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Novel Subsethood Measures for Totally Dependent-Neutrosophic Sets and Their Usage in Multiple Attribute Decision-Making

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Abstract: Multiple attribute decision-making (MADM) models are accepted as powerful tools for evaluating alternatives when the decision-analysts should consider more than one attribute while reaching a decision. The decision-makers who are consulted need a scale for expressing their judgments, experiences, or opinions. The fuzzy logic and its contemporary versions can supply different kinds of scales to allow the decision-makers to state their ideas. A recent version is totally dependent-neutrosophic (picture fuzzy) sets which include independently assignable elements: positive, neutral, negative, and refusal membership degrees. In this study, we aim to contribute to the literature of the totally dependent-neutrosophic sets by (i) proposing three new subsethood measures dedicatedly developed for totally dependent-neutrosophic sets for the first time in the literature and (ii) showing their applicability in a decision-making case study including a novel totally dependent-neutrosophic version of EDAS (Evaluation Based on Distance from Average Solution) method which is extended differently from the existing ones. To validate the proposed method, a comparative analysis with the existing totally dependent-neutrosophic MADM methods is provided. As a result, the proposed Subsethood Measure-based Totally Dependent-Neutrosophic Version of EDAS (SM-TDN-EDAS) method involving fewer steps than others gave similar rankings.

Keywords: Totally dependent-neutrosophic sets, subsethood measure, EDAS, multiple criteria evaluation, fuzzy numbers.

1. Introduction

In multiple attribute decision-making (MADM) problems in which the decision analysts do not have enough or proper cardinal data such as cost of investment, sales profit, market share at a certain time, etc., the linguistic terms are used by the decision-makers while expressing their preferences, opinions, or feelings. Linguistic terms are often defined in different fuzzy environments. Zadeh [1] initiated the concept of fuzzy sets as a symbolization and representation tool for quantifying human judgments. In the traditional definition of fuzzy sets, there is just membership degree (μ_A) which takes a value between 0 and 1. In general, the membership degree is a measure of the optimism or agreement level of judgment. Thus, it has a positive meaning.

For decades, various scholars have defined several fuzzy sets for smoothing the symbolization of the vagueness and ambiguity which are hidden in the human subconscious. Atanassov [2] initiated the concept of intuitionistic fuzzy sets (IFS) and add a new element into set definition: non-membership degree (υ_A). This novel item puts a level of resilience into the representation of judgments since the decision-maker can state his/her pessimistic view or disagreement level. So, non-membership degree exposes a negative meaning. Accordingly, Atanassov [2] also introduced a new measure regarding hesitancy which has a neutral meaning: $\pi_A = 1 - \mu_A - \upsilon_A$. Therefore, IFS can cope with three dimensions of judgments (membership, non-membership, and hesitancy). In real life, we can represent these degrees with yes, no, and abstain. However, the hesitancy degree in IFSs depends on the others so that the decision-maker cannot independently assign any value for that.

After the development of IFS, some other extensions such as Pythagorean fuzzy sets [3], q-Rung orthopair fuzzy sets [4], neutrosophic sets [5], spherical fuzzy sets [6], etc. have been introduced. From a different perspective, Cuong and Kreinovich [7] defined picture fuzzy sets as the generalization of fuzzy sets and IFSs. However, Smarandache, for the first time, renamed it by "totally dependent-neutrosophic set (TDNS)" [8]. In this paper, we use "totally dependent-neutrosophic set (TDNS)" instead of "picture fuzzy set (PFS)".

A TDNS is characterized by three independently assignable degrees expressing the positive membership, the neutral membership (which is equivalent to hesitancy degree), and the negative membership (which means non-membership). The sole constraint regarding these three degrees is that their sum must not exceed 1. The remaining part is called refusal degree and it represents the decision maker's choice of refusing to share his/her preference.

For illustration, Cuong [9] gives the voting process as an example of TDNS for clarifying the elements defined: the voters may be divided into four groups of those who: vote for the candidate, abstain, vote against the candidate, and refusal of the voting, i.e., casting a veto. Garg [10] gives another example. When a decision analyst consults a certain decision-maker regarding a certain topic, then he/she may state that 0.3 is the possibility that statement is true, 0.4 is the possibility that statement is false and 0.2 is the possibility that he/she is not sure of it. This issue cannot be handled by fuzzy sets or IFSs. This declaration of preference can be well-defined by TDNS as $(\mu, \eta, v) = (0.3, 0.2, 0.4)$ where μ is the positive membership degree, η is the neutral membership degree, and v is the negative membership degree. As seen, their sum is 0.9 and the remaining part is called refusal degree which is equal to (1-0.9=) 0.1. Formally, the refusal degree is defined as $\pi = 1 - \mu - \eta - v$. More formal definitions and operations are explained in Section 2. As seen from the examples, TDNS has greater representation power than IFS, neutrosophic sets, or other extensions since it exposes an additional fourth component, namely refusal degree. TDNS is the only fuzzy set definition that can address this issue.

The subsethood measure (or inclusion measure) indicates the degrees of quantitative extensions of the qualitative set inclusion relation. In classical set theory, since either a crisp set A is a subset of a crisp set B or vice versa, subsethood measure should be two-valued: 0 and 1. Fuzzy subsethood measures determine the degree to which a fuzzy set contains another fuzzy set within the range of [0, 1]. This notion fuzzifies classical fuzzy set containment which is a crisp property: a fuzzy set B contains a fuzzy set A if $\mu_A \leq \mu_B$. Kosko [11] argues that if this inequality holds for all but just a few elements, one can still consider A to be a subset of B to some degree. Many researchers such as Kosko [11], Sanchez [12], and Young [13] define several axioms for developing subsethood measures. As seen from Section 2, even though there are attempts to stating subsethood measures for various fuzzy sets, there is no proposition for TDNS. As the first contribution of this study to the existing literature, we have developed subsethood measures for TDNS and we proved that they satisfy the required axiomatic properties.

