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F-Centroidal Mean Labeling of Graphs Obtained From Paths

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Abstract: A function f is called an F-centroidal mean labeling of a graph $G(V,E)$ with p vertices and q edges if $f: V(G) \rightarrow \{1,2,3,\cdots,q+1\}$ is injective and the induced function $f^*: E(G) \rightarrow \{1,2,3,\cdots,q\}$ defined as

$$f^*(uv) = \left\lfloor \frac{2\left[f(u)^2 + f(u)f(v) + f(v)^2\right]}{3\left[f(u)+f(v)\right]} \right\rfloor,$$

for all $uv \in E(G)$, is bijective. A graph that admits an F-centroidal mean labeling is called an F-centroidal mean graph. In this paper, we have discussed the F-centroidal meanness of the graph $P_n(X_1, X_2, \cdots X_n)$, the twig graph $TW(P_n)$, the graph $P_n \circ S_m$ for $m \leq 4$, planar grid $P_m \times P_n$ for $m \leq 3$, the ladder graph L_n, the graph $P_n \circ K_2$, the graph P_a^b for $a \geq 2$ and $b \leq 3$, the middle graph of the path, total graph of the path and the square graph of the path, the splitting graph of the path and the graph $P(1,2,\cdots,n-1)$.

Key Words: Labeling, F-centroidal mean labeling, F-centroidal mean graph, Smarandachely F-centroidal mean labeling.

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§1. Introduction

Throughout this paper, by a graph we mean a finite, undirected and simple graph. Let $G(V,E)$ be a graph with p vertices and q edges. For notations and terminology, we follow [8]. For a detailed survey on graph labeling, we refer [7].

Path on n vertices is denoted by P_n. The graph $P_n(X_1, X_2, \ldots X_n)$, is a tree obtained from a path on n vertices by attaching X_i pendent vertices at each i^{th} vertex of the path, for $1 \leq i \leq n$. A Twig $TW(P_n), n \geq 4$ is a graph obtained from a path by attaching exactly two pendant vertices to each internal vertices of the path P_n. The graph $G \circ S_m$ is obtained from G by attaching m pendant vertices to each vertex of G. Let G_1 and G_2 be any two graphs with p_1 and p_2 vertices respectively. Then the Cartesian product $G_1 \times G_2$ has p_1p_2 vertices which are $\{(u,v) : u \in G_1, v \in G_2\}$ and the edges are obtained as follows: (u_1,v_1) and (u_2,v_2) are adjacent in $G_1 \times G_2$ if either $u_1 = u_2$ and v_1 and v_2 are adjacent in G_2 or u_1 and u_2 are adjacent

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in G_1 and $v_1 = v_2$. The product $P_m \times P_n$ and is called a planar grid and $P_2 \times P_n$ is called a ladder, denoted by L_n.

Let a and b be integers such that $a \geq 2$ and $b \geq 2$. Let $y_1, y_2, \ldots y_a$ be the $'a'$ fixed vertices. Connect y_i and y_{i+1} by means of b internally disjoint paths P_i^j of length $'i+1'$ each, for $1 \leq i \leq a-1$ and $1 \leq j \leq b$. The resulting graph embedded in the plane is denoted by P_a^b. The middle graph $M(G)$ of a graph G is the graph whose vertex set is $\{v : v \in V(G)\} \cup \{e : e \in E(G)\}$ and the edge set is $\{e_1e_2 : e_1, e_2 \in E(G)$ and e_1 and e_2 are adjacent edges of $G\} \cup \{ve : v \in V(G), e \in E(G)$ and e is incident with $v\}$. The total graph $T(G)$ of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and two vertices are adjacent if and only if either they are adjacent vertices of G or adjacent edges of G or one is a vertex of G and the other one is an edge incident on it. Square of a graph G, denoted by G^2, has the vertex set as in G and two vertices are adjacent in G^2 if they are at a distance either 1 or 2 apart in G. For each vertex v of the graph G, take a new vertex v' to these vertices of G adjacent to v. The graph thus obtained is called the splitting graph G and it is denoted by $S'(G)$. An arbitrary super subdivision $P(m_1, m_2, \cdots, m_{n-1})$ of a path P_n is a graph obtained by replacing each i^{th} edge of P_n by identifying its end vertices of the edge with a partition of K_{2,m_i} having 2 elements, where m_i is any positive integer.

Durai Baskar and Arockiaraj defined the F-harmonic mean labeling [6] and discussed its meanness of some standard graphs. The concept of F-geometric mean labeling was introduced by Durai Baskar and Arockiaraj [5] and it was developed [4]. The concept of F-root square mean labeling was introduced by Arockiaraj et al., [1] and they studied the F-root square mean labeling of some standard graphs [2]. Durai Baskar and Manivannan were introduced F-heronian mean labeling [3]. Motivated by the works of so many authors in the area of graph labeling, we introduced a new type of labeling called an F-centroidal mean labeling.

A function f is called an F-centroidal mean labeling of a graph $G(V, E)$ with p vertices and q edges if $f : V(G) \rightarrow \{1, 2, 3, \cdots, q+1\}$ is injective and the induced function $f^* : E(G) \rightarrow \{1, 2, 3, \cdots, q\}$ defined by

$$f^*(uv) = \left\lfloor \frac{2\,[f(u)^2 + f(u)f(v) + f(v)^2]}{3\,[f(u) + f(v)]} \right\rfloor,$$

for all $uv \in E(G)$, is bijective. Otherwise, it is called a Smarandachely F-centroidal mean labeling of G if there is a number $k \in \{1, 2, 3, \cdots, q\}$ such that the inverse f^{-*} of f^* holds with $|f^{-*}(k)| \geq 2$. A graph that admits an F-centroidal mean labeling is called an F-centroidal mean graph.

