



# Anti Neutrosophic multi fuzzy ideals of $\gamma$ near ring

K. Hemabala <sup>1,\*</sup> and B. Srinivasa Kumar <sup>2</sup>

1. Research Scholar, Mathematics department, Koneru Lakshmaiah Education Foundation, Vaddeswaram, Guntur- 522502, Andhra Pradesh, India.

hemaram.magi@gmail.com

2. Assoc. Prof., Mathematics department, Koneru Lakshmaiah Education Foundation, Vaddeswaram, Guntur- 522502, Andhra Pradesh, India

sk\_bhavirisetty@kluniversity.in

\* Correspondence: sk\_bhavirisetty@kluniversity.in

**Abstract:** The theory of anti neutrosophic multi fuzzy ideals of  $\gamma$  near ring is dispensed in this work and various algebraic properties such as intersection, union of anti neutrosophic multi fuzzy ideals of  $\gamma$  near ring are examined. Further we examined the direct anti product of anti neutrosophic multi fuzzy ideals of  $\gamma$  near ring and also we proved the homomorphic images and pre images of anti neutrosophic multi fuzzy ideals of  $\gamma$  near ring.

**Keywords:** Anti neutrosophic fuzzy set,  $\gamma$  near ring, anti neutrosophic multi fuzzy set, anti neutrosophic multi fuzzy ideal of  $\gamma$  near ring, anti product of anti neutrosophic multi fuzzy ideals of  $\gamma$  near ring.

## 1. Introduction

In 1965, Zadeh[25] proposed the notion of fuzzy set. Later A.Rosenfeld[16] developed fuzzy groups. The numerous authors like Bh.Satyanarayana[3,4,5] proposed the concept of fuzzy  $\gamma$  near ring. The authors S. Ragamai, Y. Bhargavi, T. Eswarlal[19] developed theory of fuzzy and  $L$  fuzzy ideals of  $\gamma$  near ring. Later the properties of  $\gamma$  near ring in multi fuzzy sets were extended by K. Hemabala and Srinivasa kumar[13]. After the theory of fuzzy sets, Florentin Smarandache[7,8] established as a new field of philosophy which is a neutrosophic theory, in 1995. The main base of neutrosophic logic is neutrosophy that includes indeterminacy. It is an agumentation of fuzzy set and intuitionstic fuzzy set. In neutrosophic logic each proposition is estimated by three components T,I,F. The neutrosophic set theory have seen great triumph in several fields such as image processing ,medical diagnosis, robotic, decision making problem and so on. I. Arockiarani[3] extended the theory of neutrosophic fuzzy set. A.Solairaju and S.Thiruveni[2] verified the algebraic properties of fuzzy neutrosophic set in near rings. In fuzzy neutrosophic set, the three components T,I,F can take single values between 0 and 1. There is some ambiguity irrespective of the distance to the element is. The neutrosophic fuzzy set theory on its own is not sufficient to study real world problems. F. Smarandache[9] developed notion of neutrosophic multi sets, an extension of neutrosophic set, in 2016. Authors like Vakkas Ulucay and Memet sahin[23] verified the concepts of neutrosophic multi fuzzy set in groups and verified the group properties. We carry the neutrosophic multi fuzzy notion in  $\gamma$  near ring and hence some properties of algebra are verified.

## 2. Preliminaries:

Basic definitions of fuzzy set, multi fuzzy set, neutrosophic set and neutrosophic multi set,  $\gamma$  near ring are presenting in this section. Fuzzy set can take a single value between [0,1]

### 2.1 Definition:

Let  $E$  be a non empty set and  $\tilde{A}$  be a fuzzy set over  $E$  is defined by[25]

$$\tilde{A} = \{\tilde{A}(a) / a \in E\} \text{ where } \tilde{A}: E \rightarrow [0,1].$$

## 2.2 Definition:

Let  $E$  be a non empty set and  $M_f$  be a multi fuzzy set over  $E$  is defined as [20,21]

$$M_f = \{ \langle a, M_f^1(a), M_f^2(a), M_f^3(a), \dots, M_f^i(a) \rangle : a \in E \} \text{ where } M_f^n: E \rightarrow [0,1] \text{ for all } n \in \{1, 2, \dots, i\} \text{ and}$$

$$a \in E$$

## 2.3 Definition:

Let  $E$  be a non empty set then neutrosophic fuzzy set  $\tilde{N}$  [7] in  $E$  is defined as

$$\tilde{N} = \{ \langle a, \check{t}_{\tilde{N}}(a), \check{i}_{\tilde{N}}(a), \check{f}_{\tilde{N}}(a) \rangle : a \in E \text{ and } \check{t}_{\tilde{N}}(a), \check{i}_{\tilde{N}}(a), \check{f}_{\tilde{N}}(a) \in [0,1] \}$$

Where  $\check{t}_{\tilde{N}}(a)$  is the truth membership function,  $\check{i}_{\tilde{N}}(a)$  is the indeterminacy membership function

and  $\check{f}_{\tilde{N}}$  falsity membership function and  $0 \leq \check{t}_{\tilde{N}}(x) + \check{i}_{\tilde{N}}(x) + \check{f}_{\tilde{N}}(x) \leq 1$ .

## 2.4 Definition:

Let  $E$  be a non empty set. A neutrosophic multi fuzzy set  $\tilde{N}$  on  $E$  can be defined as follows

$$\tilde{N} = \{ \langle a, (\check{t}_{\tilde{N}}^1(a), \check{t}_{\tilde{N}}^2(a), \dots, \check{t}_{\tilde{N}}^i(a)), (\check{i}_{\tilde{N}}^1(a), \check{i}_{\tilde{N}}^2(a), \dots, \check{i}_{\tilde{N}}^i(a)), (\check{f}_{\tilde{N}}^1(a), \check{f}_{\tilde{N}}^2(a), \dots, \check{f}_{\tilde{N}}^i(a)) \rangle : a \in E \}$$

Where  $\check{t}_{\tilde{N}}^1(a), \check{t}_{\tilde{N}}^2(a), \dots, \check{t}_{\tilde{N}}^i(a): E \rightarrow [0,1]$

$$\check{i}_{\tilde{N}}^1(a), \check{i}_{\tilde{N}}^2(a), \dots, \check{i}_{\tilde{N}}^i(a): E \rightarrow [0,1]$$

$$\check{f}_{\tilde{N}}^1(a), \check{f}_{\tilde{N}}^2(a), \dots, \check{f}_{\tilde{N}}^i(a): E \rightarrow [0,1]$$

$$0 \leq \sup \check{t}_{\tilde{N}}^n(a) + \sup \check{i}_{\tilde{N}}^n(a) + \sup \check{f}_{\tilde{N}}^n(a) \leq 1 \quad \text{for } n=1 \text{ to } i$$

$(\check{t}_{\tilde{N}}^1(a), \check{t}_{\tilde{N}}^2(a), \dots, \check{t}_{\tilde{N}}^i(a))$ ,  $(\check{i}_{\tilde{N}}^1(a), \check{i}_{\tilde{N}}^2(a), \dots, \check{i}_{\tilde{N}}^i(a))$ ,  $(\check{f}_{\tilde{N}}^1(a), \check{f}_{\tilde{N}}^2(a), \dots, \check{f}_{\tilde{N}}^i(a))$  are the sequences of truth membership values, indeterminacy membership values and falsity membership values. In addition  $i$  is called the dimension of neutrosophic multi fuzzy set  $\tilde{N}$  denoted by  $d(\tilde{N})$ .

The sequence of truth membership values are arranged in decreasing order, but the corresponding indeterminacy membership and falsity membership values may not be in any order.

