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Complex neutrosophic N-soft sets: A new model with applications

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Abstract: In this paper we establish the notion of complex single-valued neutrosophic N-soft set. It improves the traits of three general models, namely, single-valued neutrosophic sets, single-valued neutrosophic soft sets and single-valued neutrosophic N-soft sets, in such way that it makes two dimensional ambiguous information and parameterized grading evaluation compatible. We explain the modeling abilities of complex single-valued neutrosophic N-soft sets and investigate some of their fundamental properties. Moreover, the intended approach hinges on rational attributes to support the choice of the most suitable solution. The proposed method is explicated through an example from the islamic banking industry. We also perform a comparative analysis with respect to the neutrosophic TOPSIS method.

Keywords: Complex single-valued neutrosophic set, N-soft set, TOPSIS method, MAGDM.

1 Introduction

A fascinating research article by Smarandache [29] has attracted the attention of many researchers since 1998. Neutrosophic sets ($\mathcal{N}Ss$) had been born that year. They are based on formal logic that contemplates the nature, origin, and scope of objectivities with their relations for numerous intellectual spectra. The neutrosophic theory comprises probability, set theory, logics, and statistics. As such it copes with real life events characterized by degree of satisfaction, dissatisfaction and indeterminacy. It is therefore acknowledged to provide a generalization of both classic set, fuzzy set, intuitionistic fuzzy sets, interval-valued intuitionistic fuzzy sets and Pythagorean fuzzy sets [38, 17, 33]. Neutrosophic-inspired sets are classified into many subclasses like interval-valued neutrosophic sets, single-valued neutrosophic sets ($SV\mathcal{N}Ss$), and the subclass known as simplified neutrosophic set. The $SV\mathcal{N}Ss$ were introduced by Wang and Smarandache [31, 30]. They can be characterized by three real valued functions whose values are taken from the unit closed interval $[0, 1]$, therefore it is more convenient and applicable in many areas of science and engineering. After Wang and Smarandache, the single-valued neutrosophic environment has been scrutinized extensively. For example, Ye [34] provided a correlation coefficient between $SV\mathcal{N}Ss$ which became a useful tool for decision making, and Akram and Luqman [6] illustrated the concept of $SV\mathcal{N}Ss$ with the flavor of hypergraphs.

Another breakthrough was Ramot et al. [26] who extended the 1-dimensional fuzzy perspective [38] to 2-dimensional phenomena. The resulting model was called complex fuzzy sets. This new perspective prompted many authors to adapt existing models to the complex spirit. Thus complex intuitionistic fuzzy sets [15] and complex Pythagorean fuzzy sets [37], which are precisely related to multi-attribute decision making (MAGDM) phenomena, were soon developed.

The two aforementioned expressions of vagueness were made compatible by Ali and Smarandache [13]. These authors put forward the notion of complex neutrosophic set under the influence of both neutrosophic sets [29] and complex fuzzy sets [26].

In MAGDM problems, the opinions of people are not invariably expressed through binary evaluations. It is often easier to bring up decisions using non-binary evaluations, specifically in the case of qualitative information such as the perceived performance of banking industry, people's morality, hospital assistance, etc. Hence, Fatimah et al. [21] firstly presented N-soft sets and applied them on decision making methods based on non-binary evaluations. N-soft sets extended the scope of soft sets [25] whose foundation is that any alternative can be characterized by a selected list of attributes. Many real examples were given [11, 21]. Stimulated from the novel concept of N-soft set, Akram et al. [5] solved decision making problems using the hybrid combination of fuzzy set with N-soft set that improves the performance of fuzzy soft sets [10]. Further, Akram et al. [9] presented the novel idea of intuitionistic fuzzy N-soft sets ($IFNS_fSs$), Pythagorean fuzzy N-soft sets ($PFNS_fSs$) have been introduced by Zhang [39] in 2020, and recently the multi-fuzzy N-soft set model has been presented alongside its applications to decision-making [22]. This proves that N-soft sets are a trendy topic and that the model is amenable to hybridization from many standpoints including rough set theory [11] and hesitancy [4] in addition to the ideas discussed above.

The theoretical models called neutrosophic soft sets ($\mathcal{N}S_fSs$) and single-valued neutrosophic soft sets ($SV\mathcal{N}S_fSs$) were put forward by Maji [40] and Jana et al. [23], respectively. The parametrized nature of the attributes that characterizes soft set theory is combined with neutrosophic information and the possibilities of these new models are discussed in detail. Ashraf and Butt [16] and Riaz et al. [27] first established a theoretical model for neutrosophic N-soft sets ($\mathcal{N}NS_fSs$). They made applications to business and the medical field supported by the TOPSIS method, respectively. Moreover, Sahin et al. [28] used the framework of ($SV\mathcal{N}S_fSs$) for the development of a TOPSIS method which helped to find the most suitable supplier for a production industry. In 2015, Ye [36] introduced single-valued neutrosophic linguistic numbers ($SV\mathcal{N}LNs$) as an extension of intuitionistic linguistic numbers and further set theoretical description for single-valued neutrosophic linguistic-TOPSIS method. More recently, Akram et al. [7, 8] have presented new decision making methods.

In this manuscript we present a quite general model known as complex single-valued neutrosophic N-soft set ($CSV\mathcal{N}S_fS$). It describes the possibility that the parameterized nature of the universe may be complex single-valued neutrosophic, which comprises functions for satisfaction degree, hesitancy degree and dissatisfaction degree whose values are taken from the complex unit circle. The hesitancy degree and ordered grades endow the $CSV\mathcal{N}S_fS$ with excellent qualities, so much so that this model dominates over the existing $C\mathcal{N}Ss$, $\mathcal{N}NS_fSs$ and $SV\mathcal{N}Ss$.

The motivation for this paper depends upon the following elements:

1. The $\mathcal{N}NS_fSs$ and $IFNS_fSs$ have the ability to express situations including an indeterminacy part with ordered grades, but they are not designed to deal with two dimensional ambiguity in the parametric information.
2. Moreover, $SV\mathcal{N}Ss$ and $C\mathcal{N}Ss$ can tackle the hesitancy degree in human judgment with periodic terms, but they cannot assist us in the decision making problems based on non-binary evaluations or ranking systems.
3. These limitations encouraged us to present the idea of $CSV\mathcal{N}S_fS$ which competently handles the phase term of 2-dimensional problems with ordered grades, indeterminacy, hesitancy and incomplete figures in their decisions.

The practical contribution of this article is the formalization of the $CSV\mathcal{N}S_fS$-TOPSIS technique for solving MAGDM problems that require the use of $CSV\mathcal{N}S_f$ information. For this purpose, we define some basic notions and the $CSV\mathcal{N}S_fSSs$ and $CSV\mathcal{N}S_f$ averaging and geometric operators. These operators allow us to combine the decisions according to the performance of the alternatives and the weightage of the relevant attributes and experts. We also define score and accuracy function sof $CSV\mathcal{N}S_fNs$ for the sake of $CSV\mathcal{N}S_f$-PIS and $CSV\mathcal{N}S_f$-NIS. Finally, we can sort out the alternatives using a revised closeness index whose values are totally based upon the normalized Euclidean distance.

The authenticity of the presented technique is verified by a numerical example that concerns the monitoring performance of the Islamic banking industry on the basis of the CAMELS rating system. Moreover, a comparison of the proposed model with the $SV\mathcal{N}$-TOPSIS method substantiates the accuracy and reliability of the results and of our novel technique. For further useful notions related to N-soft sets not discussed in the paper, the readers are referred to [1, 2, 12]

The arrangement of this paper is as follows. Section 2 contains some basic definitions related to the proposed model. In Section 3 we describe the main features of the presented theory with some operations and properties. Section 4 presents the score function, accuracy function and some aggregation operators related to $CSV\mathcal{N}NS_fNs$. Section 5, gives a brief description for the $CSV\mathcal{N}NS_f$-TOPSIS method with a specific algorithm. Section 6, models a MAGDM problem and applies the proposed technique to find a solution. Section 7 comprises the comparison analysis with the $CSV\mathcal{N}$-TOPSIS method. In Section 8, we come to the conclusion with some ideas for future research works.

2 Preliminaries

Definition 1. *[29] A neutrosophic set ($\mathcal{N}S$) Ψ on a universe of discourse $\mathbb{U}$ has the form:*

$$\Psi = \langle u, \mathbb{T}_\Psi(u), \mathbb{I}_\Psi(u), \mathbb{F}_\Psi(u) : u \in \mathbb{U}\rangle,$$

where, $\mathbb{T}_\Psi(u)$, $\mathbb{I}_\Psi(u)$ and $\mathbb{F}_\Psi(u)$ are degree of satisfaction, degree of indeterminacy and degree of dissatisfaction, respectively, belongs to non-standard interval $]^-0, 1^+[$, for every $u \in \mathbb{U}$.

Definition 2. *[31] A single-valued neutrosophic set ($SV\mathcal{N}S$) Ψ on a universe of discourse $\mathbb{U}$ has the form*

$$\Psi = \langle u, \mathbb{T}_\Psi(u), \mathbb{I}_\Psi(u), \mathbb{F}_\Psi(u) : u \in \mathbb{U}\rangle,$$

where $\mathbb{T}_\Psi(u), \mathbb{I}_\Psi(u), \mathbb{F}_\Psi(u) : U \to [0, 1]$ are the degree of truthness, degree of hesitancy and degree of falsity, respectively, without any condition on the sum of $\mathbb{T}_\Psi(u)$, $\mathbb{I}_\Psi(u)$ and $\mathbb{F}_\Psi(u)$ for all $u \in \mathbb{U}$. The triplet $(\mathbb{T}_\Psi, \mathbb{I}_\Psi, \mathbb{F}_\Psi)$ is called single-valued neutrosophic number ($SV\mathcal{N}N$).

Definition 3. *[13] A complex single-valued neutrosophic set ($CSV\mathcal{N}S$) Ψ, on the universe $\mathbb{U}$ is defined as:*

$$\Psi = \langle u, \mathbb{T}_\Psi(u), \mathbb{I}_\Psi(u), \mathbb{F}_\Psi(u) : u \in \mathbb{U}\rangle,$$

where $\mathbb{T}_\Psi(u) = p_\Psi(u)e^{i2\pi t_\Psi(u)}$, $\mathbb{I}_\Psi(u) = q_\Psi(u)e^{i2\pi\omega_\Psi(u)}$ and $\mathbb{F}_\Psi(u) = r_\Psi(u)e^{i2\pi f_\Psi(u)}$, denote the degree of truthness, degree of hesitancy and degree of falsity, respectively, without any conditions on the sum of amplitude terms $p_\Psi(u)$, $q_\Psi(u)$, $r_\Psi(u) : U \to [0, 1]$ or the phase terms $t_\Psi(u)$, $\omega_\Psi(u)$, $f_\Psi(u) : U \to [0, 1]$ for all $u \in \mathbb{U}$. The triplet $(p_\Psi(u)e^{i2\pi t_\Psi(u)}, q_\Psi(u)e^{i2\pi\omega_\Psi(u)}, r_\Psi(u)e^{i2\pi f_\Psi(u)})$ is called complex single-valued neutrosophic number ($CSV\mathcal{N}N$).

Definition 4. *[25] Let $\mathbb{U}$ be a non-empty set and K be a set of parameters and $Y \subseteq K$. A soft set S_fS over $\mathbb{U}$ is a pair (Φ, Y), where $\Phi : K \to P(\mathbb{U})$ is a set-valued function defined as:*

$$(\Phi, Y) = \{\langle y_w, \Phi(y_w)\rangle | y_w \in Y, \Phi(y_w) \in P(\mathbb{U})\}.$$

Definition 5. *Let $\mathbb{U}$ be a non-empty set and K be a set of parameters and $Y \subseteq K$. A complex single-valued neutrosophic soft set $CSV\mathcal{N}S_fS$ over $\mathbb{U}$ is a pair (Φ, Y), where $\Phi : K \to \mathbb{P}(CSV\mathcal{N}S)$ is a set-valued function defined as:*

$$\begin{aligned}(\Phi, Y) &= \{\langle y_w, \Phi(y_w)\rangle | y_w \in Y, \Phi(y_w) \in \mathbb{P}(CSV\mathcal{N}S)\}\\ &= \{\langle y_w, (u_s, (\mathbb{T}_{ws}, \mathbb{I}_{ws}, \mathbb{F}_{ws}))\rangle\}\\ &= \{\langle y_w, (u_s, (p_{ws}e^{i2\pi t_{ws}}, q_{ws}e^{i2\pi\omega_{ws}}, r_{ws}e^{i2\pi f_{ws}}))\rangle\},\end{aligned}$$

where $\mathbb{P}(CSV\mathcal{N}S)$ is the collection of all subsets of $CSV\mathcal{N}Ss$ over the non-empty set $\mathbb{U}$ and $p_{ws}, t_{ws}, q_{ws}, \omega_{ws}, r_{ws}, f_{ws} \in [0, 1]$.

Definition 6. *[21]Let $\mathbb{U}$ be a non-empty set and K be a set of parameters and $Y \subseteq K$. Let $H = \{0, 1, 2, \ldots, N-1\}$ be a set of ordered grades with $N \in \{2, 3, \ldots\}$. A triple (Φ, Y, N) is called N-soft set (NS_fS) over $\mathbb{U}$ if Φ is a mapping define as $\Phi : Y \to 2^{\mathbb{U}\times H}$, that is there exist a unique pair $(u_s, h^s_w) \in \mathbb{U} \times H$ such that $(u_s, h^s_w) \in \Phi(y_w)$, where $u_s \in \mathbb{U}, h^s_w \in H$.*

3 Complex single-valued neutrosophic N-soft sets

Definition 7. *Let $\mathbb{U}$ be a non-empty set and K be a set of parameters with $Y \subseteq K$. Let $H = \{0, 1, 2, \ldots, N-1\}$ be a set of ordered grades with $N \in \{2, 3, \ldots\}$. A triple (Φ_Ψ, Y, N) is called a complex single-valued neutrosophic N-soft set $(CSV\mathcal{N}NS_fS)$ on Y, if (Φ, Y, N) is an NS_fS on $\mathbb{U}$, and $\Phi_\Psi : Y \to 2^{\mathbb{U}\times H} \times \mathbb{CSV\mathcal{N}N}$ is a mapping, which is defined as:*

$$\begin{aligned}\Phi_\Psi(y_w) &= \{\langle(\Phi(y_w), \Psi(y_w))\rangle : y_w \in Y\},\\ &= \{\langle((u_s, h^s_w), (\mathbb{T}_{ws}, \mathbb{I}_{ws}, \mathbb{F}_{ws}))\rangle\},\\ &= \{\langle((u_s, h^s_w), (p_{ws}e^{i2\pi t_{ws}}, q_{ws}e^{i2\pi\omega_{ws}}, r_{ws}e^{i2\pi f_{ws}}))\rangle\},\end{aligned}$$

where $\Phi : Y \to 2^{\mathbb{U}\times H}$, $\Psi : Y \to \mathbb{CSV\mathcal{N}N}$, and $\mathbb{CSV\mathcal{N}N}$ denotes the collection of all complex single-valued neutrosophic numbers of $\mathbb{U}$, h^s_w denotes the rank of parameter for the alternative y_w and $p_{ws}, t_{ws}, q_{ws}, \omega_{ws}, r_{ws}, f_{ws} \in [0, 1]$, with no conditions on their sum.

Example 1. *Let $\mathbb{U} = \{\mathbb{U}_1 =$ Emirates, $\mathbb{U}_2 =$ Eithad Airways, $\mathbb{U}_3 =$ Turkish airlines, $\mathbb{U}_4 =$ Flynas $\}$ be the set of airlines from Pakistan to Turkey and $Y = \{Y_1 =$ Price , $Y_2 =$ Entertainment, $Y_3 =$ luxuries, $Y_4 =$ Safety $\}$ be the characteristics which are experienced by the passengers and then passengers assigned ratings to these airlines. These ratings are aggregated by the experts and form a 6-soft set given Table 1, where*

0 means 'very Bad'

1 means 'Bad'

2 means 'Ok'

3 means 'Good'

4 means 'Great'

5 means 'Excellent'

Table 1: 6-soft set evaluated by experts

$Y/\mathbb{U}$	$\mathbb{U}_1$	$\mathbb{U}_2$	$\mathbb{U}_3$	$\mathbb{U}_4$
Y_1	3	5	0	1
Y_2	1	4	2	0
Y_3	2	1	4	3
Y_4	5	0	1	2

For handling the alternatives with fuzziness property related to parameters, we need $CSV\mathcal{N}NS_fSs$. Therefore, authorities defined grading criteria, given in Table 2, for the evaluation of airlines under the environment of $CSV\mathcal{N}NS_fSs$, where Table 2 is evaluated from the following criteria:

$$\begin{aligned}&when\ h^s_w = 0, && -4.00 \leq S(\Psi) < -3.30,\\ &when\ h^s_w = 1, && -3.30 \leq S(\Psi) < -2.20,\\ &when\ h^s_w = 2, && -2.20 \leq S(\Psi) < -1.00,\\ &when\ h^s_w = 3, && -1.00 \leq S(\Psi) < 0.20,\\ &when\ h^s_w = 4, && 0.20 \leq S(\Psi) < 1.20,\\ &when\ h^s_w = 5, && 1.20 \leq S(\Psi) \leq 2.000.\end{aligned}$$

Table 2: Grading criteria for $CSV\mathcal{N}6SS$

h_z^w/J	degree of truthness		degree of indeterminacy		degree of falsity	
grades	p_w	$2\pi t_w$	q_w	$2\pi\omega_w$	r_w	$2\pi f_w$
$h_w^s = 0$	$[0.00, 0.15)$	$[0.0, 0.3\pi)$	$(0.90, 1.00]$	$[1.8\pi, 2.0\pi]$	$(0.90, 1.00]$	$[1.8\pi, 2.0\pi]$
$h_w^s = 1$	$[0.15, 0.30)$	$[0.3\pi, 0.6\pi)$	$(0.70, 0.90]$	$[1.4\pi, 1.8\pi)$	$(0.70, 0.90]$	$[1.4\pi, 1.8\pi)$
$h_w^s = 2$	$[0.30, 0.50)$	$[0.6\pi, 1.0\pi)$	$(0.50, 0.70]$	$[1.0\pi, 1.4\pi)$	$(0.50, 0.70]$	$[1.0\pi, 1.4\pi)$
$h_w^s = 3$	$[0.50, 0.70)$	$[1.0\pi, 1.4\pi)$	$(0.30, 0.50]$	$[0.6\pi, 1.0\pi)$	$(0.30, 0.50]$	$[0.6\pi, 1.0\pi)$
$h_w^s = 4$	$[0.70, 0.90)$	$[1.4\pi, 1.8\pi)$	$(0.15, 0.30]$	$[0.3\pi, 0.6\pi)$	$(0.15, 0.30]$	$[0.3\pi, 0.6\pi)$
$h_w^s = 5$	$[0.90, 1.00]$	$[1.8\pi, 2\pi]$	$[0.00, 0.15]$	$[0.0, 0.3\pi)$	$[0.00, 0.15]$	$[0.0, 0.3\pi)$

Using the prescribed information, the $CSV\mathcal{N}6S_fS$, shown in 3, is defined as:

$$\begin{aligned}\Phi_\Psi(Y_1) &= \{((\mathbb{U}_1, 3), (0.60e^{i1.26\pi}, 0.35e^{i0.68\pi}, 0.4e^{i0.84\pi})), ((\mathbb{U}_2, 5), (0.95e^{i1.92\pi}, 0.05e^{i0.12\pi}, 0.12e^{i0.26\pi})),\\ &\quad ((\mathbb{U}_3, 0), (0.06e^{i0.14\pi}, 0.95e^{i1.92\pi}, 0.97e^{i1.96\pi})), ((\mathbb{U}_4, 1), (0.24e^{i0.50\pi}, 0.86e^{i1.70\pi}, 0.87e^{i1.72\pi}))\},\end{aligned}$$

$$\begin{aligned}\Phi_\Psi(Y_2) &= \{((\mathbb{U}_1, 1), (0.17e^{i0.40\pi}, 0.75e^{i1.48\pi}, 0.81e^{i1.66\pi})), ((\mathbb{U}_2, 4), (0.81e^{i1.66\pi}, 0.22e^{i0.42\pi}, 0.25e^{i0.48\pi})),\\ &\quad ((\mathbb{U}_3, 2), (0.36e^{i0.74\pi}, 0.58e^{i1.18\pi}, 0.54e^{i1.10\pi})), ((\mathbb{U}_4, 0), (0.08e^{i0.20\pi}, 0.96e^{i1.94\pi}, 0.98e^{1.98\pi}))\},\end{aligned}$$

$$\begin{aligned}\Phi_\Psi(Y_3) &= \{((\mathbb{U}_1, 2), (0.32e^{i0.70\pi}, 0.55e^{i1.12\pi}, 0.52e^{i1.06\pi})), ((\mathbb{U}_2, 1), (0.2e^{i0.42\pi}, 0.76e^{i1.54\pi}, 0.78e^{i1.58\pi})),\\ &\quad ((\mathbb{U}_3, 4), (0.75e^{i1.52\pi}, 0.17e^{i0.36\pi}, 0.20e^{i0.38\pi})), ((\mathbb{U}_4, 3), (0.69e^{i1.36\pi}, 0.41e^{i0.84\pi}, 0.48e^{0.94\pi}))\},\end{aligned}$$

$$\begin{aligned}\Phi_\Psi(Y_4) &= \{((\mathbb{U}_1, 5), (0.98e^{i1.94\pi}, 0.01e^{i0.04\pi}, 0.1e^{0.24i\pi})), ((\mathbb{U}_2, 0), (0.03e^{i0.10\pi}, 0.91e^{i1.84\pi}, 0.93e^{1.88i\pi})),\\ &\quad ((\mathbb{U}_3, 1), (0.21e^{i0.46\pi}, 0.79e^{i1.60\pi}, 0.83e^{i1.68\pi})), ((\mathbb{U}_4, 2), (0.38e^{i0.78\pi}, 0.59e^{i1.20\pi}, 0.55e^{i1.12\pi}))\}.\end{aligned}$$

