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Reliability Measures in Neutrosophic Soft Graphs

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Abstract: This paper introduces the concept of reliable nodes in neutrosophic soft graphs by evaluating path-based parameters. A reliable node is defined as one which is least susceptible to changes that are quantized by the indeterminacy and falsity values in a neutrosophic tuple. A new path measure called farness and three novel reliability measures which make use of the same are presented. Farness is defined in terms of a novel score function. The first, proximity reliability of a node, computes the farness of a node to its neighbours. The second, intermediate reliability of a node, computes the fraction of paths of minimal farness that pass through it. The third, crisis reliability of a node, is a hybrid of the two previously defined. It considers the farness of a node to its neighbours taking into account the farness of the neighbours to other nodes in the graph.
Keywords: Neutrosophic soft graphs, Strong arcs, Score functions, Proximity reliability, Intermediate reliability, Crisis reliability.

1. Introduction

Graphs have been used to model and solve real-world problems in social and information systems [1]. In the field of computer science, graphs are used to represent networks of communication, data organization, computational devices, and the flow of computation to name a few. Graphs have also been used extensively to model scenarios for path-based applications. For example, shortest path problems used in route planning model the real world as a graph with the nodes representing destinations and the edges representing connections between destinations through some mode of transport. Some prevalent algorithms which make use of the graph model are Dijkstra's shortest path algorithm and the Floyd Warshall algorithm [2].

An edge connecting two nodes in a classical graph is binary in nature: It either exists or it doesn't. Therefore, stochastic optimization problems cannot be modelled using classical graphs. An extended version of the classical set is the fuzzy set where the elements have a value ranging from 0 to 1 indicating the degree of membership. Zadeh [3] introduced the degree of membership/truth (T) in 1965 and defined the fuzzy set. The concept of fuzziness in graph theory was described by Kaufmann [4] using the fuzzy relation. Rosenfeld [5] introduced some concepts such as bridges, cycles, paths, trees, the connectedness of fuzzy graphs and described some of the properties of the fuzzy graph. Samanta and Pal [6] and Rashmanlou and Pal [7] presented the concept of irregular and regular fuzzy graphs. Intuitionistic fuzzy sets (IFS) consider not only the membership grade (degree) but also independent membership grade and non-membership grade for any entity. The only requirement is that the sum of non-membership and membership degree values be no greater than one. The idea of the intuitionistic fuzzy set (IFS) as a modified version of the classical fuzzy set was introduced by Atanassov [8–10]. The idea of the IFS relation and intuitionistic fuzzy graphs (IFG) was presented by Shannon and Atanassov [11]. In real-world problems, uncertainties due to inconsistent and indeterminate information about a problem cannot be represented properly by the fuzzy graph or IFG. To overcome this situation, a new concept was introduced which is called the neutrosophic sets. Smarandache [12] introduced the degree of indeterminacy/neutrality (I) as an independent component in 1995 and defined the neutrosophic set on three components (T, I, F) = (Truth, Indeterminacy, Falsity). Neutrosophic soft graphs [13] based on the soft set theory [14] is a parameterized family of neutrosophic graphs. The class of all neutrosophic soft graphs is denoted by NS(G*).

The concept of centrality measures in graphs has been given a lot of attention as well. Nodal centrality measures are used to quantify the influence of a node with respect to other nodes within the network. Some of the more well-known centrality measures include the degree centrality [15] eigenvector centrality [16], closeness centrality [17], and betweenness centrality [18]. The utilization of centrality measures on networks to identify influential nodes can lead to a more comprehensive understanding of the dynamics and behaviour of real-world systems. Past applications of the four well-known centralities, along with various generalizations of the measures, on real-world networks include the Internet, transportation systems and social systems. One of the most recent works in this area is that of Heatmap centrality [19]. The heatmap centrality compares the distance of a node with the average sum of the distance of its adjacent nodes in order to identify influential nodes within the network. The readers can use the ideas in [24-25] to add more reliability measures. The readers can use the applications in [26-33] to extend the ideas presented.

The motivation behind this paper was to develop path-based measures for neutrosophic soft graphs to identify important nodes. The new measures developed would be a natural extension of centrality measures to the neutrosophic domain. A new path measure for neutrosophic soft graphs is presented in this work. The concept of reliability, an extension of centrality to neutrosophic soft graphs, is defined. Three reliability measures based on the newly introduced path measure are also elucidated.

