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A Novel Approach to the Algebraic Structure of Neutrosophic SuperHyper Algebra

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Abstract. Hyperalgebras and BCI algebras extend classical algebraic structures, and these specific structures offer tools and frameworks for studying various operations and logic in algebra. Many authors continued their research in hyperstructures to explore different logical algebras. SuperHyper Algebra is one of the significant advancements in algebra which has been developed recently. In this article, we propose a generalized concept, namely, SuperHyper BCI-Algebra, and investigate some of its properties. We also define SuperHyper Subalgebra, and some of its characteristics are examined. Finally, we extend our vision to Neutrosophic SuperHyper BCI-Algebra.

Keywords: Hyper BCI-Algebra, SuperHyper operation, SuperHyper Groupoid, SuperHyper BCI-Algebra, SuperHyper Subalgebra and Neutrosophic SuperHyper BCI-Algebra.

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1. **Introduction**

The conceptual frameworks of fuzzy logic and fuzzy sets have been widely applied in several situations that involve uncertainty. This idea was first proposed by L. Zadeh [1]. As a result of an element's ambiguity or partial belongingness to the set, fuzzy sets are effective at dealing with uncertainty. Fuzzy set theory does not provide for hesitation or ambiguity in membership degrees. Atanassov [2] initiated the idea of intuitionistic fuzzy sets to include uncertainty in membership degrees. Still, some scenarios cannot address issues with incomplete data. Smarandache [3] developed neutrosophic set theory, a key factor dealing with indeterminacy. Sets containing elements that have independent degrees of truth, indeterminate and false memberships over the unit interval -]0, 1[+ are called Neutrosophic sets. Many applications of Neutrosophic logic have been developed, especially in Decision-Making difficulties. The following articles highlight the theoretical developments of Neutrosophic logic [4, 5], and its applications [6–9].

The investigation of BCK-algebras was initiated by K. Iseki in 1996 [10, 11], which extended the concepts of set-theoretic difference and propositional calculus. The algebraic representations of the set difference and its properties from set theory and the implicational functor from logical systems comprise the class of logical algebras known as BCI-algebras. They are highly related to numerous logical algebras and partially ordered commutative monoids. The combinators B, C, K, and I in combinatory logic are the origins of their names [12]. F. Marty [13] proposed the hyperstructure theory (also known as multialgebras), and numerous researchers have studied hyper BCK-algebras. The hyperstructure theory has now been applied to a wide range of mathematical structures, with applications in both pure and applied mathematics. Then, various researchers developed this new field. Many significant results emerged throughout the following decades, but one of the most blooming hyperstructures has been described since the 1970s. In [14], hyperstructures were applied to BCI-algebras and defined as hyper BCI-algebra as a generalization of a BCK-algebra and various related characteristics were examined. Hyper BCK-algebras is a natural evolution from classical BCK-algebras. In a standard BCK-algebra, combining two elements yields another element, whereas in a Hyper BCK-algebra, combining two elements results in a set. Also, many results have been studied on hyper BCK-algebras [15–17]. Recently, the area of hyperstructure theory has received a lot of attention. Hyper BCK/BCI-algebras were applied to various mathematical fields, such as topology, functional analysis, coding theory, group theory, etc. Since then, a significant research on the theory of BCK-algebras has been published in the literature. [18–21].

In [22], the authors extended the concept of fuzzy hyper BCK-subalgebras by introducing the concept of fuzzy BCK-subalgebras and formulated the notion of extendable fuzzy BCK-subalgebras. Also, the fuzzification of the implicative hyper BCK-ideals and their attributes are explored in [23]. In [24], the notion of Intuitionistic fuzzy hyper BCK-Ideals of Hyper BCK-Algebra was introduced. A study on intuitionistic fuzzy Lie sub-superalgebras and intuitionistic fuzzy ideals of Lie superalgebras was published in [25]. Many researchers have also investigated the intuitionistic fuzzification of ideals and subalgebras in BCK/BCI-algebras [26–30].

