

NEUTROSOPHIC SYSTEMS WITH APPLICATIONS

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“Neutrosophic Systems with Applications” has been created for publications on advanced studies in neutrosophy, neutrosophic set, neutrosophic logic, neutrosophic probability, neutrosophic statistics that started in 1995 and their applications in any field, such as the neutrosophic structures developed in algebra, geometry, topology, etc. The submitted papers should be professional, in good English, containing a brief review of a problem and obtained results.

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea $\langle A \rangle$ together with its opposite or negation $\langle \text{anti}A \rangle$ and with their spectrum of neutralities $\langle \text{neut}A \rangle$ in between them (i.e., notions or ideas supporting neither $\langle A \rangle$ nor $\langle \text{anti}A \rangle$). The $\langle \text{neut}A \rangle$ and $\langle \text{anti}A \rangle$ ideas together are referred to as $\langle \text{non}A \rangle$.

Neutrosophy is a generalization of Hegel's dialectics (the last one is based on $\langle A \rangle$ and $\langle \text{anti}A \rangle$ only). According to this theory every idea $\langle A \rangle$ tends to be neutralized and balanced by $\langle \text{anti}A \rangle$ and $\langle \text{non}A \rangle$ ideas - as a state of equilibrium.

In a classical way $\langle A \rangle$, $\langle \text{neut}A \rangle$, $\langle \text{anti}A \rangle$ are disjointed two by two. But, since in many cases the borders between notions are vague, imprecise, Sorites, it is possible that $\langle A \rangle$, $\langle \text{neut}A \rangle$, $\langle \text{anti}A \rangle$ (and $\langle \text{non}A \rangle$ of course) have common parts two by two, or even all three of them as well.

Neutrosophic Set and Neutrosophic Logic are generalizations of the fuzzy set and respectively fuzzy logic (especially of intuitionistic fuzzy set and respectively intuitionistic fuzzy logic). In neutrosophic logic a proposition has a degree of truth (T), a degree of indeterminacy (I), and a degree of falsity (F), where T, I, F are standard or non-standard subsets of $] -0, 1+ [$.

Neutrosophic Probability is a generalization of the classical probability and imprecise probability.

Neutrosophic Statistics is a generalization of classical statistics.

What distinguishes neutrosophic from other fields is the $\langle \text{neut}A \rangle$, which means neither $\langle A \rangle$ nor $\langle \text{anti}A \rangle$.

$\langle \text{neut}A \rangle$, which of course depends on $\langle A \rangle$, can be indeterminacy, neutrality, tie game, unknown, contradiction, ignorance, imprecision, etc.

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The Use of Neutrosophic Methods of Operation Research in the Management of Corporate Work

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Abstract: The science of operations research is one of the modern sciences that have made a great revolution in all areas of life through the methods provided by it, suitable and appropriate to solve most of the problems that were facing researchers, scholars and those interested in the development of societies, and the most beneficiaries of this science were companies and institutions that are looking for scientific methods that help them manage their work so that they achieve the greatest profit and the lowest cost, and one of the important methods that have been used in the management of companies we offer in this research two methods, Dynamic programming method. This method has been used in many practical matters and helped decision-makers in companies to achieve a maximum profit and less cost by formulating the reality of the state of the company and the data provided by decision-makers with a dynamic mathematical model that is solved using methods of solving dynamic models and we will provide in this research an example of this through the issue of choosing the optimal investment for the budget of a company so that it achieves a maximum profit, and the method of programming with integers: the method that provided these companies with solutions with integer values suitable for the nature of its work, through the use of the binary integer in the formulation of the appropriate mathematical model on the one hand, and on the other hand, the use of the binary integer variable helped to convert some nonlinear models that lead to some practical problems into linear models, and it should be noted here that in the previous two methods there is something indeterminable because we must make a decision in choosing or not choosing something, but the optimal solution that we will get remains A specific value because we are building the mathematical model for any realistic issue through the data provided by those responsible for the work and these data are calculated quantities and therefore they are uncertain values because their validity depends on the circumstances surrounding the work environment, they may be exposed to increase or decrease, and therefore the optimal solution on which the company will base its decision is suitable for specific values and any change in them can cause the company an uncalculated loss, so in this research we will use the concepts of neutrosophic science, the branch of science founded by the American scientist Florentin Smarandache in 1995 based on his belief that there is no absolute truth, a science that is interested in the study of ideas and concepts that are neither true nor false, but just in-between, and we will take the data (calculated quantities) neutrosophic values that are specified or unspecified values are any set close to the calculated quantities, then the resulting mathematical model is a neutrosophic model and the optimal solution has neutrosophic values and thanks to the indefinite uncertainty that these values have, companies from the development of appropriate plans for all circumstances and thus achieve the greatest profit and the lowest cost, and we will clarify the above through two issues, the issue of optimal designation of a warehouse site, which we will formulate the mathematical model of using the neutrosophic integer programming method - and the issue of capital budget, which we will present in two different forms, we use in the first form the neutrosophic integer programming method and in the second the neutrosophic dynamic programming method.

Keywords: Operations Research; Neutrosophic Integer Programming; Binary Integer Variable; Neutrosophic Dynamic Programming; Neutrosophic Science; Neutrosophic Linear Mathematical Model; Neutrosophic Nonlinear Mathematical Model; The Problem of Locating a Repository; The Problem of Capital Budget.

1. Introduction

The splendor of any science is completed through its scientific methods that help solve practical issues facing us in our daily lives and work on the development of societies, and one of the important methods presented by the science of operations research is the method of programming with integers, which depends on building mathematical models using the binary correct variable, which enabled researchers to convert some nonlinear mathematical models into linear mathematical models, in addition to that when solving these models, beam compounds were the optimal solution correct values that meet the nature of the issue under study and do not need to rotate fractional values for the optimal solution to obtain correct values as we used to do previously, the greatness of the science of operations research is when it meets with the concepts of neutrosophic science, the latest and most important thing that science has offered in our time, the science that brought values out of determination to indeterminacy, this margin of freedom can be greatly benefited from when the methods of operations research are reformulated using its concepts, a complement to what we have presented from research in which we formulated some methods of process research according to the concepts of neutrosophic [1–21]. Due to the importance of the dynamic programming method, we also presented a paper entitled Neutrosophic Dynamic Programming, through which we explained how to formulate dynamic models using the concepts of neutrosophic science [22]. In another paper, we used the neutrosophic integer programming method entitled Neutrosophic Mathematical Model of the product mixture problem using the binary integer variable [23]. In this research, we will use the method of dynamic neutrosophic programming and the method of neutrosophic integer programming to help companies manage their work in a scientific manner that enables them to exploit the resources available to them in an ideal way that achieves the greatest profit and the least loss by formulating two issues: the question of optimal designation of a warehouse site and the question of capital budget using the concepts of neutrosophic science. It should be noted that these two issues were studied using classical values, see reference [24, 25].

2. Discussion

Since the aim of this research is to help decision-makers in companies and institutions to make the optimal decision to ensure the greatest profit and the lowest cost, we must study the issue at hand a good study through which we can determine the data that are affected by the surrounding conditions, and then we take these neutrosophic data values any unspecified values take the form

$Na_i = a_i + \varepsilon_i$ where ε_i is the indeterminacy on the data can take one of the forms $[\lambda_{i1}, \lambda_{i2}]$ أو

$\{\lambda_{i1}, \lambda_{i2}\}$. Or----- otherwise, which is any neighborhood containing the value a_i .

We clarify the above through the following problems:

2.1 The first issue

An executive in one of the companies asked an expert in the science of operations research to help him obtain an optimal solution through which to achieve the lowest cost of transportation and operation of warehouses he wants to establish in order to expand the company's work and provided him with information through which the expert formulated the following issue: The text of the problem according to the concepts of neutrosophic science: A retail company plans to expand its activities in a specific area by establishing two new warehouses, the following Table 1 shows the potential locations, the number of customers and the possibility of meeting the demand for the sites where (*) has been placed in the event that the site can meet the customer's request and put (x) the opposite and code Nc_{ij} For the cost of transferring one unit from site i to customer j he got the following Table:

Table 1. Transportation cost in case of location selection.

Customer site	B_1	B_2	B_3	B_4
A_1	* Nc_{11}	* Nc_{12}	\times	* Nc_{14}
A_2	* Nc_{21}	* Nc_{22}	* Nc_{23}	* Nc_{24}
A_3	\times	* Nc_{32}	* Nc_{33}	* Nc_{34}
Customer orders	D_1	D_2	D_3	D_4

Table 2 shows the following information available for each of the candidate locations for warehouses.

Table 2. Operation information.

Information site	Operating cost per unit (monetary unit)	Initial Invested Capital (Monetary Unit)	Site Capacity
first	Np_1	k_1	A_1
second	Np_2	k_2	A_2
third	Np_3	k_3	A_3

It is required to choose suitable locations for warehouses that make the total costs of investment, operation and transportation as small as possible.

Building the mathematical model: Each site has a fixed capital cost independent of the quantity stored in the warehouse referred to that site and also has a variable cost proportional to the quantity transported, and therefore the total cost of establishing and operating the warehouse is a non-linear function of the stored quantity and using binary integer variables can be formulated the issue of determining the location of the warehouse in a program with integers where we assume that the binary integer variable δ_i Symbolizes the decision to choose the site or not to choose it in other words

$$\delta_i = \begin{cases} 1 & \text{if we chose the site } i \\ 0 & \text{otherwise} \end{cases}$$

Suppose that x_{ij} is the quantity transferred from site i to customer j , so the constraint expressing the ability of the first site to meet the requests is as follows:

$$x_{11} + x_{12} + x_{14} \leq A_1 \delta_1$$

When $\delta_1 = 1$, the first location with capacity A_1 is chosen. The quantity transported from the first site cannot exceed the capacity of that site A_1 When $\delta_1 = 0$ the non-negative variables

$x_{11}, x_{12}, x_{14} = 0$ directly, indicating that it is not possible to ship from the first location

In a similar way, we obtain the following two constraints for the second and third signatories.

$$x_{21} + x_{22} + x_{23} + x_{24} \leq A_2 \delta_2$$

$$x_{31} + x_{33} + x_{34} \leq A_3 \delta_3$$

To choose exactly two locations, we need the following restriction:

$$\delta_1 + \delta_2 + \delta_3 = 2$$

As δ_1 can take one of the values of 0 or 1 only, the new constraint will force two variables from among the three variables, δ_i to be equal to one

The restrictions for customer requests can be written as follows:

first customer $x_{11} + x_{21} = D_1$

Second customer $x_{12} + x_{22} + x_{32} = D_2$

Third customer $x_{23} + x_{33} = D_3$

Forth customer $x_{14} + x_{24} + x_{34} = D_4$

To write the objective function, we note that the total cost of investment, operation and transportation for the first site is as follows:

$$k_1\delta_1 + Np_1(x_{11} + x_{12} + x_{14}) + Nc_{11}x_{11} + Nc_{12}x_{12} + Nc_{14}x_{14}$$

When we do not choose the first site, variable $\delta_1 = 0$ And that forces the variables

$$x_{11}, x_{12}, x_{14} = 0$$

In a similar way, the cost functions of the second and third sites can be written, and thus the full formulation of the issue of assigning the location of the warehouse is reduced to the following correct mixed program: Z is meant to be made minimal

$$Z = k_1\delta_1 + Np_1(x_{11} + x_{12} + x_{14}) + Nc_{11}x_{11} + Nc_{12}x_{12} + Nc_{14}x_{14} + k_2\delta_2 + Np_2(x_{21} + x_{22} + x_{23} + x_{24}) + Nc_{21}x_{21} + Nc_{22}x_{22} + Nc_{23}x_{23} + Nc_{24}x_{24} + k_3\delta_3 + Np_3(x_{32} + x_{33} + x_{34}) + Nc_{32}x_{32} + Nc_{33}x_{33} + Nc_{34}x_{34}$$

considering the following restrictions:

$$\begin{aligned} x_{11} + x_{12} + x_{14} &\leq A_1\delta_1 \\ x_{21} + x_{22} + x_{23} + x_{24} &\leq A_2\delta_2 \\ x_{31} + x_{33} + x_{34} &\leq A_3\delta_3 \\ \delta_1 + \delta_2 + \delta_3 &= 2 \\ x_{11} + x_{21} &= D_1 \\ x_{12} + x_{22} + x_{32} &= D_2 \\ x_{23} + x_{33} &= D_3 \\ \delta_i &\text{ true variable for } i = 1, 2, 3 \\ x_{ij} &\geq 0 ; i = 1, 2, 3 \text{ and } j = 1, 2, 3, 4 \end{aligned}$$

2.2 The second issue

The second request addressed by the executive was how I can choose the appropriate projects to operate a limited capital available in the company through a number of projects presented, through the information provided by the official in charge of managing the company, the expert formulated the following issue:

The issue of the capital budget: A company plans to disburse its capital during the T_j periods. where $j = 1, 2, \dots, n$, and there is A_i A proposed project where $i = 1, 2, \dots, m$ versus a limited capital B_j Available for investment in period j and when choosing any project i becomes in need of a certain capital in each period j we denote it Na_{ij} . It is a neutrosophic value, the value of each project is measured in terms of the liquidity flow corresponding to the project in each period minus the value

of inflation, and this is called net present value (NPV), we denote it Nv_i . Accordingly, the following Table 3 can be organized:

Table 3. Return on Investment during periods.

period project	T_1	T_2	-----	T_n
A_1	Na_{11}	Na_{12}	-----	Na_{1n}
A_2	Na_{21}	Na_{22}		Na_{2n}
-----	-----	-----	-----	-----
A_m	Na_{m1}	Na_{m2}	-----	Na_{mn}
Limited capital	B_1	B_2	-----	B_n

What is required in this problem is to select the right projects that maximize the total value (NPV) of all selected projects. Formulation of the mathematical model:

Here we assume a binary integer variable x_j . It takes the value one if the project j is selected and takes the value zero if the project j is not selected

$$x_i = \begin{cases} 1 & \text{if we chose project } i \\ 0 & \text{otherwise} \end{cases}$$

Then the objective function is given by the following relationship:

$$Z = \sum_{i=1}^m Nv_i x_i$$

Then the objective function is given by the following relationship:

$$\sum_{i=1}^m Na_{ij} x_i \leq B_j \quad ; j = 1, \dots, n$$

Accordingly, we get the following mathematical model:

Find the maximum value of the function:

$$Z = \sum_{i=1}^m Nv_i x_i$$

Considering the following restrictions:

$$\sum_{i=1}^m Na_{ij} x_i \leq B_j \quad ; j = 1, \dots, n$$

x_i A binary variable takes one of the values 0 or 1 for all values of $i = 1, \dots, m$ in the previous two issues, we got models with integers that have special methods of solution. This research cannot be presented and we will present them in later research using the concepts of neutrosophic science.

2.3 The third issue

in the reference [25] an example of a dynamic programming problem, which is the capital budget problem using classical values, is presented. In this paper, we will reformulate the general model of this problem using neutrosophic values, and we will apply this to the example given in the reference so that we can compare the use of classical values and neutrosophic values using the optimal solution that we will get.

Another presentation of the problem of the neutrosophic capital budget in reference [25] the following question was raised: the budget of a company (5 million monetary units) wants to be distributed to four different investment projects, if you know that each investment has a certain return corresponding to each amount of one million and is shown in the attached table, it is required to find the optimal distribution of the budget if (5 million and 4 million) between the four projects in a way that obtains the greatest profit

Table 5. Periodic amounts classic values.

Investment projects Invested amounts	$F_1(x_1)$	$F_2(x_2)$	$F_3(x_3)$	$F_4(x_4)$
0	0	0	0	0
1	0.56	0.5	0.3	0.4
2	0.9	0.82	0.5	0.66
3	1.3	1.1	0.8	0.84
4	1.56	1.3	1.00	0.96
5	1.8	1.5	1.24	1.06

This issue is a dynamic programming issue that was addressed using the tables method and the summary table for the final solution was as follows:

Table 6. Final table of solution and classic values.

Z_4	x_4	$F_4(x_4)$	Z_3	$G_3^*(Z_3)$	$G_4(Z_4)$
5	0	0	5	2.12	2.12
	1	0.40	4	1.80	2.20
	2	0.66	3	1.40	2.06
	3	0.84	2	1.06	1.90
	4	0.96	1	0.56	1.52
	5	1.06	0	0	1.06
4	0	0	4	1.80	1.80
	1	0.40	3	1.40	1.80
	2	0.66	2	1.06	1.72
	3	0.84	1	0.56	1.40
	4	0.96	0	0	0.96

From the table, we find that the maximum return if (5) million is invested equals to $G_4^*(Z_4) = 2.20$

This value corresponds to $x_4 = 1$, $x_3 = 0$, $Z_3 = 4$ and therefore from the relations $Z_i = Z_{i-1} + x_i$

We find the following distribution:

$$G_4^*(Z_4) = 2.20 \Rightarrow Z_4 = 5 \Rightarrow \begin{cases} x_4 = 1 \\ x_3 = 0 \\ x_2 = 1 \\ x_1 = 3 \end{cases}$$

$x_4 = 1$ و $x_4 = 0$:

From the table, we find that the maximum return if (4) million is invested equals $G_4^*(Z_4) = 1.80$

This value corresponds to , $x_4 = 1$ or $x_4 = 0$ and the distribution is as follows:

$$G_4^*(Z_4) = 1.80 \Rightarrow Z_4 = 4 \Rightarrow \begin{cases} x_4 = 0 \\ or \\ x_4 = 1 \end{cases} \Rightarrow \begin{cases} \begin{cases} x_4 = 0 \\ x_3 = 0 \\ x_2 = 1 \\ x_1 = 3 \end{cases} \\ or \\ \begin{cases} x_4 = 1 \\ x_3 = 0 \\ x_2 = 1 \\ x_1 = 2 \end{cases} \end{cases}$$

To clarify the extent to which neutrosophic values affect the final result of the solution, that is, on the maximum profit of the company In the beginning, we formulate the general model for this problem, neutrosophic science concepts, and accordingly we determine the data that are affected by the surrounding conditions of the work environment and here we take the values that express the profit from investing an amount of amounts in one of the projects neutrosophic values because they are the values that are affected by the surrounding conditions

We get the following problem:

The problem of the distribution of the neutrosophic capital budget: the budget of the company is M monetary unit that you want to distribute to n of different investment projects, B_1, B_2, \dots, B_n if you know that for every amount invested in a project, there is a profit (i.e. the profit is related to the amount invested into the project), $NF_1(x_1), NF_2(x_2), \dots, NF_n(x_n)$, shown in the attached table, it is required to find the optimal distribution of the budget between n projects so that the company gets the greatest profit

Table 4. Return on Investment by amount used.

