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Novel Neutrosophic Objects Within Neutrosophic Topology $(N(X), \tau)$

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Abstract. This essay intends to introduce and study many new neutrosophic objects within neutrosophic topologies $(N(X), \tau)$, such as the neutrosophic point, the neutrosophic quasi-concomitant, the neutrosophic quasi neighborhood, the neutrosophic ideals, the neutrosophic local function, the neutrosophic closure operator, the generated neutrosophic topology, and quite a few of theorems, corollaries, examples related the above- mentioned concepts.

Keywords: Neutrosophic Topology $(N(X), \pi)$; Neutrosophic Point; Neutrosophic Quasi-Concomitant; Neutrosophic Quasi Neighborhood; Neutrosophic Ideals; Neutrosophic Local Function.

1 Introduction

The reformulations of all scientific fields in the perspective of the existence of indeterminacy were by the paradigm shift man F. Smarandache [1-3]. As of 1995 so far, he redefined almost all branches of knowledge, setting up the neutrosophic theory through the implication of the indeterminacy part into all components, elements, operations, thoughts, opinions,etc. [10-13].

The topological space took its share of that evolution, the eminent scientist who led changing the topological spaces into the frame of neutrosophic theory was A.A. Salama [4-6], with the collaboration of F. Smarandache, later dozens of mathematicians who were interested in the topological field joined them [7-9]. This paper comes as a point in the sea of this scientific promotion, it is not the first nor the last in the field of neutrosophic topological spaces. This article is organized by dedicating section two to new neutrosophic notions which are presented in this paper for the first time, Section three goes for demonstrates the neutrosophic ideal, neutrosophic local function, and generated neutrosophic topology, while section four encloses the basic structure of generated neutrosophic topology, eventually section five contains the conclusions and recommendations.

2. New Notions in Neutrosophic Topological Spaces $(N(X), \tau)$

2.1 Definition (Neutrosophic point):

Suppose $(N(X), \tau)$ to be neutrosophic topological space, a neutrosophic point $x \in N(X)$ is denoted by $x_{\langle \lambda_1, \lambda_2, \lambda_3 \rangle}$, where $0^- \leq \lambda_1, \lambda_2, \lambda_3 \leq 1^+$. A neutrosophic point $x_{\langle \lambda_1, \lambda_2, \lambda_3 \rangle}$ is belonging to a neutrosophic set $A \subseteq N(X)$ iff $\lambda_1 \leq A(x_{\lambda_1}), \lambda_2 \leq A(x_{\lambda_2}), \lambda_3 \leq A(x_{\lambda_3})$ and symbolized by $x_{\langle \lambda_1, \lambda_2, \lambda_3 \rangle} \in_N A$.

2.2 Definition (Neutrosophic Quasi-concomitant):

A neutrosophic set $A \in N(X)$ is said to be neutrosophic quasi-concomitant to another neutrosophic set B if there exists a neutrosophic point $x_{\langle \lambda_1, \lambda_2, \lambda_3 \rangle} \in_N N(X)$ such that the following conditions are hold together:

$$A(x_{\lambda_1}) + B(x_{\lambda_1}) > 1^+_{x_{\lambda_1}},$$
$$A(x_{\lambda_2}) + B(x_{\lambda_2}) > 1^+_{x_{\lambda_2}},$$

$A(x_{\lambda_3}) + B(x_{\lambda_3}) > 1^+_{x_{\lambda_3}}$,

And it is symbolized by $A\ qc\ B$, for any two neutrosophic sets $A \& B$, $A\ qc\ B \Leftrightarrow B\ qc\ A$.

2.3 Definition (Neutrosophic quasi neighborhood):

A neutrosophic set A in a neutrosophic topological space $(N(X),\tau)$ is called a neutrosophic quasi neighborhood of a neutrosophic point $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$ iff there exist a neutrosophic open set $\mu \subseteq A$ such that $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}\ qn\ \mu$. The set of all neutrosophic quasi neighborhood of $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}\ in\ (N(X),\tau)$ is symbolized by $NQN(x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle})$.

2.4 Definition (Accumulation neutrosophic point):

A neutrosophic point $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$ is called an accumulation neutrosophic point of a neutrosophic set A in the neutrosophic topological space $(N(X),\tau)$ iff the following condition holds:

Any neutrosophic quasi neighborhood of $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$ is neutrosophic quasi concomitant,

If $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \in_N A$, any quasi neighborhood of $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$ and A are quasi concomitant at some neutrosophic point $y_{\langle t_1,t_2,t_3\rangle}$ such that $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \neq y_{\langle t_1,t_2,t_3\rangle}$.

Note *

In the above definition of accumulation neutrosophic point, if only the condition 1 holds, then $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$ is called an adherence neutrosophic point of A.

Note **

It is obvious that any neutrosophic point $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$ is belonging to the closure of a neutrosophic set A (i.e. $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \in_N Ncl(A)$) iff for every quasi neighborhood B of $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$, $B\ qc\ A$.

