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A Neutrosophic Bézier Curve Model using Control Points

Maziah Mahmud[1,2*], Abd. Fatah Wahab[2], Mohammad Izat Emir Zulkifly[3] and Said Broumi[4,5]
[1]Faculty of Computer Science and Mathematics, Universiti Teknologi MARA Cawangan Kelantan; maziah740@uitm.edu.my
[2]Faculty of Ocean Engineering Technology and Informatics, Universiti Terengganu Malaysia; fatah@umt.edu.my
[3]Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia; izatemir@utm.my
[4]Laboratory of Information processing, Faculty of Science Ben M'Sik, University Hassan II, B.P 7955, Sidi Othman, Casablanca, Morocco; broumisaid78@gmail.com
[5]Regional Center for the Professions of Education and Training (C.R.M.E.F), Casablanca-Settat, Morocco; broumisaid78@gmail.com
*Correspondence: maziah740@uitm.edu.my;

Abstract. Uncertain and ambiguous data is usually presented as a data point. When dealing with uncertain data, it is challenging to deal with incompleteness, imprecision, and incertitude; therefore, various mathematical models were developed to resolve issues concerning uncertainty points. To overcome this difficulty, a mathematical model with redefined control points characterised by three important components of the neutrosophic set was produced. The neutrosophic set can then be translated by creating models based on the neutrosophic theory and its relationships to produce control points. Hence, this paper represents a curve generated by combining neutrosophic control points with the Bézier basic functions using their relation as a Neutrosophic Bézier Curve. With an illustration example, we show how to visualise neutrosophic data sets into Neutrosophic Bézier Curve and their relationship.

Keywords: Neutrosophic; Spline curve; Bézier curve; control point, data visualization

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1. Introduction

Computer Aided Graphic Design (CAGD) is to create three-dimensional curve models and visualisations. The design process is about identifying and solving environmental problems to realise the need to improve lifestyle in the age of technology. Work on this topic has been going on for several years [35]. Data points are also collected from physical objects or environments. Once data is collected using various specialised tools and procedures, such as echo sounding and data error, some information loss or inaccuracy occurs. Spline modelling is one of the simplest and most powerful three-dimensional object creation methods. Spline modelling also enables users to create designs faster than conventional modelling techniques. The basis of this new method for designing the Bézier Curve has been introduced by [2] This method allows the curve to fit the control polygon by moving a point of the Bézier Curve. The framework of this new method for designing the Bézier Curve has been constructed.

Defining and interpreting accurate data from actual events and scenarios dealing with ambiguous data is challenging. [3] discovered a fuzzy set to handle uncertainty data in 1965 as an extension of the classical notion set. These are well studied and documented in the literature [3–9]. Then, [10, 11] introduced an intuitionistic fuzzy set in 1983. On the other hand, the data point obtained is difficult to understand as it is affected by noise and certainty. Several studies and reviews have been undertaken to explore the uncertainty problem by considering data modelling and data reduction problems, as stated in [13–15]. To represent real data points, curves and the surface is necessary, as mentioned in [12]. Hence, using the concept of fuzzy set theory, fuzzy number, fuzzy point and fuzzy relation, a new data point defined as fuzzy point relation has represented uncertainty data. Therefore, considerable research has been conducted to explore the Fuzzy Spline Model to visualise a fuzzy data set geometrically [16]. [17] describe Type-2 Fuzzy Bézier Curve Modeling and [18] implement the Interval Type-2 Fuzzy Logic System Model in Measuring the Index Value of the Underground Economy in Malaysia from 2001 to 2010. Later, [19] introduced a new concept of intuitionistic fuzzy sets with geometric modelling called the Intuitionistic Fuzzy Bézier Curve model using intuitionistic sets. They develop a spline model for data problems that involve an intuitionistic set. The intuitionistic data problem set was converted into a point relationship and blended with a spline to be visualised geometrically by curve and surface.

