ON SOME OF THE SMARANDACHE'S PROBLEMS

Krassimir T. Atanassov

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PREFACE

In 1996 the author wrote reviews for "Zentralblatt für Mathematik" for books [1] and [2] and this was him first contact of with the Smarandache's problems.

In [1] Florentin Smarandache formulated 105 unsolved problems, while in [2] C. Dumitrescu and V. Seleacu formulated 140 unsolved problems. The second book contains almost all problems from [1], but now every one problem has unique number and by this reason the author will use the numeration of the problems from [2]. Also, in [2] there are some problems, which are not included in [1]. On the other hane, there are problems from [1], which are not included in [2]. One of them is Problem 62 from [1], which is included here under the same number.

In the summer of 1998 the author found the books in his library and for a first time tried to solve a problem from them. After some attempts one of the problems was solved and this was a power impulse for the next research. In the present book are collected the 27 problems solved by the middle of February 1999.

The bigger part of the problems discussed in the present book (22 in number) are related to different sequences. For each of them the form of the $n-$th member is determined and for all of them except 4 problems - the form of the $n-$th partial sum. Four of the problems are proved; modifications of two of the problems are formulated; counterexamples to two of the problems are constructed.

When the text was ready, the author received from "Zentralblatt für Mathematik" Charles Ashbacher's book [8] for reviewing. The author read immediatelly the book [8] and he was delighted to see that only five of the problems on which he had worked are discussed there and that the approach to these problems is different in both books. Reading [8], the author understood that there are other books related to the Smarandache's problems [9-13], which he had not known up to the moment.

The author hopes to prove some other problems from [1] and [2] in future, but there are problems, for which it is not clear whether they will be solved in the next years or will share the fate of Fermat's Last Theorem.

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§1. ON THE 4-th SMARANDACHE'S PROBLEM [1]

The 4-th problem from [2] (see also 18-th problem from [1]) is the following:

Smarandache deconstructive sequence:

$$\underbrace{1,\ 23,\ 456,\ 789}\underbrace{1,\ 23456,\ 789}\underbrace{123,\ 456789}\underbrace{1,\ 23456789},$$

$$\underbrace{123456789},\ \underbrace{123456789}1,\ \ldots$$

Let the n-th term of the above sequence be a_n. Then we can see that the first digits of the first nine members are, respectively: 1, 2, 4, 7, 2, 7, 4, 2, 1. Let us define the function ω as follows:

r	$\omega(r)$
0	1
1	1
2	2
3	4
4	7
5	2
6	7
7	4
8	2
9	1

[1] see also
K. Atanassov, On the 4-th Smarandache's problem. *Notes on Number Theory and Discrete Mathematics*, Vol. 5 (1999), No. 1, 33-35.

Here we shall use the arithmetic function ψ, discussed shortly in §16 and detailed in the author's paper [3] .

Now, we can prove that the form of the n-th member of the above sequence is

$$a_n = \overline{b_1 b_2 ... b_n},$$

where

$$b_1 = \omega(n - [\frac{n}{9}])$$

$$b_2 = \psi(\omega(n - [\frac{n}{9}]) + 1)$$

$$\dots$$

$$b_n = \psi(\omega(n - [\frac{n}{9}]) + n - 1).$$

Every natural number n can be represented in the form

$$n = 9q + r,$$

where $q \geq 0$ is a natural number and $r \in \{1, 2, ..., 9\}$.

We shall prove by induction that the forms of nine sequential members $a_{n+1}, a_{n+2}, ..., a_{n+9}$, where $n = 9q + r$, are the following:

$$a_{9q+1} = \underbrace{\underbrace{12...9}\underbrace{12...9}...\underbrace{12...9}}_{q \text{ times}}1$$

$$a_{9q+2} = \underbrace{\underbrace{23...1}\underbrace{23...1}...\underbrace{23...1}}_{q \text{ times}}23$$

$$a_{9q+3} = \underbrace{\underbrace{45...3}\underbrace{45...3}...\underbrace{45...3}}_{q \text{ times}}456$$

$$a_{9q+4} = \underbrace{\underbrace{78...6}\underbrace{78...6}...\underbrace{78...6}}_{q \text{ times}}7891$$

$$a_{9q+5} = \underbrace{\underbrace{23...1}\underbrace{23...1}...\underbrace{23...1}}_{q \text{ times}}23456$$

$$a_{9q+6} = \underbrace{\underbrace{78...6}\underbrace{78...6}...\underbrace{78...6}}_{q \text{ times}}789123$$

$$a_{9q+7} = \underbrace{\underbrace{45...3}\underbrace{45...3}...\underbrace{45...3}}_{q \text{ times}}4567891$$

$$a_{9q+8} = \underbrace{\underbrace{23...1}\underbrace{23...1}...\underbrace{23...1}}_{q \text{ times}}23456789$$

$$a_{9q+9} = \underbrace{\underbrace{12...9}\underbrace{12...9}...\underbrace{12...9}}_{q+1 \text{ times}}$$