3. Neutrosophic Ideal, Neutrosophic Local Function and Generated Neutrosophic Topology

3.1 Definition (Neutrosophic Ideal):

Suppose that $\pi, v \in N(X)$, A nonempty collection of neutrosophic sets I of $N(X)$ is called ideal on $N(X)$ if and only if

$\pi \in_N I$ and $v \subseteq \pi \Longrightarrow v \in I$ [heredity],

$\pi \in_N I$ and $v \in_N I \Longrightarrow \pi \cup v \in_N I$ [finite additivity].

3.2 Definition (Neutrosophic Local Function):

Let $(N(X),\tau)$ be a neutrosophic topological space and I be neutrosophic ideal on $N(X)$. Let A be any neutrosophic set of $N(X)$. Then the neutrosophic local function $A^*(I,\tau)$ of A is the union of all neutrosophic points $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$, such that if $\mu \in_N NQN(x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle})$ and $I_1 \in I$ then there is at least one $y_{\langle t_1,t_2,t_3\rangle} \in_N N(X)$ for which $\mu(y_{t_1}) + A(y_{t_1}) - 1^+_{y_{t_1}} > I_1(y_{t_1})$, $\mu(y_{t_2}) + A(y_{t_2}) - 1^+_{y_{t_2}} > I_1(y_{t_2})$, $\mu(y_{t_3}) + A(y_{t_3}) - 1^+_{y_3} > I_1(y_{t_3})$. therefore, any $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \notin_N A^*(I,\tau)$ [i.e. implies to $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$ is not contained in the neutrosophic set A, i.e. $\lambda_1 > A(x_{\lambda_1})$, $\lambda_2 > A(x_{\lambda_2})$, $\lambda_3 > A(x_{\lambda_3})$] implies there is at least one $\mu \in_N NQN(x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle})$ such that for every $y_{\langle t_1,t_2,t_3\rangle} \in_N N(X)$, $\mu(y_{t_1}) + A(y_{t_1}) - 1^+_{y_{t_1}} \leq I_1(y_{t_1})$, $\mu(y_{t_2}) + A(y_{t_2}) - 1^+_{y_{t_2}} \leq I_1(y_{t_2})$, $\mu(y_{t_3}) + A(y_{t_3}) - 1^+_{y_3} \leq I_1(y_{t_3})$ for some $I_1 \in I$. The symbols $A^*\ or\ A^*(I)$ will be written as abbreviate instead of $A^*(I,\tau)$.

3.3 Note:

The neutrosophic empty set and the neutrosophic universal set on $N(X)$ is denoted by $0_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}$ and $1_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}$, respectively.

3.4 Example

The simplest neutrosophic ideals on $N(X)$ are $\left\{0_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}\right\}$ and $H(N(X))$, the set of all neutrosophic sets of $N(X)$ (hereafter, if necessary, $H(N(X))$ will carry the same meaning). Obviously, $I=\left\{0_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}\right\} \Leftrightarrow A^*(I,\tau)=cl(A)$, for any neutrosophic set A of $N(X)$ and $I=H\big(N(X)\big) \Leftrightarrow A^*(I,\tau)=0_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}$.

3.5 Theorem

Let $(N(X),\tau)$ be a neutrosophic topological space and I_1, I_2 are two neutrosophic ideals on $N(X)$. Then for any neutrosophic sets $A, B\ of\ N(X)$, the following mathematical phrases are true:

$A \subseteq B \Rightarrow A^*(I_1,\tau) \subseteq B^*(I_1,\tau)$.

$I_1 \subseteq I_2 \Rightarrow A^*(I_2,\tau) \subseteq A^*(I_1,\tau)$.

$A^* = cl(A^*) \subseteq cl(A)$.

$(A^*)^* \subseteq A^*$.

$$(A \cup B)^* = A^* \cup B^*.$$

$I_1 \in I \Rightarrow (A \cup I_1)^* = A^*$.

Proof.

Since $A \subseteq B$ this implies that $A_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}} \le B_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}$ for every $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \in_N N(X)$, therefore and by Definition 3.2, $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \in_N A^*$implies that $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \in_N B^*$, which complete the proof of (1).

Suppose $I_1 \subseteq I_2 \Rightarrow A^*(I_2) \subseteq A^*(I_1)$,as there may be other neutrosophic sets which belong to I_2 so that for a neutrosophic point $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$, $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \in_N A^*(I_1)$ but $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$ may not be contained in $A^*(I_2)$.

Since the empty neutrosophic set $\left\{0_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}\right\}$ is contained in any neutrosophic ideal $I_1\ on\ N(X)$ (i.e. $\left\{0_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}\right\} \subseteq I_1$),[therefore by (2) and because of the reality that the simplest neutrosophic ideal on $N(X)$ is $\left\{0_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}\right\}$. Obviously, $A^*\left(\left\{0_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}\right\}\right)=cl(A)$, for any neutrosophic set A of $N(X)$], $A^*(I_1) \subseteq A^*\left(\left\{0_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}\right\}\right)=cl(A)$, for any neutrosophic set A of $N(X)$. Suppose, $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \in_N\ cl(A^*)$.So there is at least one $y_{\langle t_1,t_2,t_3\rangle} \in_N N(X)$ such that $\mu(y_{t_1})+A^*(y_{t_1})>1^+_{y_{t_1}}$, $\mu(y_{t_2})+A^*(y_{t_2})>1^+_{y_{t_2}}$, $\mu(y_{t_3})+A^*(y_{t_3})>1^+_{y_3}$ for each neutrosophic quasi neighborhood μ of $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$. Hence $A^*(y_{\langle t_1,t_2,t_3\rangle}) \ne 0_{y_{\langle t_1,t_2,t_3\rangle}}$. Let $f_{\langle f_1,f_2,f_3\rangle}=A^*(y_{\langle t_1,t_2,t_3\rangle})$. Clearly, $y_f \in_N A^*$ and $f_{f_1}+\mu(y_{t_1})>1^+_{y_{t_1}}$, $f_{f_2}+\mu(y_{t_2})>1^+_{y_{t_2}}$, $f_{f_3}+\mu(y_{t_3})>1^+_{y_3}$, so that μ is also neutrosophic quasi neighborhood of y_f.

