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A Hybrid Intelligent Parameter Tuning Approach for COVID-19 Time Series Modeling and Prediction

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Abstract

A novel hybrid intelligent approach for tuning the parameters of Interval Type-2 Intuitionistic Fuzzy Logic System (IT2IFLS) is introduced for the modeling and prediction of coronavirus disease 2019 (COVID-19) time series. COVID-19 is known to be a virus caused by Severe Acute Respiratory Syndrome Coronavirus 2 (SARSCoV-2) with a huge negative impact on human, work and world economy. Globally, more than 100 million people have been infected with over two million deaths and it is not certain when the pandemic will end. Predicting the trend of the COVID-19 therefore becomes an important and challenging task. Many approaches ranging from statistical approaches to machine learning methods have been formulated and applied for the prediction of the disease. In this work, the sliding mode control learning algorithm is used to adjust the parameters of the antecedent parts of IT2IFLS system while the gradient descent backpropagation is adopted to tune the consequent parameters in a hybrid manner. The results of the hybrid intelligent learning model are compared with results of single learning models using sliding mode control and gradient descent algorithms and found to provide good performance in terms of Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE) especially in noisy environments. The type-2 hybrid model also outperforms its type-1 counterparts in the different problem instances.

Keywords: Interval type-2 intuitionistic fuzzy set, Gradient descent algorithm, Sliding mode control algorithm, Intuitionistic fuzzy index.

1 | Introduction

Coronavirus disease 2019, popularly known as COVID-19, is an illness caused by a novel coronavirus called Severe Acute Respiratory Syndrome Coronavirus 2 (SARSCoV-2). This novel coronavirus, designated as 2019-nCoV emerged in Wuhan, China in December, 2019. On January 30, 2020, the World Health Organization (WHO) declared the COVID-19 outbreak a global health emergency and on March 11, 2020, it was declared a global pandemic. As of April 13, 2020, COVID-19 has been recognized in 196 countries with a total of 1,876,707 laboratory confirmed cases, 435,591 recovered and 116,789 death cases [1].

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As of 5th February 2021, the number of confirmed cases worldwide had risen to 104,165,066 with over 2,265,354 deaths as per WHO COVID-19 Dashboard. COVID-19 has therefore become a huge challenge the world over; affecting peoples' lives and work. Many countries were locked-down due to the devastating effects of COVID-19 with the aim of slowing down the spread of the virus. It behooves therefore on researchers to provide effective and reliable prediction models to COVID-19 pandemic. So far, many datamining methodologies have been applied for the prediction of COVID-19 pandemic. For instance, Muhammed et al. [2] recently developed data mining models for the prediction of COVID-19 infected patients recovery using epidemiological data set of COVID-19 patients of South Korea. The decision tree, support vector machine, naïve Bayes, logistic regression, random forest, and K-nearest neighbour algorithms were applied. Ayyoubzadeh et al. [3] predicted the incidence of COVID-19 in Iran. Linear Regression and Long Short-Term Memory (LSTM) models were used with data obtained from the Google Trends website to estimate the number of positive COVID-19 cases. Their models were evaluated using 10-fold cross validation with Root Mean Squared Error (RMSE) as the performance metric. Ardabili et al. [4] have presented a comparative analysis of machine learning and soft computing models to predict the COVID-19 outbreak as an alternative to Susceptible-Infected-Recovered (SIR) and Susceptible Exposed Infections Removed (SEIR) models. Agbelusi and Olayemi [5] developed a predictive model for the mortality rate of patients infected with COVID-19 in Nigeria using data mining techniques available in Waikato Environment for Knowledge Analysis (WEKA). Martin et al. [6] proposed a COVID-19 diagnostic model based on plithogenic cognitive maps. Matta and Saraf [7] developed a machine learning model to predict whether a patient is suffering from COVID-19 or not.

The COVID-19 time series data are highly complex, nonlinear and uncertain [8]. The COVID-19 data therefore present itself as a challenging, yet interesting prediction problem. With the levels of uncertainties in COVID-19 data, methodologies that can handle these uncertainties must be employed. In the literature, fuzzy logic comes in handy as a concept that can adequately model these uncertainties [9]. So far, type-1 Fuzzy Sets (FSs) have been adopted for the prediction of COVID-19 time series. For instance, Dhiman and Sharma [10] presented fuzzy logic inference for identification and prevention of COVID-19. Al-Qaness et al. [11] utilized Adaptive Neuro-Fuzzy Inference System (ANFIS) with parameters tuned with Flower Pollination (FPA) and Salp Swarm Algorithm (SSA) to predict confirmed cases of COVID-19 in China. Van Tinh [12] applied fuzzy time series model with particle swarm optimization for COVID-19 prediction. Fong et al. [13] used a hybridized deep learning and fuzzy rule induction for the analysis of COVID-19. Fatima et al. [14] used Internet of Things (IoT) coupled with fuzzy inference system for monitoring COVID-19 while Verma et al. [15] adopted arima and fuzzy time series models for COVID-19 prediction. Arora et al. [16] proposed a fuzzy based COVID-19 decision making system using individual's symptoms and parameters. The proposed system provided good results with 97.2% accuracy. Ly [17] presented a study on the prediction of COVID-19 in the United Kingdom using ANFIS.

However, type-1 fuzzy sets may not solve the uncertainty problem completely because once the Membership Function (MF) of type-1 FS is specified, the uncertainty disappears leaving a precise value. An Intuitionistic Fuzzy Set (IFS) introduced by Atanassov [18] provides some flexibility about fuzzy sets thus allowing domain experts to express more uncertainty about a fuzzy set by defining separate MFs and Non-Membership Functions (NMFs) for an element in a set. Authors have adopted the concept of IFS in uncertainty modelling of COVID-19 data. For instance, Kozae et al. [19], adopted intuitionistic fuzzy distance to determine those infected with COVID-19. In Traneva and Tranev [20], multilayered intuitionistic fuzzy intercriteria analysis is conducted on some key disease indicators to determine the mortality from COVID-19 in European Union. The same authors in Traneva and Tranev [21], proposed a two-way intuitionistic fuzzy analysis of variance for evaluating the spread of COVID-19 cases in Europe. Eyo et al. [22] utilised rule-based Intuitionistic Fuzzy Logic System (IFLS) for the analysis of COVID-19 time series in Nigeria. Similar to type-1 FS, the IFS of type-1 could not handle uncertainty well. Atanassov and Gargov [23] extended IFS to Interval-Valued IFS (IVIFS) where upper membership and upper non-membership of the set add up to 1. In Eyoh et al. [24], a rule-based IT2IFLS is proposed. However, for IT2IFS, upper membership and lower non-membership lie between 0 and 1, Similarly, lower membership and upper non-membership also lie within the boundary of 0 and 1 which makes IT2IFS different from