However, previous studies on Fuzzy Bézier Curve blended with spline functions only involve fuzzy and intuitionistic sets, while a problem involving a neutrosophic set has not yet been extensively developed. Therefore, this paper discusses and introduces a new model of the Neutrosophic Bézier Curve to represent data visualised with spline functions in geometric modelling. Neutrosophic sets (NSs) proposed by [20–25] which is a generalisation of fuzzy sets and intuitionistic fuzzy set, is a powerful tool to deal with incomplete, indeterminate and inconsistent information which exist in the real world. Neutrosophic set theory is a fabulous mathematical technique that can be applied to various fields. Uncertain data was analysed and visualised using a new type of geometric modelling more on neutrosophic. In addition, [26] conducted research on neutrosophic data problems to produce the Bézier curve and Bézier surface. However, [26] only used Neutrosophic Data with Basic Spline to generate three distinct curves. They did not clearly explain the relationship between neutrosophic point relation and basic spline. Furthermore, they did not introduce or identify the CAGD characteristics. As a result, this research did not meet the properties of CAGD, which are data prediction and accessible design.

Such a result, a Neutrosophic Bézier Curve model will be introduced in this paper. The Neutrosophic set can then be translated by creating models based on the neutrosophic theory and its relationships. The curve is generated by combining Neutrosophic control points with the Bézier basic functions and using their relation to represent the curve. Lastly, this new model is visualised using numerical examples of neutrosophic data sets with randomly selected membership values. Based on the visualization's findings, it is anticipated that the evaluation and analysis process will be simpler to carry out and have significant advantages in several areas, particularly the issue of uncertainty in the representation of real problems.

2. **Model Construction Method**

2.1. *Bézier Curve*

Pierre Bézier has derived the mathematical basis of curves and surface techniques from geometrical considerations as in [27, 28]. Later, around the 1970s, Forrest (1972) and Gordon and Reisenfeld (1974) found the connection between the work of Bézier and the classical Bernstein polynomials. They discovered that the Bernstein polynomials are the basis functions for Bézier curves and surfaces. The curve is necessary and inevitable for representing data points [29]. However, the nature of the data point obtained is difficult to understand, process and describe as it is affected by noise and uncertainty. Usually, data with uncertainty characteristics will be ignored or removed from a data set, disregarding its effect on the resulting curve and surface. Hence, the evaluation and analysis process will be incomplete. If there exists an element of uncertainty, the data set should be filtered so that it can be used to generate a curve of a model that wants to be investigated. Therefore an appropriate approach is needed to visualise and overcome this problem.

2.2. *Neutrosophic Set*

This section will begin with a summary of laws in neutrosophic sets as defined in [30]. Neutrosophic Set as an expansion of Intuitionistic Fuzzy Set where in Intuitionistic Fuzzy Set, the com- ponents T known as membership, I known as inconsistency and F known as non-membership are restricted either $t+i+f=1$ or $t^2+f^2\leq 1$, if T,I,F are all reduced to the points t,i,f respectively, or $sup\ T + sup\ I + sup\ F = 1$ if T,I,F are subsets of $[0,1]$. But in Neutrosophic Set, there is no restriction on T(truth-membership), I(indeterminacy-membership), F(false-membership) other than they are subsets of $]^-0,1^+[$ thus, $^-0 \leq inf\ T + inf\ I + inf\ F \leq sup\ T + sup\ I + sup\ F \leq 3^+$ [30] .

Definition 1. *[30]Let E be a universe of discourse, and W a set included in E. An element x from E is noted with respect to the set W as $x(T,I,F)$. x belongs to W and define as follows: true value in the set denoted as t, indeterminate value in the set as i and false value in the set as f, where t varies in T_W, i varies in I_W, f varies in F_W. T_W, I_W, F_W are functions depending on many known or unknown* *parameters.* T_W, I_W, F_W *are real standard or non-standard subset of* $]^{-}0, 1^{+}[$. *That is*

$$T_W : X[0,1]$$

$$I_W : X[0,1]$$

$$F_W : X[0,1]$$

$$\text{where } ^{-}0 \leq T_W + I_W + F_W \leq 3^{+}$$

Definition 2. *Let a crisp set* M *is fixed and let* $B^* \subset M$. *A Neutrosophic set* B^* *in* M *is an object of the following*

$$B^* = \{(x, \mu_B(x), \gamma_B(x), \pi_B(x))|x \in X\} \quad (1)$$

where functions $\mu_B : X \to [0,1], \gamma_B : X \to [0,1], \pi_B : X \to [0,1]$ *define the degree of membership, the degree of non membership and the degree of indeterminate of the element* $x \in M$ *to the set* B^**, respectively and for every* $x \in M$.

$$0 \leq \mu_B(x) + \gamma_B(x) + \pi_B(x) \leq 3$$

where $\mu_B(x) + \gamma_B(x) + \pi_B(x)$ *are independent membership of element* $x \in M$ *to set* B^*.