To show our measures' applicability in real-life MADM problem-solving issues, we have integrated the concept of subsethood measure in a well-known MADM method, namely EDAS (Evaluation Based on Distance from Average Solution). EDAS method was firstly presented by Keshavarz Ghorabaee et al. [14] for searching the distances between each alternative and average solution. EDAS is very similar to TOPSIS and VIKOR, but they take the distances between each alternative and positive/negative ideal alternatives as a decision criterion: the best alternative among the set of alternatives should be as distant as possible from the negative ideal alternative and as close as possible to the positive ideal one [15]. EDAS cancels the phase of obtaining the ideal solutions which might be complex by considering the distance between each alternative and average solution that can be easily found from the current data in the problem.

For enriching the representation power of EDAS, some extensions including TDNS have been proposed in the literature. For example, Zhang et al. [16] developed a TDNS-based EDAS with newly defined operations and illustrated its application in green supplier selection while Liang et al. [17] integrated TDNS-based EDAS and ELECTRE methods for cleaner production evaluation in gold mines. Similarly, Li et al. [18] defined totally dependent-neutrosophic (picture fuzzy) ordered weighted interaction averaging operator and totally dependent-neutrosophic (picture fuzzy) hybrid ordered weighted interaction averaging operator and used them in TDNS-based EDAS. Ping et al. [19] combined TDNS-based EDAS with quality function deployment and showed its application in an illustrative example. Tirmikcioglu Cinar [20] applied TDNS-based EDAS method for team leader selection for an audit firm. To the best of our knowledge, the literature does not have any integration of subsethood measures and EDAS until now. This study's second contribution is this integration proposition to ease the mathematical operations of EDAS/TDNS-based EDAS and smooth the complexity.

As a summarization, it can be stated that this study proposes some subsethood measures for TDNS for the first time in the literature and their usability is shown in a novel TDNS extension of EDAS. The rest of the paper is organized as follows. Section 2 gives the preliminaries of TDNS and its operations, and the extensive literature survey's results on subsethood measure definitions for various fuzzy set environments. In Section 3, the definitions of three novel subsethood measures are detailed and it is proven that the proposed measures satisfy the required properties. In Section 4, novel subsethood measure-based totally dependent-neutrosophic (picture fuzzy) extension of EDAS (SM-TDN-EDAS) is explained step-by-step. To demonstrate the new extension's usability, the results of a case study are shared in Section 5. Section 6 concludes the study with the findings and further research potential.

2. Preliminaries

In this chapter, the details of TDNS and operations defined on it are given. Then, the results of an extensive literature survey on subsethood measures for various fuzzy sets are stated.

2.1. Totally dependent-neutrosophic set

Cuong and Kreinovich [7] presented TDNS theory which is a generalization of Zadeh's fuzzy set theory and Atanassov's IFS theory and gave basic operations on TDNSs. A TDNS is defined with the help of the degree of positive membership, the degree of neutral membership, the degree of negative membership, and the degree of refusal membership mappings such that the sum of these components is equal to 1. Essentially, fundamental structures of TDNS have enough application to carry out situations requiring opinions of humans, which is comprising answer types such as yes, no, abstain, and refusal.

Definition 1. [9,21] Let X be a universal set. Then a TDNS A on X is defined as follows:

$$A = \{(x, \mu_A(x), \eta_A(x), \nu_A(x)) \mid x \in X\} \tag{1}$$

where μ_A, η_A, ν_A are mapping from X to $[0,1]$. For all $x \in X$, $\mu_A(x)$ is called positive membership degree of $x \in A$, $\eta_A(x)$ is called neutral membership degree of $x \in A$ and $\nu_A(x)$ is negative membership degree of $x \in A$. Also, μ_A, η_A, ν_A satisfy the following condition:

$$0 \leq \mu_A(x) + \eta_A(x) + \nu_A(x) \leq 1, \forall x \in X \tag{2}$$

and $\pi(x) = 1 - \mu_A(x) - \eta_A(x) - \nu_A(x)$ is called refusal membership degree of x in A.

We denote by $TDNS(X)$ the collection of TDNSs on X. Cuong [9] defined the subsethood, equality, union, intersection, and complement for every two TDNSs A and B as follow:

1. $A \subseteq B$ if $\forall x \in X, \mu_A(x) \leq \mu_B(x),\ \eta_A(x) \leq \eta_B(x),\ \nu_A(x) \geq \nu_B(x)$;
2. $A = B$ iff $A \subseteq B$ and $B \subseteq A$;
3. $A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x)), \min(\eta_A(x), \eta_B(x)), \min(\nu_A(x), \nu_B(x))\rangle \mid\ x \in X\}$
4. $A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x)), \min(\eta_A(x), \eta_B(x)), \max(\nu_A(x), \nu_B(x))\rangle \mid\ x \in X\}$
5. $A^c = \{\langle x, \nu_A(x), \eta_A(x), \mu_A(x)\rangle \mid\ x \in X\}$.

For all $A, B \in TDNS(X)$, Cuong [9] presented normalized Hamming distance measure by extending distance measure for IFS.

$$d_1(A,B) = \left[\frac{1}{n}\sum_{i=1}^{n}\left(\left(\mu_A(x_i) - \mu_B(x_i)\right)^2 + \left(\eta_A(x_i) - \eta_B(x_i)\right)^2 + (\nu_A(x_i) - \nu_B(x_i))^2\right)\right]^{\frac{1}{2}} \tag{3}$$

for all $A, B \in TDNS(X)$, Van Dinh et al. [22] introduced some distance measures for TDNSs as follow:

$$d_2(A,B) = \frac{1}{n}\sum_{i=1}^{n}(\max\{|\mu_A(x_i) - \mu_B(x_i)|, |\eta_A(x_i) - \eta_B(x_i)|, |\nu_A(x_i) - \nu_B(x_i)|\}), \tag{4}$$

$$d_3(A,B) = \left[\sum_{i=1}^{n}\left(\max\left\{\left(\mu_A(x_i) - \mu_B(x_i)\right)^2, \left(\eta_A(x_i) - \eta_B(x_i)\right)^2, (\nu_A(x_i) - \nu_B(x_i))^2\right\}\right)\right]^{\frac{1}{2}}. \tag{5}$$

2.2. Subsethood measures for different fuzzy environments

The subsethood measure (also called inclusion measure or degree) indicates the degrees of quantitative extensions of the qualitative set inclusion relation. In classical set theory, since either a crisp set A is a subset of a crisp set B or vice versa, subsethood measure should be two-valued. Fuzzy subsethood measures determine the degree to which a fuzzy set contains another between 0 and 1. Many researchers have studied different subsethood measures for fuzzy sets, IFS, and neutrosophic sets.

