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A Study on Fundamentals of Refined Intuitionistic Fuzzy Set with Some Properties

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ABSTRACT

Zadeh conceptualized the theory of fuzzy set to provide a tool for the basis of the theory of possibility. Atanassov extended this theory with the introduction of intuitionistic fuzzy set. Smarandache introduced the concept of refined intuitionistic fuzzy set by further subdivision of membership and non-membership value. The meagerness regarding the allocation of a single membership and non-membership value to any object under consideration is addressed with this novel refinement. In this study, this novel idea is utilized to characterize the essential elements e.g. subset, equal set, null set, and complement set, for refined intuitionistic fuzzy set. Moreover, their basic set theoretic operations like union, intersection, extended intersection, restricted union, restricted intersection, and restricted difference, are conceptualized. Furthermore, some basic laws are also discussed with the help of an illustrative example in each case for vivid understanding.

1. Introduction

Zadeh [1] introduced the concept of fuzzy set for the first time in 1965 which covers all weak aspects of the classical set theory. In fuzzy set, the membership value is allocated from the interval [0, 1] to all the elements of the universe under consideration. Zadeh [2] used his own concept as a basis for a theory of possibility. Dubois and Prade [3, 4] established relationship between fuzzy sets and probability theories and also derive monotonicity property for algebraic operations performed between random set-valued variables. Ranking fuzzy numbers in the setting of possibility theory was done by Dubois and Prade [5]. This concept was used by Liang et al. in data analysis, similarities measures in fuzzy sets were discussed by Beg and Ashraf [6-8]. Set difference and symmetric difference of fuzzy sets were established by Vemuri et al., after that, Neog and

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Sut [9] extended the work to complement of an extended fuzzy set. A lot of work is done by researchers in fuzzy mathematics and its hybrids [10-16].

In some real life situations, the values are in the form of intervals due to which it is hard to allocate a membership value to the element of the universe of discourse. Therefore, the concept of interval-valued is introduced which proved a very powerful tool in this area.

In 1986, Atanassov [17, 18] introduced the concept of intuitionistic fuzzy set in which the membership value and non-membership value is allocated from the interval [0,1] to all the elements of the universe under consideration. It is the generalization of the fuzzy set. The invention of intuitionistic fuzzy set proved very important tool for researchers. Ejegwa et al. [19] discussed about operations, algebra, model operators and normalization on intuitionistic fuzzy sets. Szmidt and Kacprzyk [20] gave geometrical representation of an intuitionistic fuzzy set is a point of departure for our proposal of distances between intuitionistic fuzzy sets and also discussed properties. Szmidt and Kacprzyk [21] also discussed about non-probabilistic-type entropy measure for these sets and used it for geometrical interpretation of intuitionistic fuzzy sets. Proposed measure in terms of the ratio of intuitionistic fuzzy cardinalities was also defined and discussed. Ersoy and Davvaz [22] discussed the basic definitions and properties of intuitionistic fuzzy Γ- hyperideals of a Γ- semi-hyperring with examples are introduced and described some characterizations of Artinian and Noetherian Γ- semi hyper ring. Bustince and Burillo [23] proved that vague sets are intuitionistic fuzzy sets. A lot of work is done by researchers in intuitionistic fuzzy environment and its hybrids [24-33].

In 2019, Smarandache defined the concept of refined intuitionistic fuzzy set [34]. In this paper, we extend the concept to refined intuitionistic fuzzy set and defined some fundamental concepts and aggregation operations of refined intuitionistic fuzzy set.

Imprecision is a critical viewpoint in any decision making procedure. Various tools have been invented to deal with the uncertain environment. Perhaps the important tool in managing with imprecision is intuitionistic fuzzy sets. Besides, the most significant thing is that in real life scenario, it is not sufficient to allocate a single membership and non-membership value to any object under consideration. This inadequacy is addressed with the introduction of refined intuitionistic fuzzy set. Having motivation from this novel concept, essential elements, set theoretic operations and basic laws are characterized for refined intuitionistic fuzzy set in this work.

The remaining article is outlined in such a way that the Section 2 recalls some basic definitions along with illustrative example. Section 3 explains basic notions of Refined Intuitionistic Fuzzy Set (RIFS) including subset, equal set, null set and complement set along with their examples for the clear understanding. Section 4 explains the aggregation operations of RIFS with the help of example, Section 5 gives some basic laws of RIFS and in the last, Section 6 concludes the work and gives the future directions.

2. Preliminaries

In this section, some basic concepts of Fuzzy Set (FS), Intuitionistic Fuzzy Set (IFS) and RIFS are discussed.

Let us consider $\breve{U}$ be a universal set, N be a set of natural numbers, $\breve{I}$ represent the interval $[0,1]$, T_{η}^{ω} denotes the degree of sub-truth of type $\omega = 1,2,3,\dots,\alpha$ and F_{η}^{λ} denotes the degree of sub-falsity of type $\lambda = 1,2,3,\dots,\beta$ such that α and β are natural numbers. An illustrative example is considered to understand these entire basic concepts throughout the paper.

Definition 1. [1, 2] The fuzzy set $\breve{\eta}_f = \{< \breve{\delta}, \alpha_{\breve{\eta}_f}(\breve{\delta}) > | \breve{\delta} \in \breve{U}\}$ on $\breve{U}$ such that $\alpha_{\breve{\eta}_f}(\breve{\delta}): \breve{U} \rightarrow \breve{I}$ where $\alpha_{\breve{\eta}_f}(\breve{\delta})$ describes the membership of $\breve{\delta} \in \breve{U}$.

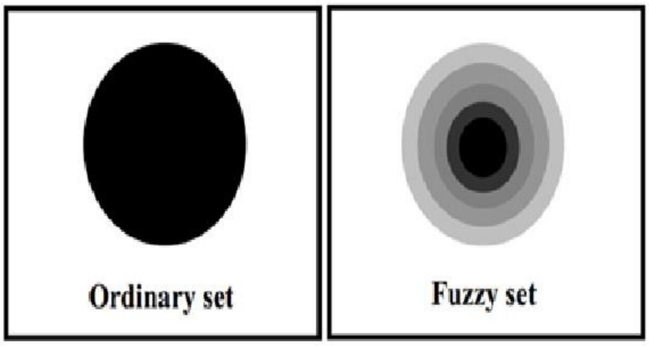

Fig. 1. Representation of fuzzy set.