An F-centroidal mean labeling of cycle C_4 is given in Figure 1.

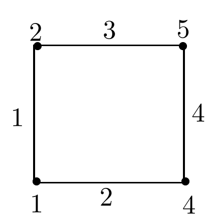

Figure 1 An F-centroidal mean labeling labeling of C_4

In this paper, we have discussed the F-centroidal meanness of the graph $P_n(X_1, X_2, \cdots X_n)$, the twig graph $TW(P_n)$, the graph $P_n \circ S_m$ for $m \leq 4$, planar grid $P_m \times P_n$ for $m \leq 3$, the ladder graph L_n, the graph $P_n \circ K_2$, the graph P_a^b for $a \geq 2$ and $b \leq 3$, the middle graph of the path, total graph of the path and the square graph of the path, the splitting graph of the path and the graph $P(1, 2, \cdots, n-1)$.

§2. Main Results

Theorem 2.1 *The graph $P_n(X_1, X_2, \cdots X_n)$ is an F-centroidal mean graph, for $1 \leq X_i \leq 3$ and $|X_i - X_{i+1}| \leq 1$, for $1 \leq i \leq n-1$.*

Proof Let $u_1, u_2, \cdots, u_n$ be the vertices of the path P_n. Let $v_i^{(1)}, v_i^{(2)}, \cdots, v_i^{(X_i)}$ be the pendant vertices attached at u_i, for $1 \leq i \leq n$.

Define $f : V(P_n(X_1, X_2, \cdots X_n)) \rightarrow \{1, 2, 3, \cdots, \sum\limits_{i=1}^{n} X_i + n\}$ as follows:

$$f(v_i^{(1)}) = \begin{cases} 2, & X_1 = 1, \\ 1, & X_1 \neq 1. \end{cases}$$

For $2 \leq i \leq n$,

$$f(v_i^{(1)}) = \begin{cases} \sum\limits_{k=1}^{i-1} X_k + i, & X_i = 2, 3, \\ \sum\limits_{k=1}^{i-1} X_k + i + 1, & X_i = 1. \end{cases}$$

For $1 \leq i \leq n$,

$$f(v_i^{(j)}) = \begin{cases} f(v_i^{(1)}) + 2, & j = 2 \\ f(v_i^{(1)}) + 3, & X_i = 3 \text{ and } j = 3 \end{cases}$$

and

$$f(u_i) = \begin{cases} f(v_i^{(1)}) + 1, & X_i = 2, 3, \\ f(v_i^{(1)}) - 1, & X_i = 1. \end{cases}$$

Then the induced edge labeling f^* is obtained as follows:

For $1 \leq i \leq n-1$,

$$f^*(u_i u_{i+1}) = \begin{cases} f(u_i) + 1, & X_i = 1, 2, \\ f(u_i) + 2, & X_i = 3 \end{cases}$$

and $f^*(v_1^{(1)} u_1) = 1$.

For $1 \le i \le n$,

$$f^*(v_i^{(1)}u_i) = \begin{cases} f(v_i^{(1)})+1, & X_i = 2,3, \\ f(v_i^{(1)})-1, & X_i = 1 \end{cases}$$

and

$$f^*(v_i^{(j)}u_i) = \begin{cases} f(u_i), & X_i = 2,3 \text{ and } j = 2, \\ f(u_i)+1, & X_i = 3 \text{ and } j = 3. \end{cases}$$

Hence f is an F-centroidal mean labeling of $P_n(X_1, X_2, \cdots X_n)$. Thus the graph $P_n(X_1, X_2, \cdots X_n)$ is an F-centroidal mean graph, for $1 \le X_i \le 3$ and $|X_i - X_{i+1}| \le 1$, for $1 \le i \le n-1$. □

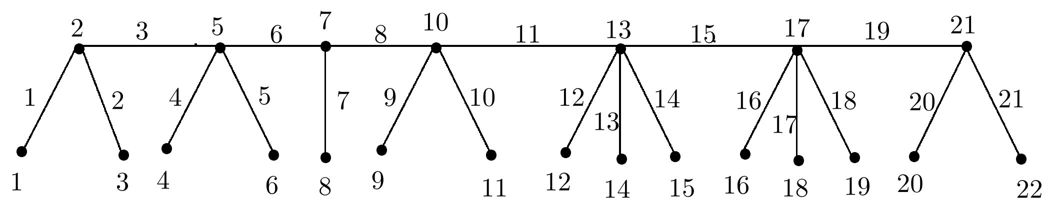

Figure 2 An F-centroidal mean labeling of $P_n(2, 2, 1, 2, 3, 3, 2)$

Corollary 2.2 *The twig graph $TW(P_n)$ of the path P_n is an F-centroidal mean graph, for $n \ge 4$.*

Theorem 2.3 *The graph $P_n \circ S_m$ is an F-centroidal mean graph, for $n \ge 1$ and $m \le 4$.*

Proof Let $v_1, v_2, v_3, \cdots, v_n$ be the vertices of the path P_n and $u_1^{(i)}, u_2^{(i)}, u_3^{(i)}, \cdots, u_m^{(i)}$ be the pendant vertices at each v_i, for $1 \le i \le n$.

Case 1. $m = 4$.

