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Supplier Selection Using Fuzzy AHP Method and D-Numbers

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Success in supply begins with the right choice of suppliers and in the long run is directly related to how suppliers are managed, because suppliers have a significant impact on the success or failure of a company. Multi-criteria decisions are approaches that deal with ranking and selecting one or more suppliers from a set of suppliers. Multi-criteria decisions provide an effective framework for comparing suppliers based on the evaluation of different criteria. The present research is applied based on the purpose and descriptive-survey based on the nature and method of the research. In the present study, two library and field methods have been used to collect information. According to the objectives of this study, suppliers will be evaluated using two methods of fuzzy hierarchical analysis with D-numbers. In order to better understand these two methods, a case study is presented in which suppliers are ranked using two methods and then the results are compared with each other. For manufacturing companies, 4 categories of parts were considered and based on the classification, the suppliers of the manufacturing company were evaluated and analyzed. In the results of suppliers of type A and B components in hierarchical analysis, D and fuzzy methods have many differences in the evaluation and ranking of suppliers, and this shows the lack of expectations of experts in D and fuzzy analysis. On the other hand, in type C and D components, the classification and ranking of suppliers have been matched in two ways and shows that the opinions in the evaluation of these suppliers are the same.

1. Introduction

In recent years, much attention has been paid to the importance of selecting suppliers and supply chain management to allocate orders. Thus, in this regard, it focuses on identifying the key factors affecting the optimal selection of suppliers in the supply chain in industries [1]. Also, with the acceleration of the process of globalization and the increasing facilitation of communication, the manager's perception of the environment becomes more complex, uncertain and ambiguous [2]. Existence of numerous and unstable information and variables affecting the consequences of the decision, challenges the manager to make the right and fast decision. Although human beings have always faced the challenge of decision making, it is no exaggeration to say that the subject of decision making has never been so complicated [3]. Therefore, along with the growth of human knowledge, various thinkers have addressed the issue of decisions and methods that can make this process easier and safer. One of the most important multi-criteria decisions that has attracted the attention of researchers in the organization is the choice of

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supplier in the supply chain of the organization [4]. This is due to the fact that in the current competitive environment, the process of effective selection of suppliers is very important in the success of any production organization [5]. In fact, success in procurement begins with the right choice of suppliers and in the long run depends directly on how suppliers are managed, because suppliers have a significant impact on the success or failure of a company [6]. Choosing the right supplier requires consideration of several criteria. Many decision makers or experts choose suppliers based on their own experiences and tastes, which are purely subjective and personal. Multi-criteria decisions are approaches that deal with ranking and selecting one or more suppliers from a set of suppliers. Multi-criteria decisions provide an effective framework for comparing suppliers based on the evaluation of different criteria [7]. At present, in order to solve the problem of evaluating supplier performance according to one criterion or determining the importance of a number of criteria with high accuracy, multi-criteria decision-making vocabulary is used by both researchers and experts [8]. On the other hand, multiple criteria decision making technique and besides multiple objective decision making can consider several goals in order of the decision maker's priority. In multi-objective planning, the decision maker has the ability to formulate conflicting goals in the form of a linear equation under the objective function and on the other hand to formulate real constraints such as purchase budget, capacity, etc. under the constraints of suppliers. Solving this model can determine the amount of materials received from each supplier in a way that provides the maximum amount of optimization and also covers the amount of aspirations for each goal [9]. The combination of these two techniques can create a model that takes into account different ideals while considering different criteria. For more than two decades, supply chain management and the supplier selection process have received considerable attention in the literature. Many factories and industry owners have been looking for ways to partner with suppliers to increase their management performance and competitiveness on the global stage. The quality of the supplier base affects the competitiveness of companies. The continuity of the relationship between suppliers and industry owners causes the company's supply chain to be a serious and strong obstacle in the way of competitors. Also, establishing a long-term relationship with the supplier will reduce the costs of the supplier and reduce the costs of the supplier will lead to a reduction in the costs of the organization (employer) (mutual benefit). On the other hand, a stable relationship causes the supplier to follow the rules and standards of the employer and the organization uses the facilities available to the suppliers such as engineering technical facilities (benefit to the organization). Therefore, the decision to select the best supplier for supply chain management is essential [10]. One of the most important issues in designing a supply chain is the issue of supplier selection. The complexity of this issue is in fact because each of the suppliers meets part of the buyer criteria, and the choice between them is in fact a Multiple Criteria Decision Making (MCDM) that requires a structured and systematic approach, and without it an important decision is likely to fail. With the help of computers, decision-making techniques have become very acceptable in all areas of the decision-making process. Therefore, the application of multi-criteria decision making methods for users, due to the mathematical complexity, has become very easy to implement. Decision making is the process of finding the best option from a range of available options. In fact, choosing the right set of suppliers to work with is crucial to a company's success, and the emphasis on supplier selection has been emphasized for many years [11]. There are different techniques and methods for making multiple fuzzy criteria that have different advantages and disadvantages over each other. A supply chain is a series of organizations involved in the production and delivery of a product or service. This chain starts with raw material suppliers and continues to the end customer. Supply chain management is one of the effective and efficient approaches that reduces production costs and waiting time. This attitude facilitates the provision of better customer service and ensures the opportunity for effective monitoring of transportation systems, inventory and distribution networks. In this way, the organization can exceed the expectations and demands of customers. Today, organizations are facing customers who want high product diversity, low costs, high quality and fast response. Organizations are well aware that they need an efficient supply chain to be able to compete in today's global marketplace and interconnected network economy [12]. Many experienced companies believe that choosing a supplier is the most important activity of an organization. Also, since the performance of suppliers has a major impact on the success or failure of a chain, selecting a supplier is now considered a strategic task. As a result, wrong decisions in choosing suppliers will have many negative consequences and losses for the company. Therefore, finding the right methods to select the right suppliers, which are the most important components of the supply chain, is very important. On the other hand, because raw materials and parts are the most important part of a company's costs, proper purchasing management is of considerable importance to the efficiency, effectiveness and profitability of an organization. On the other hand, today, due to new concepts of supply chain management and similar cases that lead to partnerships with suppliers and the company's close relationship with suppliers, suppliers and customers are no longer recognized as competitors of the organization. Rather, they are members of a core set called the supply chain, each of which aims to maximize profits and increase the productivity of the entire chain. Nosrati and Jafari Eskandari [13] in their research to design a supply chain network considering sustainability. The supply chain network model is considered to be uncertain and includes uncertain parameters (demand, shipping costs, and operating activity) that exist, which is a pessimistic possibility to control the model through robust optimization method. Therefore, by considering the conflicting goals of the supply chain network, including minimizing the total network costs and minimizing the amount of greenhouse gas emissions, the community-based multi-objective decision-making methods and refrigeration simulation algorithm have been used to solve the model. The results of T-Test statistical test on the means of the first, second, and computational objective functions show that there is a significant difference between the means of computational time. Sensitivity analysis performed on some parameters of the model also shows that reducing network costs and reducing greenhouse gas emissions increases the supply capacity and reduces the discount period for the purchase of raw materials.

Qasbeh [14] in his research stated that the key to success in the competition scene is to focus more on the main activities and goals of the organization. Since the 1980s, many managers of large organizations have decided to outsource activities that are not of strategic importance to the organization.

In their research, Shafia et al. [15] presented a new framework for evaluating suppliers by considering risk factors using decision-making techniques and two-level data envelopment analysis approach. In the first step, the criteria of the hierarchical analysis process were weighted with the opinion of experts and then used the data envelopment analysis approach to evaluate.

Mardani [16] stated in their research that frequent discussions related to supply chain sustainability events show that companies with a global presence are trying to improve the environmental, social and economic outcomes of global supply chains. They proposed sustainable supply chain management to improve the results of sustainability in supply chains.

Ghadimi [17] stated in his research that in the last two decades, the issue of sustainable supply chain has attracted the attention of many academics and professionals. In this regard, resources, maintenance and recycling, as well as their pairs (i.e., resources and maintenance, maintenance and recycling) have provided a platform for the exchange of technical, economic, institutional and policy aspects to help move societies towards sustainability.

Rifaki [18] stated in his research that the supply chain plays an important role in today's global economy. Therefore, in order to closely pursue sustainable business, a dynamic understanding of the issues affecting sustainability in supply chains must be formed. However, this field of research is still unknown due to limited theoretical knowledge and practical application.

2. Research Method

The approach of the present research is quantitative and qualitative according to the intended objectives. Therefore, the present research is of an applied type. Also, the present research is a field research in terms of implementation. Because in this research, the relationships between variables are expressed in the form of decision model, using fuzzy techniques and D numbers and variables are observed, measured and described, so the type of research method is descriptive-analytical. According to the objectives of this study, supplier evaluation will be evaluated using two methods of fuzzy hierarchical analysis and D-numbers. First, using the common and widely used method of multi-criteria decision making, namely the Analytical Hierarchy Process (AHP) using mathematics based on fuzzy sets, a method has been proposed to select the suppliers of a supply chain. Then this problem is evaluated again by combining the two methods of AHP and D numbers. Finally, in order to achieve the desired results, the results of these two methods will be compared with each other. In order to better understand these two methods, a case study is presented in which suppliers are ranked using two methods and then the results are compared with each other.

2.1. Basic Concepts in Dempster-Scheffer Theory or Belief Function

The detection framework in Dempster-Scheffer theory is a set of two by two separated elements or propositions, and if the set $x=\{x_1, x_2,\dots,x_n\}$is a set of elements or propositions, the sample space or detection framework is displayed as $\Omega = 2^x$. This set is a set of all sub-sets of *X* as follows:
$\Omega = \{\{X_1\}, \{X_2\},\dots,\{X_n\},\{X_1, X_2\},\dots, \{X_1, X_2,\dots,X_n\}\}$.
If A1 = {X1}, A2 = {X2},... are sets belonging to the detection frame, the probability mass function or the detection function of the set Ai on the detection frame is displayed as m (Ai), which has the following conditions:

$$\mathrm{m(Ai)} \geq 0 \;, \qquad \mathrm{Ai} \in \Omega$$

$$\mathrm{m}(\emptyset) = 0$$

$$\sum_{A\in 2^x} \mathrm{m(Ai)} = 1\,.$$

The most accurate belief that can be obtained from the correctness or occurrence of set A from the framework of diagnosis and based on the available evidence is called belief function. This function is the sum of the mass of probabilities determined for the elements in set A and is calculated as follows:

$$\mathrm{bel(A)} = \sum_{\mathrm{A_i}} \mathrm{m(b)}.$$

Contrary to the probability theory, *bel* (A) = 0 means lack of evidence about set A; While p (A) = 0 means the impossibility of this set, while bel (A) = 1 means the certainty of the occurrence of event A and it is similar to the probability p (A) = 1, which means the certainty of the correctness of the set A.
The maximum possible belief for the correctness of set A, which is determined on the basis of evidence, is called the possibility function. This function is the sum of the total probability masses of the existing elements of the detection framework with zero inter section by set A. It is defined as follows:

$$\mathrm{pl(A)} = \sum_{\mathrm{A_i}} \mathrm{m(b)}.$$

The probability value of set A can be defined as the complement of not being belief of A, or in other words, lack of evidence showing A is true:

$$\mathrm{pl(A)} = 1 - \mathrm{bel(\sim A)}.$$

$pl(\mathrm{A}) = 0$ means that the set A is impossible or similarly $p(\mathrm{A}) = 0$. Also $pl(\mathrm{A}) = 0$is equal to $bel(\sim A) = 1$. This means that if event A is impossible based on the evidence, then A is certainly not true. The degree of uncertainty or degree of doubt in determining the magnitude of belief and possibility based on available evidence is the distance between belief in the occurrence or correctness of set A and unbelief in the occurrence or inaccuracy of set A in the context of diagnosis and is defined as follows:

$$\mathrm{U} = 1 - \mathrm{bel(A)} - \mathrm{bel(\sim A)}.$$

Suppose A∈Ω, set A is defined according to the above definitions and using the belief sizes $bel(A), U(A)$ and $bel(\sim A)$ as follows:

$$\mathrm{s} = \{(\mathrm{bel(A)}, \mathrm{u(A)}, \mathrm{bel(\sim A)}/\mathrm{A} \in \Omega\}.$$

So that for each set A of the detection framework, and $bel(\sim A)$∈ [0,1] and U (A) and $bel(A)$and their sum for A∈Ω is as follows:

$$0 \leq \mathrm{bel(A)} + \mathrm{u(A)} + \mathrm{bel(\sim A)} \leq 1.$$

Hence, according to Dempster-Schaffer theory, the generated D numbers will be as follows:
For the discrete set Ω= {b1, b2, …, bi, …, bn} such that bi ϵ R and bi ≠ bj if i ≠ j, a special form of numbers is expressed as follows:

D({b1}) = v1,
D({b2}) = v2,
D({bi}) = vi,
D({bn}) = vn.

Or more simply D = {(b1, v1), (b2, v2),…, (bi, vi),… (bn, vn)} such that vi> 0 and if two numbers D, D1 and D2 exist, they will be as follows:

$$D_1 = \left\{\left(b_1^1, v_1^1\right), \ldots, \left(b_i^1, v_i^1\right), \ldots, \left(b_n^1, v_n^1\right)\right\},$$

$$D_2 = \left\{\left(b_1^2, v_1^2\right), \ldots, \left(b_j^2, v_j^2\right), \ldots, \left(b_m^2, v_m^2\right)\right\}.$$

The combination of D1 and D2 is shown and calculated as follows:

$$b(b)=v,$$
$$b=\frac{b_i^1+b_j^2}{2},$$
$$v=\frac{V_i^1+V_j^2}{2}/C,$$
$$\begin{cases}\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\frac{V_i^1+V_j^2}{2}\right), & \sum_{i=1}^{n}V_i^1=1 \quad \text{and} \sum_{j=1}^{m}V_j^2=1;\\ \sum_{j=1}^{m}\sum_{i=1}^{n}\left(\frac{V_i^1+V_j^2}{2}\right)+\sum_{j=1}^{m}\left(\frac{V_c^1+V_j^2}{2}\right), & \sum_{i=1}^{n}V_i^1< \quad \text{and} \sum_{j=1}^{m}V_j^2=1;\\ \sum_{j=1}^{m}\sum_{i=1}^{n}\left(\frac{V_i^1+V_j^2}{2}\right)+\sum_{i=1}^{n}\left(\frac{V_i^1+V_c^2}{2}\right), & \sum_{i=1}^{n}V_i^1< \quad \text{and} \sum_{j=1}^{m}V_j^2=1;\\ \sum_{j=1}^{m}\sum_{i=1}^{n}\left(\frac{V_i^1+V_j^2}{2}\right)+\sum_{j=1}^{m}\left(\frac{V_c^1+V_j^2}{2}\right) & \\ +\sum_{i=1}^{n}\left(\frac{V_i^1+V_j^2}{2}\right)+\frac{V_c^1+V_c^2}{2} & \sum_{i=1}^{n}V_i^1< \quad \text{and} \sum_{j=1}^{m}V_j^2<1;\end{cases}$$

such that $V_c^1=1-\sum_{i=1}^{n}V_i^1,\ V_c^2=1-\sum_{i=1}^{m}V_i^2$.

It should be noted that hybrid operations do not maintain corporate property, so D numbers can be combined correctly and efficiently:

$$(D1\oplus D2)\oplus D3\neq(D1\oplus D3)\oplus D2\neq(D2\oplus D3)\oplus D1.$$

If D = {(b1, v1), (b2, v2),…, (bi, vi),… (bn, vn)} is a D number, the consensus operator D is defined as follows:

$$I(D)=\sum_{i=1}^{n}b_i v_i.$$

3. Findings

3.1. Evaluation of Suppliers Based on AHP Method with Theory D

To evaluate suppliers based on approach D in AHP method, we perform the following steps: In this section, 8 expert opinions will be evaluated and analyzed based on three criteria of cost, time and quality, and based on the collected opinions; first the opinions will be evaluated. We will scale and then formulate a decision matrix in which experts present their views to each supplier at a brainstorming session. According to the evaluation of suppliers for the classification of parts, this section evaluates and weighs the indicators based on the average opinions of experts, which is the final weight from the experts' point of view (*Table 1* values are calculated based on the percentage of importance).

Table 1. *Weight of criteria from the perspective of experts.*

	C1	C2	C3		C1	C2	C3
Expert 1	*0.0778*	*0.4868*	*0.4355*	**Expert 5**	*0.0993*	*0.5109*	*0.3897*
Expert 2	*0.1694*	*0.4431*	*0.3875*	**Expert 6**	*0.2411*	*0.2101*	*0.5488*
Expert 3	*0.3278*	*0.2611*	*0.4111*	**Expert 7**	*0.1186*	*0.6123*	*0.2691*
Expert 4	*0.1146*	*0.4798*	*0.4057*	**Expert 8**	*0.4002*	*0.233*	*0.3668*

Hence the display of D numbers for A1 is as shown in *Table 2*.

According to *Table 2*, evaluations are performed for the other 25 suppliers. According to the evaluation, in the next step, the combination of D numbers will be done. Therefore, based on the following relation, the numbers will be combined as *Table 3*.

$$DA1 = D11 + D12 + D13 + D14 + D15 + D16 + D17 + D18.$$

Table 2. *Display of D numbers.*

A1	D numbers
Expert 1	*D11= [(0.56,0.4355), (0.66,0.4868), (0.28,0.0778)]*
Expert 2	*D12= [(0.25,0.3875), (0.4,0.4431), (0.2,0.1694)]*
Expert 3	*D13= [(0.09,0.4111), (0.71,0.2611), (0.41,0.3278)]*
Expert 4	*D14= [(0.46,0.4057), (0.61,0.4798), (0.24,0.1146)]*
Expert 5	*D15= [(0.33,0.3897), (0.65,0.5109), (0.43,0.0993)]*
Expert 6	*D16= [(0.34,0.5488), (0.45,0.2110), (0.46,0.2411)]*
Expert 7	*D17= [(0.08,0.2691), (0.72,0.6123), (0.25,0.1186)]*
Expert 8	*D18= [(0.43,0.3668), (0.82,0.2330), (0.45,0.4002)]*

According to the accepted evaluation, the suppliers' ranking for category A parts is as shown in *Table 3*.

Table 3. *Ranking of suppliers of category A parts.*

Suppliers	A1	A2	A3	A4	A5	A6
I (D)	*0.3869*	*0.2886*	*0.3420*	*0.2024*	*0.2032*	*0.3640*
ranking	*4*	*8*	*7*	*12*	*11*	*5*
Suppliers	**A7**	**A8**	**A9**	**A10**	**A11**	**A12**
I (D)	*0.2483*	*0.3981*	*0.4378*	*0.3908*	*0.2716*	*0.3616*
ranking	*10*	*2*	*1*	*3*	*9*	*6*

As can be seen, supplier A9 with a weight of 0.4378 was in the first place and supplier A8 with a weight of 0.3981 were in the second place. Also, the ranking of suppliers of category B parts is as shown in *Table 4*.

Table 4. *Ranking of suppliers of parts category B.*

Suppliers	A13	A14	A15	A16	A17	A18
I (D)	*0.3526*	*0.3077*	*0.3377*	*0.2326*	*0.3826*	*0.3829*
ranking	*3*	*5*	*4*	*6*	*1*	*2*

According to the evaluation performed on the category B parts, supplier A17 with a weight of 0.3826 was in the first place and supplier a18 with a weight of 0.3829 were in the second place. The rating of suppliers of category C parts is as shown in *Table 5*.

Table 5. *Ranking of suppliers of parts category C.*

Suppliers	A19	A20	A21	A22
I (D)	*0.4916*	*0.4892*	*0.3038*	*0.3731*
ranking	*1*	*2*	*4*	*3*

For category C, suppliers A19 with a weight of 0.4916 were ranked first and A20 with a weight of 0.4892 were ranked second. The ranking of suppliers of D parts is as shown in *Table 6*.

Also, suppliers were classified for type D components and all suppliers were evaluated and analyzed by D numbers in the hierarchical analysis method. In the next step, suppliers were ranked using the fuzzy AHP method approach.

Table 6. *Ranking of suppliers of parts category D.*

Suppliers	A23	A24	A25
I (D)	*0.5851*	*0.5400*	*0.4441*
ranking	*1*	*2*	*3*

3.2. Evaluation and Ranking of Suppliers Based on F-AHP Method

In this section, 25 suppliers identified for 4 types of parts required for supply in manufacturing companies will be evaluated and analyzed based on the fuzzy AHP method, which are in three steps as shown in *Figs. (1)-(3)*.

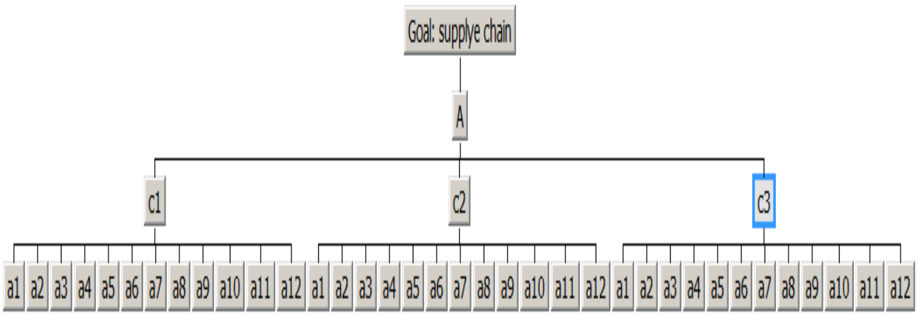

Fig. 1. *Step1: cluster the levels in expert choice software.*

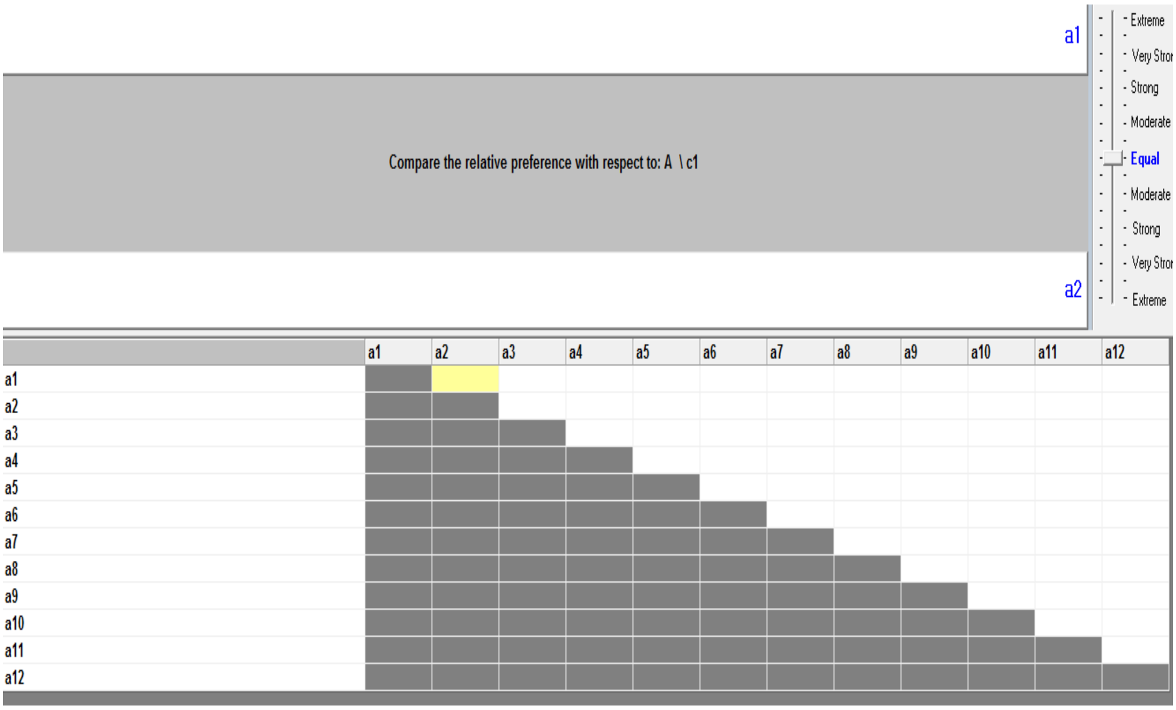

Fig. 2. *Step 2: matrix of pairwise comparison of indicators based on the mode of expert opinions.*

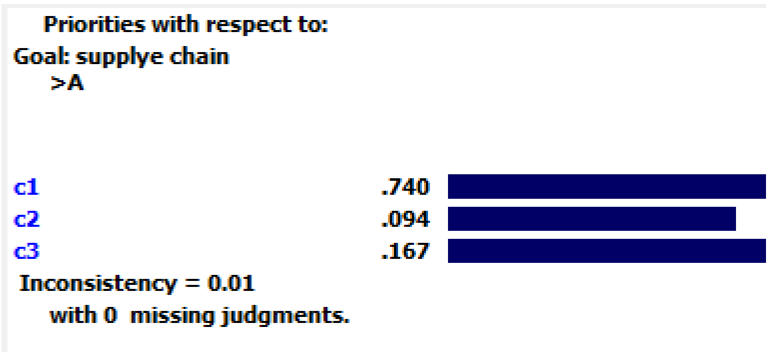

Fig. 3. *Step 3: obtain the weight of the indicators.*

As can be seen, the cost index with a weight of 0.740 was in the first place and the delivery time index with a weight of 0.167 was in the second place and the quality index with a weight of 0.094 were in the third place.