Example 1. Hamna wants to purchase a dress for farewell party event of her university. She expected to purchase such dress which meets her desired requirements according to the event. Let $\breve{U} = \{\breve{B}_1, \breve{B}_2, \breve{B}_3, \breve{B}_4\}$, be different well-known brands of clothes in Pakistan such that

$\breve{B}_1 =$ Ideas Gul Ahmad;
$\breve{B}_2 =$ Khaadi;
$\breve{B}_3 =$ Nishat Linen;
$\breve{B}_4 =$ Junaid jamshaid.

Then fuzzy set $\breve{\eta}_f$ on the universe $\breve{U}$ is written in such a way that $\breve{\eta}_f = \left\{ \begin{matrix} < \breve{B}_1, 0.45 >, < \breve{B}_2, 0.57 >, \\ < \breve{B}_3, 0.6 >, < \breve{B}_4, 0.64 > \end{matrix} \right\}$.

Definition 2. [18]. An IFS $\breve{\eta}_{IFS}$ on $\breve{U}$ is given by $\breve{\eta}_{IFS} = \{< \breve{\delta}, T_{\breve{\eta}}(\breve{\delta}), F_{\breve{\eta}}(\breve{\delta}) > | \breve{\delta} \in \breve{U}\}$,

where $T_{\breve{\eta}}(\breve{\delta}), F_{\breve{\eta}}(\breve{\delta}): \breve{U} \rightarrow P([0,1])$, respectively, with the condition $sup\, T_{\breve{\eta}}(\breve{\delta}) + sup\, F_{\breve{\eta}}(\breve{\delta}) \leq 1$.

Example 2. Consider the illustrative example, and then the intuitionistic fuzzy set $\breve{\eta}_{IFS}$ on the universe $\breve{U}$ is given as $\breve{\eta}_{IFS} = \{< \breve{B}_1, 0.75, 0.14 >, < \breve{B}_2, 0.57, 0.2 >, < \breve{B}_3, 0.6, 0.3 >, < \breve{B}_4, 0.64, 0.16 >\}$.

Definition 3. [34] A RIFS $\breve{\eta}_{RIFS}$ on $\breve{U}$ is given by $\breve{\eta}_{RIFS} = \left\{< \breve{\delta}, T_{\breve{\eta}}^{\omega}(\breve{\delta}), F_{\breve{\eta}}^{\lambda}(\breve{\delta}) > : \ \omega \in N_1^{\alpha}, \ \lambda \in N_1^{\beta}, \alpha + \beta \geq 3, \ \breve{\delta} \in \breve{U} \right\}$, where $\alpha, \beta \in \breve{I}$ such that $T_{\breve{\eta}}^{\omega}, F_{\breve{\eta}}^{\lambda} \subseteq \breve{I}$, respectively, with the condition

$$\sum_{\omega=1}^{\alpha} sup\ T_{\breve{\eta}}^{\omega}(\breve{\delta}) + \sum_{\lambda=1}^{\beta} sup\ F_{\breve{\eta}}^{\lambda}(\breve{\delta}) \leq 1.$$

It is denoted by $(\breve{\delta}, \breve{G})$, where $\breve{G} = (T_{\breve{\eta}}^{\omega}, F_{\breve{\eta}}^{\lambda})$.

Example 3. Consider the illustrative example, then the RIFS $\breve{\eta}_{RIFS}$ can be written in such a way that

$\breve{\eta}_{RIFS} = \{< \breve{B}_1, (0.5, 0.4), (0.3, 0.25) >, < \breve{B}_2, (0.35, 0.3), (0.15, 0.1) >,$
$< \breve{B}_3, (0.35, 0.25), (0.3, 0.2) >, < \breve{B}_4, (0.6, 0.1), (0.12, 0.2) >\}.$

3. Basic Notions of RIFS

In this section, some basic notions of subset, equal sets, null set and complement set for RIFS are introduced.

Definition 4. Refined intuitionistic fuzzy subset

Let $\breve{\eta}_{1_{RIFS}} = (\breve{\delta}, \breve{G}_1)$ and $\breve{\eta}_{2_{RIFS}} = (\breve{\delta}, \breve{G}_2)$ be two RIFS, then $\breve{\eta}_{1_{RIFS}} \subseteq \breve{\eta}_{2_{RIFS}}$, if

$$\sum_{\omega=1}^{\alpha} sup\ T_{\breve{\eta}_1}^{\omega}(\breve{\delta}) \le \sum_{\omega=1}^{\alpha} sup\ T_{\breve{\eta}_2}^{\omega}(\breve{\delta}),\ \sum_{\lambda=1}^{\beta} sup\ F_{\breve{\eta}_1}^{\lambda}(\breve{\delta}) \ge \sum_{\lambda=1}^{\beta} sup\ F_{\breve{\eta}_2}^{\lambda}(\breve{\delta})\ \ \forall\ \ \breve{\delta} \in \breve{U}.$$

Remark 1. If

$$\sum_{\omega=1}^{\alpha} sup\ T_{\breve{\eta}_1}^{\omega}(\breve{\delta}) < \sum_{\omega=1}^{\alpha} sup\ T_{\breve{\eta}_2}^{\omega}(\breve{\delta}),\ \sum_{\lambda=1}^{\beta} sup\ F_{\breve{\eta}_1}^{\lambda}(\breve{\delta}) > \sum_{\lambda=1}^{\beta} sup\ F_{\breve{\eta}_2}^{\lambda}(\breve{\delta})\ \ \forall\ \ \breve{\delta} \in \breve{U}.$$

Then it is denoted by $(\breve{\delta}, \breve{G}_1) \subset (\breve{\delta}, \breve{G}_2)$.

