MARIUS COMAN

THE MATH ENCYCLOPEDIA OF SMARANDACHE TYPE NOTIONS

Vol. I. NUMBER THEORY

2013

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Educational Publishing

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Education Publishing
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EAN: 9781599732527
ISBN: 978-1-59973-252-7

INTRODUCTION

About the works of Florentin Smarandache have been written a lot of books (he himself wrote dozens of books and articles regarding math, physics, literature, philosophy). Being a globally recognized personality in both mathematics (there are countless functions and concepts that bear his name), it is natural that the volume of writings about his research is huge.

What we try to do with this encyclopedia is to gather together as much as we can both from Smarandache's mathematical work and the works of many mathematicians around the world inspired by the Smarandache notions. Because this is too vast to be covered in one book, we divide encyclopedia in more volumes.

In this first volume of encyclopedia we try to synthesize his work in the field of number theory, one of the great Smarandache's passions, *a surfer on the ocean of numbers*, to paraphrase the title of the book *Surfing on the ocean of numbers – a few Smarandache notions and similar topics*, by Henry Ibstedt.

We quote from the introduction to the Smarandache'work "On new functions in number theory", Moldova State University, Kishinev, 1999: "The performances in current mathematics, as the future discoveries, have, of course, their beginning in the oldest and the closest of philosophy branch of nathematics, the number theory. Mathematicians of all times have been, they still are, and they will be drawn to the beaty and variety of specific problems of this branch of mathematics. Queen of mathematics, which is the queen of sciences, as Gauss said, the number theory is shining with its light and attractions, fascinating and facilitating for us the knowledge of the laws that govern the macrocosm and the microcosm".

We are going to structure this volume of encyclopedia in six parts: the first will cover *the Smarandache type sequences and series* (obviously, among them there are the well-known sequences of numbers obtained through concatenation but also numerous other sequences), the second part will cover *the Smarandache type functions and constants*, the third part will cover *the conjectures on Smarandache notions and the conjectures on number theory due to Florentin Smarandache*, the fourth part will cover *the theorems on Smarandache notions and the theorems on number theory due to Florentin Smarandache*, the fifth part will cover *the criteria, formulas and algorithms for computing due to Florentin Smarandache* and the sixth part will cover *the unsolved problems regarding Smarandache notions and the open problems on number theory due to Florentin Smarandache.*

Obviously, the division into these chapters has mostly the role to organise the matters treated, not to delineate them one from another, because all are related; for instance, a function treated in chapter about functions may create a sequence treated in chapter about sequences or a conjecture about primes treated in the chapter about primes may involve a diophantine equation, though these ones have their own chapter. Similarly, we presented some conjectures, theorems and problems on sequences or functions in the chapters dedicated to definition of the latter, while we presented other conjectures, theorems and problems on the same sequences or functions in separate chapters; we could say we had a certain vision doing so (for instance that we wanted to keep a proportion between the sizes of the sections treating different sequences or functions and not to interrupt the definitions between two related sequences or functions by a too large suite of problems) but it would not be entirely true: the truth is that a work, once started, gets its own life and one could say that almost it dictates you to obey its internal order.

In the book *Smarandache Notions* (editors Seleacu and Bălăcenoiu), Henry Ibstedt made a very interesting classification of Smarandache sequences in: recursive; non-recursive; obtained through concatenation of terms, elimination of terms, arrangement of terms, permutation of terms or mixed operations. Many other classifications are possible (for instance Amarnath Murthy and Charles Ashbacher classified them, in the book *Generalized partitions and new ideas on number theory and Smarandache sequences*, in *accomodative sequences* (if all natural numbers can be expressed as the sum of distinct elements of the sequence) or *semi-accomodative sequences* (if all natural numbers can be expressed as the sum or difference of distinct elements of the sequence). That's why, as it can be seen above, we simply classified them into two groups: concatenated or non-concatenated. We have listed for the each studied sequence the first few terms and also we have mentioned the article from OEIS (*On-Line Encyclopedia of Integer Sequences*) where can be found more of these terms.

We emphasize that the work is not exhaustive (though is called "encyclopedia") because, as we said before, the volume of works about Smarandache type notions is huge and the study of all these thousand of sources is a task virtually insurmontable; moreover, the number of Smarandache type sequences and functions continues to grow, while the study of those already known continues to be deepened. But, of course, each new edition of this encyclopedia will be more complete (if the phrase "more complete" is not a pleonasm).

We let aside many Smarandache type notions (which are constructed using concepts like rings, groups, groupoids) to be treated in a further volume of this encyclopedia regarding *Algebra*; we also didn't include many proofs of the theorems and just made reference to the articles or books where these can be found.

All the comments of substance (on number theory) from this book (beside the ones from the *Annex B: A proposal for a new Smarandache type notion*, which is our unique and exclusive contribution of substance to this work) belong to Florentin Smarandache (they are extracted from his works), unless is expressly indicated by footnote another source; our comments are only explanatory or descriptive. Sometimes, if the meaning of the sentence is clear, we will refer to Florentin Smarandache using the initials F.S.

We structured the work using numbered *Definitions, Theorems, Conjectures, Notes* and *Comments*, in order to facilitate an easier reading but also to facilitate references to a specific paragraph. We divided the *Bibliography* in two parts, *Writings by Florentin Smarandache* (indexed by the name of books and articles) and *Writings on Smarandache notions* (indexed by the name of authors). For some papers that appear in bibliography we just made reference to Arxiv, an well-known archive for scientific articles, though they were published in other math journals too (for instance, to refer to the article that presented for the first time the Smarandache function, we simply mentioned "*A function in the number theory*, Arxiv”, though this research paper was for the first time published in 1980 in a review published by University of Timisoara from Romania).

We also have, at the end of this book, an *Afterword* about *an infinity of problems concerning the Smarandache function* and two annexes, *Annex A* which contains a *list of few types of numbers named after Florentin Smarandache*, where we present few types of such numbers which are largely known as Smarandache numbers, Smarandache consecutive numbers, Smarandache-Wellin numbers, Smarandache-Radu duplets, Smarandache-Fibonacci triplets etc. and *Annex B* which contains *a proposal for a new Smarandache type notion*.

Because any work of proportions contains errors, especially one dedicated to number theory, a very refined field of mathematics, and this encyclopedia will probably not be an exception to the rule; the possible mistakes due to our misunderstanding of concepts treated will be removed in a later edition of the book.

We hope that mathematicians who wrote about Smarandache notions will not be offended if we mistakely attributed a proof of a theorem to another mathematician than the one that has the precedence (our references refers to the works were we found these proofs and generally this work intends to give an overview on Smarandache type notions, not to establish the paternity on these), also will not be offended if we omitted to mention an important sequence, function, theorem, conjecture. All these more than possible but probable errors will be straighten, at request, in a future edition (as we said above, the volume of works on Smarandache type notions is huge, and “we” are only one, *id est* me). Finally, we hope that we have not treated the same problem in same way in different chapters (and we stop here with concerns because ultimately this is a book not a contract to cover all possible clauses).

At the risk to appear redundant, we wrote in footnotes the complete reference every time, without resorting to references like *op. cit.*, *idem*, *ibidem*: a book, like for instance *Only problems, not solutions!*, is structured differently in an edition from 1993 than in an edition from 2000.

Because the most of Smarandache sequences are sequences of integers we will consider this implicitly and we will mention expressly only if it is the case of other type of sequence (for instance of rational numbers).

To not remove readers by a large variety of mathematical simbols, we replaced them, when this was possible, with verbal expressions; also, to accustom the readers with the symbols of operations accepted as input by the major math programs as Wolfram Alpha or commonly used by the major sites of number theory like OEIS, we use for multiplication the symbol “*” and for the rise to a power the symbol “^”. Therefore, we understand, in this paper, the numbers denoted by “abc” as the numbers obtained by the method of concatenation, where a, b, c are digits, and the numbers denoted by “a*b*c” as the products of the numbers a, b, c.

We also used the term *deconcatenation* to refer to the inverse operation than concatenation (for instance, the number 561 admits to be deconcatenated in three ways, into the sets of numbers {5, 6, 1}, {5, 61} and {56, 1}, but not, for example, into the set of numbers {5, 16}); therefore, instead to define an operation like “partition of a number into groups of digits *that are* (or *that form*) primes” (*e.g.* the partition of the number 1729 into 17 and 29) we define it like “deconcatenation of a number into a set of primes”.

We mention that, beside the universal known basic operations, we use in this book the following functions and operations: *the factorial*: the factorial of n (or n factorial) is the product of all positive integers smaller or equal to n, written as n! = 1*2*3* … *n, for instance 6! = 1*2*3*4*5*6 = 720; *the double factorial function*, written as n!!, which has the following values: n!! = 1*3*5*…*(n – 2)*n if n is odd respectively n!! = 2*4*6*…*(n – 2)*n if m is even (respectively, by convention, n!! = 1 if n = 0); *the congruence modulo*: m is congruent modulo x with n and it is noted m ≡ n (mod x) if the remainder of the division of m by x is equal to the remainder of the division of n by x, for instance 17 ≡ 5 (mod 3); *the primorial*: this function is usually (and in this book too) met with two different definitions: the n-th primorial number is the product of the first n primes (it is noted p_n#) and the primorial of the positive integer n (it is noted n#) is the product of all primes less than or equal to n.

We also mention that we used both syntagms "non-null natural numbers" and "positive integers" to designate the same thing, *i.e.* the set of natural numbers without zero, and both the sintagms "natural numbers" and "non-negative integers" to designate the same thing, *i.e.* the set of positive integers plus zero.

We noted gcd (m, n) the greatest common divisor of m and n; we noted max{m, n} the maximum value from the values of m and n and max{p: p prime, p divides n} the maximum value of p, under certain conditions (in this case, the condition that p is prime and p divides n); we also used the notation min{f(x)} for the minimum value the function f(x) can have etc. We noted, *exempli gratia*, with abs{m – n} the absolute value of the subtraction of integers m, n and with [x – y] the integer value of the subtraction of real numbers x, y.

We also mention that we understand through "proper divisors of n" all the positive divisors of n other than n itself (but including the number 1). Also, because not all the sources understand the same thing through the syntagms "inferior part of x" or "superior part of x", implicitly through the arithmetic symbols assigned to them, to avoid any possible confusion, we didn't use symbols (the specific brackets "open" up or down), but wrote, when was the case, in formulas or definitions, "in words": "the inferior part of x, *i.e.* the largest integer n less than or equal to x" respectively "the superior part of x, *i.e.* the smallest integer n greater than or equal to x".

We noted with σ(n) or sigma(n) *the divisor function* (the sum of the positive divisors of n, including 1 and n), with τ(n) or tau(n) *the (Dirichlet) divisor function*, *i.e.* the number of all positive divisors of n (including 1 and n), with π(n) sau pi(n) *the prime counting function* (the number of primes smaller than or equal to n), with ω(n) or omega(n) the number of distinct prime factors of n and with φ(n) or phi(n) *the Euler's totient*, *i.e.* the number of positive integers smaller than or equal to n which are coprime with n (we used the notations customary in many math programs like Wolfram Alpha); we also noted with R(n) the reversal of the positive integer n, *i.e.* the number formed by the same digits, in reverse order.

We hope that professional mathematicians who know of course these symbols, functions and operations will not be offended by these explanations, as this book is not addressed only to them but also to young aspirants. This encyclopedia is both for researchers that will have on hand a tool that will help them "navigate" in the universe of Smarandache type notions and for young math enthusiasts: many of them will be attached by this wonderful branch of mathematics, number theory, reading the works of Florentin Smarandache.

The author

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PART ONE
Smarandache type sequences, series and functions

Chapter I. Sequences and series of numbers obtained through concatenation

(1) The Smarandache consecutive numbers sequence[1]

Definition:

S_n is defined as the sequence obtained through the concatenation of the first n positive integers.

The first ten terms of the sequence (A007908 in OEIS):

1, 12, 123, 1234, 12345, 123456, 1234567, 12345678, 123456789, 12345678910.

Notes:

1. The *Smarandache consecutive number sequence* has the following special property: thus far there is no prime number known in this sequence, though there have been checked the first about 40 thousand terms[2].
2. The problem of the number of primes contained by this sequence was raised by Florentin Smarandache since 1979.[3]
3. Generalizing the problem, F.S. asked how many primes are among the terms of the *consecutive sequence* if this is considered in an arbitrary numeration base B; in base 3, for instance, the terms of the sequence are 1, 12, 1210, 121011, 12101112 (…), which are equivalent to the following decimal numbers: 1, 5, 48, 436, 3929 (…)[4]. The computer programs used for finding these numbers, for numeration bases until 10, showed that these numbers are very rare. For instance, no prime was found among the first thousand of these terms for the numeration base 4.

Comment[5]:

From the *Smarandache consecutive number sequence* it can be formed the series defined as the sum from $n = 1$ to $n = \infty$ of the numbers $1/S_n$. The series $1 + 1/12 + 1/123 + 1/1234 + \ldots$ is convergent to a value greater than one and smaller than or equal to 10/9.

(2) The reverse sequence

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[1] The name of this sequence was generalized (*Smarandache consecutive numbers sequences*) for all the sequences obtained through concatenation of consecutive numbers of a certain type: the sequence S_n of the numbers obtained through concatenation of first n primes (named *Smarandache-Wellin sequence*); the sequence S_n of the numbers obtained through concatenation of first n squares etc. Many sequences of this type were studied by F.S., hwo revealed an important feature common to all of them: they all contain a small number of primes. See the article *Consecutive number sequences* from Weisstein, Eric W., *CRC Concise Encyclopedia of Mathematics*, CRC Press, 1999, p. 310.

[2] According to article *Consecutive number sequences* from the on-line math encyclopedia Wolfram Math World.

[3] Student Conference, University of Craiova, Department of Mathematics, April 1979, "Some problems in number theory" by Florentin Smarandache, cited by F.S. in *Only problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, p. 18.

[4] The sequence A048435 in OEIS.

[5] Majumdar, A.A.K., *Wandering in the world of Smarandache numbers*, InProQuest, 2010, Chapter 1: *Some Smarandache sequences*, Section 1.12: *Series involving Smarandache sequences*.

Definition:

S_n is defined as the sequence obtained through the concatenation of the first n positive integers, in reverse order.

The first ten terms of the sequence (A000422 in OEIS):

1, 21, 321, 4321, 54321, 654321, 7654321, 87654321, 987654321, 10987654321.

Note: The primes appear very rare among the terms of this sequence too: until now there are only two known, corresponding to n = 82 (a number having 155 digits) şi n = 37765 (a number having 177719 digits).

Theorem[6]:

Element number n of the base 10 reverse sequence is not square-free if n is congruent to 0 or 8 modulo 9.

(3) The concatenated odd sequence

Definition:

S_n is defined as the sequence obtained through the concatenation of the first n odd numbers (the n-th term of the sequence is formed through the concatenation of the odd numbers from 1 to 2*n – 1).

The first ten terms of the sequence (A019519 in OEIS):

1, 13, 135, 1357, 13579, 1357911, 135791113, 13579111315, 1357911131517, 135791113151719.

Notes:

1. F.S. conjectured that there exist an infinity of prime terms of this sequence.
2. The terms of this sequence are primes for the following values of n: 2, 10, 16, 34, 49, 2570 (the term corresponding to n = 2570 is a number with 9725 digits); there is no other prime term known though where checked the first about 26 thousand terms of this sequence.[7]

Theorem[8]:

Let n be the number of the element in the concatenated odd sequence.

a) If n is congruent to 3 modulo 5, then element number n is evenly divisible by 5.

b) If n is congruent to 0 modulo 3, then element number n is evenly divisible by 3 and if n is congruent to 1 or 2 modulo 3, then element number n is congruent to 1 modulo 3.

(4) The concatenated even sequence

Definition:

S_n is defined as the sequence obtained through the concatenation of the first n even numbers (the n-th term of the sequence is formed through the concatenation of the even numbers from 1 to 2*n).

The first ten terms of the sequence (A019520 in OEIS):

2, 24, 246, 2468, 246810, 24681012, 2468101214, 246810121416, 24681012141618, 2468101214161820.

Notes:

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[6] For the proof of this theorem, see Ashbacher, Charles, *Smarandache Sequences, stereograms and series*, Hexis, Phoenix, 2005, p. 38-39.

[7] According to article *Consecutive number sequences* from the on-line math encyclopedia Wolfram Math World.

[8] For the proof of this theorem, see Ashbacher, C., *Smarandache Sequences, stereograms and series*, Hexis, Phoenix, p. 44-46.

1. Any term of this sequence can't be, obviously, prime. In the case of this sequence is studied the primality of the numbers obtained through the division of its terms by 2: 1, 12, 123, 1234, 123405, 1240506, 1234050607 (…).
2. F.S. conjectured that there is no any term of this sequence which is a perfect square.
3. H. Ibsted has not found any perfect square among the first 200 terms of this sequence.[9]
4. A.A.K. Majumdar proved that none of the terms of the subsequence ES(2*n – 1) is a perfect square or higher power of an integer greater than one.[10]

(5) The concatenated prime sequence

Definition:

S_n is defined as the sequence obtained through the concatenation of the first n primes.

The first ten terms of the sequence (A019518 in OEIS):

2, 23, 235, 2357, 235711, 23571113, 2357111317, 235711131719, 23571113171923, 2357111317192329.

Notes:

1. The terms of this sequence are known as Smarandache-Wellin numbers[11]. Also, the Smarandache-Wellin numbers which are primes are named Smarandache-Wellin primes. The first three such numbers are 2, 23 şi 2357; the fourth is a number with 355 digits and there are known only 8 such primes. The 8 known values of n for which through the concatenation of the first n primes we obtain a prime number are 1, 2, 4, 128, 174, 342, 435, 1429. The computer programs not yet found, until n = 10^4, another such a prime.[12]
2. F.S. conjectured that there exist an infinity of prime terms of this sequence.[13]

Comment[14]:

The concatenated odd, even and prime sequences are particular cases of so-called "G add-on sequence" defined in the following way: let G = $\{g_1, g_2, \ldots, g_k, \ldots\}$ be an ordered set of positive integers with a given property G; then the corresponding G add-on sequence is defined through formula SG = $\{a_i: a_1 = g_1, a_k = a_{k-1}*10\text{^}(1 + \log_{10}(g_k)) + g_k, k \geq 1\}$.

(6) The back concatenated prime sequence

Definition:

S_n is defined as the sequence obtained through the concatenation of the first n primes, in reverse order.

The first ten terms of the sequence (A038394 in OEIS):

2, 32, 532, 7532, 117532, 13117532, 1713117532, 191713117532, 23191713117532, 2923191713117532.

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[9] See Ibstedt, Henry, *Computer analysis of number sequences*, American Research Press, 1998, Chapter V: *Smarandache concatenated sequences*, Section 4: *The Smarandache even sequence*.

[10] See Majumdar, A.A.K., *Wandering in the world of Smarandache numbers*, InProQuest, 2010, Chapter 1: *Some Smarandache sequences*, Section 1.3: *Smarandache even sequence*. The Smarandache odd sequence is sometimes named with the acronym OS while the even sequence is sometimes named ES.

[11] After the names of F.S. and mathematician Paul R. Wellin.

[12] According to article *Smarandache-Wellin number* from the on-line math encyclopedia Wolfram Math World.

[13] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Definition 17.

[14] F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequences 229-231.

(7) The concatenated square sequence

Definition:

S_n is defined as the sequence obtained through the concatenation of the first n squares: 1(2^2)(3^2)…(n^2).

The first ten terms of the sequence (A019521 in OEIS):

1, 14, 149, 14916, 1491625, 149162536, 14916253649, 1491625364964, 149162536496481, 149162536496481100.

Notes:

1. The third term, the number 149, it is the only prime from the first about 26 thousand terms of this sequence.[15]
2. F.S. raised the problem of the number of the terms of this sequence which are perfect squares.[16]

Conjecture:

There is no term of the Smarandache concatenated square sequence which is perfect square.[17]

(8) The concatenated cubic sequence

Definition:

S_n is defined as the sequence obtained through the concatenation of the first n cubes: 1(2^3)(3^3)…(n^3).

The first ten terms of the sequence (A019521 in OEIS):

1, 18, 1827, 182764, 182764125, 182764125216, 182764125216343, 182764125216343512, 182764125216343512729, 1827641252163435127291000.

Notes:

1. There were not found prime terms of this sequence, though there were checked the first about 22 terms.[18]
2. F.S. raised the problem of the number of the terms of this sequence which are perfect cubes.[19]

Conjecture:

There is no term of the Smarandache concatenated cubic sequence which is perfect cube.[20]

(9) The sequence of triangular numbers

Definition:

S_n is defined as the sequence obtained through the concatenation of the first n triangular numbers: 1(2^3)(3^3)…(n^3).

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[15] According to article *Consecutive number sequences* from the on-line math encyclopedia Wolfram Math World.

[16] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Definition 21.

[17] Ashbacher, C., *Smarandache Sequences, stereograms and series*, Hexis, Phoenix, p. 63.

[18] According to article *Consecutive number sequences* from the on-line math encyclopedia Wolfram Math World.

[19] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Definition 22.

[20] Ashbacher, C., *Smarandache Sequences, stereograms and series*, Hexis, Phoenix, p. 65.

The first ten terms of the sequence (A078795 in OEIS):

1, 13, 136, 13610, 1361015, 136101521, 13610152128, 1361015212836, 136101521283645, 13610152128364555.

Notes:

1. The triangular numbers are a subset of the polygonal numbers (which are a subset of figurate numbers) constructed with the formula T(n) = (n*(n + 1))/2 = 1 + 2 + 3 +… + n.
2. The only two known primes from this sequence (among the first about 5000 terms) are 13 and 136101521.

(10) The symmetric numbers sequence

Definition:

S_n is defined as the sequence obtained through the concatenation in the following way: if n is odd, the n-th term of the sequence is obtained through concatenation 123…(m-1)m(m-1)…321, where m = (n + 1)/2; if n is even, the n-th term of the sequence is obtained through concatenation 123…(m-1)mm(m-1)…321, unde m = n/2.

The first ten terms of the sequence (A007907 in OEIS):

1, 11, 121, 1221, 12321, 123321, 1234321, 12344321, 123454321, 1234554321, 12345654321.

Notes:

1. F.S. raised the problem of the numbers of the terms of this sequence which are primes.
2. Generalizing the problem, F.S. asked how many primes are among the terms of the *symmetric sequence* if this is considered in an arbitrary numeration base B.

Comment:

The prime numbers among the terms of this sequence "may not be as rare as the primes in the consecutive sequence, for all the numbers in this sequence are odd."[21]

Theorem[22]:

If p is an odd prime in the base 3 symmetric sequence, then the index must be of the form 4*k + 1.

(11) The antisymmetric numbers sequence

Definition:

S_n is defined as the sequence obtained through the concatenation in the following way: 12…(n)12…(n).

The first ten terms of the sequence (A019524 in OEIS):

11, 1212, 123123, 12341234, 1234512345, 123456123456, 12345671234567, 1234567812345678, 123456789123456789.

Note: There is no term of this sequence which can be prime, no matter in what numeration base is this sequence considered. In the case of this sequence is studied the primality of the numbers of the form 12…(n)12…(n) ± 1.

(12) The mirror sequence

Definition:

21 Ashbacher, C., *Smarandache Sequences, stereograms and series*, Hexis, Phoenix, p. 14.

22 For the proof of this theorem, see Ashbacher, C., *Smarandache Sequences, stereograms and series*, Hexis, Phoenix, p. 18-19.

S_n is defined as the sequence obtained through the concatenation in the following way: n(n – 1)…32123…(n – 1)n.

The first ten terms of the sequence (A007942 in OEIS):

1, 212, 32123, 4321234, 543212345, 65432123456, 7654321234567, 876543212345678, 98765432123456789, 1098765432123456778910.

Notes:

1. F.S. raised the problem of the numbers of the terms of this sequence which are primes.
2. Generalizing the problem, F.S. asked how many primes are among the terms of the *symmetric sequence* if this is considered in an arbitrary numeration base B. The computer programs used for finding these primes, for the numeration bases up to ten, not found any prime among the first 500 terms of the sequence for the numeration base 6.

Theorems:

1. If B > 2 is odd, then all of the elements in the base B mirror sequence are odd.[23]
2. If the base B > 2 is even, then the parity of the elements in the base B Mirror Sequence alternate, with the elements of even index being even and the elements of odd index odd. [24]

(13) The "n concatenated n times" sequence

Definition:

The sequence S_n defined as the sequence of the numbers obtained concatenating n times the number n.

The first ten terms of the sequence (A000461 in OEIS):

1, 22, 333, 4444, 55555, 666666, 7777777, 88888888, 999999999, 10101010101010101010.

Note: There is no term of this sequence which can be prime, all terms of the sequence being repdigit numbers, therefore multiples of repunit numbers.

(14) The permutation sequence[25]

Definition:

The sequence S_n defined as the sequence of numbers obtained through concatenation and permutation in the following way: 13…(2n – 3)(2n – 1)(2n)(2n – 2)(2n – 4)…42.

The first seven terms of the sequence (A007943 în OEIS):

12, 1342, 135642, 13578642, 13579108642, 135791112108642, 1357911131412108642, 13579111315161412108642.

Notes:

1. There is obviously no term of this sequence which can be prime. In the case of this sequence is studied the primality of the numbers obtained through the division of its

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[23] For the proof of this theorem, see Ashbacher, C., *Smarandache Sequences, stereograms and series*, Hexis, Phoenix, p. 23-25.

[24] For the proof of this theorem, see Ashbacher, C., *Smarandache Sequences, stereograms and series*, Hexis, Phoenix, p. 25.

[25] The sequence is named *Smarandache permutation sequence* by Ashbacher, C., *Smarandache Sequences, stereograms and series*, Hexis, Phoenix, p. 28; Majumdar, A.A.K., *Wandering in the world of Smarandache numbers*, InProQuest, 2010, p. 23. Other sources (Wolfram Math World, OEIS) understand through the name *Smarandache permutation sequence* another sequence, *i.e.* the sequence obtained concatenating ascendent sequences of odd numbers with descending sequences of even numbers: 1, 2, 1, 3, 4, 2, 1, 3, 5, 6, 4, 2 (…).

terms by 2: 6, 671, 67821, 6789321 (…), or the primality of the numbers of the form 13…(2n – 3)(2n – 1)(2n)(2n – 2)(2n – 4)…42 ± 1.

2. There is no term of this sequence which can be perfect square, because every term of this sequence, beside the first one, is divisible by 2 but not by 2^2.

(15) The constructive set of digits 1 and 2 sequence[26]

Definition:

The sequence S of the numbers obtained through concatenation of the digits 1 and 2, defined in the following way: (i) the digits 1 and 2 belong to S; (ii) if a and b belong to S, then ab belong to S too; (iii) only elements obtained by rules (i) and (ii) applied a finite number of times belong to S.

The first twenty-five terms of the sequence:

1, 2, 11, 12, 21, 22, 111, 112, 121, 211, 212, 221, 222, 1111, 1112, 1121, 1122, 1211, 1212, 1221, 1222, 2111, 2112, 2121, 2122.

Comment:

There are 2^k numbers of k digits in the sequence, for k = 1, 2, 3, …

(16) The generalized constructive set sequence[27]

Definition:

The sequence S obtained generalizing the previous sequence, so the sequence of the numbers obtained through concatenation of the distinct digits $d_1, d_2, \ldots, d_m$, where $1 \le m \le 9$, defined in the following way: (i) the digits $d_1, d_2, \ldots, d_m$ belong to S; (ii) if a and b belong to S, then ab belongs to S too; (iii) only elements obtained by rules (i) and (ii) applied a finite number of times belong to S.

Comments:

1. There are 2^k numbers of k digits in the sequence, for k = 1, 2, 3, …
2. All digits d_i can be replaced by numbers as large as we wantm and also m can be as large as we want.

Theorem[28]:

The series defined as the sum from n = 1 to n = ∞ of the fractions $1/a_n$^r, where $\{a_n\}$ is a sequence constructed according to the definition (*generalised constructive set*) and r is a positive number, is convergent if r > log m and divergent if r ≤ log m.

(17) The pierced chain sequence

Definition:

The sequence obtained in the following way: the first term of the sequence is 101 and every next term is obtained through concatenation of the previous term with the group of digits 0101.

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[26] F.S., *Only problems, no solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 6. See also F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, where, in Sequence 115, is defined the constructive set of digits 1, 2 and 3. For a study of the constructive set of digits 1 and 2, see Atanassov, Krassimir T., *On some of the Smarandache's problems*, American Research Press, 1999, p. 50-51; for a study of the constructive set of digits 1, 2 and 3, see the same book, p. 51.

[27] F.S., *Only problems, no solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 8. See also F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 116. See also Atanassov, Krassimir T., *On some of the Smarandache's problems*, American Research Press, 1999, p. 51-56.

[28] The theorem is enunciated and proved by Gou Su, see the article „On the generalised constructive set", *Research on Smarandache problems in number theory (Collected papers)*, Hexis, 2004.

The first seven terms of the sequence (A031982 in OEIS):

101, 1010101, 10101010101, 101010101010101, 1010101010101010101, 10101010101010101010101, 101010101010101010101010101.

Note: Because, of course, all terms of this sequence are divisible by 101, the problem raised by F.S. is how many from the numbers obtained through the division of the terms of the sequence by 101 are primes or squarefree numbers.

Theorem[29]:

There are no primes obtained through the division of the terms of the sequence by 101.

(18) The concatenated Fibonacci sequence

Definition:

The sequence obtained through concatenation of the terms of Fibonacci sequence[30].

The first ten terms of the sequence (A019523 in OEIS):

1, 11, 112, 1123, 11235, 112358, 11235813, 1123581321, 112358132134, 11235813213455.

Notes:

1. From the first 800 terms of this sequence only two are primes, the second and the fourth (respectively the numbers 11 and 1123).
2. Florentin Smarandache raised the problem if there exist any term of this sequence (beside 1) which is a Fibonacci number.[31]

(19) The circular sequence[32]

Definition:

The sequence S_n constructed through concatenation and permutation in the following way[33]:

The first twenty terms of the sequence (A001292 in OEIS):

1, 12, 21, 123, 231, 312, 1234, 2341, 3412, 4123, 12345, 23451, 34512, 45123, 51234, 123456, 234561, 345612, 456123, 561234.

Note: The problem raised by F.S. is how many from the terms of the sequence are primes or powers of integers.[34] Another problem raised is to find the probability for which the

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[29] The theorem is proved by Kenichiro Kashihara, *Comments and topics on Smarandache notions and problems*, Erhus University Press, 1996, p. 7-8. For other theorems concerning this sequence see Majumdar, A.A.K., *Wandering in the world of Smarandache numbers*, InProQuest, 2010, Chapter 1: *Some Smarandache sequences*, Section 1.11: *Smarandache pierced chain sequence*.

[30] The Fibonacci numbers are the numbers defined by the recurrence relation $F(n) = F(n-1) + F(n-2)$.

[31] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Definition 23. Charles Ashbacher conjectured that there is no such a term of the sequence: *Smarandache Sequences, stereograms and series*, Hexis, Phoenix, p. 66.

[32] Named *Smarandache circular sequence* by F.S., *Only problems, no solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 4. Other sources (Majumdar, A.A.K., *Wandering in the world of Smarandache numbers*, InProQuest, 2010, p. 26) use the name *Smarandache circular sequence* refering to *The Smarandache consecutive numbers sequence* (1, 12, 123, 1234, …).

[33] For a formula for the n-th term of the sequence, see Vassilev-Missana, Mladen and Atanassov, Krassimir, *Some Smarandache problems*, Hexis, 2004, Chapter 1: *On some Smarandache's problems*, Section 1: *On the 2-nd Smarandache's problem*.

[34] Kashihara conjectured that the sequence contains no powers of integers. See Kashihara, K., *Comments and topics on Smarandache notions and problems*, Erhus University Press, 1996, p. 25.

trailing digit of a term is equal to c, where c belongs to the set {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}.[35]

(20) The back concatenated sequences[36]

The back concatenated odd sequence[37]:

The first ten terms of the sequence (A038395 in OEIS):

1, 31, 531, 7531, 97531, 1197531, 131197531, 15131197531, 1715131197531, 191715131197531.

The back concatenated even sequence:

The first ten terms of the sequence (A038396 in OEIS):

2, 42, 642, 8642, 108642, 12108642, 1412108642, 161412108642, 18161412108642, 2018161412108642.

The back concatenated square sequence:

The first ten terms of the sequence (A038397 in OEIS):

1, 41, 941, 16941, 2516941, 362516941, 49362516941, 6449362516941, 816449362516941, 100816449362516941.

The back concatenated cubic sequence:

The first ten terms of the sequence (A019522 in OEIS):

1, 18, 1827, 182764, 182764125, 182764125216, 182764125216343, 182764125216343512, 182764125216343512729, 1827641252163435127291000.

The back concatenated Fibonacci sequence:

The first ten terms of the sequence (A038399 in OEIS):

1, 11, 211, 3211, 53211, 853211, 13853211, 2113853211, 342113853211, 55342113853211.

(21) The concatenated S-Sequence

Definition:

The sequence obtained generalizing the Smarandache concatenated sequences defined in the following way: let $s_1, s_2, \ldots, s_n$ be a sequence of integers noted with S; then s_1, s_1s_2, $s_1s_2s_3$, …, $s_1s_2s_3\ldots s_n$ is named Concatenated S-Sequence.

Note: Florentin Smarandache raised the problem of the number of the terms of the Concatenated S-Sequence which belong to the initial sequence.[38]

(22) The generalized palindrome sequence[39]

Definition:

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[35] For a deeper study of this sequence see Ripà, Marco, *Patterns related to the Smarandache circular sequence primality problem*, Unsolved Problems in Number Theory, Logic, and Criptography.

[36] Beside the *back concatenated prime sequence* which is treated *supra*, F.S. defined many other back concatenated sequences; here are listed few of them. See F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Definitions 17, 18, 19, 21, 22, 23.

[37] For a recursion formula for general term of this sequence and theorems about it see Junzhuang, Li and Nianliang, Wang, *On the Smarandache back concatenated odd sequences*, in Wenpeng, Zhang, *et al.* (editors), *Research on Smarandache problems in number theory (vol. 2)*, Hexis, 2005.

[38] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Definition 20.

[39] F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 254.

The sequence of numbers which are called Generalized Smarandache Palindromes (GSP) and are defined as follows: numbers of the form $a_1 a_2 \ldots a_n a_n \ldots a_2 a_1$, with $n \geq 1$, where $a_1, a_2, \ldots, a_n$ are positive integers of various number of digits.

Example:

The number 1256767251 is a GSP of type ABCCBA because can be deconcatenated into the set of numbers {1, 25, 67, 67, 25, 1}.

Conjecture:

There are infinitely many primes which are GSP.

(23) The Smarandache n2*n sequence[40]

Definition:

The n-th term of the sequence a(n) is obtained concatenating the numbers n and 2*n.

The first fifteen terms of the sequence (A019550 in OEIS):

12, 24, 36, 48, 510, 612, 714, 816, 918, 1020, 1122, 1224, 1326, 1428, 1530.

Note:

Because obviously every element of this sequence a(n) is divisible by 6*n, in the case of this sequence is studied the primality of the numbers a(n)/6*n.

Conjecture:

The sequence a(n)/6*n contains infinitely many primes.

(24) The Smarandache nn^2 sequence[41]

Definition:

The n-th term of the sequence a(n) is obtained concatenating the numbers n and n^2.

The first fifteen terms of the sequence (A053061 in OEIS):

11, 24, 39, 416, 525, 636, 749, 864, 981, 10100, 11121, 12144, 13169, 14196, 15225.

Theorem:

The Smarandache nn^2 sequence contains no perfect squares.

Definition:

The sequence a(n)/n is called *the reduced Smarandache nn^2 sequence*.

The first fifteen terms of the reduced Smarandache nn^2 sequence (A061082 in OEIS):

11, 12, 13, 104, 105, 106, 107, 108, 109, 1010, 1011, 1012, 1013, 1014, 1015.

Conjecture:

There are infinitely many primes in the reduced Smarandache nn^2 sequence.

Definition:

The sequence a(n) obtained concatenating the numbers n and n^m is called *the Smarandache nn^m sequence*.

Theorem:

The Smarandache nn^m sequence, for any value of m, contains only one prime, the number 11.

Definition:

The sequence a(n)/n is called *the reduced Smarandache nn^m sequence*.

Question:

How many terms in this sequence are prime?

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[40] Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, p. 125.

[41] Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, p. 130.

(25) The Smarandache nk*n generalized sequence[42]

Definition:

The n-th term of the sequence a(n) is obtained concatenating all of the numbers n, 2*n, 3*n, …, n*n.

The first eight terms of the sequence (A053062 in OEIS):

1, 24, 369, 481216, 510152025, 61218243036, 7142128354249, 816243240485664.

Question:

How many from the numbers a(n)/n are primes?

(26) The Smarandache breakup perfect power sequences

Definition:

The n-th term of the sequence is defined as the smallest positive integer which, by concatenation with all previous terms, forms a perfect power.

The Smarandache breakup square sequence (A051671 in OEIS):

4, 9, 284, 61209, 14204828164, 4440027571600000000001, …

Example: 284 belongs to the sequence because 49284 = 222^2.

The Smarandache breakup cube sequence (A061109):

1, 6, 6375, 34623551127976881, …

Example: 6375 belongs to the sequence because 166375 = 55^3.

(27) The Smarandache breakup prime sequence

Definition:

The n-th term of the sequence is defined as the smallest positive integer which, by concatenation with all previous terms, forms a prime.

The Smarandache breakup prime sequence (A048549 in OEIS):

2, 23, 233, 2333, 23333, 2333321, 233332117, 2333321173, 233332117313, …

(28) The Smarandache power stack sequences

Definition:

The n-th term of the sequence is defined as the positive integer obtained by concatenating all the powers of k from k^0 to k^n.

The Smarandache power stack sequence for k = 2:

1, 12, 124, 1248, 12416, 1241632, 124163264…

The Smarandache power stack sequence for k = 3:

1, 13, 139, 13927, 1392781, 1392781243…

(29) The Smarandache left-right and right-left sequences

Definition 1[43]:

The sequence of positive integers obtained starting with 1 and concatenating alternatively on the left and on the right the next numbers.

The first ten terms of the sequence (A053063 in OEIS):

[42] Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, p. 125.

[43] Russo, Felice, *On a problem concerning the Smarandache left-right sequences*, Smarandache Notions Journal, vol. 14, 2004.

1, 21, 213, 4213, 42135, 642135, 6421357, 86421357, 864213579, 10864213579.

Definition 2:

The sequence of positive integers obtained starting with 1 and concatenating alternatively on the right and on the left the next numbers.

The first ten terms of the sequence (A053064 in OEIS):

1, 12, 312, 3124, 53124, 531246, 7531246, 75312468, 975312468, 97531246810.

Definition 3:

The sequence of positive integers obtained starting with 2 and concatenating alternatively on the left and on the right the next primes.

The first nine terms of the sequence (A053065 in OEIS):

2, 32, 325, 7325, 732511, 13732511, 1373251117, 191373251117, 19137325111723.

Definition 4:

The sequence of positive integers obtained starting with 2 and concatenating alternatively on the right and on the left the next primes.

The first nine terms of the sequence (A053066 in OEIS):

2, 23, 523, 5237, 115237, 11523713, 1711523713, 171152371319, 23171152371319.

Questions:

1. How many terms of this sequences are prime numbers?
2. How many terms are additive primes?[44]
3. Is the number of the primes in these sequences finite?