IVIFS. According to Eyoh et al. [25], IT2IFS can be used to capture concepts not possible with IVIFS. This presents IVIFS as a special case of IT2IFS [26] and [27]. The IT2IFSs have been applied successfully to solve many real-world problems such as clustering [27], regression problems [28], [29] and [30], transportation problems [31] and [32], risk assessment [33], load forecasting [34], resource allocation [35], time series forecasting [9] and [36], system identification problems [25] and [37] and water management [38] to mention but a few. Other studies involving IT2IFS include Singh and Garg [39], where different types of distances between T2IFSs are proposed. In Dan et al. [40], certain properties of IT2IFS are presented. Li et al. [41] introduced grey relational bidirectional projection method based on trapezoidal type-2 intuitionistic fuzzy numbers while Demiralp and Haçat [42] put forward an ordering method of c-control charts with IT2IFS.

1.1 | Research Gap and Motivation

The IT2IFS has gained widespread attention in recent years with significant and promising results in diverse problem domains and characteristics. Many algorithms have also been adopted for the optimization of the parameters of IT2IFLS for solving many application problems. These algorithms include Gradient Descent (GD) [9], [28], [29] and [30] where IT2IFLS is used for regression and time series predictions. Luo et al. [36] presented an evolving Recurrent Interval Type-2 Intuitionistic Fuzzy Neural Network (eRIT2IFNN) with the parameters of the model adjusted using Extended Kalman Filter (EKF) for online learning and time series prediction. The decoupled version of EKF is applied to tune the parameters of IT2IFLS in Eyoh et al. [44] and [45]. Recently, Sliding Mode Control (SMC) learning algorithm is used to learn the parameters of IT2IFLS [37] and [46] with application to system identification problems. A hybrid model of GD and EKF has been reported in the literature for training IT2IFLS for identification problems [25]. However, both approaches are derivative-based which are computationally intensive because they involve computing the partial derivatives of the parameters and convergence may be slow [25] and [43]. To reduce the computational cost and speed up convergence, we are motivated to integrate both derivative (GD) and derivative-free (SMC) approaches for adjusting the parameters of IT2IFLS. The rationale behind this approach is that the antecedent parameters of the model are highly nonlinear and for optimization problems, computing the gradient of the cost function in each step for nonlinear parameters is difficult and chain rule must be used [47]. Also, convergence of nonlinear parameters may sometimes be very slow leading to non-convergence of the solution [47]. Therefore, using SMC, a gradient-free parameter tuning approach is more appropriate for the non-linear antecedent parameters. On the other hand, using GD approach for tuning the consequent parameters is more practical as the parameters are linear. Moreover, the two algorithms of SMC and GD are based upon well-established mathematical background for training FLSs [48].

1.2 | Main Contributions

The contributions of this paper are as follows: 1) the analysis of COVID-19 time series using IT2IFLS, 2) the optimization of the parameters of IT2IFLS with a novel hybrid intelligent learning algorithm of SMC and GD. To the best knowledge of the authors, no work has been done on COVID-19 time series using intuitionistic fuzzy approaches of type-2 and no learning of the parameters of IT2IFLS has been carried out using SMC and GD in a hybrid manner.

1.3 | Paper Organization

The rest of the paper is organized as follows: Section 2 provides some basic definitions for IFS. Section 3 describes the IT2IFLS. In Section 4, the methodologies for the realization of the hybrid learning apparatus are given and in Section 5, the update rules are derived. Section 6 provides a brief description of the COVID-19 datasets and the experimental set-up for the study while performance evaluation is carried out in Section 7. The conclusion is drawn in Section 8.

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2 | Preliminaries

2.1 | Type-1 Intuitionistic Fuzzy Set

Definition 1. [18]. An intuitionistic fuzzy set is composed of MF and NMF and defined as: $A^* = (x\text{: } \mu_{A^*}(x)$; $\nu_{A^*}(x) \mid x \in X)$. It satisfies the condition that: $0 \leq \mu_{A^*}(x) + \nu_{A^*}(x) \leq 1$.

There exists another component called the hesitation index, π, such that $\pi(x) = 1 - (\mu_{A^*}(x) + \nu_{A^*}(x))$. Obviously, $0 \leq \pi(x) \leq 1$. Authors in [49], [50] and [51] have proposed methods for the formulation of MFs and NMFs of IFS. The Gaussian intuitionistic MFs and NMFs defined in [49] are adopted for this study and are expressed as in *Eqs. (1)* and *(2)*:

$$\mu_{ik}(x_i) = \exp\left(-\frac{(x_i - c_{ik})^2}{2\sigma^2}\right) - \pi. \tag{1}$$

$$\nu_{ik}(x_i) = 1 - \exp\left(-\frac{(x_i - c_{ik})^2}{2\sigma^2}\right). \tag{2}$$

Where x is the input, c is the center and σ is the standard deviation. Unlike the traditional FS, the IFS is not necessarily a complementary set.

2.2 | Interval Type-2 Intuitionistic Fuzzy Set

Definition 2. [24]. An IT2IFS is characterized by fuzzy MFs and fuzzy NMFs defined as

$$A^* = (\bar{\mu}_{\tilde{A}^*}(x), \underline{\mu}_{\tilde{A}^*}(x), \bar{\nu}_{\tilde{A}^*}(x), \underline{\nu}_{\tilde{A}^*}(x)).$$

Where $\underline{\mu}_{\tilde{A}^*}(x) :\ X \rightarrow (0,1)$, $\bar{\mu}_{\tilde{A}^*}(x) : X \rightarrow (0,1)$, $\underline{\nu}_{\tilde{A}^*}(x) :\ X \rightarrow (0,1)$, $\bar{\nu}_{\tilde{A}^*}(x) : X \rightarrow (0,1)$ such that $0 \leq \bar{\mu}_{\tilde{A}^*}(x) + \underline{\nu}_{\tilde{A}^*}(x) \leq 1$ and $0 \leq \underline{\mu}_{\tilde{A}^*}(x) + \bar{\nu}_{\tilde{A}^*}(x) \leq 1\ \forall x \in X$.