Suppose $(\breve{\delta}, \breve{G}_1^i) \subset (\breve{\delta}, \breve{G}_2^i)$ be two families of RIFS, then $(\breve{\delta}, \breve{G}_1^i)$ is called family of refined intuitionistic fuzzy subset of $(\breve{\delta}, \breve{G}_2^i)$, if $\breve{G}_1^i \subset \breve{G}_2^i$ and

$$\sum_{\omega=1}^{\alpha} sup\ T_{\breve{\eta}_1}^{\omega}(\breve{\delta}) < \sum_{\omega=1}^{\alpha} sup\ T_{\breve{\eta}_2}^{\omega}(\breve{\delta}), \sum_{\lambda=1}^{\beta} sup\ F_{\breve{\eta}_1}^{\lambda}(\breve{\delta}) > \sum_{\lambda=1}^{\beta} sup\ F_{\breve{\eta}_2}^{\lambda}(\breve{\delta}), \forall\, \breve{\delta} \in \breve{U}.$$

We denote it by $(\breve{\delta}, \breve{G}_1^i) \subset (\breve{\delta}, \breve{G}_2^i) \forall\ \ i = 1, 2, 3, \dots, n.$

Example 4. Consider the illustrative example, let $\breve{\eta}_{1_{RIFS}}$ and $\breve{\eta}_{2_{RIFS}}$ be two RIFS such that

$\breve{\eta}_{1_{RIFS}} = \{< \breve{B}_1, (0.35, 0.1), (0.22, 0.19) >, < \breve{B}_2, (0.25, 0.03), (0.15, 0.19) >,$
$< \breve{B}_3, (0.2, 0.1), (0.2, 0.24) >, < \breve{B}_4, (0.3, 0.4), (0.06, 0.04) >\},$

and
$\breve{\eta}_{2_{RIFS}} = \{< \breve{B}_1, (0.38, 0.11), (0.2, 0.14) >, < \breve{B}_2, (0.45, 0.04), (0.1, 0.14) >,$
$< \breve{B}_3, (0.3, 0.2), (0.01, 0.06) >, < \breve{B}_4, (0.31, 0.41), (0.01, 0.011) >\}.$

Then from above equations, it is clear that $\breve{\eta}_{1_{RIFS}} \subseteq \breve{\eta}_{2_{RIFS}}$.

Definition 5. Equal refined intuitionistic fuzzy sets

Let $\breve{\eta}_{1_{RIFS}} = (\breve{\delta},\ \breve{G}_1)$ and $\breve{\eta}_{2_{RIFS}} = (\breve{\delta},\ \breve{G}_2)$ be two RIFS, then $\breve{\eta}_{1_{RIFS}} = \breve{\eta}_{2_{RIFS}}$, if $\breve{\eta}_{1_{RIFS}} \subseteq \breve{\eta}_{2_{RIFS}}$ and $\breve{\eta}_{2_{RIFS}} \subseteq \breve{\eta}_{1_{RIFS}}$.

Example 5. Consider the illustrative example, let $\breve{\eta}_{1_{RIFS}}$ and $\breve{\eta}_{2_{RIFS}}$ be two RIFS such that

$\breve{\eta}_{1_{RIFS}} = (\breve{\delta}, \breve{G}_1) = \{ < \breve{B}_1, (0.4, 0.5), (0.03, 0.04) >, < \breve{B}_2, (0.5,0.4), (0.05,0.04) >,$
$< \breve{B}_3, (0.5,0.2), (0.01,0.06) >, < \breve{B}_4, (0.3,0.4), (0.06,0.04) > \},$

and
$\breve{\eta}_{2_{RIFS}} = (\breve{\delta}, \breve{G}_2) = \{ < \breve{B}_1, (0.4, 0.5), (0.03, 0.04) >, < \breve{B}_2, (0.5,0.4), (0.05,0.04) >,$
$< \breve{B}_3, (0.5,0.2), (0.01,0.06) >, < \breve{B}_4, (0.3,0.4), (0.06,0.04) > \}.$

Then from above equations, it is clear that $\breve{\eta}_{1_{RIFS}} = \breve{\eta}_{2_{RIFS}}$.

Definition 6. Null refined intuitionistic fuzzy set

Let RIFS $(\breve{\delta},\ \breve{G})$ is said to be null RIFS if

$$\sum_{\omega=1}^{\alpha} sup\ T_{\breve{\eta}}^{\omega}(\breve{\delta}) = 0,\ \sum_{\lambda=1}^{\beta} sup\ F_{\breve{\eta}}^{\lambda}(\breve{\delta}) = 0, \quad \forall\ \ \breve{\delta}\ \in \breve{U}.$$

It is denoted by $(\breve{\delta},\ \breve{G})_{null}$.

Example 6. Consider the illustrative example, the null RIFS is given as

$(\breve{\delta}, \breve{G}) = \{ < \breve{B}_1, (0, 0), (0, 0) >, < \breve{B}_2, (0,0), (0,0) >,$

$< \breve{B}_3, (0,0), (0,0) >, < \breve{B}_4, (0,0), (0,0) > \}.$

Definition 7. Complement of refined intuitionistic fuzzy set

The complement of RIFS$(\breve{\delta}, \breve{G})$ is denoted by $(\breve{\delta}, \breve{G}^c)$ and is defined that if

$$\sum_{\omega=1}^{\alpha} sup\ T_{\breve{\eta}^c}^{\omega}(\breve{\delta}) = \sum_{\lambda=1}^{\beta} sup\ F_{\breve{\eta}}^{\lambda}(\breve{\delta}), \quad \sum_{\lambda=1}^{\beta} sup\ F_{\breve{\eta}^c}^{\lambda}(\breve{\delta}) = \sum_{\omega=1}^{\alpha} sup\ T_{\breve{\eta}}^{\omega}(\breve{\delta}), \quad \forall\ \ \breve{\delta}\ \in \breve{U}.$$

Remark 2. The complement of family of RIFS$(\breve{\delta}, \breve{G}^c\ )$ is denoted by $(\breve{\delta}, \breve{G}^c)$ and is defined in a way that if

$$\sum_{\omega=1}^{\alpha} sup\ T_{\breve{\eta}_i^c}^{\omega}(\breve{\delta}) = \sum_{\lambda=1}^{\beta} sup\ F_{\breve{\eta}}^{\lambda}(\breve{\delta}), \sum_{\lambda=1}^{\beta} sup\ F_{\breve{\eta}_i^c}^{\lambda}(\breve{\delta}) = \sum_{\omega=1}^{\alpha} sup\ T_{\breve{\eta}}^{\omega}(\breve{\delta}), \forall\ i = 1,2,3, \dots, n.$$

Example 7. Consider the illustrative example, if there is a RIFS $\breve{\eta}_{RIFS}$ given as

$\breve{\eta}_{RIFS} = \{ <\breve{B}_1, (0.2, 0.1), (0.3, 0.35)>, <\breve{B}_2, (0.05,0.34), (0.45,0.04)>,$
$<\breve{B}_3, (0.01,0.6), (0.1,0.02)>, <\breve{B}_4, (0.3,0.04), (0.12,0.2)> \}.$

Then the complement of RIFS$\breve{\eta}_{RIFS}$ given as

$\breve{\eta}_{RIFS} = \{ <\breve{B}_1 (0.3, 0.35), (0.2, 0.1)>, <\breve{B}_2, (0.45,0.04), (0.05,0.34)>,$
$<\breve{B}_3, (0.1,0.02), (0.01,0.6)>, <\breve{B}_4, (0.12,0.2), (0.3,0.04)> \}.$

4. Aggregation Operators of RIFS

In this section, union, intersection, extended intersection, restricted union, restricted intersection and restricted difference of RIFS is defined with the help of illustrative example.

