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E-Super Arithmetic Graceful Labelling of Some Special Classes of Cubic Graphs Related to Cycles

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Abstract: We introduce a new concept called E-Super arithmetic graceful graphs.A (p, q) - graph G is said to be E-Super arithmetic graceful if there exists a bijection f from $V(G) \cup E(G)$ to $\{1, 2, \cdots, p+q\}$ such that $f(E(G)) = \{1, 2, \cdots, q\}$, $f(V(G)) = \{q+1, q+2, \cdots, q+p\}$ and the induced mapping f^* given by $f^*(uv) = f(u)+f(v)-f(uv)$ for $uv \in E(G)$ has the range $\{p+q+1, p+q+2, \cdots, p+2q\}$. In this paper we prove that $W(C_n)$,$D(C_{2n})$, $D_1(C_{2n})$, $D_2(C_{4n})$ are E-Super arithmetic graceful.

Key Words: E-Super arithmetic graceful graph, Smarandachely edge magic, $W(C_n)$, $D(C_{2n})$, $D_1(C_{2n})$, $D_2(C_{4n})$.

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§1. Introduction

Acharya and Hegde [1] have defined $(k, d)-$ arithmetic graphs. Let G be a graph with q edges and let k and d be positive integers.A labelling f of G is said to be $(k, d)-$ arithmetic if the vertex labels are distinct nonnegative integers and the edge labels induced by $f(x) + f(y)$ for each edge xy are $k, k+d, k+2d, \cdots, k+(q-1)d$. The case where $k = 1$ and $d = 1$ was called additively graceful by Hegde [3].

A labelling of $G(V, E)$ is said to be E-Super if $f(E(G)) = \{1, 2, 3, \cdots, |E(G)|\}$. A labelling of $G(V, E)$ is said to be E-Super if $f(E(G)) = \{1, 2, 3, \cdots, |E(G)|\}$. Marimuthu and Balakrishnan [5] defined a graph $G(V, E)$ to be edge magic graceful if there exists a bijection f from $V(G)\cup E(G)$ to $\{1, 2, \cdots, p+q\}$ such that $|f(u)+f(v)-f(uv)|$ is a constant for all edges uv of G. Otherwise, it is said to be *Smarandachely edge magic*, i.e., $|\{|f(u)+f(v)-f(uv)|, uv \in E(G)\}| \geq 2$.

We introduce a new concept called E-Super arithmetic graceful graphs. We define a graph $G(p, q)$ to be *E-Super arithmetic graceful* if there exists a bijection f from $V(G) \cup E(G)$ to $\{1, 2, \cdots, p+q\}$ such that $f(E(G)) = \{1, 2, \cdots, q\}$, $f(V(G)) = \{q+1, q+2, \cdots, q+p\}$ and the induced mapping f^* given by $f^*(uv) = f(u)+f(v)-f(uv)$ for $uv \in E(G)$ has the range $\{p+q+1, p+q+2, \cdots, p+2q\}$. In this paper, we prove that graphs $W(C_n)$,$D(C_{2n})$, $D_1(C_{2n})$, $D_2(C_{4n})$ are E-Super arithmetic graceful.

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§2. Preliminaries

Definition 2.1 *Let C_n denote the cycle for $n \geq 3$. Let $W(C_n)$ denote the graph with vertices $\{u_1, u_2, \cdots, u_n\}$ and $\{v_1, v_2, \cdots, v_n\}$ and edges $\{u_iu_{i+1}\}$,$\{u_iv_i\}$ and $\{v_iv_{i+1}\}$ where addition is modulo n.*

$W(C_n)$ is a cubic graph.

Illustration 2.1 The cubic graph $W(C_4)$ is shown in Fig.2.1.

