J. Fuzzy. Ext. Appl. Vol. 1, No. 1 (2020) 27–41

J. Fuzzy. Ext. Appl

Journal of Fuzzy Extension & Applications

www.journal-fea.com

Application of Transportation Problem under Pentagonal Neutrosophic Environment

Sapan Kumar Das*

Department of Revenue, Ministry of Finance, Govt. of India.

P A P E R I N F O	A B S T R A C T
Chronicle: *Received: 04 September 2019* *Revised: 09 December 2019* *Accepted: 15 January 2020* **Keywords:** *Transportation Problem.* *Pentagonal Neutrosophic Numbers.* *Linear Programming*.	The paper talks about the pentagonal Neutrosophic sets and its operational law. The paper presents the $\alpha_N, \beta_N, \lambda_N -$cut of single valued pentagonal Neutrosophic numbers and additionally introduced the arithmetic operation of single-valued pentagonal Neutrosophic numbers. Here, we consider a transportation problem with pentagonal Neutrosophic numbers where the supply, demand and transportation cost is uncertain. Taking the benefits of the properties of ranking functions, our model can be changed into a relating deterministic form, which can be illuminated by any method. Our strategy is easy to assess the issue and can rank different sort of pentagonal Neutrosophic numbers. To legitimize the proposed technique, some numerical tests are given to show the adequacy of the new model.

1. Introduction

For survival of our life there is a need to move the item from various sources to various goals. Due to high population, it is very challenging to company, how to send the product to numbers of costumers or origins to a numbers of warehouse or store in a minimizing cost. This kind of issue is called Transportation Problem (TP) and it is an exceptional sort of Linear Programming (LP) problem where the organization's primary goal is limiting the expense. On account of wide application i.e. production planning, health sector, inventory control, network system etc., TP have consistently made separate space for analysts. An outline has attracted *Fig.1* which is speaks to connection among supply and demand.

Hitchcock [1] pioneered the basic transportation problem. This kind of traditional issue can be as a direct programming issue and afterward tackled simplex strategy. This type of classical problem can be modelled as a linear programming problem and then solved simplex method. A primal simplex method to transportation problem was solved by Dantzig & Thapa [2]. Transportation Problem is a different type of structure therefore simplex method is not suitable for finding the objectives. Due to some drawback in simplex method for solving TP, a new Initial Basic Feasible Solution (IBFS) method was developed. By using the IBFS, there are three type of methods (1) north-west corner (NWC), (2) least-

* Corresponding author
E-mail address: cool.sapankumar@gmail.com
DOI: 10.22105/jfea.2020.246633.1001

cost method, (3) vogel's approximation method. In classical TP the decision makers knows the values of supply, demand and transportation cost i.e. the decision makers consider the crisp numbers. However, in our day to day life applications, the decision makers may not be known precisely to all the parameters of transportation problem due to some uncontrolled factor. To overcome this uncontrolled factor, fuzzy decision making method is introduced.

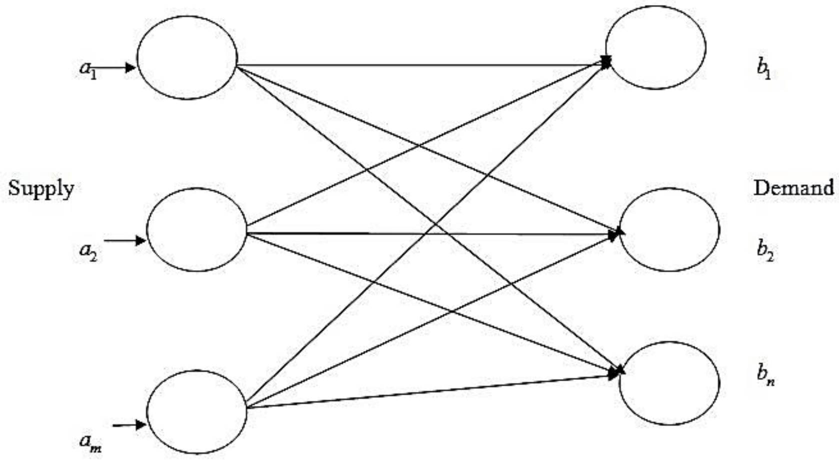


Fig. 1. *LP Transportation problem.*

The basic concepts of fuzzy set theory was introduced by Zadeh [3] in 1965. Since, several researchers have carried out investigation on fuzzy transportation problem (FTP). A Fuzzy Linear Programming (FLP) problem was proposed by Zimmermann [4] and he has proved that the method was always very effective. Subsequently, Zimmermann FLP model has developed to solve different fuzzy transportation problems. Chanas et al. [5] considered FTP where supply and demand are fuzzy numbers and transportation cost is crisp number. Das et al. [6] proposed a fully fuzzy LP problem where all the parameters are trapezoidal fuzzy numbers and that method extend to solve Fully Fuzzy TP (FFTP). Dinagar and Palanivel [7] proposed a FFTP where demand, supply, and transportation costs are trapezoidal fuzzy numbers. Kaur and Kumar [8] have introduced an algorithmic for solving the fuzzy transportation problem. Fuzzy zero pint method for solving FFTP problem was proposed by Pandian and Natrajan [9] in which supply, demand and transportation cost are trapezoidal fuzzy numbers. Kundu et al. [10] introduced a solid transportation model with crisp and rough costs. Some other researchers [11-19] also have studied this general transportation problem in a fuzzy environment. Maheswari and Ganesan [20] proposed fully fuzzy transportation problem using pentagonal fuzzy numbers.