Table 3: The $CSV\mathcal{N}6S_fS$ $(\Phi_\Psi, Y, 6)$

$(\Phi_\Psi, Y, 6)$	$\mathbb{U}_1$	$\mathbb{U}_2$	$\mathbb{U}_3$	$\mathbb{U}_4$
Y_1	$(3, (0.60e^{i1.26\pi}, 0.35e^{i0.68\pi}, 0.40e^{i0.84\pi}))$	$(5, (0.95e^{i1.92\pi}, 0.05e^{i0.12\pi}, 0.12e^{i0.26\pi}))$	$(0, (0.06e^{i0.14\pi}, 0.95e^{i1.92\pi}, 0.97e^{i1.96\pi}))$	$(1, (0.24e^{i0.50\pi}, 0.86e^{i1.70\pi}, 0.87e^{i1.72\pi}))$
Y_2	$(1, (0.17e^{i0.40\pi}, 0.75e^{i1.48\pi}, 0.81e^{i1.66\pi}))$	$(4, (0.81e^{i1.66\pi}, 0.22e^{i0.42\pi}, 0.25e^{i0.48\pi}))$	$(2, (0.36e^{i0.74\pi}, 0.58e^{i1.18\pi}, 0.54e^{i1.10\pi}))$	$(0, (0.08e^{i0.20\pi}, 0.96e^{i1.94\pi}, 0.98e^{i1.98\pi}))$
Y_3	$(2, (0.32e^{i0.70\pi}, 0.55e^{i1.12\pi}, 0.52e^{i1.06\pi}))$	$(1, (0.20e^{i0.42\pi}, 0.76e^{i1.54\pi}, 0.78e^{i1.58\pi}))$	$(4, (0.75e^{i1.52\pi}, 0.17e^{i0.36\pi}, 0.20e^{i0.38\pi}))$	$(3, (0.69e^{i1.36\pi}, 0.41e^{i0.84\pi}, 0.48e^{0.94\pi}))$
Y_4	$(5, (0.98e^{i1.94\pi}, 0.01e^{i0.04\pi}, 0.10e^{0.24i\pi}))$	$(0, (0.03e^{i0.10\pi}, 0.91e^{i1.84\pi}, 0.93e^{1.88i\pi}))$	$(1, (0.21e^{i0.46\pi}, 0.79e^{i1.60\pi}, 0.83e^{i1.68\pi}))$	$(2, (0.38e^{i0.78\pi}, 0.59e^{i1.20\pi}, 0.55e^{i1.12\pi}))$

Definition 8. *A $CSV\mathcal{N}S_fS(\Phi_\Psi, Y, N)$ over a non-empty set $\mathbb{U}$ is said to be efficient where (Φ, Y, N) is an NS_fS, if $\Phi_\Psi(y_w) = \langle(u_s, N - 1), 1, 0, 0\rangle$ for some $y_w \in Y, u_s \in \mathbb{U}$.*

Example 2. *Let $(\Phi_\Psi, Y, 6)$ be $CSV\mathcal{N}6S_fS$, as in Example 1. From Table 3, it is clear that Example 1 is not efficient.*

Definition 9. *Let (Φ_Ψ, Y, N_1) and (χ_A, C, N_2) be two $CSV\mathcal{N}S_fSs$ on a universe of discourse $\mathbb{U}$. Then, they are said to be equal if and only if $\Phi = \chi$, $\Psi = A$, $Y = C$ and $N_1 = N_2$.*

Definition 10. *Let (Φ_Ψ, Y, N) be a $CSV\mathcal{N}S_fS$ on $\mathbb{U}$. The weak complement of $CSV\mathcal{N}S_fS$ is defined as the weak complement of the N-soft set (Φ, Y, N), that is, any N-soft set such that $\Phi^c(y_w) \cap \Phi(y_w) = \emptyset$ for all $y_w \in Y$. The weak complement of $CSV\mathcal{N}S_fS$ of (Φ_Ψ, Y, N) is represented as (Φ_Ψ^c, Y, N).*

Example 3. *Let $(\Phi_\Psi, Y, 6)$ be $CSV\mathcal{N}6S_fS$, as in Example 1. The weak complement (Φ_Ψ^c, Y, N) is given in Table 4.*

Table 4: A weak complement of the $CSV\mathcal{N}6S_fS$ $(\Phi_\Psi, Y, 6)$

$(\Phi_\Psi^c, Y, 6)$	$\mathbb{U}_1$	$\mathbb{U}_2$	$\mathbb{U}_3$	$\mathbb{U}_4$
Y_1	$(5, (0.60e^{i1.26\pi}, 0.35e^{i0.68\pi}, 0.40e^{i0.84\pi}))$	$(4, (0.95e^{i1.92\pi}, 0.05e^{i0.12\pi}, 0.12e^{i0.26\pi}))$	$(1, (0.06e^{i0.14\pi}, 0.95e^{i1.92\pi}, 0.97e^{i1.96\pi}))$	$(3, (0.24e^{i0.50\pi}, 0.86e^{i1.70\pi}, 0.87e^{i1.72\pi}))$
Y_2	$(4, (0.17e^{i0.40\pi}, 0.75e^{i1.48\pi}, 0.81e^{i1.66\pi}))$	$(1, (0.81e^{i1.66\pi}, 0.22e^{i0.42\pi}, 0.25e^{i0.48\pi}))$	$(3, (0.36e^{i0.74\pi}, 0.58e^{i1.18\pi}, 0.54e^{i1.10\pi}))$	$(5, (0.08e^{i0.20\pi}, 0.96e^{i1.94\pi}, 0.98e^{i1.98\pi}))$
Y_3	$(4, (0.32e^{i0.70\pi}, 0.55e^{i1.12\pi}, 0.52e^{i1.06\pi}))$	$(3, (0.20e^{i0.42\pi}, 0.76e^{i1.54\pi}, 0.78e^{i1.58\pi}))$	$(0, (0.75e^{i1.52\pi}, 0.17e^{i0.36\pi}, 0.20e^{i0.38\pi}))$	$(5, (0.69e^{i1.36\pi}, 0.41e^{i0.84\pi}, 0.48e^{0.94\pi}))$
Y_4	$(0, (0.98e^{i1.94\pi}, 0.01e^{i0.04\pi}, 0.10e^{0.24i\pi}))$	$(2, (0.03e^{i0.10\pi}, 0.91e^{i1.84\pi}, 0.93e^{1.88i\pi}))$	$(3, (0.21e^{i0.46\pi}, 0.79e^{i1.60\pi}, 0.83e^{i1.68\pi}))$	$(3, (0.38e^{i0.78\pi}, 0.59e^{i1.20\pi}, 0.55e^{i1.12\pi}))$

Table 6: The complex single-valued neutrosophic complement (Φ_{Φ^c}, Y, N) of the $CSV\mathcal{N}6S_fS$

$(\Phi_{\Psi^c}, Y, 6)$	$\mathbb{U}_1$	$\mathbb{U}_2$	$\mathbb{U}_3$	$\mathbb{U}_4$
Y_1	$(3,(0.40e^{i0.84\pi}, 0.65e^{i1.32\pi}, 0.60e^{i1.26\pi}))$	$(5,(0.12e^{i0.26\pi}, 0.95e^{i1.88\pi}, 0.95e^{i1.92\pi}))$	$(0,(0.97e^{i1.96\pi}, 1.05e^{i1.92\pi}, 0.06e^{i0.14\pi}))$	$(1,(0.24e^{i0.50\pi}, 0.14e^{i0.24\pi}, 0.87e^{i1.72\pi}))$
Y_2	$(1,(0.81e^{i1.66\pi}, 0.25e^{i0.52\pi}, 0.17e^{i0.40\pi}))$	$(4,(0.25e^{i0.48\pi}, 0.88e^{i1.58\pi}, 0.81e^{i1.66\pi}))$	$(2,(0.54e^{i1.10\pi}, 0.42e^{i0.08\pi}, 0.36e^{i0.74\pi}))$	$(0,(0.98e^{1.98\pi}, 0.04e^{i0.06\pi}, 0.08e^{i0.20\pi}))$
Y_3	$(2,(0.52e^{i1.06\pi}, 0.45e^{i0.88\pi}, 0.32e^{i0.70\pi}))$	$(1,(0.78e^{i1.58\pi}, 0.24e^{i0.46\pi}, 0.20e^{i0.42\pi}))$	$(4,(0.20e^{i0.38\pi}, 0.83e^{i1.64\pi}, 0.75e^{i1.52\pi}))$	$(3,(0.48e^{0.94\pi}, 0.59e^{i1.16\pi}, 0.69e^{i1.36\pi}))$
Y_4	$(5,(0.10e^{0.24i\pi}, 0.09e^{i1.96\pi}, 0.98e^{i1.94\pi}))$	$(0,(0.93e^{1.88i\pi}, 0.09e^{i0.16\pi}, 0.03e^{i0.10\pi}))$	$(1,(0.83e^{i1.68\pi}, 0.21e^{i0.40\pi}, 0.21e^{i0.46\pi}))$	$(2,(0.55e^{i1.12\pi}, 0.41e^{i0.80\pi}, 0.38e^{i0.78\pi}))$

Definition 11. *Let (Φ_Ψ, Y, N) be a $CSV\mathcal{N}NS_fS$ on $\mathbb{U}$. The Strong complement of $CSV\mathcal{N}NS_fS$, denoted as (Φ'_Ψ, Y, N), is defined as:*

$$\Phi'(y_w) = \begin{cases} h^s_w - 1, & \textit{if } h^s_w = (N-1) - h^s_w, \\ (N-1) - h^s_w, & \textit{otherwise,} \end{cases}$$

for all $y_w \in Y$ and $u_s \in \mathbb{U}$, satisfying the condition $(\Phi_\Psi, Y, N) \cap (\Phi'_\Psi, Y, N) = \emptyset$.

Example 4. *Let $(\Phi_\Psi, Y, 6)$ be $CSV\mathcal{N}6S_fS$, then the strong complement $(\Phi'_\Psi, Y, 6)$ of Example 1 is given in Table 5 such that $(\Phi_\Psi, Y, 6) \cap (\Phi'_\Psi, Y, 6) = \emptyset$.*

Table 5: Strong complement of $(\Phi_\Psi, Y, 6)$

$(\Phi_\Psi, Y, 6)$	$\mathbb{U}_1$	$\mathbb{U}_2$	$\mathbb{U}_3$	$\mathbb{U}_4$
Y_1	$(2,(0.60e^{i1.26\pi}, 0.35e^{i0.68\pi}, 0.40e^{i0.84\pi}))$	$(0,(0.95e^{i1.92\pi}, 0.05e^{i0.12\pi}, 0.12e^{i0.26\pi}))$	$(5,(0.06e^{i0.14\pi}, 0.95e^{i1.92\pi}, 0.97e^{i1.96\pi}))$	$(4,(0.24e^{i0.50\pi}, 0.86e^{i1.70\pi}, 0.87e^{i1.72\pi}))$
Y_2	$(4,(0.17e^{i0.40\pi}, 0.75e^{i1.48\pi}, 0.81e^{i1.66\pi}))$	$(1,(0.81e^{i1.66\pi}, 0.22e^{i0.42\pi}, 0.25e^{i0.48\pi}))$	$(3,(0.36e^{i0.74\pi}, 0.58e^{i1.18\pi}, 0.54e^{i1.10\pi}))$	$(5,(0.08e^{i0.20\pi}, 0.96e^{i1.94\pi}, 0.98e^{i1.98\pi}))$
Y_3	$(3,(0.32e^{i0.70\pi}, 0.55e^{i1.12\pi}, 0.52e^{i1.06\pi}))$	$(4,(0.20e^{i0.42\pi}, 0.76e^{i1.54\pi}, 0.78e^{i1.58\pi}))$	$(1,(0.75e^{i1.52\pi}, 0.17e^{i0.36\pi}, 0.20e^{i0.38\pi}))$	$(2,(0.69e^{i1.36\pi}, 0.41e^{i0.84\pi}, 0.48e^{0.94\pi}))$
Y_4	$(0,(0.98e^{i1.94\pi}, 0.01e^{i0.04\pi}, 0.10e^{0.24i\pi}))$	$(5,(0.03e^{i0.10\pi}, 0.91e^{i1.84\pi}, 0.93e^{1.88i\pi}))$	$(4,(0.21e^{i0.46\pi}, 0.79e^{i1.60\pi}, 0.83e^{i1.68\pi}))$	$(3,(0.38e^{i0.78\pi}, 0.59e^{i1.20\pi}, 0.55e^{i1.12\pi}))$

Proposition 12. *A strong complement of $CSV\mathcal{N}NS_fS$ is also a weak complement but week complement may or may not be strong complement.*

Proof. The proof is straight forward from the definitions of strong complement and weak complement. □

Definition 13. *Let (Φ_Ψ, Y, N) be a $CSV\mathcal{N}S_fS$ on $\mathbb{U}$. The complex single-valued neutrosophic complement of $CSV\mathcal{N}S_fS$ is denoted as (Φ_{Ψ^c}, Y, N) and is defined as*

$$\Phi_{\Psi^c}(y_w) = \langle (u_s, h^s_w, (\mathbb{F}_{ws}, 1 - \mathbb{I}_{ws}, \mathbb{T}_{ws})) \rangle = \langle (u_s, h^s_w, (r_{ws}e^{i2\pi f_{ws}}, (1-q_{ws})e^{i2\pi(1-\omega_{ws})}, p_{ws}e^{i2\pi t_{ws}})) \rangle.$$

Example 5. *Let $(\Phi_\Psi, Y, 6)$ be $CSV\mathcal{N}6S_fS$, as in Example 1. The complex single-valued neutrosophic complement (Φ_{Ψ^c}, Y, N), is given in Table 6.*

Definition 14. *Let (Φ_Ψ, Y, N) be a $CSV\mathcal{N}S_fS$ on $\mathbb{U}$. $(F^c_{\mathcal{J}^c}, Z, N)$ is referred to as a weak complex single-valued neutrosophic complement of $((\Phi_\Psi)^c, Y, N)$ if and only if (Φ^c_Ψ, Y, N) is a weak complement and (Φ_{Ψ^c}, Y, N) is a complex single-valued neutrosophic complement of (Φ_Ψ, Y, N).*

Example 6. *Let $(\Phi_\Psi, Y, 6)$ be $CSV\mathcal{N}6S_fS$, as in Example 1. The weak complex single-valued neutrosophic complement $(\Phi^c_{\Phi^c}, Y, N)$, is given in Table 7.*

Table 7: The weak complex single-valued neutrosophic complement $(\Phi^c_{\Psi^c}, Y, 6)$ of the $CSV\mathcal{N}6S_fS$

$(\Phi^c_{\Psi^c}, Y, 6)$	$\mathbb{U}_1$	$\mathbb{U}_2$	$\mathbb{U}_3$	$\mathbb{U}_4$
Y_1	$(5,(0.40e^{i0.84\pi}, 0.65e^{i1.32\pi}, 0.60e^{i1.26\pi}))$	$(4,(0.12e^{i0.26\pi}, 0.95e^{i1.88\pi}, 0.95e^{i1.92\pi}))$	$(1,(0.97e^{i1.96\pi}, 1.05e^{i1.92\pi}, 0.06e^{i0.14\pi}))$	$(3,(0.24e^{i0.50\pi}, 0.14e^{i0.24\pi}, 0.87e^{i1.72\pi}))$
Y_2	$(4,(0.81e^{i1.66\pi}, 0.25e^{i0.52\pi}, 0.17e^{i0.40\pi}))$	$(1,(0.25e^{i0.48\pi}, 0.88e^{i1.58\pi}, 0.81e^{i1.66\pi}))$	$(3,(0.54e^{i1.10\pi}, 0.42e^{i0.08\pi}, 0.36e^{i0.74\pi}))$	$(5,(0.98e^{1.98\pi}, 0.04e^{i0.06\pi}, 0.08e^{i0.20\pi}))$
Y_3	$(4,(0.52e^{i1.06\pi}, 0.45e^{i0.88\pi}, 0.32e^{i0.70\pi}))$	$(3,(0.78e^{i1.58\pi}, 0.24e^{i0.46\pi}, 0.20e^{i0.42\pi}))$	$(0,(0.20e^{i0.38\pi}, 0.83e^{i1.64\pi}, 0.75e^{i1.52\pi}))$	$(5,(0.48e^{0.94\pi}, 0.59e^{i1.16\pi}, 0.69e^{i1.36\pi}))$
Y_4	$(0,(0.10e^{0.24i\pi}, 0.09e^{i1.96\pi}, 0.98e^{i1.94\pi}))$	$(2,(0.93e^{1.88i\pi}, 0.09e^{i0.16\pi}, 0.03e^{i0.10\pi}))$	$(3,(0.83e^{i1.68\pi}, 0.21e^{i0.40\pi}, 0.21e^{i0.46\pi}))$	$(3,(0.55e^{i1.12\pi}, 0.41e^{i0.80\pi}, 0.38e^{i0.78\pi}))$

Definition 15. *Let (Φ_Ψ, Y, N) be a $CSV\mathcal{N}NS_fS$ on $\mathbb{U}$, then the strong complex single-valued neutrosophic complement $((\Phi_\Psi)', Y, N)$ is defined as a strong complement (Φ'_Ψ, Y, N) and a complex single-valued neutrosophic complement (Φ_{Ψ^c}, Y, N) of (Φ_Ψ, Y, N), defined as:*

$\Phi'_{\Psi^c}(y_w) =$

$$\begin{cases} (h^s_w - 1, (r_{ws}e^{i2\pi f_{ws}}, (1-q_{ws})e^{i2\pi(1-\omega_{ws})}, p_{ws}e^{i2\pi t_{ws}})) & \textit{if } h^s_w = (N-1) - h^s_w, \\ ((N-1) - h^s_w, (r_{ws}e^{i2\pi f_{ws}}, (1-q_{ws})e^{i2\pi(1-\omega_{ws})}, p_{ws}e^{i2\pi t_{ws}})) & \textit{otherwise,} \end{cases}$$

for all $y_w \in Y$ and $u_s \in \mathbb{U}$.

Example 7. *Let $(\Phi_\Psi, Y, 6)$ be $CSV\mathcal{N}6S_fS$ on $\mathbb{U}$, then the strong single-valued neutrosophic complement (Φ'_{Ψ^c}, Y, N), of $(\Phi_\Psi, Y, 6)$ arranged in Table 3, is calculated in Table 8.*

Table 8: Strong single-valued neutrosophic complement of $(\Phi_\Psi, Y, 6)$

$(\Phi'_{\Psi^c}, Y, 6)$	$\mathbb{U}_1$	$\mathbb{U}_2$	$\mathbb{U}_3$	$\mathbb{U}_4$
Y_1	$(2,(0.40e^{i0.84\pi},0.65e^{i1.32\pi},0.60e^{i1.26\pi}))$	$(0,(0.12e^{i0.26\pi},0.95e^{i1.88\pi},0.95e^{i1.92\pi}))$	$(5,(0.97e^{i1.96\pi},1.05e^{i1.92\pi},0.06e^{i0.14\pi}))$	$(4,(0.24e^{i0.50\pi},0.14e^{i0.24\pi},0.87e^{i1.72\pi}))$
Y_2	$(4,(0.81e^{i1.66\pi},0.25e^{i0.52\pi},0.17e^{i0.40\pi}))$	$(1,(0.25e^{i0.48\pi},0.88e^{i1.58\pi},0.81e^{i1.66\pi}))$	$(3,(0.54e^{i1.10\pi},0.42e^{i0.08\pi},0.36e^{i0.74\pi}))$	$(5,(0.98e^{1.98\pi},0.04e^{i0.06\pi},0.08e^{i0.20\pi}))$
Y_3	$(3,(0.52e^{i1.06\pi},0.45e^{i0.88\pi},0.32e^{i0.70\pi}))$	$(4,(0.78e^{i1.58\pi},0.24e^{i0.46\pi},0.20e^{i0.42\pi}))$	$(1,(0.20e^{i0.38\pi},0.83e^{i1.64\pi},0.75e^{i1.52\pi}))$	$(2,(0.48e^{0.94\pi},0.59e^{i1.16\pi},0.69e^{i1.36\pi}))$
Y_4	$(0,(0.10e^{0.24i\pi},0.09e^{i1.96\pi},0.98e^{i1.94\pi}))$	$(5,(0.93e^{1.88i\pi},0.09e^{i0.16\pi},0.03e^{i0.10\pi}))$	$(4,(0.83e^{i1.68\pi},0.21e^{i0.40\pi},0.21e^{i0.46\pi}))$	$(3,(0.55e^{i1.12\pi},0.41e^{i0.80\pi},0.38e^{i0.78\pi}))$

Proposition 16. *Let $((\Phi_\Psi)^c, Y, N)$ and $((\Phi_\Psi)', Y, N)$ be weak and strong complex single-valued neutrosophic complement of $CSV\mathcal{N}NS_fS$ (Φ_Ψ, Y, N), then*

1 $((\Phi^c)^c_\Psi, Y, N) \neq (\Phi_\Psi, Y, N)$,

2 $(((\Phi_\Psi)^c)^c, Y, N) \neq (\Phi_\Psi, Y, N)$,

3 $((\Phi')'_\Psi, Y, N) \left\{ \begin{array}{ll} = (\Phi_\Psi, Y, N) & \textit{if } N \textit{ is even} \\ \neq (\Phi_\Psi, Y, N) & \textit{if } N \textit{ is odd.} \end{array} \right\}.$

4 $([(\Phi_\Psi)']', Y, N) \left\{ \begin{array}{ll} = (\Phi_\Psi, Y, N) & \textit{if } N \textit{ is even} \\ \neq (\Phi_\Psi, Y, N) & \textit{if } N \textit{ is odd.} \end{array} \right\}.$

Proof. The proof is straight forward from the definitions. □

Definition 17. *Let $\mathbb{U}$ be a non-empty set and (Φ_Ψ, Y, N_1) and (χ_A, C, N_2) be $CSV\mathcal{N}_1S_fS$ and $CSV\mathcal{N}_2S_fS$ on $\mathbb{U}$, respectively, their restricted intersection is defined as $(L_M, G, O) = (\Phi_\Psi, Y, N_1)\hat{\cap}(\chi_A, C, N_2)$, with $L_M = \Phi_\Psi\hat{\cap}\chi_A$, $G = Y \cap C$, $O = \min(N_1, N_2)$, i.e., $\forall x_w \in G$, $u_s \in \mathbb{U}$ we have*

$$\begin{aligned} & L_M(x_w) = \langle (h^s_w, (\mathbb{T}_{ws}, \mathbb{I}_{ws}, \mathbb{F}_{ws}))\rangle, \\ = \;& \langle (\min(h^{1s}_w, h^{2s}_w), \min(\mathbb{T}^1_{ws}, \mathbb{T}^2_{ws}), \max(\mathbb{I}^1_{ws}, \mathbb{I}^2_{ws}), \max(\mathbb{F}^1_{ws}, \mathbb{F}^2_{ws}))\rangle, \\ = \;& \langle (\min(h^{1s}_w, h^{2s}_w), \min(p^1_{ws}, p^2_{ws})e^{i2\pi\min(t^1_{ws}, t^2_{ws})}, \max(q^1_{ws}, q^2_{ws})e^{i2\pi\max(\omega^1_{ws}, \omega^2_{ws})}, \max(r^1_{ws}, r^2_{ws})e^{i2\pi\max(f^1_{ws}, f^2_{ws})})\rangle, \end{aligned}$$

where $(h^{1s}_w, (\mathbb{T}^1_{ws}, \mathbb{I}^1_{ws}, \mathbb{F}^1_{ws})) = (h^{1s}_w, (p^1_{ws}e^{i2\pi t^1_{ws}}, q^1_{ws}e^{i2\pi\omega^1_{ws}}, r^1_{ws}e^{i2\pi f^1_{ws}})) \in \Phi_\Psi$ and $(h^{2s}_w, (\mathbb{T}^2_{ws}, \mathbb{I}^2_{ws}, \mathbb{F}^2_{ws}))$ $= (h^{2s}_w, (p^2_{ws}e^{i2\pi t^2_{ws}}, q^2_{ws}e^{i2\pi\omega^2_{ws}}, r^2_{ws}e^{i2\pi f^2_{ws}})) \in \chi_A$.

