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2-Pseudo Neighbourly Irregular Intuitionistic Fuzzy Graphs

S.Ravi Narayanan

Department of Mathematics

Karpaga Vinayaga College of Engineering and Technology, Chennai, Tamil Nadu, India

S.Murugesan

PG and Research Department of Mathematics, Sri S.R.N.M. College, Sattur- 626 203, Tamil Nadu, India

E-mail: ravimathsdept@gmail.com, satturmuruges@gmail.com

Abstract: In this paper, 2-pseudo neighbourly irregular intuitionistic fuzzy graph, 2-pseudo neighbourly totally irregular intuitionistic fuzzy graph are introduced and compared through various examples. A necessary and sufficient condition under which they are equivalent is provided. 2- pseudo neighbourly irregularity on some intuitionistic fuzzy graphs whose underlying crisp graphs are a cycle C_n, a Bi-star graph $B_{n,m}$, Sub($B_{n,m}$), and a path P_n are studied.

Key Words: Degree of a vertex in an intuitionistic fuzzy graph, d_2-degree of a vertex in an intuitionistic fuzzy graph, total d_2-degree, pseudo degree, pseudo total degree,neighbourly irregular intuitionistic fuzzy graph.

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§1. Introduction

The first definition of fuzzy graph was introduced by Kaufmann [9] in 1975, based on Zadeh's fuzzy relations in 1965 ([17]). Atanassov [4] introduced the concept of intuitionistic fuzzy (IF) relations and Intuitionistic Fuzzy Graphs (IFGs). Parvathi and Karunambigai [12] introduced the concept of IFG elaborately and analyzed its components. S. Ravi Narayanan and S. Murugesan [13] introduced Pseudo Regular Intuitionistic Fuzzy Graphs. A. Nagoor Gani, R. Jahir Hussain and S. Yahya Mohamed [11] introduced Neighbourly Irregular Intuitionistic Fuzzy Graphs. Articles [4, 11, 12, 13] motivated us to introduce 2- pseudo neighbourly irregular intuitionistic fuzzy graph, 2- pseudo neighbourly totally irregular intuitionistic fuzzy graph and analyze some of its properties.

In Section 2, we review some basic concepts and definitions. Section 3 deals with 2-pseudo neighbourly irregular intuitionistic fuzzy graphs and 2-pseudo neighbourly totally irregular intuitionistic fuzzy graphs. Comparative study between them is made and necessary and sufficient condition is provided. Section 4 deals with 2-pseudo neighbourly irregularity on cycle with some

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specific membership function. Section 5 deals with 2-pseudo neighbourly irregularity on bi-star graph $B_{n,m}$ with some specific membership function. Section 6 deals with 2-pseudo neighbourly irregularity on subdivision of bi-star graph with some specific membership function. Section 7 deals with 2-pseudo neighbourly irregularity on a path with some specific membership function.

Throughout this paper, the vertices takes the membership value $A = (\mu_1, \gamma_1)$ and the edges takes the membership values $B = (\mu_2, \gamma_2)$.

§2. Preliminaries

We present some known definitions related to fuzzy graphs and intuitionistic fuzzy graphs for ready reference to go through the work presented in this paper.

Definition 2.1([6]) *A fuzzy graph $G : (\sigma, \mu)$ is a pair of functions (σ, μ), where $\sigma : V \to$[0,1] is a fuzzy subset of a non empty set V and $\mu : V \times V \to$[0, 1] is a symmetric fuzzy relation on σ such that for all u, v in V, the relation $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ is satisfied. A fuzzy graph G is called complete fuzzy graph if the relation $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ is satisfied.*

Definition 2.2([3]) *An intuitionistic fuzzy graph with underlying set V is defined to be a pair $G = (V, E)$ where*

(1) *$V = \{v_1, v_2, v_3, \cdots, v_n\}$ such that $\mu_1 : V \to [0, 1]$ and $\gamma_1 : V \to [0, 1]$ denote the degree of membership and non-membership of the element $v_i \in V$, $i = 1, 2, 3, \cdots, n$, such that $0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1$;*

(2) *$E \subseteq V \times V$, where $\mu_2 : V \times V \to [0, 1]$ and $\gamma_2 : V \times V \to [0, 1]$ are such that $\mu_2(v_i, v_j) \leq \min\{\mu_1(v_i), \mu_1(v_j)\}$ and $\gamma_2(v_i, v_j) \leq \max\{\gamma_1(v_i), \gamma_1(v_j)\}$ and $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1$ for every $(v_i, v_j) \in E$, $i, j = 1, 2, \cdots, n$.*