## 2.5 Definition:

Let  $\mathcal{L}$  and  $\mathcal{R}$  neutrosophic multi fuzzy sets

where  $\mathcal{L} = \{(\check{t}_{\mathcal{L}}^1(a), \check{t}_{\mathcal{L}}^2(a), \dots, \check{t}_{\mathcal{L}}^i(a)), (\check{i}_{\mathcal{L}}^1(a), \check{i}_{\mathcal{L}}^2(a), \dots, \check{i}_{\mathcal{L}}^i(a)), (\check{f}_{\mathcal{L}}^1(a), \check{f}_{\mathcal{L}}^2(a), \dots, \check{f}_{\mathcal{L}}^i(a))\}$  and

$\mathcal{R} = \{(\check{t}_{\mathcal{R}}^1(a), \check{t}_{\mathcal{R}}^2(a), \dots, \check{t}_{\mathcal{R}}^i(a)), (\check{i}_{\mathcal{R}}^1(a), \check{i}_{\mathcal{R}}^2(a), \dots, \check{i}_{\mathcal{R}}^i(a)), (\check{f}_{\mathcal{R}}^1(a), \check{f}_{\mathcal{R}}^2(a), \dots, \check{f}_{\mathcal{R}}^i(a))\}$  then we have

the following relations and operations

- 1)  $\mathcal{L} \subseteq \mathcal{R}$  iff  $\check{t}_{\mathcal{L}}^n(a) \leq \check{t}_{\mathcal{R}}^n(a)$ ,  $\check{i}_{\mathcal{L}}^n(a) \geq \check{i}_{\mathcal{R}}^n(a)$ ,  $\check{f}_{\mathcal{L}}^n(a) \geq \check{f}_{\mathcal{R}}^n(a)$ ,  $a \in E$  and  $n=1$  to  $i$ .
- 2)  $\mathcal{L} = \mathcal{R}$  iff  $\check{t}_{\mathcal{L}}^n(a) = \check{t}_{\mathcal{R}}^n(a)$ ,  $\check{i}_{\mathcal{L}}^n(a) = \check{i}_{\mathcal{R}}^n(a)$ ,  $\check{f}_{\mathcal{L}}^n(a) = \check{f}_{\mathcal{R}}^n(a)$ ,  $a \in E$  and  $n=1$  to  $i$ .
- 3)  $\mathcal{L} \cup \mathcal{R} = \{a, \max(\check{t}_{\mathcal{L}}^n(a), \check{t}_{\mathcal{R}}^n(a)), \min(\check{i}_{\mathcal{L}}^n(a), \check{i}_{\mathcal{R}}^n(a)), \min(\check{f}_{\mathcal{L}}^n(a), \check{f}_{\mathcal{R}}^n(a))\}$ ,  $a \in E$  and  $n=1$  to  $i$ .
- 4)  $\mathcal{L} \cap \mathcal{R} = \{a, \min(\check{t}_{\mathcal{L}}^n(a), \check{t}_{\mathcal{R}}^n(a)), \max(\check{i}_{\mathcal{L}}^n(a), \check{i}_{\mathcal{R}}^n(a)), \max(\check{f}_{\mathcal{L}}^n(a), \check{f}_{\mathcal{R}}^n(a))\}$ ,  $a \in E$  and  $n=1$  to  $i$ .

## 2.6 Definition:

A non empty set  $E$  with the binary operations '+' (addition) and '.' (multiplication) is called a near ring [3] if the following conditions hold:

- 1)  $(E, +)$  is a group
- 2)  $(E, \cdot)$  is a semigroup
- 3)  $(e_1 + e_2) \cdot e_3 = e_1 \cdot e_3 + e_2 \cdot e_3$  for all  $e_1, e_2, e_3 \in E$

To be precise, it is called right near ring. Since it satisfies the right distributive law. But the word near ring is intended to mean right near ring. We use  $gh$  instead of  $g \cdot h$

A  $\gamma$  near ring  $M$  is a triple  $(M, +, \gamma)$  where

- 1)  $(M, +)$  is a group
- 2)  $\gamma$  is a non empty set of binary operations on  $M$  such that  $\tau \in \gamma$ ,  $(M, +, \tau)$  is a near ring.
- 3)  $e_1 \tau (e_2 \sigma e_3) = (e_1 \tau e_2) \sigma e_3$  for all  $e_1, e_2, e_3 \in M$  and  $\tau, \sigma \in \gamma$ .

### 3. Anti Neutrosophic multi fuzzy set of $\mathcal{Y}$ near ring

In this section, we introduce the definition of anti neutrosophic multi fuzzy sets of  $\mathcal{Y}$  near ring. We proved that union of two anti neutrosophic multi fuzzy ideals of  $\mathcal{L}$  and  $\mathcal{R}$  is an anti neutrosophic multi fuzzy ideal. We also prove that the intersection of two anti neutrosophic multi fuzzy ideals of  $\mathcal{L}$  and  $\mathcal{R}$  is also an anti neutrosophic multi fuzzy ideal.

#### 3.1 Definition:

A neutrosophic multi fuzzy set

$\mathcal{L} = \{(\check{t}_{\mathcal{L}}^1(a), \check{t}_{\mathcal{L}}^2(a), \dots, \check{t}_{\mathcal{L}}^i(a)), (\check{i}_{\mathcal{L}}^1(a), \check{i}_{\mathcal{L}}^2(a), \dots, \check{i}_{\mathcal{L}}^i(a)), (\check{f}_{\mathcal{L}}^1(a), \check{f}_{\mathcal{L}}^2(a), \dots, \check{f}_{\mathcal{L}}^i(a))\}$  in a  $\mathcal{Y}$  near ring

$M$  is called anti neutrosophic multi fuzzy sub  $\mathcal{Y}$  near ring of  $M$  if

- 1)  $\check{t}_{\mathcal{L}}^n(a - b) \leq \max(\check{t}_{\mathcal{L}}^n(a), \check{t}_{\mathcal{L}}^n(b)),$   
 $\check{i}_{\mathcal{L}}^n(a - b) \geq \min(\check{i}_{\mathcal{L}}^n(a), \check{i}_{\mathcal{L}}^n(b)),$   
 $\check{f}_{\mathcal{L}}^n(a - b) \geq \min(\check{f}_{\mathcal{L}}^n(a), \check{f}_{\mathcal{L}}^n(b)), n = 1 \text{ to } i.$
- 2)  $\check{t}_{\mathcal{L}}^n(a \tau b) \leq \max(\check{t}_{\mathcal{L}}^n(a), \check{t}_{\mathcal{L}}^n(b)),$   
 $\check{i}_{\mathcal{L}}^n(a \tau b) \geq \min(\check{i}_{\mathcal{L}}^n(a), \check{i}_{\mathcal{L}}^n(b)),$   
 $\check{f}_{\mathcal{L}}^n(a \tau b) \geq \min(\check{f}_{\mathcal{L}}^n(a), \check{f}_{\mathcal{L}}^n(b)), n = 1 \text{ to } i.$

#### 3.2 Definition:

Let  $M$  be a  $\mathcal{Y}$  near ring. An anti neutrosophic multi fuzzy set  $\mathcal{L}$  in a  $\mathcal{Y}$  near ring  $M$  is called anti neutrosophic multi fuzzy left (resp. right) ideal of  $M$  if for all  $a, b, \rho_1, \rho_2 \in M, \tau \in \mathcal{Y}, n = 1, 2, \dots, i$

- 1)  $\check{t}_{\mathcal{L}}^n(a - b) \leq \max(\check{t}_{\mathcal{L}}^n(a), \check{t}_{\mathcal{L}}^n(b)),$   
 $\check{i}_{\mathcal{L}}^n(a - b) \geq \min(\check{i}_{\mathcal{L}}^n(a), \check{i}_{\mathcal{L}}^n(b)),$

$$\check{f}_{\mathcal{L}}^n(a - b) \geq \min(\check{f}_{\mathcal{L}}^n(a), \check{f}_{\mathcal{L}}^n(b))$$

$$2) \quad \check{t}_{\mathcal{L}}^n(b + a - b) \leq \check{t}_{\mathcal{L}}^n(a),$$

$$\check{i}_{\mathcal{L}}^n(b + a - b) \geq \check{i}_{\mathcal{L}}^n(a),$$

$$\check{f}_{\mathcal{L}}^n(b + a - b) \geq \check{f}_{\mathcal{L}}^n(a),$$

$$3) \quad \check{t}_{\mathcal{L}}^n(\rho_1 \tau(a + \rho_2) - \rho_1 \tau \rho_2) \leq \check{t}_{\mathcal{L}}^n(a),$$

$$\check{i}_{\mathcal{L}}^n(\rho_1 \tau(a + \rho_2) - \rho_1 \tau \rho_2) \geq \check{i}_{\mathcal{L}}^n(a),$$

$$\check{f}_{\mathcal{L}}^n(\rho_1 \tau(a + \rho_2) - \rho_1 \tau \rho_2) \geq \check{f}_{\mathcal{L}}^n(a),$$

[resp. right

$$\check{t}_{\mathcal{L}}^n(a \tau \rho_1) \leq \check{t}_{\mathcal{L}}^n(a)$$

$$\check{i}_{\mathcal{L}}^n(a \tau \rho_1) \geq \check{i}_{\mathcal{L}}^n(a),$$

$$\check{f}_{\mathcal{L}}^n(a \tau \rho_1) \geq \check{f}_{\mathcal{L}}^n(a)]$$

$\mathcal{L}$  is called a anti neutrosophic multi fuzzy ideal of  $M$  if  $\mathcal{L}$  both left and right anti neutrosophic multi fuzzy ideal of  $M$ .