The IT2IFS is defined in this study using type-2 Gaussian function described with uncertain standard deviation (see *Fig. 1*) and *Eqs. (3)* to *(6)* describe the upper MF, lower MF, upper NMF and lower NMF respectively.

$$\bar{\mu}_{ik}(x_i) = \exp\left(-\frac{(x_i - c_{ik})^2}{2\bar{\sigma}^2_{2,ik}}\right) - \pi. \tag{3}$$

$$\underline{\mu}_{ik}(x_i) = \exp\left(-\frac{(x_i - c_{ik})^2}{2\bar{\sigma}^2_{1,ik}}\right) - \pi. \tag{4}$$

$$\bar{\nu}_{ik}(x_i) = 1 - \exp\left(-\frac{(x_i - c_{ik})^2}{2\bar{\sigma}^2_{1,ik}}\right). \tag{5}$$

$$\underline{\nu}_{ik}(x_i) = 1 - \exp\left(-\frac{(x_i - c_{ik})^2}{2\bar{\sigma}^2_{2,ik}}\right). \tag{6}$$

Where π is the intuitionistic fuzzy index and lies between 0 and 1. Thus for IT2IFSs, two footprints of uncertainties suffice which are defined as *Eqs. (7)* and *(8)* [25]:

$$\mathrm{FOU}_{\mu}(\tilde{A}^{*})=\bigcup_{\forall x\in X}\left[\underline{\mu}_{\tilde{A}^{*}}(x),\bar{\mu}_{\tilde{A}^{*}}(x)\right]. \tag{7}$$

$$\mathrm{FOU}_{\nu}(\tilde{A}^{*})=\bigcup_{\forall x\in X}\left[\underline{\nu}_{\tilde{A}^{*}}(x),\bar{\nu}_{\tilde{A}^{*}}(x)\right]. \tag{8}$$

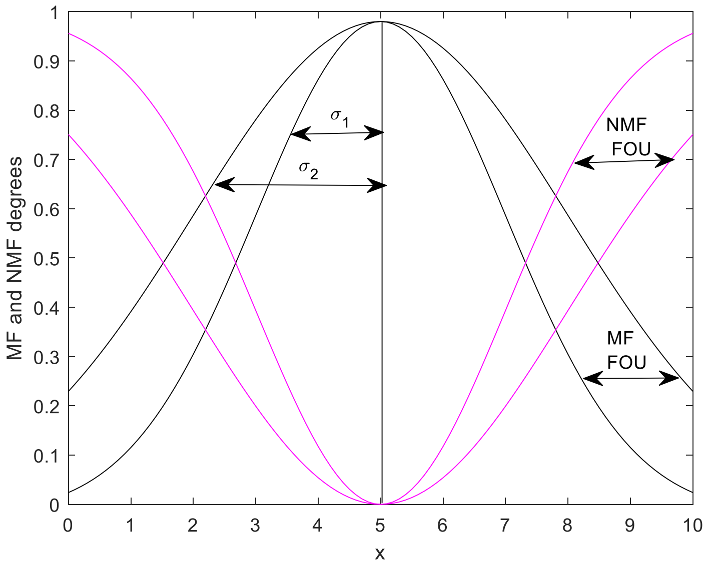

Fig. 1. Interval type-2 intuitionistic fuzzy set with uncertain standard deviation [30].

Eqs. (8) and *(9)* represent FOUs for MFs and NMFs respectively. Any system that utilizes IT2IFS either in the antecedent and/or consequent is called an IT2IFLS.

3 | Interval Type-2 Intuitionistic Fuzzy Logic System

Shown in *Fig. 2* is the block diagram of IT2IFLS comprising the intuitionistic-fuzzifier, rule base, inference engine, type-reducer and defuzzifier while *Fig. 3* shows the structure of the IT2IFLS with five layers.

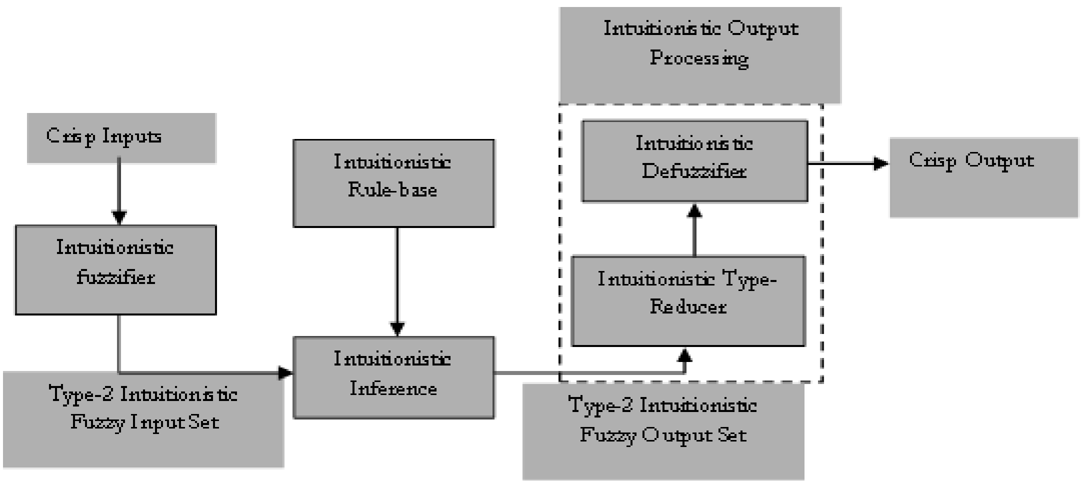

Fig. 2. Block diagram of IT2IFLS [30].