Due to some complications, insufficient information, multiple sources of data arises in our real-life problem; it is not always possible to use fuzzy numbers. The fuzzy sets mainly consider the membership functions. Intuitionistic Fuzzy Sets (IFS) is an extension version of fuzzy sets and it can be used to solve them. IFS has been proposed by Atanassov [24] and it's take care both mixture of membership function and non-membership function. Since, several researchers [25-30] considered the IFS for solving TP. Aggarwal and Gupta [31] studied the sensitivity analysis of intuitionistic fuzzy solid transportation problem. Singh and Yadav [32] introduced a novel solution for solving fully Intuitionistic Fuzzy Transportation Problem (IFTP) in which demand, supply and transportation cost are considered intuitionistic triangular fuzzy numbers. In that paper, they obtaining both negative solutions for variables and objective functions instead of positive transportations cost. After the shortcomings of

Singh and Yadav paper, Mahmoodirad et al. [33] proposed a method for fully IFTP in which demand, supply and transportation cost are considered intuitionistic fuzzy numbers.

In genuine situation, we regularly experience with deficient and uncertain data where it isn't conceivable to speak to the data just by the methods for membership function and non-membership function. To manage such circumstances, Smarandache [36] in 1988, introduced the structure of Neutrosophic Set (NS) which is higher version of both fuzzy and intuitionistic fuzzy. Neutrosophy set might be described by three autonomous degrees, i.e. (i) truth-membership degree (T), (ii) indeterminacy membership degree (I), and (iii) falsity membership degree (F). Later, Wang et al. [37] introduced a Single Value Neutrosophic Set (SVNS) problem for solving a practical problem. Ye [38] introduced the Trapezoidal Neutrosophic Set (TrNS) by combining the concept of Trapezoidal Fuzzy Numbers (TrFN) and SVNS. To take the advantages of beauty of NS, several researchers [39-45] proposed different method for solving LP problem under Neutrosophic environment. Das and Dash [46] proposed a modified solution of Neutrosophic LP problem. Recently, Das and Chakraborty [47] proposed a new approach for solving LP problem in pentagonal Neutrosophic environment.

Motivation. Neutrosophic sets always plays a vital role in uncertainty environment. Before going to discussion the motivation of our paper, we demonstrate the different author's research work towards the TP under mixed constraints.

Table 1. *Significance influences of the different authors towards under various environment.*

Author	**Year**	**Main Contribution**
Korukoglu and Balli [50]	*2011*	*Crisp environment.*
Bharati [51] environment	*2019*	*Trapezoidal intuitionistic fuzzy.*
Ahmad and Adhami [52]	*2018*	*Neutrosophic fuzzy environment.*
Singh and Yadav [32]	*2016*	*Traingular fully intuitionistic fuzzy.*
Ebrahimnejad & Verdegay [25] Environment	*2017*	*Trapezoidal intuitionistic fuzzy.*
Mahaswari & Ganesan[20]	*2018*	*Pentagonal fuzzy number.*
Srinivasan et al. [53]	*2020*	*Triangular fuzzy number.*

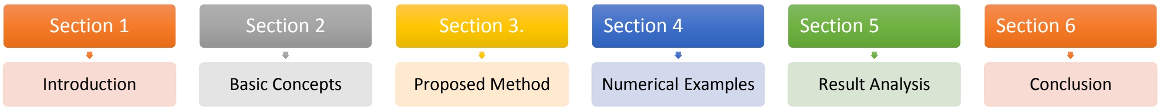
From the above discussion on TP which are readily available in our literature, there are no current techniques for solving pentagonal TP under Neutrosophic condition. In this way, there is have to setup another technique for pentagonal Neutrosophic transportation issue. This total situation has persuaded us to build up another strategy for illuminating TP with pentagonal Neutrosophic numbers. Just because, we build up a calculation and applied, all things considered, issue. The primary commitments of the paper as follows:

We characterize Neutrosophic Transportation Problem (NTP) issue in which the supply, demand and transportation cost are taken as pentagonal Neutrosophic numbers.

- This model assists with settling another arrangement of issue with pentagonal Neutrosophic numbers.
- In our literature of pentagonal Neutrosophic numbers, we will in general present a recently evolved scoring function.
- By using our recently scoring function, the pentagonal Neutrosophic TP is changing over into its crisp TP.

– To best our insight, it would be the primary strategy to unravel the PNTP. Consequently, in our paper direct relationship with relative system doesn't rise.

The rest of the paper is organized as the following way:



2. Preliminaries

In this section, we call back the some definitions and basic concepts which are pivotal in this paper. The well-defined definitions are referred [47, 49] throughout the paper.