### 3.1 Theorem:

Let  $\mathcal{L}$  and  $\mathcal{R}$  anti neutrosophic multi fuzzy left ideal of  $M$ . Then  $\mathcal{L} \cup \mathcal{R}$  is a anti neutrosophic multi fuzzy left ideal of  $M$ .

#### Proof:

Let  $\mathcal{L}$  and  $\mathcal{R}$  anti neutrosophic multi fuzzy left ideal of  $M$ .

Let  $a, b, \rho_1, \rho_2 \in M, \tau \in \gamma$

$$1) \quad \check{t}_{\mathcal{L} \cup \mathcal{R}}^n(a - b) = \max\{\check{t}_{\mathcal{L}}^n(a - b), \check{t}_{\mathcal{R}}^n(a - b)\}$$

$$\leq \max\{\{\max(\check{t}_{\mathcal{L}}^n(a), \check{t}_{\mathcal{L}}^n(b)), \max(\check{t}_{\mathcal{R}}^n(a), \check{t}_{\mathcal{R}}^n(b))\}\}$$

$$\leq \max\{\{\max(\check{t}_{\mathcal{L}}^n(a), \check{t}_{\mathcal{R}}^n(a)), \max(\check{t}_{\mathcal{L}}^n(b), \check{t}_{\mathcal{R}}^n(b))\}\}$$

$$\leq \max(\check{t}_{\mathcal{L} \cup \mathcal{R}}^n(a), \check{t}_{\mathcal{L} \cup \mathcal{R}}^n(b))$$

$$\check{i}_{\mathcal{L} \cup \mathcal{R}}^n(a - b) = \min\{\check{i}_{\mathcal{L}}^n(a - b), \check{i}_{\mathcal{R}}^n(a - b)\}$$

$$\geq \min\{\{\min(\check{i}_{\mathcal{L}}^n(a), \check{i}_{\mathcal{L}}^n(b)), \min(\check{i}_{\mathcal{R}}^n(a), \check{i}_{\mathcal{R}}^n(b))\}\}$$

$$\geq \min\{\{\min(\check{i}_{\mathcal{L}}^n(a), \check{i}_{\mathcal{R}}^n(a)), \min(\check{i}_{\mathcal{L}}^n(b), \check{i}_{\mathcal{R}}^n(b))\}\}$$

$$\geq \min(\check{i}_{\mathcal{L} \cup \mathcal{R}}^n(a), \check{i}_{\mathcal{L} \cup \mathcal{R}}^n(b))$$

$$\check{f}_{\mathcal{L} \cup \mathcal{R}}^n(a - b) = \min\{\check{f}_{\mathcal{L}}^n(a - b), \check{f}_{\mathcal{R}}^n(a - b)\}$$

$$\geq \min\{\{\min(\check{f}_{\mathcal{L}}^n(a), \check{f}_{\mathcal{L}}^n(b)), \min(\check{f}_{\mathcal{R}}^n(a), \check{f}_{\mathcal{R}}^n(b))\}\}$$

$$\geq \min\{\{\min(\check{f}_{\mathcal{L}}^n(a), \check{f}_{\mathcal{R}}^n(a)), \min(\check{f}_{\mathcal{L}}^n(b), \check{f}_{\mathcal{R}}^n(b))\}\}$$

$$\geq \min(\check{f}_{\mathcal{L} \cup \mathcal{R}}^n(a), \check{f}_{\mathcal{L} \cup \mathcal{R}}^n(b))$$

$$2) \quad \check{t}_{\mathcal{L} \cup \mathcal{R}}^n(b + a - b) = \max\{\check{t}_{\mathcal{L}}^n(b + a - b), \check{t}_{\mathcal{R}}^n(b + a - b)\}$$

$$\leq \max\{\check{t}_{\mathcal{L}}^n(a), \check{t}_{\mathcal{R}}^n(a)\}$$

$$\leq \check{t}_{\mathcal{L} \cup \mathcal{R}}^n(a)$$

$$\check{i}_{\mathcal{L} \cup \mathcal{R}}^n(b + a - b) = \min\{\check{i}_{\mathcal{L}}^n(b + a - b), \check{i}_{\mathcal{R}}^n(b + a - b)\}$$

$$\geq \min\{\check{i}_{\mathcal{L}}^n(a), \check{i}_{\mathcal{R}}^n(a)\}$$

$$\geq \check{i}_{\mathcal{L} \cup \mathcal{R}}^n(a)$$

$$\check{f}_{\mathcal{L} \cup \mathcal{R}}^n(b + a - b) = \min\{\check{f}_{\mathcal{L}}^n(b + a - b), \check{f}_{\mathcal{R}}^n(b + a - b)\}$$

$$\geq \min\{\check{f}_{\mathcal{L}}^n(a), \check{f}_{\mathcal{R}}^n(a)\}$$

$$\geq \check{f}_{\mathcal{L} \cup \mathcal{R}}^n(a)$$

$$\begin{aligned} 3. \quad & \check{t}_{\mathcal{L} \cup \mathcal{R}}^n((\rho_1 \tau(a + \rho_2) - \rho_1 \tau \rho_2)) \\ &= \max\{\check{t}_{\mathcal{L}}^n((\rho_1 \tau(a + \rho_2) - \rho_1 \tau \rho_2)), \check{t}_{\mathcal{R}}^n((\rho_1 \tau(a + \rho_2) - \rho_1 \tau \rho_2))\} \\ &\leq \max\{\check{t}_{\mathcal{L}}^n(a), \check{t}_{\mathcal{R}}^n(a)\} \\ &\leq \check{t}_{\mathcal{L} \cup \mathcal{R}}^n(a) \end{aligned}$$

$$\begin{aligned} & \check{i}_{\mathcal{L} \cup \mathcal{R}}^n((\rho_1 \tau(a + \rho_2) - \rho_1 \tau \rho_2)) \\ &= \min\{\check{i}_{\mathcal{L}}^n((\rho_1 \tau(a + \rho_2) - \rho_1 \tau \rho_2)), \check{i}_{\mathcal{R}}^n((\rho_1 \tau(a + \rho_2) - \rho_1 \tau \rho_2))\} \\ &\geq \min\{\check{i}_{\mathcal{L}}^n(a), \check{i}_{\mathcal{R}}^n(a)\} \\ &\geq \check{i}_{\mathcal{L} \cup \mathcal{R}}^n(a) \end{aligned}$$

$$\begin{aligned} & \check{f}_{\mathcal{L} \cup \mathcal{R}}^n((\theta_1 \tau(\mathfrak{x} + \theta_2) - \theta_1 \tau \theta_2)) \\ &= \min\{\check{f}_{\mathcal{L}}^n((\theta_1 \tau(\mathfrak{x} + \theta_2) - \theta_1 \tau \theta_2)), \check{f}_{\mathcal{R}}^n((\theta_1 \tau(\mathfrak{x} + \theta_2) - \theta_1 \tau \theta_2))\} \\ &\geq \min\{\check{f}_{\mathcal{L}}^n(a), \check{f}_{\mathcal{R}}^n(a)\} \\ &\geq \check{f}_{\mathcal{L} \cup \mathcal{R}}^n(a) \end{aligned}$$

$\therefore \mathcal{L} \cup \mathcal{R}$  is a anti neutrosophic multi fuzzy left ideal of  $M$ .

### 3.2 Theorem:

Let  $\mathcal{L}$  and  $\mathcal{R}$  anti neutrosophic multi fuzzy right ideal of  $M$  then  $\mathcal{L} \cup \mathcal{R}$  is a anti neutrosophic multi fuzzy right ideal of  $M$ .

#### Proof:

Let  $\mathcal{L}$  and  $\mathcal{R}$  neutrosophic multi fuzzy right ideal of  $M$ .