The rest of this paper is as follows. Section 2 lists the preliminaries required for the study of reliability measures in neutrosophic soft graphs. Section 3 introduces the concept of reliability and proximity, intermediate and crisis reliabilities with examples. Section 4 illustrates a real-world application of the reliability measures. Section 5 concludes the paper.

2. Preliminaries

2.1. Definition [20]

A neutrosophic graph is defined as a pair $G = (V, E)$ where:

1. V = {$v1, v2, \ldots, vn$} such that $T = V \rightarrow [0,1]$, $I = V \rightarrow [0,1]$ and $F = V \rightarrow [0,1]$ denotes the degree of truth-membership function, indeterminacy function and falsity-membership function, respectively and
2. $0 \leq TA(x) + IA(x) + FA(x) \leq 3$

2.2 Definition [21]

Let U be an initial universe set and E a set of parameters or attributes with respect to U. Let P(U) denote the power set of U and A $\subseteq$ E. A pair (F, A) is called a soft set over U, where F is a mapping given by F: A $\rightarrow$P(U). In other words, a soft set (F, A) over U is a parameterized family of subsets of

U. For e∈A, F(e) may be considered as the set of e-elements or e-approximate elements of the soft sets (F, A).

2.3 Definition [22]
Let U be an initial universe and P be the set of all parameters. ϱ(U) denotes the set of all neutrosophic sets of U. Let A be a subset of P. A pair (J, A) is called a neutrosophic soft set over U. Let ϱ(V) denotes the set of all neutrosophic sets of V and ϱ(E) denotes the set of all neutrosophic sets of E.
2.4 Definition [23]
A neutrosophic soft graph G = (G∗, J, K, A) is an ordered four tuple, if it satisfies the following conditions:

(i) G∗ = (V, E) is a neutrosophic graph
(ii) A is a non-empty set of parameters
(iii) (J, A) is a neutrosophic soft set over V
(iv) (K, A) is a neutrosophic soft set over E,
(v) (J(e), K(e)) is a neutrosophic graph of G∗, then

$$T_{K(e)}(xy) \leq \{T_{J(e)}(x) \wedge T_{J(e)}(y)\},$$
$$I_{K(e)}(xy) \leq \{I_{J(e)}(x) \wedge I_{J(e)}(y)\},$$
$$F_{K(e)}(xy) \leq \{F_{J(e)}(x) \vee F_{J(e)}(y)\}, such\ that$$
$$0 \leq T_{K(e)}(xy) + I_{K(e)}(xy) + F_{K(e)}(xy) \leq 3\ for\ all\ e \in A\ and\ x, y \in V.$$

2.5 Definition [23]
Consider a neutrosophic graph G. Let (u, v) be any arc in G. An arc (u, v) is said to be strong arc, if $T_{K(e)}(u,v) \geq T^{\infty}_{K(e)}(u,v)$ and $I_{K(e)}(u,v) \geq I^{\infty}_{K(e)}(u,v)$and $F_{K(e)}(u,v) \geq F^{\infty}_{K(e)}(u,v)$.
2.6 Definition [23]
Consider a neutrosophic graph G. Let vi , vj be any two vertices in G and if they are connected means of a path then the strength of that path is defined as $(min_{i,j}T_{K(e)}(vi,vj), min_{i,j}I_{K(e)}(vi,vj), max_{i,j}F_{K(e)}(vi,vj))$ where $min_{i,j}T_{K(e)}(vi,vj)$ is the$T_{K(e)}$- strength of weakest arc and $min_{i,j}I_{K(e)}(vi,vj)$is the $I_{K(e)}$- strength of weakest arc and is the $max_{i,j}F_{K(e)}(vi,vj)$ is the $F_{K(e)}$- strength of strong arc.