2.3. *Neutrosophic Number and Neutrosophic Point Relation*

Prior research in Fuzzy systems (FSs) and Intuitionistic Fuzzy systems (IFSs) discussed the result in uncertainty. Still, these methods cannot be successfully solved when decently, unacceptable, and decision-maker declaration is uncertain. Therefore, some theories are mandatory for solving the problem with uncertainty. Hence, the Neutrosophic Sets (NSs) reflect three membership which is truth membership, indeterminacy membership and falsity membership will introduce named as Neutrosophic Curve. Furthermore, Neutrosophic Set is more practical and can solve the data than FSs and IFSs, which are involved with inconsistent, incomplete and uncertain data.

The concept of Neutrosophic Set is used to develop Neutrosophic Point Relation. Generally, neutrosophic point relations are data sets defined on a universal set which are Cartesian products of $X \times Y$ that are mapping from $X \to Y$. It represents the strength of the association between elements of the two sets. Neutrosophic Point Relation is defined and used as a converter from the definition of Neutrosophic data points to introduce Neutrosophic Control Point.

Definition 3. *Let* R *and* S *be a space points with non-empty sets and* $r, s \subseteq \mathrm{R} \times \mathrm{S}$*, then Neutrosophic point relation is defined as*

$$\mathbb{M}^* = ((r_i, s_i), \mu_{R\times S}(r_i, s_i), \gamma_{R\times S}(r_i, s_i), \pi_{R\times S}(r_i, s_i)|(\mu_{R\times S}(r_i, s_i), \gamma_{R\times S}(r_i, s_i), \pi_{R\times S}(r_i, s_i)) \in \mathrm{R} \times \mathrm{S} \quad (2)$$

where (r_i, s_i) *is a point relation and* M *is a neutrosophic point relation space on* $\mathrm{R} \times \mathrm{S}$ *and functions* $\mu_B : X \to [0,1], \gamma_B : X \to [0,1], \pi_B : X \to [0,1]$ *define truth membership, indeterminacy membership and falsity membership respectively.*

$$0 \leq \mu_B(x) + \gamma_B(x) + \pi_B(x) \leq 3$$

Definition 4. *Let* $r, s \subseteq \mathrm{R} \times \mathrm{S}$ *with*

$$\tilde{M} = \{(r_i, y_i) \,|y_i \in (0,1)\} \text{ and } \tilde{N} = \{(s_i, y_i) \,|y_i \in (0,1)\} \tag{3}$$

represent two neutrosophic points. Then

$$\widetilde{S} = \{((r_i, s_i), \mu_{R\times S}(r_i, s_i), \gamma_{R\times S}(r_i, s_i), \pi_{R\times S}(r_i, s_i)) \,|0 \leq \mu_{R\times S}(x) + \gamma_{R\times S}(x) + \pi_{R\times S}(x) \leq 3\} \tag{4}$$

is a neutrosophic point relation on $\widetilde{M}$ *and* $\widetilde{N}$ *if*

$$\mu_s(r_i, s_i) \leq \mu_M(r_i), \forall (r_i, s_i) \in R \times S,$$

$$\gamma_s(r_i, s_i) \leq \gamma_M(r_i), \forall (r_i, s_i) \in R \times S$$

$$\pi_s(r_i, s_i) \leq \pi_M(r_i), \forall (r_i, s_i) \in R \times S$$

and

$$\mu_s(r_i, s_i) \leq \mu_N(r_i) \forall (r_i, s_i) \in R \times S$$

$$\gamma_s(r_i, s_i) \leq \gamma_N(r_i), \forall (r_i, s_i) \in R \times S$$

$$\pi_s(r_i, s_i) \leq \pi_N(r_i), \forall (r_i, s_i) \in R \times S$$

Neutrosophic point relation is a subset of the Cartesian product of a set that can be used to represent the data with a connection between variables, attributes or quantities. It can also visualize into the spline the dependencies and correlations of variables.