Definition 8. Union of two RIFS

The union of two RIFS $(\breve{\delta},\ \breve{G}_1)$ and $(\breve{\delta},\ \breve{G}_2)$is denoted by $(\breve{\delta},\ \breve{G}_1) \cup (\breve{\delta},\ \breve{G}_2)$ and it isdefined as $(\breve{\delta},\ \breve{G}_1) \cup (\breve{\delta},\ \breve{G}_2) = (\breve{\delta},\ \breve{\Upsilon})$, where $\breve{\Upsilon} = \breve{G}_1 \cup \breve{G}_2$, and truth and falsemembership of $(\breve{\delta},\ \breve{\Upsilon})$ is defined in such a way that

$$T_{\breve{\Upsilon}}(\breve{\delta}) = \max\left(\sum_{\omega=1}^{\alpha} \sup\ T^{\omega}_{\breve{\eta}_1}(\breve{\delta}), \sum_{\omega=1}^{\alpha} \sup\ T^{\omega}_{\breve{\eta}_2}(\breve{\delta})\right),$$

$$F_{\breve{\Upsilon}}(\breve{\delta}) = min\left(\sum_{\lambda=1}^{\beta} sup\, F^{\lambda}_{\breve{\eta}_1}(\breve{\delta}), \sum_{\lambda=1}^{\beta} sup\, F^{\lambda}_{\breve{\eta}_2}(\breve{\delta})\right).$$

Remark 3. The union of two families of RIFS $(\breve{\delta},\ \breve{G}^i_1)$ and $(\breve{\delta},\ \breve{G}^i_2)$is denoted by $(\breve{\delta},\ \breve{G}^i_1) \cup (\breve{\delta},\ \breve{G}^i_2)$ and it is defined as $(\breve{\delta},\ \breve{G}^i_1) \cup (\breve{\delta},\ \breve{G}^i_2) = (\breve{\delta},\ \breve{\Upsilon}^i)$, where $\breve{\Upsilon}^i = \breve{G}^i_1 \cup \breve{G}^i_2,\ i = 1,2,3,\dots,n$, and truth and false membership of $(\breve{\delta},\ \breve{\Upsilon}^i)$ is defined in such a way that

$$T_{\breve{\Upsilon}^i}(\breve{\delta}) = max\left(\sum_{\omega=1}^{\alpha} sup\ T^{\omega}_{\breve{\eta}_1}(\breve{\delta}), \sum_{\omega=1}^{\alpha} sup\ T^{\omega}_{\breve{\eta}_2}(\breve{\delta})\right),$$

$$F_{\breve{\Upsilon}^i}(\breve{\delta}) = min\left(\sum_{\lambda=1}^{\beta} sup\, F^{\lambda}_{\breve{\eta}_1}(\breve{\delta}), \sum_{\lambda=1}^{\beta} sup\, F^{\lambda}_{\breve{\eta}_2}(\breve{\delta})\right).$$

Example 8. Consider the illustrative example, suppose that

$(\breve{\delta},\ \breve{G}_1) = \{ <\breve{B}_1,$ *(0.13,0.19),(0.24, 0.1)*$>, <\breve{B}_2, (0.2,0.25), (0.15,0.24)>,$

$<\breve{B}_3, (0.1,0.36), (0.34,0.12)>, <\breve{B}_4, (0.16,0.14), (0.23,0.37)> \},$

and

$(\breve{\delta},\ \breve{G}_2) = \{ <\breve{B}_1,$ *(0.2,0.3),(0.3, 0.15)*$>,$

$< \breve{B}_2, (0.32,0.38), (0.1,0.04) >$,
$< \breve{B}_3, (0.01,0.16), (0.5,0.2) >, < \breve{B}_4, (0.26,0.15), (0.12,0.2) > \}$,

be two RIFS. Then the union of RIFS $(\breve{\delta},\ \breve{G}_1)$ and $(\breve{\delta},\ \breve{G}_2)$is given as

$(\breve{\delta},\ \breve{Y}), = \{ < \breve{B}_1, (0.2, 0.3),(0.24,0.1) >, < \breve{B}_2, (0.32,0.38), (0.1,0.04) >$,
$< \breve{B}_3, (0.1,0.36), (0.34,0.12) >, < \breve{B}_4, (0.26,0.15), (0.12,0.2) > \}$.

Definition 9. Intersection of two RIFS

The intersection of two RIFS $(\breve{\delta},\ \breve{G}_1)$ and $(\breve{\delta},\ \breve{G}_2)$is denoted by $(\breve{\delta},\ \breve{G}_1) \cap (\breve{\delta},\ \breve{G}_2)$ and it is defined as $(\breve{\delta},\ \breve{G}_1) \cap (\breve{\delta},\ \breve{G}_2) = (\breve{\delta},\ \breve{Y})$, where $\breve{Y} = \breve{G}_1 \cap \breve{G}_2$, and truth and falsemembership of $(\breve{\delta},\ \breve{Y})$ is defined in such a way that

$$T_{\breve{Y}}(\breve{\delta}) = min\left(\sum_{\omega=1}^{\alpha} sup\ T^{\omega}_{\breve{\eta}_1}(\breve{\delta}), \sum_{\omega=1}^{\alpha} sup\ T^{\omega}_{\breve{\eta}_2}(\breve{\delta})\right),$$

$$F_{\breve{Y}}(\breve{\delta}) = max\left(\sum_{\lambda=1}^{\beta} sup\ F^{\lambda}_{\breve{\eta}_1}(\breve{\delta}), \sum_{\lambda=1}^{\beta} sup\ F^{\lambda}_{\breve{\eta}_2}(\breve{\delta})\right).$$