Table 9: The $CSF5S_fS(\chi_A, C, 5)$

	$\mathbb{U}_1$	$\mathbb{U}_2$	$\mathbb{U}_5$
Y_1	$(0,(0.12e^{i0.23\pi},0.91e^{i1.84\pi},0.96e^{i1.96\pi}))$	$(1,(0.21e^{i0.42\pi},0.77e^{i1.50\pi},0.82e^{i1.66\pi}))$	$(0,(0.05e^{i0.14\pi},0.87e^{i1.72\pi},0.88e^{i1.80\pi}))$
Y_2	$(2,(0.42e^{i0.82\pi},0.51e^{i1.04\pi},0.56e^{i1.10\pi}))$	$(4,(0.88e^{i1.78\pi},0.09e^{i0.16\pi},0.06e^{i0.10\pi}))$	$(4,(0.90e^{i1.84\pi},0.11e^{i0.20\pi},0.14e^{i0.26\pi}))$
Y_3	$(3,(0.81e^{i1.64\pi},0.17e^{i0.36\pi},0.19e^{i0.40\pi}))$	$(3,(0.83e^{i1.68\pi},0.27e^{i0.56\pi},0.30e^{i0.58\pi}))$	$(1,(0.26e^{i0.48\pi},0.72e^{i1.42\pi},0.75e^{i1.52\pi}))$
Y_4	$(4,(0.95e^{i1.80\pi},0.012e^{i0.02\pi},0.10e^{0.22i\pi}))$	$(3,(0.70e^{i1.42\pi},0.26e^{i0.54\pi},0.31e^{0.64i\pi}))$	$(2,(0.49e^{i1.02\pi},0.61e^{i1.24\pi},0.59e^{i1.16\pi}))$

Example 8. *The restricted intersection (L_M, G, O) of $(\Phi_\Psi, Y, 6)$ and $(\chi_A, C, 5)$, given in Table 3 and Table 9, arranged in 10.*

Table 10: The restricted intersection $(L_M, G, 5)$

$(L_M, G, 56)$	$\mathbb{U}_1$	$\mathbb{U}_2$
Y_1	$(0,(0.12e^{i0.23\pi},0.91e^{i1.84\pi},0.96e^{i1.96\pi}))$	$(1,(0.21e^{i0.42\pi},0.77e^{i1.50\pi},0.82e^{i1.66\pi}))$
Y_2	$(1,(0.17e^{i0.40\pi},0.75e^{i1.48\pi},0.81e^{i1.66\pi}))$	$(4,(0.81e^{i1.66\pi},0.22e^{i0.42\pi},0.25e^{i0.48\pi}))$
Y_3	$(2,(0.32e^{i0.70\pi},0.55e^{i1.12\pi},0.52e^{i1.06\pi}))$	$(1,(0.20e^{i0.42\pi},0.76e^{i1.54\pi},0.78e^{i1.58\pi}))$
Y_4	$(4,(0.95e^{i1.80\pi},0.012e^{i0.02\pi},0.10e^{0.22i\pi}))$	$(0,(0.03e^{i0.10\pi},0.91e^{i1.84\pi},0.93e^{1.88i\pi}))$

Definition 18. *Let (Φ_Ψ, Y, N_1) and (χ_A, C, N_2) be $CSV\mathcal{N}_1S_fS$ and $CSV\mathcal{N}_2S_fS$ on $\mathbb{U}$, respectively, their extended intersection is defined as $(\mathcal{D}_Q, T, \mathfrak{S}) = (\Phi_\Psi, Y, N_1)\check{\cap}(\chi_A, C, N_2)$, with $\mathcal{D}_Q = \Phi_\Psi\check{\cap}\chi_A$, $T = Y \cup C$, $\mathfrak{S} = \max(N_1, N_2)$, that is, $\forall x_w \in T$ and $u_s \in \mathbb{U}$, we have*

$$\mathcal{D}_Q(x_w) = \begin{cases} (h^{1s}_w, (\mathbb{T}^1_{ws}, \mathbb{I}^1_{ws}, \mathbb{F}^1_{ws})), & \textit{if } x_w \in Y - C, \\ (h^{2s}_w, (\mathbb{T}^2_{ws}, \mathbb{I}^2_{ws}, \mathbb{F}^2_{ws})), & \textit{if } x_w \in C - Y, \\ \Big(\min(h^{1s}_w, h^{2s}_w), \min(p^1_{ws}, p^2_{ws})e^{i2\pi\min(t^1_{ws}, t^2_{ws})}, \max(q^1_{ws}, q^2_{ws})e^{i2\pi\max(\omega^1_{ws}, \omega^2_{ws})}, \max(r^1_{ws}, r^2_{ws})e^{i2\pi\max(f^1_{ws}, f^2_{ws})}\Big), & \textit{if } x_w \in C \cap Y \end{cases}$$

where $(h_w^{1s}, (\mathbb{T}_{ws}^1, \mathbb{I}_{ws}^1, \mathbb{F}_{ws}^1)) = (h_w^{1s}, (p_{ws}^1 e^{i2\pi t_{ws}^1}, q_{ws}^1 e^{i2\pi \omega_{ws}^1}, r_{ws}^1 e^{i2\pi f_{ws}^1})) \in \Phi_\Psi$ *and* $(h_w^{2s}, (\mathbb{T}_{ws}^2, \mathbb{I}_{ws}^2, \mathbb{F}_{ws}^2)) = (h_w^{2s}, (p_{ws}^2 e^{i2\pi t_{ws}^2}, q_{ws}^2 e^{i2\pi \omega_{ws}^2}, r_{ws}^2 e^{i2\pi f_{ws}^2})) \in \chi_A$.

Example 9. *The extended intersection* $(\mathcal{D}_Q, T, 6)$ *of* $(\Phi_\Psi, Y, 6)$ *and* $(\chi_A, C, 5)$, *given in Table 3 and Table 9, arranged in 11.*

Table 11: The extended intersection$(\mathcal{D}_Q, T, \mathfrak{S})$

$(\mathcal{D}_Q, T, \mathfrak{S})$	$\mathbb{U}_1$	$\mathbb{U}_2$	$\mathbb{U}_3$	$\mathbb{U}_4$
Y_1	$(0, (0.12e^{i0.23\pi}, 0.91e^{i1.84\pi}, 0.96e^{i1.96\pi}))$	$(1, (0.21e^{i0.42\pi}, 0.77e^{i1.50\pi}, 0.82e^{i1.66\pi}))$	$(0, (0.06e^{i0.14\pi}, 0.95e^{i1.92\pi}, 0.97e^{i1.96\pi}))$	$(1, (0.24e^{i0.50\pi}, 0.86e^{i1.70\pi}, 0.87e^{i1.72\pi}))$
Y_2	$(1, (0.17e^{i0.40\pi}, 0.75e^{i1.48\pi}, 0.81e^{i1.66\pi}))$	$(4, (0.81e^{i1.66\pi}, 0.22e^{i0.42\pi}, 0.25e^{i0.48\pi}))$	$(2, (0.36e^{i0.74\pi}, 0.58e^{i1.18\pi}, 0.54e^{i1.10\pi}))$	$(0, (0.08e^{i0.20\pi}, 0.96e^{i1.94\pi}, 0.98e^{1.98\pi}))$
Y_3	$(2, (0.32e^{i0.70\pi}, 0.55e^{i1.12\pi}, 0.52e^{i1.06\pi}))$	$(1, (0.20e^{i0.42\pi}, 0.76e^{i1.54\pi}, 0.78e^{i1.58\pi}))$	$(4, (0.75e^{i1.52\pi}, 0.17e^{i0.36\pi}, 0.20e^{i0.38\pi}))$	$(3, (0.69e^{i1.36\pi}, 0.41e^{i0.84\pi}, 0.48e^{0.94\pi}))$
Y_4	$(4, (0.95e^{i1.80\pi}, 0.012e^{i0.02\pi}, 0.10e^{0.22i\pi}))$	$(0, (0.03e^{i0.10\pi}, 0.91e^{i1.84\pi}, 0.93e^{1.88i\pi}))$	$(1, (0.21e^{i0.46\pi}, 0.79e^{i1.60\pi}, 0.83e^{i1.68\pi}))$	$(2, (0.38e^{i0.78\pi}, 0.59e^{i1.20\pi}, 0.55e^{i1.12\pi}))$
	$\mathbb{U}_5$			
Y_1	$(0, (0.05e^{i0.14\pi}, 0.87e^{i1.72\pi}, 0.88e^{i1.80\pi}))$			
Y_2	$(4, (0.90e^{i1.84\pi}, 0.11e^{i0.20\pi}, 0.14e^{i0.26\pi}))$			
Y_3	$(1, (0.26e^{i0.48\pi}, 0.72e^{i1.42\pi}, 0.75e^{i1.52\pi}))$			
Y_4	$(2, (0.49e^{i1.02\pi}, 0.61e^{i1.24\pi}, 0.59e^{i1.16\pi}))$			

Definition 19. *Let* $\mathbb{U}$ *be a non-empty set and* (Φ_Ψ, Y, N_1) *and* (χ_A, C, N_2) *be* $CSV\mathcal{N}_1S_fS$ *and* $CSV\mathcal{N}_2S_fS$ *on* $\mathbb{U}$, *respectively, their restricted union is defined as* $(\mathbb{L}_\mathbb{M}, \mathfrak{G}, \mathfrak{O}) = (\Phi_\Psi, Y, N_1)\hat{\cup}(\chi_A, C, N_2)$, *with* $\mathbb{L}_\mathbb{M} = \Phi_\Psi \hat{\cup} \chi_A$, $\mathfrak{G} = Y \cap C$, $\mathfrak{O} = \max(N_1, N_2)$, *i.e.,* $\forall x_w \in \mathfrak{G}$, $u_s \in \mathbb{U}$ *we have*

$$\begin{aligned}
\mathbb{L}_\mathbb{M}(x_w) &= \langle (h_w^s, (\mathbb{T}_{ws}, \mathbb{I}_{ws}, \mathbb{F}_{ws})) \rangle, \\
&= \langle (\min(h_w^{1s}, h_w^{2s}), \min(\mathbb{T}_{ws}^1, \mathbb{T}_{ws}^2), \max(\mathbb{I}_{ws}^1, \mathbb{I}_{ws}^2), \max(\mathbb{F}_{ws}^1, \mathbb{F}_{ws}^2)) \rangle, \\
&= \langle (\max(h_w^{1s}, h_w^{2s}), \max(p_{ws}^1, p_{ws}^2) e^{i2\pi \max(t_{ws}^1, t_{ws}^2)}, \min(q_{ws}^1, q_{ws}^2) e^{i2\pi \min(\omega_{ws}^1, \omega_{ws}^2)}, \min(r_{ws}^1, r_{ws}^2) e^{i2\pi \min(f_{ws}^1, f_{ws}^2)}) \rangle,
\end{aligned}$$

where $(h_w^{1s}, (\mathbb{T}_{ws}^1, \mathbb{I}_{ws}^1, \mathbb{F}_{ws}^1)) = (h_w^{1s}, (p_{ws}^1 e^{i2\pi t_{ws}^1}, q_{ws}^1 e^{i2\pi \omega_{ws}^1}, r_{ws}^1 e^{i2\pi f_{ws}^1})) \in \Phi_\Psi$ *and* $(h_w^{2s}, (\mathbb{T}_{ws}^2, \mathbb{I}_{ws}^2, \mathbb{F}_{ws}^2)) = (h_w^{2s}, (p_{ws}^2 e^{i2\pi t_{ws}^2}, q_{ws}^2 e^{i2\pi \omega_{ws}^2}, r_{ws}^2 e^{i2\pi f_{ws}^2})) \in \chi_A$.

Example 10. *The restricted union* (L_M, G, O) *of* $(\Phi_\Psi, Y, 6)$ *and* $(\chi_A, C, 5)$, *given in Table 3 and Table 9, arranged in 12.*

Table 12: Restricted union $(\mathbb{L}_\mathbb{M}, \mathfrak{G}, \mathfrak{O})$

$(\mathbb{L}_\mathbb{M}, \mathfrak{G}, \mathfrak{O})$	$\mathbb{U}_1$	$\mathbb{U}_2$
Y_1	$(3, (0.60e^{i1.26\pi}, 0.35e^{i0.68\pi}, 0.40e^{i0.84\pi}))$	$(5, (0.95e^{i1.92\pi}, 0.05e^{i0.12\pi}, 0.12e^{i0.26\pi}))$
Y_2	$(2, (0.42e^{i0.82\pi}, 0.51e^{i1.04\pi}, 0.56e^{i1.10\pi}))$	$(4, (0.88e^{i1.78\pi}, 0.09e^{i0.16\pi}, 0.06e^{i0.10\pi}))$
Y_3	$(3, (0.81e^{i1.64\pi}, 0.17e^{i0.36\pi}, 0.19e^{i0.40\pi}))$	$(3, (0.83e^{i1.68\pi}, 0.27e^{i0.56\pi}, 0.30e^{i0.58\pi}))$
Y_4	$(5, (0.98e^{i1.94\pi}, 0.01e^{i0.04\pi}, 0.10e^{0.24i\pi}))$	$(3, (0.70e^{i1.42\pi}, 0.26e^{i0.54\pi}, 0.31e^{0.64i\pi}))$

Definition 20. *Let* (Φ_Ψ, Y, N_1) *and* (χ_A, C, N_2) *be* $CSV\mathcal{N}_1S_fS$ *and* $CSV\mathcal{N}_2S_fS$ *on* $\mathbb{U}$, *respectively, their extended union is defined as* $(\mathcal{P}_\mathcal{Q}, \mathcal{T}, \mathfrak{B}) = (\Phi_\Psi, Y, N_1)\check{\cup}(\chi_A, C, N_2)$, *with* $\mathcal{P}_\mathcal{Q} = \Phi_\Psi \check{\cup} \chi_A$, $\mathcal{T} = Y \cup C$, $\mathfrak{B} = \max(N_1, N_2)$, *that is,* $\forall x_w \in T$ *and* $u_s \in \mathbb{U}$, *we have*

$$\mathcal{P}_\mathcal{Q}(x_w) = \begin{cases} (h_w^{1s}, (\mathbb{T}_{ws}^1, \mathbb{I}_{ws}^1, \mathbb{F}_{ws}^1)), & \textit{if } x_w \in Y - C, \\ (h_w^{2s}, (\mathbb{T}_{ws}^2, \mathbb{I}_{ws}^2, \mathbb{F}_{ws}^2)), & \textit{if } x_w \in C - Y, \\ \Big(\max(h_w^{1s}, h_w^{2s}), \max(p_{ws}^1, p_{ws}^2) e^{i2\pi \max(t_{ws}^1, t_{ws}^2)}, \min(q_{ws}^1, q_{ws}^2) e^{i2\pi \min(\omega_{ws}^1, \omega_{ws}^2)}, \min(r_{ws}^1, r_{ws}^2) e^{i2\pi \min(f_{ws}^1, f_{ws}^2)} \Big), & \textit{if } x_w \in C \cap Y \end{cases}$$

where $(h_w^{1s}, (\mathbb{T}_{ws}^1, \mathbb{I}_{ws}^1, \mathbb{F}_{ws}^1)) = (h_w^{1s}, (p_{ws}^1 e^{i2\pi t_{ws}^1}, q_{ws}^1 e^{i2\pi \omega_{ws}^1}, r_{ws}^1 e^{i2\pi f_{ws}^1})) \in \Phi_\Psi$ *and* $(h_w^{2s}, (\mathbb{T}_{ws}^2, \mathbb{I}_{ws}^2, \mathbb{F}_{ws}^2)) = (h_w^{2s}, (p_{ws}^2 e^{i2\pi t_{ws}^2}, q_{ws}^2 e^{i2\pi \omega_{ws}^2}, r_{ws}^2 e^{i2\pi f_{ws}^2})) \in \chi_A$.

Example 11. *The extended union* (L_M, G, O) *of* $(\Phi_\Psi, Y, 6)$ *and* $(\chi_A, C, 5)$, *given in Table 3 and Table 9, arranged in 13.*

Table 13: Extended union $(\mathcal{P}_{\mathcal{Q}}, \mathcal{T}, \mathfrak{B})$

$(\mathcal{P}_{\mathcal{Q}}, \mathcal{T}, \mathfrak{B})$	$\mathbb{U}_1$	$\mathbb{U}_2$	$\mathbb{U}_3$	$\mathbb{U}_4$
Y_1	$(3, (0.60e^{i1.26\pi}, 0.35e^{i0.68\pi}, 0.40e^{i0.84\pi}))$	$(5, (0.95e^{i1.92\pi}, 0.05e^{i0.12\pi}, 0.12e^{i0.26\pi}))$	$(0, (0.06e^{i0.14\pi}, 0.95e^{i1.92\pi}, 0.97e^{i1.96\pi}))$	$(1, (0.24e^{i0.50\pi}, 0.86e^{i1.70\pi}, 0.87e^{i1.72\pi}))$
Y_2	$(2, (0.42e^{i0.82\pi}, 0.51e^{i1.04\pi}, 0.56e^{i1.10\pi}))$	$(4, (0.88e^{i1.78\pi}, 0.09e^{i0.16\pi}, 0.06e^{i0.10\pi}))$	$(2, (0.36e^{i0.74\pi}, 0.58e^{i1.18\pi}, 0.54e^{i1.10\pi}))$	$(0, (0.08e^{i0.20\pi}, 0.96e^{i1.94\pi}, 0.98e^{1.98\pi}))$
Y_3	$(3, (0.81e^{i1.64\pi}, 0.17e^{i0.36\pi}, 0.19e^{i0.40\pi}))$	$(3, (0.83e^{i1.68\pi}, 0.27e^{i0.56\pi}, 0.30e^{i0.58\pi}))$	$(4, (0.75e^{i1.52\pi}, 0.17e^{i0.36\pi}, 0.20e^{i0.38\pi}))$	$(3, (0.69e^{i1.36\pi}, 0.41e^{i0.84\pi}, 0.48e^{0.94\pi}))$
Y_4	$(5, (0.98e^{i1.94\pi}, 0.01e^{i0.04\pi}, 0.10e^{0.24i\pi}))$	$(3, (0.70e^{i1.42\pi}, 0.26e^{i0.54\pi}, 0.31e^{0.64i\pi}))$	$(1, (0.21e^{i0.46\pi}, 0.79e^{i1.60\pi}, 0.83e^{i1.68\pi}))$	$(2, (0.38e^{i0.78\pi}, 0.59e^{i1.20\pi}, 0.55e^{i1.12\pi}))$
	$\mathbb{U}_5$			
Y_1	$(0, (0.05e^{i0.14\pi}, 0.87e^{i1.72\pi}, 0.88e^{i1.80\pi}))$			
Y_2	$(4, (0.90e^{i1.84\pi}, 0.11e^{i0.20\pi}, 0.14e^{i0.26\pi}))$			
Y_3	$(1, (0.26e^{i0.48\pi}, 0.72e^{i1.42\pi}, 0.75e^{i1.52\pi}))$			
Y_4	$(2, (0.49e^{i1.02\pi}, 0.61e^{i1.24\pi}, 0.59e^{i1.16\pi}))$			

Now we discuss some properties and their proofs.

