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K-algebras on Quadripartitioned Single Valued Neutrosophic Sets

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ABSTRACT

Quadripartitioned Single Valued Neutrosophic (QSVN) set is a powerful structure where we have four components: Truth-T, Falsity-F, Unknown-U and Contradiction-C. And also it generalizes the concept of fuzzy, initutionstic and single valued neutrosophic set. In this paper we have proposed the concept of K-algebras on QSVN, level subset of QSVN and studied some of the results. In addition to this we have also investigated the characteristics of QSVN K-subalgebras under homomorphism.

1. Introduction

Dar and Akram [12] proposed a novel logical algebra known as K-algebra. The algebraic structure of a group G which K-algebra was built on should have a right identity element and satisfy the properties of non-commutative and non-associative. Furthermore this group G is of the type where each non-identity element is not of order 2 and K-algebra was built by adjoining the induced binary operation on G [11, 12, 13]. Zadeh's fuzzy set theory [22] was a powerful framework which deals the concept of uncertainty, imprecision and also it represented by membership function which lies in a unit interval of [0,1]. Fuzzy K-algebra was introduced by Akram et al. [2, 3, 5] and also they established this in a wide-reaching way

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through other researchers. Later Atanassov [9] introduced the concept of intuitionistic fuzzy set in 1983. It has an additional degree called the degree of nonmembership. Intuitionistic fuzzy K-subalgebras was proposed by Akram et al. [4, 6]. Intuitionistic fuzzy Ideals of BCK-Algebras was proposed by Jun and Kim [14].

Neutrosophic set which is a generalization of fuzzy set and intuitionistic fuzzy set was introduced by Smarandache [20] in 1998. Along with membership and non-membership function neutrosophic set has one more extra component called indeterminacy membership function. Also all the values of these three components lie in the real standard or non-standard subset of unit interval]−0,1+[where −0 = 0 − ϵ, 1+ = 1 + ϵ, ϵ is an infinitesimal number. In neutrosophic set theory algebraic structures were studied in soft topological K-algebras [7]. Agboola and Davvaz [1] presented the introduction to neutrosophic BCI/BCK algebras. Smarandache and Wang et al. [21] introduced single-valued neutrosophic set which plays a vital place in many real life problems and it takes the values from the subset of [0,1]. Akram et al. [8] studied K-algebras on single valued neutrosophic sets and also discussed homomorphisms between the single valued neutrosophic K-subalgebras. Belnap [10] introduced the concept of four valued logic that is the information are represented by four components T, F, None, Both which denote true, false, neither true nor false, both true and false, respectively. Based on this concept, Smarandache proposed four numerical valued neutrosophic logic where indeterminacy is splitted into two terms known as Contradiction (C) and Unknown (U). Chatterjee et al. [19] introduced Quadripartitioned Single Valued Neutrosophic (QSVN) set in which we have four components T, C, U, and F, respectively, and also it lies in the real unit interval of [0,1]. K. Mohana and M. Mohanasundari [15, 17] studied the concept of Quadripartitioned Single Valued Neutrosophic Relations (QSVNR) as well as some properties of quadripartitioned single valued neutrosophic rough sets and its axiomatic characterizations. Under QSVN environment multicriteria decision making problems has been discussed in [16, 18].

In this paper Section 2 deals with the basic definitions of QSVN set and the concept of K-algebras on single valued neutrosophic set. Section 3 discusses about K-algebras on QSVN, level subset of QSVN and also studies some of the results. Section 4 defines the homomorphism of quadripartitioned single valued neutrosophic K-algebras, characteristic and fully invariant K-subalgebras. Section 5 concludes the paper.

2. Preliminaries

This section deals with the basic definitions of QSVNS and K-algebra of single valued neutrosophic set that helps us to study the rest of the paper.

Definition 1. [19]. Let X be a non-empty set. A quadripartitioned neutrosophic set A over X characterizes each element x in X by a truth-membership function T_A, a contradiction membership function C_A, an ignorance – membership function U_A, and a falsity membership function F_A such that for each $x \in X, T_A, C_A, U_A, F_A \in [0,1]$ and $0 \le T_A(x)+C_A(x)+U_A(x)+F_A(x) \le 4$. When X is discrete A is represented as,

$$A = \sum_{i=1}^{n} \left\langle T_A(x_i), C_A(x_i), U_A(x_i), F_A(x_i) \right\rangle / x_i, x_i \in X.$$

However, when the universe of discourse is continuous A is represented as $A=\int_X \langle T_A(x),C_A(x),U_A(x),F_A(x)\rangle / x, x\in X$.

Definition 2. [19]. Consider two QSVNS A and B over X. A is said to be contained in B, denoted by $A\subseteq B$ *iff* $T_A(x)\leq T_B(x), C_A(x)\leq C_B(x), U_A(x)\geq U_B(x)$ *and* $F_A(x)\geq F_B(x)$.

Definition 3. [19]. The complement of a QSVNS A is denoted by A^C and is defined as,

$$A^C=\sum_{i=1}^{n}\langle F_A(x_i),U_A(x_i),C_A(x_i),T_A(x_i)\rangle / x_i, x_i\in X,$$

$$T_{A^C}(x_i)=F_A(x_i), C_{A^C}(x_i)=U_A(x_i), U_{A^C}(x_i)=C_A(x_i) \text{ and } F_{A^C}(x_i)=T_A(x_i), x_i\in X.$$

Definition 4. [19]. The union of two QSVNS A and B is denoted by $A\cup B$ and is defined as,

$$A\cup B=\sum_{i=1}^{n}\langle T_A(x_i)\vee T_B(x_i), C_A(x_i)\vee C_B(x_i), U_A(x_i)\wedge U_B(x_i), F_A(x_i)\wedge F_B(x_i)\rangle / x_i, x_i\in X.$$

Definition 5. [19]. The intersection of two QSVNS A and B is denoted by $A\cap B$ and is defined as,

$$A\cap B=\sum_{i=1}^{n}\langle T_A(x_i)\wedge T_B(x_i), C_A(x_i)\wedge C_B(x_i), U_A(x_i)\vee U_B(x_i), F_A(x_i)\vee F_B(x_i)\rangle / x_i, x_i\in X.$$

