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The neutrosophic differentials calculus

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Abstract: the purpose of this article is to study the neutrosophic differentials and rules of the neutrosophic derivative, Where the neutrosophic differentiable is defined, and properties of neutrosophic differentiation are introduced, where we discussed how to find the derivatives of addition, subtraction, multiplication, and division of two neutrosophic functions. Also, derivative of composite neutrosophic functions is studied by method of chain rule, in addition to studying derivatives of inverse neutrosophic trigonometric functions, differentiation of implicit neutrosophic functions, logarithmic neutrosophic differentiation, higher order neutrosophic derivatives, and differentiation of parametric neutrosophic functions. Where detailed examples were given to clarify each case.

Keywords: the neutrosophic differentials; neutrosophic functions; indeterminacy; derivative neutrosophic functions.

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1. Introduction

As an alternative to the existing logics, Smarandache proposed the Neutrosophic Logic to represent a mathematical model of uncertainty, vagueness, ambiguity, imprecision, undefined, unknown, incompleteness, inconsistency, redundancy, contradiction, where the concept of neutrosophy is a new branch of philosophy introduced by Smarandache [3-13]. He presented the definition of the standard form of neutrosophic real number and conditions for the division of two neutrosophic real numbers to exist, he defined the standard form of neutrosophic complex number, and found root index $n \geq 2$ of a neutrosophic real and complex number [2-4], studying the concept of the Neutrosophic probability [3-5], the Neutrosophic statistics [4][6], and professor Smarandache entered the concept of preliminary calculus of the differential and integral calculus, where he introduced for the first time the notions of neutrosophic mereo-limit, mereo-continuity, mereoderivative, and mereo-integral [1-8]. Madeleine Al- Taha presented results on single valued neutrosophic (weak) polygroups [9]. Edalatpanah proposed a new direct algorithm to solve the neutrosophic linear programming where the variables and right-hand side represented with triangular neutrosophic numbers [10]. Chakraborty used pentagonal neutrosophic number in networking problem, and Shortest Path Problem [11-12]. Y.Alhasan studied the concepts of neutrosophic complex numbers and the general exponential form of a neutrosophic complex [7-

14]. On the other hand, M.Abdel-Basset presented study in the science of neutrosophic about an approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number [15]. Also the neutrosophic integrals are introduced by Y.Alhasan [16-17].

Paper consists of 5 sections. In 1th section, provides an introduction, in which neutrosophic science review has given. In 2th section, some definitions and examples of neutrosophic real number neutrosophic. The 3th section frames the neutrosophic differentiable and rules of the neutrosophic derivative, properties of neutrosophic differentiation are introduced, where we discussed how to find the derivatives of addition, subtraction, multiplication, and division of two neutrosophic functions, derivative of composite neutrosophic functions is studied by method of chain rule. The 4th section introduces the derivatives of inverse neutrosophic trigonometric functions, differentiation of implicit neutrosophic functions, logarithmic neutrosophic differentiation, higher order neutrosophic derivatives, and differentiation of parametric neutrosophic functions. In 5th section, a conclusion to the paper is given.

2. Preliminaries

2.1. Neutrosophic Real Number [4]

Suppose that w is a neutrosophic number, then it takes the following standard form: $w = a + bI$ where a , b are real coefficients, and I represent indeterminacy, such $0.\mathrm{I} = 0$ and $\mathrm{I}^n = \mathrm{I}$, for all positive integers n.

2.2. Division of neutrosophic real numbers [4]

Suppose that w_1, w_2 are two neutrosophic numbers, where

$$w_1 = a_1 + b_1 I \ , \qquad w_2 = a_2 + b_2 I$$

To find $(a_1 + b_1 I) \div (a_2 + b_2 I)$, we can write:

$$\frac{a_1 + b_1 I}{a_2 + b_2 I} \equiv x + yI$$

where x and y are real unknowns.

$$a_1 + b_1 I \equiv (a_2 + b_2 I)(x + yI)$$

$$a_1 + b_1 I \equiv a_2 x + (b_2 x + a_2 y + b_2 y)I$$

by identifying the coefficients, we get

$$a_1 = a_2 x$$

$$b_1 = b_2 x + (a_2 + b_{2)} y$$

We obtain unique one solution only, provided that:

$$\begin{vmatrix} a_2 & 0 \\ b_2 & a_2 + b_2 \end{vmatrix} \neq 0 \quad \Rightarrow \quad a_2(a_2 + b_2) \neq 0$$

Hence: $a_2 \neq 0 \ \ and \ a_2 \neq -b_2$ are the conditions for the division of two neutrosophic real numbers to exist.

Then:

$$\frac{a_1 + b_1 I}{a_2 + b_2 I} = \frac{a_1}{a_2} + \frac{a_2 b_1 - a_1 b_2}{a_2(a_2 + b_2)} . I$$

3. The neutrosophic differentials

Definition3.1
Let $f: D_f \subseteq R \to R_f \cup \{I\}$, if:

$$\lim_{h+h_0 I \to 0+0I} \frac{f(x+h+h_0 I) - f(x,I)}{h+h_0 I}$$

exist, then we say that the function $f(x,I)$ is differentiable with respect to x and it is given by the formula:

$$\acute{f}(x,I) = \lim_{h+h_0 I \to 0+0I} \frac{f(x+h+h_0 I) - f(x,I)}{h+h_0 I}$$

Where $h + h_0 I$ is amount of indetermined small change in x, and h, h_0 are real numbers, while $I =$ indeterminacy.