Investment projects Invested amounts	B_1	B_2	-----	B_n
1	Nc_{11}	Nc_{12}	-----	Nc_{1n}
2	Nc_{21}	Nc_{22}	-----	Nc_{2n}
-----	-----	-----	-----	-----
m	Nc_{m1}	Nc_{m2}	-----	Nc_{mn}

After studying the data of this issue, we note that it can be formulated in the form of a dynamic programming problem corresponding to the case of one dimension and we mean the criterion that affects the decision-making process at a certain stage of obtaining the optimal solution and one

influencer and we mean by it the goal follower and here we note that the goal is the only one which is to achieve the greatest profit so we can represent it with the following relationships:

$$Z_i = NF_i(x_i) ; i = 1, 2, \dots, n$$

Z_i is the function that expresses the state reached by the problem at a certain partial point x_i is decision variable in stage i

The relationship between the state in a given phase $i-1$ and the next phase i is written as follows:

$$Z_i = g_i(x_i, Z_{i-1})$$

These relationships represent constraints

The goal function is written according to the partial goal dependencies in the previous stages with the following relationship:

$$NG_i(Z_i) = NF_i(x_i, Z_{i-1}, Z_i) ; i = 1, 2, \dots, n$$

This function takes an optimal value at each stage and this value is given by the following

$$\text{relationship: } NG_n^*(Z_n) = \underbrace{\text{opt}}_{x_1 \rightarrow x_n} [NF_1(x_1, Z_{i-1}, Z_i) + \dots + NF_n(x_n, Z_{n-1}, Z_n)]$$

Which can be expressed as follows:

$$NG_n^*(Z_n) = \underbrace{\text{opt}}_{x_n} [NF_n(x_n, Z_{n-1}, Z_n) + NG_{n-1}^*(Z_{n-1})]$$

whereas

$NG_n^*(Z_n)$ – optimal goal function at stage n

$NG_{n-1}^*(Z_{n-1})$ – optimal goal function at stage $n - 1$

they be solved as follows:

$$NG_i^*(Z_i) = \underbrace{\text{opt}}_{x_i} NF_i[(x_i, Z_{i-1}, Z_i)] ; i = 1, 2, \dots, n \quad (1)$$

Restrictions:

$$Z_i = g_i(x_i, Z_{i-1}) ; i = 1, 2, \dots, n \quad (2)$$

The relationships,1. and restrictions,2. It includes all one-dimensional, one-effect neutrosophic dynamic programming problems and is solved in the form of tables that we will illustrate by solving the following example: Example: (We will take some neutrosophic values) The budget of the company $M = \$ 5$ million wants to spread it among four projects B_1, B_2, B_3, B_4 of the various investment projects, if you know that for every million investment in a project, there is a profit (that is, the profit is related to the amount that is invested into the project), shown in the attached table, it is required to find the optimal distribution of the budget between the four projects so that the company achieves the greatest profit

Table 7. Return on Investment with neutrosophic values.

Investment projects Invested amounts	B_1	B_2	B_3	B_4
0	0	0	0	0
1	[0.52, 0.57]	0.5	0.3	0.4
2	0.9	[0.8, 0.85]	0.5	0.66
3	1.3	1.1	[0.79, 0.82]	0.84
4	1.56	1.3	1.00	[0.9, 1.3]
5	1.8	1.5	1.24	[1.05, 1.2]

This problem is formulated with a dynamic programming program and the optimal solution is found using the following relationships: We denote $NF_i(x_i)$ -function at each stage (the value of the profit returned from the use of the amount x_i in project, x_1, x_2, x_3, x_4 is the amount of money distributed over the four projects

$$Z_i = Z_{i-1} + x_i$$

Solution equations:

$$NG_1^*(Z_1) = NF_1(x_1)$$

$$NG_2^*(Z_2) = \underset{x_2}{\text{Max}} [NF_2(x_2) + NG_1^*(Z_1)]$$

$$NG_3^*(Z_3) = \underset{x_3}{\text{Max}} [NF_3(x_3) + NG_2^*(Z_2)]$$

$$NG_4^*(Z_4) = \underset{x_4}{\text{Max}} [NF_4(x_4) + NG_3^*(Z_3)]$$

1- Solution using tables

2- Table Z_1

Table 8. Z_1 table.

Z_1	x_1	$G_1^*(Z_1)$	$Z_0 = 0$ $Z_1 = Z_0 + x_1$
0	0	0	
1	1	[0.52, 0.57]	
2	2	0.9	
3	3	1.3	
4	4	1.56	
5	5	1.8	

1- Table Z_2 where $Z_2 = Z_1 + x_2$

Table 9. Z_2 table.

Z_2	x_2	$NF_2(x_2)$	Z_1	$NG_1^*(Z_1)$	$NG_2(Z_2)$
0	0	0	0	0	0
1	0	0	1	[0.52 , 0.57]	[0.52 , 0.57]
	1	0.5	0	0	0.5
2	0	0	2	0.9	0.9
	1	0.5	1	[0.52 , 0.57]	[1.02 , 1.07]
	2	[0.8 , 0.85]	0	0	[0.8 , 0.85]
3	0	0	3	1.3	1.3
	1	0.5	2	0.9	1.4
	2	[0.8 , 0.85]	1	[0.52 , 0.57]	[1.32 , 1.42]
	3	1.1	0	0	1.1
4	0	0	4	1.56	1.56
	1	0.5	3	1.3	1.8
	2	[0.8 , 0.85]	2	0.9	[1.7 , 1.75]
	3	1.1	1	[0.52 , 0.57]	[1.62 , 1.67]
	4	1.3	0	0	1.3
5	0	0	5	1.8	1.8
	1	0.5	4	1.56	2.06
	2	[0.8 , 0.85]	3	1.3	[2.1 , 2.15]
	3	1.1	2	0.9	2.0
	4	1.3	1	[0.52 , 0.57]	[1.82 , 1.87]
	5	1.5	0	0	1.5

The values colored yellow represent the maximum values of profit returned for each amount in which the second project is supplied, so we write a table, Z_2 the abbreviation that shows the optimal values.

Table 10. Optimal values of Z_2 .

Z_2	x_2	$NG_2^*(Z_2)$	$Z_2 = Z_1 + x_2$
0	0	0	
1	0	[0.52 , 0.57]	
2	1	[1.02 , 1.07]	
3	2	[1.4 , 1.42]	
4	2	[1.7 , 1.75]	
5	2	[2.1 , 2.15]	

2- Z_3 table where $Z_3 = Z_2 + x_3$

Table 11. Z_3 table.

Z_3	x_3	$NF_3(x_3)$	Z_2	$NG_2^*(Z_2)$	$NG_3(Z_3)$
0	0	0	0	0	0
1	0	0	1	[0.52 , 0.57]	[0.52 , 0.57]
	1	0.3	0	0	0.3
2	0	0	2	[1.02 , 1.07]	[1.02 , 1.07]
	1	0.3	1	[0.52 , 0.57]	[0.82 , 0.87]
	2	0.5	0	0	0.5
3	0	0	3	[1.4 , 1.42]	[1.4 , 1.42]
	1	0.3	2	[1.02 , 1.07]	[1.32 , 1.37]
	2	0.5	1	[0.52 , 0.57]	[1.02 , 1.07]
	3	[0.79 , 0.82]	0	0	[0.79 , 0.82]
4	0	0	4	[1.7 , 1.75]	[1.7 , 1.75]
	1	0.3	3	[1.4 , 1.42]	[1.7 , 1.72]
	2	0.5	2	[1.02 , 1.07]	[1.52 , 1.57]
	3	[0.79 , 0.82]	1	[0.52 , 0.57]	[1.31 , 1.39]
	4	1.0	0	0	1.0
5	0	0	5	[2.1 , 2.15]	[2.1 , 2.15]
	1	0.3	4	[1.7 , 1.75]	[2 , 2.05]
	2	0.5	3	[1.4 , 1.42]	[1.9 , 1.92]
	3	[0.79 , 0.82]	2	[1.02 , 1.07]	[1.81 , 1.89]
	4	1.0	1	[0.52 , 0.57]	[1.52 , 1.57]
	5	1.24	0	0	1.24

The values colored yellow represent the maximum values of profit payable for each amount in which the second project is supplied, so we write a table Z_3 . The abbreviation that shows the optimal values.

Table 12. Optimal values.

Z_3	x_3	$NG_3^*(Z_3)$	$Z_3 = Z_2 + x_3$
0	0	0	
1	0	[0.52 , 0.57]	
2	0	[1.02 , 1.07]	
3	0	[1.4 , 1.42]	
4	0	[1.7 , 1.75]	
5	0	[2.1 , 2.15]	

3- Z_3 table where $Z_4 = Z_3 + x_4$

Table 13. Z_4 table.

Z_4	x_4	$NF_4(x_4)$	Z_3	$NG_3^*(Z_3)$	$NG_4(Z_4)$
0	0	0	0	0	0
1	0	0	1	[0.52 , 0.57]	[0.52 , 0.57]
	1	0.4	0	0	0.4
2	0	0	2	[1.02 , 1.07]	[1.02 , 1.07]
	1	0.4	1	[0.52 , 0.57]	[0.92 , 0.97]
	2	0.66	0	0	0.66
3	0	0	3	[1.4 , 1.42]	[1.4 , 1.42]
	1	0.4	2	[1.02 , 1.07]	[1.42 , 1.47]
	2	0.66	1	[0.52 , 0.57]	[1.18 , 1.23]
	3	0.84	0	0	0.84
4	0	0	4	[1.7 , 1.75]	[1.7 , 1.75]
	1	0.4	3	[1.4 , 1.42]	[1.8 , 1.82]
	2	0.66	2	[1.02 , 1.07]	[1.68 , 1.73]
	3	0.84	1	[0.52 , 0.57]	[1.36 , 1.41]
	4	[0.9 , 1.3]	0	0	[0.9 , 1.3]
5	0	0	5	[2.1 , 2.15]	[2.1 , 2.15]
	1	0.4	4	[1.7 , 1.75]	[2.1 , 2.15]
	2	0.66	3	[1.4 , 1.42]	[2.06 , 2.08]
	3	0.84	2	[1.02 , 1.07]	[1.86 , 1.91]
	4	[0.9 , 1.3]	1	[0.52 , 0.57]	[1.42 , 1.87]
	5	[1.05 , 1.2]	0	0	[1.05 , 1.2]

The values colored yellow represent the maximum values of profit returned for each amount in which the second project is supplied, so we write a table, Z_4 . The abbreviation that shows the optimal values

Table 14. the optimal values of Z_4 .

Z_4	x_4	$NG_4^*(Z_4)$	$Z_4 = Z_3 + x_4$
0	0	0	
1	0	[0.52 , 0.57]	
2	0	[1.02 , 1.07]	
3	1	[1.42 , 1.47]	
4	1	[1.8 , 1.82]	
5	0 or 1	[2.1 , 2.15]	

Conclusion of the optimal solution: Note that the maximum profit value when investing the amount (5) available in the company is achieved at two values of x_4 . Therefore, the optimal distribution is as follows:

$$NG_4^*(Z_4) = [2.1, 2.15] \Rightarrow Z_4 = 5 \Rightarrow \begin{cases} x_4 = 0 \\ or \\ x_4 = 1 \end{cases} \Rightarrow \begin{cases} \begin{cases} x_4 = 0 \\ x_3 = 0 \\ x_2 = 2 \\ x_1 = 3 \end{cases} \\ or \\ \begin{cases} x_4 = 1 \\ x_3 = 0 \\ x_2 = 2 \\ x_1 = 2 \end{cases} \end{cases}$$

To obtain a maximum profit of a value belonging to the range, **[2.1, 2.15]** an amount of (5) million can be distributed in one of the two forms:

Figure 1: We provide the first project with (3) million dollars and the second project with (2) million dollars and cancel investment in the third and fourth projects Figure 2: We provide the first project (2) million dollars, the second project (2) million dollars, the fourth project (1) million dollars, and cancel the investment in the third project to obtain a maximum profit of a value belonging to the range **[1.8, 1.82]** The amount of (4) million can be distributed as follows: We provide the first project with (1) million dollars, the second project with (2) million dollars, and the fourth project with (1) million dollars, and cancel the investment in the third project

$$NG_4^*(Z_4) = [1.8, 1.82] \Rightarrow Z_4 = 4 \Rightarrow \begin{cases} x_4 = 1 \\ x_3 = 0 \\ x_2 = 2 \\ x_1 = 1 \end{cases}$$

Note: When calculating Z_4 We can only calculate for (4) and (5) million, i.e. sufficiency in the following table:

Table 15. Table of optimal values for Z_4 with the short solution.

Z_4	x_4	$NF_4(x_4)$	Z_3	$NG_3^*(Z_3)$	$NG_4(Z_4)$
4	0	0	4	[1.7, 1.75]	[1.7, 1.75]
	1	0.4	3	[1.4, 1.42]	[1.8, 1.82]
	2	0.66	2	[1.02, 1.07]	[1.68, 1.73]
	3	0.84	1	[0.52, 0.57]	[1.36, 1.41]
	4	[0.9, 1.3]	0	0	[0.9, 1.3]
5	0	0	5	[2.1, 2.15]	[2.1, 2.15]
	1	0.4	4	[1.7, 1.75]	[2.1, 2.15]
	2	0.66	3	[1.4, 1.42]	[2.06, 2.08]
	3	0.84	2	[1.02, 1.07]	[1.86, 1.91]
	4	[0.9, 1.3]	1	[0.52, 0.57]	[1.42, 1.87]
	5	[1.05, 1.2]	0	0	[1.05, 1.2]

Comparison for the investment of (5) million:

Table 16. comparison in case the amount is (5) millions.

Projects \ Type of profit values	Optimal distribution of amount in case that some profit values are neutrosophic values		Optimal distribution of amount in case that some profit values are classical values
Forth project	$x_4 = 1$ or $x_4 = 0$		$x_4 = 1$
Third project	$x_3 = 0$	$x_3 = 0$	$x_3 = 0$
Second project	$x_2 = 2$	$x_2 = 2$	$x_2 = 1$
First project	$x_1 = 3$	$x_1 = 1$	$x_1 = 3$
Maximal general profit	$NG_4^*(Z_4) = [2.1, 2.15]$		$G_4^*(Z_4) = 2.20$

Comparison for the investment of (4) million:

Table 17. comparison in case the amount is (4) millions.

Projects \ Type of profit values	Optimal distribution of amount in case that some profit values are neutrosophic values		Optimal distribution of amount in case that some profit values are classical values	
Forth project	$x_4 = 1$		$x_4 = 0$ or $x_4 = 1$	
Third project	$x_3 = 0$		$x_3 = 0$	$x_3 = 0$
Second project	$x_2 = 2$		$x_2 = 1$	$x_2 = 1$
First project	$x_1 = 1$		$x_1 = 3$	$x_1 = 2$
Maximal general profit	$NG_4^*(Z_4) = [1.8, 1.82]$		$G_4^*(Z_4) = 1.80$	

3. Conclusion and Results

In this research, we presented a study whose importance lies in the following points:

1. Obtaining a general formula for the issue of establishing new warehouses that help companies and institutions expand their work at the lowest cost by choosing the ideal locations for expansion points
2. Obtaining a general formula for the issue of distributing the companies' capital budget through the ideal investment for them in selected projects within a group of proposed projects so that the company achieves the maximum profit.
3. Clarifying the role of the binary integer in converting some nonlinear models into linear models and also obtain solutions with integers appropriate to the nature of the problems under study.
4. Focusing on the need to use the concepts of neutrosophic science to obtain optimal solutions suitable for all circumstances surrounding the work environment.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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A Solution Technique of Transportation Problem in Neutrosophic Environment

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Abstract: In this article, we presented a ranking system based on the sign distance in a new direction such that to compare different single-valued neutrosophic numbers (SVN-numbers), a decision maker has the chance of flexibility. We also developed some important definitions, like level- (α, β, γ) neutrosophic point, and some properties related to the sign distance ranking function. Finally, the sign distance ranking function is applied to solving the neutrosophic transportation problem (NTP) with SVN-number converted into a transportation problem (TP) with crisp data, and to illustrate the appropriateness of this function we gave two numerical illustrations.

Keywords: SVN-numbers; Level-1 neutrosophic point; Sign distance ranking function; NTP.

1. Introduction

In the year 1941, the fundamental transportation diagram on crisp numbers with transportation constraints was initiated. Recently, transportation parameters such as unit transportation cost, demand and supply have been unstable and inconsistent because of different unrestrained arguments. The law and hypothesis of crisp set (CS) theory are inappropriate to manage those problems because they only imply that whatever elements are either 0 or 1 if they belong to a set or not, respectively. In this situation, fuzzy sets (FS) [5] and intuitionistic fuzzy sets (IFS) [6] are suitable to handle those problems. For this reason, fuzzy and intuitionistic transportation problems [1, 2, 3, 4] were introduced and solved. The generalization of CS, FS, and IFS is called neutrosophic set (NS), which is classified by three independent components: truth (T), indeterminacy (I) and falsity (F), and Smarandache [7, 8] introduced the notion of NS to handle uncertainty more accurately. The indeterministic part of inconsistent information plays a significant role in creating proper reason, which is not considered in IFS theory. Due to this, we introduced the NTP with SVN-numbers and solved it.

Here the idea of SVN-numbers is designed in a new way, and the ordering of SVN-numbers is described by λ -weighted sign distance. To establish the sign distance ranking function, we follow different ranking approaches in different fields. S. Abbasbandy, B. Asady [20] and Yao, Wu [19] define

sign distance to determine the ordering of FS. De, Das [12] and Deng Feng et.al. [10] applied ranking function based on value and ambiguity to define ordering of IFS. Suresh Mohan et.al [11] use magnitude to determine the ranking of IFS. Qiang and Zhong [9] proposed score and accuracy function to define the ordering of IFS. Bera and Mahapatra [13] use values to determine the ordering of SVN-numbers. Deli and Subas [18] define ranking of SVN-numbers on the basis of value and ambiguity. Peng et.al [17] initiated the score, accuracy, and certainty function and applied this function to determine the ordering of SVN-numbers. Ye, J [14] also use the score and accuracy functions to describe the ordering of SVN-numbers.

In this article, let us remember some essential definitions from the preliminary section. In section 3, the idea of level (α, β, γ) –neutrosophic points is introduced, which is the generalisation of level α -fuzzy points [15], and we develop a sign distance ranking function to compare different types of neutrosophic numbers and establish some important properties of the sign distance ranking function. In section 4, we introduced NTP, and applied the sign distance methodology, converting NTP with single valued trapezoidal or triangular neutrosophic numbers (SVTN-numbers or SVTrN-numbers) to the transportation problems with crisp data, and then using VAM and MODI method we get initial and optimal solutions of the mentioned NTP. In the final, the proposed work is briefly described.

2. Preliminaries

Let us remind ourselves of a few significant definitions that are essential for developing the leading concept in this article.

Definition 2.1. [13] When an SVN-Set \tilde{M} is defined on \mathfrak{R} and is presented as:

$\tilde{M} = \langle [m_1, n_1, \sigma_1, \delta_1], [m_2, n_2, \sigma_2, \delta_2], [m_3, n_3, \sigma_3, \delta_3] \rangle$, where $\sigma_i > 0, \delta_i > 0$ are the extensions of left and right, respectively and $[m_i, n_i]$ are the modal intervals of T, I, and F components respectively, for $i = 1, 2, 3$, then \tilde{M} is said to be SVN-number. The three neutrosophic components are drawn as follows.

$$T_{\tilde{M}}(\xi) = \begin{cases} \frac{1}{\sigma_1}(\xi - m_1 + \sigma_1) & m_1 - \sigma_1 \leq \xi \leq m_1 \\ 1 & m_1 \leq \xi \leq n_1 \\ \frac{1}{\delta_1}(n_1 - \xi + \delta_1) & n_1 \leq \xi \leq n_1 + \delta_1 \\ 0 & \text{otherwise} \end{cases}$$

$$I_{\tilde{M}}(\xi) = \begin{cases} \frac{1}{\sigma_2}(m_2 - \xi) & m_2 - \sigma_2 \leq \xi \leq m_2 \\ 0 & m_2 \leq \xi \leq n_2 \\ \frac{1}{\delta_2}(\xi - n_2) & n_2 \leq \xi \leq n_2 + \delta_2 \\ 1 & \text{otherwise} \end{cases}$$

$$F_{\tilde{M}}(\xi) = \begin{cases} \frac{1}{\sigma_3}(m_3 - \xi) & m_3 - \sigma_3 \leq \xi \leq m_3 \\ 0 & m_3 \leq \xi \leq n_3 \\ \frac{1}{\delta_3}(\xi - n_3) & n_3 \leq \xi \leq n_3 + \delta_3 \\ 1 & \text{otherwise} \end{cases}$$

Respectively

In parametric form, the three pairs of SVN-number \tilde{M} are $(T^l_{\tilde{M}}, T^u_{\tilde{M}}), (I^l_{\tilde{M}}, I^u_{\tilde{M}}), (F^l_{\tilde{M}}, F^u_{\tilde{M}})$, of functions $T^l_{\tilde{M}}(\varepsilon), T^u_{\tilde{M}}(\varepsilon), I^l_{\tilde{M}}(\varepsilon), I^u_{\tilde{M}}(\varepsilon), F^l_{\tilde{M}}(\varepsilon), F^u_{\tilde{M}}(\varepsilon), \varepsilon \in [0, 1]$, and satisfies the following conditions.