2.1. Definition: Neutrosophic Set (NS)

A set $\tilde{N}_M$ is identified as a Neutrosophic set if $\tilde{N}_M = \{\langle x; [\theta_{\tilde{N}_M}(x), \varphi_{\tilde{N}_M}(x), \sigma_{\tilde{N}_M}(x)]\rangle : x \in X\}$, where $\theta_{\tilde{N}_M}(x): X \to]-0,1+[$ is declared as the truthness function, $\varphi_{\tilde{N}_M}(x): X \to]-0,1+[$ is declared as the hesitation function, and $\sigma_{\tilde{N}_M}(x): X \to]-0,1+[$ is declared as the falseness function.

$\theta_{\tilde{N}_M}(x), \varphi_{\tilde{N}_M}(x)$ & $\sigma_{\tilde{N}_M}(x)$ displays the following relation:

$$-0 \leq \text{Sup}\,\{\theta_{\tilde{N}_M}(x)\} + \text{Sup}\,\{\varphi_{\tilde{N}_M}(x)\} + \text{Sup}\,\{\sigma_{\tilde{U}_M}(x)\} \leq 3+.$$

2.2. Definition: Single-Valued Neutrosophic Set (SNS)

A set $\tilde{N}_M$ in the definition 2.3 is called as a SNS $(\widetilde{SN}_M)$ if x is a single-valued independent variable. $\widetilde{SNA} = \{\langle x; [\theta_{\widetilde{SN}_M}(x), \varphi_{\widetilde{SN}_M}(x), \sigma_{\widetilde{SN}_M}(x)]\rangle : x \in X\}$, $\theta_{\widetilde{SN}_M}(x), \varphi_{\widetilde{SN}_M}(x)$& $\sigma_{\widetilde{SN}_M}(x)$ signified the notion of correct, indefinite and incorrect memberships function, respectively.

2.3. Definition: Single-Valued Pentagonal Neutrosophic Number (SPNN)

A SPNN $(\tilde{M})$ is defined as $\widetilde{SPN} = \langle [(m^1, n^1, o^1, p^1, q^1); \mu], [(m^2, n^2, o^2, p^2, q^2); \theta], [(m^3, n^3, o^3, p^3, q^3); \eta] \rangle$, where $\mu, \theta, \eta \in [0,1]$. The truth membership function$(\mu_{\widetilde{SPN}}): \mathbb{R} \to [0, \mu]$, the hesitant membership function $(\theta_{\widetilde{SPN}}): \mathbb{R} \to [\theta, 1]$ and the false membership function $(\eta_{\widetilde{SPN}}): \mathbb{R} \to [\eta, 1]$ are given as:

$$\mu_{\widetilde{SPN}}(x) = \begin{cases} \mu_{\widetilde{Ssl1}}(x) m^1 \leq x < n^1 \\ \mu_{\widetilde{Ssl2}}(x)\, n^1 \leq x < o^1 \\ \quad \mu \qquad x = o^1 \\ \mu_{\widetilde{Ssr2}}(x)\, o^1 \leq x < p^1 \\ \mu_{\widetilde{Ssr1}}(x) p^1 \leq x < q^1 \\ \quad 0 \quad otherwise \end{cases}, \quad \theta_{\widetilde{SPN}}(x) = \begin{cases} \theta_{\widetilde{Ssl1}}(x) \;\; m^2 \leq x < n^2 \\ \theta_{\widetilde{Ssl2}}(x) \;\; n^2 \leq x < o^2 \\ \quad \vartheta \qquad x = o^2 \\ \theta_{\widetilde{Ssr2}}(x) \;\; o^2 \leq x < p^2 \\ \theta_{\widetilde{Ssr1}}(x)\, p^2 \leq x < q^2 \\ \quad 1 \quad otherwise \end{cases}.$$

$$\eta_{\widetilde{SPN}}(x) = \begin{cases} \eta_{\widetilde{Ssl1}}(x) m^3 \leq x < n^3 \\ \eta_{\widetilde{Ssl2}}(x)\, n^3 \leq x < o^3 \\ \vartheta \qquad x = o^3 \\ \eta_{\widetilde{Ssr2}}(x)\, o^3 \leq x < p^3 \\ \eta_{\widetilde{Ssr1}}(x) p^3 \leq x < q^3 \\ 1 \quad otherwise \end{cases}.$$

2.4. Score and Accuracy Function

Let us consider a single valued Pentagonal Neutrosophic Numbers (PNN) as $\tilde{F}_{Pen} = (F_1, F_2, F_3, F_4, F_5; \pi, \sigma, \rho)$, The primary application of score function is to drag the judgment of conversion of PNN to crisp number. Also, the mean of the PNN components is $\frac{(F_1+F_2+F_3+F_4+F_5)}{5}$ and score value of the membership portion is $\frac{\{2+\pi-\rho-\sigma\}}{3}$.