Note:

1) The tangent slop to $f(x,I)$ at $x_0 = a + bI$ is $m_I = \acute{f}(a+bI)$.
2) The equation of the tangent to $f(x,I)$ at $x_0 = a + bI$ is:

$$y - f(a+bI) = \acute{f}(a+bI)(x - a - bI)$$

where a, b are real numbers, while $I =$ indeterminacy.

Example3.1
Differentiate $f(x,I) = Ix^2$ with respect to x using definition, and find an equation of the tangent line to the curve at $x_0 = 3 + 3I$.

Solution:

$$\acute{f}(x,I) = \lim_{h+h_0 I \to 0+0I} \frac{f(x+h+h_0 I) - f(x,I)}{h+h_0 I}$$

$$\acute{f}(x,I) = \lim_{h+h_0 I \to 0+0I} \frac{I(x+h+h_0 I)^2 - Ix^2}{h+h_0 I}$$

$$= \lim_{h+h_0 I \to 0+0I} \frac{I(x^2 + 2(h+h_0 I)x + (h+h_0 I)^2) - Ix^2}{h+h_0 I}$$

$$= \lim_{h+h_0 I \to 0+0I} \frac{Ix^2 + 2(h+h_0 I)xI + (h+h_0 I)^2 I - Ix^2}{h+h_0 I}$$

$$= \lim_{h+h_0 I \to 0+0I} \frac{2(h+h_0 I)xI + (h+h_0 I)^2 I}{h+h_0 I}$$

$$= \lim_{h+h_0 I \to 0+0I} \frac{(h+h_0 I)[2xI + (h+h_0 I)I]}{h+h_0 I}$$

$$= \lim_{h+h_0 I \to 0+0I} [2xI + (h+h_0 I)I]$$

$$\Longrightarrow \qquad \acute{f}(x,I) = 2xI$$

Finding the tangent equation:

$$m_I = \acute{f}(3+3I) = 2I(3+3I) = 12I$$

$$f(3+3I) = I(3+3I)^2 = 24I$$

Then:

$$y - f(a+bI) = \acute{f}(a+bI)(x-a-bI)$$

$$y - 24I = 12I\,(x-3-3I)$$

$$y = 12I\,x - 72I + 24I$$

$$y = 12I\,x - 48I$$

Example3.2
Differentiate $f(x,I) = sin(5x+3I)$ with respect to x using definition.
Solution:

$$\acute{f}(x,I) = \lim_{h+h_0I \to 0+0I} \frac{f(x+h+h_0I) - f(x,I)}{h+h_0I}$$

$$\acute{f}(x,I) = \lim_{h+h_0I \to 0+0I} \frac{sin(5(x+h+h_0I)+3I) - sin(5x+3I)}{h+h_0I}$$
$$= \lim_{h+h_0I \to 0+0I} \frac{sin\big(5x+3I+5(h+h_0I)\big) - sin(5x+3I)}{h+h_0I}$$

$$= \lim_{h+h_0I \to 0+0I} \frac{cos\left(5x+3I+\frac{5}{2}(h+h_0I)\right) sin(\frac{5}{2}(h+h_0I))}{\frac{h+h_0I}{2}}$$

$$= \lim_{h+h_0I \to 0+0I} cos\left(5x+3I+\frac{5}{2}(h+h_0I)\right) \lim_{h+h_0I \to 0+0I} \frac{sin(\frac{5}{2}(h+h_0I))}{\frac{h+h_0I}{2}}$$

$$= \lim_{h+h_0I \to 0+0I} cos\left(5x+3I+\frac{5}{2}(h+h_0I)\right) \lim_{h+h_0I \to 0+0I} \frac{5\,sin(\frac{5}{2}(h+h_0I))}{\frac{5(h+h_0I)}{2}}$$

$$\Rightarrow \qquad \acute{f}(x,I) = 5\,cos(5x+3I)$$

Example3.3
Differentiate $f(x,I) = \sqrt{3Ix+4I}$ with respect to x using definition.
Solution:

$$\acute{f}(x,I) = \lim_{h+h_0I \to 0+0I} \frac{f(x+h+h_0I) - f(x,I)}{h+h_0I}$$

$$\acute{f}(x,I) = \lim_{h+h_0I \to 0+0I} \frac{\sqrt{3I(x+h+h_0I)+4I} - \sqrt{3Ix+4I}}{h+h_0I}$$

$$= \lim_{h+h_0I\to 0+0I} \frac{\sqrt{3I(x+h+h_0I)+4I}-\sqrt{3Ix+4I}}{h+h_0I} \frac{\sqrt{3I(x+h+h_0I)+4I}+\sqrt{3Ix+4I}}{\sqrt{3I(x+h+h_0I)+4I}+\sqrt{3Ix+4I}}$$

$$= \lim_{h+h_0I\to 0+0I} \frac{3I(x+h+h_0I)+4I-3Ix-4I}{(h+h_0I)\left(\sqrt{3I(x+h+h_0I)+4I}+\sqrt{3Ix+4I}\right)}$$

$$= \lim_{h+h_0I\to 0+0I} \frac{3Ix+3I(h+h_0I)+3Ix}{(h+h_0I)\left(\sqrt{3I(x+h+h_0I)+4I}+\sqrt{3Ix+4I}\right)}$$

$$= \lim_{h+h_0I\to 0+0I} \frac{3I(h+h_0I)}{(h+h_0I)\left(\sqrt{3I(x+h+h_0I)+4I}+\sqrt{3Ix+4I}\right)}$$

$$= \lim_{h+h_0I\to 0+0I} \frac{3I}{\left(\sqrt{3I(x+h+h_0I)+4I}+\sqrt{3Ix+4I}\right)}$$

$$\Rightarrow \qquad \acute{f}(x,I) = \frac{3I}{2\sqrt{3Ix+4I}}$$

3.1 The rules of the neutrosophic derivative

We can prove each of the following, using the Definition3.1:

1) $\frac{d}{dx}(c+dI) = 0+0I$; where c, d are real numbers, while $I =$ indeterminacy.