Now $y_f \in_N A^*$ implies there is at least one $x'_{\langle\lambda_1{}',\lambda_2{}',\lambda_3{}'\rangle} \in_N N(X)$ such that $v\left(x'_{\lambda_1{}'}\right)+A\left(x'_{\lambda_1{}'}\right)-1^+_{x'_{\lambda_1{}'}}>I_1(x'_{\lambda_1{}'})$, $v\left(x'_{\lambda_2{}'}\right)+A\left(x'_{\lambda_2{}'}\right)-1^+_{x'_{\lambda_2{}'}}>I_1(x'_{\lambda_2{}'})$, $v\left(x'_{\lambda_3{}'}\right)+A\left(x'_{\lambda_3{}'}\right)-1^+_{x'_{\lambda_3{}'}}>I_1(x'_{\lambda_3{}'})$, for each $v \in NQN(x'_{\langle\lambda_1{}',\lambda_2{}',\lambda_3{}'\rangle})$ and $I_1 \in I$. This is also true for μ.

So there is at least one $x''_{\langle\lambda_1{}'',\lambda_2{}'',\lambda_3{}''\rangle} \in_N N(X)$ such that $\mu\left(x''_{\lambda_1{}''}\right)+A\left(x''_{\lambda_1{}''}\right)-1^+_{x''_{\lambda_1{}''}}>I_1(x''_{\lambda_1{}''})$, $\mu\left(x''_{\lambda_2{}''}\right)+A\left(x''_{\lambda_2{}''}\right)-1^+_{x''_{\lambda_2{}''}}>I_1(x''_{\lambda_2{}''})$, $v\left(x''_{\lambda_3{}''}\right)+A\left(x''_{\lambda_3{}''}\right)-1^+_{x''_{\lambda_3{}''}}>I_1(x''_{\lambda_3{}''})$, for each $I_1 \in I$, and. Since μ is an arbitrary neutrosophic quasi neighborhood of $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$, therefore $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \in_N A^*$. Hence, $A^* \subseteq cl(A^*) \subseteq cl(A)$.

By (3), we have $A^{**} = cl((A^*)^*) \subseteq cl(A^*) = A^*$.

Suppose, $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \notin_N A^* \cup B^*$, i.e. $\lambda_1 > (A^* \cup B^*)(x_{\lambda_1}) = max\{A^*(x_{\lambda_1}), B^*(x_{\lambda_1})\}$, $\lambda_2 > (A^* \cup B^*)(x_{\lambda_2}) = max\{A^*(x_{\lambda_2}), B^*(x_{\lambda_2})\}$, $\lambda_3 > (A^* \cup B^*)(x_{\lambda_3}) = max\{A^*(x_{\lambda_3}), B^*(x_{\lambda_3})\}$. So $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$ is not contained in both A^* and B^*. This implies there is at least one neutrosophic quasi neighborhood μ_1 of $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$ such that for every $y_{\langle t_1,t_2,t_3\rangle} \in_N N(X)$, $\mu_1(y_{t_1}) + A(y_{t_1}) - 1^+_{y_{t_1}} > I_2(y_{t_1})$, $\mu_1(y_{t_2}) + A(y_{t_2}) - 1^+_{y_{t_2}} > I_2(y_{t_2})$, $\mu_1(y_{t_3}) + A(y_{t_3}) - 1^+_{y_{t_3}} > I_2(y_{t_3})$, for some $I_2 \in I$ and similarly there is at least one neutrosophic quasi neighborhood μ_2 of $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$ such that for every $y_{\langle t_1,t_2,t_3\rangle} \in_N N(X)$, $\mu_2(y_{t_1}) + B(y_{t_1}) - 1^+_{y_{t_1}} > I_3(y_{t_1})$, $\mu_2(y_{t_2}) + B(y_{t_2}) - 1^+_{y_{t_2}} > I_3(y_{t_2})$, $\mu_2(y_{t_3}) + B(y_{t_3}) - 1^+_{y_{t_3}} > I_3(y_{t_3})$, for some $I_3 \in I$. Let $\mu = \mu_1 \cap \mu_2$. So μ is also a neutrosophic quasi neighborhood of $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$ and $\mu(y_{t_1}) + (A \cup B)(y_{t_1}) - 1^+_{y_{t_1}} \leq (I_2 \cup I_3)(y_{t_1})$, $\mu(y_{t_2}) + (A \cup B)(y_{t_2}) - 1^+_{y_{t_2}} \leq (I_2 \cup I_3)(y_{t_2})$, $\mu(y_{t_3}) + A(y_{t_3}) - 1^+_{y_3} \leq I_1(y_{t_3})$, for every $y_{\langle t_1,t_2,t_3\rangle} \in_N N(X)$. Therefore, by finite additivity of neutrosophic ideal, as $I_2 \cup I_3 \in I$, $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \notin_N (A \cup B)^*$. Hence $(A \cup B)^* \subseteq A^* \cup B^*$. Clearly, both A and $B \subseteq A \cup B$ implies $A^* \cup B^* \subseteq (A \cup B)^*$ and this complete the prove of (5).