Let  $a, b, \rho_1, \rho_2 \in M, \tau \in \gamma$

$$1. \quad \check{t}_{\mathcal{L} \cup \mathcal{R}}^n(a - b) = \max\{\check{t}_{\mathcal{L}}^n(a - b), \check{t}_{\mathcal{R}}^n(a - b)\}$$

$$\leq \max\{\{\max(\check{t}_{\mathcal{L}}^n(a), \check{t}_{\mathcal{L}}^n(b)), \max(\check{t}_{\mathcal{R}}^n(a), \check{t}_{\mathcal{R}}^n(b))\}\}$$

$$\leq \max\{\{\max(\check{t}_{\mathcal{L}}^n(a), \check{t}_{\mathcal{R}}^n(a)), \max(\check{t}_{\mathcal{L}}^n(b), \check{t}_{\mathcal{R}}^n(b))\}\}$$

$$\leq \max(\check{t}_{\mathcal{L} \cup \mathcal{R}}^n(a), \check{t}_{\mathcal{L} \cup \mathcal{R}}^n(b))$$

$$\check{i}_{\mathcal{L} \cup \mathcal{R}}^n(a - b) = \min\{\check{i}_{\mathcal{L}}^n(a - b), \check{i}_{\mathcal{R}}^n(a - b)\}$$

$$\geq \min\{\{\min(\check{i}_{\mathcal{L}}^n(a), \check{i}_{\mathcal{L}}^n(b)), \min(\check{i}_{\mathcal{R}}^n(a), \check{i}_{\mathcal{R}}^n(b))\}\}$$

$$\geq \min\{\{\min(\check{i}_{\mathcal{L}}^n(a), \check{i}_{\mathcal{R}}^n(a)), \min(\check{i}_{\mathcal{L}}^n(b), \check{i}_{\mathcal{R}}^n(b))\}\}$$

$$\geq \min(\check{i}_{\mathcal{L} \cup \mathcal{R}}^n(a), \check{i}_{\mathcal{L} \cup \mathcal{R}}^n(b))$$

$$\check{f}_{\mathcal{L} \cup \mathcal{R}}^n(a - b) = \min\{\check{f}_{\mathcal{L}}^n(a - b), \check{f}_{\mathcal{R}}^n(a - b)\}$$

$$\geq \min\{\{\min(\check{f}_{\mathcal{L}}^n(a), \check{f}_{\mathcal{L}}^n(b)), \min(\check{f}_{\mathcal{R}}^n(a), \check{f}_{\mathcal{R}}^n(b))\}\}$$

$$\geq \min\{\{\min(\check{f}_{\mathcal{L}}^n(a), \check{f}_{\mathcal{R}}^n(a)), \min(\check{f}_{\mathcal{L}}^n(b), \check{f}_{\mathcal{R}}^n(b))\}\}$$

$$\geq \min(\check{f}_{\mathcal{L} \cup \mathcal{R}}^n(a), \check{f}_{\mathcal{L} \cup \mathcal{R}}^n(b))$$

$$2. \quad \check{t}_{\mathcal{L} \cup \mathcal{R}}^n(b + a - b) = \max\{\check{t}_{\mathcal{L}}^n(b + a - b), \check{t}_{\mathcal{R}}^n(b + a - b)\}$$

$$\leq \max\{\check{t}_{\mathcal{L}}^n(a), \check{t}_{\mathcal{R}}^n(a)\}$$

$$\leq \check{t}_{\mathcal{L} \cup \mathcal{R}}^n(a)$$

$$\check{i}_{\mathcal{L} \cup \mathcal{R}}^n(b + a - b) = \min\{\check{i}_{\mathcal{L}}^n(b + a - b), \check{i}_{\mathcal{R}}^n(b + a - b)\}$$

$$\geq \min\{\check{i}_{\mathcal{L}}^n(a), \check{i}_{\mathcal{R}}^n(a)\}$$

$$\geq \check{i}_{\mathcal{L} \cup \mathcal{R}}^n(a)$$



$$\check{f}_{\mathcal{L} \cup \mathcal{R}}^n(b + a - b) = \min\{\check{f}_{\mathcal{L}}^n(b + a - b), \check{f}_{\mathcal{R}}^n(b + a - b)\}$$

$$\geq \min\{\check{f}_{\mathcal{L}}^n(a), \check{f}_{\mathcal{R}}^n(a)\}$$

$$\geq \check{f}_{\mathcal{L} \cup \mathcal{R}}^n(a)$$

$$3. \quad \check{t}_{\mathcal{L} \cup \mathcal{R}}^n(a \tau \rho_1) = \max\{\check{t}_{\mathcal{L}}^n(a \tau \rho_1), \check{t}_{\mathcal{R}}^n(a \tau \rho_1)\}$$

$$\leq \max\{\check{t}_{\mathcal{L}}^n(a), \check{t}_{\mathcal{R}}^n(a)\}$$

$$\leq \check{t}_{\mathcal{L} \cup \mathcal{R}}^n(a)$$

$$\check{i}_{\mathcal{L} \cup \mathcal{R}}^n(a \tau \rho_1) = \min\{\check{i}_{\mathcal{L}}^n(a \tau \rho_1), \check{i}_{\mathcal{R}}^n(a \tau \rho_1)\}$$

$$\geq \min\{\check{i}_{\mathcal{L}}^n(a), \check{i}_{\mathcal{R}}^n(a)\}$$

$$\geq \check{i}_{\mathcal{L} \cup \mathcal{R}}^n(a)$$

$$\check{f}_{\mathcal{L} \cup \mathcal{R}}^n(a \tau \rho_1) = \min\{\check{f}_{\mathcal{L}}^n(a \tau \rho_1), \check{f}_{\mathcal{R}}^n(a \tau \rho_1)\}$$

$$\geq \min\{\check{f}_{\mathcal{L}}^n(a), \check{f}_{\mathcal{R}}^n(a)\}$$

$$\geq \check{f}_{\mathcal{L} \cup \mathcal{R}}^n(a)$$

$\therefore \mathcal{L} \cup \mathcal{R}$  is a anti neutrosophic multi fuzzy right ideal of  $M$ .

### 3.3 Theorem:

Let  $\mathcal{L}$  and  $\mathcal{R}$  anti neutrosophic multi fuzzy ideal of  $M$  then  $\mathcal{L} \cup \mathcal{R}$  is a anti neutrosophic multi fuzzy ideal of  $M$ .

Proof: It is clear.

### 3.4 Theorem:

Let  $\mathcal{L}$  and  $\mathcal{R}$  anti neutrosophic multi fuzzy left ideal of  $M$  and then  $\mathcal{L} \cap \mathcal{R}$  is a anti neutrosophic multi fuzzy left ideal of  $M$ .

Proof:

Let  $\mathcal{L}$  and  $\mathcal{R}$  neutrosophic multi fuzzy left ideal of  $\xi$ .

$$\text{Let } a, b, \rho_1, \rho_2 \in M, \tau \in \gamma$$

$$\begin{aligned} 1. \quad \check{t}_{\mathcal{L} \cap \mathcal{R}}^n(a - b) &= \min\{\check{t}_{\mathcal{L}}^n(a - b), \check{t}_{\mathcal{R}}^n(a - b)\} \\ &\leq \min\{\{\max(\check{t}_{\mathcal{L}}^n(a), \check{t}_{\mathcal{L}}^n(b)), \max(\check{t}_{\mathcal{R}}^n(a), \check{t}_{\mathcal{R}}^n(b))\}\} \\ &\leq \max\{\{\min(\check{t}_{\mathcal{L}}^n(a), \check{t}_{\mathcal{R}}^n(a)), \min(\check{t}_{\mathcal{L}}^n(b), \check{t}_{\mathcal{R}}^n(b))\}\} \\ &\leq \max(\check{t}_{\mathcal{L} \cap \mathcal{R}}^n(a), \check{t}_{\mathcal{L} \cap \mathcal{R}}^n(b)) \end{aligned}$$