2) $\frac{d}{dx}[(a+bI)x+c+dI] = a+bI$; where c, d are real numbers, while $I =$ indeterminacy.

3) $\frac{d}{dx}[(a+bI)x^n] = n(a+bI)x^{n-1}; n\ is$ real number.

4) $\frac{d}{dx}\left[e^{(a+bI)x+c+dI}\right] = (a+bI)e^{(a+bI)x+c+dI}$

5) $\frac{d}{dx}(c+dI)^x = (c+dI)^x\ ln(c+dI)$

Where $c > 0\,, d > 0\ and\ I \geq 0\ \ or\ c > 0\,, d < 0\ and\ I \leq 0$

6) $\frac{d}{dx}\left[log_{a+bI}\, x\right] = \frac{1}{x\ln(a+bI)}$

Where $a > 0\,, b > 0\ and\ I \geq 0\ \ or\ a > 0\,, b < 0\ and\ I \leq 0$

7) $\frac{d}{dx}[\ln((a+bI)x+c+dI)] = \frac{a+bI}{(a+bI)x+c+dI}$

8) $\frac{d}{dx}\left[\sqrt{(a+bI)x+c+dI}\right]=\frac{a+bI}{2\sqrt{(a+bI)x+c+dI}}$

9) $\frac{d}{dx}[sin((a+bI)x+c+dI)]=(a+bI)cos((a+bI)x+c+dI)$

10) $\frac{d}{dx}[cos((a+bI)x+c+dI)]=-(a+bI)sin((a+bI)x+c+dI)$

11) $\frac{d}{dx}[tan((a+bI)x+c+dI)]=(a+bI)sec^2((a+bI)x+c+dI)$

12) $\frac{d}{dx}[cot((a+bI)x+c+dI)]=-(a+bI)csc^2((a+bI)x+c+dI)$

13) $\frac{d}{dx}[sec((a+bI)x+c+dI)]=(a+bI)sec((a+bI)x+c+dI)tan((a+bI)x+c+dI)$

14) $\frac{d}{dx}[csc((a+bI)x+c+dI)]=-(a+bI)csc((a+bI)x+c+dI)cot((a+bI)x+c+dI)$

Proof (3):

$$\frac{d}{dx}[(a+bI)x^n]=\lim_{h+h_0I\to 0+0I}\frac{f(x+h+h_0I)-f(x,I)}{h+h_0I}$$

$$=\lim_{h+h_0I\to 0+0I}\frac{(x+h+h_0I)^n-(a+bI)x^n}{h+h_0I}$$

$$=\lim_{h+h_0I\to 0+0I}\frac{\left[(a+bI)x^n+n(a+bI)x^{n-1}(h+h_0I)+\frac{n(n-1)}{2!}(a+bI)x^{n-2}(h+h_0I)^2+\cdots+n(a+bI)x(h+h_0I)^{n-1}+(h+h_0I)^n\right]-(a+bI)x^n}{h+h_0I}$$

$$=\lim_{h+h_0I\to 0+0I}\left[\frac{n(a+bI)x^{n-1}(h+h_0I)+\frac{n(n-1)}{2!}(a+bI)x^{n-2}(h+h_0I)^2+\cdots+n(a+bI)x(h+h_0I)^{n-1}+(h+h_0I)^n}{h+h_0I}\right]$$

$$=\lim_{h+h_0I\to 0+0I}\left[n(a+bI)x^{n-1}+\frac{n(n-1)}{2!}(a+bI)x^{n-2}(h+h_0I)+\cdots+n(a+bI)x(h+h_0I)^{n-2}+(h+h_0I)^{n-1}\right]$$

$$=n(a+bI)x^{n-1}+0+\cdots\cdots+0+0$$

$$=n(a+bI)x^{n-1}$$

Example3.1.1

1) $\frac{d}{dx}(5-6I)=0+0I=0$

2) $\frac{d}{dx}[(4+2I)x-8I]=4+2I$

3) $\frac{d}{dx}[(7+3I)x^4]=(28+12I)x^3$

4) $\frac{d}{dx}\left[e^{(3+I)x+5I}\right]=(3+I)e^{(3+I)x+5I}$

5) $\frac{d}{dx}(5+2I)^x=(5+2I)^x\,ln(5+2I)\ ;\ case\ I\geq 0$

6) $\frac{d}{dx}(3-I)^x=(3-I)^x\,ln(3-I)\,;case\ I\leq 0$

7) $\frac{d}{dx}\left[ln\big((3+2I)x+6+7I\big)\right]=\frac{3+2I}{(3+2I)x+6+7I}$

8) $\frac{d}{dx}\left[\sqrt{(5+4I)x+9+I}\right]=\frac{5+4I}{2\sqrt{(5+4I)x+9+I}}$

9) $\frac{d}{dx}\left[sin\big((6-2I)x+9I\big)\right]=(6-2I)cos\big((6-2I)x+9I\big)$

10) $\frac{d}{dx}\left[cos\big((3-3I)x+2-I\big)\right]=(-3+3I)sin\big((3-3I)x+2-I\big)$

11) $\frac{d}{dx}[tan((8+9I)x+6I)]=(8+9I)sec^2((8+9I)x+6I)$

12) $\frac{d}{dx}\left[csc\big((3-4I)x+6+I\big)\right]=(-3+4I)csc\big((3-4I)x+6+I\big)cot\big((3-4I)x+6+I\big)$