(30) The Smarandache sequences of happy numbers

Definition 1[45]:

The sequence of numbers obtained concatenating the *happy numbers*[46].

The first eight terms of the Smarandache sequence of happy numbers (A053064 in OEIS)[47]:

1, 17, 1710, 171013, 17101319, 1710131923, 171013192328, 17101319232831.

Definition 2 :

The sequence of numbers obtained concatenating the *happy numbers*.

The first eight terms of the reversed Smarandache H-sequence (A071827 in OEIS):

1, 71, 1071, 131071, 19131071, 2319131071, 282319131071, 31282319131071.

Comments:

1. There are only 3 primes in the first 1000 terms of the H-sequence, i.e. SH(2) = 17, SH(5) = 17101319 and SH(43), a number with 108 digits.
2. There are 1429 happy numbers in the first 10000 terms of the H-sequence.
3. There are 8 primes in the reversed H-sequence.

Questions:

1. How many terms of the H-sequence or of the reversed H-sequence are primes? Are there infinitely many?
2. How many terms are happy numbers?

Chapter II. Other sequences and series

[44] An *additive prime* is a prime number with the property that the sum of its digits is a prime too.

[45] Gupta, Shyam Sunder, *Smarandache sequence of happy numbers*, Smarandache Notions Journal, vol. 13, 2002.

[46] A *happy number* is a number with the property that, through the iterative summation of the squares of its digits, it is eventually obtained the number 1; *e.g.* 7 is a happy number because 7^2 = 49, 4^2 + 9^2 = 97, 9^2 + 7^2 = 130, 1^2 + 3^2 + 0^2 = 10, 1^2 + 0^2 = 1. The numbers which don't have this property are called unhappy numbers. The first few happy numbers (sequence A007770 in OEIS): 1, 7, 10, 13, 19, 23, 28, 31, …

[47] Also named with the acronym *Smarandache H-sequence*.

(1) The Smarandache Quotient sequence

Definition[48]:

The sequence of positive integers k with the property that they are the smallest positive integers so that the product n*k is a factorial number, where n integer, n ≥ 1.

The first twenty terms of the sequence (A007672 in OEIS):

1, 1, 2, 6, 24, 1, 720, 3, 80, 12, 3628800, 2, 479001600, 360, 8, 45, 20922789888000, 40, 6402373705728000, 6.

Comments:

1. The sequence contains an infinity of factorial numbers.[49]
2. The sequence contains an infinity of primes, perfect squares and perfect cubes.[50]

(2) The (non-cocatenated) permutation sequence

Definition:

The sequence S_n defined in the following way: the first term is 1, the second term is 2, then alternates sequences of ascending odd numbers with sequences of descending even numbers.[51]

The first thirty terms of the sequence (A004741 in OEIS):

1, 2, 1, 3, 4, 2, 1, 3, 5, 6, 4, 2, 1, 3, 5, 7, 8, 6, 4, 2, 1, 3, 5, 7, 9, 10, 8, 6, 4, 2.

(3) The deconstructive sequence

Definition:

The sequence obtained in the following way: the first term is 1 and then every term of the sequence it will have one more digit than the previous one, while the digits scroll from 1 to 9 and then they are repetead cyclically.[52]

The first seven terms of the sequence (A007923 in OEIS):

1, 23, 456, 7891, 23456, 789123, 4567891, 23456789, 123456789, 1234567891.

Properties[53]:

1. The trailing digits of the terms of this sequence follow the sequence:
   1, 3, 6, 1, 6, 3, 1, 9, 9, 1, 3, 6, 1, 6, 3, 1, 9, 9, 1, ...
2. The leading digit of the n-th term of this sequence is given by the formula n*(n + 1)/2 (mod 9).

(4) The generic digital sequence

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[48] The number k is named *Smarandache Quotient*. See also *infra*, Part Two, Chapter I, Section (1): *Smarandache function*. F.S. named this sequence *The factorial quotients*. See *Only problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 45.

[49] Kashihara, K., *Comments and topics on Smarandache notions and problems*, Erhus University Press, 1996, p. 16.

[50] Ashbacher, C., *Pluckings from the tree of Smarandache sequences and functions*, American Research Press, 1998, p. 8.

[51] Some sources (Wolfram Math World, OEIS) name this sequence *Smarandache permutation sequence*. We name it *Smarandache (non-concatenated) permutation sequence* to distinguish it from the sequence 12, 1342, 135642, 13578642…(see *supra*) which has the consacrated name *Smarandache permutation sequence*.

[52] For a study of this sequence, see Atanassov, Krassimir T., *On some of the Smarandache's problems*, American Research Press, 1999, p. 7-11.

[53] Kashihara, K., *Comments and topics on Smarandache notions and problems*, Erhus University Press, 1996, Chapter 1: *Some comments and problems on Smarandache notions*.

Definition:

The generic sequence (a particular case of the sequences of sequences) defined in the following way: in any numeration base B, for any sequence of integer or rational numbers $s_1, s_2, s_3, \ldots$ and for any digit C, $0 \le C \le B - 1$, is constructed a new sequence of integers which associates to s_1 the number of digits C of s_1, in numeration base B, to s_2 the number of digits C of s_2, in numeration base B, and so on. [54]

Examples:

1. We consider the sequence of primes in base 10 and the digit C = 1. The number of times the digit 1 appears in this sequence is: 0, 0, 0, 0, 2, 1, 1, 1, 0, 0, 1, 0 (…).[55]
2. We consider the sequence of factorials in base 10 and the digit C = 0. The number of times the digit 0 appears in this sequence is: 0, 0, 0, 0, 0, 1, 1, 2, 2, 1, 3.[56]
3. We consider the sequence n^n in base 10 and the digit C = 5. The number of times the digit 5 appears in this sequence is: 0, 0, 0, 1, 1, 1, 1, 0, 0, 0.[57]

(5) The generic construction sequence

Definition:

The generic sequence (a particular case of the sequences of sequences) defined in the following way: in any numeration base B, for any sequence of integer or rational numbers $s_1, s_2, s_3, \ldots$ and for any digits $C_1, C_2, \ldots, C_k$ ($k < B$), is constructed a new sequence of integers so that every of its terms $Q_1 < Q_2 < Q_3 < \ldots$ is constituted only from the digits $C_1, C_2, \ldots, C_k$ (plus every from these digits must be used) and corresponds to a term s_i from the initial sequence.[58]

Examples:

1. We consider the sequence of primes in base 10 and the digits $C_1 = 1$ and $C_2 = 7$. The sequence of numbers constituted only from these digits (every from these digits must be used) is: 17, 71 (…).[59]
2. We consider the sequence of multiples of primes 3 in base 10 and the digits $C_1 = 0$ and $C_2 = 1$. The sequence of numbers constituted only from these digits (every from these digits must be used) is: 1011, 1101, 1110, 10011, 10101, 10110, 11001, 11010, 11100 (…).[60]

(6) The digital sum sequence

Definition:

The sequence $d_S(n)$ defined as the sum of the digits of n.[61]

The first forty terms of the sequence (A007953 in OEIS):

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

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54 The sequence is named by F.S. (*Only problems, no solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 4), *Digital sequences*.

55 The sequence is named by F.S. *The Digit-1 prime sequence*.

56 The sequence is named by F.S. *The Digit-0 factorial sequence*.

57 The sequence is named by F.S. *The Digit-5 n^n sequence*.

58 The sequence is named by F.S. (*Only problems, no solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 4), *Construction sequences*.

59 The sequence is named by F.S. *The Digit-1-7-only prime sequence*.

60 The sequence is named by F.S. *The Digit-0-1-only multiple of 3 sequence*.

61 For the formula of the n-the term of this sequence, see Atanassov, Krassimir T., *On some of the Smarandache's problems*, American Research Press, 1999, p. 12-15.

(7) The digital product sequence

Definition:

The sequence $d_P(n)$ defined as the product of the digits of n.[62]

The first forty terms of the sequence (A007954 in OEIS):

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 0, 3, 6, 9, 12, 15, 18, 21, 24, 27.

(8) The divisor products sequence

Definition:

The sequence $P_d(n)$ defined as the product of the positive divisors of n.[63]

The first thirty terms of the sequence (A007955 in OEIS):

1, 2, 3, 8, 5, 36, 7, 64, 27, 100, 11, 1728, 13, 196, 225, 1024, 17, 5832, 19, 8000, 441, 484, 23, 331776, 125, 676, 729, 21952, 29, 810000.

Properties[64]:

1. The sequence obviously contains an infinite number of primes: if p is prime, then $P_d(p) = p$.
2. The sequence contains an infinite number of the forms p^k, where p is prime.
3. $P_d(n) = n$ only if n = 1 or n is prime; for any composite number, $P_d(n) > n$.
4. For a prime p, p^m belongs to P_d only if there is some integer k such that k*(k + 1)/2 = m.

(9) The proper divisor products sequence

Definition:

The sequence $p_d(n)$ defined as the product of the proper divisors of n.

The first thirty terms of the sequence (A007956 in OEIS):

1, 1, 1, 2, 1, 6, 1, 8, 3, 10, 1, 144, 1, 14, 15, 64, 1, 324, 1, 400, 21, 22, 1, 13824, 5, 26, 27, 784, 1, 27000.

Notes:

1. If n is prime, then $p_d(n) = 1$.
2. "Numbers of the form $p_d(n) = n$ may well be called *Smarandache amicable numbers*, after the usual *amicable numbers*".[65]

(10) The square complements sequence

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[62] For a deeper study of this sequence, see Vassilev-Missana, Mladen and Atanassov, Krassimir, *Some Smarandache problems*, Hexis, 2004, Chapter 1: *On some Smarandache's problems*, Section 4: *On the 17-th Smarandache's problem*.

[63] For a deeper study of this sequence and of the following one, see Vassilev-Missana, Mladen and Atanassov, Krassimir, *Some Smarandache problems*, Hexis, 2004, Chapter 1: *On some Smarandache's problems*, Section 5: *On the 20-th and the 21-st Smarandache's problems*.

[64] Kashihara, K., *Comments and topics on Smarandache notions and problems*, Erhus University Press, 1996, Chapter 1: *Some comments and problems on Smarandache notions*.

[65] Kashihara, K., *Comments and topics on Smarandache notions and problems*, Erhus University Press, 1996, Chapter 1: *Some comments and problems on Smarandache notions*, p. 10. A pair of *amicable numbers* consists in two numbers that have the following relation: the sum of the proper divisors of one of them is equal to the other number: for instance [220, 284] is such a pair because the sum of proper divisors of 220 equals 284 while the sum of proper divisors of 284 equals 220.

Definition:

The sequence of the numbers k with the property that k is the smallest integer so that n*k is a perfect square.

The first forty terms of the sequence:

1, 2, 3, 1, 5, 6, 7, 2, 1, 10, 11, 3, 13, 14, 15, 1, 17, 2, 19, 5, 21, 22, 23, 6, 1, 26, 3, 7, 29, 30, 31, 2, 33, 34, 35, 1, 37, 38, 39, 10.

Properties:

All the terms of the sequence are squarefree. "The Smarandache square somplement sequence is the set of all square-free numbers. Moreover, each element of the set appears an infinite number of times."[66]

(11) The cube complements sequence[67]

Definition:

The sequence of the numbers k with the property that k is the smallest integer so that n*k is a perfect cube.

The first thirty terms of the sequence (A048798 in OEIS):

1, 4, 9, 2, 25, 36, 49, 1, 3, 100, 121, 18, 169, 196, 225, 4, 289, 12, 361, 50, 441, 484, 529, 9, 5, 676, 1, 98, 841, 900.

Properties:

All the terms of the sequence are cubefree. This sequence is the set of all cubefree numbers. Moreover, every number in the sequence appears an infinite number of times.

(12) The m-power complements sequence

Definition:

The sequence of the numbers k with the property that k is the smallest integer so that n*k is a perfect m-power ($m \geq 2$)[68].

Properties:

All the terms of the sequence are m-power free.

(13) The double factorial complements sequence

Definition:

The sequence of the numbers k with the property that k is the smallest integer so that n*k is a double factorial.

The first twenty-five terms of the sequence (A007919 in OEIS):

1, 1, 1, 2, 3, 8, 15, 1, 105, 192, 945, 4, 10395, 46080, 1, 3, 2027025, 2560, 34459425, 192, 5, 3715891200, 13749310575, 2, 81081.

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[66] For the proof of this property, see Kashihara, K., *Comments and topics on Smarandache notions and problems*, Erhus University Press, 1996, Chapter 1: *Some comments and problems on Smarandache notions*, p. 10.

[67] The function that generates the numbers from this sequence is sometimes named *the Smarandache cubic complementary function*. For the properties of this function see Popescu, Marcela and Nicolescu, Mariana, *About the Smarandache complementary cubic function*, Smarandache Notions Journal, vol. 7, no. 1-2-3, 1996. For a generalization of Smarandache complementary functions see *infra*, Part Two, Chapter 1, Section (17): *The Smarandache complementary functions*.

[68] For a study of this sequence, see Atanassov, Krassimir T., *On some of the Smarandache's problems*, American Research Press, 1999, p. 16-21.

(14) The prime additive complements sequence[69]

Definition:

The sequence of the numbers k with the property that k is the smallest integer so that n + k is a prime.[70]

The first thirty terms of the sequence (A007920 in OEIS):

2, 1, 0, 0, 1, 0, 1, 0, 3, 2, 1, 0, 1, 0, 3, 2, 1, 0, 1, 0, 3, 2, 1, 0, 5, 4, 3, 2, 1, 0.

Notes[71]:

1. F.S. asked the following questions: Is it possible to have k as large as we want k, k – 1, k – 2, k – 3, ... , 2, 1 (where k is odd), included in this sequence? Is it possible to have k as large as we want k, k – 1, k – 2, k – 3, ... , 2, 1 (where k is even), included in this sequence?[72] Is the sequence convergent or divergent?
2. F.S. conjectured that the sequence is divergent.
3. F.S. also defined (and raised the same questions from above) *the prime nearest complements sequence, i.e.* the sequence formed from the numbers k with the property that, for $n \geq 1$, the absolute value of k is minimal and n + k is prime. The terms of this sequence are: $\{1, 0, 0, \pm1, 0, \pm1, 0, -1, \pm2, 1, 0, \pm1, 0, -1, \pm2, \ldots\}$.

(15) The double factorial sequence

Definition:

The sequence of the numbers k with the property that k is the smallest integer so that k!! is a multiple of n.

The first forty terms of the sequence (A007922 in OEIS):

1, 2, 3, 4, 5, 6, 7, 4, 9, 10, 11, 6, 13, 14, 5, 6, 17, 12, 19, 10, 7, 22, 23, 6, 15, 26, 9, 14, 29, 10, 31, 8, 11, 34, 7, 12, 37, 38, 13, 10.

(16) The "primitive numbers of power 2" sequence[73]

Definition:

The sequence of the numbers $S_2(n)$ with the property that $S_2(n)$ is the smallest integer for which 2^n divides $S_2(n)!$.

The first forty terms of the sequence (A007843 in OEIS):

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[69] The function that generates the numbers from this sequence is sometimes named *the Smarandache prime complementary function*. For the properties of this function see Popescu, Marcela and Seleacu, Vasile, *About the Smarandache complementary prime function*, Smarandache Notions Journal, vol. 7, no. 1-2-3, 1996. For a generalization of Smarandache complementary functions see *infra*, Part Two, Chapter 1, Section (17): *The Smarandache complementary functions*.

[70] For a deeper study of this sequence, see Vassilev-Missana, Mladen and Atanassov, Krassimir, *Some Smarandache problems*, Hexis, 2004, Chapter 1: *On some Smarandache's problems*, Section 8: *On the 46-th Smarandache's problem*.

[71] Kashihara, K., *Comments and topics on Smarandache notions and problems*, Erhus University Press, 1996, Chapter 1: *Some comments and problems on Smarandache notions*, p. 14.

[72] Maohua Le proved that, for k an arbitrary large positive integer, the Smarandache prime additive complements sequence include the decreasing sequence k, k – 1, …, 1, 0. See *On the Smarandache prime additive sequence*, in Seleacu, V., Bălăcenoiu, I. (editors), *Smarandache Notions (Book series)*, vol. 10, American Research Press, 1999.

[73] See *Only problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 3, F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Problem 47. See also F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 68.

1, 2, 4, 4, 6, 8, 8, 8, 10, 12, 12, 14, 16, 16, 16, 16, 18, 20, 20, 22, 24, 24, 24, 26, 28, 28, 30, 32, 32, 32, 32, 32, 34, 36, 36, 38, 40, 40, 40, 42.

Properties:

1. This is the sequence of even numbers, each number being repetead as many times as its exponent (of power 2) is.
2. This is one of irreductible functions, noted $S_2(k)$, which helps to calculate the Smarandache function.

(17) The "primitive numbers of power 3" sequence[74]

Definition:

The sequence of the numbers $S_3(n)$ with the property that $S_3(n)$ is the smallest integer for which 3^n divides $S_3(n)$!.

The first forty terms of the sequence (A007844 in OEIS):

1, 3, 6, 9, 9, 12, 15, 18, 18, 21, 24, 27, 27, 27, 30, 33, 36, 36, 39, 42, 45, 45, 48, 51, 54, 54, 54, 57, 60, 63, 63, 66, 69, 72, 72, 75, 78, 81, 81, 81, 81.

Properties:

1. This is the sequence of multiples of 3, each number being repetead as many times as its exponent (of power 3) is.
2. This is one of irreductible functions, noted $S_3(k)$, which helps to calculate the Smarandache function.

(18) The generalized primitive numbers (of power p) sequence[75]

Definition:

The sequence of the numbers $S_p(n)$ with the property that $S_p(n)$ is the smallest integer for which p^n divides $S_p(n)$!, where p is prime.

Properties:

1. This is the sequence of multiples of p, each number being repetead as many times as its exponent (of power p) is.
2. These are irreductible functions, noted $S_p(k)$, for any prime number p, which helps to calculate the Smarandache function.

(19) The cube free sieve sequence

Definition:

From the set of positive integers except 0 and 1 take off all multiples of 2^3, 3^3, 5^3 and so on: take off all multiples of all cubic primes.[76]

The first forty terms of the sequence (A004709 in OEIS):

2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 25, 26, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47.

Properties:

All the terms of the sequence are cubefree.

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[74] See F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 69.

[75] See F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 70. For a study of this sequence, see Ibstedt, Henry, *Computer analysis of number sequences*, American Research Press, 1998, Chapter III: *Non-recursive sequences*, Section 1: *Smarandache primitive numbers*.

[76] For a deeper study of this sequence and of the following one, see Vassilev-Missana, Mladen and Atanassov, Krassimir, *Some Smarandache problems*, Hexis, 2004, Chapter 1: *On some Smarandache's problems*, Section 6: *On the 25-th and the 26-th Smarandache's problems*.

(20) The m-power free sieve

Definition:

From the set of positive integers except 0 and 1 take off all multiples of 2^m, 3^m, 5^m and so on: take off all multiples of all m-power primes ($m \geq 2$).

Properties:

All the terms of the sequence are m-power free.

(21) The inferior prime part sequence[77]

Definition:

The sequence $P_P(n)$ defined as the sequence of numbers with the property that they are the largest primes smaller than or equal to n.

The first forty terms of the sequence (A007917 in OEIS):

2, 3, 3, 5, 5, 7, 7, 7, 7, 11, 11, 13, 13, 13, 13, 17, 17, 19, 19, 19, 19, 23, 23, 23, 23, 23, 23, 29, 29, 31, 31, 31, 31, 31, 31, 37, 37, 37, 37, 41.

(22) The superior prime part sequence[78]

Definition:

The sequence $P_P(n)$ defined as the sequence of numbers with the property that they are the smallest primes greater than or equal to n.[79]

The first forty terms of the sequence (A007918 in OEIS):

2, 2, 2, 3, 5, 5, 7, 7, 11, 11, 11, 11, 13, 13, 17, 17, 17, 17, 19, 19, 23, 23, 23, 23, 29, 29, 29, 29, 29, 29, 31, 31, 37, 37, 37, 37, 37, 37, 41, 41.

(23) The inferior square part sequence[80]

Definition:

The sequence of numbers with the property that they are the largest squares smaller than or equal to n.

The first forty terms of the sequence (A048761 in OEIS):

0, 1, 1, 1, 4, 4, 4, 4, 4, 9, 9, 9, 9, 9, 9, 9, 16, 16, 16, 16, 16, 16, 16, 16, 16, 25, 25, 25, 25, 25, 25, 25, 25, 25, 25, 25, 36, 36, 36, 36.

(24) The superior square part sequence[81]

Definition:

The sequence of numbers with the property that they are the smallest squares greater than or equal to n.[82]

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[77] See F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 38.

[78] See F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 39.

[79] For a study of Superior and Inferior prime part sequences, see Atanassov, Krassimir T., *On some of the Smarandache's problems*, American Research Press, 1999, p. 22-26.

[80] See F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 40.

[81] See F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 41. The Sequences 42-43 from the same book define the inferior and the superior cube part.

[82] For a study of Superior and Inferior square part sequences, see Atanassov, Krassimir T., *On some of the Smarandache's problems*, American Research Press, 1999, p. 27-32.

The first forty terms of the sequence (A048761 in OEIS):

0, 1, 4, 4, 4, 9, 9, 9, 9, 9, 16, 16, 16, 16, 16, 16, 16, 25, 25, 25, 25, 25, 25, 25, 25, 25, 36, 36, 36, 36, 36, 36, 36, 36, 36, 36, 36, 49, 49, 49.

(25) The inferior factorial part sequence[83]

Definition:

The sequence $F_P(n)$ defined as the sequence of numbers with the property that they are the largest factorials smaller than or equal to n.

The first thirty terms of the sequence (A048674 in OEIS):

1, 2, 2, 2, 2, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 24, 24, 24, 24, 24, 24, 24.

(26) The superior factorial part sequence[84]

Definition:

The sequence $f_P(n)$ defined as the sequence of numbers with the property that they are the smallest factorials greater than or equal to n.[85]

The first forty terms of the sequence (A048675 in OEIS):

1, 2, 6, 6, 6, 6, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 24, 120, 120, 120, 120, 120, 120, 120, 120, 120, 120, 120, 120, 120, 120, 120, 120.

(27) The irrational root sieve sequence

Definition:

From the set of positive integers greater than 1 take off all multiples of all square primes.

The first forty terms of the sequence:

2, 3, 5, 6, 7, 10, 11, 13, 14, 15, 17, 19, 21, 22, 23, 26, 29, 30, 31, 33, 34, 35, 37, 38, 39, 41, 42, 43, 46, 47, 51, 53, 55, 57, 58, 59, 61, 62, 65, 66.

Properties:

The terms of the sequence are all natural numbers those m-roots, for any $m \geq 2$, are irrational.

(28) The odd sieve sequence

Definition:

The sequence obtained in the following way: subtract 2 from all primes and obtain a temporary sequence; choose all odd numbers that do not belong to the temporary sequence.[86]

The first forty terms of the sequence (A007921 in OEIS):

7, 13, 19, 23, 25, 31, 33, 37, 43, 47, 49, 53, 55, 61, 63, 67, 73, 75, 79, 83, 85, 89, 91, 93, 97, 103, 109, 113, 115, 117, 119, 121, 123, 127, 131, 133, 139, 141, 143, 145.

(29) The binary sieve sequence[87]

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[83] See F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 44.

[84] See F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 45.

[85] For a study of Superior and Inferior prime part sequences, see Atanassov, Krassimir T., *On some of the Smarandache's problems*, American Research Press, 1999, p. 33-37.

[86] For explicit formulae for the n-th term of this sequence and theorems, see Vassilev-Missana, Mladen and Atanassov, Krassimir, *Some Smarandache problems*, Hexis, 2004, Chapter 1: *On some Smarandache's problems*, Section 7: *On the 28-th Smarandache's problem*.

Definition:

The sequence obtained in the following way: start to count on the natural numbers set and, at any step from 1, delete every 2-nd numbers, delete, from the remaining ones, every 4-th numbers and so on, delete, from the remaining ones, every (2^k)-th numbers, where k = 1, 2, ...

The first forty terms of the sequence (A007950 in OEIS):

1, 3, 5, 9, 11, 13, 17, 21, 25, 27, 29, 33, 35, 37, 43, 49, 51, 53, 57, 59, 65, 67, 69, 73, 75, 77, 81, 85, 89, 91, 97, 101, 107, 109, 113, 115, 117, 121, 123, 129, 131.

(30) The consecutive sieve sequence[88]

Definition:

The sequence obtained in the following way: from the natural numbers set: keep the first number and delete one number out of 2 from all remaining numbers; keep the first remaining number and delete one number out of 3 from the next remaining numbers; keep the first remaining number and delete one number out of 4 from the next remaining numbers and so on, for step k ($k \geq 2$), keep the first remaining number and delete one number out of k from the next remaining numbers.

The first thirty terms of the sequence (A007952 in OEIS):

0, 1, 3, 5, 9, 11, 17, 21, 29, 33, 41, 47, 57, 59, 77, 81, 101, 107, 117, 131, 149, 153, 173, 191, 209, 213, 239, 257, 273, 281.

Property:

This sequence is much less dense than the prime number sequence, and their ratio tends to p_n/n as n tends to infinity.

(31) The Smarandache-Fibonacci triplets sequence[89]

Definition:

The sequence obtained in the following way: the integer n is such one that $S(n) = S(n-1) + S(n-2)$, where S(k) is the Smarandache function.

The first fifteen terms of the sequence (A015047 in OEIS):

11, 121, 4902, 26245, 32112, 64010, 368140, 415664, 2091206, 2519648, 4573053, 7783364, 79269727, 136193976, 321022289.

Notes:

1. The Smarandache function S(n) is defined as the smallest integer S(n) such that S(n)! is divisible by n. [90]
2. The Fibonacci recurence formula is $F_n = F_{n-1} + F_{n-2}$, for $n \geq 2$ and $F_0 = F_1 = 1$.
3. The numbers that form the numbers from this sequence are known as Smarandache-Fibonacci triplets.

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[87] Similarly is defined the *Smarandache n-ary sieve sequence*. For instance, if S_n denote the Smarandache n-ary sieve sequence, S_2 is the binary sieve sequence and S_3 is the sequence {1, 2, 4, 5, 7, 8, 10, 11, 14, 16, 17, 19, 20…}.

[88] For other related sieve sequences, like *Trinary sieve sequence*, *n-ary power sieve sequence*, *k-ary consecutive sieve sequence*, *General-sequence sieve*, see F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, p. 19-21.

[89] F.S., Definitions, *solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Problem 10. See also Ibstedt, H., *Surphing on the ocean of numbers – a few Smarandache notions and similar topics*, Erhus University Press, Vail, 1997, Chapter II: *On Smarandache functions*.

[90] See *infra*, Part Two, Chapter I, Section (1): *The Smarandache function*.

Properties:

1. It is not known whether this sequence has infinitely or finitely many terms. [91]
2. The largest known number from this sequence is 19448047080036.[92]

Observation:

Apart from the case n = 26245, all the (known) terms of this sequence have a common property: from the three numbers which form a Smarandache-Fibonacci triplet, one is two times a prime number while the other two are prime numbers. Henry Ibstedt raised the following question: is the case n = 26245 the only different case?[93]

(32) The Smarandache-Radu duplets sequence[94]

Definition:

The sequence obtained in the following way: the integer n is such one that between S(n) and S(n + 1) there is no prime, where S(n) and S(n +1) are included, where S(k) is the Smarandache function.

The first fifteen terms of the sequence (A015048 in OEIS):

224, 2057, 265225, 843637, 6530355, 24652435, 35558770, 40201975, 45388758, 46297822, 67697937, 138852445, 157906534, 171531580, 299441785.

Notes:

1. The Smarandache function S(n) is defined as the smallest integer S(n) such that S(n)! is divisible by n. [95]
2. The numbers from this sequence are known as Smarandache-Radu duplets.

Properties:

1. It is not known whether this sequence has infinitely or finitely many terms.[96]
2. The largest known number from this sequence is 270329975921205253634707051822848570391313.[97]

(33) The Smarandache prime product sequence[98]

Definition:

The sequence of primes of the form $p_n\# \pm 1$, where $p_n\#$ is the product of the first n primes.

The first seven terms of the sequence (A034386 in OEIS)[99]:

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[91] Henry Ibstedt and Charles Ashbacher independently conjectured that are infinitely many terms.
[92] Found by H. Ibstedt. See Begay, Anthony, *Smarandache ceil functions*, Smarandache Notions Journal.
[93] See Ibstedt, H., *Surphing on the ocean of numbers – a few Smarandache notions and similar topics*, Erhus University Press, Vail, 1997, Chapter II: *On Smarandache functions*.
[94] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Problem 11.
[95] See *infra*, Part Two, Chapter I, Section (1): *The Smarandache function*.
[96] I.M. Radu and Henry Ibstedt conjectured that are infinitely many terms. See Ibstedt, H., *Surphing on the ocean of numbers – a few Smarandache notions and similar topics*, Erhus University Press, Vail, 1997, Chapter II: *On Smarandache functions*, Section 2: *Radu's problem*. In other words, Radu's problem can be formulated this way: "show that, except for a finite set of numbers, there exists at least one prime number between S(n) and S(n + 1)". See Radu I.M., *Proposed problem*, Ibstedt, H., *Base solution (the Smarandache function)*, Ibstedt, H., *On Radu's problem*, all three articles in Smarandache Notions Journal, vol. 7, no. 1-2-3, 1996.
[97] Found by H. Ibstedt. See Begay, Anthony, *Smarandache ceil functions*, Smarandache Notions Journal.
[98] For a study of this sequence, see Ibstedt, Henry, *Computer analysis of number sequences*, American Research Press, 1998, Chapter II: *Recursive integer sequences*.
[99] The eighth term of the sequence has 154 digits.

2, 3, 7, 31, 211, 2311, 200560490131.

Notes:

1. It is not known if the number of the terms of this sequence is infinite.
2. The primes of this type are known under the acronym *PPS primes*[100] but they are also known under the name *primorial primes*[101]. The programs *PrimeGrid*, *Open PFGW* and others are searching for PPS primes: the biggest prime known of the form $p_n\# - 1$ is the number 1098133# – 1 (a number with more than 450 thousand digits) and the biggest prime known of the form $p_n\# + 1$ is the number 392113# + 1 (a number with more than 150 thousand digits).

(34) The Smarandache friendly pairs set

Definition:

The set of pairs of natural numbers [m, n], where m < n, with the property that the product m*n is equal to the sum of all natural numbers from m to n (m and n are included).

Example:

[3, 6] is such a pair because 3*6 = 3 + 4 + 5 + 6.

First four Smarandache friendly pairs:

[1, 1], [3, 6], [15, 35], [85, 204].

Notes:

1. There is an infinity of Smarandache friendly pairs (they are known under the acronym SFP).
2. If [m, n] is a Smarandache friendly pair, then [2*n + m, 5*n + 2*m – 1] it will be too such a pair.

Question[102]:

Is there an infinity of primes q for every prime p such that [p, q] is a Smarandache friendly pair?

Definitions[103]:

1. If the sum of any set of consecutive terms of a sequence is a divisor of the product of the first and the last number of the set then this pair is called a *Smarandache under-friendly pair* with respect to the sequence.
2. If the sum of any set of consecutive terms of a sequence is a multiple of the product of the first and the last number of the set then this pair is called a *Smarandache over-friendly pair* with respect to the sequence.

(35) The Smarandache friendly prime pairs set

Definition:

The set of pairs of primes [p, q], where p < q, with the property that the product p*q is equal to the sum of all primes from p to q (p and q are included).

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[100] Majumdar, A.A.K., *Wandering in the world of Smarandache numbers*, InProQuest, 2010, Chapter 1(1.4): *Smarandache prime product sequence*.

[101] *Claims on primorial primes*, Turker Ozsari, Arxiv.

[102] Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, Chapter 2: *Smarandache sequences*, Section 5: *Smarandache friendly numbers and a few more sequences*.

[103] Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, Chapter 2: *Smarandache sequences*, Section 5: *Smarandache friendly numbers and a few more sequences*.

Example:

[7, 53] is such a pair because 7*53 = 7 + 11 + 13 + 17 + 19 + 23 + 29 + 31 + 37 + 43 + 47 + 53.

The five known Smarandache friendly prime pairs (sequence A176914 in OEIS):

[2, 5], [3, 13], [5, 31], [7, 53], [3536123, 128541727].

Notes:

1. There are only five known Smarandache friendly prime pairs (they are known under the acronym SFPP), discovered by mathematicians Philip Gibbs and Felice Russo.
2. It is not known if there is an infinity of Smarandache friendly prime pairs.
3. It is not known if for every prime p there is a prime q such that [p, q] is a Smarandache friendly prime pair.[104]

(36) The 3n-digital subsequence

Definition[105]:

The sequence of numbers that can be partitioned into two groups such that the second is three times biger than the first.

The first fifteen terms of the sequence (A019551 in OEIS):

13, 26, 39, 412, 515, 618, 721, 824, 927, 1030, 1133, 1236, 1339, 1442, 1545.

(37) The 4n-digital subsequence

Definition[106]:

The sequence of numbers that can be partitioned into two groups such that the second is four times biger than the first.

The first fifteen terms of the sequence (A019552 in OEIS):

14, 28, 312, 416, 520, 624, 728, 832, 936, 1040, 1144, 1248, 1352, 1456, 1560.

(38) The 5n-digital subsequence

Definition[107]:

The sequence of numbers that can be partitioned into two groups such that the second is five times biger than the first.

The first fifteen terms of the sequence (A019553 in OEIS):

15, 210, 315, 420, 525, 630, 735, 840, 945, 1050, 1155, 1260, 1365, 1470, 1575.

(39) The crescendo and decrescendo subsequences

The crescendo sequence:

The type of sequence of sequences constructed in the following way:

1,
1, 2,
1, 2, 3, … *(sequence A002260 in OEIS)*.

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[104] For more questions about these pairs of primes see Russo, Felice, *On a problem concerning the Smarandache friendly prime pairs*, Smarandache Notions Journal; see also Gibbs, Philip, *A fifth Smarandache friendly prime pair*, Vixra.

[105] F.S., *Considerations on new functions in number theory*, Arxiv.

[106] F.S., *Considerations on new functions in number theory*, Arxiv.

[107] F.S., *Considerations on new functions in number theory*, Arxiv.

The descrescendo sequence:

The type of sequence of sequences constructed in the following way:
1,
2, 1,
3, 2, 1, … *(sequence A004736 in OEIS)*.

(40) The crescendo and decrescendo pyramidal subsequences

The crescendo pyramidal sequence:

The type of sequence of sequences constructed in the following way:
1,
1, 2, 1,
1, 2, 3, 2, 1, … *(sequence A004737 in OEIS)*.

The descrescendo pyramidal sequence:

The type of sequence of sequences constructed in the following way:
1,
2, 1, 2,
3, 2, 1, 2, 3… *(sequence A004738 in OEIS)*.

(41) The crescendo and decrescendo symmetric subsequences

The crescendo symmetric sequence:

The type of sequence of sequences constructed in the following way:
1, 1,
1, 2, 2, 1,
1, 2, 3, 3, 2, 1, … *(sequence A004739 in OEIS)*.

The descrescendo symmetric sequence:

The type of sequence of sequences constructed in the following way:
1, 1,
2, 1, 1, 2,
3, 2, 1, 1, 2, 3… *(sequence A004737 in OEIS)*.

(42) The permutation subsequences

Definition:

The type of sequence of sequences constructed in the following way:
1, 2,
1, 3, 4, 2,
1, 3, 5, 6, 4, 2… *(sequence A004741 in OEIS)*.[108]

(43) The Smarandache bases of numeration sequences[109]

The Smarandache prime base sequence[110]:

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[108] For formulas for the general term of these sequences of subsequences (40-43) see F.S., *Considerations on new functions in number theory*, Arxiv.

[109] For the properties of the natural numbers written in the following Smarandache bases of numeration see F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Definitions 11, 12, 14, 30, 31, 32.

[110] F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 58. F.S., *Only problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 90. Studying this sequence, F.S. shows

On the set of natural numbers is defined the following infinite base: $p_0 = 1$ and p_k is the k-th prime number for $k \geq 1$.

The first twelve terms of the sequence (A007924 in OEIS):

0, 1, 10, 100, 101, 1000, 1001, 10000, 10001, 10010, 10100, 100000, 100001.

The Smarandache square base sequence[111]:

On the set of natural numbers is defined the following infinite base: $s_k = k^2$ for $k \geq 0$.

The first twelve terms of the sequence:

0, 1, 2, 3, 10, 11, 12, 13, 20, 100, 101, 102, 103, 110.

The Smarandache cubic base sequence (A007094 in OEIS):

On the set of natural numbers is defined the following infinite base: $s_k = k^3$ for $k \geq 0$.

The first twelve terms of the sequence:

0, 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13.

The Smarandache factorial base sequence[112]:

On the set of natural numbers is defined the following infinite base: : $f_k = k!$ for $k \geq 1$.

The first twelve terms of the sequence (A007623 in OEIS):

0, 1, 10, 11, 20, 21, 100, 101, 110, 111, 120, 121.

The Smarandache double factorial base sequence[113]:

On the set of natural numbers is defined the following infinite base: : $df_k = k!!$

The first twelve terms of the sequence (A019513 in OEIS):

1, 10, 100, 101, 110, 200, 201, 1000, 1001, 1010, 1100, 1101.

The Smarandache triangular base sequence[114]:

On the set of natural numbers is defined the following infinite base: : $t_k = k*(k + 1)/2$ for $k \geq 1$.

The first twelve terms of the sequence (A000462 in OEIS):

1, 2, 10, 11, 12, 100, 101, 102, 110, 1000, 1001, 1002.

(44) The multiplicative sequence[115]

Definition:

The sequence obtained in the following way: if m_1 and m_2 are the first two terms of the sequence, then m_k, for $k \geq 3$, is the smallest number equal to the product of two previous distinct terms.

Comment:

All terms of rank greater than or equal to 3 are divisible by m_1 and m_2.

The first twenty terms of the sequence for the particular case $m_1 = 2$, $m_2 = 3$:

2, 3, 6, 12, 18, 24, 36, 48, 54, 72, 96, 108, 144, 162, 192, 216, 288, 324, 384, 432.

Theorem[116]:

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that any number can be written as a sum of prime numbers or as a sum of prime numbers plus 1. See also Kashihara, K., *Comments and topics on Smarandache notions and problems*, Erhus University Press, 1996, p. 33.

111 F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 59. The Sequence 60 from the same book defines in an analogous way *the m-power base sequence*.

112 F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 61.

113 F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 62.

114 F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 63. The sequence 64 from the same book defines *the generalizerd base sequence*.

115 F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Definition 26.

116 Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, Chapter 2: *Smarandache sequences*, Section 10: *The sum of the reciprocals of the Smarandache multiplicative sequence*.

The limit of the sum of the reciprocals of the terms of the multiplicative sequence exists for all initial terms m_1 and m_2. The sum of the reciprocals of the multiplicative sequence with initial terms m_1 and m_2 is $S = 1/((m_1 - 1)*(m_2 - 1)) + 1/m_1 + 1/m_2$.

(45) The non-multiplicative general sequence[117]

Definition:

The sequence obtained in the following way: let $m_1, m_2 \ldots, m_k$ be the first k terms of the sequence, where $k \geq 2$. Then m_i, for $i \geq k + 1$, is the smallest number not equal to the product of k previous distinct terms.

(46) The non-arithmetic progression sequence[118]

Definition:

The sequence defined in the following way: if m_1 and m_2 are the first two terms of the sequence, then m_k, for $k \geq 3$, is the smallest number such that no 3-term arithmetic progression is in the sequence. Generalization: the same initial conditions, but with no i-term arithmetic progression in the sequence, for a given $i \geq 3$.