Definition 2.3([8]) *If $v_i, v_j \in V \subseteq G$, the μ-strength of connectedness between two vertices v_i and v_j is defined as $\mu_2^{\infty}(v_i, v_j) = sup\{\mu_2^k(v_i, v_j) : k = 1, 2, \cdots, n\}$ and γ-strength of connectedness between two vertices v_i and v_j is defined as $\gamma_2^{\infty}(v_i, v_j) = inf\{\gamma_2^k(v_i, v_j) : k = 1, 2, \cdots, n\}$.*

If u and v are connected by means of paths of length k then $\mu_2^k(u, v)$ is defined as sup $\{\mu_2(u, v_1) \wedge \mu_2(v_1, v_2) \wedge \cdots \wedge \mu_2(v_{k-1}, v) : (u, v_1, v_2, \cdots, v_{k-1}, v) \in V\}$ and $\gamma_2^k(u, v)$ is defined as $inf\{\gamma_2(u, v_1) \wedge \gamma_2(v_1, v_2) \wedge \cdots \wedge \gamma_2(v_{k-1}, v) : (u, v_1, v_2, \cdots, v_{k-1}, v) \in V\}$.

Definition 2.4([8]) *Let $G : (A, B)$ be an intuitionistic fuzzy graph on $G^*(V, E)$. Then the degree of a vertex $v_i \in G$ is defined by $d(v_i) = (d_{\mu_1}(v_i), d_{\gamma_1}(v_i))$, where $d_{\mu_1}(v_i) = \sum \mu_2(v_i, v_j)$ and $d_{\gamma_1}(v_i) = \sum \gamma_2(v_i, v_j)$, for $(v_i, v_j) \in E$ and $\mu_2(v_i, v_j) = 0$ and $\gamma_2(v_i, v_j) = 0$ for $(v_i, v_j) \notin E$.*

Definition 2.5([8]) *Let $G : (A, B)$ be an intuitionistic fuzzy graph on $G^*(V, E)$. Then the total degree of a vertex $v_i \in G$ is defined by $td(v_i) = (td_{\mu_1}(v_i), td_{\gamma_1}(v_i))$, where $td_{\mu_1}(v_i) = d\mu_1(v_i) + \mu_1(v_i)$ and $td_{\gamma_1}(v_i) = d\gamma_1(v_i) + \gamma_1(v_i)$.*

Definition 2.6([13]) *Let $G : (A, B)$ be an intuitionistic fuzzy graph. The membership pseudo degree of a vertex $u \in G$ is defined as $d_{(a)}\mu_1(u) = \frac{t_\mu}{d_i}$ where t_μ is the sum of membership degrees of vertices incident with vertex u. The non-membership pseudo degree of a vertex $u \in G$ is* *defined as* $d_{(a)}\gamma_1(u) = \frac{t_\gamma}{d_i}$ *where* t_γ *is the sum of non-membership degrees of vertices incident with vertex* u *and* d_i *is the total number of edges incident with the vertex* u*. The pseudo degree of a vertex* $u \in G$ *is defined as* $d_{(a)}(u) = (d_{(a)}\mu_1(u), d_{(a)}\gamma_1(u))$*.*

Definition 2.7([13]) *Let* $G : (A, B)$ *be an intuitionistic fuzzy graph. The pseudo total degree of a vertex* $u \in G$ *is defined as* $td_{(a)}(u) = (td_{(a)}\mu_1(u), td_{(a)}\gamma_1(u))$ *where* $td_{(a)}\mu_1(u) = d_{(a)}\mu_1(u) + \mu_1(u)$ *and* $td_{(a)}\gamma_1(u) = d_{(a)}\gamma_1(u) + \gamma_1(u)$*. It can also be defined as* $td_{(a)}(u) = d_{(a)}(u) + A(u)$*.*

Definition 2.8([13]) *Let* $G : (A, B)$ *be an intuitionistic fuzzy graph. The membership* d_2 *-pseudo degree of a vertex* $u \in G$ *is defined as* $d_{(a)(2)}\mu_1(u) = \frac{\sum d_{(2)\mu_1}(u)}{d_i}$*. The non-membership* d_2*-pseudo degree of a vertex* $u \in G$ *is defined as* $d_{(a)(2)}\gamma_1(u) = \frac{\sum d_{(2)\gamma_1}(u)}{d_i}$ *where* d_i *is the number of edges incident with the vertex* u*. The* d_2 *- pseudo degree of a vertex* u *is defined as* $d_{(a)(2)}(u) = (d_{(a)(2)}\mu_1(u), d_{(a)(2)}\gamma_1(u))$*.*