3. Reliability Measures

3.1 *Definition (Strong-arced graph)*
Consider a Neutrosophic graph $G*$. The underlying strong-arced graph G' of $G*$ is defined as the spanning subgraph of $G*$ with only strong arcs as edges. A strong-arc graph needn't be connected even if $G*$ is connected.
3.2 *Definition (Score function of the strength of a path)*
Consider a neutrosophic soft graph H(e) corresponding to a parameter e. Consider a path in the graph from u to v. Let the strength of the path be represented as a tuple (T, I, F). Then, the score of the strength of the path is given by the function:

$$Str(u,v) = (1 + \mathrm{t} + \mathrm{i} - \mathrm{f})/2$$

3.3 Definition (farness)
Consider a neutrosophic graph G. The farness of a node v in G is defined as

$$\sum_{u!=v} Str(u,v)$$

3.4 Definition (Reliability)

Reliability can be considered as an extension of centrality to neutrosophic graphs. A reliable node is one that is least susceptible to changes that are quantized by the indeterminacy and falsity values in a neutrosophic tuple. Reliability talks about the robustness of the system, and its configuration to avert failure. It gives the designer of a system scope to focus on unreliable nodes. In the context of real-world applications, it is the node that remains intact/functional to a large extent.

3.5 Definition (Proximity Reliability)

Consider a strong-arced graph G`(e) of a neutrosophic soft graph H(e) corresponding to a parameter e. The proximity reliability for node v in G′(e) denoted as Pr(v), is defined as the reciprocal of farness, where farness is defined as the sum of the strength of the minimised strong arcs between node v_i and all other nodes in the network [5]. Generally, the proximity reliability is a measure of how fast data spreads from the node v_i, by taking into consideration that a node is close to all nodes in the network and not just to its neighbours. In other words, proximity reliability denotes the connectivity of the network.

$$\Pr(v) = \frac{1}{\sum_{u!=v} Str(u,v)}$$

Algorithm

1. Begin
2. For all edges in the graph do:
   a. Check if the edge is a strong arc
   b. If true, add the edge to the list of strong arcs
3. For all the strong arcs in the graph do:
   a. Obtain all the paths from one vertex of the edge to the other
   b. Obtain the aggregate tuple of (T, I, F) values using definition 2.6
   c. Assess the strength of the path by applying the formula Str(.) to these aggregate tuples
   d. Retain the maximum strength reliable paths
4. For each vertex in the strong-arced graph calculate the following:
   i. $\Pr(v)$
   ii. Number of unreachable vertices
5. Sort the resultant tuples $t = (\Pr(v), number\ of\ unreachable\ vertices)$ according to min(t[1]) and min(t[0]) (for tuples with the same t[1] values).
6. End.

3.6 Examples for Proximity Reliability

Consider two parameters describing the universe U: e1, e2.

Consider the graph H(e1) shown in Figure 1.

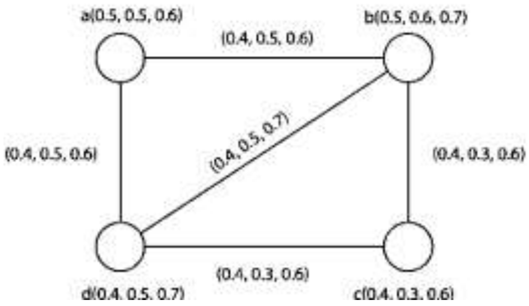

Figure 1

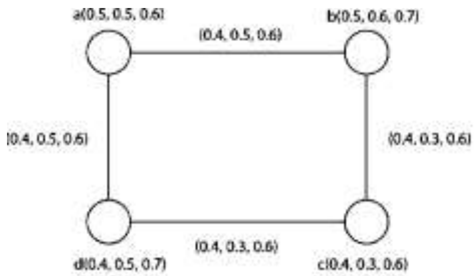

Figure 2

1. Obtain underlying strong-arced graph.

AB: (0.4,0.5,0.6) A-D-C-B: (0.4,0.3,0.6) A-D-B: (0.4,0.5,0.7) By property, AB is a strong arc.	AD: (0.4,0.5,0.6) A-B-C-D: (0.4,0.3,0.6) A-B-D: (0.4,0.5,0.7) By property, AD is a strong arc.	BC: (0.4,0.3,0.6) B-A-D-C: (0.4,0.3,0.6) B-D-C: (0.4,0.3,0.7) By property, BC is a strong arc.	BD: (0.4,0.5,0.7) B-A-D: (0.4,0.5,0.6) B-C-D: (0.4,0.3,0.6) By property, BD is not a strong arc.	CD: (0.4,0.3,0.6) C-B-A-D: (0.4,0.3,0.6) C-B-D: (0.4,0.3,0.7) By property, CD is a strong arc.

