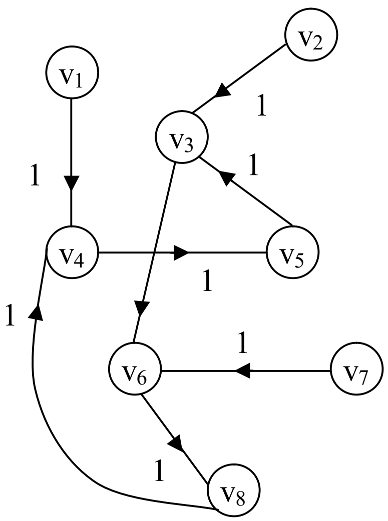


Figure 3.103

We see there are no loops only edges and this type III MOD graph G_2 is an edge shuffled type III MOD graph of G.

For only the edges are changed.

Now we find P^3 in the following.

$$P^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}.$$

We now proceed onto give the MOD natural neutrosophic directed graph G_2 of type III in the following.

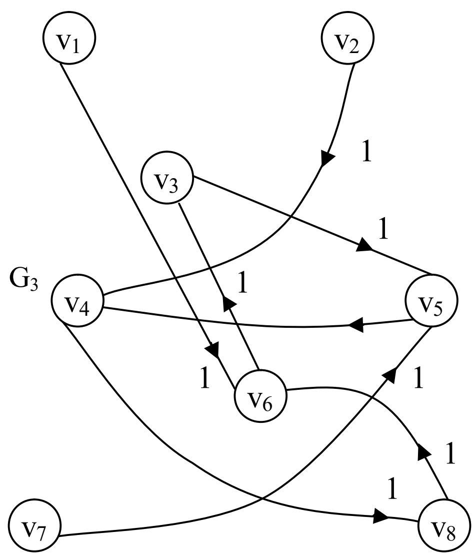


Figure 3.104

We see the MOD type III graph has no loops only the edges are reshuffled from the original G_1.

Next we find the type III MOD matrix P^4;

$$P^4 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let G_4 be the MOD type III natural neutrosophic graph associated with the matrix P^4.

G_4 =

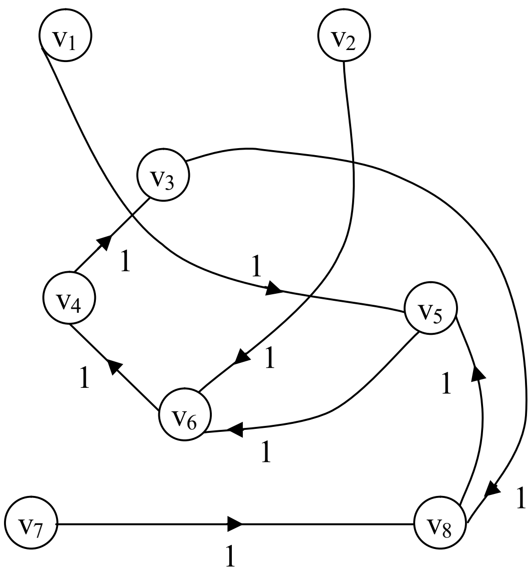


Figure 3.105

This also has no loops, only the edges are reshuffled. We can say that after finite number of product we may get $P^t = P$ so the original type III MOD natural neutrosophic directed graph is obtained.

Thus if a MOD directed graph G takes edge weights from {0, 1} and the vertex set from Z_n^I we define G to be a type III MOD natural neutrosophic directed graph.

Next we proceed onto describe and develop the notion of type III MOD natural neutrosophic finite complex number directed graph in the following.

Let G be a type III MOD natural neutrosophic directed graph, if the edge weights are taken from {0, 1} and vertex set from $C^I(Z_n)$ then we call or define G be a MOD natural neutrosophic directed finite complex number graph of type III.

We will illustrate this situation by some examples.

Example 3.30: Let K be the type III MOD natural neutrosophic finite complex number directed graph with edge weights from {0, 1} and vertex set from $C^I(Z_9)$ given by the following figure.

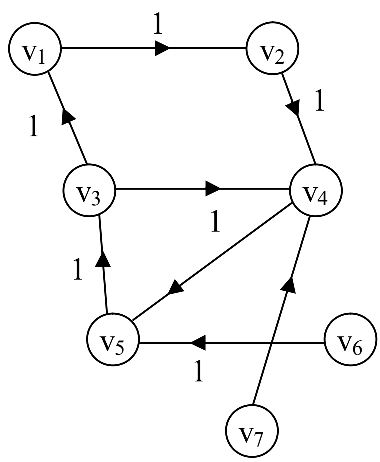


Figure 3.106

Here $v_1 = 3 + 4i_F$, $v_2 = I^C_{3+3k}$, $v_3 = I^C_{4+k}$ $I^C_6 = v_4$ $v_5 = 7$,
$v_6 = 3$, $v = I^C_0$.

The MOD connection matrix B associated with K is as follows:

$$B = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \left[\begin{array}{ccccccc} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{array}\right] \end{array}.$$

We now find B^2 in the following.

$$B^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \left[\begin{array}{ccccccc} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{array}\right] \end{array}.$$

We give the MOD directed graph K_2 of type III associated with B^2 in the following.

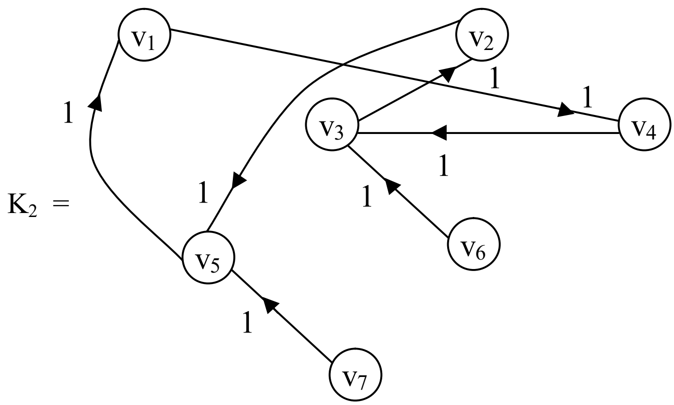


Figure 3.107

Now we find B^3 in the following:

$$B^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let K^3 be the type III MOD natural neutrosophic directed complex number graph associated with the matrix B^3.

K_3 =

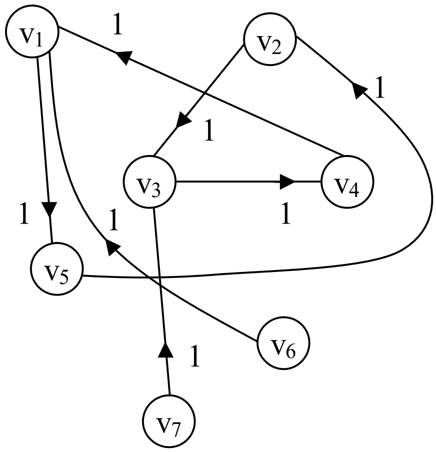


Figure 3.108

We see this type III MOD natural neutrosophic complex number directed graph has no loops only the edges are reshuffled in K_3.

Finally we find B^4 in the following.

$$B^4 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

We now give in the following type III MOD natural neutrosophic finite complex number directed graph K_4 associated with the MOD matrix B^4.

$K_4 =$

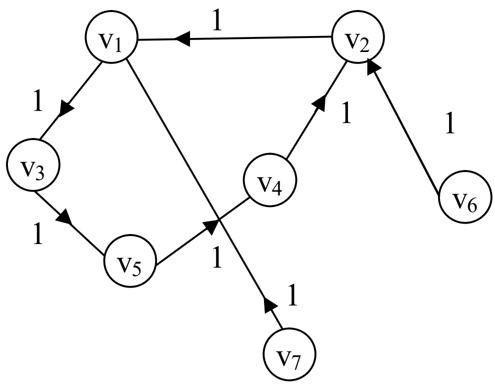


Figure 3.109

The reader is expected to prove that there exist $0 < t < \infty$ such that $B^t = B$ their by implying $K_t = K$.

Next we proceed onto describe and develop MOD natural neutrosophic-neutrosophic type III directed graphs using edge weights from {1, 0} and vertex elements from $\langle Z_n \cup I\rangle_I$.

Let G be a type III MOD natural neutrosophic-neutrosophic directed graph with vertex set from $\langle Z_n \cup I\rangle_I$ and edge sets from {0, 1}.

We will illustrate this situation by some examples.

Example 3.31: Let S be the type III MOD natural neutrosophic-neutrosophic directed graph with edge weights from {0, 1} and vertex set from $\langle Z_n \cup I\rangle_I$ given by the following figure.

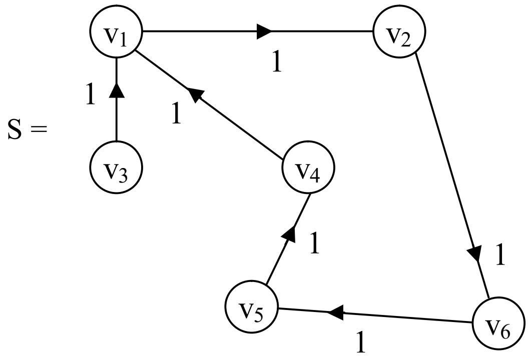


Figure 3.110

The MOD matrix associated with the MOD directed graph S of type III is as follows.

$$P = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \left[\begin{array}{cccccc} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \end{array}.$$

We find P^2 in the following

$$P^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \left[\begin{array}{cccccc} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{array}\right] \end{array}.$$

The MOD type III directed graph S_2 associated with the MOD matrix P^2 is as follows:

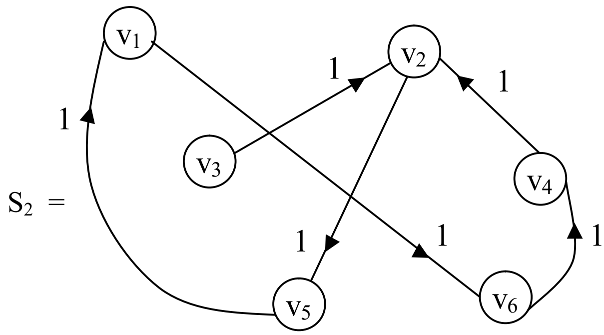


Figure 3.111

We next find P^4 in the following;

$$P^4 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

The MOD type III directed graph S_4 associated with P^4 is as follows:

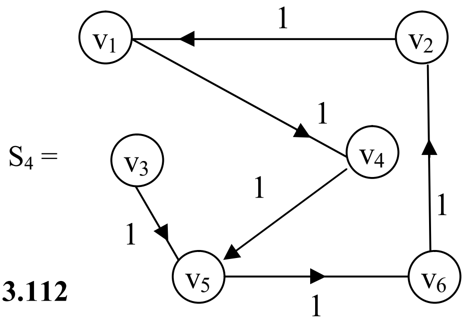


Figure 3.112

Interested reader can find P^t ($t > 0$) and the corresponding MOD type III graph S_t related with P^t.

Find a m such that $P^m = P$.

We see this S_t for no t gives a loop only edges from 0 to 1.

Next we proceed onto describe and develop the MOD type III natural neutrosophic dual number directed graph with edge weights from {0, 1} and vertex set from $\langle Z_n \cup g\rangle_I$.

We will describe this situation by some examples.

Example 3.32: Let V be the type III MOD natural neutrosophic dual number directed graph with edge weights from {0, 1} and vertex set from $\langle Z_{10} \cup g\rangle_I$ given by the following figure.

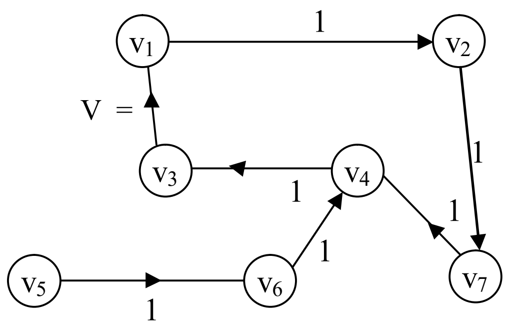


Figure 3.113

$v_1 = 5$, $v_2 = 8g$, $v_3 = 9g + 5$, $v_4 = I_g^g$, $v_5 = I_{9g}^g$ $v_6 = I_0^g$ and $v_7 = I_{3g+4}^g$

Let W be the MOD type III matrix associated with the graph V, given in the following:

$$W = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \left[\begin{array}{ccccccc} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{array}\right] \end{array}.$$

Now we find W^2 in the following;

$$W^2 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \left[\begin{array}{ccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}.$$

Let V_2 be the type III MOD natural neutrosophic dual number graph associated with W^2;

$V_2 =$

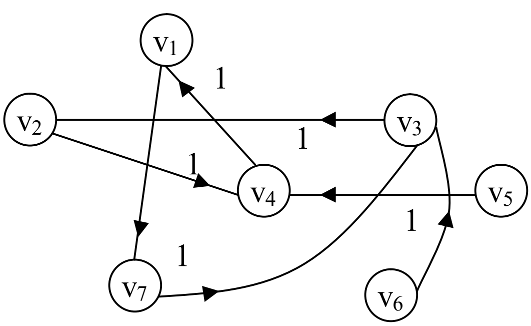


Figure 3.114

We see this MOD type III graph V_2 has no loops only edges and the edges are just reshuffled from V.

Next we find W^3 be the matrix product of W associated with the graph K.

$$W^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let V_3 be the type III MOD directed natural neutrosophic special dual number graph associated with W^3.

V_3 =

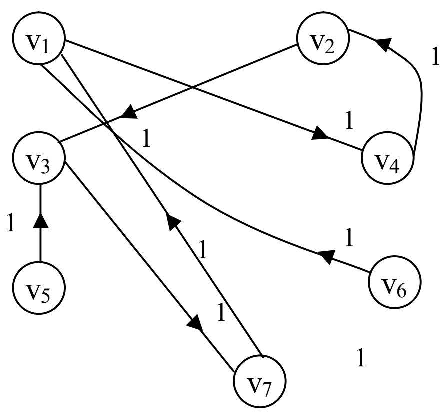


Figure 3.115

This type III MOD graph also is only a graph got by reshuffling the edges of V_1.

Next we find W^4 in the following and using W^4 we give the type III MOD directed graph V_4;

$$W^4 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let V_4 be the type III MOD directed graph given by the following figure:

V_4 =

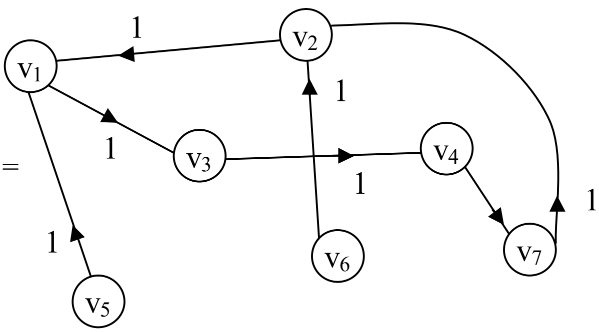


Figure 3.116

Interested reader can find any power of W say W^t and find the MOD directed graph V_t (t a large number).

Further the reader is left with the task of finding a suitable $t > 0$ such that $W^t = W$.

Next we proceed onto describe the MOD type III natural neutrosophic special dual like number directed graph by an example.

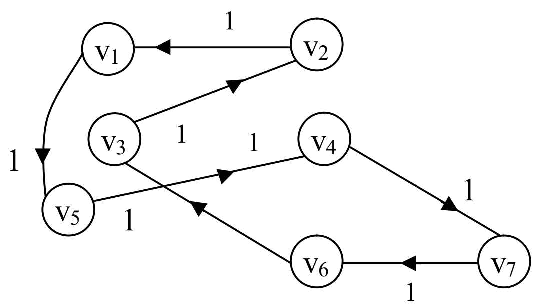
Let G be a type III MOD natural natural neutrosophic directed graph with edge weights from $\langle Z_n \cup h\rangle_I$.

Then we define G to be a type III MOD natural neutrosophic special dual like number directed graph.

We will illustrate this situation by an example or two.

Example 3.33: Let B be the type III MOD natural neutrosophic special dual like number directed graph with edge weights from $\langle Z_{12} \cup h\rangle_I$ given by the following figure:

B =



$$v_1 = 11 + 3h,\ v_2 = I_{10}^{h},\ v_3 = I_0^h + I_2^h + 7h,$$
$$v_4 = 9 + 7h,\ v_5 = 4h,\ v_6 = 8,\ v_7 = I_{10h}^{h}$$

Figure 3.117

Let M be the MOD type III natural neutrosophic special dual like number matrix associated with the type III MOD natural neutrosophic graph B.

$$M = \begin{array}{c|ccccccc} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \\ \hline v_1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ v_2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ v_3 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ v_4 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ v_5 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ v_6 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ v_7 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array}.$$

We now find M^2 in the following.

$$M^2 = \begin{array}{c|ccccccc} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \\ \hline v_1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ v_2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ v_3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ v_4 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ v_5 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ v_6 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ v_7 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{array}.$$

Let B_2 be the MOD natural neutrosophic special dual like number directed graph of type III associated with the matrix M^2.

B_2 =

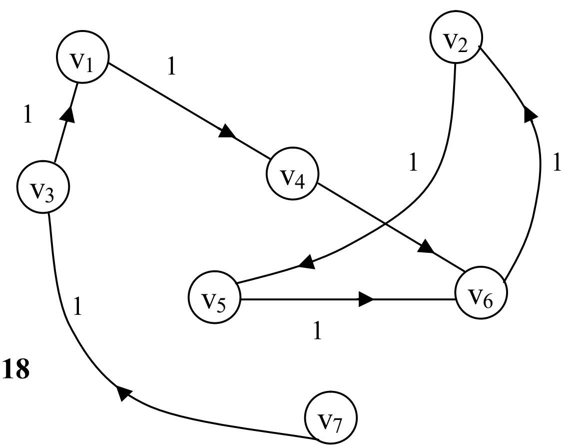


Figure 3.118

We see this graph B_2 is nothing but reshuffled edges of B.

Next we find M^3 in the following.

$$M^3 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let B_3 be the MOD special dual like number natural neutrosophic directed graph type III associated with the MOD type III matrix M^3. This figure is given in the following.

$B_3 =$

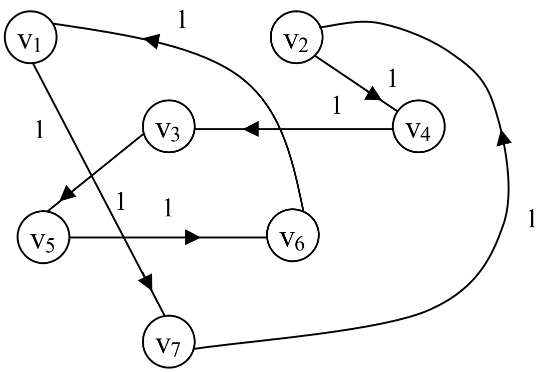


Figure 3.119

Likewise interested reader can find any power of M say M^t, $t > 2$ and the corresponding type III MOD directed graph B_t. Certainly we will have $M^t = M$ for some $t > 2$.

Interested reader can construct more examples.

Now we proceed onto describe and develop type III MOD natural neutrosophic special quasi dual number directed graphs.

Let G be the type III MOD natural neutrosophic directed graph. If the vertex set is from $\langle Z_n \cup k\rangle_I$ and edge set from {0, 1} then we define G to be the MOD natural neutrosophic special quasi dual number directed type III graph.

We will describe then by some examples.

Example 3.34: Let S be the type III MOD natural neutrosophic special quasi dual number directed graph with vertex set from $\langle Z_6 \cup k\rangle_I$ and edge sets from {0, 1} given by the following figure:

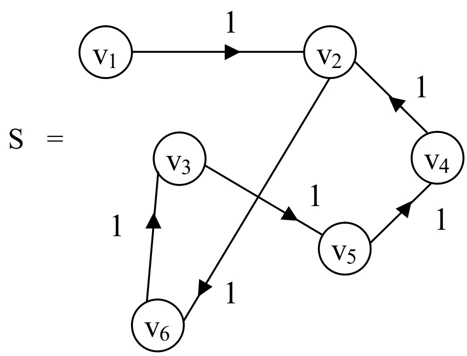


Figure 3.120

$v_1 = 3k$, $v_2 = 4 + 5k$, $v_3 = I^k_{3k}$, $v_4 = I^k_{2+4k} + I^k_0 + 2 + k$, $v_5 = 4$
$v_6 = I^k_0 + 3$.

Let N be the MOD type III matrix associated with the graph S which is as follows.

$$N = \begin{array}{c|cccccc} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\ \hline v_1 & 0 & 1 & 0 & 0 & 0 & 0 \\ v_2 & 0 & 0 & 0 & 0 & 0 & 1 \\ v_3 & 0 & 0 & 0 & 0 & 1 & 0 \\ v_4 & 0 & 1 & 0 & 0 & 0 & 0 \\ v_5 & 0 & 0 & 0 & 1 & 0 & 0 \\ v_6 & 0 & 0 & 1 & 0 & 0 & 0 \end{array}.$$

Now we find N^2 in the following;

$$N^2 = \begin{array}{c|cccccc} & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \\ \hline v_1 & 0 & 0 & 0 & 0 & 0 & 1 \\ v_2 & 0 & 0 & 1 & 0 & 0 & 0 \\ v_3 & 0 & 0 & 0 & 1 & 0 & 0 \\ v_4 & 0 & 0 & 0 & 0 & 0 & 1 \\ v_5 & 0 & 1 & 0 & 0 & 0 & 0 \\ v_6 & 0 & 0 & 0 & 0 & 1 & 0 \end{array}.$$

Let S_2 be the type III MOD special quasi dual number natural neutrosophic directed graph associated with N^2.

This graph can be realized as the reshuffled edges of the MOD type III graph S in the following.

$S_2 =$

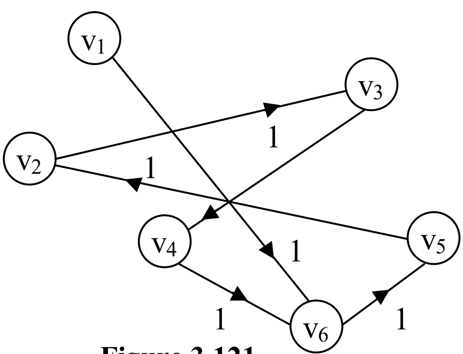


Figure 3.121

We find N^4 is the following.

$$N^4 = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let S_4 be the type III MOD natural neutrosophic special quasi dual number directed graph given by the following figure.

S_4 =

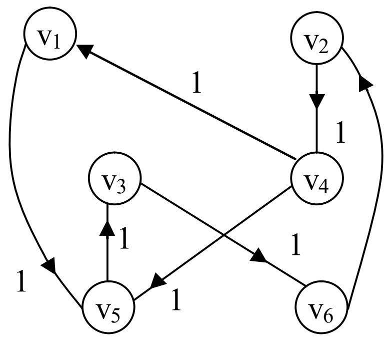


Figure 3.122

Interested reader can find all the powers of N and prove $N^t = N$ for same $t > 2$.

Now having seen all types of MOD natural neutrosophic directed graphs of type III. We now proceed onto describe MOD interval directed graphs and MOD interval natural neutrosophic directed graphs using $[0, n)$ and $^I[0, n)$ respectively.

Let G be a MOD directed graph with edge weight from $[0, n)$ or $^I[0, n)$ then we see if M is the associated matrix then

$M^t = M$ or (0) in general may not be possible further M^n for all $n \in Z^+$ may be district even as $n \to \infty$.

So these graphs may not be much useful in general applications.

We can take only subsets in ${}^I[0, n)$ which are MOD natural neutrosophic interval number which forms a finite semigroup. Only in such case we can find useful or practical applications of these graphs.

We will illustrate this situation by some examples.

Example 3.35: Let G be the MOD interval directed graph with edge weights from the set

$$S = \{ I_{0,}^{[0,6)g},\ I_{2g}^{[0,6)g},\ 1,\ I_{3g}^{[0,6)g},\ 3\ I_{1.5g}^{[0,6)g},\ I_{2.6g}^{[0,6)g} \} \subseteq {}^I[0,6)g.$$

S is closed under product. The diagram of G is as follows.

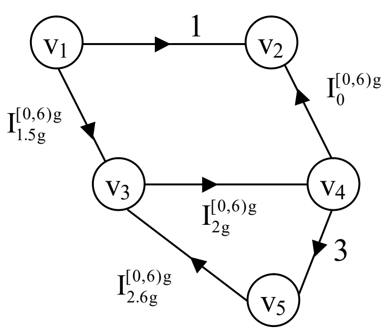
G =


Figure 3.123

The MOD matrix M associated with MOD graph is as follows:

$$M = \begin{array}{c|ccccc} & v_1 & v_2 & v_3 & v_4 & v_5 \\ \hline v_1 & 0 & 1 & I_{1.5g}^{[0,6)g} & 0 & 0 \\ v_2 & 0 & 0 & 0 & 0 & 0 \\ v_3 & 0 & 0 & 0 & I_{2g}^{[0,6)g} & 0 \\ v_4 & 0 & I_0^{[0,6)g} & 0 & 0 & 3 \\ v_5 & 0 & 0 & 0 & I_{2.6g}^{[0,6)g} & 0 \end{array}.$$

We now find M^2 in the following.

$$M^2 = \begin{array}{c|ccccc} & v_1 & v_2 & v_3 & v_4 & v_5 \\ \hline v_1 & 0 & 0 & 0 & I_0^{[0.6)g} & 0 \\ v_2 & 0 & 0 & 0 & 0 & 0 \\ v_3 & 0 & I_0^{[0,6)g} & 0 & 0 & I_{2g}^{[0,6)g} \\ v_4 & 0 & 0 & 0 & I_{2.6g}^{[0.6)g} & 0 \\ v_5 & 0 & I_0^{[0,6)g} & 0 & 0 & 0 \end{array}.$$

We can proceed onto find powers of M and the corresponding MOD interval graphs.

However we wish to keep on record the following.

i) Finding finite subsemigroup of special natural neutrosophic numbers in [0,n) other than Z_n^I happens to be a very difficult or even to be more realistic is an open conjecture.

ii) Another restriction is we may find elements like $I_2^{[0,5)}$ and $I_{2.5}^{[0,5)}$ to be natural neutrosophic zero divisors but $I_{2.5}^{[0,5)} \times I_{2.5}^{[0,5)} = I_{1.25}^{[0,5)}$ and so on. Will $I_{2.5}^{[0,5)}$, $I_{1.25}^{[0,5)}$ be torsion elements or torsion free elements of ${}^{I}[0,5)$?

So at this juncture we stop discussing about such MOD interval graphs.

We now proceed onto suggest a few problems for the interested reader.

Problems:

1. Let G be the MOD directed with edge weights from Z_8^I. Let P be the associated matrix of G.

   a) Find P^t, $t \geq 2$ and the corresponding MOD directed graphs G_t.
   b) Can $P^t = P$ for a $t > 2$?
   c) Can $P^t = (0)$ for a $t > 2$?
   d) Obtain any other special feature enjoyed by this G.

2. Let G be a MOD natural neutrosophic graph given by the following figure with vertex set from Z_7^I or by labeling.

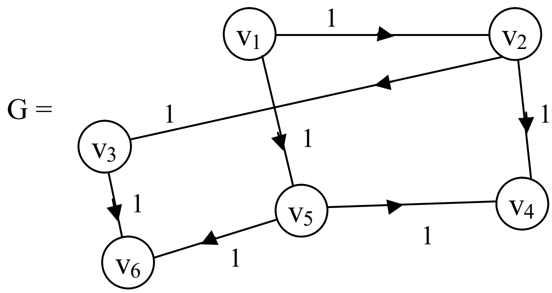


Figure 3.124

where $v_1 = 3$, $v_2 = 0$, $v_3 = 6$, $v_4 = 5$, $v_5 = 2$ and $v_6 = 1$.

i) Find all special features associated with this labeled graph.
ii) If M is matrix associated with G find M^2, M^3 and M^4 and the corresponding MOD graphs G_2, G_3 and G_4 respectively.

3. Let H be a MOD natural neutrosophic finite complex number graph with vertex set from $C^I(Z_5)$ given by the following figure, $v_i \in C(Z_5)$.

H =

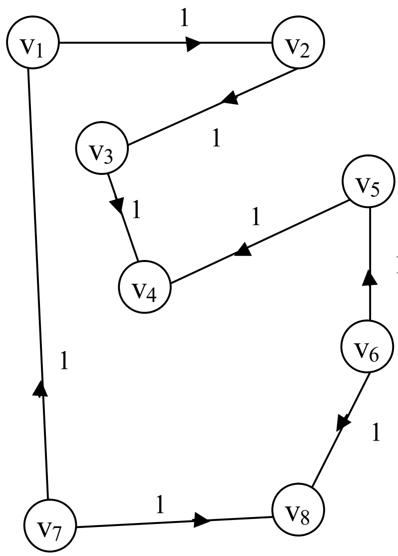


Figure 3.125

i) Find the MOD matrix M associated with H.
ii) Find M^2, M^3, M^6 and M^8 and the corresponding MOD graphs.
iii) How many distinct graph with same set of edges as that of H but different labelings from $C^I(Z_5)$ be done?

4. Let V be the MOD natural neutrosophic-neutrosophic graph with vertex set from $\langle Z_{10} \cup I\rangle_I$ $v_i \in \langle Z_{10} \cup I\rangle_I$.

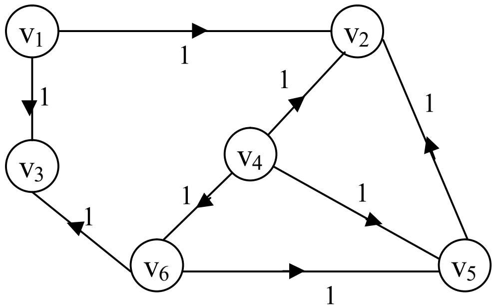
$V =$ 

Figure 3.126

Show by giving labeling from $\langle Z_{10} \cup I\rangle_I$ for V we have many special and innovative applications of these new MOD neutrosophic graphs.

i) Find the matrix M associated with V.
ii) Find M^3, M^2 and M^5 and the corresponding graphs V_3, V_2 and V_5 respectively.