Sinha and Dougherty [23] presented the axiomatic structure of subsethood measure for fuzzy sets. Young [13] introduced different axioms of the definition of subsethood measure for fuzzy sets from axioms of Sinha and Dougherty [23]. Fan et al. [24] and Guoshun and Yunsheng [25] defined new different subsethood measures for fuzzy sets. Bustince et al. [26] defined strong S-subsethood measures for interval-valued fuzzy sets (IVFS). Vlachos and Sergiadis [27] and Takáč [28,29] presented different subsethood measures for IVFS. Rickard et al. [30] introduced subsethood measure for Type-2 fuzzy sets and generalized Type-n fuzzy sets.

Liu and Xiong [31] proposed the definition of subsethood measure for IFS. Cornelis and Kerre [32] introduced a different framework of subsethood measure for IFSs by considering the subsethood degree to be in the unit square $[0,1]^2$. Grzegorzewski and Mrowka [33] presented subsethood measure for IFSs based on the Hamming distance measure. Zhang et al. [34] defined subsethood measure for IFSs and IVFSs. Xie et al. [35] gave a new axiomatic definition and some inclusion measures for IFSs. Zhang et al. [36] introduced another new axiomatic definition and presented inclusion measure for IFSs.

Şahin and Küçük [37] proposed subsethood measure for single-valued neutrosophic sets (SVNSs) while Şahin and Karabacak [38] presented a subsethood measure for interval-valued neutrosophic sets (IVNS). Ji and Zhang [39] introduced a subsethood measure for IVNSs based on the Hausdorff distance measure. Zhang and Wang [40] proposed an inclusion measure for hesitant fuzzy sets (HFSs). Finally, Aydoğdu [41] introduced the very first subsethood measure for TDNSs (PFSs) as a conference proceeding for the first time in the literature.

3. Novel subsethood measures for TDNS

This In this section, we propose axioms of the definition of subsethood measure for TDNSs and some new subsethood measures for TDNSs based on the distance measures of TDNSs. To establish the subsethood degree to which A belongs to B, we use the distance between TDNSs A and A ∩ B. d_1, d_2, and d_3 distance measures are given in Eqs. (3-5) in Chapter 2.1.

Definition 2. Let X be a universe of discourse. A mapping $S: TDNS(X) \times TDNS(X) \to [0,1]$ is called subsethood measure if it satisfies the following properties. For all $A, B, C \in TDNS(X)$,

1. $S(A,B) = 1$ iff $A \subseteq B$,
2. $S(A,A^c) = 1 \Leftrightarrow \mu_A(x) \le \nu_A(x)$,
3. $S(A,B) = 0$ if $A = \langle x,1,0,0\rangle$ and $B = \langle x,0,0,1\rangle$,
4. If $A \subseteq B \subseteq C$, then $S(C,A) \le S(B,A)$ and $S(C,A) \le S(C,B)$.

The following theorem gives the subsethood measures based on distance measures.

Theorem: Let X be a universe of discourse. For $A, B \in TDNS(X)$, the mappings

$$S_1(A,B) = 1 - \frac{1}{\sqrt{2}} d_1(A, A \cap B) \tag{6}$$

$$S_2(A,B) = 1 - d_2(A, A \cap B) \tag{7}$$

$$S_3(A,B) = 1 - \frac{1}{\sqrt{n}} d_3(A, A \cap B) \tag{8}$$

are subsethood measures for TDNSs.

Proof: In order that $S_i(A,B)$ $(i = 1,2,3)$ to be described as a subsethood measure for TDNSs, it must satisfy the properties of Definition 2. For simplicity, we only prove that $S_1(A,B)$ satisfies these properties. $S_2(A,B)$ and $S_3(A,B)$ may also be shown in the same fashion.

Let $A = \{(x, \mu_A(x), \eta_A(x), \nu_A(x)) \mid x \in X\}$ and $B = \{(x, \mu_B(x), \eta_B(x), \nu_B(x)) \mid x \in X\}$ be two TDNSs. Since $A^c = \{(x, \nu_A(x), \eta_A(x), \mu_A(x)) \mid x \in X\}$, we have $A \cap A^c = \{(x, \min(\mu_A(x), \mu_{A^c}(x) = \nu_A(x)), \min(\eta_A(x), \eta_{A^c}(x) = \eta_A(x)), \max(\nu_A(x), \nu_{A^c}(x) = \mu_A(x))) \mid x \in X\}$.

1. Let $A \subseteq B$, then $A \cap B = \{(x, \mu_A(x), \eta_A(x), \nu_A(x)) \mid x \in X\}$.

$$S_1(A,B) = 1 - \frac{1}{\sqrt{2}} d_1(A, A \cap B)$$
$$= 1 - \frac{1}{\sqrt{2}}\left[\frac{1}{n}\sum_{i=1}^{n}\left(\left(\mu_A(x_i) - \mu_{A\cap B}(x_i)\right)^2 + \left(\eta_A(x_i) - \eta_{A\cap B}(x_i)\right)^2 + \left(\nu_A(x_i) - \nu_{A\cap B}(x_i)\right)^2\right)\right]^{\frac{1}{2}}$$
$$= 1.$$

Conversely, suppose that $S_1(A,B) = 1$, then $d_1(A, A \cap B) = 0$. So $\mu_A(x) = \mu_{A\cap B}(x)$, $\eta_A(x) = \eta_{A\cap B}(x)$ and $\nu_A(x) = \nu_{A\cap B}(x)$. Because of the definition of intersection and inclusion of TDNSs, TDNS A is a subset of TDNS B.