One of the developing fields during the last few decades is the study of the algebraic properties of neutrosophic logic. In [31], Neutrosophic BCI/BCK algebra was introduced. Neutrosophic subalgebra and ideals in BCK/BCI algebra were extensively discussed in [32–36]. Theoretical aspects concerned with introducing the concept of NeutroHyperGroups and presenting their basic properties and examples were discussed in [37]. The conceptions of the BMBJ-Neutrosophic Hyper-BCK-Ideals, MBJ-neutrosophic hyper BCK-ideal and MBJ neutrosophic strong and weak hyper BCK-ideal were investigated in [38, 39]. In [40], Smarandache initiated the most generalized algebra called SuperHyper Algebras, and in [41], Smarandache defined a SuperHyperGraph (SHG) and added the SuperVertices in classical HyperGraph. The most generalized algebra, SuperHyper Algebra, and numerous variations of Hyper structures were the inspiration for this study. In this paper,

- A generalized concept SuperHyper Algebra was extended to SuperHyper groupoids.
- A hybridization of BCI algebra and SuperHyper Algebra is explored as SuperHyper BCI-Algebra over a SuperHyper groupoids.
- A framework including SuperHyper subalgebras, and Neutrosophic SuperHyper BCI-algebras are discussed

The structure of the paper is organized as follows: In the preliminaries section, we present the fundamental concepts that are pertinent to this study. In the next section, we define SuperHyper groupoid and SuperHyper BCI-Algebra and investigate some of its characteristics. In section 4, SuperHyper subalgebra and its properties were discussed. Further, we generalize our perspective to Neutrosophic SuperHyper BCI-Algebra. Finally, we have summarized our key findings and suggested avenues for future research.

2. **Preliminaries**

Definition 2.1. [3] For any subset U of S, a neutrosophic set U on S is of the form $U = \{(\alpha, T_U(\alpha), I_U(\alpha), F_U(\alpha))|\alpha \in S\}$, where $T_U, I_U, F_U : S \to [0,1]$ represents, truth, indeterminacy and falsity membership functions respectively and $0 \leq T_U(\alpha) + I_U(\alpha) + F_U(\alpha) \leq 3$.

Definition 2.2. [14] Consider a not empty set B with the binary operation '$\circ$' and the constant 0. If the ensuing axioms are true, then $(B, \circ, 0)$ is known as a BCI-algebra,

(1) $((\upsilon_1 \circ \upsilon_2) \circ (\upsilon_1 \circ \upsilon_3)) \circ (\upsilon_3 \circ \upsilon_2) = 0$
(2) $(\upsilon_1 \circ (\upsilon_1 \circ \upsilon_2)) \circ \upsilon_2 = 0$
(3) $\upsilon_1 \circ \upsilon_1 = 0$
(4) $\upsilon_1 \circ \upsilon_2 = 0$ and $\upsilon_2 \circ \upsilon_1 = 0 \implies \upsilon_1 = \upsilon_2$

$\forall\, \upsilon_1, \upsilon_2, \upsilon_3 \in B$

Definition 2.3. [14] Consider a non-empty set B and $\Diamond : B \times B \to P^*(B)$ where $P^*(B)$ represents the power set of $B \setminus \{0\}$. Let P, $Q \subseteq B$, then the notation $P \Diamond Q$ is the collection $\bigcup\limits_{p \in P, q \in Q} p \Diamond q$. Then $(B, \Diamond)$ is said to be a hyper groupoid and $\Diamond$ is called a Hyperoperation on B. Also $r \ll s$ denotes $0 \in r \Diamond s$ and for any two subsets P, Q of B, $P \ll Q$ means that $\forall\, p \in P$, $\exists\, q \in Q$ such that $p \ll q$.