Define $f : V(P_n \circ S_4) \to \{1, 2, 3, \ldots, 5n\}$ as follows:

$$\begin{aligned} f(v_1) &= 2, \\ f(v_i) &= 5i-2, \text{ for } 1 \le i \le n, \\ f(u_1^{(1)}) &= 1, \\ f(u_1^{(i)}) &= 5i-5, \text{ for } 2 \le i \le n, \\ f(u_2^{(1)}) &= 3, \\ f(u_2^{(i)}) &= 5i-3, \text{ for } 2 \le i \le n, \\ f(u_3^{(i)}) &= 5i-1, \text{ for } 1 \le i \le n, \\ f(u_4^{(i)}) &= 5i+1, \text{ for } 1 \le i \le n-1, \\ f(u_4^{(n)}) &= 5n. \end{aligned}$$

Then the induced edge labeling f^* is obtained as follows:

$$f^*(v_iv_{i+1}) = 5i, \text{ for } 1 \le i \le n-1,$$
$$f^*(v_iu_1^{(i)}) = 5i-4, \text{ for } 1 \le i \le n,$$
$$f^*(v_iu_2^{(i)}) = 5i-3, \text{ for } 1 \le i \le n,$$
$$f^*(v_iu_3^{(i)}) = 5i-2, \text{ for } 1 \le i \le n$$
$$f^*(v_iu_4^{(i)}) = 5i-1, \text{ for } 1 \le i \le n.$$

Case 2. $1 \le m \le 3$.

By Theorem 2.1, the results follows in this case.

Hence f is an F-centroidal mean labeling of $P_n \circ S_m$, for $n \ge 1$ and $m \le 4$. Thus the graph $P_n \circ S_m$ is an F-centroidal mean graph, for $n \ge 1$ and $m \le 4$. □

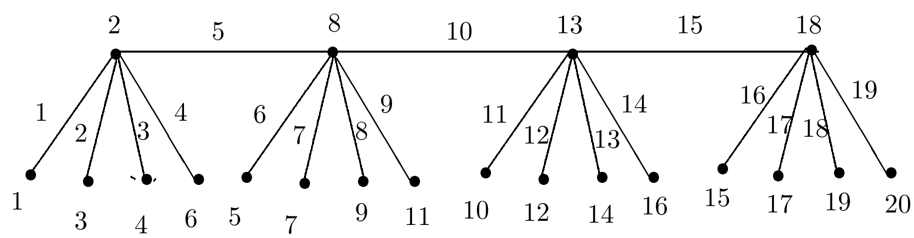

Figure 3 An F-centroidal mean labeling of $P_4 \circ S_4$

Theorem 2.4 *The planar grid $P_m \times P_n$, is an F-centroidal mean graph, for $m \le 3$ and $n \ge 2$.*

Proof Let $V(P_m \times P_n) = \{v_{ij} : 1 \le i \le m,\ 1 \le j \le n\}$ and $E(P_m \times P_n) = \{v_{ij}v_{(i+1)j} : 1 \le i \le m-1,\ 1 \le j \le n\} \cup \{v_{ij}v_{i(j+1)} : 1 \le i \le m,\ 1 \le j \le n-1\}$ be the vertex set and edge set of the graph $P_m \times P_n$.

Case 1. $m = 2$.

Define $f : V(P_2 \times P_n) \to \{1, 2, 3, \cdots, 3n-1\}$ as follows:

$$f(v_{ij}) = i + 3(j-1), \text{ for } 1 \le i \le 2 \text{ and } 1 \le j \le n.$$

Then the induced edge labeling f^* is obtained as follows:

$$f^*(v_{1j}v_{2j}) = 3j-2, \text{ for } 1 \le i \le n,$$
$$f^*(v_{ij}v_{i(j+1)}) = i + 3j - 2, \text{ for } 1 \le i \le 2 \text{ and } 1 \le j \le n-1.$$

Case 2. $m = 3$.

Define $f : V(P_3 \times P_n) \to \{1, 2, 3, \cdots, 5n-2\}$ as follows:

$$f(v_{i1}) = i, \text{ for } 1 \leq i \leq 3,$$

$$f(v_{i2}) = \begin{cases} i+4, & i = 1, \\ i+5, & 2 \leq i \leq 3, \end{cases}$$

$$f(v_{ij}) = i + 5(j-1), \text{ for } 1 \leq i \leq 3 \text{ and } 3 \leq j \leq n.$$

Then the induced edge labeling f^* is obtained as follows:

$$f^*(v_{i1}v_{(i+1)1}) = i, \text{ for } 1 \leq i \leq 2,$$

$$f^*(v_{i1}v_{i2}) = i + 2, \text{ for } 1 \leq i \leq 3,$$

$$f^*(v_{ij}v_{(i+1)j}) = \begin{cases} 2i + 3(j-1), & 1 \leq i \leq 2 \text{ and } j = 2, \\ i + 5(j-1), & 1 \leq i \leq 2 \text{ and } 3 \leq j \leq n, \end{cases}$$

$$f^*(v_{ij}v_{i(j+1)}) = i + 5j - 3, \text{ for } 1 \leq i \leq 3 \text{ and } 2 \leq j \leq n-1.$$

Hence the graph $P_m \times P_n$ admits an F-centroidal mean labeling. Thus the graph $P_m \times P_n$ is an F-centroidal meangraph for $m \leq 3$.