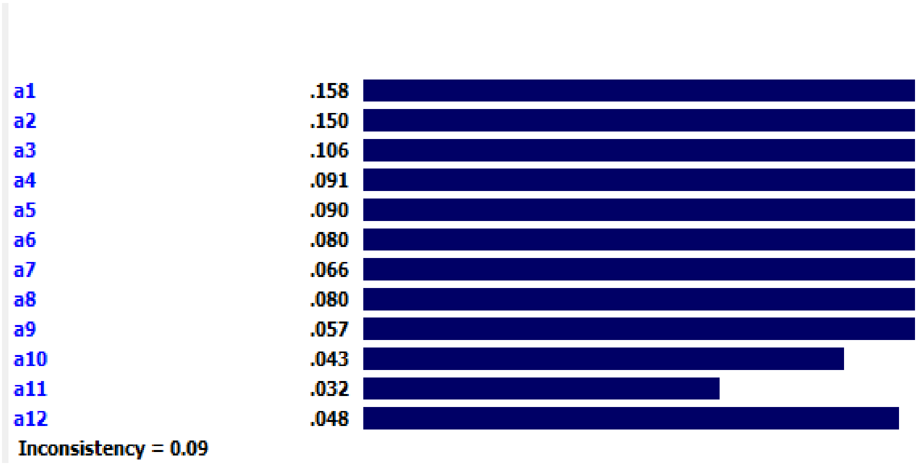

Fig. 4. *Supplier ratings for category A components.*

According to *Fig. 4*, Supplier A1 with a weight of 0.158 was in the first place and supplier A2 with a weight of 0.150 was in the third place and A3 with a weight of 0.106 was in the third place. Also, the sensitivity analysis of indicators and suppliers is as shown in *Fig. 5*.

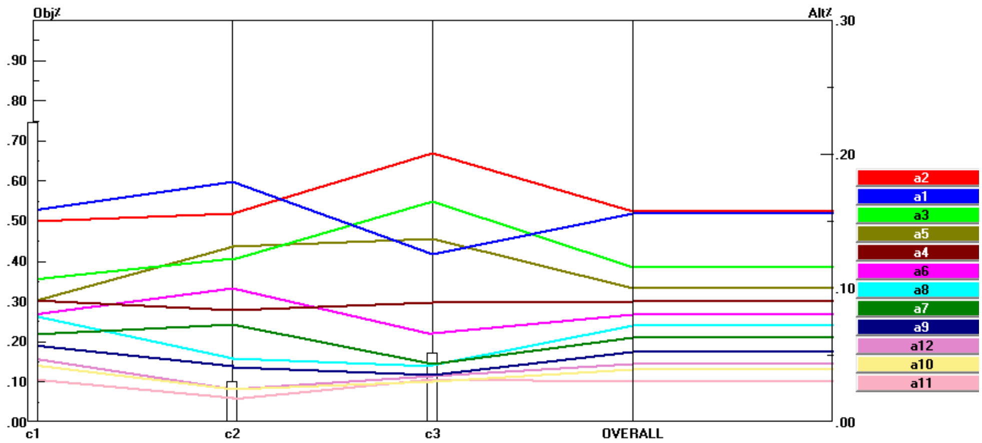

Fig. 5. *The sensitivity analysis of indicators and suppliers for category A.*

Supplier ratings for Type B components are shown in *Fig. 6*.

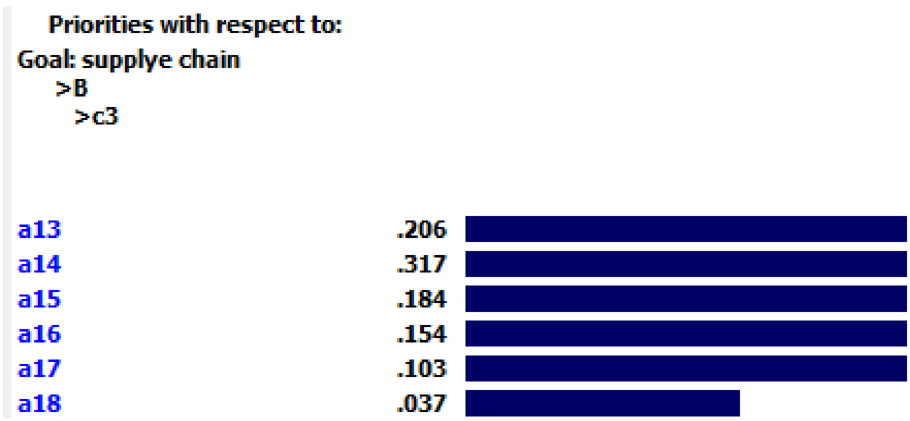

Fig. 6. *Supplier ratings for category B components.*

As can be seen, supplier 14 with a weight of 0.317 was in the first place and supplier A13 with a weight of 0.206 were in the second place. The sensitivity analysis of the assessment is as shown in *Fig. 7*.

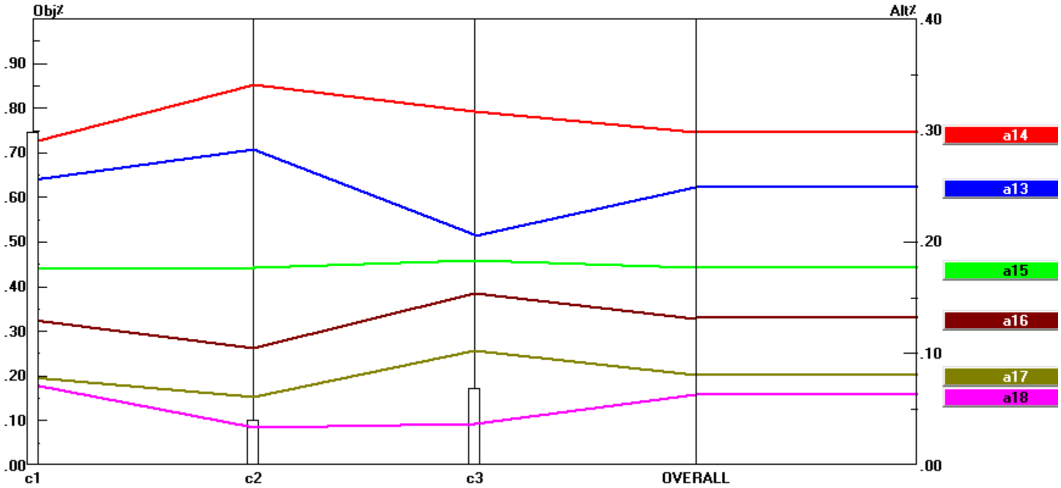

Fig. 7. *The sensitivity analysis of indicators and suppliers for category B.*

The evaluation for category C parts is shown in *Fig. 8*.

Fig. 8. *The evaluation for category C parts.*

According to the evaluation, supplier A19 with a weight of 0.473 was in the first place and A20 with a weight of 0.332 were in the second place. The sensitivity analysis indicators and suppliers for category C is as shown in *Fig. 9*.

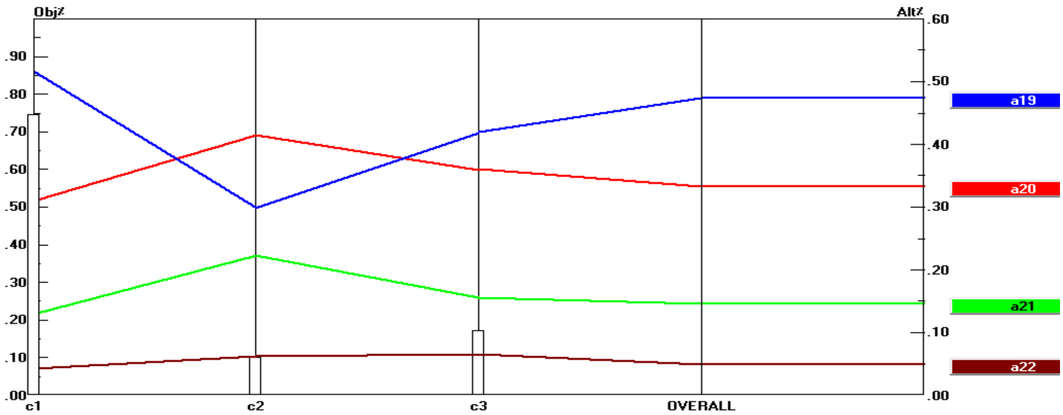

Fig. 9. *The sensitivity analysis of indicators and suppliers for category C.*

Then, for the category of type D parts, the evaluation is as *Fig. 10*.

Fig. 10. *The evaluation for category D parts.*

We also have a sensitivity analysis performed for category D which is shown in *Fig. 11*.

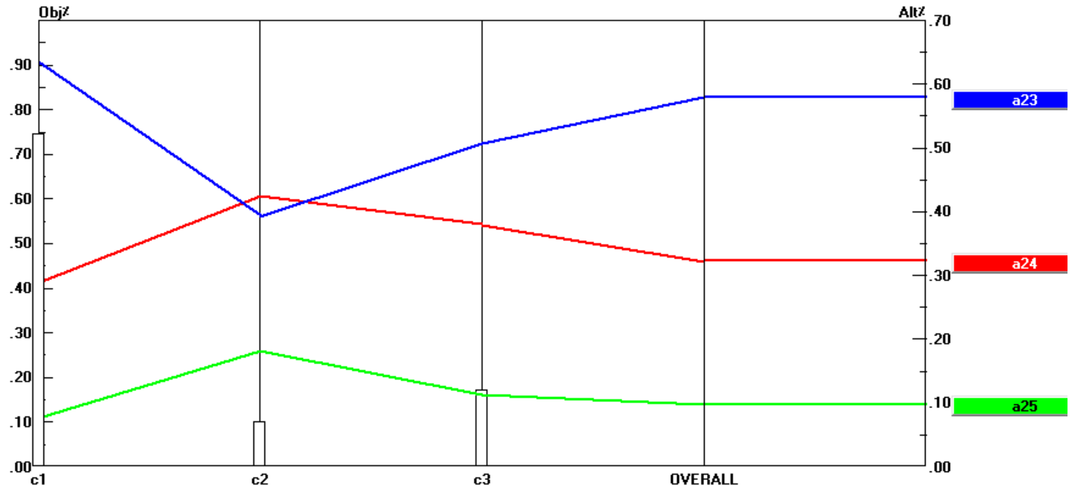

Fig. 11. *The sensitivity analysis of indicators and suppliers for category D.*

As shown in *Fig. 11*, suppliers A23 with a weight of 0.579 were in first place and A24 with a weight of 0.322 were in third place.

4. Conclusion

Today, the demands of customers along with the advancement of technology, are widely and constantly changing. This has caused the life cycle of products to be shorter and business organizations must launch a variety of products with desirable features to attract customer attention and satisfaction [19]. For this reason, in order to stay competitive, most organizations consider outsourcing the product parts to suppliers who have the technology and special ability in that field in their management, and design and produce the main parts themselves. They pay. This solution requires effective communication with suppliers and has made the issue of selecting and evaluating suppliers an important principle in the supply chain [20]. In evaluating suppliers, the most important criteria that have the greatest impact on this process must first be identified. In previous studies, criteria and indicators such as price, quality, and delivery time have been considered important in evaluating and selecting suppliers [21].Wang [22] concluded from customer research that price and quality, delivery time, and performance history are important factors. Therefore, based on two models of hierarchical analysis with D and fuzzy numbers in the evaluation of the supply chain of the manufacturing company was discussed. Therefore, 4 categories of parts were considered for manufacturing companies and based on the classification; the suppliers of the manufacturing company were evaluated and analyzed. In the results obtained from suppliers of type A and B components in the hierarchical analysis of D and fuzzy methods, there are many differences in the evaluation and ranking of suppliers, and this shows the lack of expectations of experts in D and fuzzy analysis. On the other hand, in type C and D components, the classification and ranking of suppliers have been matched in two ways and it has been shown that the opinions in the evaluation of these suppliers are the same. Like any other research, conducting this research was faced with many obstacles and problems, some of which were eliminated and some others changed the direction of the research or limited the application of the results. These restrictions include:

- Some of the contracts between the manufacturing company and the suppliers of raw materials are related to previous years, which make the price and other influential factors of these suppliers different from other suppliers that have signed a contract this year and makes it influential in choosing suppliers.
- Due to price fluctuations and market demand, it is possible to change the company's production volume. Therefore, what is considered in this study as the technical requirements of the product is without considering the product development.
- Due to the current currency situation, some suppliers are not willing to cooperate with the company due to the export of their products, which can complicate the research process and affect the choice of supplier by the company.

Considering that so far, the selection of suppliers has been done according to the needs of the company and in order to meet it, based on the intuitive judgments of experts, and the experts used to compare suppliers based on their own judgments. It is suggested that from now on, using the results of this study, the selection of suppliers in this company and other similar companies be done by collecting the required information of the models in a systematic and scientific manner. During the different stages of this research, new points were realized and as the research progressed, more ambiguities were created in front of the researcher, which due to the existing limitations, their study requires more research. Therefore, for the research of future researchers who intend to work in this field, some topics are suggested:

- To increase accuracy and reduce uncertainty in prioritizing criteria and suppliers and allocating the optimal order amount to each supplier, it is suggested to combine this model with neural network models and fuzzy logic and compare it with the results of this study.
- It is suggested to provide a comprehensive model related to similar organizations and large companies by examining other similar companies that covers all the criteria of the companies involved.
- It is suggested that the indicators be tested in similar companies based on the conceptual model or structural model in order to identify the supply framework.
- Using the gray approach to develop the accuracy of the answers obtained
- Using the heuristic factor analysis approach to identify customers' technical requirements.
- Use of fuzzy Delphi approach in order to identify the technical requirements of the product.

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A New Approach For Ranking Of Intuitionistic Fuzzy Numbers

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ABSTRACT

The concept of an intuitionistic fuzzy number (I F N) is of importance for representing an ill-known quantity. Ranking fuzzy numbers plays a very important role in the decision process, data analysis and applications. The concept of an intuitionistic fuzzy number (IFN) is of importance for quantifying an ill-known quantity. Ranking of intuitionistic fuzzy numbers plays a vital role in decision making and linear programming problems. Also, ranking of intuitionistic fuzzy numbers is a very difficult problem. In this paper, a new method for ranking intuitionistic fuzzy number is developed by means of magnitude for different forms of intuitionistic fuzzy numbers. In Particular ranking is done for trapezoidal intuitionistic fuzzy numbers, triangular intuitionistic fuzzy numbers, symmetric trapezoidal intuitionistic fuzzy numbers, and symmetric triangular intuitionistic fuzzy numbers. Numerical examples are illustrated for all the defined different forms of intuitionistic fuzzy numbers. Finally some comparative numerical examples are illustrated to express the advantage of the proposed method.

1. Introduction

Atanassov [1] introduced the concept of Intuitionistic Fuzzy Sets (IFS) which is a generalization of the concept of fuzzy set. In IFS the degree of non-membership denoting the non-belonging of an element to a set is explicitly specified along with the degree of membership.

In many real world problems, due to insufficiency in the information available, the evaluation of membership values is not possible up to our satisfaction. Also the evaluation of non –membership values is not always possible and there remains an indeterministic part in which hesitation survives. A fuzzy number plays a vital role in representation of such unknown quantity. Following this concept, the generalized concept of intuitionistic Fuzzy Number (IFN) introduced by Grzegrorzewski [5] in 2003 receives high attention and different definitions of IFN's have been proposed. Grzegrorzewski [6] defined two families of metrics in the space of IFNs and proposed a ranking method of IFNs.

Mitchell [9] interpreted an IFN as an ensemble of fuzzy numbers and introduced a ranking method. Wang [18] gave the definition of intuitionistic trapezoidal fuzzy number and interval intuitionistic

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trapezoidal fuzzy number. Based on expected values, score functions and accuracy function of intuitionistic trapezoidal fuzzy numbers a new kind of ranking was proposed by Wang et al. in 2009. They also developed the Hamming distance of intuitionistic trapezoidal fuzzy numbers and Intuitionistic Trapezoidal Fuzzy Weighted Arithmetic Averaging (ITFWAA) operator, then proposed multi-criteria decision-making method with incomplete certain information based on intuitionistic trapezoidal fuzzy number.

In 2011, Salim Rezvani defined a new ranking technique for trapezoidal intuitionistic fuzzy numbers based on value-index and ambiguity –index of trapezoidal intuitionistic fuzzy numbers. Similar value-index and ambiguity – index based ranking method for triangular intuitionistic fuzzy numbers was given by Li et al. [7] in 2010. Li [8] proposed a ranking order relation of TIFN using lexicographic technique. Nayagam et al. [12] introduced TIFNs of special type and described a method to rank them which seems to be unrealistic. Nehi [11] put forward a new ordering method for TIFNs in which two characteristic values for IFN.

Symmetric trapezoidal intuitionistic fuzzy numbers are ranked with a special ranking function which has been applied to solve a class of linear programming problems in which the data parameters are symmetric trapezoidal intuitionistic fuzzy number by Parvathi et al. [14] in 2012. Dubey et al. in 2011 developed a ranking technique for special form of triangular intuitionistic fuzzy numbers.

This paper is organized as follows. In Section 2 some preliminary definitions and concepts regarding intuitionistic fuzzy numbers were presented. In Section 3, we define the magnitude of different forms of trapezoidal and triangular intuitionistic fuzzy numbers. Section 4 is devoted to the illustration of some numerical examples for the concepts defined in the Section 3 and also contains the comparative study of results obtained by the proposed method with other existing ranking methods. Section 5 concludes the paper by giving some advantages of the proposed method over other methods.

2. Preliminaries

Definition 1. [1] An IFS A in X is given by

$$A = \{(x, \mu_A(x), \nu_A(x)), x \in X\},$$

where the functions $\mu_A(x)$: $X \rightarrow [0, 1]$ and $\nu_A(x)$: $X \rightarrow [0, 1]$ define, respectively, the degree of membership and degree of non-membership of the element $x \in X$ to the set A, which is a subset of X, and for every $x \in X$, $0 \le \mu_A(x) + \nu_A(x) \le 1$.

Obviously, every fuzzy set has the form $\{(x, \mu_A(x), \mu_{A^c}(x)), x \in X\}$.

For each IFS A in X, we will call $\Pi_A(x) = 1 - \mu(x) - \nu(x)$ the intuitionistic fuzzy index of x in A. It is obvious that $0 \le \Pi_A(x) \le 1, \forall x \in X$.

Definition 2. [11]. An IFS $A = \{(x, \mu_A(x), \gamma_A(x) | x \in X)\}$ is called IF-normal, if there exist at least two points $x_0, x_1 \in X$ such that $\mu_A(x_0) = 1, \gamma_A(x_1) = 1$, It is easily seen that given intuitionistic fuzzy set A is IF-normal if there is at least one point that surely belongs to A and at least one point which does not belong to A.

Definition 3. [11]. An IFS $A = \{(x, \mu_A(x), \gamma_A(x) | x \in X)\}$ of the real line is called IF-convex, if

$$\forall x_1, x_2 \in \mathbb{R}, \forall \lambda \in [0,1],$$
$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \mu_A(x_1) \wedge \mu_A(x_2),$$
$$\gamma_A(\lambda x_1 + (1-\lambda)x_2) \geq \gamma_A(x_1) \wedge \gamma_A(x_2).$$

Thus A is IF –convex if its membership function is fuzzy convex and its non-membership function is fuzzy concave.

Definition 4. [11]. An IFS $A = \{(x, \mu_A(x), \gamma_A(x) | x \in X)\}$of the real line is called an IFN if

- A is IF-normal,
- A is IF-convex,
- μ_A is upper semicontinuous and γ_A is lower semicontinuous,
- $A = \{(x \in X \,| \gamma_A(x) < 1\}$ is bounded.

Definition 5. [11]. A is a trapezoidal intuitionistic fuzzy number with parameters $b_1 \leq a_1 \leq b_2 \leq a_2 \leq a_3 \leq b_3 \leq a_4 \leq b_4$ and denoted by $A = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)$. In this case we will give

$$\mu_A(x) = \begin{cases} 0 & ; x < a_{1,} \\ \dfrac{x - a_1}{a_2 - a_1} & ;\ a_1 \leq x \leq a_2 \\ 1 & ; a_2 \leq x \leq a_3 \\ \dfrac{x - a_4}{a_3 - a_4} & ; a_3 \leq x \leq a_4 \\ 0 & ; a_4 < x. \end{cases}$$

$$\gamma_A(x) = \begin{cases} 0 & ; x < b_{1,} \\ \dfrac{x - b_2}{b_1 - b_2} & ;\ b_1 \leq x \leq b_2 \\ 1 & ; b_2 \leq x \leq b_3 \\ \dfrac{x - b_3}{b_4 - b_3} & ; b_3 \leq x \leq b_4 \\ 0 & ; b_4 < x. \end{cases}$$

In the above definition, if we let$b_2 = b_3$ ($and\ hence\ a_2 = a_3$), then we will get a triangular intuitionistic fuzzy number with parameters $b_1 \leq a_1 \leq (b_2 = a_2 = a_3 = b_{3)} \leq a_4 \leq b_4$ and denoted by $A = (b_1, a_1, b_2, a_4, b_4)$.

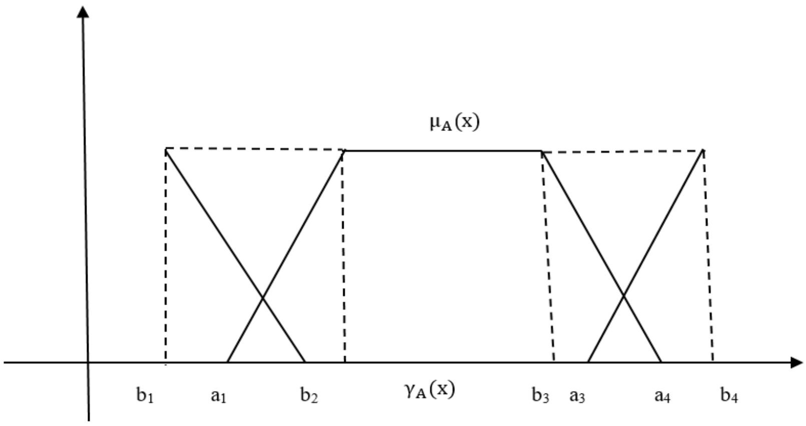

Fig. 1. *Trapezoidal intuitionistic fuzzy number.*

Definition 6. [7]. A TIFN $\tilde{a} = (\underline{a}, a, \overline{a}; w_{\tilde{a}}, u_{\tilde{a}})$ is a special IF set on the real number set R, whose membership function and non-membership function are defined as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{w_{\tilde{a}}(x - \underline{a})}{(a - \underline{a})} & \text{if } \underline{a} \le x < a \\ w_{\tilde{a}} & \text{if } x = a \\ \dfrac{w_{\tilde{a}}(\overline{a} - x)}{(\overline{a} - a)} & \text{if } a < x \le \overline{a} \\ 0 & \text{if } x < \underline{a} \text{ or } x > \overline{a} \end{cases}$$

$$v_{\tilde{a}}(x) = \begin{cases} \dfrac{[a - x + u_{\tilde{a}}(x - \underline{a})]}{(a - \underline{a})} & \text{if } \underline{a} \le x < a \\ u_{\tilde{a}} & \text{if } x = a \\ \dfrac{[x - a + u_{a}(\overline{a} - x)]}{(\overline{a} - a)} & \text{if } a < x \le \overline{a} \\ 0 & \text{if } x < \underline{a} \text{ or } x > \overline{a} \end{cases}$$

Where the values $w_{\tilde{a}}$ and $u_{\tilde{a}}$ represent the maximum degree of membership and the minimum degree of non-membership, respectively, such that they satisfy the conditions $0 \le w_{\tilde{a}} \le 1, 0 \le u_{\tilde{a}} \le 1, 0 \le w_{\tilde{a}} + u_{\tilde{a}} \le 1$.

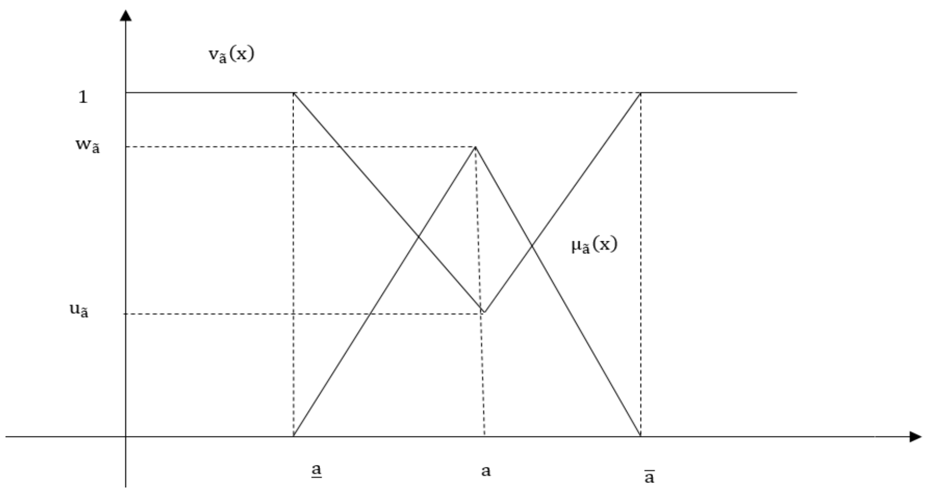

Fig. 2. *Triangular intuitionistic fuzzy number.*

Definition 7. [14]. An IFN $\tilde{A}$ in R is said to be a symmetric trapezoidal intuitionistic fuzzy numbers if there exists real numbers a_1, a_2, h, h' where $a_1 \leq a_2, h \leq h'$ and $h, h' > 0$ such that the membership and non-membership functions are as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-(a_1-h)}{h} & ; x \in [a_1-h, a_1] \\ 1 & ; x \in [a_1, a_2] \\ \dfrac{a_2+h-x}{h} & ; x \in [a_2, a_2+h] \\ 0 & ; \text{otherwise} \end{cases}$$

$$v_{\tilde{A}}(x) = \begin{cases} \dfrac{(a_1-x)}{h'} & ; x \in [a_1-h', a_1] \\ 0 & ; x \in [a_1, a_2] \\ \dfrac{x-a_2}{h'} & ; x \in [a_2, a_2+h'] \\ 1 & ; \text{otherwise} \end{cases}$$

Definition 8. [17]. A Generalized Triangular Intuitionistic Fuzzy Number (GTIFN) $\tilde{\tau}_a = \langle (a, l_\mu, r_\mu; w_a), (a, l_\gamma, r_\gamma; u_a) \rangle$ is a special intuitionistic fuzzy set on a real number set $\Re$, whose membership function and non-membership functions are defined as follows:

$$\mu_{\tilde{\tau}_a}(x) = \begin{cases} \dfrac{x-a+l_\mu}{l_\mu} w_a & ; a-l_\mu \leq x < a \\ w_a & ; x = a \\ \dfrac{a+r_\mu-x}{r_\mu} w_a & ; a < x \leq a+r_\mu \\ 0 & ; \text{otherwise} \end{cases}$$

$$v_{\tilde{\tau}_a}(x) = \begin{cases} \dfrac{(a-x)+u_a(x-a+l_\gamma)}{l_\gamma} & ; a-l_\gamma \leq x < a \\ u_a & ; x = a \\ \dfrac{(x-a)+u_a(a+r_\gamma-x)}{r_\gamma} & ; a < x \leq a+r_\gamma \\ 1 & ; \text{otherwise} \end{cases}$$

Where $l_\mu, r_\mu, l_\gamma, r_\gamma$ are called the spreads of membership and non-membership functions, respectively and a is called mean value. w_a and u_a represent the maximum degree of membership and minimum degree of non-membership respectively such that they satisfy the conditions $0 \leq w_a \leq 1, 0 \leq u_a \leq 1$ and $0 \leq w_a + u_a \leq 1$.