Definition 6. [12]. Let $(G,\cdot,\Theta,e)$ be a group in which each non-identity element is not of order 2. Then a K-algebra is a structure Ꝁ= $(G,\cdot,\Theta,e)$ on a group G in which induced binary operation $\odot: G\times G\rightarrow G$ is defined b $\odot(x,y)=x\odot y=xy^{-1}$y and satisfies the following axioms:

$$(x\odot y)\odot(x\odot z)=\left(x\odot\left((e\odot z)\odot(e\odot y)\right)\right)\odot x,$$

$$x\odot(x\odot y)=\left(x\odot(e\odot y)\right)\odot x,$$

$$x\odot x=e,$$

$$x\odot e=x,$$

$$e\odot x=x^{-1}, \text{ for all } x,y,z\in G.$$

Definition 7. [8]. A single-valued neutrosophic set $A=(T_A,I_A,F_A)$ in a K-algebra Ꝁ is called a single-valued neutrosophic K-subalgebra of Ꝁ if it satisfies the following conditions:

$$T_A(s\odot t)\geq \min\{T_A(s),T_A(t)\},$$

$$I_A(s\odot t)\geq \min\{I_A(s),I_A(t)\},$$

$$F_A(s\odot t)\leq \max\{F_A(s),F_A(t)\}, \text{ for all } s,t\in G.$$

Note that $T_A(e)\geq T_A(s), I_A(e)\geq I_A(s), F_A(e)\leq F_A(s), for\ all\ s\in G$.

3. Quadripartitioned Single Valued Neutrosophic K-Algebras

Definition 8. A quadripartitioned single valued neutrosophic set $X=(T_X,C_X,U_X,F_X)$ in a K-algebra Ҡ is called a quadripartitioned single valued neutrosophic K-subalgebra of Ҡ if it satisfies the following conditions:

$$T_X(e)\geq T_X(u), C_X(e)\geq C_X(u), U_X(e)\leq U_X(u), F_X(e)\leq F_X(u) \text{ for all } u\in G.$$

$$T_X(u \odot v)\geq \min\{T_X(u),T_X(v)\},$$

$$C_X(u \odot v)\geq \min\{C_X(u),C_X(v)\},$$

$$U_X(u \odot v)\leq \max\{U_X(u),U_X(v)\},$$

$$F_X(u \odot v)\leq \max\{F_X(u),F_X(v)\} \text{ for all } u,v\in G.$$

Example 1. Let $G=\{e,g,g^2,g^3,g^4\}$ is the cyclic group of order 5 in a K-algebra Ҡ= $(G,\cdot,\odot,e)$. The Cayley's table for ⊙ is given as follows.

$\odot$	e	g	g^2	g^3	g^4
e	e	g^4	g^3	g^2	g
g	g	e	g^4	g^3	g^2
g^2	g^2	g	e	g^4	g^3
g^3	g^3	g^2	g	e	g^4
g^4	g^4	g^4	g^3	g	e

We define a quadripartitioned single valued neutrosophic set $X=(T_X,C_X,U_X,F_X)$ in K-algebra as follows:

$T_X(e)=0.5,\quad C_X(e)=0.7,\quad U_X(e)=0.3,\quad F_X(e)=0.5$
$T_X(u)=0.2,\quad C_X(u)=0.4,\quad U_X(u)=0.5,\quad F_X(u)=0.8$

for all $u\neq e\in G$. Clearly, it shows that $X=(T_X,C_X,U_X,F_X)$ is a quadripartitioned single valued neutrosophic K-algebras of Ҡ.

Proposition 1. If $X=(T_X,C_X,U_X,F_X)$ denotes a quadripartitioned single valued neutrosophic K-algebras of Ҡ then,

a) $(\forall\, u,v\in G),(T_X(u\odot v)=T_X(v)\Rightarrow T_X(u)=T_X(e))$

$(\forall\, u,v\in G),(T_X(u)=T_X(e)\Rightarrow T_X(u\odot v)\geq T_X(v))$;

b) $(\forall\, u,v\in G),(C_X(u\odot v)=C_X(v)\Rightarrow C_X(u)=C_X(e))$

$(\forall\, u, v \in G), \big(C_X(u) = C_X(e) \Rightarrow C_X(u \odot v) \geq C_X(v)\big);$

$c)$ $(\forall\, u, v \in G), (U_X(u \odot v) = U_X(v) \Rightarrow U_X(u) = U_X(e))$

$(\forall\, u, v \in G), \big(U_X(u) = U_X(e) \Rightarrow U_X(u \odot v) \leq U_X(v)\big);$

$d)$ $(\forall\, u, v \in G), (F_X(u \odot v) = F_X(v) \Rightarrow F_X(u) = F_X(e))$

$(\forall\, u, v \in G), \big(F_X(u) = F_X(e) \Rightarrow F_X(u \odot v) \geq F_X(v)\big);$

Proof. We only prove (a) and (c). (b) and (d) proved in a similar way.

(a) First we assume that $T_X(u \odot v) = T_X(v)\ \forall\, u, v \in G$. Put $v = e$ and use (iii) of ***Definition 6*** we get $T_X(u) = T_X(u \odot e) = T_X(e)$. Let for $u, v\ \in G$ be such that $T_X(u) = T_X(e)$ then$T_X(u \odot v) \geq min\{T_X(u), T_X(v)\} = min\{T_X(e), T_X(v)\} = T_X(v)$.

Now to prove (c) consider that $U_X(u \odot v) = U_X(v)\ \forall\, u, v \in$ G. Put $v = e$ and use (iii) of ***Definition 6***, we have $U_X(u) = U_X(u \odot e) = U_X(e)$. Let for $u, v\ \in G$ be such that $U_X(u) = U_X(e)$ then, $U_X(u \odot v) \leq max\{U_X(u), U_X(v)\} = max\{U_X(e), U_X(v)\} = U_X(v)$. Hence the proof.

Definition 9. Let $X = (T_X, C_X, U_X, F_X)$ be a quadripartitioned single valued neutrosophic set in a K-algebra of Ҟ and let $(\lambda, \mu, \vartheta, \xi) \in [0,1] \times [0,1] \times [0,1] \times [0,1]$ with $\lambda + \mu + \vartheta + \xi \leq 4$. Then the sets,

$X_{(\lambda,\mu,\vartheta,\xi)} = \{u \in G |\ T_X(u) \geq \lambda,\ C_X(u) \geq \mu,\ U_X(u) \leq \vartheta,\ F_X(u) \leq \xi\},$

$(\lambda, \mu, \vartheta, \xi) X_{(\lambda,\mu,\vartheta,\xi)} = U(T_X, \lambda) \cap U'(C_X, \mu) \cap L(U_X, \vartheta) \cap L'(F_X, \xi)$

are called $(\lambda, \mu, \vartheta, \xi)$ level subsets of quadripartitioned single valued neutrosophic set X.