For $n = 2$, an F-centroidal mean labeling of $P_2 \times P_4$ is as shown in Figure 4. □

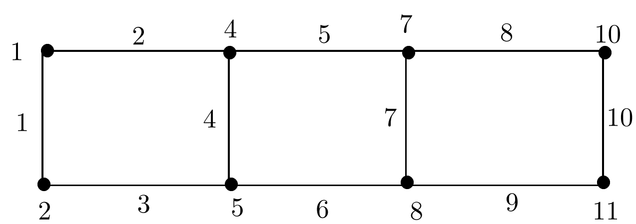

Figure 4 An F-centroidal mean labeling of $P_2 \times P_4$

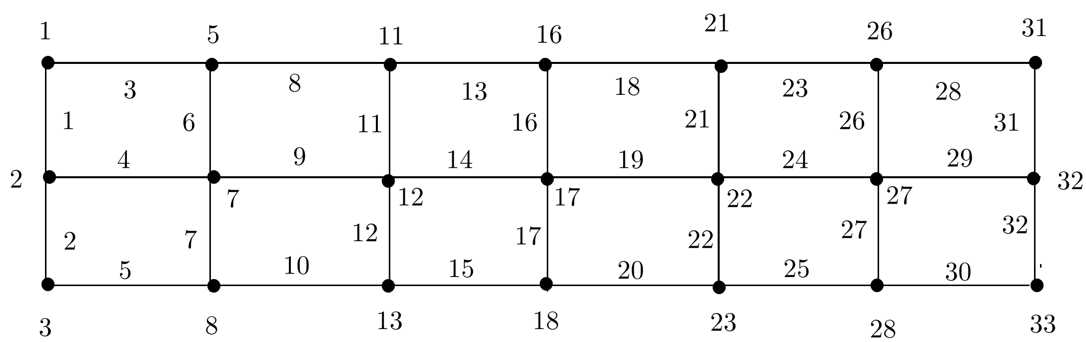

Figure 5 An F-centroidal mean labeling of $P_3 \times P_7$

Corollary 2.5 *Every Ladder graph $L_n = P_2 \times P_n$ is an F-centroidal mean graph for $n \geq 2$.*

Theorem 2.6 *The graph $P_n \circ K_2$ is an F-centroidal mean graph for $n \geq 1$.*

Proof Let $v_1, v_2, v_3, \cdots, v_n$ be the vertices of the path P_n and $u_i^{(1)}, u_i^{(2)}$ be the vertices of i^{th} copy of K_2 attached with v_i, for $1 \leq i \leq n$. Define $f : V(P_n \circ K_2) \to \{1, 2, 3, \cdots, 4n\}$ as follows:

$$f(v_i) = 4i - 2, \text{ for } 1 \leq i \leq n,$$
$$f(u_i^{(1)}) = 4i - 3, \text{ for } 1 \leq i \leq n,$$
$$f(u_i^{(2)}) = 4i, \text{ for } 1 \leq i \leq n.$$

Then the induced edge labeling f^* is obtained as follows:

$$f^*(v_i v_{i+1}) = 4i, \text{ for } 1 \leq i \leq n-1,$$
$$f^*(u_i^{(1)} u_i^{(2)}) = 4i - 2, \text{ for } 1 \leq i \leq n,$$
$$f^*(u_i^{(1)} v_1) = 4i - 3, \text{ for } 1 \leq i \leq n,$$
$$f^*(u_i^{(2)} v_i) = 4i - 1, \text{ for } 1 \leq i \leq n.$$

Hence f is an F-centroidal mean labeling of $P_n \circ K_2$ for $n \geq 1$. Thus the graph $P_n \circ K_2$ is an F-centroidal mean graph, for $n \geq 1$. $\square$

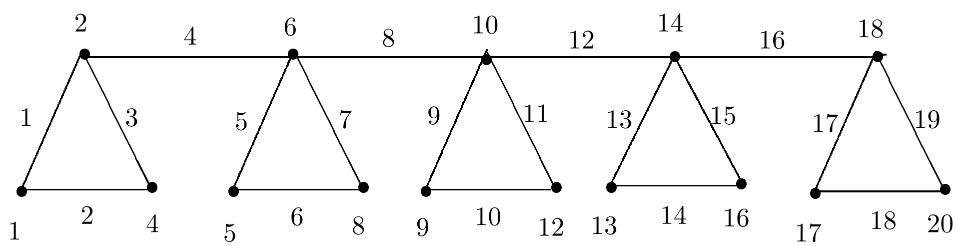

Figure 6 An F-centroidal mean labeling of $P_5 \circ K_2$

Theorem 2.7 *The graph P_a^b is an F-centroidal mean graph, for $a \geq 2$ and $b \leq 3$.*

Proof Let $y_i, x_{ij1}, x_{ij2}, \ldots x_{iji}, y_{i+1}$ be the vertices of the path P_i^j, where $1 \leq i \leq a-1$ and $1 \leq j \leq b$. Let $V(P_a^b) = \{y_i : 1 \leq i \leq a\} \cup \bigcup_{i=1}^{a-1} \bigcup_{j=1}^{b} \{x_{ijk} : 1 \leq k \leq i\}$ and $E(P_a^b) = \bigcup_{i=1}^{a-1} \{y_i x_{ij1} : 1 \leq i \leq b\} \cup \bigcup_{i=1}^{a-1} \bigcup_{j=1}^{b} \{x_{ijk} x_{ij(k+1)} : 1 \leq k \leq i-1\} \cup \bigcup_{i=1}^{a-1} \{x_{iji} y_{i+1} : 1 \leq j \leq b\}$ be the vertex set and edge set of the graph P_a^b.

Case 1. $b = 2$.