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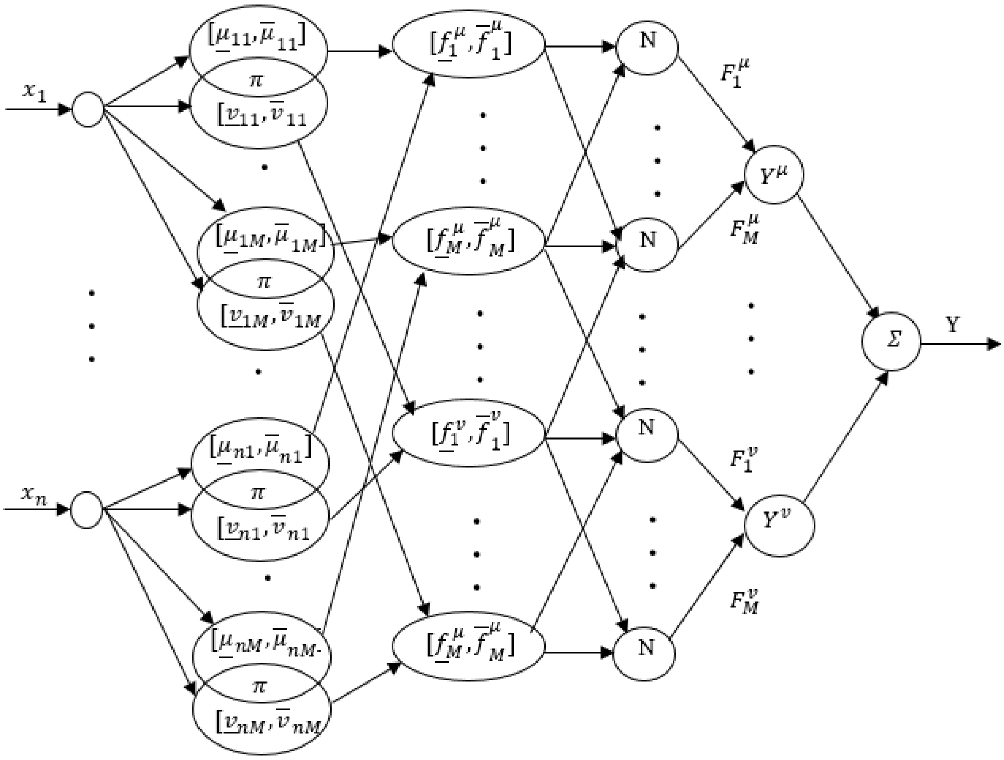

Fig. 3. Structure of IT2IFLS [52].

3.1 | Fuzzification

The external inputs are propagated forward and the intuitionistic fuzzifier accepts these crisp input vectors, translates them into intuitionistic MFs and NMFs. The intersection of the MF and NMF in *Fig. 3* represents the intuitionistic fuzzy index (π). The Gaussian function is adopted for the definition of the IT2IFS because it is differentiable at all points, thus, making it quite suitable for these optimization problems.

3.2 | Rules

The IT2IFLS rule structure can be expressed as in *Eq. (9)*. The rule representation of IT2IFLS is similar to the classical IT2FLS. However, for IT2IFLS, the IT2IFS are utilized.

$$R_k : \text{if } x_i \text{ is } \tilde{A}^*_{ik} \text{ and} \ldots \text{and } x_n \text{ is } \tilde{A}^*_{nk} \text{ then } y_k = \sum_{i=1}^{n} w_{ik} x_i + b_k. \quad (9)$$

Where $\tilde{A}^*_{ik}, \ldots, \tilde{A}^*_{nk}$ are IT2IFSs and y_k is the output of the k[th] rule. IT2IFLS uses IT2IFSs in the rule base.

The rule in *Eq. (9)* may further be decomposed into two parts, one for MF and the other for NMF as shown in *Eqs. (10)* and *(11)* respectively. For the MFs, the rule in *Eq. (9)* becomes:

$$R^{\mu}_k : \text{if } x_i \text{ is } \tilde{A}^{*\mu}_{ik} \text{ and } \ldots \text{ and } x_n \text{ is } \tilde{A}^{*\mu}_{nk} \text{ then } y^{\mu}_k = \sum_{i=1}^{n} w^{\mu}_{ik} x_i + b^{\mu}_k. \quad (10)$$

For NMFs, the rule becomes:

$$R^{v}_k : \text{if } x_i \text{ is } \tilde{A}^{*v}_{ik} \text{ and} \ldots \text{and } x_n \text{ is } \tilde{A}^{*v}_{nk} \text{ then } y^{v}_k = \sum_{i=1}^{n} w^{v}_{ik} x_i + b^{v}_k. \quad (11)$$

where y^{μ}_k is the MF output and y^{v}_k is the NMF outputs of the k^{th} rule, w and b are the consequent parameters.

The inferencing procedure for IT2IFLS can be approached either as Mamdani or TSK. In this work, the TSK inferencing known as A2-CO is adopted. A2-CO inferencing requires that the antecedents (A) of the rule be IT2IFSs while the consequents (C) of the rule be linear functions of the inputs. With the TSK-inferencing, the computationally intensive type-reduction is circumvented and the output is directly computed.

3.3 | Defuzzification

The final output of IT2IFLS is computed as a weighted average of both MF and NMF's outputs [44]. Thus, the computationally complicated type-reduction procedure is by-passed to directly compute the outputs of IT2IFLS. The final output of IT2IFLS is defined as follows [24]:

$$y=(1-\beta)\sum_{k=1}^{M}\tilde{f}_k^{\mu}y_k^{\mu}+\beta\sum_{k=1}^{M}\tilde{f}_k^{\nu}y_k^{\nu},$$

where

$$\tilde{f}_k^{\mu}=\frac{\left(\underline{f}_k^{\mu}+\overline{f}_k^{\mu}\right)}{\sum_{k=1}^{M}\underline{f}_k^{\mu}+\sum_{k=1}^{M}\overline{f}_k^{\mu}}, \tag{12}$$

$$\tilde{f}_k^{\nu}=\frac{\left(\underline{f}_k^{\nu}+\overline{f}_k^{\nu}\right)}{\sum_{k=1}^{M}\underline{f}_k^{\nu}+\sum_{k=1}^{M}\overline{f}_k^{\nu}}.$$

Where $\tilde{f}_k^{\mu}$ and $\tilde{f}_k^{\nu}$ are normalized firing signals for MFs and NMFs respectively and utilises the "prod" t-norm to specify the firing strength such that:

$$\underline{f}_k^{\mu}(x)=\prod_{i=1}^{n}\underline{\mu}_{\tilde{A}_{ik}^{*}}(x_i),$$
$$\overline{f}_k^{\mu}(x)=\prod_{i=1}^{n}\overline{\mu}_{\tilde{A}_{ik}^{*}}(x_i),$$
$$\underline{f}_k^{\nu}(x)=\prod_{i=1}^{n}\underline{\nu}_{\tilde{A}_{ik}^{*}}(x_i),$$
$$\overline{f}_k^{\nu}(x)=\prod_{i=1}^{n}\overline{\nu}_{\tilde{A}_{ik}^{*}}(x_i).$$

where $\left(\underline{f}_k^{\mu},\overline{f}_k^{\mu}\right)$ and $\left(\underline{f}_k^{\nu},\overline{f}_k^{\nu}\right)$ are the firing strengths for MF and NMFs respectively, $\prod$ is the "prod" operator, y_k^{μ} and y_k^{ν} are the outputs of the k^th^ rule of IT2IFLS, with β as the user defined parameter such that $0\le\beta\le 1$. The value of β influences the contribution of MF and NMF values to the final output. It follows that:

$$y=\begin{cases}\text{MF only} & \text{if } \beta=0\\ \text{NMF only} & \text{if } \beta=1\\ \text{MF and NMF} & \text{if } 0<\beta<1\end{cases}.$$

This implies that the MF alone contributes to the final output if the value of β is 0 and NMF alone contributes to the final output if β is 1. When the value of β lies between 0 and 1, then both MFs and NMFs simultaneously contribute to the final output of IT2IFLS.