And also the set $X_{(\lambda,\mu,\vartheta,\xi)} = \{u \in G |\ T_X(u) > \lambda,\ C_X(u) > \mu,\ U_X(u) < \vartheta,\ F_X(u) < \xi\}$ is known as strong level subset of X.

Note. The set of all $(\lambda, \mu, \vartheta, \xi) \in Im(T_X) \times Im(C_X) \times Im(U_X) \times Im(F_X)$ is known as image of $X = (T_X, C_X, U_X, F_X)$.

Proposition 2. If $X = (T_X, C_X, U_X, F_X)$ is a quadripartitioned single valued neutrosophic K-algebra of Ҟ then the level subsets,

$U(T_X, \lambda) = \{u \in G |\ T_X(u) \geq \lambda\}, U'(C_X, \mu) = \{u \in G |\ C_X(u) \geq \mu\},$

$L(U_X, \vartheta) = \{u \in G |\ U_X(u) \leq \vartheta\}, L'(F_X, \xi) = \{u \in G |\ F_X(u) \leq \xi\}$

are K-subalgebras of Ҟ for every $(\lambda, \mu, \vartheta, \xi) \in Im(T_X) \times Im(C_X) \times Im(U_X) \times Im(F_X) \subseteq [0,1]$

where $Im(T_X), Im(C_X), Im(U_X)$ *and* $Im(F_X)$ are sets of values $T(X), C(X), U(X)$ *and* $F(X)$, respectively.

Proof. Let $X = (T_X, C_{X,} U_X, F_X)$ be a quadripartitioned single valued neutrosophic set in a K-algebra of Ꝁ and$(\lambda, \mu, \vartheta, \xi) \in Im(T_X) \times Im(C_X) \times Im(U_X) \times Im(F_X)$ be such that $U(T_X, \lambda) \neq \emptyset$, $U'(C_X, \mu) \neq \emptyset$, $L(U_X, \vartheta) \neq \emptyset$ *and* $L'(F_X, \xi) \neq \emptyset$. We have to show that U, U', L *and* L' are level K-subalgebras. Let for $u, v \in U(T_X, \lambda)$, $T_X(u) \geq \lambda$ *and* $T_X(v) \geq \lambda$. Then from ***Definition 8*** we get $T_X(u \odot v) \geq min\{T_X(u), T_X(v)\} \geq \lambda$. It shows that $u \odot v \in U(T_X, \lambda)$. Hence $U(T_X, \lambda)$ is a level K-subalgebra of Ꝁ. Similarly, we can prove for $U'(C_X, \mu), L(U_X, \vartheta)$ *and* $L'(F_X, \xi)$.

Theorem 1. Let $X = (T_X, C_{X,} U_X, F_X)$ be a quadripartitioned single valued neutrosophic set in a K-algebra of Ꝁ. Then $X = (T_X, C_{X,} U_X, F_X)$ is a quadripartitioned single valued neutrosophic K-subalgebra of Ꝁ if and only if $X_{(\lambda,\mu,\vartheta,\xi)}$ is a K-subalgebra of Ꝁ for every $(\lambda, \mu, \vartheta, \xi) \in Im(T_X) \times Im(C_X) \times Im(U_X) \times Im(F_X)$ with $\lambda + \mu + \vartheta + \xi \leq 4$.

Proof. First assume tha$X_{(\lambda,\mu,\vartheta,\xi)}$t is a K-subalgebra of Ꝁ. If the conditions in ***Definition 8*** fail, then there exist $s, t \in G$ such that,

$$T_X(s \odot t) < \min\{T_X(s), T_X(t)\},$$

$$C_X(s \odot t) < \min\{C_X(s), C_X(t)\},$$

$$U_X(s \odot t) > \max\{U_X(s), U_X(t)\},$$

$$F_X(s \odot t) > \max\{F_X(s), F_X(t)\}.$$

Now let $\lambda_1 = \frac{1}{2}(T_X(s \odot t) + min\{T_X(s), T_X(t)\})$, $\mu_1 = \frac{1}{2}(C_X(s \odot t) + min\{C_X(s), C_X(t)\})$,

$\vartheta_1 = \frac{1}{2}(U_X(s \odot t) + max\{U_X(s), U_X(t)\})$, $\xi_1 = \frac{1}{2}(F_X(s \odot t) + max\{F_X(s), F_X(t)\})$.
Now we have,

$$T_X(s \odot t) < \lambda_1 < \min\{T_X(s), T_X(t)\},$$

$$C_X(s \odot t) < \mu_1 < \min\{C_X(s), C_X(t)\},$$

$$U_X(s \odot t) > \vartheta_1 > \max\{U_X(s), U_X(t)\},$$

$$F_X(s \odot t) > \xi_1 > \max\{F_X(s), F_X(t)\}.$$

This implies that $s, t \in X_{(\lambda,\mu,\vartheta,\xi)}$ *and* $s \odot t \notin X_{(\lambda,\mu,\vartheta,\xi)}$ which is a contradiction. This proves that the conditions of ***Definition 8*** is true. Hence $X = (T_X, C_{X,} U_X, F_X)$ is a quadripartitioned single valued neutrosophic K-subalgebra of Ꝁ.

Now assume that $X = (T_X, C_{X,} U_X, F_X)$ be a quadripartitioned single valued neutrosophic K-subalgebra of Ꝁ. Let for $(\lambda, \mu, \vartheta, \xi) \in Im(T_X) \times Im(C_X) \times Im(U_X) \times Im(F_X)$ with $\lambda + \mu + \vartheta + \xi \leq 4$ such that $X_{(\lambda,\mu,\vartheta,\xi)} \neq \emptyset$. Let $u, v \in X_{(\lambda,\mu,\vartheta,\xi)}$ be such that,

$$T_X(u) \geq \lambda, T_X(v) \geq \lambda',$$

$C_X(u) \geq \mu, C_X(v) \geq \mu'$,

$U_X(u) \leq \vartheta, U_X(v) \leq \vartheta'$,

$F_X(u) \leq \xi, F_X(v) \leq \xi'$.

Now assume that $\lambda \leq \lambda', \mu \leq \mu', \vartheta \geq \vartheta'$ *and* $\xi \geq \xi'$. It follows from ***Definition 8*** that,

$T_X(u \odot v) \geq \lambda = \min\{T_X(u), T_X(v)\}$,

$C_X(u \odot v) \geq \mu = \min\{C_X(u), C_X(v)\}$,

$U_X(u \odot v) \leq \vartheta = \max\{U_X(u), U_X(v)\}$,

$F_X(u \odot v) \leq \xi = \max\{F_X(u), F_X(v)\}$.