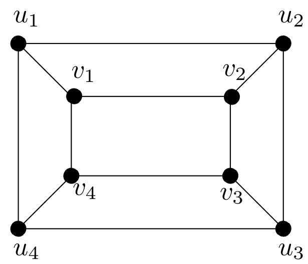

Fig.2.1

Definition 2.2 *Let C_{2n}, $n \geq 2$ denote the even cycle with $2n$ vertices $\{u_1, u_2, \cdots, u_{2n}\}$. By drawing n diagonals suitably we obtain cubic graphs related to even cycles. $D(C_{2n})$ denotes the cubic graph with vertices $\{u_1, u_2, \cdots, u_{2n}\}$ and edges $\{u_iu_{i+1} | i = 1, 2, \cdots, 2n,$ where $u_{2n+1} = u_1\}$ and $\{u_iu_{n+i} | i = 1, 2, \cdots, n\}$, $D(C_{2n})$ has $2n$ vertices and $3n$ edges. Particularly, $D(C_4)$ is the complete graph K_4.*

Illustration 2.2 The cubic graph $D(C_8)$ is shown in Fig.2.2.

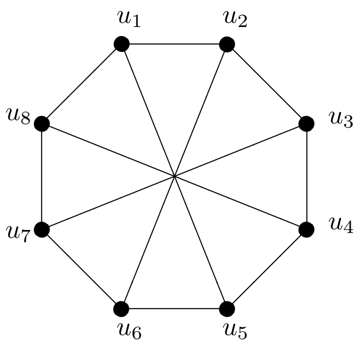

Fig.2.2

Definition 2.3 *$D_1(C_{2n})$ denotes the cubic graph with vertices $\{u_1, u_2, \cdots, u_{2n}\}$ and edges $\{u_iu_{i+1} | i = 1, 2, \cdots, 2n$ where $u_{2n+1} = u_1\}$, u_1u_{n+1} and $\{u_iu_{2n+2-i} | i = 2, 3, \cdots, n\}$. $D_1(C_{2n})$ is a cubic graph with $2n$ vertices and $3n$ edges.*

Illustration 2.3 The cubic graph $D_1(C_6)$ is shown in Fig.2.3.

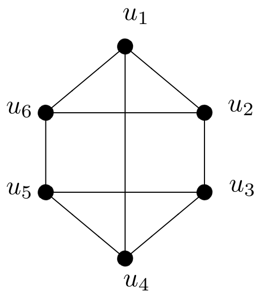

Fig.2.3

Definition 2.4 *$D_2(C_{4n})$ denotes the cubic graph with vertices $\{u_1, u_2, \cdots, u_{4n}\}$ and edges $\{u_iu_{i+1}|i = 1, 2, \cdots, 4n$ where $u_{n+1} = u_1\}$,$\{u_iu_{3n+1+i}|i = 1, 2, \cdots, n\}$ and $\{u_iu_{5n+1-i}|i = n+1, n+2, \cdots, 2n\}$. $D_2(C_{4n})$ has $4n$ vertices and $6n$ edges.*

Illustration 2.4 The cubic graph $D_2(C_{12})$ is shown in Fig.2.4.

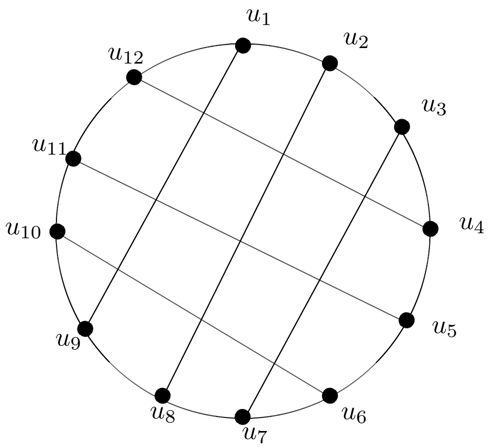

Fig.2.4

§3. Main Results

Theorem 3.1 *$W(C_n)$ is E-Super arithmetic graceful for all $n \geq 3$.*

Proof $W(C_n)$ has $2n$ vertices and $3n$ edges. Define $f : V \cup E \longrightarrow \{1, 2, \ldots, 5n\}$ as follows:

$f(u_i) = 3n + i, \quad i = 1, 2, \cdots, n,$

$f(v_i) = 4n + i, \quad i = 1, 2, \cdots, n,$

$f(u_iu_{i+1}) = n + i, \quad i = 1, 2, \cdots, n$ where $u_{n+1} = u_1$,

$f(u_iv_i) = i, \quad i = 1, 2, \cdots, n,$

$f(v_iv_{i+1}) = 2n + i, \quad i = 1, 2, \cdots, n$ where $v_{n+1} = v_1$.

Clearly, f is a bijection and $f^*(E(W(C_n))) = \{5n + 1, \cdots, 8n\}$. Therefore, $W(C_n)$ is E-Super arithmetic graceful for $n \geq 3$. □

Example 3.2 A E-Super arithmetic graceful labelling of $W(C_5)$ is shown in Fig.3.1.

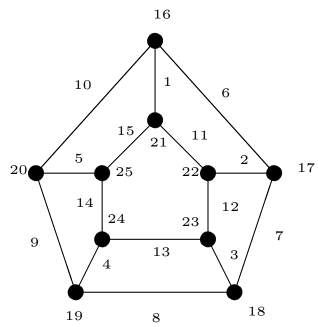

Fig.3.1

Theorem 3.3 *$D(C_{2n})$ is E-Super arithmetic graceful for all $n \geq 2$.*

Proof Let $\{u_1, u_2, \ldots, u_{2n}\}$ be the vertices of $D(C_n)$. Define $f : V \cup E \longrightarrow \{1, 2, \cdots, 5n\}$ as follows:

$f(u_i) = 3n + i, \quad i = 1, 2, \cdots, 2n,$

$f(u_i u_{i+1}) = i, \quad i = 1, 2, \cdots, 2n$ where $u_{2n+1} = u_1$,

$f(u_i u_{n+i}) = 2n + i, \quad i = 1, 2, \cdots, n.$

Clearly, f is a bijection and $f^*(E(D(C_{2n}))) = \{5n + 1, \cdots, 8n\}$. Therefore $D(C_{2n})$ is E-Super arithmetic graceful for $n \geq 2$. □

Example 3.4 An E-Super arithmetic graceful labelling of $D(C_6)$ is shown in Fig.3.2.

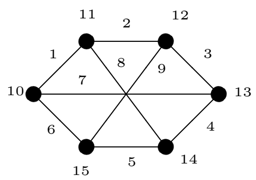

Fig.3.2

Theorem 3.5 *$D_1(C_{2n})$ for $n \geq 3$ is E-Super arithmetic graceful.*

Proof Let $u_1, u_2, \cdots, u_{2n}$ be the vertices of $D_1(C_{2n})$. Define $f : V \cup E \longrightarrow \{1, 2, \ldots, 5n\}$ as follows:

$f(u_i) = 3n + i, \quad i = 1, 2, \cdots, 2n,$

$f(u_i u_{i+1}) = i, \quad i = 1, 2, \cdots, 2n$ where $u_{2n+1} = u_1$,

$f(u_1 u_{n+1}) = 2n + 1,$

$f(u_i u_{2n+2-i}) = 2n + i, \quad i = 2, 3, \cdots, n.$

Clearly, f is a bijection and

$$f^*(E(D_1(C_{2n}))) = \{5n + 1, \cdots, 8n\}.$$

Therefore, $E(D_1(C_{2n}))$ is E-Super arithmetic graceful for $n \geq 3$. □

Example 3.6 An E-Super arithmetic graceful labelling of $D_1(C_8)$ is shown in Fig.3.3.