Remark 4. The intersection of two families of RIFS $(\breve{\delta},\ \breve{G}_1^i)$ and $(\breve{\delta},\ \breve{G}_2^i)$is denoted by $(\breve{\delta},\ \breve{G}_1^i) \cap (\breve{\delta},\ \breve{G}_2^i)$ and it is defined as$(\breve{\delta},\ \breve{G}_1^i) \cap (\breve{\delta},\ \breve{G}_2^i) = (\breve{\delta},\ \breve{Y}^i)$, where $\breve{Y}^i = \breve{G}_1^i \cap \breve{G}_2^i,\ i = 1,2,3,\dots,n$, and truth and false membership of $(\breve{\delta},\ \breve{Y}^i)$ is defined in such a way that

$$T_{\breve{Y}^i}(\breve{\delta}) = min\left(\sum_{\omega=1}^{\alpha} sup\ T^{\omega}_{\breve{\eta}_1}(\breve{\delta}), \sum_{\omega=1}^{\alpha} sup\ T^{\omega}_{\breve{\eta}_2}(\breve{\delta})\right),$$

$$F_{\breve{Y}^i}(\breve{\delta}) = max\left(\sum_{\lambda=1}^{\beta} sup\ F^{\lambda}_{\breve{\eta}_1}(\breve{\delta}), \sum_{\lambda=1}^{\beta} sup\ F^{\lambda}_{\breve{\eta}_2}(\breve{\delta})\right).$$

Example 9. Consider the illustrative example, suppose that

$(\breve{\delta},\ \breve{G}_1) = \{ < \breve{B}_1, (0.13, 0.19),(0.24,0.1) >$,

$< \breve{B}_2, (0.2,0.25), (0.15,0.24) >$,
$< \breve{B}_3, (0.1,0.36), (0.34,0.12) >, < \breve{B}_4, (0.16,0.14), (0.23,0.37) > \}$,

and

$(\breve{\delta},\ \breve{G}_2) = \{ < \breve{B}_1, (0.2,0.3),(0.3, 0.15) >$,

$< \breve{B}_2, (0.32,0.38), (0.1,0.04) >$,
$< \breve{B}_3, (0.01,0.16), (0.5,0.2) >, < \breve{B}_4, (0.26,0.15), (0.12,0.2) > \}$,

be two RIFS. Then the intersection of RIFS $(\breve{\delta},\ \breve{G}_1)$ and $(\breve{\delta},\ \breve{G}_2)$is given as

$(\breve{\delta},\ \breve{Y}) = \{ < \breve{B}_1, (0.13, 0.19),(0.24, 0.1)>,$

$< \breve{B}_2, (0.2,0.25), (0.15,0.24) >,$
$< \breve{B}_3, (0.01,0.16), (0.5,0.2) >, < \breve{B}_4, (0.16,0.14), (0.23,0.37) > \}.$

Definition 10. Extended intersection of two RIFS

The intersection of two RIFS $(\breve{\delta},\ \breve{G}_1)$ and $(\breve{\delta},\ \breve{G}_2)$is denoted by $(\breve{\delta},\ \breve{G}_1) \cap_\varepsilon (\breve{\delta},\ \breve{G}_2)$ and it is defined as $(\breve{\delta},\ \breve{G}_1) \cap_\varepsilon (\breve{\delta},\ \breve{G}_2) = (\breve{\delta},\ \breve{Y})$, where $\breve{Y} = \breve{G}_1 \cup \breve{G}_2$, and truth and falsemembership of $(\breve{\delta},\ \breve{Y})$ is defined in such a way that

$$T_{\breve{Y}}(\breve{\delta}) = min\left(\sum_{\omega=1}^{\alpha} sup\ T^{\omega}_{\breve{\eta}_1}(\breve{\delta}), \sum_{\omega=1}^{\alpha} sup\ T^{\omega}_{\breve{\eta}_2}(\breve{\delta})\right),$$

$$F_{\breve{Y}}(\breve{\delta}) = max\left(\sum_{\lambda=1}^{\beta} sup\, F^{\lambda}_{\breve{\eta}_1}(\breve{\delta}), \sum_{\lambda=1}^{\beta} sup\, F^{\lambda}_{\breve{\eta}_2}(\breve{\delta})\right).$$

Remark 5. The extended intersection of two families of RIFS $(\breve{\delta},\ \breve{G}_1^i)$ and $(\breve{\delta},\ \breve{G}_2^i)$is denoted by $(\breve{\delta},\ \breve{G}_1^i) \cap_\varepsilon (\breve{\delta},\ \breve{G}_2^i)$ and it isdefined as$(\breve{\delta},\ \breve{G}_1^i) \cap_\varepsilon (\breve{\delta},\ \breve{G}_2^i) = (\breve{\delta},\ \breve{Y}^i)$, where $\breve{Y}^i = \breve{G}_1^i \cup \breve{G}_2^i,\ i = 1,2,3, \ldots, n$, and truth and false membership of $(\breve{\delta},\ \breve{Y}^i)$ is defined in such a way that

$$T_{\breve{Y}^i}(\breve{\delta}) = min\left(\sum_{\omega=1}^{\alpha} sup\ T^{\omega}_{\breve{\eta}_1}(\breve{\delta}), \sum_{\omega=1}^{\alpha} sup\ T^{\omega}_{\breve{\eta}_2}(\breve{\delta})\right),$$

$$F_{\breve{Y}^i}(\breve{\delta}) = max\left(\sum_{\lambda=1}^{\beta} sup\, F^{\lambda}_{\breve{\eta}_1}(\breve{\delta}), \sum_{\lambda=1}^{\beta} sup\, F^{\lambda}_{\breve{\eta}_2}(\breve{\delta})\right).$$

Example 10. Consider the illustrative example, suppose that

$(\breve{\delta},\ \breve{G}_1) = \{ < \breve{B}_1, (0.13, 0.19),(0.24,0.1)>,$

$< \breve{B}_2, (0.2,0.25), (0.15,0.24) >,$
$< \breve{B}_3, (0.1,0.36), (0.34,0.12) > \},$

and

$(\breve{\delta}, \breve{G}_2) = \{< \breve{B}_3, (0.01,0.16), (0.5,0.2) >, < \breve{B}_4, (0.26,0.15), (0.12,0.2) > \},$

be two RIFS. Then the extended intersection of RIFS $(\breve{\delta},\ \breve{G}_1)$ and $(\breve{\delta},\ \breve{G}_2)$is given as