(47) The non-geometric progression sequence[119]

Definition:

The sequence defined in the following way: if m_1 and m_2 are the first two terms of the sequence, then m_k, for $k \geq 3$, is the smallest number such that no 3-term geometric progression is in the sequence. Generalization: the same initial conditions, but with no i-term geometric progression in the sequence, for a given $i \geq 3$.

(48) The "wrong numbers" sequence[120]

Definition:

The sequence of "wrong numbers" which are defined in the following way: the number $n = a_1a_2\ldots a_k$, consisted of at least two digits, with the property that the sequence $a_1, a_2, \ldots, a_k, b_{k+1}, b_{k+2}, \ldots$(where b_{k+i} is the product of the previous k terms, for any $i \geq 1$), contains n as its term.

Comment:

F.S. conjectured that no number is wrong; therefore, this sequence is empty.

(49) The "impotent numbers" sequence[121]

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[117] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Definition 34.

[118] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Definition 35. F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 232. For a study of this sequence, see Ibstedt, Henry, *Computer analysis of number sequences*, American Research Press, 1998, Chapter II: *Recursive integer sequences*.

[119] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Definition 43. F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 217.

[120] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Definition 27.

[121] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Definition 28.

Definition:

The sequence of "impotent numbers" which are defined in the following way: a number n those proper divisors product is less than n.

Comment:

The terms of this sequence are the primes and the squares of primes.

The first twenty terms of the sequence (A000430 in OEIS):

2, 3, 4, 5, 7, 9, 11, 13, 17, 19, 23, 25, 29, 31, 37, 41, 43, 47, 49, 53, 59.

(50) The "simple numbers" sequence[122]

Definition:

The sequence of "simple numbers" which are defined in the following way: a number n those proper divisors product is less than or equal to n.[123]

Theorem[124]:

The terms of this sequence can be only primes, squares of primes, cubes of primes or semiprimes.

The first twenty terms of the sequence (A007964 in OEIS):

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 17, 19, 21, 22, 23, 25.

(51) The square product sequence[125]

Definition:

The sequence defined in the following way: $S_n = 1 + s_1*s_2*\ldots*s_n$, where s_k is the k-th square number.

The first nine terms of the sequence:

2, 5, 27, 577, 14401, 518401, 25401601, 1625702401, 131681894401.

Comment:

F.S. raised the question: how many terms of this sequence are primes?

Note:

The sequence defined above (*i.e.* 1^2*2^2*3^2*…*n^2 + 1, where n ≥ 1) is sometimes called the *Smarandache square product sequence of the first kind* and named with the acronym $SPS_1(n)$ while the sequence defined as 1^2*2^2*3^2*…*n^2 – 1, where n ≥ 1, is called the *Smarandache square product sequence of the second kind* and named with the acronym $SPS_2(n)$.[126]

(52) The cubic product sequence[127]

Definition:

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[122] F.S., *Only problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 23.

[123] For three different explicit representations for the n-th term of the sequence, see Vassilev-Missana, Mladen and Atanassov, Krassimir, *Some Smarandache problems*, Hexis, 2004, Chapter 1: *On some Smarandache's problems*, Section 3: *On the 15-th Smarandache's problem*.

[124] Ashbacher, C., *Collection of problems on Smarandache notions*, Erhus University Press, 1996, p. 20.

[125] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Problem 22. For a study of this sequence, see Ibstedt, Henry, *Computer analysis of number sequences*, American Research Press, 1998, Chapter II: *Recursive integer sequences*.

[126] See Majumdar, A.A.K., *Wandering in the world of Smarandache numbers*, InProQuest, 2010, Chapter 1: *Some Smarandache sequences*, Section 1.5: *Smarandache square product sequence*.

[127] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Definition 36 and Problem 23.

The sequence defined in the following way: $C_n = 1 + c_1 * c_2 * \ldots * c_n$, where c_k is the k-th cubic number.

The first nine terms of the sequence (A019514 in OEIS):

2, 3, 13, 289, 34561, 24883201, 125411328001, 5056584744960001, 1834933472251084800001.

Comment:

F.S. raised the question: how many terms of this sequence are primes?[128]

Note:

The square product and the cubic product sequences defined above were generalized resulting *Smarandache higher power product sequence of the first kind* respectively *Smarandache higher power product sequence of the second kind* named with the acronyms $HPPS_1(n)$ the one defined as 1^m*2^m*…*n^m + 1, where $n \geq 1$, $m > 3$, respectively $HPPS_2(n)$ the one defined as 1^m*2^m*…*n^m – 1, where $n \geq 1$, $m > 3$.[129]

(53) The factorial product sequence[130]

Definition:

The sequence defined in the following way: $F_n = 1 + f_1 * f_2 * \ldots * f_n$, where f_k is the k-th factorial number.

The first nine terms of the sequence (A019515 in OEIS):

2, 9, 217, 13825, 1728001, 373248001, 128024064001, 65548320768001, 47784725839872001.

Comment:

F.S. raised the question: how many terms of this sequence are primes?

(54) The Smarandache recurrence type sequences[131]

1. The general term of the sequence is:

The smallest number, strictly greater than the previous one, which is the sum of squares of two previous distinct terms of the sequence, for given first two terms.

The first sixteen terms of the sequence for first two terms 1 and 2 (A008318 in OEIS):

1, 2, 5, 26, 29, 677, 680, 701, 842, 845, 866, 1517, 458330, 458333, 458354, 459005.

2. The general term of the sequence is:

The smallest number which is the sum of squares of previous distinct terms of the sequence.

The first sixteen terms of the sequence (A008319 in OEIS):

1, 1, 2, 4, 5, 6, 16, 17, 18, 20, 21, 22, 25, 26, 27, 29.

3. The general term of the sequence is:

The smallest number, strictly greater than the previous one, which is not the sum of squares of two previous distinct terms of the sequence, for given first two terms.

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[128] For a study of primality of the terms of the *Smarandache cubic product sequence* and generally of the *Smarandache power product sequences* see Le, Maohua and Wu, Kejian, *The primes in Smarandache power product sequences*, Smarandache Notions Journal, vol. 9, no. 1-2-3, 1998.

[129] See Majumdar, A.A.K., *Wandering in the world of Smarandache numbers*, InProQuest, 2010, Chapter 1: *Some Smarandache sequences*, Section 1.6: *Smarandache higher power product sequences*.

[130] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Definition 37 and problem 24.

[131] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Definition 42. F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequences 194-202. See also Bencze, Mihály, *Smarandache recurrence type sequences*, Smarandache Notions Journal, vol. 11, no. 1-2-3, 2000.

The first sixteen terms of the sequence for first two terms 1 and 2 (A004439 in OEIS):
1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 14, 15, 16, 18, 19, 21.

4. The general term of the sequence is:
The smallest number which is not the sum of squares of previous distinct terms of the sequence.
The first sixteen terms of the sequence (A008321 in OEIS):
1, 2, 3, 6, 7, 8, 11, 12, 15, 16, 17, 18, 19, 20, 21, 22.

5. The general term of the sequence is:
The smallest number, strictly greater than the previous one, which is the sum of cubes of two previous distinct terms of the sequence, for given first two terms.
The first ten terms of the sequence for first two terms 1 and 2 (A008322 in OEIS):
1, 2, 9, 730, 737, 389017001, 389017008, 389017729, 400315554, 400315561.

6. The general term of the sequence is:
The smallest number which is the sum of cubes of previous distinct terms of the sequence.
The first sixteen terms of the sequence (A019511 in OEIS):
1, 1, 2, 8, 9, 10, 512, 513, 514, 520.

7. The general term of the sequence is:
The smallest number, strictly greater than the previous one, which is not the sum of cubes of two previous distinct terms of the sequence, for given first two terms.
The first sixteen terms of the sequence for first two terms 1 and 2 (A031980 in OEIS):
1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17.

8. The general term of the sequence is:
The smallest number which is not the sum of cubes of previous distinct terms of the sequence.
The first sixteen terms of the sequence (A019511 in OEIS):
1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 18.

(55) The Smarandache partition type sequences[132]

1. The general term of the sequence, a(n), is:
The number of times in which n can be written as a sum of non-null squares, disregarding terms order.
Example: a(9) = 4 because 9 = 1^2 + 1^2 + 1^2 + 1^2 + 1^2 + 1^2 + 1^2 + 1^2 + 1^2 = 1^2 + 1^2 + 1^2 + 1^2 + 1^2 + 2^2 = 1^2 + 2^2 + 2^2 = 3^2.
The first thirty terms of the sequence (A001156 in OEIS):
1, 1, 1, 1, 2, 2, 2, 2, 3, 4, 4, 4, 5, 6, 6, 6, 8, 9, 10, 10, 12, 13, 14, 14, 16, 19, 20, 21, 23, 26.

2. The general term of the sequence, a(n), is:
The number of times in which n can be written as a sum of non-null cubes, disregarding terms order.
The first thirty terms of the sequence (A003108 in OEIS):
1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 3, 4, 4, 4, 5, 5, 5.

(56) The square residues sequence[133]

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[132] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Problem 15. F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequences 203-215.

[133] F.S., *Only problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 63. See also See F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 75.

Definition:

The general term of the sequence, a(n), is the largest square free number which divides n.

The first twenty-five terms of the sequence (A007947 in OEIS):

1, 2, 3, 2, 5, 6, 7, 2, 3, 10, 11, 6, 13, 14, 15, 2, 17, 6, 19, 10, 21, 22, 23, 6, 5.

(57) The cubic residues sequence[134]

Definition:

The general term of the sequence, a(n), is the largest cube free number which divides n.

The first twenty-five terms of the sequence (A007948 in OEIS):

1, 2, 3, 4, 5, 6, 7, 4, 9, 10, 11, 12, 13, 14, 15, 4, 17, 18, 19, 20, 21, 22, 23, 12, 25.

(58) The exponents of power 2 sequence[135]

Definition:

The general term of the sequence, $e_2(n)$, is the largest exponent of power 2 which divides n.

The first thirty terms of the sequence (A007814 in OEIS):

0, 1, 0, 2, 0, 1, 0, 3, 0, 1, 0, 2, 0, 1, 0, 4, 0, 1, 0, 2, 0, 1, 0, 3, 0, 1, 0, 2, 0, 1.

(59) The exponents of power 3 sequence[136]

Definition:

The general term of the sequence, $e_3(n)$, is the largest exponent of power 3 which divides n.

The first thirty terms of the sequence (A007949 in OEIS):

0, 0, 1, 0, 0, 1, 0, 0, 2, 0, 0, 1, 0, 0, 1, 0, 0, 2, 0, 0, 1, 0, 0, 1, 0, 0, 3, 0, 0, 1.

(60) The unary sequence[137]

Definition:

The general term of the sequence, u(n), is equal to 11…1, where the digit 1 is repetead p_n times, p_n being the n-th prime.

The first seven terms of the sequence (A031974 in OEIS):

11, 111, 11111, 1111111, 11111111111, 1111111111111, 11111111111111111.

Note:

F.S. raised the question: is there an infinite number of primes belonging to this sequence?[138]

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[134] F.S., *Only problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 64. See also F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 76; the Sequence 77 from this book defines in an analogous way *the m-power residues sequences*. For a study of this sequence, see Ibstedt, Henry, *Computer analysis of number sequences*, American Research Press, 1998, Chapter III: *Non-recursive sequences*, Section 3: *Smarandache m-power residues*.

[135] F.S., *Only problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 66. See also F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 78.

[136] F.S., *Only problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 67. See also F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 79; the Sequence 80 from this book defines in an analogous way *the exponents of power p sequences*. For a study of these sequences, see Atanassov, Krassimir T., *On some of the Smarandache's problems*, American Research Press, 1999, p. 38-49.

[137] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Problem 44.

(61) The Smarandache periodic sequences[139]

The subtraction periodic sequences[140]:

Definition: Let c be a positive integer; start with the positive integer n and let R(n) be its digital reverse. Put n_1 be the absolute value of the number (R(n) – c) and let $R(n_1)$ be its digital reverse and so on. It is obtained eventually a repetition.

Exemple: For c = 1 and n = 52 the sequence is: 52, 24, 41, 13, 30, 02, 19, 90, 08, 79, 96, 68, 85, 57, 74, 46, 63, 35, 52, …

Comment: In the example above the repetition occurs after 18 steps, and the length of the repeating cycle is 18.

The multiplication periodic sequences[141]:

Definition: Let c > 1 be a positive integer; start with the positive integer n, multiply each digit x of n by c and replace that digit by the last digit of c*x to give n_1 and so on. It is obtained eventually a repetition.

Exemple: For c = 7 and n = 68 the sequence is: 68, 26, 42, 8468, …

Comment: Integers with digits that are all equal to 5 are invariant under the given operation after one iteration.

The mixed composition periodic sequences[142]:

Definition: Let n be a two-digit number; add the digits and, if the sum is greater than 10, add them again; also take the absolute value of their difference: these are the first and second digits of n_1; repeat the operation.

Exemple: For n = 75 the sequence is: 75, 32, 51, 64, 12, 31, 42, 62, 84, 34, 71, 86, 52, 73, 14, 53, 82, 16, 75, …

Comment: There are no invariants in this case.

The two-digit periodic sequence[143]:

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[138] For a study of primality of the terms of the *Smarandache unary sequence* see Le, Maohua and Wu, Kejian, *A note on the primes in Smarandache power product sequences*, Smarandache Notions Journal, vol. 9, no. 1-2-3, 1998.

[139] F.S. defines (see *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 238) the *general periodic sequence* as follows: let S be a finite set, and f a function defined for all elements of S with values in S; then the general term a(n) of this sequence is defined as: a(1) = f(s), where s is an element of S; a(2) = f(a(1)) = f(f(s)) and so on. F.S. noted that there will always be a periodic sequence whenever is repetead the composition of the function f with itsealf more times than card(S), accordingly to the box principle of Dirichlet. See Ibstedt, Henry, *Smarandache continued fractions*, in Seleacu, V., Bălăcenoiu, I. (editors), *Smarandache Notions (Book series)*, vol. 10, American Research Press, 1999, for a study about the relation between Smarandache periodic sequences, Smarandache continued fractions and quadratic equations.

[140] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Theorem 31. For a study of this sequence, see Ibstedt, Henry, *Computer analysis of number sequences*, American Research Press, 1998, Chapter IV: *Periodic sequences*, Section 4: *The Smarandache subtraction periodic sequence*.

[141] For a study of this sequence, see Ibstedt, Henry, *Computer analysis of number sequences*, American Research Press, 1998, Chapter IV: *Periodic sequences*, Section 5: *The Smarandache multiplication periodic sequence*.

[142] For a study of this sequence, see Ibstedt, Henry, *Computer analysis of number sequences*, American Research Press, 1998, Chapter IV: *Periodic sequences*, Section 6: *The Smarandache mixed composition periodic sequence*.

[143] F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequences 239-244 (F.S. also defines here the *n-digit periodic sequence* and studies the 3-digit, 4-digit, 5-digit and 6-digit periodic sequences). For a study of this sequence, see Ibstedt, Henry, *Computer analysis of number sequences*,

Definition: Let n_1 be an integer of at most two digits and $R(n_1)$ its digital reverse; its's defined n_2 as the absolute value of the number $n_1 - R(n_1)$, n_3 as the absolute value of the number $n_2 - R(n_2)$ and so on; if the number n has one digit only, is considered its reverse as n*10 (for example 5, which is 05, reversed will be 50).

Comment: This sequence is periodic, except the case when the two digits are equal. The iteration always produces a loop of length 5, which starts on the second or the third term of the sequence, and the period is 9, 81, 63, 27, 45 or a cyclic permutation thereof.

(62) The Smarandache pseudo-primes sequences[144]

The pseudo-primes of first kind sequence:

Definition: A number is pseudo-prime of first kind if some permutation of its digits is a prime number, including the identity permutation.

The first fifteen terms of the sequence (A007933 in OEIS):

2, 3, 5, 7, 11, 13, 14, 16, 17, 19, 20, 23, 29, 30, 31.

The pseudo-primes of second kind sequence:

Definition: A number is pseudo-prime of second kind if is composite and some permutation of its digits is a prime number.

The first fifteen terms of the sequence (A007935 in OEIS):

14, 16, 20, 30, 32, 34, 35, 38, 50, 70, 74, 76, 91, 92, 95.

The pseudo-primes of third kind sequence:

Definition: A number is pseudo-prime of third kind if its reversal, when leading zeros are omitted, is prime.

The first fifteen terms of the sequence (A095179 in OEIS):

14, 16, 20, 30, 32, 34, 35, 38, 50, 70, 74, 76, 91, 92, 95.

Note: F.S. conjectured that there exist infinite many pseudo-primes of third kind which are primes.

(63) The Smarandache pseudo-squares sequences[145]:

The pseudo-square of first kind sequence

Definition: A number is pseudo-square of first kind if some permutation of its digits is a perfect square, including the identity permutation.

The first fifteen terms of the sequence (A007936 in OEIS):

1, 4, 9, 10, 16, 18, 25, 36, 40, 46, 49, 52, 61, 63, 64.

The pseudo-square of second kind sequence

Definition: A number is pseudo-square of second kind if is composite and some permutation of its digits is a perfect square.

The first fifteen terms of the sequence (A007937 in OEIS):

10, 18, 40, 46, 52, 61, 63, 90, 94, 106, 108, 112, 136, 148, 160.

The pseudo-square of third kind sequence

American Research Press, 1998, Chapter IV: *Periodic sequences*, Section 2: *The two-digit Smarandache periodic sequence*, and Section 3: *The Smarandache n-digit periodic sequence*.

[144] F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequences 81-83.

[145] F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequences 86-88. The Sequences 89-91 from this book define, in an analogous way, the pseudo-cubes of first, second and third kind and the Sequences 92-94 defines the pseudo-m-powers of the first, second and third kind.

Definition: A number is pseudo-square of third kind if some nontrivial permutation of its digits is a perfect square.

The first fifteen terms of the sequence (A007938 in OEIS):

10, 18, 40, 46, 52, 61, 63, 90, 94, 100, 106, 108, 112, 121, 136.

(64) The Smarandache pseudo-factorials sequences[146]:

The pseudo-factorials of first kind sequence

Definition: A number is pseudo-factorial of first kind if some permutation of its digits is a factorial number, including the identity permutation.

The first fifteen terms of the sequence (A007926 in OEIS):

1, 2, 6, 10, 20, 24, 42, 60, 100, 102, 120, 200, 201, 204, 207.

The pseudo-factrorials of second kind sequence

Definition: A number is pseudo-factorial of second kind if is non-factorial and some permutation of its digits is a factorial number.

The first fifteen terms of the sequence:

10, 20, 42, 60, 100, 102, 200, 201, 204, 207, 210, 240, 270, 402, 420.

The pseudo-factorials of third kind sequence:

Definition: A number is pseudo-factorial of third kind if some nontrivial permutation of its digits is a factorial number.

The first fifteen terms of the sequence (A007927 in OEIS):

10, 20, 42, 60, 100, 102, 200, 201, 204, 207, 210, 240, 270, 402, 420.

Conjecture:

F.S. conjectured that there are no pseudo-factorials of third kind to be also factorial numbers, which means that the pseudo-factorils of the second kind set and the pseudo-factorials of the third kind set coincide.

(65) The Smarandache pseudo-divisors sequences[147]:

The pseudo-divisors of first kind sequence

Definition: A number is a pseudo-divisor of first kind of n if some permutation of its digits is a divisor of n, including the identity permutation.

The first fifteen terms of the sequence:

1, 10, 100, 1, 2, 10, 20, 100, 200, 1, 3, 10, 30, 100, 300.

The pseudo-divisors of second kind sequence

Definition: A number is pseudo-divisor of second kind of n if is a non-divisor of n and some permutation of its digits is a divisor of n.

The first fifteen terms of the sequence:

10, 100, 10, 20, 100, 200, 10, 30, 100, 300, 10, 20, 40, 100, 200.

The pseudo-divisors of third kind sequence:

Definition: A number is a pseudo-divisor of third kind of n if some nontrivial permutation of its digits is a divisor of n.

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[146] F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequences 95-97.

[147] F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequences 98-100. The Sequences 101-103 from the same book define the pseudo-odd numbers of the first kind (some permutation of digits is odd number, including the identity permutation), of the second and of the third kind; the Sequence 104 defines the pseudo-triangular numbers (some permutation of digits is a triangular number); the Sequences 105-107 define the pseudo-even numbers of the first kind (some permutation of digits is even number, including the identity permutation), of the second and of the third kind. The Sequences 108-113 define in an analogous way the pseudo-multiples of first, second and third kind.

The first fifteen terms of the sequence:

10, 100, 10, 20, 100, 200, 10, 30, 100, 300, 10, 20, 40, 100, 200.

Properties:

Any integer has an infinity of pseudo-divisors of first kind and of the third kind because 1 divides any integer.

(66) The Smarandache almost primes sequences[148]:

The almost primes of first kind sequence

Definition: Let $a(1) \geq 2$ and, for $n \geq 1$, $a(n+1)$ is the smallest number that is not divisible by any of the previous terms of the sequence $a(1), a(2), \ldots, a(n)$.

Example for a(1) = 10:

10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 23, 25, 27, 29, 31, 35, 37, 41, …

Comment: If one starts by a(1) = 2 it obtains the complete prime sequence and only it. If one starts by a(1) > 2, it obtains after a rank r, where a(r) = p(a(1))^2, with p(x) the strictly superior prime part of x, *i.e.* the largest prime strictly less than x, the prime sequence: between a(1) and a(r), the sequence contains all prime numbers of this interval and some composite numbers; from a(r+1) and up, the sequence contains all prime numbers greater than a(r) and no composite numbers.

The almost primes of second kind sequence

Definition: Let $a(1) \geq 2$ and, for $n \geq 1$, $a(n+1)$ is the smallest number that is coprime with all of the previous terms of the sequence $a(1), a(2), \ldots, a(n)$.

Example for a(1) = 10:

10, 11, 13, 17, 19, 21, 23, 29, 31, 37, 41, 43, 47, 53, 57, 61, 67, 71, 73, ...

Comment: This second kind sequence merges faster to the prime numbers than the first kind sequence.

(67) The square roots sequence[149]

Definition:

The general term of the sequence, $s_q(n)$, is the superior integer part of square root of n.

The first thirty terms of the sequence (A000196 in OEIS):

0, 1, 1, 1, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 4, 4, 4, 4, 4, 5, 5, 5, 5, 5.

Comment:

This sequence is the natural sequence, where each number is repetead 2*n + 1 times, because between n^2 (included) and (n + 1)^2 (excluded) there are (n + 1)^2 – n^2 different numbers.

(68) The cubical roots sequence[150]

Definition:

The general term of the sequence, $c_q(n)$, is the superior integer part of cubical root of n.

The first thirty terms of the sequence (A048766 in OEIS):

0, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3.

Comment:

[148] F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequences 84-85.

[149] F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 117.

[150] F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 118.

This sequence is the natural sequence, where each number is repetead 3*n^2 + 3*n + 1 times, because between n^3 (included) and (n + 1)^3 (excluded) there are (n + 1)^3 – n^3 different numbers.

(69) The m-power roots sequence[151]

Definition:

The general term of the sequence, $m_q(n)$, is the superior integer part of m-power root of n.

Comment:

This sequence is the natural sequence, where each number is repetead (n + 1)^m – n^m times.

(70) The no-prime-digit sequence[152]

Definition:

The terms of this sequence contain no digits which are primes.

The first thirty-five terms of the sequence (A019516 in OEIS):

0, 1, 4, 6, 8, 9, 10, 11, 1, 1, 14, 1, 16, 1, 18, 19, 0, 1, 4, 6, 8, 9, 0, 1, 4, 6, 8, 9, 40.

Comment:

F.S. raised the question if there is any number which occurs infinitely many times in this sequence (for instance 1, or 4, or 6, or 11). Igor Shparlinski showed that, if n has already occurred, then, for instance, n3, n33, n333 etc. gives infinitely many repetitions of the number.

(71) The no-square-digit sequence[153]

Definition:

The terms of this sequence contain no digits which are squares.

The first thirty terms of the sequence (A031976 in OEIS):

2, 3, 5, 6, 7, 8, 2, 3, 5, 6, 7, 8, 2, 2, 22, 23, 2, 25, 26, 27, 28, 2, 3, 3, 32, 33, 3, 35, 36, 37.

(72) The Smarandache prime-digital subsequence[154]

Definition:

The terms of this sequence are primes that contain only digits which are also primes.

The first twenty terms of the sequence (A019546 in OEIS):

2, 3, 5, 7, 23, 37, 53, 73, 223, 227, 233, 257, 277, 337, 353, 373, 523, 557, 577, 727.

Comments:

1. Charles Ashbacher[155] conjectured that this sequence is infinite. Henry Ibstedt proved that this conjecture is true.[156]

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[151] F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 119. For a study of this sequence, see Atanassov, Krassimir T., *On some of the Smarandache's problems*, American Research Press, 1999, p. 58-61.

[152] F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 219.

[153] F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 220.

[154] F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 235. This sequence is sometimes named with the acronym SPDS.

[155] Ashbacher, Charles, *Collection of problems on Smarandache notions*, Erhus University Press, 1996, Conjecture 5.

[156] Ibstedt, Henry, *Computer analysis of number sequences*, American Research Press, 1998, Chapter II: *Recursive integer sequences*, Serction 4: *The Smarandache prime-digital sub-sequence*.

2. C. Ashbacher[157] raised the question, which he related to the problem of infinity of the set SPDS, how many *repunit primes*[158] exist.
3. C. Ashbacher[159] also conjectured that the limit of the sequence SPDSN(n)/π(n) is 0 as n tends to infinity, where SPDSN(n) represents the number of elements of SPSD(n) not exceeding n and π(n) represents the number of primes not exceeding n.

(73) The Smarandache prime-partial-digital sequence[160]

Definition:

The sequence of prime numbers which admit a deconcatenation into a set of primes.

Exemple:

The number 241 belongs to this sequence because admits the deconcatenation into the set of numbers {2, 41} which are both primes.

The first twenty terms of the sequence (A019549 in OEIS):

23, 37, 53, 73, 113, 137, 173, 193, 197, 211, 223, 227, 229, 233, 241, 257, 271, 277, 283, 293.

Comments:

Charles Ashbacher conjectured that this sequence is infinite; because SPPDS includes SPDS, the proof that SPDS is infinite implies that SPPDS is also infinite, and Henry Ibstedt proved that SPDS is indeed infinite.[161]

(74) The square-partial-digital subsequence[162]

Definition:

The sequence of square integers which admit a deconcatenation into a set of square integers.

Exemple:

The number 256036 (= 506^2) belongs to this sequence because admits the deconcatenation into the set of numbers {256 (= 16^2), 0, 36 (= 6^2)}, which are all three perfect squares.

Comment:

Charles Ashbacher proved that SSPDS is infinite.[163]

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[157] Ashbacher, Charles, *Collection of problems on Smarandache notions*, Erhus University Press, 1996, Unsolved problem 3.

[158] Primes that contain only digit 1. That are only 5 such primes known, having 2, 19, 23, 317 respectively 1031 digits 1 (sequence A004023 in OEIS); a necessary but not sufficient condition for a repunit to be prime is that the number of its digits (of 1) to be prime.

[159] Ashbacher, Charles, *Collection of problems on Smarandache notions*, Erhus University Press, 1996, Unsolved problem 4. The conjecture was proved: see Shang, Songye; Su, Juanli, *On the Smarandache prime-digital subsequence sequences*, Scientia Magna, Dec 1, 2008.

[160] Ashbacher, Charles, *Collection of problems on Smarandache notions*, Erhus University Press, 1996, Definition 32. This sequence is sometimes named with the acronym SPPDS. This sequence could as well be defined as the sequence of primes formed by concatenating other primes and treated to the chapter regarding concatenated sequences, but from obvious reasons (is related with the previous treated sequence) we treated it here.

[161] For a study of this sequence, see Ibstedt, Henry, *Computer analysis of number sequences*, American Research Press, 1998, Chapter I: *Partition sequences*.

[162] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Definition 50. F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 234. This sequence is sometimes named with the acronym SSPDS.

Questions:

1. The number 441 belongs to SSPDS and its square 194481 also belongs to the SSPDS. Can another example of integers m, m^2, m^4, all belonging to SSPDS, be found?
2. It is relatively easy to find two consecutive squares in SSPDS, *e.g.* 144 (= 12^2) and 169 (= 13^2). Does the SSPDS contain three or more consecutive squares as well? What is the maximum length?

(75) The Erdős-Smarandache numbers sequence[164]

Definition:

The sequence of Erdős-Smarandache numbers which are defined in the following way: solutions of the diophantine equation P(n) = S(n), where P(n) is the largest prime factor which divides n, and S(n) is the Smarandache function.

The first twenty-five terms of the sequence:

2, 3, 5, 6, 7, 10, 11, 13, 14, 15, 17, 19, 20, 21, 22, 23, 26, 28, 29, 30, 31, 33, 34, 35, 37.

(76) The Goldbach-Smarandache table sequence[165]

Definition:

The general term of the sequence, t(n), is the largest even number such that any other even number not exceeding it is the sum of two of the first n odd primes.

The first twenty terms of the sequence (A007944 in OEIS):

6, 10, 14, 18, 26, 30, 38, 42, 42, 54, 62, 74, 74, 90, 90, 90, 108, 114, 114, 134.

Comments:

1. This sequence helps to better understand Goldbach's Conjecture[166]: if t(n) is unlimited, then the conjecture is true; if t(n) is constant after a certain rank, then the conjecture is false.
2. The sequence also gives how many times an even number is written as a sum of two odd primes, and in what combinations.

Problems[167]:

1. All of the values known from this sequence are congruent to 2 modulo 4. Is that true for every term in the sequence?
2. How many primes does it take to represent all even numbers less than 2*n as sums of two primes from that set?

(77) The Smarandache-Vinogradov table sequence[168]

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[163] For a study of this sequence, see Ibstedt, Henry, *Computer analysis of number sequences*, American Research Press, 1998, Chapter I: *Partition sequences*. Here is also a study of the Smarandache cube-partial-digital subsequence.

[164] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Problem 44.

[165] F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 121.

[166] Which states that every number that is greater than 2 is the sum of three primes; note that Golbach considered the number 1 to be a prime – the majority of mathematicians from today don't; note also that the conjecture is equivalent with the statement that all positive even integers greater than 4 can be expressed as the sum of two primes.

[167] Kashihara, K., *Comments and topics on Smarandache notions and problems*, Erhus University Press, 1996, Chapter 1: *Some comments and problems on Smarandache notions*, p. 20.

[168] F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 122.

Definition:

The general term of the sequence, v(n), is the largest odd number such that any odd number greater than or equal to 9 not exceeding it is the sum of three of the first n odd primes.

The first twenty terms of the sequence (A007962 in OEIS):

9, 15, 21, 29, 39, 47, 57, 65, 71, 93, 99, 115, 129, 137, 143, 149, 183, 189, 205, 219.

Comments:

1. This sequence helps to better understand Goldbach's Conjecture: if v(n) is unlimited, then the conjecture is true; if v(n) is constant after a certain rank, then the conjecture is false.
2. Vinogradov proved in 1937 that any sufficiently large odd number is a sum of three primes. Mathematicians J.R. Chen şi T.Z. Wang showed in 1989 that the number is enough to be greater than 10^43000.
3. The sequence also gives in how many different combinations an odd number is written as a sum of three odd primes, and in what combinations.
4. The general term of the sequence, v(n), is smaller than or equal to $3*p_n$, where p_n is the n-th odd prime.
5. The table is also generalized for the sum of m primes and how many times a number is written as a sum of m primes (m > 2).

Problems[169]:

1. Examine the congruence of the terms of this sequence and determine if there is a pattern.
2. How many primes are needed to represent all odd numbers smaller than 3*n as sums of three primes?

(78) The Smarandache-Vinogradov sequence[170]

Definition:

Let G = $\{g_1, g_2, \ldots g_k, \ldots\}$ be an ordered set of positive integers with a given property G. Then the corresponding G add-on sequence is defined through formula:

SG = $\{a_i: a_1 = g_1, a = a_k*10^{\wedge}(1 + \log_{10}(g_k)) + g_k, k \geq 1\}$.

Note: The sequence is deduced from the Smarandache-Vinogradov table.

(79) The Smarandache paradoxist numbers sequence[171]

Definition:

The sequence of numbers (called "Smarandache paradoxist numbers") which don't belong to any of the Smarandache defined sequences of numbers.

Dilemma:

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[169] Kashihara, K., *Comments and topics on Smarandache notions and problems*, Erhus University Press, 1996, Chapter 1: *Some comments and problems on Smarandache notions*, p. 20.

[170] F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 123. Kashihara defines different sequences under the names Goldbach-Smarandache and Vinogradov-Smarandache, respectivelly Smarandache-Goldbach and Smarandache-Vinogradov (note the different order in listing of the names): see Kashihara, K., *Comments and topics on Smarandache notions and problems*, Erhus University Press, 1996, p. 19-21.

[171] F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 124. See also in the same book sequences 125-126 which define *The non-Smarandache numbers* and *The paradox of Smarandache numbers*.

If a number k doesn't belong to any of the Smarandache defined numbers, then k is a Smarandache paradoxist number, therefore k belongs to a Smarandache defined sequence of numbers (because Smarandache paradoxist numbers is also in the same category) – contradiction. Is the Smarandache paradoxist number sequence empty?[172]

(80) Sequences involving the Smarandache function[173]

Definition 1:

Let $\{a_n\}$ be the sequence defined in the following way: $a_0 = 1$, $a_1 = 2$ and $a_{n+1} = a_{S(n)} + S(a_n)$ for $n > 1$, where $S(n)$ is the Smarandache function[174].

Question 1:

Are there infinitely many pairs of integers (m, n), with $m \neq n$, such that $a_m = a_n$?

Conjecture: There are infinitely many such pairs.

Question 2:

Is there a number M such that $a_n < M$ for all $n > 0$?

Theorem: There is no such number M.

Definition 2:

An A-sequence is an integer sequence $1 \leq a_1 < a_2 <\ldots$ such that no element a_i is the sum of a set of distinct elements of the sequence that does not contain a_i.

Question 3:

Is it possible to construct an A-sequence $a_1, a_2,\ldots$ such that $S(a_1), S(a_2),\ldots$ is also an A-sequence?

Theorem: There are infinitely many such A-sequences.

Question 4:

For how many values of k is there a set of numbers $n, n + 1, n + 2, n + 3,\ldots, n + k$ such that $S(n), S(n + 1),\ldots, S(n + k)$ is a complete system of residues modulo $k + 1$?

Conjecture: The numbers of such integers k is finite.

Theorem: There is no limit to the size of n where $a_1, a_2,\ldots, a_n$ is a complete system of residue systems modulo n and $S(a_1), S(a_2),\ldots, S(a_n)$ is also a complete system of residues modulo n.[175]

Theorem: If there is a sequence of primes $p_1, p_2,\ldots, p_k$ such that the primes are all in arithmetic progression, then $S(p_1), S(p_2),\ldots, S(p_k)$ is also in arithmetic progression.[176]

(81) The Smarandache perfect sequence[177]

Definition:

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172 Another paradox, about the natural numbers, is the "interesting numbers paradox": if exists a set of uninteresting natural numbers, than one is the smaller one from them, an enough quality to make this number interesting. For instance, the number 11630 was the smaller integer which didn't appear in any of the sequences from OEIS in june 2009 (see Nathaniel Johnston, *11630 is the first uninteresting number*); now, it appears in 6 sequences from OEIS, and another number is "the smallest uninteresting number".

173 Ashbacher, C., *Pluckings from the tree of Smarandache sequences and functions*, American Research Press, 1998, p. 15-41.

174 The Smarandache function is defined *infra*, Part Two, Chapter I, Section (1).

175 The proof is based on Dirichlet's Theorem: let $d > 2$ and $a \neq 0$ be two numbers relatively prime to each other. Then the sequence $a, a + d, a + 2*d, a + 3*d,\ldots$ contains an infinite numbers of primes.

176 The longest arithmetic progression of primes known to date has 26 terms.

177 Ibstedt, H., *Mainly natural numbers – a few elementary studies on Smarandache sequences and other number problems*, American Research Press, 2003, Chapter V: *The Smarandache partial perfect additive sequence*.

A Smarandache perfect fp sequence is defined in the following way: if f_p is a p-ary relation on $\{a_1, a_2, a3, \ldots\}$ and $f_p(a_i, a_{i+1}, a_{i+2}, \ldots, a_{i+p-1}) = f_p(a_j, a_{j+1}, a_{j+2}, \ldots, a_{j+p-1})$ for all a_i, a_j and all $p > 1$, then $\{a_n\}$ is called a Smarandache perfect f_p sequence.

Note:

If the defining relation is not satisfied for all a_i, a_j or all p then $\{a_n\}$ may qualify as a Smarandache partial perfect f_p sequence.

(82) The partial perfect additive sequence[178]

Definition:

A particular case of Smarandache partial perfect sequence, defined in the following way: $a_{2*k+1} = a_{k+1} - 1$, $a_{2*k+2} = a_{k+1} + 1$ for $k \geq 1$, with $a_1 = a_2 = 1$.

(83) The Smarandache A-sequence[179]

Definition:

An infinite Smarandache sequence a(n) of positive integers $1 \leq a(1) \leq a(2) \leq a(3) \leq \ldots$ is called an *A-sequence* if a(k) cannot be expressed as the sum of two or more distinct earlier terms of the sequence.

(84) The Smarandache B2-sequence

Definition:

An infinite Smarandache sequence b(n) of positive integers $1 \leq b(1) \leq b(2) \leq b(3) \leq \ldots$ is called an *B2-sequence* if all pairwise sums $b(i) + b(j)$, $i \leq j$, are distinct.

(85) The Smarandache C-sequence

Definition:

An infinite Smarandache sequence c(n) of positive integers $1 \leq c(1) \leq c(2) \leq c(3) \leq \ldots$ is said to be a *nonaveraging sequence* or a *C-sequence* if it contains no three terms in arithmetic progression. That is, $c(i) + c(j) \neq c(k)$ for any three distinct terms c(i), c(j) and c(k) forming the sequence.

(86) The Smarandache uniform sequences[180]

Definition:

Let n be an integer not equal to 0 and $d_1, d_2, \ldots, d_r$ digits in a base B (of course, $r < B$). Then the multiples of n, written with digits $d_1, d_2, \ldots, d_r$ only (but all r of them), in base B, increasingly ordered, are called the *Smarandache uniform sequence*.

Examples (in base 10):

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[178] Ibstedt, H., *Mainly natural numbers – a few elementary studies on Smarandache sequences and other number problems*, American Research Press, 2003, Chapter V: *The Smarandache partial perfect additive sequence*.

[179] The sequences treated in the sections (83)-(85) are defined by Felice Russo; see R., Felice, *A set of new Smarandache functions, sequences and conjectures in number theory*, American Research Press, 2000, Chapter II: *A set of new Smarandache-type notions in number theory*. The author presents in this book, Chapter III: *A set of new Smarandache sequences*, yet a lot of Smarandache type sequences: *Smarandache repetead digit sequence with 1-endpoints*, *Smarandache alternate consecutive and reverse sequence* etc.

[180] Smith, Sylvester, *A set of conjectures on Smarandache sequences*, Smarandache Notions Journal.