Definition 2.4. [14] A hyper groupoid $(B, \Diamond)$ with a constant element 0 is called a hyper BCI-algebra if it satisfies the following conditions:

(H1) $(\zeta \Diamond \eta) \Diamond (\eta \Diamond \theta) \ll \zeta \Diamond \eta$
(H2) $(\zeta \Diamond \eta) \Diamond \theta = (\zeta \Diamond \theta) \Diamond \eta$
(H3) $\zeta \ll \zeta$
(H4) $\zeta \ll \eta$ and $\eta \ll \zeta \implies \zeta = \eta$
(H5) $0 \Diamond (0 \Diamond \zeta) \ll \zeta$

$\forall\, \zeta, \eta, r \in B$

Notation 1. *[40] $P_*^n(B)$ denotes the k^{th} powerset of the set B and none of $P^i(B), i = 1, 2, ..., k$ contain the empty set Φ.*

Definition 2.5. [40] A classical - type Binary SuperHyper Operation $\circ^*_{(2,k)}$ is defined as follows: $\circ^*_{(2,k} : B^2 \to P_*^k(B)$, where $P_*^k(B)$ is the kth - power set of the set B, with no empty-set Φ.

3. SuperHyper BCI-Algebra

In this section, we define SuperHyper BCI-Algebra and investigate some of its characteristics. The following notation generalizes the notation $x \in B$ to the power sets. This notation will help to define algebraic structure in SuperHyper theory.

Notation 2. *For a set B and an element τ , $\tau \prec B$ denotes $\tau \in B$. By induction, for a collection $\mathcal{C} \in P_*^n(B)$ and an element τ, $\tau \prec \mathcal{C}$ denotes $\tau \prec \mathcal{X}$ for some $\mathcal{X} \in \mathcal{C}$.*

The following example provides an overview of the above notation.

Example 3.1. Let $B = \{0, 1, 2, 3\}$ and $\mathcal{C} = \{\{\{0\}, \{1, 3\}\}, \{\{1\}, \{4, 3\}\}, \{\{1, 3\}\}\} \in P_*^3(B)$ then $0 \prec \mathcal{C}$, since $0 \in \{0\}$, $0 \prec \{\{0\}\} \in \mathcal{C}$. Similarly $1 \prec \mathcal{C}$, $3 \prec \mathcal{C}$ and $4 \prec \mathcal{C}$ but $2 \not\prec \mathcal{C}$ since 2 is not in any of the collection of $\mathcal{C}$.

Smarandache [40] has introduced the algebraic operations in SuperHyper theory. Here, an algebraic operation in SuperHyper groupoid was investigated.

Definition 3.2. Let S denote a non-empty set and $\oslash$ be a SuperHyper operation on S defined as a function $\oslash : S \times S \to P_*^n(S)$. Here $P_*^n(S)$ denotes the nth powerset of the set $S\backslash\Phi$. For any two subsets X and Y of S, $X \oslash Y$ is denotes the collection $\bigcup\limits_{\mathfrak{x}\in X, \mathfrak{y}\in Y} \mathfrak{x} \oslash \mathfrak{y}$ and for any two collection $\mathcal{C}$ and $\mathcal{D}$ of $P_*^i(S)$, $\mathcal{C} \oslash \mathcal{D}$ denotes the collection $\bigcup\limits_{\mathcal{X}\in\mathcal{C}, \mathcal{Y}\in\mathcal{D}} \mathcal{X} \oslash \mathcal{Y}$, $\forall i = 1, 2, ..., n$.

The set S with a SuperHyper operation $\oslash$ and all those above said notations is called as SuperHyper groupoid and denoted by $(S, \oslash)$.

Notation 3. *Furthermore, we say $\tau \ll \rho$ if $0 \prec \tau \oslash \rho \in P_*^{n-1}(S)$. For all $\mathcal{X}$, $\mathcal{Y} \subseteq S$, $\mathcal{X} \ll \mathcal{Y}$ represents that $\forall\, \mathfrak{x} \in \mathcal{X}$, $\exists\, \mathfrak{y} \in \mathcal{Y}$ such that $\mathfrak{x} \ll \mathfrak{y}$ and for every $\mathcal{C}$ and $\mathcal{D} \in P_*^i(S)$, $\mathcal{C} \ll \mathcal{D}$ means that for every $\mathcal{X} \in \mathcal{C}$, $\exists\, \mathcal{Y} \in \mathcal{D}$ such that $\mathcal{X} \ll \mathcal{Y}$, $\forall i = 1, 2, ..., n$.*

Definition 3.3. A SuperHyper groupoid $(S, \oslash)$ that contains a constant 0 is described as a SuperHyper BCI-algebra under the following conditions:

(SH1) $(\kappa \oslash \mu) \oslash (\lambda \oslash \mu) \ll \kappa \oslash \lambda$

(SH2) $(\kappa \oslash \lambda) \oslash \mu = (\kappa \oslash \mu) \oslash \lambda$

(SH3) $\kappa \ll \kappa$

(SH4) $\kappa \ll \lambda$ and $\lambda \ll \kappa \implies \kappa = \lambda$

(SH5) $0 \oslash (0 \oslash \kappa) \ll \kappa$

for every $\kappa, \lambda, \mu \in S$

Example 3.4. Let $S = \{0, 1\}$ and $\oslash : S \times S \to P^2_*(S)$. Consider the table below:

$\oslash$	0	1
0	{{0}, {0,1}}	{{0},{1}}
1	{{1}}	{{0},{1},{0,1}}

Then $(S, \oslash)$ is a SuperHyper BCI-algebra.

The examples below illustrate the existence of SuperHyper BCI-algebra.

Example 3.5. Suppose we have a hyper BCI-Algebra $(S, \Diamond, 0)$. A SuperHyper operation $\oslash$ on S is defined as $\tau \oslash \rho = P^{n-1}_*(\tau \Diamond \rho)$ for all $\tau, \rho \in S$. Here $(S, \oslash)$ forms a SuperHyper BCI-algebra.

Example 3.6. Let $\oslash : S \times S \to P^n_*(S)$ be a SuperHyper operation on $S = [0, \infty)$. Then we define $(\tau \oslash \rho)$ as

$$(\tau \oslash \rho) = \begin{cases} P^{n-1}_*[0, \tau], & \text{if } \tau \leq \rho \\ P^{n-1}_*(0, \rho], & \text{if } \tau > \rho \neq 0 \\ P^{n-1}_*\{\tau\} = \{\tau\}, & \text{if } \rho = 0 \end{cases}$$

for all $\tau, \rho \in S$. Then $(S, \oslash)$ is a SuperHyper BCI-algebra.

The following theorem discusses some characteristics of SuperHyper BCI-algebra.

Proposition 3.7. *In any SuperHyper BCI-algebra, the following holds.*

(i) $\mu \ll 0 \implies \mu = 0$

(ii) $0 \prec \mu \oslash (\mu \oslash 0)$

(iii) $\mu \ll \mu \oslash 0$

(iv) $0 \oslash (\mu \oslash \lambda) \ll \lambda \oslash \mu$

(v) $\mathcal{X} \ll \mathcal{X}$

(vi) $\mathcal{X} \subseteq \mathcal{Y} \implies \mathcal{X} \ll \mathcal{Y}$

(vii)$\mathcal{X} \ll P^i_*(\{0\}) \implies \mathcal{X} = P^i_*(\{0\}) \,\forall \mathcal{X} \subseteq P^{i-1}_*(S)$

(viii) $\mu \oslash 0 \ll P^n_*(\{\lambda\}) \implies \mu \ll \lambda$

(ix) $\mu \oslash \lambda = P^n_*(\{0\}) \implies (\mu \oslash \kappa) \oslash (\lambda \oslash \kappa) = P^n_*(\{0\})$ *and* $\mu \oslash \kappa \ll \lambda \oslash \kappa$

(x) $\mathcal{X} \oslash P^i_*(\{0\}) = P^n_*(\{0\}) \implies \mathcal{X} = P^i_*(\{0\}) \forall \mathcal{X} \subseteq P^{i-1}_*(\{0\})$

(xi) $(\mathcal{X} \oslash \mathcal{Y}) \oslash \mathcal{C} = (\mathcal{X} \oslash \mathcal{C}) \oslash \mathcal{Y}$

for all $\mu, \lambda, \kappa \in S$ *and for all non-empty subsets* $\mathcal{X}, \mathcal{Y}$ *and* $\mathcal{C}$ *of* $P^i_*(S), i = 1, 2, ...n$

Proof :