4 | Hybrid Learning Methodology

In this research, a novel hybrid learning approach for the optimization of IT2IFLS parameters is introduced. The ensuing algorithm is used for evaluating COVID-19 time series in five countries. Here, the consequent parameters are tuned using the GD backpropagation and the antecedent parameters are tuned with SMC learning methodology.

4.1 | Sliding Mode Control Learning Algorithm

Although the SMC can be applied to both linear and nonlinear systems, in this study, it is applied to tune the non-linear parameters. The error which is the difference between the actual output and the output of the IT2IFLS can be defined as in *Eq. (13)*.

$$e(t)=y^{a}(t)-y(t).$$

The zero value of the error coordinate can be specified as a time varying sliding surface [48] defined as:

$$S(e(t))=e(t)=y^{a}(t)-y(t)=0.$$

Which guarantees a system in a sliding mode to be on the sliding surface such that the predicted output using the IT2IFLS will follow the actual output signal for all time $t>t_h$, where th is the hitting time for $e(t)=0$.

Definition 3. A sliding motion will be on a sliding manifold $S(e(t))=e(t)=0$ after a time th if the condition $S(t)\dot{S}(t)=e(t)\dot{e}(t)<0$ is valid $\forall t$ in some nontrivial semi open subinterval of time of the form $[t,t_h)\subset(-\infty,t_h)$ [53] and [54].

4.2 | Gradient Descent Learning Algorithm

To adjust the consequent parameters (weight and bias) of the IT2IFLS, GD algorithm is executed. For a single output, the cost function is expressed as:

$$E = \frac{1}{2}(y^{a} - y)^2.$$

Where y^a is the actual output and y is the IT2IFLS output. The generic GD update rule is as follows:

$$\theta_{ik}(t+1) = \theta_{ik}(t) - \gamma\frac{\partial E}{\partial \theta_{ik}}.$$

Where θ is the generic parameter to be updated and γ is the learning rate.

5 | Parameter Update

In this study, the parameter update is achieved using two different methods, namely SMC and GD in a hybrid manner. The non-linear antecedent parameters namely center c, lower standard deviation σ1 and upper standard deviation σ2 are updated using SMC learning algorithm while the linear parameters (weight (w) and bias (b)) are updated using GD.

The update rules for the consequent parameters (w and b) using GD are as follows [29]:

$$w_{ik}(t+1) = w_{ik}(t) - \gamma \frac{\partial E}{\partial w_{ik}}. \tag{14}$$

$$b_k(t+1) = b_k(t) - \gamma \frac{\partial E}{\partial b_k}. \tag{15}$$

Where γ must be carefully chosen as a large value may lead to instability. On the other hand, small value may lead to slow learning. The derivatives in *Eqs. (14)* and *(15)* are computed as follows:

$$\frac{\partial E}{\partial w_{ik}} = \frac{\partial E}{\partial y}\frac{\partial y}{\partial y_k}\frac{\partial y_k}{\partial w_{ik}} = \sum_{k=1}^{M}\frac{\partial E}{\partial y}\left[\frac{\partial y}{\partial y_k^{\mu}}\frac{\partial y_k^{\mu}}{\partial w_{ik}} + \frac{\partial y}{\partial y_k^{v}}\frac{\partial y_k^{v}}{\partial w_{ik}}\right] = (y^{a}(t) - y(t)) * \left[(1-\beta)\left(\frac{\underline{f}_k^{\mu}}{\sum_{k=1}^{M}\underline{f}_k^{\mu} + \sum_{k=1}^{M}\overline{f}_k^{\mu}} + \frac{\overline{f}_k^{\mu}}{\sum_{k=1}^{M}\underline{f}_k^{\mu} + \sum_{k=1}^{M}\overline{f}_k^{\mu}}\right) + \beta\left(\frac{\underline{f}_k^{v}}{\sum_{k=1}^{M}\underline{f}_k^{v} + \sum_{k=1}^{M}\overline{f}_k^{v}} + \frac{\overline{f}_k^{v}}{\sum_{k=1}^{M}\underline{f}_k^{v} + \sum_{k=1}^{M}\overline{f}_k^{v}}\right)\right] * x_i,$$

$$\frac{\partial E}{\partial b_k} = \frac{\partial E}{\partial y}\frac{\partial y}{\partial y_k}\frac{\partial y_k}{\partial b_k} = \sum_{k=1}^{M}\frac{\partial E}{\partial y}\left[\frac{\partial y}{\partial y_k^{\mu}}\frac{\partial y_k^{\mu}}{\partial b_k} + \frac{\partial y}{\partial y_k^{v}}\frac{\partial y_k^{v}}{\partial b_k}\right] = (y^{a}(t) - y(t)) * \left[(1-\beta)\left(\frac{\underline{f}_k^{\mu}}{\sum_{k=1}^{M}\underline{f}_k^{\mu} + \sum_{k=1}^{M}\overline{f}_k^{\mu}} + \frac{\overline{f}_k^{\mu}}{\sum_{k=1}^{M}\underline{f}_k^{\mu} + \sum_{k=1}^{M}\overline{f}_k^{\mu}}\right) + \beta\left(\frac{\underline{f}_k^{v}}{\sum_{k=1}^{M}\underline{f}_k^{v} + \sum_{k=1}^{M}\overline{f}_k^{v}} + \frac{\overline{f}_k^{v}}{\sum_{k=1}^{M}\underline{f}_k^{v} + \sum_{k=1}^{M}\overline{f}_k^{v}}\right)\right] * 1.$$

The user-defined parameter, β, update also follows the GD procedure as defined in [29]

$$\beta(t+1) = \beta_{ik}(t) - \gamma \frac{\partial E}{\partial \beta_{ik}}. \tag{16}$$