1. Multiples of 7 written with digit 1 only:
   111111, 111111, 111111, 111111, 111111, 111111, 111111, 111111, 111111…
2. Multiples of 7 written with digit 2 only:
   222222, 222222222222, 222222222222222222, 222222222222222222222222…
3. Multiples of 79365 written with digit 5 only:
   555555, 555555555555, 555555555555555555, 555555555555555555555555…

Note:

For some cases, the *Smarandache uniform sequence* may be empty (impossible): *e.g.* the multiples of 79365 written with digit 6 only (because any multiple will end in 0 or 5).

(87) The Smarandache operation sequences[181]

Definition:

Let E be an ordered set of elements, E = $\{e_1, e_2, \ldots\}$ and O a set of binary operations well-defined for these elements. Then: a_1 is an element of E and $a_{n+1} = \min\{e_1O_1e_2O_2\ldots O_ne_{n+1}\} > a_n$, for $n > 1$, where all O_i are operations belonging to O, is called the *Smarandache operation sequence*.

Example:

Let E be the natural numbers set and O be formed by the four arithmetic operations: addition, subtraction, multiplication and division. Then $a_1 = 1$ and $a_{n+1} = \min\{1O_12O_2\ldots O_{98}99\} > a_n$, for $n > 1$, where all O_i are elements of $\{+, -, *, /\}$, chosen in a convenient way.

(88) The repeatable reciprocal partition of unity sequence[182]

Definition:

For n > 0, the *Smarandache repeatable reciprocal partition of unity* for n, noted with the acronym SRRPS(n), is the set of all sets of n natural numbers such that the sum of the reciprocals is 1, algebraic formulated SRRPS(n) = {x: x = $(a_1, a_2, \ldots, a_n)$, where the sum from r = 1 to r = n of the numbers $1/a_r$ is equal to 1}.

Examples:

If we note with $f_{RP}(n)$ the order of the set SRRPS(n), we have:

1. SRRPS(1) = {(1)}, $f_{RP}(n) = 1$;
2. SRRPS(2) = {(2, 2)}, $f_{RP}(2) = 1$;
3. SRRPS(3) = {(3, 3, 3), (2, 3, 6), (2, 4, 4)}, $f_{RP}(3) = 3$;
4. SRRPS(4) = {(4, 4, 4, 4), (2, 4, 6, 12), (2, 3, 7, 42), (2, 4, 5, 20), (2, 6, 6, 6), (2, 4, 8, 8), (2, 3, 12, 12), (4, 4, 3, 6), (3, 3, 6, 6), (2, 3, 10, 15), (2, 3, 9, 18)}, $f_{RP}(4)$ = 14.

Theorem[183]:

Let m be a member of SRRPS(n), say m = a_k, from $(a_1, a_2, \ldots, a_n)$ and by definition the sum from k = 1 to k = n of the numbers $1/a_k$ is equal to 1. Then m contributes $[(\tau(m) + 1)/2]$ elements to SRRPS(n + 1), where $\tau(m)$ is the number of divisors of m.

(89) The distinct reciprocal partition of unity sequence[184]

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[181] Smith, Sylvester, *A set of conjectures on Smarandache sequences*, Smarandache Notions Journal.

[182] Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, Chapter 1: *Smarandache partition functions*, Section 1: *Smarandache partition sets, sequences and functions*.

[183] For the proof of the theorem see Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, p. 14-15.

Definition:

For n > 0, the *Smarandache distinct reciprocal partition of unity* set, noted with the acronym SDRPS(n), is the set SRRPS(n) where the element of each set of size n must be unique, algebraic formulated SDRPS(n) = {x: x = (a_1, a_2, …, a_n), where the sum from r = 1 to r = n of the numbers 1/a_r is equal to 1 and a = a < = > i = j}.

Examples:

If we note with f_{DP}(n) the order of the set SDRPS(n), we have:

1. SDRPS(3) = {(2, 3, 6)}, f_{DP}(3) = 1;
2. SDRPS(4) = {(2, 4, 6, 12), (2, 3, 7, 42), (2, 4, 5, 20), (2, 3, 10, 15), (2, 3, 9, 18)}, f_{DP}(4) = 5.

Definition:

The *Smarandache distinct reciprocal partition of unity sequence* is the sequence of numbers f_{DP}(n).

Theorem[185]:

The following inequality is true: f_{DP}(n) ≥ Σ + (n^2 – 5*n + 8)/2, where Σ is the sum from k = 3 to k = n – 1 of the numbers f_{DP}(k) and n > 3.

(90) The Smarandache Pascal derived sequences[186]

Definition:

Starting with any sequence S_b = {b_1, b_2, …}, called *the base sequence*, a Smarandache Pascal derived sequence S_d = {d_1, d_2, …} is defined as follows: $d_1 = b_1$, $d_2 = b_1 + b_2$, $d_3 = b_1 + 2*b_2 + b_3$, $d_4 = b_1 + 3*b_2 + 3*b_3 + b_4$, …

Examples:

1. Let S_b be the set of positive integers {1, 2, 3, 4, …}; then S_d = {1, 3, 8, 20, …}. Let T_n be the general term of the sequence:
   Properties:
   (i) $T_n = 4*(T_{n-1} – T_{n-2})$ for n > 2;
   (ii) T_n = (n + 1)*2^(n – 2).
2. Let S_b be the set of odd integers {1, 3, 5, 7, …}; then S_d = {1, 4, 12, 32, …}.
3. Let S_b be the set of Bell numbers[187] {1, 1, 2, 5, 15, 52, 203, …}. Then S_d is identically with S_b.[188]
4. Let S_b be the set of Fibonacci numbers {1, 1, 2, 3, 5, 8, 13, …}; then S_d = {1, 2, 5, 13, 34, 89, 233…}. Let S_d be the base sequence; then S_{dd} = {1, 3, 10, 35, 125, 450, 1625, 5875, 21250, …}.
   Property: $T_{2*n-1} \equiv T_{2*n} \equiv 0$ (mod 5^n).

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184 Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, Chapter 1: *Smarandache partition functions*, Section 1: *Smarandache partition sets, sequences and functions*.

185 For the proof of the theorem see Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, p. 12.

186 Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, Chapter 2: *Smarandache sequences*, Section 2: *Smarandache Pascal derived sequences*.

187 *Bell numbers*, named after mathematician Eric Temple Bell, are the natural numbers which satisfy the following relation of recurrence expressed with the binomial coefficients: B(0) = 1, B(1) = 1 and B(n + 1) is equal to the sum of the first n terms, each one multiplicated with C(n, k), where k takes values from 0 to n.

188 The Bell numbers sequence is identically with the Smarandache factor partitions sequence (SFP) for the squarefree numbers.

(91) The Smarandache sigma divisor prime sequence[189]

Definition:

The sequence of the primes p with the property that p divides the sum of all primes less than or equal to p.

The five known terms of the sequence (A007506 in OEIS):

2, 5, 71, 369119, 415074643.

Examples:

(i) The number 5 is an element of this sequence because 5 divides 2 + 3 + 5 = 10.

(ii) The number 71 is an element of this sequence because 71 divides 2 + 3 +…+ 67 + 71 = 639.

Note: There are not any other elements known (all the primes less than 10^12 were checked) beside these five ones.

Question:

Is this sequence infinite?

(92) The Smarandache smallest number with n divisors sequence[190]

Definition:

The sequence of numbers which are the smallest numbers with exactly n divisors.

The first twenty terms of the sequence (A005179 in OEIS):

1, 2, 4, 6, 16, 12, 64, 24, 36, 48, 1024, 60, 4096, 192, 144, 120, 65536, 180, 262144, 240.

Conjectures:

1. The $T_n + 1$ sequence contains infinitely many primes (where T_n is the general term of the *Smarandache smallest number with n divisors sequence*).
2. The number 7 is the only Mersenne prime in the sequence $T_n + 1$.
3. The $T_n + 1$ sequence contains infinitely many perfect squares.

(93) The Smarandache summable divisor pairs set[191]

Definition:

The set of ordered pairs [m, n] with the property that $\tau(m) + \tau(n) = \tau(m + n)$.

Examples of Smarandache summable divisor pairs:

[2, 10], [3, 5], [4, 256], [8, 22].

Conjectures:

1. There are infinitely many SSDPs.
2. For every integer m there exists an integer n such that [m, n] is a SSDP.

(94) The Smarandache integer part of x^n sequences[192]

Definition 1 :

The Smarandache integer part of π^n is the sequence of numbers:

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[189] Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, p. 87.

[190] Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, p. 87.

[191] Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, p. 88.

[192] Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, p. 88.

[π^1], [π^2], [π^3], …

The first thirteen terms of the sequence (001672 in OEIS):

1, 3, 9, 31, 97, 306, 961, 3020, 9488, 29809, 93648, 294204, 924269.

Definition 2 :

The Smarandache integer part of e^n is the sequence of numbers:

[e^1], [e^2], [e^3], …

The first thirteen terms of the sequence (000149 in OEIS):

1, 2, 7, 20, 54, 148, 403, 1096, 2980, 8103, 22026, 59874, 162754, 442413.

(95) The Smarandache sigma product of digits natural sequence [193]

Definition:

The n-th term of this sequence is defined as the sum of the products of the digits of all the numbers from 1 to n.

The first twenty terms of the sequence (061076 in OEIS):

1, 3, 6, 10, 15, 21, 28, 36, 45, 45, 46, 48, 51, 55, 60, 66, 73, 81, 90, 90.

Subsequence 1: *The Smarandache sigma product of digits odd sequence*:

The first twenty terms of the sequence (061077 in OEIS):

1, 4, 9, 16, 25, 26, 29, 34, 41, 50, 52, 58, 68, 82, 100, 103, 112, 127, 148, 175.

Subsequence 2: *The Smarandache sigma product of digits even sequence*:

The first twenty terms of the sequence (061078 in OEIS):

2, 6, 12, 20, 20, 22, 26, 32, 40, 40, 44, 52, 64, 80, 80, 86, 98, 116, 140, 140.

(96) The Smarandache least common multiple sequence [194]

Definition:

The n-th term of this sequence is the least common multiple of the natural numbers from 1 to n.

The first fifteen terms of the sequence (003418 in OEIS):

1, 1, 2, 6, 12, 60, 60, 420, 840, 2520, 2520, 27720, 27720, 360360, 360360.

(97) The Smarandache reverse auto correlated sequences [195]

Definition:

Let $\{a_1, a_2, \ldots\}$ be a sequence; then the n-th term b_n of the *Smarandache reverse auto correlated sequence* $\{b_1, b_2, \ldots\}$ is defined in the following way: b_n is the sum from k = 1 to k = n of the numbers a_n*a_{n-k+1}.

The first three terms of the sequence:

$b_1 = a_1$^2, $b_2 = 2*a_1*a_2$, $b_3 = a_2$^2 + $2*a_1*a_3$.

(98) The Smarandache forward reverse sum sequence[196]

[193] Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, p. 88-89.

[194] Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, p. 88-89.

[195] Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, p. 94.

[196] Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, p. 114.

Definition:

The n-th term of the sequence T_n is equal to $T_{n-1} + R(T_{n-1})$, where $R(T_{n-1})$ is the number formed reversing the digits of T_{n-1}.

The first fifteen terms of the sequence (A001127 in OEIS):

1, 2, 4, 8, 16, 77, 154, 605, 1111, 2222, 4444, 8888, 17776, 85547, 160105.

Conjectures:

1. There are infinitely many palindromes in this sequence.
2. The number 16 is the only square in this sequence.

(99) The Smarandache reverse multiple sequence[197]

Definition:

The sequence of numbers that are multiples of their reversals; palindromes and multiples of ten are considered trivial and are not included.

The first ten terms of the sequence (A031877 in OEIS):

8712, 9801, 87912, 98901, 879912, 989901, 8799912, 9899901, 87128712, 87999912.

Properties:

1. This sequence is infinite.
2. There are two families of numbers in this sequence, one derived from 8712 and one derived from 9801; each family is constructed by placing 9's in the middle.
3. The number formed by concatenation of two terms of this sequence derived from the same family is also a member of that family.

(100) The Smarandache symmetric perfect power sequences [198]

Definition:

The sequence of numbers that are simultaneously m-th power and palindromic.

Smarandache symmetric perfect square sequence:

{1, 4, 9, 121, 484, 14641, …}

Smarandache symmetric perfect cube sequence:

{1, 8, 343, 1331, …}

Theorem:

The Smarandache symmetric perfect m-th power sequence has infinitely many terms for m = 1, 2, 3 and 4.

Conjecture:

The Smarandache symmetric perfect m-th power sequence has infinitely many terms for all values of m.

(101) The Smarandache Fermat additive cubic sequence[199]

Definition:

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[197] Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, p. 114.

[198] Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, p. 124.

[199] Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, p. 127. According to the authors, "the name of Fermat is included in the description to relate it to the fact that though the sum of two cubes can not yield a third cube, the sum of more than two cubes can be a third cube (3^3 + 4^3 + 5^3 = 6^3)".

The terms of the sequence are the perfect cubes that have the property that the sum of the cubes of their digits is also a perfect cube.

The first four terms of the sequence (A061212 in OEIS):

1, 8, 474552, 27818127.

Examples:

(i) 474552 = 78^3 and 4^3 + 7^3 + 4^3 + 5^3 + 5^3 + 2^3 = 729 = 9^3.

(ii) 27818127 = 303^3 and the sum of cubes of digits equals 1728 =12^3.

Theorems:

1. The Smarandache Fermat additive cubic sequence contains an infinite number of terms.
2. The number (10^(n + 2) – 4)^3 is a member of the Smarandache Fermat additive cubic sequence when n can be expressed in the form 4*((10^(3*k – 1)/27) – 1, where k positive integer. The sum of the cubes of the digits will then equal (6*10^k)^3.

(102) The Smarandache patterned sequences

Definition:

Sequences of numbers which follow a certain pattern, obtained through a certain arithmetic operation from a *root sequence* of numbers which also follow a certain pattern.[200]

Smarandache patterned perfect square sequences[201]:

b_n = 169, 17689, 1776889, 177768889, … is obtained from the root sequence:

a_n = 13, 133, 1333, 13333, … ($b_n = a_n$^2);

b_n = 1156, 111556, 11115556, 1111155556, … is obtained from the root sequence:

a_n = 34, 334, 3334, 3334, … ($b_n = a_n$^2);

Smarandache patterned perfect cube sequences[202]:

b_n = 1003303631331, 1000330036301331, 1000033000363001331, … is obtained from:

a_n = 10011, 100011, 1000011, 1000011,… ($b_n = a_n$^3);

b_n = 912673, 991026973, 999100269973, 999910002699973, … is obtained from:

a_n = 97, 997, 9997, 99997,… ($b_n = a_n$^3).

Smarandache patterned fourth power sequences[203]:

b_n = 96059601, 996005996001, 9996000599960001, … is obtained from:

a_n = 99, 999, 9999, 99999,… ($b_n = a_n$^4).

(103) The Smarandache prime generator sequence[204]

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[200] Our definition is vaque; for the original definition of these sequences see Murthy, Amarnath, *Exploring some new ideas on Smarandache type sets, functions and sequences*, Smarandache Notions Journal, vol. 11, no. 1-2-3, 2000.

[201] Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, p. 110. Obviously the sequences presented here are just few examples. In the cited book, the authors give more examples and formulas for the general term of each from these sequences.

[202] Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, p. 121.

[203] Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, p. 140. The authors give examples for *Smarandache patterned fifth power sequences*, *Smarandache patterned sixth power sequences*, *Smarandache patterned seventh power sequences* and *Smarandache patterned eighth power sequences* also, together with formulas for the general term and open problems.

[204] Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, p. 143.

Note:

Are considered the recursive sequences of numbers formed in the following way: T_1 is a prime and $T_{n+1} = k*T_n + 1$, where k is the smallest number yielding a prime.[205]

Examples:

1. For $T_1 = 2$, the sequence is (A061092 in OEIS):
   2, 3, 7, 29, 59, 709, 2837, 22697, 590123, 1180247, …
2. For $T_1 = 5$, the sequence is (A059411 in OEIS):
   5, 11, 23, 47, 283, 1699, 20389, 244669, 7340071, …

Definition:

Starting with the first prime, 2, the first prime not included in the sequence which starts with $T_1 = 2$ is 5. Then starting with 5, the first prime not included in the sequence which starts with $T_1 = 5$ is 13. Then starting with 13 the process is repeated. The Smarandache prime generator sequence is constructed using the first terms of these sequences.

The first twenty terms of the Smarandache prime generator sequence (A061303 in OEIS):

2, 5, 13, 17, 19, 31, 37, 41, 43, 61, 67, 71, 73, 79, 89, 97, 101, 109, 113, 127.

Conjecture:

The Smarandache prime generator sequence is finite.

(104) The Smarandache LCM ratio sequences

Definition:

Let $lcm(x_1, x_2, …, x_t)$ denote the least common multiple of positive integers $x_1, x_2, …, x_t$. Let r be a positive integer, r > 1. For any positive integer n, let T(r, n) = lcm(n, n + 1,…, n + r – 1)/lcm(1, 2, …, r), then the sequences SLR(r) = {T(r, n)} is called Smarandache LCM ratio sequences of degree r.

Theorems[206]:

1. T(2, n) = n*(n + 1)/2;
2. T(3, n) = n*(n + 1)*(n + 2)/6 if n is odd and
   T(3, n) = n*(n + 1)*(n + 2)/12 if n is even;
3. T(4, n) = n*(n + 1)*(n + 2)*(n + 3)/24 if n is not congruent to 0(mod 3) and
   T(4, n) = n*(n + 1)*(n + 2)*(n + 3)/72 if n is congruent to 0(mod 3);
4. T(6, n) = n*(n + 1)*…*(n + 5)/7200 if n ≡ 0, 15 mod 20;
   T(6, n) = n*(n + 1)*…*(n + 5)/720 if n ≡ 1, 2, 6, 9, 13, 14, 17, 18 mod 20;
   T(6, n) = n*(n + 1)*…*(n + 5)/3600 if n ≡ 5, 10 mod 20 and
   T(6, n) = n*(n + 1)*…*(n + 5)/1440 if n ≡ 3, 4, 7, 8, 11, 12, 16, 19 mod 20.

[205] It has been proved that for every prime p there is a prime of the form k*p + 1.

[206] For the proof of the Theorems 1-3 see Le, Maohua, *Two formulas for Smarandache LCM ratio sequences*, Smarandache Notions Journal, vol. 14, 2004; for the proof of theorem 4 and other theorems see Ting, Wang, *Two formulas for Smarandache LCM ratio sequences*, Scientia Magna, vol. 1, no. 1, 2005.

PART TWO
Smarandache type functions and constants

Chapter I. Smarandache type functions

(1) The Smarandache function[207]

Definition:

The function S(n) defined on the set of positive integers with values in the set of positive integers with the property that S(n) is the smallest number so that S(n)! is divisible by n.[208]

Example:

S(8) = 4 because 1!, 2!, 3! Are not divisible by 8 but 4! is divisible by 8.

Definition:

The numbers generated by this function are called Smarandache numbers.

The first thirty Smarandache numbers (A002034 in OEIS)[209]:

1, 2, 3, 4, 5, 3, 7, 4, 6, 5, 11, 4, 13, 7, 5, 6, 17, 6, 19, 5, 7, 11, 23, 4, 10, 13, 9, 7, 29, 5.

Properties:

1. Max {p: p prime and p divides n} $\leq$ S(n) $\leq$ n for any positive integer n.
2. S(m*n) does not always equal S(m)*S(n): the Smarandache function is not multiplicative.

Theorems[210]:

1. *A characterization of a prime number*: Let p be an integer greater than 4. Then p is prime if and only if S(p) = p.
2. *A formula to calculate the number of primes less than or equal to n*: If n is an integer, n $\geq$ 4, then π(n), the number of prime numbers less than or equal to n, is equal to one less than the sum, from k = 2 to k = n, of the numbers m, where m is the smallest integer greater than or equal to S(k)/k.
3. If p and q are distinct primes, then S(p*q) = max{p, q}.
4. Let n = p_1*p_2*…*p_k, where all p_i are distinct primes; then S(n) = max{p_1, p_2, …, p_k}.
5. If p is prime, then S(p^2) = 2*p.
6. If p is prime, then S(p^k) = n*p, where n $\leq$ k.
7. Let p be an arbitrary prime and n $\geq$ 1. Then, it is possible to find a number k such that S(p^k) = n*p.
8. For any integer n $\geq$ 0, it is possible to find another integer m such that S(m) = n!.

[207] The research paper presenting for the first time this function, *A function in the number theory* , was published by Florentin Smarandache in 1980. Since then, hundreds of articles have been written about the properties of the Smarandache function. For a history of this function see Dumitrescu, Constantin, *A brief history of the Smarandache function*, Smarandache Function Journal, vol. 2-3, 1993. See also Ashbacher, Charles, *An introduction to the Smarandache function*, Erhus University Press, 1995.

[208] See also *supra*, Part One, Chapter II, Section (1): *The Smarandache Quotient sequence*.

[209] For a computer algorithm for the calculation of S(n), see Ibstedt, Henry, *Computer analysis of number sequences*, American Research Press, 1998, Chapter III: *Non-recursive sequences*, Section 2: *The Smarandache function S(n)*.

[210] For the proof of the Theorems 1-2, see Ruiz, S.M. and Perez, M., *Properties and problems related to the Smarandache type functions*, Arxiv. For the proof of Theorems 3-11, see Ashbacher, Charles, *An introduction to the Smarandache function*, Erhus University Press, S(p^k) 1995, p. 8-14, 30-32.

9. Let $S^k(n)$ be used to represent k iterations of the function S, *i.e.* S(S(…S(n)…)). Then: if n = 1, $S^k(n)$ is undefined for k > 1; if n > 1, $S^k(n)$ = m, where m is 4 or prime, for all k sufficiently large.
10. There is no number k such that, for every number n > 1, $S^k(n)$ = m, where m is a fixed point of the function S.
11. If m > 2 is a fixed point of the function S, then there are infinitely many n such that $S^k(n)$ = m.

Problems[211]:

1. Study the Dirichlet series: sum from n = 1 to n = ∞ of the numbers S(n)/n^s.
2. Let OS(n) be the number of integers 1 ≤ k ≤ n such that S(k) is odd and ES(n) be the number of integers 1 ≤ k ≤ n such that S(k) is even. Determine the limit when n tends to ∞ of the number OS(n)/ES(n).

Definition:

The series defined as the sum, for n ≥ 2, of the numbers 1/S(n)^m are called Smarandache harmonic series.[212]

(2) The Smarandache double factorial function

Definition:

The function Sdf(n) defined on the set of positive integers with values in the set of positive integers with the property that Sdf(n) is the smallest number so that Sdf(n)!! is divisible by n.

The first twenty-five values of the function Sdf(n):

1, 2, 3, 4, 5, 6, 7, 4, 9, 10, 11, 6, 13, 14, 5, 6, 17, 12, 19, 10, 7, 22, 23, 6, 15.

Theorems[213]:

1. Sdf(p) = p, where p is any prime number.
2. For any even squarefree number n, Sdf(n) = 2*max{p_1, p_2 , …, p_k}, where p_1, p_2 , …, p_k are the prime factors of n.
3. For any composite squarefree odd number n, Sdf(n) = max{p_1, p_2 , …, p_k}, where p_1, p_2 , …, p_k are the prime factors of n.
4. The series defined as the sum from n = 1 to n = ∞ of the numbers 1/Sdf(n) diverges.
5. The series defined as the sum from n = 1 to n = ∞ of the numbers Sdf(n)/n diverges.
6. The Sdf function is not additive, that is Sdf(n + m) ≠ Sdf(m) + Sdf(n) for gcd(m, n) = 1.
7. The Sdf function is not multiplicative, that is Sdf(n*m) ≠ Sdf(m)*Sdf(n) for gcd(m, n) = 1.
8. Sdf(n) ≤ n.
9. Sdf(n) ≥ 1 for n ≥ 1.
10. 0 ≤ Sdf(n)/n ≤ 1 for n ≥ 1.

Problem:

Given any n ≥ 1, how many times does n appear in this sequence?

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[211] Kashihara, K., *Comments and topics on Smarandache notions and problems*, Erhus University Press, 1996, p. 15.

[212] For a study of these series see Tabirca, Sabin and Tabirca, Tatiana, *The convergence of Smarandache harmonic series*, Smarandache Notions Journal, vol. 9, no. 1-2-3, 1998 and also Luca, Florian, *On the divergence of the Smarandache harmonic series*, Smarandache Notions Journal, vol. 10, no. 1-2-3, 1999.

[213] R., Felice, *A set of new Smarandache functions, sequences and conjectures in number theory*, American Research Press, 2000, Chapter IV: *An introduction to the Smarandache double factorial function*.

(3) The Smarandache near-to-primorial function[214]

Definition:

The function Sntp(n) defined on the set of positive integers with values in the set of primes with the property that Sntp(n) is the smallest prime such that either p# – 1, p# or p# + 1 is divisible by n.[215]

Note: Sntp(n) is undefined for squareful integers.

(4) The Smarandache-Kurepa function[216]

Definition[217]:

The function SK(p) defined on the set of primes with values in the set of positive integers with the property that SK(p) is the smallest number so that !SK(p) is divisible by p, where !SK(p) = 0! + 1! + 2! + … + (p – 1)!.

The first twenty values of the function Smarandache-Kurepa (A049041 in OEIS):

2, 4, 6, 6, 5, 7, 7, 12, 22, 16, 55, 54, 42, 24, 25, 86, 97, 133, 64, 94, 72.

(5) The Smarandache-Wagstaff function[218]

Definition[219]:

The function SW(p) defined on the set of primes with values in the set of positive integers with the property that SW(p) is the smallest number so that W(SW(p)) is divisible by p, where W(p) = 1! + 2! + … + p!.

The first twenty values of the function Smarandache-Wagstaff:

2, 4, 5, 12, 19, 24, 32, 19, 20, 20, 7, 57, 6, 83, 15, 33, 38, 9, 23, 70.

(6) The Smarandache ceil functions of n-th order

Definition:

The function $S_k(n)$ defined on the set of positive integers with values in the set of positive integers with the property that $S_k(n)$ is the smallest number so that $S_k(n)$^k is divisible by n.[220]

The first fifteen values of the Smarandache ceil function of the second order $S_2(n)$:

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[214] For the properties of this function see Mudge, M.R., *The Smarandache near-to-primorial (SNTP) function*, and Asbacher, Charles, *A note on the Smarandache near-to-primorial function*, both articles in Smarandache Notions Journal, vol. 7, no. 1-2-3, 1996.

[215] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Problem 8.

[216] See Mudge, M.R., *Introducing the Smarandache-Kurepa and the Smarandache-Wagstaff functions*, Smarandache Notions Journal, vol. 7, no. 1-2-3, 1996.

[217] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Problem 4.

[218] See Mudge, M.R., *Introducing the Smarandache-Kurepa and the Smarandache-Wagstaff functions*, Smarandache Notions Journal, vol. 7, no. 1-2-3, 1996.

[219] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Problem 5.

[220] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Problem 6. See also Ibstedt, H., *Surphing on the ocean of numbers – a few Smarandache notions and similar topics*, Erhus University Press, Vail, 1997, Chapter II: *On Smarandache functions*, Section 3: *The Smarandache ceil function*.

2, 4, 3, 6, 10, 12, 5, 9, 14, 8, 6, 20, 22, 15, 12.

The first fifteen values of the Smarandache ceil function of the third order $S_3(n)$:

2, 2, 3, 6, 4, 6, 10, 6, 5, 3, 14, 4, 6, 10, 22.[221]

(7) The Smarandache primitive functions

Definition[222]:

The function $S_p(n)$ defined on the set of positive integers with values in the set of positive integers with the property that $S_p(n)!$ is the smallest number so that $S_p(n)!$ is divisible by p^n, where p is prime.[223]

Example:

$S_3(4)$ = 9, because 9! is divisible by 3^4, and this is the smallest number with this property .

Note: These functions help us compute Smarandache function.

(8) The Smarandache functions of the first kind

Definition[224]:

The functions S_n defined on the set of positive integers with values in the set of positive integers in the following way:

(i) if n = u^r (whith u = 1 or u = p being a prime number), then $S_n(a)$ = k, where k is the smallest positive integer such that k! is a multiple of u^(r*a);

(ii) if n = p(1)^r(1)*p(1)^r(2)*…*p(t)^r(t), then $S_n(a)$ = max $\{S_{p(j)\text{^}r(j)}(a)\}$, where 1 ≤ j ≤ t.

(9) The Smarandache functions of the second kind

Definition[225]:

The functions S^k defined on the set of positive integers with values in the set of positive integers in the following way: $S^k(n) = S_n(k)$ for k positive integer, where S_n are the Smarandache functions of the first kind.

(10) The Smarandache functions of the third kind

Definition[226]:

The functions $S_{a(n)}(b(n))$, where $S_{a(n)}$ is the Smarandache function of the first kind and the sequences a(n) and b(n) are different from the following situations:

(i) a(n) = 1 and b(n) = n for n positive integer;

(ii) a(n) = n and b(n) = n for n positive integer.

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[221] For the first few values of the Smarandache ceil functions of fourth, fifth and sixth order see Begay, Anthony, *Smarandache ceil functions*, Smarandache Notions Journal.

[222] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Theorem 11.

[223] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Problem 6.

[224] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Theorem 12.

[225] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Theorem 13.

[226] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Theorem 14.

(11) The pseudo-Smarandache function

Definition[227]:

The function Z(n) defined on the set of positive integers with values in the set of positive integers with the property that Z(n) is the smallest number so that the number 1 + 2 + … + Z(n) is divisible by n.

The first thirty pseudo-Smarandache numbers (A011772 in OEIS):

1, 3, 2, 7, 4, 3, 6, 15, 8, 4, 10, 8, 12, 7, 5, 31, 16, 8, 18, 15, 6, 11, 22, 15, 24, 12, 26, 7, 28, 15, 30, 63, 11, 16, 14, 8, 36, 19, 12, 15.

Properties:

1. $Z(n) \geq 1$ for any n natural.
2. It is not always the case that $Z(n) < n$.
3. Z(m + n) does not always equal Z(m) + Z(n): the pseudo-Smarandache function is not additive.
4. Z(m*n) does not always equal Z(m)*Z(n): the pseudo-Smarandache function is not multiplicative.

Theorems[228]:

1. If p is a prime greater than 2, then Z(p) = p – 1.
2. If x equals any natural number, p equals a prime number greater than 2, and b equals p^x, then Z(b) = b – 1.
3. If x equals 2 to any natural power, then Z(x) = 2*x – 1.
4. Z(p^k) = p^k – 1 for any prime p greater than 2.
5. If n is composite, then Z(n) = max{Z(m): m divides n}.
6. The series defined as the sum from n = 1 to n = ∞ of the numbers 1/Z(n) is divergent.
7. Abs{Z(n + 1) – Z(n)} is unbounded.

9. Given any integer k, $k \geq 2$, the equation Z(k*n) = n has an infinite number of solutions.
10. Given any fixed integer k, $k \geq 2$, the equation k*Z(n) = n has an infinite number of solutions.
11. Given any integer k, $k \geq 2$, the equation Z(n + 1)/Z(n) = k has solutions.
12. The ratio Z(2*n)/Z(n) is unbounded above.

(12) The pseudo-Smarandache function of first kind

Definition[229]:

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[227] The pseudo-Smarandache function was defined by K. Kashihara (who mentioned that he thought to a function analogous with the Smarandache function, with similar definition but where multiplication is replaced by summation). See also Ibstedt, H., *Surphing on the ocean of numbers – a few Smarandache notions and similar topics*, Erhus University Press, Vail, 1997, Chapter II: *On Smarandache functions*, Section 4: *The Smarandache pseudo function Z(n)*.

[228] For the proof of theorems 1-6, see K. Kashihara, *Comments and topics on Smarandache notions and problems*, Erhus University Press, 1996, Chapter 2: *The pseudo-Smarandache function* and also Gorski, David, *The pseudo-Smarandache function*, Smarandache Notions Journal. For the proof of theorems 7-8 see Ashbacher, C., *Pluckings from the tree of Smarandache sequences and functions*, American Research Press, 1998, p. 55-56. For the proof of Theorems 9-10 see Majumdar, A.A.K., *Wandering in the world of Smarandache numbers*, InProQuest, 2010, Chapter 4: *The pseudo Smarandache function*, Section 4.4.: *Miscellaneous topics*.

[229]The pseudo-Smarandache functions of first and second kind were defined by A.S. Muktibodh and S.T. Rathod, *Pseudo-Smarandache functions of first and second kind*.

The function $Z_1(n)$ defined on the set of positive integers with values in the set of positive integers with the property that $Z_1(n)$ is the smallest number so that the number 1^2 + 2^2 + … + $Z_1(n)$^2 is divisible by n.

The first fifteen values of the function $Z_1(n)$:

1, 3, 4, 7, 2, 4, 3, 15, 13, 4, 5, 8, 6, 3, 4.

Properties:

1. $Z_1(n) = 1$ only if $n = 1$.
2. $Z_1(n) \geq 1$ for any n natural.
3. $Z_1(p) \leq p$ for p prime.
4. If $Z_1(p) = n$ and $p \neq 3$, then $p > n$.

Theorem[230]: If p is prime, $p \geq 5$, then $Z_1(p) = (p - 1)/2$.

(13) The pseudo-Smarandache function of second kind

Definition:

The function $Z_2(n)$ defined on the set of positive integers with values in the set of positive integers with the property that $Z_1(n)$ is the smallest number so that the number 1^3 + 2^3 + … + $Z_1(n)$^3 is divisible by n.

The first fifteen values of the function $Z_2(n)$:

1, 3, 2, 3, 4, 3, 6, 7, 2, 4, 10, 3, 12, 7, 5.

(14) The Smarandache multiplicative one function[231]

Definition:

The function f defined on the set of positive integers with values in the set of positive integers with the property that, for any a and b with gcd(a, b) = 1, f(a*b) = max{f(a), f(b)}, *i.e.* it reflects the main property of the Smarandache function.

Examples:

Few functions that are S-multiplicative: the Smarandache function defined as S(n) = min{k: n divides k!} and the Erdős function defined as f(n) = max{p: p prime, p divides n}.

(15) The inferior and the superior f–part of x[232]

Definition:

The strictly increasing functions f defined on the set of natural numbers with values in the set of natural numbers defined in the following way: if x is an element of the set of real numbers, then the *inferior f–part of x* is the smallest k such that $f(k) \leq x < f(k + 1)$ and the *superior f–part of x* is the smallest k such that $f(k) < x \leq f(k + 1)$.

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230 For the proof of this theorem, see Mukthibodh, A.S. and Rathod, S.T., *Pseudo-Smarandache functions of first and second kind*, p. 5.

231 F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Definition 2. This function is also met under the name *Smarandache-multiplicative function* or *S-multiplicative function*: see F.S., *Considerations on new functions in number theory*, Arxiv, F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 312, Tabirca, Sabin, *About Smarandache-multiplicative functions*, Smarandache Notions Journal, vol. 11, no. 1-2-3, 2000.

232 F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Definition 3. See also F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequences 36-37.

Note: Particular cases of this function are: inferior/superior prime part, inferior/superior square part, inferior/superior factorial part etc.[233]

(16) The inferior and the superior fractional f–part of x[234]

Definition:

The strictly increasing functions f defined on the set of natural numbers with values in the set of natural numbers defined in the following way: if x is an element of the set of real numbers, then the *inferior fractional f–part of x* is the number x – f(x), where f(x) is the *inferior f–part of x*, defined above, and the *superior fractional f–part of x* is the number f(x) – x, where f(x) is the *superior f–part of x*, defined above.

Note: Particular cases of this function are: fractional prime part, fractional square part, fractional cubic part, fractional factorial part etc.

(17) The Smarandache complementary functions

Definition[235]:

The strictly increasing function g defined on the set A with values in the set A defined in the following way: let "~" be a given internal law on A. Then we say that f, where f is a function also defined on the set A with values in set A, is complementary with respect to the function g and the internal law "~" if f(x) is the smallest k such that there exists z, where z belongs to the set A, so that $x \sim k = g(z)$.

Note: Particular cases of this function are: square complementary function, cubic complementary function, m-power complementary function, prime complementary function etc.[236]

(18) The functional Smarandache iteration of first kind

Definition[237]:

Let f be a function defined on the set A with values in the set A defined in the following way: $f(x) \leq x$ for all x and $\min\{f(x)\} \geq m_0 \neq -\infty$. Let f have $p \geq 1$ fix points $m_0 \leq x_1 \leq x_2 \leq \ldots \leq x_p$ [the point x is called fix is f(x) = x]. Then $SI1_f(x)$ is the smallest number of iterations k such that f(f(…f(x)…)), iterated k times, is constant.

Example:

Let n > 1 be an integer and τ(n) be the number of positive divisors of n. Then $SI1_\tau(n)$ is the smallest number of iterations k such that τ(τ(...τ(n)...)), iterated k times, is equal to 2, because τ(n) < n for n > 2 and the fix points of the function τ are 1 and 2. Thus $SI1_\tau(6) = 3$, because τ(τ(τ(6))) = τ(τ(4)) = τ(3) = 2 = constant.

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233 See *supra*, Part One, Chapter II, the homonymous sequences.

234 F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Definition 4. See also F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequences 46-51.

235 F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Definition 5.

236 See *supra*, Part One, Chapter II, the following sequences: *The square complements sequence*, *The cube complements sequence*, *The m-power complements sequence*, *The prime additive sequence*.

237 For the definitions of functional Smarandache iterations of the first, second and third kind see F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Definitions 6, 7, 8. See also Ruiz, S.M. and Perez, M., *Properties and problems related to the Smarandache type functions*, Arxiv.

(19) The functional Smarandache iteration of second kind

Definition:

Let g be a function defined on the set A with values in the set A such that $g(x) > x$ for all x and let $b > x$. Then $SI2_g(x,b)$ is the smallest number of iterations k such that $g(g(\ldots g(x)\ldots))$, iterated k times, is greater than or equal to b.

Example:

Let $n > 1$ be an integer and $\sigma(n)$ be the number of positive divisors of n. Then $SI2_\sigma(n,b)$ is the smallest number of iterations k such that $\sigma(\sigma(...\sigma(n)...))$, iterated k times, is greater than or equal to b, because $\sigma(n) > n$ for $n > 1$. Thus $SI2_\sigma(4,11) = 3$, because $\sigma(\sigma(\sigma(4))) = \sigma(\sigma(7)) = \sigma(8) = 15 \geq 11$.

(20) The functional Smarandache iteration of third kind

Definition:

Let h be a function with values in the set A such that $h(x) < x$ for all x and let $b < x$. Then $SI3_h(x,b)$ is the smallest number of iterations k such that $h(h(\ldots h(x)\ldots))$, iterated k times, is smaller than or equal to b.

Example:

Let n be an integer and gd(n) be the greatest positive divisor of n less than n. Then $gd(n) < n$ for $n > 1$. Thus $SI3_{gd}(60,3) = 4$, because $gd(gd(gd(gd(60)))) = gd(gd(gd(30))) = gd(gd(15)) = gd(5) = 1 \leq 3$.

(21) The Smarandache prime function[238]

Definition:

Let P be a function defined on the set of natural numbers with values in the set {0, 1}. Then $P(n) = 0$ if p is prime and $P(n) = 1$ otherwise.

Example:

$P(2) = P(3) = P(5) = P(7) = P(11) = \ldots = 0$ whereas $P(0) = P(1) = P(4) = P(6) = ... = 1$.

Generalization:

Let P_k, where $k \geq 2$, be a function defined on the set of natural numbers with values in the set {0,1}. Then $P_k(n_1, n_2, \ldots, n_k) = 0$ if $n_1, n_2, \ldots, n_k$ are all prime numbers and $P_k(n_1, n_2, \ldots, n_k) = 1$ otherwise.