2.4. *Neutrosophic Control Point Relations*

Neutrosophic spline model in the context of geometric modeling results when each coefficient geometry spline model redefined through neutrosophic fuzzy approach until produced a form of control points. A Bézier curve is a curve that is determined by its control polygon. Bézier curve is a parametric curve used in computer graphics and related fields. The Bézier curve is a parametric curve $B(t)$ that is a polynomial function of the parameter, t. The polynomial degree depends on the number of points used to define the curve. This paper employs neutrosophic control point relation using the neutrosophic point relation we introduced in the previous section and produces an approximating curve. The approximating curve does not pass through the interior points but is attracted to them. This section discussed blending Neutrosophic Control Point Relation with Bézier function to produce Neutrosophic Bézier Curves. Next, the curve is generated with the blending and recursive processes. The Neutrosophic Control Points are defined as follows:

Definition 5. *Neutrosophic control point relation can be defined as a set of $n+1$ points that shows positions and coordinates of a location and is used to describe a curve which is denoted by*

$$\tilde{C}_{PR_i} = \left\{\tilde{C}_{PR_0}, \tilde{C}_{PR_1}, \ldots, \tilde{C}_{PR_s}\right\} \tag{5}$$

and can be written as

$$\left\{\left(\left(p_i, q_i\right), \mu_{p\times q}\left(p_i, q_i\right)\right)_1, \left(\left(p_i, q_i\right), \mu_{p\times q}\left(p_i, q_i\right)\right)_2, \ldots, \left(\left(p_i, q_i\right), \mu_{p\times q}\left(p_i, q_i\right)\right)_n\right\}$$

where the neutrosophic control point relation is also control the shape of a curve.

2.5. *Neutrosophic Bézier Model*

In a previous study, [31] had come out with the design and tuning of fuzzy control surfaces with Bézier functions. Hence, [32] and [33] use fuzzy set theory, uncertainty data and technique of interpolation to build rational Bézier curve and followed by [34] whose used Bézier curve modeling to interpret intuitionistic data problem. The idea of constructing Neutrosophic Bézier Model starts with the new Neutrosophic Control Point. let $\widetilde{C}_{PR}$ be a Neutrosophic Control Point Relations defined by Neutrosophic Point Relation and $B(t)$ be a Bézier curve with parameter, t, hence by blending it, Neutrosophic Bézier Curve is defined as follow.

$$\widetilde{B}(t) = \sum_{i=0}^{n} \widetilde{C}_{PR} B_{n,i}(t), \ \ 0 \le t \le 1 \tag{6}$$

$$\text{w}ith$$

$$\widetilde{B^{\mu}}(t) = \sum_{i=0}^{n} \widetilde{C}_{PR} B_{n,i}(t), \ \ 0 \le t \le 1$$

$$\widetilde{B^{\lambda}}(t) = \sum_{i=0}^{n} \widetilde{C}_{PR} B_{n,i}(t), \ \ 0 \le t \le 1$$

$$\widetilde{B^{\pi}}(t) = \sum_{i=0}^{n} \widetilde{C}_{PR} B_{n,i}(t), \ \ 0 \le t \le 1$$

with Bernstein polynomials or blending function,
$B_{n,i}(t) = \begin{pmatrix} n \\ i \end{pmatrix}^{i} t^i (1-t)^{n-i}$ where $\begin{pmatrix} n \\ i \end{pmatrix} = \frac{n!}{i!(n-i)!}$ are the binomial coefficients.
For degree of n Neutrosophic Bézier also can be written as

$$\widetilde{B}(t) = \widetilde{C}_{PR_0} B_{n,0} + \widetilde{C}_{PR_1} B_{n,1} + \ldots + \widetilde{C}_{PR_n} B_{n,d} \tag{7}$$

3. Results

This paper uses cubic Bézier curve approximation to show how Neutrosophic Bézier curve will represent in the graph and illustrate Neutrosophic Bézier Model.

3.1. *Example 1*

Let $C_0^* = (1,3), C_1^* = (3,6), C_2^* = (6,2)$ and $C_3^* = (9,5)$ be an Neutrosophic Control Point Relation. Hence, truth-membership, indeterminacy-membership, false-membership is summarized as follows:

TABLE 1. Examples of some NCP and its respective degrees.

NCP	truth-membership, $\mu_C (C_i^*)$	indeterminacy-membership, $v_C (C_i^*)$	false-membership, $\pi_C (C_i^*)$
C_0^*	0.3	0.6	0.1
C_1^*	0.8	0.1	0.1
C_2^*	0.7	0.1	0.2
C_3^*	0.2	0.4	0.4