Theorem 21. *Let (Φ_Ψ, Y, N_1) be a $CSV\mathcal{N}NS_fS$ over a non-empty set $\mathbb{U}$. Then,*

1 $(\Phi_\Psi, Y, N_1)\check{\cap}(\Phi_\Psi, Y, N_1) = (\Phi_\Psi, Y, N_1)$

2 $(\Phi_\Psi, Y, N_1)\hat{\cap}(\Phi_\Psi, Y, N_1) = (\Phi_\Psi, Y, N_1)$

3 $(\Phi_\Psi, Y, N_1)\check{\cup}(\Phi_\Psi, Y, N_1) = (\Phi_\Psi, Y, N_1)$

4 $(\Phi_\Psi, Y, N_1)\hat{\cup}(\Phi_\Psi, Y, N_1) = (\Phi_\Psi, Y, N_1)$

Proof. 1.

$$R.H.S = (\Phi_\Psi, Y, N_1)\check{\cap}(\Phi_\Psi, Y, N_1), \tag{1}$$

where the extended intersection of two $CSV\mathcal{N}NS_fSs$ is calculated as:

$$(\mathcal{D}_Q, T, \mathfrak{S}) = (\Phi_\Psi, Y, N_1)\check{\cap}(\Phi_\Psi, Y, N_1), \tag{2}$$

with $T = Y \cup Y$, $\mathfrak{S} = \max(N_1, N_1)$ and

$$\mathcal{D}_Q(x_w) = \begin{cases} (h_w^{1s}, (\mathbb{T}_{ws}^1, \mathbb{I}_{ws}^1, \mathbb{F}_{ws}^1)), & \text{if } x_w \in Y - Y, \\ (h_w^{1s}, (\mathbb{T}_{ws}^1, \mathbb{I}_{ws}^1, \mathbb{F}_{ws}^1)), & \text{if } x_w \in Y - Y, \\ (\min(h_w^{1s}, h_w^{1s}), (\min(\mathbb{T}_{ws}^1, \mathbb{T}_{ws}^1), \max(\mathbb{I}_{ws}^1, \mathbb{I}_{ws}^1), \max(\mathbb{F}_{ws}^1, \mathbb{F}_{ws}^1))), & \text{if } x_w \in Y \cap Y. \end{cases}$$

Case 1 : If $x_w \in Y - Y = \emptyset$,

$$\mathcal{D}_Q(x_w) = \Phi_\Psi(x_w). \tag{3}$$

Case 2 : If $x_w \in Y - Y = \emptyset$,

$$\mathcal{D}_Q(x_w) = \Phi_\Psi(x_w). \tag{4}$$

Case 3 : If $x_w \in Y \cap Y = Y$,

$$\begin{aligned} \mathcal{D}_Q(x_w) &= (\min(h_w^{1s}, h_w^{1s}), (\min(\mathbb{T}_{ws}^1, \mathbb{T}_{ws}^1), \max(\mathbb{I}_{ws}^1, \mathbb{I}_{ws}^1), \max(\mathbb{F}_{ws}^1, \mathbb{F}_{ws}^1))), \\ &= (h_w^{1s}, (\mathbb{T}_{ws}^1, \mathbb{I}_{ws}^1, \mathbb{F}_{ws}^1)), \\ &= \Phi_\Psi(x_w). \end{aligned} \tag{5}$$

From Equations 2, 3, 4 and 5, $(\mathcal{D}_Q, T, \mathfrak{S}) = (\Phi_\Psi, Y, N_1)$ and further Eq.1 implies $(\Phi_\Psi, Y, N_1)\check{\cap}(\Phi_\Psi, Y, N_1) = (\Phi_\Psi, Y, N_1)$.

2.

$$R.H.S = (\Phi_\Psi, Y, N_1)\hat{\cap}(\Phi_\Psi, Y, N_1), \tag{6}$$

where the restricted intersection of two $CSV\mathcal{N}NS_fSs$ is calculated as:

$$(L_M, G, O) = (\Phi_\Psi, Y, N_1)\hat{\cap}(\Phi_\Psi, Y, N_1), \tag{7}$$

with $G = Y \cap Y = Y$, $O = \min(N_1, N_1) = N_1$ and

$$\begin{aligned} L_M(x_w) &= (\min(h_w^{1s}, h_w^{1s}), (\min(\mathbb{T}_{ws}^1, \mathbb{T}_{ws}^1), \max(\mathbb{I}_{ws}^1, \mathbb{I}_{ws}^1), \max(\mathbb{F}_{ws}^1, \mathbb{F}_{ws}^1))), \\ &= (h_w^{1s}, (\mathbb{T}_{ws}^1, \mathbb{I}_{ws}^1, \mathbb{F}_{ws}^1)), \\ &= \Phi_\Psi(x_w), \end{aligned} \tag{8}$$

clearly, from Equations 6, 7 and 8, we get the required result.

3.

$$R.H.S = (\Phi_\Psi, Y, N_1)\check{\cup}(\Phi_\Psi, Y, N_1), \tag{9}$$

where the extended union of two $CSV\mathcal{N}NS_fSs$ is calculated as:

$$(\mathcal{P}_{\mathcal{Q}}, \mathcal{T}, \mathfrak{B}) = (\Phi_\Psi, Y, N_1)\check{\cup}(\Phi_\Psi, Y, N_1), \tag{10}$$

with $\mathcal{T} = Y \cup Y$, $\mathfrak{B} = \max(N_1, N_1)$ and

$$\mathcal{P}_{\mathcal{Q}}(x_w) = \begin{cases} (h_w^{1s}, (\mathbb{T}_{ws}^1, \mathbb{I}_{ws}^1, \mathbb{F}_{ws}^1)), & \text{if } x_w \in Y - Y, \\ (h_w^{1s}, (\mathbb{T}_{ws}^1, \mathbb{I}_{ws}^1, \mathbb{F}_{ws}^1)), & \text{if } x_w \in Y - Y, \\ (\max(h_w^{1s}, h_w^{1s}), (\max(\mathbb{T}_{ws}^1, \mathbb{T}_{ws}^1), \min(\mathbb{I}_{ws}^1, \mathbb{I}_{ws}^1), \min(\mathbb{F}_{ws}^1, \mathbb{F}_{ws}^1))), & \text{if } x_w \in Y \cap Y. \end{cases}$$

Case 1 : If $x_w \in Y - Y = \emptyset$,

$$\mathcal{P}_{\mathcal{Q}}(x_w) = \Phi_\Psi(x_w). \tag{11}$$

Case 2 : If $x_w \in Y - Y = \emptyset$,

$$\mathcal{P}_{\mathcal{Q}}(x_w) = \Phi_\Psi(x_w). \tag{12}$$

Case 3 : If $x_w \in Y \cap Y = Y$,

$$\begin{aligned} \mathcal{P}_{\mathcal{Q}}(x_w) &= (\max(h_w^{1s}, h_w^{1s}), (\max(\mathbb{T}_{ws}^1, \mathbb{T}_{ws}^1), \min(\mathbb{I}_{ws}^1, \mathbb{I}_{ws}^1), \min(\mathbb{F}_{ws}^1, \mathbb{F}_{ws}^1))), \\ &= (h_w^{1s}, (\mathbb{T}_{ws}^1, \mathbb{I}_{ws}^1, \mathbb{F}_{ws}^1)), \\ &= \Phi_\Psi(x_w). \end{aligned} \tag{13}$$

From Equations 9, 10, 11, 12 and 13, we get $(\Phi_\Psi, Y, N_1)\check{\cup}(\Phi_\Psi, Y, N_1) = (\Phi_\Psi, Y, N_1)$.

4.
$$R.H.S = (\Phi_\Psi, Y, N_1)\hat{\cup}(\Phi_\Psi, Y, N_1), \tag{14}$$

where the restricted union of two $CSV\mathcal{N}NS_fSs$ is calculated as:

$$(\mathbb{L}_{\mathbb{M}}, \mathfrak{G}, \mathfrak{O}) = (\Phi_\Psi, Y, N_1)\hat{\cup}(\Phi_\Psi, Y, N_1), \tag{15}$$

with $\mathfrak{G} = Y \cap Y = Y$, $\mathfrak{O} = \max(N_1, N_1) = N_1$ and

$$\begin{aligned} \mathbb{L}_{\mathbb{M}}(x_w) &= (\max(h_w^{1s}, h_w^{1s}), (\max(\mathbb{T}_{ws}^1, \mathbb{T}_{ws}^1), \min(\mathbb{I}_{ws}^1, \mathbb{I}_{ws}^1), \min(\mathbb{F}_{ws}^1, \mathbb{F}_{ws}^1))), \\ &= (h_w^{1s}, (\mathbb{T}_{ws}^1, \mathbb{I}_{ws}^1, \mathbb{F}_{ws}^1)), \\ &= \Phi_\Psi(x_w), \end{aligned} \tag{16}$$

clearly, from Equations 14, 15 and 16, we get the required result.

□

Theorem 22. *Let (Φ_Ψ, Y, N_1) and (χ_A, C, N_2) be $CSV\mathcal{N}N_1S_fS$ and $CSV\mathcal{N}N_2S_fS$, respectively, over the same universe $\mathbb{U}$, then the absorption properties hold:*

1. $((\Phi_\Psi, Y, N_1)\check{\cup}(\chi_A, C, N_2))\hat{\cap}(\Phi_\Psi, E, N_1) = (\Phi_\Psi, Y, N_1)$
2. $(\Phi_\Psi, Y, N_1)\check{\cup}((\chi_A, C, N_2)\hat{\cap}(\Phi_\Psi, E, N_1)) = (\Phi_\Psi, Y, N_1)$
3. $((\Phi_\Psi, Y, N_1)\hat{\cap}(\chi_A, C, N_2))\check{\cup}(\Phi_\Psi, E, N_1) = (\Phi_\Psi, Y, N_1)$
4. $(\Phi_\Psi, Y, N_1)\hat{\cap}((\chi_A, C, N_2)\check{\cup}(\Phi_\Psi, E, N_1)) = (\Phi_\Psi, Y, N_1)$

Proof. 1. Let the extended union of $CSV\mathcal{N}N_1S_fS$ (Φ_Ψ, Y, N_1) and $CSV\mathcal{N}N_2S_fS$ (χ_A, C, N_2), be

$$(\mathcal{P}_{\mathcal{Q}}, \mathcal{T}, \mathfrak{B}) = (\Phi_\Psi, Y, N_1)\check{\cup}(\chi_A, C, N_2),$$

with $\mathcal{T} = Y \cup C$, $\mathfrak{B} = \max(N_1, N_2)$ and
$\mathcal{P}_{\mathcal{Q}}(x_w) = (h_w^s, (\mathbb{T}_{ws}, \mathbb{I}_{ws}, \mathbb{F}_{ws})) =$

$$\begin{cases} (h_w^{1s}, (\mathbb{T}_{ws}^1, \mathbb{I}_{ws}^1, \mathbb{F}_{ws}^1)), & \text{if } x_w \in Y - C, \\ (h_w^{2s}, (\mathbb{T}_{ws}^2, \mathbb{I}_{ws}^2, \mathbb{F}_{ws}^2)), & \text{if } x_w \in C - Y, \\ (\max(h_w^{1s}, h_w^{2s}), (\max(\mathbb{T}_{ws}^1, \mathbb{T}_{ws}^2), \min(\mathbb{I}_{ws}^1, \mathbb{I}_{ws}^2), \min(\mathbb{F}_{ws}^1, \mathbb{F}_{ws}^2))), & \text{if } x_w \in Y \cap C. \end{cases} \tag{17}$$

Now, consider the restricted intersection of $(\mathcal{P}_{\mathcal{Q}}, \mathcal{T}, \mathfrak{B})$ and (Φ_Ψ, Y, N_1), that is defined as

$$(L_M, G, O) = (\mathcal{P}_{\mathcal{Q}}, \mathcal{T}, \mathfrak{B})\hat{\cap}(\Phi_\Psi, Y, N_1),$$

with $G = \mathcal{T} \cap Y$, $O = \min(\mathfrak{B}, N_1) = N_1$ and

$$L_M(x_w) = (\min(h_w^s, h_w^{1s}), (\min(\mathbb{T}_{ws}, \mathbb{T}_{ws}^1), \max(\mathbb{I}_{ws}, \mathbb{I}_{ws}^1), \max(\mathbb{F}_{ws}, \mathbb{F}_{ws}^1))), \tag{18}$$

for all $x_w \in G = Y \cap C$, so that $x_w \in W$, $x_w \in C$. If $x_w \in W$, then there are three cases.

Case 1: if $x_w \in Y - C$, using Equations 17 and 18, we get,

$$\begin{aligned} L_M(x_w) &= (\min(h_w^{1s}, h_w^{1s}), (\min(\mathbb{T}_{ws}^1, \mathbb{T}_{ws}^1), \max(\mathbb{I}_{ws}^1, \mathbb{I}_{ws}^1), \max(\mathbb{F}_{ws}^1, \mathbb{F}_{ws}^1))) \\ &= (h_w^{1s}, \mathbb{T}_{ws}^1, \mathbb{F}_{ws}^1) \\ &= \Phi_\Psi(x_w) \end{aligned} \tag{19}$$

Case 2: if $x_w \in C - Y$, since $x_w \in G = Y \cap C$ implies $x_w \in Y$, therefore, this case is omitted.

Case 3: if $x_w \in C \cap Y$, using Equations 17 and 18, we get,

$$\begin{aligned} L_M(x_w) &= (\min(\max(h_w^{1s}, h_w^{2s}), h_w^{1s}), (\min(\max(\mathbb{T}_{ws}^1, \mathbb{T}_{ws}^2), \mathbb{T}_{ws}^1), \max(\min(\mathbb{I}_{ws}^1, \mathbb{T}_{ws}^2), \mathbb{I}_{ws}^1), \max(\min(\mathbb{F}_{ws}^1, \mathbb{T}_{ws}^2), \mathbb{F}_{ws}^1))) \\ &= (h_w^{1s}, \mathbb{T}_{ws}^1, \mathbb{F}_{ws}^1) \\ &= \Phi_\Psi(x_w) \end{aligned} \tag{20}$$

Thus from Equations 19 and 20, we get $((\Phi_\Psi, E, N_1)\check{\cup}(\chi_A, C, N_2))\hat{\cap}(\Phi_\Psi, E, N_1) = (\Phi_\Psi, E, N_1)$.

2. proofs of 2, 3 and 4 are same as above.

□

Theorem 23. *Let (Φ_Ψ, Y, N_1), (χ_A, C, N_2) and $(\Upsilon_\kappa, \varrho, N_3)$ be any three $CSV\mathcal{N}N_1S_fS$, $CSV\mathcal{N}N_2S_fS$, and $CSV\mathcal{N}N_3S_fS$, and over the same universe $\mathbb{U}$, then the following properties hold:*

1 $(\Phi_\Psi, Y, N_1)\check{\cup}(\chi_A, C, N_2) = (\chi_A, C, N_2)\check{\cup}(\Phi_\Psi, Y, N_1)$,
2 $(\Phi_\Psi, Y, N_1)\hat{\cup}(\chi_A, C, N_2) = (\chi_A, C, N_2)\hat{\cup}(\Phi_\Psi, Y, N_1)$,
3 $(\Phi_\Psi, Y, N_1)\check{\cap}(\chi_A, C, N_2) = (\chi_A, C, N_2)\check{\cap}(\Phi_\Psi, Y, N_1)$,
4 $(\Phi_\Psi, Y, N_1)\hat{\cap}(\chi_A, C, N_2) = (\chi_A, C, N_2)\hat{\cap}(\Phi_\Psi, Y, N_1)$,
5 $((\Phi_\Psi, Y, N_1)\check{\cup}(\chi_A, C, N_2))\check{\cup}(\Upsilon_\kappa, \varrho, N_3) = (\Phi_\Psi, Y, N_1)\check{\cup}(((\chi_A, C, N_2))\check{\cup}(\Upsilon_\kappa, \varrho, N_3))$,
6 $((\Phi_\Psi, Y, N_1)\hat{\cup}(\chi_A, C, N_2))\hat{\cup}(\Upsilon_\kappa, \varrho, N_3) = (\Phi_\Psi, Y, N_1)\hat{\cup}(((\chi_A, C, N_2))\hat{\cup}(\Upsilon_\kappa, \varrho, N_3))$,
7 $((\Phi_\Psi, Y, N_1)\check{\cap}(\chi_A, C, N_2))\check{\cap}(\Upsilon_\kappa, \varrho, N_3) = (\Phi_\Psi, Y, N_1)\check{\cap}(((\chi_A, C, N_2))\check{\cap}(\Upsilon_\kappa, \varrho, N_3))$,
8 $((\Phi_\Psi, Y, N_1)\hat{\cap}(\chi_A, C, N_2))\hat{\cap}(\Upsilon_\kappa, \varrho, N_3) = (\Phi_\Psi, Y, N_1)\hat{\cap}(((\chi_A, C, N_2))\hat{\cap}(\Upsilon_\kappa, \varrho, N_3))$,
9 $(\Phi_\Psi, Y, N_1)\check{\cup}((\chi_A, C, N_2)\hat{\cap}(\Upsilon_\kappa, \varrho, N_3)) = ((\Phi_\Psi, Y, N_1)\check{\cup}(\chi_A, C, N_2))\hat{\cap}((\Phi_\Psi, Y, N_1)\check{\cup}(\Upsilon_\kappa, \varrho, N_3))$,
10 $(\Phi_\Psi, Y, N_1)\check{\cap}((\chi_A, C, N_2)\hat{\cup}(\Upsilon_\kappa, \varrho, N_3)) = ((\Phi_\Psi, Y, N_1)\check{\cap}(\chi_A, C, N_2))\hat{\cup}((\Phi_\Psi, Y, N_1)\check{\cup}(\Upsilon_\kappa, \varrho, N_3))$,
11 $(\Phi_\Psi, Y, N_1)\hat{\cup}((\chi_A, C, N_2)\check{\cap}(\Upsilon_\kappa, \varrho, N_3)) = ((\Phi_\Psi, Y, N_1)\hat{\cup}(\chi_A, C, N_2))\check{\cap}((\Phi_\Psi, Y, N_1)\hat{\cup}(\Upsilon_\kappa, \varrho, N_3))$,
12 $(\Phi_\Psi, Y, N_1)\hat{\cap}((\chi_A, C, N_2)\check{\cup}(\Upsilon_\kappa, \varrho, N_3)) = ((\Phi_\Psi, Y, N_1)\hat{\cap}(\chi_A, C, N_2))\check{\cup}((\Phi_\Psi, Y, N_1)\hat{\cap}(\Upsilon_\kappa, \varrho, N_3))$.

4 Complex single-valued neutrosophic N-soft number

Definition 24. *Let $\Phi_\Psi(y_w) = ((u_s, h_w^s), (p_{ws}e^{i2\pi t_{ws}}, q_{ws}e^{i2\pi\omega_{ws}}, r_{ws}e^{i2\pi f_{ws}}))$ be a $CSV\mathcal{N}NS_fS$. Then the complex single-valued neutrosophic N-soft number $(CSV\mathcal{N}NS_fN)$ is defined as:*

$$\alpha_{ws} = (h_w^s, p_{ws}e^{i2\pi t_{ws}}, q_{ws}e^{i2\pi\omega_{ws}}, r_{ws}e^{i2\pi f_{ws}}),$$

Definition 25. *Consider a $CCSV\mathcal{N}NS_fN$ $\alpha_{ws} = (h_w^s, p_{ws}e^{i2\pi t_{ws}}, q_{ws}e^{i2\pi\omega_{ws}}, r_{ws}e^{i2\pi f_{ws}})$. The score function $S(\alpha_{ws})$ is:*

$$S(\alpha_{ws}) = \frac{h_w^s}{N-1} + (p_{ws} - q_{ws} - r_{ws}) + [t_{ws} - \omega_{ws} - f_{ws}], \tag{21}$$

where $S(\alpha_{ws}) \in [-4, 3]$. The accuracy function $A(\alpha_{ws})$ is:

$$A(\alpha_{ws}) = \frac{h_w^s}{N-1} + (p_{ws} + q_{ws} + r_{ws}) + [t_{ws} + \omega_{ws} + f_{ws}] \tag{22}$$

where $A(\alpha_{ws}) \in [0, 7]$, respectively.