Define $f : V(P_a^2) \to \{1, 2, 3, \cdots, (a-1)(a+2)+1\}$ as follows:

$$f(y_1) = 1,$$
$$f(y_i) = (i-1)(i+2)+1, \text{ for } 2 \leq i \leq a,$$
$$f(x_{1j1}) = j+1, \text{ for } 1 \leq j \leq 2 \text{ and for } 2 \leq i \leq a-1,$$

$$f(x_{ijk}) = (i-1)(i+2) + 2k + j - 1, \text{ for } 1 \leq k \leq i \text{ and } 1 \leq j \leq 2.$$

Then the induced edge labeling f^* is obtained as follows:

$$\begin{aligned} f^*(y_1 x_{1j1}) &= j, \text{ for } 1 \leq j \leq 2, \\ f^*(x_{1j1} y_2) &= j + 2, \text{ for } 1 \leq j \leq 2, \\ f^*(y_i x_{ij1}) &= (i-1)(i+2) + j, \text{ for } 2 \leq i \leq a-1 \text{ and } 1 \leq j \leq 2, \\ f^*(x_{ijk} x_{ij(k+1)}) &= (i-1)(i+2) + j + 2k, \text{ for } 2 \leq i \leq a-1, \ 1 \leq k \leq i-1 \text{ and } 1 \leq j \leq 2, \\ f^*(x_{iji} y_{i+1}) &= i(i+3) + j - 2, \text{ for } 2 \leq i \leq a-1 \text{ and } 1 \leq j \leq 2. \end{aligned}$$

Case 2. $b = 3$.

Define $f : V(P_a^3) \to \{1, 2, 3, \cdots, \frac{3(a-1)(a+2)}{2} + 1\}$ as follows:

$$f(y_1) = 1, \ f(y_2) = 5, \ f(x_{111}) = 2, \ f(x_{1j1}) = 4j - 5, \text{ for } 2 \leq j \leq 3,$$

$$f(y_i) = \frac{3(i-1)(i+2)}{2} + 1, \text{ for } 3 \leq i \leq a, \ f(x_{21k}) = \begin{cases} 4k+5, & k = 1, \\ 4k+4, & k = 2, \end{cases}$$

$$f(x_{22k}) = \begin{cases} 7k+6, & k = 1, \\ 7k-4, & k = 2, \end{cases} \quad f(x_{23k}) = \begin{cases} 5k+6, & k = 1, \\ 5k+4, & k = 2. \end{cases}$$

For $3 \leq i \leq a-1$,

$$f(x_{ij1}) = \begin{cases} \frac{3(i-1)(i+2)}{2} + j + 1, & 1 \leq j \leq 2, \\ \frac{3(i-1)(i+2)}{2} + 2j, & j = 3 \text{ and} \end{cases}$$

$$f(x_{ijk}) = \begin{cases} \frac{3(i-1)(i+2)}{2} + 2j + 3k - 1, & 1 \leq j \leq 2, \ 2 \leq k \leq i-1, \text{ and } k \text{ is even}, \\ \frac{3(i-1)(i+2)}{2} + 3k - 2, & j = 3, \ 2 \leq k \leq i-1 \text{ and } k \text{ is even}, \\ \frac{3(i-1)(i+2)}{2} + 2j + 3k - 3, & 1 \leq j \leq 3, \ 2 \leq k \leq i-1 \text{ and } k \text{ is odd}, \\ \frac{3(i-1)(i+2)}{2} + 3k - 1, & j = 1, k = i \text{ and } k \text{ is odd}, \\ \frac{3(i-1)(i+2)}{2} + 3k + j - 1, & j = 2, k = i \text{ and } k \text{ is odd}, \\ \frac{3(i-1)(i+2)}{2} + 3k + j, & j = 3, k = i \text{ and } k \text{ is odd}, \\ \frac{3(i-1)(i+2)}{2} + 3k + j, & 1 \leq j \leq 2, k = i, \text{ and } k \text{ is even}, \\ \frac{3(i-1)(i+2)}{2} + 3k - 1, & j = 3, k = i \text{ and } k \text{ is even}. \end{cases}$$

Then the induced edge labeling f^* is obtained as follows:

$$f^*(y_1x_{1j1}) = \begin{cases} j, & 1 \le j \le 2, \\ 5, & j = 3, \end{cases}$$

$$f^*(y_ix_{ij1}) = \begin{cases} \frac{3(i-1)(i+2)}{2} + j, & j = 1 \text{ and } 2 \le i \le a-1, \\ \frac{3(i-1)(i+2)}{2} + j + 1, & j = 2 \text{ and } i = 2, \\ \frac{3(i-1)(i+2)}{2} + j - 1, & j = 3 \text{ and } i = 2, \\ \frac{3(i-1)(i+2)}{2} + j, & j = 2, 3 \text{ and } 3 \le i \le a-1, \end{cases}$$

$$f^*(x_{1j1}y_2) = \begin{cases} 3, & j = 1, \\ 2j, & 2 \le j \le 3, \end{cases} \qquad f^*(x_{2j2}y_3) = \begin{cases} 14, & j = 1, \\ 13, & j = 2, \\ 15, & j = 3, \end{cases}$$

$f^*(x_{2j1}x_{2j2}) = j + 9$ for $1 \le j \le 3$ and $3 \le i \le a - 1$,

$$f^*(x_{ijk}x_{ij(k+1)}) = \begin{cases} \frac{3(i-1)(i+2)}{2} + 3k + 2(j-1) + 1, & 1 \le k \le i-1, \\ & \text{and } 1 \le j \le 2, \\ \frac{3(i-1)(i+2)}{2} + 3k + 2, & 1 \le k \le i-1, \\ & \text{and } j = 3, \end{cases}$$

$$\text{and } f^*(x_{iji}y_{i+1}) = \begin{cases} \frac{3i(i+3)}{2} + j - 3, & 1 \le j \le 3 \text{ and } i \text{ is odd}, \\ \frac{3i(i+3)}{2} + j - 2, & 1 \le j \le 2 \text{ and } i \text{ is even}, \\ \frac{3i(i+3)}{2} - 2, & j = 3 \text{ and } i \text{ is even}. \end{cases}$$