Definition 9. [13]. A TIFN is an intuitionistic fuzzy set in R with the following membership function $\mu_A(x)$ and non-membership function $\vartheta_A(x)$

$$\mu_A(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & , a_1 \leq x \leq a_2 \\ \dfrac{x-a_3}{a_2-a_3} & , a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases}$$

$$\vartheta_A(x)=\begin{cases}\dfrac{a_2-x}{a_2-a_1'} & ,a_1'\le x\le a_2\\ \dfrac{x-a_2}{a_3'-a_2} & ,a_2\le x\le a_3'\\ 1 & ,\text{otherwise.}\end{cases}$$

Where $a_1' \le a_1 \le a_2 \le a_3 \le a_3'$ and $\mu_A(x)+\vartheta_A(x)\le 1$ or $\mu_A(x)=\vartheta_A(x)$ for all $x\in R$. This TIFN is denoted by $A=(a_1,a_2,a_3;a_1',a_2,a_3')$.

Definition 10. [18]. Let $\tilde{a}=\langle([a,b,c,d];\mu_{\tilde{a}}),([a_1,b,c,d_1];\gamma_{\tilde{a}})\rangle$ be a trapezoidal intuitionistic fuzzy number whose membership and non-membership is given by

$$\mu_{\tilde{a}}=\begin{cases}\dfrac{x-a}{b-a}\mu_{\tilde{a}}, & a\le x<b\\ 1, & b\le x\le c\\ \dfrac{d-x}{d-c}\mu_{\tilde{a}}, & c<x\le d\\ 0 & ,\text{otherwise}\end{cases}$$

$$\gamma_{\tilde{a}}=\begin{cases}\dfrac{b-x+\gamma_{\tilde{a}}(x-a_1)}{b-a} & ,a_1\le x<b\\ 0 & ,\ b\le x\le c\\ \dfrac{x-c+\gamma_{\tilde{a}}(d_1-x)}{d_1-c} & ,\ c<x\le d_1\\ 1 & ,\text{otherwise.}\end{cases}$$

Where $0\le\ \mu_{\tilde{a}}\le 1, 0\le\gamma_{\tilde{a}}\le 1, \mu_{\tilde{a}}+\gamma_{\tilde{a}}\le 1, a,b,c,d\in R$. When $b=c$, the intuitionistic trapezoidal fuzzy number becomes intuitionistic triangular fuzzy number.

3. New Approach for Ranking of Intuitionistic Fuzzy Numbers

In this section we define the concept of magnitude of an intuitionistic fuzzy number and discussed various methods for ranking the different forms of triangular intuitionistic fuzzy numbers and trapezoidal intuitionistic fuzzy numbers by means of magnitude**.**

Definition 11. Let $A=(b_1,a_1,b_2,a_2,a_3,b_3,a_4,b_4)$ be a Trapezoidal intuitionistic fuzzy number we define magnitude as follows:

$$\text{Mag}(A)=\frac{1}{2}\int_0^1(f_A(x)+g_A(x)+h_A(x)+k_A(x))\,f(r)dr. \quad (1)$$

where (r) is a non-negative and increasing weighting function on $[0,1]$ with

$$f(0)=0,\qquad f(1)=1 \text{ and } \int_0^1 f(r)dr=\frac{1}{2}.$$

In this paper we assume $f(r)=r$ for our convenience, we get magnitude of A as

$$\mathrm{Mag(A)} = \frac{1}{12}\left(a_1 + b_1 + a_4 + b_4 + 2(a_2 + a_3 + b_2 + b_3)\right). \quad (2)$$

Using this definition of $Mag(A)$, we define the ranking procedure of any two trapezoidal intuitionistic fuzzy numbers as follows:

- $Mag(A) > Mag\ (B) iff\ A \succ B$.
- $Mag(A) < Mag\ (B) iff\ A \prec B$.
- $Mag(A) = Mag\ (B) iff\ A \backsim B$.

Remark 1. If $A = (b_1, a_1, b_2, a_2, a_3, b_3, a_4, b_4)\ and\ B = (b_1', a_1', b_2', a_2', a_3', b_3', a_4', b_4')$ be any two trapezoidal intuitionistic fuzzy numbers, then $Mag(A + B) = Mag\ A + Mag\ B$.

Definition 12. We define magnitude of a symmetric trapezoidal intuitionistic fuzzy number, $A = (b_1, a_1, a_2, a_2, a_3, a_3, a_4, b_4)$ using *Eq. (1)* as

$$\mathrm{Mag(A)} = \frac{1}{12}\left(a_1 + b_1 + a_4 + b_4 + 4(a_2 + a_3)\right). \quad (3)$$

Remark 2. For any two symmetric trapezoidal intuitionistic fuzzy numbers $A = (a_1, a_2, h, h, a_1, a_2, h', h'), B = (a_1, a_2, k, k, a_1, a_2, k', k')$, we have

$$\mathrm{Mag(A)} = \mathrm{Mag(B)}. \quad (4)$$

Remark 3. For any symmetric trapezoidal intuitionistic fuzzy number

$$\mathrm{A} = (-a_1, a_1, h, h, -a_1, a_1, h', h'),\ \ \mathrm{Mag(A)} = 0. \quad (5)$$

Definition 13. For a trapezoidal intuitionistic fuzzy number $A = (a_1, a_2, \alpha, \beta, a_1, a_2, \alpha', \beta')$

$$\mathrm{Mag(A)} = \frac{1}{12}\left(\beta - \alpha + 6(a_1 + a_2) + 2(\beta' - \alpha'\right). \quad (6)$$

Definition 14. Let $A = (a_1, a_2, h, h, a_1, a_2, h', h')$ be a symmetric trapezoidal intuitionistic fuzzy number. Then its magnitude defined by

$$\mathrm{Mag(A)} = \frac{1}{2}(\mathrm{a_1 + a_2}).. \quad (7)$$

Definition 15. Let $A = (a_1, b_1, c_1, d_1; a_1', b_1, c_1, d_1')$ be a trapezoidal intuitionistic fuzzy number, then

$$\mathrm{Mag(A)} = \frac{1}{12}(\mathrm{a_1 + d_1 + 2(a_1' + d_1') + 3(b_1 + c_1)}). \quad (8)$$

Definition 16. If $A = (\overline{a}, a, \underline{a}; w_a, u_a)$ is a triangular intuitionistic fuzzy number , then

$$\mathrm{Mag(A)} = \frac{1}{12}\left[\frac{4a - 2(\overline{a} + \underline{a}) + 3w_a(\underline{a} + \overline{a})}{w_a} + \frac{2(\overline{a} + a + \underline{a}) - 3u_a(a + \overline{a})}{(1 - u_a)}\right] \quad (9)$$

Definition 17. Let $A = ((a, l_\mu, r_\mu; w_a), (a, l_\gamma, r_\gamma; u_a))$ be a triangular intuitionistic fuzzy number. Then

$$\mathrm{Mag\ (A)} = \frac{1}{12}\left\{\left(\frac{6aw_a - 3w_a(l_\mu - r_\mu) + 2(l_\mu - r_\mu)}{w_a} + \frac{6(a - au_a) + 3u_a(l_\gamma - r_\gamma) + 2(r_\gamma - l_\gamma)}{(1 - u_a)}\right)\right\}. \quad (10)$$

Definition 18. Let $A = \langle [a, b, c, d]; \mu_a, \gamma_a \rangle$ be a trapezoidal intuitionistic fuzzy number, then

$$\mathrm{Mag(A)} = \frac{1}{12}\left\{\frac{2(b - a + c - d) + 3\mu_a(a + d)}{\mu_a} + \frac{2(a + d) + (b + c) - 3\gamma_a(a + d)}{1 - \gamma_a}\right\}. \quad (11)$$

Definition 19. Consider a triangular intuitionistic fuzzy number of the form $A = (a_1, a_2, a_3; a_1', a_2, a_3')$, then

$$\mathrm{Mag(A)} = \frac{1}{12}\{\mathrm{a_1 + a_3 + 6a_2 + 2(a_1' + a_3')}\} \quad (12)$$

4. Numerical Examples

This section illustrates some examples for comparative analysis of various existing ranking methods

Example 1. Consider two trapezoidal intuitionistic fuzzy numbers as follows: A= (0.2,0.4,0.6,0.8,0.11,0.12,0.13,0.15) and B=(0,0.1,0.2,0.3,0.4,0.5,0.6,0.7). In [11], Nehi used characteristic values of membership or non-membership functions to rank trapezoidal intuitionistic fuzzy numbers. The ranking procedure depends on the value of ‘k’. As ‘k’ varies in the interval$(0, \infty)$, the ranking also varies which leads to an unreasonable result. This can be seen from the following example.

Table 1. *Calculation of* $c_{\mu}^{k}(A)$.

a_1	a_2	a_3	a_4	b_1	b_2	b_3	b_4	k	$c_{\mu}^{k}(A)$
0.4	0.8	0.11	0.13	0.2	0.6	0.12	0.15	1	0.392
0.4	0.8	0.11	0.13	0.2	0.6	0.12	0.15	2	0.408

Table 2. *Calculation of* $c_{\mu}^{k}(B)$.

a_1	a_2	a_3	a_4	b_1	b_2	b_3	b_4	k	$c_{\mu}^{k}(B)$
0.1	0.3	0.4	0.6	0	0.2	0.5	0.7	1	0.350
0.2	0.4	0.5	0.7	0.1	0.3	0.6	0.8	2	0.450

From the table, we see that when k=1, $A > B$ and when k=2, $B > A$

Example 2. Consider two symmetric trapezoidal intuitionistic fuzzy numbers A=(23,25,1,1;23,25,3,3) and B = (5,7,2,2; 5,7,4,4) as in [15]. Here the ranking of STIFNs are obtained by a special ranking function by considering all the parameters of both membership and non- membership functions of given STIFNs. The values obtained by this method are similar to the proposed method.

Example 3. Consider two trapezoidal intuitionistic fuzzy numbers of the forms A = (0.2,0.3,0.4,0.5; 0.1,0.3,0.4,0.6) and B = (0.1,0.2,0.3,0.4; 0,0.2,0.3,0.5) discussed in [16]. Rezvani used value index of membership and non-membership functions separately to rank trapezoidal intuitionistic fuzzy numbers.

Example 4. Consider three triangular intuitionistic fuzzy numbers as below A=(0.592,0.774,0.910;0.6,0.4) , B=(0.769,0.903,1;0.4,0.5) and C=(0.653,0.849,0.956;0.5,0.2) as given in [7]. In the paper [7] Li used ratio ranking method to rank triangular intuitionistic fuzzy numbers and applied it to multi attribute decision making problem In the case of ration ranking method, the raking differs on the choice of λ. For the above IFN's we have

Table 3. *Ranking of IFN's for values of* λ.

S.No	λ	Ranking results
1	$[0, 0.1899)$	$A > C > B$
2	$(0.1899,0.9667)$	$C > A > B$
3	$(0.9667,1]$	$C > B > A$

So this leads to a conflicted state which yields an unreasonable result.

Example 5. Consider the same IFN's as in *example 4* and ranking developed in [8]. Here the ranking is done by the extended additive weighted method using the value-index and ambiguity-index. For the above numbers, we have the following ranking results as tabulated below from [8].

Table 4. *Ranking of IFN's for values of* λ.

S.No.	λ	Ranking results
1	[0,0.793]	$C > A > B>$
2	(0.793,1]	$A > C > B$

From the above table, we see that the ranking differs on the basis of given weight λ.

Example 6. Consider the two Generalized triangular fuzzy intuitionistic numbers $\tilde{\tau}_a = \big((5,1,2;0.6),(5,1.5,2.6;0.3)\big)$ and $\tilde{\tau}_b = \big((6,2,1;0.6),(6,2.1,1.5;0.4)\big)$ in [17]. If we use $R_{\mu(}\tilde{\tau}_a)$ to rank these numbers we obtain $\tilde{\tau}_a < \tilde{\tau}_b$. But when we rank in terms of $R_\gamma(\tilde{\tau}_a)$, we get $\tilde{\tau}_a > \tilde{\tau}_b$. Hence the ranking of generalized triangular intuitionistic fuzzy numbers varies with the use of membership and non-membership value in ranking. This is an unreasonable result. Therefore the proposed method which uses both membership and non-membership values as a whole is suitable for ranking such GTIFN's.

Example 7. Consider the two triangular intuitionistic fuzzy numbers as follows: A = {(14,15,17;0.9),(10,15,18;0)} and B = {(25,30,34;0.9),(23,30,38;0)}as in [4]. In this paper, Dubey used the concept of value and ambiguity of a triangular intuitionistic fuzzy numbers to rank the above numbers. The ranking obtained in [4] is similar to the proposed method.

Example 8. Consider 5 set of trapezoidal intuitionistic fuzzy number as in [18].

$$\widetilde{a_1} = \langle[0.407,0.539,0.683,0.814];0.727,0.21\rangle.$$

$$\widetilde{a_2} = \langle[0.547,0.679,0.810,0.942];0.705,0.230\rangle.$$

$$\widetilde{a_3} = \langle[0.424,0.572,0.704,0.868];0.697,0.252\rangle.$$

$$\widetilde{a_4} = \langle[0.392,0.557,0.724,0.902];0.639,0.280\rangle.$$

$$\widetilde{a_5} = \langle[0.411,0.555,0.699,0.831];0.812,0.137\rangle.$$

In [18] ranking is done based on the comparison of score function values and accuracy function values of integrated intuitionistic fuzzy numbers. The ranking here in [18] differs from our proposed method.

Example 9. Consider two triangular intuitionistic fuzzy numbers as below $\tilde{A} = \{(2.68,3,3.71);(2.2,3,4.67)\}$ and $B = \{(2.75,6,9.375);(2.38,6,16.2)\}$ as in [13]. In this paper ranking is done by using the score function and the result obtained is similar to the proposed method.

The following table gives a comparative analysis of various ranking methods so far defined in intuitionistic fuzzy setting with the proposed method.

Table 5. *Comparative analysis of different ranking methods.*

S.No	Intuitionistic Fuzzy Numbers	Existing Method	Proposed Method
1	*A=(0.2,0.4,0.6,0.8,0.11,0.12,0.13,0.15)* *B=(0,0.1,0.2,0.3,0.4,0.5,0.6,0.7)*	$c_\mu^k(A) = 0.392$; $c_\mu^k(B) = 0.35$, $A \succ B$ [11]	$Mag(A) = 0.35$ $Mag(B) = 0.35$ $A \sim B$
2	*A=(23,25,1,1;23,25,3,3)* *B=(5,7,2,2;5,7,4,4)*	$\Re(A) = 49$ $\Re(B) = 13$, $A \succ B$ [15]	$Mag\ (A) = 24$; $Mag(B) = 6$, $A \succ B$
3	*A=(0.2,0.3,0.4,0.5;0.1,0.3,0.4,0.6)* *B= (0.1,0.2,0.3,0.4;0,0.2,0.3,0.5)*	$v_\mu(A) = 0.35$; $v_\mu(B) = 0.25$, $A \succ B$ [16]	$Mag(A) = 0.35$; $Mag(B) = 0.25$ $A \succ B$
4	*A=(0.592,0.774,0.910;0.6,0.4)* *B=(0.769,0.903,1;0.4,0.5)* *C=(0.653,0.849,0.956;0.5,0.2)*	$R(A, \lambda) = 0.4321$; $R(B, \lambda) = 0.3455$; $R(C, \lambda) = 0.3858$ $A \succ C \succ B$ [7]	$Mag(A) = 0.8282$; $Mag(B) = 0.9688$; $Mag(C) = 0.9322$, $B \succ C \succ A$
5	*A=(0.592,0.774,0.910;0.6,0.4)* *B=(0.769,0.903,1;0.4,0.5)* *C=(0.653,0.849,0.956;0.5,0.2)*	$v_\lambda(A) = 0.276$; $v_\lambda(B) = 0.224$; $v_\lambda(C) = 0.534$, $C > A > B$ [8]	$Mag(A) = 0.828$; $Mag(B) = 0.969$; $Mag(C) = 0.932$, $B > C > A$
6	$\tau_a = \big((5,1,2; 0.6), (5,1.5,2.6; 0.3)\big)$ $\tau_b = \big((6,2,1; 0.6), (6,2.1,1.5; 0.4)\big)$	$R_\gamma(\tau_a) = 3.98$, $R_\gamma(\tau_b) = 3.51$, $A \succ B$ [17]	$Mag(\tau_a) = 5.12$, $Mag\ (\tau_b) = 5.96$, $A \prec B$
7	$A = \{(14,15,17; 0.9), (10,15,18; 0)\}$ $B = \{(25,30,34; 0.9), (23,30,38; 0)\}$	$F(A, \lambda) = 13.76$; $F(B, \lambda) = 27.47$; $A \succ B$ [4]	$Mag(A) = 15.28$; $Mag(B) = 31.74$ $A \succ B$
8	$\widetilde{a_1} = \langle [0.407,0.539,0.683,0.814]; 0.727,0.21 \rangle$ $\widetilde{a_2} = \langle [0.547,0.679,0.810,0.942]; 0.705,0.230 \rangle$ $\widetilde{a_3} = \langle [0.424,0.572,0.704,0.868]; 0.697,0.252 \rangle$ $\widetilde{a_4} = \langle [0.392,0.557,0.724,0.902]; 0.639,0.280 \rangle$ $\widetilde{a_5} = \langle [0.411,0.555,0.699,0.831]; 0.812,0.137 \rangle$	$S(\widetilde{a_1}) = 0.236$; $S(\widetilde{a_2}) = 0.261$; $S(\widetilde{a_3}) = 0.206$; $S(\widetilde{a_4}) = 0.153$; $S(\widetilde{a_5}) = 0.353$; $a_5 \succ a_2 \succ a_1 \succ a_3 \succ a_4$ [18]	$Mag(\widetilde{a_1}) = 0.611$; $Mag(\widetilde{a_2}) = 0.745$; $Mag(\widetilde{a_3}) = 0.640$; $Mag(\widetilde{a_4}) = 0.642$; $Mag(\widetilde{a_5}) = 0.625$; $a_2 \succ a_4 \succ a_3 \succ a_5 \succ a_1$
9	$\tilde{A} = \{(2.68,3,3.71); (2.2,3,4.67)\}$ *and* $\tilde{B} = \{(2.75,6,9.375); (2.38,6,16.2)\}$	$S(\tilde{A}) = 3.2175$ $S(\tilde{B}) = 7.645$ $\tilde{A} \prec \tilde{B}$ [13]	$Mag(\tilde{A}) = 3.1772$ $Mag(\tilde{B}) = 7.12$ $\tilde{A} \prec \tilde{B}$

5. Conclusions

In many of the existing ranking methods, ranking is done either by considering the membership or non-membership values only. But in the newly proposed method the ranking is done directly by taking both membership and non-membership values in a single formula. This ranking procedure is very simple and time consuming compared to the existing methods. We also illustrated the advantages of our method by means of suitable examples. The proposed ranking technique can be applied to multi-criteria decision making problems, linear programming problems, assignment problems, transportation, some management problems and industrial problems which are our future research works.

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Application of Transportation Problem under Pentagonal Neutrosophic Environment

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PAPER INFO	ABSTRACT
Chronicle: *Received: 04 September 2019* *Revised: 09 December 2019* *Accepted: 15 January 2020* **Keywords:** *Transportation Problem. Pentagonal Neutrosophic Numbers. Linear Programming*.	The paper talks about the pentagonal Neutrosophic sets and its operational law. The paper presents the $\alpha_N, \beta_N, \lambda_N -$ cut of single valued pentagonal Neutrosophic numbers and additionally introduced the arithmetic operation of single-valued pentagonal Neutrosophic numbers. Here, we consider a transportation problem with pentagonal Neutrosophic numbers where the supply, demand and transportation cost is uncertain. Taking the benefits of the properties of ranking functions, our model can be changed into a relating deterministic form, which can be illuminated by any method. Our strategy is easy to assess the issue and can rank different sort of pentagonal Neutrosophic numbers. To legitimize the proposed technique, some numerical tests are given to show the adequacy of the new model.

1. Introduction

For survival of our life there is a need to move the item from various sources to various goals. Due to high population, it is very challenging to company, how to send the product to numbers of costumers or origins to a numbers of warehouse or store in a minimizing cost. This kind of issue is called Transportation Problem (TP) and it is an exceptional sort of Linear Programming (LP) problem where the organization's primary goal is limiting the expense. On account of wide application i.e. production planning, health sector, inventory control, network system etc., TP have consistently made separate space for analysts. An outline has attracted *Fig.1* which is speaks to connection among supply and demand.

Hitchcock [1] pioneered the basic transportation problem. This kind of traditional issue can be as a direct programming issue and afterward tackled simplex strategy. This type of classical problem can be modelled as a linear programming problem and then solved simplex method. A primal simplex method to transportation problem was solved by Dantzig & Thapa [2]. Transportation Problem is a different type of structure therefore simplex method is not suitable for finding the objectives. Due to some drawback in simplex method for solving TP, a new Initial Basic Feasible Solution (IBFS) method was developed. By using the IBFS, there are three type of methods (1) north-west corner (NWC), (2) least-

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cost method, (3) vogel's approximation method. In classical TP the decision makers knows the values of supply, demand and transportation cost i.e. the decision makers consider the crisp numbers. However, in our day to day life applications, the decision makers may not be known precisely to all the parameters of transportation problem due to some uncontrolled factor. To overcome this uncontrolled factor, fuzzy decision making method is introduced.

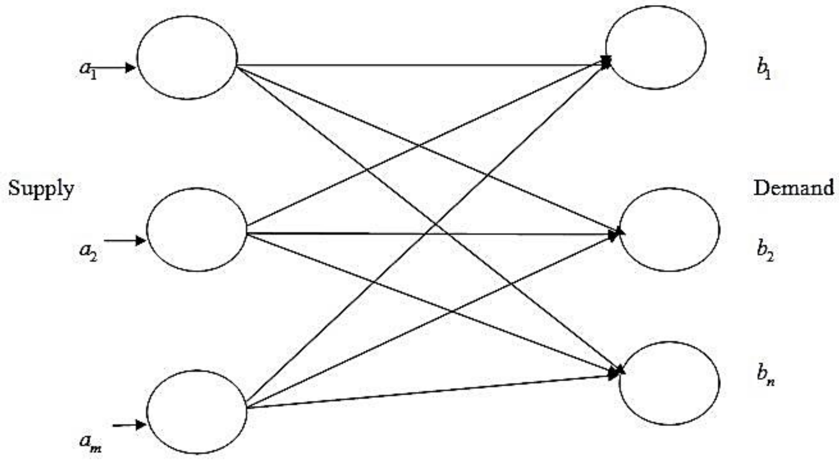


Fig. 1. *LP Transportation problem.*

The basic concepts of fuzzy set theory was introduced by Zadeh [3] in 1965. Since, several researchers have carried out investigation on fuzzy transportation problem (FTP). A Fuzzy Linear Programming (FLP) problem was proposed by Zimmermann [4] and he has proved that the method was always very effective. Subsequently, Zimmermann FLP model has developed to solve different fuzzy transportation problems. Chanas et al. [5] considered FTP where supply and demand are fuzzy numbers and transportation cost is crisp number. Das et al. [6] proposed a fully fuzzy LP problem where all the parameters are trapezoidal fuzzy numbers and that method extend to solve Fully Fuzzy TP (FFTP). Dinagar and Palanivel [7] proposed a FFTP where demand, supply, and transportation costs are trapezoidal fuzzy numbers. Kaur and Kumar [8] have introduced an algorithmic for solving the fuzzy transportation problem. Fuzzy zero pint method for solving FFTP problem was proposed by Pandian and Natrajan [9] in which supply, demand and transportation cost are trapezoidal fuzzy numbers. Kundu et al. [10] introduced a solid transportation model with crisp and rough costs. Some other researchers [11-19] also have studied this general transportation problem in a fuzzy environment. Maheswari and Ganesan [20] proposed fully fuzzy transportation problem using pentagonal fuzzy numbers.