2. If $\mu_A(x) \le \nu_A(x)$, then

$$A \cap A^c = \{(x, \min(\mu_A(x), \nu_A(x)), \min(\eta_A(x), \eta_A(x)), \max(\nu_A(x), \mu_A(x))) \mid x \in X\}$$
$$= \{(x, \mu_A(x), \eta_A(x), \nu_A(x))\} \ .$$

Thus,

$$S_1(A,A^c) = 1 - \frac{1}{\sqrt{2}} d_1(A, A \cap A^c)$$

$$= 1 - \frac{1}{\sqrt{2}} \left[\frac{1}{n} \sum_{i=1}^{n} \left(\left(\mu_A(x_i) - \mu_{A\cap A^c}(x_i)\right)^2 + \left(\eta_A(x_i) - \eta_{A\cap A^c}(x_i)\right)^2 + \left(\nu_A(x_i) - \nu_{A\cap A^c}(x_i)\right)^2 \right) \right]^{\frac{1}{2}}$$

$$= 1 - \frac{1}{\sqrt{2}} \left[\frac{1}{n} \sum_{i=1}^{n} \left(\left(\mu_A(x_i) - \mu_A(x_i)\right)^2 + \left(\eta_A(x_i) - \eta_A(x_i)\right)^2 + \left(\nu_A(x_i) - \nu_A(x_i)\right)^2 \right) \right]^{\frac{1}{2}} = 1$$

3. For $A = \{\langle x, 1,0,0 \rangle\}$ and $B = \{\langle x, 0,0,1 \rangle\}$, we have $A \cap B = \{\langle x, 0,0,1 \rangle\}$. Hence

$$S_1(A,B) = 1 - \frac{1}{\sqrt{2}} d_1(A, A \cap B)$$

$$= 1 - \frac{1}{\sqrt{2}} \left[\frac{1}{n} \sum_{i=1}^{n} ((1-0)^2 + (0-0)^2 + (0-1)^2) \right]^{\frac{1}{2}}$$

$$= 0$$

4. To prove that $S_1(C,A) \le S_1(B,A)$, it suffices to show $d_1(C, C \cap A) \ge d_1(B, B \cap A)$. Since $A \subseteq B \subseteq C$, $\mu_A(x) \le \mu_B(x) \le \mu_C(x)$, $\eta_A(x) \le \eta_B(x) \le \eta_C(x)$ and $\nu_A(x) \ge \nu_B(x) \ge \nu_C(x)$. We get

$$d_1(C, C \cap A) = \left[\frac{1}{n} \sum_{i=1}^{n} \left(\left(\mu_C(x_i) - \mu_{C\cap A}(x_i)\right)^2 + \left(\eta_C(x_i) - \eta_{C\cap A}(x_i)\right)^2 + \left(\nu_C(x_i) - \nu_{C\cap A}(x_i)\right)^2 \right) \right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{n} \sum_{i=1}^{n} \left(\left(\mu_C(x_i) - \mu_A(x_i)\right)^2 + \left(\eta_C(x_i) - \eta_A(x_i)\right)^2 + \left(\nu_C(x_i) - \nu_A(x_i)\right)^2 \right) \right]^{\frac{1}{2}}$$

$$\ge \left[\frac{1}{n} \sum_{i=1}^{n} \left(\left(\mu_B(x_i) - \mu_A(x_i)\right)^2 + \left(\eta_B(x_i) - \eta_A(x_i)\right)^2 + \left(\nu_B(x_i) - \nu_A(x_i)\right)^2 \right) \right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{n} \sum_{i=1}^{n} \left(\left(\mu_B(x_i) - \mu_{B\cap A}(x_i)\right)^2 + \left(\eta_B(x_i) - \eta_{B\cap A}(x_i)\right)^2 + \left(\nu_B(x_i) - \nu_{B\cap A}(x_i)\right)^2 \right) \right]^{\frac{1}{2}}$$

$$= d_1(B, B \cap A).$$

Similarly, it can be shown that $S(C,A) \le S(C,B)$.

4. Subsethood measure-based totally dependent-neutrosophic set extension of EDAS (SM-TDN-EDAS)

EDAS is a distance-based MADM method like TOPSIS and VIKOR. The distances between each alternative and positive/negative ideal alternatives are computed and operationalized by the mentioned methods and then these distance measures are accepted as a criterion for reaching a decision about the rankings of alternatives. They include steps that are dedicated to obtaining or generating a positive and a negative ideal solution. In EDAS these probably complex and confusing steps are eliminated because the distance between alternative and the average solution is considered. Therefore, decision-analyst does not need to generate positive/negative ideal solutions but to compute the average performance scores of each attribute. Traditional EDAS uses two distinct measures: positive distance from average (PDA) and negative distance from average (NDA). Naturally, the decision reached should be based on higher positive distance and lower negative distance.

TDNS is one of the recent fuzzy concepts that can be used in MADM analysis in representing human judgments, opinions, or expertise. After a brief literature review, studies extending various MADM approaches into TDNS environment are exemplified and summarized in Table A1. In the first column, the studies are given while the second column shows the study's methodology which includes extension(s) of the MADM approach(es) under TDNS and the third column depicts the application of the study. As seen from the table, VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje), EDAS, and TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) extensions have the majority. There are also TDNS extensions of ARAS, TODIM, MABAC, MULTIMOORA, and PROMETHEE II in a few studies. Also, it is found that there is no proposition integrating subsethood measure and EDAS under any kind of fuzzy sets as well as TDNSs.

This study has extended EDAS method under the totally dependent-neutrosophic (picture fuzzy) environment in a different manner from existing extensions that are summarized in Table A1 as a contribution to the literature. In this novelty, we propose to use the subsethood measures as decision criteria rather than PDA and NDA. Indeed, our basic aim is to show the applicability of subsethood measures in a MADM problem-solving methodology under TDNS environment. Additionally, it is seen that the EDAS method's mathematical part is smoothed since the calculation complexity is reduced by replacing the idea of measuring distances to the average solution with calculating subsethood degree to the average solution in which includes just one operation. Also, TDNS may provide a higher independence possibility to the decision-makers since it is allowed to express independent degrees for positive, negative, and hesitancy preferences. The refusal degrees can also be calculated as a fourth element.