Definition 26. *Let $\alpha_{ws} = (h_w^s, p_{ws}e^{i2\pi t_{ws}}, q_{ws}e^{i2\pi\omega_{ws}}, r_{ws}e^{i2\pi f_{ws}})$ and $\alpha_{ls} = (h_l^s, p_{ls}e^{i2\pi t_{ls}}, q_{ls}e^{i2\pi\omega_{ls}}, r_{ls}e^{i2\pi f_{ls}})$ be two $CSV\mathcal{N}NS_fNs$*

1. *If* $S_{\alpha_{ws}} < S_{\alpha_{ls}}$*, then* $\alpha_{ws} \prec \alpha_{ls}$ *(*α_{ws} *is inferior to* α_{ls}*),*
2. *If* $S_{\alpha_{ws}} = S_{\alpha_{ls}}$*, then*
   i $A_{\alpha_{ws}} < A_{\alpha_{ls}}$*, then* $\alpha_{ws} \prec \alpha_{ls}$ *(*α_{ws} *is inferior to* α_{ls}*),*
   ii $A_{\alpha_{ws}} = A_{\alpha_{ls}}$*, then* $\alpha_{ws} \sim \alpha_{ls}$ *(*α_{ws} *is equivalent to* α_{ls}*).*

Definition 27. *Let* $\alpha_{ws} = (h^s_w, p_{ws}e^{i2\pi t_{ws}}, q_{ws}e^{i2\pi\omega_{ws}}, r_{ws}e^{i2\pi f_{ws}})$ *and* $\alpha_{ls} = (h^s_l, p_{ls}e^{i2\pi t_{ls}}, q_{ls}e^{i2\pi\omega_{ls}}, r_{ls}e^{i2\pi f_{ls}})$ *be two* $CSV\mathcal{N}NS_fNs$ *and* $\beta > 0$*. Some operation for* $CSV\mathcal{N}NS_fNs$ *are*

$$\begin{aligned}
\beta\alpha_{ws} &= \Big(h^s_w, [1-(1-p_{ws})^\beta]e^{i2\pi[1-(1-t_{ws})^\beta]}, q^\beta_{ws}e^{i2\pi\omega^\beta_{ws}}, r^\beta_{ws}e^{i2\pi f^\beta_{ws}}\Big),\\
\alpha^\beta_{ws} &= \Big(h^s_w, p^\beta_{ws}e^{i2\pi t^\beta_{ws}}, [1-(1-q_{ws})^\beta]e^{i2\pi[1-(1-\omega_{ws})^\beta]}, [1-(1-r_{ws})^\beta]e^{i2\pi[1-(1-f_{ws})^\beta]}\Big),\\
\alpha_{ws}\oplus\alpha_{ls} &= \Big(\max(h^s_w,h^s_l), (p_{ws}+p_{ls}-p_{ws}p_{ls})e^{i2\pi(t_{ws}+t_{ls}-t_{ws}t_{ls})}, (q_{ws}q_{ls})e^{i2\pi(\omega_{ws}\omega_{ls})}, (r_{ws}r_{ls})e^{i2\pi(f_{ws}f_{ls})}\Big),\\
\alpha_{ws}\otimes\alpha_{ls} &= \Big(\min(h^s_w,h^s_l), (p_{ws}p_{ls})e^{i2\pi(t_{ws}t_{ls})}, (q_{ws}+q_{ls}-q_{ws}q_{ls})e^{i2\pi(\omega_{ws}+\omega_{ls}-\omega_{ws}\omega_{ls})}, (r_{ws}+r_{ls}-r_{ws}r_{ls})e^{i2\pi(f_{ws}+f_{ls}-f_{ws}f_{ls})}\Big).
\end{aligned}$$

Definition 28. *Let* $\alpha_{ws} = (h^s_w, p_{ws}e^{i2\pi t_{ws}}, q_{ws}e^{i2\pi\omega_{ws}}, r_{ws}e^{i2\pi f_{ws}})$ *and* $\alpha_{ls} = (h^s_l, p_{ls}e^{i2\pi t_{ls}}, q_{ls}e^{i2\pi\omega_{ls}}, r_{ls}e^{i2\pi f_{ls}})$ *be two* $CSV\mathcal{N}NS_fNs$ *and* $\beta > 0$*, then the following properties hold:*

1. $\alpha_{ws}\oplus\alpha_{ls} = \alpha_{ls}\oplus\alpha_{ws}$,
2. $\alpha_{ws}\otimes\alpha_{ls} = \alpha_{ls}\otimes\alpha_{ws}$,
3. $\beta\alpha_{ws}\oplus\beta\alpha_{ls} = \beta(\alpha_{ls}\oplus\alpha_{ws}), \beta > 0$,
4. $\beta_1\alpha_{ws}\oplus\beta_1\alpha_{ws} = (\beta_1+\beta_2)\alpha_{ws}, \beta_1, \beta_2 > 0$,
5. $\alpha^\beta_{ws}\otimes\alpha^\beta_{ls} = (\alpha_{ls}\otimes\alpha_{ws})^\beta, \beta > 0$,
6. $\alpha^{\beta_1}_{ws}\otimes\alpha^{\beta_1}_{ws} = \alpha^{(\beta_1+\beta_2)}_{ws}. \beta_1, \beta_2 > 0.$

Proof. 1.

$$\begin{aligned}
\alpha_{ws}\oplus\alpha_{ls} &= \Big(\max(h^s_w,h^s_l), (p_{ws}+p_{ls}-p_{ws}p_{ls})e^{i2\pi(t_{ws}+t_{ls}-t_{ws}t_{ls})}, (q_{ws}q_{ls})e^{i2\pi(\omega_{ws}\omega_{ls})}, (r_{ws}r_{ls})e^{i2\pi(f_{ws}f_{ls})}\Big),\\
&= \Big(\max(h^s_l,h^s_w), (p_{ls}+p_{ws}-p_{ls}p_{ws})e^{i2\pi(t_{ls}+t_{ws}-t_{ls}t_{ws})}, (q_{ls}q_{ws})e^{i2\pi(\omega_{ls}\omega_{ws})}, (r_{ls}r_{ws})e^{i2\pi(f_{ls}f_{ws})}\Big),\\
&= \alpha_{ls}\oplus\alpha_{ws}.
\end{aligned}$$

2.

$$\begin{aligned}
\alpha_{ws}\otimes\alpha_{ls} &= \Big(\min(h^s_w,h^s_l), (p_{ws}p_{ls})e^{i2\pi(t_{ws}t_{ls})}, (q_{ws}+q_{ls}-q_{ws}q_{ls})e^{i2\pi(\omega_{ws}+\omega_{ls}-\omega_{ws}\omega_{ls})}, (r_{ws}+r_{ls}-r_{ws}r_{ls})e^{i2\pi(f_{ws}+f_{ls}-f_{ws}f_{ls})}\Big)\\
&= \Big(\min(h^s_l,h^s_w), (p_{ls}p_{ws})e^{i2\pi(t_{ls}t_{ws})}, (q_{ls}+q_{ws}-q_{ls}q_{ws})e^{i2\pi(\omega_{ls}+\omega_{ws}-\omega_{ls}\omega_{ws})}, (r_{ls}+r_{ws}-r_{ls}r_{ws})e^{i2\pi(f_{ls}+f_{ws}-f_{ls}f_{ws})}\Big)\\
&= \alpha_{ls}\otimes\alpha_{ws}.
\end{aligned}$$

3.

$$\begin{aligned}
\beta\alpha_{ws}\oplus\beta\alpha_{ls} &= \Big(h^s_w, [1-(1-p_{ws})^\beta]e^{i2\pi[1-(1-t_{ws})^\beta]}, q^\beta_{ws}e^{i2\pi\omega^\beta_{ws}}, r^\beta_{ws}e^{i2\pi f^\beta_{ws}}) \oplus (h^s_l, [1-(1-p_{ls})^\beta]e^{i2\pi[1-(1-t_{ls})^\beta]},\\
&\quad q^\beta_{ls}e^{i2\pi\omega^\beta_{ls}}, r^\beta_{ls}e^{i2\pi f^\beta_{ls}}\Big)\\
&= \Big(\max(h^s_w,h^s_l), ([1-(1-p_{ws})^\beta]+[1-(1-p_{ls})^\beta]-[1-(1-p_{ws})^\beta][1-(1-p_{ls})^\beta])\\
&\quad e^{i2\pi([1-(1-t_{ws})^\beta]+[1-(1-t_{ls})^\beta]-[1-(1-t_{ws})^\beta][1-(1-t_{ls})^\beta])}, (q^\beta_{ws}q^\beta_{ls})e^{i2\pi(\omega^\beta_{ws}\omega^\beta_{ls})}, (r^\beta_{ws}r^\beta_{ls})e^{i2\pi(q^\beta_{ws}q^\beta_{ls})}\Big)\\
&= \Big(\max(h^s_w,h^s_l), [1-(1-p_{ws}+p_{ls}-p_{ws}p_{ls})^\beta]e^{i2\pi[1-(1-t_{ws}+t_{ls}-t_{ws}t_{ls})^\beta]}, (q_{ws}q_{ls})^\beta e^{i2\pi(\omega_{ws}\omega_{ls})^\beta},\\
&\quad (r_{ws}r_{ls})^\beta e^{i2\pi(f_{ws}f_{ls})^\beta}\Big)\\
&= \beta\Big(\max(h^s_w,h^s_l), (p_{ws}+p_{ls}-p_{ws}p_{ls})e^{2\pi(p_{ws}+p_{ls}-p_{ws}p_{ls})}, (q_{ws}q_{ls})e^{2\pi(\omega_{ws}\omega_{ls})}, (r_{ws}r_{ls})e^{2\pi(f_{ws}f_{ls})}\Big)\\
&= \beta\left(\alpha_{ws}\oplus\alpha_{ls}\right).
\end{aligned}$$

Similarly, we can prove 4, 5 and 6.

□

Definition 29. *Let $\alpha_{ws} = (h^s_w, p_{ws}e^{i2\pi t_{ws}}, q_{ws}e^{i2\pi\omega_{ws}}, r_{ws}e^{i2\pi f_{ws}})$ $(w = 1, 2, \ldots, k)$ be a collection of $CSV\mathcal{N}NS_fNs$ and ν_w be the weight vectors of α_{ws} with $\nu_w > 0$ and $\sum_{w=1}^{k}\nu_w = 1$. The complex single-valued neutrosophic N-soft weighted average operator $(CSV\mathcal{N}NS_fWA)$ is a mapping $CSV\mathcal{N}NS_fWA : \mathcal{J}^k \to \mathcal{J}$, where $\mathcal{J}$ is the set of $CSV\mathcal{N}NS_fNs$, defined as follows:*

$$CSV\mathcal{N}NS_fWA(\alpha_{1s}, \alpha_{2s}, \ldots, \alpha_{ls}) = (\nu_1\alpha_{1s} \oplus \nu_2\alpha_{2s} \oplus \ldots \oplus \nu_k\alpha_{ks})$$
$$= \Big(\max_{w=1}^{k}(h^s_w), [1 - \Pi^k_{w=1}(1 - p_{ws})^{\nu_w}]e^{i2\pi[1- \Pi^k_{w=1}(1 - t_{ws})^{\nu_w}]}, [\Pi^k_{w=1}(q_{ws})^{\nu_w}]e^{i2\pi[\Pi^k_{w=1}(\omega_{ws})^{\nu_w}]}, [\Pi^k_{w=1}(r_{ws})^{\nu_w}]e^{i2\pi[\Pi^k_{w=1}(f_{ws})^{\nu_w}]}\Big).$$

Definition 30. *Let $\alpha_{ws} = (h^s_w, p_{ws}e^{i2\pi t_{ws}}, q_{ws}e^{i2\pi\omega_{ws}}, r_{ws}e^{i2\pi f_{ws}})$ $(w = 1, 2, \ldots, k)$ be a collection of $CSV\mathcal{N}NS_fNs$ and ν_w be the weight vectors of α_{ws} with $\nu_w > 0$ and $\sum_{w=1}^{k}\nu_w = 1$. The complex single-valued neutrosophic N-soft ordered weighted average operator $(CSV\mathcal{N}NS_fOWA)$ is a mapping $CSV\mathcal{N}NS_fOWA : \mathcal{J}^k \to \mathcal{J}$, where $\mathcal{J}$ is the set of $CSV\mathcal{N}NS_fNs$, defined as follows:*

$$\begin{aligned}
&CSV\mathcal{N}NS_fOWA(\alpha_{1s}, \alpha_{2s}, \ldots, \alpha_{ls})\\
&= \big(\nu_1\alpha_{\varrho(1s)} \oplus \nu_2\alpha_{\varrho(2s)} \oplus \ldots \oplus \nu_k\alpha_{\varrho(ks)}\big)\\
&= \Big(\max_{w=1}^{k}(h^s_\varrho(w)), [1 - \Pi^k_{w=1}(1 - p_{\varrho(ws)})^{\nu_w}]e^{i2\pi[1- \Pi^k_{w=1}(1 - t_{\varrho(ws)})^{\nu_w}]}, [\Pi^k_{w=1}(q_{\varrho(ws)})^{\nu_w}]e^{i2\pi[\Pi^k_{w=1}(\omega_{\varrho(ws)})^{\nu_w}]},\\
&[\Pi^k_{w=1}(r_{\varrho(ws)})^{\nu_w}]e^{i2\pi[\Pi^k_{w=1}(f_{\varrho(ws)})^{\nu_w}]}\Big).
\end{aligned}$$

where, $\varrho(ws)$ is a permutation ordered by $\alpha_{\varrho(ws)} \geq \alpha_{\phi(vs)}$, for all $w < v$, $(w, v = 1, 2, \ldots, k)$ and $(s = 1, 2, \ldots, t)$.

Definition 31. *Let $\alpha_{ws} = (h^s_w, p_{ws}e^{i2\pi t_{ws}}, q_{ws}e^{i2\pi\omega_{ws}}, r_{ws}e^{i2\pi f_{ws}})$ $(i = 1, 2, \ldots, l)$ be a collection of $CSV\mathcal{N}NS_fNs$ and ν_w be the weight vectors of α_{ws} with $\nu_w > 0$ and $\sum_{w=1}^{k}\nu_w = 1$. The single-valued neutrosophic N-soft weighted geometric operator $(CSV\mathcal{N}NS_fWG)$ is a mapping $CSV\mathcal{N}NS_fWG : \mathcal{J}^k \to \mathcal{J}$, where $\mathcal{J}$ is the set of $CSV\mathcal{N}NS_fNs$, defined as follows:*

$$CSV\mathcal{N}NS_fWG(\alpha_{1s}, \alpha_{2s}, \ldots, \alpha_{ks}) = (\alpha_{1s}^{\nu_1} \otimes \alpha_{2s}^{\nu_2} \otimes \ldots \otimes \alpha_{ks}^{\nu_k})$$
$$= \Big(\min_{w=1}^{k}(h^s_w), [\Pi^k_{w=1}(p_{ws})^{\nu_w}]e^{[\Pi^k_{w=1}(t_{ws})^{\nu_w}]}, [1 - \Pi^k_{w=1}(1 - q_{ws})^{\nu_w}]e^{[1- \Pi^k_{w=1}(1 - \omega_{ws})^{\nu_w}]}, [1 - \Pi^k_{w=1}(1 - r_{ws})^{\nu_w}]e^{[\Pi^k_{w=1}(1 - f_{ws})^{\nu_w}]}\Big).$$

Definition 32. *Let $\alpha_{ws} = (h^s_w, p_{ws}e^{i2\pi t_{ws}}, q_{ws}e^{i2\pi\omega_{ws}}, r_{ws}e^{i2\pi f_{ws}})$ $(i = 1, 2, \ldots, l)$ be a collection of $CSV\mathcal{N}NS_fNs$ and ν_w be the weight vectors of α_{ws} with $\nu_w > 0$ and $\sum_{w=1}^{k}\nu_w = 1$. The single-valued neutrosophic N-soft ordered weighted geometric operator $(CSV\mathcal{N}NS_fOWG)$ is a mapping $CSV\mathcal{N}NS_fOWG : \mathcal{J}^K \to \mathcal{J}$, where $\mathcal{J}$ is the set of $CSV\mathcal{N}NS_fNs$, defined as follows:*

$$\begin{aligned}
&CSV\mathcal{N}NS_fOWG(\alpha_{1s}, \alpha_{2s}, \ldots, \alpha_{ks})\\
&= \big(\alpha_{\varrho(1s)^{\nu_1}} \otimes \alpha_{\varrho(2s)^{\nu_2}} \otimes \ldots \otimes \alpha_{\varrho(ks)^{\nu_k}}\big)\\
&= \Big(\min_{i=1}^{l}(h^s_\varrho(w)), [\Pi^k_{w=1}(p_{\varrho(ws)})^{\nu_w}]e^{i2\pi[\Pi^k_{w=1}(t_{\varrho(ws)})^{\nu_w}]}, [1 - \Pi^k_{w=1}(1 - q_{\varrho(ws)})^{\nu_w}]e^{i2\pi[1- \Pi^k_{w=1}(1 - \omega_{\varrho(ws)})^{\nu_w}]},\\
&[1 - \Pi^k_{w=1}(1 - r_{\varrho(ws)})^{\nu_w}]e^{i2\pi[1- \Pi^k_{w=1}(1 - q_{\varrho(ws)})^{\nu_w}]}\Big),
\end{aligned}$$

where, $\varrho(ws)$ is a permutation ordered by $\alpha_{\varrho(ws)} \geq \alpha_{\phi(vs)}$, for all $w < v$, $(w, v = 1, 2, \ldots, k)$ and $(s = 1, 2, \ldots, t)$.

5 Complex single-valued neutrosophic N-soft TOPSIS method

In this section, we developed methodology for TOPSIS method under the framework of $CSV\mathcal{N}NS_fSs$ for solving multi-attribute group decision making (MAGDM) problem. For the optimal solution of the MADM problem, TOPSIS method specifically used ideal solutions of that problem. Consider a MAGDM problem with $\mathbb{U} = \{\mathbb{U}_1, \mathbb{U}_2, \mathbb{U}_3, \ldots, \mathbb{U}_t\}$ and $Y = \{Y_1, Y_2, Y_3, \ldots, Y_k\}$ be the set of alternative and attributes decided by the experts $\widetilde{Z}_1, \widetilde{Z}_2, \widetilde{Z}_3, \ldots, \widetilde{Z}_f$, where the experts weight vector for this MAGDM problem is $\nu = (\nu_1, \nu_2, \nu_3, \ldots, \nu_k)^T$. The procedure for $CSV\mathcal{N}NS_f$-TOPSIS method is as follows:

5.1 Organizing the complex single-valued neutrosophic N-soft decision matrix

After studied the MADM problem properly, decision makers use rating system for assigning rank to each alternative, parallel to each semantic term, relative to the attributes that indeed form a NS_fS. Further, decision making panel associate $CSV\mathcal{N}NS_fN$ corresponding to each rank (ordered grade) by defining grading criteria related to the aptitude of the MADM problem. Therefore, a complex single-valued neutrosophic N-soft decision matrix $(CSV\mathcal{N}NS_fDM)$ $\mathbb{H} = (\mathbb{H}^{(j)}_{ws})_{(s\times w)}$ is organized as follow:

$$\mathbb{H}^{(j)} = \begin{pmatrix} ({h^1_1}^{(j)}, \mathbb{T}^{(j)}_{11}, \mathbb{I}^{(j)}_{11}, \mathbb{F}^{(j)}_{11}) & ({h^1_2}^{(j)}, \mathbb{T}^{(j)}_{12}, \mathbb{I}^{(j)}_{12}, \mathbb{F}^{(j)}_{12}) & \dots & ({h^1_k}^{(j)}, \mathbb{T}^{(j)}_{1k}, \mathbb{I}^{(j)}_{1k}, \mathbb{F}^{(j)}_{1k}) \\ ({h^2_1}^{(j)}, \mathbb{T}^{(j)}_{21}, \mathbb{I}^{(j)}_{21}, \mathbb{F}^{(j)}_{21}) & ({h^2_2}^{(j)}, \mathbb{T}^{(j)}_{22}, \mathbb{I}^{(j)}_{22}, \mathbb{F}^{(j)}_{22}) & \dots & ({h^2_m}^{(j)}, \mathbb{T}^{(j)}_{2k}, \mathbb{I}^{(j)}_{2k}, \mathbb{F}^{(j)}_{2k}) \\ \vdots & \vdots & \ddots & \vdots \\ ({h^t_1}^{(j)}, \mathbb{T}^{(j)}_{t1}, \mathbb{I}^{(j)}_{t1}, \mathbb{F}^{(j)}_{t1}) & ({h^t_2}^{(j)}, \mathbb{T}^{(j)}_{t2}, \mathbb{I}^{(j)}_{t2}, \mathbb{F}^{(j)}_{t2}) & \dots & ({h^t_k}^{(j)}, \mathbb{T}^{(j)}_{tk}, \mathbb{I}^{(j)}_{tk}, \mathbb{F}^{(j)}_{tk}) \end{pmatrix},$$

where, $\mathbb{H}^{(j)}_{ws} = ((h^j_i)^{(j)}, \mathbb{T}^{(j)}_{ws}, \mathbb{I}^{(j)}_{ws}, \mathbb{F}^{(j)}_{ws}) = (h^s_w, p^{(j)}_{ws}e^{i2\pi t^{(j)}_{ws}}, q^{(j)}_{ws}e^{i2\pi\omega^{(j)}_{ws}}, r^{(j)}_{ws}e^{i2\pi f^{(j)}_{ws}})$, $s = \{1, 2, 3, \dots, t\}$, $j = \{1, 2, 3, \dots, f\}$, and $w = \{1, 2, 3, \dots, k\}$.