Due to some complications, insufficient information, multiple sources of data arises in our real-life problem; it is not always possible to use fuzzy numbers. The fuzzy sets mainly consider the membership functions. Intuitionistic Fuzzy Sets (IFS) is an extension version of fuzzy sets and it can be used to solve them. IFS has been proposed by Atanassov [24] and it's take care both mixture of membership function and non-membership function. Since, several researchers [25-30] considered the IFS for solving TP. Aggarwal and Gupta [31] studied the sensitivity analysis of intuitionistic fuzzy solid transportation problem. Singh and Yadav [32] introduced a novel solution for solving fully Intuitionistic Fuzzy Transportation Problem (IFTP) in which demand, supply and transportation cost are considered intuitionistic triangular fuzzy numbers. In that paper, they obtaining both negative solutions for variables and objective functions instead of positive transportations cost. After the shortcomings of

Singh and Yadav paper, Mahmoodirad et al. [33] proposed a method for fully IFTP in which demand, supply and transportation cost are considered intuitionistic fuzzy numbers.

In genuine situation, we regularly experience with deficient and uncertain data where it isn't conceivable to speak to the data just by the methods for membership function and non-membership function. To manage such circumstances, Smarandache [36] in 1988, introduced the structure of Neutrosophic Set (NS) which is higher version of both fuzzy and intuitionistic fuzzy. Neutrosophy set might be described by three autonomous degrees, i.e. (i) truth-membership degree (T), (ii) indeterminacy membership degree (I), and (iii) falsity membership degree (F). Later, Wang et al. [37] introduced a Single Value Neutrosophic Set (SVNS) problem for solving a practical problem. Ye [38] introduced the Trapezoidal Neutrosophic Set (TrNS) by combining the concept of Trapezoidal Fuzzy Numbers (TrFN) and SVNS. To take the advantages of beauty of NS, several researchers [39-45] proposed different method for solving LP problem under Neutrosophic environment. Das and Dash [46] proposed a modified solution of Neutrosophic LP problem. Recently, Das and Chakraborty [47] proposed a new approach for solving LP problem in pentagonal Neutrosophic environment.

Motivation. Neutrosophic sets always plays a vital role in uncertainty environment. Before going to discussion the motivation of our paper, we demonstrate the different author's research work towards the TP under mixed constraints.

Table 1. *Significance influences of the different authors towards under various environment.*

Author	Year	Main Contribution
Korukoglu and Balli [50]	*2011*	*Crisp environment.*
Bharati [51]	*2019*	*Trapezoidal intuitionistic fuzzy.*
environment		
Ahmad and Adhami [52]	*2018*	*Neutrosophic fuzzy environment.*
Singh and Yadav [32]	*2016*	*Traingular fully intuitionistic fuzzy.*
Ebrahimnejad & Verdegay [25]	*2017*	*Trapezoidal intuitionistic fuzzy.*
Environment		
Mahaswari & Ganesan[20]	*2018*	*Pentagonal fuzzy number.*
Srinivasan et al. [53]	*2020*	*Triangular fuzzy number.*

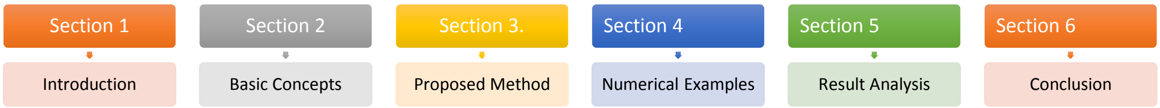
From the above discussion on TP which are readily available in our literature, there are no current techniques for solving pentagonal TP under Neutrosophic condition. In this way, there is have to setup another technique for pentagonal Neutrosophic transportation issue. This total situation has persuaded us to build up another strategy for illuminating TP with pentagonal Neutrosophic numbers. Just because, we build up a calculation and applied, all things considered, issue. The primary commitments of the paper as follows:

We characterize Neutrosophic Transportation Problem (NTP) issue in which the supply, demand and transportation cost are taken as pentagonal Neutrosophic numbers.

- This model assists with settling another arrangement of issue with pentagonal Neutrosophic numbers.
- In our literature of pentagonal Neutrosophic numbers, we will in general present a recently evolved scoring function.
- By using our recently scoring function, the pentagonal Neutrosophic TP is changing over into its crisp TP.

– To best our insight, it would be the primary strategy to unravel the PNTP. Consequently, in our paper direct relationship with relative system doesn't rise.

The rest of the paper is organized as the following way:



2. Preliminaries

In this section, we call back the some definitions and basic concepts which are pivotal in this paper. The well-defined definitions are referred [47, 49] throughout the paper.

2.1. Definition: Neutrosophic Set (NS)

A set $\tilde{N}_M$ is identified as a Neutrosophic set if $\tilde{N}_M = \{\langle x; [\theta_{\tilde{N}_M}(x), \varphi_{\tilde{N}_M}(x), \sigma_{\tilde{N}_M}(x)]\rangle : x \in X\}$, where $\theta_{\tilde{N}_M}(x): X \to]-0,1+[$ is declared as the truthness function, $\varphi_{\tilde{N}_M}(x): X \to]-0,1+[$ is declared as the hesitation function, and $\sigma_{\tilde{N}_M}(x): X \to]-0,1+[$ is declared as the falseness function.

$\theta_{\tilde{N}_M}(x), \varphi_{\tilde{N}_M}(x)$ & $\sigma_{\tilde{N}_M}(x)$ displays the following relation:

$$-0 \leq \mathrm{Sup}\,\{\theta_{\tilde{N}_M}(x)\} + \mathrm{Sup}\,\{\varphi_{\tilde{N}_M}(x)\} + \mathrm{Sup}\,\{\sigma_{\tilde{U}_M}(x)\} \leq 3+.$$

2.2. Definition: Single-Valued Neutrosophic Set (SNS)

A set $\tilde{N}_M$ in the definition 2.3 is called as a SNS $(\widetilde{SN}_M)$ if x is a single-valued independent variable. $\widetilde{SNA} = \{\langle x; [\theta_{\widetilde{SN}_M}(x), \varphi_{\widetilde{SN}_M}(x), \sigma_{\widetilde{SN}_M}(x)]\rangle : x \in X\}$, $\theta_{\widetilde{SN}_M}(x), \varphi_{\widetilde{SN}_M}(x)$& $\sigma_{\widetilde{SN}_M}(x)$ signified the notion of correct, indefinite and incorrect memberships function, respectively.

2.3. Definition: Single-Valued Pentagonal Neutrosophic Number (SPNN)

A SPNN $(\tilde{M})$ is defined as $\widetilde{SPN} = \langle [(m^1, n^1, o^1, p^1, q^1); \mu], [(m^2, n^2, o^2, p^2, q^2); \theta], [(m^3, n^3, o^3, p^3, q^3); \eta] \rangle$, where $\mu, \theta, \eta \in [0,1]$. The truth membership function$(\mu_{\widetilde{SPN}}): \mathbb{R} \to [0, \mu]$, the hesitant membership function $(\theta_{\widetilde{SPN}}): \mathbb{R} \to [\theta, 1]$ and the false membership function $(\eta_{\widetilde{SPN}}): \mathbb{R} \to [\eta, 1]$ are given as:

$$\mu_{\widetilde{SPN}}(x) = \begin{cases} \mu_{\widetilde{Ssl1}}(x) m^1 \leq x < n^1 \\ \mu_{\widetilde{Ssl2}}(x)\, n^1 \leq x < o^1 \\ \mu \qquad x = o^1 \\ \mu_{\widetilde{Ssr2}}(x)\, o^1 \leq x < p^1 \\ \mu_{\widetilde{Ssr1}}(x) p^1 \leq x < q^1 \\ 0 \quad otherwise \end{cases}, \quad \theta_{\widetilde{SPN}}(x) = \begin{cases} \theta_{\widetilde{Ssl1}}(x) \;\; m^2 \leq x < n^2 \\ \theta_{\widetilde{Ssl2}}(x) \;\; n^2 \leq x < o^2 \\ \vartheta \qquad x = o^2 \\ \theta_{\widetilde{Ssr2}}(x) \;\; o^2 \leq x < p^2 \\ \theta_{\widetilde{Ssr1}}(x)\, p^2 \leq x < q^2 \\ 1 \quad otherwise \end{cases}.$$

$$\eta_{\widetilde{SPN}}(x) = \begin{cases} \eta_{\widetilde{Ssl1}}(x) m^3 \leq x < n^3 \\ \eta_{\widetilde{Ssl2}}(x)\, n^3 \leq x < o^3 \\ \vartheta \quad\quad x = o^3 \\ \eta_{\widetilde{Ssr2}}(x)\, o^3 \leq x < p^3 \\ \eta_{\widetilde{Ssr1}}(x) p^3 \leq x < q^3 \\ 1 \quad otherwise \end{cases}.$$

2.4. Score and Accuracy Function

Let us consider a single valued Pentagonal Neutrosophic Numbers (PNN) as $\tilde{F}_{Pen} = (F_1, F_2, F_3, F_4, F_5; \pi, \sigma, \rho)$, The primary application of score function is to drag the judgment of conversion of PNN to crisp number. Also, the mean of the PNN components is $\frac{(F_1+F_2+F_3+F_4+F_5)}{5}$ and score value of the membership portion is $\frac{\{2+\pi-\rho-\sigma\}}{3}$.

Thus, for a P.N.N $\tilde{F}_{Pen} = (F_1, F_2, F_3, F_4, F_5; \pi, \sigma, \rho)$. Score function is scaled as $\widetilde{SC}_{Pen} = \frac{1}{15}(F_1 + F_2 + F_3 + F_4 + F_5) \times \{2 + \pi - \rho - \sigma\}$. Accuracy function is scaled as $\widetilde{AC}_{Pen} = \frac{1}{15}(F_1 + F_2 + F_3 + F_4 + F_5) \times \{2 + \pi - \sigma\}$. Here, $\widetilde{SC}_{Pen} \in R$, $\widetilde{AC}_{Pen} \in R$.

2.5. Relationship between any Two PNN

Let us consider any two pentagonal Neutrosophic number defined as follows:

$\mathrm{F}_{\mathrm{Pen1}} = (\pi_{\mathrm{Pen1}}, \sigma_{\mathrm{Pen1}}, \rho_{\mathrm{Pen1}})$,$\mathrm{F}_{\mathrm{Pen2}} = (\pi_{\mathrm{Pen2}}, \sigma_{\mathrm{Pen2}}, \rho_{\mathrm{Pen2}})$.

$\mathrm{SC}_{\mathrm{Pen1}} > \mathrm{SC}_{\mathrm{Pen2}}$, $\mathrm{F}_{\mathrm{Pen1}} > \mathrm{F}_{\mathrm{Pen2}}$,

$\mathrm{SC}_{\mathrm{Pen1}} < \mathrm{SC}_{\mathrm{Pen2}}$, $\mathrm{F}_{\mathrm{Pen1}} < \mathrm{F}_{\mathrm{Pen2}}$,

$\mathrm{SC}_{\mathrm{Pen1}} = \mathrm{SC}_{\mathrm{Pen2}}$, $\mathrm{F}_{\mathrm{Pen1}} = \mathrm{F}_{\mathrm{Pen2}}$.

Then,

$\mathrm{AC}_{\mathrm{Pen1}} > \mathrm{AC}_{\mathrm{Pen2}}$, $\mathrm{F}_{\mathrm{Pen1}} > \mathrm{F}_{\mathrm{Pen2}}$,

$\mathrm{AC}_{\mathrm{Pen1}} < \mathrm{AC}_{\mathrm{Pen2}}$, $\mathrm{F}_{\mathrm{Pen1}} < \mathrm{F}_{\mathrm{Pen2}}$,

$\mathrm{AC}_{\mathrm{Pen1}} = \mathrm{AC}_{\mathrm{Pen2}}$, $\mathrm{F}_{\mathrm{Pen1}} = \mathrm{F}_{\mathrm{Pen2}}$.

2.6. Basic Operations

Let $\widetilde{F_1} = < (c_1, c_2, c_3, c_4, c_5); \pi_{\widetilde{p_1}}, \mu_{\widetilde{p_1}}, \sigma_{\widetilde{p_1}} >$ and $\widetilde{F_2} = <(d_1, d_2, d_3, d_4, d_5); \pi_{\widetilde{p_2}}, \mu_{\widetilde{p_2}}, \sigma_{\widetilde{p_2}}>$ be two IPFNs and $\alpha \geq 0$. Then the following operational relations hold:

$\widetilde{F_1} + \widetilde{F_2} = < (c_1+d_1, c_2+d_2, c_3+d_3, c_4+d_4, c_5+d_5); \max\{\pi_{\widetilde{p_1}}, \pi_{\widetilde{p_2}}\}, \min\{\mu_{\widetilde{p_1}}, \mu_{\widetilde{p_2}}\}, \min\{\sigma_{\widetilde{p_1}}, \sigma_{\widetilde{p_2}}\} >$,

$\widetilde{F_1}$- $\widetilde{F_2}$= < (c_1- d_5, c_2-d_4, c_3-d_3, c_4-d_2, c_5-d_1); max{$\pi_{\widetilde{p_1}}, \pi_{\widetilde{p_2}}$}, min{$\mu_{\widetilde{p_1}}$, $\mu_{\widetilde{p_2}}$},min{ $\sigma_{\widetilde{p_1}}$, $\sigma_{\widetilde{p_2}}$} >,

$\widetilde{F_1} \times \widetilde{F_2}$= < ($c_1d_1, c_2d_2, c_3d_3, c_4d_4, c_5d_5$); $\pi_{\widetilde{p_1}}$ $\pi_{\widetilde{p_2}}$, $\mu_{\widetilde{p_1}}$+, $\mu_{\widetilde{p_2}} - \mu_{\widetilde{p_1}}$ $\mu_{\widetilde{p_2}}$, $\sigma_{\widetilde{p_1}} + \sigma_{\widetilde{p_2}} - \sigma_{\widetilde{p_1}}\sigma_{\widetilde{p_2}}$ >,

$\alpha\widetilde{F_1}$= <($\alpha c_1, \alpha c_2, \alpha c_3, \alpha c_4, \alpha c_5$);$1-(1-\pi_{\widetilde{p_1}})^{\alpha}, {\mu_{\widetilde{p_1}}}^{\alpha}, {\sigma_{\widetilde{p_1}}}^{\alpha}$)>,

$\widetilde{F_1}^{\alpha}$= <(${c_1}^{\alpha}, {c_2}^{\alpha}, {c_3}^{\alpha}, {c_4}^{\alpha}, {c_5}^{\alpha}$);${\pi_{\widetilde{p_1}}}^{\alpha}$, $(1-\mu_{\widetilde{p_1}})^{\alpha}, (1-\sigma_{\widetilde{p_1}})^{\alpha}$ >.

3. Neutrosophic Transportation Problem

Assume that there are *s* number of sources and *t* destinations. Mathematically, the NTP may be stated as:

$$\text{Min } Z = \sum_{i=1}^{s}\sum_{j=1}^{t} \tilde{c}_{ij}^{N} y_{ij}. \tag{1}$$

Subject to constraints

$$\sum_{j=1}^{t} y_{ij} = \tilde{p}_i^{N}, i = 1, 2, ..., s. \tag{2}$$

$$\sum_{i=1}^{s} y_{ij} = \tilde{q}_i^{N}, j = 1, 2, ..., t. \tag{3}$$

$$y_{ij} \geq 0 \quad \forall i, j.$$

For the above mathematical model of PNTP, we defined the following notations:

- *s & t* is the number of sources and destination being indexed by *i & j*.
- $\tilde{p}_i^N = (p_i^{N1}, p_i^{N2}, p_i^{N3}, p_i^{N4}, p_i^{N5}; \theta_i^N, \sigma_i^N, \mu_i^N)$ is the PNN for the items supplied by source i.
- $\tilde{q}_j^N = (q_j^{N1}, q_j^{N2}, q_j^{N3}, p_j^{N4}, p_j^{N5}; \theta_j^N, \sigma_j^N, \mu_j^N)$ is the PNN for the items demanded by destination j.
- $\tilde{c}_{ij}^N = (c_{ij}^{N1}, c_{ij}^{N2}, c_{ij}^{N3}, c_{ij}^{N4}, c_{ij}^{N5}; \theta_{ij}^N, \sigma_{ij}^N, \mu_{ij}^N)$ is the PNN for the items sending one unit from source i to destination j.
- $\tilde{y}_{ij}^N = (y_{ij}^{N1}, y_{ij}^{N2}, y_{ij}^{N3}, y_{ij}^{N4}, y_{ij}^{N5}; \theta_{ij}^N, \sigma_{ij}^N, \mu_{ij}^N)$ is the PNN cost from sources to destination.

The steps of the proposed method are as follows:

Step 1. Considering the pentagonal Neutrosophic parameters and variables, the *problem (3)* may be written as:

$$\text{Min } Z=\sum_{i=1}^{s}\sum_{j=1}^{t}(c_{ij}^{N1},c_{ij}^{N2},c_{ij}^{N3},c_{ij}^{N4},c_{ij}^{N5};\theta_{ij}^{N},\sigma_{ij}^{N},\mu_{ij}^{N})\otimes(y_{ij}^{N1},y_{ij}^{N2},y_{ij}^{N3},y_{ij}^{N4},y_{ij}^{N5};\theta_{ij}^{N},\sigma_{ij}^{N},\mu_{ij}^{N}) \quad (4)$$

Subject to constraints

$$\sum_{j=1}^{t}(y_{ij}^{N1},y_{ij}^{N2},y_{ij}^{N3},y_{ij}^{N4},y_{ij}^{N5};\theta_{ij}^{N},\sigma_{ij}^{N},\mu_{ij}^{N})=(p_{i}^{N1},p_{i}^{N2},p_{i}^{N3},p_{i}^{N4},p_{i}^{N5};\theta_{i}^{N},\sigma_{i}^{N},\mu_{i}^{N}), i=1,2,...,s. \quad (5)$$

$$\sum_{i=1}^{s}(y_{ij}^{N1},y_{ij}^{N2},y_{ij}^{N3},y_{ij}^{N4},y_{ij}^{N5};\theta_{ij}^{N},\sigma_{ij}^{N},\mu_{ij}^{N})=(q_{j}^{N1},q_{j}^{N2},q_{j}^{N3},p_{j}^{N4},p_{j}^{N5};\theta_{j}^{N},\sigma_{j}^{N},\mu_{j}^{N}), j=1,2,...,t$$
$$y_{ij}^{N1},y_{ij}^{N2},y_{ij}^{N3},y_{ij}^{N4},y_{ij}^{N5};\theta_{ij}^{N},\sigma_{ij}^{N},\mu_{ij}^{N}\geq 0 \quad \forall i,j. \quad (6)$$

Step 2. Here, we confirmed whether the available model is balanced or not, i.e., demand=supply (or) $\sum_{i=1}^{s}\tilde{q}_{j}^{N}=\sum_{j=1}^{t}\tilde{p}_{i}^{N}$. If not, then add dummy variables on row or column and make it balanced model.

Step 3. With the help of accuracy function $\Re$, we transform the supply, demand and transportation cost as the following model:

$$\text{Min } Z=\Re(\sum_{i=1}^{s}\sum_{j=1}^{t}(c_{ij}^{N1},c_{ij}^{N2},c_{ij}^{N3},c_{ij}^{N4},c_{ij}^{N5};\theta_{ij}^{N},\sigma_{ij}^{N},\mu_{ij}^{N})\otimes(y_{ij}^{N1},y_{ij}^{N2},y_{ij}^{N3},y_{ij}^{N4},y_{ij}^{N5};\theta_{ij}^{N},\sigma_{ij}^{N},\mu_{ij}^{N})). \quad (7)$$

Subject to constraints

$$\Re(\sum_{j=1}^{t}(y_{ij}^{N1},y_{ij}^{N2},y_{ij}^{N3},y_{ij}^{N4},y_{ij}^{N5};\theta_{ij}^{N},\sigma_{ij}^{N},\mu_{ij}^{N}))=\Re((p_{i}^{N1},p_{i}^{N2},p_{i}^{N3},p_{i}^{N4},p_{i}^{N5};\theta_{i}^{N},\sigma_{i}^{N},\mu_{i}^{N})), i=1,2,...,s. \quad (8)$$

$$\Re(\sum_{i=1}^{s}(y_{ij}^{N1},y_{ij}^{N2},y_{ij}^{N3},y_{ij}^{N4},y_{ij}^{N5};\theta_{ij}^{N},\sigma_{ij}^{N},\mu_{ij}^{N}))=\Re(q_{j}^{N1},q_{j}^{N2},q_{j}^{N3},p_{j}^{N4},p_{j}^{N5};\theta_{j}^{N},\sigma_{j}^{N},\mu_{j}^{N}), j=1,2,...,t$$
$$y_{ij}^{N1},y_{ij}^{N2},y_{ij}^{N3},y_{ij}^{N4},y_{ij}^{N5};\theta_{ij}^{N},\sigma_{ij}^{N},\mu_{ij}^{N}\geq 0 \quad \forall i,j. \quad (9)$$

Step 4. After using our new accuracy function, we get PNTP into crisp transportation problem.

Step 5. Now find initial basic feasible solution.

- To determine the penalty, subtraction between smallest unit and next to smallest unit in the row (column).
- Identify the largest penalty in row/column, and make the allotment in the cell having the least unit cost.
- If the largest penalty arises in more than one row/column, then select topmost row/left side column.
- When the rows (column) have zero supply and demand until (m+n-1), then stop. Otherwise go to first line.

Step 6. Substitute the all y_{ij} in the objective function, we get the transportation cost.

4. Numerical Example

In our literature study, we got there is no method to solve PNTP. There is a lot of scope in this area to develop new method. We take the advantages in the field of pentagonal Neutrosophic area and we focus to start a developing new algorithm for solving PNTP. The main limitations in between fuzzy and Neutrosophic numbers is that fuzzy numbers taken only membership function (truth degree) however, Neutrosophic number taken truth, indeterminacy & falsity degree. In this segment, we consider another strategy to solve PNLP problem and compare with fuzzy pentagonal LP problem. To prove the relevance and proficiency of our proposed strategy, we consider the fuzzy problem which introduced by [20, 47].

Example 1. (Real-life problem) [47]. In Odisha, India have a company named M/s. Ashirivad dress pvt. Ltd. and the organisation has three plants for delivering dress. The dresses ought to be transport to three ware house under pentagonal Neutrosophic numbers. The conditions of transportation problem are presented in *Table 2*. As the problem should be PNN therefore, the decision-maker considers the confirmation degree of pentagonal number is (1,0,0).

Table 2. *Cost of unit for pentagonal Neutrosophic transportation problem*.

Ware house				**Supply**
Factories	**Bhubaneswar**	**Cuttack**	**Rourkela**	
Asha	*(5,10,13,14,18;1,0,0)*	*(1,2,3,4,5;1,0,0)*	*(2,6,8,10,14;1,0,0)*	*(2,11,23,34,45;1,0,0)*
Omm	*(3,4,5,6,7;1,0,0)*	*(1,5,6,7,11;1,0,0)*	*(1,4,5,9,16;1,0,0)*	*(10,47,52,65,76;1,0,0)*
Disha	*(3,6,9,12,15;1,0,0)*	*(2,5,7,8,8;1,0,0)*	*(1,1,1,1,1;1,0,0)*	*(3,18,56,76,87;1,0,0)*
Demand	*(11,16,51,67,75;1,0,0)*	*(20,40,60,80,100;1,0,0)*	*(15,30,45,75,110;1,0,0)*	

Step 1. Now using our new ranking function, the issue of PNTP is converting into crisp transportation problem. The model is now available in *Table 3*.

Table 3. *The defuzzified pentagonal Neutrosophic transportation problem.*

Ware house				**Supply**
Factories	**Bhubaneswar**	**Cuttack**	**Rourkela**	
Asha	*12*	*3*	*8*	*25*
Omm	*5*	*6*	*7*	*50*
Disha	*9*	*6*	*1*	*48*
Demand	*44*	*60*	*55*	

Step 2. To check whether the model is balanced or not.

Supply $\sum a_i = 23 + 50 + 48 = 121.$

Demand $\sum b_i = 44 + 60 + 55 = 159.$

Table 4. *Balanced transportation problem.*

Ware house Factories	Bhubaneswar	Cuttack	Rourkela	Supply
Asha	*12*	*3*	*8*	*23*
Omm	*5*	*6*	*7*	*50*
Disha	*9*	*6*	*1*	*48*
Demand	*44*	*60*	*55*	

Step 3. We use our algorithm (Step 5, Line 1) for finding the initial basic feasible solution.

Table 5. *Initial penalties allocation.*

Ware house Factories	Bhubaneswar	Cuttack	Rourkela	Supply	Penalty
Asha	*12*	*3*	*8*	*23*	*5*
Omm	*5*	*6*	*7*	*50*	*1*
Disha	*9*	*6*	*1*	*48*	*5*
Dummy	*0*	*0*	*0*	*38*	*0*
Demand	*44*	*60*	*55*		
Penalty	*5*	*3*	*1*		

Table 6. *After strike out of 3rd Row the penalties allocation.*

Ware house Factories	Bhubaneswar	Cuttack	Rourkela	Supply	Penalty
Asha	*12*	*3*	*8*	*23*	*5*
Omm	*5*	*6*	*7*	*50*	*1*
Disha	*9*	*6*	*1*	*48*	*5*
Dummy	*0*	*0*	*0*	--	--
Demand	*6*	*60*	*55*		
Penalty	*4*	*3*	*6*		

Table 7. *3rd Penalties allocation.*

Ware house Factories	Bhubaneswar	Cuttack	Rourkela	Supply	Penalty
Asha	*12*	*3*	*8*	*23*	*5*
Omm	*5*	*6*	*7*	*50*	*1*
Disha	*9*	*6*	*1*	--	--
Dummy	*0*	*0*	*0*	--	--
Demand	*6*	*60*	*7*		
Penalty	*7*	*3*	*1*		

Table 8. *4th Penalties allocation.*

Ware house Factories	Bhubaneswar	Cuttack	Rourkela	Supply	Penalty
Asha	*12*	*3*	*8*	*23*	*5*
Omm	*5*	*6*	*7*	*44*	*1*
Disha	*9*	*6*	*1*	--	--
Dummy	*0*	*0*	*0*	--	--
Demand	--	*60*	*7*		
Penalty	--	*3*	*1*		

Table 9. *Final allocation.*

Ware house Factories	Bhubaneswar	Cuttack	Rourkela	Supply	Penalty
Asha	*12*	*3*	*8*	--	--
Omm	*5*	*6*	*7*	*7*	*7*
Disha	*9*	*6*	*1*	--	--
Dummy	*0*	*0*	*0*	--	--
Demand	--	--	*7*		
Penalty	--	--	*7*		

The maximum penalty occurs, 7 in row 2.