Thus, for a P.N.N $\tilde{F}_{Pen} = (F_1, F_2, F_3, F_4, F_5; \pi, \sigma, \rho)$. Score function is scaled as $\widetilde{SC}_{Pen} = \frac{1}{15}(F_1 + F_2 + F_3 + F_4 + F_5) \times \{2 + \pi - \rho - \sigma\}$. Accuracy function is scaled as $\widetilde{AC}_{Pen} = \frac{1}{15}(F_1 + F_2 + F_3 + F_4 + F_5) \times \{2 + \pi - \sigma\}$. Here, $\widetilde{SC}_{Pen} \in R$, $\widetilde{AC}_{Pen} \in R$.

2.5. Relationship between any Two PNN

Let us consider any two pentagonal Neutrosophic number defined as follows:

$$F_{Pen1} = (\pi_{Pen1}, \sigma_{Pen1}, \rho_{Pen1}), F_{Pen2} = (\pi_{Pen2}, \sigma_{Pen2}, \rho_{Pen2}).$$

$$SC_{Pen1} > SC_{Pen2}, F_{Pen1} > F_{Pen2},$$

$$SC_{Pen1} < SC_{Pen2}, F_{Pen1} < F_{Pen2},$$

$$SC_{Pen1} = SC_{Pen2}, F_{Pen1} = F_{Pen2}.$$

Then,

$$AC_{Pen1} > AC_{Pen2}, F_{Pen1} > F_{Pen2},$$

$$AC_{Pen1} < AC_{Pen2}, F_{Pen1} < F_{Pen2},$$

$$AC_{Pen1} = AC_{Pen2}, F_{Pen1} = F_{Pen2}.$$

2.6. Basic Operations

Let $\widetilde{F_1}$= < $(c_1, c_2, c_3, c_4, c_5)$;$\pi_{\widetilde{p_1}}, \mu_{\widetilde{p_1}}, \sigma_{\widetilde{p_1}}$> and $\widetilde{F_2}$=<$(d_1, d_2, d_3, d_4, d_5)$;$\pi_{\widetilde{p_2}}, \mu_{\widetilde{p_2}}, \sigma_{\widetilde{p_2}}$> be two IPFNs and $\alpha \geq 0$. Then the following operational relations hold:

$$\widetilde{F_1}+\widetilde{F_2} = < (c_1+d_1, c_2+d_2, c_3+d_3, c_4+d_4, c_5+d_5);\ \max\{\pi_{\widetilde{p_1}}, \pi_{\widetilde{p_2}}\}, \min\{\mu_{\widetilde{p_1}}, \mu_{\widetilde{p_2}}\}, \min\{\sigma_{\widetilde{p_1}}, \sigma_{\widetilde{p_2}}\} >,$$

$\widetilde{F_1}$- $\widetilde{F_2}$= < (c_1- d_5, c_2-d_4, c_3-d_3, c_4-d_2, c_5-d_1); max{$\pi_{\widetilde{p_1}}, \pi_{\widetilde{p_2}}$}, min{$\mu_{\widetilde{p_1}}$, $\mu_{\widetilde{p_2}}$},min{ $\sigma_{\widetilde{p_1}}$, $\sigma_{\widetilde{p_2}}$} >,

$\widetilde{F_1} \times \widetilde{F_2}$= < ($c_1d_1, c_2d_2, c_3d_3, c_4d_4, c_5d_5$); $\pi_{\widetilde{p_1}}\ \pi_{\widetilde{p_2}}, \mu_{\widetilde{p_1}}+, \mu_{\widetilde{p_2}} - \mu_{\widetilde{p_1}}\ \mu_{\widetilde{p_2}}, \sigma_{\widetilde{p_1}} + \sigma_{\widetilde{p_2}} - \sigma_{\widetilde{p_1}}\sigma_{\widetilde{p_2}}$ >,

$\alpha\widetilde{F_1}$= <($\alpha c_1, \alpha c_2, \alpha c_3, \alpha c_4, \alpha c_5$);$1-(1-\pi_{\widetilde{p_1}})^{\alpha}, {\mu_{\widetilde{p_1}}}^{\alpha}, {\sigma_{\widetilde{p_1}}}^{\alpha}$)>,

$\widetilde{F_1}^{\alpha}$= <(${c_1}^{\alpha}, {c_2}^{\alpha}, {c_3}^{\alpha}, {c_4}^{\alpha}, {c_5}^{\alpha}$);${\pi_{\widetilde{p_1}}}^{\alpha}, (1-\mu_{\widetilde{p_1}})^{\alpha}, (1-\sigma_{\widetilde{p_1}})^{\alpha}$ >.