In this novel extension, there are 5 steps explained below.

Step 1. Decision-makers (e=1,…,k) are asked to express their judgments about alternatives' (i=1,…,m) performances with respect to attributes (j=1,…,n). So, after collecting data from decision-makers, there will be k decision matrices $(X^1, X^2, \dots, X^k)$ in hand. The judgments are aggregated via an aggregation operator defined for TDNS. In this step, the decision-makers can be weighted according to their expertise (ω_e). $\langle \mu_{ij}^e, \eta_{ij}^e, v_{ij}^e \rangle$ depicts the linguistic evaluation of e^{th} decision-maker and $\langle \mu_{ij}, \eta_{ij}, v_{ij} \rangle$ represents the aggregated performance evaluation. For obtaining aggregated decision matrix (Eq. 10), the totally dependent-neutrosophic (picture fuzzy) weighted averaging (TDNWA) operator (Eq. 9) defined by Zhang et al. [16] is utilized.

$$\begin{aligned} X^{agg} &= PFWA_\omega(X^1, X^2, \dots, X^k) = \bigoplus_{e=1}^{k} \omega_e X^e = \langle \mu_{ij}, \eta_{ij}, v_{ij} \rangle \\ &= \left\{ 1 - \prod_{e=1}^{k} \left(1 - \mu_{ij}^e\right)^{\omega_e}, \prod\nolimits_{e=1}^{k} \left(\eta_{ij}^e\right)^{\omega_e}, \prod\nolimits_{e=1}^{k} \left(v_{ij}^e\right)^{\omega_e} \right\} \end{aligned} \tag{9}$$

$$X^{agg} = \begin{bmatrix} \langle \mu_{11}, \eta_{11}, v_{11} \rangle & \cdots & \langle \mu_{1n}, \eta_{1n}, v_{1n} \rangle \\ \vdots & \ddots & \vdots \\ \langle \mu_{m1}, \eta_{m1}, v_{m1} \rangle & \cdots & \langle \mu_{mn}, \eta_{mn}, v_{mn} \rangle \end{bmatrix} \tag{10}$$

Step 2. The attributes included in any decision problem can be cost or benefit type. In order to convert any cost attribute to a benefit one, the positive and negative membership degrees should be replaced while the neutral membership degree keeps its value. This is called normalization.

After normalization, the weights of attributes representing the importance and significance of the attribute should be considered. There are 4 possibilities: *(i)* When the weights are already known as prior information, they can be used directly; *(ii)* When the decision-makers' preferences are important for the decision problem in hand, their expertise can be consulted and the subjective weights may be calculated via different approaches such as Analytic Hierarchy Process (AHP) or Analytic Network Process (ANP), etc.; *(iii)* When the subjectivity is not desired with the purpose of eliminating manipulation risk that may be originated from the decision-makers or when there is not enough time for data collection, the objective weights can be computed from the current data by referring to the methods such as entropy-based approaches or maximizing standard deviation method; *(iv)* If required, a mixture of objective and subjective methods can be used.

Independent from the methodology used, the weighted normalized decision matrix is obtained via Eq. (11) where w_j represents the weight of attribute *j*. For this weighting process, we utilized the weighting formula proposed by Jovcic et al. [42].

$$\langle \mu_{ij}^{w}, \eta_{ij}^{w}, v_{ij}^{w} \rangle = w_j * \langle \mu_{ij}, \eta_{ij}, v_{ij} \rangle = \langle 1-(1-\mu_{ij})^{w_j}, \eta_{ij}^{w_j}, (\eta_{ij}+v_{ij})^{w_j} - (\eta_{ij})^{w_j} \rangle \tag{11}$$

Step 3. The basic distinctive feature of EDAS is the consideration of average scores rather than positive or negative ideals. In this step, the TDN average scores of each attribute will be obtained. For this purpose, all the weighted aggregated performance scores depicted in columns are averaged. Firstly, the addition operation is used iteratively as given in Eq. (12).

$$\langle \mu_{1j}^{w}, \eta_{1j}^{w}, v_{1j}^{w} \rangle + \langle \mu_{2j}^{w}, \eta_{2j}^{w}, v_{2j}^{w} \rangle = \langle 1-\left(1-\mu_{1j}^{w}\right)\left(1-\mu_{2j}^{w}\right), \eta_{1j}^{w}\eta_{2j}^{w}, \left(\eta_{1j}^{w}+v_{ij}^{w}\right)\left(\eta_{2j}^{w}+v_{2j}^{w}\right) - \eta_{1j}^{w}\eta_{2j}^{w} \rangle \tag{12}$$

The sum of the overall TDN numbers is represented by $\langle \mu_{ij}^{sum}, \eta_{ij}^{sum}, v_{ij}^{sum} \rangle$ for each attribute *j*. Then, multiplication by a scalar ($\lambda = 1/m > 0$) operation is used (Eq. 13). The mathematical operations are defined by Jovcic et al [42].

$$\begin{aligned} \widetilde{AV} &= \langle \mu_{j}^{AV}, \eta_{j}^{AV}, v_{j}^{AV} \rangle = \frac{1}{m} * \langle \mu_{ij}^{sum}, \eta_{ij}^{sum}, v_{ij}^{sum} \rangle \\ &= \langle 1-(1-\mu_{ij}^{sum})^{\frac{1}{m}}, (\eta_{ij}^{sum})^{\frac{1}{m}}, (\eta_{ij}^{sum}+v_{ij}^{sum})^{\frac{1}{m}} - (\eta_{ij}^{sum})^{\frac{1}{m}} \rangle \end{aligned} \tag{13}$$