(22) The Smarandache coprime function[239]

Definition:

[238] For a deeper study of this function and of the following one see Ruiz, S.M., *Applications of Smarandache functions, and prime and coprime functions*, American Research Press, 2002. For theorems on this function and on the following one, see Vassilev-Missana, Mladen and Atanassov, Krassimir, *Some Smarandache problems*, Hexis, 2004, Chapter 1: *On some Smarandache's problems*, Section 11: *On four prime and coprime functions*.

[239] This function and the previous one are met, in the paper F.S., *Considerations on new functions in number theory*, Arxiv, under the abreviations *S-prime function* and *S-coprime function*. In the book F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequences 285-286, these two functions are analogously defined but called *Anti-prime function* and *Anti-coprime function*. In the book *Collected Papers*, vol. II, Moldova State University, Kishinev, 1997, p. 137, they are simply called *Prime function* and *Anti-prime* function.

Let P_k, where $k \geq 2$, be a function defined on the set of natural numbers with values in the set {0,1}. Then $P_k(n_1, n_2,\ldots, n_k) = 0$ if $n_1, n_2,\ldots, n_k$ are coprime numbers and $P_k(n_1, n_2,\ldots, n_k) = 1$ otherwise.

(23) The smallest power function

Definition:

SP(n) is the smallest number m such that m^k is divizible by n, where $k \geq 2$ is given.

The first twenty values of the function SP(n) for k = 2 (sequence A019554 in OEIS):

1, 2, 3, 2, 5, 6, 7, 4, 3, 10, 11, 6, 13, 14, 15, 4, 17, 6, 19, 10.

Properties[240]:

1. If p is prime, then SP(p) = p.
2. If r is squarefree, then SP(r) = r.
3. If $(p_1\text{^}s_1)*\ldots*(p_k\text{^}s_k)$ and all $s_i \leq p_i$, then SP(n) = n.

(24) The residual function[241]

Definition:

Let L be a function defined on the set of integers with values in the set of integers. Then $L(x, m) = (x + C_1)\ldots(x + C_{F(m)})$, m = 2, 3, 4, …, where C_i, $1 \leq i \leq F(m)$, forms a reduced set of residues mod m, $m \geq 2$, x is an integer, and F is Euler's totient.

Example:

For x = 0 is obtained the following sequence *(A001783 in OEIS)*: $L(m) = C_1\ldots C_{F(m)}$, where m = 2, 3, 4, …:

1, 2, 3, 24, 5, 720, 105, 2240, 189, 3628800, 385, 479001600, 19305, 896896, 2027025...

Property[242]:

The following congruence is true: $(x + C_1)\ldots(x + C_{F(m)}) \equiv x\text{^}F(m) – 1 \pmod m$.

Comment:

The residual function is important because it generalizes the classical theorems by Wilson, Fermat, Euler, Wilson, Gauss, Lagrange, Leibnitz, Moser, and Sierpinski all together.

(25) The Smarandacheian complements[243]

Definition:

Let g be a strictly increasing function defined on the set A and let "~" be an internal given law on A. Then the function f defined on the set A with values in the set A is a smarandacheian complement with respect to the function g and the internal law "~" if f(x) is the smallest k such that there exist a z in A so that x ~ k = g(z).

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240 For more properties of this function, see F.S., *Considerations on new functions in number theory*, Arxiv.

241 F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 35. For lemmas and theorems regarding this function see F.S., *A numerical function in the congruence theory*, Arxiv.

242 For the proof of this property, see Kashihara, K., *Comments and topics on Smarandache notions and problems*, Erhus University Press, 1996, Chapter 1: *Some comments and problems on Smarandache notions*, p. 11.

243 F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 52. The Sequences 53-57 from the same book define the following notions: *The square complements sequence*; *The cube complements sequence*; *The m-power complements sequence*; *The double factorial complements sequence*; *The prime additive complements sequence*, which are treated *supra*, Part one, Chapter II.

(26) The increasing repetead compositions[244]

Definition:

Let g be a function defined on the set of natural numbers with values in the set of natural numbers, such that g(n) > n for all natural n. An increasing repetead composition related to g and a given positive number m is defined in the following way: the function F_g defined on the set of natural numbers with values in the set of natural numbers, $F_g(n) = k$, where k is the smallest integer such that g(…g(n)…) ≥ m (where g is composed k times).

Note:

F.S. suggest the study of F_s, where s is the function that associates to each positive integer n the sum of its positive divisors.

(27) The decreasing repetead compositions

Definition:

Let g be a non-constant function defined on the set of natural numbers with values in the set of natural numbers, such that g(n) ≤ n for all natural n. A decreasing repetead composition related to g is defined in the following way: the function F_s defined on the set of natural numbers with values in the set of natural numbers, $F_s(n) = k$, where k is the smallest integer such that g(…g(n)…) = constant (where g is composed k times).

Note:

F.S. suggest the study of F_d, where d is the function that associates to each positive integer n the number of its positive divisors. Same for π(n), the number of primes not exceeding n, p(n), the largest prime factor of n and ω(n), the number of distinct prime factors of n.

(28) The back and forth factorials (the Smarandacheials)[245]

Definition:

Let n > k ≥ 1 be two integers. Then *the Smarandacheial* is defined as $!n!_k = \Pi$, where Π is the product for 0 < abs{n – k*i} ≤ n of the numbers (n – k*i).

Example:

In the case k = 1 is obtained:
$!n!_1$ = n*(n – 1)*(n – 2)*…*1*(– 1)*(– 2)*…*(– n + 2)*(– n + 1)*(– n) = (– 1)^n*n!^2;
Thus !5! = 5*(5 – 1)*(5 – 2)*(5 – 3)*(5 – 4)*(5 – 6)*(5 – 7)*(5 – 8)*(5 – 9)*(5 – 10) = -14400.
The sequence is: 4, -36, 576, -14400, 518400, -25401600, 1625702400, -131681894400, 13168189440000, -1593350922240000, 229442532802560000 (…).

Notes:

In the case k = 2 is obtained $!n!_2$ = (-1)^((n + 1)/2)*(n!!)^2 for n odd and $!n!_2$ = (-1)^(n/2)*(n!!)^2 for n even. The sequence is: 9, 64, -225, -2304, 11025, 147456, -893025, -14745600, 108056025, 2123366400 (…).

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[244] F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 151.

[245] F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 290. See also *Back and forth factorials*, Arizona State University, Special Collections (article available on Vixra), where F.S. defines *the Smarandacheial* and *the generalized Smarandacheial*. See also F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequences 316-325, for the definition of *the back and forth summants*, a function related to *Smarandacheials* and Bencze, Mihály, *Smarandache summands*, Smarandache Notions Journal.

In the case k = 3 is obtained the sequence: -8, 40, 324, 280, -2240, -26244, -22400, 246400, 3779136, 3203200, -44844800 (…).
In the case k = 4 is obtained the sequence: -15, 144, 105, 1024, 945, -14400, -10395, -147456, -135135, 2822400, 2027025 (…).
In the case k = 5 is obtained the sequence: -24, -42, 336, 216, 2500, 2376, 4032, -52416, -33264, -562500, -532224, -891072, 16039296 (…).

(29) The Smarandache infinite products[246]

Definition:

Let a(n) be any from the Smarandache type sequences and functions. Then *the infinite product* is defined as the product from n = 1 to n = ∞ of the numbers 1/a(n).

Note: Many of these infinite products lead to interesting constants.

(30) The Smarandache-simple function[247]

Definition:

$S_p(n)$ = min{m: m natural, p^n divides m!), defined for fixed primes p.

Properties[248]:

1. For any prime p and any positive integer k, let $S_p(k)$ denote the smallest positive integer such that p^k divides $S_p(k)$! Then, for any p and k, p divides $S_p(k)$.
2. In the conditions mentioned in the above property, $k*(p – 1) < S_p(k) \le k*p$.

(31) The duals of few Smarandache type functions[249]

Definition:

József Sándor defined the *dual arithmetic functions* as follows: Leg g be a function defined on the set of positive integers with values in the set of non-null integers having the property that for each n ≥ 1 there exists at least a k ≥ 1 such that g(k) divides n.

A dual of Smarandache function:

Putting in the definition above g(k) = k! is obtained a dual of Smarandache function, denoted by S_*; then $S_*(n)$ = max{m: m natural, m! divides n).

A dual of pseudo-Smarandache function:

Putting in the definition above g(k) = k*(k + 1)/2 is obtained a dual of pseudo-Smarandache function, denoted by Z_*; then $Z_*(n)$ = max{m: m natural, m*(m + 1)/2 divides n).

A dual of Smarandache-simple function:

Is denoted by S_{p*} and $S_{p*}(n)$ = max{m: m natural, m! divides p^n).

A dual of Smarandache ceil function[250]:

Is denoted by S_{k*} and $S_{k*}(n)$ = max{m: m natural, m^k divides n).[251]

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[246] F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 314.
[247] Sándor, József, *On additive analogues of certain arithmetic functions*, Smarandache Notions Journal, vol. 14, 2004.
[248] Le, Maohua, *On Smarandache simple functions*, in Seleacu, V., Bălăcenoiu, I. (editors), *Smarandache Notions (Book series)*, vol. 10, American Research Press, 1999.
[249] Sándor, József, *On additive analogues of certain arithmetic functions*, Smarandache Notions Journal, vol. 14, 2004, Sándor, József, *On certain generalizations of the Smarandache function*, Smarandache Notions Journal, vol. 11, no. 1-2-3, 2000, Sándor, József, *On a dual of pseudo-Smarandache function*, Smarandache Notions Journal, vol. 13, no. 1-2-3, 2002.
[250] It was introduced by Lu Yaming, see *On a dual function of the Smarandache ceil function*, in Wenpeng, Zhang, *et al.* (editors), *Research on Smarandache problems in number theory (vol. 2)*, Hexis, 2005.

(32) Generalizations of Smarandache function

Note:

The Smarandache function is the well known function that gives a criterion for primality and is related with many other functions, *i.e.* the function S(n) defined on the set of positive integers with values in the set of positive integers with the property that S(n) is the smallest number so that S(n)! is divisible by n. Many mathematicians constructed analogously defined functions[252]:

Definition 1[253]:

Let f be an arithmetical function defined on the set of positive integers with values in the set of positive integers with the property that for each positive integer n there exist at least a positive integer k such that n divides f(k). Let F_f be a function defined on the set of positive integers with values in the set of positive integers with the property that $F_f(n)$ = min{k: k natural, n divides f(k)}. Since every subset of natural numbers is well ordered, is clearly that $F_f(n) \geq 1$ for all n positive integers.

Examples:

(i) Let id(k) = k for all $k \geq 1$. Then $F_{id}(n) = n$;

(ii) Let f(k) = k!. Then $F_!(n) = S(n)$, the Smarandache function;

(iii) Let f(k) = k*(k + 1)/2. Then $F_f(n) = Z(n)$, the pseudo-Smarandache function;

(iiii) Let f(k) = p_k!, where p_k denotes the k-th prime number. Then $F_f(n)$ = min{k: k positive integer, n divides p_k!}.

Note:

Analogously are defined the functions F_φ and $F\sigma$, where φ is the Euler's totient and σ the divisor function.

Definition 2:

Let A be a nonvoid set of the set of natural numbers, having the property that for each $n \geq 1$ there exists k belonging to A such that n divides k!. Then is introduced the following function: $S_A(n)$ = min{k: k belongs to A, n divides k!}.

Examples:

(i) Let A be equal to the set of positive integers; then $S_N(n) \equiv S(n)$, the Smarandache function;

(ii) Let A be equal to the set of odd positive integers; it's obtained a new Smarandache type function;

(iii) Let A be equal to the set of even positive integers; it's obtained a new Smarandache type function;

(iiii) Let A be equal to the set of prime numbers P; then $S_P(n)$ = min{k: k belongs to P, n divides k!}.[254]

(33) The Smarandache counter[255]

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[251] See *supra*, this chapter, Section (6) for the definition of *Smarandache ceil functions of n-th order*.

[252] We present here just few of them. See also Hungenbühler, Norbert and Specker, Ernst, *A generalization of the Smarandahe function to several variables*, Integers: Electronic Journal of Combinatorial Number Theory, 6(2006).

[253] See Sándor, József, *On certain generalizations of the Smarandache function*, Smarandache Notions Journal, vol. 11, no. 1-2-3, 2000, for the functions defined here by Definitions 1 and 2.

[254] For more about this function, see *infra*, Part four, Chapter I: *Theorems on the Smarandache type function P(n)*.

[255] Ashbacher, C., *Pluckings from the tree of Smarandache sequences and functions*, American Research Press, 1998, p. 8-9. See also Kashihara, K., *Comments and topics on Smarandache notions and problems*, Erhus University Press, 1996, p. 24.

Definition:

The Smarandache counter C(a, b), for any a decimal digit and b integer, is the number of times a appears as a digit in b.

Note:

F.S. raised the following question: what is the value of C(1, n!) and C(1, n^n)?

(34) The pseudoSmarandache totient function[256]

Definition:

Zt(n) is the smallest integer m such that the sum from k = 1 to k = m of the numbers φ(k) is divisible by n.

Theorems:

1. Zt(n) is not additive and not multiplicative.
2. Zt(n) > 1 for n > 1.
3. The series defined as the sum from n = 1 to n = ∞ of the numbers 1/Zt(n) diverges.
4. The series defined as the sum from n = 1 to n = ∞ of the numbers Zt(n)/n diverges.
5. The series defined as the sum from k = 1 to k = Zt(n) of the numbers φ(k) is greater than or equal to n.
6. Zt(n) is greater than or equal to floor(π*(n/3)^(1/2)), where the floor function floor(x) designates the largest integer smaller than or equal to x.
7. It is not always the case that Zt(n) < n.

(35) The pseudoSmarandache squarefree function[257]

Definition:

Zw(n) is the smallest integer m such that m^n is divisible by n, that is the value of m such that m^n/n is an integer.

Theorems:

1. Zw(p) = p, where p is any prime number.
2. Xw(p^a*q^b*s^c*…) = p*q*s*…, where p, q, s are distinct primes.
3. Zw(n) = n if and only if n is squarefree.
4. Zw(n) ≤ n.
5. Zw(n) ≥ 1 for n ≥ 1.
6. Zw(p^k) = p for k ≥ 1 and p any prime.
7. The series defined as the sum from n = 1 to n = ∞ of the numbers 1/Zw(n) diverges.
8. The series defined as the sum from n = 1 to n = ∞ of the numbers Zw(n)/n diverges.

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[256] This function, analogous to the pseudoSmarandache function, is defined by Felice Russo: see R., Felice, *A set of new Smarandache functions, sequences and conjectures in number theory*, American Research Press, 2000, Chapter I: *On some new Smarandache functions in number theory*, Section I.1.: *PseudoSmarandache totient function*.

[257] This function, analogous to the pseudoSmarandache function, is defined by Felice Russo: see R., Felice, *A set of new Smarandache functions, sequences and conjectures in number theory*, American Research Press, 2000, Chapter I: *On some new Smarandache functions in number theory*, Section I.2.: *PseudoSmarandache squarefree function*.

9. The function Zw(n) is multiplicative, that is if gcd(m, n) = 1 then Zw(m*n) = Zw(m)*Zw(n).
10. The function Zw(n) is not additive, that is that Zw(m + n) ≠ Zw(m) + Zw(n).

(36) The Smarandache Zeta function[258]

Definition:

Sz(s) is the sum from n = 1 to n = ∞ of the numbers 1/a(n)^s, where s natural.

(37) The Smarandache sequence density

Definition:

S_δ is the limit when n tends to ∞ of the number A(n)/n, where A(n) is the number of terms not exceeding n in a Smarandache sequence a(n), strictly increasing and composed of nonnegative integers.

(38) The Smarandache generating function

Definition:

Sf(x) is the sum of the numbers a(n)*x^n.

(39) The Smarandache totient function

Definition:

St(n) is equal to φ(a(n)), that is the number of positive integers smaller than or equal to a(n) which are relatively prime to a(n).

(40) The Smarandache divisor function

Definition:

Sd(n) is equal to τ(a(n)), that is the number of positive divisors of a(n), where a(n) is any Smarandache sequence.

(41) The additive analoque of few Smarandache functions[259]

Definition 1:

The *additive analogue of the Smarandache function* is defined as S(x) = min{m: m natural, x ≤ m!}, where x belongs to the set of real numbers, x > 1.

Definition 2:

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[258] The functions treated in the Sections (36)-(40) are defined by Felice Russo: see R., Felice, *A set of new Smarandache functions, sequences and conjectures in number theory*, American Research Press, 2000, Chapter II: *A set of new Smarandache-type notions in number theory*. Here, the author defines yet many other functions like *Smarandache continued radical*, *Smarandache Euler-Mascheroni sum*, *Smarandache-Chebyshev function*, *Smarandache Gaussian sum*, *Smarandache Dirichlet beta function*, *Smarandache Mobius function*, *Smarandache Mertens function*, *Smarandache Dirichlet eta function*, *Smarandache Dirichlet lambda function* etc.

[259] Sándor, József, *Geometric theorems, diophantine equations, and arithmetic functions*, American Research Press, 2002, p. 171. See also Sándor, József, *On an additive analoque of the function S*, Notes Number Th. Discr. Math. 7(2001), no. 3; Sándor, József, *On additive analoques of certain arithmetic functions*, Smarandache Notions Journal, vol. 14, 2004; Yuan, Yi and Wenpeng, Zhang, *Mean value of the additive analoque of Smarandache function*, Scientia Magna, vol. 1, no. 1, 2005.

The *additive analogue of the dual of the Smarandache function*[260] is defined as $S_*(x) = \max\{m: m \text{ natural}, m! \le x\}$, where x belongs to the set of real numbers, $x > 1$.

Properties:

1. $S(x) = S_*(x) + 1$, if $k! < x < (k + 1)!$, where $k \ge 1$ and $S(x) = S_*(x)$, if $x = (k + 1)!$, where $k \ge 1$, therefore $S_*(x) + 1 \ge S(x) \ge S_*(x)$.
2. $S_*(x)$ is surjective and an increasing function.

Theorem:

S(x) is asymptotically equal to $(\log x)/(\log \log x)$, when x tends to ∞.

Definition 3:

The *additive analogue of the Smarandache simple function*[261] [which is defined for fixed primes p as $S_p(n) = \min\{m: m \text{ natural}, p^n \text{ divides } m!)\}$] is defined as $S_p(x) = \min\{m: m \text{ natural}, p^x \le m!\}$, where x belongs to the set of real numbers, $x > 1$.

Definition 4:

The *additive analogue of the dual of Smarandache simple function*[262] [which is defined for fixed primes p as $S_{p*}(n) = \max\{m: m \text{ natural}, m! \text{ divides } p^n)\}$] is defined as $S_{p*}(x) = \max\{m: m \text{ natural}, m! \le p^x\}$, where x belongs to the set of real numbers, $x > 1$.

(42) The Smarandache P and S persistence of a prime[263]

Definition 1:

Let X be any n-digits prime number, $X = x_1x_2x_3\ldots x_n$. Reiterating the operation $X + x_1*x_2*x_3*\ldots*x_n$, is eventually obtained a composite number; the number of steps required for X to collapse into a composite number is called the Smarandache P-persistence of the prime X.

Examples:

1. For X = 43 is obtained 43 + 4*3 = 55, a composite number, so the Smarandache P-persistence of the prime 43 is 1 (only one step was required to obtain a composite number).
2. For X = 23 is obtained 23 + 2*3 = 29, then, reiterating, 29 + 2*9 = 47 and 47 + 4*7 = 75, a composite number, so the Smarandache P-persistence of the prime 23 is 3 (three steps were required to obtain a composite number).

Definition 2:

Let X be any n-digits prime number, $X = x_1x_2x_3\ldots x_n$. Reiterating the operation $X + x_1 + x_2 + x_3 + \ldots + x_n$, is eventually obtained a composite number; the number of steps required for X to collapse into a composite number is called the Smarandache S-persistence of the prime X.

Example:

For X = 277 is obtained 277 + 2 + 7 + 7 = 293, a prime number; reiterating the operation is obtained 293 + 2 + 9 + 3 = 307, also a prime, 307 + 3 + 0 + 7 = 317, also a prime, and eventually 317 + 3 + 1 + 7 = 328, finally a composite number, so the Smarandache S-persistence of the prime 277 is 4 (four steps were required to obtain a composite number).

(43) Smarandache type multiplicative functions[264]

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[260] See *supra*, this chapter, Section (31) for the definition of the *dual of the Smarandache function*.
[261] See *supra*, this chapter, Section (30) for the definition of the *Smarandache simple function*.
[262] See *supra*, this chapter, Section (31) for the definition of the *dual of the Smarandache simple function*.
[263] Russo, Felice, *The Smarandache P and S peristence of a prime*, Smarandache Notions Journal.
[264] Bottomley, Henry, *Some Smarandache-type multiplicative functions*, Smarandache Notions Journal.

Note:

The following functions are multiplicative in the sense that, for any two coprime positive integers a, b, the following relation is true: f(a*b) = f(a)*f(b).

Definitions:

1. $A_m(n)$ is the number of solutions to the equation x^m ≡ 0(mod n);
2. $B_m(n)$ is the largest m-th power dividing n;
3. $C_m(n)$ is the m-th root of the largest m-th power dividing n;
4. $D_m(n)$ is the m-th power free part of n;
5. $E_m(n)$ is the smallest number x, x > 0, such that n*x is a perfect m-th power (Smarandache m-th power complements);
6. $F_m(n)$ is the smallest m-th power divisible by n divided by the largest m-th power which divides n;
7. $G_m(n)$ is the m-th root of the smallest m-th power divisible by n divided by the largest m-th power which divides n;
8. $H_m(n)$ is the smallest m-th power divisible by n;
9. $J_m(n)$ is the m-th root of the smallest m-th power divisible by n (Smarandache ceil function of m-th order);
10. $K_m(n)$ is the largest m-th power-free number dividing n (Smarandache m-th power residues);
11. $L_m(n)$ is the number obtained dividing n by the largest squarefree divisor of n.

First few values, for m = 2, of the functions above[265]:

$A_m(n)$: 1, 1, 1, 2, 1, 1, 1, 2, 3, 1, 1, 2, 1, 1, 1, 4, 1, 3, 1, 2, … (sequence A000188);
$B_m(n)$: 1, 1, 1, 4, 1, 1, 1, 4, 9, 1, 1, 4, 1, 1, 1, 16, 1, 9, 1, … (sequence A008833);
$C_m(n)$: 1, 1, 1, 2, 1, 1, 1, 2, 3, 1, 1, 2, 1, 1, 1, 4, 1, 3, 1, 2, … (sequence A000188);
$D_m(n)$: 1, 2, 3, 1, 5, 6, 7, 2, 1, 10, 11, 3, 13, 14, 15, 1, 17, … (sequence A007913);
$E_m(n)$: 1, 2, 3, 1, 5, 6, 7, 2, 1, 10, 11, 3, 13, 14, 15, 1, 17, … (sequence A007913);
$F_m(n)$: 1, 4, 9, 1, 25, 36, 49, 4, 1, 100, 121, 9, 169, 196, … (sequence A055491);
$G_m(n)$: 1, 2, 3, 1, 5, 6, 7, 2, 1, 10, 11, 3, 13, 14, 15, 1, 17, … (sequence A007913);
$H_m(n)$: 1, 4, 9, 4, 25, 36, 49, 16, 9, 100, 121, 36, 169, 196, … (sequence A053143);
$J_m(n)$: 1, 2, 3, 2, 5, 6, 7, 4, 3, 10, 11, 6, 13, 14, 15, 4, 17, 6, … (sequence A019554);
$K_m(n)$: 1, 2, 3, 2, 5, 6, 7, 2, 3, 10, 11, 6, 13, 14, 15, 2, 17, 6, … (sequence A007947);
$L_m(n)$: 1, 2, 3, 2, 5, 6, 7, 2, 3, 10, 11, 6, 13, 14, 15, 2, 17, 6, … (sequence A003557).

Comment:

Between the functions defined above are the following relashinships: $B_m(n)$ = $C_m(n)$^m; n = $B_m(n)$* $D_m(n)$; $F_m(n)$ = $D_m(n)$* $E_m(n)$; $F_m(n)$ = $G_m(n)$^m; $F_m(n)$; $H_m(n)$ = n* $E_m(n)$; $H_m(n)$ = $B_m(n)$* $F_m(n)$; $H_m(n)$ = $J_m(n)$^m; n = $K_m(n)$* $L_m(n)$.

(44) The Smarandache factor partition function[266]

Definition:

Let α_1, α_2, …, α_r be a set of natural numbers and p_1, p_2, …, p_3 a set of arbitrary primes. The Smarandache factor partition (SFP) of α_1, α_2, …, α_r, f(α_1, α_2, …, α_r), is defined as the number of ways in which the number n = p_1^α_1* p_2^α_2*…* p_2^α_2 can be expressed as the product of its' divisors.

[265] The first values of all these functions, for m = 2, 3 and 4, are listed in OEIS.

[266] Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, Chapter 1: *Smarandache partition functions*, Section 4: *Generalizations of partition function, introduction of the Smarandache factor partition*. In this book (Chapter 1) the authors introduced yet other Smarandache type functions like *Smarandache star function* and raised a lot of open problems and conjectures on the factor/reciprocal partition theory.

Example:

For the set of primes, (2, 3), f(1, 2) = 4 as n = 2^1*3^2 = 18 and n = 18 = 2*9 = 3*6 = 2*3*3.

Theorem[267]:

Definition: For a positive integer n let τ(n) and f(n) be the number of distinct divisors and the Smarandache factor partitions respectively. If n is the smallest number satisfying τ(n) = f(n) = r for some r, then n is called a *Balu number*.

Enunciation: The number 36 is the largest Balu number (in other words, the three Balu numbers known to date, *i.e.* 1, 16, 36, are the only three Balu numbers).[268]

(45) Smarandache fitorial and supplementary fitorial functions[269]

Definition:

The *Smarandache fitorial*, denoted by FI(n), is defined as the product of all the numbers relatively prime to and less than n.

Examples:

FI(6) = 1*5 = 5; FI(7) = 6! = 620; FI(12) = 1*5*7*11 = 385.

Definition:

The *Smarandache fitorial*, denoted by FI(n), is defined as the product of all the numbers relatively prime to and less than n.

Examples:

FI(6) = 1*5 = 5; FI(7) = 6! = 620; FI(12) = 1*5*7*11 = 385.

Definition:

The *Smarandache supplementary fitorial*, denoted by SFI(n), is defined as the product of all the numbers less than or equal to n which are not relatively prime to n.

Examples:

SFI(6) = 2*3*4*6 = 144; SFI(7) = 7; SFI(12) = 2*3*4*6*8*9*10*12 = 1244160.

Properties:

1. FI(n)*SFI(n) = n!;
2. SFI(p) = p and Fi(p) = (p – 1)! if and only if p is prime.

Theorem[270]:

For large values of n, SFI(2^n)/FI(2^n) ≈ (π/2)^(1/2).

(46) The Smarandache reciprocal function[271]

Definition:

The function $S_c(n)$ defined in the following way: $S_c(n)$ = x, where x + 1 does not divide n! and, for every y < x, y divides n!.

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[267] For the proof of the theorem see Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, Chapter 2: *Smarandache sequences*, Section 1: *On the largest Balu numberand some SFP equations*.

[268] Maohua Le proved before that there are only finitely many Balu numbers; see Le, Maohua, *On the Balu numbers*, Smarandache Notions Journal, vol. 12, no. 1-2-3, 2001.

[269] Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, Chapter 3: *Miscellaneous topics*, Section 8: *Smarandache fitorial and supplementary fitorial functions*.

[270] Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, p. 172.

[271] Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, Chapter 3: *Miscellaneous topics*, Section 10: *Smarandache reciprocal function and an elementary inequality*.

Theorem[272]:

If $S_c(n) = x$ and $n \neq 3$, then $x + 1$ is the smallest prime greater than n.

(47) The sumatory function associated to Smarandache function[273]

Definition:

The sumatory function F(n) associated to Smarandache function is defined as the sum of the numbers S(d), where S is the Smarandache function and d divides n.

Chapter II. Constants involving the Smarandache function

(1) The first constant of Smarandache[274]

Definition:

Let S(n) be the Smarandache function, *i.e.* the smallest integer such that S(n)! is divisible by n. Then the series defined as the sum from n = 2 to n = ∞ of the numbers 1/S(n)! is convergent to a number s1 between 0**.**000 and 0**.**717.

Note:

The fact that the sum is convergent is proved using the following theorem: for n > 10, S(n)! > n.[275]

(2) The second constant of Smaraendache[276]

Definition:

Let S(n) be the Smarandache function. Then the series defined as the sum from n = 2 to n = ∞ of the numbers S(n)/n! is convergent to an irrational number s2.

Theorem:

The sum that defines the second constant of Smarandache is convergent.[277]

(3) The third constant of Smarandache[278]

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[272] Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, p. 175.

[273] Andrei, M., *et al.*, *Some considerations concerning the sumatory function associated to Smarandache function*,
Smarandache Notions Journal, vol. 7, no. 1-2-3, 1996.

[274] For the definitions of the first, second, third and fourth constant of Smarandache see F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Theorems 15, 16, 17, 18. See also Sándor, József, *On the irrationality of certain constants related to the Smarandache function*, in Seleacu, V., Bălăcenoiu, I. (editors), *Smarandache Notions (Book series)*, vol. 10, American Research Press, 1999, for the comments about the proof of irrationality of few constants of Smarandache. See Cojocaru, Ion and Cojocaru, Sorin, *The first constant of Smarandache*, Smarandache Notions Journal, vol. 7, no. 1-2-3, 1996.

[275] For the proof of this theorem, see Ashbacher, C., *Smarandache Sequences, stereograms and series*, Hexis, Phoenix, p. 95.

[276] See Cojocaru, Ion and Cojocaru, Sorin, *The second constant of Smarandache*, Smarandache Notions Journal, vol. 7, no. 1-2-3, 1996.

[277] For the proof of this theorem, see Ashbacher, C., *Smarandache Sequences, stereograms and series*, Hexis, Phoenix, p. 103.

[278] See Cojocaru, Ion and Cojocaru, Sorin, *The third and fourth constants of Smarandache*, Smarandache Notions Journal, vol. 7, no. 1-2-3, 1996.

Definition:

Let S(n) be the Smarandache function. Then the series defined as the sum from n = 2 to n = ∞ of the numbers 1/S(2)*S(3)*…*S(n) is convergent to a number s3 between 0.71 and 1.01.

Theorem:

The sum that defines the third constant of Smarandache is convergent.[279]

(4) The fourth constant of Smarandache

Definition:

Let S(n) be the Smarandache function. Then the series defined as the sum from n = 2 to n = ∞ of the numbers n^x/S(2)*S(3)*…*S(n), where x ≥ 2, is convergent to a number s4.

Note:

The number s4 is different for different values of x, so it designates a set of constants.

Theorem[280]:

The sum that defines the fourth constant of Smarandache is convergent for any value of x ≥ 2.

(5) Other Smarandache constants[281]

Theorems[282]: Let S_n be the Smarandache function. Then:

1. The series defined as the sum from n = 2 to n = ∞ of the numbers ((–1)^(n – 1))*(S_n/n!) is convergent to an irrational number s5.
2. The series defined as the sum from n = 2 to n = ∞ of the numbers S_n/(n + 1)! is convergent to an irrational number s6 greater than e^(-3/2) and smaller than 1/2.
3. The series defined as the sum from n = k to n = ∞ of the numbers S_n/(n + k)!, where k is a natural number, is convergent to a number s7.
4. The series defined as the sum from n = k to n = ∞ of the numbers S_n/(n – k)!, where k is a nonzero natural number, is convergent to a number s8.
5. The series defined as the sum from n = 2 to n = ∞ of the numbers 1/Σ, where Σ is the sum from i = 2 to i = n of the numbers S_i!/i, is convergent to a number s9.
6. The series defined as the sum from n = 2 to n = ∞ of the numbers 1/(S_n*S_n!^(1/x)), where x > 1, is convergent to a number s10.
7. The series defined as the sum from n = 2 to n = ∞ of the numbers 1/(S_n*(S_n – 1)!^(1/x)), where x > 1, is convergent to a number s11.
8. Let f be a function defined on the set of positive integers with values in the set of real numbers which satisfies the condition f(n) ≤ c/((τ(n!))*n^x – τ((n – 1)!)), where c and x are given constants, greater than 1, and τ(n) is the number of positive divisors of n. Then the series defined as the sum from 1 to ∞ of the numbers f(S(n)) is convergent to a number s12.

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[279] For the proof of this theorem, see Ashbacher, C., *Smarandache Sequences, stereograms and series*, Hexis, Phoenix, p. 105.

[280] For the proof of this theorem, see Ashbacher, C., *Smarandache Sequences, stereograms and series*, Hexis, Phoenix, p. 107.

[281] Many Smarandache constants are defined in F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Theorems 15-30.

[282] For the proof of these theorems see Ashbacher, C., *Smarandache Sequences, stereograms and series*, Hexis, Phoenix, p. 109-118. For even more theorems about constants involving Smarandache function see the same book, p. 119-132.

9. The series defined as the sum from $n = 1$ to $n = \infty$ of the numbers $1/\Pi^n$, where Π is the product from $k = 2$ to $k = n$ of the numbers $S_k!$, is convergent to a number s13.
10. The series defined as the sum from $n = 1$ to $n = \infty$ of the numbers $1/(S_n!*(S_n!)^{(1/2)}*(\log S_n)^p)$, where $p > 1$, is convergent to a number s14.
11. The series defined as the sum from $n = 1$ to $n = \infty$ of the numbers $(2^n)/S(2^n)!$ is convergent to a number s15.
12. The series defined as the sum from $n = 1$ to $n = \infty$ of the numbers $S_n/(n^{(p + 1)})$, where p is a real number greater than 1, is convergent to a number s16 (when $0 \le n \le 1$, the series diverges).

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PART THREE
Conjectures on Smarandache notions and conjectures on number theory due to Florentin Smarandache

Chapter I. Conjectures on Smarandache notions

(1) Conjectures on Smarandache function

Conjecture 1 (Tutescu's Conjecture[283]*)*:
The diophantine equation S(n) = S(n + 1) has no solutions. This conjecture was checked up to n = 10^9.[284]

Conjecture 2 (Radu's Conjecture):
The diophantine equation S(n) + S(n + 1) = S(n + 2) has infinitely many solutions.

Conjecture 3[285]:
There are infinitely many pairs of Fibonacci numbers (F_i, F_j) such that S(F_i) = F_j.
Note: If F_i is prime, then clearly (F_i, F_i) is a solution but it is not known if there are infinitely many Fibonacci numbers that are also primes.

(2) Conjectures on pseudo-Smarandache function[286]

Conjecture 1:
The diophantine equation Z(x) = Z(x + 1) has no solutions.

Conjecture 2:
For any given positive number r there exists an integer s, such that the absolute value of Z(s) – Z(s + 1) is greater than r.

Conjecture 3:
Abs{Z(n + 1)/Z(n)} is unbounded.

Conjecture 4:
There are infinitely many integers n such that Z(τ(n)) = τ(Z(n)), where τ(n) is the number of positive divisors of n.

Conjecture 5:
Let $Z^k(n)$ represent the repeated application of the pseudo-Smarandache function k times Z(Z(…Z(n)…)); question: are there any integers n such that there is not some k for which $Z^k(n)$ = 3? Conjecture: there is no value of n for which the repeated application of the pseudo-Smarandache does not lead to 3.

(3) Conjectures on Smarandache double factorial function[287]

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283 Tutescu, L., *On a conjecture concerning the Smarandache function*, Abstracts of Papers presented to the Amer. Math. Soc., 17, 583, 1996. For Conjectures 1-2 see also Ruiz, S.M. and Perez, M., *Properties and problems related to the Smarandache type functions*, Arxiv.

284 According to article *Smarandache function* from the on-line math encyclopedia Wolfram Math World.

285 Ashbacher, Charles, *An introduction to the Smarandache function*, Erhus University Press, 1995, p. 41.

286 For the first two conjectures, see F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Problem 29. For Conjectures 3-5, see Ashbacher, C., *Pluckings from the tree of Smarandache sequences and functions*, American Research Press, 1998, p. 55, 76, 78.

287 See Russo, Felice, *A set of new Smarandache functions, sequences and conjectures in number theory*, American Research Press, 2000, Chapter IV: *An introduction to the Smarandache double factorial function*.

Conjecture 1:

The sum from n = 1 to n = ∞ of the numbers Sdf(n) is asymptotically equal to a*n^b where a and b are close to 0.8834… and 1.759… respectively.

Conjecture 2:

The sum from n = 1 to n = ∞ of the numbers 1/Sdf(n) is asymptotically equal to a*n^b where a and b are close to 0.9411… and 0.49… respectively.

Conjecture 3:

The function Sdf(n)/n is not distributed uniformly in the interval [0, 1].

Conjecture 4:

For any arbitrary real number r > 0, there is some number n ≥ 1 such that Sdf(n)/n < r.

Conjecture 5:

The equations Sdf(n + 1)/Sdf(n) = k respectively Sdf(n)/Sdf(n + 1) = k , where k is any positive integer and n > 1 for the first equation don't admit solutions.

(4) Conjecture involving irrational and transcendental numbers

Enunciation:

Let a(n) be a Smarandache sequence, different from u(n) = 1…1, where 1 is repetead p_n times, where p_n is the n-th prime. Then the concatenation 0.a(1)a(2) …a(n)… is an irrational number and, even more, 0.a(1)a(2) …a(n)… is a transcendental number.[288]

(5) Conjecture on Smarandache function average[289]

Enunciation:

Let SA be the Smarandache function average. Then SA(n) = 2*n/ln n, for n > 1.

Note: S. Tabirca and T. Tabirca proved that SA(n) ≤ 3*n/8 + 1/4 + 2/n for n > 5 and SA(n) ≤ 21*n/72 + 1/12 – 2/n for n > 23.

(6) Conjecture on pseudo-Smarandache function and palindromes[290]

Enunciation:

Let Z(n) be the pseudo-Smarandache function.[291] There are some palindromic numbers n such that Z(n) is also palindromic: Z(909) = 404, Z(2222) = 1111. Let $Z^k(n)$ = Z(Z(Z(…(n)…))), where function Z is executed k times and $Z^0(n)$ is, by convention, n. What is the largest value of Z(n) such that, for some n, $Z^k(n)$ is a palindrome for all k = 0, 1, 2, …, m?

Note: A number is called *palindromic number* or *palindrome* if it reads the same forwards and backwards.

Conjecture:

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[288] See *infra*, Part Four, Chapter 1, Section (11): *Theorem on the Smarandache concatenated power decimals*, for the proof of irrationality of few such numbers. See also Luca, Florian, *On the Smarandache irrationality conjecture*, Smarandache Notions Journal, vol. 11, no. 1-2-3, 2000, for the proof of irrationality of other such types of numbers.

[289] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Problem 28. See also Finch, Steven R., *The average value of the Smarandache function*, in Seleacu, V., Bălăcenoiu, I. (editors), *Smarandache Notions (Book series)*, vol. 10, American Research Press, 1999.

[290] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Theorem 1.

[291] The smallest number Z(n) such that 1 + 2 + 3 + …+ Z(n) is divisible by n.

Charles Ashbacher conjectured that there is no a largest value of Z(n) such that, for some n, $Z^k(n)$ is a palindrome for all k = 0, 1, 2, …, m.