The minimum c_{ij} in this row is c_{23} =7.

The maximum allocation in this cell is min (7,7)=7.

It is also satisfy the supply of row 2 (Omm) and demand in column 3 (Rourkela).

Table 10. *Initial basic feasible solution.*

Ware house Factories	Bhubaneswar	Cuttack	Rourkela	Supply
Asha	*12*	*3*	*8*	*23*
Omm	*5*	*6*	*7*	*50*
Disha	*9*	*6*	*1*	*48*
Dummy	*0*	*0*	*0*	*38*
Demand	*44*	*60*	*55*	

The minimum transportation cost is obtained as:

$$\text{Min} = (23\times 3) + (6\times 5) + (37\times 6) + (7\times 7) + (48\times 1) + (38\times 0) = 418.$$

Here, the number of allocated cells=6 which is equal to m+n-1=4+3-1=6

Therefore, this solution is non-degenerate.

Table 11. *Comparison of proposed method with existing method of example 1.*

	Transportation Cost
Proposed Method	*418*
Existing Method [20]	*418*

Example 2 [47]. Consider the pentagonal Neutrosophic numbers (supply, demand & transportation cost) are presented in *Table 12*.

Table 12. *Input data for pentagonal transportation problem.*

Factories	Bhubaneswar	Cuttack	Supply
Protein	*(0.2,0.4,0.5,0.6,0.7)*	*(0.3,0.2,0.6,0.5,0.1)*	*(0.1,0.2,0.5,0.4,0.3)*
Calories	*(0.7,0.8,0.6,0.9,0.1)*	*(0.2,0.3,0.5,0.7,0.1)*	*(0.2,0.3,0.5,0.8,0.9)*
Demand	*(0.8,0.7,0.5,0.3,0.2)*	*(0.2,0.3,0.4,0.1,0.2)*	

Here, the decision-makers consider the degree of each pentagonal number is (1,0,0).

After executing the steps of our algorithm, we get the initial basic feasible solutions presented in *Table 13*. This is balanced transportation problem.

Table 13. *Final initial basic feasible solution.*

Factories	Bhubaneswar	Cuttack	Supply
Protein	*0.48*	*0.34*	*0.3*
Calories	*0.62*	*0.36*	*0.54*
Demand	*0.5*	*0.34*	

The minimum transportation cost is obtained as:

$$\text{Min} = (0.3 \times 0.48) + (0.2 \times 0.62) + (0.34 \times 0.36) = 0.3904.$$

Here, the number of allocated cells=3 which is equal to m+n-1=2+2-1=3

Therefore, this solution is non-degenerate.

Table 14. *Comparison of proposed method with existing method of example 2.*

	Transportation cost
Proposed Method	*0.3904*
Existing Method [20]	*0.41*

5. Analysis and Observation of the Proposed Model

5.1. Observation

Due to non-availability of pentagonal transportation problem under Neutrosophic environment, there is no direct comparison made in this paper. Therefore, we consider pentagonal Neutrosophic transportation problem under fuzzy environment for comparison our result. Hence, we compared our proposed method with the existing method [20].

For *Example 1* the pentagonal Neutrosophic transportation cost of IBFS is 418, which is exactly the transportation cost of fuzzy pentagonal numbers. In *Example 2*, the pentagonal Neutrosophic transportation problem is 0.3904, which is not exactly the cost of fuzzy pentagonal transportation problem i.e. 0.41. The decision-makers always want to minimize the cost of transportation when supplying the materials. Thusly, we can say that our proposed technique under Neutrosophic environment is always better than the other existing method. We also depicted our result along with the existing method results in graphical representation i.e. *Figs. (1)-(2)*.

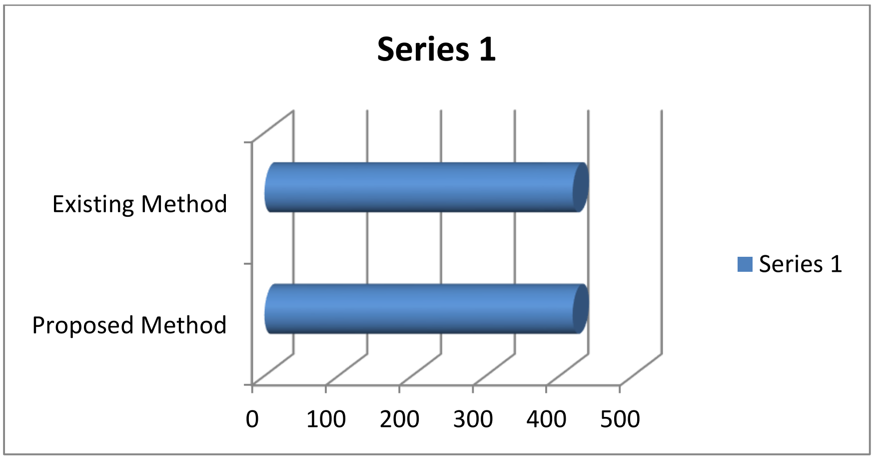


Fig. 1. *Graphical analysis of our proposed method with existing method of example 1.*

From the above analysis of both tabular form and graphical form, we can finalise that our proposed method is better to the existing method. Further, we can also claimed that our transportation cost obtained by our proposed method always lie within region of Neutrosophic sets.

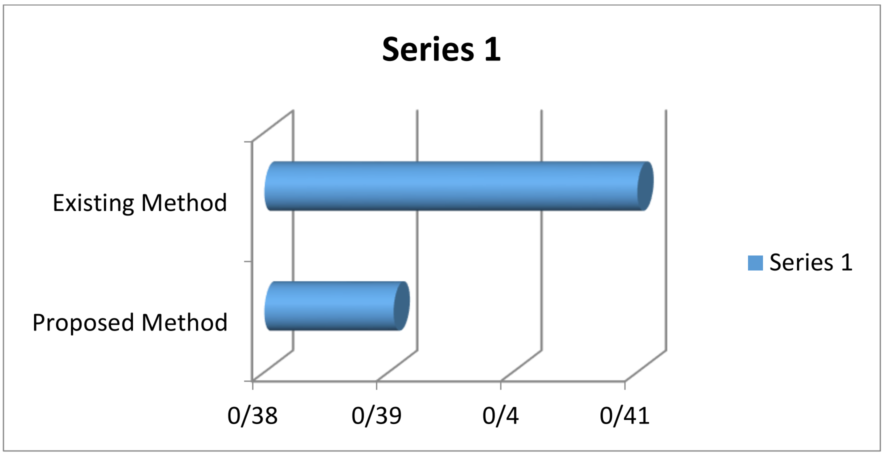


Fig. 2. *Graphical analysis of proposed work with existing work of example 2.*

5.2. Advantages of the Proposed Model

The pentagonal fuzzy numbers were widely applied in transportation problem to get the minimum cost. However, the decision-makers always consider the truth degree of pentagonal fuzzy numbers, which is the main drawback. In real-life problem, the decision-makers always want the clarity data means truth degree, indeterminacy degree and falsity degree. Neutrosophic sets consider the degree of truth, indeterminacy and falsity and we take the advantage of the properties of Neutrosophic sets, we develop a new algorithm for solving pentagonal Neutrosophic transportation problem. We proposed a new score function of Neutrosophic pentagonal numbers and also developed a new technique for getting initial basic feasible solutions. In our problem, our transportation cost is always minimizing then other existing method and minimization the cost is required for decision-makers. We also solved our problem in LP model by using LINGO 18 version or MATLAB, we get the same result.

In the above conversation, we can infer that our proposed calculation is another approach to deal with the vulnerability and indeterminacy in the transportation issue.

6. Conclusions

The transportation problem is one of the most popular optimization problems in operation research. The main objective of this problem is finding the minimum cost of transportation to supplier and demand. In this paper, Neutrosophic transportation problem has been solved under pentagonal Neutrosophic numbers. We also developed a score function and applied to find the IBFS. In the computation point of view, our proposed method is very easier when applied in real-life problem. Further comparative analysis is done with fuzzy pentagonal transportation problem. Also, the proposed algorithm has less computational complexity and saves time. By comparing our method with fuzzy method, we can conclude that our method can handle any type of uncertainties arise in real-life situation and it is very simple & efficient than other uncertainty. In addition to our proposed method, it will be extend in application of pentagonal assignment problem, pentagonal linear fractional programming and pentagonal job scheduling problem.

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Precise Services and Supply Chain Prioritization in Manufacturing Companies Using Cost Analysis Provided in a Fuzzy Environment

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ABSTRACT

In recent years, management and, consequently, supply chain performance measurement, has attracted the attention of a large number of managers and researchers in the field of production and operations management. In parallel with the evolution of organizations from a single approach to a network and supply chain approach, performance measurement systems have also changed and moved towards network and supply chain performance measurement. Therefore, in order to face the storm of great change and transformation and not give in to the wave of competitive aggression, organizations have long had one thing in common, and that is to focus approaches and focus efforts towards achieving results. Results that lead to a competitive advantage and are more effective and decisive in the performance indicators of the organization, including earning more. In this study, in order to identify and prioritize the factors affecting the supply chain in manufacturing companies, using indicators such as cost, timely delivery and procurement time to evaluate the supply chain efficiency is considered. And performance evaluation was performed at the manufacturer level. Therefore, in order to evaluate the performance of the supply chain using the AHP integration approach and the DEA method approach in the fuzzy environment, the suppliers and suppliers of the manufacturing company were evaluated and ranked in terms of performance.

1. Introduction

In recent years, management and, consequently, supply chain performance measurement, has attracted the attention of a large number of managers and researchers in the field of production and operations management [1]. In parallel with the evolution of organizations from a single approach to network approach and supply chain, performance measurement systems have also changed and moved towards measuring network performance and supply chain [2]. This attitude is rooted in the thinking of a system in which the efficiency of any production system does not depend only on the optimal functioning of a subsystem, and all subsystems must work diligently to achieve the pre-drawn goals [1]. Supply chain management is one of the components of competitive strategies for organizational productivity and profitability. Managers in many industries, especially those in the manufacturing sector, try to better

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manage the supply chain and evaluate its performance [3]. Therefore, it is important to evaluate and track the performance of its supply chain because several organizations are involved in this chain [4]. Many critical and complex barriers may distract current performance measurement systems from providing significant assistance to improve and expand supply chain management. Due to this inherent complexity, it is necessary to select the appropriate criteria for evaluating the performance of the supply chain [5]. In this chapter, while presenting the problem statement, the topics related to the necessity of conducting research, the theoretical framework of research (model and definition of variables), research objectives, research questions, research scope and limitations are also described. To deal with the storm of change and massive transformation and not to give in to the wave of competitive aggression, organizations have long had one thing in common, and that is to focus approaches and focus all efforts on achieving results; Results that lead to competitive advantage and are more effective and decisive in the performance indicators of the organization, including earning more revenue. Knowing that we are in the age of information and competition between organizations, and every organization to create a new way to transform its organization to surpass its competitors and maintain and gain a competitive advantage. As well as the important role that efficiency plays in the development of societies; examining all its dimensions, especially in the form of mathematical analysis, as a criterion for measuring performance is inevitable [6]. Manufacturing organizations need a high degree of flexibility in order to maintain a competitive advantage as well as to operate in an ever-changing dynamic environment. The success of organizations depends on their ability to deliver outputs. Optimal presentation of products according to criteria such as cost, quality, performance, delivery, flexibility and innovation depends on the ability of the organization to manage the flow of materials, information, etc. inside and outside the organization [7]. Supply chain evaluation is done using different methods. Data envelopment analysis as a non-parametric method is based on linear programming technique and compares the efficiency of different units. Wen et al. [8] provide evidence and reasons that data envelopment analysis is a good way to manage supply chain. Data envelopment analysis can have multi-cell inputs and outputs and uses quantitative and qualitative indicators [8]. Data envelopment analysis is a method to evaluate the performance of organizations in the private and public sectors. The reason for using data analysis as a way to evaluate performance is the complex nature of the relationships between multiple inputs and outputs in activities [9].

In this paper, indicators such as cost, timely delivery and procurement time are considered to evaluate the efficiency of the supply chain and performance evaluation is done at the manufacturer level, while usually looking at the supply chain as a system and overview. This means that performance appraisal indicators are measured for the manufacturer (second level of the chain) and in relation to the supplier and the customer, and the overall supply chain is maintained. Therefore, in this study, we seek to answer the question, how to identify and prioritize the factors affecting the supply chain in manufacturing companies using Data Envelopment Analysis (DEA) in a fuzzy environment? The analytical models proposed to evaluate supply chain efficiency include a variety of techniques, from simple rhythmic scoring methods to complex mathematical scheduling, and from definitive evaluation models to models under uncertainty conditions. Recently, various methods have been proposed to address the issue of supply chain efficiency assessment. 16 categories of these methods are presented by Estampe et al. [10]. To evaluate the efficiency of the supply chain, various indicators are measured in categories such as cost, time, profit, level of service and. Thomas and Griffin [11] equated transportation with more than half the cost of a supply chain and used it for evaluation.

Lee and Billington [12] consider the level of customer satisfaction in companies with customers from all over the world as an important factor and point out that the strategies adopted will not be very costly in order to achieve customer satisfaction. Most existing studies are based on evaluating the supply chain efficiency of a comprehensive evaluation index system. However, most of these methods use the individuals themselves to calculate the weight of the indicators in the evaluation process. Due to personal opinions, the weight of the indicators cannot be measured accurately [13].

To reduce the inaccuracy of the index weight, which is increased by the decision maker's personal opinion, data envelopment analysis is used as a non-parametric method to evaluate supply chain efficiency. The main feature of overlay analysis is that it can measure performance when there are multiple inputs and outputs. Wang and Wang [14] presented a data envelopment analysis model using indicators such as cost, on-time delivery, profit, and production time cycle. Given that some of the indicators measured in the supply chain, especially costs are not definitive and indices of uncertainty are seen in them, the use of uncertainty methods such as fuzzy logic seems appropriate [15]. Until now, uncertainty methods have rarely been used to evaluate the supply chain [14]. The methods of uncertainty used can be referred to uneven sets [15]. In this reference, by developing a rugged set of indicators such as cost, number of employees, production flexibility and level of service have been used to evaluate efficiency. In this research, fuzzy data envelopment analysis is used to evaluate the efficiency of supply chains, which has not been used in previous research. In this paper, the supply chain is considered as a whole and a system that the inputs and outputs of the fuzzy data envelopment analysis model are the same as the manual inputs and outputs of the supply chain. Evaluation indicators are measured at the manufacturer level and to maintain the integrity of the supply chain, the indicators are measured for the manufacturer and by maintaining its relationship with suppliers and customers. In this paper, the cost index is considered fuzzy due to the uncertainty present during the measurement. To deal with the uncertainty environment created by the cost index area, fuzzy set theory is used as a method to deal with uncertainty environments. Considering that no conceptual model is presented in this research, then there is no hypothesis in this research, but the assumptions for conducting the research are as follows:

- Information received from suppliers is fuzzy uncertainty.
- Suppliers are evaluated in the company's supply chain list and are accepted in the initial and technical evaluation.

Saleh and Shafiei [16], in a study entitled "Performance evaluation using envelopment analysis of three-level data" state that, attention to organizational performance evaluation in recent years has led to the development of several frameworks and methodologies, each of which has provided a wide range of benefits. One of the appropriate methods in calculating the efficiency of data envelopment analysis is that despite some limitations, it is a powerful methodology that allows managers to determine the efficiency of the organization under their management compared to other units. In the real world, we encounter different situations that follow a hierarchical structure with decentralized decisions. In this research, the efficiency of supply chains that have a hierarchical structure will be evaluated and a three-level model of data envelopment analysis will be presented by selecting appropriate indicators.

Koushki and Mashayekhi Nezamabadi [17], in a study entitled "A method of network data envelopment analysis to evaluate supply chains and its application in pharmacy" state that, data envelopment analysis is a non-parametric technique based on mathematical programming to evaluate the performance of heterogeneous decision-making units. Many units have a multi-stage structure in which the output of one stage is the input of the next stage. A supply chain, which includes several members such as supplier and manufacturer, has a multi-step process. In this paper, for the first time, network methods for achieving maximum productivity in supply chains, which are considered as a multi-stage system, are introduced. Such a view provides management concepts to improve the efficiency of the supply chain as well as the productivity of each member.

Mousavi and Ahmadzadeh [18], in a study entitled "Study and evaluation of supply chain efficiency using data envelopment analysis (Case study: Amol paper companies)" state that, rapid progress and development, rapid environmental changes, and awareness of new developments and approaches to achieve efficiency and effectiveness in organizations have become essential. In recent years, supply chain management and performance evaluation has received more attention in the business administration of organizations. In this study, the operational performance of seven supply chains operating in the same industry and having different key companies and relatively similar suppliers and distributors has been evaluated using data envelopment analysis method. In order to evaluate the supply chain in this research, indicators such as direct costs, manpower, and depreciation have been considered.

Hosseinzadeh Seljooghi and Rahimi [19], in a study entitled "Evaluation of efficiency and efficiency at the scale of the supply chain of the Iranian resin industry with the model of definitive and fuzzy data envelopment analysis" state, the fuzzy DEA model is used based on the cut-off approach to measure efficiency and determine the supply chain scale efficiency. The proposed ideas have been used to evaluate the efficiency and efficiency of the supply chain scale of 27 resin production companies. In evaluating with definitive data, 6 companies are network efficient; while in the case of fuzzy data, three companies are network efficient. These companies have managed and coordinated the flow of materials between several organizations and within the organization in the most optimal way and with regard to environmental issues.

Samuelinko stated in 2013 that the competitive nature of the business environment requires the awareness of productivity-based organizations of the relative level of effectiveness and efficiency of their competitors. This indicates, firstly, the need for an effective mechanism that allows the discovery of appropriate productivity models to improve overall organizational performance, and secondly, the need for a feedback mechanism that allows the evaluation of different productivity models to select the most appropriate model. In this article, we focus on organizations that consider the state of the internal organizational environment (for example, likely to represent a resource-oriented perspective) and external (for example, likely to represent a positioning perspective) in formulating their strategies. We propose and test a DEA-based Decision Support System (DSS) that aims to evaluate and manage the relative performance of such organizations [20].

Singh stated in 2014 that manpower in an organization is an important and fundamental asset. Qualified personnel have unique academic and managerial abilities in specific disciplines and individual capabilities that can perform many of the different marketing and research tasks required in any organization because they are, in fact, the creditors of the organization's performance. They forgive. Therefore, designing rational methods for assessing the capability of personnel during employment is crucial. The methods that are commonly used for decision making in identifying functional characteristics, including their heavy tasks, include methods such as Delphi and decision matrix, Hierarchical Analysis Process (AHP), and so on. The AHP method converts experts' qualitative theory into quantitative values and creates a decision matrix. In this paper, in this study, the Data Push Analysis method is investigated to establish the internal weights of alternative methods by comparing two-by-two comparison matrices in AHP for a three-property system to measure personnel performance at levels. Login to the management hierarchy is used. Several expert judgments have been made to determine the weight of the features. In conclusion, the SUPER EFFICIENCY DEA (or DEA-AHP combination method) is proposed in this paper as an alternative to traditional weight derivation methods in AHP [21].

Comelli et al. [22] have proposed an approach for evaluating production planning in supply chains. They noted that production planning evaluations are usually based on physical parameters such as inventory level and demand satisfaction. They found it useful to add financial valuation to classical models. They applied an ABC method to estimate the cash flow of supply chain production planning.

In 2016, Lim stated that supplier selection is an important issue that supply chain managers have faced for many years. Choosing the right suppliers is no longer as easy as choosing (based on the price) they offer. There are many quantitative and qualitative criteria that must be considered. Therefore, there is an urgent need for an approach that can meet these criteria. In addition, as supply chains become increasingly important today, it is important to consider the risks of inadequate supply in evaluating suppliers. This research presents an approach that focuses mainly on data envelopment analysis to analyze and compare the relative performance of suppliers. Because data envelopment analysis can only cover quantitative features, the Analytic Hierarchy Process (AHP) is used to aid qualitative analysis. Risks are also considered in the evaluation of suppliers. The purpose of the proposed approach is to provide a comprehensive approach to addressing the issue of supplier selection [23].

Liang et al. [24] identified two barriers to supply chain evaluation and its members in the form of multiple indicators that determine member performance and the existence of conflict between chain members. They showed that the classical DEA model could not perform as well as the mosque due to the presence of intermediate indicators, so in their research they have developed several DEA-based models in which intermediate indicators are integrated in performance evaluation. They developed their model as a two-chain, seller-buyer model. They considered two different modes. The first mode is that one chain acts as the leader and the second chain follows it. The leader is evaluated using member results. The second case is in the form of a partnership in which an attempt is made to maximize the joint efficiency of the two chains, which is considered as their average efficiency. In this case, both supply chains are evaluated simultaneously.

In his research, Chen [25] divided supply chain evaluation criteria into two main categories: quantitative and qualitative. Quantitative indicators include cost and resource use, and qualitative indicators include quality, flexibility, visibility, trust and innovation. He then states the measurement criteria for each of these seven categories of indicators and then uses the AHP technique to identify the most important indicators for the electronics industry. He also made suggestions for other industries.

Easton et al. [26] evaluated the evaluation of purchasing sector efficiency in the supply chain. They pointed out that it is very difficult to measure the efficiency of the procurement department and compare that efficiency with other departments of procurement, and attributed this difficulty to the lack of acceptable measurement criteria and appropriate methods to integrate these criteria and provide an overall efficiency. They developed a DEA model to evaluate purchasing efficiency in the petrochemical industry.

2. Methodology

In this study, according to the parts intended to provide an efficient supply chain, first, according to the conditions governing the production of these parts, all suppliers in this field are identified and we put one of the basic and serious principles in the list of suppliers with contract priority. In the supply of these parts, the reduction of risk arises from the selection of the supplier, which in the event of a mistake will incur irreparable losses, which will lead to the failure of the project. In order to conclude a contract for the supply of these parts, it is necessary to prove the efficiency of the supplier in the first stage and to be ranked according to the rank in which they are placed in the next step. In order to evaluate the efficiency of suppliers, it is necessary to measure the input to output ratio of each supplier, and for this issue, according to the main source of this research, the Super Efficiency DEA method has been used. Therefore, it can be said that this research is applied based on the purpose and descriptive-survey based on the nature and method of research. The data collected to solve the research model are related to the years 2019-2020. In the present study, two library and field methods have been used to collect information. In order to collect information in this research, first the documentary method will be used. In order to study and obtain more information in order to know more precisely the subject of research and use the findings of research in this field, the researcher to study and study academic dissertations, foreign and Iranian books, Persian and English journals and textbooks Some professors pay. This research is in the field of measuring the efficiency of supply chains of a manufacturing company and examines the separation of efficient and inefficient chains, determining the appropriate pattern for inefficient units, as well as how to allocate resources optimally. The present study is conducted to investigate the efficiency of supply chains in manufacturing and industrial companies.

2.1. Identify Supplier Evaluation Indicators

In the first step of the research, after reviewing and identifying the suppliers, the first phase of the evaluation begins by selecting appropriate indicators for evaluation. In this section, after reviewing the written scientific texts, the evaluation indicators were identified as *Table 1*.

Table 1. *Supplier survey indicators.*

Row	Description of the index
1	*Price product*
2	*Place of delivery*
3	*Quality systems certifications*
4	*After sales service indicators*
5	*Customization capability*
6	*Product quality*
7	*Ability to reduce costs*
8	*Packing*

In this study, the verbal variables to determine the importance of the indicators are fuzzified according to the triangular fuzzy numbers in *Table 2* and *Fig. 1*.

Table 2. *Fuzzification of verbal variables in Delphi technique*.

Verbal Variables	Triangular Fuzzy Numbers
Very little importance	*(0.25,0,0)*
Low importance	*(0.5,0.25,0)*
Medium importance	*(0.75,0.5,0.25)*
Important	*(1,0.75,0.5)*
Very important	*(1,1,0.75)*

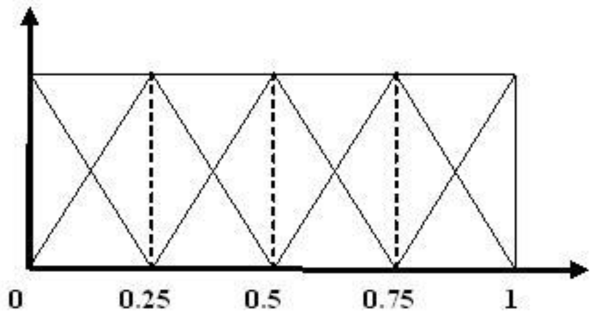

Fig. 1. Triangular fuzzy numbers.

3. Findings

3.1. Introducing the Company's Suppliers

According to the scope of work of the manufacturing company and also the studies carried out in accordance with the executive instructions of the company, 10 suppliers have been selected as the final candidate supply chain for evaluation and transfer of supply of parts. Suppliers are as follows:

- Sepehr Ryan Sanhat Company (Symbol: A).
- Cheese Company (symbol: B).
- Parsian Sazeh Sepahan Company (Symbol: C).
- Techno Sanat Company (symbol: D).
- Peyman Sanat Company (Symbol: E).
- Tractor Manufacturing Company (Symbol: F).
- Ataco Company (Symbol: G).
- Sarco Company (symbol: H).
- Beshl Motor Company (Symbol: I).
- Iran Casting Company (Symbol: K).