5. Let M be the MOD natural neutrosophic-neutrosophic graph G with vertex set from $\langle Z_n \cup I\rangle_I$ and having t number of vertices $v_1, v_2, \ldots, v_t \in \langle Z_n \cup I\rangle_I$, $t < n$ (t fixed).

   i) How many such graphs can be constructed with same set of edges?
   ii) Study when $t = 7$.
   iii) Study when $t = 20$.
   iv) Find the $t \times t$ matrix M associated with G.
   v) Can these graphs be used in labeling indeterminate vertices?

6. Let G be the MOD natural neutrosophic dual number graph with vertex set from $\langle Z_{12} \cup I_g\rangle_I$ given by the following figure $v_i \in \langle Z_{12} \cup g\rangle_I$; $1 \leq i \leq 7$.

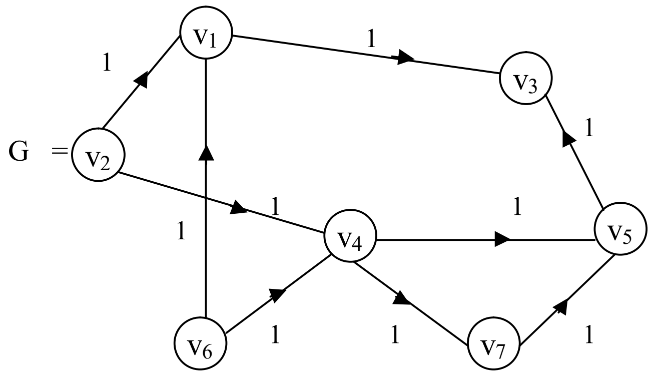


Figure 3.127

i) How many graphs can be got with varying vertices v_1, …, v_7 but edges remaining the same?
ii) Prove these MOD dual number graphs will be a powerful tool when the dual concept is involved.
iii) Find M associated with G and calculate M^2, M^3, M^4, M^6 and M^8 and the corresponding graphs.

7. Enumerate all benefits in using MOD natural neutrosophic dual number graphs with vertex set from $\langle Z_n \cup g\rangle_I$.

8. Let S be the MOD natural neutrosophic special dual like number graph with vertex set from $\langle Z_{10}^I \cup h\rangle_I$ with 9 vertices, $v_1, v_2, \ldots, v_9 \in \langle Z_n \cup h\rangle_I$ given in the following. The edge weights are taken from $\langle Z_{10}^I \cup h\rangle_I$.

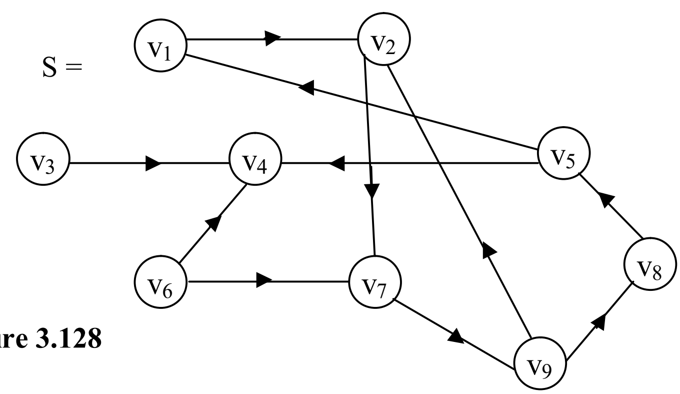


Figure 3.128

i) Find the MOD matrix P associated with S.
ii) Find P^2, P^3, P^4 and P^8 and the corresponding MOD graphs S_2, S_3, S_4 and S_8 respectively.
iii) How many labeled graphs (different) can be got using different edges weights from $\langle Z_{10}^{I} \cup h\rangle_I$?

9. Obtain all special features enjoyed by MOD natural neutrosophic special dual like number graphs with entries from $\langle Z_n \cup h\rangle_I$.

10. Let P be the MOD natural neutrosophic special quasi dual number graph with vertex set from $\langle Z_9 \cup k\rangle_I$ given by the following figure, $v_i \in \langle Z_9 \cup k\rangle_I$; with any set of edge weights from $\langle Z_9 \cup k\rangle_I$.

P =

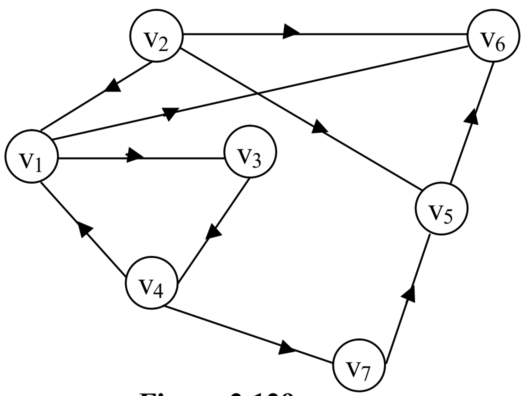


Figure 3.129

i) Find the MOD natural neutrosophic special quasi dual number matrix M associated with P.
ii) Find M^2, M^4, M^{16} and M^8 and their respective MOD graphs P_2, P_4, P_{16} and P_8.
iii) How many graphs isomorphic to P can be drawn using different sets of labeling from $\langle Z_9 \cup k\rangle_I$?

12. Let W be the MOD natural neutrosophic special quasi dual number graph given in figure 3.129 with vertex set and edge weights from $\{\langle Z_{10} \cup k\rangle_I$.

Study questions (i) to (iii) of problem (11) for this W.

13. What are the special and distinct features enjoyed by type I MOD natural neutrosophic directed graphs with edge weights from Z_n^I ?

14. Let G be the type I MOD natural neutrosophic directed graph with edge weights from Z_{10}^I given by the following figure.

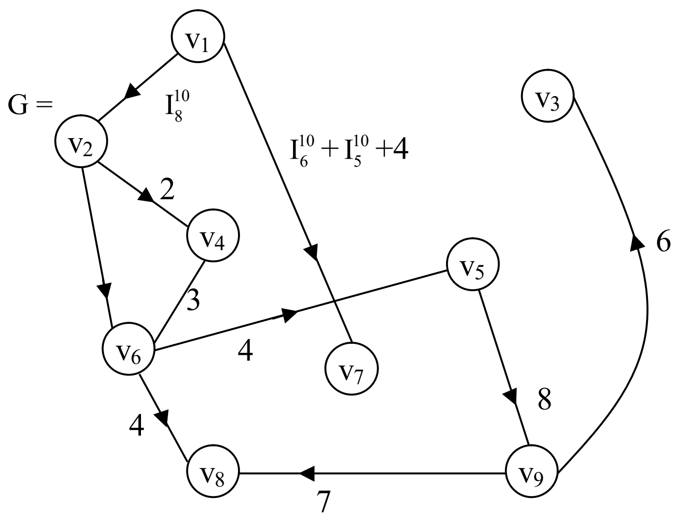


Figure 3.130

i) Find M related with G.
ii) Find M^2, M^3, …, M^t, $2 \leq t < \infty$ and find the corresponding type I MOD natural neutrosophic directed graphs G_2, G_3, …, G_t.
iii) Which of the graphs G_2, G_3, G_4, …, G_t have loops?

iv) Can we say only odd power of M have loops?
v) Can we say only even power of M have loops?
vi) Can $M^t = M$? or $M^t = (0)$ $(2 \leq t < \infty)$?

15. Does these exist a type I MOD natural neutrosophic directed graph G with edge weights from Z_n^I so that if M is the MOD matrix M associated with G then none of the type I MOD graphs associated with matrices $M^2, M^3, \ldots, M^t$, have loops?

    If so characterize them.

16. Does these exist type I MOD natural neutrosophic directed graphs G with edge weights from Z_n^I so that for M the matrix associated with G have all MOD graphs associated with matrices $M^2, M^3, \ldots, M^t$ have loops? Characterize them.

17. Let P be the type I MOD natural neutrosophic finite complex number directed graph with edge weights from $C^I(Z_8)$ given by the following figure.

P =

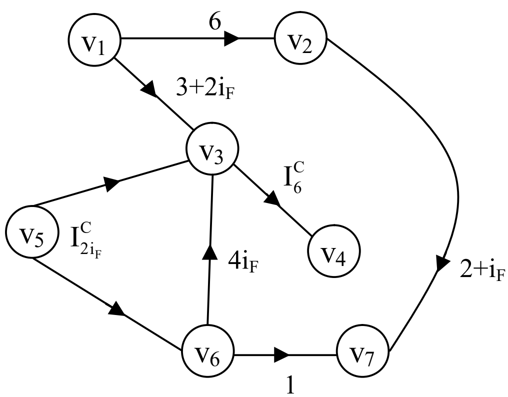


Figure 3.131

i) Find M the type I MOD natural neutrosophic matrix associated with P.

ii) If M^2, M^3, M^4, M^5 and M^6 be the type I MOD matrices find the type I MOD natural neutrosophic complex number directed graphs P_2, P_3, P_4, P_5 and P_6 respectively.
iii) Which powers of M contribute to loops?
iv) Which powers of M will have no loop?
v) Enumerate all special features enjoyed by these type I MOD graphs.

18. Let V be the type I MOD natural neutrosophic-neutrosophic directed with edge weights from $\langle Z_{12} \cup I\rangle_I$ given by the following figure:

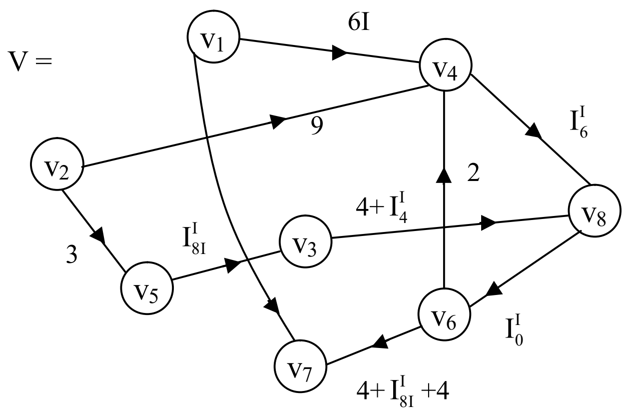


Figure 3.132

i) Find the type I MOD natural neutrosophic-neutrosophic matrix M associated with V.
ii) Find all M^t; $2 \leq t < \infty$, which has no loops associated with V_t the corresponding type I MOD graphs.
iii) Can $M^t = (0)$ for $2 \leq t < \infty$?
iv) Can $M^t = M$ for any t, $2 \leq t < \infty$?

19. Obtain all special features associated with type I MOD natural neutrosophic-neutrosophic directed graphs with edge weights from $\langle Z_n \cup I\rangle_I$.

20. Let H be the MOD natural neutrosophic dual number directed graph of type I given by the following figure with edge weights from $\langle Z_{11} \cup g\rangle_I$;

H =

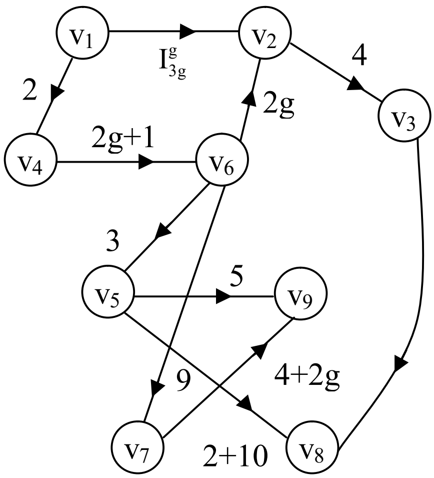


Figure 3.133

i) Let M be the type I MOD dual number matrix associated with H:
   a) Find M^2, M^3, …, M^t and the corresponding type I MOD natural neutrosophic dual number graphs H_2, H_3, …, H_t respectively.
   b) Can M^t for any t; $2 \le t < \infty$ have loops?
   c) Can M^t for some t; $2 \le t < \infty$ have only edges?

ii) Enumerate all special features enjoyed by type I MOD natural neutrosophic dual number directed graphs G with edge weights from $\langle Z_n \cup g\rangle_I$.

21. Let G be the MOD natural neutrosophic special dual like number directed graph of type I with edge weights from $\langle Z_n \cup h\rangle_I$.

    i) Obtain all special features enjoyed by M.

    ii) Can every G_t have loops? (G_t is the type I MOD graph associated with matrix M^t where M is the matrix associated with G)?

    iii) Enumerate all special features associated with G.

    iv) Study G when n = 10.

22. Let G be the type I MOD natural neutrosophic special quasi dual number directed graph with edge weights from $\langle Z_n \cup k\rangle_I$. Let M be the MOD matrix associated with G.

    Study questions (i) to (iii) of problem (21) for this G.

23. Obtain all special and distinct features associated with type II MOD natural neutrosophic directed graphs with edge weights and vertex set from Z_n^I or $\langle Z_n \cup I\rangle_I$ or $\langle Z_n \cup g\rangle_I$ or $C^I(Z_n)$ or $\langle Z_n \cup h\rangle_I$ or $\langle Z_n \cup k\rangle_I$.

24. What are the distinct properties when edge weights and labeling of vertices are from the same set?

25. Let B be the MOD natural neutrosophic directed graph of type II with edge weights and vertices from Z_9^I.

    i) If M is the matrix associated with B when will B be the such that $M^t = M$ or $M^t = (0)$.
    ii) For what values of edge weights of B. M^r will have loops r > 2.
    iii) What values of edge weight of B M^r will have no loops?
    iv) Can $M^t = M^r$ $(t \neq r)$ $2 \geq t, r$?
    v) Obtain any other special feature enjoyed by B.

26. Study questions (i) to (v) of problem (25) for the type II MOD natural neutrosophic directed graph H with edge weights from Z_{15}^{I} and $v_i \in Z_{15}^{I}$, $1 \leq i \leq 8$.

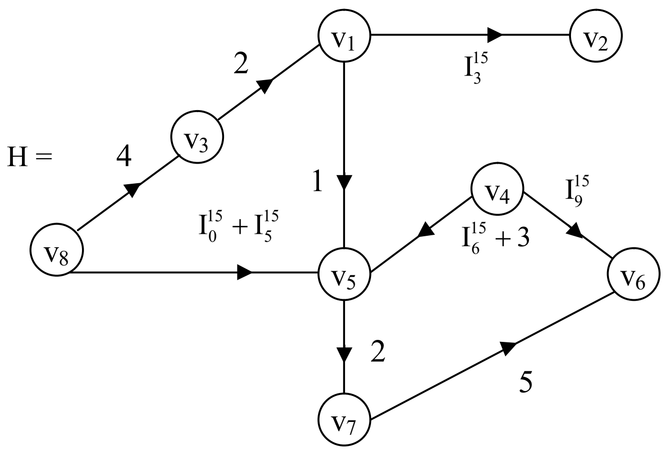


Figure 3.134

27. Obtain all special features associated with type II MOD natural neutrosophic finite complex number directed graphs with vertex set and edge weights from $C^I(Z_n)$; $2 \leq n < \infty$.

28. Let G be the type II MOD natural neutrosophic finite complex number directed graph with edge weights and vertex set from $C^I(Z_{12})$ with M the MOD matrix associated with G and G given in the following figure:

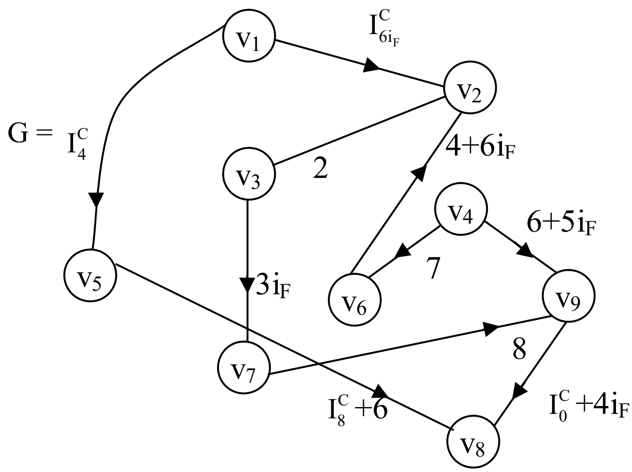


Figure 3.135

$v_i \in C^I(Z_{12})$; $1 \le i \le 9$.

i) Find M^2, M^3, M^4, M^5, M^6, M^7 and their respective type II MOD directed graphs G_2, G_3, G_4, G_5, G_6 and G_7 respectively.
   a. Which of the G_i's have loops $2 \le i \le 7$?
   b. Which of the G_i's have only edges, $2 \le i \le 7$?

ii) Can $M^t = M$; $2 \le t < \infty$?

iii) Can $M^t = (0)$; $2 \le t < \infty$?

iv) Obtain any special and interesting feature associated with this G.

27. Let V be the type II MOD natural neutrosophic-neutrosophic directed graph with edge weights and vertex set from $\langle Z_{10} \cup I\rangle_I$ given by the following figure:

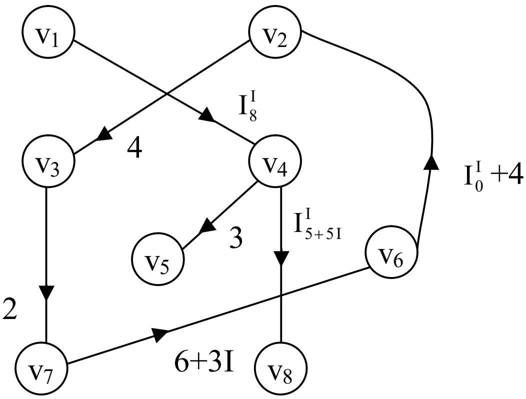
$V =$ 

Figure 3.136

Let N be the type II MOD matrix associated with V.

i) Find the least t so that the associated graph of N^t will have loops.

ii) What is the largest value of t so the associated of N^t has no edges?

iii) Can $N^t = N$ for, $2 < t < \infty$?

iv) Can $N^t = (0)$ for, $2 \leq t < \infty$?

v) Give all special features associated with this V.

30. Let X be the type II MOD natural neutrosophic dual number directed graph with edge weights and vertex set from $\langle Z_{10} \cup g\rangle_I$ given by the following figure:

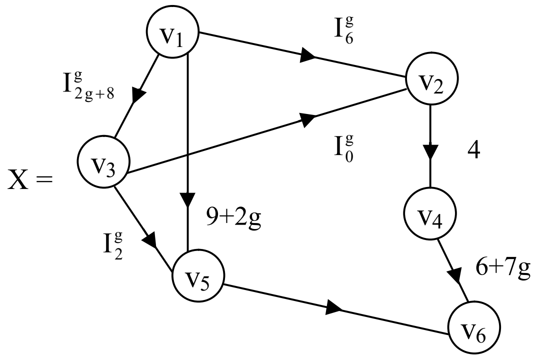


Figure 3.137

Let Y be the type II MOD matrix associated with X.

Study questions (i) to (v) of problem (29) for this X and related matrix Y.

31. Let W be the type II MOD natural neutrosophic special dual like number directed graph with edge weights and vertex set from $\langle Z_{15} \cup g\rangle_I$.

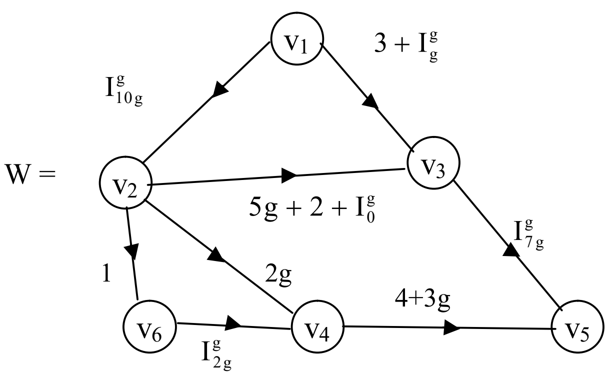


Figure 3.138

Let S be the type II MOD matrix associated with W.

i) Study questions (i) to (v) of problem (29) for this W and S.

ii) Obtain any other special property associated with type II MOD natural neutrosophic special dual like number directed graph with edge weights and vertex set from $\langle Z_n \cup h\rangle_I$.

32. Let P be the type II MOD natural neutrosophic special quasi dual number directed graph with edge weights and vertex set from $\langle Z_{18} \cup k\rangle_I$. Let L be the type II MOD matrix associated with P.
P is given by the following figure:

P =

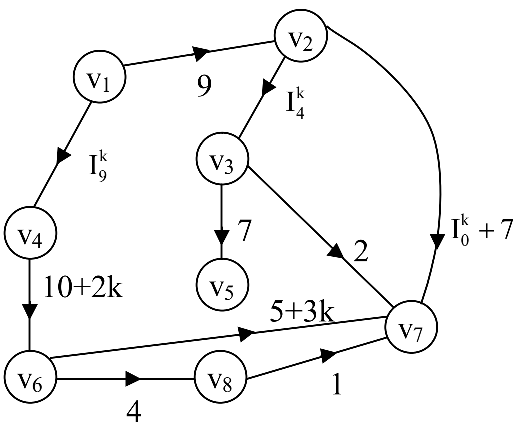


Figure 3.139

i) Study questions (i) to (v) of problem (29) for this P and L.

ii) Obtain any other special feature associated with P and L.

33. Enumerate all special and interesting features associated with type III MOD natural neutrosophic directed graph with edge weights from {0, 1} and vertex set from Z_n^I.

34. If H be the type III MOD natural neutrosophic directed graph with edge weights from {0, 1} and vertex set from Z_{10}^I given by the figure, $v_i \in Z_{10}^I$, $1 \leq i \leq 9$.

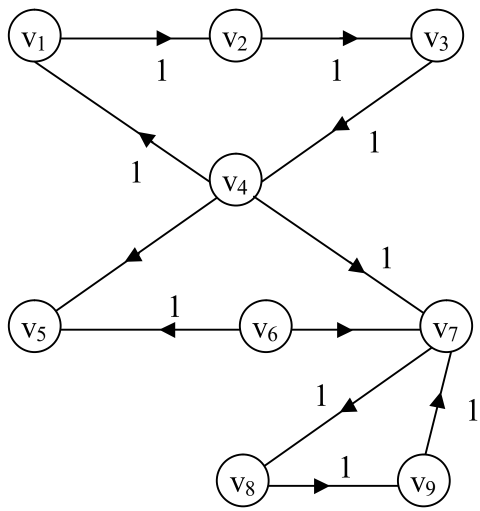
H = 

Figure 3.140

Let M be the type III MOD matrix of the graph H.

i) Can $M^t = M$ for, $2 \leq t < \infty$?

ii) Can MOD graph associated with M^t; $2 \leq t < \infty$ have loops?

iii) Obtain all special features enjoyed by M and H.

35. Let P be a type III MOD natural neutrosophic complex number directed graph with vertex set from $C^I(Z_{16})$ and edge weights from {0, 1} given by the following figure:

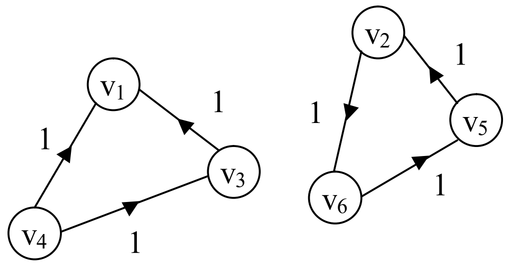
P = 

Figure 3.141

Let D be the MOD matrix associated with it.

Study questions (i) to (iii) of problem 34 in case of this P.

36. Study type III MOD natural neutrosophic-neutrosophic directed graphs and enumerate all the special features enjoyed by it.

37. Can MOD natural neutrosophic graphs be used in semi automaton?

38. Can MOD natural neutrosophic graphs using $C^I(Z_5)$ give any special features different from those constructed using Z_5^I?

39. Compare MOD graphs which are labeled using Z_n^I with those labeled with $\langle Z_n \cup I\rangle_I$. Prove these MOD graphs contribute to several types of labeling.

40. Show that all these type I MOD directed graph of natural neutrosophic entries from Z_n^I or $C^I(Z_n)$ or $\langle Z_n \cup I\rangle_I$ or $\langle Z_n \cup g\rangle_I$ or $\langle Z_n \cup h\rangle_I$ or $\langle Z_n \cup k\rangle_I$ can be used in MOD Cognitive Maps models.

i) Prove this has more advantages over FCMs and NCMs.
ii) Give any other feature which is interesting about these graphs.

41. Can these type I MOD natural neutrosophic directed graphs be used in networking?

42. Enumerate all special applications that can be made using these type I MOD natural neutrosophic directed graphs.

43. Research on MOD interval graphs with edge sets and vertex sets from [0,n) happens to be at a dormant stage (study and analyse them).

44. Find all semigroups in ${}^{I}[0,10)$ of finite order under $\times$ apart from Z_{19} and Z_{19}^{I}.

45. Can ${}^{I}[0,24)$ have finite semigroups under $\times$ apart from Z_{24} and Z_{24}^{I}?

46. Find all semigroups of finite order in $C^{I}[0,13)$. Using those elements of these semigroups construct type II MOD natural neutrosophic interval finite complex number directed graphs.

47. For ${}^{I}[0, 7)$ do we have finite order subsemigroups under product other than Z_7 and Z_7^{I}

48. Obtain all special and innovative properties enjoyed by MOD type II natural neutrosophic dual number directed graphs with edge weights and vertex set from $\langle Z_{15} \cup g\rangle_I$ with MOD type II natural neutrosophic special quasi dual number directed graphs with edge weights from $\langle Z_{15} \cup k\rangle_I$.

49. Specify the difference between type I MOD natural neutrosophic directed graphs using Z_{19}^{I} and type II MOD natural neutrosophic directed graphs using Z_{19}^{I}.

50. Compare type I MOD natural neutrosophic-neutrosophic directed graphs using $\langle Z_{12} \cup I\rangle_I$ and type III MOD natural neutrosophic neutrosophic directed graphs.

51. Classify and compare the special features of using vertex set from Z_n^I or $\langle Z_n \cup I\rangle_I$ for labeling.

52. Prove the method of labeling using $C^I(Z_n)$ or $\langle Z_n \cup g\rangle_I$ are much more advantageous in comparison with labeling with natural numbers.

Chapter Four

MOD BIPARTITE GRAPHS

In this chapter we for the first time carry out a systematic study of the MOD bipartite graphs built using Z_n for edge weights. MOD dual number bipartite graphs using $\langle Z_n \cup g\rangle$, for edge weights.

MOD special dual like number bipartite graph with edge weights from $\langle Z_n \cup h\rangle$.

MOD special quasi dual number bipartite graph with edge weights from $\langle Z_n \cup k\rangle$.

Likewise in case using edge weights from $C(Z_n)$ and $\langle Z_n \cup I\rangle$, we call them as MOD finite complex number edge weights bipartite graph and MOD neutrosophic edge weights bipartite graph respectively.

All these will be described by examples.

Example 4.1: Let G be a MOD bipartite graph with edge weights from Z_9 given by the following figure.

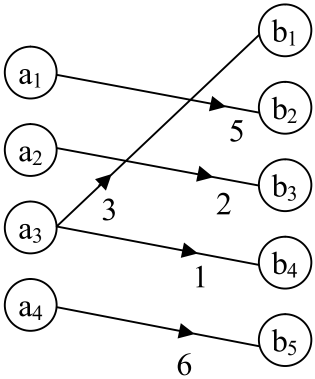


Figure 4.1

Example 4.2: Let H be the bipartite directed graph with edge weights from Z_{11}.

H is a MOD bipartite directed graph given by the following figure.

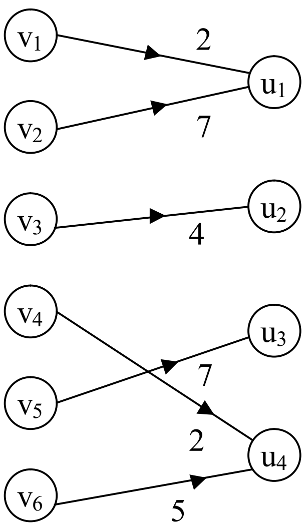


Figure 4.2

Example 4.3: Let S be the MOD directed bipartite graph with edge weights from Z_4 given by the following figure:

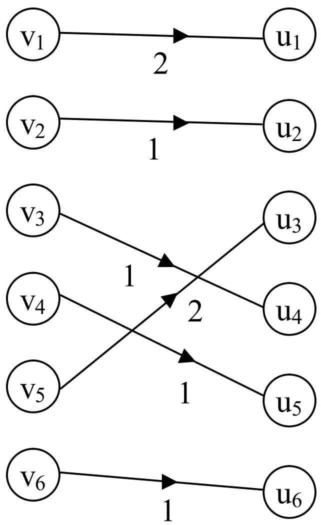


Figure 4.3

Let G be the bipartite graph with edge weights from Z_n.

We define G to be the MOD bipartite directed graph.

We describe MOD rectangular matrix given in the following.

Example 4.4: Let

$$M_1 = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \\ a_{17} & a_{18} & a_{19} & a_{20} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{bmatrix} \text{ where } a_i \in Z_{12};\ 1 \leq i \leq 24;$$

M is MOD rectangular matrix.

Example 4.5: Let

$$S = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 \\ a_6 & a_7 & a_8 & a_9 & a_{10} \\ a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \end{bmatrix}$$

be the MOD rectangular matrix with entries $a_i \in Z_{23}$, $1 \leq i \leq 15$.

We will describe the MOD connection matrix associated with MOD bipartite directed graphs by some examples.

Example 4.6: Let G be the MOD bipartite directed graph with entries from Z_{10} given by the following figure.