(7) Conjecture on Smarandache deconstructive sequence[292]

Enunciation:

The Smarandache deconstructive sequence contains infinitely many primes.

(8) Conjectures on Smarandache odd sequence[293]

Conjecture 1:

Except for the trivial case of n = 1, there are no numbers in the Smarandache odd sequence that are also Fibonacci numbers.

Conjecture 2:

Except for the trivial case of n = 1, there are no numbers in the Smarandache odd sequence that are also Lucas numbers.

(9) Conjectures on Smarandache even sequence[294]

Note:

Up through the number 2468101214161820222426283032, just one element of even sequence (ES) was found to be twice a prime (2468101214 = 2*1234050607).

Conjecture 1:

There are other values of n such that ES(n) = 2*p for p a prime.

Conjecture 2:

The only number in the Smarandache even sequence and a Fibonacci number is the trivial case of n = 2.

Conjecture 3:

The only number in the Smarandache even sequence and a Lucas number is the trivial case of n = 2.

Chapter II. Conjectures on primes due to Smarandache

(1) Generalizations of Andrica's Conjecture[295]

Enunciation:

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292 Ashbacher, C., *Pluckings from the tree of Smarandache sequences and functions*, American Research Press, 1998, p. 11.

293 Ashbacher, C., *Pluckings from the tree of Smarandache sequences and functions*, American Research Press, 1998, p. 12.

294 Ashbacher, C., *Pluckings from the tree of Smarandache sequences and functions*, American Research Press, 1998, p. 13.

295 F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Problem 1. F.S., *Six conjectures which generalize or are related to Andrica's Conjecture*, Arxiv. Also see Perez, M.L., *Five Smarandache conjectures on primes*, Smarandache Notions Journal. Also see R., Felice, *A set of new Smarandache functions, sequences and conjectures in number theory*, American Research Press, 2000, Chapter V: *On some Smarandache conjectures and unsolved problems*. For more about the conjecture named after mathematician Dorin Andrica [which states that, for p_n the n-th prime number, the inequality (p_{n+1})^(1/2) – (p_n)^(1/2) < 1 holds] see the articles *Andrica's Conjecture* and *Smarandache constants* from the on-line math encyclopedia Wolfram Math World.

The equation $p_{n+1}^x - p_n^x = 1$, where p_n is the n-th prime, has a unique solution between 0.5 and 1; the maximum solution occurs for n = 1, *i.e.* $3^x - 2^x = 1$ when x = 1 and the minimum solution occurs for n = 31, *i.e.* $127^x - 113^x = 1$ when $x = 0.567148\ldots = a_0$. Thus, Andrica's conjecture $A_n = p_{n+1}^{(1/2)} - p_n^{(1/2)} < 1$ is generalised to:

(i) $B_n = p_{n+1}^a - p_n^a < 1$, where $a < a_0$;

(ii) $C_n = p_{n+1}^{(1/k)} - p_n^{(1/k)} < 2/k$, where $k \geq 2$;

(iii) $D_n = p_{n+1}^a - p_n^a < 1/n$, where $a < a_0$ and n big enough, n = n(a), holds for infinitely many consecutive primes. Questions: is this still available for $a < a_0 < 1$? Is there any rank n_0 depending on a and n such that this relation is verified for all $n \geq n_0$?

(iiii) $p_{n+1}/p_n \leq 5/3$, and the maximum occurs at n = 2.[296]

Note: The number 0.567148130202017714646846875533482564586790249388… is called the Smarandache constant.[297]

(2) Generalizations of Goldbach's and de Polignac's Conjectures[298]

A.Odd numbers

1. Any odd integer n can be expressed as a combination of three primes as follows:

(i) As a sum of two primes minus another prime: n = p + q – r, where p, q, r are all prime numbers (do not include the trivial solution: p = p + q – q when p is prime. Questions: Is this conjecture equivalent with Goldbach's Conjecture (any odd integer greater than or equal to 9 is the sum of three primes)? Is the conjecture true when all three prime numbers are different? In how many ways can each odd integer be expressed as above?

(ii) As a prime minus another prime and minus again another prime: n = p – q – r, where p, q, r are all prime numbers. Questions: Is this conjecture equivalent with Goldbach's Conjecture? Is the conjecture true when all three prime numbers are different? In how many ways can each odd integer be expressed as above? (Vinogradov proved in 1937 that every sufficiently large odd number k is the sum of three odd primes).

2. Any odd integer n can be expressed as a combination of five primes as follows:

(i) As a sum of four primes minus another prime: n = p + q + r + t – u, where p, q, r, t, u are all prime numbers (do not include the solution u equal to one of other four primes). Questions: Is the conjecture true when all five prime numbers are different? In how many ways can each odd integer be expressed as above?

(ii) As a sum of three primes minus another two primes: n = p + q + r – t – u, where p, q, r, t, u are all prime numbers (do not include the solutions t or u equal to one of

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296 This conjecture (iiii) has been proved to be true by Jozsef Sandor, *On a conjecture of Smarandache on prime numbers*, Smarandache Notions Journal.

297 For more decimals of the Smarandache constant see the sequence A038458 in OEIS.

298 F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Problem 2. Also see Perez, M.L., *More Smarandache conjectures on primes' summation*, Smarandache Notions Journal. Also see R., Felice, *A set of new Smarandache functions, sequences and conjectures in number theory*, American Research Press, 2000, Chapter V: *On some Smarandache conjectures and unsolved problems*. For more about the two conjectures named after mathematicians Christian Goldbach [which states that every number that is greater than 2 is the sum of three primes; note that Golbach considered the number 1 to be a prime – the majority of mathematicians from today don't; note also that the conjecture is equivalent with the statement that all positive even integers greater than 4 can be expressed as the sum of two primes] and Alphonse de Polignac [which states that every even number is the difference of two consecutive primes in infinitely many ways] see the articles *de Polignac's Conjecture* and *Golbach Conjecture* from the on-line math encyclopedia Wolfram Math World.

other three primes). Questions: Is the conjecture true when all five prime numbers are different? In how many ways can each odd integer be expressed as above?

(iii) As a sum of two primes minus another three primes: $n = p + q - r - t - u$, where p, q, r, t, u are all prime numbers (do not include the solutions r, t or u equal to one of other two primes). Questions: Is the conjecture true when all five prime numbers are different? In how many ways can each odd integer be expressed as above?

(iiii) As a prime minus another four primes: $n = p - q - r - t - u$, where p, q, r, t, u are all prime numbers (do not include the solution p equal to one of other four primes). Questions: Is the conjecture true when all five prime numbers are different? In how many ways can each odd integer be expressed as above?

B. Even numbers

1. Any even integer n can be expressed as a combination of two primes as follows:

(i) As a difference of two primes: $n = p - q$, where p, q are both prime numbers. Questions: Is it equivalent with Goldbach conjecture that every even number greater than 4 is the sum of two odd primes? In how many ways can each even integer be expressed as above?

2. Any even integer n can be expressed as a combination of four primes as follows:

(i) As a sum of three primes minus another prime: $n = p + q + r - t$, where p, q, r, t are all primes. Questions: Is the conjecture true when all four prime numbers are different? In how many ways can each odd integer be expressed as above?

(ii) As a sum of two primes minus another two primes: $n = p + q - r - t$, where p, q, r, t are all primes. Questions: Is the conjecture true when all four prime numbers are different? In how many ways can each odd integer be expressed as above?

(iii) As a prime minus a sum of three primes: $n = p - q - r - t$, where p, q, r, t are all primes. Questions: Is the conjecture true when all four prime numbers are different? In how many ways can each odd integer be expressed as above?

C. General conjecture

Let $k \geq 3$, and $1 \leq s < k$, be integers. Then:

(i) If k is odd, any odd integer can be expressed as a sum of $k - s$ primes (first set) minus a sum of s primes (second set), such that the primes of the first set is different from the primes of the second set. Questions: Is the conjecture true when all k prime numbers are different? In how many ways can each odd integer be expressed as above?

(ii) If k is even, any even integer can be expressed as a sum of $k - s$ primes (first set) minus a sum of s primes (second set), such that the primes of the first set is different from the primes of the second set. Questions: Is the conjecture true when all k prime numbers are different? In how many ways can each even integer be expressed as above?

(3) Conjecture on Gaussian primes[299]

Definition:

Let ω numbers be a + b*ω, where ω is a complex n-th root of unity, ω^(n – 1) + ω^(n – 2)+...+ 1 = 0, which enjoy unique factorization. The units are: ±1, ±ω, ±ω^2,…, ±ω^(n – 1).

[299] F.S., *Only problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 59.

Conjecture:

The configurations of ω primes are symmetric of the 2n regular polygon.

Note:

This is a generalization of Einstein's integers.

(4) Conjecture on the difference between two primes[300]

Enunciation:

There are not, for any even integer n, two primes those difference is equal to n.

(5) Conjecture on a Silverman problem[301]

Notes:

1. Daniel Silverman raised the problem if the product from n = 1 to n = m of the numbers $(p_n + 1)/(p_n – 1)$, where p_n is the n-th prime, is an integer for any other value of m beside the values 1, 2, 3, 4, 8.
2. F.S. conjectured that the number R_m, where R_m is the product from n = 1 to n = m of the numbers $(p_n + k)/(p_n – k)$, is an integer for a finite number of values of m and there is an infinite number of values of k for which no R_m is an integer.

(6) Conjecture on twin primes involving the pseudo-twin primes[302]

Enunciation:

Let p be a positive integer. Then p and p + 2 are twin primes if and only if (p – 1)!*((1/p + 2/(p + 2)) + 1/p + 1/(p + 2) is an integer.

Definition:

Let p be a positive integer. Then p and p + 2 are *pseudo-twin primes* if and only if ((p – 1)! + 1)/p + ((p + 1)! + 1)/(p + 2) is an integer.

Note:

If p and p + 2 are classic twin primes, then they are also pseudo-twin primes, for by Wilson's Theorem, both the first and second terms are integers.

Problem:

Are there pseudo-twin primes that are not classic twin primes?

Chapter III. Conjectures on Diophantine equations due to Smarandache

(1) Generalization of Catalan’s Conjecture[303]

Enunciation:

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[300] F.S., *Only problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 44.

[301] F.S., *Only problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 61.

[302] Kashihara, K., *Comments and topics on Smarandache notions and problems*, Erhus University Press, 1996, p. 22.

[303] F.S., *Only Problems, not Solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 54. F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 138, *Prime equation conjecture*. For more about the Catalan’s Conjecture see the article *Catalan’s Conjecture* from the on-line math encyclopedia Wolfram Math World. For a study of the diophantine equation proposed by F.S. see See Ibstedt, H., *Surphing on the ocean of numbers – a few Smarandache notions and similar topics*, Erhus University Press, Vail, 1997, Chapter IV: *Diophantine equations*.

Let k be a non-zero integer. There are only a finite number of solutions in integers p, q, x, y, each greater than 1, of the equation $x^p - y^q = k$.

Notes:

1. Eugène Charles Catalan conjectured in nineteenth century that the only solution of the diophantine equation $x^p - y^q = 1$ is the solution [x, y, p, q] = [3, 2, 2, 3].
2. J.W.S. Cassels conjectured in 1953 that, if exist, there are only a finite number of solutions in integers of the equation $x^p - y^q = 1$. Robert Tijdeman proved this in 1976.
3. Preda Mihăilescu proved the Catalan's Conjecture in 2002.

(2) Conjecture proved by Florian Luca[304]

Enunciation:

Let a, b, c be three integers with $a*b \neq 0$. Then the equation $a*x^y + b*y^x = c*z^n$, with $x, y, z \geq 2$ and $\gcd(x, y) \geq 1$, has finitely many solutions [x, y, z, n].

(3) Conjecture on diophantine equation $y = 2*x_1*x_2*\ldots*x_n + 1$

Enunciation[305]:

Let n be integer, $n \geq 2$. The diophantine equation $y = 2*x_1*x_2*\ldots*x_n + 1$ has an infinity of solutions of primes.

Examples: 691 = 2*3*5*23 + 1, where k = 4 or 647 = 2*17*19 + 1, where k = 3.

Problems[306]:

1. Find all n such that $p_m = p_1*p_2*\ldots*p_n + 1$, where all are prime and m > n.
2. Is there a solution for the $m = 2*n$, $m = n^2$ and $m = n*(n + 1)/2$ cases?
3. Find the solution of $y = 2*x_1*x_2*\ldots*x_k + 1$ for all k such that the product $x_1*x_2*\ldots*x_k$ is the smallest. Does this equation have a solution for all k natural numbers?

Chapter IV. Other conjectures due to Smarandache

(1) Conjecture on an Erdős' open problem [307]

Description:

In one of his books, Paul Erdős proposed the following problem: "The integer n is called a barrier for an arithmetic function f if $m + f(m) \leq n$ for all $m < n$. Question: are there infinitely many barriers for $x*\omega(n)$, for some x greater than 0 (where $\omega(n)$ is the number

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304 F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Problem 20. See also Luca, Florian, *Products of factorials in Smarandache type expressions* and Luca, Florian, *Perfect powers in Smarandache type expressions*, both articles in Smarandache Notions Journal, vol. 8, no. 1-2-3, 1997.

305 F.S., *Only problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 34. F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 139, *Generalized prime equation conjecture*. For a study of the diophantine equation proposed by F.S. see See Ibstedt, H., *Surphing on the ocean of numbers – a few Smarandache notions and similar topics*, Erhus University Press, Vail, 1997, Chapter IV: *Diophantine equations*.

306 Kashihara, K., *Comments and topics on Smarandache notions and problems*, Erhus University Press, 1996, p. 23.

307 F.S., *On an Erdős' open problem*, in *Collected Papers*, vol. I (second edition), InfoLearnQuest, 2007.

of distinct prime factors of n)?". Based on some results regarding this question (four lemmas), F.S. conjectured that there is a finite number of barriers, for all x > 0.

Lemma 1:

If x > 1, there are two barriers only: n = 1 and n = 2 (trivial barriers).

Lemma 2:

There is an infinity of numbers which can not be barriers for x*ω(n), for any x > 0.

Lemma 3:

For all x between 0 (exclusive) and 1 (inclusive) there are nontrivial barriers for x*ω(n).

Lemma 4:

Let n be a number between 1 (inclusive) and $p_1*\ldots*p_r*p_{r+1}$ (inclusive) and x between 0 (exclusive) and 1 (inclusive). Then n is a barrier if and only if R(n) is verified for m belonging to the set {n – 1, n – 2, …, n – r + 1}.

(2) Conjecture on the difference between a cube and a square[308]

Enunciation:

There are infinitely many numbers that cannot be expressed as the difference between a cube and a square (in absolute value). These numbers are called *Smarandache bad numbers*.

Examples:

1. The following numbers can be written as the difference between a cube and a square (so they are not *Smarandache bad numbers*): 1 = abs{2^3 – 3^2}; 2 = abs{3^3 – 5^2}; 3 = abs{1^3 – 2^2}; 4 = abs{5^3 – 11^2}; 8 = {1^3 – 3^2} etc.
2. The following numbers are *probable Smarandache bad numbers*: 5, 6, 7, 10, 13, 14 etc.

[308] Bencze, Mihály, *Smarandache relashionships and subsequences*, Smarandache Notions Journal.

PART FOUR
Theorems on Smarandache notions and theorems on number theory due to Florentin Smarandache

Chapter I. Theorems on Smarandache type notions

(1) Theorems on Smarandache function

Theorem 1[309]:

Let NS(k), k ≥ 1, be a generic expression for sequences of integers expressed in functional form.[310] Let NS(k) = k*(k + 1)/2, the k-th triangular number. Then, there are infinitely many integers k such that S(SN(k)) = k.

Theorem 2[311]:

There is no composite number k such that S(k*(k + 1)/2) = k.

Theorem 3[312]:

It is not possible to find a number n such that S(n)*S(n + 1) = n.

Theorem 4[313]:

Given the values of the Smarandache function S(1) = 0, S(2) = 2, S(3) = 3, S(4) = 4, S(5) = 5,…, construct the number r by concatenating the values in the following way: 0**.**02345…The number r is irrational.

Theorem 5[314]:

Let p be any prime number; then S((p^p)^n) = p^(n + 1) – p^n + p.

Theorem 6[315]:

The following inequalities are true for a, b, n positive integers:

(i) S(a*b) ≤ S(a) + S(b);
(ii) S(a*b) ≤ a*S(b);
(iii) S(n^2) ≤ 2*S(n) ≤ n for n > 4, n even.

Theorem 7[316]:

The series defined as the sum from n ≥ 1 of the numbers x^n/S(1)*S(2)*…*S(n) converges absolutely for every x.

Theorem 8[317]:

Let S(m) = min{k: k natural, m divides k!} be the Smarandache function, let k = 1, 2, …, n and a_k and b_k belonging to set of non-null natural numbers. Then we have the following

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[309] Ashbacher, C., *Pluckings from the tree of Smarandache sequences and functions*, American Research Press, 1998, p. 37-38.

[310] For example, Fibonacci numbers, Lucas numbers, triangular numbers can be placed in functional form.

[311] Ashbacher, C., *Pluckings from the tree of Smarandache sequences and functions*, American Research Press, 1998, p. 38.

[312] Ashbacher, C., *Pluckings from the tree of Smarandache sequences and functions*, American Research Press, 1998, p. 43.

[313] Ashbacher, C., *Pluckings from the tree of Smarandache sequences and functions*, American Research Press, 1998, p. 59.

[314] Ruiz, S.M., *A result obtained using Smarandache function*, in Seleacu, V., Bălăcenoiu, I. (editors), *Smarandache Notions (Book series)*, vol. 10, American Research Press, 1999.

[315] Sándor, József, *On an inequality for the Smarandache function*, in Seleacu, V., Bălăcenoiu, I. (editors), *Smarandache Notions (Book series)*, vol. 10, American Research Press, 1999.

[316] Luca, Florian, *On a series involving S(1)*S(2)*…*S(n)*, in Seleacu, V., Bălăcenoiu, I. (editors), *Smarandache Notions (Book series)*, vol. 10, American Research Press, 1999.

[317] Bencze, Mihály, *A new inequality for the Smarandache function*, in Seleacu, V., Bălăcenoiu, I. (editors), *Smarandache Notions (Book series)*, vol. 10, American Research Press, 1999.

inequality: $S(\Pi) \leq \Sigma$, where Π is the product from $k = 1$ to $k = n$ of the numbers $(a_k!)^\wedge b_k$ and Σ is the sum from $k = 1$ to $k = n$ of the numbers $a_k * b_k$.

Theorem 8[318]:

Let $S(m) = \min\{k: k \text{ natural}, m \text{ divides } k!\}$ be the Smarandache function. Then we have the following inequality: $S(\Pi) \leq \Sigma$, where Π is the product from $k = 1$ to $k = m$ of the numbers m_k and Σ is the sum from $k = 1$ to $k = m$ of the numbers $S(m_k)$.

Theorem 9[319]:

The following inequality is true, for p prime and n natural: $(p - 1)*n + 1 \leq S(p^\wedge n) \leq p*n$.

Theorem 10:

The following statement is true, for p prime and n natural: $S(p^\wedge n) = p*(n - m)$ for a particular m, where $0 \leq m \leq [(n - 1)/p]$.

Theorem 11[320]:

The following inequality is true: $S(p^\wedge a) \leq S(q^\wedge a)$ for $p \leq q$ primes and a nonnegative integer.

Theorem 12:

The following inequality is true: $S(n) \leq S(n - S(n))$.

Theorem 13:

The following inequality is true: $S(p^\wedge x) \leq S(p^\wedge y)$, for p prime and $x \leq y$, where x, y, nonnegative integers.

Theorem 14:

The following inequality is true: $S(p^\wedge a)/(p^\wedge a) \leq S(p^\wedge(a + 1))/((p^\wedge(a + 1))$, for a nonegative integer.

Theorem 15:

The following inequality is true: $S(m*n) \leq m*S(n)$ for all positive integers m, n.

Theorem 16:

The following inequality is true: $\max\{S(m), S(n)\} \leq m*S(n)$ for all positive integers m, n.

Theorem 17:

The following inequality is true: $S(m*n) \geq \max\{S(m), S(n)\}$ for all positive integers m, n.

Theorem 18:

The following inequality is true: $S((m!)^\wedge n)) \leq m*n$ for all positive integers m, n.

Theorem 19:

The following inequality is true: $S(p! \pm 1) > S(p!)$ for p prime.

Theorem 20:

The inferior limit, when n tends to ∞, from $S(n)/n$ is equal to 0 and the superior limit, when n tends to ∞, from $S(n)/n$ is equal to 1.

Theorem 21:

The inferior limit, when n tends to ∞, from $S(n + 1)/S(n)$ is equal to 0 and the superior limit, when n tends to ∞, from $S(n + 1)/S(n)$ is equal to $+\infty$.

Theorem 22:

The inferior limit, when n tends to ∞, from $[S(n + 1) - S(n)]$ is equal to $-\infty$ and the superior limit, when n tends to ∞, from $[S(n + 1) - S(n)]$ is equal to $+\infty$.

Theorem 23:

The inferior limit, when n tends to ∞, from $S(\sigma(n))/n$, where $\sigma(n)$ is the divisor function, is equal to 0.

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[318] Bencze, Mihály, *An inequality for the Smarandache function*, in Seleacu, V., Bălăcenoiu, I. (editors), *Smarandache Notions (Book series)*, vol. 10, American Research Press, 1999.

[319] For the proof of the Theorems 9-10, see Grønås, Pål, *A note on S(p^r)*, Smarandache Function Journal, vol. 2-3, 1993.

[320] For Theorems 11-25, see Sándor, József, *Geometric theorems, diophantine equations, and arithmetic functions*, American Research Press, 2002, p. 123-134.

Theorem 24:

The inferior limit, when n tends to ∞, from S(φ(n))/n, where φ(n) is the Euler's totient, is equal to 0.

Theorem 25:

The inferior limit, when n tends to ∞, from S(S(n))/n is equal to 0 and max{S(S(n))/n: n natural} is equal to 1.

Theorem 26[321]:

The positive integer n is a solution of equation S(n)^2 + S(n) = k*n, where k is a fixed positive integer, if and only if one of the following conditions is satisfied:

(i) n = 1 for k = 2;
(ii) n = 4 for k = 5;
(iii) n = p*(p + 1) for k = 1, where p is a prime with p > 3;
(iiii) n = p*(p + 1)/k for k > 1, where p is a prime with p ≡ -1(mod k).

Theorem 27[322]:

For any positive nitger k, k ≥ 1, the equation $S(m_1) + S(m_2) +\ldots+S(m) = S(m_1 + m_2 +\ldots+m_k)$ has an infinity of positive integer solutions.

(2) Theorems on Smarandache function of a set

Definition[323]:

Let A be a nonvoid set of positive integers having the following property: for each n ≥ 1, there exist at least a k belonging to A such that n divides k!. Then the Smarandache function of a set is defined as $S_A(n)$ = min{k: k belongs to A, n divides k!}. When A = P = set of prime numbers, the arithmetic function obtained is P(n) = min{p: p prime, n divides p!}.[324] When A = Q = set of squares, the arithmetic function obtained is Q(n) = min{m^2: n divides (m^2)!}.

Theorem 1[325]:

Let p be prime such that m^2 < p < (m + 1)^2. Then Q(p) = (m + 1)^2.

Theorem 2:

Let p be prime and k positive integer. Then Q(p^k) = ([(k*p)^(1/2)] + 1)^2 for p > k.

Theorem 3:

If p < q are primes, then Q(p*q) = ([q^(1/2)] + 1)^2.

(3) Theorems on pseudo-Smarandache function

Theorem 1[326]:

Given the values of the pseudo-Smarandache function Z(1) = 1, Z(2) = 3, Z(3) = 2, Z(4) = 7, Z(5) = 4,…, construct the number r by concatenating the values in the following way: 0.13274…The number r is irrational.

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321 Chen, Rongi and Le, Maohua, *On the functional equation S(n)^2 + S(n) = k*n*, Smarandache Notions Journal, vol. 11, no. 1-2-3, 2000.

322 Yaming, Lu, *On the solutions of an equation involving the Smarandache function*, Scientia Magna, vol. 2, no. 1, 2006.

323 Sándor, József, *Geometric theorems, diophantine equations, and arithmetic functions*, American Research Press, 2002, p. 152.

324 See *infra*, this chapter, Section 5.

325 For Theorems 1-3, see Sándor, József, *Geometric theorems, diophantine equations, and arithmetic functions*, American Research Press, 2002, p. 152-153.

326 For the proof of the Theorems 1-7, see Ashbacher, C., *Pluckings from the tree of Smarandache sequences and functions*, American Research Press, 1998, p. 59-74.

Theorem 2:

There are infinitely many solutions to the equation Z(n) = S(n).

Theorem 3:

The series defined as the sum from k = 1 to k = ∞ of the numbers 1/(Z(n) +S(n)) is divergent.

Theorem 4:

There are infinitely many integers n such that Z(n) = φ(n), where φ is Euler's totient.

Theorem 5:

There are infinitely many composite integers n such that Z(n) = φ(n).

Theorem 6:

There are infinitely many solutions to the expression Z(n) + φ(n) = n.

Theorem 7:

The only solutions to the equation Z(n) + τ(n) = n, where n > 0 and τ(n) is the number of positive divisors of n, are 1, 8 and 9.

Theorem 8[327]:

The alternating iteration Z(…(φ(Z(φ(n)))…) ultimately leads to one of the following five 2-cycles: 2 - 3, 8 - 15, 128 - 255, 32768 - 65535, 2147483648 - 4294967295.

Theorem 9[328]:

The following inequality is true: Z(n) > τ(n) for all integers n > 120.

Theorem 10:

The equation Z(n) + φ(n) = τ(n) has no solution.

Theorem 11:

The following inequality is true: Z(n) + φ(n) > τ(n) for any integer n, n ≥ 1.

Theorem 12:

The equation Z(n) + τ (n) = n has the only solution n = 56.

Theorem 13:

The only solutions of the equation Z(n) = σ(n) are n = 2^k, where k ≥ 1 and σ(n) is the divisor function.

Theorem 14:

The equation Z(S(n)) = Z(n) has an infinite number of solutions.

Theorem 15:

The equation S(Z(n)) = S(n) has an infinite number of solutions.

Theorem 16:

The equation S(Z(n)) = Z(n) has an infinite number of solutions.

Theorem 17:

The equation Z(S(n)) = S(n) has no solution.

Theorem 18[329]:

If we note with $\Delta_{S,Z}(n)$ the absolute value of the number S(Z(n)) – Z(S(n)), then the following statements are true: the inferior limit, when n tends to ∞, of the number $\Delta_{S,Z}(n)$ is smaller than or equal to 1; the superior limit, when n tends to ∞, of the number $\Delta_{S,Z}(n)$ is equal to +∞.

(4) Theorems on Smarandache double factorial function

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[327] Ibstedt, H., *Mainly natural numbers – a few elementary studies on Smarandache sequences and other number problems*, American Research Press, 2003, Chapter IV: *The alternating iteration of the Euler φ function followed by the Smarandache Z function*.

[328] For the proof of the Theorems 9-17 see Majumdar, A.A.K., *Wandering in the world of Smarandache numbers*, InProQuest, 2010, Chapter 4: *The pseudo Smarandache function*, Section 4.4.: *Miscellaneous topics*.

[329] Sándor, József, *Geometric theorems, diophantine equations, and arithmetic functions*, American Research Press, 2002, p. 156.

Theorem 1[330]:

The equation Sdf(n)/n = 1has an infinite number of solutions.

Theorem 2:

The even (odd respectively) numbers are invariant under the application of Sdf function, namely Sdf(even) = even and Sdf(odd) = odd.

Theorem 3:

The diophantine equation Sdf(n) = Sdf(n + 1) doesn't admit solutions.

Theorem 4[331]:

The equation Sdf(n) + φ(n) = n, where φ is Euler's totient, has only four positive integer solutions, they are 8, 18, 27 and 125.

(5) Theorems on Smarandache type function P(n)[332]

Definition:

P(n) is the function defined analogously with Smarandache function in the following way: let P be equal to the set of prime numbers; then P(n) = min{k: k belongs to P, n divides k!}

Theorem 1:

For each prime p one has P(p) = p, and, if n is squarefree, then P(n) is equal to the greatest prime divisor of n.

Theorem 2:

One has the inequality P(p^2) ≥ 2*p + 1. If q = 2*p + 1 is prime, then P(p^2) = q. More generally, P(p^m) ≥ m*p + 1 for all primes p and all integers m. there is equality, if m*p + 1 is prime.

Theorem 3:

One has, for all n, m ≥ 1, S(n) ≤ P(n) ≤ 2*S(n) – 1 and P(n*m) ≤ 2*(P(n) + P(m)) – 1, where S(n) is the Smarandache function.

(6) Theorem on Smarandache type function C(n)[333]

Definition:

C(n) is the function defined analogously with Smarandache function in the following way: let C(n, k) be the binomial coefficient, i.e. C(n, k) = n*(n – 1)*…*(n – k + 1)/1*2*…*k = n!/(k!*(n – k)!)) for 1 ≤ k ≤ n; then C(n) = max{k: 1 ≤ k < n – 1, n divides C(n, k)}

Theorem:

C(n) is the greatest totient[334] of n which is less then or equal to n – 2.

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[330] See, for Theorems 1-3, Russo, Felice, *A set of new Smarandache functions, sequences and conjectures in number theory*, American Research Press, 2000, Chapter IV: *An introduction to the Smarandache double factorial function*.

[331] Yuan, Xia, *On the Smarandache double factorial function*, in Wenpeng, Zhang (editor), *Research on number theory and Smarandache notions (Proceedings of the sixth international conference on number theory and Smarandache notions)*, Hexis, 2010.

[332] See Sándor, József, *On certain generalizations of the Smarandache function*, Smarandache Notions Journal, vol. 11, no. 1-2-3, 2000, where this function is defined; also see *supra*, Part Two, Chapter I, Section (32): *Generalizations of Smarandache function*.

[333] Sándor, József, *Geometric theorems, diophantine equations, and arithmetic functions*, American Research Press, 2002, p. 169.

[334] A totient of n is a number k such that gcd(k, n) = 1.

(7) Theorems on a dual of Smarandache function[335]

Definition:

The dual of the Smarandache function $S_*(n)$ is defined as $S_*(n) = \max\{m: m$ natural, $m!$ divides $n)$.

Theorem 1[336]:

For any integer n, $n \geq 1$, the following inequality is true: $1 \leq S_*(n) \leq S(n) \leq n$.

Theorem 2:

For any integer n, $n \geq p$, where p is any prime, $p > 2$, the following equality is true: $S_*(n! + (p - 1)!) = p - 1$.

Theorem 3:

For any integer n and a, where $1 \leq a \leq n$, the following inequality is true: $S_*(n*(n - 1)*\ldots*(n - a + 1) \geq a$.

Theorem 4:

For any integer n, $n \geq 1$, the following statement is true: $S_*((2*n)!*(2*n + 2)!)$ is equal to $2*n + 2$, if $2*n + 3$ is a prime and is greater than or equal to $2*n + 3$, if $2*n + 3$ is not a prime.

Theorem 5:

For any integer n, $n \geq 1$, the following inequality is true: $S_*((2*n + 1)!*(2*n + 3)!) \geq 2*(n + 2)$.

(8) Theorems on a dual of pseudo-Smarandache function[337]

Definition:

A dual of the pseudo-Smarandache function, $Z_*(n)$, is the function defined in the following way: $Z_*(n) = \max\{m: m$ natural, $m*(m + 1)/2$ divides $n)$.

Theorem 1[338]:

Let q be a prime such that $p = 2*q - 1$ is a prime too. Then $Z_*(p*q) = p$.

Theorem 2:

For all $n \geq 1$ the following inequality is true: $1 \leq Z_*(n) \leq Z(n)$.

Theorem 3:

All solutions of equation $Z_*(n) = Z(n)$ can be written in the form $n = r*(r + 1)/2$, where r is a non-null natural number.

Theorem 4:

For all n the following inequality is true: $Z_*(n) \leq ((8*n + 1)^{(1/2)} - 1)/2$.

Theorem 5:

For all a, $b \geq 1$ the following inequality is true: $Z_*(a*b) \geq \max\{Z_*(a), Z_*(b)\}$.

Theorem 6:

For any integer k, $k \geq 1$, the following equality is true: $Z_*(k*(k + 1)/2)) = k$.

Theorem 7:

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335 See *supra*, Part Two, Chapter I, Section (31) for the definitions of *the duals of few Smarandache type functions*.

336 For the proof of Theorems 1-5 see Majumdar, A.A.K., *Wandering in the world of Smarandache numbers*, InProQuest, 2010, Chapter 3: *The Smarandache function*, Section 3.2.1: *The Smarandache dual function*.

337 See *supra*, Part Two, Chapter I, Section (31) for the definitions of *the duals of few Smarandache type functions*.

338 For the proof of the Theorems 1-7 see Sándor, József, *On a dual of pseudo-Smarandache function*, Smarandache Notions Journal, vol. 13, no. 1-2-3, 2002 and Majumdar, A.A.K., *Wandering in the world of Smarandache numbers*, InProQuest, 2010, Chapter 4: *The pseudo Smarandache function*, Section 4.3.1: *The pseudo Smarandache dual function*.

For any p prime, $p \geq 3$, and k integer, $k \geq 1$, the following statement is true: $Z_*(p^k)$ is equal to 2 if $p = 3$ and is equal to 1 if $p \neq 3$.

(9) Theorems on Smarandache ceil function[339]

Definition:

The ceil function, denoted $S_k(n)$, is the function defined on the set of positive integers with values in the set of positive integers with the property that $S_k(n)$ is the smallest number so that $S_k(n)^k$ is divisible by n.

Theorem 1:

$S_k(n)$ is a multiplicative function.

Theorem 2:

$S_{k+1}(n)$ divides $S_k(n)$.

Theorem 3:

There exists k so that $S_k(n!) = p\#$, where p is the largest prime dividing n and p# denotes the product all of primes less than or equal to p.

(10) Theorems on Smarandache sequences

Theorem 1[340]:

There are no integers m, n and k such that $SPS(n) = m^k$.[341]

Theorem 2[342]:

The fixed points of SSC(n) are 1 and all numbers where every prime factor is to the first power.[343]

Theorem 3[344]:

There is no quadruple (m, m + 1, m + 2, m + 3) such that all four are fixed points of SSC(n).

Theorem 4[345]:

If the number p = 123456…k belongs to SCS, where p is prime, then $k \equiv 1 \pmod 3$.[346]

Theorem 5[347]:

For any positive integer n, $n > 1$, S_n (where S_n denotes the Smarandache n-ary sieve) contains infinitely many composite numbers.

Theorem 6[348]:

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[339] See, for the enunciation and proof of the Theorems 1-3, Ibstedt, H., *Surphing on the ocean of numbers – a few Smarandache notions and similar topics*, Erhus University Press, Vail, 1997, Chapter II: *On Smarandache functions*, Section 3: *The Smarandache ceil function*. See also *supra*, Part Two, Chapter I, Section (6): *The Smarandache ceil functions of n-th order*.

[340] Ashbacher, C., *Collection of problems on Smarandache notions*, Erhus University Press, 1996, p. 8.

[341] SPS is an acronym for *Smarandache permutation sequence*, *i.e.* the sequence 12, 1342, 135642, 13578642…; see *supra*, Part One, Chapter I, Section (14).

[342] Ashbacher, C., *Collection of problems on Smarandache notions*, Erhus University Press, 1996, p. 10.

[343] SSC is an acronym for *Smarandache square complements*, *i.e.* the sequence 1, 2, 3, 1, 5, 6, 7…; see *supra*, Part One, Chapter II, Section (10).

[344] Ashbacher, C., *Collection of problems on Smarandache notions*, Erhus University Press, 1996, p. 11.

[345] Ashbacher, C., *Collection of problems on Smarandache notions*, Erhus University Press, 1996, p. 56.

[346] SCS is an acronym for *Smarandache consecutive sequence*, *i.e.* the sequence 1, 12, 123, 1234…; see *supra*, Part One, Chapter I, Section (1).

[347] Le, Maohua, *On the Smarandache n-ary sieve*, in Seleacu, V., Bălăcenoiu, I. (editors), *Smarandache Notions (Book series)*, vol. 10, American Research Press, 1999. For the definition of the *Smarandache n-ary sieve sequence* see *supra*, Part one, Chapter II, Section (29).

For any positive integer m, m > 1, there exist infinitely many m-powers which are Smarandache pseudo-m-powers of third kind.

Theorem 7[349]:

The density of GSPs in positive integers is approximatively 0.11.

Theorem 8[350]:

There are no numbers in the Smarandache odd sequence there are also Fibonacci or Lucas numbers, except for the cases OS(1) = F(1) = F(2) = L(1) = 1, OS(2) = F(7) = 13.

Theorem 10[351]:

There are no numbers in the Smarandache even sequence that are also Fibonacci or Lucas numbers, except for the case ES(1) = F(3) = 2.

Theorem 11[352]:

There are no terms in the Smarandache prime product sequence that are squares or higher powers of an integer greater than 1.

Theorem 12:

There are no numbers in the Smarandache prime product sequence there are also Fibonacci or Lucas numbers, except for the cases PPS(1) = F(4) = L(2) = 3 and PPS(2) = L(4) = 7.

Theorem 13[353]:

There are no terms in the Smarandache square product sequence that are squares, cubes or higher powers of an integer greater than 1.

Theorem 14:

There are no numbers in the Smarandache square product of the first kind and of the second kind sequences there are also Fibonacci or Lucas numbers, except for the cases $SPS_1(1)$ = F(3) = 2, $SPS_1(2)$ = F(5) = 5, respectively $SPS_2(2)$ = F(4) = L(2) = 3.

Theorem 15[354]:

There are no terms in the Smarandache higher power product sequences that are squares of an integer greater than 1.

Theorem 16:

If we define with 1^m*2*m*…*n^m + 1 the Smarandache higher power product sequence of the first kind, then: if m is not a number of the form m = 2^k for some integer k ≥ 1, then the sequence $HPPS_1(n)$ contains only one prime, namely $HPPS_1(1)$ = 2.

Theorem 17:

If we define with 1^m*2*m*…*n^m – 1 the Smarandache higher power product sequence of the second kind, then: if both m and 2^m – 1 are primes, then the sequence $HPPS_2(n)$ contains only one prime, namely $HPPS_2(2)$ = 2^m – 1; otherwise, the sequence contains no prime.

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348 Le, Maohua, *On Smarandache pseudo-powers of third kind*, in Seleacu, V., Bălăcenoiu, I. (editors), *Smarandache Notions (Book series)*, vol. 10, American Research Press, 1999. For the definition of the *Smarandache pseudo-m-powers of the third kind* see *supra*, Part one, Chapter II, Section (63).

349 Ashbacher, Charles and Neirynck Lori, *The density of generalized Smarandache palindromes*, Smarandache Notions Journal. For the definition of Generalized Smarandache Palindromes (GSPs) see *supra*, Part One, Chapter I, Section (22).

350 For the proof of the Theorems 8-19 see Majumdar, A.A.K., *Wandering in the world of Smarandache numbers*, InProQuest, 2010, Chapter 1: *Some Smarandache sequences*.

351 For the definitons of *Smarandache odd* and *even sequences*, see *supra*, Part One, Chapter I, Sections (3)-(4).

352 For the definition of *Smarandache prime product sequence*, see *supra*, Part One, Chapter II, Section (33).

353 For the definition of *Smarandache square product sequence*, see *supra*, Part One, Chapter II, Section (51).

354 For the definition of *Smarandache higher power product sequences*, see *supra*, Part One, Chapter II, Section (52).

Theorem 18[355]:

There are no terms in the Smarandache consecutive sequence CS(n) and Smarandache reverse sequence RS(n) that are Fibonacci and Lucas numbers, except for the cases CS(1) = F(1) = F(2) = L(1) = 1 and CS(3) = L(10) = 123 respectively RS(1) = F(1) = F(2) = L(1) = 1 and RS(2) = F(8) = 21.