This shows that $u \odot v \in X_{(\lambda,\mu,\vartheta,\xi)}$. Hence $X_{(\lambda,\mu,\vartheta,\xi)}$ is a K-subalgebra of Ҝ.

Theorem 2. Let $X = (T_X, C_X, U_X, F_X)$ be a quadripartitioned single valued neutrosophic K-subalgebra and $(\lambda_1, \mu_1, \vartheta_1, \xi_1), (\lambda_2, \mu_2, \vartheta_2, \xi_2) \in Im(T_X) \times Im(C_X) \times Im(U_X) \times Im(F_X)$ with $\lambda_i + \mu_i + \vartheta_i + \xi_i \leq 4$ for $i = 1,2$. Then $X_{(\lambda_1,\mu_1,\vartheta_1,\xi_1)} = X_{(\lambda_2,\mu_2,\vartheta_2,\xi_2)}$ if $(\lambda_1, \mu_1, \vartheta_1, \xi_1) = (\lambda_2, \mu_2, \vartheta_2, \xi_2)$.

Proof. When $(\lambda_1, \mu_1, \vartheta_1, \xi_1) = (\lambda_2, \mu_2, \vartheta_2, \xi_2)$ then the result is obvious for $X_{(\lambda_1,\mu_1,\vartheta_1,\xi_1)} = X_{(\lambda_2,\mu_2,\vartheta_2,\xi_2)}$. Conversely assume that $X_{(\lambda_1,\mu_1,\vartheta_1,\xi_1)} = X_{(\lambda_2,\mu_2,\vartheta_2,\xi_2)}$. Since $(\lambda_1, \mu_1, \vartheta_1, \xi_1) \in Im(T_X) \times Im(C_X) \times Im(U_X) \times Im(F_X)$ there exists $u \in G$ such that $T_X(u) = \lambda_1$, $C_X(u) = \mu_1, U_X(u) = \vartheta_1$ *and* $F_X(u) = \xi_1$. This implies that $u \in X_{(\lambda_1,\mu_1,\vartheta_1,\xi_1)} = X_{(\lambda_2,\mu_2,\vartheta_2,\xi_2)}$. Hence $\lambda_1 = T_X(u) \geq \lambda_2, \mu_1 = C_X(u) \geq \mu_2, \vartheta_1 = U_X(u) \leq \vartheta_2$ *and* $\xi_1 = F_X(u) \leq \xi_2$. Also $(\lambda_2, \mu_2, \vartheta_2, \xi_2) \in Im(T_X) \times Im(C_X) \times Im(U_X) \times Im(F_X)$ there exists $v \in G$ such that $T_X(v) = \lambda_2$, $C_X(v) = \mu_2, U_X(v) = \vartheta_2$ *and* $F_X(v) = \xi_2$. This implies that $v \in X_{(\lambda_2,\mu_2,\vartheta_2,\xi_2)} = X_{(\lambda_1,\mu_1,\vartheta_1,\xi_1)}$. Hence $\lambda_2 = T_X(v) \geq \lambda_1, \mu_2 = C_X(v) \geq \mu_1, \vartheta_2 = U_X(v) \leq \vartheta_1$ *and* $\xi_2 = F_X(v) \leq \xi_1$. Hence $(\lambda_1, \mu_1, \vartheta_1, \xi_1) = (\lambda_2, \mu_2, \vartheta_2, \xi_2)$.

Theorem 3. Let I be a K-subalgebra of K-algebra Ҝ. Then there exists a quadripartitioned single valued neutrosophic K-subalgebra $X = (T_X, C_X, U_X, F_X)$ of K-algebra Ҝ such that $X = (T_X, C_X, U_X, F_X) = I$ for some $\lambda, \mu \in (0,1]$ *and* $\vartheta, \xi \in [0,1)$.

Proof. Let $X = (T_X, C_X, U_X, F_X)$ be a quadripartitioned single valued neutrosophic set in K-algebra Ҝ given by,

$$T_X(u) = \begin{cases} \lambda \in (0,1], & \text{if } u \in I \\ 0, & \text{otherwise} \end{cases}$$

$$C_X(u) = \begin{cases} \mu \in (0,1], & \text{if } u \in I \\ 0, & \text{otherwise} \end{cases}$$

$$U_X(u) = \begin{cases} \vartheta \in [0,1), & \text{if } u \in I \\ 0, & \text{otherwise} \end{cases}$$

$$\mathrm{F_X(u)} = \begin{cases} \xi \in [0,1), & \text{if } u \in I \\ 0, & \text{otherwise} \end{cases}.$$

Let $u, v \in G$. If $u, v \in I$, then $u \odot v \in I$ and so,

$$\mathrm{T_X(u \odot v) \geq \min\{T_X(u), T_X(v)\}},$$

$$\mathrm{C_X(u \odot v) \geq \min\{C_X(u), C_X(v)\}},$$

$$\mathrm{U_X(u \odot v) \leq \max\{U_X(u), U_X(v)\}},$$

$$\mathrm{F_X(u \odot v) \leq \max\{F_X(u), F_X(v)\}}.$$

Suppose $u \notin I \ or \ v \notin I$ then,

$$\mathrm{T_X(u) = 0 \text{ or } T_X(v), C_X(u) = 0 \text{ or } C_X(v), U_X(u) = \ 0 \text{ or } U_X(v) \text{ and } F_X(u) = 0 \text{ or } F_X(v)}.$$

It implies that,

$$\mathrm{T_X(u \odot v) \geq \min\{T_X(u), T_X(v)\}},$$

$$\mathrm{C_X(u \odot v) \geq \min\{C_X(u), C_X(v)\}},$$

$$\mathrm{U_X(u \odot v) \leq \max\{U_X(u), U_X(v)\}},$$

$$\mathrm{F_X(u \odot v) \leq \max\{F_X(u), F_X(v)\}}.$$

Hence $X = (T_X, C_{X,} U_X, F_X)$ is a quadripartitioned single valued neutrosophic K-subalgebra of Ҡ.

Consequently $X_{(\lambda,\mu,\vartheta,\xi)} = I$

Theorem 4. Let Ҡ be a K-algebra. Let a chain of K-subalgebras: $X_0 \subset X_1 \subset X_2 \subset \cdots \subset X_n = G$. Then the level K-subalgebras of the quadripartitioned single valued neutrosophic K-subalgebra remains same as the K-subalgebras of this chain.