Definition 2.9([13]) *Let* $G : (A, B)$ *be an intuitionistic fuzzy graph. Then the* d_2*-pseudo total degree of a vertex* $u \in V$ *is defined as* $td_{(a)(2)}(u) = (td_{(a)(2)}\mu_1(u), td_{(a)(2)}\gamma_1(u))$*, where* $td_{(a)(2)}\mu_1(u) = d_{(a)(2)}\mu_1(u) + \mu_1(u)$ *and* $td_{(a)(2)}\gamma_1(u) = d_{(a)(2)}\gamma_1(u) + \gamma_1(u)$*. Also it can be defined as* $td_{(a)(2)}(u) = d_{(a)(2)}(u) + A(u)$ *where* $A(u) = (\mu_1(u), \gamma_1(u))$*.*

Definition 2.10([11]) *Let* $G : (A, B)$ *be an intuitionistic fuzzy graph. Then* G *is said to be neighbourly irregular intuitionistic fuzzy graph if every two adjacent vertices have distinct degrees.*

Definition 2.11([14]) *Let* $G : (A, B)$ *be an intuitionistic fuzzy graph. If* $d_{(a)}(v) = (r_1, r_2)$ *and* $d_{(a)(2)}(v) = (c_1, c_2)$*, then* G *is said to be* $((r_1, r_2), 2, (c_1, c_2))$*- pseudo regular intuitionistic fuzzy graph.*

§3. 2-Pseudo Neighbourly Irregular Intuitionistic Fuzzy Graphs

In this section, 2-pseudo neighbourly irregular and 2-pseudo neighbourly totally irregular intuitionistic fuzzy graphs are defined. A necessary and sufficient condition under which they are equivalent is provided.

Definition 3.1 *Let* $G : (A, B)$ *be a connected intuitionistic fuzzy graph. Then* G *is said to be 2-pseudo neighbourly irregular intuitionistic fuzzy graph if every two adjacent vertices of* G *have distinct* d_2*-pseudo degrees.*

Example 3.2 Consider an intuitionistic fuzzy graph on $G^* : (V, E)$.

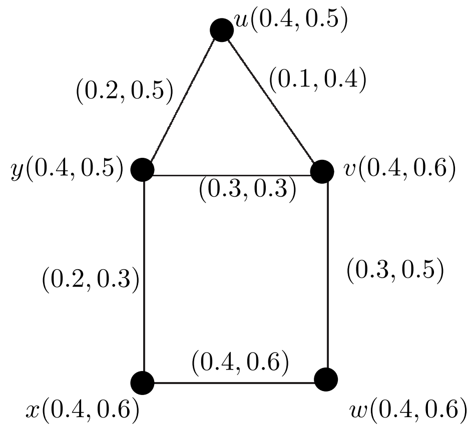

Figure 1

Here, $d_{(a)(2)}(u) = (0.3, 0.55)$, $d_{(a)(2)}(v) = (0.33, 0.83)$, $d_{(a)(2)}(w) = (0.4, 0.8)$, $d_{(a)(2)}(x) = (0.35, 0.75)$ and $d_{(a)(2)}(y) = (0.37, 0.87)$.

So, every two adjacent vertices have distinct d_2-pseudo degrees. Hence G is 2-pseudo neighbourly irregular intuitionistic fuzzy graph.

Definition 3.3 *If every two adjacent vertices of an intuitionistic fuzzy graph $G : (A, B)$ have distinct d_2 -pseudo total degrees, then G is said to be 2-pseudo neighbourly totally irregular intuitionistic fuzzy graph.*

Example 3.4 Consider an intuitionistic fuzzy graph on $G^* : (V, E)$.