$(\breve{\delta},\ \breve{Y}) = \{ < \breve{B}_1, (0.13,0.19),(0.24,0.1)>, < \breve{B}_2, (0.2,0.25), (0.15,0.24) >,$
$< \breve{B}_3, (0.01,0.16), (0.5,0.2) >, < \breve{B}_4, (0.26,0.15), (0.12,0.2) > \}.$

Definition 11. Restricted union of two RIFS

The restricted union of two RIFS $(\breve{\delta},\ \breve{G}_1)$ and $(\breve{\delta},\ \breve{G}_2)$is denoted by $(\breve{\delta},\ \breve{G}_1) \cup_R (\breve{\delta},\ \breve{G}_2)$ and it isdefined as $(\breve{\delta},\ \breve{G}_1) \cup_R (\breve{\delta},\ \breve{G}_2) = (\breve{\delta},\ \breve{\Upsilon})$, where $\breve{\Upsilon} = \breve{G}_1 \cap_R \breve{G}_2$, and truth and falsemembership of $(\breve{\delta},\ \breve{\Upsilon})$ is defined in such a way that

$$T_{\breve{\Upsilon}}(\breve{\delta}) = max\left(\sum_{\omega=1}^{\alpha} sup\ T^{\omega}_{\breve{\eta}_1}(\breve{\delta}), \sum_{\omega=1}^{\alpha} sup\ T^{\omega}_{\breve{\eta}_2}(\breve{\delta})\right).$$

$$F_{\breve{\Upsilon}}(\breve{\delta}) = min\left(\sum_{\lambda=1}^{\beta} sup\ F^{\lambda}_{\breve{\eta}_1}(\breve{\delta}), \sum_{\lambda=1}^{\beta} sup\ F^{\lambda}_{\breve{\eta}_2}(\breve{\delta})\right).$$

Remark 6. The restricted union of two families of RIFS $(\breve{\delta},\ \breve{G}_1^i)$ and $(\breve{\delta},\ \breve{G}_2^i)$is denoted by

$(\breve{\delta},\ \breve{G}_1^i) \cup_R (\breve{\delta},\ \breve{G}_2^i)$ and it is defined as $(\breve{\delta},\ \breve{G}_1^i) \cup_R (\breve{\delta},\ \breve{G}_2^i) = (\breve{\delta},\ \breve{\Upsilon}^i)$, where $\breve{\Upsilon}^i = \breve{G}_1^i \cap_R \breve{G}_2^i,\ i = 1,2,3,\dots,n$, and truth and false membership of $(\breve{\delta},\ \breve{\Upsilon}^i)$ is defined in such a way that

$$T_{\breve{\Upsilon}^i}(\breve{\delta}) = max\left(\sum_{\omega=1}^{\alpha} sup\ T^{\omega}_{\breve{\eta}_1}(\breve{\delta}), \sum_{\omega=1}^{\alpha} sup\ T^{\omega}_{\breve{\eta}_2}(\breve{\delta})\right),$$

$$F_{\breve{\Upsilon}^i}(\breve{\delta}) = min\left(\sum_{\lambda=1}^{\beta} sup\ F^{\lambda}_{\breve{\eta}_1}(\breve{\delta}), \sum_{\lambda=1}^{\beta} sup\ F^{\lambda}_{\breve{\eta}_2}(\breve{\delta})\right).$$

Example 11. Consider the illustrative example, suppose that

$(\breve{\delta},\ \breve{G}_1) = \{ < \breve{B}_1, (0.13, 0.19), (0.24, 0.1) >,$

$< \breve{B}_2, (0.2,0.25), (0.15,0.24) >,$
$< \breve{B}_3, (0.1,0.36), (0.34,0.12) > \},$

and

$(\breve{\delta}, \breve{G}_2) = \{< \breve{B}_3, (0.01,0.16), (0.5,0.2) >, < \breve{B}_4, (0.26,0.15), (0.12,0.2) > \},$

be two RIFS. Then the restricted union of RIFS $(\breve{\delta},\ \breve{G}_1)$ and $(\breve{\delta},\ \breve{G}_2)$is given as

$(\breve{\delta}, \breve{\Upsilon}) = \{< \breve{B}_3, (0.1,0.36), (0.34,0.12) > \}.$

Definition 12. Restricted intersection of two RIFS

The restricted intersection of two RIFS $(\breve{\delta},\ \breve{G}_1)$ and $(\breve{\delta},\ \breve{G}_2)$is denoted by $(\breve{\delta},\ \breve{G}_1) \cap_R (\breve{\delta},\ \breve{G}_2)$ and it isdefined as $(\breve{\delta},\ \breve{G}_1) \cap_R (\breve{\delta},\ \breve{G}_2) = (\breve{\delta},\ \breve{\Upsilon})$, where $\breve{\Upsilon} = \breve{G}_1 \cap_R \breve{G}_2$, and truth and false membership of $(\breve{\delta},\ \breve{\Upsilon})$ is defined in such a way that

$$T_{\breve{Y}}(\breve{\delta}) = min\left(\sum_{\omega=1}^{\alpha} sup\ T_{\breve{\eta}_1}^{\omega}(\breve{\delta}), \sum_{\omega=1}^{\alpha} sup\ T_{\breve{\eta}_2}^{\omega}(\breve{\delta})\right),$$

$$F_{\breve{Y}}(\breve{\delta}) = max\left(\sum_{\lambda=1}^{\beta} sup\ F_{\breve{\eta}_1}^{\lambda}(\breve{\delta}), \sum_{\lambda=1}^{\beta} sup\ F_{\breve{\eta}_2}^{\lambda}(\breve{\delta})\right).$$

Remark 7. The restricted intersection of two families of RIFS $(\breve{\delta},\ \breve{G}_1^i)$ and $(\breve{\delta},\ \breve{G}_2^i)$is denoted by