Step 4. Rather than measuring the negative and positive distances from the average solution, this study proposes the usage of subsethood degrees. In this step, each alternative's subsethood degree to the average solution will be measured. For this purpose, one of the subsethood measures proposed in this study can be used alternately. They are rewritten with the appropriate notions in Eqs. (14-16). Suppose $\tilde{A}_i = \langle \mu_{ij}^{w}, \eta_{ij}^{w}, v_{ij}^{w} \rangle$ shows the TDN evaluation scores of alternative *i* and $\widetilde{AV} = \langle \mu_{j}^{AV}, \eta_{j}^{AV}, v_{j}^{AV} \rangle$ represents the average solution,

$$S_1\left(\tilde{A}_i, \widetilde{AV}\right) = 1 - \frac{1}{\sqrt{2}} d_1\left(\tilde{A}_i, \tilde{A}_i \cap \widetilde{AV}\right) \tag{14}$$

$$S_2\left(\tilde{A}_i, \widetilde{AV}\right) = 1 - d_2\left(\tilde{A}_i, \tilde{A}_i \cap \widetilde{AV}\right) \tag{15}$$

$$S_3\left(\tilde{A}_i, \widetilde{AV}\right) = 1 - \frac{1}{\sqrt{n}} d_3\left(\tilde{A}_i, \tilde{A}_i \cap \widetilde{AV}\right) \tag{16}$$

where

$$d_1\left(\tilde{A}_i, \tilde{A}_i \cap \widetilde{AV}\right) = \left[\frac{1}{n}\sum_{i=1}^{n}\left(\begin{array}{c} \left(\mu_{ij}^{w} - \min\left(\mu_{ij}^{w}, \mu_{j}^{AV}\right)\right)^2 \\ +\left(\eta_{ij}^{w} - \min\left(\eta_{ij}^{w}, \eta_{j}^{AV}\right)\right)^2 \\ +\left(v_{ij}^{w} - \max\left(v_{ij}^{w}, v_{j}^{AV}\right)\right)^2 \end{array}\right)\right]^{\frac{1}{2}} \tag{17}$$

$$d_2\left(\tilde{A}_i, \tilde{A}_i \cap \widetilde{AV}\right) = \frac{1}{n}\sum_{i=1}^{n}\left(\max\left\{\begin{array}{c} \left|\mu_{ij}^{w} - \min\left(\mu_{ij}^{w}, \mu_{j}^{AV}\right)\right|, \\ \left|\eta_{ij}^{w} - \min\left(\eta_{ij}^{w}, \eta_{j}^{AV}\right)\right|, \\ \left|v_{ij}^{w} - \max\left(v_{ij}^{w}, v_{j}^{AV}\right)\right| \end{array}\right\}\right) \tag{18}$$

$$d_3\left(\tilde{A}_i, \tilde{A}_i \cap \widetilde{AV}\right) = \left[\sum_{i=1}^{n}\left(\max\left\{\begin{array}{c} \left(\mu_{ij}^{w} - \min\left(\mu_{ij}^{w}, \mu_{j}^{AV}\right)\right)^2, \\ \left(\eta_{ij}^{w} - \min\left(\eta_{ij}^{w}, \eta_{j}^{AV}\right)\right)^2, \\ \left(v_{ij}^{w} - \max\left(v_{ij}^{w}, v_{j}^{AV}\right)\right)^2 \end{array}\right\}\right)\right]^{\frac{1}{2}} \tag{19}$$

Step 5. The decision-makers expect that the best alternative should have the lowest possibility of being a subset of the average solution since the average solution does not represent the ideal solution but a mean one. So, it is required that the subsethood measure between the best alternative and the average solution should be the lowest one. Thus, the alternatives are ranked in ascending order of their subsethood measures against average solution and it is decided that the alternative with the minimum subsethood measure is the best one.

5. A hypothetical application

In this study, we have aimed to develop a novel TDNS version of EDAS with the integration of subsethood degree instead of distances between alternatives and average solution. We have also tried to keep the computations totally dependent-neutrosophic (picture fuzzy) until the very end of the steps. The proposed SM-TDN-EDAS is here applied in a real case. This case is taken from Jovcic, et al. [42]. They used a TDNS version of ARAS (Additive Ratio Assessment) method to the freight distribution concept selection problem for a tire manufacturing company in the Czech Republic. They considered 5 experts' evaluations on 3 alternatives with respect to 23 sub-criteria under four main criteria. In order to show the applicability of our method proposition of SM-TDN-EDAS, we chose the environmental main criterion which includes 5 sub-criteria, namely air pollution, noise pollution, the effect on public health, energy consumption, and vehicle utilization. The alternatives are freight distribution by own transport fleet, freight distribution by the 3PL provider, and freight distribution by combining own transport fleet and 3PL services. They collected the data from experts and found the aggregated decision matrix of X^{agg} as given in Table 1. Here we have the aggregated decision matrix so that we did not apply TDNWA operator just for this case.

Table 1. Aggregated decision matrix (X^{agg})

	C_1			C_2			C_3			C_4			C_5		
A_1	0.2	0.4	0.2	0.4	0.2	0.2	0	0.6	0.2	0.8	0.2	0	0	0.2	0.8
A_2	0.4	0.4	0	0.2	0.4	0.2	0.2	0.4	0	0	0.2	0.8	0.8	0.2	0
A_3	0.4	0.4	0	0.2	0.6	0.2	0.2	0.4	0.2	0	0.4	0.6	0.4	0.4	0.2

The weights of attributes (w_j) are provided as 0.2593, 0.0963, 0.1333, 0.1407, 0.3704. There is no need for normalization since all the attributes have benefit features. The weighted matrix is found by operating Eq. (11) and is given in Table 2. For illustration purposes, the weighting of the first alternatives' scores concerning the first criterion is given as follows:

$$\langle \mu_{11}^{w}, \eta_{11}^{w}, \nu_{11}^{w} \rangle = 0.2593 * \langle 0.2,0.4,0.2 \rangle =$$
$$\langle 1-(1-0.2)^{0.2593}, 0.4^{0.2593}, (0.4+0.2)^{0.2593} - 0.4^{0.2593} \rangle = \langle 0.0562,0.7885,0.0874 \rangle$$