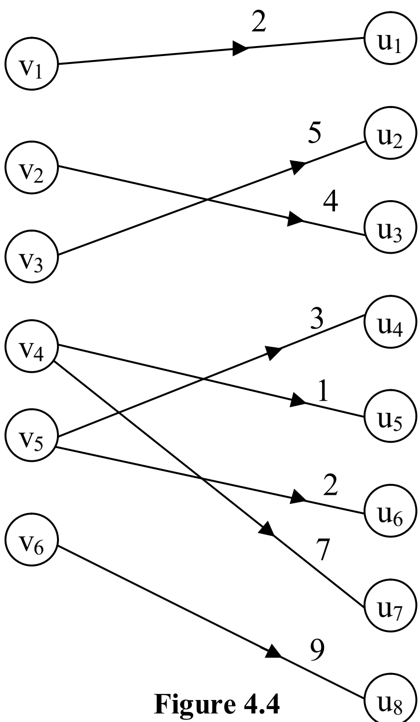


Figure 4.4

Let M be the MOD connection matrix or relational matrix associated with G given in the following

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array}\begin{array}{c} \begin{array}{cccccccc} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 & u_7 & u_8 \end{array} \\ \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 7 & 0 \\ 0 & 0 & 0 & 3 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 \end{bmatrix} \end{array}.$$

Example 4.7: Let H be the MOD directed bipartite graph with edge weights from Z_7 given by the figure. Let N be the MOD relational matrix associated with H is given the following.

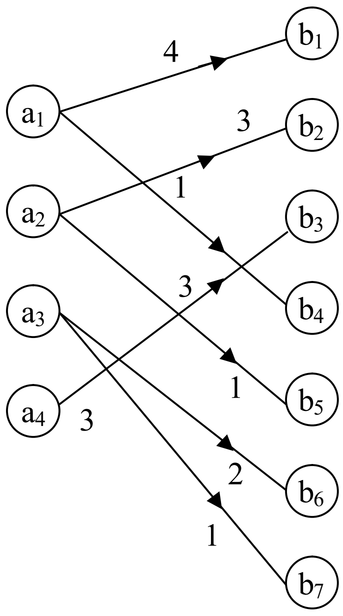


Figure 4.5

Let N be the MOD relational matrix

$$N = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \\ a_4 \end{array} \begin{array}{c} \begin{array}{ccccccc} b_1 & b_2 & b_3 & b_4 & b_5 & b_6 & b_7 \end{array} \\ \begin{bmatrix} 4 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 3 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}.$$

Let G be the MOD directed bipartite graph with edge weights from $\langle Z_n \cup g\rangle$; G is defined as the MOD directed bipartite dual number graph.

We give examples of them.

Example 4.8: Let G be the MOD directed dual number bipartite graph given by the following figure with edge weights from $\langle Z_6 \cup g\rangle$

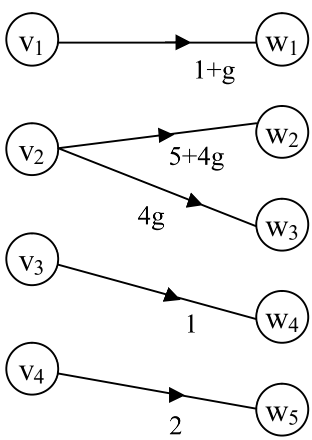


Figure 4.6

Example 4.9: Let H be the MOD directed bipartite graph with edge weights from $\langle Z_{13} \cup g\rangle$ given by the following figure.

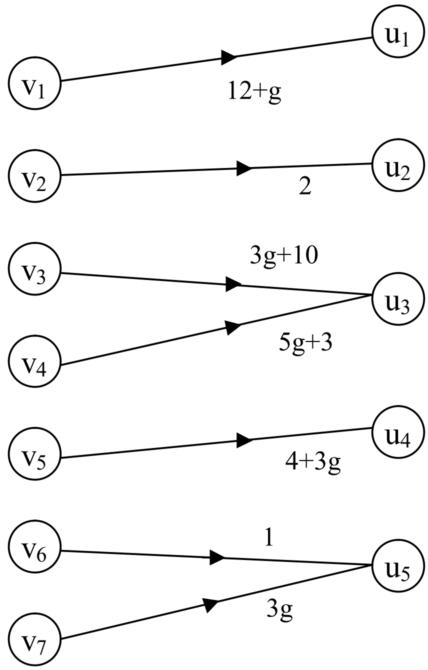


Figure 4.7

Example 4.10: Let W be a MOD directed bipartite graph with edge weights from $C(Z_9)$.

We call W as the MOD directed finite complex number graph which is given by the following figure:

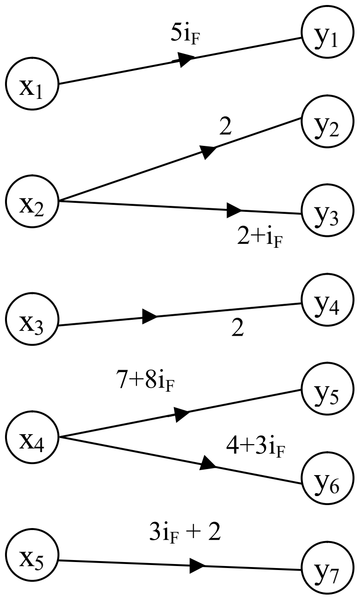


Figure 4.8

Thus if G be a MOD bipartite directed graph which takes its edge weights from $C(Z_n)$ then we define G to be a MOD bipartite directed finite complex number graph.

We will give one more example of this.

Example 4.11: Let G be the MOD bipartite directed graph with edge weights from $C(Z_n)$ given by the following figure:

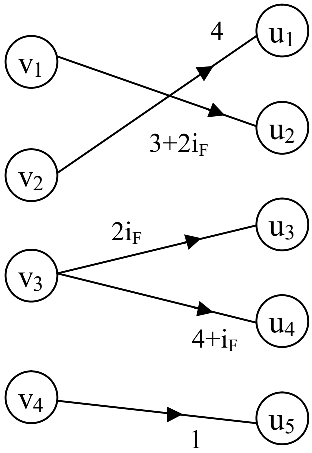


Figure 4.9

Next we proceed onto describe MOD bipartite directed neutrosophic graphs.

Let G be the MOD bipartite directed graph.

If G takes the edge values from $\langle Z_n \cup I\rangle$ then we define G to be a MOD bipartite directed neutrosophic graph.

We will give one or two examples of MOD directed bipartite neutrosohic graph in the following.

Example 4.12: Let G be a MOD directed bipartite graph with edge weights from $\langle Z_6 \cup I\rangle$ given by the following figure:

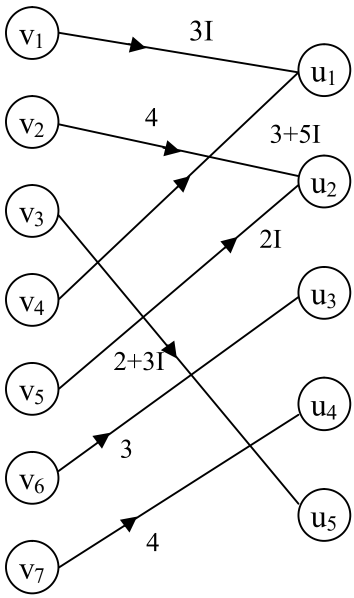


Figure 4.10

Example 4.13: Let G be the MOD directed neutrosophic bipartite graph with edge weights from $\langle Z_7 \cup I\rangle$ given by following figure:

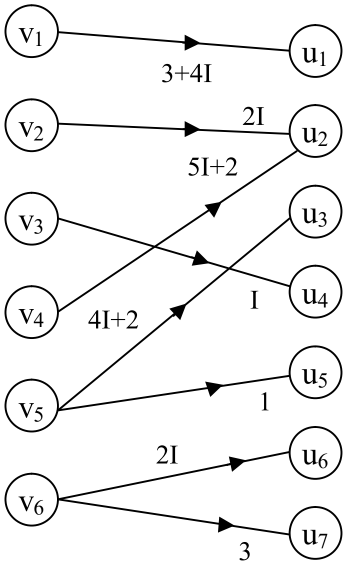


Figure 4.11

Next we proceed onto describe the notion of MOD bipartite special dual like number directed graph.

Let G be a MOD directed bipartite graph with edge weights from $\langle Z_n \cup h\rangle$.

Then we define G to be a MOD bipartite directed special dual like number graph.

We will describe this situation by some examples.

Example 4.14: Let S be the MOD directed bipartite special dual like number graph with edge weights from $\langle Z_{10} \cup h\rangle$ given by the following figure:

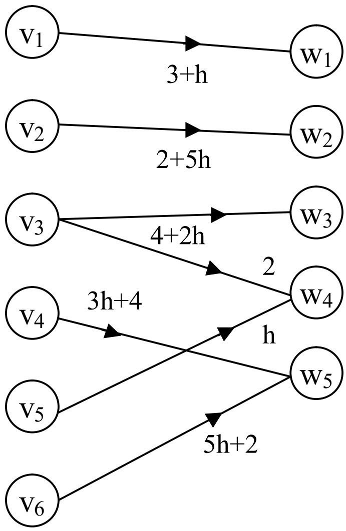


Figure 4.12

Example 4.15: Let P be the MOD directed bipartite special dual like number graph given by the following figure with edge weights from $\langle Z_8 \cup h\rangle$.

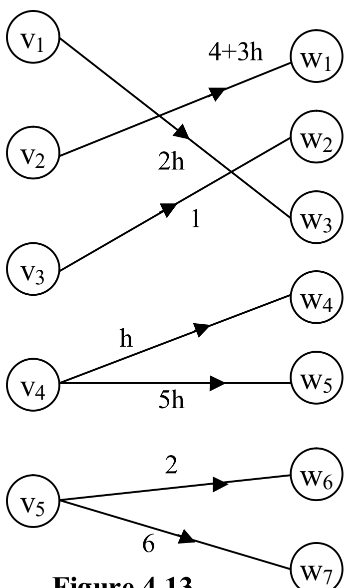


Figure 4.13

Next we just give examples of a MOD bipartite directed special quasi dual number graph in the following.

Example 4.16: Let G be a MOD bipartite directed graph with edge weights from $\langle Z_{12} \cup k\rangle$ given by the following figure.

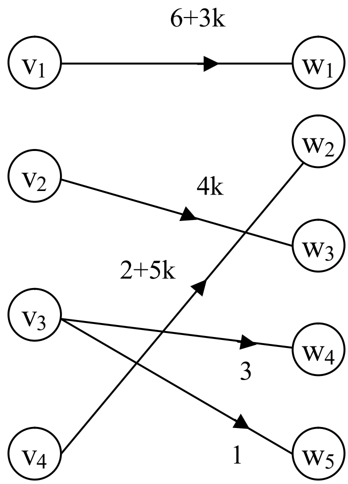


Figure 4.14

Thus if G a MOD directed bipartite graph with edge weights from $\langle Z_n \cup k\rangle$ then we define G to be a MOD directed bipartite special quasi dual number graphs.

Example 4.17: Let G be a MOD directed bipartite graph with edge weights from $\langle Z_7 \cup k\rangle$ given by the following figure.

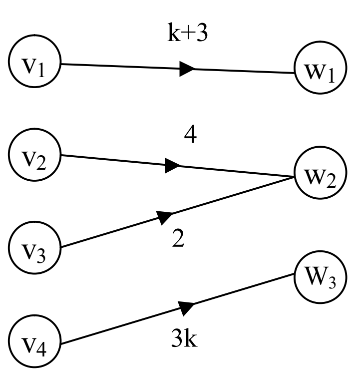


Figure 4.15

Now having seen MOD bipartite graphs with edge weights from different sets we can define any MOD n-partite graph $n \geq 3$ with edge weights from Z_n or $\langle Z_n \cup g\rangle$ or $\langle Z_n \cup h\rangle$ or $\langle Z_n \cup k\rangle$ or $C(Z_n)$ or $\langle Z_n \cup I\rangle$.

These situations will be appropriately denoted by some examples.

Just we give one example of a special type of MOD linked graph.

Example 4.18: Let K be a MOD 3 linked graph with edge weights from Z_{10} given by the following figure:

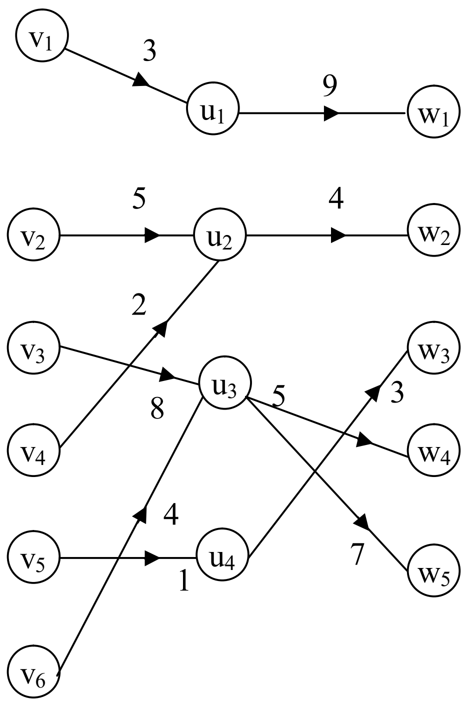


Figure 4.16

We see this is a special type of MOD 3-linked graph.

We can howeer have any MOD n-linked graph $n \geq 3$.

We now describe a MOD n-partite graph $n \geq 3$ by some example.

Example 4.19: Let G be a MOD 3 partite graph with edge weights from Z_{15} by the following figure:

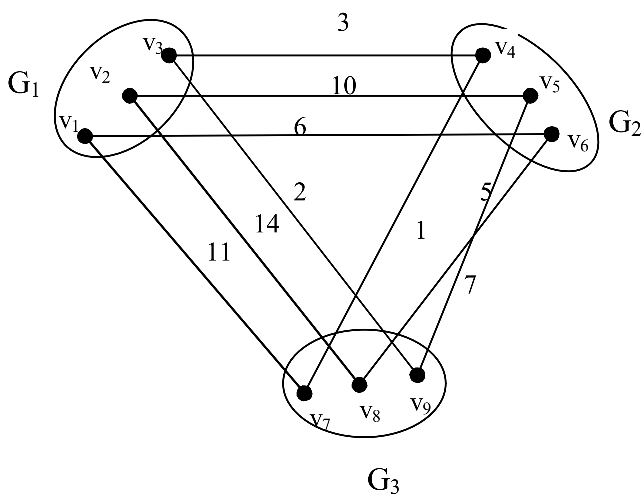


Figure 4.17

Example 4.20: Let G be the MOD 4 partite graph given by the following figure with edge weights from Z_{12}.

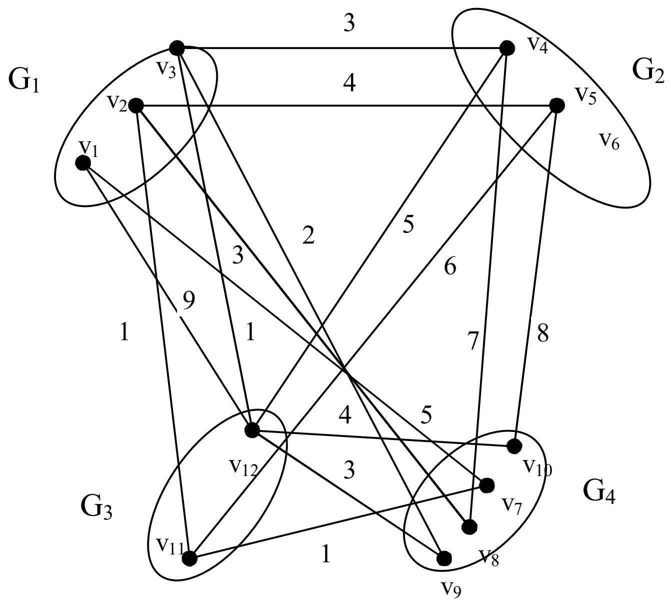


Figure 4.18

Now in the case MOD n partite graph we have an advantage $n \geq 3$.

For we can use more than one edge weight set.

We will illustrate this situation from the following example.

Example 4.21: Let G be a MOD 3-partite mixed edge weight graph given by the following figure.

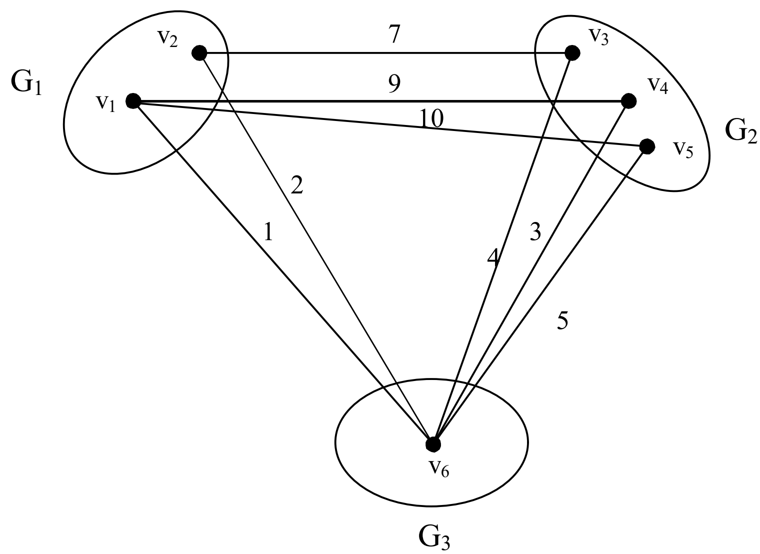


Figure 4.19

The edge weights from G_1 to G_2 are from Z_{11}, the edge weights from G_1 to G_3 are from Z_3 and the edge weights from G_2 to G_3 are from Z_6.

Thus one of the uniqueness of these MOD 3-partite mixed edge weights graphs is for the MOD 3-partite graph the edge weights can maximum be taken from 3 sets.

In this case the three sets are Z_{11}, Z_3 and Z_6.

We give one more example of this situation.

Example 4.22: Let G be a MOD 4-partite mixed edge weights graph given by the following figure:

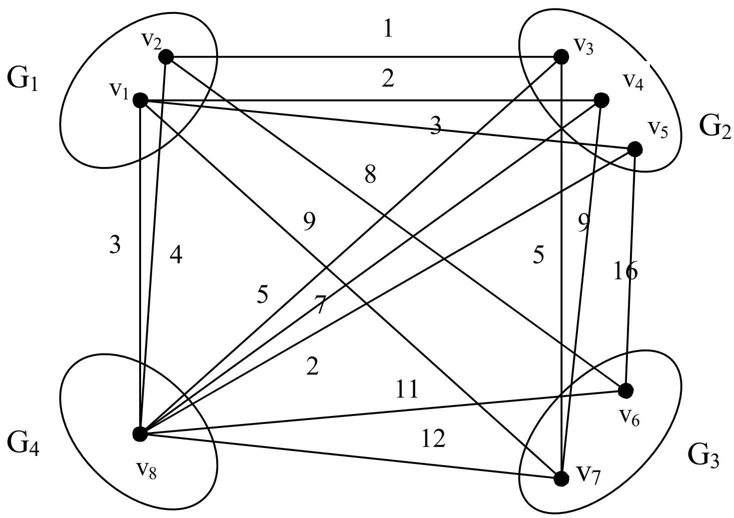


Figure 4.20

The edge weights from the six sets Z_{10}, Z_4, Z_5, Z_8, Z_{13} and Z_{17}.

Here for G_1 to G_3 the edge weights are from Z_{10}, from G_1 to G_2 the edge weights are from Z_4 from G_1 to G_4 the edge weights are from Z_5 from G_4 to G_2 the edge weights are from Z_8 from G_4 to G_3 edge weights are from Z_{13} and the edge weights are from Z_{17} for G_2 to G_3.

This is the way MOD 4-partite mixed edge weights graphs are constructed.

Now it is important to note that we can for some sets G_i to G_j have the same edge weight sets so that we need not have each of the sets to have distinct edge weights.

This situation is illustrated by an example or two.

Example 4.23: Let G be the MOD 3 partite mixed edge weights graph which is given by the following figure:

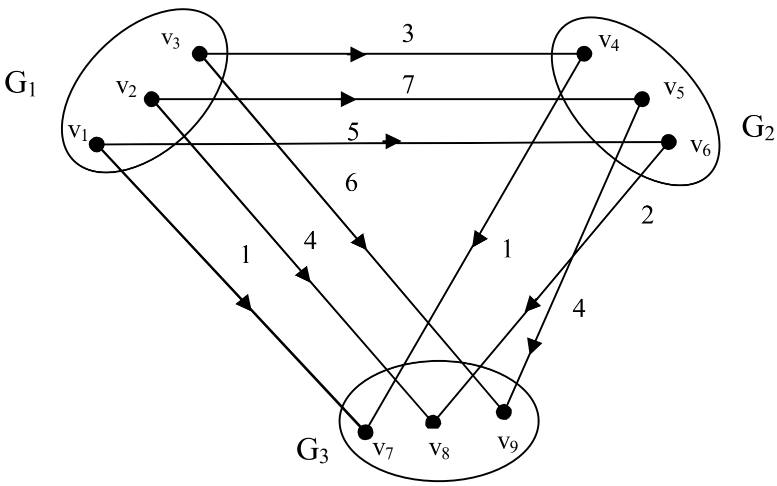


Figure 4.21

The edge weights of G_1 to G_3 is from Z_8, the edge weights of G_1 to G_2 is also from the set Z_8 and the edge weights from G_2 to G_3 is from Z_5.

Thus we see for G_1 to G_3 and G_1 to G_2 the edge weights are from the same set Z_8 where as for G_2 to G_3 the edge weights are from Z_5.

Thus we see a MOD 3-partite mixed edge weights can have the edge weights only from two sets Z_8 and Z_5.

We give yet one more example.

Example 4.24: Let G be the MOD 4-partite mixed edge weights graph given by the following figure:

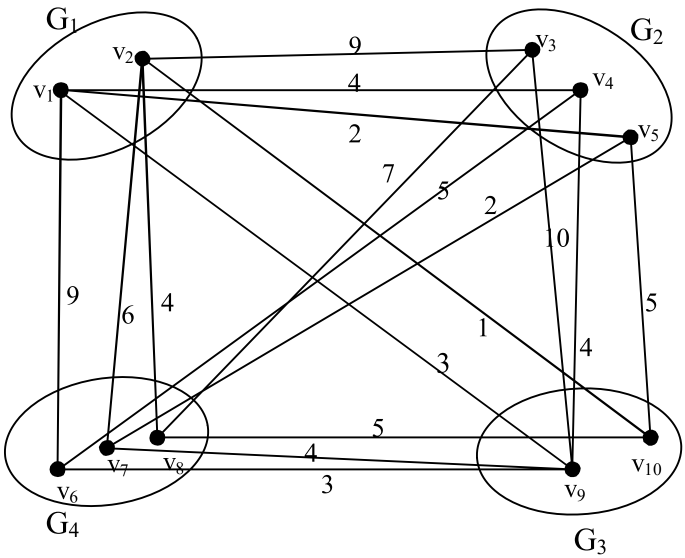


Figure 4.22

The edge weights from G_1 to G_2 are taken from Z_{10}.

The edge weights from G_1 to G_3 and G_2 to G_3 are also taken from Z_{11} whereas the edge weights from G_3 to G_4 and G_1 to G_4 are taken from Z_{11}.

The edge weights of G_2 to G_4 are taken from Z_{10}.

Thus the edge weights are from Z_6, Z_{11} and Z_{10}.

Thus if G a MOD n-partite graph we can have atlest two edge weight sets then we define G to be a MOD n-partite mixed edge weight graph.

Next we proceed onto show the MOD n-partite graphs can have edge weights from Z_n and $\langle Z_n \cup I\rangle$ and $C(Z_n)$ and so on.

We as a misnomer call them also as MOD n-partite mixed edge graphs only.

We will illustrate this situation by an example or two.

Example 4.25: Let G be a MOD 4-partite mixed edge weights graphs given by the following figure:

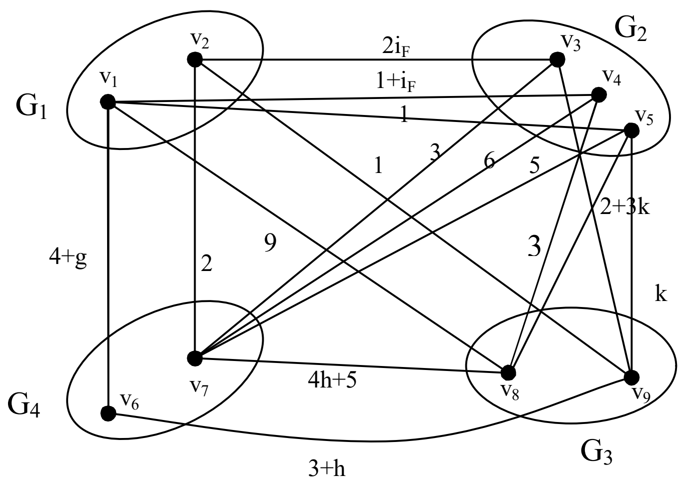


Figure 4.23

We see G_1 to G_4 takes the edge weights from $\langle Z_5 \cup g\rangle$.

The edge weights are taken from $C(Z_3)$ from G_1 to G_2. The edge weights from G_2 to G_3 are from $\langle Z_6 \cup k\rangle$.

From G_4 to G_3 the edge weights are taken from $\langle Z_8 \cup h\rangle$.

From G_2 to G_4 edge weights are taken from Z_7.

For G_1 to G_3 the edge weights are taken from Z_{12}.

We give yet another example of the MOD 3-partite mixed edge weights graph given by the following figure.

Example 4.26: Let G be the MOD 3-partite mixed edge weights graph given by the following figure.

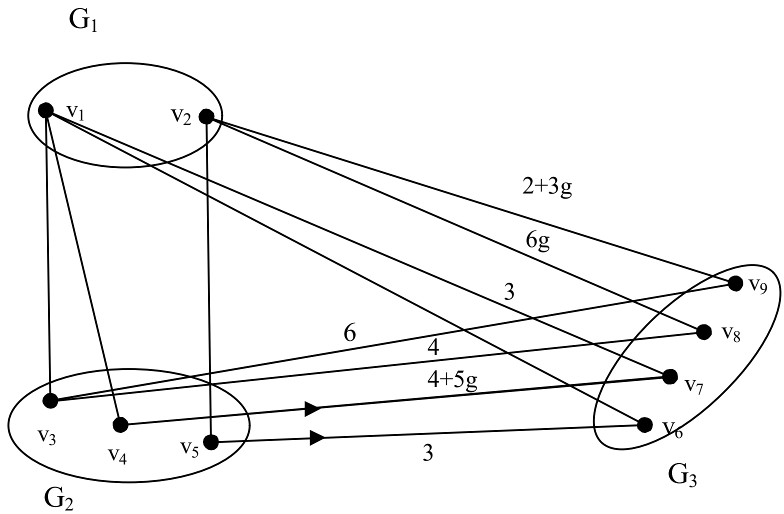


Figure 4.24

The edge weights are from $C(Z_9)$ and $\langle Z_8 \cup h\rangle$. From G_1 to G_2 the edge weights from $C(Z_9)$.

From G_1 to G_3 and G_2 to G_4 the edge weights are from $\langle Z_8 \cup g\rangle$.

Example 4.27: Let G be the MOD 5 partite mixed edge weight graph given by the following figure:

The edge weights are from $C(Z_3)$, $C(Z_7)$, $\langle Z_6 \cup g\rangle$, $\langle Z_{12} \cup h\rangle$ and $\langle Z_{12} \cup I\rangle$ given by the following figure:

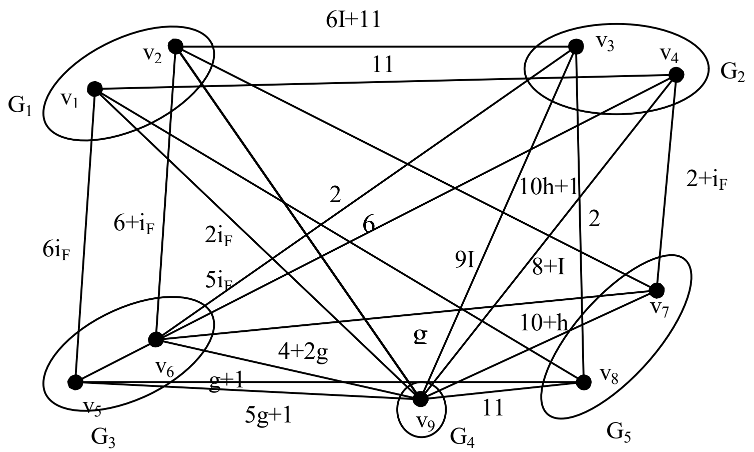


Figure 4.25

The edge weights from G_1 to G_2 is from $\langle Z_{12} \cup I\rangle$.

The edge weights from G_1 to G_3 is from $C(Z_7)$.

The edge weights from G_1 to G_4 is from $C(Z_3)$. The edge weights from G_1 to G_5 are taken from $\langle Z_{12} \cup h\rangle$.

The edge weights from G_2 to G_5 is taken from $C(Z_3)$.

The edge weights from G_2 to G_4 are taken from $\langle Z_{12} \cup I\rangle$. The edge weights from G_2 to G_3 are taken from $C(Z_7)$.

The edge weights from G_3 to G_4 are taken from $\langle Z_6 \cup g\rangle$.

The edge weight from G_3 to G_5 are taken from $\langle Z_6 \cup g\rangle$. The edge weights from G_4 to G_5 are taken from $\langle Z_{12} \cup h\rangle$.

Thus one can get many such examples.

Clearly this innovative idea of MOD n-partite mixed edge weights graph will be a boon to researchers in this area.

Next we proceed onto describe and develop the notion of MOD bipartite graphs of type I where the edge weights as well as the verties are from Z_n or $C(Z_n)$ or $\langle Z_n \cup I\rangle$ or $\langle Z_n \cup g\rangle$ or $\langle Z_n \cup h\rangle$ or $\langle Z_n \cup k\rangle$ or used in the mutually exclusive sense by some examples.