Hence f is an F-centroidal mean labeling of P_a^b for $a \ge 2$ and $b \le 3$. Thus the graph P_a^b is an F-centroidal mean graph, for $a \ge 2$ and $b \le 3$. □

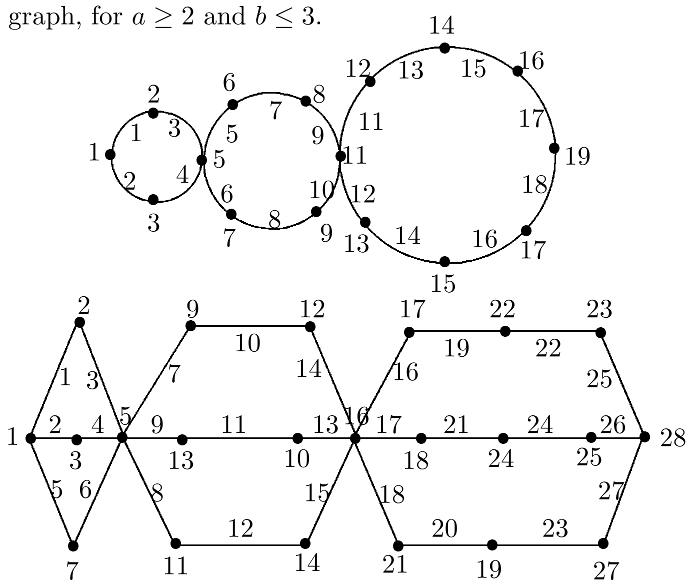

Figure 7 An F-centroidal mean labeling of P_4^2 and P_4^3

Theorem 2.8 *The middle graph $M(P_n)$ of a path P_n is an F-centroidal mean graph.*

Proof Let $V(P_n) = \{v_1, v_2, v_3, \cdots, v_n\}$ and $E(P_n) = \{e_i = v_iv_{i+1} : 1 \leq i \leq n-1\}$ be the vertex set and edge set of the path P_n. Then,

$$V(M(P_n)) = \{v_1, v_2, v_3, \ldots, v_n, e_1, e_2, e_3, \cdots, e_{n-1}\},$$
$$E(M(P_n)) = \{v_ie_i, e_iv_{i+1} : 1 \leq i \leq n-1\} \cup \{e_ie_{i+1} : 1 \leq i \leq n-2\}.$$

Define $f : V(M(P_n)) \to \{1, 2, 3, \cdots, 3n-3\}$ as follows:

$$f(v_i) = \begin{cases} 1, & \text{for } i = 1, \\ 3i-3, & \text{for } 2 \leq i \leq n, \end{cases}$$
$$f(e_i) = 3i-1, \text{ for } 1 \leq i \leq n-1.$$

Then the induced edge labeling f^* is obtained as follows:

$$f^*(v_ie_i) = 3i-2, \text{ for } 1 \leq i \leq n-1,$$
$$f^*(e_iv_{i+1}) = 3i-1, \text{ for } 1 \leq i \leq n-1,$$
$$f^*(e_ie_{i+1}) = 3i, \text{ for } 1 \leq i \leq n-2.$$

Hence f is an F-centroidal mean labeling of the graph $M(P_n)$. Thus the graph $M(P_n)$ is an F-centroidal mean graph. □

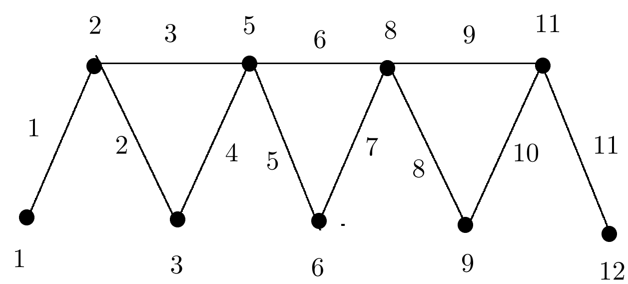

Figure 8 An F-centroidal mean labeling of $M(P_5)$

Theorem 2.9 *The total graph $T(P_n)$ of a path P_n is an F-centroidal mean graph for $n \geq 1$.*

Proof Let $V(P_n) = \{v_1, v_2, v_3, \cdots, v_n\}$ and $E(P_n) = \{e_i = v_iv_{i+1} : 1 \leq i \leq n-1\}$ be the vertex set and edge set of the path P_n. Then $V(T(P_n)) = \{v_1, v_2, v_3, \cdots, v_n, e_1, e_2, e_3, \cdots, e_{n-1}\}$ and $E(T(P_n)) = \{v_iv_{i+1}, e_iv_i, e_iv_{i+1} : 1 \leq i \leq n-1\} \cup \{e_ie_{i+1} : 1 \leq i \leq n-2\}$.