3. Neutrosophic Transportation Problem

Assume that there are *s* number of sources and *t* destinations. Mathematically, the NTP may be stated as:

$$\text{Min } Z = \sum_{i=1}^{s}\sum_{j=1}^{t}\tilde{c}_{ij}^{N} y_{ij}. \tag{1}$$

Subject to constraints

$$\sum_{j=1}^{t} y_{ij} = \tilde{p}_i^{N}, i = 1, 2, ..., s. \tag{2}$$

$$\sum_{i=1}^{s} y_{ij} = \tilde{q}_i^{N}, j = 1, 2, ..., t. \tag{3}$$

$$y_{ij} \geq 0 \quad \forall i, j.$$

For the above mathematical model of PNTP, we defined the following notations:

- *s & t* is the number of sources and destination being indexed by *i & j*.
- $\tilde{p}_i^N = (p_i^{N1}, p_i^{N2}, p_i^{N3}, p_i^{N4}, p_i^{N5}; \theta_i^N, \sigma_i^N, \mu_i^N)$ is the PNN for the items supplied by source i .
- $\tilde{q}_j^N = (q_j^{N1}, q_j^{N2}, q_j^{N3}, p_j^{N4}, p_j^{N5}; \theta_j^N, \sigma_j^N, \mu_j^N)$ is the PNN for the items demanded by destination j .
- $\tilde{c}_{ij}^N = (c_{ij}^{N1}, c_{ij}^{N2}, c_{ij}^{N3}, c_{ij}^{N4}, c_{ij}^{N5}; \theta_{ij}^N, \sigma_{ij}^N, \mu_{ij}^N)$ is the PNN for the items sending one unit from source i to destination j .
- $\tilde{y}_{ij}^N = (y_{ij}^{N1}, y_{ij}^{N2}, y_{ij}^{N3}, y_{ij}^{N4}, y_{ij}^{N5}; \theta_{ij}^N, \sigma_{ij}^N, \mu_{ij}^N)$ is the PNN cost from sources to destination.

The steps of the proposed method are as follows:

<u>*Step 1.*</u> Considering the pentagonal Neutrosophic parameters and variables, the *problem (3)* may be written as:

$$\text{Min } Z=\sum_{i=1}^{s}\sum_{j=1}^{t}(c_{ij}^{N1},c_{ij}^{N2},c_{ij}^{N3},c_{ij}^{N4},c_{ij}^{N5};\theta_{ij}^{N},\sigma_{ij}^{N},\mu_{ij}^{N})\otimes(y_{ij}^{N1},y_{ij}^{N2},y_{ij}^{N3},y_{ij}^{N4},y_{ij}^{N5};\theta_{ij}^{N},\sigma_{ij}^{N},\mu_{ij}^{N}) \quad (4)$$

Subject to constraints

$$\sum_{j=1}^{t}(y_{ij}^{N1},y_{ij}^{N2},y_{ij}^{N3},y_{ij}^{N4},y_{ij}^{N5};\theta_{ij}^{N},\sigma_{ij}^{N},\mu_{ij}^{N})=(p_{i}^{N1},p_{i}^{N2},p_{i}^{N3},p_{i}^{N4},p_{i}^{N5};\theta_{i}^{N},\sigma_{i}^{N},\mu_{i}^{N}), i=1,2,...,s. \quad (5)$$

$$\sum_{i=1}^{s}(y_{ij}^{N1},y_{ij}^{N2},y_{ij}^{N3},y_{ij}^{N4},y_{ij}^{N5};\theta_{ij}^{N},\sigma_{ij}^{N},\mu_{ij}^{N})=(q_{j}^{N1},q_{j}^{N2},q_{j}^{N3},p_{j}^{N4},p_{j}^{N5};\theta_{j}^{N},\sigma_{j}^{N},\mu_{j}^{N}), j=1,2,...,t$$
$$y_{ij}^{N1},y_{ij}^{N2},y_{ij}^{N3},y_{ij}^{N4},y_{ij}^{N5};\theta_{ij}^{N},\sigma_{ij}^{N},\mu_{ij}^{N}\geq 0 \quad \forall i,j. \quad (6)$$

<u>*Step 2.*</u> Here, we confirmed whether the available model is balanced or not, i.e., demand=supply (or) $\sum_{i=1}^{s}\tilde{q}_{j}^{N}=\sum_{j=1}^{t}\tilde{p}_{i}^{N}$. If not, then add dummy variables on row or column and make it balanced model.

<u>*Step 3.*</u> With the help of accuracy function $\Re$, we transform the supply, demand and transportation cost as the following model:

$$\text{Min } Z=\Re(\sum_{i=1}^{s}\sum_{j=1}^{t}(c_{ij}^{N1},c_{ij}^{N2},c_{ij}^{N3},c_{ij}^{N4},c_{ij}^{N5};\theta_{ij}^{N},\sigma_{ij}^{N},\mu_{ij}^{N})\otimes(y_{ij}^{N1},y_{ij}^{N2},y_{ij}^{N3},y_{ij}^{N4},y_{ij}^{N5};\theta_{ij}^{N},\sigma_{ij}^{N},\mu_{ij}^{N})). \quad (7)$$