Example 4.28: Let G be a MOD bipartite graph of type I with edge weights and vertex set from Z_{15} given by the following figure:

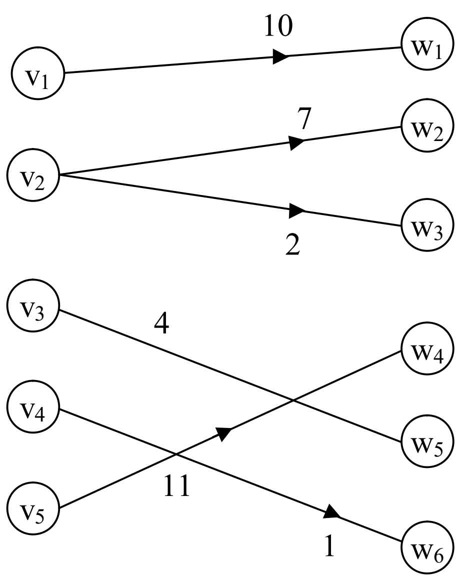


Figure 4.26

$v_1, v_2, v_3, v_4, v_5 \in Z_{15}$ and w_1, w_2, w_3, w_4, w_5 and $w_6 \in Z_{15} \setminus \{v_1, v_2, v_3, v_4, v_5\}$.

The min demand in this case is
$\{v_1, \ldots, v_5\} \cap \{w_1, \ldots, w_6\} = \phi$ and both are subsets of Z_{15}.

We give yet another example.

Example 4.29: Let G be the MOD bipartite graph of type I with edge weights and vertex set from Z_9 given by the following figure.

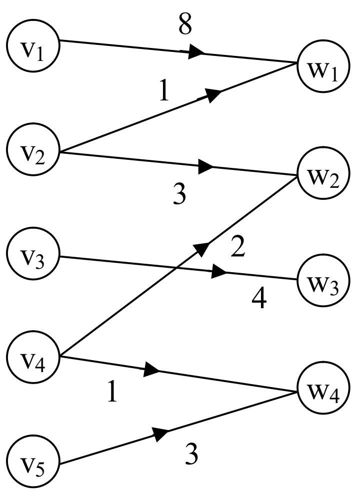


Figure 4.27

where $v_1 = 0$, $v_2 = 4$, $v_3 = 5$, $v_4 = 6$ and $v_5 = 7$ all elements of Z_9.

$w_1 = 2$ $w_2 = 2$, $w_3 = 3$ and $w_4 = 8$ all are elements of Z_9.

It is pertinent to record that the edge weights can be the same for any pairs of vertices.

This is evident from the MOD graph in the figure 4.27.

Next we provide examples of MOD bipartite finite complex number graphs of type I.

Example 4.30: Let G be a MOD bipartite finite complex number graph of type I with edge weights and vertex set from $C(Z_5)$ given by the following figure:

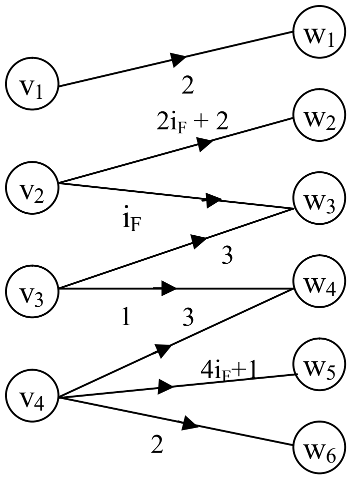


Figure 4.28

$v_1 = 2 + i_F$, $v_2 = 0$, $v_3 = 4$ and $v_4 = 1$

$w_1 = 2 + 4i_F$, $w_2 = 3$, $w_3 = 4i_F$, $w_4 = i_F$, $w_5 = 2i_F$ and $w_6 = 1 + i_F$

Clearly $\{v_1, v_2, v_3, v_4\} \cap \{w_1, w_2, w_3, w_4, w_5, w_6\} = \phi$ and both are subsets of $C(Z_5)$.

Example 4.31: Let G be a MOD bipartite finite complex number graph of type I with edge weights from $C(Z_{12})$ and vertex set from $C(Z_{12})$ given by the following figure:

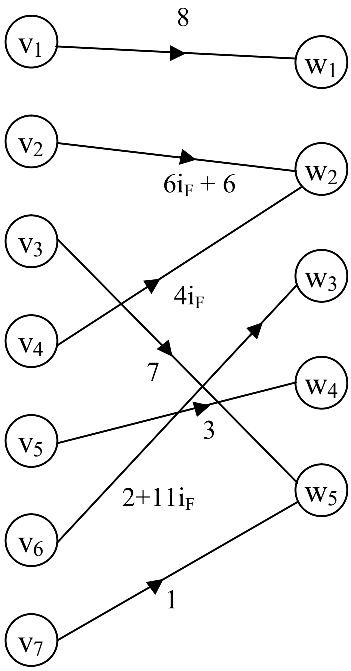


Figure 4.29

$v_i \in Z_{12}$; $1 \leq i \leq 7$ and $w_j \in C(Z_{12}) \setminus Z_{12}$; $1 \leq j \leq 5$.

Next we proceed onto describe the MOD bipartite neutrosophic graph of type I by some examples.

Example 4.32: Let G be a MOD bipartite neutrosophic graph of type I with edge weights and vertex set from $\langle Z_{10} \cup I\rangle$ given by the following figure:

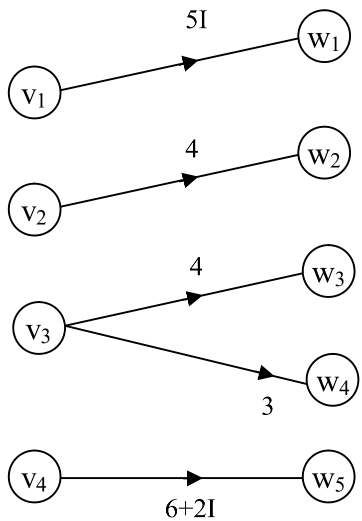


Figure 4.30

$v_i \in Z_{10}I$; $1 \leq i \leq 4$ and $w_j \in \langle Z_{10} \cup I\rangle \setminus Z_{10}I$; $1 \leq j \leq 5$.

Example 4.33: Let G be a bipartite neutrosophic graph of type I given by the following figure with edge weights and vertex set from $\langle Z_{17} \cup I\rangle$.

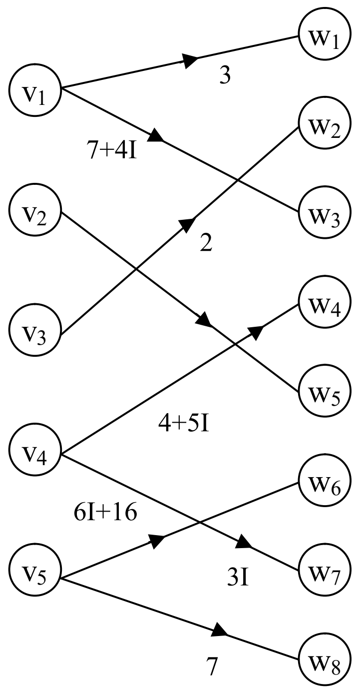


Figure 4.31

$v_i \in Z_{17}$ and $w_j \in Z_{17}I$; $1 \leq i \leq 5$ and $1 \leq j \leq 8$.

Thus if G is a MOD bipartite graph type I with edge weights and vertex set entries from $\langle Z_n \cup I\rangle$ then we define G to be a MOD bipartite neutrosophic graph of type I.

Next we proceed onto describe MOD bipartite dual number graphs of type I the following example.

Example 4.34: Let G be a bipartite dual number graph of type I with edge weights from $\langle Z_{16} \cup g\rangle$ and vertex set from $\langle Z_{16} \cup g\rangle$ given by the following figure:

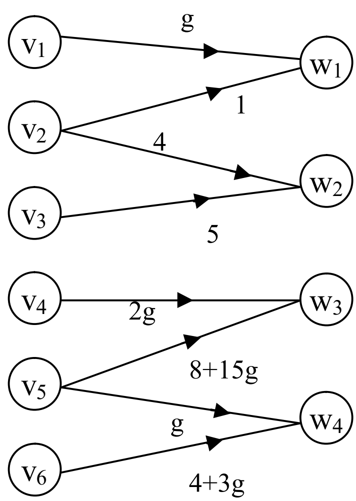


Figure 4.32

$v_i \in Z_{16}g$ and $w_j \in \langle Z_{16} \cup g\rangle \setminus Z_{16}g$, $1 \leq i \leq 6$ and $1 \leq j \leq 4$.

Example 4.35: Let G be a MOD bipartite dual number graph of type I with edge weight and vertex set from $\langle Z_{11} \cup g\rangle$ given by the following figure:

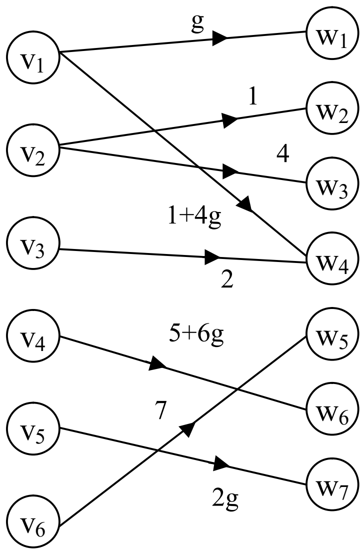


Figure 4.33

$V_i \in Z_{11}$ and $w_j \in \langle Z_{11} \cup g\rangle \setminus Z_{11}$ $1 \leq i \leq 6$ and $1 \leq j \leq 7$.

Thus if G is a MOD directed bipartite graph of type I with edge weights and vertex set from $\langle Z_n \cup g\rangle$ then we define G to be a MOD bipartite dual number graph of type I we have seen examples of them.

Now we proceed onto describe by examples MOD bipartite special dual like number graph of type I and MOD bipartite special quasi dual number graph of type I.

The definition of these graphs can be made analogous to MOD directed bipartite dual number graphs of type I.

Example 4.36: Let H be a MOD bipartite special dual like number graph of type I with edge weights and vertex set from $\langle Z_{10} \cup h\rangle$ given by the following figure:

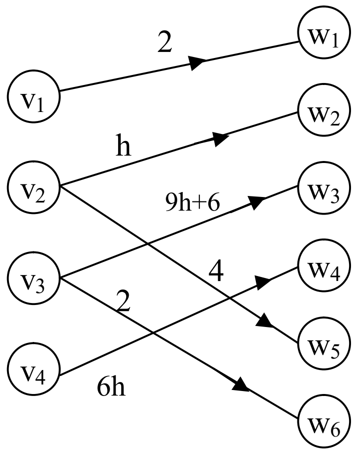


Figure 4.34

$v_i \in Z_{10}h$ and $w_j \in \langle Z_{10} \cup h\rangle \setminus Z_{10}h$ $1 \leq i \leq 4$ and $1 \leq j \leq 6$.

Example 4.37: Let G be a MOD bipartite special quasi dual number graph of type I with edge weights from $\langle Z_{12} \cup k\rangle$ given by the following figure:

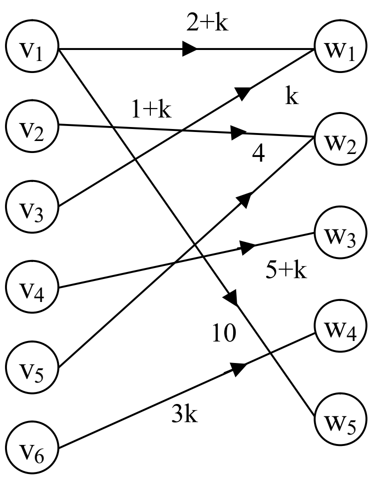


Figure 4.35

The edge weights are from $\langle Z_{12} \cup k\rangle$ and the vertices v_i's takes their values from Z_{12} and w_j take their values from $Z_{12}k$; $1 \leq i \leq 6$ and $1 \leq j \leq 5$.

Interested reader can study these structures with more examples and apply them to MOD RMs models of different types discussed in [69].

Now we proceed onto describe MOD bipartite graphs of type II. Clearly in case of MOD bipartite graphs of type II they take vertexes from any two different sets and the edge weights from any one of those two sets.

We will first illustrate this situation by some examples.

Example 4.38: Let G be the MOD bipartite graph of type II with edge weight from Z_{10} and vertex sets from Z_{10} and Z_7 given by the following figure:

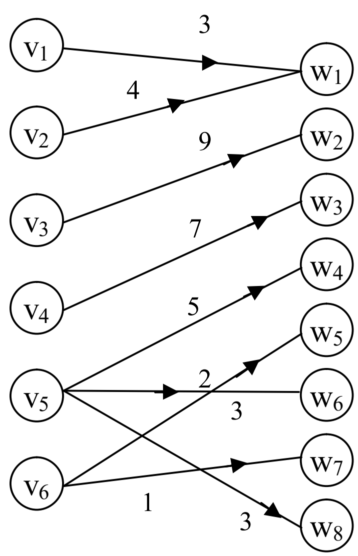


Figure 4.36

$v_i \in Z_0$; $1 \leq i \leq 6$ and $w_j \in Z_7$ $1 \leq j \leq 8$.

Example 4.39: Let H be the MOD bipartite graphof type II with vertex weights from the sets Z_9 and Z_{15} and edge weights from Z_{15} given by the following figure:

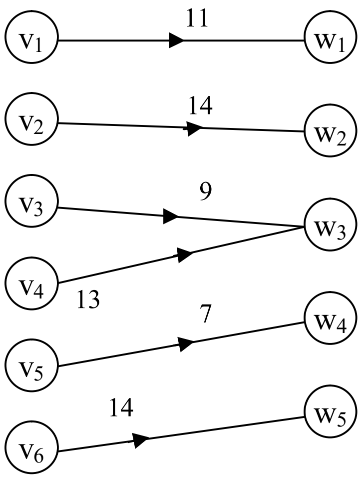


Figure 4.37

$v_i \in Z_9$ and $w_j \in Z_{15}$; $1 \leq i \leq 6$ and $1 \leq j \leq 5$.

Example 4.40: Let V be the MOD bipartite graph of type II with vertex sets from Z_{11} and $C(Z_5)$ and edge sets from Z_{11} given by the following figure:

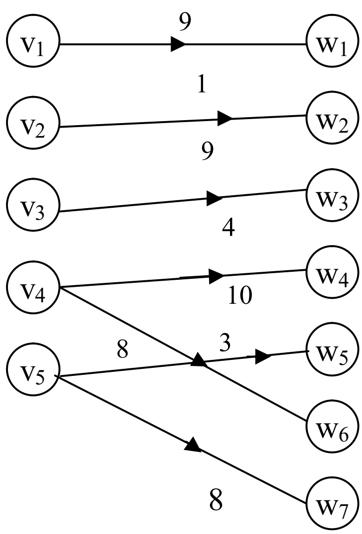


Figure 4.38

$v_i \in Z_{11}$ and $w_j \in C(Z_5)$ $1 \leq i \leq 5$ and $1 \leq j \leq 7$.

Example 4.41: Let G be a MOD bipartite graph of type II with edge weights from $\langle Z_9 \cup g\rangle$ and vertex sets from $\langle Z_8 \cup I\rangle$ and $\langle Z_9 \cup g\rangle$ given by the following figure:

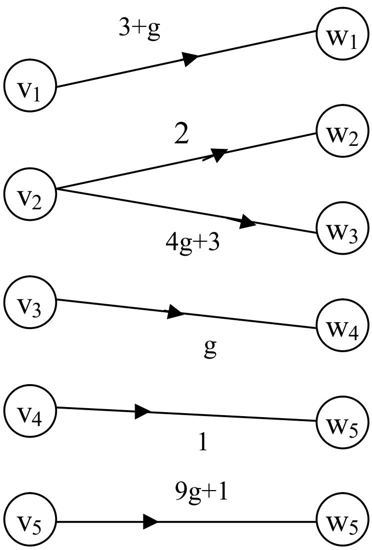


Figure 4.39

$v_i \in \langle Z_0 \cup g\rangle$; $1 \leq i \leq 5$ and $w_j \in \langle Z_8 \cup I\rangle$; $1 \leq i \leq 6$.

Example 4.42: Let S be the MOD bipartite graph of type II with vertex sets from $\langle Z_5 \cup k\rangle$ and $C(Z_{10})$ and edge sets from $C(Z_{10})$ given by the following figure:

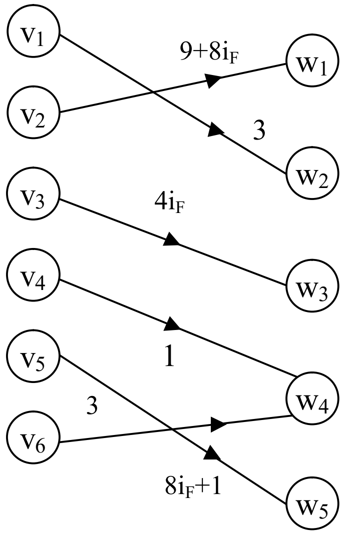


Figure 4.40

$v_i \in \langle Z_5 \cup k\rangle$; $1 \leq i \leq 6$ and $w_j \in C(Z_{10})$; $1 \leq j \leq 5$.

Example 4.43: Let M be the MOD bipartite graph of type II with edge weights from $\langle Z_4 \cup h\rangle$ and vertex sets from Z_{10} and $\langle Z_4 \cup h\rangle$ given by the following figure.

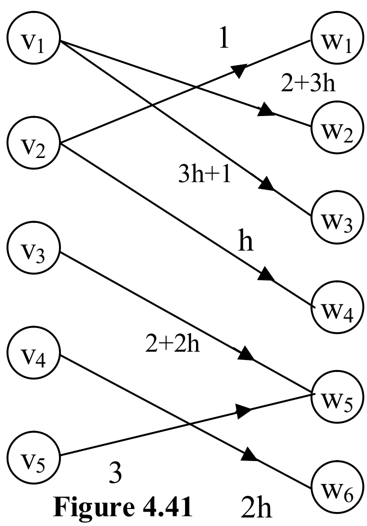


Figure 4.41

$v_i \in \langle Z_4 \cup h\rangle$; and $w_j \in Z_{10}$ $1 \leq j \leq 5$ and $1 \leq j \leq 6$.

Example 4.44: Let W be a MOD bipartite graph by type II with vertex sets from $C(Z_{12})$ and $\langle Z_{16} \cup h\rangle$ and edge sets from $C(Z_{12})$ given by the following figure.

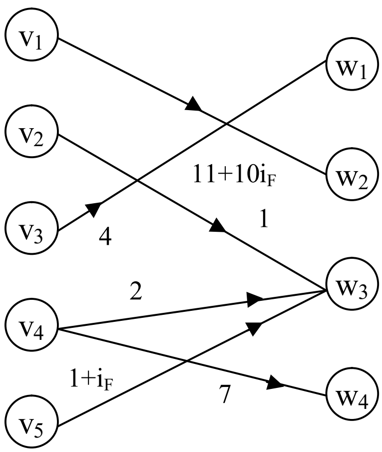


Figure 4.42

$v_i \in \langle Z_{16} \cup h\rangle$ and $w_j \in C(Z_{12})$; $1 \leq j \leq 5$ and $1 \leq j \leq 4$.

Next we proceed onto describe type III MOD bipartite graphs.

We call a MOD bipartite graph G to be a type III if the vertex sets are distinct and edge weights are also taken from a different set.

We will describe this situation by some examples.

Example 4.45: Let G be a MOD bipartite graph of type III with edge weights from $C(Z_7)$ and vertex sets from Z_{12} and Z_{19} given by the following figure:

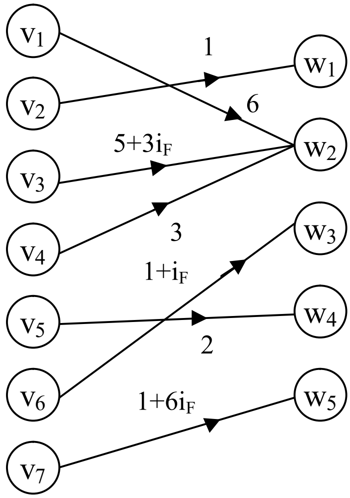


Figure 4.43

$v_i \in Z_{12}$; $1 \leq i \leq 7$ and $w_j \in Z_{19}$; $1 \leq j \leq 5$.

Example 4.46: Let G be a MOD bipartite graph of type III with edge weights from $\langle Z_5 \cup I\rangle$ and vertex from $\langle Z_{10} \cup I\rangle$ and $\langle Z_7 \cup I\rangle$ given by the following figure.

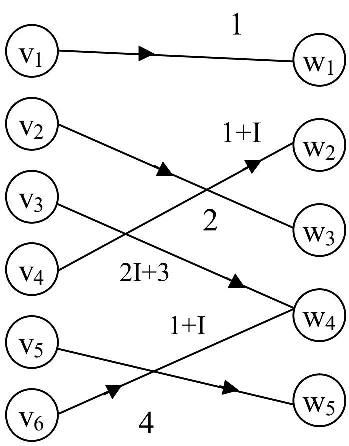


Figure 4.44

$v_i \in \langle Z_{10} \cup I\rangle$ and $w_j \in \langle Z_7 \cup I\rangle$ $1 \leq i \leq 6$ and $1 \leq j \leq 5$.

Example 4.47: Let K be the MOD bipartite graph of type III with edge weights from $\langle Z_7 \cup g\rangle$ with vertex sets from $\langle Z_6 \cup k\rangle$ and $\langle Z_{12} \cup h\rangle$ given by the following figure

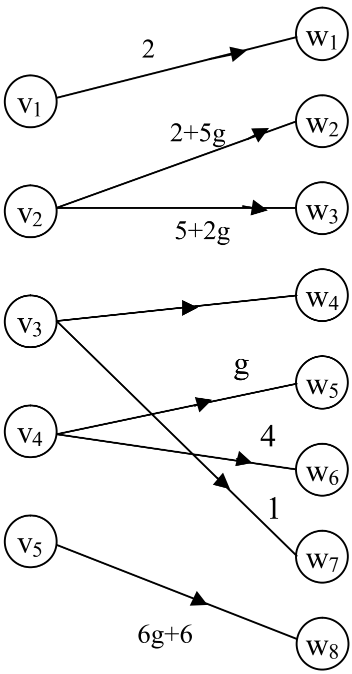


Figure 4.45

$v_i \in \langle Z_6 \cup k\rangle$; $1 \leq i \leq 5$ and $w_j \in \langle Z_{12} \cup h\rangle$; $1 \leq j \leq 8$.

Having seen some new types of MOD bipartite graphs we are sure these will surely find applications in MOD mathematical models.

Now we proceed onto describe MOD n-partite graphs of different types by some examples.

If a MOD n-partite graph takes the edge weights and vertex sets from the same set we call that MOD n-partite graph to be of type I MOD n-partite graph.

Example 4.48: Let G be a MOD 3-partite graph of type I with edge weights and vertex set from Z_{12} given by the following figure:

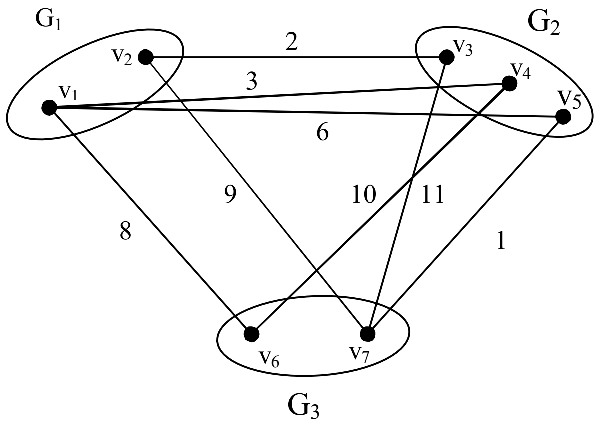


Figure 4.46

All v_i's are distinct and take values of from Z_{12}; $1 \leq i \leq 7$.

Example 4.49: Let H be a MOD 4-partite graph of type I with edge weights and vertex set from $C(Z_{10})$ given by the following figure:

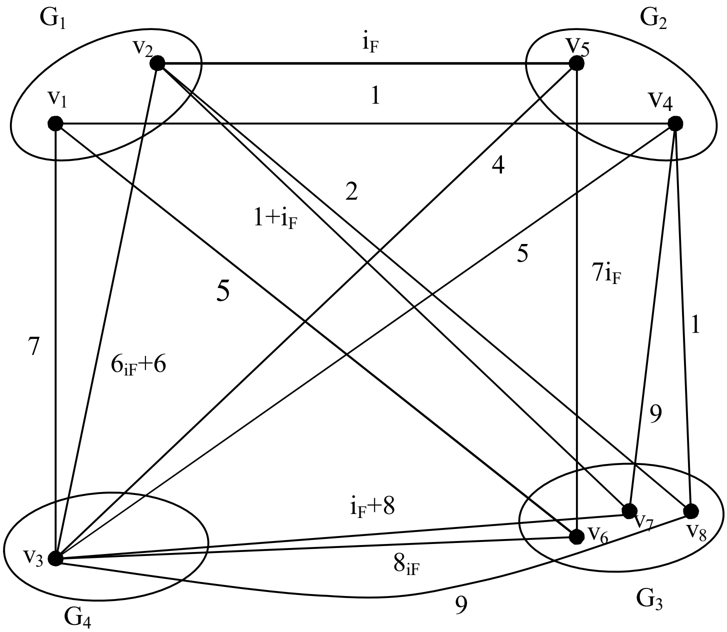


Figure 4.47

$v_i \in C(Z_{10})$; $1 \le i \le 8$ and all of them are distinct.

Example 4.50: Let H be a MOD 3-partite graph of type I with edge weights and vertex set from $\langle Z_{13} \cup I\rangle$ given by the following figure:

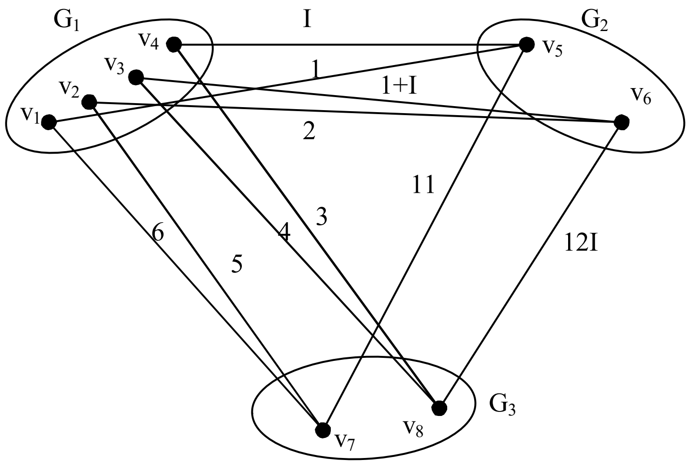


Figure 4.48

All the $v_i \in \langle Z_{12} \cup I\rangle$; $1 \leq i \leq 8$ and are distinct.

Example 4.51: Let V be the MOD 3-partite graph of type I with edge weights from $\langle Z_9 \cup k\rangle$ given by the following figure:

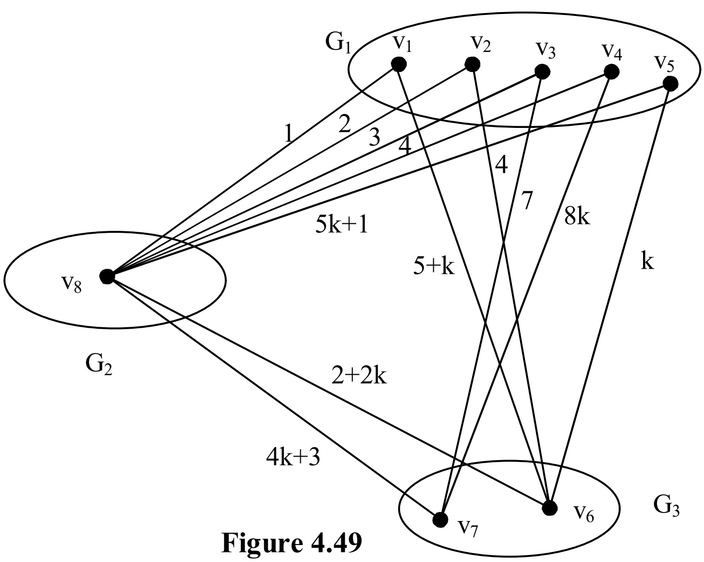


Figure 4.49

$v_i \in \langle Z_9 \cup k\rangle$; $1 \leq i \leq 8$ and each v_I is distinct.

Next we proceed onto describe MOD n-partite graphwith edge weights from one set and the vertex sets from another set which we choose to call as MOD n-partite graph of type II.

Example 4.52: Let S be the MOD 4-partite graph of type II with edge weights from Z_{15} and vertex set elements from Z_9 given by the following figure:

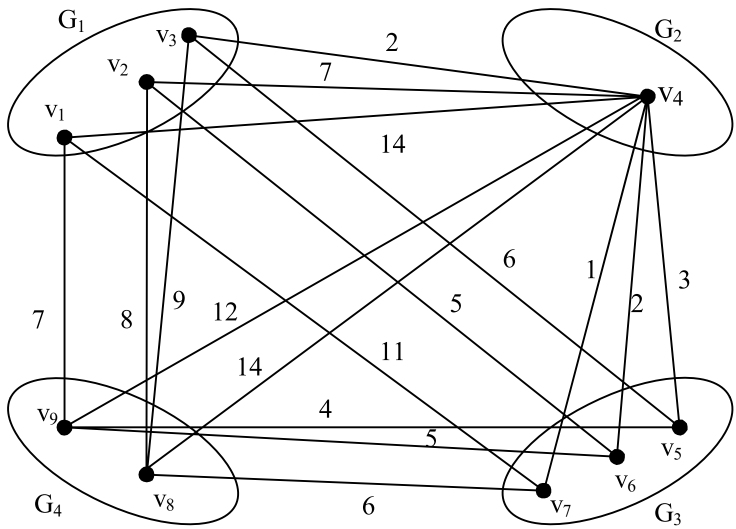


Figure 4.50

$v_I \in Z_9$; $1 \leq i \leq 9$.