Proof. Let $\{\lambda_i | i = 0,1,\ldots,n\}, \{\mu_i | i = 0,1,\ldots,n\}$ be finite decreasing sequences and $\{\vartheta_i | i = 0,1,\ldots,n\}$, $\{\xi_i | i = 0,1,\ldots,n\}$ be finite increasing sequences in $[0,1]$ such that $\lambda_k + \mu_k + \vartheta_k + \xi_k \leq 4$ for $k = 0,1,2,\ldots,n$. Let $X = (T_X, C_{X,} U_X, F_X)$ be a quadripartitioned single valued neutrosophic set in Ҡ defined by $T_X(X_0) = \lambda_0$, $C_X(X_0) = \mu_0, U_X(X_0) = \vartheta_0 \ and \ F_X(X_0) = \xi_0$,

$$T_X(X_i \backslash X_{i-1}) = \lambda_i, C_X(X_i \backslash X_{i-1}) = \mu_i, U_X(X_i \backslash X_{i-1}) = \vartheta_i \ and \ F_X(X_i \backslash X_{i-1}) = \xi_i \ for \ 0 < i \leq n.$$

We have t $u \odot v \in X_{i-1}$o prove that $X = (T_X, C_{X,} U_X, F_X)$ is a quadripartitioned single valued neutrosophic K-subalgebra of Ҡ. Let $u, v \in G$. If $u, v \in X_i \backslash X_{i-1}$ then it implies that $T_X(u) = \lambda_i = T_X(v)$, $C_X(u) = \mu_i = C_X(v)$, $U_X(u) = \vartheta_i = U_X(v)$ and$F_X(u) = \xi_i = F_X(v)$. Since each X_i is a K-subalgebra, we get $u \odot v \in X_i$. So that either $u \odot v \in X_i \backslash X_{i-1}$ or. In any of the above case it follows that,

$$T_X(u \odot v) \geq \lambda_i = \min\{T_X(u), T_X(v)\},$$

$$C_X(u \odot v) \geq \mu_i = \min\{C_X(u), C_X(v)\},$$

$$U_X(u \odot v) \leq \vartheta_i = \max\{U_X(u), U_X(v)\},$$

$$F_X(u \odot v) \leq \xi_i = \max\{F_X(u), F_X(v)\}.$$

For $k > l$ if $u \in X_k \backslash X_{k-1}$ and $v \in X_l \backslash X_{l-1}$ then,

$$T_X(u) = \lambda_k, T_X(v) = \lambda_l,$$

$$C_X(u) = \mu_k, C_X(v) = \mu_{l,}$$

$$U_X(u) = \vartheta_k, U_X(v) = \vartheta_l,$$

$$F_X(u) = \xi_k, F_X(v) = \xi_l,$$

and $u \odot v \in X_k$ because X_k is a K-subalgebra and $X_l \subset X_k$. It follows that,

$$T_X(u \odot v) \geq \lambda_k = \min\{T_X(u), T_X(v)\},$$

$$C_X(u \odot v) \geq \mu_k = \min\{C_X(u), C_X(v)\},$$

$$U_X(u \odot v) \leq \vartheta_k = \max\{U_X(u), U_X(v)\},$$

$$F_X(u \odot v) \leq \xi_k = \max\{F_X(u), F_X(v)\}.$$

Hence $X = (T_X, C_{X,} U_X, F_X)$ is a quadripartitioned single valued neutrosophic K-subalgebra of Ꝃ and all its non-empty level subsets are level K-subalgebras of Ꝃ. Since $Im(T_X) = \{\lambda_0, \lambda_1, \dots, \lambda_n\}$, $Im(C_X) = \{\mu_0, \mu_1, \dots, \mu_n\}$, $Im(U_X) = \{\vartheta_0, \vartheta_1, \dots, \vartheta_n\}$ and $Im(F_X) = \{\xi_0, \xi_1, \dots, \xi_n\}$. Therefore, the level K-subalgebras of $X = (T_X, C_{X,} U_X, F_X)$ are given by the chain of K-subalgebras:

$$U(T_X, \lambda_0) \subset U(T_X, \lambda_1) \subset \cdots \subset U(T_X, \lambda_n) = G,$$

$$U'(C_X, \mu_0) \subset U'(C_X, \mu_1) \subset \cdots \subset U'(C_X, \mu_n) = G,$$

$$L(U_X, \vartheta_0) \subset L(U_X, \vartheta_1) \subset \cdots \subset L(U_X, \vartheta_n) = G,$$

$$L'(F_X, \xi_0) \subset L'(F_X, \xi_1) \subset \cdots \subset L'(F_X, \xi_n) = G,$$

respectively. Indeed,

$$U(T_X, \lambda_0) = \{u \in G |\ T_X(u) \geq \lambda_0\} = X_0,$$

$$U'(C_X, \mu_0) = \{u \in G |\ C_X(u) \geq \mu_0\} = X_0,$$

$$L(U_X, \vartheta_0) = \{u \in G |\ U_X(u) \leq \vartheta_0\} = X_0,$$

$L'(F_X, \xi_0) = \{u \in G | \, F_X(u) \le \xi_0\} = X_0.$

Now we have to prove that,

$U(T_X, \lambda_i) = X_i, U'(C_X, \mu_i) = X_i, L(U_X, \vartheta_i) = X_i$ *and* $L'(F_X, \xi_i) = X_i$ *for* $0 < i \le n.$ Clearly $X_i \subseteq U(T_X, \lambda_i), X_i \subseteq U'(C_X, \mu_i), X_i \subseteq L(U_X, \vartheta_i)$ *and* $X_i \subseteq L'(F_X, \xi_i)$. If $u \in U(T_X, \lambda_i)$ then $T_X(u) \ge \lambda_i$ and so $u \notin A_k$ *for* $k > i$. Hence $T_X(u) \in \{\lambda_0, \lambda_1, \dots, \lambda_i\}$ which shows that $u \in X_k$ *for* $k \le i$, since $X_k \subseteq X_{i.}$ It follows that $u \in X_i$. Consequently $U(T_X, \lambda_i) = X_i$ for some $0 < i \le n$. Similarly, it is proved for $U'(C_X, \mu_i) = X_i$. Now if $v \in L(U_X, \vartheta_i)$ then $U_X(v) \le \vartheta_i$ and so $v \notin X_k$ *for some* $i \le k$. Thus $U_X(u) \in \{\vartheta_0, \vartheta_1, \dots, \vartheta_i\}$ which shows that $u \in X_l$ *for some* $l \le i$, since $X_l \subseteq X_{i.}$ It follows that $v \in X_i$. Consequently, $L(U_X, \vartheta_i) = X_i$ for some $0 < i \le n$. Similarly, it is proved for $L'(F_X, \xi_i) = X_i$. Hence the proof.