$(\breve{\delta},\ \breve{G}_1^i) \cap_R (\breve{\delta},\ \breve{G}_2^i)$ and it isdefined as $(\breve{\delta},\ \breve{G}_1^i) \cap_R (\breve{\delta},\ \breve{G}_2^i) = (\breve{\delta},\ \breve{Y}^i)$, where$\breve{Y}^i = \breve{G}_1^i \cap_R \breve{G}_2^i,\ i = 1,2,3,\dots,n$, and truth and false membership of $(\breve{\delta},\ \breve{Y}^i)$ is defined in such a way that

$$T_{\breve{Y}^i}(\breve{\delta}) = min\left(\sum_{\omega=1}^{\alpha} sup\ T_{\breve{\eta}_1}^{\omega}(\breve{\delta}), \sum_{\omega=1}^{\alpha} sup\ T_{\breve{\eta}_2}^{\omega}(\breve{\delta})\right),$$

$$F_{\breve{Y}^i}(\breve{\delta}) = max\left(\sum_{\lambda=1}^{\beta} sup\ F_{\breve{\eta}_1}^{\lambda}(\breve{\delta}), \sum_{\lambda=1}^{\beta} sup\ F_{\breve{\eta}_2}^{\lambda}(\breve{\delta})\right).$$

Example 12. Consider the illustrative example, suppose that

$(\breve{\delta},\ \breve{G}_1) = \{ < \breve{B}_1,$ *(0.13, 0.19),(0.24,0.1)*$>$,

$< \breve{B}_2, (0.2,0.25), (0.15,0.24) >,$
$< \breve{B}_3, (0.1,0.36), (0.34,0.12) > \},$

and

$(\breve{\delta}, \breve{G}_2) = \{< \breve{B}_2, (0.32,0.38), (0.1,0.04) >, < \breve{B}_4, (0.26,0.15), (0.12,0.2) > \},$

be two RIFS. Then the restricted intersection of RIFS $(\breve{\delta},\ \breve{G}_1)$ and $(\breve{\delta},\ \breve{G}_2)$is given as

$(\breve{\delta}, \breve{Y}) = \{< \breve{B}_2, (0.2,0.25), (0.15,0.24) > \}.$

Definition 13. Restricted difference of two RIFS

The restricted difference of two RIFS $(\breve{\delta},\ \breve{G}_1)$ and $(\breve{\delta},\ \breve{G}_2)$is denoted by $(\breve{\delta},\ \breve{G}_1) -_R (\breve{\delta},\ \breve{G}_2)$ and it is defined as $(\breve{\delta},\ \breve{G}_1) -_R (\breve{\delta},\ \breve{G}_2) = (\breve{\delta},\ \breve{Y})$, where $\breve{Y} = \breve{G}_1 -_R \breve{G}_2$.

Example 13. Consider the illustrative example, suppose that

$(\breve{\delta},\ \breve{G}_1) = \{ < \breve{B}_1,$ *(0.13, 0.19),(0.24,0.1)*$>$,

$< \breve{B}_2, (0.2,0.25), (0.15,0.24) >,$
$< \breve{B}_3, (0.1,0.36), (0.34,0.12) >, < \breve{B}_4, (0.16,0.14), (0.23,0.37) > \},$

and

$(\breve{\delta},\ \breve{G}_2) = \{ < \breve{B}_1, (0.2,0.3),(0.3,\ 0.15) >, < \breve{B}_2, (0.32,0.38), (0.1,0.04) >,$

$< \breve{B}_3, (0.01,0.16), (0.5,0.2) > \},$

be two RIFS. Then the restricted difference of RIFS $(\breve{\delta},\ \breve{G}_1)$ and $(\breve{\delta},\ \breve{G}_2)$is given as

$(\breve{\delta}, \breve{Y}\ ) = \{< \breve{B}_4, (0.16,0.14), (0.23,0.37) > \}.$

5. Some Basic Laws of RIFS

In this section, we prove some basic fundamental laws including idempotent law, identity law, domination law, De-Morgan law and commutative law with the help of illustrative example.

5.1. Idempotent Law

$(\breve{\delta},\breve{G}) \cup (\breve{\delta},\breve{G}) = (\breve{\delta},\breve{G}) = (\breve{\delta},\breve{G}) \cup_R (\breve{\delta},\breve{G}).$

$(\breve{\delta},\breve{G}) \cap (\breve{\delta},\breve{G}) = (\breve{\delta},\breve{G}) = (\breve{\delta},\breve{G}) \cap_\varepsilon (\breve{\delta},\breve{G}).$

Example 14. To prove (1) law, we consider illustrative example. For this, suppose that

$(\breve{\delta},\ \breve{G}) = \{ < \breve{B}_1, (0.13,\ 0.19),(0.24,0.1) >,$

$< \breve{B}_2, (0.2,0.25), (0.15,0.24) >,$
$< \breve{B}_3, (0.1,0.36), (0.34,0.12) >, < \breve{B}_4, (0.16,0.14), (0.23,0.37) > \}.$

One can observe

$(\breve{\delta},\ \breve{G}) \cup (\breve{\delta},\breve{G}) = \{ < \breve{B}_1, (0.13,\ 0.19),(0.24,0.1) >,$

$< \breve{B}_2, (0.2,0.25), (0.15,0.24) >,$
$< \breve{B}_3, (0.1,0.36), (0.34,0.12) >, < \breve{B}_4, (0.16,0.14), (0.23,0.37) > \}$

$= (\breve{\delta},\breve{G}) = (\breve{\delta},\breve{G}) \cup_R (\breve{\delta},\breve{G}).$

Similarly, we can prove (2).

5.2. Identity Law

$(\breve{\delta},\breve{G}) \cup \breve{\emptyset} = (\breve{\delta},\breve{G}) = (\breve{\delta},\breve{G}) \cup_R \breve{\emptyset}.$

$(\breve{\delta},\breve{G}) \cap \breve{U} = (\breve{\delta},\breve{G}) = (\breve{\delta},\breve{G}) \cap_\varepsilon \breve{U}.$

Example 15. To prove (1) law, we consider illustrative example. For this, suppose that

$(\breve{\delta},\ \breve{G}) = \{ < \breve{B}_1, (0.13,\ 0.19),(0.24,0.1) >,$

$< \breve{B}_2, (0.2,0.25), (0.15,0.24) >$,
$< \breve{B}_3, (0.1,0.36), (0.34,0.12) >, < \breve{B}_4, (0.16,0.14), (0.23,0.37) > \}$.

One can observe

$(\breve{\delta},\ \breve{G}) \cup \breve{\emptyset} = \{ < \breve{B}_1,$ *(0.13, 0.19),(0.24,0.1)*>,

$< \breve{B}_2, (0.2,0.25), (0.15,0.24) >$,
$< \breve{B}_3, (0.1,0.36), (0.34,0.12) >, < \breve{B}_4, (0.16,0.14), (0.23,0.37) > \}$

$= (\breve{\delta}, \breve{G}) = (\breve{\delta}, \breve{G}) \cup_R \breve{\emptyset}$.