Example 4.53: Let G be the 3-partite graph of type II with edge weights from $C(Z_{10})$ and vertex set from $\langle Z_{12} \cup k\rangle$ given by the following figure:

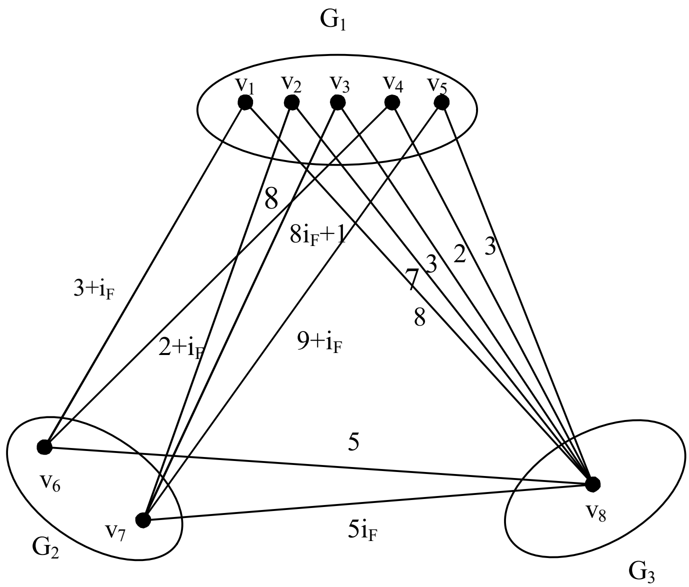


Figure 4.51

$v_i \in \langle Z_{12} \cup k\rangle;\ 1 \leq i \leq 8.$

Example 4.54: Let G be a MOD 4-partite graph of type II with edge weights from $\langle Z_{12} \cup I\rangle$ and vertex set from $C(Z_5)$; given by the following figure:

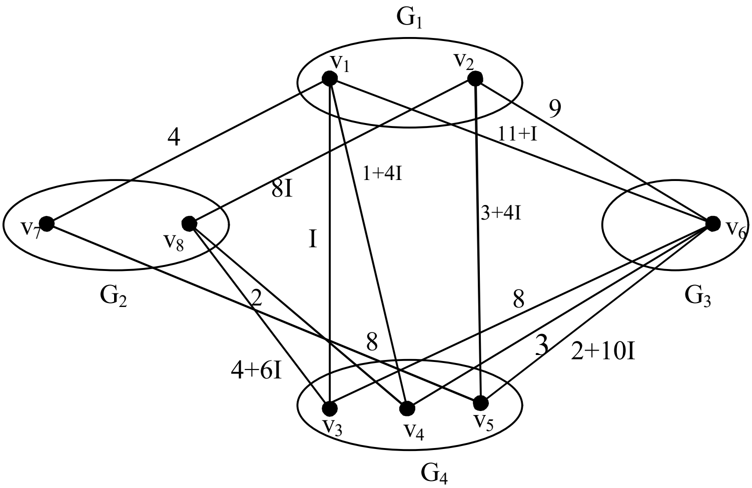


Figure 4.52

$v_i \in C(Z_{15})$; $1 \leq i \leq 8$.

Example 4.55: Let G be a MOD 3-partite graph of type II with edge weights from Z_{20} and vertex set from $\langle Z_7 \cup h\rangle$.

This given by the following example.

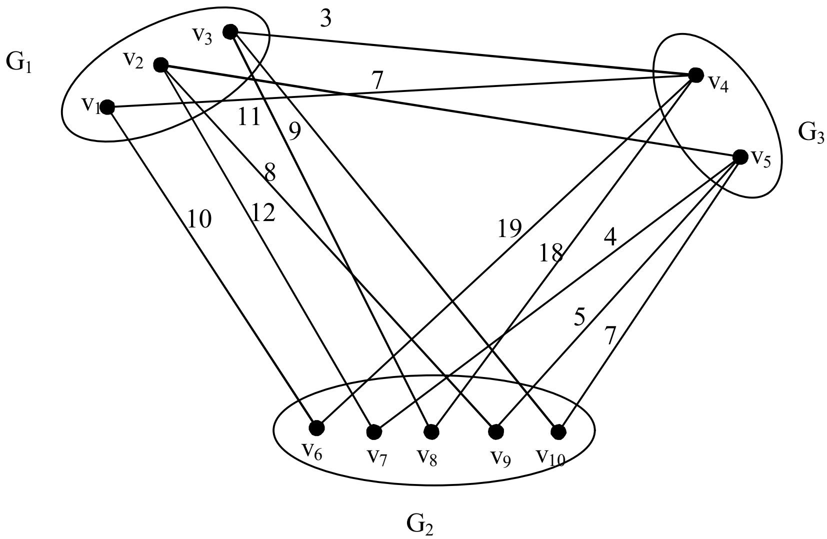


Figure 4.53

$v_i \in \langle Z_7 \cup h\rangle$; $1 \leq i \leq 10$.

Next we proceed onto define MOD n-partite graph of type III.

We call a MOD n-partite graph to be of type III if the edge weights are from a set and the vertex sets are n-distint in number.

This will be described by the following examples.

Example 4.56: Let G be a MOD 3-partite graph of type III with vertex set from Z_{12}, Z_9 and Z_{15} and edge weights from $C(Z_{10})$ given by the following figure:

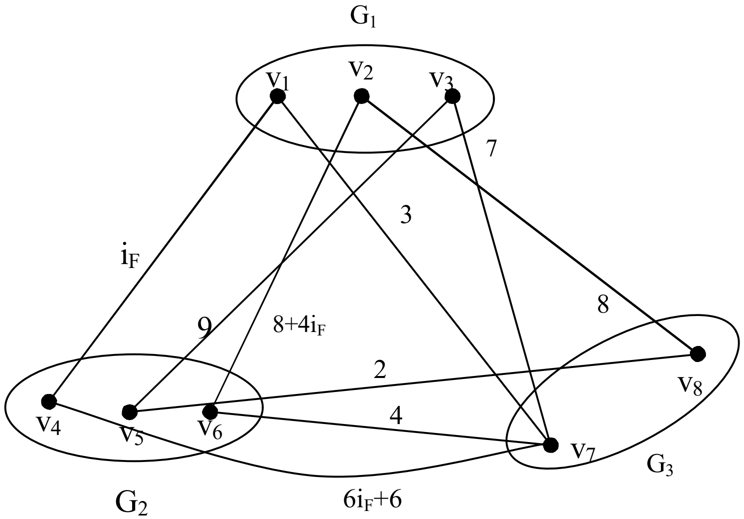


Figure 4.54

The vertex set of G_1 from Z_{12}, $v_1, v_2, v_3 \in Z_{12}$, the vertex set of G_2 from Z_9; that is $v_4, v_5, v_6 \in Z_9$ and the vertex of G_3 from Z_{15} that is $v_7, v_8 \in Z_{15}$.

The edge weights are from $C(Z_{10})$.

Example 4.57: Let G be a MOD 4 partite graph of type III with vertex set from $C(Z_4)$, $\langle Z_7 \cup I\rangle$, $\langle Z_{11} \cup g\rangle$ and $\langle Z_{11} \cup h\rangle$.

The edge weights are from Z_{15}.

The figure associated with G is as follows:

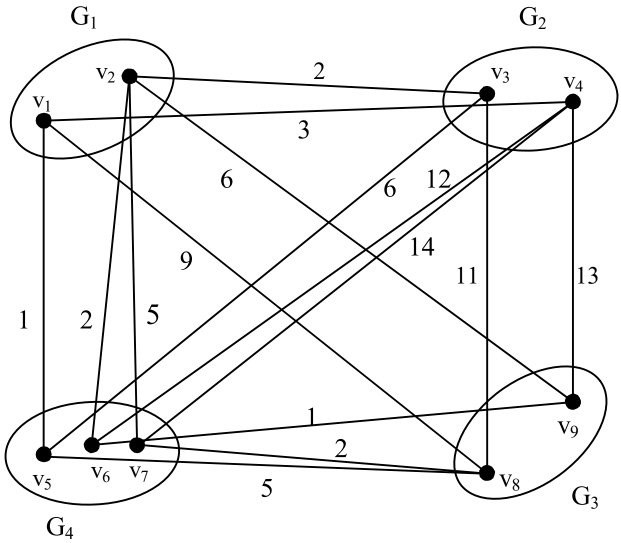


Figure 4.55

v_1, v_2 of G_1 belongs to $C(Z_4)$ v_3 and v_4 of G_2 belongs to $\langle Z_7 \cup I\rangle$ v_8m $v_9 \in G_3$ and $v_8, v_9 \in \langle Z_{11} \cup g\rangle$ and $v_5, v_6, v_7 \in G_4$; $v_5, v_6, v_7 \in \langle Z_{11} \cup h\rangle$.

Thus we can have MOD n-partite graph of type III.

Interested reader can give more examples of the MOD-n-partite graphs of all the 3 types.

We now give the description of MOD natural neutrosophic bipartite graphs first by examples.

Example 4.58: Let G be a MOD natural neutrosophic directed bipartite graph with edge weights from Z_9^I.

We call G to be the MOD directed bipartite natural neutrosophic graph which given by the following figure:

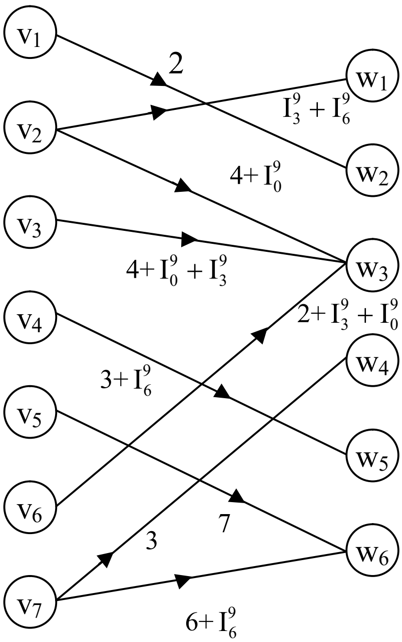


Figure 4.56

Clearly the edge weights are from Z_9^I.

Example 4.59: Let V be the MOD natural neutrosophic directed bigraph with edge weights from Z_{11}^I given by the following figure:

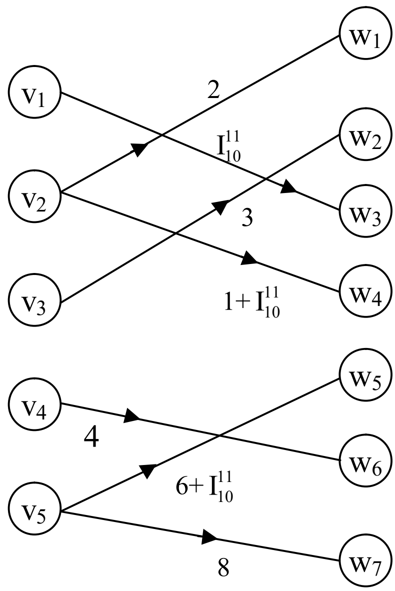


Figure 4.57

Example 4.60: Let W be the MOD natural neutrosophic directed bigraph of type I with edge weights and vertex set from Z_{15}^{I} given by the following figure.

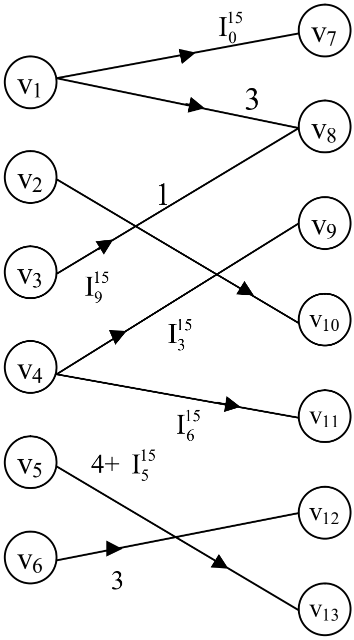


Figure 4.58

Clearly $v_i \in Z_{15}^I$; $1 \leq i \leq 15$.

Example 4.61: Let S be the MOD natural neutrosophic bipartite graph of type I with vertex set and edge weights from $C^I(Z_6)$.

S is also known as the MOD natural neutrosophic finite complex number type I graph given by the following figure:

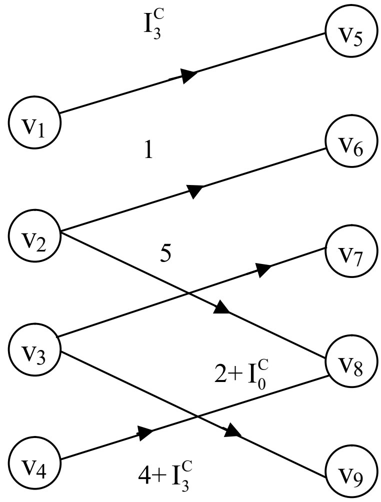


Figure 4.59

$$v_i \in C^I(Z_6);\ 1 \leq i \leq 9.$$

We just describe MOD natural neutrosophic bipartite graphs with edge weights from $C^I(Z_n)$, $\langle Z_n \cup I\rangle$, $\langle Z_n \cup g\rangle$, $\langle Z_n \cup h\rangle$ and $\langle Z_n \cup k\rangle$ by some examples.

Example 4.62: Let G be the MOD natural neutrosophic finite complex number bipartite graph with edge weights from $C^I(Z_{10})$ given by the following figure :

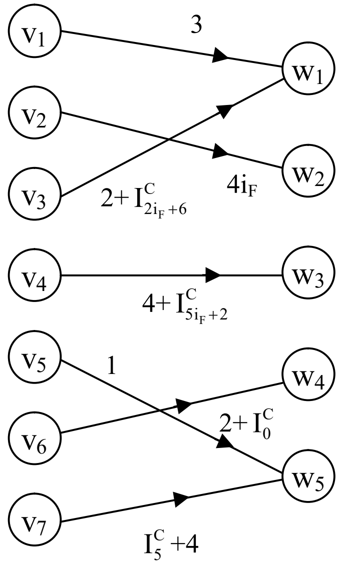


Figure 4.60

Example 4.63: Let W be the MOD natural neutrosophic finite complex number bipartite graph with edge weights from $C^I(Z_{15})$ given by the following figure:

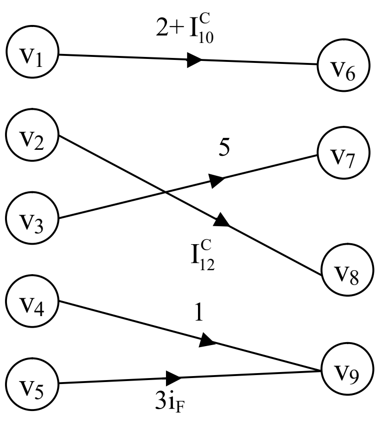


Figure 4.61

Next we proceed onto describe MOD natural neutrosophic-neutrosophic edge weights from $\langle Z_n \cup I\rangle_I$ by some examples.

Example 4.64: Let W be the MOD directed bipartite natural neutrosophic-neutrosophic graph with edge weights from $\langle Z_6 \cup I\rangle_I$ given by the following figure:

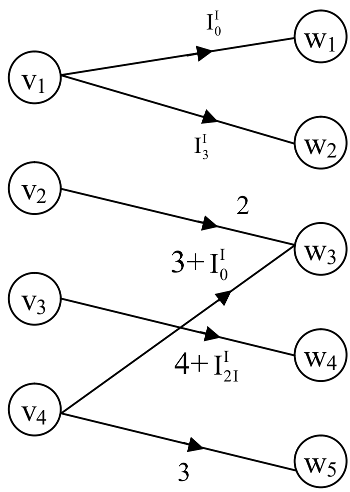


Figure 4.62

Example 4.65: Let S be a MOD bipartite natural neutrosophic-neutrosophic graph with edge weights from $\langle Z_{11} \cup I\rangle_I$ given by the following figure:

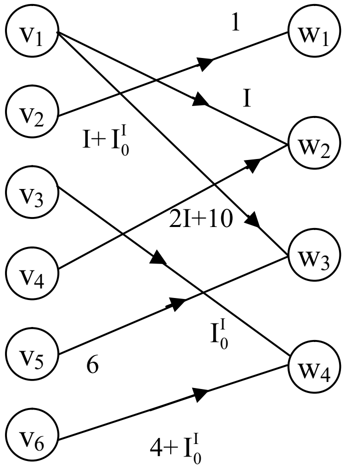


Figure 4.63

Example 4.66: Now we proceed onto describe MOD natural neutrosophic dual number bipartite graph with edge weights from $\langle Z_{10} \cup g\rangle_I$ given by the following figure.

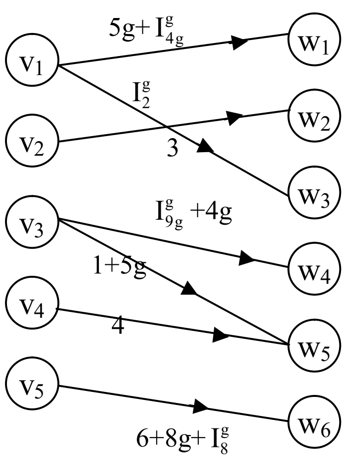


Figure 4.64

Example 4.67: Let S be a MOD natural neutrosophic bipartite dual number graph with edge weights from $\langle Z_7 \cup g\rangle_I$ given by the following figure:

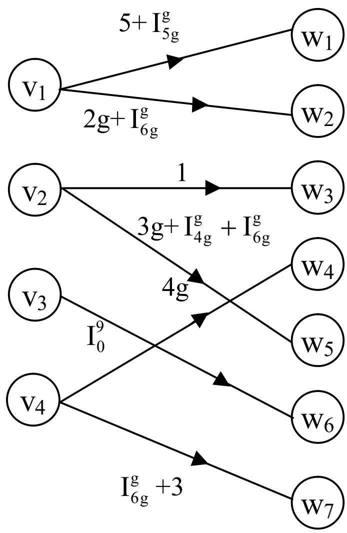


Figure 4.65

Next we proceed onto describe MOD natural neutrosophic special dual like number bipartite graph with edge weights from $\langle Z_n \cup h\rangle_I$ in the following examples.

Example 4.68: Let G be the MOD natural neutrosophic special quasi dual number bipartite graph with edge weights from $\langle Z_4 \cup h\rangle_I$ given by the following figure:

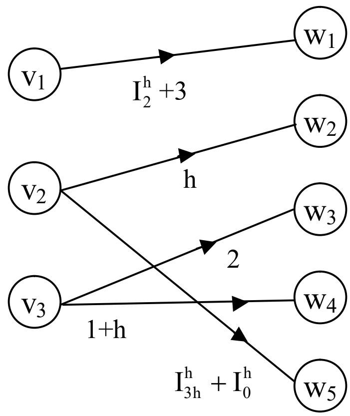


Figure 4.66

Example 4.69: Let G be the MOD natural neutrosophic special dual like number bipartite graph with edge weights from $\langle Z_{12} \cup h\rangle_I$ given by the following figure:

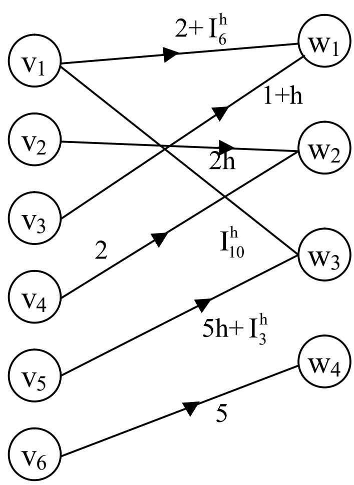


Figure 4.67

If G is MOD natural neutrosophic bipartite graph taking edge weights from $\langle Z_n \cup k\rangle_I$; then we define G to be a MOD natural neutrosophic special quasi dual number bipartite graph.

We will illustrate this situation by some examples.

Example 4.70: Let S be a MOD natural neutrosophic special quasi dual number bipartite graph with edge weights from $\langle Z_{13} \cup k\rangle_I$ given by the following figure.

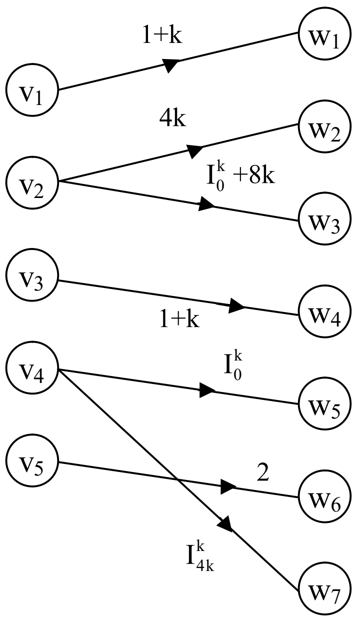


Figure 4.68

Example 4.71: Let H be the MOD natural neutrosophic special quasi dual number bipartite graph with edge weights from $\langle Z_9 \cup k\rangle_I$ given by the following figure:

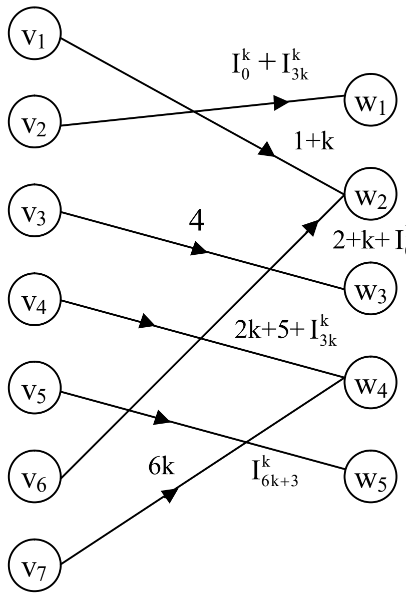


Figure 4.69

Having seen MOD bipartite graph of natural neutrosophic numbers we now proceed onto describe MOD natural neutrosophic bipartite graph of type I by some examples.

Example 4.72: Let G be the MOD natural neutrosophic bipartite graph of type I with edge weights and vertex set from Z_9^I given by the following figure:

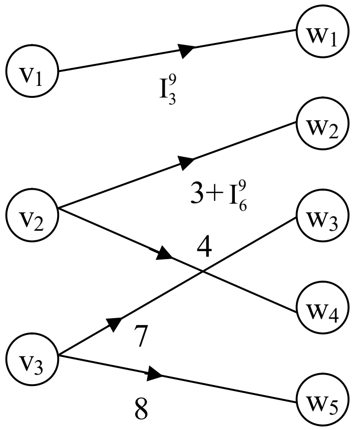


Figure 4.70

$v_i \in Z_9^I$ and $w_j \in Z_9^I$; $1 \leq i \leq 3$ and $1 \leq j \leq 5$.

Example 4.73: Let W be the MOD natural neutrosophic bipartite graph of type I with vertex set and edge weights from Z_{15}^I given by the following figure:

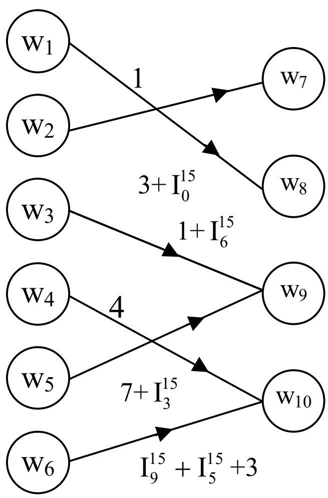


Figure 4.71

Example 4.74: We proceed onto describe the MOD natural neutrosophic bipartite finite complex number of type I with vertex set and edge weights from $C^I(Z_{12})$ given by the following figure:

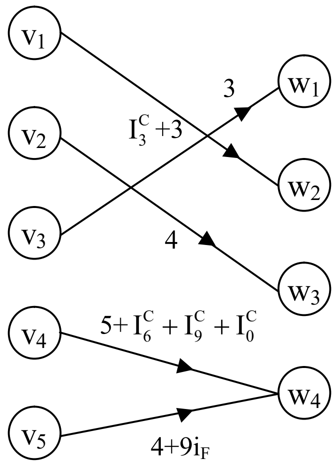


Figure 4.72

Example 4.75: Let G be the MOD natural neutrosophic finite complex number bipartite graph of type I given by the following figure with vertex set and edge weights $C^I(Z_{18})$.

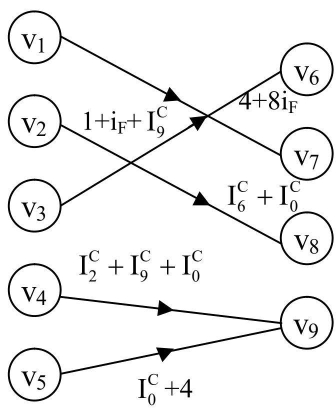


Figure 4.73

Thus if G is a MOD natural neutrosophic bipartite graph of type I with edge weights and vertex set from $C^I(Z_n)$ then we define G to be a MOD natural neutrosophic finite complex number bipartite of type I with edge weights and vertex set from $C^I(Z_n)$.

Next we proceed onto describe and develop the notion of MOD natural neutrosophic bipartite dual number graph of I which will take edge weights and vertex set from $\langle Z_n \cup g\rangle_I$.

Example 4.76: Let H be a natural neutrosophic dual number bipartite graph of type I with vertex set and edge weights from $\langle Z_{10} \cup g\rangle_I$ given by the following figure:

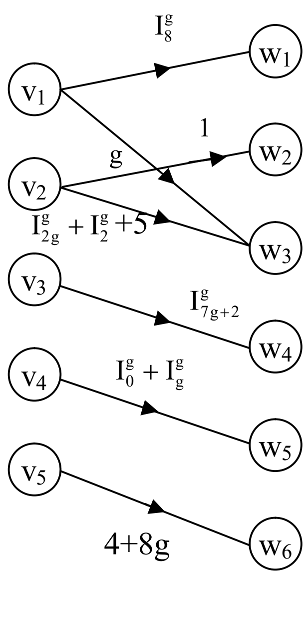


Figure 4.74

Example 4.77: Let S be the MOD natural neutrosophic dual number bipartite graph of type I with edge weights and vertex set from $\langle Z_{16} \cup g\rangle_I$ given by the following figure:

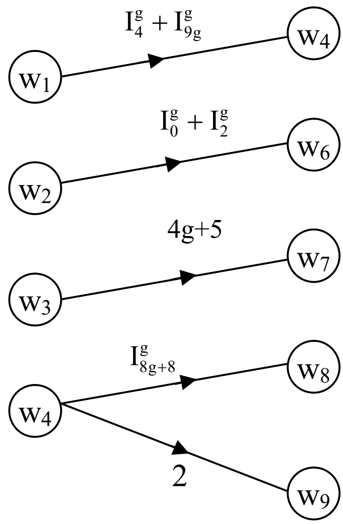


Figure 4.75

Next we proceed onto describe the MOD natural neutrosophic special dual like number bipartite graph of type I with edge weights and vertex set from $\langle Z_n \cup h\rangle_I$.

We proceed to give examples of them.

Example 4.78: Let G be the MOD natural neutrosohic special quasi dual like number bipartite graph of type I with edge weights and vertex set from $\langle Z_{12} \cup h\rangle_I$ given by the following figure:

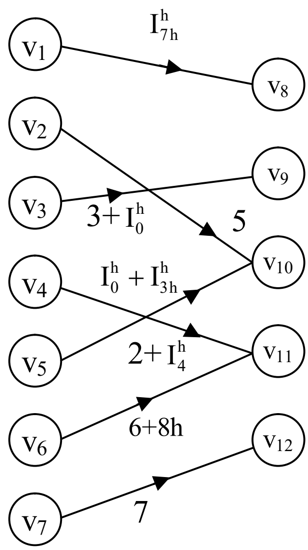


Figure 4.76

Example 4.79: Let H be the MOD natural neutrosophic special dual like number directed bipartite graph with edge weights and vertex set from $\langle Z_7 \cup h\rangle_I$ given by the following figure:

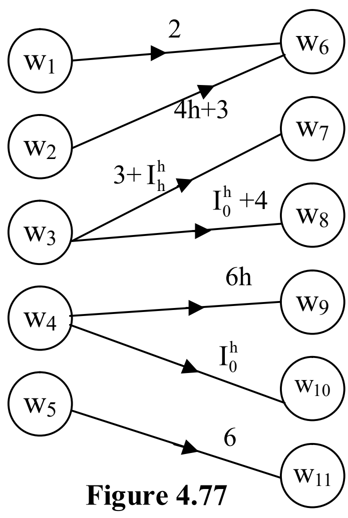


Figure 4.77

Next we proceed onto develop the notion of MOD natural neutrosophic special quasi dual number bipartite graph using edge weights and vertex sets from $\langle Z_n \cup k\rangle_I$ by some examples.

Example 4.80: Let P be the MOD natural neutrosophic special quasi dual number partite graph with edge weights and vertex set from $\langle Z_{15} \cup k\rangle_I$.

It is given by the following figure:

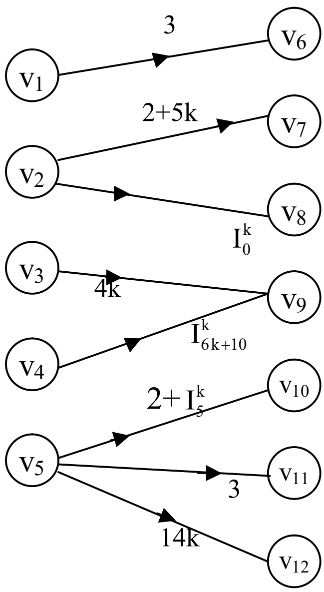


Figure 4.78

Example 4.81: Let M be the MOD natural neutrosophic special quasi dual number bipartite graph of type I with edge weights and vertex set from $\langle Z_{11} \cup k\rangle_I$ given by the following figure:

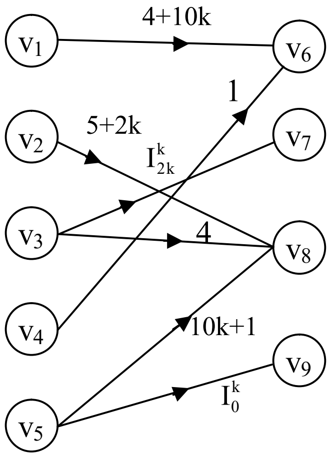


Figure 4.79

Next we proceed onto describe the notion of MOD natural neutrosophic bipartite graph of type II which takes edge weights from two sets Z_n^I and Z_m^I and vertex elements are either from Z_n^I or Z_m^I.

We will describe this situation by an example or two.