- (i) $T_{\tilde{M}}^l, I_{\tilde{M}}^u, F_{\tilde{M}}^u$ are continuous, bounded and monotone increasing function.
- (ii) $T_{\tilde{M}}^u, I_{\tilde{M}}^l, F_{\tilde{M}}^l$ are continuous, bounded and monotone decreasing function.
- (iii) $T_{\tilde{M}}^l(\varepsilon) \leq T_{\tilde{M}}^u(\varepsilon), I_{\tilde{M}}^l(\varepsilon) \geq I_{\tilde{M}}^u(\varepsilon), F_{\tilde{M}}^l(\varepsilon) \geq F_{\tilde{M}}^u(\varepsilon)$

Definition 2.2. [13]: A SVN-number \tilde{M} defined in 2.1 is converted into a SVTN-number if all modal intervals of \tilde{M} are same

Thus $\tilde{M} = \langle [m_1, n_1, \sigma_1, \delta_1], [m_1, n_1, \sigma_2, \delta_2], [m_1, n_1, \sigma_3, \delta_3] \rangle$ is a SVTN-number whose membership functions are given below:

$$T_{\tilde{M}}(\xi) = \begin{cases} \frac{1}{\sigma_1}(\xi - m_1 + \sigma_1) & m_1 - \sigma_1 \leq \xi \leq m_1 \\ 1 & m_1 \leq \xi \leq n_1 \\ \frac{1}{\delta_1}(n_1 - \xi + \delta_1) & n_1 \leq \xi \leq n_1 + \delta_1 \\ 0 & \text{otherwise} \end{cases}$$

$$I_{\tilde{M}}(\xi) = \begin{cases} \frac{1}{\sigma_2}(m_1 - \xi) & m_1 - \sigma_2 \leq \xi \leq m_1 \\ 0 & m_1 \leq \xi \leq n_1 \\ \frac{1}{\delta_2}(\xi - n_1) & n_1 \leq \xi \leq n_1 + \delta_2 \\ 1 & \text{otherwise} \end{cases}$$

$$F_{\tilde{M}}(\xi) = \begin{cases} \frac{1}{\sigma_3}(m_1 - \xi) & m_1 - \sigma_3 \leq \xi \leq m_1 \\ 0 & m_1 \leq \xi \leq n_1 \\ \frac{1}{\delta_3}(\xi - n_1) & n_1 \leq \xi \leq n_1 + \delta_3 \\ 1 & \text{otherwise} \end{cases}$$

respectively.

Let us consider two SVTN-numbers $\tilde{N} = \langle [m, n, \sigma_1, \delta_1], [m, n, \sigma_2, \delta_2], [m, n, \sigma_3, \delta_3] \rangle$ and $\tilde{N}_2 = \langle [p, q, \mu_1, v_1], [p, q, \mu_2, v_2] \rangle$

Then for $k (\neq 0) \in \mathfrak{R}$

- (i) $\tilde{N}_1 \oplus \tilde{N}_2 = \langle [m + p, n + q, \sigma_1 + \mu_1, \delta_1 + v_1], [m + p, n + q, \sigma_2 + \mu_2, \delta_2 + v_2], [m + p, n + q, \sigma_3 + \mu_3, \delta_3 + v_3] \rangle$
- (ii) $\tilde{N}_1 \ominus \tilde{N}_2 = \langle [m - q, n - p, \sigma_1 + v_1, \delta_1 + \mu_1], [m - q, n - p, \sigma_2 + v_2, \delta_2 + \mu_2], [m - q, n - p, \sigma_3 + v_3, \delta_3 + \mu_3] \rangle$
- (iii) $k\tilde{N}_1 = \langle ([km, kn, k\sigma_1, k\delta_1], [km, kn, k\sigma_2, k\delta_2], [km, kn, k\sigma_3, k\delta_3]) \rangle$ for $k > 0$
 $k\tilde{N}_1 = \langle ([kn, km - k\delta_1, -k\sigma_1], [kn, km - k\delta_2, -k\sigma_2], [kn, km - k\delta_3, -k\sigma_3]) \rangle$ for $k < 0$

The graphical representation of SVTN-number $\tilde{N} = [0.4, 0.9, 0.3, 0.6], [0.5, 1, 0.4, 0.8], [0.8, 1.6, 0.3, 0.6] \rangle$ is given below:

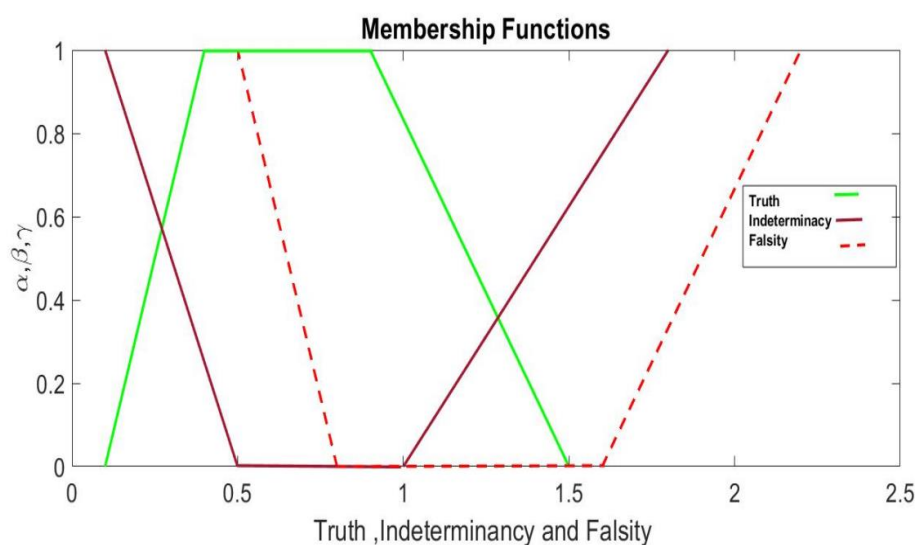


Figure1. The graphical representation of Single valued trapezoidal neutrosophic number (SVTN) \tilde{N} .

Definition 2.3. [18, 13] The SVN-number \tilde{M} defined in 2.1 is turned into SVTrN-number if all three modal intervals of \tilde{M} are decreased into modal points and three modal points are equal. Thus $\tilde{M} = [m_1, \sigma_1, \delta_1], [m_1, \sigma_2, \delta_2], [m_1, \sigma_3, \delta_3]$ is a SVTrN-number whose membership functions are given below.

$$T_{\tilde{M}}(\xi) = \begin{cases} \frac{1}{\sigma_1}(\xi - m_1 + \sigma_1) & m_1 - \sigma_1 \leq \xi \leq m_1 \\ 1 & \xi = m_1 \\ \frac{1}{\delta_1}(m_1 - \xi + \delta_1) & m_1 \leq \xi \leq m_1 + \delta_1 \\ 0 & \text{otherwise} \end{cases}$$

$$I_{\tilde{M}}(\xi) = \begin{cases} \frac{1}{\sigma_2}(m_1 - \xi) & m_1 - \sigma_2 \leq \xi \leq m_1 \\ 0 & \xi = m_1 \\ \frac{1}{\delta_2}(\xi - m_1) & m_1 \leq \xi \leq m_1 + \delta_2 \\ 1 & \text{otherwise} \end{cases}$$

$$F_{\tilde{M}}(\xi) = \begin{cases} \frac{1}{\sigma_3}(m_1 - \xi) & m_1 - \sigma_3 \leq \xi \leq m_1 \\ 0 & \xi = m_1 \\ \frac{1}{\delta_3}(\xi - m_1) & m_1 \leq \xi \leq m_1 + \delta_3 \\ 1 & \text{otherwise} \end{cases}$$

respectively.

Let us consider two SVTrN-numbers $\tilde{N}_1 = \langle [m, \sigma_1, \delta_1], [m, \sigma_2, \delta_2], [m, \sigma_3, \delta_3] \rangle$ and $\tilde{N}_2 = \langle [p, \mu_1, v_1], [p, \mu_2, v_2], [p, \mu_3, v_3] \rangle$. Then for $k (\neq 0) \in \Re$

- (i) $\tilde{N}_1 \oplus \tilde{N}_2 = \langle [m + p, \sigma_1 + \mu_1, \delta_1 + v_1], [m + p, \sigma_2 + \mu_2, \delta_2 + v_2], [m + p, \sigma_3 + \mu_3, \delta_3 + v_3] \rangle$
- (ii) $\tilde{N}_1 \ominus \tilde{N}_2 = \langle [m - p, \sigma_1 + v_1, \delta_1 + \mu_1], [m - p, \sigma_2 + v_2, \delta_2 + \mu_2], [m - p, \sigma_3 + v_3, \delta_3 + \mu_3] \rangle$
- (iii) $k\tilde{N}_1 = \langle [km, k\sigma_1, k\delta_1], [km, k\sigma_2, k\delta_2], [km, k\sigma_3, k\delta_3] \rangle$ for $k > 0$

$$k\tilde{N}_1 = \langle ([km, -k\delta_1, -k\sigma_1], [km, -k\delta_2, -k\sigma_2], [km, -k\delta_3, -k\sigma_3]) \rangle \text{ for } k < 0$$

The graphical representation of SVTrN-number $\tilde{N} = [1.6, 0.9, 0.9], [1.5, .5, .8], [1, .9, 1.1] \rangle$ is given below:

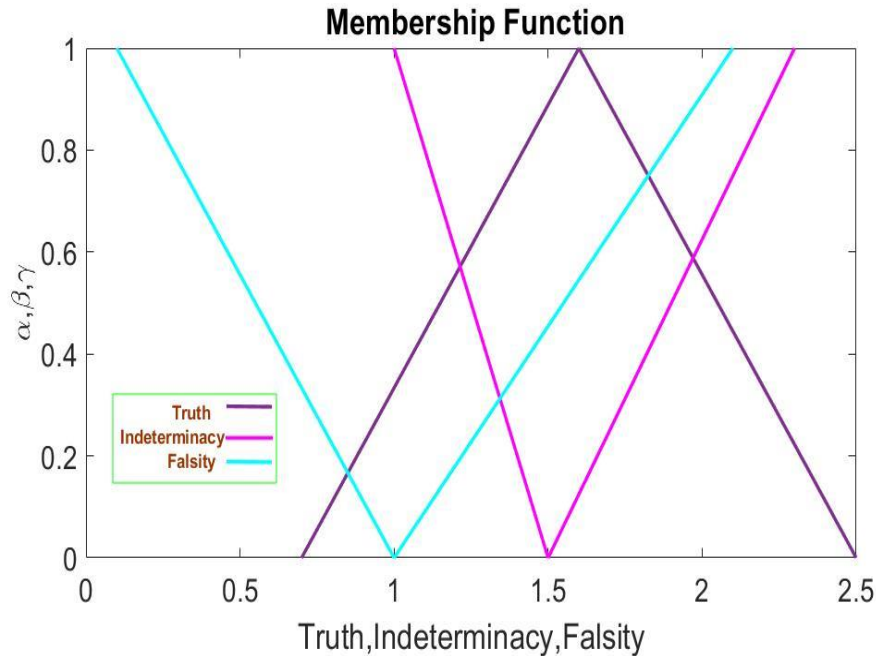


Figure 2. The graphical representation of Single valued triangular neutrosophic number(SVTrN) \tilde{N} .

Definition 2.4. [18, 21] Let $\tilde{M} = \langle [m_1, n_1, \sigma_1, \delta_1], [m_2, n_2, \sigma_2, \delta_2], [m_3, n_3, \sigma_3, \delta_3] \rangle$ be an SVN number. Then (α, β, γ) cut of \tilde{M} is designed by $\tilde{M}^{(\alpha, \beta, \gamma)}$ and described as $\{\xi \in X : T_{\tilde{M}}(\xi) \geq \alpha, I_{\tilde{M}}(\xi) \leq \beta, F_{\tilde{M}}(\xi) \leq \gamma\}$ for $0 \leq \alpha, \beta, \gamma \leq 3$.

Thus, (α, β, γ) cut of \tilde{M} for truth, indeterminacy, and falsity components is given by closed intervals a $[L_{\tilde{M}}(\alpha), R_{\tilde{M}}(\alpha)] = [m_1 - \sigma_1 + \sigma_1\alpha, n_1 + \delta_1 - \delta_1\alpha]$, $[L'_{\tilde{M}}(\beta), R'_{\tilde{M}}(\beta)] = [m_2 - \sigma_2\beta, n_2 + \delta_2\beta]$ and $[L''_{\tilde{M}}(\gamma), R''_{\tilde{M}}(\gamma)] = [m_3 - \sigma_3\gamma, n_3 + \delta_3\gamma]$ respectively, for $\alpha, \beta, \gamma \in [0, 1]$.

Where $L_{\tilde{M}}, R'_{\tilde{M}}, R''_{\tilde{M}}$ are monotonic non-decreasing continuous functions, and $R_{\tilde{M}}, L'_{\tilde{M}}, L''_{\tilde{M}}$ are monotonic nonincreasing continuous functions in their respectively intervals.

3. The Ranking Method of Neutrosophic Number on \mathfrak{R}

Here, the idea of sign distance between SVN-numbers is introduced in a new way, together with the development of its characteristics, and using this sign distance method, ordering between SVN-numbers is constructed.

Definition 3.1.

- (i) The single valued neutrosophic set $\tilde{a}_\alpha (0 \leq \alpha \leq 1)$ of \mathfrak{R} is said to be a level α neutrosophic point for truth membership if

$$X\tilde{a}_\alpha(\xi) = \begin{cases} \alpha & \xi = \tilde{a} \\ 0 & \xi \neq \tilde{a} \end{cases}$$

Let us consider $N_t(1, 0, 0)$ to be the set of all level 1 neutrosophic points for truth membership.

- (ii) The single valued neutrosophic set \tilde{a}_β ($0 \leq \beta \leq 1$ of \mathfrak{R} is said to be a level β neutrosophic point for indeterminacy membership if

$$X\tilde{a}_\beta(\xi) = \begin{cases} \beta & \xi = \tilde{a} \\ 1 & \xi \neq \tilde{a} \end{cases}$$

Let us consider $N_i(1,0,0)$ to be the set of all level 1 neutrosophic points for indeterminacy membership.

- (iii) The single valued neutrosophic set \tilde{a}_γ ($0 \leq \gamma \leq 1$ of \mathfrak{R} is said to be a level γ neutrosophic point for falsity membership

$$X\tilde{a}_\gamma(\xi) = \begin{cases} \gamma & \xi = \tilde{a} \\ 1 & \xi \neq \tilde{a} \end{cases}$$

Let us consider $N_f(1,0,0)$ to be the set of all level 1 neutrosophic points for falsity membership.

Definition 3.2. A neutrosophic number must satisfy the following conditions:

- (i) A neutrosophic number \tilde{M} is normal i.e., $\exists \xi_0 \in \mathfrak{R}$ such that $T_{\tilde{M}}(\xi_0) = 1$ and $I_{\tilde{M}}(\xi_0) = F_{\tilde{M}}(\xi_0) = 0$.
- (ii) A neutrosophic number \tilde{M} is convex for $T_{\tilde{M}}(\xi)$ i.e. $\forall \xi_1, \xi_2 \in \mathfrak{R}$ and $k \in [0,1]$ such that $T_{\tilde{M}}(k\xi_1 + (1-k)\xi_2) \geq \min(T_{\tilde{M}}(\xi_1), T_{\tilde{M}}(\xi_2))$.
- (iii) A neutrosophic number \tilde{M} is concave for $I_{\tilde{M}}(\xi)$ and $F_{\tilde{M}}(\xi)$ i.e. $\forall \xi_1, \xi_2 \in \mathfrak{R}$ and $k \in [0,1]$ such that $I_{\tilde{M}}(k\xi_1 + (1-k)\xi_2) \geq \max(I_{\tilde{M}}(\xi_1), I_{\tilde{M}}(\xi_2))$ and $F_{\tilde{M}}(k\xi_1 + (1-k)\xi_2) \geq \max(F_{\tilde{M}}(\xi_1), F_{\tilde{M}}(\xi_2))$.

Let us consider the set of whole SVN-numbers of the form $\langle [m_1, n_1, \sigma_1, \delta_1], [m_2, n_2, \sigma_2, \delta_2], [m_3, n_3, \sigma_3, \delta_3] \rangle$ satisfying the above definition 3.3 [(i),(ii),(iii)] is named N_N .

Let $\Gamma = N_N \cup N_t(1,0,0) \cup N_i(0,1,0) \cup N_f(0,0,1)$. If $\tilde{a}_1 \in N_N \cup N_t(1,0,0) \cup N_i(0,1,0) \cup N_f(0,0,1)$ then each left and right edge of (α, β, γ) -cut of \tilde{a}_1 is equal to a . Therefore, each (α, β, γ) -cut of \tilde{a}_1 are equal to $[a, a] = a, \forall \alpha, \beta, \gamma \in [0,1]$.

Property 3.1. For any $\tilde{N}_1, \tilde{N}_2 \in \Gamma$

- (i) The (α, β, γ) -cut of $\tilde{N}_1 \oplus \tilde{N}_2$ are respectively $[L_{\tilde{N}_1}(\alpha) + L_{\tilde{N}_2}(\alpha), R_{\tilde{N}_1}(\alpha) + R_{\tilde{N}_2}(\alpha)]$, $[L'_{\tilde{N}_1}(\beta) + L'_{\tilde{N}_2}(\beta), R'_{\tilde{N}_1}(\beta) + R'_{\tilde{N}_2}(\beta)]$ and $[L''_{\tilde{N}_1}(\gamma) + L''_{\tilde{N}_2}(\gamma), R''_{\tilde{N}_1}(\gamma) + R''_{\tilde{N}_2}(\gamma)]$, and
- (ii) The (α, β, γ) -cut of $\tilde{N}_1 \ominus \tilde{N}_2$ are respectively $[L_{\tilde{N}_1}(\alpha) - L_{\tilde{N}_2}(\alpha), R_{\tilde{N}_1}(\alpha) - R_{\tilde{N}_2}(\alpha)]$, $[L'_{\tilde{N}_1}(\beta) - L'_{\tilde{N}_2}(\beta), R'_{\tilde{N}_1}(\beta) - R'_{\tilde{N}_2}(\beta)]$ and $[L''_{\tilde{N}_1}(\gamma) - L''_{\tilde{N}_2}(\gamma), R''_{\tilde{N}_1}(\gamma) - R''_{\tilde{N}_2}(\gamma)]$

Property 3.2. For any $\tilde{M} \in \Gamma$ and $\tilde{a}_1 \in N_N \cup N_t(1,0,0) \cup N_i(0,1,0) \cup N_f(0,0,1)$. The (α, β, γ) -cut of $\tilde{a}_1 \odot \tilde{M}$ are respectively

- (i) $[aL_{\tilde{M}}(\alpha), aR_{\tilde{M}}(\alpha)], [aL'_{\tilde{M}}(\beta), aR'_{\tilde{M}}(\beta)]$ and $[aL''_{\tilde{M}}(\gamma), aR''_{\tilde{M}}(\gamma)]$ if $a > 0$, and
- (ii) $[aR_{\tilde{M}}(\alpha), aL_{\tilde{M}}(\alpha)], [aR'_{\tilde{M}}(\beta), aL'_{\tilde{M}}(\beta)]$ and $[aR''_{\tilde{M}}(\gamma), aL''_{\tilde{M}}(\gamma)]$ if $a < 0$.