The update rules using SMC for antecedent parameters are as follows [54]:

$$\dot{c}_{ik} = \dot{x}_i + (x_i - c_{ik})\alpha_1 \operatorname{sgn}(e). \tag{17}$$

$$\dot{\underline{\sigma}}_{ik}^{\mu} = -\left(\underline{\sigma}_{ik} + \frac{(\underline{\sigma}_{ik})^3}{(x_i - c_{ik})^2}\right)\alpha_1 \operatorname{sgn}(e). \tag{18}$$

$$\dot{\overline{\sigma}}_{ik}^{\mu} = -\left(\overline{\sigma}_{ik} + \frac{(\overline{\sigma}_{ik})^3}{(x_i - c_{ik})^2}\right)\alpha_1 \operatorname{sgn}(e). \tag{19}$$

For the NMFs, the lower NMF standard deviation is updated using the value of the upper MF standard deviation while the upper NMF standard deviation is updated using the value of the lower MF standard deviation [37] and [55], that is:

$$\dot{\underline{\sigma}}_{ik}^{\nu} = -\left(\overline{\sigma}_{ik} + \frac{(\overline{\sigma}_{ik})^3}{(x_i - c_{ik})^2}\right)\alpha_1 \operatorname{sgn}(e). \tag{20}$$

$$\dot{\underline{\sigma}}_{ik}^{\nu} = -\left(\overline{\sigma}_{ik} + \frac{(\overline{\sigma}_{ik})^3}{(x_i - c_{ik})^2}\right)\alpha_1 \operatorname{sgn}(e). \tag{21}$$

Algorithm 1. Hybrid IT2IFLS-SMC+GD learning approach.

Input: training data (x_i, y_i^a).

(1) Initialize weight (w), bias (b), center (c), standard deviation (σ), hesitation index (π), user defined parameter (β) and learning rate (γ).

(2) Initialize training epoch to unity.

(3) Initialize training data, (x_1, y_1^a).

(4) Propagate the training data through the IT2IFLS.

(5) Tune the weights, bias and the user defined parameter of the hybrid model using *Eqs. (14) - (16)* respectively.

(6) Compute the output of IT2IFLS using *Eq. (12)*.

(7) Compute the model error using *Eq. (13)*.

(8) Tune the center of IT2IFLS using *Eq. (17)*.
(9) Tune the standard deviation of MF using *Eqs. (18)* and *(19)*.
(10) Tune the standard deviation of NMF using *Eqs. (20)* and *(21)*.
(11) Pick the next training data. If training data ≤ total number of training data, go to step 4 else increment training epoch by 1.
(12) If maximum epoch is reached END; else.
(13) Go to step 4.
Output: Prediction error.

6 | Data Preprocessing and Experimental Setup

The COVID-19 dataset for the analysis is downloaded from Kaggle https://www.kaggle.com/antgoldbloom/covid19-data-from-john-hopkins-university obtained from the COVID-19 data repository of Center for Systems Science and Engineering (CSSE) at Johns Hopkins University (JHU). This work adopts COVID-19 confirmed cases for five of the most affected regions globally as at November 6, 2020. The most affected regions and their COVID-19 confirmed cases are USA (10,903,889), India (8,814,579), Brazil (5,848,959), Russia (1,887,836) and France (1,867,721) respectively. The corresponding death cases for each region are also adopted for the study. The data is made available in both raw and convenient forms and covers the period of January 23, 2020 to November 6, 2020.

This work uses the convenient form representation of the COVID-19 data and each set consists of 297 instances. Shown in *Fig. 4* is the trend of the COVID-19 confirmed cases for USA, India, Brazil, Russia and France while *Fig. 5* represents the associated death cases from COVID-19 for the five countries. For experimental analysis, each COVID-19 dataset is scaled to lie between 0 and 1 using the min-max normalization. As artificial neural network forms an integral part of the IT2IFLS, normalizing the data aids in smooth learning process and improves network performance in terms of prediction accuracy. Each of the COVID-19 data is modeled as a time series using the input-output generating vector: [y(t) y(t-1) y(t-2); y(t+1)], where y(t+1) is the one-day ahead output to be predicted. For each of the COVID-19 time series case, 10 simulation runs are conducted with terminating condition set to 100 epochs. The initial user-defined parameter is selected as 0.5 while the learning rate is set at 0.01.

7 | Performance Evaluation

The performances of IT2IFLS-SMC+GD are measured using two of the metrics for prediction problems namely: RMSE and Mean Absolute Error (MAE) defined as:

$$\mathrm{RMSE} = \sqrt{\frac{1}{T}\sum_{i=1}^{T}(y^{a} - y)^2}.$$

$$\mathrm{MAE} = \frac{1}{T}\sum_{i=1}^{T}|y^{a} - y|.$$

Where T is the total number of instances, y^a denote the actual output and y is the predicted output of IT2IFLS. For all experiments, three IT2IFSs are used in the antecedents. To aid comparison, the individual learning algorithms are also used to tune the parameters of the IT2IFLS. That is, IT2IFLS-GD (SMC) where GD (SMC) is used to tune the consequent and antecedent parameters respectively. The first sets of experiments are conducted to evaluate the performance of IT2IFLS-SMC+GD on COVID-19 confirmed cases in the five selected regions. In the second experiments, the confirmed cases are combined to obtain a 297 by 1485 data samples and then modelled as time series.

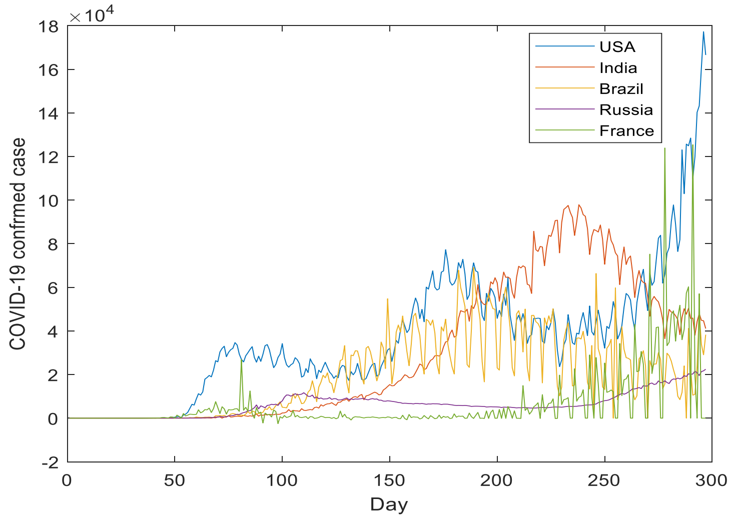

Fig. 4. COVID-19 confirmed case.