When $q = 0$ the validity of the above assertion is obvious. Let us assume that for some natural number q, a_{9q+1}, a_{9q+2}, ... a_{9q+9} have the above forms. Then for a_{9q+10}, a_{9q+11}, ... a_{9q+18} we obtain the following representations, taking $p = q + 1$:

$$a_{9q+10} = a_{9p+1} = \underbrace{\underbrace{12...9}\underbrace{12...9}...\underbrace{12...9}}_{p \text{ times}}1$$

$$a_{9q+11} = a_{9p+2} = \underbrace{\underbrace{23...1}\underbrace{23...1}...\underbrace{23...1}}_{p \text{ times}}23$$

$$a_{9q+12} = a_{9p+3} = \underbrace{\underbrace{45...3}\underbrace{45...3}...\underbrace{45...3}}_{p \text{ times}}456$$

$$a_{9q+13} = a_{9p+4} = \underbrace{\underbrace{78...6}\underbrace{78...6}...\underbrace{78...6}}_{p \text{ times}}7891$$

$$a_{9q+14} = a_{9p+5} = \underbrace{\underbrace{23...1}\underbrace{23...1}...\underbrace{23...1}}_{p \text{ times}}23456$$

$$a_{9q+15} = a_{9p+6} = \underbrace{\underbrace{78...6}\underbrace{78...6}...\underbrace{78...6}}_{p \text{ times}}789123$$

$$a_{9q+16} = a_{9p+7} = \underbrace{\underbrace{45...3}\underbrace{45...3}...\underbrace{45...3}}_{p \text{ times}}4567891$$

$$a_{9q+17} = a_{9p+8} = \underbrace{\underbrace{23...1}\underbrace{23...1}...\underbrace{23...1}}_{p \text{ times}}23456789$$

$$a_{9q+18} = a_{9p+9} = \underbrace{\underbrace{12...9}\underbrace{12...9}...\underbrace{12...9}}_{p+1 \text{ times}}$$

To the above sequence $\{a_n\}_{n=1}^{\infty}$ we can juxtapose the sequence $\{\psi(a_n)\}_{n=1}^{\infty}$ for which we can prove (as above) that its basis is $[1, 5, 6, 7, 2, 3, 4, 8, 9]$.

The problem can be generalized, e.g., to the following form:

Study the sequence $\{a_n\}_{n=1}^{\infty}$, which $s-th$ member has the form

$$a_s = \overline{b_1 b_2 ... b_{s.k}},$$

where $b_1 b_2 ... b_{s.k} \in \{1, 2, ..., 9\}$ and

$$b_1 = \omega'(s - [\frac{s}{9}])$$

$$b_2 = \psi(\omega'(s - [\frac{s}{9}]) + 1)$$

$$...$$

$$b_{s.k} = \psi(\omega'(s - [\frac{s}{9}]) + s.k - 1),$$

and here

r	$\omega'(r)$
1	1
2	$\psi(k+1)$
3	$\psi(3k+1)$
4	$\psi(6k+1)$
5	$\psi(10k+1)$
6	$\psi(15k+1)$
7	$\psi(21k+1)$
8	$\psi(28k+1)$
9	$\psi(36k+1)$

For example, when $k = 2$:

$$\underbrace{12,\quad 3456,\quad 789}\underbrace{123,\quad 456789}\underbrace{12,\quad 3456789}\underbrace{123,\quad 456789}\underbrace{123456,\qquad}$$

$$\underbrace{\qquad 789}\underbrace{123456789}\underbrace{12,\quad 3456789}\underbrace{123456789},$$

$$\underbrace{123456789}\underbrace{123456789},\ldots$$

To the last sequence $\{a_n\}_{n=1}^{\infty}$ we can juxtapose again the sequence $\{\psi(a_n)\}_{n=1}^{\infty}$ for which we can prove (as above) that its basis is $[3,9,3,6,3,6,9,8,9]$.

§2. ON THE 16-th SMARANDACHE'S PROBLEM [2]

The 16-th problem from [2] (see also 21-st problem from [1]) is the following:

Digital sum:

$$\underbrace{0,1,2,3,4,5,6,7,8,9},\underbrace{1,2,3,4,5,6,7,8,9,10},\underbrace{2,3,4,5,6,7,8,9,10,11},$$

$$\underbrace{3,4,5,6,7,8,9,10,11,12},\underbrace{4,5,6,7,8,9,10,11,12,13},$$

$$\underbrace{5,6,7,8,9,10,11,12,13,14},\ldots \qquad (1)$$

($d_s(n)$ is the sum of digits.)
Study this sequence.

First we shall note that function d_s is the first step of another arithmetic (digital) function φ, discussed in details in the author's paper [3] and shortly - in §16.