13) $\frac{d}{dx}\left[log_{3+5I}\,x\right]=\frac{1}{x\ln(3+5I)}\ ;\ case\ I\geq 0$

3.2 Properties of neutrosophic differentiation:

3.2.1 Derivative of sum or difference of neutrosophic functions.

Suppose that $f(x,I)$ and $g(x,I)$ are any two differentiable neutrosophic functions, then:

$$\frac{d}{dx}[f(x,I)\ \pm g(x,I)]=\frac{d}{dx}[f(x,I)\,]\pm\frac{d}{dx}[g(x,I)]$$

Proof:

$\frac{d}{dx}[f(x,I)+g(x,I)]=$

$$=\lim_{h+h_0I\to 0+0I}\frac{f(x+h+h_0I)\pm g(x+h+h_0I)-[f(x,I)+g(x,I)]}{h+h_0I}$$

$$= \lim_{h+h_0I\to 0+0I} \frac{[f(x+h+h_0I)-f(x,I)] \pm [g(x+h+h_0I)-g(x,I)]}{h+h_0I}$$

$$= \lim_{h+h_0I\to 0+0I} \left[\frac{[f(x+h+h_0I)-f(x,I)]}{h+h_0I} \pm \frac{[g(x+h+h_0I)-g(x,I)]}{h+h_0I}\right]$$

$$= \lim_{h+h_0I\to 0+0I} \frac{[f(x+h+h_0I)-f(x,I)]}{h+h_0I} \pm \lim_{h+h_0I\to 0+0I} \frac{[g(x+h+h_0I)-g(x,I)]}{h+h_0I}$$

$$= \frac{d}{dx} f(x,I) \pm \frac{d}{dx} g(x,I)$$

Example3.2.1

1) $\frac{d}{dx}[3Ix^3 + tan((8+9I)x)] = 9Ix^2 + (8+9I)sec^2((8+9I)x)$

2) $\frac{d}{dx}\left[8Ix + ln\left((3+2I)x\right)\right] = 8I + \frac{3+2I}{(3+2I)x}$

3.2.2 Derivative of product of a neutrosophic constant & neutrosophic function

$$\frac{d}{dx}[(c+dI)f(x,I)] = (c+dI)\frac{d}{dx}[f(x,I)]$$

where c, d are real numbers, while $I =$ indeterminacy.

Proof:

$$\frac{d}{dx}[(c+dI)f(x,I)] = \lim_{h+h_0I\to 0+0I} \frac{(c+dI)f(x+h+h_0I)-(c+dI)f(x,I)}{h+h_0I}$$

$$= \lim_{h+h_0I\to 0+0I} (c+dI)\left[\frac{f(x+h+h_0I)-f(x,I)}{h+h_0I}\right]$$

$$= (c+dI) \lim_{h+h_0I\to 0+0I} \left[\frac{f(x+h+h_0I)-f(x,I)}{h+h_0I}\right]$$

$$= (c+dI)\frac{d}{dx}[f(x,I)]$$

3.2.3 Derivative of product of two neutrosophic functions

$$\frac{d}{dx}[f(x,I).g(x,I)] = f(x,I)\frac{d}{dx}[g(x,I)] + g(x,I)\frac{d}{dx}[f(x,I)]$$

Proof:

$$\frac{d}{dx}[f(x,I).g(x,I)] =$$

$$= \lim_{h+h_0I\to 0+0I} \frac{f(x+h+h_0I).g(x+h+h_0I) - f(x,I).g(x,I)}{h+h_0I}$$

$$= \lim_{h+h_0I\to 0+0I} \frac{f(x+h+h_0I).g(x+h+h_0I) - f(x+h+h_0I)g(x,I) + f(x+h+h_0I)g(x,I) - f(x,I).g(x,I)}{h+h_0I}$$

$$= \lim_{h+h_0I\to 0+0I} \left[f(x+h+h_0I)\frac{g(x+h+h_0I)-g(x,I)}{h+h_0I} + g(x,I)\frac{f(x+h+h_0I)-f(x,I)}{h+h_0I}\right]$$

$$= \lim_{h+h_0I\to 0+0I} f(x+h+h_0I) \lim_{h+h_0I\to 0+0I} \frac{g(x+h+h_0I)-g(x,I)}{h+h_0I} + \lim_{h+h_0I\to 0+0I} g(x,I) \lim_{h+h_0I\to 0+0I} \frac{f(x+h+h_0I)-f(x,I)}{h+h_0I}$$

$$= f(x,I)\frac{d}{dx}[g(x,I)] + g(x,I)\frac{d}{dx}[f(x,I)]$$

Example3.2.2

1) $\frac{d}{dx}[-7Ix^2 sin((8+9I)x)] = -14x.sin\big((8+9I)x\big) - 119Icos\big((8+9I)x\big)$

2) $\frac{d}{dx}\Big[2Ix\sqrt{(5+4I)x+9+I}\Big] = 2I\sqrt{(5+4I)x+9+I} + \frac{9Ix}{\sqrt{(5+4I)x+9+I}}$

3.2.3 Derivative of quotient of two neutrosophic functions

$$\frac{d}{dx}\left[\frac{f(x,I)}{g(x,I)}\right] = \frac{f(x,I)\frac{d}{dx}[g(x,I)] - g(x,I)\frac{d}{dx}[f(x,I)]}{(g(x,I))^2}$$