3.2. Introducing the Experts of the Research

In this research, in order to evaluate the indicators and select them, using the opinion of the company's experts, the specifications of the experts are as *Table 3*.

Table 3. *The Specifications of the experts.*

Row	Side	Work Experience	Education	Age
1	*plan and program manager*	*18 years*	*MA*	*54 years*
2	*Supply management*	*20 years*	*Bachelor*	*48 years*
3	*Procurement manager*	*10 years*	*Bachelor*	*38 years*
4	*Quality assurance management*	*20 years*	*Bachelor*	*48 years*
5	*Market research and development management*	*10 years*	*Doctorate*	*35 years*
6	*Engineering management*	*23 years*	*Bachelor*	*45 years*
7	*Laboratory management*	*20 years*	*MA*	*55 years*

3.3. Identification, Refining and Screening of Input and Output Indicators with Fuzzy Technique

First, based on the research literature and specialized interviews, a set of input and output indicators of DMUs has been identified. Fuzzy technique was used for screening and final confirmation of the indicators. The indicators are symbolized in *Table 4*.

Table 4. *Symbolization of indicators.*

Symbol	Description of the Index
i1	*price product*
i2	*Place of delivery*
i3	*Quality systems certifications*
i4	*After sales service indicators*
i5	*Customization capability*
O1	*Product quality*
O2	*Ability to reduce costs*
O3	*Packing*

The views of seven experts to measure the importance of the indicators related to each of the input and output indicators are as *Table 5*.

Table 5. *Experts' views about each indicator.*

Symbol	Expert 1	Expert 2	Expert 3	Expert 4	Expert 5	Expert 6	Expert 7
i1	*very much*	*much*	*much*	*very much*	*medium*	*medium*	*medium*
i2	*very much*	*medium*	*medium*	*very much*	*much*	*very much*	*much*
i3	*very much*	*very much*	*much*	*very much*	*much*	*much*	*much*
i4	*very much*	*very much*	*medium*	*very much*	*much*	*medium*	*much*
i5	*much*	*very much*	*medium*	*much*	*very much*	*medium*	*very much*
O1	*medium*	*medium*	*medium*	*medium*	*very much*	*much*	*very much*
O2	*much*	*very much*	*much*	*much*	*very much*	*much*	*very much*
O3	*very much*	*much*	*medium*	*very much*	*very much*	*much*	*very much*

The collected data are fuzzy evaluated according to the *Table 5*. The fuzzy values of the experts' point of view are shown in *Table 6*.

Table 6. *Fuzzified values of the seven experts' views about each indicator.*

Symbol	**Expert 1**			**Expert 2**			**Expert 3**			**Expert 4**		
i1	*1*	*1*	*0.75*	*1*	*0.75*	*0.5*	*1*	*0.75*	*0.5*	*1*	*1*	*0.75*
i2	*1*	*1*	*0.75*	*0.75*	*0.5*	*0.25*	*0.75*	*0.5*	*0.25*	*1*	*1*	*0.75*
i3	*1*	*1*	*0.75*	*1*	*1*	*0.75*	*1*	*0.75*	*0.5*	*1*	*1*	*0.75*
i4	*1*	*1*	*0.75*	*1*	*1*	*0.75*	*0.75*	*0.5*	*0.25*	*1*	*1*	*0.75*
i5	*1*	*0.75*	*0.5*	*1*	*1*	*0.75*	*0.75*	*0.5*	*0.25*	*1*	*0.75*	*0.5*
O1	*1*	*0.75*	*0.5*	*1*	*1*	*0.75*	*0.75*	*0.5*	*0.25*	*1*	*0.75*	*0.5*
O2	*1*	*1*	*0.75*	*1*	*0.75*	*0.5*	*1*	*0.75*	*0.5*	*1*	*1*	*0.75*
O3	*0.75*	*0.5*	*0.25*	*1*	*0.75*	*0.5*	*0.75*	*0.5*	*0.25*	*0.75*	*0.5*	*0.25*

Continue of table 6

Symbol	**Expert 5**			**Expert 6**			**Expert 7**		
i1	*0.75*	*0.5*	*0.25*	*0.75*	*0.5*	*0.25*	*0.75*	*0.5*	*0.25*
i2	*1*	*0.75*	*0.5*	*1*	*1*	*0.75*	*1*	*0.75*	*0.5*
i3	*1*	*0.75*	*0.5*	*1*	*0.75*	*0.5*	*1*	*0.75*	*0.5*
i4	*1*	*0.75*	*0.5*	*0.75*	*0.5*	*0.25*	*1*	*0.75*	*0.5*
i5	*1*	*1*	*0.75*	*0.75*	*0.5*	*0.25*	*1*	*1*	*0.75*
O1	*1*	*1*	*0.75*	*1*	*0.75*	*0.5*	*1*	*1*	*0.75*
O2	*1*	*1*	*0.75*	*1*	*1*	*0.75*	*1*	*1*	*0.75*
O3	*1*	*0.75*	*0.5*	*1*	*0.75*	*0.5*	*1*	*0.75*	*0.5*

In the next step, the fuzzy average of expert opinions is calculated. In the following work is used to defuzzificate and determine the importance of input and output indicators. The fuzzy mean and the definite value of the values related to the indicators are shown in *Table 7*. Since the definite value of all values is greater than 0.5, all indices are confirmed.

Table 7. *The fuzzy average of experts' opinions and the definite amounts of the indicators' values.*

Symbol	**Description of the index**	**Fuzzy average**	**Definite amount**
i1	*Price product*	*(0.46,0.71,0.89)*	*0.70*
i2	*Place of delivery*	*(0.54.0.79.0.93)*	*0.77*
i3	*Quality systems certifications*	*(0.61,0.86,1)*	*0.84*
i4	*After sales service indicators*	*(0.54.0.79.0.93)*	*0.77*
i5	*Customization capability*	*(0.54.0.79.0.93)*	*0.77*
O1	*Product quality*	*(0.57,0.82,0.96)*	*0.80*
O2	*Ability to reduce costs*	*(0.68,0.93,1)*	*0.90*
O3	*Packing*	*(0.39,0.64,0.89)*	*0.64*

3.4. Pairewise Comparison of Suppliers Based on Input and Output Indicators

In this step, according to the identification of input and output indicators of each supplier, we prioritize suppliers using pairwise comparison based on each indicator.

3.4.1. Prioritization of suppliers based on product quality index

According to the identified quality index, by forming a pairwise comparison matrix by 7 experts, the matrix shown in *Table 8* was formed.

Table 8. *Prioritization of suppliers based on product quality index.*

	A	B	C	D	E	F	G	H	I	j
A	1.00	2.50	0.75	1.33	0.40	0.40	0.75	0.75	6.00	0.40
B	0.40	1.00	0.17	1.33	2.50	1.33	0.40	1.33	0.75	0.40
C	1.33	6.00	1.00	0.40	0.75	1.33	2.50	6.00	1.33	0.40
D	0.75	0.75	2.50	1.00	0.40	2.50	0.75	0.75	1.33	0.17
E	2.50	0.40	1.33	2.50	1.00	0.40	0.17	6.00	0.75	0.75
F	2.50	0.75	0.75	0.40	2.50	1.00	0.17	0.40	1.33	1.33
G	1.33	2.50	0.40	1.33	6.00	6.00	1.00	0.75	0.75	2.50
H	1.33	0.75	0.17	1.33	0.17	2.50	1.33	1.00	0.75	2.50
I	0.17	1.33	0.75	0.75	1.33	0.75	1.33	1.33	1.00	0.40
j	2.50	2.50	2.50	6.00	1.33	0.75	0.40	0.40	2.50	1.00

Table 9. *Ranking of suppliers based on the product quality index.*

A	B	C	D	E	F	G	H	I	j
0.940241	0.737199	1.383006	0.840422	0.971642	0.835959	1.568282	0.863876	0.785027	1.436823

3.4.2. Prioritization of suppliers based on cost reduction capability index

According to the cost reduction capability index, by forming a pair comparison matrix by 7 experts, the matrix shown in *Table 10* was formed.

Table 10. *Prioritization of suppliers based on cost reduction capability index.*

	A	B	C	D	E	F	G	H	I	j
A	1.00	0.75	1.33	0.17	2.50	1.33	0.40	0.40	1.33	0.75
B	1.33	1.00	0.40	0.75	0.17	2.50	1.33	6.00	1.33	0.40
C	0.75	2.50	1.00	0.75	2.50	0.75	0.75	1.33	2.50	2.50
D	6.00	1.33	1.33	1.00	0.75	1.33	1.33	1.33	1.33	2.50
E	0.40	6.00	0.40	1.33	1.00	0.40	2.50	6.00	1.33	1.33
F	0.75	0.40	1.33	0.75	2.50	1.00	0.75	1.33	2.50	2.50
G	2.50	0.75	1.33	0.75	0.40	1.33	1.00	6.00	1.33	2.50
H	2.50	0.17	0.75	0.75	0.17	0.75	0.17	1.00	2.50	2.50
I	0.75	0.75	0.40	0.75	0.75	0.40	0.75	0.40	1.00	0.75
j	1.33	2.50	0.40	0.40	0.75	0.40	0.40	0.40	1.33	1.00

Table 11. *Ranking of suppliers based on cost reduction capability index.*

A	B	C	D	E	F	G	H	I	j
0.785027	*0.966482*	*1.32341*	*1.513835*	*1.298745*	*1.167063*	*1.349283*	*0.705432*	*0.639226*	*0.713375*

3.4.3. Prioritization of suppliers based on the packaging index

According to the packing index, by forming a pairwise comparison matrix by 7 experts, the matrix shown in *Table 12* was formed.

Table 12. *Prioritization of suppliers based on packing index.*

	A	B	C	D	E	F	G	H	I	j
A	*1.00*	*0.40*	*1.33*	*0.17*	*1.33*	*0.75*	*2.50*	*0.40*	*0.75*	*6.00*
B	*2.50*	*1.00*	*0.40*	*1.33*	*0.75*	*1.33*	*1.33*	*0.40*	*2.50*	*2.50*
C	*0.75*	*2.50*	*1.00*	*0.75*	*1.33*	*1.33*	*2.50*	*2.50*	*6.00*	*0.75*
D	*6.00*	*0.75*	*1.33*	*1.00*	*0.75*	*1.33*	*1.33*	*1.33*	*1.33*	*2.50*
E	*0.75*	*1.33*	*0.75*	*1.33*	*1.00*	*2.50*	*1.33*	*1.33*	*0.40*	*0.17*
F	*1.33*	*0.75*	*0.75*	*0.75*	*0.40*	*1.00*	*0.75*	*1.33*	*2.50*	*2.50*
G	*0.40*	*0.75*	*0.40*	*0.75*	*0.75*	*1.33*	*1.00*	*0.17*	*0.75*	*0.40*
H	*2.50*	*2.50*	*0.40*	*0.75*	*0.75*	*0.75*	*6.00*	*1.00*	*0.75*	*0.75*
I	*1.33*	*0.40*	*0.17*	*0.75*	*2.50*	*0.40*	*1.33*	*1.33*	*1.00*	*1.33*
j	*0.17*	*0.40*	*1.33*	*0.40*	*6.00*	*0.40*	*2.50*	*1.33*	*0.75*	*1.00*

Table 13. *Ranking of suppliers based on the packaging index.*

A	B	C	D	E	F	G	H	I	j
0.912444	*1.160865*	*1.530042*	*1.429193*	*0.885467*	*1.03468*	*0.582534*	*1.135376*	*0.83152*	*0.856852*

3.4.4. Prioritization of suppliers based on product price index

According to the input indices identified for each supplier, based on the price index of the pairwise comparison by 7 experts, the matrix shown in *Table 14* was formed.

Table 14. *Prioritization of suppliers based on product price index.*

	A	B	C	D	E	F	G	H	I	G
A	*1.00*	*0.40*	*1.33*	*1.33*	*0.17*	*2.50*	*2.50*	*1.33*	*1.33*	*0.40*
B	*2.50*	*1.00*	*0.75*	*0.75*	*2.50*	*0.75*	*1.33*	*6.00*	*2.50*	*2.50*
C	*0.75*	*1.33*	*1.00*	*2.50*	*0.75*	*1.33*	*0.17*	*1.33*	*1.33*	*0.75*
D	*0.75*	*1.33*	*0.40*	*1.00*	*2.50*	*0.40*	*0.40*	*6.00*	*2.50*	*2.50*
E	*6.00*	*0.40*	*1.33*	*0.40*	*1.00*	*0.75*	*1.33*	*1.33*	*1.33*	*1.33*
F	*0.40*	*1.33*	*0.75*	*2.50*	*1.33*	*1.00*	*2.50*	*2.50*	*1.33*	*1.33*
G	*0.40*	*0.75*	*6.00*	*2.50*	*0.75*	*0.40*	*1.00*	*6.00*	*2.50*	*1.33*
H	*0.75*	*0.17*	*0.75*	*0.17*	*0.75*	*0.40*	*0.17*	*1.00*	*2.50*	*1.33*
I	*0.75*	*0.40*	*0.75*	*0.40*	*0.75*	*0.75*	*0.40*	*0.40*	*1.00*	*2.50*
j	*2.50*	*0.40*	*1.33*	*0.40*	*0.75*	*0.75*	*0.75*	*0.75*	*0.40*	*1.00*

Table 15. *Ranking of suppliers based on the product price Index.*

A	B	C	D	E	F	G	H	I	j
0.937908	*1.629309*	*0.942915*	*1.196231*	*1.117384*	*1.309392*	*1.390389*	*0.551527*	*0.677084*	*0.763713*

3.4.5. Prioritization of suppliers based on the place of delivery index

Table 16. *Prioritization of suppliers based on the place of delivery index.*

	A	B	C	D	E	F	G	H	I	j
A	*1.00*	*0.75*	*2.50*	*1.33*	*0.75*	*1.33*	*1.33*	*2.50*	*6.00*	*1.33*
B	*1.33*	*1.00*	*2.50*	*1.33*	*2.50*	*2.50*	*1.33*	*0.17*	*1.33*	*0.75*
C	*0.40*	*0.40*	*1.00*	*0.75*	*1.33*	*2.50*	*1.33*	*6.00*	*1.33*	*2.50*
D	*0.75*	*0.75*	*1.33*	*1.00*	*1.33*	*1.33*	*1.33*	*6.00*	*0.40*	*2.50*
E	*1.33*	*0.40*	*0.75*	*0.75*	*1.00*	*0.75*	*0.40*	*0.40*	*0.75*	*6.00*
F	*0.75*	*0.40*	*0.40*	*0.75*	*1.33*	*1.00*	*0.50*	*0.75*	*0.75*	*2.50*
G	*0.75*	*0.75*	*0.75*	*0.75*	*2.50*	*2.00*	*1.00*	*6.00*	*0.75*	*1.33*
H	*0.40*	*6.00*	*0.17*	*0.17*	*2.50*	*1.33*	*0.17*	*1.00*	*2.50*	*1.33*
I	*0.17*	*0.75*	*0.75*	*2.50*	*1.33*	*1.33*	*1.33*	*0.40*	*1.00*	*1.33*
j	*0.75*	*1.33*	*0.40*	*0.40*	*0.17*	*0.40*	*0.75*	*0.75*	*0.75*	*1.00*

Table 17. *Supplier rating.*

A	B	C	D	E	F	G	H	I	j
1.521917	*1.199633*	*1.267077*	*1.267077*	*0.833588*	*0.780947*	*1.252381*	*0.811244*	*0.885467*	*0.582534*

3.4.6. Prioritization of suppliers based on the quality system certification index

Table 18. *Prioritization of suppliers based on the quality system certification index.*

	A	B	C	D	E	F	G	H	I	G
A	1.00	0.40	0.40	0.40	0.75	0.75	0.17	0.40	0.17	0.75
B	2.50	1.00	1.33	0.75	0.75	2.50	0.40	0.40	0.75	1.33
C	2.50	0.75	1.00	1.33	0.40	0.17	0.17	0.17	0.40	0.75
D	2.50	1.33	0.75	1.00	0.75	2.50	2.50	2.50	6.00	1.33
E	1.33	1.33	2.50	1.33	1.00	1.33	1.33	2.50	2.50	6.00
F	1.33	0.40	6.00	0.40	0.75	1.00	1.33	1.33	0.75	2.50
G	6.00	2.50	6.00	0.40	0.75	0.75	1.00	6.00	0.40	1.33
H	2.50	2.50	6.00	0.40	0.40	0.75	0.17	1.00	0.40	1.33
I	6.00	1.33	2.50	0.17	0.40	1.33	2.50	2.50	1.00	1.33
j	1.33	0.75	1.33	0.75	0.17	0.40	0.75	0.75	0.75	1.00

Table 19. *Supplier rating.*

A	B	C	D	E	F	G	H	I	j
0.444337	0.971642	0.517925	1.725803	1.818304	1.12335	1.517601	0.912444	1.309392	0.699696

3.4.7. Prioritization of suppliers based on after-sales service indicators

Table 20. *Prioritization of suppliers based on after-sales service indicators.*

	A	B	C	D	E	F	G	H	I	G
A	1.00	0.40	1.33	1.33	0.17	2.50	2.50	1.33	1.33	0.40
B	2.50	1.00	0.75	0.75	2.50	0.75	1.33	6.00	2.50	2.50
C	0.75	1.33	1.00	2.50	0.75	1.33	0.17	1.33	1.33	0.75
D	0.75	1.33	0.40	1.00	2.50	0.40	0.40	6.00	2.50	2.50
E	6.00	0.40	1.33	0.40	1.00	0.75	1.33	1.33	1.33	1.33
F	0.40	1.33	0.75	2.50	1.33	1.00	2.50	2.50	1.33	1.33
G	0.40	0.75	6.00	2.50	0.75	0.40	1.00	6.00	2.50	1.33
H	0.75	0.17	0.75	0.17	0.75	0.40	0.17	1.00	2.50	1.33
I	0.75	0.40	0.75	0.40	0.75	0.75	0.40	0.40	1.00	2.50
j	2.50	0.40	1.33	0.40	0.75	0.75	0.75	0.75	0.40	1.00

Table 21. *Supplier rating.*

A	B	C	D	E	F	G	H	I	j
0.937908	*1.629309*	*0.942915*	*1.196231*	*1.117384*	*1.309392*	*1.390389*	*0.551527*	*0.677084*	*0.763713*

3.4.8. Prioritization of suppliers based on customization indicators

Table 22. *Prioritization of suppliers based on customization indicators.*

	A	B	C	D	E	F	G	H	I	j
A	*1.00*	*0.40*	*1.33*	*0.17*	*1.33*	*0.75*	*2.50*	*0.40*	*0.75*	*6.00*
B	*2.50*	*1.00*	*0.40*	*1.33*	*0.75*	*1.33*	*1.33*	*0.40*	*2.50*	*2.50*
C	*0.75*	*2.50*	*1.00*	*0.75*	*1.33*	*1.33*	*2.50*	*2.50*	*6.00*	*0.75*
D	*6.00*	*0.75*	*1.33*	*1.00*	*0.75*	*1.33*	*1.33*	*1.33*	*1.33*	*2.50*
E	*0.75*	*1.33*	*0.75*	*1.33*	*1.00*	*2.50*	*1.33*	*1.33*	*0.40*	*0.17*
F	*1.33*	*0.75*	*0.75*	*0.75*	*0.40*	*1.00*	*0.75*	*1.33*	*2.50*	*2.50*
G	*0.40*	*0.75*	*0.40*	*0.75*	*0.75*	*1.33*	*1.00*	*0.17*	*0.75*	*0.40*
H	*2.50*	*2.50*	*0.40*	*0.75*	*0.75*	*0.75*	*6.00*	*1.00*	*0.75*	*0.75*
I	*1.33*	*0.40*	*0.17*	*0.75*	*2.50*	*0.40*	*1.33*	*1.33*	*1.00*	*1.33*
J	*0.17*	*0.40*	*1.33*	*0.40*	*6.00*	*0.40*	*2.50*	*1.33*	*0.75*	*1.00*

Table 23. *Supplier rating.*

A	B	C	D	E	F	G	H	I	j
0.912444	*1.160865*	*1.530042*	*1.429193*	*0.885467*	*1.03468*	*0.582534*	*1.135376*	*0.83152*	*0.856852*

According to the final evaluation of suppliers based on input and output indicators, the final matrix of suppliers based on indicators will be as *Tables (24)-(25)*. Supplier scores based on output indicators.

Table 24. *Supplier scores based on output indicators.*

	Product quality	Reduce costs	Packaging
A	*0.94*	*0.79*	*0.91*
B	*0.74*	*0.97*	*1.16*
C	*1.38*	*1.32*	*1.53*
D	*0.84*	*1.51*	*1.43*
E	*0.97*	*1.30*	*0.89*
F	*0.84*	*1.17*	*1.03*
G	*1.57*	*1.35*	*0.58*
H	*0.86*	*0.71*	*1.14*
I	*0.79*	*0.64*	*0.83*
j	*1.44*	*0.71*	*0.86*

Table 25. *Scores of suppliers based on input indicators.*

	Price Product	Place of Delivery	Quality Systems	After Sales Service	Customization
A	*0.94*	*1.52*	*0.44*	*0.94*	*0.91*
B	*1.63*	*1.20*	*0.97*	*1.63*	*1.16*
C	*0.94*	*1.27*	*0.52*	*0.94*	*1.53*
D	*1.20*	*1.27*	*1.73*	*1.20*	*1.43*
E	*1.12*	*0.83*	*1.82*	*1.12*	*0.89*
F	*1.31*	*0.78*	*1.12*	*1.31*	*1.03*
G	*1.39*	*1.25*	*1.52*	*1.39*	*0.58*
H	*0.55*	*0.81*	*0.91*	*0.55*	*1.14*
I	*0.68*	*0.89*	*1.31*	*0.68*	*0.83*
j	*0.76*	*0.58*	*0.70*	*0.76*	*0.86*

4. Conclusion

With the increase in the number of suppliers in the supply sector of manufacturing companies, the need to have information about the capabilities, capabilities and executive records of suppliers for companies is felt more than ever. In the meantime, having a procedure and instructions that can evaluate suppliers from several different criteria and angles and can select the best supplier is more important. Therefore, in this study, after initial screening of supplier review indicators, the most important indicators were evaluated and selected. Due to the quality of the evaluation indicators, at first, all suppliers were ranked and weighted based on each index using the AHP method. Then, according to the evaluation, all suppliers were evaluated using the Super Efficiency DEA method, all suppliers, based on which the suppliers were ranked among the efficient suppliers, and an accurate evaluation can be provided in this regard. The results of comparing the manufacturing company supplier chain rankings based on AHP, FAHP, and Super Efficiency DEA methods are as *Table 26*.
According to the points obtained, the ranking of suppliers with the methods introduced is as *Table 27*.

All With the increase in the number of suppliers in the supply sector of manufacturing companies, the need to have information about the capabilities, capabilities and executive records of suppliers for companies is felt more than ever. In the meantime, having a procedure and instructions that can evaluate suppliers from several different criteria and angles and can select the best supplier is more important. Therefore, in this study, after initial screening of supplier review indicators, the most important indicators were evaluated and selected. Due to the quality of the evaluation indicators, at first, all suppliers were ranked and weighted based on each index using the AHP method. Then, according to the evaluation, all suppliers were evaluated using the Super Efficiency DEA method, all suppliers, based on which the suppliers were ranked among the efficient suppliers, and an accurate evaluation can be provided in this regard. The results of comparing the manufacturing company supplier chain rankings based on AHP, FAHP, and Super Efficiency DEA methods are as *Table 26*.

Table 26. *Comparison of supplier chain rankings.*

Row	Supplier	AHP	FAHP	Super Efficiency DEA
1	**A**	*4.75*	*0.124*	*0.992*
2	**B**	*6.59*	*0.11*	*0.969*
3	**C**	*5.2*	*0.114*	*1.693*
4	**D**	*6.83*	*0.1*	*0.998*
5	**E**	*5.78*	*0.093*	*1.187*
6	**F**	*5.55*	*0.097*	*1.122*
7	**G**	*6.13*	*0.093*	*1.992*
8	**H**	*3.96*	*0.098*	*1.273*
9	**I**	*4.39*	*0.087*	*0.972*
10	**j**	*3.66*	*0.085*	*2.066*

According to the points obtained, the ranking of suppliers with the methods introduced is as *Table 27*.

Table 27. *Scores of suppliers ranking.*

Row	Supplier	AHP	FAHP	Super Efficiency DEA
1	**A**	*7*	*1*	*8*
2	**B**	*2*	*3*	*10*
3	**C**	*6*	*2*	*3*
4	**D**	*1*	*4*	*7*
5	**E**	*4*	*7*	*5*
6	**F**	*5*	*6*	*6*
7	**G**	*3*	*8*	*2*
8	**H**	*9*	*5*	*4*
9	**I**	*8*	*9*	*9*
10	**j**	*10*	*10*	*1*

According to the study, AHP and FAHP methods in the ranking of suppliers had closer answers than data envelopment analysis. And according to the computational accuracy of data envelopment analysis methods, which is based on the input and output information of each supplier, so supplier number 10 is declared the best supplier. According to the assessments made in this study, first, key indicators regarding supply risks using the articles [23, 24, 27-29] using selection of experts from seven experts of the company, based on the risks of selecting suppliers, the most appropriate indicators have been identified using fuzzy, which in the meantime, article [23] was accepted with the highest selection of indicators and then we evaluated the suppliers. Due to the very high sensitivity in supply chain development, it is necessary for suppliers to be evaluated and selected based on all strategic indicators of the organization, so to develop this research, the following suggestions are provided:

- It is suggested that the production company form a working group consisting of executive units for accurate evaluation of suppliers and all evaluations be reviewed and selected in a multi-purpose working group.
- It is suggested that the executive instructions of the organization be updated and rewritten in accordance with the context of this research.
- It is recommended to conduct periodic evaluations of suppliers to maintain efficiency.