Table 1

2. Obtain all the shortest paths.

Reliable path from A to B: Paths: A-B and A-D-B-C (0.4,0.5,0.6) and (0.4,0.3,0.6) Applying the formula: (1+T+I-F)/2 we have, (1+0.4+0.5-0.6)/2 = 0.65 (1+0.4+0.3-0.6)/2 = 0.55 Reliable path from A to B: A-B	Reliable path from A to C: Paths: A-B-C and A-D-C (0.4,0.3,0.6) and (0.4,0.3,0.6) Applying the formula: (1+T+I-F)/2 we have, (1+0.4+0.3-0.6)/2 = 0.55 (1+0.4+0.3-0.6)/2 = 0.55 Reliable path from A to C: A-B-C and A-D-C	Reliable path from A to D: Paths: A-D and A-B-C-D (0.4,0.5,0.6) and (0.4,0.3,0.6) Applying the formula: (1+T+I-F)/2 we have, (1+0.4+0.5-0.6)/2 = 0.65 (1+0.4+0.3-0.6)/2 = 0.55 Reliable path from A to D: A-D
Reliable path from B to C: Paths: B-C and B-A-D-C (0.4,0.3,0.6) and (0.4,0.3,0.6) Applying the formula: (1+T+I-F)/2 we have, (1+0.4+0.3-0.6)/2 = 0.55 (1+0.4+0.3-0.6)/2 = 0.55 Reliable path from B to C: B-C and B-A-D-C	Reliable path from B to D: Paths: B-A-D and B-C-D (0.4,0.5,0.6) and (0.4,0.3,0.6) Applying the formula: (1+T+I-F)/2 we have, (1+0.4+0.5-0.6)/2 = 0.65 (1+0.4+0.3-0.6)/2 = 0.55 Reliable path from B to D: B-A-D	Reliable path from C to D: Paths: C-D and C-B-A-D (0.4,0.3,0.6) and (0.4,0.3,0.6) Applying the formula: (1+T+I-F)/2 we have, (1+0.4+0.3-0.6)/2 = 0.55 (1+0.4+0.3-0.6)/2 = 0.55 Reliable path from C to D: C-D and C-B-A-D

Table 2

3. **Compute the proximity reliability.**

VERTEX	Pr (VERTEX)	Number of unreachable nodes
A	$Pr(A) = \frac{1}{(0.55+0.65+0.65)} = 0.540$	0
B	$Pr(B) = \frac{1}{(0.55+0.65+0.65)} = 0.540$	0
C	$Pr(C) = \frac{1}{(0.55+0.55+0.55)} = 0.606$	0
D	$Pr(D) = \frac{1}{(0.55+0.65+0.65)} = 0.540$	0

Table 3

Hence in H(e1) C is reliable as per the proximity reliability criterion.

Now consider H(e2),

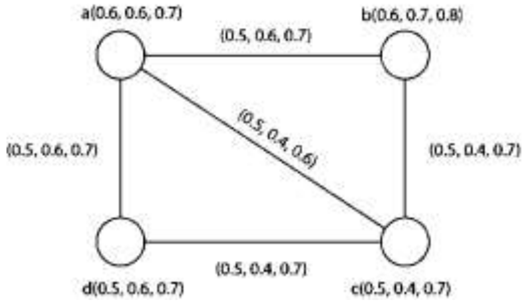

Figure 3

1. **Obtain underlying strong-arced graph.** It is the same as figure 3.
2. **Obtain all Reliable paths.**

Reliable path from A to B: A-B

Reliable path from A to C: A-B-C and A-D-C

Reliable path from A to D: A-D

Reliable path from B to C: B-A-C, B-A-D-C and B-C

Reliable path from B to D: B-A-D

Reliable path from C to D: C-D, C-B-A-D and C-A-D

3. **Compute the proximity reliability.**

VERTEX	Pr (VERTEX)	Number of unreachable nodes
A	$Pr(A) = \frac{1}{(0.7+0.6+0.7)} = 0.50$	0
B	$Pr(B) = \frac{1}{(0.7+0.6+0.7)} = 0.50$	0
C	$Pr(C) = \frac{1}{(0.6+0.6+0.6)} = 0.555$	0
D	$Pr(D) = \frac{1}{(0.7+0.65+0.7)} = 0.4878$	0

Table 4

Hence in H(e2) D is reliable as per the proximity reliability criterion.