Subject to constraints

$$\Re(\sum_{j=1}^{t}(y_{ij}^{N1},y_{ij}^{N2},y_{ij}^{N3},y_{ij}^{N4},y_{ij}^{N5};\theta_{ij}^{N},\sigma_{ij}^{N},\mu_{ij}^{N}))=\Re((p_{i}^{N1},p_{i}^{N2},p_{i}^{N3},p_{i}^{N4},p_{i}^{N5};\theta_{i}^{N},\sigma_{i}^{N},\mu_{i}^{N})), i=1,2,...,s. \quad (8)$$

$$\Re(\sum_{i=1}^{s}(y_{ij}^{N1},y_{ij}^{N2},y_{ij}^{N3},y_{ij}^{N4},y_{ij}^{N5};\theta_{ij}^{N},\sigma_{ij}^{N},\mu_{ij}^{N}))=\Re(q_{j}^{N1},q_{j}^{N2},q_{j}^{N3},p_{j}^{N4},p_{j}^{N5};\theta_{j}^{N},\sigma_{j}^{N},\mu_{j}^{N}), j=1,2,...,t$$
$$y_{ij}^{N1},y_{ij}^{N2},y_{ij}^{N3},y_{ij}^{N4},y_{ij}^{N5};\theta_{ij}^{N},\sigma_{ij}^{N},\mu_{ij}^{N}\geq 0 \quad \forall i,j. \quad (9)$$

<u>*Step 4.*</u> After using our new accuracy function, we get PNTP into crisp transportation problem.

<u>*Step 5.*</u> Now find initial basic feasible solution.

- To determine the penalty, subtraction between smallest unit and next to smallest unit in the row (column).
- Identify the largest penalty in row/column, and make the allotment in the cell having the least unit cost.
- If the largest penalty arises in more than one row/column, then select topmost row/left side column.
- When the rows (column) have zero supply and demand until (m+n-1), then stop. Otherwise go to first line.

Step 6. Substitute the all y_{ij} in the objective function, we get the transportation cost.

4. Numerical Example

In our literature study, we got there is no method to solve PNTP. There is a lot of scope in this area to develop new method. We take the advantages in the field of pentagonal Neutrosophic area and we focus to start a developing new algorithm for solving PNTP. The main limitations in between fuzzy and Neutrosophic numbers is that fuzzy numbers taken only membership function (truth degree) however, Neutrosophic number taken truth, indeterminacy & falsity degree. In this segment, we consider another strategy to solve PNLP problem and compare with fuzzy pentagonal LP problem. To prove the relevance and proficiency of our proposed strategy, we consider the fuzzy problem which introduced by [20, 47].

Example 1. (Real-life problem) [47]. In Odisha, India have a company named M/s. Ashirivad dress pvt. Ltd. and the organisation has three plants for delivering dress. The dresses ought to be transport to three ware house under pentagonal Neutrosophic numbers. The conditions of transportation problem are presented in *Table 2*. As the problem should be PNN therefore, the decision-maker considers the confirmation degree of pentagonal number is (1,0,0).

Table 2. *Cost of unit for pentagonal Neutrosophic transportation problem*.

Ware house / **Factories**	**Bhubaneswar**	**Cuttack**	**Rourkela**	**Supply**
Asha	*(5,10,13,14,18;1,0,0)*	*(1,2,3,4,5;1,0,0)*	*(2,6,8,10,14;1,0,0)*	*(2,11,23,34,45;1,0,0)*
Omm	*(3,4,5,6,7;1,0,0)*	*(1,5,6,7,11;1,0,0)*	*(1,4,5,9,16;1,0,0)*	*(10,47,52,65,76;1,0,0)*
Disha	*(3,6,9,12,15;1,0,0)*	*(2,5,7,8,8;1,0,0)*	*(1,1,1,1,1;1,0,0)*	*(3,18,56,76,87;1,0,0)*
Demand	*(11,16,51,67,75;1,0,0)*	*(20,40,60,80,100;1,0,0)*	*(15,30,45,75,110;1,0,0)*	

Step 1. Now using our new ranking function, the issue of PNTP is converting into crisp transportation problem. The model is now available in *Table 3*.

Table 3. *The defuzzified pentagonal Neutrosophic transportation problem.*

Ware house / **Factories**	**Bhubaneswar**	**Cuttack**	**Rourkela**	**Supply**
Asha	*12*	*3*	*8*	*25*
Omm	*5*	*6*	*7*	*50*
Disha	*9*	*6*	*1*	*48*
Demand	*44*	*60*	*55*	

Step 2. To check whether the model is balanced or not.

Supply $\sum a_i = 23 + 50 + 48 = 121.$

Demand $\sum b_i = 44 + 60 + 55 = 159.$

Table 4. *Balanced transportation problem.*

Ware house Factories	Bhubaneswar	Cuttack	Rourkela	Supply
Asha	*12*	*3*	*8*	*23*
Omm	*5*	*6*	*7*	*50*
Disha	*9*	*6*	*1*	*48*
Demand	*44*	*60*	*55*	

Step 3. We use our algorithm (Step 5, Line 1) for finding the initial basic feasible solution.