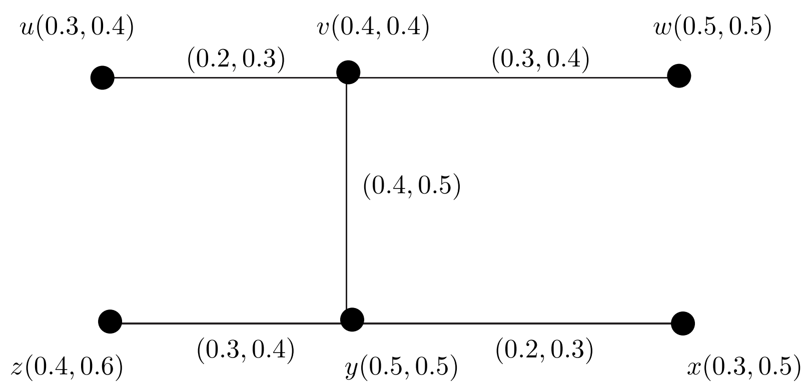

Figure 2

Here, $td_{(a)(2)}(u) = (0.8, 1.4), td_{(a)(2)}(v) = (0.87, 1.13), td_{(a)(2)}(w) = (1, 1.5)$, $td_{(a)(2)}(x) = (0.8, 1.5), td_{(a)(2)}(y) = (0.97, 1.43)$ and $td_{(a)(2)}(z) = (0.9, 1.6)$.

So, every two adjacent vertices have distinct d_2-pseudo total degrees. Hence G is 2-pseudo neighbourly totally irregular intuitionistic fuzzy graph.

Remark 3.5 A 2-pseudo neighbourly irregular intuitionistic fuzzy graph need not be a 2-pseudo neighbourly totally irregular intuitionistic fuzzy graph.

Remark 3.6 A 2-pseudo neighbourly totally irregular intuitionistic fuzzy graph need not be a 2-pseudo neighbourly irregular intuitionistic fuzzy graph.

Proposition 3.7 *If the membership value of the adjacent vertices are distinct, then* $((r_1, r_2), 2, (c_1, c_2))$*-pseudo regular intuitionistic fuzzy graph is* 2*-pseudo neighbourly totally irregular intuitionistic fuzzy graph.*

Proof The proof is obvious. □

Theorem 3.8 *Let* $G : (A, B)$ *be an intuitionistic fuzzy graph on* $G^* : (V, E)$*. If* G *is a* 2*-pseudo neighbourly irregular intuitionistic fuzzy graph and* A *is a constant function, then* G *is a* 2*-pseudo neighbourly totally irregular intuitionistic fuzzy graph.*

Proof Let $G : (A, B)$ be a 2-pseudo neighbourly irregular intuitionistic fuzzy graph. Then the d_2- pseudo degree of every two adjacent vertices are distinct. Let u and v be two adjacent vertices with distinct d_2 -pseudo degrees. This implies that $d_{(a)(2)}(u) = (k_1, k_2)$ and $d_{(a)(2)}(v) = (k_3, k_4)$, where $k_1 \neq k_3$, $k_2 \neq k_4$ and $A(u) = A(v) = (c_1, c_2)$, a constant where $c_1, c_2 \in [0, 1]$. Suppose $td_{(a)(2)}(u) = td_{(a)(2)}(v) \Rightarrow d_{(a)(2)}(u) + A(u) = d_{(a)(2)}(v) + A(v) \Rightarrow (k_1, k_2) + (c_1, c_2) = (k_3, k_4) + (c_1, c_2) \Rightarrow (k_1, k_2) = (k_3, k_4)$, which is a contradiction. So, $td_{(a)(2)}(u) \neq td_{(a)(2)}(v)$. Hence any two adjacent vertices u and v with distinct d_2- pseudo degrees have their d_2- pseudo total degrees distinct, provided A is a constant function. This is true for every pair of adjacent vertices in G. Hence G is 2-pseudo neighbourly totally irregular intuitionistic fuzzy graph. □

Theorem 3.9 *Let* $G : (A, B)$ *be an intuitionistic fuzzy graph on* $G^* : (V, E)$*. If* G *is a* 2*-pseudo neighbourly totally irregular intuitionistic fuzzy graph and* A *is a constant function, then* G *is a* 2*-pseudo neighbourly irregular intuitionistic fuzzy graph.*

Proof Let $G : (A, B)$ be a 2-pseudo neighbourly totally irregular intuitionistic fuzzy graph. Then the d_2-pseudo total degree of every two adjacent vertices are distinct. Let u and v be two adjacent vertices with d_2 -pseudo degrees (k_1, k_2) and (k_3, k_4). Then $d_{(a)(2)}(u) = (k_1, k_2)$ and $d_{(a)(2)}(v) = (k_3, k_4)$. Given that $A(u) = A(v) = (c_1, c_2)$, a constant where $c_1, c_2 \in [0, 1]$ and $td_{(a)(2)}(u) \neq td_{(a)(2)}(v)$. Since, $td_{(a)(2)}(u) \neq td_{(a)(2)}(v) \Rightarrow d_{(a)(2)}(u) + A(u) \neq d_{(a)(2)}(v) + A(v)$ $\Rightarrow (k_1, k_2) + (c_1, c_2) \neq (k_3, k_4) + (c_1, c_2) \Rightarrow (k_1, k_2) \neq (k_3, k_4) \Rightarrow d_{(a)(2)}(u) \neq d_{(a)(2)}(v)$. Hence any two adjacent vertices u and v with distinct d_2- pseudo total degrees have their d_2- pseudo degrees distinct, provided A is a constant function. This is true for every pair of adjacent vertices in G. Hence G is 2-pseudo neighbourly irregular intuitionistic fuzzy graph. □