Example 4.82: Let S be the MOD natural neutrosophic bipartite graph of type II with edge weights from Z_{12}^I and vertex sets from Z_7^I and Z_{12}^I given by the following figure:

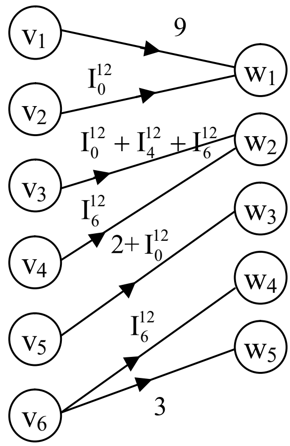


Figure 4.80

$v_i \in Z_{12}^I$ and $w_j \in Z_7^I$; $1 \leq i \leq 6$ and $1 \leq j \leq 5$.

Example 4.83: Let M be the MOD natural neutrosophic bipartite graph of type II with vertex sets from Z_{16}^I and Z_{17}^I and edge weights from Z_{17}^I given by the following figure:

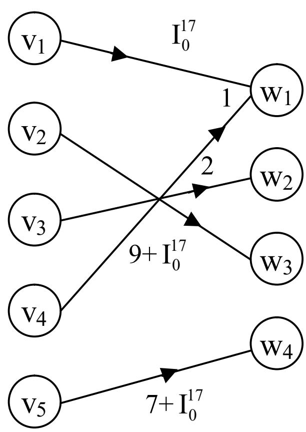


Figure 4.81

$v_i \in Z^I_{16}$ and $w_j \in Z^I_{17}$; $1 \leq i \leq 5$ and $1 \leq j \leq 4$.

Next we proceed onto describe the MOD natural neutrosophic finite complex number type II bipartite graph with edge weights from $C^I(Z_n)$ vertex sets entries from $C^I(Z_m)$ and $Z^I(Z_n)$.

We will describe this situation by some examples.

Example 4.84: Let G be the MOD natural neutosophic finite complex number bipartite graph of type II with edge weights from $C^I(Z_{10})$ and vertex sets from $C^I(Z_{10})$ and $C^I(Z_{12})$ given by the following figure:

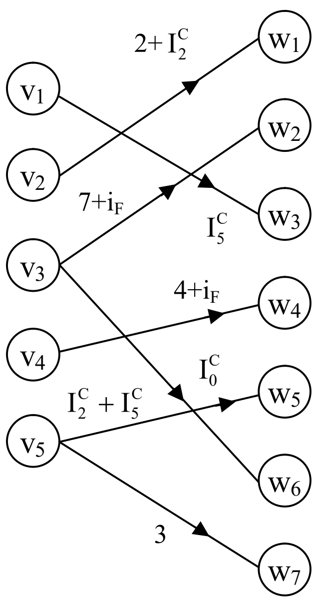


Figure 4.82

$v_i \in C^I(Z_{10})$ and $w_j \in C^I(Z_{12})$; $1 \leq i \leq 5$ and $1 \leq j \leq 7$.

Example 4.85: Let G be a MOD natural neutrosophic finite complex number directed graph of type II with edge weights from $C_I(Z_7)$ and vertex sets from $C^I(Z_9)$ and $C(Z_7^I)$ given by the following figure:

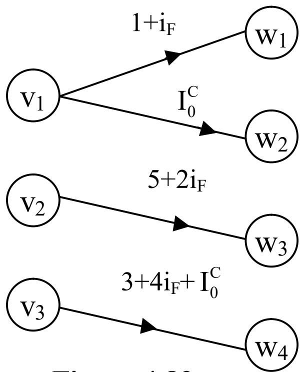


Figure 4.83

$v_1, v_2, v_3 \in C^I(Z_9)$ and $w_1, w_2, w_3, w_4 \in C^I(Z_7)$.

Example 4.86: Let G be a MOD natural neutrosophic dual number directed bipartite graph of type II with edge weights from $\langle Z_{18} \cup g\rangle_I$ and vertex sets from $\langle Z_{11} \cup g\rangle_I$ and $\langle Z_{18} \cup g\rangle_I$ given by the following figure:

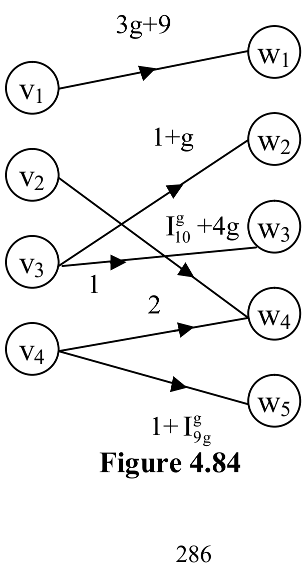


Figure 4.84

$v_i \in \langle Z_{91} \cup g\rangle_I$ and $w_j \in \langle Z_{18} \cup g\rangle_I$; $1 \leq i \leq 4$ and $1 \leq j \leq 5$.

Example 4.87: Let S be the MOD natural neutrosophic dual number directed graph of type II with edge sets from $\langle Z_5 \cup g\rangle_I$ and vertex sets from $\langle Z_5 \cup g\rangle_I$ and $\langle Z_{12} \cup g\rangle_I$ given by the following figure:

The vertices $v_1, v_2, v_3, v_4, v_5, v_6, v_7 \in \langle Z_5 \cup g\rangle_I$ and $w_j \in \langle Z_{12} \cup g\rangle_I$; $1 \leq j \leq 4$.

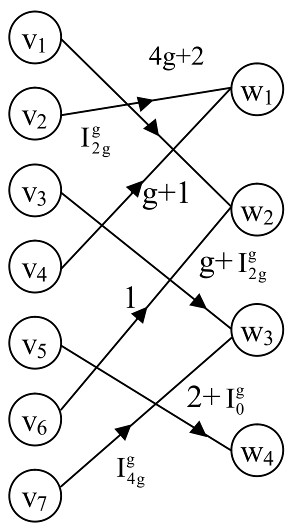


Figure 4.85

Next we proceed onto describe MOD natural neutrosophic special quasi dual number type II directed graph with edge weights from $\langle Z_8 \cup k\rangle_I$ and vertex sets from $\langle Z_{13} \cup k\rangle_I$ and $\langle Z_8 \cup k\rangle_I$ given by the following figure:

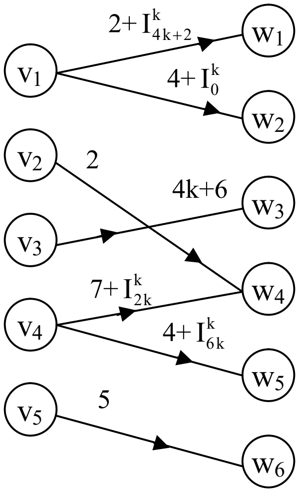


Figure 4.86

$v_i \in \langle Z_{13} \cup k\rangle_I$; $1 \leq i \leq 5$ and $w_j \in \langle Z_8 \cup k\rangle_I$; $1 \leq j \leq 6$.

Example 4.88: Let K be the MOD natural neutrosophic special quasi dual number directed graph of type II with edge weights from $\langle Z_{15} \cup k\rangle_I$ and vertex sets from $\langle Z_{15} \cup k\rangle_I$ and $\langle Z_{10} \cup k\rangle_I$ givenby the following figure:

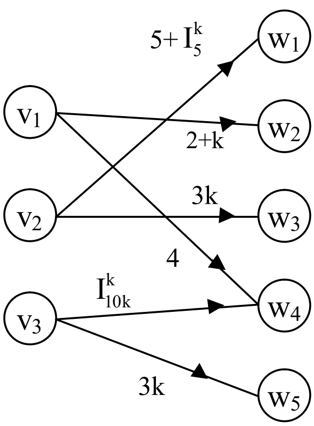


Figure 4.87

Next we proceed onto describe MOD natural neutrosophic special dual like number bipartite directed graph of type II with edge weights from $\langle Z_m \cup h\rangle_I$ and vertex sets from $\langle Z_n \cup h\rangle_I$ and $\langle Z_m \cup h\rangle_I$, $(m \neq n)$.

We will describe this sitation by some examples.

Example 4.89: Let G be the MOD natural neutrosophic special dual like number directed graph of type II with edge weights from $\langle Z_{19} \cup h\rangle_I$ and vertex sets from $\langle Z_{19} \cup h\rangle_I$ and $\langle Z_{29} \cup h\rangle_I$ given by the following figure:

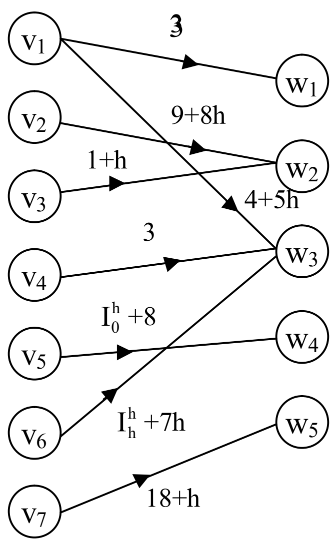


Figure 4.88

Example 4.90: Let V be the MOD natural neutrosophic special dual like number directed bipartite graph of type II with edge weights from $\langle Z_8 \cup h\rangle_I$ and vertex sets from $\langle Z_{11} \cup h\rangle_I$ and $\langle Z_8 \cup h\rangle_I$ given by the following figure:

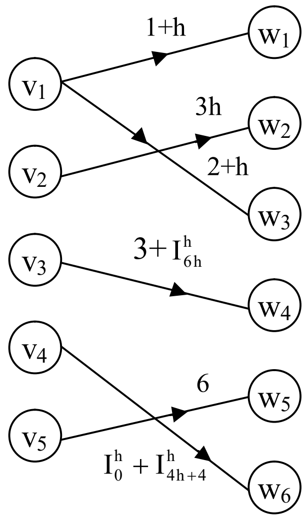


Figure 4.89

$v_i \in \langle Z_{11} \cup h\rangle_I$, $w_j \in \langle Z_8 \cup h\rangle_I$, $1 \leq i \leq 5$, $1 \leq j \leq 6$.

Next we proceed onto describe MOD natural neutrosophic neutrosophic directed bipartite graph of type II with edge weights from $\langle Z_n \cup I\rangle_I$ and vertex set from $\langle Z_n \cup I\rangle_I$ and $\langle Z_m \cup I\rangle_I$.

We will describe this by some examples.

Example 4.91: Let G be the MOD natural neutrosophic neutrosophic directed bipartite graph of type II with edge weights from $\langle Z_{10} \cup I\rangle_I$ and vertex sets from $\langle Z_{10} \cup I\rangle_I$ and $\langle Z_{17} \cup I\rangle_I$ given by the following figure:

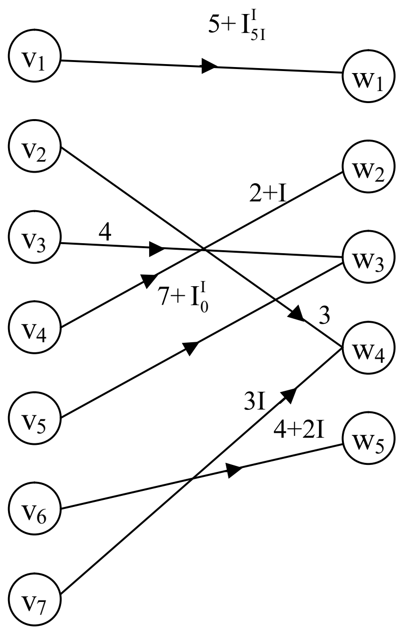


Figure 4.90

$v_i \in \langle Z_{10} \cup I\rangle_I$ $1 \leq i \leq 7$, $w_j \in \langle Z_{17} \cup h\rangle_I$ $1 \leq j \leq 5$.

Example 4.92: Let S be the MOD natural neutrosophic neutrosophic bipartite directed graph with edge weights from $\langle Z_7 \cup I\rangle_I$ and vertex sets from $\langle Z_7 \cup I\rangle_I$ and $\langle Z_{12} \cup I\rangle_I$ given by the following figure:

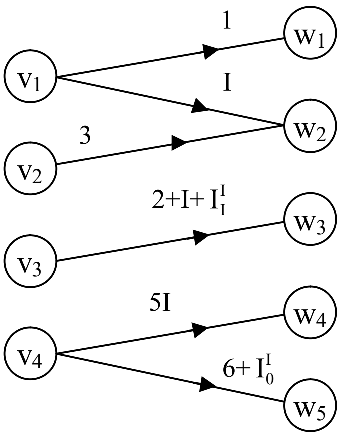


Figure 4.91

$v_i \in \langle Z_7 \cup I\rangle_I$; $1 \leq i \leq 4$ and $w_j \in \langle Z_{12} \cup I\rangle_I$; $1 \leq j \leq 5$.

Next we proceed onto describe MOD natural neutrosophic directed bipartite graph of type III which take edge weights from Z_n^I and vertex sets from Z_m^I and Z_t^I; $t \neq m$, $m \neq n$ and $n \neq t$, $2 \leq t, m, n < \infty$.

We proceed onto describe this situation by some examples.

Example 4.93: Let G be a MOD natural neutrosophic bipartite graph of type III with edge weights from Z_8^I and vertex sets from Z_{12}^I and Z_{16}^I given by the following figure:

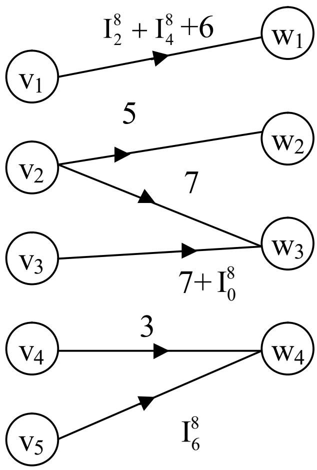


Figure 4.92

$v_i \in Z_{12}^I$; $1 \leq i \leq 5$ and $w_j \in Z_{16}^I$; $1 \leq j \leq 4$.

Example 4.94: Let G be the the MOD natural neutrosophic bipartite graph of type III with edge weights from Z_{11}^I, and vertex sets from Z_6^I and Z_{10}^I given by the following figure:

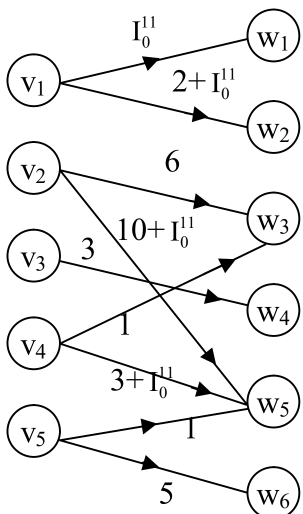


Figure 4.93

$v_i \in Z_6^I$ and $w_j \in Z_{10}^I$ $1 \leq i \leq 5$ and $1 \leq j \leq 6$.

Thus if G is a MOD natural neutrosophic bipartite graph of type III with edge weights from $C^I(Z_n)$ and vertex sets from $C^I(Z_n)$ and $C^I(Z_t)$ m, n and t distinct positive integers, then G is defined as the MOD natural neutrosophic directed bipartite finite complex number graph of type III.

We will just illustrate this situation by an example.

Example 4.95: Let G be the MOD natural neutrosophic finite complex number bipartite graph of type III with edge weights from $C^I(Z_6)$ and vertex sets from $C^I(Z_3)$ and $C^I(Z_{10})$ given by the following fiture.

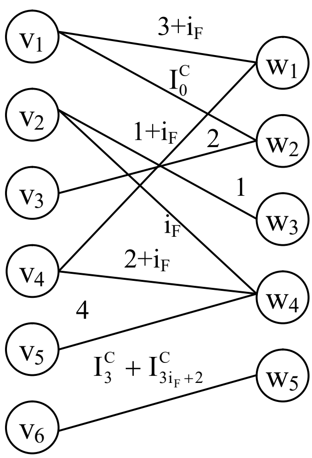


Figure 4.94

On similar line we can define MOD natural neutrosophic neutrosophic bipartite grapha of type III with edge weights and vertex sets from three different sets $\langle Z_n \cup I\rangle_I$, $\langle Z_m \cup I\rangle_I$ and $\langle Z_t \cup I\rangle_I$, m, n and t are 3 distinct finite positive integers.

This will be just illustrated by an example.

Example 4.96: Let G be the MOD natural neutrosophic neutrosophic bipartite graph of type III with edge weights from $\langle Z_{10} \cup I\rangle_I$ and vertex sets from $\langle Z_7 \cup I\rangle_I$ and $\langle Z_{12} \cup I\rangle_I$ given by the following figure:

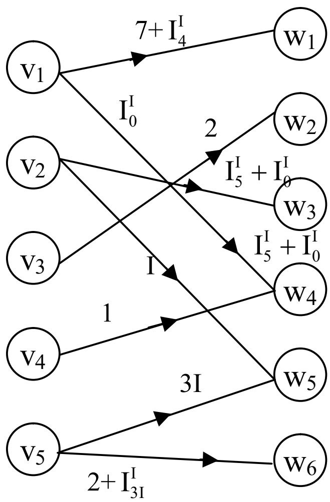


Figure 4.95

Next we proceed onto describe the MOD natural neutrosophic dual number type III directed bipartite graph by an example.

Example 4.97: Let G be the MOD natural neutrosophic dual number type III directed bipartite graph with edge weights from $\langle Z_3 \cup g\rangle_I$ and vertex sets from $\langle Z_7 \cup g\rangle_I$ and $\langle Z_{10} \cup g\rangle_I$ given by the following figure:

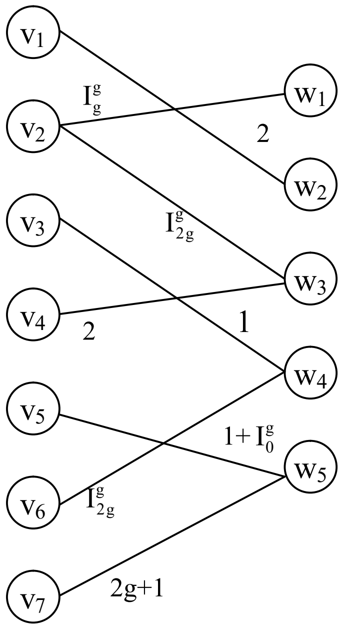


Figure 4.96

The edges are from v_i to w_j.

$v_i \in \langle Z_7 \cup g\rangle_I$; $1 \leq i \leq 7$ and $w_j \in \langle Z_{10} \cup g\rangle_I$; $1 \leq i \leq 5$.

Example 4.98: Let G be the MOD natural neutrosophic special quasi dual number bipartite graph of type III with edge weights from $\langle Z_9 \cup k\rangle_I$ and vertex sets from $\langle Z_4 \cup k\rangle_I$ and $\langle Z_{10} \cup k\rangle_I$ given by the following figure:

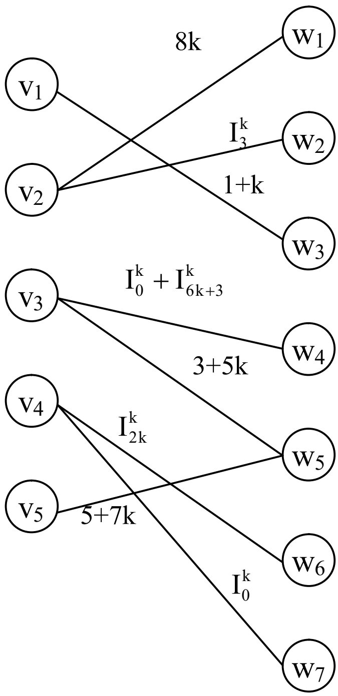


Figure 4.97

$v_i \in \langle Z_4 \cup k\rangle_I$; $1 \leq i \leq 5$ and $w_j \in \langle Z_{10} \cup k\rangle_I$ $1 \leq j \leq 7$.

Next we describe by an example the MOD natural neutrosophic special dual like number bipartite graph of type III by an example.

Example 4.99: Let H be the MOD natural neutrosophic special dual like number bipartite graph of type III with edge weights from $\langle Z_{10} \cup h\rangle_I$ and vertex set from $\langle Z_7 \cup h\rangle_I$ and $\langle Z_{12} \cup h\rangle_I$ given by the following figure:

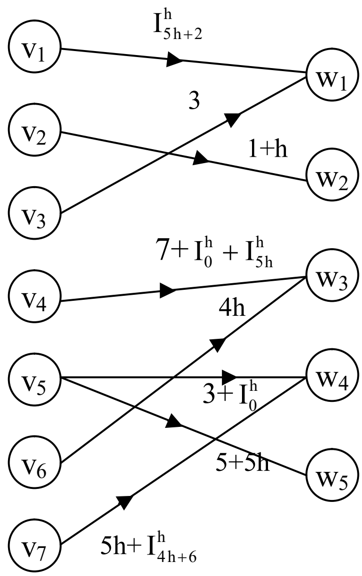


Figure 4.98

$v_i \in \langle Z_7 \cup h\rangle_I$; $1 \leq i \leq 7$ and $w_j \in \langle Z_{12} \cup h\rangle_I$; $1 \leq j \leq 5$.

Now having seen examples of MOD natural neutrosophic bipartite graphs of type III.

We now proceed onto describe the MOD n-partite natural neutrosophic graphs.

We will illustrate this situation by some examples.

A MOD n-partite natural neutrosophic graph if the edge weights are from Z^I_n; $2 \leq m < \infty$ which is described by some examples.

Example 4.100: Let G be a MOD 3 partite natural neutrosophic graph with edge weights from Z_{10}^{I} given by the following figure:

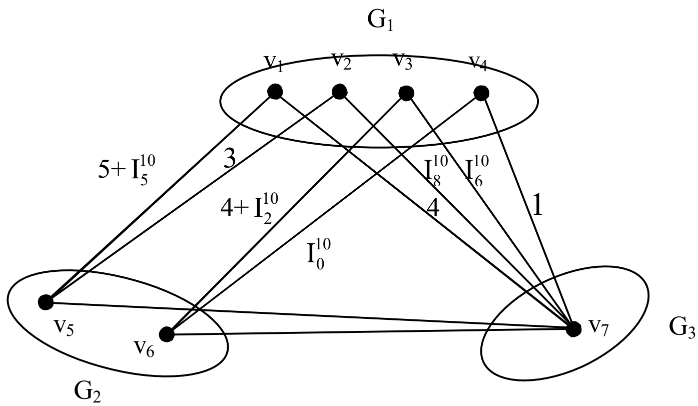


Figure 4.99

Example 4.101: Let G be the MOD natural neutrosophic 4-partite graph with edge weights from Z_{13}^{I} given by the following figure:

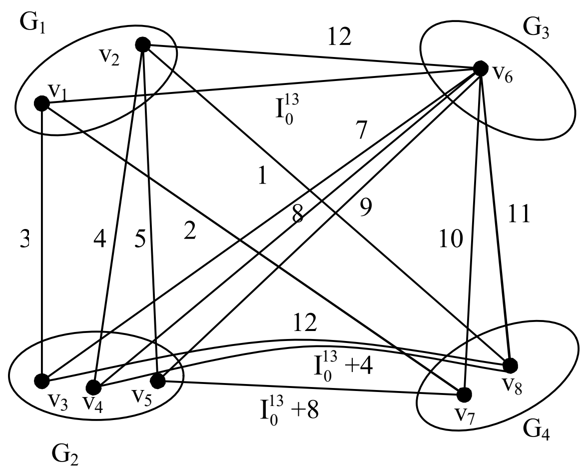


Figure 4.100

Example 4.102: Let G be the MOD 5 partite natural neutrosophic finite complex number graph with edge weights from $C^I(Z_6)$, given by the following figure:

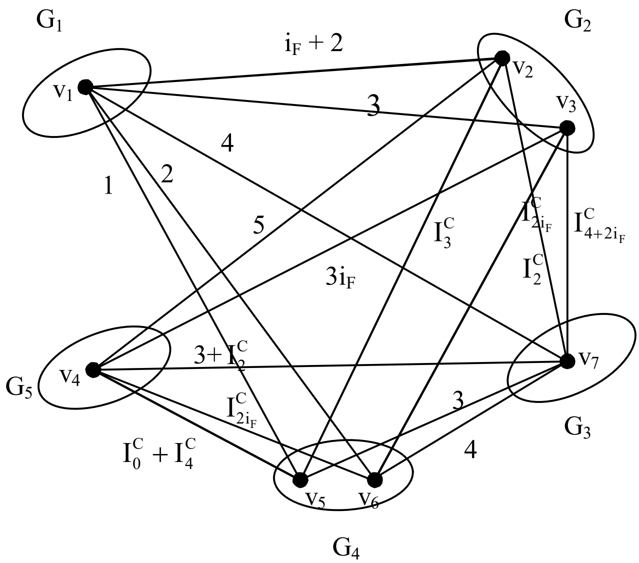


Figure 4.101

Example 4.103: Let H be MOD natural neutrosophic 4-partite finite complex number graph with edge weights from $C^I(Z_{10})$ by the following figure:

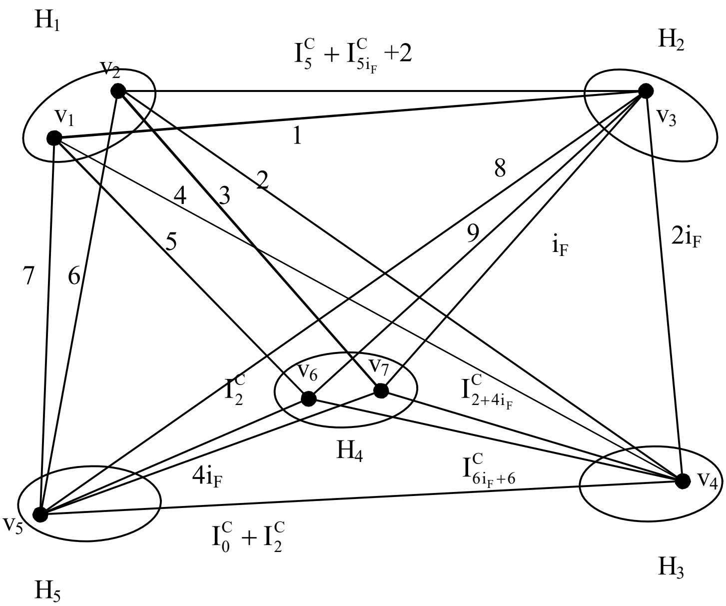


Figure 4.102

Example 4.104: Let G be the MOD 3-partite natural neutrosophic dual number graph with edge weights from $\langle Z_6 \cup h\rangle_I$ given by the following figure:

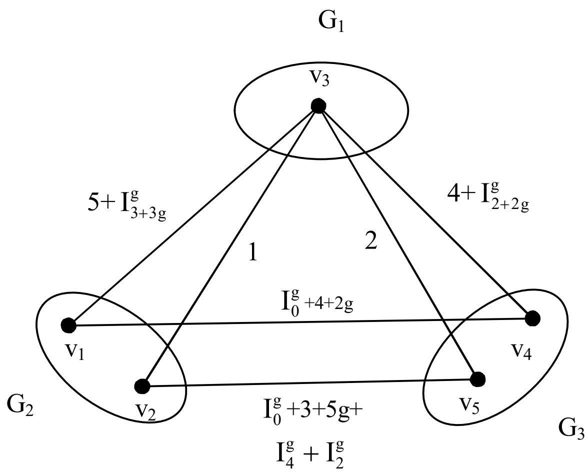


Figure 4.103

Example 4.105: Let H be the MOD 4-partite natural neutrosophic dual number graph with edge weight from $\langle Z_9 \cup g\rangle_I$ given by the following figure:

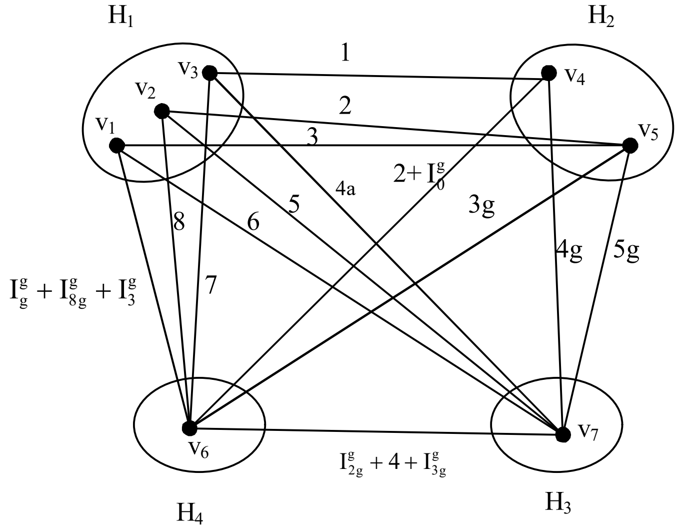


Figure 4.104

Example 4.106: Let P be the MOD 6-partite natural neutrosophic - neutrosophic graph with edge weights from $\langle Z_8 \cup I\rangle_I$ given by the following figure:

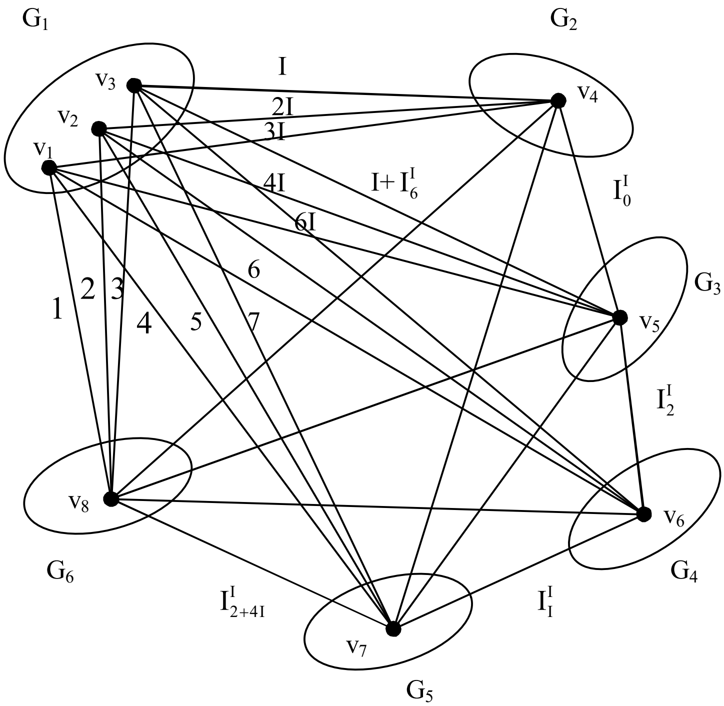


Figure 4.105

Example 4.107: Let V be the MOD 3-partite natural neutrosophic-neutrosophic graph with edge weights from $\langle Z_{12} \cup I\rangle_I$ given by the following figure:

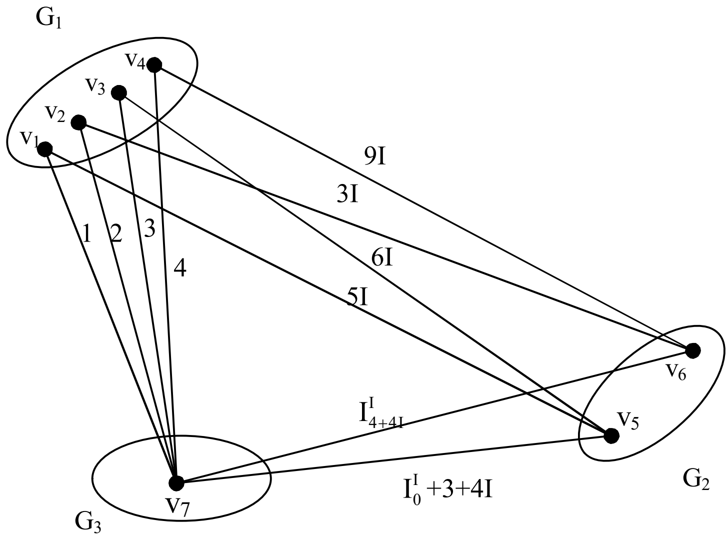


Figure 4.106

Example 4.108: Let W be the MOD 4-partite natural neutrosophic dual number graph with edge weights from $\langle Z_{14} \cup g \rangle_I$ given by the following figure:

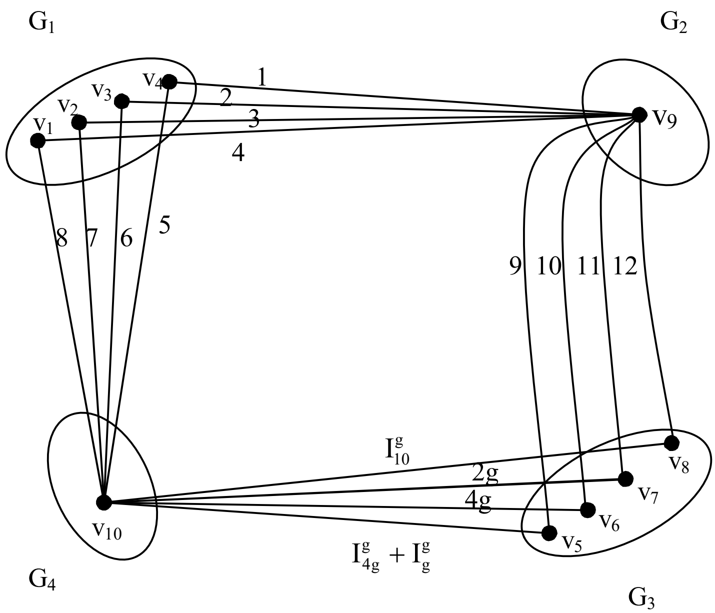


Figure 4.107

Example 4.109: Let M be the MOD 5-partite natural neutrosophic dual number graph.