$$\begin{aligned} \check{i}_{\mathcal{L} \cap \mathcal{R}}^n(a - b) &= \max\{\check{i}_{\mathcal{L}}^n(a - b), \check{i}_{\mathcal{R}}^n(a - b)\} \\ &\geq \max\{\{\min(\check{i}_{\mathcal{L}}^n(a), \check{i}_{\mathcal{L}}^n(b)), \min(\check{i}_{\mathcal{R}}^n(a), \check{i}_{\mathcal{R}}^n(b))\}\} \\ &\geq \min\{\{\max(\check{i}_{\mathcal{L}}^n(a), \check{i}_{\mathcal{R}}^n(a)), \max(\check{i}_{\mathcal{L}}^n(b), \check{i}_{\mathcal{R}}^n(b))\}\} \\ &\geq \min(\check{i}_{\mathcal{L} \cap \mathcal{R}}^n(a), \check{i}_{\mathcal{L} \cap \mathcal{R}}^n(b)) \end{aligned}$$

$$\begin{aligned} \check{f}_{\mathcal{L} \cap \mathcal{R}}^n(a - b) &= \max\{\check{f}_{\mathcal{L}}^n(a - b), \check{f}_{\mathcal{R}}^n(a - b)\} \\ &\geq \max\{\{\min(\check{f}_{\mathcal{L}}^n(a), \check{f}_{\mathcal{L}}^n(b)), \min(\check{f}_{\mathcal{R}}^n(a), \check{f}_{\mathcal{R}}^n(b))\}\} \\ &\geq \min\{\{\max(\check{f}_{\mathcal{L}}^n(a), \check{f}_{\mathcal{R}}^n(a)), \max(\check{f}_{\mathcal{L}}^n(b), \check{f}_{\mathcal{R}}^n(b))\}\} \\ &\geq \min(\check{f}_{\mathcal{L} \cap \mathcal{R}}^n(a), \check{f}_{\mathcal{L} \cap \mathcal{R}}^n(b)) \end{aligned}$$

$$\begin{aligned} 2. \quad \check{t}_{\mathcal{L} \cap \mathcal{R}}^n(b + a - b) &= \min\{\check{t}_{\mathcal{L}}^n(b + a - b), \check{t}_{\mathcal{R}}^n(b + a - b)\} \\ &\leq \min\{\check{t}_{\mathcal{L}}^n(a), \check{t}_{\mathcal{R}}^n(a)\} \\ &\leq \check{t}_{\mathcal{L} \cap \mathcal{R}}^n(a) \end{aligned}$$

$$\begin{aligned} \check{i}_{\mathcal{L} \cap \mathcal{R}}^n(b + a - b) &= \max\{\check{i}_{\mathcal{L}}^n(b + a - b), \check{i}_{\mathcal{R}}^n(b + a - b)\} \\ &\geq \max\{\check{i}_{\mathcal{L}}^n(a), \check{i}_{\mathcal{R}}^n(a)\} \end{aligned}$$

$$\geq \ddot{i}_{\mathcal{L} \cap \mathcal{R}}^n(a)$$

$$\ddot{f}_{\mathcal{L} \cap \mathcal{R}}^n(b + a - b) = \max\{\ddot{f}_{\mathcal{L}}^n(b + a - b), \ddot{f}_{\mathcal{R}}^n(b + a - b)\}$$

$$\geq \max\{\ddot{f}_{\mathcal{L}}^n(a), \ddot{f}_{\mathcal{R}}^n(a)\}$$

$$\geq \ddot{f}_{\mathcal{L} \cap \mathcal{R}}^n(a)$$

$$3. \quad \ddot{t}_{\mathcal{L} \cap \mathcal{R}}^n((\rho_1 \tau(a + \rho_2) - \rho_1 \tau \rho_2))$$

$$= \min\{\ddot{t}_{\mathcal{L}}^n((\rho_1 \tau(a + \rho_2) - \rho_1 \tau \rho_2)), \ddot{t}_{\mathcal{R}}^n((\rho_1 \tau(a + \rho_2) - \rho_1 \tau \rho_2))\}$$

$$\leq \min\{\ddot{t}_{\mathcal{L}}^n(a), \ddot{t}_{\mathcal{R}}^n(a)\}$$

$$\leq \ddot{t}_{\mathcal{L} \cap \mathcal{R}}^n(a)$$

$$\ddot{i}_{\mathcal{L} \cap \mathcal{R}}^n((\rho_1 \tau(a + \rho_2) - \rho_1 \tau \rho_2))$$

$$= \max\{\ddot{i}_{\mathcal{L}}^n((\rho_1 \tau(a + \rho_2) - \rho_1 \tau \rho_2)), \ddot{i}_{\mathcal{R}}^n((\rho_1 \tau(a + \rho_2) - \rho_1 \tau \rho_2))\}$$

$$\geq \max\{\ddot{i}_{\mathcal{L}}^n(a), \ddot{i}_{\mathcal{R}}^n(a)\}$$

$$\geq \ddot{i}_{\mathcal{L} \cap \mathcal{R}}^n(a)$$

$$\ddot{f}_{\mathcal{L} \cap \mathcal{R}}^n((\theta_1 \tau(x + \theta_2) - \theta_1 \tau \theta_2))$$

$$= \max\{\ddot{f}_{\mathcal{L}}^n((\theta_1 \tau(x + \theta_2) - \theta_1 \tau \theta_2)), \ddot{f}_{\mathcal{R}}^n((\theta_1 \tau(x + \theta_2) - \theta_1 \tau \theta_2))\}$$

$$\geq \max\{\ddot{f}_{\mathcal{L}}^n(a), \ddot{f}_{\mathcal{R}}^n(a)\}$$

$$\geq \ddot{f}_{\mathcal{L} \cap \mathcal{R}}^n(a)$$

$\therefore \mathcal{L} \cap \mathcal{R}$  is a anti neutrosophic multi fuzzy left ideal of  $M$ .

### 3.5 Theorem:

Let  $\mathcal{L}$  and  $\mathcal{R}$  anti neutrosophic multi fuzzy right ideal of  $\mathbf{M}$  then  $\mathcal{L} \cap \mathcal{R}$  is a anti neutrosophic multi fuzzy right ideal of  $\mathbf{M}$ .

**Proof:**

Let  $\mathcal{L}$  and  $\mathcal{R}$  neutrosophic multi fuzzy right ideal of  $\mathbf{M}$ .

Let  $a, b, \rho_1, \rho_2 \in M, \tau \in \gamma$

$$\begin{aligned}
 1. \quad \check{t}_{\mathcal{L} \cap \mathcal{R}}^n(a - b) &= \min\{\check{t}_{\mathcal{L}}^n(a - b), \check{t}_{\mathcal{R}}^n(a - b)\} \\
 &\leq \min\{\{\max(\check{t}_{\mathcal{L}}^n(a), \check{t}_{\mathcal{L}}^n(b)), \max(\check{t}_{\mathcal{R}}^n(a), \check{t}_{\mathcal{R}}^n(b))\}\} \\
 &\leq \max\{\{\min(\check{t}_{\mathcal{L}}^n(a), \check{t}_{\mathcal{R}}^n(a)), \min(\check{t}_{\mathcal{L}}^n(b), \check{t}_{\mathcal{R}}^n(b))\}\} \\
 &\leq \max(\check{t}_{\mathcal{L} \cap \mathcal{R}}^n(a), \check{t}_{\mathcal{L} \cap \mathcal{R}}^n(b)) \\
 \check{i}_{\mathcal{L} \cap \mathcal{R}}^n(a - b) &= \max\{\check{i}_{\mathcal{L}}^n(a - b), \check{i}_{\mathcal{R}}^n(a - b)\} \\
 &\geq \max\{\{\min(\check{i}_{\mathcal{L}}^n(a), \check{i}_{\mathcal{L}}^n(b)), \min(\check{i}_{\mathcal{R}}^n(a), \check{i}_{\mathcal{R}}^n(b))\}\} \\
 &\geq \min\{\{\max(\check{i}_{\mathcal{L}}^n(a), \check{i}_{\mathcal{R}}^n(a)), \max(\check{i}_{\mathcal{L}}^n(b), \check{i}_{\mathcal{R}}^n(b))\}\} \\
 &\geq \min(\check{i}_{\mathcal{L} \cap \mathcal{R}}^n(a), \check{i}_{\mathcal{L} \cap \mathcal{R}}^n(b)) \\
 \check{f}_{\mathcal{L} \cap \mathcal{R}}^n(a - b) &= \max\{\check{f}_{\mathcal{L}}^n(a - b), \check{f}_{\mathcal{R}}^n(a - b)\} \\
 &\geq \max\{\{\min(\check{f}_{\mathcal{L}}^n(a), \check{f}_{\mathcal{L}}^n(b)), \min(\check{f}_{\mathcal{R}}^n(a), \check{f}_{\mathcal{R}}^n(b))\}\} \\
 &\geq \min\{\{\max(\check{f}_{\mathcal{L}}^n(a), \check{f}_{\mathcal{R}}^n(a)), \max(\check{f}_{\mathcal{L}}^n(b), \check{f}_{\mathcal{R}}^n(b))\}\} \\
 &\geq \min(\check{f}_{\mathcal{L} \cap \mathcal{R}}^n(a), \check{f}_{\mathcal{L} \cap \mathcal{R}}^n(b)) \\
 2. \quad \check{t}_{\mathcal{L} \cap \mathcal{R}}^n(b + a - b) &= \min\{\check{t}_{\mathcal{L}}^n(b + a - b), \check{t}_{\mathcal{R}}^n(b + a - b)\} \\
 &\leq \min\{\check{t}_{\mathcal{L}}^n(a), \check{t}_{\mathcal{R}}^n(a)\} \\
 &\leq \check{t}_{\mathcal{L} \cap \mathcal{R}}^n(a)
 \end{aligned}$$