(i) Let $\mu \ll 0$, then $0 \prec \mu \oslash 0$. By (SH3), $0 \ll 0$, so that $0 \prec 0 \oslash (\mu \oslash 0)$. Also, $(0 \oslash 0) \oslash (\mu \oslash 0) \ll 0 \oslash \mu$, which implies $0 \ll 0 \oslash \mu$. Hence, $0 \prec 0 \oslash (0 \oslash \mu)$. Now, by (SH5), $0 \prec 0 \oslash (0 \oslash \mu) \ll \mu$.Then $0 \ll x$. Therefore by (SH4), $\mu = 0$

(ii) Since $\mu \ll \mu$, $0 \prec (\mu \oslash 0) \oslash (\mu \oslash 0) = (\mu \oslash (\mu \oslash 0)) \oslash 0$. Then $\exists$, a λ from $\mu \oslash (\mu \oslash 0)$ such that $\lambda \ll 0$. By (i) $\lambda = 0$. Hence $0 \prec \mu \oslash (\mu \oslash 0)$.

(iii) By (ii) it follows.

(iv) Since $\lambda \ll \lambda \implies 0 \oslash (\mu \oslash \lambda) \subseteq (\lambda \oslash \lambda) \oslash (\mu \oslash \lambda) \ll \lambda \oslash \mu$

(v) Since $\mu \ll \mu \implies \mathcal{X} \ll \mathcal{X}\ \forall, \mathcal{X} \in P_*^1(S)$. By induction it follwos that $\mathcal{X} \ll \mathcal{X}, \forall \mathcal{X} \in P_*^i(S), i = 2, ...n$

(vi) Trivial.

(vii) Let $\mathcal{X} \ll P_*^i\{0\}$ and let $\mu \in \mathcal{X}$. Then $\mu \ll 0 \implies \mu = 0$. Therefore $\mathcal{X} = P_*^i\{0\}$.

(viii) We know that $0 \prec (\mu \oslash 0) \oslash \lambda$= $(\mu \oslash \lambda) \oslash 0$, so there exists $\mu \prec \mu \oslash \lambda$ such that $0 \prec \mu \oslash 0$, ie, $\mu \ll 0$, which implies that $\mu = 0 \prec \mu \oslash \lambda$ by(i). Hence $\mu \ll \lambda$.

(ix) Let $\lambda \ll \kappa$ .Then $(\mu \oslash \kappa) \oslash 0 \subseteq (\mu \oslash \kappa) \oslash (\mu \oslash \lambda) \ll \mu \oslash \lambda$ (by SH1).Hence, $(\mu \oslash \kappa) \oslash 0 \ll \mu \oslash \lambda$. This means that for each $\mathfrak{a} \prec \mu \oslash \kappa$, $\exists\ \mathfrak{b} \prec (\mu \oslash \lambda)$ such that $\mathfrak{a} \oslash 0 \ll \{\mathfrak{b}\}$. Hence by (vii), $\mathfrak{a} \oslash \mathfrak{b}$.Hence $\mu \oslash \kappa \ll \mu \oslash \lambda$.

(x) It follows from (i).

(xi) It follows from (SH2)

4. SuperHyper Subalgebra

In this segment, we define SuperHyper subalgebra and examine a few of its characteristics.

Definition 4.1. Let $(S, \oslash)$ denotes the SuperHyper BCI-Algebra and $S' \subset S$ such that $0 \in S'$. If S' is a SuperHyper BCI-Algebra corresponding to the SuperHyper operation $\oslash$ on S, then S' is called as SuperHyper subalgebra of S.

Theorem 4.2. *Let S' be the subset of a SuperHyper BCI-algebra $(S, \oslash)$ such that $S' \neq \emptyset$. Then S' is a SuperHyper subalgebra of S iff the restricted map $\oslash|_{S'} : S' \times S' \to P_*^n(S')$ is a binary SuperHyper operation.*

Proof: $(\Rightarrow)$ Obvious.

$(\Leftarrow)$ It is easy to verify (SH1), (SH2),(SH3),(SH4) & (SH5). Hence it is need to show that $0 \in S'$. Since $\oslash|S'$ is a binary SuperHyper operation, $\tau \oslash \rho \subseteq P_*^{n-1}(S')\ \forall \tau, \rho \in S'$. Then $\tau \ll \tau\ \forall \tau \in S'$, we have $0 \prec \tau \oslash \tau$. Hence $0 \prec P_*^{n-1}(S')$ ie., $0 \in S'$.