The edge weights from $\langle Z_7 \cup g\rangle_I$ given by the following figure:

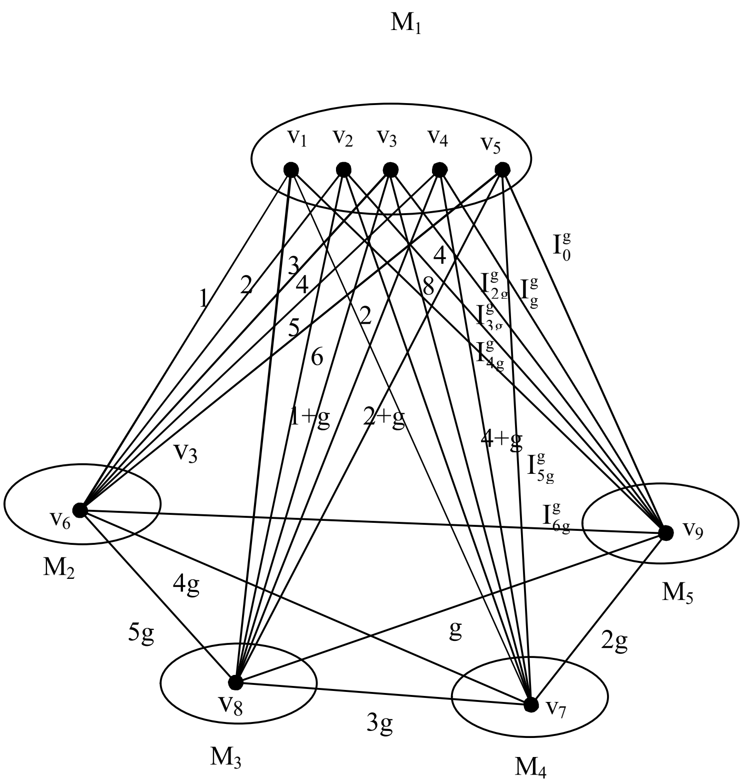


Figure 4.108

Example 4.110: Let P be the MOD 3-partite natural neutrosophic special quasi dual number graph with edge weights from $\langle Z_9 \cup k \rangle_I$ given by the following figure:

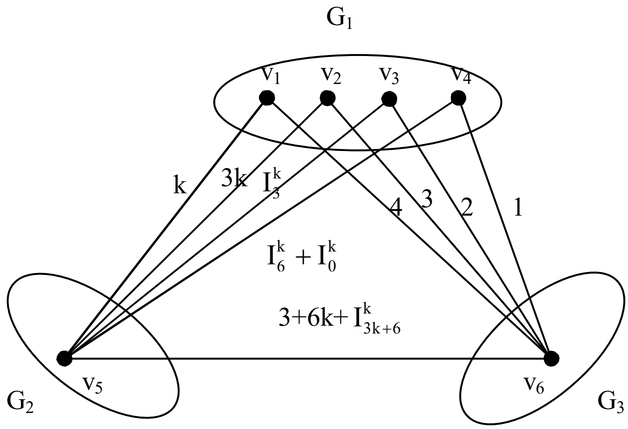


Figure 4.109

Example 4.111: Let S be the MOD 4-partite natural neutrosophic special dual like number graph with edge weights from $\langle Z_{10} \cup h\rangle_I$ given by the following figure:

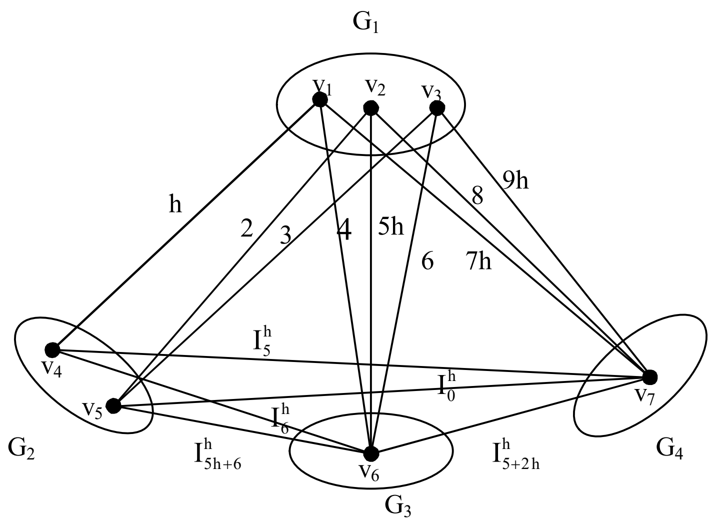


Figure 4.110

Interested reader is left with the task of working with MOD natural neutrosophic n-partite graphs with edge weights from Z_m^I $\langle Z_m \cup g\rangle_I$ or $C^I(Z_m)$ or $\langle Z_m \cup h\rangle_I$ or $\langle Z_m \cup I\rangle_I$ or $\langle Z_m \cup k\rangle_I$.

Next we proceed onto develop and describe type I MOD - n partite natural neutrosophic graphs by examples.

A MOD n-partite natural neutrosophic graph G is said to be of type I of the edge weights and vertex sets are from Z_n^I or $C^I(Z_n)$ or $\langle Z_n \cup I\rangle_I$ or $\langle Z_n \cup g\rangle_I$ or $\langle Z_n \cup h\rangle_I$ or $\langle Z_n \cup k\rangle_I$.

We will describe each of these situation by one example each.

Example 4.112: Let G be a 4-partite natural neutrosophic graph of type I with vertex set and edge set from Z_8^I given by the following figure:

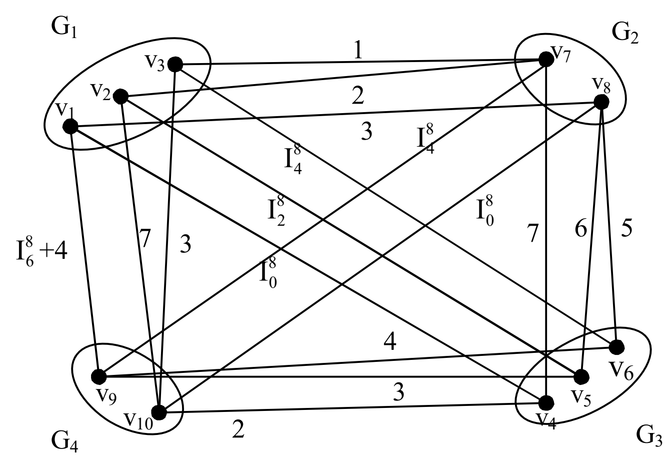


Figure 4.111

$v_i \in Z_8^I$; $1 \leq i \leq 10$.

Example 4.113: Let G be the MOD natural neutrosophic 3-partite graph of type I with edge weights and vertex set from Z_{12}^{I} given by the following figure:

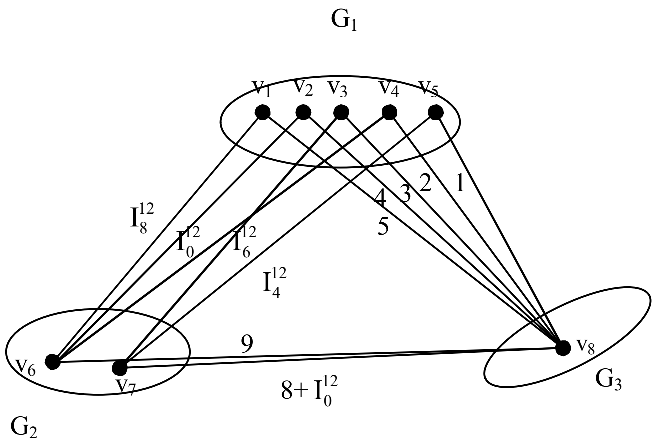


Figure 4.112

$$v_i \in Z_{12}^{I};\ 1 \leq i \leq 8.$$

Next we give example of a MOD n-partite natural neutrosophic finite complex number graph of type I which takes edge weights and vertex sets from $C^I(Z_n)$.

Example 4.114: Let G be a MOD 4-partite natural neutrosohic finite complex number graph of type I with edge weights and vertex set from $C^I(Z_{10})$ given by the following figure:

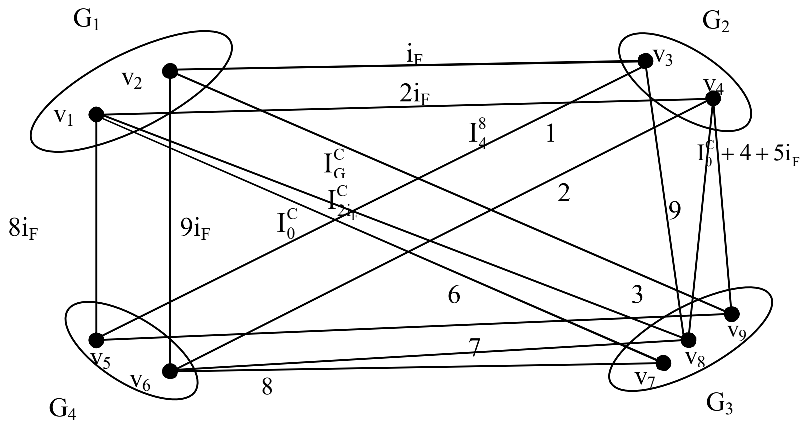


Figure 4.113

$v_i \in C^I(Z_{10})$; $1 \leq i \leq 9$.

Example 4.115: Let V be a MOD 3-partite finite complex natural neutrosophic type I graph with edge weights and vertex set from $C^I(Z_{13})$ given by the following figure:

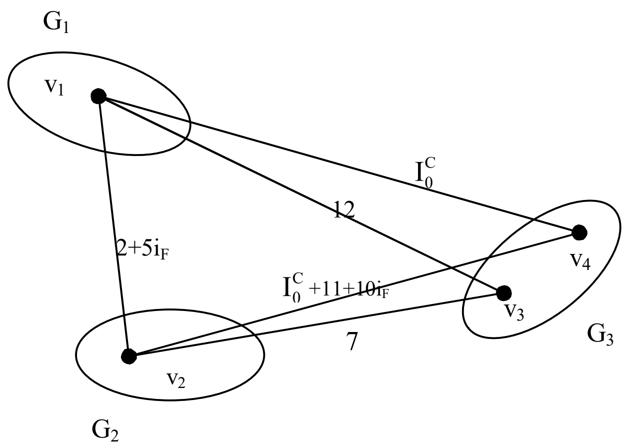


Figure 4.114

$$v_i \in C^I(Z_{13});\ 1 \leq i \leq 4.$$

Next we define a MOD n-partite natural neutrosophic graph of type I to be MOD n-partite natural neutrosophic neutrosophic graph of type I if edge weights and vertex sets are from $\langle Z_n \cup I\rangle_I$.

We will illustrate this situation by an example.

Example 4.116: Let G be a MOD 4-partite natual neutrosophic neutrosophic graph of type I with edge weights from $\langle Z_{10} \cup I\rangle_I$ and vertex sets from $\langle Z_{10} \cup I\rangle_I$ given by the following figure:

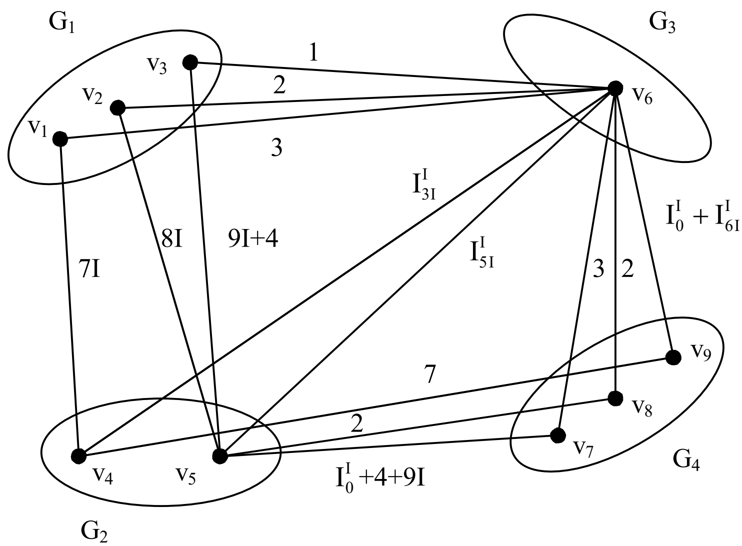


Figure 4.115

Next we define a MOD n-partite natural neutrosophic graph of type I with edge weights and vertex sets from $\langle Z_m \cup g\rangle_I$ to be a MOD n-partite natural neutrosohic dual number graph of type I with edge weights and vertex sets from $\langle Z_m \cup g\rangle_I$.

We will illustrate this situation by an example .

Example 4.117: Let G be a MOD 5-partite natural neutrosophic dual number graph of type I with edge weights and vertex sets from $\langle Z_9 \cup g\rangle_I$ given by the following figure:

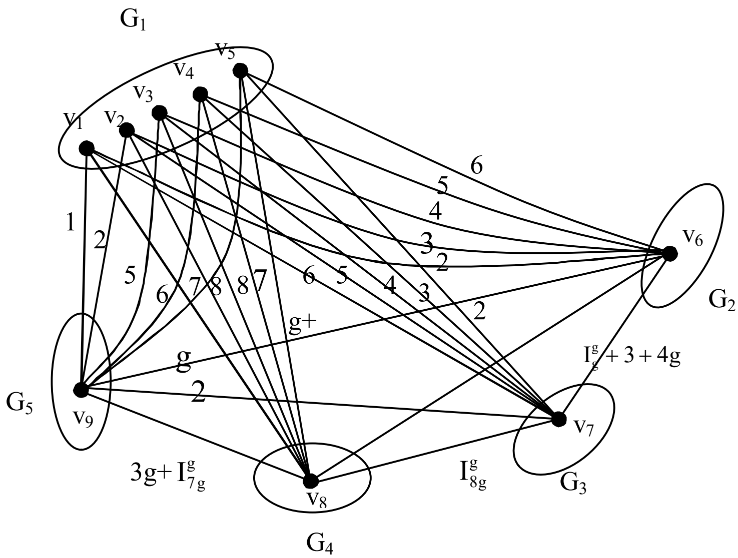


Figure 4.116

Next we proceed onto define G to be a MOD n-partite natural neutrosophic special dual like number graph of type I as the vertex sets and edge weights are from $\langle Z_n \cup h \rangle_I$.

We will illustrate this situation by some example.

Example 4.118: Let G be a MOD 3-partite natural neutrosohic special dual like number type I graph with edge weights and vertex sets from $\langle Z_{12} \cup h \rangle_I$ given by the following figure:

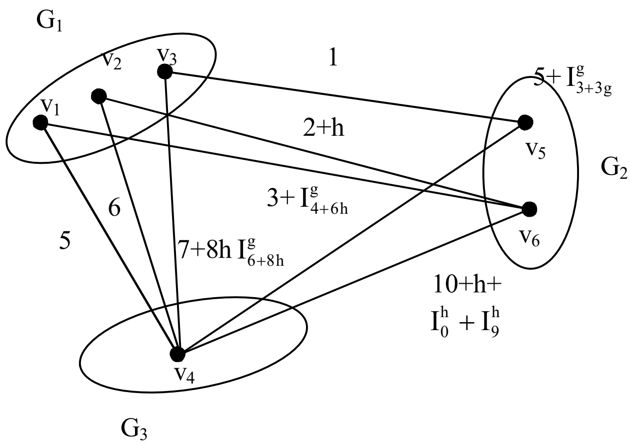


Figure 4.117

Likewise we can define MOD n-partite natural neutrosophic special quasi dual number type I graph with edge weights and vertex sets from $\langle Z_m \cup k\rangle_I$.

We will illustrate this situation by an example.

Example 4.119: Let G be a MOD 3-partite natural neutrosophic special quasi dual number graph of type I with edge weights and vertex sets from $\langle Z_9 \cup k\rangle_I$ given by the following figure:

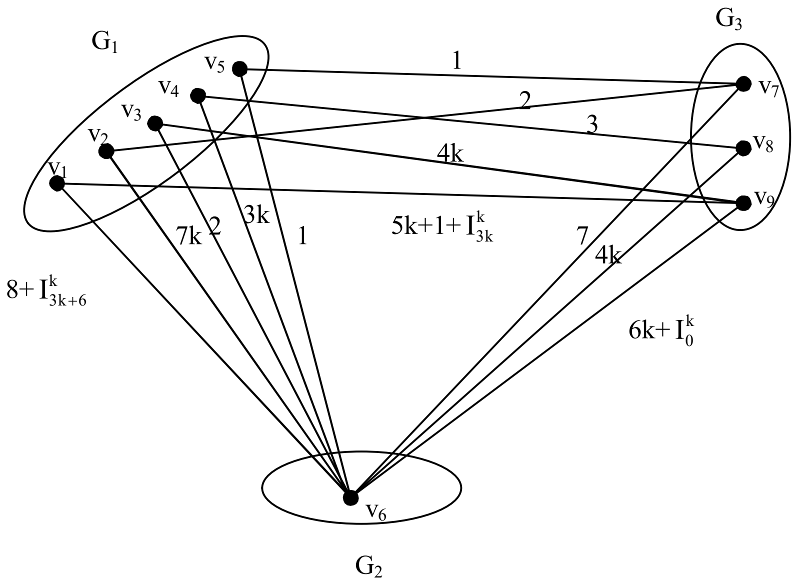


Figure 4.118

$$v_i \in \langle Z_9 \cup k \rangle_I;\ 1 \leq i \leq 9.$$

Next we proceed of to define type II MOD n-partite natural neutrosophic graphs.

Let G be a MOD n-partite natural neutrosophic graph G is said to be a type II graph if edge weights and vertex sets are from two distinct sets.

This situation is describe by some examples.

Example 4.120: Let G be a MOD 4-partite natural neutrosophic finite complex number graph of type II with edge weights from Z_{10}^{I} and vertex set from $C^{I}(Z_8)$.

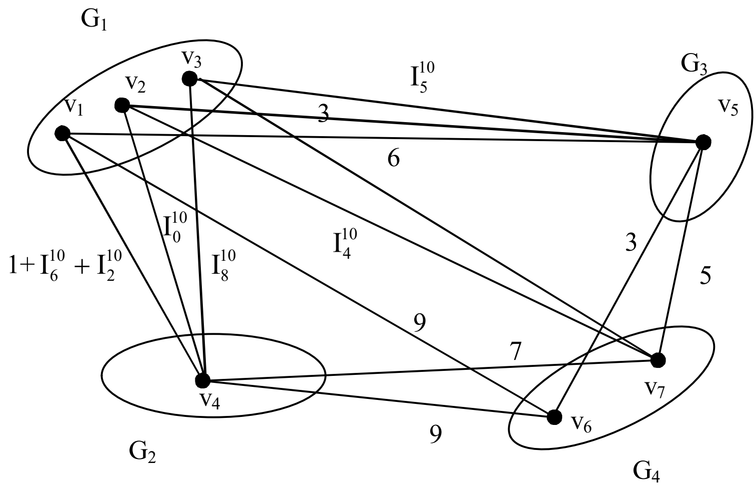


Figure 4.119

$$v_i \in C^{I}(Z_8);\ 1 \leq i \leq 7.$$

Example 4.121: Let G be a MOD 3-partite natural neutrosophic graph of type II with edge weights from $\langle Z_8 \cup I\rangle_I$ and vertex set from Z_{12}^{I} given by the following figure:

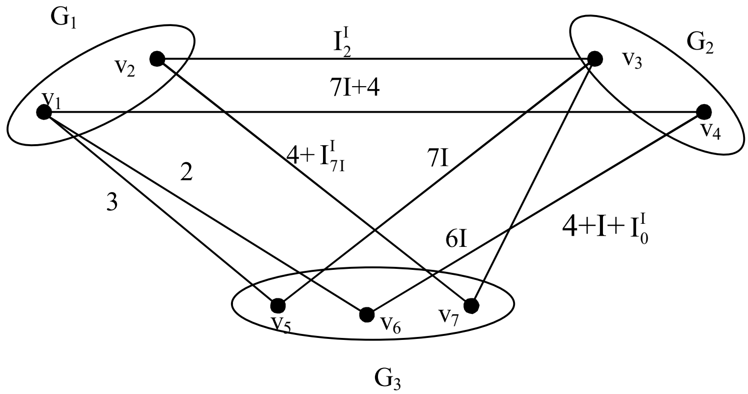


Figure 4.120

$$v_i \in Z_{12}^I;\ 1 \le i \le 7.$$

Example 4.122: Let G be a MOD-4 partite graph of natural neutrosophic numbers of type II with edge weights from $\langle Z_9 \cup g\rangle_I$ and vertex set from $\langle Z_{16} \cup k\rangle_I$ given by the following figure:

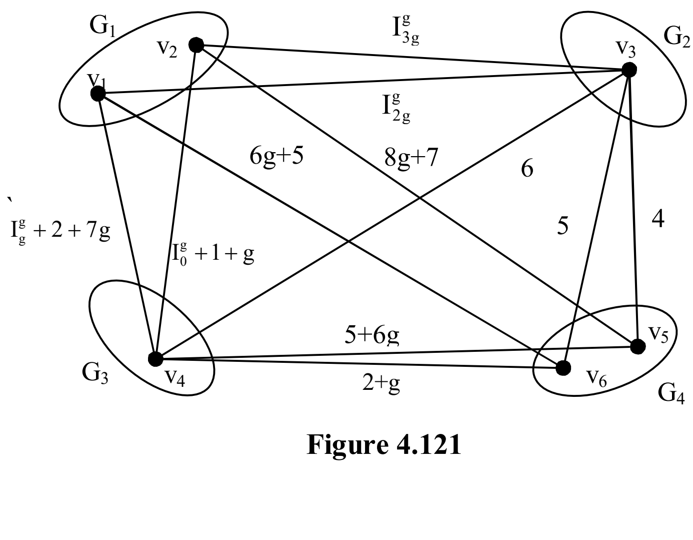


Figure 4.121

Example 4.123: Let P be a MOD 5-partite natural neutrosophic number graph of type II with edge weights from $C^I(Z_8)$ and vertex set from $\langle Z_7 \cup h\rangle_I$ given by the following figure:

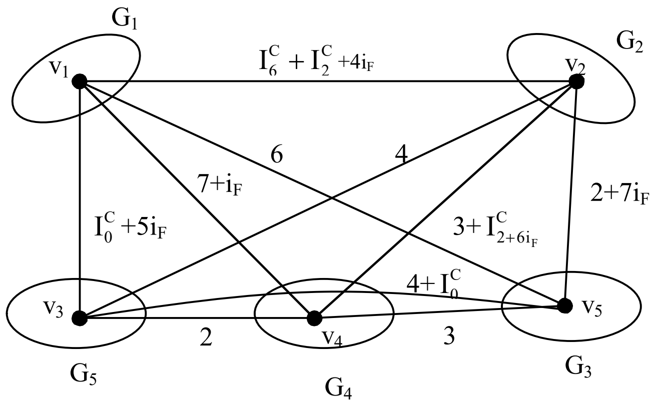


Figure 4.122

$$v_i \in \langle Z_7 \cup h\rangle_I \ ; 1 \leq i \leq 5.$$

Next we proceed onto describe MOD n-partite natural neutrosophic graphs of type III which has edge weights and vertex sets to be from more than two sets.

We will illustrate these situation by some examples.

Example 4.124: Let G be a MOD natural neutrosophic 4 partite graph of type III with edge weights from Z_{10}^I and vertex set entries from $\langle Z_5 \cup I\rangle_I$, $C^I(Z_9)$ and $\langle Z_{12} \cup g\rangle_I$; given by the following figure:

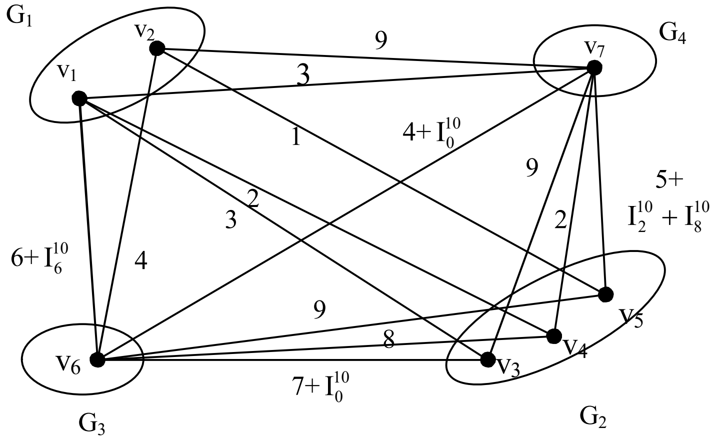


Figure 4.123

$v_1, v_2 \in Z_7^I$; $v_6 \in \langle Z_5 \cup I\rangle_I$ $v_7 \in C^I(Z_9)$ and v_3, v_4 and $v_5 \in \langle Z_{12} \cup g\rangle_I$.

Example 4.125: Let G be a MOD 3-partite natural neutrosophic type III graph with edge weights from $C^I(Z_4)$ and vertex sets from Z_6^I, $\langle Z_{10} \cup k\rangle_I$ and $\langle Z_3 \cup g\rangle_I$ given by the following figure:

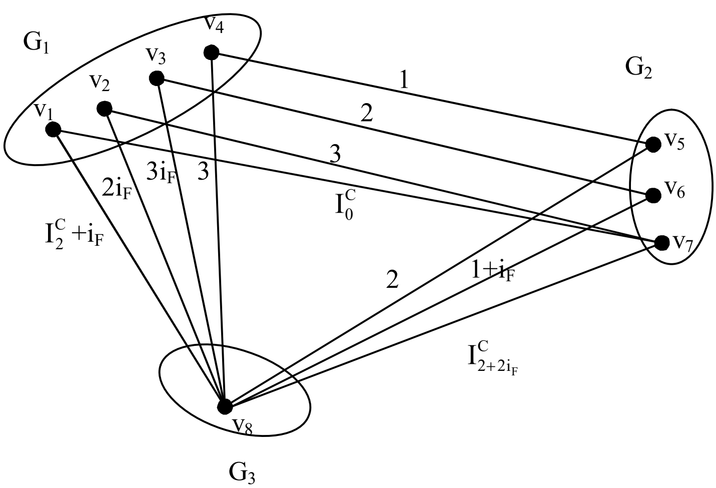


Figure 4.124

v_1, v_2, v_3 and $v_4 \in Z_6^I$, $v_8 \in \langle Z_{10} \cup k\rangle_I$ and $v_5, v_6, v_7 \in \langle Z_3 \cup g\rangle_I$.

Interested reader can build any number of MOD natural neutrosophi n-partite graphs of type III using any one of the set Z_m^I, $C^I(Z_m)$, $\langle Z_t \cup I\rangle_I$, $\langle Z_s \cup g\rangle_I$, $\langle Z_r \cup k\rangle_I$ and $\langle Z_p \cup h\rangle_I$.