Similarly, we can prove (2).

5.3. Domination Law

$(\breve{\delta}, \breve{G}) \cup \breve{U} = \breve{U} = (\breve{\delta}, \breve{G}) \cup_R \breve{U}$.

$(\breve{\delta}, \breve{G}) \cap \breve{\emptyset} = \breve{\emptyset} = (\breve{\delta}, \breve{G}) \cap_\varepsilon \breve{\emptyset}$.

Example 16. To prove (1) law, we consider illustrative example. For this, suppose that

$(\breve{\delta},\ \breve{G}) = \{ < \breve{B}_1,$ *(0.13, 0.19), (0.24,0.1)*>,

$< \breve{B}_2, (0.2,0.25), (0.15,0.24) >$,
$< \breve{B}_3, (0.1,0.36), (0.34,0.12) >, < \breve{B}_4, (0.16,0.14), (0.23,0.37) > \}$.

One can observe

$(\breve{\delta},\ \breve{G}) \cup \breve{U} = \{ < \breve{B}_1,$ *(0.13, 0.19),(0.24,0.1)*>,

$< \breve{B}_2, (0.2,0.25), (0.15,0.24) >$,
$< \breve{B}_3, (0.1,0.36), (0.34,0.12) >, < \breve{B}_4, (0.16,0.14), (0.23,0.37) > \} \cup \breve{U}$

$= \breve{U} = (\breve{\delta}, \breve{G}) \cup_R \breve{U}$.

Similarly, we can prove (2).

5.4. De-Morgan Law

$\left((\breve{\delta}, \breve{G}_1) \cup (\breve{\delta}, \breve{G}_2)\right)^c = (\breve{\delta}, \breve{G}_1)^c \cap_\varepsilon (\breve{\delta}, \breve{G}_2)^c$.

$\left((\breve{\delta}, \breve{G}_1) \cap_\varepsilon (\breve{\delta}, \breve{G}_2)\right)^c = (\breve{\delta}, \breve{G}_1)^c \cup (\breve{\delta}, \breve{G}_2)^c$.

Example 17. To prove (1) law, we consider illustrative example. For this, suppose that L.H.S is

$\left(\breve{\delta},\breve{G}_1\right) \cup \left(\breve{\delta},\breve{G}_2\right) = \{<\breve{B}_1, (0.2, 0.3),(0.24,0.1)>,$

$<\breve{B}_2,(0.32,0.38),(0.1,0.04)>,$
$<\breve{B}_3,(0.1,0.36),(0.34,0.12)>, <\breve{B}_4,(0.26,0.15),(0.12,0.2)>\}.$

Then

$\left(\left(\breve{\delta},\breve{G}_1\right) \cup \left(\breve{\delta},\breve{G}_2\right)\right)^c = \{<\breve{B}_1, (0.24,0.1), (0.2,0.3)>,$

$<\breve{B}_2,(0.1,0.04),(0.32,0.38)>,$
$<\breve{B}_3,(0.34,0.12),(0.1,0.36)>, <\breve{B}_4,(0.12,0.2),(0.26,0.15)>\}.$

Now consider R.H.S.

$\left(\breve{\delta},\breve{G}_1\right)^c \cap_\varepsilon \left(\breve{\delta},\breve{G}_2\right)^c = \{<\breve{B}_1, (0.24,0.1), (0.2,0.3)>,$

$<\breve{B}_2,(0.1,0.04),(0.32,0.38)>,$
$<\breve{B}_3,(0.34,0.12),(0.1,0.36)>, <\breve{B}_4,(0.12,0.2),(0.26,0.15)>\}.$

From this, it is clear that L.H.S.=R.H.S. Similarly, we can prove (2).

5.5. Commutative Law

$\left(\breve{\delta},\breve{G}_1\right) \cup \left(\breve{\delta},\breve{G}_2\right) = \left(\breve{\delta},\breve{G}_2\right) \cup \left(\breve{\delta},\breve{G}_1\right).$

$\left(\breve{\delta},\breve{G}_1\right) \cup_R \left(\breve{\delta},\breve{G}_2\right) = \left(\breve{\delta},\breve{G}_2\right) \cup_R \left(\breve{\delta},\breve{G}_1\right).$

$\left(\breve{\delta},\breve{G}_1\right) \cap \left(\breve{\delta},\breve{G}_2\right) = \left(\breve{\delta},\breve{G}_2\right) \cap \left(\breve{\delta},\breve{G}_1\right).$

$\left(\breve{\delta},\breve{G}_1\right) \cap_\varepsilon \left(\breve{\delta},\breve{G}_2\right) = \left(\breve{\delta},\breve{G}_2\right) \cap_\varepsilon \left(\breve{\delta},\breve{G}_1\right).$

Example 18. To prove (1) law, we consider illustrative example. For this, suppose that

L.H.S:

$\left(\breve{\delta},\breve{G}_1\right) \cup \left(\breve{\delta},\breve{G}_2\right) = \{<\breve{B}_1, (0.2, 0.3),(0.24,0.1)>,$

$<\breve{B}_2,(0.32,0.38),(0.1,0.04)>,$
$<\breve{B}_3,(0.1,0.36),(0.34,0.12)>, <\breve{B}_4,(0.26,0.15),(0.12,0.2)>\}.$

R.H.S:

$\left(\breve{\delta},\breve{G}_2\right) \cup \left(\breve{\delta},\breve{G}_1\right) = \{<\breve{B}_1, (0.2, 0.3), (0.24,0.1)>,$

$<\breve{B}_2,(0.32,0.38),(0.1,0.04)>,$
$<\breve{B}_3,(0.1,0.36),(0.34,0.12)>, <\breve{B}_4,(0.26,0.15),(0.12,0.2)>\}.$

From above equation, we meet the required result. Similarly, we can prove the remaining.

6. Conclusion

In this article, the basic fundamentals of refined intuitionistic fuzzy Set (RIFS) i.e. RIF subset, Equal RIFS, Complement of RIFS, Null RIFS and aggregation operators i.e. union, intersection, restricted intersection, extended union, extended intersection and restricted difference of two RIFS is defined. All these fundamentals are explained using an illustrative example. Further extension can be sought through developing similarity measures for comparison purposes.

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