It is obvious that $I_1 \in I$ implies $I^* = 0_x$ so that $(A \cup I)^* = A^* \cup I^* = A^*$.

3.6 Definition

A neutrosophic closure operator $\psi: H(N(X)) \longrightarrow H(N(X))$ is defined by

$\psi\left(0_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}\right) = 0_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}$.

$A \in H(N(X)) \Rightarrow A \subseteq \psi(A)$.

$A, B \in H(N(X)) \Rightarrow \psi(A \cup B) = \psi(A) \cup \psi(B)$.

$A \in H(N(X)) \Rightarrow \psi(\psi(A)) = \psi(A)$.

Obviously, $\{A: \psi(A) = A\}$ constitutes as a collection of neutrosophic closed sets for a neutrosophic topology on $N(X)$.

3.7 Theorem

Let $D: H(N(X)) \longrightarrow H(N(X))$ be a function such that:

$D\left(0_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}\right) = 0_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}$.

$D(A \cup B) = D(A) \cup D(B)$.

$$D(D(A)) \subseteq D(A)$$

Where A, B are any neutrosophic sets of $N(X)$. Then $\psi: H(N(X)) \longrightarrow H(N(X))$ defined by $\psi(A) = A \cup D(A)$ is a neutrosophic closure operator. Clearly, D does not necessarily coincide with neutrosophic derived set operator in the generated neutrosophic operator.

3.8 Theorem

$*: H(N(X)) \longrightarrow H(N(X))$ satisfies all the required condition for D.

Proof.

Since $0^*_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}} = 0_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}$, $(A \cup B)^* = A^* \cup B^*$ and $(A^*)^* \subseteq A^*$, the proof is complete.

Let $(N(X), \tau)$ be a neutrosophic topological space and I be a neutrosophic ideal on $N(X)$. Let us define $cl^*(A) = A \cup A^*$ for any neutrosophic set A of $N(X)$. Clearly, $cl^*$is a neutrosophic closure operator. Let $\tau^*(I)$ be the neutrosophic topology generated by cl^*, i.e., $\tau^*(I) = \{A: cl^*(A^c) = A^c\}$, A^c will denote the complement of A.

Now, let $I = \left\{0_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}\right\} \Rightarrow cl^*(A) = A \cup A^* = A \cup cl(A) = cl(A)$, for every $A \in H(N(X))$. So, $\tau^*\left(\left\{0_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}\right\}\right) = \tau$. Again, let $I = H(N(X)) \Rightarrow cl^*(A) = A$, because $A^* = 0_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}$, for every $A \in H(N(X))$. So, $\tau^*(H(N(X))$ is the neutrosophic discrete topology on $N(X)$. Since $\left\{0_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}\right\}$ and $H(N(X))$ are two extreme neutrosophic ideals on $N(X)$, therefore for any neutrosophic ideal I on $N(X)$ we have

$\left\{0_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}\right\} \subseteq I \subseteq H(N(X))$. So we can conclude by theorem 3.5 (2), $\tau^*\left(\left\{0_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}\right\}\right) \subseteq \tau^*(H(N(X)))$, i.e. $\tau \subseteq \tau^*(I) \subseteq$ neutrosophic discrete topology, for any neutrosophic ideal I on X. In particular, we have, for any two neutrosophic ideals I_1 and I_2 on $N(X)$, $I_1 \subseteq I_2 \Longrightarrow \tau^*(I_1) \subseteq \tau^*(I_2)$.

3.9 Theorem

Let τ_1, τ_2 be two neutrosophic topologies on $N(X)$. Then for any neutrosophic ideal I on $N(X)$, $\tau_1 \subseteq \tau_2$ implies that:

$A^*(\tau_2, I) \subseteq A^*(\tau_1, I)$, for every $A \in H(N(X))$.

$\tau_1^* \subseteq \tau_2^*$.

Proof.

Since every τ_1 neutrosophic quasi neighborhood of any neutrosophic point $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$ is also a τ_2 neutrosophic quasi neighborhood of $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$. Therefore, $A^*(\tau_2, I) \subseteq A^*(\tau_1, I)$ as there may be other τ_2 neutrosophic quasi neighborhood of $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$ where the condition for $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \in_N A^*(\tau_2, I)$ may not hold true, although $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \in_N A^*(\tau_1, I)$. Clearly, $\tau_1^*(I) \subseteq \tau_2^*(I)$ as $A^*(\tau_2, I) \subseteq A^*(\tau_1, I)$.

3.10 Theorem

Suppose $A^{D^*}$is the neutrosophic derived set of A in τ^*neutrosophic topology, then, $A^{D^*} \subseteq A^D$ and $A^{D^*} \subseteq A^*$for all neutrosophic set A of $N(X)$.