Definition 3.3. For any $\lambda \in [0,1]$ and $\tilde{N}_1, \tilde{N}_2 \in \Gamma$, the sign distance of \tilde{N}_1, \tilde{N}_2 is designed by $d_\lambda(\tilde{N}_1, \tilde{N}_2)$ and described by $d(\tilde{N}_1, \tilde{N}_2) = \lambda d_T(\tilde{N}_1, \tilde{N}_2) + (1-\lambda)d_I(\tilde{N}_1, \tilde{N}_2) + (1-\lambda)d_F(\tilde{N}_1, \tilde{N}_2)$ where

$d_T(\tilde{N}_1, \tilde{N}_2) = \frac{1}{2} \int_0^1 (L_{\tilde{N}_1}(\alpha) + R_{\tilde{N}_1}(\alpha) - L_{\tilde{N}_2}(\alpha) - R_{\tilde{N}_2}(\alpha))f(\alpha)d\alpha$, the sign distance of \tilde{N}_1 and \tilde{N}_2 for truth component.

$d_I(\tilde{N}_1, \tilde{N}_2) = \frac{1}{2} \int_0^1 (L'_{\tilde{N}_1}(\beta) + R'_{\tilde{N}_1}(\beta) - L'_{\tilde{N}_2}(\beta) - R'_{\tilde{N}_2}(\beta))g(\beta)d\beta$, the sign distance of \tilde{N}_1 and \tilde{N}_2 for indeterminacy component, and

$d_F(\tilde{N}_1, \tilde{N}_2) = \frac{1}{2} \int_0^1 (L''_{\tilde{N}_1}(\gamma) + R''_{\tilde{N}_1}(\gamma) - L''_{\tilde{N}_2}(\gamma) - R''_{\tilde{N}_2}(\gamma))h(\gamma)d\gamma$, the sign distance of \tilde{N}_1 and \tilde{N}_2 for falsity component.

Where $\lambda \in [0, 1]$ is weight of sign distance which indicates the decision maker's choice of information. If $\lambda \in [0, 0.5)$, then the decision-makers behaviour indicates pessimistic behaviour in the direction of negativity and uncertainty; if $\lambda \in (0.5, 1]$, then the decision-makers behaviour indicates optimistic behaviour in the direction of positivity and certainty, and if $\lambda = 0.5$, then the decision-makers behaviour indicates indifference between certainty and uncertainty.

Here $f(\alpha), g(\beta), h(\gamma) \in [0, 1]$ and $f(0) = 0, g(1) = 0 = h(1)$, and $f(\alpha)$ is continuous monotonic non decreasing function of $\alpha \in [0, 1]$, and $g(\beta), h(\gamma)$ are continuous monotone non-increasing functions of β, γ respectively for $\beta, \gamma \in [0, 1]$.

Without loss of generality, from now on we will take the value of $f(\alpha), g(\beta)$, and $h(\gamma)$ are $\alpha, 1-\beta$, and $1-\gamma$ respectively through the paper.

Corollary 3.4. For any two two SVTN-numbers $\tilde{N}_1 = \langle [m, n, \sigma_1, \delta_1], [m, n, \sigma_2, \delta_2], [m, n, \sigma_3, \delta_3] \rangle$ and $\tilde{N}_2 = \langle [p, q, \mu_1, \vartheta_1], [p, q, \mu_2, \vartheta_2], [p, q, \mu_3, \vartheta_3] \rangle$, the λ -weighted sign distance of \tilde{N}_1 and \tilde{N}_2 is described as $d_\lambda(\tilde{N}_1, \tilde{N}_2) = \frac{1}{12} [\{(3m + 3n - \sigma_1 + \delta_1)\lambda + (3m + 3n - \sigma_2 + \delta_2)(1 - \lambda) + (3m + 3n - \sigma_3 + \delta_3)(1 - \lambda) + \{(3p + 3q - \mu_1 + \vartheta_1)\lambda + (3p + 3q - \mu_2 + \vartheta_2)(1 - \lambda) + (3p + 3q - \mu_3 + \vartheta_3)(1 - \lambda)\}]$.

Corollary 3.5.: For any two two SVTrN-numbers numbers $\tilde{N}_1 = \langle [m, \sigma_1, \delta_1], [m, n, \sigma_2, \delta_2], [m, \sigma_3, \delta_3] \rangle$ and $\tilde{N}_2 = \langle [p, \mu_1, \vartheta_1], [p, \mu_2, \vartheta_2], [p, \mu_3, \vartheta_3] \rangle$ the λ -weighted sign distance of \tilde{N}_1 and \tilde{N}_2 is described as

$$d_\lambda(\tilde{N}_1, \tilde{N}_2) = \frac{1}{12} [\{(6m + -\sigma_1 + \delta_1)\lambda + (6m - \sigma_2 + \delta_2)(1 - \lambda) + (6m - \sigma_3 + \delta_3)(1 - \lambda) + \{(6p - \mu_1 + \vartheta_1)\lambda + (6p - \mu_2 + \vartheta_2)(1 - \lambda) + (6p - \mu_3 + \vartheta_3)(1 - \lambda)\}]$$

Property 3.3. For any two PROPERTY 3.3. For any $\tilde{N}_1, \tilde{N}_2 \in \Gamma, d_\lambda(\tilde{N}_1, \tilde{N}_2) = d_\lambda(\tilde{N}_1, \tilde{0}_1) - d_\lambda(\tilde{N}_2, \tilde{0}_1)$.

Proof. From definition 3.3 easily ,it can be proved

PROPERTY 3.4 For any $k \in \Re$ and $\tilde{N}_1, \tilde{N}_2 \in \Gamma$,

- (i) $d_\lambda(\tilde{N}_1 \oplus \tilde{N}_2, \tilde{0}_1) = d_\lambda(\tilde{N}_1, \tilde{0}_1) + d_\lambda(\tilde{N}_2, \tilde{0}_1)$.
- (ii) $d_\lambda(k\tilde{N}_1, k\tilde{N}_2) = kd_\lambda(\tilde{N}_1, \tilde{N}_2)$,
- (iii) $d_\lambda(\tilde{N}_1, \tilde{N}_2)$ is constant or monotone increasing or decreasing according as $d_T(\tilde{N}_1, \tilde{N}_2) = d_I(\tilde{N}_1, \tilde{N}_2) + d_F(\tilde{N}_1, \tilde{N}_2)$ or $d_T(\tilde{N}_1, \tilde{N}_2) > d_I(\tilde{N}_1, \tilde{N}_2) + d_F(\tilde{N}_1, \tilde{N}_2)$ or $d_T(\tilde{N}_1, \tilde{N}_2) < d_I(\tilde{N}_1, \tilde{N}_2) + d_F(\tilde{N}_1, \tilde{N}_2)$.

Proof (i) and (ii) are obvious by the definition of 3.4.

(iii) Here, $d_\lambda(\tilde{N}_1, \tilde{N}_2) = \lambda d_T(\tilde{N}_1, \tilde{N}_2) + (1 - \lambda)d_I(\tilde{N}_1, \tilde{N}_2) + (1 - \lambda)d_F(\tilde{N}_1, \tilde{N}_2)$

$$\frac{d}{d_\lambda}(d_\lambda(\tilde{N}_1, \tilde{N}_2)) = d_T(\tilde{N}_1, \tilde{N}_2) - d_I(\tilde{N}_1, \tilde{N}_2) - d_F(\tilde{N}_1, \tilde{N}_2)$$

So, $\frac{d}{d_\lambda}(d_\lambda(\tilde{N}_1, \tilde{N}_2)) = >, < 0$ according as $d_T(\tilde{N}_1, \tilde{N}_2) = >, < = d_I(\tilde{N}_1, \tilde{N}_2) + d_F(\tilde{N}_1, \tilde{N}_2)$. This clear the fact.

Property 3.5. for any four SVN-number $\tilde{N}_1, \tilde{N}_2, \tilde{N}_3, \tilde{N}_4$ and $\tilde{0}_1 \in N_t(1,0,0) \cup N_i(0,1,0) \cup N_f(0,1,0)$.

- (i) $d(\tilde{N}_1 \oplus \tilde{N}_2, \tilde{N}_3 \oplus \tilde{N}_4) = d(\tilde{N}_1, \tilde{N}_3) + d(\tilde{N}_2, \tilde{N}_4) = d(\tilde{N}_1, \tilde{N}_4) + d(\tilde{N}_2, \tilde{N}_3) = d(\tilde{N}_1, \tilde{0}_1) + d(\tilde{N}_2, \tilde{0}_1) - d(\tilde{N}_3, \tilde{0}_1) - d(\tilde{N}_4, \tilde{0}_1)$.
- (ii) $d(\tilde{N}_1 \ominus \tilde{N}_2, \tilde{N}_3 \ominus \tilde{N}_4) = d(\tilde{N}_1, \tilde{N}_3) - d(\tilde{N}_2, \tilde{N}_4) = d(\tilde{N}_1, \tilde{N}_2) - d(\tilde{N}_3, \tilde{N}_4) = d(\tilde{N}_1, \tilde{0}_1) + d(\tilde{N}_4, \tilde{0}_1) - d(\tilde{N}_2, \tilde{0}_1) - d(\tilde{N}_3, \tilde{0}_1)$.

Proof. Using definition 3.3 and property 3.3 we can easily prove the properties (i) and (ii) .

Definitio3.6. For any two SVN- numbers \tilde{N}_1, \tilde{N}_2 we define the ranking of \tilde{N}_1, \tilde{N}_2 by

$$\begin{aligned} d_\lambda(\tilde{N}_1, \tilde{N}_2) &> 0 \text{ iff } d_\lambda(\tilde{N}_1, \tilde{0}_1) > d_\lambda(\tilde{N}_2, \tilde{0}_1) \text{ iff } \tilde{N}_2 < \tilde{N}_1 \\ d_\lambda(\tilde{N}_1, \tilde{N}_2) &< 0 \text{ iff } d_\lambda(\tilde{N}_1, \tilde{0}_1) < d_\lambda(\tilde{N}_2, \tilde{0}_1) \text{ iff } \tilde{N}_1 < \tilde{N}_2 \\ d_\lambda(\tilde{N}_1, \tilde{N}_2) &= 0 \text{ iff } d_\lambda(\tilde{N}_1, \tilde{0}_1) = d_\lambda(\tilde{N}_2, \tilde{0}_1) \text{ iff } \tilde{N}_1 \approx \tilde{N}_2 \end{aligned}$$

Then the order \lesssim is formulated as $\tilde{N}_1 \lesssim \tilde{N}_2$ iff $\tilde{N}_1 \approx \tilde{N}_2$ or $\tilde{N}_1 < \tilde{N}_2$.

Property 3.6. The relation \lesssim fulfil the three principles of partial order in Γ .Thus, for each $\tilde{N}_1, \tilde{N}_2, \tilde{N}_3 \in \Gamma$ the principles of partial order are as follows:

- (i) $\tilde{N}_1 \lesssim \tilde{N}_1$ (reflexive law)
- (ii) $\tilde{N}_1 \lesssim \tilde{N}_2$ and $\tilde{N}_1 \lesssim \tilde{N}_2 \Rightarrow \tilde{N}_1 \approx \tilde{N}_2$ (antisymmetric law)
- (iii) $\tilde{N}_1 \lesssim \tilde{N}_2$ and $\tilde{N}_2 \lesssim \tilde{N}_3 \Rightarrow \tilde{N}_1 \lesssim \tilde{N}_3$ (transitive law)

Proof (i),(ii), and (iii) are obvious by the Definitions 3.3 and 3.6.

Property 3.7. The relation \lesssim fulfil all the conditions of the law and trichotomy in Γ .

Proof It can be easily proved that by the Definition-3.3 and 3.6.

Remark 3.1. By the property 3.5 and 3.6, we can say that \lesssim is total ordering in Γ

Remark 3.2. $\tilde{N}_1 \approx \tilde{N}_2$ may not imply $\tilde{N}_1 = \tilde{N}_2$ and hence the relation \approx may not equal to the relation $=$.

Property 3.8. if $\tilde{N}_1, \tilde{N}_2, \tilde{N}_3, \tilde{N}_4$ be four SVN-numbers, then

- (i) $\tilde{N}_2 < \tilde{N}_1 \Rightarrow \tilde{N}_2 \oplus \tilde{N}_3 < \tilde{N}_1 \oplus \tilde{N}_3$
- (ii) $\tilde{N}_2 < \tilde{N}_1$ and $\tilde{N}_4 < \tilde{N}_2 \Rightarrow \tilde{N}_3 \oplus \tilde{N}_4 < \tilde{N}_1 \oplus \tilde{N}_2$
- (iii) $\tilde{N}_2 < \tilde{N}_1 \Rightarrow \tilde{N}_2 \ominus \tilde{N}_3 < \tilde{N}_1 \ominus \tilde{N}_3$
- (iv) $\tilde{N}_1 < \tilde{N}_3$ and $\tilde{N}_2 < \tilde{N}_4 \Rightarrow \tilde{N}_1 \ominus \tilde{N}_4 < \tilde{N}_3 \ominus \tilde{N}_2$ and $\tilde{N}_2 \ominus \tilde{N}_3 < \tilde{N}_4 \ominus \tilde{N}_1$.

Proof. (i) By the definition 3.6. $\tilde{N}_2 < \tilde{N}_1$ iff $d_\lambda(\tilde{N}_1, \tilde{N}_2) > 0$

By property 3.4 and $d_\lambda(\tilde{N}_1, \tilde{N}_2) > 0, d_\lambda(\tilde{N}_3, \tilde{N}_3) = 0$ we have $d_\lambda(\tilde{N}_1 \oplus \tilde{N}_3, \tilde{N}_2 \oplus \tilde{N}_3) = d_\lambda(\tilde{N}_1, \tilde{N}_2) + d_\lambda(\tilde{N}_4, \tilde{N}_4) > 0$. Therefore $\tilde{N}_2 \oplus \tilde{N}_3 < \tilde{N}_1 \oplus \tilde{N}_3$.

In a similar way, (ii), (iii), and (iv) can be proved.

4. Neutrosophic Transportation Problem

This section presents a specific type of linear programming problem (LPP) is called TP, which have a unique solution technique separate from general solution. TP is connected to our real-life daily activities. The goal of TP is to minimise the cost of transporting a single commodity from a number of sources to different destinations. The ability of manufacture at each source, the requirements at each destinations, and the cost of transportation of the goods from different sources to several destinations are known. It assumed that the cost of transportation on a given route is directly proportional to the number of units transported. But in current situation, because of different uncontrolled arguments, transportation parameters such as supply, demand, and unit transportation cost are unpredictable. Here we consider the neutrosophic transportation problem of SVTN-numbers and desire to define a methodology and solve. For simplicity, let us take $\lambda = 0.96$ to calculate the sign distance SVTN-numbers from 0 to 1 in the remaining paper because $\lambda = 0.96 \in (0.5, 1]$ shows that the decision maker's behaviour indicates optimistic in the direction of positivity and certainty.

First of all, we described TP with crisp data. Let us consider there being r sources, each source possessing a_i units of a certain product, whereas there are s destinations (r may or may not equal to s) with destinations j requiring b_j units. Let c_{ij} be the one unit of transportation cost of the product from the source to j^{th} destination, and x_{ij} be the amount to be shipped from the i^{th} source to j^{th} destination. Then the mathematical formulation of TP is given below as follows:

$$\text{Minimum } Z = \sum_{i=1}^r \sum_{j=1}^s x_{ij} c_{ij}$$

Subject to,

$$\sum_{j=1}^s x_{ij} = a_i$$

$$\sum_{i=1}^r x_{ij} = b_j$$

and $x_{ij} \geq 0$ for $i = 1, 2, \dots, r$ and $j = 1, 2, \dots, s$.

Where, the total availabilities $\sum a_i$ may or may not equal to the total requirements $\sum b_j$ for $i = 1, 2, \dots, r$ and $j = 1, 2, \dots, s$.

Now we have to describe NTP with SVN-numbers in two cases.

Case-1: NTP with supplies or capacities and demands or requirements in terms of SVN-numbers.

Minimizing the total cost of transportation (shipping)

$$Z = \sum_{i=1}^r \sum_{j=1}^s x_{ij} c_{ij} \text{ (objective function)}$$

Subject to,

$$\sum_{j=1}^s x_{ij} = \tilde{a}_i$$

$$\sum_{i=1}^r x_{ij} = \tilde{b}_j$$

and $x_{ij} \geq 0$ for $i = 1, 2, \dots, r$ and $j = 1, 2, \dots, s$.

Where, c_{ij} are cost values with crisp data, and $\sum \tilde{a}_i$ and $\sum \tilde{b}_j$ are the supplies and demands respectively in terms of SVN-numbers for $i = 1, 2, \dots, r$ and $j = 1, 2, \dots, s$.

Case-2: NTP with cost values in terms of SVN-numbers

Minimizing the total cost of transportation (shipping)

$$Z = \sum_{i=1}^r \sum_{j=1}^s x_{ij} \tilde{c}_{ij} \text{ (objective function)}$$

Subject to,

$$\sum_{j=1}^s x_{ij} = \tilde{a}_i$$

$$\sum_{i=1}^r x_{ij} = \tilde{b}_j$$

and $x_{ij} \geq 0$ for $i = 1, 2, \dots, r$ and $j = 1, 2, \dots, s$.

Where, \tilde{c}_{ij} are cost values in terms of SVN-numbers, and $\sum \tilde{a}_i$ and $\sum \tilde{b}_j$ are the supplies and demands respectively with crisp data for $i = 1, 2, \dots, r$ and $j = 1, 2, \dots, s$.

Methodology

Step 1. In a neutrosophic transportation problem with SVN-numbers, the SVN-numbers are converted to crisp numbers by using the sign distance method, and if we see the problem is unbalanced, i.e., total supply is less or greater than total demand, then we introduce a dummy row or a dummy column to balanced the neutrosophic transportation problem.

Step 2. VAM method is applied to determine the initial basic feasible solution.

Step 3. Finally, the MODI method is used to determine the optimal solution.

5. Numerical Example

In this section, we give two examples of NTP with SVTN-numbers. In the first examples, we take NTP with requirements and capacities in terms of SVTN-numbers, and in second example, we take NTP with transportation costs in terms of SVTN-numbers.

Examples 5.1 Let us consider five factories F_1, F_2, F_3, F_4, F_5 of a company from which goods have to be transported to the seven warehouses $W_1, W_2, W_3, W_4, W_5, W_6, W_7$. The warehouses are situated at varying distances from the factories, from where supplies are transported to them ; the transportation costs from the factories to warehouse thus naturally vary from Rs.3 to Rs.19 per unit, and the company desires to minimise these transportation costs. In the form of a 5×7 transportation problem with capacities and requirements in SVTN-numbers per unit, as shown in the given table below.