3.7 Definition (Intermediate Reliability)

Consider a strong-arced graph G`(e) of a neutrosophic soft graph H(e) corresponding to a parameter e. The intermediate reliability of a vertex v in G`(e) is defined as the fraction of all the Reliable paths between any two vertices that passes through v. Mathematically it is defined as follows:

$$Int(v) = \frac{Number\ of\ shortest\ paths\ passing\ through\ v}{Total\ number\ of\ shortest\ paths\ between\ any\ two\ vertices}$$

Inferences:

1. The higher the value of $Int(v)$, the more reliable is the node.
2. $0 \leq Int(v) \leq 1$

Algorithm:

1. Begin
2. For all edges in the graph do:
   a. Check if the edge is a strong arc
   b. If true, add the edge to the list of strong arcs
3. For all the strong arcs in the graph do:
   a. Obtain all the paths from one vertex of the edge to the other
   b. Obtain the aggregate tuple of (T, I, F) values using definition 2.6
   c. Assess the strength of the path by applying the formula Str(.) to these aggregate tuples
   d. Retain the maximum strength reliable paths
4. For each vertex in the strong-arced graph calculate Int(v).
5. End.

Note: Intermediate reliability doesn't maintain a count of the number of unreachable nodes. The reason is that Intermediate reliability is a relative measure, it is the fraction of Reliable paths passing through a particular vertex *relative* to the number of paths present.

3.8 Definition **Sufficient criteria for G` = G***

Consider a neutrosophic graph G* based on parameter e. The underlying strong-arced graph G` is the same as G* if the following criteria are met:

1. G` has the same number of nodes as G*.
2. G` has the same number of edges as G*.
3. Every edge in G* connecting nodes u and v is constructed as:

   (Min(T(u), T(v)), Min(I(u), I(v)), Max(F(u), F(v))**)**

Proof: The proof here is direct as every edge constructed using condition 3 will be a strong arc. Every path in consideration will reduce to **(**Min(T(u), T(v)), Min(I(u), I(v)), Max(F(u), F(v))**)** of all the edges along the path, which is the basis for constructing an edge in the first place.

3.9 Examples for Intermediate Reliability:

Consider the same universe as described in Example 3.6.

Compute intermediate reliability for H(e1).

Total number of Reliable paths: 9

$Int(A) = 3/9$, $Int(B) = 2/9$, $Int(C) = 0/9$, $Int(D) = 2/9$

Hence in H(e1), A is the intermediate reliable node.

Compute intermediate reliability for H(e2).

Total number of Reliable paths: 11

$Int(A) = 5/11$, $Int(B) = 2/11$, $Int(C) = 0/11$, $Int(D) = 2/11$

Hence in H(e2), A is the intermediate reliable node.

3.10 Definition (Crisis Reliability)

Consider a strong-arced graph $G'(e)$ of a neutrosophic soft graph $H(e)$ corresponding to a parameter e. The crisis reliability of a vertex v in $G'(e)$ is defined as the difference between the farness of v and the average farness of the neighbors of v. When the strong-arced graph contains unreachable vertices resulting in un-connectedness, then, we take the maximum number of unreachable neighbors when computing crisis reliability. The most reliable node is the one with minimum farness and the minimum number of unreachable neighbors.

Algorithm:

Crisis Reliability:

1. Begin
2. For each parameter produced in the neutrosophic soft graph
    a. For all edges in the graph
        i. Find all paths between the vertices of the selected edge and reduce the path to a neutrosophic tuple
        ii. If the current edge has larger or equal T, I value and smaller of equal F value than all the known path reduces, then the current edge is <u>strong</u>
    b. For each vertex in the graph
        i. For every other vertex
            1. find all the paths
            2. compute the path cost for each path
            3. obtain the max of all computed path costs to be the cost of reaching that vertex
            4. note the number of unreachable neighbors
        ii. Farness = Sum all the path costs for reaching every other vertex
    c. Reliability score
        i. For each vertex
            1. compute the <u>neighbor farness</u> by computing the average of farness of neighbors
            2. Score is computed as the difference between the farness of the current vertex and the neighbor farness
            3. note the maximum number of unreachable neighbors

3. For each vertex in the graph, find the minimum reliability score and the maximum number of unreachable neighbors as an aggregate across all the parameters produced neutrosophic graph generated and tabulate the result.
4. Sort the tabulated result by the number of disconnected vertices. Within the same number of disconnected nodes, sort by lower heatmap value.
5. Vertex at the top of the tabulated result is the most reliable during a crisis.
6. End

3.11 Examples for Crisis Reliability

Consider two parameters describing the universe U: e1, e2.