Proof:

$$\frac{d}{dx}\left[\frac{f(x,I)}{g(x,I)}\right] = \lim_{h+h_0I\to 0+0I} \frac{\frac{f(x+h+h_0I)}{g(x+h+h_0I)} - \frac{f(x,I)}{g(x,I)}}{h+h_0I}$$

$$= \lim_{h+h_0I\to 0+0I} \frac{f(x+h+h_0I).g(x,I) - f(x,I).g(x,I) - f(x,I).g(x+h+h_0I) + f(x,I).g(x,I)}{(h+h_0I)g(x,I).g(x+h+h_0I)}$$

$$= \lim_{h+h_0I\to 0+0I} \left[\frac{g(x,I)\frac{f(x+h+h_0I)-f(x,I)}{h+h_0I} - f(x,I)\frac{g(x+h+h_0I)-g(x,I)}{h+h_0I}}{g(x,I).g(x+h+h_0I)}\right]$$

$$= \frac{\lim_{h+h_0I\to 0+0I}\left[g(x,I)\frac{f(x+h+h_0I)-f(x,I)}{h+h_0I}\right] - \lim_{h+h_0I\to 0+0I}\left[f(x,I)\frac{g(x+h+h_0I)-g(x,I)}{h+h_0I}\right]}{\lim_{h+h_0I\to 0+0I}[g(x,I).g(x+h+h_0I)]}$$

$$= \frac{\lim_{h+h_0I\to 0+0I} g(x,I) \lim_{h+h_0I\to 0+0I} \frac{f(x+h+h_0I)-f(x,I)}{h+h_0I} - \lim_{h+h_0I\to 0+0I} f(x,I) \lim_{h+h_0I\to 0+0I} \frac{g(x+h+h_0I)-g(x,I)}{h+h_0I}}{\lim_{h+h_0I\to 0+0I} g(x,I). \lim_{h+h_0I\to 0+0I} g(x+h+h_0I)}$$

$$= \frac{d}{dx}\left[\frac{f(x,I)}{g(x,I)}\right] = \frac{f(x,I)\frac{d}{dx}[g(x,I)] - g(x,I)\frac{d}{dx}[f(x,I)]}{g(x,I).g(x,I)}$$

$$= \frac{d}{dx}\left[\frac{f(x,I)}{g(x,I)}\right] = \frac{f(x,I)\frac{d}{dx}[g(x,I)] - g(x,I)\frac{d}{dx}[f(x,I)]}{(g(x,I))^2}$$

Example3.2.3

1) $\frac{d}{dx}\left[\frac{e^{(3+I)x+5I}}{(3+4I)x}\right] = \frac{(3+I)(3+4I)xe^{(3+I)x+5I} - (3+4I)e^{(3+I)x+5I}}{(3+4I)^2x^2}$

$$= \frac{(3+I)xe^{(3+I)x+5I} - e^{(3+I)x+5I}}{(3+4I)x^2}$$

$$= (\frac{1}{3} - \frac{4}{21}I)\left[\frac{(3+I)xe^{(3+I)x+5I} - e^{(3+I)x+5I}}{x^2}\right]$$

2) $\frac{d}{dx}\left[\frac{5I}{(1+I)x}\right] = \frac{-5I}{(1+I)x^2} = \left(-5 - \frac{5}{2}I\right)\frac{1}{x^2}$

3.3 Derivative of composite neutrosophic functions

Chain Rule:
if $y = f(u, I)$ $and u = g(x, I)$, then:

$$\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx} \quad \Rightarrow \quad \frac{dy}{dx} = \acute{f}(u, I).\acute{g}(x, I)$$

Remarks:

1) $\frac{d}{dx}\left[f(g(x,I))\right] = f'(g(x,I)).g'(x,I)$
2) $\frac{d}{dx}\left[f\left(g\left(h(x,I)\right)\right)\right] = f'\left(g\left(h(x,I)\right)\right).g'\left(h(x,I)\right).h'(x,I)$
3) $\frac{d}{dx}[f(x,I)]^n = n[f(x,I)]^{n-1}.[f'(x,I)]\ ; n \in R - \{0,1\}$

Example3.3.1

1) $\frac{d}{dx}\left((2+I)x^2 + 2Ix - 5 + 6I\right)^7 = 7\left((2+I)x^2 + 2Ix - 5 + 6I\right)^6\left((4+2I)x + 2I\right)$

2) $\frac{d}{dx}sin^5\left((3+4I)x + 5I\right) = 4(3+4I)sin^4\left((3+4I)x + 5I\right)\left(cos\left((3+4I)x + 5I\right)\right)$

$$= (12 + 16I)sin^4\left((3+4I)x + 5I\right)\left(cos\left((3+4I)x + 5I\right)\right)$$

3) $\frac{d}{dx}\left[\sqrt{tan\left((6+4I)x - 2 + 7I\right)}\right] = \frac{(6+4I)sec^2\left((6+4I)x - 2 + 7I\right)}{2\sqrt{tan\left((6+4I)x - 2 + 7I\right)}}$

$$= \frac{(3+2I)sec^2\left((6+4I)x - 2 + 7I\right)}{\sqrt{tan\left((6+4I)x - 2 + 7I\right)}}$$

Example3.3.2

Find $\frac{dy}{dx}$ of each the following:

a) $y = f(t,I) = 3It^2 + 5 - 6I$, $t = g(x,I) = sin\big((6+4I)x - 2 + 7I\big) + tan\big((6+4I)x - 2 + 7I\big)$

Solution:

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} \quad \Rightarrow \quad \frac{dy}{dx} = \acute{f}(t,I).\acute{g}(x,I)$$

$$= 6It \,.\Big((6+4I)cos\big((6+4I)x - 2 + 7I\big) + (6+4I)sec^2\big((6+4I)x - 2 + 7I\big)\Big)$$

$$= 6I(6+4I)t \,.\Big(cos\big((6+4I)x - 2 + 7I\big) + sec^2\big((6+4I)x - 2 + 7I\big)\Big)$$

$$= 60Isin\big((6+4I)x - 2 + 7I\big) + tan\big((6+4I)x - 2 + 7I\big) \,.\Big(cos\big((6+4I)x - 2 + 7I\big) + sec^2\big((6+4I)x - 2 + 7I\big)\Big)$$

b) $y = f(t,I) = (t + 1 - 4I)^2$, $t = g(x,I) = \sqrt{(4-2I)x + 5I}$

Solution:

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} \quad \Rightarrow \quad \frac{dy}{dx} = \acute{f}(t,I).\acute{g}(x,I)$$

$$= 2(t + 1 - 4I)\frac{4-2I}{2\sqrt{(4-2I)x + 5I}}$$

$$= \frac{(4-2I)(t + 1 - 4I)}{\sqrt{(4-2I)x + 5I}}$$

$$= \frac{(4-2I)\left(\sqrt{(4-2I)x + 5I} + 1 - 4I\right)}{\sqrt{(4-2I)x + 5I}}$$

$$= \frac{(4-2I)\sqrt{(4-2I)x + 5I} + (4-2I)(1-4I)}{\sqrt{(4-2I)x + 5I}}$$

$$= (4-2I) + \frac{4-10I}{\sqrt{(4-2I)x + 5I}}$$

4. Derivatives of inverse neutrosophic trigonometric functions

9) $$\frac{d}{dx}[sin^{-1}((a+bI)x + c + dI)] = \frac{a+bI}{\sqrt{1 - \big((a+bI)x + c + dI\big)^2}}$$

10) $$\frac{d}{dx}[cos^{-1}((a+bI)x + c + dI)] = -\frac{a+bI}{\sqrt{1 - \big((a+bI)x + c + dI\big)^2}}$$

11) $\frac{d}{dx}[tan^{-1}((a+bI)x+c+dI)] = \frac{a+bI}{1+\left((a+bI)x+c+dI\right)^2}$

12) $\frac{d}{dx}[cot^{-1}((a+bI)x+c+dI)] = -\frac{a+bI}{1+\left((a+bI)x+c+dI\right)^2}$

13) $\frac{d}{dx}[sec^{-1}((a+bI)x+c+dI)] = \frac{a+bI}{|(a+bI)x+c+dI|\sqrt{\left((a+bI)x+c+dI\right)^2-1}}$

14) $\frac{d}{dx}[csc^{-1}((a+bI)x+c+dI)] = -\frac{a+bI}{|(a+bI)x+c+dI|\sqrt{\left((a+bI)x+c+dI\right)^2-1}}$

Proof (1):

$$y = sin^{-1}\left((a+bI)x+c+dI\right)$$

$$sin\, y = (a+bI)x+c+dI$$

$$\Rightarrow cos\, y\frac{dy}{dx} = a+bI$$

$$\frac{dy}{dx} = \frac{a+bI}{cos\, y}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{a+bI}{\sqrt{1-sin^2 y}} = \frac{a+bI}{\sqrt{1-\left((a+bI)x+c+dI\right)^2}}$$

Note:
In the same way we can prove the rest of the rules.

Example4.1

1) $\frac{d}{dx}tan^{-1}\left((5+6I)x+4-7I\right) = \frac{5+6I}{1+\left((5+6I)x+4-7I\right)^2}$

2) $\frac{d}{dx}\left[(4-2I)x^2+sec^{-1}\left((1+4I)x-3I\right)\right] = (8-4I)x+\frac{1+4I}{|(1+4I)x-3I|\sqrt{\left((1+4I)x-3I\right)^2-1}}$

3) $\frac{d}{dx}[(-4Ix+6-I)cos^{-1}(9Ix+5-3I)] = -4Icos^{-1}(9Ix+5-3I)-\frac{9I(-4Ix+6-I)}{\sqrt{1-(9Ix+5-3I)^2}}$

$$= -4Icos^{-1}(9Ix+5-3I)+\frac{36Ix-45I}{\sqrt{1-(9Ix+5-3I)^2}}$$

4.1 Differentiation of implicit neutrosophic functions

$y = f(x,I)$ can be directly expressed as a function of (x,I), such functions are known as explicit neutrosophic functions. But the relations of the form $f(x,y,I) = 0+0I,\ y$ is not directly expressed as a function of (x, I) and also it is not easily solvable for y, In such case functions are known as implicit neutrosophic functions.

To find differentiation of implicit neutrosophic functions, we follow the following steps:

- Differentiate two sides of the given equation with respect to (x, I).
- We isolate d on one side and the other terms on the other side to get the following equation:

$$\varphi(x, y, I)\frac{dy}{dx} = \omega(x, y, I)$$

Hence:

$$\frac{dy}{dx} = \frac{\omega(x, y, I)}{\varphi(x, y, I)}$$

Example4.1.1

If $3Ixy^3 - (2 + 5I)x^2y = 7Ix + 3 + 8I$, find $\frac{dy}{dx}$.

Solution:

$$\frac{d}{dx}(3Ixy^3 - (2 + 5I)x^2y) = \frac{d}{dx}(7Ix + 3 + 8I)$$

$$9Ixy^2\frac{dy}{dx} - (4 + 10I)xy - (2 + 5I)x^2\frac{dy}{dx} = 7I$$

$$(9Ixy^2 - (2 + 5I)x^2)\frac{dy}{dx} = (4 + 10I)xy - 3Iy^3 + 7I$$

$$\frac{dy}{dx} = \frac{(4 + 10I)xy - 3Iy^3 + 7I}{9Ixy^2 - (2 + 5I)x^2}$$

Example4.1.2

If $(3 + 5I)xy - siny = (5 + I)y - 1 + 2I$, find $\frac{dy}{dx}$.