Proof.

Since $\tau \subseteq \tau^*$. Therefore, $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \in_N A^{D^*}$ implies that every neutrosophic quasi neighborhood of $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$ in neutrosophic topology τ^* is neutrosophic quasi concomitant with $A \Longrightarrow x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \in_N A^D$, so that $A^{D^*} \subseteq A^*$.

Again, for any neutrosophic point $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \in_N A^{D^*}$ implies $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \in_N cl^*(A) = A \cup A^*$, i.e. $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \in_N A$ or $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \in_N A^*$ or both. Now, if $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \in_N A$, then for any neutrosophic quasi neighborhood μ of $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$ in neutrosophic topology τ^* , there exists $y_{\langle t_1,t_2,t_3\rangle} \in_N N(X)$ such that $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \neq y_{\langle t_1,t_2,t_3\rangle}$ and $A(y_{t_1}) + \mu(y_{t_1}) > 1^+_{y_{t_1}}$, $A(y_{t_2}) + \mu(y_{t_2}) > 1^+_{y_{t_2}}$, $A(y_{t_3}) + \mu(y_{t_3}) > 1^+_{y_3}$, this implies $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$ is accumulation neutrosophic point of the neutrosophic set A' such that

$$A'(y_{\langle t_1,t_2,t_3\rangle}) = \begin{cases} A(y_{\langle t_1,t_2,t_3\rangle}) & if \quad y_{t_1} \neq x_{\lambda_1}, y_{t_2} \neq x_{\lambda_2}, y_{t_3} \neq x_{\lambda_3} \\ t_1 & if \quad y_{t_1} = x_{\lambda_1} \ and \ t_1 < \lambda_1 \\ t_2 & if \quad y_{t_2} = x_{\lambda_2} \ and \ t_2 < \lambda_2 \\ t_3 & if \quad y_{t_3} = x_{\lambda_3} \ and \ t_3 < \lambda_3 \end{cases}$$

Obviously, $A' \subseteq A$, so that $(A')^* \subseteq A^*$ also $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \notin_N A'$. Hence, $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \in_N (A')^*$, because $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \in_N cl^*(A') = A' \cup (A')^*$. So, $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \in_N A^*$, therefore, $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \in_N A^{D^*} \Longrightarrow x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \in_N A^* \Longrightarrow A^{D^*} \subseteq A^*$.

3.11 Definition

A neutrosophic set μ of $N(X)$ is called neutrosophic closed and discrete if and only if $\mu^D = 0_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}$.

3.12 Theorem

Let $(N(X), \tau)$ be a neutrosophic topology space with I a fuzzy ideal on $N(X)$. Then,

$I_1 \in I$ is closed and discrete in $(N(X), \tau^*)$.

$A^* = cl(A - I_1)$ for every $I_1 \in I$ and for any neutrosophic set A of $N(X)$, where $A - I_1$ is the neutrosophic set defined by $(A - I_1)(x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}) = max\left\{A(x_{\lambda_1}) - I_1(x_{\lambda_1}), A(x_{\lambda_2}) - I_1(x_{\lambda_2}), A(x_{\lambda_3}) - I_1(x_{\lambda_3}), 0_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}\right\}$, for every $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \in_N N(X)$.

Proof.

$I_1 \in I \Longrightarrow I_1^* = 0_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}$, therefore by theorem 3.10, $I_1^{D^*} = 0_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}$.

The proof is clear from the definition of neutrosophic local function and the closure of a neutrosophic set.

The above theorem characterizes a useful fact about the construction of different neutrosophic ideals in relation with the original neutrosophic topology and the generated neutrosophic topology. The following examples show some cases where the two neutrosophic topologies τ and τ^* on $N(X)$ are equal.

3.12 Example:

If I_1 be a neutrosophic ideal on $N(X)$ such that $A^D \subseteq cl(A - I_1)$ for every $I_1 \in I$ and for any neutrosophic set A of $N(X)$, then it is clear that $A^D \subseteq A^*$ so that $cl(A) = cl^*(A)$. Therefore, $\tau = \tau^*$.

Again, if I_1 be such that $A^D = (A - I_1)^D$ for every $I_1 \in I$, then it is obvious that $\tau = \tau^*$.

Also, $A^D = A^*$, for a neutrosophic ideal I on $N(X)$ implies $\tau = \tau^*$.