Consider the graph H(e1):

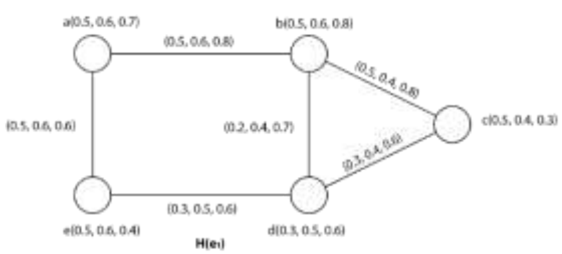

Figure 4

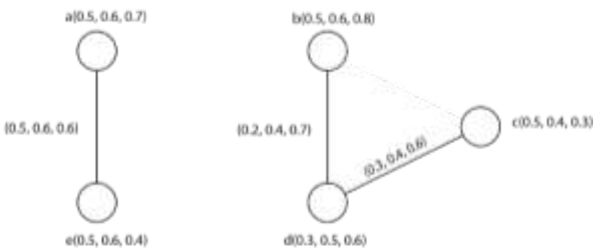

Figure 5

1. Find strong arcs.

Test if ab(0.5, 0.6, 0.8) is strong a-e-d-b: (0.2, 0.4,0 .7) a-e-d-c-b: (0.3, 0.4, 0.8) ab is not a strong arc.	Test if bc(0.5, 0.4, 0.8) is strong b-d-c: (0.2, 0.4, 0.7) b-a-e-d-c: (0.3, 0.4, 0.8) bc is not a strong arc.	Test if cd(0.3, 0.4, 0.6) is strong c-b-d: (0.2, 0.4, 0.8) c-b-a-e-d: (0.3, 0.4, 0.8) cd is a strong arc.
Test if de(0.3, 0.5, 0.6) is strong d-b-a-e: (0.2, 0.4, 0.8) d-c-b-a-e: (0.3, 0.4, 0.8) de is a strong arc.	Test if ea(0.5, 0.6, 0.6) is strong e-d-b-a: (0.2, 0.4, 0.8) e-d-c-b-a: (0.3, 0.4, 0.8) ea is a strong arc.	Test if bd(0.2, 0.4, 0.7) is strong b-a-e-d: (0.3, 0.5, 0.8) b-c-d: (0.3, 0.4, 0.8) bd is not a strong arc.

Table 5

The underlying strong-arced graph is shown in Figure 5.

2. Obtain farness measures of each vertex.

<u>Vertex a:</u> To b: Unreachable To c: Unreachable To d: Unreachable To e: a-e (0.5, 0.6, 0.6) Farness = (1+0.5+0.6-0.6)/2 = 0.75 Farness(a) = 0.75 Unreachable neighbors(a) = 3	<u>Vertex b:</u> To a: Unreachable To c: b-d-c (0.2, 0.4, 0.7) Farness = (1+0.2+0.4-0.7)/2 = 0.45 To d: b-d (0.2, 0.4, 0.7) Farness = (1+0.2+0.4-0.7)/2 = 0.45 To e: Unreachable Farness(b) = 0.9 Unreachable neighbors(b) = 2	<u>Vertex c:</u> To a: Unreachable To b: c-d-b (0.2, 0.4, 0.7) Farness = (1+0.2+0.4-0.7)/2 = 0.45 To d: c-d (0.3, 0.4, 0.6) Farness = (1+0.3+0.4-0.6)/2 = 0.55 To e: Unreachable Farness(c) = 1.0 Unreachable neighbors(c) = 2
<u>Vertex d:</u> To a: Unreachable	<u>Vertex e:</u> To a: e-a (0.5, 0.6, 0.6) Farness = (1+0.5+0.6-0.6)/2 = 0.75	

To b: d-b (0.2, 0.4, 0.7) Farness = (1+0.2+0.4-0.7)/2 = 0.45 To c: d-c (0.3, 0.4, 0.6) Farness = (1+0.2+0.4-0.7)/2 = 0.55 To e: Unreachable Farness(d) = 1.0 Unreachable neighbors(d) = 2	To b: Unreachable To c: Unreachable To d: Unreachable Farness(e) = 0.75 Unreachable neighbors(e) = 3	