	W_1	W_2	W_3	W_4	W_5	W_6	W_7	Capacity
F_1	4	8	7	9	10	13	14	\tilde{S}_1
F_2	3	6	6	9	11	15	17	\tilde{S}_2
F_3	8	7	6	8	9	7	13	\tilde{S}_3
F_4	7	12	13	14	13	14	16	\tilde{S}_4
F_5	11	15	16	18	16	18	19	\tilde{S}_5

Required \tilde{d}_1 \tilde{d}_2 \tilde{d}_3 \tilde{d}_4 \tilde{d}_5 \tilde{d}_6 \tilde{d}_7

Where,

$$\tilde{S}_1 = \langle [20, 23, 5, 6], [20, 23, 7, 8], [20, 23, 3, 9] \rangle, \tilde{S}_2 = \langle [15, 17, 3, 7], [15, 17, 5, 9], [15, 17, 2, 8] \rangle$$

$$\tilde{S}_3 = \langle [10, 24, 2, 5], [10, 24, 3, 10], [10, 24, 5, 8] \rangle, \tilde{S}_4 = \langle [7, 18, 4, 9], [7, 18, 10, 2], [7, 18, 6, 12] \rangle$$

$$\tilde{S}_5 = \langle [7, 23, 4, 10], [7, 23, 3, 8], [7, 23, 1, 9] \rangle, \tilde{d}_1 = \langle [9, 17, 2, 7], [9, 17, 1, 6], [9, 17, 5, 10] \rangle$$

$$\tilde{d}_2 = \langle [18, 19, 3, 6], [18, 19, 4, 9], [18, 19, 5, 5] \rangle, \tilde{d}_3 = \langle [27, 29, 3, 8], [27, 29, 6, 8], [27, 29, 3, 5] \rangle$$

$$\tilde{d}_4 = < [25, 28, 13, 6], [25, 28, 4, 7], [25, 28, 2, 9] >, \tilde{d}_5 = < [8, 9, 2, 7], [8, 9, 3, 6], [8, 9, 7, 7] > \\ \tilde{d}_6 = < [10, 11, 3, 5], [10, 11, 7, 8], [10, 11, 4, 3] >, \tilde{d}_7 = < [19, 20, 6, 5], [19, 20, 4, 4], [19, 20, 3, 4] >$$

Solution:

Step-1: First of all, applying sign distance method to convert the supplies and demands in terms of SVTN-numbers to crisp data, we get an unbalanced transportation problem with crisp data given below.

	W_1	W_2	W_3	W_4	W_5	W_6	W_7	Capacity
F_1	3	8	7	4	5	1	8	11.28
F_2	5	2	9	5	3	6	7	8.67
F_3	4	3	6	2	10	9	8	9.11
F_4	10	7	3	6	7	4	5	6.89
F_5	1	6	8	3	8	10	7	8.35

Required 7.19 9.88 14.97 13.25 4.83 5.62 10.06

Since in the above transportation problem, the total demand is greater than the total supply, we introduce a dummy row s_6 origin to balance the neutrosophic transportation problem given in the following below:

	W_1	W_2	W_3	W_4	W_5	W_6	W_7	Capacity
F_1	3	8	7	4	5	1	8	11.28
F_2	5	2	9	5	3	6	7	8.67
F_3	4	3	6	2	10	9	8	9.11
F_4	10	7	3	6	7	4	5	6.89
F_5	1	6	8	3	8	10	7	8.35
F_6	0	0	0	0	0	0	0	21.5

Required 7.19 9.88 14.97 13.25 4.83 5.62 10.06

Step-2: In this step, we get the initial solution by using the VAM method, given in the below table

	W_1	W_2	W_3	W_4	W_5	W_6	W_7	Capacity
F_1	3	8	1.47 7	4.19 4	5	5.62 1	8	11.28
F_2	5	8.67 2	9	5	3	6	7	8.67
F_3	4	1.21 3	6	7.9 2	10	9	8	9.11
F_4	10	7	6.89 3	6	7	4	5	6.89
F_5	7.19 1	6	8	1.16 3	8	10	7	8.35
F_6	0	0	6.61 0	0	4.83 0	0	10.06 0	21.5

Required 7.19 9.88 14.97 13.25 4.83 5.62 10.06

Therefore, the initial transportation cost is Rs. 100.78.

Step-3: In this step, we obtain the optimal solution of NTP by using the MODI method, and we get an alternative optimal solution that is given in the following tables:

First optimal solution table:

	W_1	W_2	W_3	W_4	W_5	W_6	W_7	Capacity				
F_1	3	8	4.19	7	1.47	4	5	5.62	1	8	11.28	
F_2	5	8.67	2	9	5	3	6			7	8.67	
F_3	4	1.21	3	6	7.9	2	10	9		8	9.11	
F_4	10	7	6.89	3	6	7	4			5	6.89	
F_5	7.19	1	6	8	1.16	3	8	10		7	8.35	
F_6	0	0	8.08	0	0	3.36	0	0		10.06	0	21.5
Required	7.19	9.88	14.97	13.25	4.83	5.62	10.06					

Second optimal solution table:

	W_1	W_2	W_3	W_4	W_5	W_6	W_7	Capacity	
F_1				2.98		2.68		5.62	11.28
	3	8	7		4	5	1	8	
F_2	5	8.67							8.67
		2	9	5	3	6	7		
F_3			6	9.11					9.11
	4	3			2	10	9	8	
F_4			6.89						6.89
	10	7	3	6	7	4	5		
F_5	7.19			1.16					8.35
		1			3				
		6	8		8	10	7		
F_6		1.21	8.08			3.36		10.06	21.5
	0	0	0	0	0	0	0	0	
Required	7.19	9.88	14.97	13.25	4.83	5.62	10.06		

Therefore, the unique optimum transportation cost is Rs. 97.84.

Example 5.2. There are three sources or origin which store a given product. These sources supply these products to five dealers and the cost of transportation from one source to one dealer varies. In the form of a 3×5 neutrosophic transportation problem with cost values in terms of SVTN-numbers per unit and the supply of sources, the demands are shown in the below table.

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	\tilde{c}_{11}	\tilde{c}_{12}	\tilde{c}_{13}	\tilde{c}_{14}	\tilde{c}_{15}	30
S_2	\tilde{c}_{21}	\tilde{c}_{22}	\tilde{c}_{23}	\tilde{c}_{24}	\tilde{c}_{25}	50
S_3	\tilde{c}_{31}	\tilde{c}_{32}	\tilde{c}_{33}	\tilde{c}_{34}	\tilde{c}_{35}	75
Demand	10	20	33	40	52	

Where,

$$\tilde{c}_{11} = \langle [3,4,5,6], [3,4,4,7], [3,4,5,8] \rangle$$

$$\tilde{c}_{12} = \langle [13,25,6,9], [13,25,5,8], [13,25,2,7] \rangle$$

$$\tilde{c}_{13} = \langle [12,22,4,7], [12,22,2,5], [12,22,3,9] \rangle$$

$$\tilde{c}_{14} = \langle [5,7,6,9], [5,7,4,5], [5,7,7,8] \rangle$$

$$\tilde{c}_{15} = \langle [10,15,5,8], [10,15,3,8], [10,15,3,7] \rangle$$

$$\tilde{c}_{21} = \langle [1,4,3,7], [1,4,4,6], [1,4,5,6] \rangle$$

$$\tilde{c}_{22} = \langle [7,14,6,6], [7,14,7,8], [7,14,6,9] \rangle$$

$$\tilde{c}_{23} = \langle [9,16,5,7], [9,16,3,6], [9,16,2,5] \rangle$$

$$\tilde{c}_{24} = \langle [2,5,4,6], [2,5,2,7], [2,5,2,4] \rangle$$

$$\tilde{c}_{25} = \langle [1,3,7,8], [1,3,5,6], [1,3,6,9] \rangle$$

$$\tilde{c}_{31} = \langle [4,7,5,7], [4,7,4,5], [4,7,7,8] \rangle$$

$$\tilde{c}_{32} = \langle [15,18,3,9], [15,18,7,8], [15,18,2,7] \rangle$$

$$\tilde{c}_{33} = \langle [3,12,4,9], [3,12,2,9], [3,12,3,8] \rangle$$

$$\tilde{c}_{34} = \langle [10,21,9,11], [10,21,9,12], [10,21,7,8] \rangle$$

$$\tilde{c}_{35} = \langle [18,28,7,10], [18,28,8,11], [18,28,10,3] \rangle$$

Solution:

Step-1: First of all, applying sign distance method to convert the cost values in terms of SVTN-numbers to crisp data, we get a balanced TP with crisp data given below

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	1.92	10.15	9.11	3.37	6.77	30
S_2	1.47	5.47	6.68	2.00	1.13	50
S_3	3.03	9.08	4.34	8.24	12.22	75

Demand 10 20 33 40 52

Step-2: Here, applying VAM method, we obtain an initial basic feasible solution (IBF-solution) as given in the following table:

	D_1	D_2	D_3	D_4	D_5	Supply
S_1				28	2	30
	1.92	10.15	9.11	3.37	6.77	
S_2					50	50
	1.47	5.47	6.68	2.00	1.13	
	10	20	33	12		
S_3						75
	3.03	9.08	4.34	8.24	12.22	
Demand	10	20	33	40	52	

Therefore, the initial cost is Rs. 618.40.

Step-3: Now, we get the optimal solution by applying the MODI method, which is equal to the IBF-solution, and hence the optimal transportation cost is Rs. 618.40.

6. Conclusion

In this article, we define level- (α, β, γ) neutrosophic points which is the generalisation of level- α fuzzy point. We designed the sign sign distance ranking methodology in a new way and also established some important properties of this method. Finally, we discussed NTP, and using the

proposed methodology, some neutrosophic transportation problems with SVTN-numbers were solved. In the future, the method can be applied to several fields like assignment problems, game theory, multi-criteria decision-making problems, inventory, and so on.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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An Intelligent Decision Support Model for Optimal Selection of Machine Tool under Uncertainty: Recent Trends

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Abstract: Many scholars have been interested in the subject of machine tool selection as a result of the growing number of different machines and the continuous advancement of technology associated with these machines. The selection of an unsuitable machine tool may lead to a variety of issues, including limitations on production capacities and productivity indicators when taking into account both time and money from an industrial and practical perspective. The present strategy of selecting machine tools, known as multi-criteria decision-making (MCDM), relies on the subjective viewpoint the vast majority of the time. When selecting an appropriate machining tool, however, it is necessary to take both the subjective and objective points of view into consideration. This is due to the fact that the objective assessment accurately reflects the performance of the machine tools. As a result, the purpose of this work is to provide a strategy for selecting machine tools that are based on an innovative hybrid MCDM framework. The study was conducted under a neutrosophic environment and using triangular neutrosophic numbers (TNNs). In the beginning, the CRiteria Importance through InterCriteria Correlation (CRITIC) method is used to assess and prioritize the criteria set for the study. Then, the Additive Ratio Assessment (ARAS) method is applied to evaluate and rank four machine tools that were selected and used as alternatives in the study. The results indicate that the criteria of maximum spindle speed and linkage accuracy are the most important in determining the best machine tool. Also, the results indicate that the best alternative among the four tools used is FIDLA GTF-28. As a result, the requirements and priorities for research in the future are highlighted.

Keywords: Machine tool selection; Decision support model; Neutrosophic MCDM; CRITIC method; ARAS method.

1. Introduction

Manufacturing sectors are experiencing many problems such as globalization and quickly changing market demand, which result in greater manufacturing needs and more complex components. These issues have resulted in higher manufacturing requirements and more complicated products [1]. Because of this, it is essential for the continued profitability of businesses to choose suitable machining equipment for the various production activities they must do. In large-scale mechanical equipment, such as ships, construction machinery, railway vehicles, wind turbines, hydroelectric generators, nuclear power equipment, petrochemical equipment, and so on, machine tools are typically used because they are efficient, cost-effective, have a high value-added component, and have a complex structure [2]. This type of electromechanical equipment is known as a machine tool. Machine tools provide the key manufacturing capacity to turn raw materials into finished parts for final product assembly. When it comes to specialized machining jobs, manufacturing companies often have more than one machine tool that is capable of meeting the machining requirements. In order to pick the proper machine tools, manufacturing businesses need to examine the cost,

productivity, and company profit. If you choose the appropriate machine tool, you may cut down on the amount of time it takes to supply produced goods, increase the flexibility and quality of your output, and boost your total productivity. Inappropriate judgments regarding the selection of machines lower the return on investments, raise the expenses associated with quality and maintenance, and finally have a negative influence on customer satisfaction. Because choosing machines is both an involved and time-consuming procedure, it is essential that those in charge of making the call possess the appropriate level of expertise and experience. Researchers have been motivated to construct models that may assist decision-makers as a result of testing the machine selection issue [3]. methods for rational decision support that are based on numerical models provide a methodical approach that makes use of the existing knowledge and, in addition, offers insights that may assist the decision-maker in analysing the choices that have been reached as a consequence of using the methods.

On the other hand, if the machine tool is not appropriately chosen, it might result in damage to the machine tool. There are around 300 businesses in the United States that are capable of remanufacturing machine tools. It has been shown that the cost of machine tool downtime and maintenance brought on by failure or incorrect usage is very close to 75 percent of the cost of brand-new machine tools. There are about 2000 businesses in China that are capable of providing services for the repair and remanufacturing of machine tools. According to the findings of the study, however, the primary cause for machine tool downtime and maintenance is still due to the failure of the machine tools themselves or incorrect usage of the machine tools. As a result, when confronted with particular processing tasks, how to choose the appropriate machine tool from among the many candidates of machine tools has always been a major difficulty for enterprise decision-makers. This is due to the fact that an incorrect selection of machine tools may have the potential to negatively affect the overall performance of the manufacturing system [4].

It is typical for engineers and managers to find the process of selecting machine tools to be a challenging one [5]. This is due to the fact that there is a great deal of qualitative and quantitative aspects that need to be taken into consideration when choosing the proper machine tool. A significant amount of work and effort has been put into selecting the appropriate machine tools. The most important components of the present methodologies are the multi-criteria decision-making (MCDM) model and the optimization-based model. The best mathematical model is one that is able to deal with objective facts well, but it often misses the qualitative and subjective aspects. Because there are many different criteria for selecting machine tools, some of which are in direct opposition to one another, the quantification of unknown qualitative attribute information may be an incredibly difficult task. For instance, the amount of time spent on auxiliary tasks, clamping tasks, machining tasks, and changeover tasks might vary depending on the capabilities of the machine tool. As a result, for a successful selection of machine tools, it is necessary to find a middle ground between competing physical and intangible variables; MCDM has been shown to be helpful in finding solutions to these problems. The evaluation of qualitative characteristics by experts, on the other hand, is inherently subjective and, as a result, imprecise because of the ambiguity involved. The neutrosophic linguistic approach has been proven to be an appropriate strategy for dealing with the issue of expert assessment. This is due to the fact that the evaluation information, such as criterion weights and alternative ratings, are often stated in terms of language.

The primary contribution of this study is the development of a machine tool selection approach that incorporates expert knowledge and the actual performance of alternatives. This is achieved through the application of a proposed neutrosophic CRiteria Importance through InterCriteria Correlation (CRITIC) [6] - Additive Ratio Assessment (ARAS) [7] framework. By integrating these factors, the decision-making process yields results that closely align with real-world conditions, thereby enhancing the reliability and practicality of the evaluation outcomes.

This work makes a number of important contributions, one of the most important of which is the development of a system for selecting machine tools that makes use of both expert knowledge and actual operation information in order to deliver the most suitable machine tool for a given manufacturing job. Another contribution that was made was the development of a hybrid MCDM framework known as neutrosophic CRITIC-ARAS. This framework permits the examination of alternatives from both subjective and objective points of view and gives the decision-makers with the best possible choice.

The purpose of this work is to present a decision-making approach for the purpose of resolving the issue of alternative selection in machine tools. This method is one that can avoid the subjectivity of standard decision-making methods that are based on experience.

2. Problem Elements

In this section, the criteria used in the study are presented to select the most appropriate machine tool. The criteria used are maximum spindle speed (C_1), failure rate (C_2), utilization (C_3), linkage accuracy (C_4), maximum spindle torque (C_5), and cost (C_6). The alternatives determined namely: APEC GL-27 (A_1), FIDLA Y2K-411 (A_2), FRFQ-250-VR/A8 (A_3), and FIDLA GTF-28 (A_4). Three professionals were hired to participate in the study, and their information appears in Table 1. Also, Figure 1 presents the main objective of the study, evaluation criteria, and four machine tools used in the study.

Table 1. Particulars about professionals.

Professionals	Experience	Title	Graduation degree
Professional ₁	24	Senior engineer	Ph.D. in Mechanical Engineering
Professional ₃	12	Process engineer	B.S. in Process Engineering
Professional ₄	17	Operation manger	MSc. in Industrial Engineering

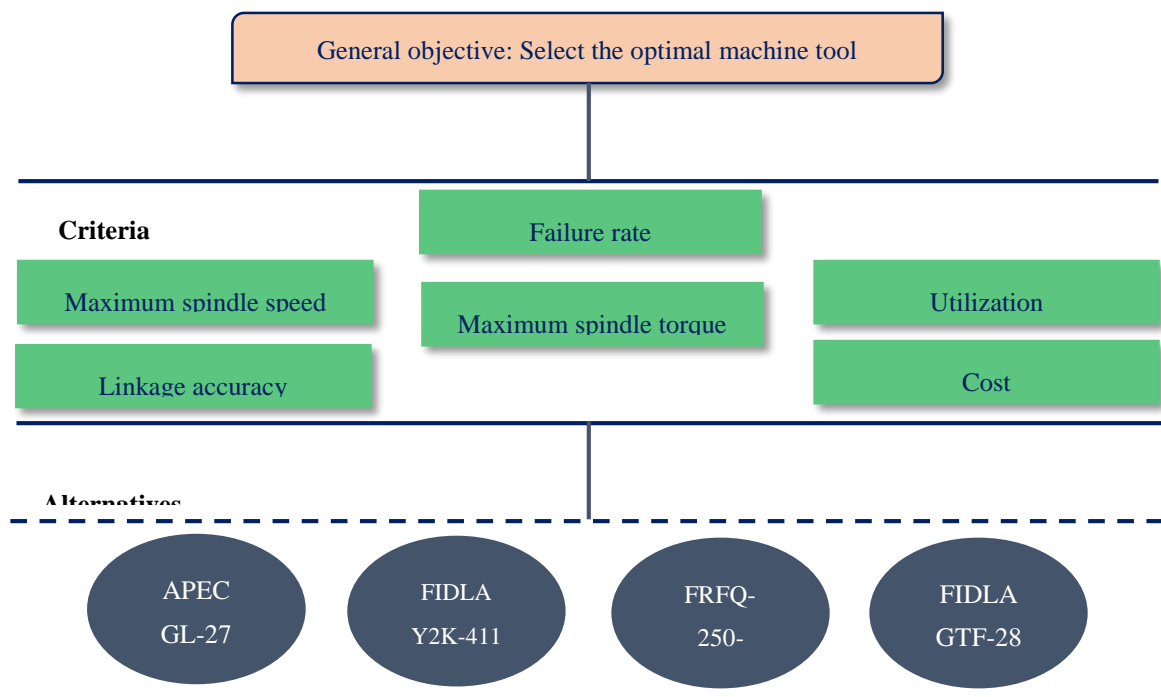


Figure 1: Objective, criteria, and alternatives of the problem.

3. Research Framework

In this section, the proposed CRITIC-ARAS methodology is presented to solve the machine tools selection problem. The proposed methodology is performed under a neutrosophic environment using TFNs. Figure 2 provides details of the proposed methodology.