Theorem 19[356]:

There are no terms in the Smarandache symmetric sequence SS(n) that are Fibonacci and Lucas numbers, except for the cases SS(1) = F(1) = F(2) = L(1) = 1 and SS(2) = L(5) = 11.

Theorem 20[357]:

The series defined as the sum from n = 1 to n = ∞ of the numbers CS(n)/RS(n) is divergent.

Theorem 21[358]:

If we note with a(n) the *inferior factorial part* of the positive integer n and with b(n) the *superior factorial part* of n[359], the the following statement is true: the series I, defined as the sum from n = 1 to n = ∞ of the numbers 1/a(n)^α, and S, defined as the sum from n = 1 to n = ∞ of the numbers 1/b(n)^α, for α any positive real number, are convergent if α > 1 and divergent if α ≤ 1.

Theorem 22[360]:

If we note with a(n) the *square complements (of n) sequence*[361], then the equation Σ = a(n*(n + 1)/2), where Σ is the sum from k = 1 to k = n of the numbers a(k), has only three solutions, they are 1, 2 and 3.

(11) Theorem on the Smarandache concatenated power decimals[362]

Definition:

For any positive integer k is defined the *Smarandache concatenated k-power decimal* α_k as follows: α_1 = 0.12345678910111213…, α_2 = 0.149162536496481100…, α_3 = 0.182764125216343…

Enunciation:

For any positive integer k, α_k is an irrational number.

(12) Theorem on Smarandache function and perf ect numbers[363]

Enunciation:

[355] For the definitons of *Smarandache consecutive* and *reverse sequences*, see *supra*, Part One, Chapter I, Sections (1)-(2).
[356] For the definiton of *Smarandache symmetric sequence*, see *supra*, Part One, Chapter I, Section (10).
[357] Majumdar, A.A.K., *Wandering in the world of Smarandache numbers*, InProQuest, 2010, Chapter 1: *Some Smarandache sequences*, Section 1.12: *Series involving Smarandache sequences*.
[358] Jie, Li, *On the inferior and superior factorial part sequences*, in Wenpeng, Zhang (editor), *Research on Smarandache problems in number theory (Collected papers)*, Hexis, 2004.
[359] See *supra*, Part One, Chapter 2, Sections (25)-(26) for the definitions of *inferior/superior factorial part of n*.
[360] Zhanhu, Li, *On an equation for the square complements*, Scientia Magna, vol. 2, no. 1, 2006.
[361] See *supra*, Part One, Chapter 2, Section (10) for the definition of *square complements sequence*.
[362] Guo, Yongdong and Le, Maohua, *Smarandache concatenated power decimals and their irrationality*, Smarandache Notions Journal, vol. 9, no. 1-2-3, 1998.
[363] Ruiz, Sebastian Martin, *Smarandache's function applied to perfect numbers*, Smarandache Notions Journal, vol. 10, no. 1-2-3, 1999.

If n is a perfect number of the form n = (2^(k – 1))*(2^k – 1), where k positive integer, and 2^k – 1 = p prime, then S(n) = p.

(13) Theorem on Smarandache function and the Dirichlet divisor function[364]

Enunciation:

For any positive integer n, the equation S(n) = τ(n) holds if and only if n = 2^(2^n – 1), where n is non-negative integer, and n = m*p^α, where m > 0 and m divides ((α_1 + 1)*(α_2 + 1)*…*(α_s + 1))!/(p^α), if α ≠ 1, p divides α + 1, 1 < s < 2^(α*p/α + 1)).

(14) Theorems on Smarandache primitive numbers of power p[365]

Definition:

Let p be a prime, n be any fixed positive integer, then S_p(n) denotes the smallest positive integer such that S_p(n)! is divisible by p^n.[366]

Theorem 1:

Let p be an odd prime, m_i be positive integer. Then the following inequality is true:
S_p(Σ_1) ≤ Σ_2, where Σ_1 is the sum from i = 1 to i = k of the numbers m_i and Σ_2 is the sum from i = 1 to i = k of the numbers S_p(m_i).

Theorem 2:

There are infinite integers m_i (i = 1, 2, …, k) satisfying the following equality:
S_p(Σ_1) = Σ_2, where Σ_1 is the sum from i = 1 to i = k of the numbers m_i and Σ_2 is the sum from i = 1 to i = k of the numbers S_p(m_i).

Chapter II. Theorems due to Smarandache

(1) A generalization of Euler's Theorem on congruences[367]

Enunciation:

Let a, m be integers and m ≠ 0. Then a^(φ(m_s) + s) ≡ a^s (mod m), where φ is Euler's totient and m_s and s are obtained by the following algorithm:

(0): {$a = a_0 d_0$; gcd(a_0, m_0) = 1 and $m = m_0 d_0$; d = 1};
(1): {$d_0 = d^1_0\, d_1$; gcd(d^1_0, m_1) = 1 and $m_0 = m_1 d_1$; d_1 = 1};
……………………………………………………………...
(s-1): {$d_{s-2} = d^1_{s-2}\, d_{s-1}$; gcd($d^1_{s-2}$, m_{s-1}) = 1 and $m_{s-2} = m_{s-1} d_{s-1}$; d_{s-1} = 1};
(s): {$d_{s-1} = d^1_{s-1}\, d_s$; gcd($d^1_{s-1}$, m_s) = 1 and $m_{s-1} = m_s d_s$; d_s = 1};

(2) Theorem on an inequality involving factorials[368]

Enunciation:

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364 Wang, Chunping and Zhao, Yanlin, *On an equation involving the Smarandache function and the Dirichlet divisor function*, in Wenpeng, Zhang (editor), *Research on number theory and Smarandache notions (Proceedings of the fifth international conference on number theory and Smarandache notions)*, Hexis, 2009.
365 Liping, Ding, *On the primitive numbers of power p and its triangle inequality*, in Wenpeng, Zhang (editor), *Research on Smarandache problems in number theory (Collected papers)*, Hexis, 2004.
366 See *supra*, Part One, Chapter Two, Sections (16)-(18).
367 F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 135. For the proof of the theorem see F.S., *A generalization of Euler's Theorem on congruences*, Arxiv.
368 F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 141.

Let n and k be positive integers. Then n! is greater than k^(n – k + 1)*Π, where Π is the sum from i = 0 to i = k – 1 of the numbers ((n – 1)/k)!

Example:

For k = 2 it is obtained n! > (2^(n – 1))*((n – 1)/2)!*(n/2)!

(3) Theorem on divisibility involving factorials[369]

Enunciation:

Let a and m be integers, m > 0. Then (a^m – a)*(m – 1)! is divisible by m.

(4) Theorem on an infinity of a set of primes[370]

Enunciation:

There exist an infinite number of primes which contain given digits, a_1, a_2, …, a_m, in the positions i_1, i_2, …, i_m, with i_1, i_2, …, $i_m \geq 0$, where the "i-th position" is the (10^i)-th digit.

Note:

If $i_m = 0$, then a_m must be odd and different from 5.

(5) General theorem of characterization of n primes simultaneously[371]

Enunciation:

Let p_{ij} ,where $1 \leq i \leq n$ and $1 \leq j \leq m_i$, be coprime integers two by two and let r_1, …, r_n and a_1, …a_n be integers such that a_i and r_i are coprime for all i. Under certain conditions[372] the following statements are equivalent:

(i) The numbers p_{ij}, where $1 \leq i \leq n$ and $1 \leq j \leq m_i$, are simultaneously prime.

(ii) (R/D)*Σ ≡ 0 (mod R/D), where R is the product from i = 1 to i = n of the numbers r_i, D is a divisor of R and Σ is the sum from i = 1 to i = n of the numbers a_i*c_i/r_i.

(6) Theorems on Carmichael's totient function conjecture[373]

Theorem 1:

The equation φ(x) = n, where φ is Euler's totient and n is a natural number, admits a finite number of solutions.

Theorem 2:

If the equation φ(x) = n has, for a n natural, an unique solution x_0, then x_0 is a multiple of the number 2^2*3^2*7^2*43^2 (note also that, if a counterexemple x_0 to the Carmichael's totient function exists, it has to be greater than 10^10000)[374].

Theorem 3:

[369] F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 142. See also Le, Maohua, *An improvement on the Smarandache divisibility theorem*, in Seleacu, V., Bălăcenoiu, I. (editors), *Smarandache Notions (Book series)*, vol. 10, American Research Press, 1999.

[370] F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 155.

[371] F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 313.

[372] See, for these conditions and more about this theorem, F.S., *A general theorem for the characterization of n prime numbers simultaneously*, Arxiv.

[373] For the proof of these theorems see F.S., *On Carmichael's conjecture*, Arxiv. Carmichael's totient function conjecture asserts that, if there is any x such that φ(x) = n, then there are at least two solutions x. For more about this conjecture see the article *Carmichael's totient function conjecture* from the on-line math encyclopedia Wolfram Math World.

[374] P. Masai and A. Vallette, *A lower bound for a counterexample to Carmichael's conjecture*.

If n is a counterexample to Carmichael's totient function conjecture, then n is a multiple of a product of a very large number of primes (but F.S. also conjectures that there is no such a counterexample).[375]

(7) Theorem inspired by Crittenden and Vanden Eynden's Conjecture[376]

Enunciation:

It is not posible to cover all positive integers with n geometrical progressions of integers.

Note:

Crittenden and Vanden Eynden's Conjecture refers to arithmetical proggresions and asserts that, if n arithmetic progressions, each having modulus at least k, include all integers from 1 to k*2^(n – k + 1), then they include all the integers.[377]

(8) Theorem which generalizes Wilson's Theorem[378]

Description:

In 1770, Wilson found the following result in number theory: "If p is prime, then (p – 1)! ≡ -1 (mod p)". Smarandache provided the following generalization of this theorem:

Enunciation:

Let m be a whole number and A be the set of the numbers of the form ±p^n, ±2*p^n, ±2^r, or 0, where p is odd prime, n natural and r belongs to the set {0, 1, 2}. Let c_1, c_2, …, $c_{\varphi(n)}$ be a reduced system of residues modulo m. Then c_1*c_2*…*$c_{\varphi(n)}$ ≡ -1 (mod m) if m belongs to the set A, respectively +1 if m doesn't belong to the set A, where φ is Euler's totient.

9) Theorems on arithmetic and geometric progressions[379]

Theorem 1:

It does not matter the way in which one partitions the set of the terms of an arithmetic progression (respectively geometric) in subsets: in at least one of these subsets there will be at least 3 terms in arithmetic progression (respectively geometric).

Theorem 2:

A set *M* , which contains an arithmetic progression (respectively geometric) infinite, not constant, preserves the property of the theorem 1. Indeed, this directly results from the fact that any partition of *M* implies the partition of the terms of the progression.

10) Theorem on the number of natural solutions of a linear equation[380]

Definition:

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[375] F.S., A property for a counterexample to Carmichael's Conjecture, in *Collected Papers*, vol. I (second edition), InfoLearnQuest, 2007.

[376] F.S., *Only problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 85. F.S., *Thirty-six unsolved problems in number theory*, Arxiv.

[377] R.J. Simpson, *On a conjecture of Crittenden and Vanden Eynden concerning coverings by arithmetic progressions*.

[378] F.S., *On a Theorem of Wilson*, in *Collected Papers*, vol. I (second edition), InfoLearnQuest, 2007.

[379] F.S., *About some progressions*, in *Collected Papers*, vol. I (second edition), InfoLearnQuest, 2007.

[380] F.S., *On solving general linear equations in the set of natural numbers*, in *Collected Papers*, vol. I (second edition), InfoLearnQuest, 2007. This article, beside proving this theorem, also gives a method for solving general linear equations on the set of natural numbers.

The equation $a_1*x_1 + \ldots + a_i*x_i + \ldots + a_n*x_n = b$, with all a_i and b integers, $a_i \neq 0$, and gcd $(a_1, \ldots, a_n) = d$, has variations of sign if there are at least two coefficients a_i, a_j with $1 \leq i, j \leq n$, such that sign $(a_i*a_j) = -1$.

Theorem:

The equation from the definition above admits an infinity of natural solutions if and only if has variations of sign.

11) Theorems on the solutions of diophantine quadratic equations[381]

Theorem 1:

The equation x^2 – y^2 = c admits integer solutions if and only if c is integer and is a multiple of number 4.

Theorem 2:

The equation x^2 – d*y^2 = c^2, where d is not a perfect square, admits an infinity of natural solutions.

Theorem 3[382]:

The equation a*x^2 – b*y^2 = c, where $c \neq 0$ and a*b = k^2 (k integer), admits a finite number of natural solutions.

Theorem 4:

If the equation a*x^2 – b*y^2 = c, where a*b $\neq$ k^2 (k integer), admits a particular nontrivial natural solution, then it admits an infinity of natural solutions.

12) Theorems on linear congruences[383]

Theorem 1:

The linear congruence $a_1*x_1 + \ldots + a_n*x_n \equiv b \pmod{m}$ has solutions if and only if gcd $(a_1, \ldots, a_n, m)$ divides b.

Theorem 2:

The congruence $a*x \equiv b \pmod{m}$, $m \neq 0$, with gcd (a, m) = d and d divides b, has d distinct solutions.

Theorem 3:

The congruence $a_1*x_1 + \ldots + a_n*x_n \equiv b \pmod{m}$, $m_1 \neq 0$, with gcd $(a_1, \ldots, a_n, m) = d$ and d divides b, has d*m^(n – 1) distinct solutions.

13) Theorem on very perfect numbers[384]

Definition:

A natural number n is named *very perfect number* if $\sigma(\sigma(n)) = 2*n$, were $\sigma(n)$ is the sum of the positive divisors of n (including 1 and n).[385]

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[381] F.S., *Existence and number of solutions of diophantine quadratic equations with two unknows in Z and N*, in *Collected Papers*, vol. I (second edition), InfoLearnQuest, 2007.

[382] For Theorems 3-4 see also F.S., *A method of solving a diophantine equation of second degree with n variables*, Arxiv.

[383] F.S., *Algorithms for solving linear congruences and systems of linear congruences*, in *Collected Papers*, vol. I (second edition), InfoLearnQuest, 2007. Beside proving these theorems, this article gives also a method for solving linear congruences and systems of linear congruences.

[384] F.S., *About very perfect numbers*, in *Collected Papers*, vol. III, Abaddaba, Oradea, 2000.

[385] A natural number is called a *perfect number* if $\sigma(n) = 2*n$; there are known in present 47 such numbers, as much as Mersenne primes known, because between the two sets is a biunivocal correspondence. It is not known yet if there exist a perfect number which is odd; it is also not known if the set of perfect numbers (implicitly the set of Mersenne primes) is infinite.

Theorem:

The square of an odd prime number can't be very perfect number.

14) Theorems on inequalities for the integer part function [386]

Theorem 1:

For any x, y > 0, we have the inequality (we note with [x], [y] etc. the integer part of the numbers x, y etc.): $[5*x] + [5*y] \geq [3*x + y] + [3*y + x]$.

Theorem 2:

If x, y, z ≥ 0, then we have the inequality $[3*x] + [3*y] + [3*z] \geq [x] + [y] + [z] + [x + y] + [x + z] + [y + z]$.

Theorem 3:

If x, y, z ≥ 0, then we have the inequality $[2*x] + [2*y] + [2*z] \leq [x] + [y] + [z] + [x + y + z]$.

Theorem 4:

If x, y ≥ 0 and n, k are integers such that n ≥ k ≥ 0, then we have the inequality $[n*x + n*y] \geq k*[x] + k*[y] + (n – k)*[x + y]$.

Note:

From the theorems above, Smarandache found the following applications concerning factorial function and divisibility:

Application 1:

For any m, n naturals, (5*m)!*(5*n)! is divisible by m!*n!*(3*m + n)!*(3*n + m)!.

Application 2:

For any a, b, c naturals, (3*a)!*(3*b)!*(3*c)! is divisible by a!*b!*c!*(a + b)!*(a + c)!*(b + c)!.

Application 3:

For any a, b, c naturals, a!*b!*c!*(a + b + c)! is divisible by (2*a)!*(2*b)!*(2*c)!.

Application 4:

For any a, b, n, k naturals with n ≥ k, (n*a)!*(n*b)! is divisible by a!^k*b!^k*(a + b)!^(n*k).

[386] F.S., *Inequalities for the integer part function*, in *Collected Papers*, vol. III, Abaddaba, Oradea, 2000. In this article are presented more theorems and applications (than the ones presented here).

PART FIVE
Criteria, formulas and algorithms for computing due to Florentin Smarandache

(1) Criterion for coprimes involving Euler's totient[387]

Enunciation:

If a, b are strictly positive coprime integers, then a^(φ(b) + 1) + b^(φ(a) + 1) ≡ a + b (mod a*b), where φ is Euler's totient.

(2) Criteria of simultaneous primality

A. Characterization of twin primes[388]:

Let p and p + 2 be positive odd integers; then the following statements are equivalent:

1. p and p + 2 are both primes;
2. (p – 1)!(3*p + 2) + 2*p + 2 is congruent to 0 (mod p*(p + 2));
3. (p – 1)!(p – 2) – 2 is congruent to 0 (mod p*(p + 2));
4. ((p – 1)! + 1)/p + (2*(p – 1)! + 1)/(p + 2) is an integer.

B. Characterization of a pair of primes[389]:

Let p and p + k be positive integers, with the property that gcd (p, p + k) = 1; then p and p + k are both primes if and only if (p – 1)!*(p + k) + (p + k – 1)!*p + 2*p + k is congruent to 0 (mod p*(p + k)).

C. Characterization of a triplet of primes[390]:

Let p – 2, p and p + 4 be positive integers, coprime two by two; then p – 2, p and p + 4 are all primes if and only if (p – 1)! + p*((p – 3)! + 1)/(p – 2) + p*((p + 3)! + 1)/(p + 4) is congruent to -1 (mod p).

D. Characterization of a quadruple of primes[391]:

Let, p p + 2, p + 6 and p + 8 be positive integers, coprime two by two; then p, p + 2, p + 6 and p + 8 are all primes if and only if p*((p – 1)! + 1)/p + 2!*((p – 1)! + 1)/(p + 2) + 6!*((p – 1)! + 1)/(p + 6) + 8!*((p – 1)! + 1)/(p + 8) is an integer.

(3) Criteria of primality derived from Wilson's Theorem[392]

Enunciations[393]:

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[387] F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 133.

[388] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Theorem 6.

[389] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Theorem 7.

[390] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Theorem 8.

[391] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Theorem 9.

[392] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, from Theorem 2 to Theorem 5. For more about the conjecture named after mathematician John Wilson [which states that (p – 1)! + 1 is a multiple of p if and only if p is a prime] see the article *Wilson's Theorem* from the on-line math encyclopedia Wolfram Math World.

[393] For a study of these criteria, see Vassilev-Missana, Mladen and Atanassov, Krassimir, *Some Smarandache problems*, Hexis, 2004, Chapter 1: *On some Smarandache's problems*, Section 10: *On four Smarandache's*

1. Let p be an integer, $p \geq 3$; then p is prime if and only if $(p - 3)!$ is congruent to $((p - 1)/2) \pmod p$;
2. Let p be an integer, $p \geq 1$; then p is prime if and only if $(p - 4)!$ is congruent to $(-1)^{(h + 1)}*r \pmod p$, where h is the smallest integer greater than or equal to p/3 and r is the smallest integer greater than or equal to $(p + 1)/6$;
3. Let p be an integer, $p \geq 5$; then p is prime if and only if $(p - 5)!$ is congruent to $r*h + ((r^2 - 1)/24) \pmod p$, where h the smallest integer greater than or equal to p/24 and $r = p - 24*h$;
4. Let $p = (k - 1)!*h + 1$ be a positive integer, $k > 5$, h being a natural number. Then p is prime if and only if $(p - k)!$ is congruent to $(-1)^t*h \pmod p$, where $t = h + q + 1$ and q the smallest integer greater than or equal to p/h.

(4) A formula to calculate the number of primes[394]

Enunciation:

If $\pi(x)$ is the number of primes less than or equal to x, then $\pi(x) = -1 + \Sigma$, where Σ is the sum from k = 2 to k = x of the numbers n, where n is the smallest integer greater than or equal to S(k)/k and S(k) is the Smarandache function.

(5) A closed expression for the generalized Pells's equation [395]

Description:

The equation $a*x^2 - b*y^2 + c = 0$, where a and b are pozitive integers, different from 0, and c is an integer different from 0, is a generalization of Pell's equation $x^2 - D*y^2 = 1$. Smarandache showed that, if the equation has an integer solution and a*b is not a perfect square, then it has an infinitude of integer solutions and found a closed expression for these solutions.

Example:

For equation $x^2 - 3*y^2 - 4 = 0$, the general solution in positive integers is: $x_n = (2 + 3^{(1/2)})^n + (2 - 3^{(1/2)})^n$ and $y_n = (1/3^{(1/2)})*(2 + 3^{(1/2)})^n + (2 - 3^{(1/2)})^n$, for all n natural, that is (2, 0), (4, 2), (14, 8), (52, 30) etc.

(5) The Romanian multiplication[396]

Description:

It is an algorithm to multiply two integers, A and B. Let k be an integer greater than or equal to 2; write A and B on two different vertical columns: c(A), respectively c(B); multiply A by k, and write the product A_1 on the column c(A); divide B by k, and write the integer part of the quotient B_1 on the column c(B) and so on with the new numbers A_1 and B_1, until we get a $B_i < k$ on the column c(B). Then: write another column c(r), on the right side of c(B), such that: for each number of column c(B), which may be a multiple of k plus the rest r (where r = 0, 1, 2, ..., k – 1), the corresponding number on c(r) will be r; multiply each number of column A by its corresponding r of c(r), and put the new

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problems. For the proof of these theorems, see F.S., *Criteria of primality*, in *Collected Papers*, vol. I (second edition), InfoLearnQuest, 2007.

394 Seagull, L., *The Smarandache Function and the number of primes up to x*, Mathematical Spectrum, University of Shielfield, vol. 28, no. 3, 1995/6, p. 53.

395 F.S., *A method to solve the diophantine equation a*x^2 – b*y^2 + c = 0*, in *Collected Papers*, vol. I (second edition), InfoLearnQuest, 2007.

396 F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 127.

products on another column c(P) on the right side of c(r); finally add all numbers of column c(P). It is obtained A*B which is equal to the sum of all numbers of c(P).

Comments:

1. Remark that any multiplication of integer numbers can be done only by multiplication with 2, 3, …, k, divisions by k, and additions.
2. This is a generalization of Russian multiplication (the case k = 2, known since Egyptian time), called by F.S. *Romanian multiplication*.
3. This multiplication is useful when k is very small, the best values being for k = 2 or k = 3; if k is greater than or equal to min{10, B}, this multiplication is trivial.

(6) Algorithm for division by k^n

Description[397]:

It is an algorithm to divide an integer A by k^n, where k and n are integers greater than or equal to 2. Write A and k^n on two different vertical columns: c(A), respectively c(k^n); divide A by k, and write the integer quotient A_1 on the column c(A); divide k^n by k, and write the quotient q_1 = k^(n – 1) on the column c(k^n) and so on with the new members A_1 and q_1, until we get q_n = 1 (= k^0) on the column c(k^n). Then: write another column c(r), on the left side of c(A), such that for each number of column c(A), which may be a multiple of k plus the rest r (where r = 0, 1, 2, ..., k – 1), the corresponding number on c(r) will be r; write another column c(P), on the left side of c(r), in the following way: the element on line i (except the last line which is 0) will be k^(n – 1); multiply each number of column c(P) by its corresponding r of c(r), and put the new products on another column c(R) on the left side of c(P); finally add all numbers of column c(R) to get the final rest R_n, while the final quotient will be stated in front of c(k^n)'s 1. Therefore, A/(k^n) = A_n and rest R_n.

Comments:

1. Remark that any division of an integer number by k can be done only by divisions to k, calculations of powers of k, multiplications with 1, 2, …, k – 1 and additions.
2. This division is useful when k is small, the best values being when k is an one-digit number and n large. If k is very big an n is very small, this division becomes useless.

[397] F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequence 128.

PART SIX
Unsolved problems regarding Smarandache notions and open problems on number theory due to Florentin Smarandache

Chapter I. Problems regarding sequences

(1)
Enunciation[398]:

Find the sequences a_n defined in the following way: for any i positive integer, there exist j, k positive integers, with the property that $i \neq j \neq k \neq i$, so that $a_i \equiv a_j \pmod{a_k}$.
Find the sequences a_n defined in the following way: for any i positive integer, there exist j, k positive integers, with the property that $i \neq j \neq k \neq i$, so that $a_j \equiv a_k \pmod{a_i}$.

(2)
Enunciation[399]:

Let N(n) be the number of terms not greater than n of the sequence $a_1, a_2, \ldots$, where this is a strictly increasing sequence of positive integers. Find the smallest k such that N(N(…N(n)…)) is constant, for a given n.

(3)
Enunciation[400]:

Let $1 \leq a_1 < a_2 < \ldots$ be an infinite sequence of integers such that any three members do not constitute an arithmetical progression.

Example:

Let be $a_n = p^{(n-1)}$, $n \geq 1$, p is an integer greater than 1; then a_n has the property of the assumption and the sum from $n \geq 1$ of the numbers $1/a_n$ is equal to a number smaller than or equal to 2, i.e. the number $1 + 1/(n-1)$.

Questions:

1. Is it always the sum from $n \geq 1$ of the numbers $1/a_n$ smaller than or equal to 2?[401]
2. Is the function $S(\{a_n\}_{n\geq 1})$ representing the sum from $n \geq 1$ of the numbers $1/a_n$ bijective (biunivocal)?
3. Analogously for geometrical progressions.

(4)
Enunciation[402]:

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[398] F.S., *Only problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 1. See also Kashihara, K., *Comments and topics on Smarandache notions and problems*, Erhus University Press, 1996, p. 24-25.

[399] F.S., *Only problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 53. See also Kashihara, K., *Comments and topics on Smarandache notions and problems*, Erhus University Press, 1996, p. 28.

[400] F.S., *Only problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 62. See also Kashihara, K., *Comments and topics on Smarandache notions and problems*, Erhus University Press, 1996, p. 30-31.

[401] Hristo Aladjov and Krassimir Atanassov showed that there is an infinite number of such sequences for which this sum is greater than 2; see *Remark on the 62-th Smarandache's problem*, Smarandache Notions Journal, vol. 11, no. 1-2-3, 2000.

[402] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Problem 20.

We consider the consecutive sequence (1, 12, 123, 1234, …) and we form the simple continued fraction 1 + (1/(12 + 1/(123 + 1/(1234 + 1/12345 + …)))). We also consider the reverse sequence (1, 21, 321, 4321, …) and we form the general continued fraction (1/(12 + 21/(123 + 321/(1234 + 4321/12345 + …)))). Calculate each of these two continued fractions. The first continued fraction is known as convergent.[403]

(5)

Enunciation[404]:

It is considered the sequence constructed by concatenating in the same manner with the *Smarandache consecutive numbers sequence* the terms of the sequence of *happy numbers*.[405]

Questions:

1. How many terms of Smarandache H-sequence are primes?
2. How many terms of Smarandache H-sequence belongs to the sequence of happy numbers?

(6)

Enunciation[406]:

We denote with SDS the Smarandache deconstructive sequence.[407]

Questions:

1. Does every element of the Smarandache deconstructive sequence ending with a 6 contain at least 3 instances of the prime 2 as a factor?
2. If we form a sequence from the elements of SDS(n) that end in 6, do the powers of 2 that divide them form a monotonically increasing sequence?
3. Let k be the largest integer such that 3^k divides n and j the largest integer such that 3^j divides SDS(n). Is it true that k is always equal to j?

(7)

Enunciation[408]:

What is the maximum value of k such that n, n + 1, n + 2, …, n + k are all Smarandache pseudo-primes of the first kind?[409]

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[403] For the definition and study of *Smarandache continued fractions* see Castillo, Jose, *Other Smarandache type functions*, Smarandache Notions Journal, vol. 9, no. 1-2-3, 1998. See also Ibstedt, H., *Mainly natural numbers – a few elementary studies on Smarandache sequences and other number problems*, American Research Press, 2003, Chapter VI: *Smarandache continued fractions*. For the proof that the continued fraction 1 + (1/(12 + 1/(123 + 1/(1234 + 1/12345 + …)))) is convergent see Ashbacher, Charles and Le, Maohua, *On the Smarandache simple continued fractions*, in Seleacu, V., Bălăcenoiu, I. (editors), *Smarandache Notions (Book series)*, vol. 10, American Research Press, 1999.

[404] The definition of *Smarandache sequence of happy number*s, or *Smarandache H-sequence*, belongs to Shyam Sunder Gupta hwo also proposed the problems presented here; see *Smarandache sequence of happy numbers*, Smarandache Notions Journal, vol. 13, no. 1-2-3, 2002.

[405] The *happy numbers* are the numbers with the following property: if you iterate the process of summing the squares of their digits this process ends in number 1. By doing this process of iteration with any integer, finally the process ends in number 1 (in the case of *happy numbers*) or into a loop (in the case of *unhappy numbers*) formed by only few possible numbers: {4, 16, 20, 37, 42, 58, 89, 145}. For the first terms of the sequence of *happy numbers* see the sequence A035497 in OEIS.

[406] Ashbacher, C., *Pluckings from the tree of Smarandache sequences and functions*, American Research Press, 1998, p. 11.

[407] For the definition of Smarandache deconstructive sequence, see *supra*, Part one, Chapter II, Section (3).

[408] For the problems (7)-(9) from this chapter, see Kashihara, K., *Comments and topics on Smarandache notions and problems*, Erhus University Press, 1996, Chapter 1: *Some comments and problems on Smarandache notions*, p. 15-18.

(8)

Enunciation[410]:

Let SPPFK(n) be the n-th member of the sequence of Smarandache pseudo-primes of the first kind. What is the largest possible difference between succesive terms, *i.e.* what is the upper bound of SPPFK(n + 1) – SPPFK(n)?

(9)

Enunciation:

Let $S_{PP}(n)$ be the number of integers $k \leq n$ such that k is a Smarandache pseudo-prime of the first kind. Determine the limit when n tends to ∞ of the numbers $S_{PP}(n)/n$.

(10)

Enunciation[411]: Study the sequences defined in the following way:

1. For k, n_i belonging to natural set, $k < n_i$, $n_0 = n$, $n_{i+1} = \max\{p: p \text{ divides } n_i – k; p \text{ is prime}\}$;
2. For k, n_i belonging to natural set, $k < n_i$, $n_0 = n$, $n_{i+1} = \max\{p: p \text{ divides } n_i/k; p \text{ is prime}\}$;
3. For k, n_i belonging to natural set, $1 \leq k \leq n_i$, $n_0 = n$, $n_{i+1} = \max\{p: p \text{ divides } n_i + k; p \text{ is prime}\}$;
4. For k, n_i belonging to natural set, $1 \leq k \leq n_i$, $n_0 = n$, $n_{i+1} = \max\{p: p \text{ divides } n_i*k; p \text{ is prime}\}$.

(11)

Enunciation[412]: Let $e_p(n)$ be the largest exponent of p which divides n; for example, if p = 3, the values of $e_p(n)$ are: {0, 0, 1, 0, 0, 1, 0, 0, 2, 0, 0, 1, 0, 0, 1, 0, 0, 2, 0, 0, 1, 0, 0,…}.

1. What is the expectation of $e_p(n)$, for any n belonging to natural set?
2. What is the value of $e_m(n)$ expressed using $e_p(n)$, $e_q(n)$, …, where m = p*q*…?

(12)

Enunciation[413]:

Prove that in the infinite Smarandache prime base sequence 1, 2, 3, 5, 7, 11, … (defined as all prime numbers proceeded by 1) any positive integer can be uniquely written with only two digits: 0 and 1 (a linear combination of distinct primes and integer 1, whose coefficients are 0 and 1 only).

Chapter II. Problems regarding Smarandache function

(1)

Enunciation[414]:

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[409] For the definition of Smarandache pseudo-primes, see *supra*, Part one, Chapter II, Section (62).

[410] Kashihara conjectured that there is no such an upper bound.

[411] Kashihara, K., *Comments and topics on Smarandache notions and problems*, Erhus University Press, 1996, p. 26-27.

[412] Kashihara, K., *Comments and topics on Smarandache notions and problems*, Erhus University Press, 1996, p. 31.

[413] This problem has been solved; see Perez, M (editor), *On some Smarandache problems*, Notes on Number Theory and Discrete Mathematics, vol. 9, Number 2, 2003, Proposed problem 2.

Given any pair of integers (m, n) where both are greater than 1 and $m \neq n$, is it always possible to find another pair of integers (p, q) such that $S(m) + S(m + 1) + ... + S(m + p) = S(n) + S(n + 1) + ...+ S(n + q)$?[415]

(2)

Enunciation[416]:

Are there integers m, n, p, k with $m \neq n$ and $p > 0$ such that $(S(m)^2 + S(m + 1)^2 +…+ S(m + p)^2)/(S(n)^2 + S(n + 1)^2 +…+ S(n + p)^2) = k$?[417]

(3)

Enunciation[418]:

How many primes have the form $S(n)S(n + 1)S(n + 2)…S(n + k)$ for a fixed integer k?

(4)

Enunciation[419]:

Is the set of integers $\{n: S(n)S(n + 1) \text{ prime}\}$ an infinite set?

(5)

Enunciation[420]:

Are there n, m positive integers, $n \neq 1 \neq m$ for which $S(n*m) = S(n)*m^k$?

(6)

Enunciation[421]:

Let A be a set of consecutive positive integers. Find the largest set of numbers $\{n, n + 1, n + 2,…\}$ such that $\{S(n), S(n + 1), S(n + 2),…\}$ is monotonic.

(7)

Enunciation[422]:

What is the smallest value of r such that $1/(S(n))^r$ is convergent?

(8)

Enunciation[423]:

How many quadruplets satisfy the relation $S(n) + S(n + 1) = S(n + 2) + S(n + 3)$?

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[414] Ashbacher, C., *Pluckings from the tree of Smarandache sequences and functions*, American Research Press, 1998, p. 29.
[415] Charles Ashbacher conjectured that the answer to this question is yes.
[416] Ashbacher, C., *Pluckings from the tree of Smarandache sequences and functions*, American Research Press, 1998, p. 30.
[417] Charles Ashbacher conjectured that the answer to this question is yes.
[418] Ashbacher, C., *Pluckings from the tree of Smarandache sequences and functions*, American Research Press, 1998, p. 31.
[419] Ashbacher, C., *Pluckings from the tree of Smarandache sequences and functions*, American Research Press, 1998, p. 32.
[420] Ashbacher, C., *Pluckings from the tree of Smarandache sequences and functions*, American Research Press, 1998, p. 33.
[421] See Ashbacher, C., *Pluckings from the tree of Smarandache sequences and functions*, American Research Press, 1998, p. 36-37, for a study of this problem.
[422] Ashbacher, C., *Pluckings from the tree of Smarandache sequences and functions*, American Research Press, 1998, p. 57.
[423] Ashbacher, Charles, *An introduction to the Smarandache function*, Erhus University Press, 1995, p. 42. The author conjectured that there are infinitely many such quadruplets.

(9)

Enunciation[424]:

How many quadruplets satisfy the relation S(n) – S(n + 1) = S(n + 2) – S(n + 3)?

(10)

Note[425]:

The value of the number S(2^k – 1) (mod k) is equal to 1 for all integers from k = 2 to k = 97, with just four exceptions, for k = 28, k = 52, k = 68 and k = 92.

Enunciation:

One can obtain a formula that gives in function of k the value S(2^k – 1) (mod k) for all positive integer values of k?

(11)

Enunciation[426]:

Let p be a positive prime and S(n) the Smarandache function. Prove that S(p^p) = p^2.

(12)

Enunciation[427]:

Prove that in between n and S(n) there exists at least a prime number.

(13)

Enunciation[428]:

Solve the following diophantine equations:

(i) x^S(x) = S(x)^x;

(ii) x^S(y) = S(y)^x;

(iii) x^S(x) + S(x) = S(x)^x + x;

(iiii) x^S(y) + S(y) = S(y)^x + x.

(14)

Enunciation[429]:

For what triplets n, n – 1, n – 2 does the Smarandache function satisfy the Fibonacci recurrence S(n) = S(n – 1) + S(n – 2)? Is there a pattern that would lead to the proof that there is an infinite family of solutions?

Note:

Solutions have been found for n = 11, 121, 4902, 26245, 32112, 64010, 368140, 415664.

(15)

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[424] Ashbacher, Charles, *An introduction to the Smarandache function*, Erhus University Press, 1995, p. 43. The author conjectured that there are infinitely many such quadruplets.

[425] Ruiz, S.M., *Applications of Smarandache function, and prime and coprime functions*, American Research Press, 2002, p. 11-14.

[426] This problem has been solved; see Perez, M (editor), *On some Smarandache problems*, Notes on Number Theory and Discrete Mathematics, vol. 9, Number 2, 2003, Proposed problem 3.

[427] This problem has been solved; see Perez, M (editor), *On some problems related to Smarandache notions*, Notes on Number Theory and Discrete Mathematics, vol. 9, Number 2, 2003, Proposed problem 1.

[428] Tutescu, Lucian and Burton, Emil, *On some diophantine equations*, Smarandache Notions Journal, vol. 7, no. 1-2-3, 1996. In this article the equation (i) is solved and there are few more proposed diophantine equations concerning Smarandache function.

[429] Ibstedt, H., *Base solution (the Smarandache function)*, Smarandache Notions Journal, vol. 7, no. 1-2-3, 1996. The author name this problem "Asbacher's problem".

Enunciation[430]:

Prove the following:

(i) $S(n) = S(n + 2)$ for only finitely many n;

(ii) $S(n) = S(n + 3)$ for only finitely many n.

Chapter III. Problems regarding pseudoSmarandache function

(1)

Enunciation[431]:

Let Z(n) be the pseudo-Smarandache function[432] and $Z^k(n) = Z(Z(Z(\ldots(n)\ldots)))$, where the function is composed k times. For a given pair of natural numbers (k, m), find all integers n such that $Z^k(n) = m$.

(2)

Enunciation:

Let Z(n) be the pseudo-Smarandache function. Is the absolute value of the numbers $Z(n + 1) – Z(n)$ bounded or ubounded? The same question for the numbers $Z(n + 1)/Z(n)$.

(3)

Enunciation:

Try to find the relationships between $Z(m + n)$ and $Z(m)$, $Z(n)$ and also between $Z(m*n)$ and $Z(m)$, $Z(n)$.

(4)

Enunciation:

Find all values of n such that: $Z(n) = Z(n + 1)$; $Z(n)$ divides $Z(n + 1)$; $Z(n + 1)$ divides $Z(n)$.

(5)

Enunciation:

For a given natural number m, how many n are there such that $Z(n) = m$?

(6)

Enunciation[433]:

The sum from k = 1 to k = n of the numbers 1/Z(k) is an integer for n = 1. Is it an integer for any other value of n?

(7)

Enunciation:

Is it the series defined as the sum from k = 1 to k = ∞ of the numbers 1/(Z(n))^2 convergent or divergent?

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430 Mullin, Albert A., *On the Smarandache function and the fixed-point theory of numbers*, Smarandache Notions Journal, vol. 7, no. 1-2-3, 1996.