4. Homomorphism of Quadripartitioned Single Valued Neutrosophic K-Algebras

Definition 10. Consider two K-algebras $Ꝁ_1 = (G_1, \cdot, \odot, e_1)$ *and* $Ꝁ_2 = (G_2, \cdot, \odot, e_2)$ and f be a function from $Ꝁ_1$ into $Ꝁ_2$. If $Y = (T_Y, C_Y, U_Y, F_Y)$ is a quadripartitioned single valued neutrosophic K-subalgebra of $Ꝁ_2$, then the preimage of $Y = (T_Y, C_Y, U_Y, F_Y)$ under f is a quadripartitioned single valued neutrosophic K-subalgebra of $Ꝁ_1$ defined by,

$$f^{-1}(T_Y)(u) = T_Y(f(u)), f^{-1}(C_Y)(u) = C_Y(f(u)),$$

$$f^{-1}(U_Y)(u) = U_Y(f(u)), f^{-1}(F_Y)(u) = F_Y(f(u)),$$

for all $u \in G$.

Definition 11. A quadripartitioned single valued neutrosophic K-subalgebra $X = (T_X, C_X, U_X, F_X)$ of a K-algebra Ꝁ is called characteristic if $T_X(f(u)) = T_X(u), C_X(f(u)) = C_X(u), U_X(f(u)) = U_X(u)$ *and* $F_X(f(u)) = F_X(u)$ for all $u \in G$ and $f \in Aut(K)$.

Definition 12. A K-subalgebra U of a K-algebra Ꝁ is said to be fully invariant if $f(U) \subseteq U$ for all $f \in End(K)$ where $End(K)$ is the set of all endomorphisms of a K-algebra Ꝁ. A quadripartitioned single valued neutrosophic K-subalgebra $X = (T_X, C_X, U_X, F_X)$ of a K-algebra Ꝁ is called fully invariant if $T_X(f(u)) \le T_X(u), C_X(f(u)) \le C_X(u), U_X(f(u)) \ge U_X(u)$ *and* $F_X(f(u)) \ge F_X(u)$ for all $u \in G$ and $f \in End(K)$.

Definition 13. Let $X_1 = (T_{X_1}, C_{X_1}, U_{X_1}, F_{X_1})$ and $X_2 = (T_{X_2}, C_{X_2}, U_{X_2}, F_{X_2})$ be two quadripartitioned single valued neutrosophic K-subalgebras of Ꝁ. Then $X_1 = (T_{X_1}, C_{X_1}, U_{X_1}, F_{X_1})$ is said to be the same type of $X_2 = (T_{X_2}, C_{X_2}, U_{X_2}, F_{X_2})$ if there exists $f \in Aut(K)$ such that $X_1 = X_2 \circ f$ i.e., $T_{X_1}(u) = T_{X_2}(f(u)), C_{X_1}(u) = C_{X_2}(f(u)), U_{X_1}(u) = U_{X_2}(f(u))$ *and* $F_{X_1}(u) = F_{X_2}(f(u))$ for all $u \in G$.

Theorem 5. Let $f: Ꝁ_1 \to Ꝁ_2$ be an epimorphism of K-algebras. If $Y = (T_Y, C_Y, U_Y, F_Y)$ is a quadripartitioned single valued neutrosophic K-subalgebra of $Ꝁ_2$, then $f^{-1}(Y)$ is a quadripartitioned single valued neutrosophic K-subalgebra of $Ꝁ_1$.

Proof. It is obvious that,

$$f^{-1}(T_Y)(e) \geq f^{-1}(T_Y)(u), f^{-1}(C_Y)(e) \geq f^{-1}(C_Y)(u),$$

$$f^{-1}(U_Y)(e) \leq f^{-1}(U_Y)(u), f^{-1}(F_Y)(e) \leq f^{-1}(F_Y)(u),$$

for all $u \in G_1. Let\ u, v \in G_1$ then,

$$f^{-1}(T_Y)(u \odot v) = T_Y(f(u \odot v)),$$

$$f^{-1}(T_Y)(u \odot v) = T_Y(f(u) \odot f(v)),$$

$$f^{-1}(T_Y)(u \odot v) \geq \min\{T_Y(f(u)), T_Y(f(v))\},$$

$$f^{-1}(T_Y)(u \odot v) \geq \min\{f^{-1}(T_Y)(u), f^{-1}(T_Y)(v)\};$$

$$f^{-1}(C_Y)(u \odot v) = C_Y(f(u \odot v)),$$

$$f^{-1}(C_Y)(u \odot v) = C_Y(f(u) \odot f(v)),$$

$$f^{-1}(C_Y)(u \odot v) \geq \min\{C_Y(f(u)), C_Y(f(v))\},$$

$$f^{-1}(C_Y)(u \odot v) \geq \min\{f^{-1}(C_Y)(u), f^{-1}(C_Y)(v)\};$$

$$f^{-1}(U_Y)(u \odot v) = U_Y(f(u \odot v)),$$

$$f^{-1}(U_Y)(u \odot v) = U_Y(f(u) \odot f(v)),$$

$$f^{-1}(U_Y)(u \odot v) \leq \max\{U_Y(f(u)), U_Y(f(v))\},$$

$$f^{-1}(U_Y)(u \odot v) \leq \max\{f^{-1}(U_Y)(u), f^{-1}(U_Y)(v)\};$$

$$f^{-1}(F_Y)(u \odot v) = F_Y(f(u \odot v)),$$

$$f^{-1}(F_Y)(u \odot v) = F_Y(f(u) \odot f(v)),$$

$$f^{-1}(F_Y)(u \odot v) \leq \max\{F_Y(f(u)), F_Y(f(v))\},$$

$$f^{-1}(F_Y)(u \odot v) \leq \max\{f^{-1}(F_Y)(u), f^{-1}(F_Y)(v)\}.$$

Hence $f^{-1}(Y)$ is a quadripartitioned single valued neutrosophic K-subalgebra of K_1.

Theorem 6. Let $f: \mathrm{K}_1 \rightarrow \mathrm{K}_2$ be an epimorphism of K-algebras. If $Y = (T_Y, C_Y, U_Y, F_Y)$ is a quadripartitioned single valued neutrosophic K-subalgebra of K_2 and $X = (T_X, C_X, U_X, F_X)$ is the preimage of Y under f. Then X is a quadripartitioned single valued neutrosophic K-subalgebra of K_1.