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Using Interval Arithmetic for Providing A MADM Approach

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The VIKOR method was developed for Multi-Criteria Decision Making (MCDM). It determines the compromise ranking list and the compromise solution obtained with the initial weights. This method focuses on ranking and selecting from a set of alternatives in the presence of conflicting criteria. It introduces the multi-criteria ranking index based on the particular measure of ‘‘closeness” to the “Ideal” solution. The aim of this paper is to extend the VIKOR method for decision making problems with interval number. The extended VIKOR method’s ranking is obtained through comparison of interval numbers and for doing the comparisons between intervals. In the end, a numerical example illustrates and clarifies the main results developed in this paper.

1. Introduction

The MCDM is the process of determining the best feasible solution according to the established criteria. Practical problems are often characterized by several non-commensurable and conflicting criteria and there may be no solution satisfying all criteria simultaneously. Thus, the solution is a set of non-inferior solutions, or a compromise solution according to the decision maker’s preferences. The compromise solution was established by Yu [1] and Zeleny [2] for a problem with conflicting criteria and it can be helping the decision makers to reach a final solution. The compromise solution is a feasible solution, which is the closest to the Ideal, and compromise means an agreement established by mutual concessions.

A MADM problem can be defined as:

	$\mathbf{C_1}$	$\mathbf{C_2}$	…	$\mathbf{C_n}$
$\mathbf{A_1}$	f_{11}	f_{12}	…	f_{1n}
$\mathbf{A_2}$	f_{21}	f_{22}	…	f_{2n}
⋮	⋮	⋮	⋮	⋮
$\mathbf{A_m}$	f_{m1}	f_{m2}	…	f_{mn}
$\boldsymbol{w}$	w_1	w_2	…	w_n

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where $A_1, A_2, \dots, A_m$ are possible alternatives among which decision makers have to choose, $C_1, C_2, \dots, C_n$ are criteria with which alternative performance is measured, fij is the rating of alternative A_i with respect to criterion C_j, w_j is the weight of criterion C_j [3–5].

In classical MCDM methods, the ratings and the weights of the criteria are known precisely, whereas in the real world, in an imprecise and uncertain environment, it is an unrealistic assumption that the knowledge and representation of a decision maker or expert are so precise. For example, human judgment including preferences is often vague and Decision Maker (DM) cannot estimate his preference with exact numerical values. In these situations, determining the exact value of the attributes is difficult or impossible. So, to describe and treat imprecise and uncertain elements present in a decision problem, fuzzy and stochastic approaches are frequently used. In the literature, in the works of fuzzy decision making [6–8], fuzzy parameters are assumed to be with known membership functions and in stochastic decision making [9–12] parameters are assumed to have known probability distributions. However, in reality to a DM it is not always easy to specify the membership function or probability distribution in an inexact environment. At least in some of the cases, the use of interval numbers may serve the purpose better. An interval number can be thought as an extension of the concept of a real number and also as a subset of the real line R [13]. However, in decision problems its use is not much attended as it merits. Recently, Jahanshahloo et al. [14] have extended TOPSIS method to solve decision making problems with interval data.

According to a comparative analysis of VIKOR and TOPSIS written by Opricovic and Tzeng [15], VIKOR method and TOPSIS method use different aggregation functions and different normalization methods. TOPSIS method is based on the principle that the optimal point should have the shortest distance from the Positive Ideal Solution (PIS) and the farthest from the Negative Ideal Solution (NIS). Therefore, this method is suitable for cautious (risk avoider) decision maker(s), because the decision maker(s) might like to have a decision which not only makes as much profit as possible, but also avoids as much risk as possible. Besides, computing the optimal point in the VIKOR is based on the particular measure of ‘‘closeness” to the PIS. Therefore, it is suitable for those situations in which the decision maker wants to have maximum profit and the risk of the decisions is less important for him. Therefore, we extend the concept of VIKOR method to develop a methodology for solving MADM problems with interval numbers. The VIKOR method is presented in the next section. In Section 3, extended VIKOR method is introduced and a new method is proposed for interval ranking on the basis of decision maker's optimistic level. In Section 4, an illustrative example is presented to show an application of extended VIKOR method. Finally, conclusion is presented.

2. VIKOR Method

The VIKOR method was introduced as one applicable technique to be implemented within MCDM problem and it was developed as a multi attribute decision making method to solve a discrete decision making problem with non-commensurable and conflicting criteria [15, 16]. This method focuses on ranking and selecting from a set of alternatives, and determines compromise solution for a problem with conflicting criteria, which can help the decision makers to reach a final solution. The multi-criteria measure for compromise ranking is developed from the LP-metric used as an aggregating function in a compromise programming method [1, 2].

Assuming that each alternative is evaluated according to each criterion function, the compromise ranking could be performed by comparing the measure of closeness to the Ideal alternative. The various m alternatives are denoted as $A_1, A_2, \dots, A_m$. For alternative A_i, the rating of the jth aspect is denoted by

f_{ij}, i.e. f_{ij} is the value of jth criterion function for the alternative A_i; n is the number of criteria. Development of the VIKOR method is started with the following form of LP-metric:

$$L_{pi} = \left\{ \sum_{j=1}^{n} \left(\frac{f_j^* - f_{ij}}{f_j^* - f_j^-} \right)^p \right\}^{\frac{1}{p}} \quad 1 \le p \le \infty; \; i = 1,2, \dots, m \tag{1}$$

In the VIKOR method $L_{1,i}$ (as S_i) and $L_{\infty,i}$ (as R_i) are used to formulate ranking measure. The solution obtained by min S_i is with a maximum group utility, and the solution obtained by min R_i is with a minimum individual regret of the ''opponent".The compromise ranking algorithm of the VIKOR method has the following steps:

Step 1. Determine the best f_j and the worst f_j values of all criterion functions $j = 1,2, \dots, n$. If the jth function represents a benefit then:

$$f_j^* = \max_i f_{ij}, f_j^- = \min_i f_{ij} \tag{2}$$

Step 2. Compute the values S_i and R_i; $i = 1,2, \dots, m$, by these relations:

$$S_i = \sum_{j=1}^{n} w_j \left(\frac{f_j^* - f_{ij}}{f_j^* - f_j^-} \right). \tag{3}$$

$$R_i = \max_i w_j \left(\frac{f_j^* - f_{ij}}{f_j^* - f_j^-} \right). \tag{4}$$

Where w_j are the weights of criteria, expressing their relative importance.

Step 3. Compute the values Q_i; $i = 1,2, \dots, m$, by the following relation:

$$Q_i = v \left(\frac{S_i - S^*}{S^- - S^*} \right) + (1 - v) \left(\frac{R_i - R^*}{R^- - R^*} \right). \tag{5}$$

Where

$$S^* = \text{Min}_i S_i, \; S^- = \text{Max}_i S_i. \tag{6}$$

$$R^* = \text{Min}_i R_i, \; R^- = \text{Max}_i R_i. \tag{7}$$

v is introduced as weight of the strategy of ''the majority of criteria" (or ''the maximum group utility"), here suppose that $v = 0.5$.

Step 4. Rank the alternatives, sorting by the values S, R and Q in decreasing order. The results are three ranking lists.

Step 5. Propose as a compromise solution the alternative′ , which is ranked the best by the measure Q (Minimum Value) if the following two conditions are satisfied:

- Acceptable advantage, $Q(A") - Q(A') \geq DQ$. where $A"$ is the alternative with second position in the ranking list by Q; $DQ = \frac{1}{m-1}$; m is the number of alternatives.
- Acceptable stability in decision making. Alternative A' must also be the best ranked by S or/and R. This compromise solution is stable within a decision making process, which could be ‘‘voting by majority rule” (when $v > 0.5$ is needed), or ‘‘by consensus” $v = 0.5$, or ‘‘with veto”($v < 0.5$). Here, v is the weight of the decision making strategy ‘‘the majority of criteria” (or ‘‘the maximum group utility”).

If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:

- Alternatives A' and $A"$ if only condition C_2 is not satisfied.
- AlternativesA', $A"$, ... ,$A^{(M)}$ if condition C_1 is not satisfied; $A^{(M)}$ is determined by the relation $Q(A^{(M)})$ _ $Q(A') < DQ$ for maximum M (the positions of these alternatives are ‘‘in closeness”).

The best alternative, ranked by Q, is the one with the minimum value of Q. The main ranking result is the compromise ranking list of alternatives, and the compromise solution with the ‘‘advantage rate”. VIKOR is an effective tool in multi-criteria decision making, particularly in a situation where the decision maker is not able, or does not know to express his/her preference at the beginning of system design. The obtained compromise solution could be accepted by the decision makers because it provides a maximum ‘‘group utility” (represented by min S) of the ‘‘majority”, and a minimum of the ‘‘individual regret” (represented by min$R$) of the ‘‘opponent”. The compromise solutions could be the basis for negotiations, involving the decision maker’s preference by criteria weights.

3. VIKOR Method with Interval Numbers

As it was said in the introduction, the interval numbers are more suitable to deal with the decision making problems in the imprecise and uncertain environment, because they are the simplest form of representing uncertainty in the decision matrix. The interval numbers require the minimum amount of information about the values of attributes. Specifying an interval for a parameter in decision matrix indicates that the parameter can take any value within the interval. Note that, the interval numbers does not indicate how probable it is to the value to be in the interval, nor does it indicate which of the many values in the interval is the most likely to occur [17]. In other way, an interval number can be thought as:

- An extension of the concept of a real number and also as a subset of the real line.
- A degenerate flat fuzzy number or fuzzy interval with zero left and right spreads.
- An α-cut of a fuzzy number [18].

So an interval number signifies the extent of tolerance or a region that the parameter can possibly take. An extensive research and wide coverage on interval arithmetic and its applications can be found in [13, 19, 20]. More information about the interval numbers and its differences with other methods of representing uncertainty such as probability and fuzzy theory can be found in [18, 21, 22].According to these facts, when determining the exact values of the attributes is difficult or impossible, it is more appropriate to consider them as interval numbers. Therefore, in the present paper, we extend the VIKOR method to solve MADM problem with interval numbers.

3.1. Interval Arithmetic

If two intervals $Ix = [x^L, x^U]$ and $Iy = [y^L, y^U]$ are given, the sum, difference, product, quotient, and additive inverse of the intervals are calculated based on the following equations [23]:

$$I = k * Ix = [kx^L, kx^U];\ k \in \mathbb{R}^+. \tag{8}$$

$$I = -Iy = [-y^U, -y^L]. \tag{9}$$

$$I = Ix + Iy = [x^L + y^L, x^U + y^U]. \tag{10}$$

$$I = Ix - Iy = [x^L - y^U, x^U - y^L]. \tag{11}$$

$$I = Ix * Iy = [\min\{x^L y^L, x^L y^U, x^U y^L, x^U y^U\}, \max\{x^L y^L, x^L y^U, x^U y^L, x^U y^U\}]. \tag{12}$$

$$I = \frac{Ix}{Iy} = \left[\min\{\frac{x^L}{y^L}, \frac{x^L}{y^U}, \frac{x^U}{y^L}, \frac{x^U}{y^U}\}, \max\{\frac{x^L}{y^L}, \frac{x^L}{y^U}, \frac{x^U}{y^L}, \frac{x^U}{y^U}\}\right]; 0 \notin Iy. \tag{13}$$

3.2. Interval Ranking

For ranking intervals, the mean value of each of the intervals is first calculated, and the rankings are then specified based on the obtained values. The mean value of $Ix = [x^L, x^U]$ is represented by $\text{me}(Ix)$, which is obtained from the following equation [23]:

$$\text{me}(Ix) = \frac{x^L + x^U}{2}. \tag{14}$$

3.3. Presentation of an Extended VIKOR Method

Suppose that a decision matrix with interval numbers has the following form:

	$\mathbf{C_1}$	$\mathbf{C_2}$	…	$\mathbf{C_n}$
$\mathbf{A_1}$	$[f_{11}^L, f_{11}^U]$	$[f_{12}^L, f_{12}^U]$	…	$[f_{1n}^L, f_{1n}^U]$
$\mathbf{A_2}$	$[f_{21}^L, f_{21}^U]$	$[f_{22}^L, f_{22}^U]$	…	$[f_{2n}^L, f_{2n}^U]$
⋮	⋮	⋮	…	⋮
$\mathbf{A_m}$	$[f_{m1}^L, f_{m1}^U]$	$[f_{m2}^L, f_{m2}^U]$	…	$[f_{mn}^L, f_{mn}^U]$
Iw	$[w_1^L, w_1^U]$	$[w_2^L, w_2^U]$	…	$[w_n^L, w_n^U]$

Where $A_1, A_2, \ldots, A_m$ are possible alternatives among which decision makers have to choose, $C_1, C_2, \ldots, C_n$ are criteria with which alternative performance are measured, If_{ij} is the rating of alternative A_i with respect to criterion C_j and is not known exactly and only we know $If_{ij} \in [f_{ij}^L, f_{ij}^U]$.

and $Iw_j = [w_j^L, w_j^U]$ is the weight of criterion C_j. The Interval VIKOR method consists of the following steps:

Step 1. Determine the PIS and NIS.

$$A^* = \{f_1^*, \dots, f_n^*\} = \left\{\left(\max_i f_{ij}^U \,\middle|\, j \in I\right) \; or \; \left(\min_i f_{ij}^L \,\middle|\, j \in J\right)\right\} \; ; \; j = 1,2,\dots,n. \quad (15)$$

$$A^- = \{f_1^-, \dots, f_n^-\} = \left\{\left(\min_i f_{ij}^L \,\middle|\, j \in I\right) \; or \; \left(\max_i f_{ij}^U \,\middle|\, j \in J\right)\right\} \; ; \; j = 1,2,\dots,n.$$

Where I is associated with benefit criteria, and J is associated with cost criteria. A^* and A^- are PIS and NIS.

Step 2. In this step compute $IS_i = [S_i^L, S_i^U]$ and $IR_i = [R_i^L, R_i^U]$ intervals below:

$$S_i^L = \sum_{j \in I} w_j^L \left(\frac{f_j^* - f_{ij}^U}{f_j^* - f_j^-}\right) + \sum_{j \in J} w_j^L \left(\frac{f_{ij}^L - f_j^*}{f_j^- - f_j^*}\right) \; ; i = 1,2, \dots m. \quad (16)$$

$$S_i^U = \sum_{j \in I} w_j^U \left(\frac{f_j^* - f_{ij}^L}{f_j^* - f_j^-}\right) + \sum_{j \in J} w_j^U \left(\frac{f_{ij}^U - f_j^*}{f_j^- - f_j^*}\right) \; ; i = 1,2, \dots m.$$

$$R_i^L = \max\left\{ w_j^L \left(\frac{f_j^* - f_{ij}^U}{f_j^* - f_j^-}\right) \middle| j \in I, w_j^L \left(\frac{f_{ij}^L - f_j^*}{f_j^- - f_j^*}\right) \middle| j \in J \right\}; i = 1,2, \dots, m \, . \quad (17)$$

$$R_i^U = \max\left\{ w_j^U \left(\frac{f_j^* - f_{ij}^L}{f_j^* - f_j^-}\right) \middle| j \in I, w_j^U \left(\frac{f_{ij}^U - f_j^*}{f_j^- - f_j^*}\right) \middle| j \in J \right\}; i = 1,2, \dots, m \, .$$

Step 3. Compute the interval $IQ_i = [Q_i^L, Q_i^U]; i = 1,2, \dots, m$, by these relations:

$$Q_i^L = v\left(\frac{S_i^L - S^*}{S^- - S^*}\right) + (1 - v)\left(\frac{R_i^L - R^*}{R^- - R^*}\right). \quad (18)$$

$$Q_i^U = v\left(\frac{S_i^U - S^*}{S^- - S^*}\right) + (1 - v)\left(\frac{R_i^U - R^*}{R^- - R^*}\right).$$

Where

$$S^* = \mathrm{Min}_i S_i^L \, , \; S^- = \mathrm{Max}_i S_i^U. \quad (19)$$

$$R^* = \mathrm{Min}_i R_i^L \, , \; R^- = \mathrm{Max}_i R_i^U. \quad (20)$$

Step 4. Based on the VIKOR method, the alternative that has minimum Q_i is the best alternative and it is chosen as compromise solution.

4. Numerical Example

In this section, we present a numerical example to illustrate how the proposed method can be used. Suppose that, there are three alternatives (A_1, A_2, A_3) and two criteria (C_1, C_2). The decision maker wants to choose an alternative that has minimum C_1 and maximumC_2. The values of decision matrix are not precise and interval numbers are used to describe and treat the uncertainty of the decision problem. The interval decision matrix is shown in *Table 1*. In this example, both criteria have similar relative importance, $Iw_1 = [0.45, 0.50], Iw_2 = [0.50, 0.55], v = 0.5$.

To solve this example using the Interval VIKOR (IVIKOR) method we go through the following steps.

Step 1. The PIS and NIS are computed by (15a) and (15b) and shown in Table 2.

Step 2. In this step, we compute $IS_i = [S_i^L, S_i^U]$ and $IR_i = [R_i^L, R_i^U]$ using *Eqs. (16)-(17)*. The result is presented in *Table 3*.

Step 3. We compute the interval $IQ_i = [Q_i^L, Q_i^U]; i = 1,2, \dots, m$, by (18a), (18b), (19) and (20). The results are shown in *Table 4*.

$$S^* = 0.3224, \ S^- = 0.6030.$$

$$R^* = 0.2172, R^- = 0.5500.$$

Final ranking is obtained as follows:

$$Q_2 = me(IQ_2) < Q_3 = me(IQ_3) < Q_1 = me(IQ_1) \quad \Rightarrow \text{Final ranking is:} A_2 > A_3 > A_1.$$

Table 1. *Interval decision matrix.*

	C_1	C_2
A_1	[0.75,1.24]	[2784,3192]
A_2	[1.83,2,11]	[3671,3857]
A_3	[4.90,5.37]	[4409,4681]

Table 2. *PIS and NIS.*

	C_1	C_2
f_j^*	0.8	4681
f_j^-	5.4	2784

Table 3. *IS and IR.*

	$IS = [S^L, S^U]$	$IR = [R^L, R^U]$
A_1	[0.3925,0.6030]	[0.3925,0.5500]
A_2	[0.3224,0.4400]	[0.2172,0.2928]
A_3	[0.4042,0.5789]	[0.4042,0.5000]

Table 4. *IQ and Q.*

	$\mathbf{IQ} = [\mathbf{Q}^{\mathbf{L}}, \mathbf{Q}^{\mathbf{U}}]$	$\mathbf{Q} = \mathbf{me}(\mathbf{IQ}) = \frac{\mathbf{Q}^{\mathbf{L}} + \mathbf{Q}^{\mathbf{U}}}{\mathbf{2}}$	**Rank**
$\mathbf{A_1}$	[0.3882,1]	0.6941	3
$\mathbf{A_2}$	[0,0.3232]	0.1616	1
$\mathbf{A_3}$	[0.4268,0.8818]	0.6543	2

The compromise solution of extended VIKOR method is A_2.

As mentioned in the introduction, Jahanshahloo et al. have extended TOPSIS method to solve decision making problems with interval data. This method uses different aggregation functions and different normalization methods. Here to make a comparison between these two methods, we solve this example using the extended TOPSIS method. Doing the introduced steps in the extended TOPSIS method, compromise solution is obtained as follows:

The ranking of extended TOPSIS is: $A_2 > A_3 > A_1$.

The compromise solution obtained by extended TOPSIS is different with the compromise solution of extended VIKOR.

These different solutions derive from differences in aggregation functions and normalization methods. Moreover, in extended TOPSIS, the interval numbers are reduced to exact values. These reductions lead to miss some information. In the extended VIKOR method by keeping interval numbers, considering the decision maker's optimism level and using the comparison of interval numbers, the compromise solution is obtained.

5. Conclusion

Because of the fact that determining the exact values of the attributes is difficult or impossible, it is more appropriate to consider them as interval numbers. In this paper, we extended the VIKOR (IVIKOR) method to MADM problem with interval numbers. This method introduced the ranking index based on particular measure of closeness to PIS. In the extended VIKOR method, we compute S, R and Q as interval numbers and to obtain the compromise solution, we need to compare interval numbers with each other. For that purpose, we utilized the interval means method.

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Fuzzy Logic in Accounting and Auditing

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Ambiguity.

A B S T R A C T

Many areas of accounting have highly ambiguous due to undefined and inaccurate terms. Many ambiguities are generated by the human mind. In the field of accounting, these ambiguities lead to the creation of uncertain information. Many of the targets and concepts of accounting with binary classification are not consistent. Similarly, the discussion of the materiality or reliability of accounting is not a two-part concept. Because there are degrees of materiality or reliability. Therefore, these ambiguities lead to the presentation information that is not suitable for decision making. Lack of attention to the issue of ambiguity in management accounting techniques, auditing procedures, and financial reporting may lead to a reduced role of accounting information in decision-making processes. Because information plays an important role in economic decision-making, and no doubt, the quality of their, including accuracy in providing it to a wide range of users, can be useful for decision-making. One of the features of the fuzzy set is that it reduces the need for accurate data in decision making. Hence this information can be useful for users.

1. Introduction

Ambiguity and imprecision in human judgments are in many scientific disciplines. Accountants in dealing with the issue of ambiguity and imprecision, they behave there is no ambiguity or it is a random [1]. In recent years, fuzzy logic has gained wide acceptance in the field of accounting and business. This acceptance is due to the ability to management in situations of ambiguity and lack of consistency that does not exist within other approaches to dual value logic. In dual value logic, the proposition is true or false. Also, accounting has ambiguous in many important respects [2]. The problem of ambiguity and imprecision in accounting and auditing is related to the rules and accounting system [3]. Ro [4] argues that the concept of materiality is not essentially two-dimensional, such as black or white and good or bad, but that there are degrees of materiality that are overlooked in accounting. However, ambiguity and imprecision is different from being random. Randomness refers to the uncertainty about the occurrence or non-occurrence of an event and is expressed in the form of probabilities. While, ambiguity and imprecision are related to the inaccuracy and lack of clarity in the definition of words, the occurrence of events and judgments [2]. Zariffard [1] argues that neglect of ambiguity and imprecision in decision models can limit the usability of accounting models due to reduced usefulness and predictive power. Therefore, it is important attention to ambiguity. The purpose of this study is to

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introduce fuzzy logic and also is examine its major applications in accounting and auditing. The importance of this research is that considering that information is the main component of any decision making today, it has economic value.

2. Literature Review

2.1. Fuzzy Set Theory

In 1965, Zadeh discussed the existence of ambiguity and fuzziness in many human systems. According to Zadeh [5], the need to be very careful in decision analysis causes the analyst to ignore some related issues and consider only a part of this relationship with the real world. Fuzzy thinking followed the objection to Aristotelian logic about the distance between logic and reality. Aristotelian logic forms the basis of classical mathematics. This logic assumes that the world is black and white or two values one or 0. Zadeh [6] proposed the theory of fuzzy sets as a method for modeling in ambiguity and uncertainty. Sets can be divided into finite sets and fuzzy sets. In finite set, is there a member in a set or not? That is, it has no more than two values, one or 0. But not in the fuzzy set.

In fact, Aristotelian logic sacrifices accuracy for ease. But the real phenomena are not just black or white, they are somewhat gray. In other words, real phenomena are always fuzzy, that is, ambiguity and imprecision [7]. Fuzzy set theory reduces the possibility of making personal judgments by expressing qualitative and subjective information, and leads to more rational decisions [8].

2.2. Definition of Fuzzy Set

Let U be a classical (or ordinary) set of objects, called the universe, whose generic elements are denoted by x. That is, U={x}. A fuzzy set A in U is characterized by a membership function $\mu_A(X)$ which associates with each element in U a real number in the interval (0–1) [9]. The fuzzy set, A, is usually denoted by the set of pairs [10].

$$A = \{(x, \mu_A(X)), x \in U\}. \tag{1}$$

For an ordinary set, A

$$\mu_A(X) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}. \tag{2}$$

When U is a finite set $\{x_1,\ldots, x_n\}$, the fuzzy set on U may also be represented as

$$A = \sum_{i=1}^{n} x_i / \mu_A(x_i). \tag{3}$$

When U is an infinite set, the fuzzy set maybe represented as

$$A = \int (x / \mu_A(x))dx. \tag{4}$$

2.3. Basic Concepts of Fuzzy Set

The complement, support, a-cut, convexity, normality and cardinality of a fuzzy set are presented in the following sections [9].

Complement of a fuzzy set. The definition of the complement of fuzzy set A is defined as

$$\mu_A(X) = 1 - \mu_A(X) \quad x \in U. \tag{5}$$

Support of a fuzzy set. Those elements which have nonzero membership grades are considered as support of that fuzzy set

$$S(A) = \{x \in U | \mu_A(X) \geq 0\}. \tag{6}$$

a-Cut of a fuzzy set. a-Cut of a fuzzy set is an ordinary set whose elements belong to fuzzy set A, at least to the degree of a

$$A\alpha = \{x \in U | \mu_A(X) \geq \alpha\}. \tag{7}$$

It is a more general case of the support of a fuzzy set. If α=0 then Aα=S(A).

Convexity of a fuzzy set. A fuzzy set is convex if

$$\mu_A(\lambda X_1 + (1-\lambda)X_2) \geq \min(\mu_A(X_1), \quad \mu_A(X_2)). \tag{8}$$

X_1 and $X_2 \epsilon U$ also $\lambda \epsilon$ (0–1).

Normality of a fuzzy set. A fuzzy set A is normal only if there are one or more x′ values such that $\mu_A(X') = 1$.

Cardinality of a fuzzy set. The cardinality of fuzzy set A evaluates the proportion of elements of U having the property A. When U is finite, it is defined as

$$|A| = \sum \mu_A(x), \quad x \in U. \tag{9}$$

For infinite U, the cardinality is defined as

$$|A| = \int_x \mu_A(x)dx. \tag{10}$$

For more details, enormous materials can be found in the literature about fuzzy set theory.