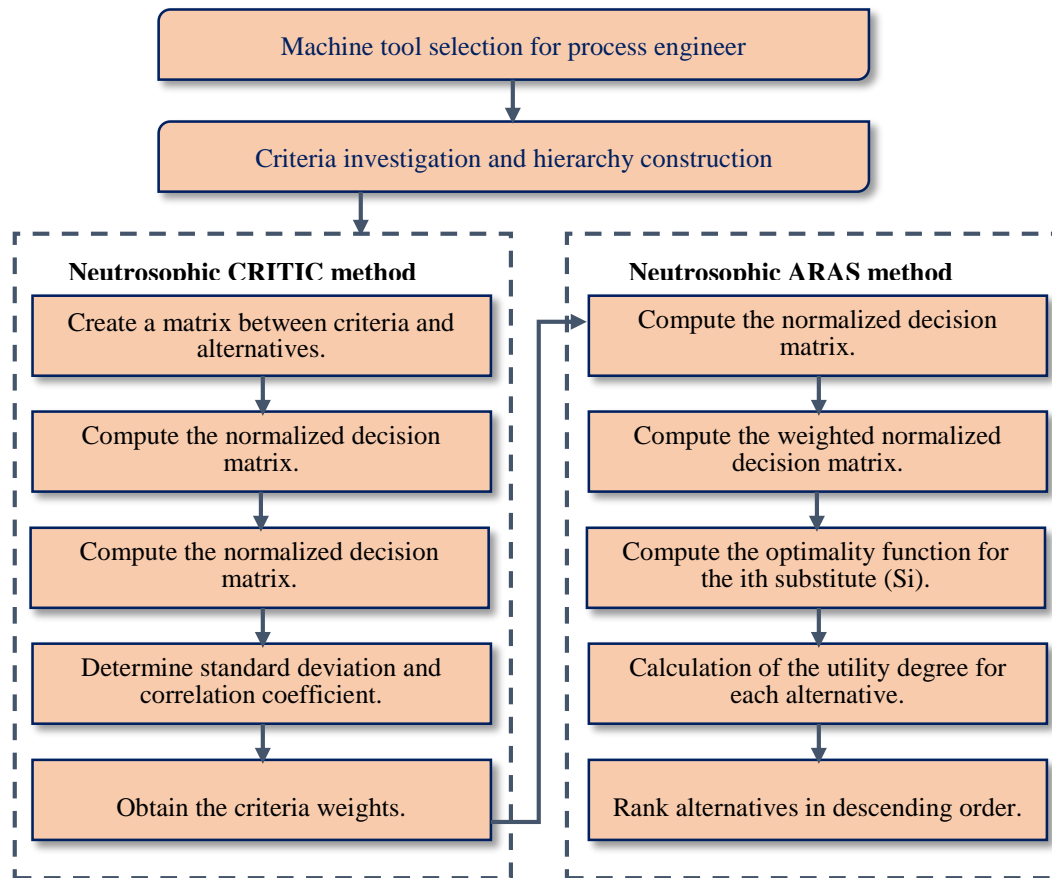


Figure 2: The main structure of the research methodology for this problem.

Step 1: The problem is studied in detail and its main aspects are identified, which consist of the main objective, criteria, and substitutes used. A set of alternatives are identified to be used in the evaluation process. The set machine tools = (A_1, A_2, \dots, A_m) having $i = 1, 2, \dots, m$ substitutes, is measured by n decision criteria of $C_j = (C_1, C_2, \dots, C_n)$, with $j = 1, 2, \dots, n$. Let $w = (w_1, w_2, \dots, w_n)$ be the vector set utilized for defining the criteria weights, $w_j > 0$ and $\sum_{j=1}^n w_j = 1$.

Step 2: A set of terms and their corresponding TFNs is defined in Table 2, to be used by the authors and professionals involved in evaluating the criteria and alternatives used.

Table 2. Linguistic variables and their equivalent TNNs for evaluating criteria and alternatives.

Linguistic variables	Abbreviations	TNNs
Absolute Low Worth	ALW	$\langle (0.1, 0.2, 0.3); 0.4, 0.1, 0.3 \rangle$
Very Low Worth	VLW	$\langle (0.2, 0.3, 0.4); 0.5, 0.1, 0.3 \rangle$
Low Worth	LOW	$\langle (0.3, 0.4, 0.5); 0.6, 0.2, 0.1 \rangle$
Modest Low Worth	MLW	$\langle (0.4, 0.5, 0.6); 0.7, 0.3, 0.2 \rangle$
Nearly Worth	NWO	$\langle (0.5, 0.6, 0.7); 0.8, 0.3, 0.3 \rangle$
Modestly High Worth	MHW	$\langle (0.6, 0.7, 0.8); 0.9, 0.4, 0.4 \rangle$

High Worth	HGW	$\langle(0.7, 0.8, 0.9); 1.0, 0.3, 0.5\rangle$
Very High Worth	VHW	$\langle(0.8, 0.9, 1.0); 1.0, 0.2, 0.3\rangle$
Absolute High Worth	AHW	$\langle(0.9, 1.0, 1.0); 1.0, 0.2, 0.2\rangle$

Step 3: Create a pairwise comparison matrix amongst the alternatives and the selected criteria by all professionals to simplify their preferences for these criteria.

Step 4: Transform the TNNs to crisp values by applying the score function according to Eq. (1).

$$S(\tilde{x}_{ij}) = \frac{1}{8} (l + m + u) \times (2 + \alpha_{\tilde{x}} - \theta_{\tilde{x}} - \beta_{\tilde{x}}) \quad (1)$$

Step 5: Calculate the normalized decision matrix for criteria according to Eq. (2).

$$x_{ij}^* = \frac{x_{ij} - \min(x_{ij})}{\max(x_{ij}) - \min(x_{ij})} \quad i = 1, 2 \dots m \text{ and } j = 1, 2 \dots n. \quad (2)$$

Step 6: Compute the standard deviation and correlation coefficient.

Step 7: Compute the quantity of information of criteria according to Eq. (3). D_j is an abbreviation that stands for "the amount or volume of information provided in the j^{th} criteria".

$$D_j = \sigma_j \cdot \sum_{k=1}^m (1 - r_{jk}) \quad (3)$$

Step 8: Obtain the criteria weights according to Eq. (4).

$$w_j = \frac{D_j}{\sum_{k=1}^m D_k} \quad (4)$$

Step 9: Create the assessment decision matrix by all professionals between the determined criteria and the available alternatives to evaluate a machine tools by using the linguistic variables, provided in Table 2.

Step 10: Compute the normalized decision matrix for the advantageous criteria according to Eq. (5), and for non-advantageous factors according Eq. (6).

$$y_{ij} = \frac{y_{ij}}{\sum_{i=0}^m y_{ij}} \quad (5)$$

$$y_{ij} = \frac{1}{y_{ij}^*} ; \quad y_{ij} = \frac{y_{ij}}{\sum_{i=0}^m y_{ij}} \quad (6)$$

Step 11: Compute the weighted assessment decision matrix by multiplying the value of the normalized decision matrix by the corresponding weights according to Eq. (7).

$$y_{ij} = w_j \times y_{ij} \quad (7)$$

Step 12: Define the values of optimality function for the i^{th} substitute according to Eq. (8).

$$S_i = \sum_{j=1}^n y_{ij}, \quad i = 1, 2 \dots m. \quad (8)$$

Step 13: Compute the utility degree for each substitute according to Eq. (9). Then, rank the substitutes in descending order according to the value of k_i .

$$k_i = \frac{S_i}{S_0}, \quad i = 1, 2 \dots m. \quad (9)$$

4. Application

4.1 Implementation of the recommended methodology

In this part, the proposed CRITIC-ARAS methodology is applied to evaluate and rank four machine tools.

Step 1: The problem was studied and its basic details were determined. In this regard, the main goal has been identified, which is to determine the best machine tool. Also, six criteria have been identified that have a direct impact on solving the problem. The six criteria used are maximum spindle speed (C_1), failure rate (C_2), utilization (C_3), linkage accuracy (C_4), maximum spindle torque (C_5), and cost (C_6). Finally, the alternatives used were identified namely: APEC GL-27 (A_1), FIDLA Y2K-411 (A_2), FRFQ-250-VR/A8 (A_3), and FIDLA GTF-28 (A_4).

Step 2: A pairwise comparison matrix amongst the alternatives and the selected criteria was created by all professionals to simplify their preferences for these criteria using linguistic terms in Table 2, as presented in Table 3, then using TNNs as exhibited in Table 4. Also, the TNNs were transformed to crisp values by applying the score function according to Eq. (1).

Step 3: The normalized decision matrix was computed for criteria according to Eq. (2), as presented in Table 5. Then, the standard deviation was computed, as presented in Table 5.

Step 4: The quantity of information of criteria was computed according to Eq. (3), as presented in Table 6. Finally, the criteria weights were obtained according to Eq. (4), as presented in Table 6 and shown in Figure 3.

Table 3. Evaluation matrix based on the selected criteria for all professionals using linguistic variables.

Criteria/ Alternatives	Professional _s					
	C_1	C_2	C_3	C_4	C_5	C_6
A_1	MLW	MLW	MHW	VLW	HGW	MHW
A_2	AHW	ALW	MLW	HGW	MLW	ALW
A_3	MLW	LOW	HGW	ALW	MLW	MHW
A_4	MHW	MHW	MHW	NOW	VLW	MLW

Table 4. Evaluation matrix based on the selected criteria for all professionals using TNNs.

Criteria/ Alternatives	Professional _s		
	C_1	C_2	C_3
A_1	$\langle(0.4, 0.5, 0.6); 0.7, 0.3, 0.2\rangle$	$\langle(0.4, 0.5, 0.6); 0.7, 0.3, 0.2\rangle$	$\langle(0.6, 0.7, 0.8); 0.9, 0.4, 0.4\rangle$
A_2	$\langle(0.9, 1.0, 1.0); 1.0, 0.2, 0.2\rangle$	$\langle(0.1, 0.2, 0.3); 0.4, 0.1, 0.3\rangle$	$\langle(0.4, 0.5, 0.6); 0.7, 0.3, 0.2\rangle$
A_3	$\langle(0.4, 0.5, 0.6); 0.7, 0.3, 0.2\rangle$	$\langle(0.3, 0.4, 0.5); 0.6, 0.2, 0.1\rangle$	$\langle(0.7, 0.8, 0.9); 1.0, 0.3, 0.5\rangle$
A_4	$\langle(0.6, 0.7, 0.8); 0.9, 0.4, 0.4\rangle$	$\langle(0.6, 0.7, 0.8); 0.9, 0.4, 0.4\rangle$	$\langle(0.6, 0.7, 0.8); 0.9, 0.4, 0.4\rangle$
Criteria	Professional _s		
	C_4	C_5	C_6
A_1	$\langle(0.2, 0.3, 0.4); 0.5, 0.1, 0.3\rangle$	$\langle(0.7, 0.8, 0.9); 1.0, 0.3, 0.5\rangle$	$\langle(0.6, 0.7, 0.8); 0.9, 0.4, 0.4\rangle$
A_2	$\langle(0.7, 0.8, 0.9); 1.0, 0.3, 0.5\rangle$	$\langle(0.4, 0.5, 0.6); 0.7, 0.3, 0.2\rangle$	$\langle(0.1, 0.2, 0.3); 0.4, 0.1, 0.3\rangle$
A_3	$\langle(0.1, 0.2, 0.3); 0.4, 0.1, 0.3\rangle$	$\langle(0.4, 0.5, 0.6); 0.7, 0.3, 0.2\rangle$	$\langle(0.6, 0.7, 0.8); 0.9, 0.4, 0.4\rangle$
A_4	$\langle(0.5, 0.6, 0.7); 0.8, 0.3, 0.3\rangle$	$\langle(0.2, 0.3, 0.4); 0.5, 0.1, 0.3\rangle$	$\langle(0.4, 0.5, 0.6); 0.7, 0.3, 0.2\rangle$

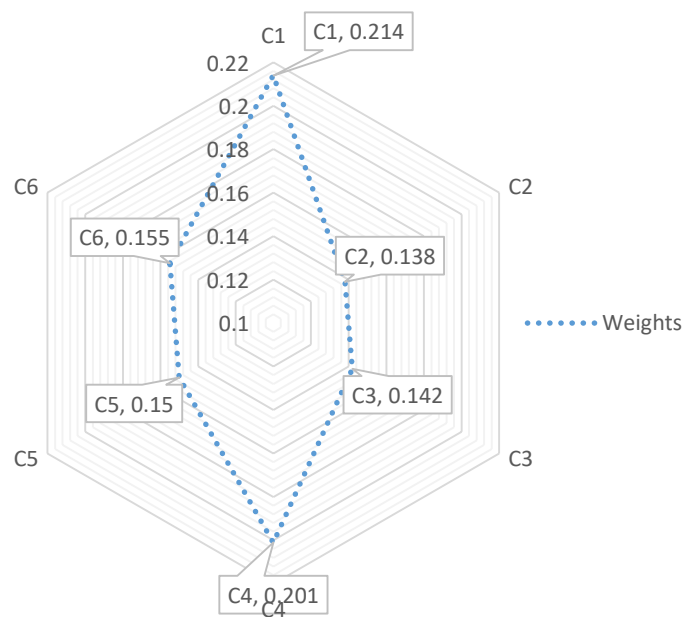
Table 5. Normalized decision matrix based on criteria by all professionals.

Criteria/Alternatives	C_1	C_2	C_3	C_4	C_5	C_6
A_1	0.0000	0.6542	0.5600	0.1691	1.0000	0.9969

A_2	1.0000	0.0000	0.0000	1.0000	0.4100	0.0000
A_3	0.0000	0.4860	1.0000	0.0000	0.4159	1.0000
A_4	0.2653	1.0000	0.5600	0.6765	0.0000	0.6480
Standard Deviation	0.4726	0.4160	0.4097	0.4600	0.4113	0.4708

Table 6. Correlation coefficient of the relationship among the criteria and final weights.

Criteria	C_1	C_2	C_3	C_4	C_5	C_6	D_j	w_j
C_1	1.0000	-0.6981	-0.8986	0.9134	-0.2873	-0.9959	3.2926	0.214
C_2	-0.6981	1.0000	0.5438	-0.3491	-0.2350	0.6316	2.1242	0.138
C_3	-0.8986	0.5438	1.0000	-0.8926	0.0162	0.8975	2.1853	0.142
C_4	0.9134	-0.3491	-0.8926	1.0000	-0.4640	-0.9443	3.0985	0.201
C_5	-0.2873	-0.2350	0.0162	-0.4640	1.0000	0.3537	2.3102	0.150
C_6	-0.9959	0.6316	0.8975	-0.9443	0.3537	1.0000	2.3808	0.155

**Figure 3.** Final weights of the selected criteria.

Step 5: The assessment decision matrix was created by all professionals between the determined criteria and the four machine tools by using the linguistic variables, provided in Table 2.

Step 6: The normalized decision matrix was computed for the advantageous criteria according to Eq. (5), and for non-advantageous factors according to Eq. (6), as presented in Table 7.

Step 7: The weighted assessment decision matrix was computed by multiplying the value of the normalized decision matrix by the corresponding weights according to Eq. (7), as presented in Table 8.

Step 8: The values of optimality function for the i^{th} substitute was defined according to Eq. (8), as presented in Table 9.

Step 9: The utility degree for each substitute was computed according to Eq. (9), as presented in Table 9. Then, the four machine tools were ranked in descending order according to the value of k_i , as presented in Table 9 and shown in Figure 4.

Table 7. Normalized evaluation matrix of four machine tools regarding criteria.

Alternatives	C_1	C_2	C_3	C_4	C_5	C_6
Optimal value	0.2894	0.2743	0.2332	0.2998	0.2775	0.2492
A_1	0.1259	0.2052	0.1943	0.1073	0.2775	0.2486
A_2	0.2894	0.0746	0.1449	0.2998	0.1724	0.0678
A_3	0.1259	0.1716	0.2332	0.0681	0.1734	0.2492
A_4	0.1693	0.2743	0.1943	0.2249	0.0993	0.1853

Table 8. Weighted normalized evaluation matrix of four machine tools regarding criteria.

Alternatives	C_1	C_2	C_3	C_4	C_5	C_6
Optimal value	0.0619	0.0378	0.0331	0.0603	0.0416	0.0386
A_1	0.0269	0.0283	0.0276	0.0216	0.0416	0.0385
A_2	0.0619	0.0103	0.0206	0.0603	0.0259	0.0105
A_3	0.0269	0.0237	0.0331	0.0137	0.0260	0.0386
A_4	0.0362	0.0378	0.0276	0.0452	0.0149	0.0287

Table 9. Final ranking of four machine tools regarding all criteria.

Alternatives	S_i	K_i	Rank
S_0	0.2734	1.0000	
A_1	0.1846	0.6751	3
A_2	0.1894	0.6929	2
A_3	0.1621	0.5928	4
A_4	0.1905	0.6967	1

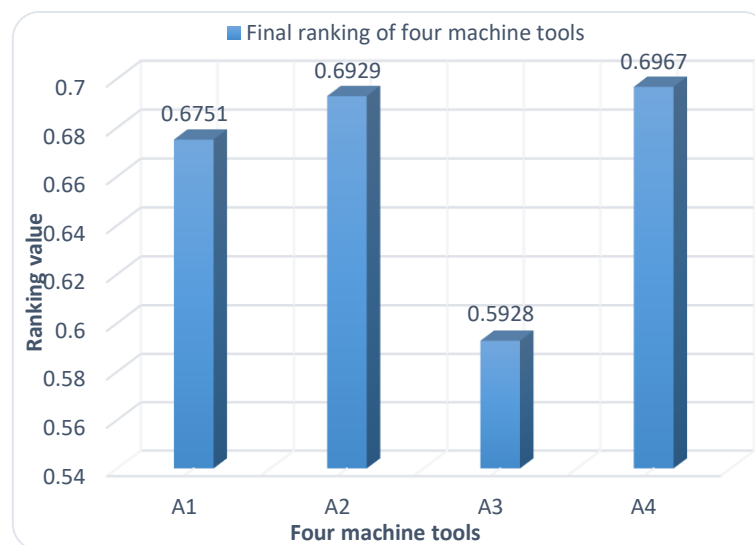


Figure 4. Final ranking of four machine tools using ARAS technique.

4.2 Discussion

In this part, the results obtained from the application of the proposed methodology CRITIC-ARAS under the neutrosophic environment are discussed.

Initially, the six criteria were evaluated using the CRITIC method. The results in Table 5 indicate that the maximum spindle speed criterion is the most influential criterion with a weight of 0.214, followed by the linkage accuracy criterion, while the failure rate criterion is the least influential with a weight of 0.138.

Also, four machine tools were evaluated and ranked as shown in Table 7. The results indicate that the FIDLA GTF-28 machine tool is the best to be used in the industry.

5. Conclusion

One of the most important decisions that must be made, which may have significant repercussions for the performance of a company, is which new machine will be introduced to the production system. In recent years, a rising number of scholars have been interested in the topic of machine selection. This is mostly due to the fact that the number of different machine tools is growing, as is the rate at which manufacturing technology is advancing. It was said that applying the MCDM approaches to the issue of machine selection is a project that requires appropriate preparation in order to be successful. This is because the judgments that go into machine selection are primarily impacted by technical and economic factors, both of which have clear evaluations easily accessible in the majority of situations. As a result, this is something that can be linked to the fact that this is the case. Comparing two machines' capacities, dimensions, levels of power consumption, and speeds, for instance, is as simple as comparing apples and oranges. Both the purchase price and the running expenses are able to be obtained in a similar fashion and included in the comparisons. The majority of the publications that were examined indicated that it was simple to use such criteria when making comparisons. In situations when some technical criteria do not have crisp values or when other non-crisp criteria are chosen, such as those relating to sustainability, maintainability, productivity, and other things of a similar kind, fuzzy representations are often used. Researchers are the most common users of fuzzy representations.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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Neutrosophic Model to Examine the Challenges Faced by Manufacturing Businesses in Adopting Green Supply Chain Practices and to Provide Potential Solutions

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Abstract: Several obstacles stand in the way of companies trying to adopt green supply-chain practices. The purpose of this research is to examine the challenges faced by the industrial industry in adopting green supply chain practices and to provide potential solutions. The information for this research was gathered via in-depth, personal conversations with manufacturing sector managers who are well-versed in green supply chain practices. The Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) technique was used in the evaluation phase to evaluate obstacles and assess efficient options for introducing green supply chain practices. The TOPSIS method is integrated with the neutrosophic set. The neutrosophic set is used to deal with uncertain data. This study used eight barriers to analysis and eight solutions to rank. We obtained collaboration as the best solution. To find places for improvement and garner support for sustainability efforts, it is helpful to work together with suppliers, customers, and other stakeholders.