Table 5. *Initial penalties allocation.*

Ware house Factories	Bhubaneswar	Cuttack	Rourkela	Supply	Penalty
Asha	*12*	*3*	*8*	*23*	*5*
Omm	*5*	*6*	*7*	*50*	*1*
Disha	*9*	*6*	*1*	*48*	*5*
Dummy	*0*	*0*	*0*	*38*	*0*
Demand	*44*	*60*	*55*		
Penalty	*5*	*3*	*1*		

Table 6. *After strike out of 3rd Row the penalties allocation.*

Ware house Factories	Bhubaneswar	Cuttack	Rourkela	Supply	Penalty
Asha	*12*	*3*	*8*	*23*	*5*
Omm	*5*	*6*	*7*	*50*	*1*
Disha	*9*	*6*	*1*	*48*	*5*
Dummy	*0*	*0*	*0*	--	--
Demand	*6*	*60*	*55*		
Penalty	*4*	*3*	*6*		

Table 7. *3rd Penalties allocation.*

Ware house Factories	Bhubaneswar	Cuttack	Rourkela	Supply	Penalty
Asha	*12*	*3*	*8*	*23*	*5*
Omm	*5*	*6*	*7*	*50*	*1*
Disha	*9*	*6*	*1*	--	--
Dummy	*0*	*0*	*0*	--	--
Demand	*6*	*60*	*7*		
Penalty	*7*	*3*	*1*		

Table 8. *4th Penalties allocation.*

Ware house Factories	Bhubaneswar	Cuttack	Rourkela	Supply	Penalty
Asha	*12*	*3*	*8*	*23*	*5*
Omm	*5*	*6*	*7*	*44*	*1*
Disha	*9*	*6*	*1*	--	--
Dummy	*0*	*0*	*0*	--	--
Demand	--	*60*	*7*		
Penalty	--	*3*	*1*		

Table 9. *Final allocation.*

Ware house Factories	Bhubaneswar	Cuttack	Rourkela	Supply	Penalty
Asha	*12*	*3*	*8*	--	--
Omm	*5*	*6*	*7*	*7*	*7*
Disha	*9*	*6*	*1*	--	--
Dummy	*0*	*0*	*0*	--	--
Demand	--	--	*7*		
Penalty	--	--	*7*		

The maximum penalty occurs, 7 in row 2.

The minimum c_{ij} in this row is c_{23} =7.

The maximum allocation in this cell is min (7,7)=7.

It is also satisfy the supply of row 2 (Omm) and demand in column 3 (Rourkela).

Table 10. *Initial basic feasible solution.*

Ware house Factories	Bhubaneswar	Cuttack	Rourkela	Supply
Asha	*12*	*3*	*8*	*23*
Omm	*5*	*6*	*7*	*50*
Disha	*9*	*6*	*1*	*48*
Dummy	*0*	*0*	*0*	*38*
Demand	*44*	*60*	*55*	

The minimum transportation cost is obtained as:

$$\text{Min} = (23\times 3) + (6\times 5) + (37\times 6) + (7\times 7) + (48\times 1) + (38\times 0) = 418.$$

Here, the number of allocated cells=6 which is equal to m+n-1=4+3-1=6

Therefore, this solution is non-degenerate.

Table 11. *Comparison of proposed method with existing method of example 1.*

	Transportation Cost
Proposed Method	*418*
Existing Method [20]	*418*

Example 2 [47]. Consider the pentagonal Neutrosophic numbers (supply, demand & transportation cost) are presented in *Table 12*.

Table 12. *Input data for pentagonal transportation problem.*

Factories	Bhubaneswar	Cuttack	Supply
Protein	*(0.2,0.4,0.5,0.6,0.7)*	*(0.3,0.2,0.6,0.5,0.1)*	*(0.1,0.2,0.5,0.4,0.3)*
Calories	*(0.7,0.8,0.6,0.9,0.1)*	*(0.2,0.3,0.5,0.7,0.1)*	*(0.2,0.3,0.5,0.8,0.9)*
Demand	*(0.8,0.7,0.5,0.3,0.2)*	*(0.2,0.3,0.4,0.1,0.2)*	

Here, the decision-makers consider the degree of each pentagonal number is (1,0,0).

After executing the steps of our algorithm, we get the initial basic feasible solutions presented in *Table 13*. This is balanced transportation problem.

Table 13. *Final initial basic feasible solution.*

Factories	Bhubaneswar	Cuttack	Supply
Protein	*0.48*	*0.34*	*0.3*
Calories	*0.62*	*0.36*	*0.54*
Demand	*0.5*	*0.34*	

The minimum transportation cost is obtained as:

$$\text{Min} = (0.3\times 0.48) + (0.2\times 0.62) + (0.34\times 0.36) = 0.3904.$$

Here, the number of allocated cells=3 which is equal to m+n-1=2+2-1=3

Therefore, this solution is non-degenerate.

Table 14. *Comparison of proposed method with existing method of example 2.*

	Transportation cost
Proposed Method	*0.3904*
Existing Method [20]	*0.41*

5. Analysis and Observation of the Proposed Model

5.1. Observation

Due to non-availability of pentagonal transportation problem under Neutrosophic environment, there is no direct comparison made in this paper. Therefore, we consider pentagonal Neutrosophic transportation problem under fuzzy environment for comparison our result. Hence, we compared our proposed method with the existing method [20].

For *Example 1* the pentagonal Neutrosophic transportation cost of IBFS is 418, which is exactly the transportation cost of fuzzy pentagonal numbers. In *Example 2*, the pentagonal Neutrosophic transportation problem is 0.3904, which is not exactly the cost of fuzzy pentagonal transportation problem i.e. 0.41. The decision-makers always want to minimize the cost of transportation when supplying the materials. Thusly, we can say that our proposed technique under Neutrosophic environment is always better than the other existing method. We also depicted our result along with the existing method results in graphical representation i.e. *Figs. (1)-(2)*.