431 The problems (1)-(5) from this Chapter are raised by K. Kashihara; see *Comments and topics on Smarandache notions and problems*, Erhus University Press, 1996, Chapter 2: *The pseudo-Smarandache function*.

432 Z(n) is the smallest number such that $1 + 2 + 3 +\ldots+ Z(n)$ is divisible by n.

433 The problems (6)-(12) from this Chapter are raised by C. Ashbacher; see *Pluckings from the tree of Smarandache sequences and functions*, American Research Press, 1998, p. 56-68.

(8)

Enunciation:

What is the smallest value of r such that the series defined as the sum from k = 1 to k = ∞ of the numbers 1/(Z(n))^r is convergent?

(9)

Enunciation:

Is there a value for k where there are only a finite number of solutions to the equation k*Z(n) = n?

(10)

Enunciation:

What is the smallest value of r such that the series defined as the sum from k = 1 to k = ∞ of the numbers 1/(Z(k) + S(k))^r is convergent?

(11)

Enunciation:

Is the series defined as the sum from k = 1 to k = ∞ of the numbers 1/(Z(k)*S(k)) convergent or divergent?

(12)

Enunciation:

Is there an infinite number of solutions to the equation Z(σ(n)) = σ(Z(n)), where σ(n) is the divisor function?

Chapter IV. Problems regarding Smarandache double factorial function

(1)

Enunciation[434]:

We note with Sdf(n) the double factorial function. Is the difference abs{Sdf(n + 1) – Sdf(n)} bounded or unbounded?

(2)

Enunciation:

For each value of n, which iteration of Sdf(n) produces always a fixed point or a cycle? For iteration is intended the repetead application of Sdf(n).

(3)

Enunciation:

Find the smallest k such that between Sdf(n) and Sdf(k + n), for n > 1, there is at least a prime.

(4)

Enunciation:

[434] The problems (1)-(9) from this Chapter are raised by Felice Russo; see *A set of new Smarandache functions, sequences and conjectures in number theory*, American Research Press, 2000, Chapter IV: *An introduction to the Smarandache double factorial function*.

Is the number 0.1232567491011…, where the sequence of digits is Sdf(n) for $n \geq 1$ an irrational or trascendental number?

(5)
Enunciation:
Are there k, n, m nonnull positive integers for which Sdf(n*m) = m^k*Sdf(n)?

(6)
Enunciation:
Are there k, n nonnull positive integers for which (Sdf(n))^k = k*Sdf(n*k)?

(7)
Enunciation:
Find all the solution for the equation Sdf(n)! = Sdf(n!).

(8)
Enunciation:
Find all the solution for the equation Sdf(n^k) = k*Sdf(n), for k > 1, n > 1.

(9)
Enunciation:
Find all the solution for the equation Sdf(n^k) = n*Sdf(k), for k > 1.

(10)
Enunciation[435]:
Let p be prime and Sdf(x) Smarandache double factorial function. Solve the diophantine equation Sdf(x) = p. How many solutions are there?

Chapter V. Problems regarding other functions

(1)
Enunciation[436]:
Let M be a number in a base b. All distinct digits of M are named *generalized period of M* (for example, if M = 104001144, its generalized period is g(M) = {0, 1, 4}). Of course, g(M) is included in {0, 1, 2, ..., b – 1}. The number of generalized periods of M is equal to the number of the groups of M such that each group contains all distinct digits of M (for example, $n_g(M) = 2$ if M = 104001144 because both groups of digits 104 respectively 001144 contain all distinct digits of M). Length of generalized period is equal to the number of its distinct digits (for example, $l_g(M) = 3$). Questions:

(i) Find n_g and l_g for p_n, n!, n^n, n^(1/n).

(ii) For a given $k \geq 1$, is there an infinite number of primes p_n or n! or n^n or n^(1/n) which have a generalized period of length k? Same question such that the number of generalized periods be equal to k.

(iii) Let $a_1, a_2, \ldots, a_h$ be distinct digits. Is there an infinite number of primes p_n or n! or n^n or n^(1/n) which have as a generalized period the set $\{a_1, a_2, \ldots, a_h\}$?

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435 This problem has been solved; see Perez, M (editor), *On some Smarandache problems*, Notes on Number Theory and Discrete Mathematics, vol. 9, Number 2, 2003, Proposed problem 5.

436 F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Sequences 129-131.

(2)

Enunciation[437]:

Is it possible to construct a function which obtains all irrational numbers? How about all transcendental numbers?

(3)

Enunciation[438]:

Let FI(n) and SFI(n) be the Smarandache fitorial and supplementary fitorial functions. Find the values of n for the following relationships to be true:

(i) FI(n) < SFI(n) and FI(n) > SFI(n);

(iii) τ(F(n)) > τ(SFI(n)) and τ(F(n)) > τ(SFI(n));

(iiii) σ(F(n)) > σ(SFI(n)) and σ(F(n)) > σ(SFI(n)).

Chapter VI. Problems regarding equations

(1)

Enunciation[439]:

Let q be a rational number, q different from {-1, 0, 1}. Solve the equation:
x*q^(1/x) + (1/x)*q^x = 2*q.

(2)

Enunciation[440]:

The equation x^3 + y^3 + z^3 = 1 has as solutions (9, l0, -12) and (-6, -8, 9). How many other nontrivial integer solutions are there?

(3)

Enunciation[441]:

Consider the following equation: (a – b*n^(1/m)) *x + c*n^(1/m)*y + q^(1/p)*z + (d + e*w)*s^(1/r) = 0, where a, b, c, d are constant integers and the m-th, p-th and r-th roots are irrational distinct numbers. What conditions must the parameters m, n, p, q, r and s accomplish such that the equation admits integer solutions (x, y, z and w being variables)?

(4)

Enunciation[442]:

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437 Kashihara, K., *Comments and topics on Smarandache notions and problems*, Erhus University Press, 1996, p. 25.

438 Murthy, Amarnath and Ashbacher, Charles, *Generalized partitions and new ideas on number theory and Smarandache sequences*, Hexis, 2005, p. 172-173. See *supra*, Part Two, Chapter 1, Section (45) for the definitions of the Smarandache fitorial and supplementary fitorial functions.

439 F.S., *Only problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 50. See also, for a discussion on this equation, Kashihara, K., *Comments and topics on Smarandache notions and problems*, Erhus University Press, 1996, p. 27-28.

440 F.S., *Only problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 60.

441 F.S., *Only problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 86.

442 F.S., *Only problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 88. The problem is inspired by a equation from a William Lowell Putnam Mathematical Competition, *i.e.* x^3 – z = 3, where z is the greater integer less then or equal to x. See also, for a discussion on this equation, Kashihara, K., *Comments and topics on Smarandache notions and problems*, Erhus University Press, 1996, p. 32.

Find all real solutions (x, y) of the equation $x^y - z = y$, where z is the greater integer less than or equal to x.

(5)

Enunciation[443]:

Solve the diophantine equation $2*x^2 - 3*y^2 = 5$.

(6)

Enunciation[444]:

Solve the diophantine equation ISPP(x) + SSPP(x) = k, where ISPP(x) is *the Inferior Smarandache Prime Part* (the largest prime less than or equal to n) and SSPP(x) is the *Superior Smarandache Prime Part* (the smallest prime greater than or equal to n).

Chapter VII. Problems regarding prime numbers

(1)

Enunciation[445]:

Find the number of primes which can be formed from the digits $a_1, a_2, \ldots, a_n$, where $a_1, a_2, \ldots, a_n$ are distinct digits of the set {0, 1, …, 9}, for a certain n, $1 \leq n \leq 9$. Generalizing, the same question is raised in the case when n is positive integer and $a_1, a_2, \ldots, a_n$ are distinct positive integers.

Comment:

The problem "can be solved quickly on a modern computer".[446]

Conjecture[447]:

Can be formed an infinity of such primes (obviously, if is allowed the repetition of the digits $a_1, a_2, \ldots, a_n$).

(2)

Enunciation[448]:

Find the number of the digits of a, where a is a certain digit between 0 and 9, contained by the n-th prime number P_n; the same question is raised for n! or for n^n and, generalizing, for a non-negative integer a.

Comment:

"The sizes P_n, n! and n^n have jumps when $n \rightarrow n + 1$, hence the analytical expressions are approximate only. Moreover, the results depend on the exact (and not approximate) value of these sizes".[449]

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[443] This equation has been solved; see Vassilev-Missana, Mladen and Atanassov, Krassimir, *Some Smarandache problems*, Hexis, 2004, Chapter 1: *On some Smarandache's problems*, Section 9: *On the 78-th Smarandache's problem*.

[444] This equation has been solved; see Perez, M (editor), *On some Smarandache problems*, Notes on Number Theory and Discrete Mathematics, vol. 9, Number 2, 2003, Proposed problem 1.

[445] F.S., *Only problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 3, F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006, Problem 154.

[446] R.K. Guy, Calgary University, Alberta, Canada, Letter to F.S., 15 november 1985, cited by F.S., *Only Problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 3.

[447] F.S., *Sequences of numbers involved in unsolved problems*, Hexis, 2006.

[448] F.S., *Only problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 3.

[449] E. Grosswald, Pennsylvania University, Philadelphia, SUA, Letter to F.S., 3 august 1985, cited by F.S., *Only Problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 3.

(3)

Enunciation[450]:

Are there, for any set of digits a_1, a_2, …,a_n, primes to contain in their writing the concatenated group a_1a_2 … a_n of these digits? The problem is raised for other bases of numeration beside 10 too, also for n! and for n^n.

Example:

For $a_1 = 0$ and $a_2 = 9$ we have the primes 109, 409, 709, 809 etc.

(4)

Enunciation[451]:

Does the sequence of numbers $d_n = (1/2)*(p_{n+1} - p_n)$, where p_n and p_{n+1} are two consecutive primes, contain an infinite number of primes? Does d_n contain an infinity of numbers of the form n! or of the form n^n?

(5)

Enunciation[452]:

If gcd (a, b) = 1, how many primes does the progression $a*p_n + b$, where n = 1, 2, …, and p_n is the n-th prime, contain? But numbers of the form n! or of the form n^n? Same questions for a^n + b, where a different from {-1, 0, 1}. Same questions for k^k + 1 and k^k – 1, where k is positive integer.

Notes on progression $a*p_n + b$ [453]:

1. For a = 1 and b = 2, we have the classical unsolved problem: "are there infinitely many twin primes?"
2. For a = 2 and b = 1, we have again a classical unsolved problem: "are there infinitely many Sophie-Germain primes?"
3. Henry Ibstedt conjectured that there are infinitely many primes in the progression $a*p_n + b$, if gcd (a, b) = 1.

Notes on progression a^n + b:

1. For a = 2 and b = -1, we have the classical unsolved problem: "are there infinitely many Mersenne primes?"
2. Kenichiro Kashihara conjectured that each element of the family of sequences a^n + b contains an infinite number of prime numbers, for gcd (a, b) = 1 and a different from {-1, 0, 1}, if a + b is odd.[454]

(6)

Enunciation[455]:

How many primes are there in the expression x^y + y^x, where gcd (x, y) = 1?

Notes:

1. This problem is called "*Smarandache expression*".

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[450] F.S., *Only problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 33.

[451] F.S., *Only problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 35. See also Kashihara, K., *Comments and topics on Smarandache notions and problems*, Erhus University Press, 1996, p. 25-26.

[452] F.S., *Only problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 51.

[453] For a study of how the function $a*p_n + b$ behaves see Ibstedt, H., *Surphing on the ocean of numbers – a few Smarandache notions and similar topics*, Erhus University Press, Vail, 1997, Chapter I: *On prime numbers*.

[454] Kashihara, K., *Comments and topics on Smarandache notions and problems*, Erhus University Press, 1996, p. 24.

[455] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Problem 20.

2. Kenichiro Kashihara announced that there are only finitely many numbers of this form which are products of factorials.
3. Florian Luca announced a lower bound for the size of the largest prime divisor of an expression of type a*x^y + b*y^x, where a*b ≠ 0, x, y ≥ 2 and gcd (x, y) = 1.

Chapter VIII. Other unsolved problems

(1)

Enunciation[456]:

Let $\tau(n)$ be the number of positive divisors of n, where n is positive integer. Find the smallest k such that $\tau(\tau(\ldots\tau(n)\ldots)) = 2$, where the function τ is applied repeatedly k times.

(2)

Enunciation[457]:

Find the maximum r such that the set {1, 2,…, r} can be partitioned into n classes such that no class contains integers x, y, z with x*y = z. Same question when x^y = z. Same question when no integer can be the sum of another integers of its class.

(3)

Enunciation[458]:

Let N = {1, 2, ..., n}. Find the maxim number of elements extracted from N such that any m from these be not an arithmetic progression (n > m > 2). Same question when the m elements must not be a geometrical progression.

(4)

Enunciation[459]:

Let f be an arithmetic function and R a k-relation among numbers. How many times can n be expressed as a sum of non-null squares, or cubes, or m-powers? How many times can n be expressed as $R(f(n_1), f(n_2), \ldots, f(n_k))$ for some k and $n_1, n_2, \ldots, n_k$ so that $n_1 + n_2 + \ldots + n_k = n$?

(5)

Enunciation[460]:

Let $\sigma(n)$ be the sum of divisors of n, $\pi(x)$ the number of primes not exceeding x, $\omega(n)$ the number of distinct prime factors of n, $\tau(n)$ the number of positive divisors of n and p(n) the largest prime factor of n. Let $f^{(k)}$ be the function f composed k times, for any function f. Find the smallest k for which:

(i) For fixed n and m, we have $\sigma^{(k)}(n) > m$;
(ii) For a fixed real number x, $x \geq 2$, we have $\pi^{(k)}(x) = 1$;
(iii) For a fixed n, we have $\omega^{(k)}(n) = 1$;
(iiii) For fixed n and m, we have $d^{(k)}(n) > m$;
(iiiii) For a fixed n, we have $p(p(\ldots(p(n) - 1)\ldots) - 1) - 1 = 1$, where the operation $p(n) - 1$ is repetead k times.

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[456] F.S., *Only problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 52.
[457] F.S., *Only problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 57.
[458] F.S., *Only problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 58.
[459] F.S., *Definitions, solved and unsolved problems, conjectures, and theorems in number theory and geometry*, Xiquan Publishing House, 2000, Definition 49 and Problem 17.
[460] F.S., *Only problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 83.

(6)

Enunciation[461]:

For any integers m and n, $n \geq 1$, $m \geq 3$, find the maximum number S(n, m) such that the set {1, 2, 3, …, n} has a subset A of cardinality S(n, m) with the property that A contains no m-term arithmetic progression. S(n, m) is called *the cardinality number*.

[461] F.S., *Only problems, not solutions!*, Xiquan Publishing House, fourth edition, 1993, Problem 156.

AFTERWORD
An infinity of problems concerning the Smarandache function

In the *Abstract* to the paper *An infinity of unsolved problems concerning a function in the number theory*[462], F.S. says: "W. Sierpinski has asserted to an international conference that if mankind lasted for ever and numbered the unsolved problems, then in the long run all these unsolved problems would be solved. The purpose of our paper is that making an infinite number of unsolved problems to prove his supposition is not true. Moreover, the author considers the unsolved problems proposed in this paper can never be all solved!"

Indeed, can be formulated an infinity of problems starting from a simple question raised by F.S. in the above mentioned paper: are there non-null and non-prime integers $a_1, a_2, \ldots, a_n$ in the relation P, so that $S(a_1), S(a_2), \ldots, S(a_n)$ are in the relation R? Where each P, R can represent one of the following number sequences: Abundant numbers, Almost perfect numbers, Amicable numbers, Bell numbers, Bernoulli numbers, Catalan numbers, Carmichael numbers, Congruent numbers, Cullen numbers, Deficient numbers, Euler numbers, Fermat numbers, Fibonacci numbers, Genocchi numbers, Harmonic numbers, Heteromenous numbers, K-hyperperfect numbers, Kurepa numbers, Lucas numbers, Lucky numbers, Mersenne numbers, Multiply perfect numbers, Perfect numbers, Polygonal numbers, Pseudoperfect numbers, Pseudoprime numbers, Pyramidal numbers, Pythagorian numbers, Stirling numbers, Superperfect numbers, Untouchable numbers, Ulam numbers, Weird numbers etc.

As the list of the sequences of numbers related by special properties is potentially infinite, here's how can you construct with just one question an infinity of unsolved problems.

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[462]F.S., *Collected Papers*, vol. III, Abaddaba, Oradea, 2000. In this paper, F.S. raised hundreds of questions concerning the Smarandache function. Some of them were given an answer but many of them still await a solution.

ANNEX A
List of twenty types of numbers named after Florentin Smarandache

(1)
Smarandache numbers

Definition:

The numbers generated by the Smarandache function, *i.e.* the least positive integers k with the property that k! is divisible by n.

The first thirty Smarandache numbers (sequence A002034 in OEIS):

1, 2, 3, 4, 5, 3, 7, 4, 6, 5, 11, 4, 13, 7, 5, 6, 17, 6, 19, 5, 7, 11, 23, 4, 10, 13, 9, 7, 29, 5.

Reference:

Part Two, Chapter 1, Section (1).

(2)
Smarandache consecutive numbers

Definition:

The numbers obtained through the concatenation of first n positive integers.

The first ten such numbers (sequence A007908 in OEIS):

1, 12, 123, 1234, 12345, 123456, 1234567, 12345678, 123456789, 12345678910.

Reference:

Part One, Chapter 1, Section (1).

(3)
Smarandache-Wellin numbers

Definition:

The numbers obtained through the concatenation of first n primes.

The first ten such numbers (sequence A019518 in OEIS):

2, 23, 235, 2357, 235711, 23571113, 2357111317, 235711131719, 23571113171923, 2357111317192329.

Reference:

Part One, Chapter 1, Section (5).

(4)
Smarandache-Fibonacci numbers

Definition:

The positive integers n with the property that S(n) = S(n – 1) + S(n – 2), where S(k) is the Smarandache function.

The first fifteen such numbers (sequence A015047 in OEIS):

11, 121, 4902, 26245, 32112, 64010, 368140, 415664, 2091206, 2519648, 4573053, 7783364, 79269727, 136193976, 321022289.

Reference:

Part One, Chapter 2, Section (31).

(5)

Smarandache-Radu numbers

Definition:

The positive integers n with the property that between S(n) and S(n + 1) there is no prime, where S(n) and S(n +1) are included, where S(k) is the Smarandache function.

The first fifteen such numbers (sequence A015048 in OEIS):

224, 2057, 265225, 843637, 6530355, 24652435, 35558770, 40201975, 45388758, 46297822, 67697937, 138852445, 157906534, 171531580, 299441785.

Reference:

Part One, Chapter 2, Section (32).

(6)
Smarandache friendly numbers

Definition:

The pairs of natural numbers [m, n], where m < n, with the property that the product m*n is equal to the sum of all natural numbers from m to n (m and n are included).

The first four such pairs of numbers:

[1, 1], [3, 6], [15, 35], [85, 204].

Reference:

Part One, Chapter 2, Section (34).

(7)
Smarandache friendly primes

Definition:

The pairs of Smarandache friendly numbers with the property that are also primes.

The five known such pairs of numbers (sequence A176914 in OEIS):

[2, 5], [3, 13], [5, 31], [7, 53], [3536123, 128541727].

Reference:

Part One, Chapter 2, Section (35).

(8)
Pseudo-Smarandache numbers

Definition:

The least positive integers k with the property that 1 + 2 +…+ k is divisible by n, which is equivalent to n divides k*(k + 1)/2.

The first thirty such numbers (sequence A011772 in OEIS):

1, 3, 2, 7, 4, 3, 6, 15, 8, 4, 10, 8, 12, 7, 5, 31, 16, 8, 18, 15, 6, 11, 22, 15, 24, 12, 26, 7, 28, 15, 30, 63, 11, 16, 14, 8, 36, 19, 12, 15.

Reference:

Part Two, Chapter 1, Section (11).

(9)
Pseudo-Smarandache numbers of first kind

Definition:

The least positive integers k with the property that 1^2 + 2^2 +…+ k^2 is divisible by n, which is equivalent to n divides k*(k + 1)*(2*k + 1)/6.

The first fifteen such numbers:

1, 3, 4, 7, 2, 4, 3, 15, 13, 4, 5, 8, 6, 3, 4.

Reference:

Part Two, Chapter 1, Section (12).

(10)
Pseudo-Smarandache numbers of second kind

Definition:

The least positive integers k with the property that 1^3 + 2^3 +…+ k^3 is divisible by n, which is equivalent to n divides k^2*(k + 1)^2/4.

The first fifteen such numbers:

1, 3, 2, 3, 4, 3, 6, 7, 2, 4, 10, 3, 12, 7, 5.

Reference:

Part Two, Chapter 1, Section (13).

(11)
Smarandache wrong numbers

Definition:

The positive integers n, where n = $a_1a_2\ldots a_k$, consisted of at least two digits, with the property that the sequence a_1, a_2, …, a_k, b_{k+1}, b_{k+2}, …(where b_{k+i} is the product of the previous k terms, for any i ≥ 1), contains n as its term.

Reference:

Part One, Chapter 2, Section (48).

(12)
Smarandache impotent numbers

Definition:

The positive integers n with the property that its proper divisors product is less than n.

The first twenty such numbers (sequence A000430 in OEIS):

2, 3, 4, 5, 7, 9, 11, 13, 17, 19, 23, 25, 29, 31, 37, 41, 43, 47, 49, 53, 59.

Reference:

Part One, Chapter 2, Section (49).

(13)
Smarandache simple numbers

Definition:

The positive integers n with the property that its proper divisors product is less than or equal to n.

The first twenty such numbers (sequence A007964 in OEIS):

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 17, 19, 21, 22, 23, 25.

Reference:

Part One, Chapter 2, Section (50).

(14)
Smarandache bad numbers

Definition:

The positive integers n with the property that cannot be expressed as the difference between a cube and a square (in absolute value).

Reference:

Part Three, Chapter 4, Section (2).

(15)
Smarandache primitive numbers

Definition:

The least positive integers k with the property that p^n divides k!, where p is prime. The least positive integers k for which 2^n divides k! are called Smarandache primitive numbers of power two, the least positive integers k for which 3^n divides k! are called Smarandache primitive numbers of power three etc.

The first forty primitive numbers of power two (sequence A007843 in OEIS):

1, 2, 4, 4, 6, 8, 8, 8, 10, 12, 12, 14, 16, 16, 16, 16, 18, 20, 20, 22, 24, 24, 24, 26, 28, 28, 30, 32, 32, 32, 32, 32, 34, 36, 36, 38, 40, 40, 40, 42.

The first forty primitive numbers of power three (sequence A007844 in OEIS):

1, 3, 6, 9, 9, 12, 15, 18, 18, 21, 24, 27, 27, 27, 30, 33, 36, 36, 39, 42, 45, 45, 48, 51, 54, 54, 54, 57, 60, 63, 63, 66, 69, 72, 72, 75, 78, 81, 81, 81, 81.

Reference:

Part One, Chapter 2, Sections (16)-(18).

(16)
Erdős-Smarandache numbers

Definition:

The numbers n which are solutions of the diophantine equation P(n) = S(n), where P(n) is the largest prime factor which divides n, and S(n) is the Smarandache function.

The first twenty-five such numbers:

2, 3, 5, 6, 7, 10, 11, 13, 14, 15, 17, 19, 20, 21, 22, 23, 26, 28, 29, 30, 31, 33, 34, 35, 37.

Reference:

Part One, Chapter 2, Section (75).

(17)
Goldbach-Smarandache numbers

Definition:

The numbers n with the property that n is the largest even number such that any other even number not exceeding it is the sum of two of the first n odd primes.

The first twenty such numbers (sequence A007944 in OEIS):

6, 10, 14, 18, 26, 30, 38, 42, 42, 54, 62, 74, 74, 90, 90, 90, 108, 114, 114, 134.

Reference:

Part One, Chapter 2, Section (76).

(18)
Smarandache-Vinogradov numbers

Definition:

The numbers n with the property that n is the largest odd number such that any odd number greater than or equal to 9 not exceeding it is the sum of three of the first n odd primes.

The first twenty such numbers (sequence A007962 in OEIS):

9, 15, 21, 29, 39, 47, 57, 65, 71, 93, 99, 115, 129, 137, 143, 149, 183, 189, 205, 219.

Reference:

Part One, Chapter 2, Section (77).

(19)

Smarandache perfect and completely perfect numbers[463]

Definition 1:

An integer n, $n \geq 1$, is called *Smarandache perfect* (or *S-perfect*) if and only if n is equal to te sum from i = 1 to i = k of the numbers $S(d_i)$, where S is the Smarandache function and $d_1 = 1, d_2, \ldots, d_i = d_k$ are the proper divisors of n.

Definition 2:

An integer n, $n \geq 1$, is called *Smarandache completely perfect* (or *completely S-perfect*) if and only if n is equal to te sum from i = 1 to i = n of the numbers $S(d_i)$, where S is the Smarandache function and $d_1 = 1, d_2, \ldots, d_i = n$ are the divisors of n.

Note:

In the same way, considering beside the Smarandache function the pseudo-Smarandache function, are defined the Z-perfect and the completely Z-perfect numbers.[464]

(20)

Smarandache Ulam numbers[465]

Definition:

The numbers obtained concatenating the Ulam numbers.[466] The first few *Smarandache Ulam numbers* (or, in other words, the first terms of the *Smarandache U-sequence*) are: 1, 12, 123, 1234, 12346, 123468, 12346811, 1234681113, 123468111316, …

Comment:

There are only two primes known in the first 3200 terms of this sequence, *i.e.* SU(22) and SU(237), where SU(n) is the n-th element of the sequence.

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463 Introduced by A.A.K. Majumdar, see *S-perfect and completely S-perfect numbers*, in Wenpeng, Zhang (editor), *Research on number theory and Smarandache notions (Proceedings of the fifth international conference on number theory and Smarandache notions)*, Hexis, 2009. The author proved that the only S-perfect numbers are 1 and 6 and the only completely perfect S-numbers are 1 and 28.

464 To find the all Z-perfect and completely Z-perfect numbers is still an open problem. The only known Z-perfect numbers less than 10^6 are 4 and 6.

465 Introduced by Shyam Sunder Gupta, see *Smarandache sequence of Ulam numbers*, in Wenpeng, Zhang (editor), *Research on number theory and Smarandache notions (Proceedings of the fifth international conference on number theory and Smarandache notions)*, Hexis, 2009.

466 An (m, n) - Ulam number is said to be a term of the sequence defined in the following way: the first term of the sequence is equal to m, the second is equal to n and the following terms are the least integers that can be expressed in an unique way as the sum of two distinct earlier terms. Here is considered the *standard Ulam sequence*, where m = 1 and n = 2: the first terms of this sequence are: 1, 2, 3, 4, 6, 8, 11, 13, 16, 18, 26, 28, 36, 38, 47, … (sequence A002858 in OEIS).

ANNEX B
A proposal for a new Smarandache type notion

Definition 1:

We call *the set of Smarandache-Coman divisors of order 1 of a composite positive integer n with m prime factors*, $n = d_1*d_2*\ldots*d_m$, where the least prime factor of n, d_1, is greater than or equal to 2, the set of numbers defined in the following way:

$SCD_1(n) = \{S(d_1 - 1), S(d_2 - 1), \ldots, S(d_m - 1)\}$, where S is the Smarandache function.

Examples:

1. The set of SC divisors of order 1 of the number 6 is $\{S(2 - 1), S(3 - 1)\} = \{S(1), S(2)\} = \{1, 2\}$, because $6 = 2*3$;
2. $SCD_1(429) = \{S(3 - 1), S(11 - 1), S(13 - 1)\} = \{S(2), S(10), S(12)\} = \{2, 5, 4\}$, because $429 = 3*11*13$.

Definition 2:

We call *the set of Smarandache-Coman divisors of order 2 of a composite positive integer n with m prime factors*, $n = d_1*d_2*\ldots*d_m$, where the least prime factor of n, d_1, is greater than or equal to 3, the set of numbers defined in the following way:

$SCD_2(n) = \{S(d_1 - 2), S(d_2 - 2), \ldots, S(d_m - 2)\}$, where S is the Smarandache function.

Examples:

1. The set of SC divisors of order 2 of the number 21 is $\{S(3 - 2), S(7 - 2)\} = \{S(1), S(5)\} = \{1, 5\}$, because $21 = 3*7$;
2. $SCD_2(2429) = \{S(7 - 2), S(347 - 2)\} = \{S(5), S(345)\} = \{5, 23\}$, because $2429 = 7*347$.

Definition 3:

We call *the set of Smarandache-Coman divisors of order k of a composite positive integer n with m prime factors*, $n = d_1*d_2*\ldots*d_m$, where the least prime factor of n, d_1, is greater than or equal to k + 1, the set of numbers defined in the following way:

$SCD_k(n) = \{S(d_1 - k), S(d_2 - k), \ldots, S(d_m - k)\}$, where S is the Smarandache function.

Examples:

1. The set of SC divisors of order 5 of the number 539 is $\{S(7 - 5), S(11 - 5)\} = \{S(2), S(6)\} = \{2, 3\}$, because $539 = 7^2*11$;
2. $SCD_6(221) = \{S(13 - 6), S(17 - 6)\} = \{S(7), S(11)\} = \{7, 11\}$, because $221 = 13*17$.

Comment:

We obviously defined the sets of numbers above because we believe that they can have interesting applications, in fact we believe that they can even make us re-think and re-consider the Smarandache function as an instrument to operate in the world of number theory: while at the beginning its value was considered to consist essentially in that to be a criterion for primality, afterwards the Smarandache function crossed a normal process of substantiation, so it was constrained to evolve in a relatively closed (even large) circle of equalities, inequalities, conjectures and theorems concerning, most of them, more or less related concepts. We strongly believe that some of the most important applications of the Smarandache function are still undiscovered. We were inspired in defining the Smarandache-Coman divisors by the passion for Fermat pseudoprimes, especially for

Carmichael numbers and Poulet numbers, by the Korselt's criterion, one of the very few (and the most important from them) instruments that allow us to comprehend Carmichael numbers, and by the encouraging results we easily obtained, even from the first attempts to relate these two types of numbers, Fermat pseudoprimes and Smarandache numbers.

Smarandache-Coman divisors of order 1 of the 2-Poulet numbers:

(See the sequence A214305 in OEIS, posted by us, for a list with Poulet numbers with two prime factors)

$SCD_1(341) = SCD_1(11*31) = \{S(11 – 1), S(31 – 1)\} = \{S(10), S(30)\} = \{5, 5\}$;
$SCD_1(1387) = SCD_1(19*73) = \{S(19 – 1), S(73 – 1)\} = \{S(18), S(72)\} = \{6, 6\}$;
$SCD_1(2047) = SCD_1(23*89) = \{S(23 – 1), S(89 – 1)\} = \{S(22), S(88)\} = \{11, 11\}$;
$SCD_1(2701) = SCD_1(37*73) = \{S(37 – 1), S(73 – 1)\} = \{S(36), S(72)\} = \{6, 6\}$;
$SCD_1(3277) = SCD_1(29*113) = \{S(29 – 1), S(113 – 1)\} = \{S(28), S(112)\} = \{7, 7\}$;
$SCD_1(4033) = SCD_1(37*109) = \{S(37 – 1), S(109 – 1)\} = \{S(36), S(108)\} = \{6, 9\}$;
$SCD_1(4369) = SCD_1(17*257) = \{S(17 – 1), S(257 – 1)\} = \{S(16), S(256)\} = \{6, 10\}$;
$SCD_1(4681) = SCD_1(31*151) = \{S(31 – 1), S(151 – 1)\} = \{S(30), S(150)\} = \{5, 10\}$;
$SCD_1(5461) = SCD_1(43*127) = \{S(43 – 1), S(127 – 1)\} = \{S(42), S(126)\} = \{7, 7\}$;
$SCD_1(7957) = SCD_1(73*109) = \{S(73 – 1), S(109 – 1)\} = \{S(72), S(108)\} = \{6, 9\}$;
$SCD_1(8321) = SCD_1(53*157) = \{S(53 – 1), S(157 – 1)\} = \{S(52), S(156)\} = \{13, 13\}$.

Comment:

It is notable how easily are obtained interesting results: from the first 11 terms of the 2-Poulet numbers sequence checked there are already foreseen few patterns.

Open problems:

1. Is for the majority of the 2-Poulet numbers the case that the two Smarandache-Coman divisors of order 1 are equal, as for the seven from the eleven numbers checked above?
2. Is there an infinity of 2-Poulet numbers for which the set of SCD of order 1 is equal to {6, 6}, the case of Poulet numbers 1387 and 2701, or with {6, 9}, the case of Poulet numbers 4033 and 7957?

Smarandache-Coman divisors of order 2 of the 2-Poulet numbers:

$SCD_2(341) = SCD_2(11*31) = \{S(11 – 2), S(31 – 2)\} = \{S(9), S(29)\} = \{6, 29\}$;
$SCD_2(1387) = SCD_2(19*73) = \{S(19 – 2), S(73 – 2)\} = \{S(17), S(71)\} = \{17, 71\}$;
$SCD_2(2047) = SCD_2(23*89) = \{S(23 – 2), S(89 – 2)\} = \{S(21), S(87)\} = \{7, 29\}$;
$SCD_2(2701) = SCD_2(37*73) = \{S(37 – 2), S(73 – 2)\} = \{S(35), S(71)\} = \{7, 71\}$;
$SCD_2(3277) = SCD_2(29*113) = \{S(29 – 2), S(113 – 2)\} = \{S(27), S(111)\} = \{9, 37\}$;
$SCD_2(4033) = SCD_2(37*109) = \{S(37 – 2), S(109 – 2)\} = \{S(35), S(107)\} = \{7, 107\}$;
$SCD_2(4369) = SCD_2(17*257) = \{S(17 – 2), S(257 – 2)\} = \{S(15), S(255)\} = \{5, 17\}$;
$SCD_2(4681) = SCD_2(31*151) = \{S(31 – 2), S(151 – 2)\} = \{S(29), S(149)\} = \{29, 149\}$;
$SCD_2(5461) = SCD_2(43*127) = \{S(43 – 2), S(127 – 2)\} = \{S(41), S(125)\} = \{41, 15\}$;
$SCD_2(7957) = SCD_2(73*109) = \{S(73 – 2), S(109 – 2)\} = \{S(71), S(107)\} = \{71, 107\}$;
$SCD_2(8321) = SCD_2(53*157) = \{S(53 – 2), S(157 – 2)\} = \{S(52), S(156)\} = \{17, 31\}$.

Comment:

In the case of SCD of order 2 of the 2-Poulet numbers there are too foreseen few patterns.

Open problems:

1. Is for the majority of the 2-Poulet numbers the case that the two Smarandache-Coman divisors of order 2 are both primes, as for the eight from the eleven numbers checked above?
2. Is there an infinity of 2-Poulet numbers for which the set of SCD of order 2 is equal to {p, p + 20*k}, where p prime and k positive integer, the case of Poulet numbers 4033 and 4681?

Smarandache-Coman divisors of order 1 of the 3-Poulet numbers:

(See the sequence A215672 in OEIS, posted by us, for a list with Poulet numbers with two prime factors)

$SCD_1(561)$	$= SCD_1(3*11*17) = \{S(2), S(10), S(16)\}$	$= \{2, 5, 6\}$;
$SCD_1(645)$	$= SCD_1(3*5*43) = \{S(2), S(4), S(42)\}$	$= \{2, 4, 7\}$;
$SCD_1(1105)$	$= SCD_1(5*13*17) = \{S(4), S(12), S(16)\}$	$= \{4, 4, 6\}$;
$SCD_1(1729)$	$= SCD_1(7*13*19) = \{S(6), S(12), S(18)\}$	$= \{3, 4, 6\}$;
$SCD_1(1905)$	$= SCD_1(3*5*127) = \{S(2), S(4), S(126)\}$	$= \{2, 4, 7\}$;
$SCD_1(2465)$	$= SCD_1(5*17*29) = \{S(4), S(16), S(28)\}$	$= \{4, 6, 7\}$;
$SCD_1(2821)$	$= SCD_1(7*13*31) = \{S(6), S(12), S(30)\}$	$= \{3, 4, 5\}$;
$SCD_1(4371)$	$= SCD_1(3*31*47) = \{S(2), S(30), S(46)\}$	$= \{2, 5, 23\}$;
$SCD_1(6601)$	$= SCD_1(7*23*41) = \{S(6), S(22), S(40)\}$	$= \{3, 11, 5\}$;
$SCD_1(8481)$	$= SCD_1(3*11*257) = \{S(2), S(10), S(256)\}$	$= \{2, 5, 10\}$;
$SCD_1(8911)$	$= SCD_1(7*19*67) = \{S(6), S(18), S(66)\}$	$= \{3, 19, 67\}$.

Open problems:

1. Is there an infinity of 3-Poulet numbers for which the set of SCD of order 1 is equal to {2, 4, 7}, the case of Poulet numbers 645 and 1905?
2. Is there an infinity of 3-Poulet numbers for which the sum of SCD of order 1 is equal to 13, the case of Poulet numbers 561 (2 + 5 + 6 = 13), 645 (2 + 4 + 7 = 13), 1729 (3 + 4 + 6 = 13), 1905 (2 + 4 + 7 = 13) or is equal to 17, the case of Poulet numbers 2465 (4 + 6 + 7 = 17) and 8481 (2 + 5 + 10 = 17)?
3. Is there an infinity of Poulet numbers for which the sum of SCD of order 1 is prime, which is the case of the eight from the eleven numbers checked above? What about the sum of SCD of order 1 plus 1, the case of Poulet numbers 2821 (3 + 4 + 5 + 1 = 13) and 4371 (2 + 5 + 23 + 1 = 31) or the sum of SCD of order 1 minus 1, the case of Poulet numbers 1105 (4 + 4 + 6 – 1 = 13), 2821 (3 + 4 + 5 – 1 = 11) and 4371 (2 + 5 + 23 – 1 = 29)?

Note: We stop here for now, because the purpose of this book is not to substantiate new concepts but to show the richness and the potential of the already largely known Smarandache notions.

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About the works of Florentin Smarandache have been written a lot of books (he himself wrote dozens of books and articles regarding math, physics, literature, philosophy). Being a globally recognized personality in both mathematics (there are countless functions and concepts that bear his name) and literature, it is natural that the volume of writings about his research is huge. What we try to do with this encyclopedia is to gather together as much as we can both from Smarandache's mathematical work and the works of many mathematicians around the world inspired by the Smarandache notions. We structured this book using numbered Definitions, Theorems, Conjectures, Notes and Comments, in order to facilitate an easier reading but also to facilitate references to a specific paragraph. We divided the Bibliography in two parts, Writings by Florentin Smarandache (indexed by the name of books and articles) and Writings on Smarandache notions (indexed by the name of authors). We treated, in this book, about 130 Smarandache type sequences, about 50 Smarandache type functions and many solved or open problems of number theory. We also have, at the end of this book, a proposal for a new Smarandache type notion, id est the concept of "a set of Smarandache-Coman divisors of order k of a composite positive integer n with m prime factors", notion that seems to have promising applications, at a first glance at least in the study of absolute and relative Fermat pseudoprimes, Carmichael numbers and Poulet numbers. This encyclopedia is both for researchers that will have on hand a tool that will help them "navigate" in the universe of Smarandache type notions and for young math enthusiasts: many of them will be attached by this wonderful branch of mathematics, number theory, reading the works of Florentin Smarandache.

ISBN 9781599732527

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