Solution:

$$\frac{d}{dx}\big((3 + 5I)xy - siny\big) = \frac{d}{dx}\big((5 + I)y - 1 + 2I\big)$$

$$(3 + 5I)y + (3 + 5I)x\frac{dy}{dx} - cosy\,\frac{dy}{dx} = (5 + I)\frac{dy}{dx}$$

$$(3 + 5I)x\frac{dy}{dx} - cosy\,\frac{dy}{dx} - (5 + I)\frac{dy}{dx} = -(3 + 5I)y$$

$$\big((3 + 5I)x - cosy - (5 + I)\big)\frac{dy}{dx} = -(3 + 5I)y$$

$$\frac{dy}{dx} = \frac{-(3 + 5I)y}{(3 + 5I)x - cosy - (5 + I)}$$

4.2 Logarithmic neutrosophic differentiation

We use the logarithmic neutrosophic differentiation for differentiating neutrosophic functions of the form $y = f(x, I)^{g(x,I)}$ and for neutrosophic function which contains product and quotient of two or more neutrosophic functions.

We will discuss the steps for solving the first case, and in the same way the second case is done.

Solution steps:

- Take logarithmic of the two sides

$$lny = lnf(x,I)^{g(x,I)}$$

$$lny = g(x,I).lnf(x,I)$$

- Now differentiate of the two sides

$$\frac{1}{y}\frac{dy}{dx} = g(x,I)\frac{d}{dx}lnf(x,I) + lnf(x,I)\frac{d}{dx}g(x,I)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = g(x,I)\frac{1}{f(x,I)}\acute{f}(x,I) + lnf(x,I)\acute{g}(x,I)$$

$$\Rightarrow \frac{dy}{dx} = y\left(g(x,I)\frac{1}{f(x,I)}\acute{f}(x,I) + lnf(x)\acute{g}(x,I)\right)$$

$$\frac{dy}{dx} = f(x,I)^{g(x,I)}\left(g(x,I)\frac{1}{f(x,I)}\acute{f}(x,I) + lnf(x)\acute{g}(x,I)\right)$$

Example4.2.1

If $y = (ln(2+3I)x + 9I)^{\sqrt{(4-2I)x}}$, find $\frac{dy}{dx}$.

Solution:

$$lny = ln(ln(2+3I)x + 9I)^{\sqrt{(4-2I)x}}$$

$$lny = \sqrt{(4-2I)x}\, ln(ln(2+3I)x + 9I)$$

$$\frac{d}{dx}lny = \frac{d}{dx}\left(\sqrt{(4-2I)x}\, ln(ln(2+3I)x + 9I)\right)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{4-2I}{2\sqrt{(4-2I)x}}\, ln(\ln(2+3I)x + 9I) + \frac{\frac{2+3I}{ln(2+3I)x}}{(ln(2+3I)x + 9I)}\sqrt{(4-2I)x}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2-I}{\sqrt{(4-2I)x}}\, ln(\ln(2+3I)x + 9I) + \frac{2+3I}{ln(2+3I)x\,(ln(2+3I)x + 9I)}\sqrt{(4-2I)x}$$

$$\frac{dy}{dx} = y\left(\frac{2-I}{\sqrt{(4-2I)x}}\, ln(\ln(2+3I)x + 9I) + \frac{2+3I}{ln(2+3I)x\,(ln(2+3I)x + 9I)}\sqrt{(4-2I)x}\right)$$

$$\frac{dy}{dx} = (ln(2+3I)x + 9I)^{\sqrt{(4-2I)x}}\left(\frac{2-I}{\sqrt{(4-2I)x}}\, ln(\ln(2+3I)x + 9I) + \frac{2+3I}{ln(2+3I)x\,(ln(2+3I)x + 9I)}\sqrt{(4-2I)x}\right)$$

Example4.2.2

If $y = \frac{(3x^2-1+6I)^4 tan^{-1}(5Ix+2+9I)}{(3-5I)x-4I}$, find $\frac{dy}{dx}$.

Solution:

$$lny = ln\left(\frac{(3x^2-1+6I)^4 tan^{-1}(5Ix+2+9I)}{(3-5I)x-4I}\right)$$

$$lny = ln\big((3x^2-1+6I)^4 tan^{-1}(5Ix+2+9I)\big) - ln\big((3-5I)x-4I\big)$$

$$lny = 4ln(3x^2-1+6I) + ln\big(tan^{-1}(5Ix+2+9I)\big) - ln\big((3-5I)x-4I\big)$$

$$\frac{d}{dx}lny = \frac{d}{dx}\Big(4ln(3x^2-1+6I) + ln\big(tan^{-1}(5Ix+2+9I)\big) - ln\big((3-5I)x-4I\big)\Big)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{12}{3x^2-1+6I} + \frac{\frac{5I}{1+(5Ix+2+9I)^2}}{tan^{-1}(5Ix+2+9I)} - \frac{3-5I}{(3-5I)x-4I}$$

$$\frac{dy}{dx} = y\left[\frac{12}{3x^2-1+6I} + \frac{5I}{(1+(5Ix+2+9I)^2)tan^{-1}(5Ix+2+9I)} - \frac{3-5I}{(3-5I)x-4I}\right]$$

$$\frac{dy}{dx} = \frac{(3x^2-1+6I)^4 tan^{-1}(5Ix+2+9I)}{(3-5I)x-4I}\left[\frac{12}{3x^2-1+6I} + \frac{5I}{(1+(5Ix+2+9I)^2)tan^{-1}(5Ix+2+9I)} - \frac{3-5I}{(3-5I)x-4I}\right]$$

4.3 Higher order neutrosophic derivatives

Let $f: D_f \subseteq R \rightarrow R_f \cup \{I\}$, then $\frac{dy}{dx} = \acute{f}(x,I)$ is also a neutrosophic function, which can be again differentiated with respect to x. The derivative of $\frac{dy}{dx}$ is denoted by $\frac{d^2y}{dx^2}$ and is called (second order derivative) of the neutrosophic function $y = f(x,I)$, $\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = f''(x,I)$ is called (third order derivative).