Remark 3.10 Let $G : (A, B)$ be an intuitionistic fuzzy graph on $G^* : (V, E)$. Theorems 3.8 and 3.9 jointly yield the following result. If A is a constant function, then G is a 2-pseudo neighbourly totally irregular intuitionistic fuzzy graph if and only if G is a 2-pseudo neighbourly irregular intuitionistic fuzzy graph.

Remark 3.11 Let $G : (A, B)$ be an intuitionistic fuzzy graph on $G^* : (V, E)$. If G is both 2-pseudo neighbourly irregular intuitionistic fuzzy graph and G is a 2-pseudo neighbourly totally irregular intuitionistic fuzzy graph. Then A need not be a constant function.

§4. 2-Pseudo Neighbourly Irregular Intuitionistic Fuzzy Graph on a Cycle with Some Specific Membership Functions

In this section, Theorems 4.1 and 4.4 provide 2-pseudo neighbourly irregularity on intuitionistic fuzzy graph $G:(A,B)$ on a cycle $G^*:(V,E)$.

Theorem 4.1 *Let $G:(A,B)$ be an intuitionistic fuzzy graph on a cycle $G^*:(V,E)$ of length n. If the values of the edges $e_1,e_2,e_3,\cdots,e_n$ are respectively $(c_1,k_1),(c_2,k_2),(c_3,k_3),\cdots,(c_n,k_n)$ such that $c_i<c_{i+1}$ and $k_i>k_{i+1}$, for $i=1,2,\cdots,n-1$, then G is a 2-pseudo neighbourly irregular intuitionistic fuzzy graph.*

Proof Let $G:(A,B)$ be an intuitionistic fuzzy graph on a cycle $G^*:(V,E)$ of length n. Let $e_1,e_2,e_3,\cdots,e_n$ be the edges of the cycle of G^* in that order. Let the values of the edges $e_1,e_2,e_3,\cdots,e_n$ be $(c_1,k_1),(c_2,k_2),(c_3,k_3),\cdots,(c_n,k_n)$ such that $c_i<c_{i+1}$ and $k_i>k_{i+1}$ for $i=1,2,\cdots,n-1$

$$\begin{aligned} d_{(2)}\mu_1(v_1) &= \{\mu_2(e_1)\wedge\mu_2(e_2)\}+\{\mu_2(e_n)\wedge\mu_2(e_{n-1})\}\\ &= \{c_1\wedge c_2\}+\{c_n\wedge c_{n-1}\}\\ &= c_1+c_{n-1}. \end{aligned}$$

$$\begin{aligned} d_{(2)}\mu_1(v_2) &= \{\mu_2(e_1)\wedge\mu_2(e_n)\}+\{\mu_2(e_2)\wedge\mu_2(e_3)\}\\ &= \{c_1\wedge c_n\}+\{c_2\wedge c_3\}\\ &= c_1+c_2. \end{aligned}$$

For $i=3,4,5,\cdots,n-1$,

$$\begin{aligned} d_{(2)}\mu_1(v_i) &= \{\mu_2(e_{i-1})\wedge\mu_2(e_{i-2})\}+\{\mu_2(e_{i+1})\wedge\mu_2(e_i)\}\\ &= \{c_{i-1}\wedge c_{i-2}\}+\{c_i\wedge c_{i+1}\}\\ &= c_{i-2}+c_i. \end{aligned}$$

$$\begin{aligned} d_{(2)}\mu_1(v_n) &= \{\mu_2(e_1)\wedge\mu_2(e_n)\}+\{\mu_2(e_{n-1})\wedge\mu_2(e_{n-2})\}\\ &= \{c_1\wedge c_n\}+\{c_{n-1}\wedge c_{n-2}\}\\ &= c_1+c_{n-2}. \end{aligned}$$