Referring to Eqs. (12-13), the average solution's performance scores with respect to each attribute are obtained. To illustrate, the average solution's performance score for attribute 1 is given:

- $\langle \mu_{11}^{w}, \eta_{11}^{w}, \nu_{11}^{w} \rangle + \langle \mu_{21}^{w}, \eta_{21}^{w}, \nu_{21}^{w} \rangle = \langle 0.0562,0.7885,0.0874 \rangle + \langle 0.1241,0.7885,0 \rangle =$ $\langle 1-(1-0.0562)(1-0.1241), 0.7885 * 0.7885, (0.7885+0.0874)(0.7885+0) - 0.7885 * 0.7885 \rangle = \langle 0.1733,0.6218,0.0689 \rangle$.
- $\langle 0.1733,0.6218,0.0689 \rangle + \langle \mu_{31}^{w}, \eta_{31}^{w}, \nu_{31}^{w} \rangle = \langle 0.1733,0.6218,0.0689 \rangle + \langle 0.1241,0.7885,0 \rangle = \langle 1-(1-0.1733)(1-0.1241), 0.6218 * 0.7885, (0.6218+0.0689)(0.7885+0) - 0.6218 * 0.7885 \rangle = \langle 0.2759,0.4903,0.0544 \rangle$.
- $\langle \mu_{1}^{AV}, \eta_{1}^{AV}, \nu_{1}^{AV} \rangle = \frac{1}{3} * \langle 0.2759,0.4903,0.0544 \rangle = \langle 1-(1-0.2759)^{\frac{1}{3}}, (0.4903)^{\frac{1}{3}}, (0.4903+0.0544)^{\frac{1}{3}} - (0.4903)^{\frac{1}{3}} \rangle = \langle 0.1020,0.7885,0.0281 \rangle$.

All the TDN values of the average solution are shown in the last row of Table 2. In the next phase, the subsethood measures of each alternative to the average solution are calculated. Eq. (14-16) defines three novel subsethood measures and we use all of them for comparison purposes. To illustrate, the first subsethood measure (Eq. 14) between $\tilde{A}_1$ and $\widetilde{AV}$ is:

$d_1(\tilde{A}_1, \tilde{A}_1 \cap \widetilde{AV}) = \Big[\frac{1}{5}((0.0562 - \min(0.0562, 0.1020))^2 + (0.7885 - \min(0.7885, 0.7885))^2 + (0.0874 - \max(0.0874, 0.0281))^2 + \cdots + (0 - \min(0, 0.2303))^2 + (0.5509 - \min(0.5509, 0.6002))^2 + (0.4491 - \max(0.4491, 0.1695))^2)\Big]^{\frac{1}{2}} = 0.0762.$

$S_1(\tilde{A}_i, \widetilde{AV}) = 1 - \frac{1}{\sqrt{2}} d_1(\tilde{A}_i, \tilde{A}_i \cap \widetilde{AV}) = 1 - \frac{1}{\sqrt{2}} * 0.0762 = 0.9461.$

Table 3 shows all the solutions including alternatives' distance values (please see Eqs.17-19) and subsethood measures for three different definitions (please see Eqs.14-16). In the last step, the alternatives are ranked in ascending order of subsethood measures: S_1, S_2, and S_3. For each measure, similar rankings of alternatives are obtained as seen from the columns of *Ranking* in Table 3. For S_1: $A_2 \succ A_1 \succ A_3$; for S_2 and S_3: $A_2 \succ A_3 \succ A_1$ which is the same ranking obtained by the original methodology. The results of other applications specified by Jovcic et al. [42] are summarized in Table 4 and it is seen that all these methods have given similar rankings. For each ranking, the most convenient alternative is found as A_2. The rankings of the other alternatives are slightly different in the various applications. For instance, in the original application, there are so many consecutive steps while our proposition of SM-TDN-EDAS includes just 5 steps. Our method's contribution to complexity reduction is obvious.

6. Conclusion and future work

Subsethood (inclusion) measures are very important components of fuzzy sets like entropy, distance, or similarity measures. In the literature, there are many subsethood measures developed for fuzzy sets, IFSs, and neutrosophic sets but there is no proposition for TDNSs. TDNS is generally accepted by the MADM field as one of the important fuzzy environments because it gives an extensive representation opportunity to the decision-maker. TDNS is defined by four elements, namely positive, negative, neutral, and refusal membership degrees and the first three elements can be independently assignable. The only rule is that the sum of these four elements should be equal to 1. In order to exploit this feature in the applications of MADM,

- for the first time in the literature, three subsethood measures were developed for TDNSs and it is proven that these definitions satisfy the required axiomatic properties;

Table 2. Weighted aggregated decision matrix ($w_j * X^{agg}$)

		C_1			C_2	
A_1	0.0562	0.7885	0.0874	0.0480	0.8564	0.0591
A_2	0.1241	0.7885	0.0000	0.0213	0.9155	0.0365
A_3	0.1241	0.7885	0.0000	0.0213	0.9520	0.0267
$\widetilde{AV}$	*0.1020*	*0.7885*	*0.0281*	*0.0303*	*0.9071*	*0.0413*
		C_3			C_4	
A_1	0.0000	0.9342	0.0365	0.2026	0.7974	0.0000
A_2	0.0293	0.8850	0.0000	0.0000	0.7974	0.2026
A_3	0.0293	0.8850	0.0492	0.0000	0.8790	0.1210
$\widetilde{AV}$	*0.0196*	*0.9011*	*0.0282*	*0.0727*	*0.8237*	*0.1036*
		C_5				
A_1	0.0000	0.5509	0.4491			
A_2	0.4491	0.5509	0.0000			

A_3	0.1724	0.7122	0.1154
$\widetilde{AV}$	*0.2303*	*0.6002*	*0.1695*

Table 3. Results

	d_1	S_1	Ranking	d_2	S_2	Ranking	d_3	S_3	Ranking	Ranking by Jovcic et al. [42]
A_1	0.0762	0.9461	2	0.0361	0.9639	3	0.1352	0.9395	3	3
A_2	0.1256	0.9112	1	0.0567	0.9433	1	0.2225	0.9005	1	1
A_3	0.0665	0.9529	3	0.0500	0.9500	2	0.1361	0.9392	2	2