4. Basic Structure of Generated Neutrosophic Topology

Let $(N(X), \tau)$ be a neutrosophic topological space and I is a neutrosophic ideal on $N(X)$. Let A be a neutrosophic quasi neighborhood of a neutrosophic point $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$ in the neutrosophic topology τ^*. Therefore, there exists $\mu \in \tau^*$ such that, $\lambda_1 + \mu(x_{\lambda_1}) > 1^+_{x_{\lambda_1}}$, $\lambda_2 + \mu(x_{\lambda_2}) > 1^+_{x_{\lambda_2}}$, $\lambda_3 + \mu(x_{\lambda_3}) > 1^+_{x_{\lambda_3}}$. And $\mu \subseteq A$. Now, $\mu \in \tau^* \Leftrightarrow \mu^c$ is closed in $\tau^* \Leftrightarrow cl^*(\mu^c) = \mu^c \Leftrightarrow (\mu^c)^* \subseteq \mu^c \Leftrightarrow \mu \subseteq \{(\mu^c)^*\}^c$. Therefore, $\lambda_1 + \mu(x_{\lambda_1}) > 1^+_{x_{\lambda_1}}$, $\lambda_2 + \mu(x_{\lambda_2}) > 1^+_{x_{\lambda_2}}$, $\lambda_3 + \mu(x_{\lambda_3}) > 1^+_{x_{\lambda_3}} \Longrightarrow \lambda_1 + \{(\mu^c)^*\}^c(x_{\lambda_1}) > 1^+_{x_{\lambda_1}}$, $\lambda_2 + \{(\mu^c)^*\}^c(x_{\lambda_2}) > 1^+_{x_{\lambda_2}}$, $\lambda_3 + \{(\mu^c)^*\}^c(x_{\lambda_3}) > 1^+_{x_{\lambda_3}} \Longrightarrow \lambda_1 > (\mu^c)^*(x_{\lambda_1})$, $\lambda_2 > (\mu^c)^*(x_{\lambda_2})$, $\lambda_3 > (\mu^c)^*(x_{\lambda_3}) \Longrightarrow x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \notin_N (\mu^c)^*$. this implies there exists at least one neutrosophic quasi neighborhood v_1 of $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$ in τ such that for every $y_{\langle t_1,t_2,t_3\rangle} \in_N N(X)$, $v_1(y_{t_1}) + \mu^c(y_{t_1}) - 1^+_{y_{t_1}} > I_1(y_{t_1})$, $v_1(y_{t_2}) + \mu^c(y_{t_2}) - 1^+_{y_{t_2}} > I_1(y_{t_2})$, $v_1(y_{t_3}) + \mu^c(y_{t_3}) - 1^+_{y_3} > I_1(y_{t_3})$, for some $I_1 \in I$, i.e. $v_1(y_{t_1}) - I_1(y_{t_1}) \leq \mu(y_{t_1})$, $v_1(y_{t_2}) - I_1(y_{t_2}) \leq \mu(y_{t_2})$, $v_1(y_{t_3}) - I_1(y_{t_3}) \leq \mu(y_{t_3})$, for every $y_{\langle t_1,t_2,t_3\rangle} \in_N N(X)$. Therefore, as v_1 is a neutrosophic quasi neighborhood of $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$ in τ, there is $v \in \tau$ such that $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$ is a neutrosophic quasi concomitant to $v \subseteq v_1$, and by heredity property of neutrosophic ideal we have $I_2 \in I$ for which $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}\ qc\ (v - I_2) \subseteq \mu$, we have

$(v - I_2)(y_{\langle t_1,t_2,t_3\rangle}) = max\left\{v(y_{\langle t_1,t_2,t_3\rangle}) - I_2(y_{\langle t_1,t_2,t_3\rangle}), 0_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}\right\}$, for every $y_{\langle t_1,t_2,t_3\rangle} \in_N N(X)$. Here for $\mu \in \tau^*$, we have $v \in \tau$ and $I_2 \in I$ such that, $(v - I_2) \subseteq \mu$. Let us denote $\beta(I, \tau) = \{v - I_2 : v \in_N \tau, I_2 \in I\}$.

4.1 Theorem

β forms a basis for the generated neutrosophic topology τ^* of the neutrosophic ideal I on $N(X)$.

Proof. Straight forward.

The following example is very important for the further results that justifies the above construction.

4.2 Example:

Let W be the neutrosophic indiscrete topology on $N(X)$, i.e. $W = \left\{0_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}, 1_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}\right\}$. So $1_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}$ is the only neutrosophic quasi neighborhood of every neutrosophic point $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$. Now, let $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \in_N A^*$ for a neutrosophic set $A \Leftrightarrow$ for each $I_1 \in I$, there is at least one $y_{\langle t_1,t_2,t_3\rangle} \in_N N(X)$ such that $1^+_{y_{t_1}} + A(y_{t_1}) - 1^+_{y_{t_1}} > I_1\left(1^+_{y_{t_1}}\right), 1^+_{y_{t_2}} + A(y_{t_2}) - 1^+_{y_2} > I_1\left(1^+_{y_{t_2}}\right), 1^+_{y_{t_3}} + A(y_{t_3}) - 1^+_{y_{t_3}} > I_1(1^+_{y_{t_3}})$, this implies for each $I_1 \in I$, $A(y_{\langle t_1,t_2,t_3\rangle}) > I_1(y_{\langle t_1,t_2,t_3\rangle})$ for at least one $y_{\langle t_1,t_2,t_3\rangle} \in_N N(X)$. So $A \notin I$. Therefore, $A^* = 1_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}$ if $A \notin I$ and $A^* = 0_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}$ if $A \in I$. This implies that we have, $cl^*(A) = A \cup A^* = 1_{x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}}$, if $A \notin I$ and $cl^*(A) = A$, if $A \in I$, for any neutrosophic set A of $N(X)$. Hence $W^* = \{\mu : \mu^c \in I\}$. Let $\tau \vee W^*(I)$ be the supremum neutrosophic topology of τ and $W^*(I)$, i.e. the smallest neutrosophic topology generated by $\tau \cup W^*(I)$. Then we have the following theorem:

4.3 Theorem

$\tau^*(I) = \tau \vee W^*(I)$

Proof.