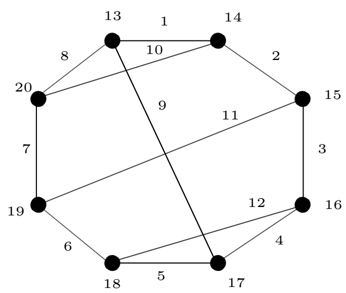

Fig.3.3

Theorem 3.7 *$D_2(C_{4n})$ for $n \geq 2$ is E-Super arithmetic graceful.*

Proof Define $f : V \cup E \longrightarrow \{1, 2, \cdots, 10n\}$ as follows:

$f(u_i) = 6n + i, \quad i = 1, 2, \cdots, 4n,$

$f(u_i u_{i+1}) = i, \quad i = 1, 2, \cdots, 4n$ where $u_{4n+1} = u_1$,

$f(u_i u_{3n+1-i}) = 4n + i, \quad i = 1, 2, \cdots, n,$

$f(u_i u_{5n+1-i}) = 4n + i, \quad i = n + 1, \cdots, 2n.$

Clearly, f is a bijection and

$$f^*(E(D_2(C_{4n}))) = \{10n + 1, 10n + 2, \cdots, 16n\}.$$

Therefore $D_2(C_{4n})$ is E-Super arithmetic graceful for $n \geq 2$. □

Example 3.8 An E-Super arithmetic graceful labelling of $D_2(C_{16})$ is shown in Fig.3.4.

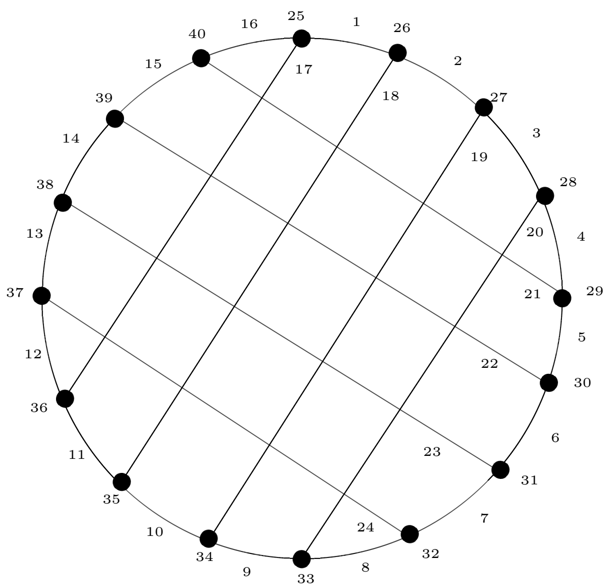

Fig.3.4

References

[1] B.D.Acharya and S.M.Hedge, Arithmetic graphs, *J. Graph Theory*, 14 (1990) 275-299.

[2] S.W.Golomb, How to number a graph, in *Graph Theory and Computing*, R.C.Reed ed., Academic press, New York(1972) 23-37.

[3] S.M.Hedge, Additively graceful graphs, *Mat. Acad,Sci.Lett.,* 12(1989) 387-390.

[4] Joseph A.Gallian, A Dynamic Survey of Graph Labelling, *The Electronic Journal of combinatorics*, DS6 (2016).

[5] A Kotzig and A.Rosa, Magic valuation of finite graphs, *Canad.Math.Bull*, 13(1970) 451-456.

[6] J.A.MacDougall, M.Miller, Slamin and W.D.Walls, Vertex - magic total labelling of graphs, *Util.Math.*61(2002) 3-21.

[7] G.Marimuthu and M.Balakrishnan, Super edge magic graceful graphs, *Information Sciences*, 287 (10)(2014) 140-151.

[8] V.Ramachandran, C.Sekar, (1,N)-arithmetic graphs, *International Journal of Computers and Applications*, Vol.38 (1) (2016) 55-59.

[9] A.Rosa, On certain valuations of the vertices of a graph, *Theory of graphs* (International Symposium, Rome, July 1966), Gordon and Breach, N.Y and Dunod Paris (1967) 349-355.