Keywords: Neutrosophic Set; Supply Chain; Green Supply Chain; Barriers; Solutions.

1. Introduction

The beginning of the Industrial Revolution in the 1800s was a pivotal time in human history. Population estimates put the world's total inhabitants at 7.8 billion, up from a mere few million before the uprising. Since the dawn of industrialization, nature has been badly affected by the unchecked use of earth's resources and the many forms of pollution that have resulted from their usage. Before the Industrial Age, the number of greenhouse emissions in the atmosphere remained relatively constant at about 250–300 parts per million (ppm), but with the advent of industrialization, it has climbed to 400 ppm (for carbon dioxide) and doubled (for methane gas). Understanding of ecological issues has increased all across the world, particularly as a result of the spread of globalization. Consequently, concerns about the items' impact on the surroundings have grown in importance [1, 2].

The development of supply chains and leadership has resulted from the rise of globalization, industrialization, and environmental consciousness in recent years. As environmental problems have worsened, conventional supply chains have been blamed for adding to the problem by producing excessive quantities of waste and emissions. The importance of ecological consciousness has risen in recent years as the effects of ecological problems on people's level of life have become more obvious. Clients with a concern for the environment have started putting pressure on supply chains to improve environmental sustainability. As a result of these shifts, businesses that wish to compete in the global market must prioritize protecting the planet.

To put it simply, the supply chain is the system through which raw materials are transformed into finished goods and then delivered to consumers. In order to address ecological issues in SCM, the notion of a "green supply chain" has evolved. As a result of rising consumer knowledge and environmental demands in the varied worldwide marketplace, modern companies are putting increased focus on ecologically conscious buying and sustainable practices throughout the supply chain. Reducing carbon dioxide (CO₂) emissions and increasing energy savings throughout the supply chain management (SCM) system is what is meant by "green supply chain management" (GSCM) [3, 4].

Companies have been compelled to use green practices at all stages of production due to the broad recognition of the relevance of sustainable supply chains in a variety of circles. As participants in the supply chain have become more concerned about global warming, the fast depletion of the earth's resources, and the rising loss of biodiversity, a green supply chain has emerged to address these issues. Environmental issues in customer and supplier interactions are assessed as part of the GSCM procedure. The use of energy is integral to the manufacturing process, and as a result, carbon dioxide gas is released into the environment, contributing to air pollution.

Many businesses have made efforts recently to switch to environmentally friendly supply chain operations. Growing attention and acknowledgment of ecological responsibility among academics and supply chain managers is largely attributable to the increasing popularity of "green" ideas and government rules requiring the implementation of GSCM practices. However, businesses have hit roadblocks that prohibit them from deploying these cutting-edge apps in full. Some difficulties may arise throughout the process of GSCM implementation. However, companies struggle to both recognize and address the challenges they face when attempting to put GSCM into practice. It is up to individual companies to better themselves in order to triumph against the opposition. While it would be ideal to remove all obstacles simultaneously, this is obviously not achievable. So, in the first phases of GSCM implementation, organizations should pinpoint the obstacles that need to be removed [5, 6].

Researchers are interested in multi-criteria decision-making (MCDM) approaches because of the enormous advantages they provide in tackling complex social and industrial challenges. Due to its better accuracy and dependability in resolving real-world difficulties, MCDM approaches have been extensively used by academics across a broad range of industries [7, 8].

Because of the inherent inconsistency and confusion associated with solutions in green supply chain selection, both linguistic and non-linguistic factors provide distinct benefits. Fang and Ye created linguistic neutrosophic numbers (LNNs) to handle MCDM situations with uncertain and contradictory linguistic information, combining the benefits of the two approaches. LNNs synthesize the benefits of both linguistic parameters and single-valued neutrosophic numbers (SVNNs) [9, 10].

2. Green Supply Chain

Evaluation and selection of green vendors, costing and procurement of substances, inbound and outbound logistics organizing, and resource shortage and demand balancing are all part of GSCM. In the manufacturing industry, essentials such as materials, parts, and outsourced procedures account for more than 60% of total revenue spent on suppliers. The results of a study done by Lo et al. showed that suppliers have an impact on the success of businesses and the long-term growth of enterprises. Research suggests that industrial processes are responsible for 45% of all greenhouse gases [11, 12].

Changes to the supply chain are a potential issue in the aftermath of COVID-19. Changes in the supply chain may have a significant influence on the availability of items and services because they lead to limitations of ingredients, parts, and completed products. Changes in the supply chain may have a negative effect on a company's bottom line by increasing the price of inputs like raw materials, shipping, and others. Customers may become unsatisfied if they cannot receive the essential products

or services as a result of supply chain interruptions, and companies may experience reputational harm if they are unable to satisfy orders or meet consumer expectations. It may be easier to deal with supply chain interruptions in a post-COVID-19 situation if vendors with the necessary expertise are identified [13, 14].

Up lately, ecological effects and loss of earth's resources were not major concerns for supply chains, which instead prioritized speedy product delivery at cheap prices. However, with a growing global population comes rising demand for already scarce assets such as water, power, metals, rocks, and land. Furthermore, as people become more aware that greenhouse gas emissions are contributing to global warming, they put increased pressure on businesses to adopt ecologically beneficial practices.

As environmental degradation accelerates, more and more people are interested in studying and implementing GSCs, which are an expansion of standard supply chains that involve steps to reduce a product's ecological impact at every stage of its existence. In order to meet the requirements of a rapidly expanding population, supply networks must improve their efficiency and raise output. Improvements in technology at the chain stage can be evaluated, and current logistics administration (including manufacturing, shipping, and inventory leadership) can be optimized, with the help of decision support tools that take into account product features such as, for example, higher dangers related to ambiguity in the market and effectiveness [15, 16].

Calculating trade-offs among financial and ecological factors and quantifying what is technically achievable are prerequisites for reducing redundancies and constructing GSCs. Our definition of eco-efficiency originates from this idea, and it's as follows: "preserving or enhancing the value of the economy while lessening the impact of business operations upon ecological systems." Therefore, an "eco-efficient approach" is one in which additional ecological damage can only be stopped at greater prices, combining the needs of ecology and the economy [17, 18].

Due to the inherent trade-offs inherent in the architecture of any Supply Chain (SC), researching eco-efficiency in GSCs necessitates the evaluation of numerous competing factors. Supply chains that include several criteria are a logical method to cope with the many facets of sustainability [19, 20].

3. Neutrosophic TOPSIS Method

For decision issues in a broad variety of domains, the TOPSIS has become a popular and commonly used MCDM approach owing to its straightforward methodology and user-friendliness. The methodology of this technique relies on measuring how far away the beneficial and detrimental ideal solutions are from the reference points used to narrow down the options under consideration. As a result, the optimal choice is the one that minimizes the distance between itself and the Positive Ideal Solution (PIS) [21]–[24]. This study employed the TOPSIS method with the single-valued neutrosophic set to rank the solution in the green supply chain. Figure 1 shows the steps of the neutrosophic TOPSIS method.

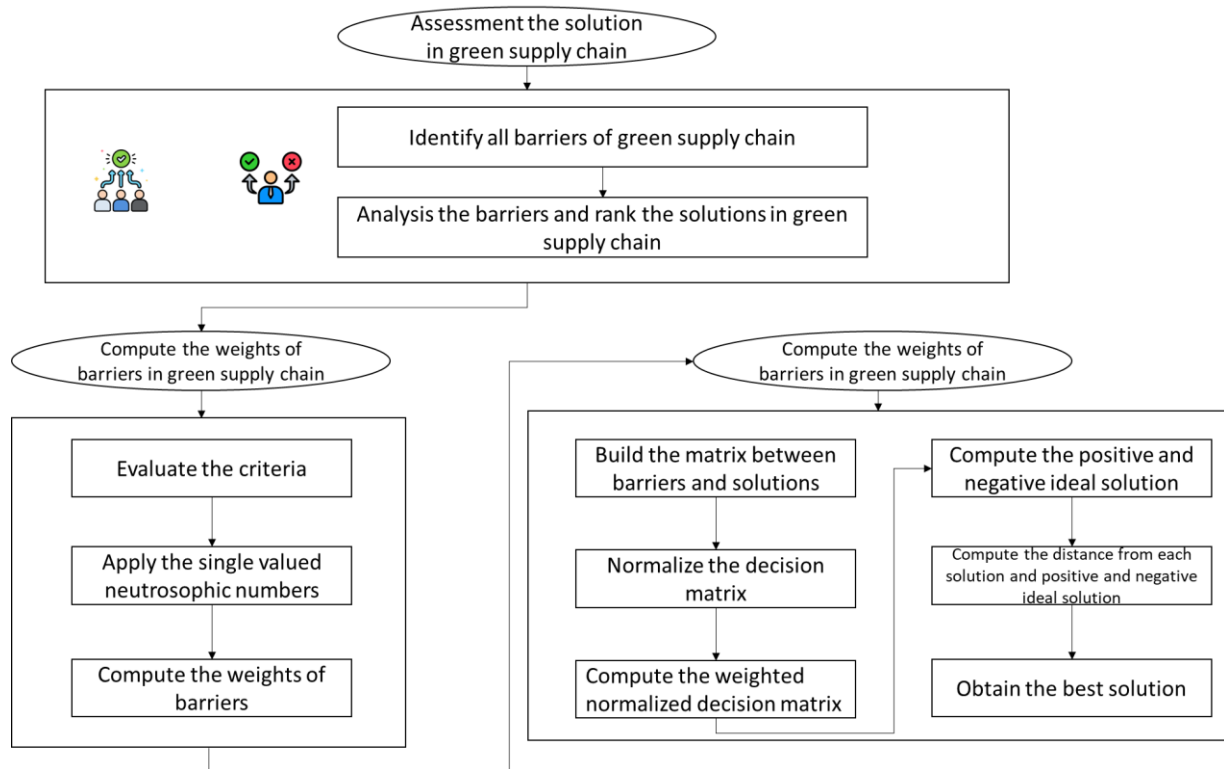


Figure 1. The phases of single valued neutrosophic TOPSIS method.

Build the matrix between barriers and solutions.

Normalize the decision matrix.

Normalize the data between barriers and solutions in green supply chain.

$$n_{ij} = \frac{a_{ij} - \min_j(a_{ij})}{\max_j(a_{ij}) - \min_j(a_{ij})} \quad (1)$$

$$n_{ij} = \frac{\max_j(a_{ij}) - a_{ij}}{\max_j(a_{ij}) - \min_j(a_{ij})} \quad (2)$$

Compute the weighted normalized decision matrix

$$t_{ij} = w_j * n_{ij} \quad (3)$$

Compute the positive and negative ideal solution

$$t_j^+ = \{t_1, t_2^+, \dots, t_n^+\} = \left\{ \max_j(t_j^+) \right\} \quad (4)$$

Compute the distance from each solution and positive and negative ideal solution

$$E_i^+ = \sqrt{\sum_{j=1}^n (t_{ij} - t_j^+)^2} \quad (5)$$

$$E_i^- = \sqrt{\sum_{j=1}^n (t_{ij} - t_j^-)^2} \quad (6)$$

Obtain the best solution

The best solution is computed by

$$S_i = \frac{E_i^-}{E_i^- + E_i^+} \quad (7)$$

4. Ranking Solutions in Green Supply Chain

This section provides the ranking of solutions to overcome the barriers in the green supply chain. We collected the barriers and solutions from previous studies. There are eight barriers and eight solutions. The barriers of this study are:

The environmental effects of their supply chain may not be well understood, and many businesses may not see the advantages of adopting sustainable practices.

Financial constraints: Some businesses may struggle to implement a green supply chain because of the time and money needed to invest in new technology, procedures, and staff training.

Employees and stakeholders may be resistant to change if implementing a green supply chain requires altering current processes and procedures.

The availability of sustainable raw materials and providers that can satisfy organizations sustainability needs is limited.

Organizations that operate in many areas or countries with varying environmental requirements may find it difficult to execute sustainable supply chain practices due to regulations and rules.

Insufficient teamwork: Different supply chain parties need to work together to implement a green supply chain. It might be difficult to successfully apply sustainable practices if there is a lack of teamwork and communication.

Organizations may apply a variety of strategies to build a green supply chain and overcome obstacles. The cost and technical are also barriers of green supply chain.

This study used eight solutions as:

In order to enhance environmental performance and establish sustainability goals, it is recommended that businesses conduct a sustainability assessment.

Create a plan for long-term sustainability that details the organization's sustainability aspirations and the steps that will be taken to realize them.

Organizations should actively involve their suppliers in order to discover sustainable options and promote the adoption of sustainable practices.

Organizations may invest in renewable energy, smart logistics, and sustainable packaging, all of which can help lessen the supply chain's negative effects on the environment.

Give your workers some education on how they may lessen the supply chain's environmental effects by learning about and implementing sustainable practices.

Working together with consumers, regulators, and NGOs may help businesses determine where they will have the most effect and how to garner public support for sustainability programs.

Businesses should make environmental responsibility a top priority and incorporate green thinking into every step of the supply chain.

Organizations should monitor and assess their environmental performance to determine whether or not they are meeting their sustainability targets and where they may make changes.

The experts and decision-makers are evaluated the barriers and solution in green supply chain. The TOPSIS method is used with the single valued neutrosophic set to rank the proposed solutions in the green supply chain. The data between barriers and solutions. Then normalize the data between barriers and solutions as shown in Table 1. Then compute the weighted normalized matrix as shown in Table 2. Then compute the positive and negative ideal solution. Then compute the distance between solutions and the positive and negative ideal solution. Then compute the closeness value as shown in Figure 2.

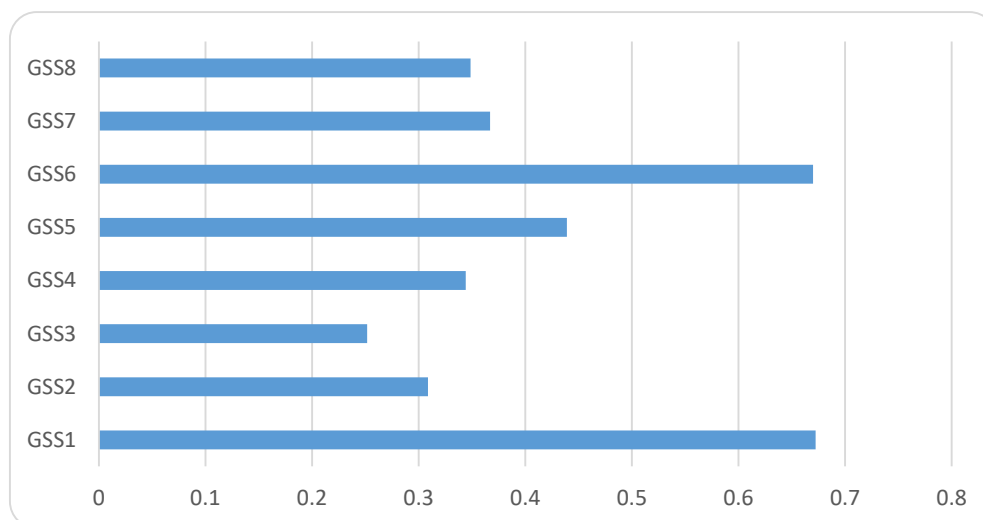
Table 1. The normalized data between barriers and solutions in green supply chain.

	GSB ₁	GSB ₂	GSB ₃	GSB ₄	GSB ₅	GSB ₆	GSB ₇	GSB ₈
GSS ₁	0.194467	0.412729	0.641527	0.28105	0.150034	0.306796	0.132317	0.435881
GSS ₂	0.30406	0.412729	0.16236	0.68244	0.513441	0.722509	0.438825	0.23344
GSS ₃	0.30406	0.412729	0.16236	0.180435	0.150092	0.306796	0.441348	0.161953
GSS ₄	0.738562	0.287344	0.16236	0.28105	0.558063	0.212845	0.438825	0.1493
GSS ₅	0.301424	0.264974	0.253859	0.43338	0.213837	0.102764	0.440885	0.378311
GSS ₆	0.30406	0.412729	0.590962	0.194983	0.213837	0.306796	0.29152	0.496992
GSS ₇	0.195208	0.288575	0.253859	0.197268	0.137285	0.306796	0.329118	0.416838
GSS ₈	0.101922	0.287344	0.176738	0.28105	0.519236	0.213593	0.122016	0.378311

Table 2. The weighted normalized data between barriers and solutions in green supply chain.

	GSB ₁	GSB ₂	GSB ₃	GSB ₄	GSB ₅	GSB ₆	GSB ₇	GSB ₈
GSS ₁	0.013187	0.04376	0.171889	0.029798	0.011161	0.032528	0.009767	0.086292
GSS ₂	0.020618	0.04376	0.043502	0.072356	0.038195	0.076604	0.032392	0.046214
GSS ₃	0.020618	0.04376	0.043502	0.019131	0.011165	0.032528	0.032578	0.032062
GSS ₄	0.050082	0.030466	0.043502	0.029798	0.041514	0.022567	0.032392	0.029557
GSS ₅	0.02044	0.028094	0.068018	0.045949	0.015907	0.010896	0.032544	0.074895
GSS ₆	0.020618	0.04376	0.15834	0.020673	0.015907	0.032528	0.021519	0.09839
GSS ₇	0.013237	0.030596	0.068018	0.020915	0.010213	0.032528	0.024294	0.082522
GSS ₈	0.006911	0.030466	0.047355	0.029798	0.038626	0.022646	0.009007	0.074895

Collaboration is the best solution ranked by the TOPSIS method. In order to find places for improvement and garner support for sustainability efforts, it is helpful to work together with suppliers, customers, and other stakeholders.

**Figure 2.** The score of eight solutions.

5. Conclusion

A green supply chain functions economically and has minimal negative effects on the environment. It entails including environmental concerns throughout the whole production process, from sourcing raw materials to final disposal. A green supply chain employs a variety of measures to lessen its negative effects on the environment. But the green supply chains have many barriers. So this paper analysis the barriers and give the various solutions and ranked them. This study used the TOPSIS method to rank the solutions and analysis the barriers. The TOPSIS method is used with the neutrosophic set to deal with uncertain data. We achieved coloration as the best solution from eight solutions. To find places for improvement and garner support for sustainability efforts, it is helpful to work together with suppliers, customers, and other stakeholders.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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Neutrosophic MCDM Methodology for Assessment Risks of Cyber Security in Power Management

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Abstract: Every day, new reports of cyberattacks on interconnected control systems emerge. The vulnerability of their communication mechanism makes similar control systems a target for malicious outsiders. Protecting sensitive data and maintaining network reliability and availability are two of the main reasons why network security is so important. Strong and dependable network security strategies use a number of safeguards to protect users and businesses from malware and cyber assaults like distributed denial of service. A safety analysis is an essential step that must precede the introduction of any security measures. There hasn't been much experience with cyberattacks on power control systems yet, therefore it's important to develop a method for thoroughly assessing the safety of power control technologies used in data transmission systems. A prior study has identified the authority control process evaluation of security and the safety level of each control stage as two of the primary obstacles to effective security assessment. For this reason, this article provides a safety risk evaluation of the communication networks of power management and control technologies (PMCT) using the neutrosophic Evaluation Based on Distance from Average Solution (EDAS) method. A prime instance of a multi-criteria decision-making (MCDM) issue is used to solve the security risk assessment in the power system. In this study, we offer an interval-valued neutrosophic version of the EDAS approach for solving the MCDM issue. The neutrosophic EDAS method is used to rank and assess the security risks in power system.