Table 6

3. Compute crisis reliability.

Farness(a) = 0.75 Unreachable neighbors(a) = 3 Neighbor Farness(a) = Farness(e) = 0.75 Unreachable neighbors from neighbors(a) = Unreachable neighbors(e) = 3 **Aggregate unreachable neighbors = max (3, 3) = 3** **Score = Farness(a) – Neighbor Farness(a) = 0.75 – 0.75 = 0**	Farness(b) = 0.9 Unreachable neighbors(b) = 2 Neighbor Farness(b) = Farness(d) = 1.0 Unreachable neighbors from neighbors(b) = Unreachable neighbors(d) = 2 **Aggregate unreachable neighbors = max (2, 2) = 2** **Score = Farness(b) – Neighbor Farness(b) = 0.9 – 1.0 = -0.1**	Farness(c) = 1.0 Unreachable neighbors(c) = 2 Neighbor Farness(c) = Farness(d) = 1.0 Unreachable neighbors from neighbors(c) = Unreachable neighbors(d) = 2 **Aggregate unreachable neighbors = max (2, 2) = 2** **Score = Farness(c) – Neighbor Farness(c) = 1.0 – 1.0 = 0**
Farness(d) = 1 Unreachable neighbors(d) = 2 Neighbor Farness(d) = Mean (Farness(b), Farness(c)) = (0.9+1.0)/2 = 0.95 Unreachable neighbors from neighbors(d) = max (Unreachable neighbors(b), Unreachable neighbors(c)) = max(2, 2) = 2 **Aggregate unreachable neighbors = max (2, 2) = 2** **Score = Farness(d) – Neighbor Farness(d) = 1-0.95 = 0.05**	Farness(e) = 0.75 Unreachable neighbors(e) = 3 Neighbor Farness(e) = Farness(a) = 0.75 Unreachable neighbors from neighbors(e) = 3 **Aggregate unreachable neighbors = max (3, 3) = 3** **Score = Farness(e) – Neighbor Farness(e) = 0.75 – 0.75 = 0**	

Table 7

Now consider the graph H(e2):

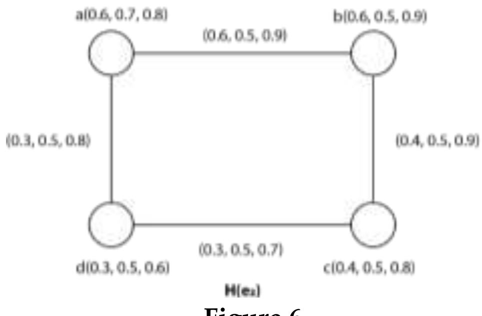

Figure 6

1. **Find strong arcs.** The strong-arc graph is the same as Figure 6.

2. Obtain farness measures of each vertex.

Vertex a:	Vertex b:	Vertex c:	Vertex d:
To b: a-b = **0.6**, To c: a-b-c = **0.5** a-d-c = **0.5**, To d: a-d = **0.5** Farness(a) = 0.6 + 0.5 + 0.5 = 1.6; Unreachable neighbors(a) = 0	To a: b-a = **0.6,** To c: b-c = **0.5**, To d: b-c-d = **0.45** b-a-d = **0.45** Farness(b) = 0.6 + 0.5 + 0.45 = 1.55; Unreachable neighbors(b) = 0	To c: c-b-a = **0.5** c-d-a = **0.5**, To b: c-b = **0.5**, To d: c-d = **0.55** Farness(c) = 0.5 + 0.5 + 0.55 = 1.55; Unreachable neighbors(c) = 0	To a: d-a = **0.5**, To b: d-c-b = **0.45,** d-a-b = **0.45**, To c: d-c = **0.55** Farness(d) = 0.5 + 0.45 + 0.55 = 1.5; Unreachable neighbors(d) = 0

Table 8

3. Compute crisis reliability.

	H(e1)		H(e2)		Aggregate	
	Score	Unreachable	Score	Unreachable	Score	Unreachable
A	0	3	0.075	0	0	3
B	-0.1	2	-0.025	0	-0.1	2
C	0	2	0.025	0	0	2
D	0.05	2	-0.075	0	-0.075	2
E	0	3	-	-	0	3

Table 9

4. Applications

We consider a neutrosophic set of five countries: Germany, China, USA, Brazil and Mexico. Suppose we want to travel between these countries through an airline journey. The airline companies aim to facilitate their passengers with high quality of services.