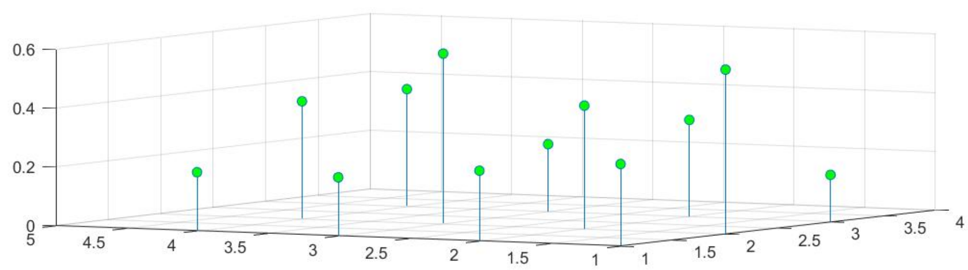

FIGURE 1. Neutrosophic Control Points

A Bézier curve is a curve that is determined by its control polygon. The Bézier curve is a parametric curve $B(t)$ that is a polynomial function of the parameter, t. Here, we will employ Neutrosophic Control Point relation using neutrosophic point relation. Figure 1 shows Neutrosophic Control Point that results from Neutrosophic Data Point in Table 1. Next, by blending it with Bézier curve, the following graphs of Neutrosophic Bézier curve are sketched for truth-membership, indeterminacy-membership, false-membership and all membership respectively.

TABLE 2. Neutrosophic Bézier curve for truth-membership

NCP	truth-membership μ_C
C_0	$\langle(1,3);0.3\rangle$
C_1	$\langle(3,6);0.8\rangle$
C_2	$\langle(6,2);0.7\rangle$
C_3	$\langle(9,5);0.2\rangle$

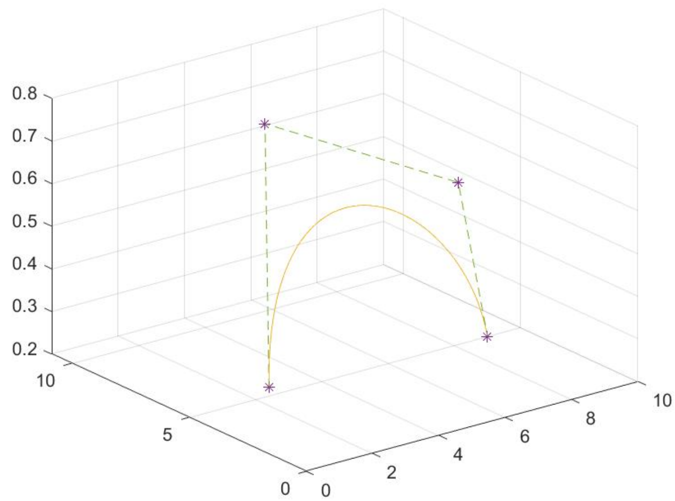

FIGURE 2. Neutrosophic Bézier curve for truth-membership

Figure 2 above is Neutrosophic Bézier curve produced from Neutrosophic Control Point with truth membership, μ_C in Table 2.

TABLE 3. Neutrosophic Bézier curve for indeterminacy-membership

NCP	indeterminacy-membership v_C
C_0	$\langle(1,3);0.6\rangle$
C_1	$\langle(3,6);0.1\rangle$
C_2	$\langle(6,2);0.1\rangle$
C_3	$\langle(9,5);0.1\rangle$

Figure 3 shows Neutrosophic Bézier curve produced from Neutrosophic Control Point with indeterminacy membership, v_C in Table 3.

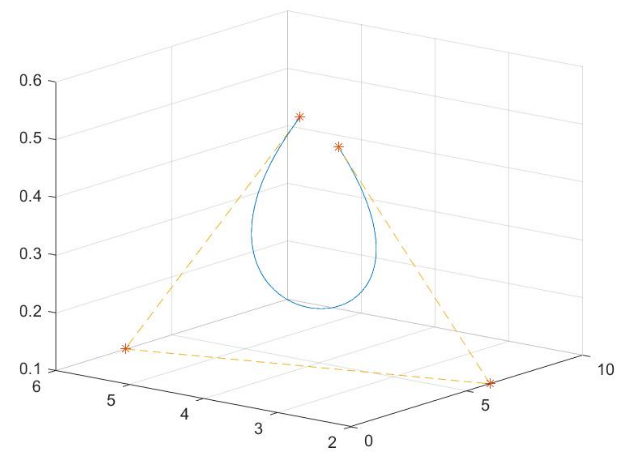

FIGURE 3. Neutrosophic Bézier curve for indeterminacy-membership

TABLE 4. Neutrosophic Bézier curve for false-membership

NCP	false-membership π_C
C_0	$\langle (1,3);0.1 \rangle$
C_1	$\langle (3,6);0.1 \rangle$
C_2	$\langle (6,2);0.2 \rangle$
C_3	$\langle (9,5);0.4 \rangle$