Proof. It is obvious that $T_X(e) \geq T_X(u), C_X(e) \geq C_X(u), U_X(e) \leq U_X(u)$ *and* $F_X(e) \leq F_X(u)$

for all $u \in G_1$. Now for any $u, v \in G_1$,

$$\mathrm{T_X(u \odot v) = T_Y(f(u \odot v))},$$

$$\mathrm{T_X(u \odot v) = T_Y(f(u) \odot f(v))},$$

$$\mathrm{T_X(u \odot v) \geq \min\{T_Y(f(u)), T_Y(f(v))\}},$$

$$\mathrm{T_X(u \odot v) \geq \min\{T_X(u), T_X(v)\}};$$

$$\mathrm{C_X(u \odot v) = C_Y(f(u \odot v))},$$

$$\mathrm{C_X(u \odot v) = C_Y(f(u) \odot f(v))},$$

$$\mathrm{C_X(u \odot v) \geq \min\{C_Y(f(u)), C_Y(f(v))\}},$$

$$\mathrm{C_X(u \odot v) \geq \min\{C_X(u), C_X(v)\}};$$

$$\mathrm{U_X(u \odot v) = U_Y(f(u \odot v))},$$

$$\mathrm{U_X(u \odot v) = U_Y(f(u) \odot f(v))},$$

$$\mathrm{U_X(u \odot v) \leq \max\{U_Y(f(u)), U_Y(f(v))\}},$$

$$\mathrm{U_X(u \odot v) \leq \max\{U_X(u), U_X(v)\}};$$

$$\mathrm{F_X(u \odot v) = F_Y(f(u \odot v))},$$

$$\mathrm{F_X(u \odot v) = F_Y(f(u) \odot f(v))},$$

$$\mathrm{F_X(u \odot v) \leq \max\{F_Y(f(u)), F_Y(f(v))\}},$$

$$\mathrm{F_X(u \odot v) \leq \max\{F_X(u), F_X(v)\}}.$$

Hence X is a quadripartitioned single valued neutrosophic K-subalgebra of $\mathcal{K}_1$.

Definition 14. Let f be a mapping from $\mathbb{K}_1$into $\mathbb{K}_2$ i.e., $f: \mathbb{K}_1 \to \mathbb{K}_2$ of K-algebras and let $X = (T_X, C_{X,} U_X, F_X)$ be a quadripartitioned single valued neutrosophic set of $\mathbb{K}_2$. The map $X = (T_X, C_{X,} U_X, F_X)$ is called the preimage of X under f if $T_X^f(u) = T_X(f(u)), C_X^f(u) = C_X(f(u)), U_X^f(u) = U_X(f(u))$ and $F_X^f(u) = F_X(f(u))$ for all $u \in G_1$.

Theorem 7. Let $f: \mathbb{K}_1 \to \mathbb{K}_2$ be an epimorphism of K-algebras. Then $X^f = (T_X^f, C_X^f, U_X^f, F_X^f)$ is a quadripartitioned single valued neutrosophic K-subalgebra of $\mathbb{K}_1$ if and only if $X = (T_X, C_X, U_X, F_X)$ is a quadripartitioned single valued neutrosophic K-subalgebra of $\mathbb{K}_2$.

Proof. Let $f: \mathbb{K}_1 \to \mathbb{K}_2$ be an epimorphism of K-algebras. First assume that $X^f = (T_X^f, C_X^f, U_X^f, F_X^f)$ is a quadripartitioned single valued neutrosophic K-subalgebra of $\mathbb{K}_1$. Then we have to prove that $X = (T_X, C_X, U_X, F_X)$ is a quadripartitioned single valued neutrosophic K-subalgebra of $\mathbb{K}_2$. Since there exists $u \in G_1$ such that $v = f(u)$ for any $v \in G_2$:

$$\mathrm{T_X(v) = T_X(f(u)) = T_X^{f(u)} \le T_X^{f(e_1)} = T_X(f(e_1)) = T_X(e_2),}$$

$$\mathrm{C_X(v) = C_X(f(u)) = C_X^{f(u)} \le C_X^{f(e_1)} = C_X(f(e_1)) = C_X(e_2),}$$

$$\mathrm{U_X(v) = U_X(f(u)) = U_X^{f(u)} \ge U_X^{f(e_1)} = U_X(f(e_1)) = U_X(e_2),}$$

$$\mathrm{F_X(v) = F_X(f(u)) = F_X^{f(u)} \ge F_X^{f(e_1)} = F_X(f(e_1)) = F_X(e_2).}$$

For any $u, v \in G_2, s, t \in G_1$ such that $u = f(s)$ and $v = f(t)$. It follows that:

$$\mathrm{T_X(u \odot v) = T_X(f(s \odot t)),}$$

$$\mathrm{T_X(u \odot v) = T_X^f(s \odot t),}$$

$$\mathrm{T_X(u \odot v) \ge \min\{T_X^f(s), T_X^f(t)\},}$$

$$\mathrm{T_X(u \odot v) \ge \min\{T_X(f(s)), T_X(f(t))\},}$$

$$\mathrm{T_X(u \odot v) \ge \min\{T_X(u), T_X(v)\};}$$

$$\mathrm{C_X(u \odot v) = C_X(f(s \odot t)),}$$

$$\mathrm{C_X(u \odot v) = C_X^f(s \odot t),}$$

$$\mathrm{C_X(u \odot v) \ge \min\{C_X^f(s), C_X^f(t)\},}$$

$$\mathrm{C_X(u \odot v) \ge \min\{C_X(f(s)), C_X(f(t))\},}$$

$$\mathrm{C_X(u \odot v) \ge \min\{C_X(u), C_X(v)\};}$$

$$U_X(u\odot v)=U_X\big(f(s\odot t)\big),$$

$$U_X(u\odot v)=U_X^f(s\odot t),$$

$$U_X(u\odot v)\le \max\{U_X^f(s),U_X^f(t)\},$$

$$U_X(u\odot v)\le \max\{U_X\big(f(s)\big),U_X\big(f(t)\big)\},$$

$$U_X(u\odot v)\le \max\{U_X(u),U_X(v)\};$$

$$F_X(u\odot v)=F_X\big(f(s\odot t)\big),$$

$$F_X(u\odot v)=F_X^f(s\odot t),$$

$$F_X(u\odot v)\le \max\{F_X^f(s),F_X^f(t)\},$$

$$F_X(u\odot v)\le \max\{F_X\big(f(s)\big),F_X\big(f(t)\big)\},$$

$$F_X(u\odot v)\le \max\{F_X(u),F_X(v)\}.$$

Hence $X=(T_X,C_X,U_X,F_X)$ is a quadripartitioned single valued neutrosophic K-subalgebra of $Ꝃ_2$. Conversely, assume that $X=(T_X,C_X,U_X,F_X)$ is a quadripartitioned single valued neutrosophic K-subalgebra of $Ꝃ_2$. Then we have to prove that $X^f=(T_X^f,C_X^f,U_X^f,F_X^f)$ is a quadripartitioned single valued neutrosophic K-subalgebra of $Ꝃ_1$. For any $u\in G_1$ we have:

$$T_X^f(e_1)=T_X\big(f(e_1)\big)=T_X(e_2)\ge T_X\big(f(u)\big)=T_X^f(u),$$

$$C_X^f(e_1)=C_X\big(f(e_1)\big)=C_X(e_2)\ge C_X\big(f(u)\big)=C_X^f(u),$$

$$U_X^f(e_1)=U_X\big(f(e_1)\big)=U_X(e_2)\le U_X\big(f(u)\big)=U_X^f(u),$$

$$F_X^f(e_1)=F_X\big(f(e_1)\big)=F_X(e_2)\le F_X\big(f(u)\big)=F_X^f(u).$$

Since X is a quadripartitioned single valued neutrosophic K-subalgebra of $Ꝃ_2$ and for any $u,v\in G_1$,

$$T_X^f(u\odot v)=T_X\big(f(u\odot v)\big),$$

$$T_X^f(u\odot v)=T_X\big(f(u)\odot f(v)\big),$$

$$T_X^f(u\odot v)\ge \min\{T_X\big(f(u)\big),T_X\big(f(v)\big)\},$$

$$T_X^f(u\odot v)\ge \min\{T_X^f(u),T_X^f(v)\};$$

$C_X^f(u⊙v) = C_X(f(u⊙v))$,

$C_X^f(u⊙v) = C_X(f(u)⊙f(v))$,

$C_X^f(u⊙v) \geq \min\{C_X(f(u)), C_X(f(v))\}$,

$C_X^f(u⊙v) \geq \min\{C_X^f(u), C_X^f(v)\}$;

$U_X^f(u⊙v) = U_X(f(u⊙v))$,

$U_X^f(u⊙v) = U_X(f(u)⊙f(v))$,

$U_X^f(u⊙v) \leq \max\{U_X(f(u)), U_X(f(v))\}$,

$U_X^f(u⊙v) \leq \max\{U_X^f(u), U_X^f(v)\}$;

$F_X^f(u⊙v) = F_X(f(u⊙v))$,

$F_X^f(u⊙v) = F_X(f(u)⊙f(v))$,

$F_X^f(u⊙v) \leq \max\{F_X(f(u)), F_X(f(v))\}$,

$F_X^f(u⊙v) \leq \max\{F_X^f(u), F_X^f(v)\}$.

Hence $X^f = (T_X^f, C_X^f, U_X^f, F_X^f)$ is a quadripartitioned single valued neutrosophic K-subalgebra of K_1.

Theorem 8. Let $X_1 = (T_{X_1}, C_{X_1}, U_{X_1}, F_{X_1})$ and $X_2 = (T_{X_2}, C_{X_2}, U_{X_2}, F_{X_2})$ be two quadripartitioned single valued neutrosophic K-subalgebras of K. Then a quadripartitioned single valued neutrosophic K-subalgebra $X_1 = (T_{X_1}, C_{X_1}, U_{X_1}, F_{X_1})$ is of the same type of quadripartitioned single valued neutrosophic K-subalgebra $X_2 = (T_{X_2}, C_{X_2}, U_{X_2}, F_{X_2})$ if and only if X_1 is isomorphic to X_2.

Proof. It is enough to prove only the necessary condition since sufficient condition holds trivially. Let $X_1 = (T_{X_1}, C_{X_1}, U_{X_1}, F_{X_1})$ be quadripartitioned single valued neutrosophic K-subalgebra having same type of $X_2 = (T_{X_2}, C_{X_2}, U_{X_2}, F_{X_2})$. Then there exists $f \in Aut(K)$ such that $T_{X_1}(u) = T_{X_2}(f(u)), C_{X_1}(u) = C_{X_2}(f(u)), U_{X_1}(u) = U_{X_2}(f(u))\ and\ F_{X_1}(u) = F_{X_2}(f(u))$ for all $u \in G$.

Let $g: X_1(K) \to X_2(K)$ be a mapping defined by $g(X_1(s)) = X_2(f(u))$ for all $u \in G$ i.e., $g\left(T_{X_1}(u)\right) = T_{X_2}(f(u)), g\left(C_{X_1}(u)\right) = C_{X_2}(f(u)), g\left(U_{X_1}(u)\right) = U_{X_2}(f(u))\ and\ g\left(F_{X_1}(u)\right) = F_{X_2}(f(u))$ for all $u \in G$. g is surjective obviously. And if $g\left(T_{X_1}(u)\right) = g\left(T_{X_1}(v)\right)$ for all $u, v \in G$ then $T_{X_2}(f(u)) = T_{X_2}(f(v))$ and we get $T_{X_1}(u) = T_{X_1}(v)$. Similarly we can prove for $C_{X_1}(u) = C_{X_1}(v), U_{X_1}(u) = U_{X_1}(v)$ and $F_{X_1}(u) = F_{X_1}(v)$.

Hence g is injective. Therefore g is a homomorphism such that for $u, v \in G$ we have:

$$g\big(T_{X_1}(u\odot v)\big) = T_{X_2}(f(u\odot v)) = T_{X_2}(f(u)\odot f(v)),$$

$$g\big(C_{X_1}(u\odot v)\big) = C_{X_2}(f(u\odot v)) = C_{X_2}(f(u)\odot f(v)),$$

$$g\big(U_{X_1}(u\odot v)\big) = U_{X_2}(f(u\odot v)) = U_{X_2}(f(u)\odot f(v)),$$

$$g\big(F_{X_1}(u\odot v)\big) = F_{X_2}(f(u\odot v)) = F_{X_2}(f(u)\odot f(v)).$$

Hence $X_1 = (T_{X_1}, C_{X_1}, U_{X_1}, F_{X_1})$ is isomorphic to $X_2 = \left(T_{X_2}, C_{X_2}, U_{X_2}, F_{X_2}\right)$.

5. Conclusion

In recent years, a new branch of logical algebra known as K-algebra applied in fuzzy set, intuitionistic fuzzy set and single valued neutrosophic set which helps us to extend the concept to K-algebra on quadripartitioned single valued neutrosophic sets. Quadripartitioned single valued neutrosophic set has four components truth,contradiction,unknown,false which helps to deal the concept of indeterminacy effectively. In this paper we defined K-algebras on quadripartitioned single valued neutrosophic sets and studied some of the results. Further the homomorphism of quadripartitioned single valued neutrosophic K-algebras, characteristic and fully invariant K-subalgebras also discussed in detail.

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