5.2 Aggregated complex single-valued neutrosophic N-soft decision matrix

As the decision makers (experts) are not equally weighted in MAGDM problems, therefore by utilizing the weightage of each expert decided by the panel we cumulate the decision of all experts and get aggregated complex single-valued neutrosophic N-soft decision matrix $(ACSV\mathcal{N}NS_fDM)$. The $CSV\mathcal{N}NS_fWA$ operator or $CSV\mathcal{N}NS_fWG$ operator are precisely used to commulate the $CSV\mathcal{N}NS_fDM$ $(\mathcal{H})$ as follows:

$$\begin{aligned} \mathcal{H}_{ws} &= CSV\mathcal{N}NS_fWA(\mathbb{H}^{(1)}_{ws}, \mathbb{H}^{(2)}_{ws}, \dots, \mathbb{H}^{(f)}_{ws}); \\ (OR) &= CSV\mathcal{N}NS_fWG(\mathbb{H}^{(1)}_{ws}, \mathbb{H}^{(2)}_{ws}, \dots, \mathbb{H}^{(f)}_{ws}); \end{aligned}$$

where, $\mathcal{H}_{ws} = (h^1_1, \mathbb{T}_{ws}, \mathbb{I}_{ws}, \mathbb{F}_{ws}) = (h^s_w, p_{ws}e^{i2\pi t_{ws}}, q_{ws}e^{i2\pi\omega_{ws}}, r_{ws}e^{i2\pi f_{ws}})$.
The $ACSV\mathcal{N}N_fSDM$ denoted as:

$$\mathcal{H} = \begin{pmatrix} (h^1_1, \mathbb{T}_{11}, \mathbb{I}_{11}, \mathbb{F}_{11}) & (h^1_2, \mathbb{T}_{12}, \mathbb{I}_{12}, \mathbb{F}_{12}) & \dots & (h^1_k, \mathbb{T}_{1k}, \mathbb{I}_{1k}, \mathbb{F}_{1k}) \\ (h^2_1, \mathbb{T}_{21}, \mathbb{I}_{21}, \mathbb{F}_{21}) & (h^2_2, \mathbb{T}_{22}, \mathbb{I}_{22}, \mathbb{F}_{22}) & \dots & (h^2_k, \mathbb{T}_{2k}, \mathbb{I}_{2k}, \mathbb{F}_{2k}) \\ \vdots & \vdots & \ddots & \vdots \\ (h^s_1, \mathbb{T}_{s1}, \mathbb{I}_{s1}, \mathbb{F}_{s1}) & (h^q_2, \mathbb{T}_{s2}, \mathbb{I}_{s2}, \mathbb{F}_{s2}) & \dots & (h^s_k, \mathbb{T}_{sk}, \mathbb{I}_{sk}, \mathbb{F}_{sk}) \end{pmatrix}.$$

5.3 Weights for parameters

To highlight the influence of the parameters in the MAGDM problem, experts judged each parameter and assign grades as the weight of the parameter. Further, $CSV\mathcal{N}NS_fNs$ are associated to each grade using the grading criteria finalized by the panel. Let $\theta^{(j)}_w = (h^{(j)}_w, \mathbb{T}^{(j)}_w, \mathbb{I}^{(j)}_w, \mathbb{F}^{(j)}_w)$ be the weight of wth parameter given by the jth expert in the MAGDM problem. Let $\theta = (\theta_1, \theta_2, \dots, \theta_k)^T = (h_w, \mathbb{T}_w, \mathbb{I}_w, \mathbb{F}_w)$ be the weight vector of attributes that is summarized, by $CSV\mathcal{N}NS_fWA$ operator or $CSV\mathcal{N}NS_fWG$ operator, as follows:

$$\begin{aligned} \theta_w &= CSV\mathcal{N}NS_fWA(\theta^{(j)}_1, \theta^{(j)}_2, \dots, \theta^{(j)}_k); \\ (OR) &= CSV\mathcal{N}NS_fWG(\theta^{(j)}_1, \theta^{(j)}_2, \dots, \theta^{(j)}_k). \end{aligned}$$

where, $\theta_w = (h_1, \mathbb{T}_w, \mathbb{I}_w, \mathbb{F}_w) = (h_w, p_we^{i2\pi t_w}, q_we^{i2\pi\omega_w}, r_we^{i2\pi f_w})$.

5.4 Aggregated weighted complex single-valued neutrosophic N-soft decision matrix

The $ACSV\mathcal{N}N_fSDM$ $\mathcal{H}$ is used within the weight vector $(\theta_1, \theta_2, \dots, \theta_k)^T$ of parameter for the formulation of aggregated weighted single-valued neutrosophic N-soft decision matrix $(AWCSV\mathcal{N}NS_fDM)$. The calculations for are performed as follows:

$$\begin{aligned} \bar{H}_{ws} &= \mathcal{H}_{ws} \otimes \theta_w \\ &= (\min((h^s_w), h_w), (\mathbb{T}_{ws}\mathbb{T}_w), (\mathbb{I}_{ws} + \mathbb{I}_w - \mathbb{I}_{ws}\mathbb{I}_w), (\mathbb{F}_{ws} + \mathbb{F}_i - \mathbb{F}_{ws}\mathbb{F}_W)) \\ &= \left(\min(h^s_w, h_w), p_{ws}p_we^{i2\pi t_{ws}t_w}, (q_{ws} + q_w - q_{ws}q_w)e^{i2\pi[\omega_{ws}+\omega_{ws}-\omega_{ws}\omega_w]}, (r_{ws} + r_w - r_{ws}r_w)e^{i2\pi[f_{ws}+f_{ws}-f_{ws}f_w]} \right) \\ &= (\bar{h}^s_w, \bar{\mathbb{T}}_{ws}, \bar{\mathbb{I}}_{ws}, \bar{\mathbb{F}}_{ws}) \\ &= (\bar{h}^s_w, \bar{p}_{ws}e^{i2\pi\bar{t}_{ws}}, \bar{q}_{ws}e^{i2\pi\bar{\omega}_{ws}}, \bar{r}_{ws}e^{i2\pi\bar{f}_{ws}}). \end{aligned}$$

The $AWCSV\mathcal{N}NS_fDM$ is:

$$\bar{H}_{ws} = \begin{pmatrix} (\bar{h}_1^1, \bar{\mathbb{T}}_{11}, \bar{\mathbb{I}}_{11}, \bar{\mathbb{F}}_{11}) & (\bar{h}_2^1, \bar{\mathbb{T}}_{12}, \bar{\mathbb{I}}_{12}, \bar{\mathbb{F}}_{12}) & \ldots & (\bar{h}_k^1, \bar{\mathbb{T}}_{1k}, \bar{\mathbb{I}}_{1k}, \bar{\mathbb{F}}_{1k}) \\ (\bar{h}_1^2, \bar{\mathbb{T}}_{21}, \bar{\mathbb{I}}_{21}, \bar{\mathbb{F}}_{21}) & (\bar{h}_2^2, \bar{\mathbb{T}}_{22}, \bar{\mathbb{I}}_{22}, \bar{\mathbb{F}}_{22}) & \ldots & (\bar{h}_k^2, \bar{\mathbb{T}}_{2k}, \bar{\mathbb{I}}_{2k}, \bar{\mathbb{F}}_{2k}) \\ \vdots & \vdots & \ddots & \vdots \\ (\bar{h}_1^s, \bar{\mathbb{T}}_{s1}, \bar{\mathbb{I}}_{s1}, \bar{\mathbb{F}}_{s1}) & (\bar{h}_2^s, \bar{\mathbb{T}}_{s2}, \bar{\mathbb{I}}_{s2}, \bar{\mathbb{F}}_{s2}) & \ldots & (\bar{h}_k^q, \bar{\mathbb{T}}_{sk}, \bar{\mathbb{I}}_{sk}, \bar{\mathbb{F}}_{sk}) \end{pmatrix}.$$

5.5 Complex single-valued neutrosophic N-soft ideal solutions

Let $\mathbb{BT}$ be the collection of benefit-type criteria and $\mathbb{CT}$ be the collection of cost-type criteria opted from the number of parameters, keeping in view the expertise of the given problem. Using these collection we are able to evaluate the complex single-valued neutrosophic positive ideal solution $CSV\mathcal{N}N_fS$-PIS and complex single-valued neutrosophic N-soft negative ideal solution $CSV\mathcal{N}NS_f$-NIS of the MAGDM problem. The $CSV\mathcal{N}NS_f$-PIS, related to the parameter Y_w, is defined as:

$$\bar{H}_w^{PIS} = \begin{cases} \max\limits_{j=1}^{s} \bar{H}_{ws}, & \text{if } Y_w \in \mathbb{BT}, \\ \min\limits_{j=1}^{s} \bar{H}_{ws}, & \text{if } Y_w \in \mathbb{CT}, \end{cases}$$

and the $CSV\mathcal{N}NS_f$-NIS is defined as:

$$\bar{H}_w^{NIS} = \begin{cases} \max\limits_{j=1}^{s} \bar{H}_{ws}, & \text{if } Y_w \in \mathbb{CT}, \\ \min\limits_{j=1}^{s} \bar{H}_{ws}, & \text{if } Y_w \in \mathbb{BT}. \end{cases}$$

The $CSV\mathcal{N}NS_f\text{-}PIS$ and $CSV\mathcal{N}NS_f\text{-}NIS$ are denoted as: $\bar{H}_w^{PIS} = (\dot{h}_w, \dot{p}_w e^{i2\pi \dot{t}_w}, \dot{q}_w e^{i2\pi\dot{\omega}_w}, \dot{r}_w e^{i2\pi \dot{f}_w})$., and $\bar{H}_w^{NIS} = (\ddot{h}_w, \ddot{p}_w e^{i2\pi \ddot{t}_w}, \ddot{q}_w e^{i2\pi\ddot{\omega}_w}, \ddot{r}_w e^{i2\pi \ddot{f}_w})$, respectively.

5.6 Formulation of normalized Euclidean distance

For evaluating the alternatives distance from the ideal solution, we can used similarity measures or distance measure. Moreover, from distance measures we used the normalized Euclidean distance. The normalized Euclidean distance of any of the alternative $\mathbb{U}_s$ from the $CSV\mathcal{N}NS_f\text{-}PIS$ is defined as:

$$d(\bar{H}_w^{PIS}, \mathbb{U}_s) = \Big(\frac{1}{7w}\sum_{w=1}^{k}[(\frac{\dot{h}_w}{N-1}) - (\frac{\bar{h}_w^s}{N-1})^2 + (\dot{p}_w - \bar{p}_{ws})^2 + (\dot{q}_w - \bar{q}_{ws})^2 + (\dot{r}_w - \bar{r}_{ws})^2 + (\dot{t}_w - \bar{t}_{ws})^2 + (\dot{\omega}_w - \bar{\omega}_{ws})^2 + (\dot{f}_w - \bar{f}_{ws})^2]\Big) \tag{23}$$

The normalized Euclidean distance between the $CSV\mathcal{N}NS_f$-NIS and any of the alternative $\mathbb{U}_s$, can be evaluated as follows:

$$d(\bar{H}_w^{NIS}, \mathbb{U}_s) = \Big(\frac{1}{7w}\sum_{w=1}^{k}[(\frac{\ddot{h}_w}{N-1}) - (\frac{\bar{h}_w^s}{N-1})^2 + (\ddot{p}_w - \bar{p}_{ws})^2 + (\ddot{q}_w - \bar{q}_{ws})^2 + (\ddot{r}_w - \bar{r}_{ws})^2 + (\ddot{t}_w - \bar{t}_{ws})^2 + (\ddot{\omega}_w - \bar{\omega}_{ws})^2 + (\ddot{f}_w - \bar{f}_{ws})^2]\Big) \tag{24}$$

5.7 Revised closeness index

In TOPSIS method, at last we left with two values related to the alternative that prescribed the distance of that particular alternative from $CSV\mathcal{N}NS_f$-PIS and $CSV\mathcal{N}NS_f$-NIS. Therefore, revised closeness index is utilized for the choice of right solution. The revised closeness index $\Lambda(\mathbb{U}_s)$ is calculated as:

$$\Lambda(\mathbb{U}_s) = \frac{d(\bar{H}_w^{PIS}, \mathbb{U}_s)}{\min\limits_{s} d(\bar{H}_w^{PIS}, \mathbb{U}_s)} - \frac{d(\bar{H}_w^{NIS}, \mathbb{U}_s)}{\max\limits_{s} d(\bar{H}_w^{NIS}, \mathbb{U}_s)}, \tag{25}$$

where, $s = 1, 2, \ldots, t$.

5.8 Identify dominant alternative

For the evaluation of dominant alternative with respect to their performance in MAGDM problem, revised closeness index related to each alternative arranged in ascending order. So that the alternative with least revised closeness index will be the required one.
For solving a MAGDM problem, the Algorithm 1 is given as:

Algorithm 1: Steps to deal MAGDM problem by $CSV\mathbb{N}N_f$-TOPSIS method

1. **Input**:
   $\mathbb{U}$: Set of alternatives,
   Y : Set of attributes,
   ν : Weight vector for experts $\widetilde{Z}_j$,
   $NS_fS : (\Phi_\Psi, Y, N)$ with $H = \{0, 1, 2, 3, \ldots, N-1\}$, $N \in \{1, 2, 3, \ldots\}$,
2. Construct the $CSV\mathbb{N}NS_fDM$ $\mathbb{H}^{(j)}$, using the input data.
3. Evaluate the $ACSV\mathbb{N}NS_fDM$ as follows:

$$\begin{aligned}\mathcal{H}_{ws} \quad = \quad & \Big(\max_{j=1}^{f}(h_w^s)^{(j)}, [1-\Pi_{j=1}^{f}(1-p_{ws}^{(j)})^{\nu_w}]e^{i2\pi[1-\Pi_{j=1}^{f}(1-t_{ws}^{(j)})^{\nu_w}]}, [\Pi_{j=1}^{f}(q_{ws}^{(j)})^{\nu_w}]e^{i2\pi[\Pi_{j=1}^{f}(\omega_{ws}^{(j)})^{\nu_w}]}, \\ & [\Pi_{j=1}^{f}(r_{ws}^{(j)})^{\nu_w}]e^{i2\pi[\Pi_{j=1}^{f}(f_{ws}^{(j)})^{\nu_w}]}\Big).\end{aligned}$$

4. Calculating the weight vector $\theta = (\theta_1, \theta_2, \ldots, \theta_k)^T$ for parameters as:

$$\begin{aligned}\theta_w \quad = \quad & \Big(\max_{j=1}^{f}(h_w)^{(j)}, [1-\Pi_{j=1}^{f}(1-p_w^{(j)})^{\nu_w}]e^{i2\pi[1-\Pi_{j=1}^{f}(1-t_w^{(j)})^{\nu_w}]}, [\Pi_{j=1}^{f}(q_w^{(j)})^{\nu_w}]e^{i2\pi[\Pi_{j=1}^{f}(\omega_w^{(j)})^{\nu_w}]}, \\ & [\Pi_{j=1}^{f}(r_w^{(j)})^{\nu_w}]e^{i2\pi[\Pi_{j=1}^{f}(f_w^{(j)})^{\nu_w}]}\Big).\end{aligned}$$

5. Compute the $AWCSV\mathbb{N}NS_fDM$ using $ACSV\mathbb{N}NS_fDM$ and the weight vector of attributes θ_w, as follows:

$$\bar{H}_{ws} \quad = \quad \left(\min(h_w^s, h_w), p_{ws}p_w e^{i2\pi t_{ws}t_w}, (q_{ws}+q_w-q_{ws}q_w)e^{i2\pi[\omega_{ws}+\omega_{ws}-\omega_{ws}\omega_w]}, (r_{ws}+r_w-r_{ws}r_w)e^{i2\pi[f_{ws}+f_{ws}-f_{ws}f_w]}\right).$$

6. Evaluate the $CSV\mathbb{N}NS_f$ PIS and $CSV\mathbb{N}NS_f$ NIS.
7. Evaluate the normalized Euclidean distance $d(\bar{H}_w^{PIS}, \mathbb{U}_s)$ and $d(\bar{H}_w^{NIS}, \mathbb{U}_s)$
8. Evaluate the revised closeness index $\Lambda(\mathbb{U}_s)$.
9. Arranged revised closeness index in ascending order.

Output: Choose the alternative with minimum revised closeness index.

6 Application

In this section, we solve a MAGDM problem using $CSV\mathcal{N}NS_f - TOPSIS$ method for analyzing the performance of Islamic banks in Pakistan with CAMELS rating system.

6.1 Monitoring performance of Islamic banking industry on the basis of CAMELS rating system.

The banks are more closely monitored other than any field of economy because of their constitution and important role in the economy of the country. Analyzing the banking system create more assurance and reliability in making both short and long term decisions, that in return give on to healthier business in the country. In banking industry, one of the flourishing institute is Islamic banking that follow the rules of Islamic Shariah and promote the Islamic principles to the transaction of financial banking. The evaluation of financial performance of Islamic banking in Pakistan using the CAMELS model and TOPSIS method is necessary for higher level of efficiency that further help to set a benchmark for the country. In this MAGDM problem, following Islamic banks are considered as alternatives:

$\mathbb{U}_1$: Bank Albarka(BA)

$\mathbb{U}_2$: Bank Islamic (BIL)

$\mathbb{U}_3$: Dubai Islamic Bank (DIB)

$\mathbb{U}_4$: Muslim Commercial Bank (MCB)

$\mathbb{U}_5$: Meezan Bank (MBL)

For this MAGDM problem, decision making panel consists of three experts $\widetilde{Z}_1, \widetilde{Z}_2, \widetilde{Z}_3$ that collected data from the official websites of the banks according to the CAMELS model. CAMELS model is generally apply to analyze the performance of the banks on the basis of five different attributes described as follow:

Y_1 : **C**apital adequacy: Experts rank the capital adequacy by checking the factors of growth plan and capacity to control financial risk and loan.

Y_2 : **A**sset quality: In this attribute the banking stability is measure whenever the bank faced loss of values of the assets.

Y_3 : **M**anagement: Experts rate this attribute by measuring the efficiency of banks while dealing with daily activities.

Y_4 : **E**arning capacity: This attribute includes the existing assets, earnings and growth of the banks, as well as to remain competitive in economy.

Y_5 : **L**iquidity: This attribute examine on the basis of the availability of adequate funds by converting assets into the cash.

We solve this MAGDM problem by following the $CSV\mathcal{N}NS_f$-TOPSIS method.

Step 1: According to these attributes each expert model 5-soft set in Table14 where

0 means 'Bad'

1 means 'Ok'

2 means 'Good'

3 means 'Great'

4 means 'Excellent'

Table 14: Initial rating by decision making experts

$Parameters$	$Alternatives$	$\widetilde{Z}_1$	$\widetilde{Z}_2$,	$\widetilde{Z}_3$
Y_1	$\mathbb{U}_1$	$****\ =4$	$***\ =3$	$**\ =2$
	$\mathbb{U}_2$	$****\ =4$	$*\ =1$	$***\ =3$
	$\mathbb{U}_3$	$***\ =3$	$**\ =2$	$*\ =1$
	$\mathbb{U}_4$	$***\ =3$	$*\ =1$	$***\ =3$
	$\mathbb{U}_5$	$****\ =4$	$\bullet\ =0$	$*\ =1$
Y_2	$\mathbb{U}_1$	$***\ =3$	$**\ =2$	$***\ =3$
	$\mathbb{U}_2$	$***\ =3$	$****\ =4$	$****\ =4$
	$\mathbb{U}_3$	$****\ =4$	$\bullet\ =0$	$**\ =2$
	$\mathbb{U}_4$	$****\ =4$	$***\ =3$	$****\ =4$
	$\mathbb{U}_5$	$****\ =4$	$*\ =1$	$**\ =2$
Y_3	$\mathbb{U}_1$	$\bullet\ =0$	$*\ =1$	$**\ =2$
	$\mathbb{U}_2$	$****\ =4$	$****\ =4$	$***\ =3$
	$\mathbb{U}_3$	$\bullet\ =0$	$**\ =2$	$*\ =1$
	$\mathbb{U}_4$	$\bullet\ =0$	$***\ =3$	$****\ =4$
	$\mathbb{U}_5$	$\bullet\ =0$	$\bullet\ =0$	$*\ =1$
Y_4	$\mathbb{U}_1$	$\bullet\ =0$	$*\ =1$	$\bullet\ =0$
	$\mathbb{U}_2$	$\bullet\ =0$	$***\ =3$	$**\ =2$
	$\mathbb{U}_3$	$*\ =1$	$**\ =2$	$***\ =3$
	$\mathbb{U}_4$	$****\ =4$	$****\ =4$	$***\ =3$
	$\mathbb{U}_5$	$\bullet\ =0$	$\bullet\ =0$	$*\ =1$
Y_5	$\mathbb{U}_1$	$**\ =2$	$\bullet\ =0$	$*\ =1$
	$\mathbb{U}_2$	$***\ =3$	$**\ =2$	$*\ =1$
	$\mathbb{U}_3$	$***\ =3$	$****\ =4$	$**\ =2$
	$\mathbb{U}_4$	$****\ =4$	$***\ =3$	$**\ =2$
	$\mathbb{U}_5$	$\bullet\ =0$	$*\ =1$	$\bullet\ =0$

To assign $CSV\mathcal{N}NS_fS$ to each rank in Table 14, experts defined grading criteria given in Table 15 and Tables 16, 17, 18 representing the decision of the experts $\widetilde{Z}_1, \widetilde{Z}_2, \widetilde{Z}_3$, respectively.