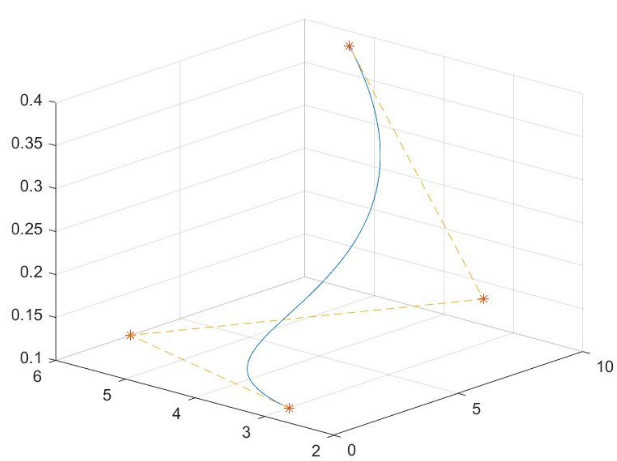

FIGURE 4. Neutrosophic Bézier curve for false-membership

Figure 4 represents Neutrosophic Bézier curve produced from Neutrosophic Control Point with false membership, π_C in Table 4.

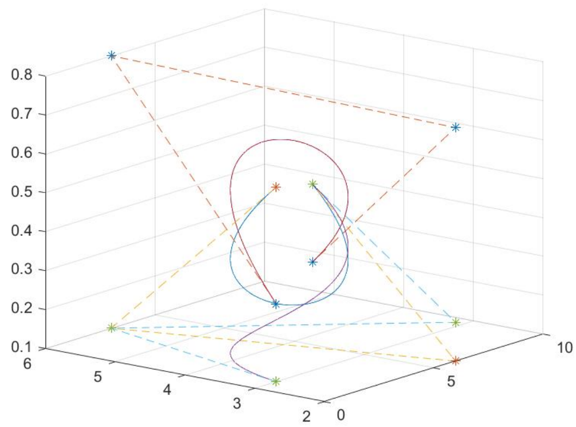

FIGURE 5. Neutrosophic Bézier curve for all membership degree

Figure 5 is the combination of all membership degrees to produce Neutrosophic Bézier Curve where

$$\widetilde{B}(t) = \sum_{i=0}^{3} \widetilde{C}_{PR} B_{3,i}(t), \quad 0 \leq t \leq 1 \tag{8}$$

with

$$\widetilde{B^{\mu}}(t) = \sum_{i=0}^{3} \widetilde{C}_{PR} B_{3,i}(t), \ 0 \leq t \leq 1$$

$$\widetilde{B^{\lambda}}(t) = \sum_{i=0}^{3} \widetilde{C}_{PR} B_{3,i}(t), \ 0 \leq t \leq 1$$

$$\widetilde{B^{\pi}}(t) = \sum_{i=0}^{3} \widetilde{C}_{PR} B_{3,i}(t), \ 0 \leq t \leq 1$$

4. **Conclusions**

A new model is proposed to represent a visualisation of neutrosophic data set which called as Neutrosophic Bézier Curve model. Neutrosophic Bézier Curve model approximation is an optimal method for modeling data with uncertainty data since it is defined by truth-membership T, indeterminacy-membership I, false-membership F. Based on this definition blended with the control point, an approximation Neutrosophic Bézier Curve has been developed. All three curves representing the data will solve complex uncertainty data in graphic design and visualisation problems.

The proposed model is described in basic terms and illustrates the final results. As a result, additional deep research with new definitions and ideas is required to depict the processes in greater detail. This generalised model must be applied to real data to achieve the intended visualisation and analysis. Organisation can use the data to increase productivity and employee satisfaction by showing the importance of various employee satisfaction in logistic services as stated in [35]. The Neutrosophic data combined with visualisation with Bézier will provide complete knowledge of the study and explain the problems studied with its reasoning. The system and the resulting model will contribute to the Neutrosophic Modeling techniques area.

This paper could also be extended to other spline models such as B-Spline and NURBS (Non-Uniform Rational B-Spline), and also in future works, especially in the development of management decision-making field, stochastic processes, stock market, remote sensing, data mining, real-time tracking, routing and wireless sensor networks.

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