Example 4.3. Let $(S, \oslash)$ be a SuperHyper BCI-algebra as in example 3.6 and let $S' = [0, \mathfrak{a}]$ for every $\mathfrak{a} \in [0, \infty)$. Then $(S', \oslash)$ is a SuperHyper subalgebra.

Proof: For any $\tau, \rho \in S'$,

$$(\tau \oslash \rho) = \begin{cases} P_*^{n-1}[0,\tau], & \text{if } \tau \leq \rho \\ P_*^{n-1}(0,\rho], & \text{if } \tau > \rho \neq 0 \\ P_*^{n-1}\{\tau\} = \{\tau\}, & \text{if } \rho = 0 \end{cases}$$

Clearly, $P_*^{n-1}[0,\tau]$, $P_*^{n-1}(0,\rho]$ and $P_*^{n-1}\{\tau\}$ are subsets of $P_*^n([0,\infty))$. Hence $\oslash|_{S'}$ is a binary SuperHyper operation from $S' \times S' \to P_*^n(S')$.

Theorem 4.4. *Consider a SuperHyper BCI-algebra $(S,\oslash)$. Then the set*

$$K(S) = \{\tau \oslash S | 0 \oslash \tau = P_*^n(\{0\})\}$$

is a SuperHyper subalgebra of S whensoever $K(S) \neq \Phi$.

Proof: Let $\tau, \rho \in K(S)$ and $\mathfrak{a} \prec \tau \oslash \rho$. Then $0 \oslash (\tau \oslash \rho) = (0 \oslash \rho) \oslash (\tau \oslash \rho) \ll 0 \oslash \tau = 0$. Therefore by 3.7 (vii) $0 \oslash (\tau \oslash \rho) = \{0\}$. Hence $\tau \oslash \rho \subseteq K(S)$. Hence by theorem 4.2 $K(S)$ is a non-empty SuperHyper subalgebra.

Theorem 4.5. *Suppose there is a SuperHyper BCI-algebra defined by $(S,\oslash)$. Then $S_1' = \{\tau \in S | \tau \oslash (\tau \oslash 0) = 0\}$ is a SuperHyper subalgebra of S whenever $S_1' \neq \Phi$.*

Proof: Proof follows from the proposition 3.7 and theorem 4.2.

Theorem 4.6. *Let $(S,\oslash)$ be a SuperHyper BCI-algebra. If $S_2' = \{\tau \in S | 0 \oslash x = P_{n-1}^*(\{0\})\}$ is non-empty then S_2' is SuperHyper subalgebra.*

Proof: Let $\tau, \rho \in S_2'$. Then $0 \oslash \tau = P_*^{n-1}(\{0\})$ and $0 \oslash \rho = P_*^{n-1}(\{0\})$. Now, $0 \oslash (\tau \oslash \rho) = (0 \oslash \rho) \oslash (\tau \oslash \rho) \ll 0 \oslash \tau = P_*^{n-1}(\{0\})$. Hence by proposition 3.7 (viii) $0 \oslash (\tau \oslash \rho) = P_*^{n-1}(\{0\})$. Therefore, for any $\mathfrak{a} \prec \tau \oslash \rho$, $0 \oslash \mathfrak{a} = P_*^{n-1}(\{0\})$. ie., $\mathfrak{a} \in S_2'$ which implies $\tau \oslash \rho \subseteq P_*^{n-1}(S_2')$.

5. Neutrosophic SuperHyper BCI-Algebra

In this section, the concept of Neutrosophic SuperHyper BCI-Algebra is introduced.