Further these special types of MOD n-partite natural neutrosophic number graphs will be a boon to any researcher who wants to use the indeterminancy concept.

In the following we suggest a few problems to the reader.

Problems

1. Obtain any of the special features associated with the MOD bipartite graphs with edge weights from Z_n.

2. Can we uses these graphs in MOD relational maps models? Justify with an illustration.

3. Let G be a MOD bipartite finite complex number graphs with edge weights from $C(Z_n)$.

   i) Compare these with MOD bipartite graphs with edge weights from Z_n.

   ii) What are the special and interesting features associated with these graphs?

4. Given Z_n and $v_1, \ldots, v_t$ and $w_1, \ldots, w_s$ the vertex sets of the MOD bipartite graphs t and s fixed; n is fixed.

   i) How many MOD bipartite graphs can be drawn using edge weights from Z_n? (t and s fixed).

      a) If edge weights in (1) do not repeat?
      b) If edge weights can repeat?

5. Let G be a MOD bipartite neutrosophic graph with edge weights from $\langle Z_n \cup I \rangle$.

   i) Compare these graphs with MOD bipartite graphs with edge weights from Z_n.

   ii) Distingish these graphs from MOD bipartite finite complex number graphs with edge weights from $C(Z_n)$.

6. Give an example of a MOD bipartite graph with edge weights from Z_9.

7. Give an example of a MOD bipartite finite complex number graph with edge weights from $C(Z_{11})$.

8. Give an example of a MOD bipartite neutrosophic graph with edge weights from $\langle Z_{10} \cup I \rangle$.

9. For MOD graphs given in examples in problems (6), (7) and (8), obtain their MOD relational (connection) matrices associated with those graphs.

10. Give an example of a MOD bipartite dual number graph with edge weights from $\langle Z_{15} \cup g \rangle$. Find the MOD relational matrix associated with it.

11. Illustrate by an example the MOD bipartite special quasi dual number graph with edge weights from $\langle Z_{18} \cup k \rangle$ which can function as a MOD relational matrix.

12. Let G be a MOD bipartite special dual like number graph with edge weights from $\langle Z_{13} \cup h \rangle$. Use the MOD relational matrix associated with G.

13. Compare the 6 MOD bipartite graphs using edge weights from Z_n, $\langle Z_n \cup h \rangle$, $\langle Z_n \cup g \rangle$, $\langle Z_n \cup k \rangle$, $\langle Z_n \cup I \rangle$ and $C^I(Z_n)$; for a particular real problem.

14. What are the advantages of using Z_n in place of $C^I(Z_n)$?

15. Enumerate the advantages of using $\langle Z_n \cup k\rangle$ in the place of $\langle Z_n \cup I\rangle$.

16. Give an example of MOD n-partite graph with edge weights from Z_m.

17. Give an example of MOD 9-partite graph with edge weights from Z_{18}.

18. What are the interesting and important features associated with these MOD n-partite graphs with edge weights from Z_m?

19. Mention any of the applications of MOD n-partite graph with edge weights from Z_t.

20. Let G be a MOD n-partite dual number graph with edge weights from $\langle Z_n \cup g\rangle$.

    Mention the special features associated with it.

21. Compare the MOD n-partite graphs with MOD n-partite dual number graphs.

    a) Can you give an application in which MOD n-partite dual number graph performance better than MOD n-partite graph?

22. Let G be a MOD n-partite finite complex number graph with edge weights from $C^I(Z_m)$.

    i) Compare this G with the MOD n-partite graph with edge weights from Z_m.

    ii) Compare this G with the MOD n-partite dual number graph with edge weights from $\langle Z_m \cup g\rangle$.

23. Give some real world applicatons of MOD n-partite dual number graph with edge weights from $\langle Z_n \cup g\rangle$.

24. Let V be the MOD n-partite special quasi dual number graph with edge weights from $\langle Z_m \cup k\rangle$.

    Mention the special and distinct features associated with this V.

25. Let P be the MOD n-partite special dual like number graph with edge weights from $\langle Z_m \cup h\rangle$. Mention the special and distinct features enjoyed by the P.

    i) Compare this P with V in problem 24.

    ii) Compare this P with G in problem 22.

    iii) Find all the special and distinct features associated with this P.

    iv) Give some special applications of this new MOD n-partite graph P.

26. Give an example of a MOD n-partite graph with mixed edge weights.

27. What are advantages of using MOD n-partite graphs with MOD n-partite graphs with mixed edge weights?

28. Use the notion of MOD n-partite mixed edge weight graphs in practical problems.

29. What are the special features associated with MOD bipartite graphs of type I?

30. Compare MOD bipartite graphs of type I with MOD bipartite graphs.

31. Give an example of MOD bipartite dual number graph of type I.

32. Let V be the MOD bipartite finite complex number type I graph.

    i) Compare V with MOD bipartite finite complex number graph.

    ii) Compare V with MOD bipartite dual number graph.

33. Let W be the MOD bipartite special quasi dual number graph of type I.

    i) Compare W with V of problem 32.

    ii) What are advantages of using W in place of MOD bipartite graph of type I with entries from Z_n.

34. Let S be the MOD bipartite special dual like number graph of type I.

    i) Compare this S with W of problem 33.

    ii) Compare this S with V of problem 32.

    iii) Give a problem in which S is best suited.

    iv) Give a problem in which W is best suited.

35. Let B be the MOD bipartite neutrosophic graph of type I.

    i) Compare this B with S in problem 34.
    ii) Compare this B with W in problem 33.

36. What are the special features associated with MOD bipartite type II graphs?

37. Compare MOD bipartite type II graphs with MOD bipartite type I graphs.

38. Compare MOD bipartite type III graphs with MOD bipartite graphs.

39. Let G be a MOD bipartitet type III graph with edge weights from Z_n, vertex sets from $\langle Z_m \cup g\rangle$, and $C(Z_p)$, n, m and p are distinct.

    i) Compare G with a MOD bipartite graph of type II when $n = m = p$.

    ii) Enumerate any advantage of using type III graphs in place of type I graphs.

40. Let B be a MOD bipartite graph of type III with edge weights from $C(Z_{10})$ and vertex sets from $\langle Z_{12} \cup g\rangle$ and $\langle Z_{15} \cup I\rangle$.

    i) Show there can only be a finite number of such B's if the number of vertices are fixed.

    ii) Show even if the number of verties are not fixed. We can have only finite such B's.

    iii) Find the number of such B's when $C(Z_{10})$ replaced by $C(Z_4)$, $\langle Z_{12} \cup g\rangle$ by $\langle Z_3 \cup g\rangle$ and $\langle Z_{15} \cup I\rangle$ by $\langle Z_5 \cup I\rangle$. Prove in case of (iii) the number of vertices is curtailed.

41. What are special features associated with type II graphs which vertex sets are $C(Z_{10})$ and Z_{17} and edge sets from $C(Z_{10})$?

42. In (41) if edge sets are from Z_{17} and vertex sets $C(Z_{10})$ and Z_{17} compare these MOD bipartite type II graphs.

43. Does there exist a real world application in which MOD bipartite type III graph is preferred to MOD bipartite type II graph?

44. Can there be a situation in which reverse in problem (43) is applicable?

45. Enumerate all special features associated with MOD n- partite graphs of type I.

46. Give a real world problem situation in which MOD n-partite type I graph is suited.

47. What are the special features associated with MOD n-partitte type II graphs?

48. Compare MOD n-partite type I graph with usual MOD n-partite graphs.

49. Describe some situations in with MOD n-partite graphs are better than MOD n-partite graph of type I graphs.

50. Give atleast a situation in which MOD n-partite graph of type I is better than MOD n-partite graph.

51. Describe by examples MOD n-partite graphs of type II.

52. Compare MOD n-partite graphs of type II with MOD n-partite graphs of type I.

53. Show by a practical illustration in which MOD n-partite graph of type II is preferred to MOD n-partite graph of type I.

54. Describe the special features enjoyed by MOD n-partite graphs of type III.

55. Give an example of a MOD of partite graph of type III.

56. Compare MOD n-partite graph of type III with MOD n-partite graph of type II.

57. Compare MOD n-partite graph of type III with MOD n-partite graph of type I.

58. Show by some practical situation in which MOD n-partite graph of type III is preferred to MOD n-partite graph of type II and type I.

59. Obtain all special features associated with MOD n-partite graphs of type I, type II and type III.

60. Give an example of a MOD natural neutrosophic bipartite graph.

61. Give an example of a MOD natural neutrosophic finite complex number bipartite graph.

62. Give an example of a MOD natural neutrosophic neutrosophic bipartite graphs with edge weights from Z_9^I.

63. What are the special and distinct features enjoyed by MOD natural neutrosophic bipartite graphs with edge weights from Z_n^I and those MOD bipartite graphs with edge weights from Z_n?

64. Give an example of a MOD natural neutrosophic neutrosophic bipartite graph G with edge weights from $\langle Z_{12} \cup I \rangle_I$.

    i) Compare this G with MOD natural neutrosophic bipartite graph H with edge weights from Z_{12}^I.

    ii) Using this G and H build MODRMs model and study them.

65. Give an example of a MOD natural neutrosophic finite complex number bipartite graph G with edge weights from $C^I(Z_{10})$.

    i) Compare this V with MOD natural neutrosophic bipartite graph G with edge weights from Z_{10}^I.

    ii) Compare the MODRMs model associated with V and G.

    iii) Prove this MODRM model given by V is more appropriate in problems which involves indeterminancy and complex qualitity.

66. Let F be a MOD neutral neutrosophic dual number bipartite graph with edge weights from $\langle Z_{18} \cup g\rangle_I$.

    i) Construct the MODRM model associated with F.

    ii) Compare this model associated with the models constructed using the MOD natural neutrosophic bipartite graph using edge weights from $\langle Z_8 \cup I\rangle_I$ and $C^I(Z_{18})$.

    iii) Obtain the special and interesting features associated witih MODRM model associated with F.

67. Let P be the MOD natural neutrosophic bipartite graph with edge weights from $\langle Z_{16} \cup k\rangle_I$.

    i) Construct the MODRM model associated with P.

    ii) Compare this model with MODRM model built using $\langle Z_{16} \cup I\rangle_I$ and $C^I(Z_{16})$.

68. Let S be a MOD 9-partite natural neutrosophic graph with edge weights from Z_{12}^I.

i) Find all special features associated with S.

ii) Study S when edge weights are from $C^I(Z_{12})$.

iii) What are the advantages of studyng these types of MOD n-partite natural neutrosophic graphs with edge weights from $\langle Z_n \cup g\rangle_I$ or $\langle Z_{12} \cup g\rangle_I$ or $\langle Z_{12} \cup h\rangle_I$ or $\langle Z_{12} \cup k\rangle_I$ or $C^I(Z_n)$?

69. Find all the special features associated with MOD natural neutrosophic bipartite graph of type I.

i) Find the MODRMs models associated with these graphs.

ii) Show these models are more appropriate when the problem involves indeterminancy and imaginary concepts.

iii) Obtain any other special feature associated with these models.

70. Give an example of a MOD natural neutrosophic bipartite graph of type I with vertex set and edge weights from $C^I(45)$.

71. Compare type I MOD natural neutrosophic bipartite graphs with MOD natural neutrosophic bipartite graphs.

72. Let G be the MOD natural neutrosophic bipartite graph of type I with edge weights and vertex set from $\langle Z_{17} \cup I\rangle_I$. Obtain all special features associated with G.

73. Study G in problem 72 if edge weights are from $\langle Z_{17} \cup g\rangle_I$.

74. Study G in problem 72 if edge weights are from $\langle Z_{17} \cup k\rangle_I$.

75. Describe with example MOD natural neutrosophic bipartite graph of type II.

76. Compare and distinguish between MOD natural neutrosophi bipartite graphs of type I and type II.

77. Construct a MODRM model using type I MOD bipartite graph.

78. Describe a MODRM model using type II MOD bipartite graph.

79. Construct using $C^I(Z_{10})$ and $\langle Z_{15} \cup g\rangle_I$ and $c^I(Z_{10})$ and $\langle Z_{18} \cup k\rangle_I$ a MOD natural neutrosophic bipartite graphs G_1 and G_2 of type II.

    i) Use G_1 and G_2 construct MODRM models.

    ii) Enumerate all the special features associated these MODRM models in general.

80. Describe MOD natural neutrosophic bipartite graphs of type III.

81. Bring out the differences between MOD natural neutrosophic bipartite graphs of type I and III.

82. Bring out the similarities and differences between MOD natural neutrosophic bipartite graphs of type II and type III.

83. Let G be a MOD natural neutrosophic bipartite graph of type III with edge weights from $C^I(Z_{12})$ and vertex sets from $\langle Z_{10} \cup I\rangle_I$ and $\langle Z_8 \cup g\rangle_I$.

    i) Find the MODRM associated with G.

    ii) How many G's can be got using these 3 sets?

iii) Find any other special feature associated with these G's.

84. Give an example of a MOD 5-partite natural neutrosophic - neutrosophic graph with edge weights from $\langle Z_{16} \cup I \rangle_I$.

 i) How many distinct MOD 5-partitte natural neutrosophic neutrosophic graphs with edge weights from $\langle Z_{16} \cup I \rangle_I$ can be obtained?

 ii) Can there be infinite number of such MOD 5-partite graphs using $\langle Z_{16} \cup I \rangle_I$? Justify your answer.

 iii) Give some applications of such graphs.

85. Study question 84 when $\langle Z_{16} \cup I \rangle_I$ is replaced by $C^I(Z_{16})$.

86. Study a MOD 6-partitte natural neutrosophic graph with edge weights from $\langle Z_{24} \cup g \rangle_I$.

87. Study question (86) when $\langle Z_{24} \cup g \rangle_I$ is replaced by $\langle Z_{10} \cup k \rangle_I$.

88. Prove for which different edge weight sets the MOD n-partite graphs also behave differently.

89. What are type I MOD n-partite graphs?

90. Distinguish between MOD n-partite graphs from MOD n-partite graphs of type I.

91. Find some interesting and appropriate applications of type I, type II and type III MOD graphs .

92. Give an example of a MOD natural neutrosophic dual number 7-partite graph G with edge weights and vertex sets from $\langle Z_{12} \cup g \rangle_I$.

i) Prove the number of vertices of G must be atmost equal to $|\langle Z_{24} \cup g\rangle_I|$.

ii) Prove the least number of vertices is 7 for this G.

iii) What is the maximum number of vertices G can have?

iv) How many such G's exist with maximum number of vertices?

v) If G has 12 vertices how many MOD 7-partite graphs can be constructed using $\langle Z_{12} \cup g\rangle_I$?

vi) Obtain any other special feature associated with G.

93. Desribe MOD natural neutrosophic n-partite graphs of type II.

94. Distinguish MOD natural neutrosophic n-partite graphs of type I and type II.

95. Give an example of a MOD natural neutrosophic 8-partite graph G of type II with edge weights fom $C^I(Z_5)$ and vertex set from $\langle Z_6 \cup I\rangle_I$.

i) Find the maximum number vertices G can have.
ii) Prove G can have 8 to be the minimum number of vertices.
iii) Given 8 to be the number vertices (fixed values in $\langle Z_6 \cup I\rangle_I$) how many distinct MOD natural neutrosophic 8-partite graphs can be constructed.
iv) Study question (iii) with 16 vertices.

96. Describe all the special features associated with MOD natural neutrosophic n-partite type III graphs.

97. Give an example of a MOD 5-partite natural neutrosophic graph of type III with edge weights from Z_{12}^{I}, vertex sets from $\langle Z_9 \cup g\rangle_I$, $C^I(14)$, $\langle Z_{19} \cup h\rangle_I$, $\langle Z_{10} \cup k\rangle_I$ and Z_{18}^{I}.

98. Describe all special properties associated with type I, type II and type III MOD n-partite graphs.

99. Give some special applications of these MOD n-partite graphs all types.

100. Give examples of MOD n-partite graphs of all these types these using the set Z_9^I $\langle Z_9 \cup I\rangle_I$, $\langle Z_9 \cup h\rangle_I$, $\langle Z_9 \cup g\rangle_I$, $C^I(Z_9)$ and $\langle Z_9 \cup k\rangle_I$ as vertex sets and edge weights.

FURTHER READING

1. Harary, F., Graph Theory, Narosa Publications (reprint, Indian edition), New Delhi, 1969.

2. Kosko, B., Fuzzy Cognitive Maps, Int. J. of Man-Machine Studies, 24 (1986) 65-75.

3. Kosko, B., Fuzzy Thinking, Hyperion, 1993.

4. Kosko, B., Hidden Patterns in Combined and Adaptive Knowledge Networks, Proc. of the First IEEE International Conference on Neural Networks (ICNN-86), 2 (1988) 377-393.

5. Kosko, B., Neural Networks and Fuzzy Systems: A Dynamical Systems Approach to Machine Intelligence, Prentice Hall of India, 1997.

6. Smarandache, F. (editor), Proceedings of the First International Conference on Neutrosophy, Neutrosophic Set, Neutrosophic Probability and Statistics, Univ. of New Mexico – Gallup, 2001.

   http://www.gallup.unm.edu/~smarandache/NeutrosophicProceedings.pdf

7. Smarandache, F. Neutrosophic Logic - Generalization of the Intuitionistic Fuzzy Logic, presented at the Special Session on

Intuitionistic Fuzzy Sets and Related Concepts, of International EUSFLAT Conference, Zittau, Germany, 10-12 September 2003. http://lanl.arxiv.org/ftp/math/papers/0303/0303009.pdf

8. Smarandache, F., Collected Papers III, Editura Abaddaba, Oradea, 2000. http://www.gallup.unm.edu/~smarandache/CP3.pdf

9. Vasantha Kandasamy, W.B., Love. Life. Lust. Loss: 101 Real Life Stories of Migration and AIDS—A Fuzzy Analysis, Tamil Nadu State AIDS Control Society, Chennai, 2003.

10. Vasantha Kandasamy, W.B., Women. Work. Worth. Womb., 101 real life stories of rural women, Tamil Nadu State AIDS Control Society, Chennai, 2005.

11. Vasantha Kandasamy, W.B., Public. Pity. Patients. Peace., Public opinion on HIV/AIDS, Tamil Nadu State AIDS Control Society, Chennai, 2006.

12. Vasantha Kandasamy, W.B., and Minor, A., Estimation of Production and Loss or Gain to Industries Using Matrices, Proc. of the National Conf. on Challenges of the 21st century in Mathematics and its allied topics, Feb. 3-4, 2001, Univ. of Mysore, 211-218.

13. Vasantha Kandasamy, W.B., and Balu, M. S., Use of Weighted Multi-Expert Neural Network System to Study the Indian Politics, Varahimir J. of Math. Sci., 2 (2002) 44-53.

14. Vasantha Kandasamy, W.B., and Indra, V., Maximizing the passengers comfort in the madras transport corporation using fuzzy programming, Progress of Mat., Banaras Hindu Univ., 32 (1998) 91-134.

15. Vasantha Kandasamy, W.B., and Karuppusamy, Environmental pollution by dyeing industries: A FAM analysis, Ultra Sci, 18(3) (2006) 541-546.

16. Vasantha Kandasamy, W.B., and Mary John, M. Fuzzy Analysis to Study the Pollution and the Disease Caused by

Hazardous Waste From Textile Industries, Ultra Sci, 14 (2002) 248-251.

17. Vasantha Kandasamy, W.B., and Ram Kishore, M. Symptom-Disease Model in Children using FCM, Ultra Sci., 11 (1999) 318-324.

18. Vasantha Kandasamy, W.B., and Pramod, P., Parent Children Model using FCM to Study Dropouts in Primary Education, Ultra Sci., 13, (2000) 174-183.

19. Vasantha Kandasamy, W.B., and Praseetha, R., New Fuzzy Relation Equations to Estimate the Peak Hours of the Day for Transport Systems, J. of Bihar Math. Soc., 20 (2000) 1-14.

20. Vasantha Kandasamy, W.B., and Uma, S. Combined Fuzzy Cognitive Map of Socio-Economic Model, Appl. Sci. Periodical, 2 (2000) 25-27.

21. Vasantha Kandasamy, W.B., and Uma, S. Fuzzy Cognitive Map of Socio-Economic Model, Appl. Sci. Periodical, 1 (1999) 129-136.

22. Vasantha Kandasamy, W.B., and Smarandache, F., Analysis of social aspects of migrant labourers living with HIV/AIDS using fuzzy theory and neutrosophic cognitive maps, Xiquan, Phoenix, 2004.

23. Vasantha Kandasamy, W.B., and Smarandache, F., Basic Neutrosophic algebraic structures and their applications to fuzzy and Neutrosophic models, Hexis, Church Rock, 2004

24. Vasantha Kandasamy, W.B., and Smarandache, F., Fuzzy and Neutrosophic analysis of Periyar's views on untouchability, Hexis, Phoenix, 2005.

25. Vasantha Kandasamy, W.B., and Smarandache, F., Fuzzy Cognitive Maps and Neutrosophic Cognitive Maps, Xiquan, Phoenix, 2003.

26. Vasantha Kandasamy, W.B., and Smarandache, F., Fuzzy Relational Equations and Neutrosophic Relational Equations, Neutrosophic Book Series 3, Hexis, Church Rock, USA, 2004.

27. Vasantha Kandasamy, W.B., and Smarandache, F., Introduction to n-adaptive fuzzy models to analyse Public opinion on AIDS, Hexis, Phoenix, 2006.

28. Vasantha Kandasamy, W.B., and Smarandache, F., Vedic Mathematics: 'Vedic' or 'mathematics' A Fuzzy and Neutrosophic Analysis, automaton, Los Angeles, 2006.

29. Vasantha Kandasamy, W.B., Smarandache, F., and Ilanthenral, K., Elementary Fuzzy matrix theory and fuzzy models for social scientists, automaton, Los Angeles, 2006.

30. Vasantha Kandasamy, W.B., and Anitha, V., Studies on Female Infanticide Problem using Neural Networks BAM-model, Ultra Sci., 13 (2001) 174-183.

31. Vasantha Kandasamy, W.B., and Indra, V., Applications of Fuzzy Cognitive Maps to Determine the Maximum Utility of a Route, J. of Fuzzy Maths, publ. by the Int. fuzzy Mat. Inst., 8 (2000) 65-77.

32. Vasantha Kandasamy, W.B., and Victor Devadoss, Identification of the maximum age group in which the agricultural labourers suffer health hazards due to chemical Pollution using fuzzy matrix, Dimension of pollution, 3 (2005) 1-55.

33. Vasantha Kandasamy, W.B., and Yasmin Sultana, FRM to Analyse the Employee-Employer Relationship Model, J. Bihar Math. Soc., 21 (2001) 25-34.

34. Vasantha Kandasamy, W.B., and Yasmin Sultana, Knowledge Processing Using Fuzzy Relational Maps, Ultra Sci., 12 (2000) 242-245.

35. Vasantha Kandasamy, W.B., Mary John, M., and Kanagamuthu, T., Study of Social Interaction and Woman Empowerment Relative to HIV/AIDS, Maths Tiger, 1(4) (2002) 4-7.

36. Vasantha Kandasamy, W.B., Neelakantan, N.R., and Ramathilagam, S., Maximize the Production of Cement Industries by the Maximum Satisfaction of Employees using Fuzzy Matrix, Ultra Science, 15 (2003) 45-56.

37. Vasantha Kandasamy, W.B., Neelakantan, N.R., and Kannan, S.R., Operability Study on Decision Tables in a Chemical Plant using Hierarchical Genetic Fuzzy Control Algorithms, Vikram Mathematical Journal, 19 (1999) 48-59.

38. Vasantha Kandasamy, W.B., Neelakantan, N.R., and Kannan, S.R., Replacement of Algebraic Linear Equations by Fuzzy Relation Equations in Chemical Engineering, In Recent Trends in Mathematical Sciences, Proc. of Int. Conf. on Recent Advances in Mathematical Sciences held at IIT Kharagpur on Dec. 20-22, 2001, published by Narosa Publishing House, (2001) 161-168.

39. Vasantha Kandasamy, W.B., Neelakantan, N.R., and Ramathilagam, S., Use of Fuzzy Neural Networks to Study the Proper Proportions of Raw Material Mix in Cement Plants, Varahmihir J. Math. Sci., 2 (2002) 231-246.

40. Vasantha Kandasamy, W.B., Pathinathan, and Narayan Murthy. Child Labour Problem using Bi-directional Associative Memories (BAM) Model, Proc. of the 9th National Conf. of the Vijnana Parishad of India on Applied and Industrial Mathematics held at Netaji Subhas Inst. of Tech. on Feb. 22-24, 2002.

41. Vasantha Kandasamy, W.B., Ramathilagam, S., and Neelakantan, N.R., Fuzzy Optimisation Techniques in Kiln Process, Proc. of the National Conf. on Challenges of the 21st century in Mathematics and its allied topics, Feb. 3-4 (2001), Univ. of Mysore, (2001) 277-287.

42. Vasantha Kandasamy, W. B. and Smarandache, F., Neutrosophic algebraic structures and neutrosophic N-algebraic structures, Hexis, Phoenix, Arizona, (2006).

43. Vasantha Kandasamy, W. B. and Smarandache, F., Smarandache Neutrosophic algebraic structures, Hexis, Phoenix, Arizona, (2006).

44. Vasantha Kandasamy, W.B., and Smarandache, F., Fuzzy Interval Matrices, Neutrosophic Interval Matrices and their Applications, Hexis, Phoenix, (2006).

45. Vasantha Kandasamy, W.B., Smarandache, F., and Ilanthenral, K., Special fuzzy matrices for social scientists, InfoLearnQuest, Ann Arbor, (2007).

46. Vasantha Kandasamy, W.B. and Smarandache, F., Interval Semigroups, Kappa and Omega, Glendale, (2011).

47. Vasantha Kandasamy, W.B. and Smarandache, F., Finite Neutrosophic Complex Numbers, Zip Publishing, Ohio, (2011).

48. Vasantha Kandasamy, W.B. and Smarandache, F., Dual Numbers, Zip Publishing, Ohio, (2012).

49. Vasantha Kandasamy, W.B. and Smarandache, F., Special dual like numbers and lattices, Zip Publishing, Ohio, (2012).

50. Vasantha Kandasamy, W.B. and Smarandache, F., Special quasi dual numbers and Groupoids, Zip Publishing, Ohio, (2012).

51. Vasantha Kandasamy, W.B. and Smarandache, F., Algebraic Structures using Subsets, Educational Publisher Inc, Ohio, (2013).

52. Vasantha Kandasamy, W.B. and Smarandache, F., Algebraic Structures using [0, n), Educational Publisher Inc, Ohio, (2013).

53. Vasantha Kandasamy, W.B. and Smarandache, F., Algebraic Structures on the fuzzy interval [0, 1), Educational Publisher Inc, Ohio, (2014).

54. Vasantha Kandasamy, W.B. and Smarandache, F., Algebraic Structures on Fuzzy Unit squares and Neutrosophic unit square, Educational Publisher Inc, Ohio, (2014).

55. Vasantha Kandasamy, W.B. and Smarandache, F., Natural Product on Matrices, Zip Publishing Inc, Ohio, (2012).

56. Vasantha Kandasamy, W.B. and Smarandache, F., Algebraic Structures on Real and Neutrosophic square, Educational Publisher Inc, Ohio, (2014).

57. Vasantha Kandasamy, W.B., Ilanthenral, K., and Smarandache, F., MOD planes, EuropaNova, (2015).

58. Vasantha Kandasamy, W.B., Ilanthenral, K., and Smarandache, F., MOD Functions, EuropaNova, (2015).

59. Vasantha Kandasamy, W.B., Ilanthenral, K., and Smarandache, F., Multidimensional MOD planes, EuropaNova, (2015).

60. Vasantha Kandasamy, W.B., Ilanthenral, K., and Smarandache, F., Natural Neutrosophic numbers and MOD Neutrosophic numbers, EuropaNova, (2015).

61. Vasantha Kandasamy, W.B., Ilanthenral, K., and Smarandache, F., Algebraic Structures on MOD planes, EuropaNova, (2015).

62. Vasantha Kandasamy, W.B., Ilanthenral, K., and Smarandache, F., MOD Pseudo Linear Algebras, EuropaNova, (2015).

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66. Vasantha Kandasamy, W.B., Ilanthenral, K., and Smarandache, F., Special Type of Fixed Points pairs of MOD Rectangular Matrix Operators, EuropaNova, (2016).

67. Vasantha Kandasamy, W.B., Smarandache, F. and Ilanthenral, K., Distance in matrices and their applications to fuzzy models and neutrosophic models, EuropaNova, (2014).

68. Vasantha Kandasamy, W.B., Smarandache, F. and Ilanthenral, K., MOD Cognitive Maps models and MOD natural neutrosophic Cognitive Maps models, EuropaNova, (2016).

69. Vasantha Kandasamy, W.B., Smarandache, F. and Ilanthenral, K., MOD Relational Maps models and MOD natural neutrosophic relational Maps models, EuropaNova, (2016).

70. Zadeh, L.A., A Theory of Approximate Reasoning, Machine Intelligence, 9 (1979) 149- 194.

71. Zhang, W.R., and Chen, S., A Logical Architecture for Cognitive Maps, Proceedings of the 2nd IEEE Conference on Neural Networks (ICNN-88), 1 (1988) 231-238.

72. Zimmermann, H.J., Fuzzy Set Theory and its Applications, Kluwer, Boston, 1988.

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On India's 60th Independence Day, Dr.Vasantha was conferred the Kalpana Chawla Award for Courage and Daring Enterprise by the State Government of Tamil Nadu in recognition of her sustained fight for social justice in the Indian Institute of Technology (IIT) Madras and for her contribution to mathematics. The award, instituted in the memory of Indian-American astronaut Kalpana Chawla who died aboard Space Shuttle Columbia, carried a cash prize of five lakh rupees (the highest prize-money for any Indian award) and a gold medal.

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In this book authors for the first time introduce, study and develop the notion of MOD graphs, MOD directed graphs, MOD finite complex number graphs, MOD neutrosophic graphs, MOD dual number graphs and MOD directed natural neutrsophic graphs. There are open conjectures which can help researchers in graph theory.

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