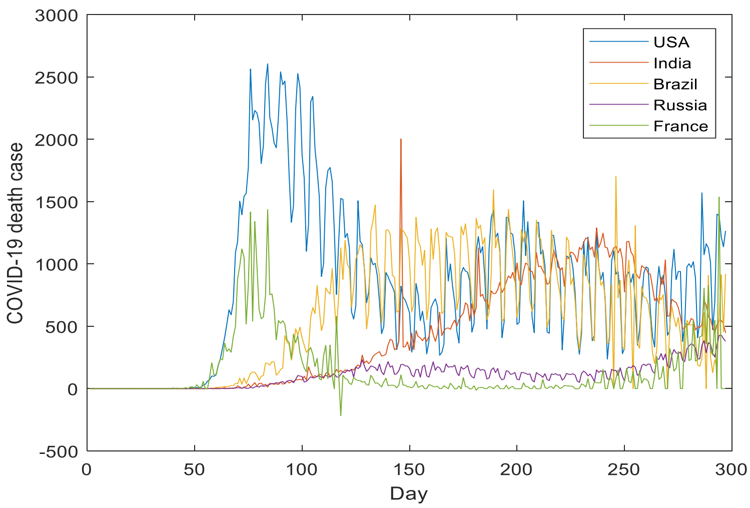

Fig. 5. COVID-19 death case.

Table 1 shows the performance of IT2IFLS-SMC+GD against IT2IFLS-GD and IT2IFLS-SMC on confirmed COVID-19 cases in the selected regions. As shown in *Table 1*, the accuracies of IT2IFLS-SMC+GD and IT2IFLS-SMC are quite similar. Howbeit, the hybrid model provides better performance than IT2IFLS-SMC in most of the cases. The GD-based learning model suffers some loss in performance in the confirmed cases for all regions except for Brazil in the noise-free dataset. For the combined dataset, the hybrid model outperforms both GD and SMC trained IT2IFLS with reduced RMSE and MAE.

Next some noise is injected into the COVID-19 confirmed cases data for the five countries and in the combined confirmed cases. Shown in *Figs. 6* and *Fig. 7* are the data points for COVID-19 combined noise free and noisy confirmed cases respectively. The aim is to study the behaviour of IT2IFLS-SMC+GD learning model under noisy condition. Here, an additive white Gaussian noise with SNR = 0dB is added to the COVID-19 confirmed cases in USA, India, Brazil, Russia and France and also in the combined confirmed cases. The SNR=0dB is chosen because it represents a high noise level.

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Table 1. Performance comparison of IT2IFLS-SMC+GD with individual learning models on COVID-19 confirmed cases.

Country	Model	RMSE(NF)	MAE(NF)	RMSE(N)	MAE(N)
USA	IT2IFLS-GD	0.1283	0.0338	0.5501	0.2391
	IT2IFLS-SMC	0.0994	0.0250	0.4329	0.1871
	IT2IFLS-SMC+GD	0.0948	0.0237	0.2908	0.1260
	IT2IFLS-GD	0.0762	0.0311	0.6456	0.2837
INDIA	IT2IFLS-SMC	0.0740	0.0302	0.5618	0.2455
	IT2IFLS-SMC+GD	0.0739	0.0303	0.3413	0.1495
	IT2IFLS-GD	0.0720	0.0288	0.5799	0.2555
BRAZIL	IT2IFLS-SMC	0.0910	0.0373	0.5549	0.2482
	IT2IFLS-SMC+GD	0.0885	0.0367	0.5412	0.2367
	IT2IFLS-GD	0.1760	0.0631	0.5920	0.2590
RUSSIA	IT2IFLS-SMC	0.1520	0.0525	0.5529	0.2459
	IT2IFLS-SMC+GD	0.1531	0.0529	0.3524	0.1543
	IT2IFLS-GD	0.0380	0.0118	0.5231	0.2457
FRANCE	IT2IFLS-SMC	0.0318	0.0099	0.5288	0.2291
	IT2IFLS-SMC+GD	0.0313	0.0096	0.2431	0.1040
	IT2IFLS-GD	0.0013	0.0007	0.5709	0.2555
Combined confirmed case	IT2IFLS-SMC	0.00096	0.00052	0.5488	0.2417
	IT2IFLS-SMC+GD	0.00084	0.00045	0.5205	0.2295

*NF = Noise Free COVID-19 data, N = Noisy COVID-19 data

Also shown in *Table 1* are the performances of the three models of IT2IFLS-GD, IT2IFLS-SMC and IT2IFLS-SMC+GD under noisy condition (SNR=0dB). The benefit of IT2IFLS-SMC+GD is apparent in *Table 1*, as noise is injected into the COVID-19 time series data. As shown in *Table 1*, with additive noise, the new hybrid IT2IFLS-SMC+GD provides significant performance improvements over the individual learning models of GD and SMC in terms of RMSE and MAE.

Based on these results, the authors conjectured that in the presence of noise and uncertainties in a system, the proposed hybrid model may stand as the best choice compared to IT2IFLS-GD and IT2IFLS-SMC.

Nevertheless, for noise-free data under investigation, IT2IFLS-SMC may suffice because of its computational simplicity. Finally, an experiment is conducted to show the performance difference between type-1 IFLS-SMC+GD and IT2IFLS-SMC+GD. This is done using COVID-19 death cases of the five selected countries and then the combined death cases of the five regions, under noise-free and noisy conditions.

Table 2 shows the prediction performance of IT2IFLS-SMC+GD against IFLS-SMC+GD. As shown in *Table 2*, the accuracy of IT2IFLS-SMC+GD is better than IFLS trained with the same hybrid algorithm on the individual region's COVID-19 death cases except for France, where the errors are very close on the noise free dataset. *Fig. 8* is an instance of the real COVID-19 death cases and the predicted death cases using IFLS-SMC+GD and IT2IFLS-SMC+GD. The inset figure within *Fig. 8* clearly shows that the prediction accuracy of IT2IFLS-SMC+GD is better compared to its type-1 counterparts as it follows the actual data more closely; using an instance of USA. However, in the combined noise free COVID-19 death cases, the performance of IFLS is comparable with that of IT2IFLS (see *Table 2*), an indication that type-1 IFLSs do model uncertainty in some cases better and that any "T2FLSs must be used when needed" [56], whether classical or intuitionistic.