After applying of this function over the set of the natural numbers, or over the above sequence, we obtain the sequence

$$\underbrace{0,1,2,3,4,5,6,7,8,9},\underbrace{1,2,3,4,5,6,7,8,9},\underbrace{10,2,3,4,5,6,7,8,9},\ldots$$

$$\underbrace{10,11,3,4,5,6,7,8,9},\ldots\underbrace{10,11,12,4,5,6,7,8,9},\ldots$$

[2] see also

K. Atanassov, On the 16-th Smarandache's problem. *Notes on Number Theory and Discrete Mathematics,* Vol. 5 (1999), No. 1, 36-38.

On the other hand, in [3] (shortly in §16) another function (ψ) is introduced. After its applying over the set of the natural numbers, or over the above sequence, we obtain the sequence

$$\underbrace{0,1,2,3,4,5,6,7,8,9},\underbrace{1,2,3,4,5,6,7,8,9},\underbrace{1,2,3,4,5,6,7,8,9},...$$

and the set $[1,2,3,4,5,6,7,8,9]$ is called a *basis* of the set of the natural numbers about ψ.

Below we shall show the form of the general term of the sequence from the Smarandache's problem. Let its members are denoted as $a_1, a_2, ..., a_n, ...$ The form of the member a_n is:

$$a_n = n - 9. \sum_{k=1}^{\infty} [\frac{n}{10^k}]. \qquad (2)$$

The validity of (2) can be proved, e.g., by induction. It is obviously valid for $n = 1$. Let us assume that for some n (2) is true. For n there are two cases.

Case 1: $n \neq \underbrace{9\,9\,...\,9}_{m \text{ times}}$ $(m \geq 1)$. Therefore

$$n + 1 \leq \underbrace{9\,9...\,9}_{m \text{ times}}$$

and

$$\sum_{k=1}^{\infty} [\frac{n}{10^k}] = \sum_{k=1}^{\infty} [\frac{n+1}{10^k}],$$

from where

$$a_{n+1} = a_n + 1 = n - 9. \sum_{k=1}^{\infty} [\frac{n}{10^k}] + 1 = (n+1) - 9. \sum_{k=1}^{\infty} [\frac{n+1}{10^k}].$$

Case 2: $n = \underbrace{9\,9\,...\,9}_{m \text{ times}}$. Therefore

$$n + 1 = 1\,\underbrace{0\,0...\,0}_{m \text{ times}}$$

and

$$a_{n+1} = 1 = 1\,\underbrace{0\,0...\,0}_{m \text{ times}} - \underbrace{9\,9\,...\,9}_{m \text{ times}} = 1\,\underbrace{0\,0...\,0}_{m \text{ times}} - 9.(1\ \underbrace{0\,0\,...\,0}_{m-1 \text{ times}}$$

$$+1\ \underbrace{0\,0\,...\,0}_{m-2 \text{ times}}\ + ...1)$$

$$= 1\,\underbrace{0\,0...\,0}_{m \text{ times}} - 9.\ \sum_{k=1}^{\infty}\ [\frac{100...0}{10^k}] = (n+1) - 9.\ \sum_{k=1}^{\infty}\ [\frac{n+1}{10^k}].$$

Therefore (2) is true.

The second important question, which must be disccused about the sequence (1), is the validity of the equality $d_s(m) + d_s(n) = d_s(m+n)$. Obviously, it is not always valid. For example

$$d_s(2) + d_s(3) = 2 + 3 = 5 = d_s(5),$$

but

$$d_s(52) + d_s(53) = 7 + 8 = 15 \neq 6 = d_s(105).$$

The following assertion is true

$$d_s(m+n) = \begin{cases} d_s(m) + d_s(n), \text{ if } d_s(m) + d_s(n) \leq 9.\max([\frac{d_s(m)}{9}], \\ \qquad [\frac{d_s(n)}{9}]) \\ d_s(m) + d_s(n) - 9.\max([\frac{d_s(m)}{9}], [\frac{d_s(n)}{9}]), \text{ otherwise} \end{cases}$$

The proof can be made again by the method of induction.

Let

$$R_k = k + (k+1) + ... + (k+9) = 10k + 45.$$

Obviously, R_k is the sum of the elements of the k-th group of (1).

Therefore, the sum of the first n members of (1) will be

$$S_n = \sum_{k=0}^{[\frac{n}{10}]-1} R_k + [\frac{n}{10}] + ([\frac{n}{10}] + 1) + ... + ([\frac{n}{10}] + n - 10.[\frac{n}{10}] - 1)$$

$$= 5.[\frac{n}{10}].([\frac{n}{10}] + 8) + (n - 10.[\frac{n}{10}]).[\frac{n}{10}] + \frac{1}{2}.(n - 10.[\frac{n}{10}]) .(n - 10.[\frac{n}{10}] - 1),$$

i.e.,

$$S_n = 5.[\frac{n}{10}].([\frac{n}{10}] + 8) + (n - 10.[\frac{n}{10}]).(\frac{n-1}{2} - 4.[\frac{n}{10}]).$$

This equality can be proved directly or by induction.