Table 4. Comparison of different TDNS (PFS)-based MADM methods [42]

Method	Ranking of Alternatives
TDNS TOPSIS [43]	$A_2 \succ A_3 \succ A_1$
TDNS EDAS [16]	$A_2 \succ A_3 \succ A_1$
TDNS MABAC [44]	$A_2 \succ A_3 \succ A_1$
TDNS VIKOR [45]	$A_2 \succ A_3 \succ A_1$
TDNS Fuzzy TODIM [46]	$A_2 \succ A_1 \succ A_3$

EDAS, a well-known MADM approach is extended into TDNS in a different manner from the existing state-of-the-art propositions, i.e., the traditional and extended versions have focused on the distance between each alternative and average solution while the proposed version called SM-TDN-EDAS considered the subsethood degree of each alternative to the average solution as a decision criterion. So, the number of mathematical operations is significantly reduced in this new version;

To validate the novel SM-TDN-EDAS method, an application is conducted, and the resulting rankings are compared with different applications' rankings. It is found that the existing methods and current study give similar rankings. So, it is clear that the proposition is robust.

The study also needs some improvements. Rather than enforcing the decision-makers to allocate directly positive, neutral, and negative membership degrees, a further study may work on providing appropriate linguistic terms which have TDN number correspondences so that the data collection process is eased and becomes more practical. Also, novel aggregation operators, entropy measures, similarity, and distance measures as well as division and subtraction operators can be defined for the concept of TDNS.

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Appendix A

Table A1. Literature overview of MADM approaches under TDNS (PFS)

Paper	MADM Methods Used	Application
Zhang. et al. [16]	TDN(PF)-EDAS for selection	A numerical example for green supplier selection
Liang, et al. [17]	Extended SWARA for subjective criteria weights; Mean-squared deviation model for objective criteria weights; TDN(PF)-EDAS for establishing difference matrix; ELECTRE III for ranking orders; Extended MABAC, EDAS for comparison	Evaluating the cleaner production performance for gold mines in China
Li, et al. [18]	Maximizing deviation method for criteria weights; TDN(PF)-EDAS for evaluation	A numerical example of selecting an optimal emergency alternative
Ping, et al. [19]	TOPSIS & Maximum Entropy Theory for Expert Weighting; TDN(PF)-EDAS for evaluation; HF-VIKOR, cloud model GRA for comparison	A numerical example of characteristic prioritization in quality function deployment
Jovcic, et al. [42]	TDN(PF)-ARAS for selection; TDN(PF)-TOPSIS, TDN(PF)-EDAS, TDN(PF)-TODIM, TDN(PF)-VIKOR, TDN(PF)-MABAC, TDN(PF)-GRA for comparison	Freight distribution concept selection problem for a tire manufacturing company in the Czech Republic
Torun and Gördebil [43]	Fuzzy TOPSIS, IF-TOPSIS, and TDN(PF)-TOPSIS for comparison	Citizens' satisfaction level from public services in Turkey
Wang, et al. [44]	Modified maximizing deviation method for criteria weighting; prospect theory-based TDN(PF)-MABAC for evaluation; TDN(PF)-MABAC, TDN(PF)-VIKOR for comparison	Risk ranking of energy performance contracting project in Shanghai, China
Wang, et al. [45]	TDN(PF)-entropy-based objective weighting of attributes; TDN(PF)-normalized projection-based VIKOR for evaluation;	Risk evaluation of construction projects in China
Wei [46]	TDN(PF)-TODIM for evaluation	A numerical example of evaluation of emerging technology commercialization
Meksavang, et al. [47]	TDN(PF)-VIKOR for evaluation; fuzzy TOPSIS, IF-VIKOR, IF-GRA for comparison	A numerical example of sustainable supplier selection case in the beef supply chain
Si, et al. [48]	TDN(PF)-VIKOR & TDN(PF)-TOPSIS for evaluation	Ranking of tiger reserve national parks in India
Sindhu, et al. [49]	Linear programming for criteria weighting; TDN(PF)-TOPSIS for evaluation	A numerical example of human resource management

Zeng, et al. [50]	TDN(PF)-TOPSIS for evaluation	A numerical example of selecting Enterprise Resource Planning System
Arya and Kumar [51]	TDN(PF)-entropy-based TDN(PF)-VIKOR and TDN(PF)-TODIM for evaluation	Numerical examples based on election forecast through opinion polls
Joshi [52]	TDN(PF)-entropy-based TDN(PF)-VIKOR for evaluation; TOPSIS, VIKOR for comparison	Numerical examples based on election forecast through opinion polls
Joshi [53]	R-Norm information measure-based TDN(PF)-VIKOR for evaluation; TDN(PF)-TODIM for comparison	A numerical example of election; A numerical example of investment alternative evaluation
Lin, et al. [54]	TDN(PF)-entropy based criteria weighting; TDN(PF)-MULTIMOORA for evaluation; TDN(PF)-TODIM for comparison	Site selection of car-sharing station in Beijing, China
Tian, et al. [55]	Improved AHP for criteria weighting; TDN(PF)-PROMETHEE II for evaluation; TDN(PF)-VIKOR for comparison	Tourism environmental impact assessment in Hubei, China
Tian and Peng [56]	Improved ANP for criteria weighting; TDN(PF)-TODIM for evaluation	Personalized tourism attraction evaluation
Gül and Aydoğdu [57]	TDN(PF)-CODAS for evaluation; CODAS, spherical fuzzy CODAS, and spherical fuzzy TOPSIS for comparison	Selecting the best green supplier in Turkey
Simic, et al. [58]	CODAS, TOPSIS, EDAS, TODIM, VIKOR, MABAC, Cross-entropy, Projection, Grey relational projection, and Grey relational analysis under TDN(PF) environment	Locating a new vehicle shredding facility in the Republic of Serbia

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