Keywords: Power Management; Risk Assessment; Cyber Security; Neutrosophic Sets; MCDM.

1. Introduction

The primary uses of information and communication technology (ICT) in today's power systems include tariffing and trading, network scheduling, oversight, and computerization grid linking of green power and electric shipping, managing power, electrical security measures, cyber security, and abundant data-based executions like (predicting) maintenance. Huge R&D efforts are now in progress in each of these fields. Power is seen as crucial to a country's development plan, yet in the present climate, utilization is inadequate and costs are on the rise [1, 2].

Therefore, sustainable power sources should be used for long-term use. And renewable sources of power, such as solar, are only accessible during daylight hours. By employing sources of clean power as part of an integrated power grid, it is crucial to provide a steady supply of electricity throughout the day. Power management and control technologies (PMCT) aim to reduce a building's power and operational costs while maintaining safe and healthy conditions for the building's inhabitants. Due to developments in electronics, computing technology, and state-of-the-art

interactions, PMCTs have been developed to improve indoor quality while conserving more electricity.

Fire alarms, security cameras, badge readers, lighting systems, etc. are just a few of the many outside sensors and solutions that make up the PMCT framework. Heating, ventilation, and air conditioning (HVAC) structures, as well as supporting power systems like microgrids and power restores, are also included. The growing focus on EMCS security has resulted in the inclusion of additional stakeholders, such as the building managers in charge of the at-risk PMCT design.

This article provides essential factors that may greatly improve data safety control rules for those now responsible for guaranteeing that PMCT has the requisite degree of cyber security procedures as part of its essential risk control programs. Protection of PMCT is complicated by the nature of many of the components utilized to deliver a wide variety of support services. Components of the PMCT that are still in use present often have a lengthy history. When element design and distribution initially started, the idea of connected devices did not yet exist [3, 4].

Unfortunately, PMCT still has a long way to go before it can be considered adequately cyber secure. Multiple advanced and intricate control networks have recently been developed using easily accessible networking technology. They still need a concerted effort and engagement from numerous stakeholders to secure them against hostile actors, despite increased understanding from a cybersecurity perspective. Organizations providing support for vital infrastructure must ensure that all operating equipment, regardless of age, is adequately protected against intrusion. Several agencies mandate rigorous cyber security for all power administration and oversight systems. An effective risk management strategy must be put in place for every business [5, 6].

The purpose of this research was to use the neutrosophic Evaluation Based on Distance from Average Solution (EDAS) method to evaluate the security threat posed by PMCT. Additionally, the neutrosophic EDAS method was applied to problems involving group decision-making in a neutrosophic environment. It is the method of choice when all of the factors being considered are of equal weight yet there is little information available to help narrow the field [7]–[10]. Figure 1 shows the security risk assessment in power system. The criteria and alternatives are collected from the power system and cyber security risk assessment then entered as input to the interval-valued neutrosophic set and EDAS multi-criteria decision making (MCDM) methodology. The assessment of security in power system has many and various criteria, so the concept of MCDM. Then we applied the steps of EDAS method to compute the weights of criteria and rank the high risks in security assessment in power system.

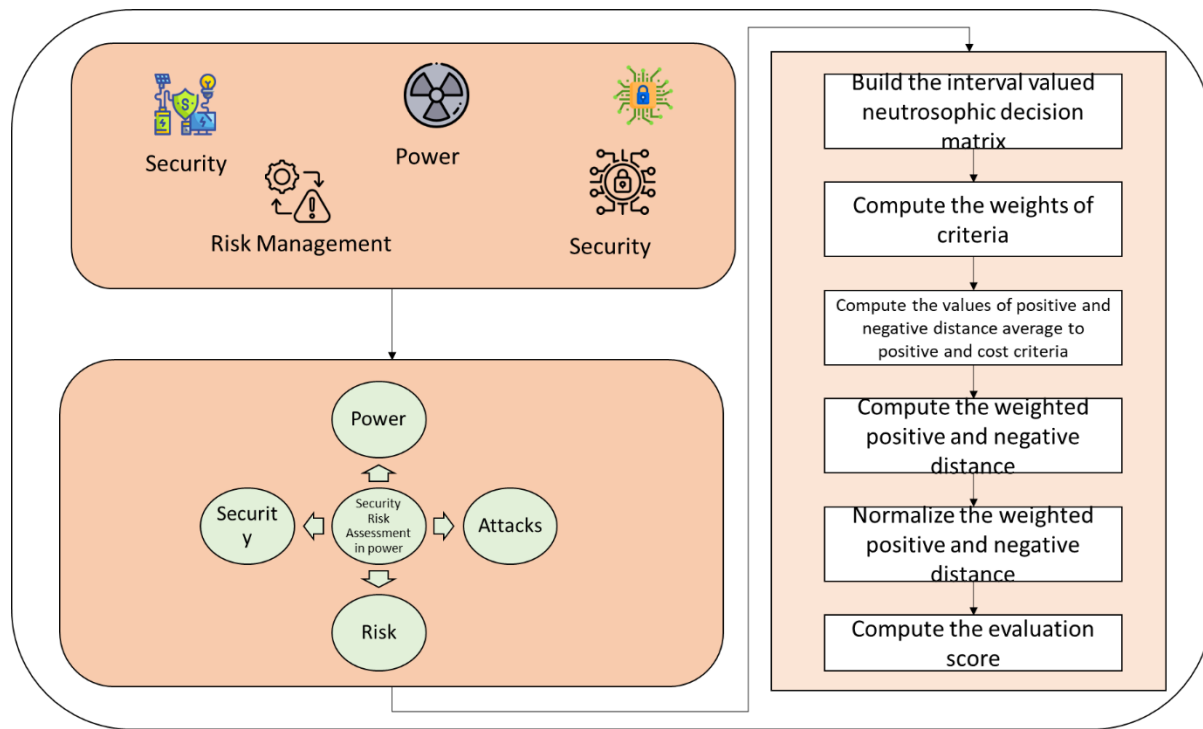


Figure 1. The security risk assessment in power system framework.

2. Power Security

Chester called the idea of power security "polysomic" and "slippery," meaning that it might stand for a number of different things at once. Various parties have various ideas on what constitutes adequate security and how to get there. The significance that various stakeholders place on various parameters. Additionally, developed and underdeveloped nations might have varying objectives and access to resources. Access to power for the impoverished in rural regions and for the fast-growing manufacturing and service industries in urban areas are two key factors in ensuring power security in the latter situation. The different perspectives may also be attributable to the scientists' educational backgrounds; for instance, scholars of politics, engineers, and complicated system analysts may see power safety from a more sovereign, strong, or resilient perspective [11, 12].

Power security is inherently fluid, since the vantage point may change depending on the time period under consideration. Analysts that focus on longer time horizons, for instance, often prioritize stability above efficiency. As a result of these divergent viewpoints and objectives, academics have been debating the future of power security and the best ways to adapt to it. The goal of any given power security study will dictate both the issues to be considered and their relative importance. As a result, it is unlikely, and probably even undesirable, that academics will settle on a single, universally accepted concept and understanding of power security.

There are several potential dangers or hazards that might cause intentional or unintentional disruptions in the flow of power. However, there are two distinct but interconnected features that customers care about securing: (1) the physical, which includes factors like available, dependable, and/or affordable power supply; and (2) the economic, which includes factors like fluctuating prices and accessibility. Since physically unstable supply or scarce resources may have an effect on market pricing, these factors are related. Supply disruption refers to the impact of low or variable prices on the physical aspect by discouraging investment in network and manufacturing facilities. Therefore, markets should be structured such that prices may serve as a go-between for suppliers and buyers and serve as an early warning system for impending shortages or surpluses [13, 14].

While numerous descriptions of power security place emphasis on the physical and economic aspects of the concept, it is less typical for such definitions to include examples such as when high prices constitute a danger to power security. In other words, many definitions emphasize angles and vantage points but fail to specify cutoffs.

3. Security Assessment

Power system management and monitoring need dynamic security evaluation (DSA). Growing requests, privatization, and innovation in the power industry force modern power systems to keep running under strained circumstances approaching their stability limitations. When things are like that, even a little change might throw the system into chaos. This highlights the critical need of conducting regular internet security audits [15, 16].

Stability difficulties caused by both minor and major disturbances are thoroughly explored during a power system's DSA. The capacity of an integrating power system to maintain synchronization after a significant disruption is known as transient stability. The most difficult of these evaluations is the transient stability assessment (TSA), which requires extensive processing time. Time-constrained online assessment for immediate management and management of operations reveals the limitations of conventional model-based techniques for performing accurate TSA.

The real-time DSA evaluates the reliability of the grid in the face of plausible disruptions. Many nonlinear differential and algebraic equations must be solved numerically in this investigation. The longer the calculation cycle, the more quickly the solutions from traditional methods become obsolete. The standard time for a computing cycle is 15-30 minutes, according to the literature. The accuracy and applicability of DSA findings, however, improve with an increase in the frequency of computing cycles [17, 18].

4. Security Risk Assessment

As data and industry advance at a fast pace, the importance of cyber security grows. As technology has progressed, however, several issues with cyber security have become apparent. About 40% of nations worldwide observe cyberattacks as a possible danger, and as a consequence, cyber security measures are realized at all stages. As the internet and computers have grown more interconnected, several online programs including online banking, online shopping, and m-commerce have been vulnerable to cyber assaults. Despite many benefits, the expanding digital world also poses serious risks to vital governmental sectors like the defense industry [19, 20].

As the number of cybercrimes continues to rise, the notion of cyber security has emerged as one of critical importance in the modern world. Due to damages caused by cyber-attacks, innovators in the information security area have found it imperative to build trustworthy and effective security solutions. Cybersecurity is a broad subject, and several definitions of it may be found in the literature. For example, "assesses adopted to safeguard a system or machine (as on the Internet) against unauthorized access or intrusion" is how Merriam-Webster defines cyber security [21, 22].

In addition, the International Telecommunications Union (ITU) determines this term as an ensemble of resources like regulations, safety ideas, safety measures, instructions, risk management methods, behaviors, learning, best practices, trust, and innovations that can be used to keep a company's or an individual's data and resources safe online. Organizational and individual resources in the cyber sphere include computers and their associated hardware and software, as well as human resources, physical facilities, networks, apps, services, and networks for communication and data. Preventing and mitigating security breaches in the cyber environment is the goal of cyber security measures [23, 24].

Computer, network, program, and data security are all part of what's known as "cyber security," a collection of practices and procedures designed to keep these things safe from harm. A firewall,

anti-virus programs, and an intrusion detection system (IDS) are required components of both securities for computers and network solutions. Information security monitoring systems allow for the detection and analysis of data breaches caused by unauthorized access, replication, modification, or destruction. Both inside and outside assaults on an organization count as security breaches.

When it comes to protecting sensitive data, implementing cyber security measures is crucial. The scholarship presents an extensive number of cybersecurity strategies. Organizations may profit greatly from a ranking of the significance of these innovations and an analysis of the requirement of having these innovations in the first place. However, the dangers posed by such innovations must also be taken into account [25, 26].

5. Interval Valued Neutrosophic EDAS MCDM Methodology

This section provides the steps of the EDAS method under the interval valued neutrosophic set to evaluate the cyber security risks in power systems [27]–[30]. This method constructs the calculations between the criteria of risks assessment and cyber-attacks network.

- 1) Build the interval valued neutrosophic decision matrix.

This step builds the decision matrix by using interval valued neutrosophic numbers between criteria and alternatives.

- 2) Average the interval valued neutrosophic decision matrix

There are more than one expert to evaluate the criteria and alternatives, so these values are combined into one matrix.

- 3) Compute the weights of criteria.
- 4) Compute the values of positive and negative distance average to positive and cost criteria.

These step specify the positive and negative criteria to compute the PD and ND values

$$PD = [pd]_{m \times n} \quad (1)$$

$$ND = [nd]_{m \times n} \quad (2)$$

$$PD_{mn} = \begin{cases} \frac{z(a_{mn} \ominus w_n)}{S(w_n)} & \text{positive criteria} \\ \frac{z(w_n \ominus a_{mn})}{S(w_n)} & \text{cost criteria} \end{cases} \quad (3)$$

$$ND_{mn} = \begin{cases} \frac{z(w_n \ominus a_{mn})}{S(w_n)} & \text{positive criteria} \\ \frac{z(a_{mn} \ominus w_n)}{S(w_n)} & \text{cost criteria} \end{cases} \quad (4)$$

Where a_{mn} is a value of decision matrix, w_n refers to the average weight, and $S(w_n)$ refers to the crisp value of average weight.

- 5) Compute the weighted positive and negative distance

$$EPD_n = \sum_{i=1}^n (e_j \otimes PD_{mn}) \quad (5)$$

$$END_n = \sum_{i=1}^n (e_j \otimes ND_{mn}) \quad (6)$$

- 6) Normalize the weighted positive and negative distance

$$NEPD_n = \frac{EPD_n}{\max(EPD_n)} \quad (7)$$

$$NEND_n = 1 - \frac{END_n}{\max(EPD_n)} \quad (8)$$

7) Compute the evaluation score

$$AS = 0.5 (NEPD_n \oplus NEND_n) \quad (9)$$

6. Application

In the past few years, cyber security concerns have become a major issue for online infrastructures. New technology techniques are the primary focus of most initiatives to strengthen cyber security. However, such safety measures capture a lot of data, which presents a significant privacy risk to the people they are meant to safeguard. Therefore, it is crucial to conduct risk assessments for cyber security tools. The aim of this section provides the assessment risks of cyber security in power management. This study used nine criteria and ten alternatives like $\{NCA_1, NCA_2, NCA_3, NCA_4, NCA_5, NCA_6, NCA_7, NCA_8, NCA_9, NCA_{10}\}$ to rank the network communication in power management. The nine criteria are proposed in Figure 2. The criteria are collected from the literature with the risk assessment security and cyber security assessment.



Figure 2. The cyber security risks criteria.

We applied the interval valued neutrosophic EDAS method to obtain the rank of risks in cyber security risks assessment. The decision makers and experts built the decision matrix by the terms in interval valued neutrosophic set. Then these terms are replaced by the interval valued neutrosophic numbers. Then compute weights of criteria as: $SRA_1 = 0.10412, SRA_2 = 0.126683, SRA_3 = 0.013866, SRA_4 = 0.145245, SRA_5 = 0.056585, SRA_6 = 0.142162, SRA_7 = 0.079966, SRA_8 = 0.131478, SRA_9 = 0.199892$. From the weights of criteria, the criterion 9 is the highest and the criterion 3 is the worst.

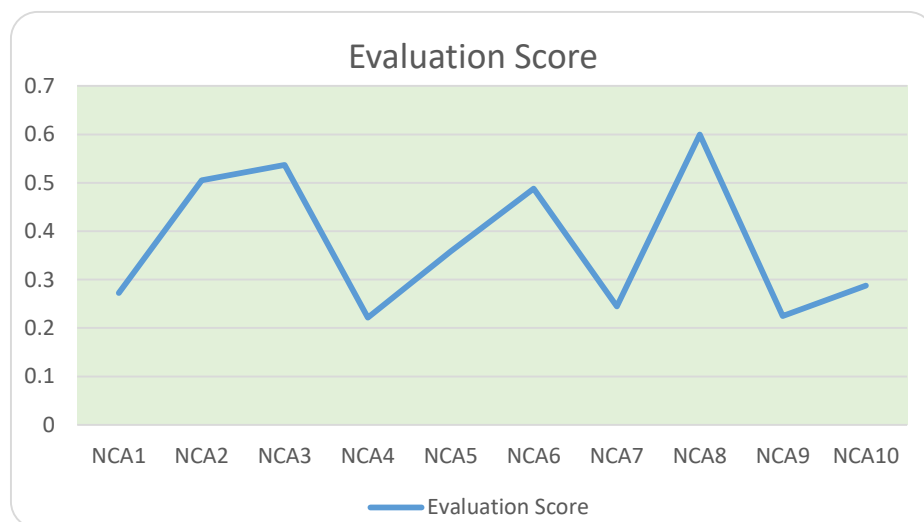
Then compute the positive and negative criteria by using Eqs. (1-4). Then compute the weighted positive and negative distance average by using Eqs. (5-6) as shown in Table 1-2. Then compute the normalized weighted positive and negative distance average by using Eqs. (7-8). Then compute the evaluation score by using Eq. (9) as shown in Figure 3. From Figure 3, the risk number eight is the highest and risk number 4 is the least.

Table 1. The weighted positive distance.

	SRA ₁	SRA ₂	SRA ₃	SRA ₄	SRA ₅	SRA ₆	SRA ₇	SRA ₈	SRA ₁₀
NCA ₁	0.020225	0	0	0	0	0	0	0.013111	0
NCA ₂	0.020225	0.041728	0	0.054132	0.039544	0.047083	0.014833	0.016247	0.010789
NCA ₃	0	0	0.000655	0	0.004337	0	0	0.013111	0
NCA ₄	0.020225	0	0.002782	0	0	0	0.014833	0.002244	0.010789
NCA ₅	0	0	0	0	0	0	0	0.013111	0.036533
NCA ₆	0	0	0.002782	0	0	0	0.002717	0	0
NCA ₇	0.020225	0.041728	0	0	0	0	0	0.013111	0.010789
NCA ₈	0	0	0.000655	0	0.006643	0.047083	0.014833	0	0.04166
NCA ₉	0	0	0.002782	0	0.011781	0	0.01282	0.016247	0.036533
NCA ₁₀	0.020225	0	0	0	0	0	0	0.013111	0

Table 2. The weighted negative distance.

	SRA ₁	SRA ₂	SRA ₃	SRA ₄	SRA ₅	SRA ₆	SRA ₇	SRA ₈	SRA ₁₀
NCA ₁	0	0.019952	0.001931	0.000374	0.006512	0.007572	0.012007	0	0.02673
NCA ₂	0	0	0.001931	0	0	0	0	0	0
NCA ₃	0.010425	0.126683	0	0.000374	0	0.007572	0.012007	0	0.02673
NCA ₄	0	0.00691	0	0.000374	0.018129	0.017687	0	0	0
NCA ₅	0.012909	0.019952	0.001931	0.018889	0.006512	0.022227	0.012007	0	0
NCA ₆	0.010425	0.00691	0	0.000374	0.018129	0.022227	0	0.050147	0.066903
NCA ₇	0	0	0.001931	0.000374	0.006512	0.004653	0.012007	0	0
NCA ₈	0.056942	0.015912	0	0.014106	0	0	0	0.050147	0
NCA ₉	0.010425	0.00691	0	0.000374	0	0.004653	0	0	0
NCA ₁₀	0	0.00691	0.001931	0.018889	0.006512	0.007572	0.012007	0	0.02673

**Figure 3.** The evaluation scores.

7. Conclusion

Hybrid structures that are interdependent on data and communication are the future of renewable power. While the use of information technology improves the administration and operation of power systems, it also increases the likelihood of cybersecurity problems. As a consequence, safeguarding our information and communication infrastructures has become more important to ensuring the reliability of our power grid. The proposed model uses the neutrosophic EDAS-based MCDM approach to address the challenge of identifying threats inside the interconnected wireless networks of energy administration and control network backbones. This paper used the interval-valued neutrosophic set to overcome the uncertain information. The experts used interval-valued neutrosophic terms to evaluate the criteria and alternatives. Then we replaced these terms with interval-valued neutrosophic numbers. Then we applied the EDAS method to nine criteria and ten alternatives. Risk number eight is the highest and risk number 4 is the least.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflict of interest

The authors declare that there is no conflict of interest in the research.

Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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