Follows from the fact that β forms a basis for τ^*.

4.4 Corollary

For any two neutrosophic ideals I_1 and I_2 on $N(X)$, $I_1 \vee I_2 = \{I_1 \cup I_2 : J_1 \in I_1, J_2 \in I_2\}$ and $I_1 \cap I_2$ are neutrosophic ideals on $N(X)$.

Proof.

It is clear and straight forward.

4.5 Theorem

Let $(N(X), \tau)$ be a neutrosophic topological space and I_1, I_2 be two neutrosophic ideals on $N(X)$. Then, for any neutrosophic set A of $N(X)$,

$$A^*(I_1 \cap I_2) = A^*(I_1) \cup A^*(I_2)$$
$$A^*(I_1 \vee I_2, \tau) = A^*(I_1, \tau^*(I_2)) \cap A^*(I_2, \tau^*(I_1))$$

Proof.

Let $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \notin_N A^*(I_1) \cup A^*(I_2)$. Then, $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \notin_N$ both $A^*(I_1)$ and $A^*(I_2)$. Now $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \notin_N A^*(I_1)$ implies there is at least one quasi neighborhood μ of $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$ (in τ) such that for every $y_{\langle t_1,t_2,t_3\rangle} \in_N N(X)$, we have $\mu(y_{t_1}) + A(y_{t_1}) - 1^+_{y_{t_1}} \le I(y_{t_1})$, $\mu(y_{t_2}) + A(y_{t_2}) - 1^+_{y_{t_2}} > I(y_{t_2})$, $\mu(y_{t_3}) + A(y_{t_3}) - 1^+_{y_3} > I(y_{t_3})$, for some $I \in I_1$.

Again, $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \notin_N A^*(I_2)$ implies there is at least one quasi neighborhood v of $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$ (in τ) such that for every $y_{\langle t_1,t_2,t_3\rangle} \in_N N(X)$, we have $v(y_{t_1}) + A(y_{t_1}) - 1^+_{y_{t_1}} \le J(y_{t_1})$, $v(y_{t_2}) + A(y_{t_2}) - 1^+_{y_{t_2}} > J(y_{t_2})$, $v(y_{t_3}) + A(y_{t_3}) - 1^+_{y_3} > J(y_{t_3})$, for some $J \in I_2$ .therefore, we have $(\mu \cap v)(y_{t_1}) + A(y_{t_1}) - 1^+_{y_{t_1}} \le (I \cap J)(y_{t_1})$, $(\mu \cap v)(y_{t_2}) + A(y_{t_2}) - 1^+_{y_{t_2}} > (I \cap J)(y_{t_2})$, $(\mu \cap v)(y_{t_3}) + A(y_{t_3}) - 1^+_{y_3} > (I \cap J)(y_{t_3})$, for every $y_{\langle t_1,t_2,t_3\rangle} \in_N N(X)$. Since $(\mu \cap v)$ is also a quasi-neighborhood of $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$ (in τ) and $I \cap J \in I_1 \cap I_2$, therefore $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \notin_N A^*(I_1 \cap I_2)$, so that $A^*(I_1 \cap I_2) \subseteq A^*(I_1) \cap A^*(I_2)$. Also, $I_1 \cap I_2$ is included in both I_1 and I_2, so by theorem (3.4/2), reverse inclusion is obvious, which completes the proof of (1).