$$\ddot{i}_{\mathcal{L} \cap \mathcal{R}}^n(b + a - b) = \max\{\ddot{i}_{\mathcal{L}}^n(b + a - b), \ddot{i}_{\mathcal{R}}^n(b + a - b)\}$$

$$\geq \max\{\ddot{i}_{\mathcal{L}}^n(a), \ddot{i}_{\mathcal{R}}^n(a)\}$$

$$\geq \ddot{i}_{\mathcal{L} \cap \mathcal{R}}^n(a)$$

$$\ddot{f}_{\mathcal{L} \cap \mathcal{R}}^n(b + a - b) = \max\{\ddot{f}_{\mathcal{L}}^n(b + a - b), \ddot{f}_{\mathcal{R}}^n(b + a - b)\}$$

$$\geq \max\{\ddot{f}_{\mathcal{L}}^n(a), \ddot{f}_{\mathcal{R}}^n(a)\}$$

$$\geq \ddot{f}_{\mathcal{L} \cap \mathcal{R}}^n(a)$$

$$3. \quad \ddot{t}_{\mathcal{L} \cap \mathcal{R}}^n(a \tau \rho_1) = \min\{\ddot{t}_{\mathcal{L}}^n(a \tau \rho_1), \ddot{t}_{\mathcal{R}}^n(a \tau \rho_1)\}$$

$$\leq \min\{\ddot{t}_{\mathcal{L}}^n(a), \ddot{t}_{\mathcal{R}}^n(a)\}$$

$$\leq \ddot{t}_{\mathcal{L} \cap \mathcal{R}}^n(a)$$

$$\ddot{i}_{\mathcal{L} \cap \mathcal{R}}^n(a \tau \rho_1) = \max\{\ddot{i}_{\mathcal{L}}^n(a \tau \rho_1), \ddot{i}_{\mathcal{R}}^n(a \tau \rho_1)\}$$

$$\geq \max\{\ddot{i}_{\mathcal{L}}^n(a), \ddot{i}_{\mathcal{R}}^n(a)\}$$

$$\geq \ddot{i}_{\mathcal{L} \cap \mathcal{R}}^n(a)$$

$$\ddot{f}_{\mathcal{L} \cap \mathcal{R}}^n(a \tau \rho_1) = \max\{\ddot{f}_{\mathcal{L}}^n(a \tau \rho_1), \ddot{f}_{\mathcal{R}}^n(a \tau \rho_1)\}$$

$$\geq \max\{\ddot{f}_{\mathcal{L}}^n(a), \ddot{f}_{\mathcal{R}}^n(a)\}$$

$$\geq \ddot{f}_{\mathcal{L} \cap \mathcal{R}}^n(a)$$

$\therefore \mathcal{L} \cap \mathcal{R}$  is a anti neutrosophic multi fuzzy right ideal of  $M$ .

### 3.6 Theorem:

Let  $\mathcal{L}$  and  $\mathcal{R}$  anti neutrosophic multi fuzzy ideal of  $M$  then  $\mathcal{L} \cap \mathcal{R}$  is also a anti neutrosophic multi fuzzy ideal of  $M$ .

**Proof:** It is clear.

#### 4. Anti Product of anti neutrosophic multi fuzzy ideals

In this section we define anti product of anti neutrosophic multi fuzzy  $\mathcal{Y}$  near ring  $M$ . We proved that anti product of anti neutrosophic multi fuzzy ideals of  $M$  is a anti neutrosophic multi fuzzy ideal of  $M$ .

##### 4.1 Definition:

Let  $\mathcal{L}$  and  $\mathcal{R}$  are two anti neutrosophic multi fuzzy ideals of  $\mathcal{Y}$  near rings  $M_1$  and  $M_2$  resp. Then the anti product of anti neutrosophic multi fuzzy subset of  $\mathcal{Y}$  near ring is defined by

$\mathcal{L}x\mathcal{R}: M_1xM_2 \rightarrow [0,1]$  such that

$$\mathcal{L}x\mathcal{R} = \{ \langle (a, b), \check{t}_{\mathcal{L}x\mathcal{R}}^n(a, b), \check{i}_{\mathcal{L}x\mathcal{R}}^n(a, b), \check{f}_{\mathcal{L}x\mathcal{R}}^n(a, b) \rangle : a \in M_1, b \in M_2 \}$$

Where  $\check{t}_{\mathcal{L}x\mathcal{R}}^n(a, b) = \max\{\check{t}_{\mathcal{L}}^n(a), \check{t}_{\mathcal{R}}^n(b)\}$

$$\check{i}_{\mathcal{L}x\mathcal{R}}^n(a, b) = \min\{\check{i}_{\mathcal{L}}^n(a), \check{i}_{\mathcal{R}}^n(b)\}$$

$$\check{f}_{\mathcal{L}x\mathcal{R}}^n(a, b) = \min\{\check{f}_{\mathcal{L}}^n(a), \check{f}_{\mathcal{R}}^n(b)\}$$

##### 4.2 Theorem:

Let  $\mathcal{L}$  and  $\mathcal{R}$  anti neutrosophic multi fuzzy left ideal of  $\mathcal{Y}$  near rings  $M_1$  and  $M_2$  then  $\mathcal{L}x\mathcal{R}$  is also a anti neutrosophic multi fuzzy left ideal of  $M_1xM_2$ .

Proof:

Let  $\mathcal{L}$  and  $\mathcal{R}$  be anti neutrosophic fuzzy left ideals of  $M_1xM_2$  respectively

Let  $(a_1, a_2), (b_1, b_2), (\rho_1, \rho_2) \in M_1xM_2$

$$\begin{aligned} 1. \quad \check{t}_{\mathcal{L}x\mathcal{R}}^n((a_1, a_2) - (b_1, b_2)) &= \check{t}_{\mathcal{L}x\mathcal{R}}^n(a_1 - b_1, a_2 - b_2) \\ &= \max(\check{t}_{\mathcal{L}}^n(a_1 - b_1), \check{t}_{\mathcal{R}}^n(a_2 - b_2)) \end{aligned}$$

$$\begin{aligned}
&\leq \max \{ \max (\ddot{t}_{\mathcal{L}}^n (a_1), \ddot{t}_{\mathcal{L}}^n (b_1)), \max [\ddot{t}_{\mathcal{R}}^n (a_2), \ddot{t}_{\mathcal{R}}^n (b_2)] \} \\
&\leq \max \{ \max [\ddot{t}_{\mathcal{L}}^n (a_1), \ddot{t}_{\mathcal{R}}^n (a_2)], \max [\ddot{t}_{\mathcal{L}}^n (b_1), \ddot{t}_{\mathcal{R}}^n (b_2)] \} \\
&\leq \max (\ddot{t}_{\mathcal{L} \times \mathcal{R}}^n (a_1, a_2), \ddot{t}_{\mathcal{L} \times \mathcal{R}}^n (b_1, b_2))
\end{aligned}$$