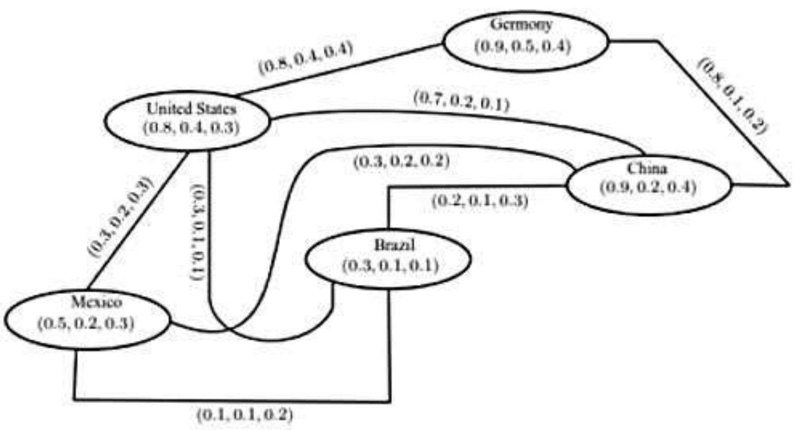

Figure 7

The reliability measures presented in this paper can be applied to the above graph to obtain the central nodes:

Proximity Reliability:

On applying proximity reliability, we obtain the following statistic:

$Pr(United\ States) =$ 0.328, Number of unreachable nodes: 0

$Pr(Mexico) =$ 0.4, Number of unreachable nodes: 0

$Pr(Germany) =$ 0.322, Number of unreachable nodes: 0

$Pr(Brazil) =$ 0.408, Number of unreachable nodes: 0

$Pr(China) =$ 0.328, Number of unreachable nodes: 0

Conclusion: 'China' is the node that is the most reliably connected to all other nodes, and is connected with every other node as well. It can thus be used as a fail-safe airport in case an airplane needs to make an emergency landing.

Intermediate Reliability:

On applying intermediate reliability, we obtain the following statistic:

Total number of Reliable paths: 10

$Int(United\ States) = 3/10$, $Int(Mexico) = 0/10$, $Int(Germany) = 0/10$, $Int(Brazil) = 0/10$

$Int(China) = 5/10$.

Conclusion: The node 'China' lies on 50% (half) of the reliable paths between the nodes. It can thus be used as a connecting terminal for long-distance flights.

Crisis Reliability:

United States: (0.175,0), Mexico: (-0.45,0), Germany: (-0.1,0), Brazil (-0.4,0), United States: (0.2,0).

Conclusion: The node 'Mexico' can reach other destinations more reliably than other nodes. Airplane companies can therefore make a strategic decision to dock planes in Mexico or to start journeys from Mexico for flights that go to multiple destinations.

5. Conclusions

In this paper, the concept of strong-arced graphs and reliability measures were introduced. Three pertinent reliability measures, namely, proximity, intermediate and crisis reliability were discussed. The first, proximity reliability of a node, computes the farness of a node to its neighbors. The second, intermediate reliability of a node, computes the fraction of paths of minimal farness that pass through it. The third, crisis reliability of a node, is a hybrid of the two previously defined. It considers the farness of a node to its neighbors taking into account the farness of the neighbors to other nodes in the graph. These reliability measures were applied to a real-world airplane application to determine the important nodes.

A summary of the new notations presented in this paper is shown below:

Notation	**Description**
$Str(u,v)$	Then, the score of the strength of the path from node u to node v.
$\mathrm{Pr}(v)$	Proximity reliability of node v.
$Int(v)$	Intermediate reliability of node v.
$Cr(v)$	Crisis reliability of node v.

6. Future Scope

All three measures make use of the same score function. A score function tailored to each measure can be developed in the future. The algorithms used in this paper find all possible paths between pairs of nodes and then only eliminate the paths which are not required. There is great scope for improvement in this area. One could try to develop a heuristics-based algorithm to improve the efficiency of finding reliable nodes and paths by eliminating certain paths. A few fields that can benefit from this research work are: supply chain management, logistics, network management and warfare planning.

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