Since $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \notin_N A^*(I_1 \vee I_2, \tau)$ implies there is at least one quasi-neighborhood μ of $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$ $(in\ \tau)$ such that, for every $y_{\langle t_1,t_2,t_3\rangle} \in_N N(X)$, $\mu(y_{t_1}) + A(y_{t_1}) - 1^+_{y_{t_1}} \le I'(y_{t_1})$, $\mu(y_{t_2}) + A(y_{t_2}) - 1^+_{y_{t_2}} > I'(y_{t_2})$, $\mu(y_{t_3}) + A(y_{t_3}) - 1^+_{y_3} > I'(y_{t_3})$, for some $I' \in I_1 \vee I_2$. Therefore, by heredity of neutrosophic ideals and considering the structure of neutrosophic open sets in generated neutrosophic topology, we can find v or v', the quasi-neighborhood of the neutrosophic point $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$ in $\tau^*(I_1)$ or $\tau^*(I_2)$ respectively, such that for every $y_{\langle t_1,t_2,t_3\rangle} \in_N N(X)$, $v(y_{t_1}) + A(y_{t_1}) - 1^+_{y_{t_1}} \le J(y_{t_1})$, $v(y_{t_2}) + A(y_{t_2}) - 1^+_{y_{t_2}} > J(y_{t_2})$, $v(y_{t_3}) + A(y_{t_3}) - 1^+_{y_3} > J(y_{t_3})$, OR $v'(y_{t_1}) + A(y_{t_1}) - 1^+_{y_{t_1}} \le I(y_{t_1})$, $v'(y_{t_2}) + A(y_{t_2}) - 1^+_{y_{t_2}} > I(y_{t_2})$, $v'(y_{t_3}) + A(y_{t_3}) - 1^+_{y_3} > I(y_{t_3})$, for some $I \in I_1$ or $J \in I_2$. This implies $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \notin_N A^*(I_2, \tau^*(I_1))$ or $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \notin_N A^*(I_1, \tau^*(I_2))$, thus we have, $A^*(I_2, \tau^*(I_1)) \cap A^*(I_1, \tau^*(I_2)) \subseteq A^*(I_1 \vee I_2, \tau)$. Conversely, let $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \notin_N A^*(I_1, \tau^*(I_2))$. This implies there is at least one quasi-neighborhood μ in $\tau^*(I_2)$ of $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$ such that for every $y_{\langle t_1,t_2,t_3\rangle} \in_N N(X)$, $\mu(y_{t_1}) + A(y_{t_1}) - 1^+_{y_{t_1}} \le I_3(y_{t_1})$, $\mu(y_{t_2}) + A(y_{t_2}) - 1^+_{y_{t_2}} > I_3(y_{t_2})$, $\mu(y_{t_3}) + A(y_{t_3}) - 1^+_{y_3} > I_3(y_{t_3})$, for some $I_3 \in I_1$ .Since μ is a $\tau^*(I_2)$ quasi-neighborhood of $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$, by heredity of neutrosophic ideals we have a quasi-neighborhood v of $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$ $(in\ \tau)$ such that for every $y_{\langle t_1,t_2,t_3\rangle} \in_N N(X)$, $v(y_{t_1}) + A(y_{t_1}) - 1^+_{y_{t_1}} \le (I \cup J)(y_{t_1})$, $v(y_{t_2}) + A(y_{t_2}) - 1^+_{y_{t_2}} > (I \cup J)(y_{t_2})$, $v(y_{t_3}) + A(y_{t_3}) - 1^+_{y_3} > (I \cup J)(y_{t_3})$, for some $I \in I_1$ and $J \in I_2$, i.e. $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle} \notin_N A^*(I_1 \vee I_2, \tau)$. Thus, $A^*(I_1 \vee I_2, \tau) \subseteq A^*(I_1, \tau^*(I_2))$. Similarly, $A^*(I_1 \vee I_2, \tau) \subseteq A^*(I_2, \tau^*(I_1))$ and this completes the proof.

An important result follows from the above theorem that $\tau^*(I_1)$ and $[\tau^*(I_1)]^*$ [in short τ^{**}] are equal for any neutrosophic ideal on $N(X)$.

4.6 Corollary

Let $(N(X), \tau)$ be a neutrosophic topological space and I_1 be a neutrosophic ideal on $N(X)$. Then, $A^*(I_1, \tau) = A^*(I_1, \tau^*)$ and $\tau^*(I_1) = [\tau^*(I_1)]^*(I_1)$.

Proof.

By putting $I_1 = I_2$ in theorem (4.5/2) we have the required result.

4.7 Corollary

Let $(N(X),\tau)$ be a neutrosophic topological space and I_1, I_2 be two neutrosophic ideals on $N(X)$. Then,

$\tau^*(I_1 \vee I_2) = [\tau^*(I_2)]^*(I_1) = [\tau^*(I_1)]^*(I_2)$,

$\tau^*(I_1 \vee I_2) = \tau^*(I_1) \vee \tau^*(I_2)$,

$\tau^*(I_1 \cap I_2) = \tau^*(I_1) \cap \tau^*(I_2)$.

Proof.

By theorem (4.5/2) the result follows.

By (i), we have, $\tau^*(I_1 \vee I_2) = [\tau^*(I_2)]^*(I_1) = [\tau^*(I_1)]^*(I_2)$ [by theorem 4.5/2). Since, $\tau \subseteq \tau^*$ for any neutrosophic ideal on $N(X)$. Therefore, $\tau^*(I_1 \vee I_2) = (\tau \vee \tau^*)(I_2) \vee \tau^*(I_1) = \tau^*(I_1) \vee \tau^*(I_2)$.

Since $I_1 \cap I_2$ is included in both I_1 and I_2, $\tau^*(I_1 \cap I_2)$ is included in both $\tau^*(I_1)$ and $\tau^*(I_2)$. Now μ is a neutrosophic open set in $\tau^*(I_1) \cap \tau^*(I_2)$, implies μ^c is neutrosophic closed set in both $\tau^*(I_1)$ and $\tau^*(I_2)$. That means $(\mu^c)^*(I_1) \subseteq \mu^c$ and $(\mu^c)^*(I_2) \subseteq \mu^c$. So, $(\mu^c)^*(I_1) \cup (\mu^c)^*(I_2) \subseteq \mu^c$. Therefore, by theorem (4.5/1), $(\mu^c)^*(I_1 \cap I_2) \subseteq \mu^c$. Hence, $\mu \in \tau^*(I_1 \cap I_2)$. This completes the proof.

Conclusion

This work contains new insight into defining many mathematical notions from corners that have not been addressed before, such as neutrosophic topological space, neutrosophic ideal, neutrosophic quasi neighborhood, and neutrosophic point $x_{\langle\lambda_1,\lambda_2,\lambda_3\rangle}$ in the neutrosophic topology τ^*. As well as, many theorems and corollaries, some examples that have support the theoretical concepts.

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