$$\begin{aligned}
\ddot{i}_{\mathcal{L} \times \mathcal{R}}^n ((a_1, a_2) - (b_1, b_2)) &= \ddot{i}_{\mathcal{L} \times \mathcal{R}}^n (a_1 - b_1, a_2 - b_2) \\
&= \min (\ddot{i}_{\mathcal{L}}^n (a_1 - b_1), \ddot{i}_{\mathcal{R}}^n (a_2 - b_2)) \\
&\geq \min \{ \min (\ddot{i}_{\mathcal{L}}^n (a_1), \ddot{i}_{\mathcal{L}}^n (b_1)), \min [\ddot{i}_{\mathcal{R}}^n (a_2), \ddot{i}_{\mathcal{R}}^n (b_2)] \} \\
&\geq \min \{ \min [\ddot{i}_{\mathcal{L}}^n (a_1), \ddot{i}_{\mathcal{R}}^n (a_2)], \min [\ddot{i}_{\mathcal{L}}^n (b_1), \ddot{i}_{\mathcal{R}}^n (b_2)] \} \\
&\geq \min (\ddot{i}_{\mathcal{L} \times \mathcal{R}}^n (a_1, a_2), \ddot{i}_{\mathcal{L} \times \mathcal{R}}^n (b_1, b_2))
\end{aligned}$$

$$\begin{aligned}
\ddot{f}_{\mathcal{L} \times \mathcal{R}}^n ((a_1, a_2) - (b_1, b_2)) &= \ddot{f}_{\mathcal{L} \times \mathcal{R}}^n (a_1 - b_1, a_2 - b_2) \\
&= \min (\ddot{f}_{\mathcal{L}}^n (a_1 - b_1), \ddot{f}_{\mathcal{R}}^n (a_2 - b_2)) \\
&\geq \min \{ \min (\ddot{f}_{\mathcal{L}}^n (a_1), \ddot{f}_{\mathcal{L}}^n (b_1)), \min [\ddot{f}_{\mathcal{R}}^n (a_2), \ddot{f}_{\mathcal{R}}^n (b_2)] \} \\
&\geq \min \{ \min [\ddot{f}_{\mathcal{L}}^n (a_1), \ddot{f}_{\mathcal{R}}^n (a_2)], \min [\ddot{f}_{\mathcal{L}}^n (b_1), \ddot{f}_{\mathcal{R}}^n (b_2)] \} \\
&\geq \min (\ddot{f}_{\mathcal{L} \times \mathcal{R}}^n (a_1, a_2), \ddot{f}_{\mathcal{L} \times \mathcal{R}}^n (b_1, b_2))
\end{aligned}$$

$$\begin{aligned}
2. \quad \ddot{t}_{\mathcal{L} \times \mathcal{R}}^n ((b_1, b_2) + (a_1, a_2) - (b_1, b_2)) &= \ddot{t}_{\mathcal{L} \times \mathcal{R}}^n (b_1 + a_1 - b_1, b_2 + a_2 - b_2) \\
&= \max (\ddot{t}_{\mathcal{L}}^n (b_1 + a_1 - b_1), \ddot{t}_{\mathcal{R}}^n (b_2 + a_2 - b_2)) \\
&\leq \max \{ \ddot{t}_{\mathcal{L}}^n (a_1), \ddot{t}_{\mathcal{R}}^n (a_2) \} \\
&\leq \ddot{t}_{\mathcal{L} \times \mathcal{R}}^n (a_1, a_2)
\end{aligned}$$

$$\begin{aligned}
\ddot{i}_{\mathcal{L} \times \mathcal{R}}^n ((b_1, b_2) + (a_1, a_2) - (b_1, b_2)) &= \ddot{i}_{\mathcal{L} \times \mathcal{R}}^n (b_1 + a_1 - b_1, b_2 + a_2 - b_2) \\
&\geq \min \{ \ddot{i}_{\mathcal{L}}^n (a_1), \ddot{i}_{\mathcal{R}}^n (a_2) \} \\
&\geq \ddot{i}_{\mathcal{L} \times \mathcal{R}}^n (a_1, a_2)
\end{aligned}$$

$$\begin{aligned}
\check{f}_{\mathcal{L} \times \mathcal{R}}^n((b_1, b_2) + (a_1, a_2) - (b_1, b_2)) &= \check{f}_{\mathcal{L} \times \mathcal{R}}^n(b_1 + a_1 - b_1, b_2 + a_2 - b_2) \\
&\geq \min\{\check{f}_{\mathcal{L}}^n(a_1), \check{f}_{\mathcal{R}}^n(a_2)\} \\
&\geq \check{f}_{\mathcal{L} \times \mathcal{R}}^n(a_1, a_2)
\end{aligned}$$

$$\begin{aligned}
3. \quad \check{t}_{\mathcal{L} \times \mathcal{R}}^n((\rho_1 \tau(a_1, a_2) + \rho_2) - \rho_1 \tau \rho_2) \\
&= \check{t}_{\mathcal{L} \times \mathcal{R}}^n(\rho_1 \tau(a_1 + \rho_2) - \rho_1 \tau \rho_2, \rho_1 \tau(a_2 + \rho_2) - \rho_1 \tau \rho_2) \\
&= \max\{\check{t}_{\mathcal{L}}^n((\rho_1 \tau(a_1 + \rho_2) - \rho_1 \tau \rho_2)), \check{t}_{\mathcal{R}}^n((\rho_1 \tau(a_2 + \rho_2) - \rho_1 \tau \rho_2))\} \\
&\leq \max\{\check{t}_{\mathcal{L}}^n(a_1), \check{t}_{\mathcal{R}}^n(a_2)\} \\
&\leq \check{t}_{\mathcal{L} \times \mathcal{R}}^n(a_1, a_2)
\end{aligned}$$

$$\begin{aligned}
\check{i}_{\mathcal{L} \times \mathcal{R}}^n((\rho_1 \tau(a_1, a_2 + \rho_2) - \rho_1 \tau \rho_2) \\
&= \check{i}_{\mathcal{L} \times \mathcal{R}}^n(\rho_1 \tau(a_1 + \rho_2) - \rho_1 \tau \rho_2, \rho_1 \tau(a_2 + \rho_2) - \rho_1 \tau \rho_2) \\
&= \min\{\check{i}_{\mathcal{L}}^n((\rho_1 \tau(a_1 + \rho_2) - \rho_1 \tau \rho_2)), \check{i}_{\mathcal{R}}^n((\rho_1 \tau(a_2 + \rho_2) - \rho_1 \tau \rho_2))\} \\
&\geq \min\{\check{i}_{\mathcal{L}}^n(a_1), \check{i}_{\mathcal{R}}^n(a_2)\} \\
&\geq \check{i}_{\mathcal{L} \times \mathcal{R}}^n(a_1, a_2)
\end{aligned}$$

$$\begin{aligned}
\check{f}_{\mathcal{L} \times \mathcal{R}}^n((\rho_1 \tau(a_1, a_2 + \rho_2) - \rho_1 \tau \rho_2) \\
&= \check{f}_{\mathcal{L} \times \mathcal{R}}^n(\rho_1 \tau(a_1 + \rho_2) - \rho_1 \tau \rho_2, \rho_1 \tau(a_2 + \rho_2) - \rho_1 \tau \rho_2) \\
&= \min\{\check{f}_{\mathcal{L}}^n((\rho_1 \tau(a_1 + \rho_2) - \rho_1 \tau \rho_2)), \check{f}_{\mathcal{R}}^n((\rho_1 \tau(a_2 + \rho_2) - \rho_1 \tau \rho_2))\} \\
&\geq \min\{\check{f}_{\mathcal{L}}^n(a_1), \check{f}_{\mathcal{R}}^n(a_2)\} \\
&\geq \check{f}_{\mathcal{L} \times \mathcal{R}}^n(a_1, a_2)
\end{aligned}$$

$\therefore \mathcal{L} \times \mathcal{R}$  is also a anti neutrosophic multi fuzzy left ideal of  $M_1 \times M_2$ .

### 4.3 Theorem:



Let  $\mathcal{L}$  and  $\mathcal{R}$  anti neutrosophic multi fuzzy right ideal of  $\mathcal{Y}$  near rings  $M_1$  and  $M_2$  then  $\mathcal{LxR}$  is also a anti neutrosophic multi fuzzy right ideal of  $M_1xM_2$ .