Define $f : V(T(P_n)) \to \{1, 2, 3, \ldots, 4(n-1)\}$ as follows:

$$f(v_1) = 1,$$
$$f(v_i) = 4i-4, \text{ for } 2 \leq i \leq n,$$
$$f(e_i) = 4i-2, \text{ for } 1 \leq i \leq n-1.$$

Then the induced edge labeling f^* is obtained as follows:

$$f^*(v_iv_{i+1}) = 4i-2, \text{ for } 1 \leq i \leq n-1,$$
$$f^*(e_ie_{i+1}) = 4i, \text{ for } 1 \leq i \leq n-2,$$
$$f^*(v_ie_i) = 4i-3, \text{ for } 1 \leq i \leq n-1,$$
$$f^*(e_iv_{i+1}) = 4i-1, \text{ for } 1 \leq i \leq n-1.$$

Hence f is an F-centroidal mean labeling of the graph $T(P_n)$. Thus the graph $T(P_n)$ is an F-centroidal mean graph. □

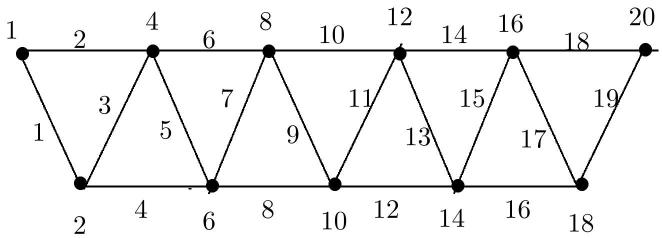

Figure 9 An F-centroidal mean labeling of $T(P_6)$

Theorem 2.10 *The square graph P_n^2 of the path P_n is an F-centroidal mean graph for $n \geq 1$.*

Proof Let $v_1, v_2, v_3, \cdots, v_n$ be the vertices of the path P_n. Define $f : V\left(P_n^2\right) \to \{1, 2, 3, \cdots, 2(n-1)\}$ as follows:

$$f(v_1) = 1,$$
$$f(v_i) = 2i-2, \text{ for } 2 \leq i \leq n.$$

Then the induced edge labeling f^* is obtained as follows:

$$f^*(v_iv_{i+1}) = 2i-1, \text{ for } 1 \leq i \leq n-1,$$
$$f^*(v_iv_{i+2}) = 2i, \text{ for } 1 \leq i \leq n-2.$$

Hence f is an F-centroidal mean labeling of the graph P_n^2. Thus the graph P_n^2 is an F-centroidal mean graph. □

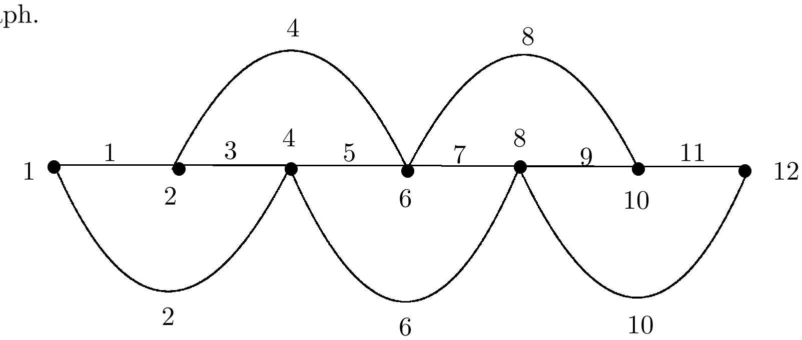

Figure 10 An F-centroidal mean labeling of P_7^2

Theorem 2.11 *The splitting graph $S'(P_n)$ is an F-centroidal mean graph for $n \geq 2$.*

Proof Let $v_1, v_2, \cdots, v_n$ be the vertices of the path P_n. Let $v_1, v_2, \cdots, v_n, v'_1, v'_2, \cdots, v'_n$ be the vertices of the graph $S'(P_n)$. Let $V(S'(P_n)) = \{v_i, v'_i : 1 \leq i \leq n\}$ and $E(S'(P_n)) = \{v_iv_{i+1}, v_iv'_{i+1}, v'_iv_{i+1} : 1 \leq i \leq n-1\}$ be the vertex set and edge set of the splitting graph $S'(P_n)$.

Case 1. n is odd.

Define $f : V(S'(P_n)) \rightarrow \{1, 2, 3, \cdots, 3n-2\}$ as follows:

$$f(v_i) = \begin{cases} 4i-3, & 1 \leq i \leq 2, \\ 3, & i = 3, \\ 3i-4, & 4 \leq i \leq n \text{ and } i \text{ is odd}, \\ 3i, & 4 \leq i \leq n \text{ and } i \text{ is even}, \end{cases}$$

$$f(v'_i) = \begin{cases} 6, & i = 1, \\ 2, & i = 2, \\ 3i-2, & 3 \leq i \leq n. \end{cases}$$

Then the induced edge labeling f^* is obtained as follows:

$$f^*(v_iv_{i+1}) = \begin{cases} i+2, & 1 \leq i \leq 2, \\ 3i-1, & 3 \leq i \leq n-1, \end{cases}$$

$$f^*(v_iv'_{i+1}) = \begin{cases} 5i-4, & 1 \leq i \leq 2, \\ 3i-2, & 3 \leq i \leq n-1 \text{ and } i \text{ is odd}, \\ 3i, & 3 \leq i \leq n-1 \text{ and } i \text{ is even}, \end{cases}$$

$$f^*(v'_iv_{i+1}) = \begin{cases} 5, & i = 1, \\ 2, & i = 2, \\ 3i, & 3 \leq i \leq n-1 \text{ and } i \text{ is odd}, \\ 3i-2, & 3 \leq i \leq n-1 \text{ and } i \text{ is even}. \end{cases}$$

Case 2. n is even.