Table 15: Grading criteria for $CSV\mathcal{N}5SS$

h_z^w/J	degree of truthness		degree of indeterminacy		degree of falsity	
grades	p_w	$2\pi t_w$	q_w	$2\pi\omega_w$	r_w	$2\pi f_w$
$h_w^s=0$	$[0.00, 0.15)$	$[0.0, 0.3\pi)$	$(0.85, 1.00]$	$(1.7\pi, 2.0\pi]$	$(0.85, 1.00]$	$(1.7\pi, 2.0\pi]$
$h_w^s=1$	$[0.15, 0.35)$	$[0.3\pi, 0.7\pi)$	$(0.65, 0.85]$	$(1.3\pi, 1.7\pi]$	$(0.65, 0.85]$	$(1.3\pi, 1.7\pi]$
$h_w^s=2$	$[0.35, 0.65)$	$[0.7\pi, 1.3\pi)$	$(0.35, 0.65]$	$(0.7\pi, 1.3\pi]$	$(0.35, 0.65]$	$(0.7\pi, 1.3\pi]$
$h_w^s=3$	$[0.65, 0.85)$	$[1.3\pi, 1.7\pi)$	$(0.15, 0.35]$	$(0.3\pi, 0.7\pi]$	$(0.15, 0.35]$	$(0.3\pi, 0.7\pi]$
$h_w^s=4$	$[0.85, 1.00)$	$[1.7\pi, 2.0\pi)$	$[0.00, 0.15)$	$[0.0, 0.3\pi)$	$[0.00, 0.15)$	$(0.0, 0.3\pi]$

Table 16: $CSV\mathcal{N}DM$ related to expert $\widetilde{Z}_1$,

	Y_1	Y_2	Y_3	Y_4	Y_5
$\mathbb{U}_1$	$(4,(0.86e^{i1.76\pi},0.08e^{i0.14\pi},0.07e^{i0.12\pi}))$	$(3,(0.71e^{i1.46\pi},0.31e^{i0.64\pi},0.29e^{i0.60\pi}))$	$(0,(0.11e^{i0.26\pi},0.91e^{i1.84\pi},0.93e^{i1.88\pi}))$	$(0,(0.12e^{i0.28\pi},0.87e^{i1.72\pi},0.86e^{i1.74\pi}))$	$(2,(0.61e^{i1.18\pi},0.41e^{i0.84\pi},0.43e^{i0.88\pi}))$
$\mathbb{U}_2$	$(4,(0.87e^{i1.78\pi},0.09e^{i0.16\pi},0.08e^{i0.14\pi}))$	$(3,(0.66e^{i1.36\pi},0.27e^{i0.56\pi},0.31e^{i0.60\pi}))$	$(4,(0.89e^{i1.74\pi},0.04e^{i0.10\pi},0.11e^{i0.24\pi}))$	$(0,(0.13e^{i0.28\pi},0.87e^{i1.72\pi},0.86e^{1.74\pi}))$	$(3,(0.67e^{i1.38\pi},0.25e^{i0.48\pi},0.23e^{i0.44\pi}))$
$\mathbb{U}_3$	$(3,(0.69e^{i1.42\pi},0.19e^{i0.40\pi},0.22e^{i0.46\pi}))$	$(4,(0.88e^{i1.72\pi},0.06e^{i0.14\pi},0.10e^{i0.18\pi}))$	$(0,(0.14e^{i0.26\pi},0.88e^{i1.74\pi},0.89e^{i1.76\pi}))$	$(1,(0.34e^{i0.64\pi},0.66e^{i1.32\pi},0.67e^{1.36\pi}))$	$(3,(0.71e^{i1.44\pi},0.24e^{i0.50\pi},0.27e^{i0.52\pi}))$
$\mathbb{U}_4$	$(3,(0.82e^{i1.78\pi},0.18e^{i0.38\pi},0.21e^{0.44i\pi}))$	$(4,(0.91e^{i1.86\pi},0.02e^{i0.02\pi},0.03e^{0.08i\pi}))$	$(0,(0.13e^{i0.28\pi},0.88e^{i1.74\pi},0.86e^{i1.74\pi}))$	$(4,(0.93e^{i1.90\pi},0.04e^{i0.06\pi},0.01e^{i0.04\pi}))$	$(4,(0.96e^{i1.94\pi},0.05e^{i0.08\pi},0.03e^{i0.04\pi}))$
$\mathbb{U}_5$	$(4,(0.87e^{i1.78\pi},0.13e^{i0.28\pi},0.12e^{0.26i\pi}))$	$(4,(0.90e^{i1.84\pi},0.07e^{i0.12\pi},0.10e^{0.22i\pi}))$	$(0,(0.02e^{i0.08\pi},0.95e^{i1.72\pi},0.97e^{i1.78\pi}))$	$(0,(0.03e^{i0.02\pi},0.96e^{i1.90\pi},0.98e^{i1.70\pi}))$	$(0,(0.05e^{i0.06\pi},0.95e^{i1.84\pi},0.94e^{1.86i\pi}))$

Table 17: $CSV\mathcal{N}DM$ related to expert $\widetilde{Z}_2$,

	Y_1	Y_2	Y_3	Y_4	Y_5
$\mathbb{U}_1$	$(3,(0.72e^{i1.46\pi},0.32e^{i0.66\pi},0.66e^{i0.68\pi}))$	$(2,(0.41e^{i0.86\pi},0.51e^{i1.04\pi},0.61e^{i1.24\pi}))$	$(1,(0.16e^{i0.36\pi},0.69e^{i1.40\pi},0.72e^{i1.46\pi}))$	$(1,(0.17e^{i0.28\pi},0.75e^{i1.52\pi},0.77e^{i1.56\pi}))$	$(0,(0.06e^{i0.08\pi},0.91e^{i1.84\pi},0.92e^{i1.86\pi}))$
$\mathbb{U}_2$	$(1,(0.19e^{i0.42\pi},0.72e^{i1.46\pi},0.75e^{i1.52\pi}))$	$(4,(0.93e^{i1.82\pi},0.12e^{i0.26\pi},0.13e^{i0.28\pi}))$	$(4,(0.88e^{i1.74\pi},0.08e^{i0.18\pi},0.10e^{i0.22\pi}))$	$(3,(0.73e^{i0.75\pi},0.23e^{i0.48\pi},0.20e^{0.38\pi}))$	$(2,(0.64e^{i1.26\pi},0.36e^{i0.74\pi},0.37e^{i0.76\pi}))$
$\mathbb{U}_3$	$(2,(0.45e^{i0.94\pi},0.46e^{i0.94\pi},0.56e^{i1.04\pi}))$	$(0,(0.09e^{i0.14\pi},0.87e^{i1.76\pi},0.86e^{i1.74\pi}))$	$(2,(0.58e^{i1.20\pi},0.37e^{i0.74\pi},0.39e^{i0.80\pi}))$	$(2,(0.59e^{i1.22\pi},0.53e^{i1.08\pi},0.44e^{0.86\pi}))$	$(4,(0.92e^{i1.82\pi},0.05e^{i0.08\pi},0.12e^{i0.22\pi}))$
$\mathbb{U}_4$	$(1,(0.32e^{i0.68\pi},0.67e^{i1.38\pi},0.69e^{1.36i\pi}))$	$(3,(0.84e^{i1.66\pi},0.16e^{i0.34\pi},0.17e^{0.36i\pi}))$	$(3,(0.83e^{i1.62\pi},0.18e^{i0.38\pi},0.19e^{i0.40\pi}))$	$(4,(0.98e^{i1.98\pi},0.10e^{i0.16\pi},0.01e^{i0.04\pi}))$	$(3,(0.81e^{i1.02\pi},0.20e^{i0.42\pi},0.19e^{i0.26\pi}))$
$\mathbb{U}_5$	$(0,(0.11e^{i0.26\pi},0.90e^{i1.82\pi},0.91e^{1.84i\pi}))$	$(1,(0.22e^{i0.46\pi},0.81e^{i1.64\pi},0.84e^{1.66i\pi}))$	$(0,(0.08e^{i0.20\pi},0.91e^{i1.80\pi},0.92e^{i1.82\pi}))$	$(0,(0.07e^{i0.18\pi},0.87e^{i1.72\pi},0.88e^{i1.74\pi}))$	$(1,(0.23e^{i0.48\pi},0.83e^{i1.68\pi},0.82e^{1.66i\pi}))$

Table 18: $CSV\mathcal{N}DM$ related to expert $\widetilde{Z}_3$,

	Y_1	Y_2	Y_3	Y_4	Y_5
$\mathbb{U}_1$	$(2,(0.62e^{i1.20\pi},0.36e^{i1.74\pi},0.39e^{i0.80\pi}))$	$(3,(0.70e^{i1.36\pi},0.26e^{i1.50\pi},0.28e^{i1.58\pi}))$	$(2,(0.59e^{1.22\pi},0.43^{i0.88\pi},0.42e^{i0.86\pi}))$	$(0,(0.86e^{i1.74\pi},0.02e^{i0.02\pi},0.03e^{i0.04\pi}))$	$(1,(0.31e^{i0.64\pi},0.69e^{i1.36\pi},0.68e^{i1.38\pi}))$
$\mathbb{U}_2$	$(3,(0.81e^{i1.66\pi},0.20e^{i0.28\pi},0.18e^{i0.28\pi}))$	$(4,(0.95e^{i1.88\pi},0.05e^{i0.08\pi},0.07e^{i0.16\pi}))$	$(3,(0.80e^{i1.64\pi},0.21e^{i0.40\pi},0.22e^{i0.46\pi}))$	$(2,(0.62e^{i1.28\pi},0.36e^{i0.74\pi},0.38e^{0.78\pi}))$	$(1,(0.34e^{i0.64\pi},0.66e^{i1.34\pi},0.67e^{i1.38\pi}))$
$\mathbb{U}_3$	$(1,(0.31e^{i0.66\pi},0.68e^{i1.38\pi},0.69e^{i1.40\pi}))$	$(2,(0.60e^{i1.22\pi},0.41e^{i0.80\pi},0.42e^{i0.84\pi}))$	$(1,(0.29e^{i0.62\pi},0.70e^{i1.42\pi},0.72e^{i1.46\pi}))$	$(3,(0.79e^{i1.62\pi},0.23e^{i0.44\pi},0.20e^{0.42\pi}))$	$(2,(0.61e^{i1.26\pi},0.39e^{i0.80\pi},0.40e^{i0.82\pi}))$
$\mathbb{U}_4$	$(3,(0.84e^{i1.72\pi},0.17e^{i0.32\pi},0.16e^{0.34i\pi}))$	$(4,(0.96e^{i1.96\pi},0.03e^{i0.08\pi},0.02e^{0.06i\pi}))$	$(4,(0.98e^{i1.93\pi},0.04e^{i0.06\pi},0.03e^{i0.04\pi}))$	$(3,(0.82e^{i1.68\pi},0.18e^{i0.34\pi},0.19e^{i0.36\pi}))$	$(2,(0.63e^{i1.22\pi},0.38e^{i0.74\pi},0.37e^{i0.72\pi}))$
$\mathbb{U}_5$	$(1,(0.27e^{i0.38\pi},0.74e^{i1.46\pi},0.73e^{1.50i\pi}))$	$(2,(0.57e^{i1.10\pi},0.45e^{i0.92\pi},0.47e^{0.96i\pi}))$	$(1,(0.25e^{i0.34\pi},0.76e^{i1.54\pi},0.78e^{i1.58\pi}))$	$(1,(0.23e^{i0.50\pi},0.79e^{i1.60\pi},0.81e^{i1.64\pi}))$	$(0,(0.30e^{i0.58\pi},0.95e^{i1.92\pi},0.96e^{1.94i\pi}))$

Step 2: The decision of all experts cumulated using the $CSV\mathcal{N}NS_fWA$ operator with $\nu = (0.33, 0.40, 0.27)^T$ be the weight vector for the experts so that we get $ACSV\mathcal{N}NS_fDM$ summarized in Table 19.

Table 19: Aggregated complex single-valued neutrosophic N-soft decision matrix

	Y_1	Y_2	Y_3
$\mathbb{U}_1$	$(4,(0.0680e^{i0.1440\pi},0.9139e^{i1.8191\pi},0.9096e^{i1.8092\pi}))$	$(3,(0.0430e^{i0.0920\pi},0.9591e^{i1.9208\pi},0.9568e^{i1.91598\pi}))$	$(2,(0.0041e^{i0.0098\pi},0.9966e^{i1.9940\pi},0.9974e^{i1.9956\pi}))$
$\mathbb{U}_2$	$(4,(0.0710e^{i0.1520\pi},0.9177e^{i1.8278\pi},0.9139e^{i1.81914\pi}))$	$(4,(0.0380e^{i0.95569\pi},0.9544e^{i1.9114\pi},0.9591e^{i1.9159\pi}))$	$(4,(0.0760e^{i0.1400\pi},0.8916e^{i1.7974\pi},0.92434e^{i1.8544\pi}))$
$\mathbb{U}_3$	$(3,(0.0040e^{i0.0860\pi},0.9425e^{i1.8885\pi},0.9474e^{i1.8979\pi}))$	$(4,(0.0720e^{i0.1360\pi},0.9045e^{i1.8192\pi},0.9212e^{i1.8355\pi}))$	$(2,(0.0053e^{i0.0098\pi},0.9954e^{i1.9900\pi},0.9958e^{i1.9909\pi}))$
$\mathbb{U}_4$	$(3,(0.0590e^{i0.1260\pi},0.9407e^{i1.8850\pi},0.9458e^{i1.8949\pi}))$	$(4,(0.0820e^{i0.0180\pi},0.8698e^{i1.6972\pi},0.8825e^{i1.7832\pi}))$	$(4,(0.0050e^{i0.0108\pi},0.9954e^{i1.9901\pi},0.9946e^{i1.9901\pi}))$
$\mathbb{U}_5$	$(4,(0.070e^{i0.1520\pi},0.9323e^{i1.8597\pi},0.9272e^{i1.8597\pi}))$	$(4,(0.0780e^{i0.1720\pi},0.9096e^{i1.8092\pi},0.9212e^{i1.8486\pi}))$	$(1,(0.0007e^{i0.0028\pi},0.9982e^{i1.9970\pi},0.9989e^{i1.9986\pi}))$

	Y_4	Y_5
$\mathbb{U}_1$	$(1,(0.0045e^{i0.0108\pi},0.9977e^{i1.9948\pi},0.9970e^{i1.9932\pi}))$	$(2,(0.0330e^{i0.0624\pi},0.9687e^{i1.9391\pi},0.9704e^{i1.9423\pi}))$
$\mathbb{U}_2$	$(3,(0.0050e^{i0.0106\pi},0.9950e^{i1.9892\pi},0.9946e^{i1.9900\pi}))$	$(2,(0.0387e^{i0.0817\pi},0.9517e^{i1.9008\pi},0.9489e^{i1.8949\pi}))$
$\mathbb{U}_3$	$(3,(0.0146e^{i0.0272\pi},0.9853e^{i1.9716\pi},0.9858e^{i1.9726\pi}))$	$(4,(0.0432e^{i0.0887\pi},0.9517e^{i1.9008\pi},0.9544e^{i1.9062\pi}))$
$\mathbb{U}_4$	$(4,(0.0904e^{i0.2024\pi},0.8916e^{i1.7650\pi},0.8486e^{i1.7397\pi}))$	$(4,(0.1084e^{i0.2349\pi},0.8987e^{i1.7832\pi},0.8825e^{i1.7832\pi}))$
$\mathbb{U}_5$	$(1,(0.0010e^{i0.0007\pi},0.9985e^{i1.9962\pi},0.9992e^{i1.9978\pi}))$	$(1,(0.0182e^{i0.00216\pi},0.9982e^{i1.9955\pi},0.9978e^{i1.9948\pi}))$

Step 3: In CAMELS model each attribute has its own worth and value that continuously change as the time passing out, therefore experts rank them and then assigned $CSV\mathcal{N}NS_fNs$ accordingly. We summarized the weights of the experts related to the attributes, are arranged in Table 20, using the $CSV\mathcal{N}NS_fWA$ operator and get the weight vector θ, given as:

$$\chi = \begin{pmatrix} (2,(0.0079e^{i0.0168\pi},0.9893e^{i1.9794\pi},0.9902e^{1.9814i\pi})) \\ (4,(0.0387e^{i0.0794\pi},0.9388e^{i1.8814\pi},0.9425e^{1.8884i\pi})) \\ (4,(0.0820e^{i0.1720\pi},0.9298e^{i1.8544\pi},0.9243e^{1.8424i\pi})) \\ (3,(0.0408e^{i0.0804\pi},0.9458e^{i1.8948\pi},0.9489e^{1.9008i\pi})) \\ (3,(0.0180e^{i0.0372\pi},0.9642e^{i1.9304\pi},0.9842e^{1.9672i\pi})) \end{pmatrix}$$

Table 20: Weights for attributes from experts

	$\widetilde{Z}_1$	$\widetilde{Z}_2$	$\widetilde{Z}_3$
Y_1	$(1,(0.20e^{i0.42\pi},0.74e^{i1.50\pi},0.76e^{i1.54\pi}))$	$(2,(0.42e^{i0.86\pi},0.38e^{i0.778\pi},0.62e^{i1.22\pi}))$	$(0,(0.09e^{i0.24\pi},0.92e^{i1.86\pi},0.95e^{i1.88\pi}))$
Y_2	$(3,(0.67e^{i1.36\pi},0.17e^{i0.36\pi},0.19e^{i0.40\pi}))$	$(4,(0.93e^{i1.88\pi},0.09e^{i0.16\pi},0.14e^{i0.26\pi}))$	$(1,(0.18e^{i0.38\pi},0.70e^{i1.42\pi},0.72e^{i1.46\pi}))$
Y_3	$(4,(0.91e^{i1.84\pi},0.13e^{i0.24\pi},0.11e^{i0.20\pi}))$	$(1,(0.16e^{i0.34\pi},0.66e^{i1.37\pi},0.68e^{i1.38\pi}))$	$(2,(0.44e^{i0.90\pi},0.40e^{i0.82\pi},0.60e^{i1.18\pi}))$
Y_4	$(3,(0.69e^{i1.40\pi},0.21e^{i0.44\pi},0.23e^{i0.48\pi}))$	$(3,(0.71e^{i1.42\pi},0.25e^{i0.52\pi},0.27e^{i0.56\pi}))$	$(3,(0.75e^{i1.53\pi},0.31e^{i0.64\pi},0.33e^{i0.68\pi}))$
Y_5	$(2,(0.40e^{i0.82\pi},0.36e^{i0.74\pi},0.64e^{i1.26\pi}))$	$(3,(0.73e^{i1.48\pi},0.29e^{i0.60\pi},0.30e^{i0.62\pi}))$	$(3,(0.77e^{i1.56\pi},0.31e^{i0.60\pi},0.26e^{i0.50\pi}))$

Step 4: The weight vector θ and $ACSV\mathcal{N}NS_fDM$ are encapsulated using the $CSV\mathcal{N}NS_fWG$ operator into $AWCSV\mathcal{N}NS_fDM$, compile in Table 21.

Table 21: Aggregated weighted complex single-valued neutrosophic N-soft decision matrix

	Y_1	Y_2	Y_3
$\mathbb{U}_1$	$(2,(0.00053e^{i0.00120\pi},0.99900e^{i1.99812\pi},0.99911e^{i1.99822\pi}))$	$(3,(0.00016e^{i0.00036\pi},0.99749e^{i1.99528\pi},0.99751e^{i1.9953\pi}))$	$(2,(0.00032e^{i0.00084\pi},0.99976e^{i1.99956\pi},0.99980e^{i1.99964\pi}))$
$\mathbb{U}_2$	$(2,(0.00055e^{i0.00126\pi},0.99912e^{i1.99822\pi},0.99916e^{i1.99832\pi}))$	$(4,(0.00014e^{i0.00032\pi},0.99720e^{i1.99474\pi},0.99764e^{i1.99530\pi}))$	$(4,(0.00062e^{i0.00120\pi},0.99239e^{i1.98524\pi},0.99427e^{i1.98852\pi}))$
$\mathbb{U}_3$	$(2,(0.00031e^{i0.00072\pi},0.99938e^{i1.99884\pi},0.99948e^{i1.99906\pi}))$	$(4,(0.00028e^{i0.00052\pi},0.99416e^{i1.98868\pi},0.99546e^{i1.9908\pi}))$	$(2,(0.00043e^{i0.00084\pi},0.99968e^{i1.99926\pi},0.99968e^{i1.99928\pi}))$
$\mathbb{U}_4$	$(2,(0.00046e^{i0.00104\pi},0.99936e^{i1.99880\pi},0.99946e^{i1.99902\pi}))$	$(4,(0.00032e^{i0.00072\pi},0.99203e^{i1.98928\pi},0.99324e^{i1.9879\pi}))$	$(4,(0.00041e^{i0.00092\pi},0.99968e^{i1.99926\pi},0.99959e^{i1.99920\pi}))$
$\mathbb{U}_5$	$(2,(0.00055e^{i0.00128\pi},0.99927e^{i1.99856\pi},0.99928e^{i1.99868\pi}))$	$(4,(0.00030e^{i0.00068\pi},0.994466e^{i1.98868\pi},0.99546e^{i1.9916\pi}))$	$(1,(0.00057e^{i0.00024\pi},0.99987e^{i1.99978\pi},0.99990e^{i1.99980\pi}))$

	Y_4	Y_5
$\mathbb{U}_1$	$(1,(0.00018e^{i0.00044\pi},0.99987e^{i1.99972\pi},0.99984e^{i1.99964\pi}))$	$(2,(0.00059e^{i0.00116\pi},0.99880e^{i1.99786\pi},0.99953e^{i1.99904\pi}))$
$\mathbb{U}_2$	$(3,(0.00020e^{i0.00044\pi},0.99973e^{i1.99942\pi},0.99972e^{i1.99948\pi}))$	$(3,(0.00069e^{i0.00152\pi},0.99827e^{i1.99652\pi},0.99919e^{i1.99826\pi}))$
$\mathbb{U}_3$	$(3,(0.00059e^{i0.00114\pi},0.99920e^{i1.99850\pi},0.99927e^{i1.99986\pi}))$	$(3,(0.00077e^{i0.00164\pi},0.99827e^{i1.99654\pi},0.99927e^{i1.9984\pi}))$
$\mathbb{U}_4$	$(3,(0.00368e^{i0.00850\pi},0.99412e^{i1.98760\pi},0.99220e^{i1.98680\pi}))$	$(3,(0.00195e^{i0.00436\pi},0.99637e^{i1.99242\pi},0.99814e^{i1.99644\pi}))$
$\mathbb{U}_5$	$(1,(0.00004e^{i0.00002\pi},0.99990e^{i1.99980\pi},1.00000e^{i1.99980\pi}))$	$(1,(0.00032e^{i0.00068\pi},0.99990e^{i1.99980\pi},0.99990e^{i2.00000\pi}))$

Step 5 The groundwork of the TOPSIS method that differentiate it from others is to evaluate the PIS and NIS that help to find out optimal solution using the tool of distance measure. The criteria evaluated for this MAGDM problem based on CAMELS model and all are related to benefit-type criteria. Therefore, the $CSV\mathcal{N}NS_f$-PIS and $CSV\mathcal{N}NS_f$-NIS, taking into account the nature of the attributes, are arranged in Table 22.