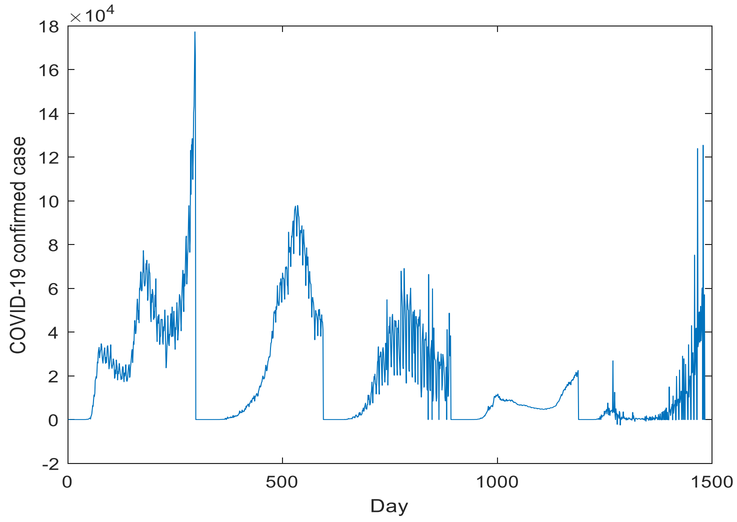

Fig. 6. Noise-free combined confirmed cases.

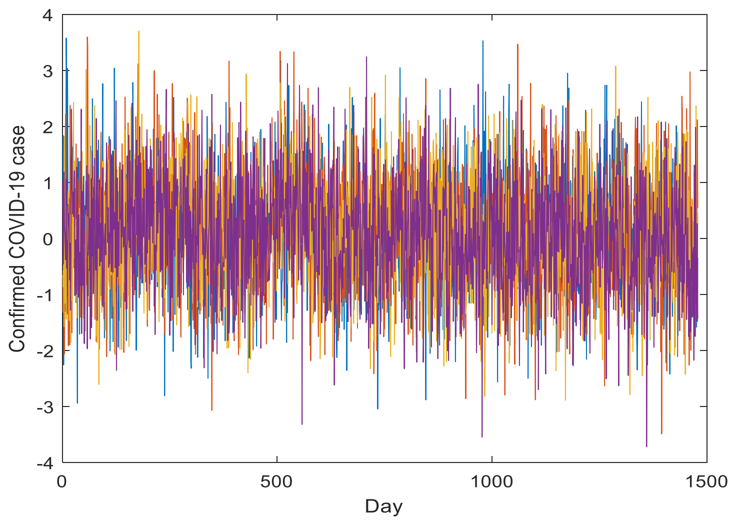

Fig. 7. Noisy combined confirmed cases.

Table 2. Performance comparison of IFLS-SMC+GD and IT2IFLS-SMC+GD on COVID-19 death cases.

Country	Model	RMSE(NF)	MAE(NF)	RMSE(N)	MAE(N)
USA	IFLS-SMC+GD	0.0366	0.0169	0.5200	0.2316
	IT2IFLS-SMC+GD	0.0229	0.0103	0.5122	0.2258
INDIA	IFLS-SMC+GD	0.0342	0.0152	0.5844	0.2593
	IT2IFLS-SMC+GD	0.0325	0.0148	0.5608	0.2465
BRAZIL	IFLS-SMC+GD	0.0862	0.0332	0.5398	0.2366
	IT2IFLS-SMC+GD	0.0619	0.0243	0.5269	0.2325
RUSSIA	IFLS-SMC+GD	0.1442	0.0497	0.5899	0.2579
	IT2IFLS-SMC+GD	0.1380	0.0362	0.5797	0.2564
FRANCE	IFLS-SMC+GD	0.0333	0.0096	0.5621	0.2486
	IT2IFLS-SMC+GD	0.0323	0.0096	0.5388	0.2347
Combined death case	IFLS-SMC+GD	0.00925	0.00399	0.54098	0.23668
	IT2IFLS-SMC+GD	0.01038	0.00401	0.5196	0.22038

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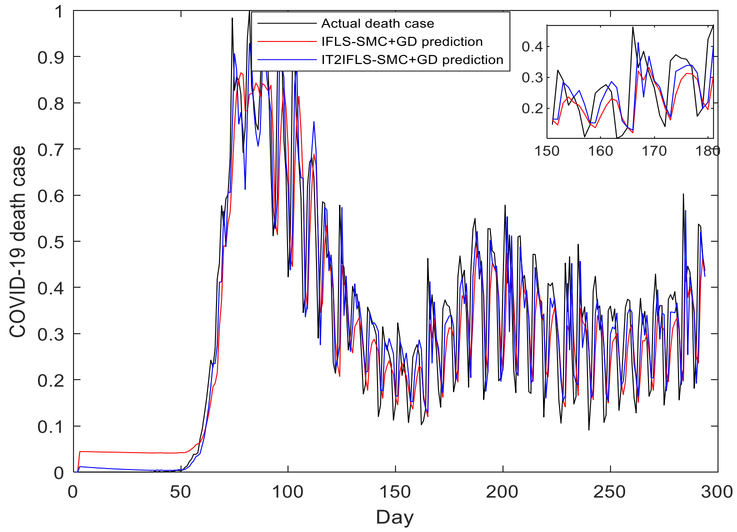

Fig. 8. IFLS-SMC+GD and IT2IFLS-SMC+GD (instance of USA).

8 | Conclusion

In this paper, a hybrid learning model comprising GD backpropagation and SMC algorithm for optimizing the parameters of IT2IFLS is introduced for the first time. The SMC learning algorithm is applied to tune the antecedent parameters due to non-linearity of these parameters while GD is used to tune the linear parameters in the consequent parts. As shown from experimental analyses, the IT2IFLS-SMC+GD learning model provides better prediction accuracies especially in noisy environment where uncertainty abounds. Analyses also reveal that although the prediction performance of IT2IFLS-SMC+GD is better than IT2IFLS-SMC especially for high level of noise, their prediction accuracies are similar for noise-free data. Thus, for noisy data such as noisy COVID-19 pandemic cases, the novel IT2IFLS hybrid learning model stands as a better prediction model compared to those of the individual models such as IT2IFLS-SMC and IT2IFLS-GD. The intuitionistic type-2 hybrid model is also shown to outperform its type-1 counterpart in many cases and clearly shows significant performance in the noisy COVID-19 instances. Overall, IT2IFLS-SMC+GD gives significant improvement when there is noise in the system compared to IT2IFLS-SMC, IT2IFLS-GD and its type-1 counterpart. In the future, we intend to adopt the general IT2IFLS and other learning models such as particle swarm optimization and simulated annealing for the analysis of the COVID-19 pandemic data.

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Conflicts of Interest

All co-authors have seen and agree with the contents of this manuscript and there is no financial interest to report. We certify that the submission is original work and is not under review at any other publication.

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