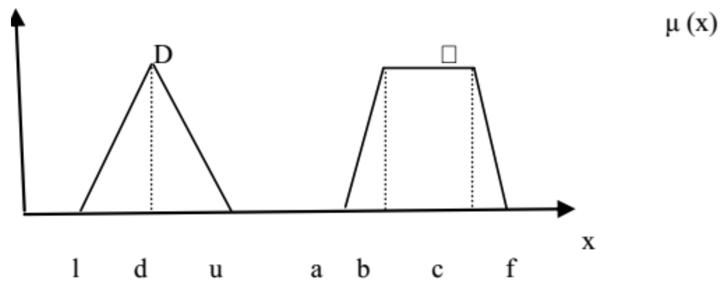

Fig. 1. *Triangular and trapezoidal fuzzy numbers.*

Fig. 1 defines a fuzzy and triangular number D as follows [8]:

D= {1, d, u} where 1, d, and u as the lower spread, the middle spread, and the upper spread. In this case the membership function $\mu_{(D)}$ X it is defined as follows:

$$\mu_D(x) = \begin{cases} 0; & x \leq 1 \\ (x-1)/(d-1); & 1 < x \leq d \\ (u-x)/(u-d); & d < x \leq u \\ 0; & x > u \end{cases}. \tag{11}$$

If the climax of the triangular number D is not unique; the fuzzy number is known as a trapezoid.

3. Fuzzy Set Theory and Accounting and Auditing

For a variety of reasons, fuzzy set theory can be of great value to accountants in practice. First, fuzzy set theory provides a mathematical framework which fuzzy concepts of accounting can be examined on a regular basis, for instance, materiality errors. Therefore, using fuzzy set theory, accountants will be able to apply fuzzy set theory in accounting. As a result, they no longer have to ignore ambiguities in accounting matters. Also, they will be able to deal with it like random events using probability theory. Accountants with ignoring the ambiguity cause inaccuracies in accounting matters [1].

In addition, unlike ordinary set theory, fuzzy set theory abandons the rule of excluding the mean and logic of two values. As a result, there will be no need for a binary classification of accounting objectives that are generally unrealistic and artificial. Many of the targets and concepts of accounting with binary classification are not consistent. For example, neutrality is not a debate of being black and white. There are different degrees of neutrality, or in the discussion of deviation analysis, controllable deviations or uncontrollable deviations are kinds of unrealistic integration. Similarly, the discussion of the materiality or reliability of accounting is not a two-part concept. Because there are degrees of materiality or reliability. One of the features of the fuzzy set is that it reduces the need for accurate data in decision making.

Recently, this theory has been used to solve accounting problems. These studies can be divided into two groups. The first group deals with audit problems such as internal control, audit sampling, and judgment of materiality. The second group deals with management accounting issues and problems such as capital budgeting, cost deviations, and strategic planning. Some applications of fuzzy sets in the audit are summarized as:

Friedlob and Schleifer [11] argue that auditors usually express risk in the form of probabilities, examining different types of audit uncertainty. Finally, they introduced the fuzzy logic-based method as a new method of examining audit uncertainty.

Pathak et al. [12] indicates that in order to reduce the costs of detecting fraud in the claims made in their insurance companies, they designed fuzzy expert systems to evaluate and express the elements related to fraud in resolving insurance claims. This system is useful for deciding whether settled insurance claims are actual or whether there is evidence of fraud.

Comunale and Sexton [13] introduced a fuzzy logic approach to assess the importance of presenting financial statements. A fuzzy logic-based approach to significance assessment can provide an expert system for significance assessment compared to traditional approaches, which are based on binary valuation; In such a way that the importance of presenting financial statements correctly can be shown between zero and one, and on the other hand, quality criteria can be considered in evaluating the importance.

Dereli et al. [14] using a fuzzy mathematical programming model, they proposed a strategic algorithm to shape the quality audit team. In this study, the fuzzy ranking method has been used to determine the adequacy of the skills and expertise of each auditor in team auditing.

De Korvin et al. [15] examined the risk of internal controls in computer accounting information systems through a fuzzy set approach. The model presented in this research is used through a risk analysis matrix in a company active in the chemical industry. This model is useful in evaluating and applying new control procedures to increase the security of the company's information systems.

Also, some applications of fuzzy sets in the management accounting [16, 17] are summarized as:

Oderanti and De Wilde [18] used the concepts of fuzzy logic and game theory to model the strategic decision-making process by business organizations based on uncertain information. In this study, competition between business organizations is considered as a game and organizations are its actors. They model their decisions through strategic actions based on uncertain information.

Cassia et al. [19] examined the development of corporate management accounting systems in providing information to facilitate the strategic decision-making process and its relationship to the shape, development and size of companies through the general mode of fuzzy logic. The results of the study indicate that 511 Italian companies are always advances in the evolution of corporate management accounting system do not meet. In other words, you can find a large number of companies with a simple organizational structure but with an advanced management accounting system.

Rangone [20] according to strategic management accounting, strategic cost management and non-financial performance metrics are introduced as strategies to overcome the limitations of traditional management accounting systems. He provided an analytical framework using fuzzy logic to establish a relationship between the effectiveness of the organization, key indicators of success and performance measurement.

Nagasawa [21] using fuzzy set theory, a model for value engineering and cost management was designed. The existence of different tools and solutions for value engineering, their prioritization as well as the related ambiguities related to them, have been expressed as reasons for the need to address fuzzy set theory in value engineering.

Nachtmann and Needy [22] through the application of fuzzy logic concepts in costing, they developed an activity-based costing system. This study demonstrates the benefits of a fuzzy activity based costing system and the stages of development and implementation in a pharmaceutical company.

Nachtmann and Needy [23] have introduced and compared methods of overcoming ambiguity and uncertainty over the input data of the activity-based costing system from the perspective of cost-benefit analysis. According to the comparison, the use of fuzzy method in activity-based costing to consider the conditions of ambiguity and uncertainty is more appropriate than the methods based on each of the standard models, distance and Monte Carlo with normal input variables.

Yuan [24] Using a fuzzy expert system, designed a model to analyze costs, activity volume and profit in ambiguous conditions by management. In this new system, unlike the traditional mode, which uses the break-even point and assumes a state of confidence, the information of experts and the concepts of fuzzy sets are used to overcome inaccuracies and ambiguities.

4. Conclusion

The purpose of this paper is to introduce fuzzy set theory in accounting and examine its relationship as a way to solve accounting problems in conditions of ambiguity. Fuzzy theory, unlike traditional quantitative methods, provides a mathematical framework for inaccurate phenomena in human systems and decision making that can be applied on a regular basis. This theory does not require accurate measurements. As a result, fuzzy theory can be invaluable to accountants, especially in times of ambiguity and when care cannot be taken. Therefore, due to the ambiguities that exist in accounting and auditing issues; accountants and auditors should not hesitate to use fuzzy set theory. One of the features of the fuzzy set is that it reduces the need for accurate data in decision making. Because today, information plays an important role in economic decision-making, and no doubt [25], the quality of their, including accuracy in providing it to a wide range of users, can be useful for decision-making [26].

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Efficiency Study with Undesirable Inputs and Outputs in DEA

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Data Envelopment Analysis (DEA) is one of the well-known methods for calculating efficiency, determining efficient boundaries and evaluating efficiency that is used in specific input and output conditions. Traditional models of DEA do not try to reduce undesirable outputs and increase undesirable inputs. Therefore, in this study, in addition to determining the efficiency of Decision-Making Units (DMU) with the presence of some undesirable input and output components, its effect has also been investigated on the efficiency limit. To do this, we first defined the appropriate production possibility set according to the problem assumptions, and then we presented a new method to determine the unfavorable performance of some input and output components in decision-making units. And we determined the impact of unfavorable inputs and outputs on the efficient boundary. We also showed the model result by providing examples for both unfavorable input and output states and solving them and determining the efficiency score and driving them to the efficient boundary by plotting those boundaries.

1. Introduction

Data envelopment analysis is one of the most popular methods for determining efficiency, and the boundary of efficiency based on the concept of condition of defective units. In 1987, Charles et al. [2] identified efficient boundaries using linear programming and used them to determine productivity. In this way, they used both output-axis and input-axis models. Although these two models are not the only ones used, they are still the most popular DEA model. Many researchers use the DEA method to determine the boundary of performance and evaluate performance [4].

Over the past two decades, DEA has established itself as the strongest and most valuable methodology [3]. In many practical issues, some inputs of decision-making units may be such that increasing these inputs increases efficiency and decreasing it reduces efficiency, such as waste recycling operations, scrap metal and glass, etc., where it is necessary to increase the undesirable inputs to improve the level of efficiency, or some of the outputs of decision-making units may be such that increasing these outputs reduces efficiency and decreases it increases efficiency. Consider the waste of a factory or the deaths of patients in hospitals and the dismissals of doctors and nurses in

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training centers, which should be reduced as an undesirable output to increase efficiency. Undesirable outputs are generally desirable products and therefore can only be reduced by reducing them. There are methods for importing undesirable outputs into the DEA that can be divided into two categories:

– Direct methods.
– Indirect methods.

In indirect methods, undesirable inputs and outputs in each unit are converted into desired inputs and outputs by a uniform descending function and then the performance of the units is evaluated using DEA standard models. Direct ones are methods that use assumptions in production possibility set so that they are used in evaluating desirable input and output.

Conventional data envelopment analysis models in most studies treated the units under evaluation as a black box, producing only a series of primary inputs and using them to produce a series of final outputs; However, with the research of the last two decades, they came to the conclusion that the obtained efficiency is not accurate without considering their internal structure, and there are ambiguities in the analysis of its efficiency, and the role of undesirable factors should also be examined.

In the real world, it is not possible to match all inputs and outputs of inefficient units based on DEA results, which Chiang Kao showed with a new model design, which is possible. Unfavorable outputs are generally desirable products and can therefore be reduced only by a concomitant reduction in the second product. To understand this concept, the price of an undesirable output shadow must be negative and the opposite for a positive output. Based on these conditions, Kao et al. [6] in a paper presented a data envelopment analysis model that allows the production units under evaluation to determine the shadow price for both favorable and unfavorable outputs to maximize the measured performance score. The proposed model satisfies the assumption of poor usability of outputs. It is also shown that there is a directional function model in a group that has been widely used in modeling adverse outputs. However, unlike conventional directional distance measures, the proposed model is able to provide performance in the range of zero and one for easy comparison between inefficiently produced units.

Cross-productivity evaluation methods have long been proposed as an option for ranking decision-making units in data envelopment analysis. Neutral reciprocity performance evaluation methods are developed in a way that is only self-interested and indifferent to other DMUs. Accordingly, in 2019, Shi et al. [19] introduced a new cross-performance evaluation method in which each DMU has a neutral attitude towards its other peer units. This is done by introducing an Ideal Virtual Border (IVF) and a Non-Ideal Virtual Border (AVF). Unlike cross-performance evaluation methods, this cross-performance evaluation method determines the set of input and output weights for each DMU. The most important operation in this study is to introduce an ideal virtual boundary and a non-ideal virtual boundary improving DMU performance by considering IVF and AVF as evaluation criteria, minimizing deviation from IVF and maximizing deviation from AVF. In 2019, Wu et al. [5] studied the environmental efficiency measurement of thermoelectric power plants using an efficient frontier DEA approach with fixed-sum undesirable output.

In 2020, Song et al. [9] Studied accident deaths as undesirable output in the production and safety evaluation in Chinese coal mines. In 2020, Walheer [1] studied the output, input, and undesirable output interconnections in data envelopment analysis: Convexity and returns-to-scale. In 2020, Yu et al. [8] assessed environmental provincial eco-efficiency in China an improved network data envelopment analysis model with undesirable output. In 2020, Gómez-Calvet et al. [7] evaluated European energy efficiency evaluation based on the use of super-efficiency under undesirable outputs in SBM models. In this research, we have presented the possibility of production in accordance with the concept of undesirable inputs and outputs. Then, in the concept of inputs and outputs, we have examined the efficiency of decision-making units and the efficiency boundary diagram with the presence of undesirable inputs and outputs by providing an example.

2. Production Possibility Set

Suppose we have n observations on n DMUs with input and output vectors (x_j, y_j) for j = 1, 2,..., n. Let $x_j = (x_1,..., x_{mj})^T$ and $y_j = (y_{1j},..., y_{sj})$. All $x_j \in R^m$ and $y_j \in R^s$ and $x_j > 0$, $y_j > 0$ *for j = 1, 2,... n.* The input matrix *X* and output matrix *Y* can be represented as $X = [x_1,..., x_j,..., x_n]$, $Y = [y_1,..., y_j,..., y_n]$.

Where *X* is an $(m \times n)$ matrix and *Y* an $(s \times n)$ matrix.

The production possibility set T is generally defined as

$$T = \{(x,y)1 \text{ x can produce y}\}. \tag{1}$$

In DEA, the production possibility set under a Variable Return to Scale (VRS) technology is constructed form the observed data (x_j, y_j) *for j = 1, 2, ..., n* as follows:

$$T = \left\{ (x, y) \middle| x \geq \sum_{j=1}^{n} \lambda_j x_j \ , \ y \leq \sum_{j=1}^{n} \lambda_j y_j \ , \lambda_j \geq 0 \ , \ \sum_{j=1}^{n} \lambda_j = 1, \ j = 1,..., n \right\}. \tag{2}$$

In the absence of undesirable factors when a DMU_O, $o \in \{1,2,..., n\}$, is under evaluation, we can use the following BCC model:

$$\begin{aligned} & \min \ \theta \\ & s.t \quad \theta x_o - X\lambda \geq 0 \\ & \qquad Y\lambda \geq y_o \ , \\ & \qquad 1^T \lambda = 1, \\ & \qquad \lambda \geq 0. \end{aligned} \tag{3}$$

Corresponding to each output *y*, *L(y)* is defined as the following:

$$L(y_j) = \{x | (x, y_j) \in T\}. \tag{4}$$

In fact, $L(y_j)$ is a function that y_j portrays to a subset of inputs so that inputs can produce y_j.

Now suppose that some inputs are undesirable so input matrix X can be represented

as $X=(X^g, X^b)^T$, where $X^g=(x^g_{1j},\dots,x^g_{m_1 j}), j=1,\dots,n$ and $X^b=(x^b_{1j},\dots,x^b_{m_1 j})$ $j=1,\dots,n$ *are* $(m_1\times n)$ *and* $(m_2\times n)$ matrixes that represent desirable (good) and undesirable (bad) inputs, respectively. And similarly, suppose that some outputs are undesirable so outputs. Matrix Y can be represented as $Y=(Y^g, Y^b)^T$, where $Y^g=(y^g_{1j},\dots,y^g_{s_1 j}), j=1,\dots,n$ and $Y^b=(y^b_{1j},\dots,y^b_{s_2 j})$, $j=1,\dots,n$ are $(s_1\times n)$ and $(s_2\times n)$ matrixes that represent. Desirable (good) and undesirable (bad) inputs, respectively.

Definition 1. Let DMU of $(x^g_1, x^b_1, y^g_1, y^b_1)$ is dominant to DMU of $(x^g_2, x^b_2, y^g_2, y^b_2)$ if $(x^g_1 \le x^g_2, x^b_1 \ge x^b_2, y^g_1 \ge y^g_2)$ and $y^b_1 \le y^b_2$ the unequal be strict at least in a component. So that,

$$\begin{pmatrix} -x^g_1 \\ x^b_1 \\ y^g_1 \\ -y^b_1 \end{pmatrix} \ge \begin{pmatrix} -x^g_2 \\ x^b_2 \\ y^g_2 \\ -y^b_2 \end{pmatrix}.$$

Definition 2. DMU_O is efficient if in T there is no DMU to be dominant over it.

We consider the properties of the Production Possibility Set as the following:

- T is convex.
- T is closed.
- The monotony property of desirable inputs and outputs. So that, $\forall u \in R^{m_1}_+, v \in R^{s_1}_+, (x^g, x^b, y^g, y^b) \in T \Rightarrow (x^g+u, x^b, y^g-v, y^b) \in T$

This is not necessarily established for undesirable factors, because in this case, T has no efficient DMU.

We can define the production possibility set T satisfying *Eq. (1)* through *Eq. (3)* by

$$T=\left\{(x^g, x^b, y^b, y^g) \;\middle|\; \begin{array}{c} x^g \ge \sum_{j=1}^{n}\lambda_j x^g_j, x^b=\sum_{j=1}^{n}\lambda_j x^b_j, y^b=\sum_{j=}^{n}\lambda_j y^b_j, y^g \le \sum_{j=1}^{n}\lambda_j y^g_j \\ \sum_{j=1}^{n}\lambda_j=1, \lambda_j \ge 0, j=1,\dots,n \end{array}\right\} \quad (5)$$

3. Measures of Efficiency Using Undesirable Factors

In input oriented data, the efficiency of the DMU under evaluation is obtained by decreasing and increasing the desirable and undesirable input, respectively. And similarly, in output oriented data, we increase desirable output and decrease the undesirable output.

3.1. Nature of the Input

Suppose $DMU_o = (x_o^g, x_o^b, y_o^g, y_o^b)$ be unit under evaluation, corresponding to the output $y_o = (y_o^g, y_o^b)$ and using *Eq. (2)* $L(y_o^g, y_o^b)$ in defined as follows:

$$L(y_o^g, y_o^b) = \left\{(x^g, x^b) \middle| (x^g, x^b, y_o^g, y_o^b) \in T\right\}. \tag{6}$$

And we consider the subset of $L(y_o^g, y_o^b)$ as:

$$\partial^p L(y_o^g, y_o^b) = \left\{(x^g, x^b) \middle| \forall (u,v) \geq 0, (u,v) \neq 0 \quad \Rightarrow \quad (x^g - u, x^b + v) \notin L(y_o^g, y_o^b)\right\}. \tag{7}$$

That $\partial^s L(y_o^g, y_o^b)$ includes all inputs of the efficient DMUs which can produce (y_o^g, y_o^b).

The model to evaluate the efficiency of DMUo with the most decrease of x_o^g and the most increase of x_o^b is as follows:

$$d_o{}^g = x_o{}^g,$$

$$d_o{}^b = x_o{}^b - x_{max}^b.$$

So that

$$(x_{max}^b)_i = Max_j \left\{x_{ij}^b\right\}.$$

Therefore, according to the definition of inefficiency we have:

$$\begin{aligned}
&\theta_o^* = Max \quad \theta_o, \\
&st. \\
&\quad \sum_{j=1}^{n} \lambda_j x_j^g + s^- = x_o^g - \theta_o d_o{}^g. \\
&\quad \sum_{j=1}^{n} \lambda_J x_j^b = x_o^b - \theta d_o{}^b. \\
&\quad \sum_{j=1}^{n} \lambda_j y_j^g - s^+ = y_o^g. \\
&\quad \sum_{j=1}^{n} \lambda_j y_j^b = y_o^b. \\
&\quad \sum_{j=1}^{n} \lambda_j = 1. \\
&\quad \lambda_j \geq 0 \qquad for \quad all \quad j = 1, ..., n.
\end{aligned} \tag{8}$$

According to the definition of production possibility set, *model (1)* is possible in this set.

Theorem 1. The DMUo in *model (8)* is efficient if and only if

- $\theta_o^* = 1$.
- All slacks are zero for all optimal solutions.

Theorem 2. If all optimal solution of *model (8)* be (θ^*, s^{-*}), then

$$(x^g - \theta^* d^g - s^{-*}, x^b - \theta^* d^b) \in \partial^p L(y_o^b, y_o^g).$$

s^- is one of optimal answers.

4. Numerical Example 1

As an example, consider seven DMUs with one desirable input, one undesirable input to produce a desirable output normalized at level 1. These DMUs were explained in *Table 1*.

Regarding *Table 1* and *Fig. 1*, it can be seen that DMUs D, E, and F are efficient and they are on the $\partial^s L(y_G^g)$. On the other hand, efficiency of other DMUs have been examined through their image on $\partial^s L(y_G^g)$. (Efficient Frontiers).

Table 1. *The inputs and outputs data for 7 DMUs.*

DMU's	x^g	x^b	y^g	$1-\theta^*$
A	3	1	1	0.33
B	2	2	1	0.5
C	1	3	1	1
D	1	5	1	1
E	2	6	1	1
F	3	7	1	1
G	4	4	1	0.43

Similar discussion can be presented for the output oriented.

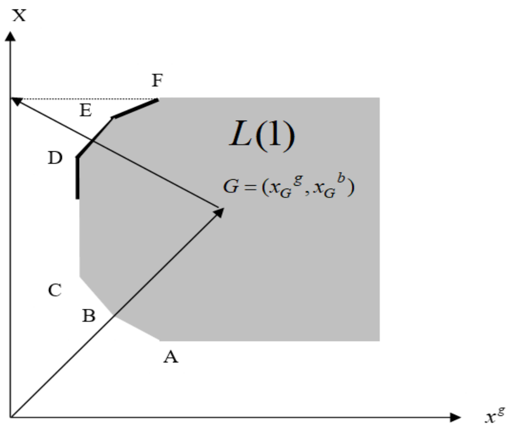

Fig. 1. *The graph of the* $L(y_G)$.

3.1. Nature of the Output

Suppose $DMU_o = (x_o^g, x_o^b, y_o^g, y_o^b)$ be unit under evaluation, corresponding to the output $x_o = (x_o^g, x_o^b)$ and using *Eq. (2)* $p(x_o^g, x_o^b)$ is defined as follows:

$$p(x_o^g, x_o^b) = \left\{ (y^g, y^b) \middle| (x_o^g, x_o^b, y^g, y^b) \in T \right\}.$$

And we consider the subset of $p(x_o^g, x_o^b)$ as:

$$\partial^p p(x_o^g, x_o^b) = \left\{ (y^g, y^b) \middle| \forall (u, v) \geq 0, (u, v) \neq 0 \quad \Rightarrow \quad (y^g + u, y^b - v) \notin p(x_o^g, x_o^b) \right\} \tag{9}$$

That $\partial^s L(y_o^g, y_o^b)$ includes all inputs of the efficient DMUs which can produce (y_o^g, y_o^b).

The model to evaluate the efficiency of DMUo with the most decrease of y_o^g and the most increase of y_o^b is as follows:

$$NE^d(x_o, y_o) = \sup \left\{ \beta \middle| y_o + \beta d \in p(x_o) \right\}.$$

where $d = (d^g, d^b)$ indicate the direction of unit under evaluation such that $d^g \in R_+^{s_1}$ and $d \in R_-^{m_2}$ leads to increase the corresponding outputs and decreasing the unconfirmed outputs.

In this research, we direct the desired outputs to the efficient boundary in a radial direction. Thus: $d^g = y_o^g$.

We also reduce the undesirable outputs in the radial direction, i.e.

$$d^I = -y_o^b.$$

Therefore, according to the definition we have:

$$\begin{aligned}
&\beta_o^* = \text{Max} \quad \beta_o, \\
&\text{st.} \\
&\quad \sum_{j=1}^{n} \lambda_j x_j^g + s^- = x_o^g. \\
&\quad \sum_{j=1}^{n} \lambda_j x_j^b = x_o^b. \\
&\quad \sum_{j=1}^{n} \lambda_j y_j^g - s^+ = y_o^g + \beta_o d_o^g. \\
&\quad \sum_{j=1}^{n} \lambda_j y_j^b = y_o^b + \beta_o d_o^b. \\
&\quad \sum_{j=1}^{n} \lambda_j = 1. \\
&\quad \lambda_j \geq 0 \qquad \text{for} \quad \text{all} \quad j = 1, \ldots, n.
\end{aligned}$$

Theorem 3. The DMUo in *model (10)* is efficient if and only if

- $\beta_o^* = 1$.
- All slacks are zero for all optimal solutions.

Theorem 3. If be optimal solution of *model (10)* in, then

$$(y_o^* + \beta_o^* d_o^g + s^{+^*}, y_o^b + \beta_o^* d_o^b) \in \partial^p p(x_o^g, x_o^b).$$

5. Numerical Example 2

We consider five decision-making units with an optimal input to produce an undesirable output and a desirable output. These decision-making units are described in *Table 2*. *Fig. 2* shows that the decision-making units D, E and F are efficient. On the other hand, other decision-making units have been examined through their image on the (efficient border) of their efficiency.

Table 2. *The inputs and outputs data for 5 DMUs by model 10.*

$DMU's$	x^g	y^g	y^b	$1-\beta^*$
A	1	4	1	1
B	1	5	2	1
C	1	5	4	1
D	1	4	5	0.25
E	1	3	3	0.5

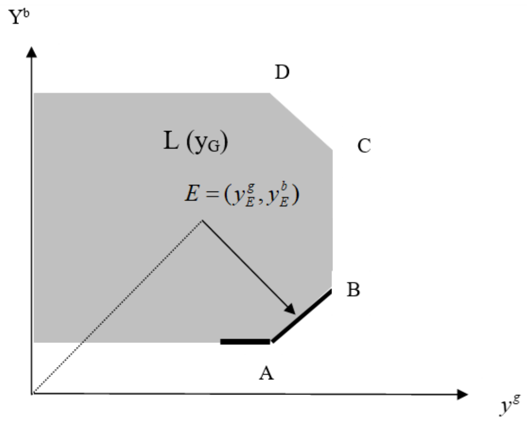

Fig. 2. *The graph of the L.*

6. Conclusion

Our proposed models in this study determine the efficiency of decision-making units, assuming that some of their input and output components may be undesirable. Numerical examples and model diagrams show that these models ensure that the presence of undesirable input and output factors is effective in determining the efficiency boundary of the decision-making units under evaluation and are compared with a unit corresponding to the efficient boundary set. By decreasing undesirable output and increasing undesirable input, the efficiency of decision-making units can improve and push them towards the efficient frontier.

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