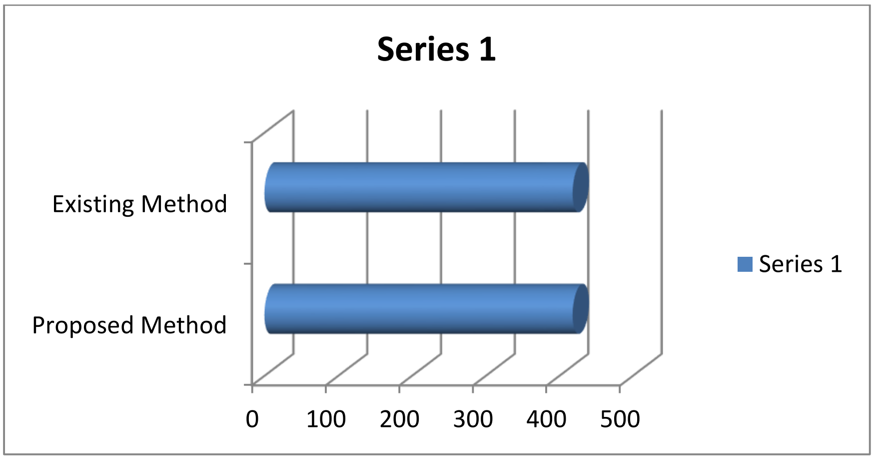


Fig. 1. *Graphical analysis of our proposed method with existing method of example 1.*

From the above analysis of both tabular form and graphical form, we can finalise that our proposed method is better to the existing method. Further, we can also claimed that our transportation cost obtained by our proposed method always lie within region of Neutrosophic sets.

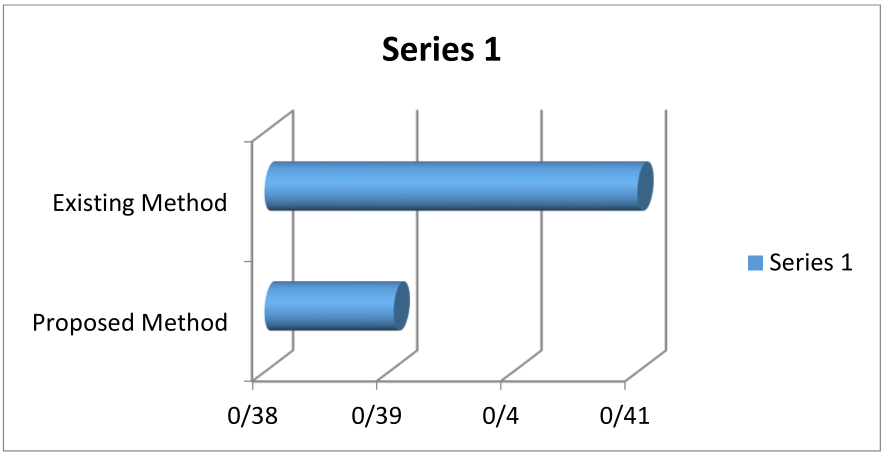


Fig. 2. *Graphical analysis of proposed work with existing work of example 2.*

5.2. Advantages of the Proposed Model

The pentagonal fuzzy numbers were widely applied in transportation problem to get the minimum cost. However, the decision-makers always consider the truth degree of pentagonal fuzzy numbers, which is the main drawback. In real-life problem, the decision-makers always want the clarity data means truth degree, indeterminacy degree and falsity degree. Neutrosophic sets consider the degree of truth, indeterminacy and falsity and we take the advantage of the properties of Neutrosophic sets, we develop a new algorithm for solving pentagonal Neutrosophic transportation problem. We proposed a new score function of Neutrosophic pentagonal numbers and also developed a new technique for getting initial basic feasible solutions. In our problem, our transportation cost is always minimizing then other existing method and minimization the cost is required for decision-makers. We also solved our problem in LP model by using LINGO 18 version or MATLAB, we get the same result.

In the above conversation, we can infer that our proposed calculation is another approach to deal with the vulnerability and indeterminacy in the transportation issue.

6. Conclusions

The transportation problem is one of the most popular optimization problems in operation research. The main objective of this problem is finding the minimum cost of transportation to supplier and demand. In this paper, Neutrosophic transportation problem has been solved under pentagonal Neutrosophic numbers. We also developed a score function and applied to find the IBFS. In the computation point of view, our proposed method is very easier when applied in real-life problem. Further comparative analysis is done with fuzzy pentagonal transportation problem. Also, the proposed algorithm has less computational complexity and saves time. By comparing our method with fuzzy method, we can conclude that our method can handle any type of uncertainties arise in real-life situation and it is very simple & efficient than other uncertainty. In addition to our proposed method, it will be extend in application of pentagonal assignment problem, pentagonal linear fractional programming and pentagonal job scheduling problem.

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