Definition 5.1. For any neutrosophic set $A =< T_A, I_A, F_A >$ in S we define the following notations.
(i) Let B be a subset of S. Then

$$T_B = inf\{T(\tau)|\tau \in B\}, I_B = inf\{I(\tau)|\tau \in B\}, F_B = inf\{F(\tau)|\tau \in B\}.$$

(ii) Let $\mathcal{Y}$ be an element in $P_i^*(S)$. Then

$$T_{\mathcal{Y}} = inf\{T(\mathcal{X})|\mathcal{X} \in \mathcal{Y}\}, I_{\mathcal{Y}} = inf\{I(\mathcal{X})|\mathcal{X} \in \mathcal{Y}\}, F_{\mathcal{Y}} = inf\{F(\mathcal{X})|\mathcal{X} \in \mathcal{Y}\}.$$

The following defines neutrosophic SuperHyper BCI-algebra.

Definition 5.2. In the realm of SuperHyper BCI-algebras, let S be the designated algebraic structure, and consider a neutrosophic set $U =< T_U, I_U, F_U >$ within S. We define U as a neutrosophic SuperHyper BCI-algebra of S when it adheres to the ensuing conditions for all elements τ and ρ in S.
$T(\tau \oslash \rho) \geq min(T(\tau), T(\rho)), I(\tau \oslash \rho) \geq min(I(\tau), I(\rho))$ & $F(\tau \oslash \rho) \leq max(F(\tau), F(\rho))$.

Example 5.3. Let $S = \{0, 1\}$ and $\oslash : S \times S \to P^2_*(S)$. Consider the table below:

$\oslash$	0	1
0	{{0}}	{{0}}
1	{{1}}	{{0},{1},{0,1}}

Then $(S, \oslash)$ is a SuperHyper BCI-algebra.We characterize a neutrosophic set A on S by

S	$T_A(\tau)$	$I_A(\tau)$	$F_A(\tau)$
0	0.71	0.63	0.18
1	0.53	0.42	0.67

Here $T(0 \oslash 1) = T(\{\{0\}\}) = 0.71 \geq min(T(0), T(0)) = 0.53$, Similarly we can verify for other values. Therefore A is a neutrosophic SuperHyper BCI-algebra.

Example 5.4. Consider the SuperHyper BCI-algebra as in example 3.4 and a neutrosophic set defined in the example 5.3. Here A is not Neutrosophic SuperHyper BCI-algebra because $T(0 \oslash 0) = T(\{\{0\}, \{0, 1\}\}) = 0.53 \ngeq min(T(0), T(0)) = 0.71$.

Proposition 5.5. *Let A be Neutrosophic SuperHyper BCI-algebra of S then the following holds. If $A =< T_A, I_A, F_A >$ then there exist τ, ρ and $\gamma \in S$ such that (i) $T_A(0) \geq T_A(\tau)$, (ii)$I_A(0) \geq I_A(\rho)$ and (iii)$F_A(0) \leq F_A(\gamma)$*

Proof:(i) By proposition 3.7 $\tau \ll 0$, then $\tau = 0$, so that $0 \not\prec \tau \oslash 0$, $\forall \tau \neq 0$. By definition $T(\tau \oslash 0) \geq min(T(\tau), T(0))$.
Case:1 Suppose $T(\tau) \leq T(0)$, then it is proved.
Case:2 If $T(\tau) > T(0), T(\tau \oslash 0) \geq T(0)$, then $\exists\ \kappa \prec \tau \oslash 0$ such that $T(\kappa) \leq T(0)$.
Similarly, we can prove for Indeterminacy and Falsity membership.

6. Conclusion

We have introduced the SuperHyper Groupoid, SuperHyper BCI algebra, SuperHyper Subalgebra, and Neutrosophic SuperHyper BCI-Algebra, which are the most extensive forms of algebras. With appropriate examples, we have discussed the characterizations of SuperHyper BCI algebra and SuperHyper subalgebras. Finally, we expanded our notion to Neutrosophic SuperHyper BCI-Algebra. These ideas will pave the way for additional theoretical research on SuperHyper theory. In [42] the extensions soft set to the HyperSoft Set, IndetermSoft Set, IndetermHyperSoft Set, and TreeSoft Set, and their practical applications are highlighted. The generalized notion of SuperHyper BCI algebra can be used to explore more hyper algebraic structures in the future, and it can be extended to rough sets, soft sets and extensions of soft sets.

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