Define $f : V(S'(P_n)) \rightarrow \{1, 2, 3, \cdots, 3n-2\}$ as follows:

$$f(v_i) = \begin{cases} 4-i, & 1 \leq i \leq 2, \\ 3i-1, & 3 \leq i \leq n \text{ and } i \text{ is odd}, \\ 3i-3, & 3 \leq i \leq n \text{ and } i \text{ is even}, \end{cases}$$

$$f(v_i') = \begin{cases} 1, & i = 1, \\ 3i-3, & 2 \leq i \leq n \text{ and } i \text{ is odd}, \\ 3i-2, & 2 \leq i \leq n \text{ and } i \text{ is even}. \end{cases}$$

Then the induced edge labeling f^* is obtained as follows:

$$f^*(v_i v_{i+1}) = 3i-1, \text{ for } 1 \leq i \leq n-1,$$

$$f^*(v_i v_{i+1}') = \begin{cases} 3i, & 3 \leq i \leq n-1 \text{ and } i \text{ is odd}, \\ 3i-2, & 3 \leq i \leq n-1 \text{ and } i \text{ is even}, \end{cases}$$

$$f(v_i' v_{i+1}) = \begin{cases} 3i-2, & 1 \leq i \leq n-1 \text{ and } i \text{ is odd}, \\ 3i, & 1 \leq i \leq n-1 \text{ and } i \text{ is even}. \end{cases}$$

Hence f is an F-centroidal mean labeling of $S'(P_n)$. Thus the splitting graph $S'(P_n)$ is an F-centroidal mean graph for $n \geq 2$. □

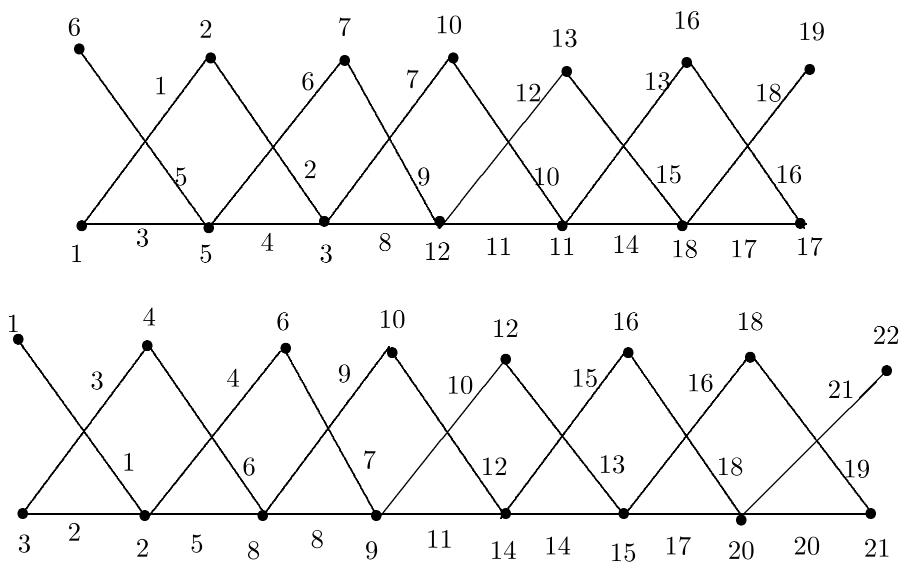

Figure 11 An F-centroidal mean labeling of $S'(P_7)$ and $S'(P_8)$

Theorem 2.12 *The graph $P(1, 2, \cdots, n-1)$ is an F-centroidal mean graph for $n \geq 2$.*

Proof Let $v_1, v_2, \cdots, v_n$ be the vertices of the path P_n and let u_{ij} be the vertices of the partition of K_{2,m_i} with cardinality m_i, $1 \leq i \leq n-1$ and $1 \leq j \leq m_i$. Define $f : V(P(1, 2, \cdots, n-1)) \to \{1, 2, 3, \cdots, n(n-1)+1\}$ as follows:

$$f(v_i) = i(i-1)+1, \text{ for } 1 \leq i \leq n,$$
$$f(u_{ij}) = i(i-1)+2j, \text{ for } 1 \leq j \leq i, \text{ and } 1 \leq j \leq n-1.$$

Then the induced edge labeling f^* is obtained as follows:

$$f^*(v_i u_{ij}) = i(i-1) + j, \text{ for } 1 \leq j \leq i \text{ and } 1 \leq i \leq n-1,$$
$$f(u_{ij} v_{i+1}) = i^2 + j, \text{ for } 1 \leq j \leq i \text{ and } 1 \leq i \leq n-1.$$

Hence f is an F-centroidal mean labeling of $P(1,2,\ldots,n-1)$. Thus the graph $P(1,2,\ldots,n-1)$ is an F-centroidal mean graph for $n \geq 2$. □

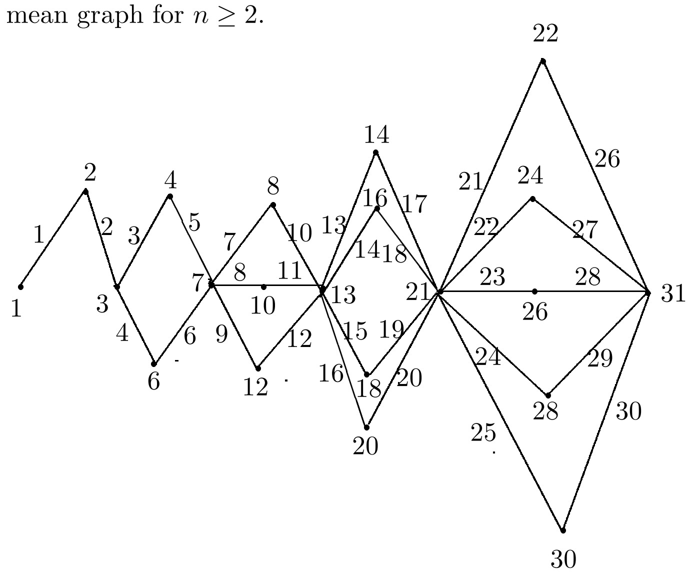

Figure 12 An F-centroidal mean labeling of $P(1,2,3,4,5)$

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