Proof:

Let  $\mathcal{L}$  and  $\mathcal{R}$  be anti neutrosophic fuzzy right ideals of  $M_1xM_2$  respectively

Let  $(a_1, a_2), (b_1, b_2), (\rho_1, \rho_2) \in M_1xM_2$

$$\begin{aligned}
 1. \quad \check{t}_{\mathcal{LxR}}^n((a_1, a_2) - (b_1, b_2)) &= \check{t}_{\mathcal{LxR}}^n(a_1 - b_1, a_2 - b_2) \\
 &= \max(\check{t}_{\mathcal{L}}^n(a_1 - b_1), \check{t}_{\mathcal{R}}^n(a_2 - b_2)) \\
 &\leq \max\{\max(\check{t}_{\mathcal{L}}^n(a_1), \check{t}_{\mathcal{L}}^n(b_1)), \max[\check{t}_{\mathcal{R}}^n(a_2), \check{t}_{\mathcal{R}}^n(b_2)]\} \\
 &\leq \max\{\max[\check{t}_{\mathcal{L}}^n(a_1), \check{t}_{\mathcal{R}}^n(a_2)], \max[\check{t}_{\mathcal{L}}^n(b_1), \check{t}_{\mathcal{R}}^n(b_2)]\} \\
 &\leq \max(\check{t}_{\mathcal{LxR}}^n(a_1, a_2), \check{t}_{\mathcal{LxR}}^n(b_1, b_2))
 \end{aligned}$$

$$\begin{aligned}
 \check{i}_{\mathcal{LxR}}^n((a_1, a_2) - (b_1, b_2)) &= \check{i}_{\mathcal{LxR}}^n(a_1 - b_1, a_2 - b_2) \\
 &= \min(\check{i}_{\mathcal{L}}^n(a_1 - b_1), \check{i}_{\mathcal{R}}^n(a_2 - b_2)) \\
 &\geq \min\{\min(\check{i}_{\mathcal{L}}^n(a_1), \check{i}_{\mathcal{L}}^n(b_1)), \min[\check{i}_{\mathcal{R}}^n(a_2), \check{i}_{\mathcal{R}}^n(b_2)]\} \\
 &\geq \min\{\min[\check{i}_{\mathcal{L}}^n(a_1), \check{i}_{\mathcal{R}}^n(a_2)], \min[\check{i}_{\mathcal{L}}^n(b_1), \check{i}_{\mathcal{R}}^n(b_2)]\} \\
 &\geq \min(\check{i}_{\mathcal{LxR}}^n(a_1, a_2), \check{i}_{\mathcal{LxR}}^n(b_1, b_2))
 \end{aligned}$$

$$\begin{aligned}
 \check{f}_{\mathcal{LxR}}^n((a_1, a_2) - (b_1, b_2)) &= \check{f}_{\mathcal{LxR}}^n(a_1 - b_1, a_2 - b_2) \\
 &= \min(\check{f}_{\mathcal{L}}^n(a_1 - b_1), \check{f}_{\mathcal{R}}^n(a_2 - b_2)) \\
 &\geq \min\{\min(\check{f}_{\mathcal{L}}^n(a_1), \check{f}_{\mathcal{L}}^n(b_1)), \min[\check{f}_{\mathcal{R}}^n(a_2), \check{f}_{\mathcal{R}}^n(b_2)]\} \\
 &\geq \min\{\min[\check{f}_{\mathcal{L}}^n(a_1), \check{f}_{\mathcal{R}}^n(a_2)], \min[\check{f}_{\mathcal{L}}^n(b_1), \check{f}_{\mathcal{R}}^n(b_2)]\} \\
 &\geq \min(\check{f}_{\mathcal{LxR}}^n(a_1, a_2), \check{f}_{\mathcal{LxR}}^n(b_1, b_2))
 \end{aligned}$$

$$\begin{aligned}
2. \quad \check{t}_{\mathcal{L} \times \mathcal{R}}^n((b_1, b_2) + (a_1, a_2) - (b_1, b_2)) &= \check{t}_{\mathcal{L} \times \mathcal{R}}^n(b_1 + a_1 - b_1, b_2 + a_2 - b_2) \\
&= \max\{\check{t}_{\mathcal{L}}^n(b_1 + a_1 - b_1), \check{t}_{\mathcal{R}}^n(b_2 + a_2 - b_2)\} \\
&\leq \max\{\check{t}_{\mathcal{L}}^n(a_1), \check{t}_{\mathcal{R}}^n(a_2)\} \\
&\leq \check{t}_{\mathcal{L} \times \mathcal{R}}^n(a_1, a_2)
\end{aligned}$$

$$\begin{aligned}
\check{i}_{\mathcal{L} \times \mathcal{R}}^n((b_1, b_2) + (a_1, a_2) - (b_1, b_2)) &= \check{i}_{\mathcal{L} \times \mathcal{R}}^n(b_1 + a_1 - b_1, b_2 + a_2 - b_2) \\
&\geq \min\{\check{i}_{\mathcal{L}}^n(a_1), \check{i}_{\mathcal{R}}^n(a_2)\} \\
&\geq \check{i}_{\mathcal{L} \times \mathcal{R}}^n(a_1, a_2)
\end{aligned}$$

$$\begin{aligned}
\check{f}_{\mathcal{L} \times \mathcal{R}}^n((b_1, b_2) + (a_1, a_2) - (b_1, b_2)) &= \check{f}_{\mathcal{L} \times \mathcal{R}}^n(b_1 + a_1 - b_1, b_2 + a_2 - b_2) \\
&\geq \min\{\check{f}_{\mathcal{L}}^n(a_1), \check{f}_{\mathcal{R}}^n(a_2)\} \\
&\geq \check{f}_{\mathcal{L} \times \mathcal{R}}^n(a_1, a_2)
\end{aligned}$$

$$\begin{aligned}
3. \quad \check{t}_{\mathcal{L} \times \mathcal{R}}^n((a_1, a_2) \tau (\rho_1, \rho_2)) &= \check{t}_{\mathcal{L} \times \mathcal{R}}^n\{(a_1 \tau \rho_1), (a_2 \tau \rho_2)\} \\
&\leq \max\{\check{t}_{\mathcal{L}}^n(a_1), \check{t}_{\mathcal{R}}^n(a_2)\} \\
&\leq \check{t}_{\mathcal{L} \times \mathcal{R}}^n(a_1, a_2)
\end{aligned}$$

$$\begin{aligned}
\check{i}_{\mathcal{L} \times \mathcal{R}}^n((a_1, a_2) \tau (\rho_1, \rho_2)) &= \check{i}_{\mathcal{L} \times \mathcal{R}}^n\{(a_1 \tau \rho_1), (a_2 \tau \rho_2)\} \\
&\geq \min\{\check{i}_{\mathcal{L}}^n(a_1), \check{i}_{\mathcal{R}}^n(a_2)\} \\
&\geq \check{i}_{\mathcal{L} \times \mathcal{R}}^n(a_1, a_2)
\end{aligned}$$

$$\begin{aligned}
\check{f}_{\mathcal{L} \times \mathcal{R}}^n((a_1, a_2) \tau (\rho_1, \rho_2)) &= \check{f}_{\mathcal{L} \times \mathcal{R}}^n\{(a_1 \tau \rho_1), (a_2 \tau \rho_2)\} \\
&\geq \min\{\check{f}_{\mathcal{L}}^n(a_1), \check{f}_{\mathcal{R}}^n(a_2)\} \\
&\geq \check{f}_{\mathcal{L} \times \mathcal{R}}^n(a_1, a_2)
\end{aligned}$$

$\therefore \mathcal{L} \times \mathcal{R}$  is also a anti neutrosophic multi fuzzy right ideal of  $M_1 \times M_2$ .

#### 4.4 Theorem:

Let  $\mathcal{L}$  and  $\mathcal{R}$  anti neutrosophic multi fuzzy ideal of  $\gamma$  near rings  $M_1$  and  $M_2$  then  $\mathcal{L} \times \mathcal{R}$  is also a anti neutrosophic multi fuzzy ideal of  $M_1 \times M_2$ .

**Proof:** It is clear

#### 5. Conclusion

To conclude, the notion of neutrosophic multi fuzzy gamma near-ring, neutrosophic multi fuzzy ideals of gamma near-rings have been discussed. The proof for the theorem that states “Union and Intersection of two neutrosophic multi fuzzy ideals of gamma near-ring is also a Neutrosophic multi fuzzy ideal of gamma near-ring” has been provided.

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