Similarly, fourth, fifth and so on. In general the n^{th} derivative of neutrosophic function $y = f(x,I)$ is denoted by $\frac{d^ny}{dx^n} = f^{(n)}(x,I)$.

Example4.3.1

Find the second derivative of $f(x,I) = cos\big((4-5I)x-7I\big)$

Solution:

$$\acute{f}(x,I) = -(4-5I)sin\big((4+5I)x-7I\big)$$

$$f''(x,I) = -(4-5I)^2 cos\big((4+5I)x-7I\big)$$

$$f''(x,I) = (-16+15I)cos\big((4+5I)x-7I\big)$$

Example4.3.2

Let $f(x,I) = 6Ix^3 - (2-I)x^2 + 5Ix + 2 - 7I$, find $f''(1-3I)$.

Solution:

$$\acute{f}(x,I) = 12Ix^2 - (4-2I)x + 5I$$

$$f''(x,I) = 24Ix - 4 + 2I$$

$$\Rightarrow f''(1-3I) = 24I(1-3I) - 4 + 2I$$

$$= 24I - 72I - 4 + 2I = -4 - 46I$$

4.4 Differentiation of parametric neutrosophic functions

Definition4.4.1

Let $y = \phi(x,I)$ neutrosophic function, it can be represented by means of some parametric equations such as $y = f(t,I)$ and $x = g(t,I)$, where t is some parameter. We call $y = \phi(x,I)$ a parametric neutrosophic functions.

Then:

$$\acute{\phi}(x,I) = \frac{\acute{f}(x,I)}{\acute{g}(x,I)} \quad \Rightarrow \quad \frac{dy}{dx} = \frac{dy/_{dt}}{dx/_{dt}}$$

Example4.4.1

Let $x = (1-3I)t^2$ and $y = (2-2I)t$, find $\frac{dy}{dx}$.

Solution:

$$\frac{dy}{dx} = \frac{dy/_{dt}}{dx/_{dt}}$$

$$= \frac{2-2I}{2(1-3I)t} = \frac{1+I}{(1-3I)t}$$

$$= (1-2I)\frac{1}{t}$$

Example4.4.2

Let $x = (4+I)\,(\theta + sin(2\theta + 4 - 6I))$ and $y = (2-2I)(1 - cos(2\theta + 4 - 6I))$, find $\frac{dy}{dx}$.

Solution:

$$\frac{dy}{dx} = \frac{dy/_{d\theta}}{dx/_{d\theta}}$$

$$= \frac{(2-2I)\,sin(2\theta + 4 - 6I)}{(4+I)\,(1 + cos(2\theta + 4 - 6I))} = \frac{2-2I}{4+I}\,\frac{sin(2\theta + 4 - 6I)}{1 + cos(2\theta + 4 - 6I)}$$

$$= \left(\frac{1}{2} - \frac{1}{2}I\right)\frac{2sin(\theta + 2 - 3I)\,cos(\theta + 2 - 3I)}{2\,cos^2(\theta + 2 - 3I)}$$

$$= \left(\frac{1}{2} - \frac{1}{2}I\right)\frac{sin(\theta + 2 - 3I)}{cos(\theta + 2 - 3I)}$$

$$= \left(\frac{1}{2} - \frac{1}{2}I\right) tan(\theta + 2 - 3I)$$

Example4.4.2

Differentiate $e^{(3+I)x+5I}$ with respect to $\sqrt{(5+4I)x+9+I}$.

Solution:

Let $u = e^{(3+I)x+5I}$ and $v = \sqrt{(5+4I)x+9+I}$

Then, the required derivative is:

$$\frac{du}{dv} = \frac{du/dx}{dv/dx}$$

$$= \frac{(3+I)e^{(3+I)x+5I}}{\frac{5+4I}{2\sqrt{(5+4I)x+9+I}}}$$

$$= \frac{6+2I}{5+4I}\sqrt{(5+4I)x+9+I}\; e^{(3+I)x+5I}$$

$$= \left(\frac{6}{5} - \frac{14}{45}I\right)\sqrt{(5+4I)x+9+I}\; e^{(3+I)x+5I}$$

5. Conclusions

The derivatives are important in our lives, such as calculating the function of velocity, displacement and acceleration as a function of time for rectilinear motion and others, and calculating any rate of change of any variable in relation to another variable or variables such as the rate of fuel consumption or the rate of decreasing or increasing any variable by changing any other. This led us to study the neutrosophic differentials for neutrosophic functions from that contain indeterminacy. Where the neutrosophic differentiable is defined, and properties of neutrosophic differentiation are introduced. In addition to studying derivative of composite neutrosophic functions, derivatives of inverse neutrosophic trigonometric functions, differentiation of implicit neutrosophic functions, logarithmic neutrosophic differentiation, higher order neutrosophic derivatives, and differentiation of parametric neutrosophic functions. The importance of this paper lies in the field of the neutrosophic integrals.

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