Table 22: $CSV\mathcal{N}NS_f$-PIS and $CSV\mathcal{N}NS_f$-NIS

$\mathbb{U}_s$	$\tilde{H}_w^{PIS}$	$\tilde{H}_w^{NIS}$
$\mathbb{U}_1$	$(2,(0.00055e^{i0.00126\pi},0.99912e^{i1.99822\pi},0.99916e^{i1.99832\pi}))$	$(2,(0.00031e^{i0.00072\pi},0.99938e^{i1.99880\pi},0.99948e^{i1.99906\pi}))$
$\mathbb{U}_2$	$(4,(0.00032e^{i0.00072\pi},0.99203e^{i1.98928\pi},0.99324e^{i1.98790\pi}))$	$(3,(0.00016e^{i0.00036\pi},0.99749e^{i1.99528\pi},0.99751e^{i1.99530\pi}))$
$\mathbb{U}_3$	$(4,(0.00062e^{i0.00120\pi},0.99239e^{i0.03400\pi},0.99427e^{i1.98852\pi}))$	$(1,(0.00570e^{i0.00024\pi},0.99987e^{i1.99970\pi},0.99989e^{1.99980\pi}))$
$\mathbb{U}_4$	$(3,(0.00368e^{i0.00850\pi},0.99412e^{i1.98760\pi},0.99220e^{1.98680\pi}))$	$(1,(0.00004e^{i0.00002\pi},0.99990e^{i1.99980\pi},1.00000e^{1.99980\pi}))$
$\mathbb{U}_5$	$(2,(0.00195e^{i0.00436\pi},0.99637e^{i1.99242\pi},0.99814e^{i1.99644\pi}))$	$(1,(0.00032e^{i0.00068\pi},0.99990e^{i1.99980\pi},0.99990e^{i2.0000\pi}))$

Step 6 For distance measure, normalized Euclidean distance is used that precisely evaluate the distance between the alternatives and the ideal solutions, simultaneously. Table 23 describe the distance of each alternative from $CSV\mathcal{N}NS_f$-PIS and $CSV\mathcal{N}NS_f$-NIS, respectively.

Table 23: Distance measures of alternatives from ideal solution

$\mathbb{U}_s$	$d(\bar{H}_w^{PIS}, \mathbb{U}_s)$	$d(\bar{H}_w^{NIS}, \mathbb{U}_s)$
$\mathbb{U}_1$	0.133746	0.059764
$\mathbb{U}_2$	0.005061	0.179298
$\mathbb{U}_3$	0.084647	0.13363
$\mathbb{U}_4$	0.003998	0.1793085
$\mathbb{U}_5$	0.174320	0.042260

Step 7: Revised closeness index is used for ranking the alternatives having the properties of closeness and far-away from the ideal solution at a time. The numeric values of revised closeness index calculated in Table **??**

Table 24: Index of alternatives

$\mathbb{U}_s$	$\Lambda(\mathbb{U}_s)$
$\mathbb{U}_1$	33.1199
$\mathbb{U}_2$	0.26594
$\mathbb{U}_3$	20.4343
$\mathbb{U}_4$	0.00000
$\mathbb{U}_5$	43.3661

Step 8: Clearly, from the values of revised closeness index we can easily highlight the bank with best performance that is actually the $\mathbb{U}_4 = MCB$ opting as best performer in Pakistan, where, the ascending order of the values of revised closeness index describe the ranks of the banks on the basis of the CAMELS model and TOPSIS method, shown in Table 25

Table 25: Ranking of alternatives

Alternative	$\mathbb{U}_1$	$\mathbb{U}_2$	$\mathbb{U}_3$	$\mathbb{U}_4$	$\mathbb{U}_5$
Ranking	4	2	3	1	5

7 Comparison

To prove the versatility of the $CSV\mathcal{N}NS_f$-TOPSIS method we compare the proposed method with $SV\mathcal{N}$-TOPSIS method [28] by solving the describe MAGDM problem of "Monitoring performance of Islamic banking industry on the basis of CAMELS rating syste" by SVN-TOPSIS method[28]. The evaluation of the problem by SVN-TOPSIS method [28] is as follows:

Step 1 For the implication of SVN-TOPSIS method on the proposed MAGDM problem we have to exclude the grading part as well as reduce the periodic terms to zero in the $CSV\mathcal{N}NS_fN$, so that experts $\widetilde{Z}_1, \widetilde{Z}_2, \widetilde{Z}_3$ assigned SVNs to each rank given in Tables 26, 27 and 28, respectively.

Table 26: $SV\mathcal{N}DM$ related to expert $\widetilde{Z}_1$,

	Y_1	Y_2	Y_3	Y_4	Y_5
$\mathbb{U}_1$	$(0.86, 0.08, 0.07)$	$(0.71, 0.31, 0.29)$	$(0.11, 0.91, 0.93)$	$(0.12, 0.87, 0.86)$	$(0.61, 0.41, 0.43)$
$\mathbb{U}_2$	$(0.87, 0.09, 0.08)$	$(0.66, 0.27, 0.31)$	$(0.89, 0.04, 0.11)$	$(0.13, 0.87, 0.86)$	$(0.67, 0.25, 0.23)$
$\mathbb{U}_3$	$(0.69, 0.19, 0.22)$	$(0.88, 0.06, 0.10)$	$(0.14, 0.88, 0.89)$	$(0.34, 0.66, 0.67)$	$(0.71, 0.24e, 0.27)$
$\mathbb{U}_4$	$(0.82, 0.18, 0.21)$	$(0.91, 0.02, 0.03)$	$(0.13, 0.88, 0.86)$	$(0.93, 0.04, 0.01)$	$(0.96, 0.05, 0.03)$
$\mathbb{U}_5$	$(0.87, 0.13, 0.12)$	$(0.90, 0.07, 0.10)$	$(0.02, 0.95, 0.97)$	$(0.03, 0.96, 0.98)$	$(0.05, 0.95, 0.94)$

Table 27: $SV\mathcal{N}DM$ related to expert $\widetilde{Z}_2$,

	Y_1	Y_2	Y_3	Y_4	
$\mathbb{U}_1$	$(0.72, 0.32, 0.66)$	$(0.41, 0.51, 0.61)$	$(0.16, 0.69, 0.72)$	$(0.17, 0.75, 0.77)$	$(0.06, 0.91, 0.92)$
$\mathbb{U}_2$	$(0.19, 0.72, 0.75)$	$(0.93, 0.12, 0.13)$	$(0.88, 0.08, 0.10)$	$(0.73, 0.23, 0.20)$	$(0.64, 0.36, 0.37)$
$\mathbb{U}_3$	$(0.45, 0.46, 0.56)$	$(0.09, 0.87, 0.86)$	$(0.58, 0.37, 0.39)$	$(0.59, 0.53, 0.44)$	$(0.92, 0.05, 0.12)$
$\mathbb{U}_4$	$(0.32, 0.67, 0.69)$	$(0.84, 0.16, 0.17)$	$(0.83, 0.18, 0.19)$	$(0.98, 0.10, 0.01)$	$(0.81, 0.20, 0.19)$
$\mathbb{U}_5$	$(0.11, 0.90, 0.91)$	$(0.22, 0.81, 0.84)$	$(0.08, 0.91, 0.92)$	$(0.07, 0.87, 0.88)$	$(0.23, 0.83, 0.82)$

Table 28: $SV\mathcal{N}DM$ related to expert $\widetilde{Z}_3$,

	Y_1	Y_2	Y_3	Y_4	Y_5
$\mathbb{U}_1$	$(0.62, 0.36, 0.39)$	$(0.70, 0.26, 0.28)$	$(0.59, 0.43, 0.42)$	$(0.86, 0.02, 0.03)$	$(0.31, 0.69, 0.68)$
$\mathbb{U}_2$	$(0.81, 0.20, 0.18)$	$(0.95, 0.05, 0.07)$	$(0.80, 0.21, 0.22)$	$(0.62, 0.36, 0.38)$	$(0.34, 0.66, 0.67)$
$\mathbb{U}_3$	$(0.31, 0.68, 0.69)$	$(0.60, 0.41, 0.42)$	$(0.29, 0.70, 0.72)$	$(0.79, 0.23, 0.20)$	$(0.61, 0.39, 0.40)$
$\mathbb{U}_4$	$(0.84, 0.17, 0.16)$	$(0.96, 0.03, 0.02)$	$(0.98, 0.04, 0.03)$	$(0.82, 0.18, 0.19)$	$(0.63, 0.38, 0.37)$
$\mathbb{U}_5$	$(0.27, 0.74, 0.73)$	$(0.57, 0.45, 0.47)$	$(0.25, 0.76, 0.78)$	$(0.23, 0.79, 0.81)$	$(0.30, 0.95, 0.96)$

Step 2 The weights of experts $\nu = (0.33, 0.40, 0.27)^T$ and averaging operator [28], we can cumulate the aggregated single-valued neutrosophic decision matrix $(ASV\mathcal{N}DM)$, as follows:

$$\mathcal{H}_{ws} = \Big([1 - \Pi_{j=1}^{f}(1 - p_{ws}^{(j)})^{\nu_w}], [\Pi_{j=1}^{f}(q_{ws}^{(j)})^{\nu_w}], [\Pi_{j=1}^{f}(r_{ws}^{(j)})^{\nu_w}]\Big).$$

The $ASVNDM$ is arranged in Table 29.

Table 29: $ASV\mathcal{N}DM$

	Y_1	Y_2	Y_3	Y_4	Y_5
$\mathbb{U}_1$	$(0.0680, 0.9139, 0.9096)$	$(0.0430, 0.9591, 0.9568)$	$(0.0041, 0.9966, 0.9974)$	$(0.0045, 0.9977, 0.9970)$	$(0.0330, 0.9687, 0.9704)$
$\mathbb{U}_2$	$(0.0710, 0.9177, 0.9139)$	$(0.0380, 0.9544, 0.9591)$	$(0.0760, 0.8916, 0.92434)$	$(0.0050, 0.9950, 0.9946)$	$(0.0387, 0.9517, 0.9489)$
$\mathbb{U}_3$	$(0.0040, 0.9425, 0.9474)$	$(0.0720, 0.9045, 0.9212)$	$(0.0053, 0.9954, 0.9958)$	$(0.0146, 0.9853, 0.9858)$	$(0.0432, 0.9517, 0.9544)$
$\mathbb{U}_4$	$(0.0590, 0.9407, 0.9458)$	$(0.0820, 0.8698, 0.8825)$	$(0.0050, 0.9954, 0.9946)$	$(0.0904, 0.8916, 0.8486)$	$(0.1084, 0.8987, 0.8825)$
$\mathbb{U}_5$	$(0.070, 0.9323, 0.9272)$	$(0.0780, 0.9096, 0.9212)$	$(0.0007, 0.9982, 0.9989)$	$(0.0010, 0.9985, 0.9992)$	$(0.0182, 0.9982, 0.9978)$

Step 3 The weights for attributes are calculated, by summarizing the experts opinion about the nature of attributes given in Table 30, as follows:

$$\theta_w = \Big([1 - \Pi_{j=1}^{f}(1 - p_w^{(j)})^{\nu_w}], [\Pi_{j=1}^{f}(q_w^{(j)})^{\nu_w}], [\Pi_{j=1}^{f}(r_w^{(j)})^{\nu_w}]\Big).$$

Thus we have,

$$\theta = \begin{pmatrix} (0.0079, 0.9893, 0.9902) \\ (0.0387, 0.9388, 0.9425) \\ (0.0820, 0.9298, 0.9243) \\ (0.0408, 0.9458, 0.9489) \\ (0.0180, 0.9642, 0.9842) \end{pmatrix}.$$

Table 30: Weights for attributes from experts

	$\widetilde{Z}_1$	$\widetilde{Z}_2$	$\widetilde{Z}_3$
Y_1	$(0.20, 0.74, 0.76)$	$(0.42, 0.38, 0.62)$	$(0.09, 0.92, 0.95)$
Y_2	$(0.67, 0.17, 0.19)$	$(0.93, 0.09, 0.14)$	$(0.18, 0.70, 0.72)$
Y_3	$(0.91, 0.13, 0.11)$	$(0.16, 0.66, 0.68)$	$(0.44, 0.40, 0.60)$
Y_4	$(0.69, 0.21, 0.23)$	$(0.71, 0.25, 0.27)$	$(0.75, 0.31, 0.33)$
Y_5	$(0.40, 0.36, 0.64)$	$(0.73, 0.29, 0.30)$	$(0.77, 0.31, 0.26)$

Step 4 The aggregated weighted single-valued neutrosophic decision matrix$(AWSV\mathcal{N}DM)$, shown in Table 31, calculated as:

$$\bar{H}_{ws} = \Big(p_{ws}p_w, (q_{ws} + q_w - q_{ws}q_w), (r_{ws} + r_w - r_{ws}r_w)\Big).$$

Table 31: $AWSV\mathcal{N}DM$

	Y_1	Y_2	Y_3	Y_4	Y_5
$\mathbb{U}_1$	$(0.00053, 0.99900, 0.99911)$	$(0.00016, 0.99749, 0.99751)$	$(0.00032, 0.99976, 0.99980)$	$(0.00018, 0.99987, 0.99984)$	$(0.00059, 0.99880, 0.99953)$
$\mathbb{U}_2$	$(0.00055, 0.99912, 0.99916)$	$(0.00014, 0.99720, 0.99764)$	$(0.00062, 0.99239, 0.99427)$	$(0.00020, 0.99973, 0.99972)$	$(0.00069, 0.99827, 0.99919)$
$\mathbb{U}_3$	$(0.00031, 0.99938, 0.99948)$	$(0.00028, 0.99416, 0.99546)$	$(0.00043, 0.99968, 0.99968)$	$(0.00059, 0.99920, 0.99927)$	$(0.00077, 0.99827, 0.99927)$
$\mathbb{U}_4$	$(0.00046, 0.99936, 0.99946)$	$(0.00032, 0.99203, 0.99324)$	$(0.00041, 0.99968, 0.99959)$	$(0.00368, 0.99412, 0.99220)$	$(0.00195, 0.99637, 0.99814)$
$\mathbb{U}_5$	$(0.00055, 0.99927, 0.99928)$	$(0.00030, 0.994466, 0.99546)$	$(0.00057, 0.99987, 0.99990)$	$(0.00004, 0.99990, 1.00000)$	$(0.00032, 0.99990, 0.99990)$

Step 5 Keeping in view the nature of data, Equation 26 and 27 is used for the evaluation of the single-valued neutrosophic positive ideal solution and negative ideal solution arranged in Table 32.

$$\bar{H}_w^{PIS} = \begin{cases} (\max_s \bar{\mathbb{T}}_{ws}, \min_s \bar{\mathbb{I}}_{ws}, \min_s \bar{\mathbb{F}}_{ws}), & \text{if } Y_w \in \mathbb{BT}, \\ (\min_s \bar{\mathbb{T}}_{ws}, \max_s \bar{\mathbb{I}}_{ws}, \max_s \bar{\mathbb{F}}_{ws}), & \text{if } Y_w \in \mathbb{CT}, \end{cases} \tag{26}$$

and

$$\bar{H}_w^{NIS} = \begin{cases} (\min_s \bar{\mathbb{T}}_{ws}, \max_s \bar{\mathbb{I}}_{ws}, \max_s \bar{\mathbb{F}}_{ws}), & \text{if } Y_w \in \mathbb{BT}, \\ (\max_s \bar{\mathbb{T}}_{ws}, \min_s \bar{\mathbb{I}}_{ws}, \min_s \bar{\mathbb{F}}_{ws}), & \text{if } Y_w \in \mathbb{CT}, \end{cases} \tag{27}$$

Table 32: $SV\mathcal{N}$-PIS and $SV\mathcal{N}$-NIS

$\mathbb{U}_1$	$(0.00055, 0.99900, 0.99911)$	$(0.00031, 0.99938, 0.99948)$
$\mathbb{U}_2$	$(0.00032, 0.99203, 0.99324)$	$(0.00014, 0.99749, 0.99764)$
$\mathbb{U}_3$	$(0.00062, 0.99239, 0.99427)$	$(0.00033, 0.99987, 0.99990)$
$\mathbb{U}_4$	$(0.00368, 0.99412, 0.99220)$	$(0.00004, 0.99990, 1.00000)$
$\mathbb{U}_5$	$(0.00195, 0.99637, 0.99814)$	$(0.00032, 0.99990, 0.99990)$

Step 6 To measure distance of alternatives from PIS and NIS, Euclidean distance used. The calculated values are given in Table 33

Table 33: Distance measures of alternatives from ideal solution

$\mathbb{U}_s$	$d(\bar{H}_w^{PIS}, \mathbb{U}_s)$	$d(\bar{H}_w^{NIS}, \mathbb{U}_s)$
$\mathbb{U}_1$	0.00935	0.00078
$\mathbb{U}_2$	0.00762	0.00660
$\mathbb{U}_3$	0.00810	0.00260
$\mathbb{U}_4$	0.00500	0.00763
$\mathbb{U}_5$	0.00890	0.00210

Step 7 The revised closeness index calculated using Equation 28 , is tabulated in Table 34 and the ranks evaluated through the index values are arranged in Table 35 in descending order, according to which $\mathbb{U}_4$ is the best performer.

$$\Lambda(\mathbb{U}_s) = \frac{d(\bar{H}_w^{NIS}, \mathbb{U}_s)}{d(\bar{H}_w^{PIS}, \mathbb{U}_s) + d(\bar{H}_w^{NIS}, \mathbb{U}_s)}, \tag{28}$$

where, $s = 1, 2, \ldots, k$.

Table 34: Revised closeness index of each alternative

Alternative	$\Lambda(\mathbb{U}_s)$
$\mathbb{U}_1$	0.0769
$\mathbb{U}_2$	0.4641
$\mathbb{U}_3$	0.2429
$\mathbb{U}_4$	0.6041
$\mathbb{U}_5$	0.1900

Table 35: Ranking in single-valued neutrosophic environment

Alternative	$\mathbb{U}_1$	$\mathbb{U}_2$	$\mathbb{U}_3$	$\mathbb{U}_4$	$\mathbb{U}_5$
Ranking	5	2	3	1	1

7.1 Discussion

1. The comparison of the $CSV\mathcal{N}NS_f$-TOPSIS method with the existing SVN-TOPSIS method have same findings for the Islamic bank as best performer in Pakistan but the consequences relevant to the ranks of other banks have no analogy given in Table 36.

Table 36: Comparison

Model	Ranks	Best Performer
$SV\mathcal{N}$-TOPSIS [28]	$\mathbb{U}_4 > \mathbb{U}_2 > \mathbb{U}_3 > \mathbb{U}_5 > \mathbb{U}_1$	$\mathbb{U}_4$
$CSV\mathcal{N}NS_f$-TOPSIS(Proposed)	$\mathbb{U}_4 > \mathbb{U}_2 > \mathbb{U}_3 > \mathbb{U}_1 > \mathbb{U}_5$	$\mathbb{U}_4$

2. The expertise of the presented methodology $CSV\mathcal{N}NS_f$-TOPSIS method to manipulate the indeterminacy degree and two dimensional information in the MAGDM problems by using the frame of $CSV\mathcal{N}NS_fSs$.
3. The presented methodology of $CSV\mathcal{N}NS_f$-TOPSIS method has potential to operate the problems of $IFNS_fSs$, being the generalization of the $IFSs$.
4. The presented model has proficiency to overcome the latest problems characterized by parameterized ordered evaluation system but the existing methods have no grip on such problems.
5. By employing $N = 2$ and periodic terms equal to zero, we switch from $CSV\mathcal{N}NNS_f$ environment to single-valued environment so that the $CSV\mathcal{N}NS_f$-TOPSIS method could sensibly handled the daily life problems under single-valued environment.

8 Conclusion

In this paper we have merged the idea of single-valued neutrosophic set with N-soft sets, and in doing so, we have initiated the idea of $CSV\mathcal{N}NS_fSs$. These sets combine the 2-dimensional single-valued neutrosophic nature of the attributes with parameterized ordered grades which demonstrates their superiority over FNS_fS, $IFNS_fS$ and $\mathcal{N}NS_fS$. A MAGDM model of TOPSIS method is extended to handle the real life problems under the frame of $CSV\mathcal{N}NS_fSs$ in which the ordered grades are assigned to each alternative as initial evaluation that are further characterized by $CSV\mathcal{N}NS_fNs$. The PIS and NIS in $CSV\mathcal{N}NS_f$-TOPSIS method have been determined by the score function which has been further employed to quantify the distance measures and the closeness index that sort the alternatives from highest to lowest rank. An example from the banking industry and the comparison with single-valued neutrosophic TOPSIS method have clarified the accuracy and superiority of the presented technique. The new model and method pioneer a promising avenue for research in the decision making arena that we have only hinted at in this paper. Moreover, the proposed $CSV\mathcal{N}NS_f$-TOPSIS method does not evaluate the relative importance of the normalized Euclidean distances. Therefore we will work for the extension of the VIKOR method under a $CSV\mathcal{N}NS_f$ environment, which might be more credible and trustworthy.

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