$$\begin{aligned} d_{(2)}\gamma_1(v_1) &= \{\gamma_2(e_1)\vee\gamma_2(e_2)\}+\{\gamma_2(e_n)\vee\gamma_2(e_{n-1})\}\\ &= \{k_1\vee k_2\}+\{k_n\vee k_{n-1}\}\\ &= k_1+k_{n-1}. \end{aligned}$$

$$\begin{aligned} d_{(2)}\gamma_1(v_2) &= \{\gamma_2(e_1)\vee\gamma_2(e_n)\}+\{\gamma_2(e_2)\vee\gamma_2(e_3)\}\\ &= \{k_1\vee k_n\}+\{k_2\vee k_3\}\\ &= k_1+k_2. \end{aligned}$$

For $i = 3, 4, 5, \cdots, n-1$,

$$\begin{aligned} d_{(2)}\gamma_1(v_i) &= \{\gamma_2(e_{i-1}) \vee \gamma_2(e_{i-2})\} + \{\gamma_2(e_{i+1}) \vee \gamma_2(e_i)\} \\ &= \{k_{i-1} \vee k_{i-2}\} + \{k_i \vee k_{i+1}\} \\ &= k_{i-2} + k_i. \end{aligned}$$

$$\begin{aligned} d_{(2)}\gamma_1(v_n) &= \{\gamma_2(e_1) \vee \gamma_2(e_n)\} + \{\gamma_2(e_{n-1}) \vee \gamma_2(e_{n-2})\} \\ &= \{k_1 \wedge k_n\} + \{k_{n-1} \wedge k_{n-2}\} \\ &= k_1 + k_{n-2}. \end{aligned}$$

Every two adjacent vertices have distinct d_2-pseudo degrees. Hence G is a 2- pseudo neighbourly irregular intuitionistic fuzzy graph. □

Remark 4.2 Even if the values of the edges $e_1, e_2, e_3, \ldots, e_n$ are respectively $(c_1, k_1), (c_2, k_2)$, $(c_3, k_3), \cdots, (c_n, k_n)$ such that $c_i < c_{i+1}$ and $k_i > k_{i+1}$ for $i = 1, 2, \cdots, n-1$ then G need not be 2- pseudo neighbourly totally irregular intuitionistic fuzzy graph.

Theorem 4.3 *Let $G : (A, B)$ be an intuitionistic fuzzy graph on a cycle $G^* : (V, E)$ of length n. If the values of the edges $e_1, e_2, e_3, \cdots, e_n$ are respectively $(c_1, k_1), (c_2, k_2), (c_3, k_3), \cdots, (c_n, k_n)$ such that $c_i > c_{i+1}$ and $k_i < k_{i+1}$, for $i = 1, 2, \cdots, n-1$, then G is a 2-pseudo neighbourly irregular intuitionistic fuzzy graph.*

Proof Let $G : (A, B)$ be an intuitionistic fuzzy graph on $G^* : (V, E)$ of length n. Let $e_1, e_2, e_3, \cdots, e_n$ be the edges of the cycle G^* in that order. Let the values of the edges $e_1, e_2, e_3, \cdots, e_n$ be respectively $(c_1, k_1)(c_2, k_2), (c_3, k_3), \cdots, (c_n, k_n)$ such that $c_i > c_{i+1}$ and $k_i < k_{i+1}$ for $i = 1, 2, \cdots, n-1$,

$$\begin{aligned} d_{(2)}\mu_1(v_1) &= \{\mu_2(e_1) \wedge \mu_2(e_2)\} + \{\mu_2(e_n) \wedge \mu_2(e_{n-1})\} \\ &= \{c_1 \wedge c_2\} + \{c_n \wedge c_{n-1}\} \\ &= c_2 + c_n. \\ d_{(2)}\mu_1(v_2) &= \{\mu_2(e_1) \wedge \mu_2(e_n)\} + \{\mu_2(e_2) \wedge \mu_2(e_3)\} \\ &= \{c_1 \wedge c_n\} + \{c_2 \wedge c_3\} \\ &= c_n + c_3. \end{aligned}$$

For $(3 \leq i \leq n-1)$,

$$\begin{aligned} d_{(2)}\mu_1(v_i) &= \{\mu_2(e_{i-1}) \wedge \mu_2(e_{i-2})\} + \{\mu_2(e_{i+1}) \wedge \mu_2(e_i)\} \\ &= \{c_{i-1} \wedge c_{i-2}\} + \{c_i \wedge c_{i+1}\} \\ &= c_{i-1} + c_{i+1}. \end{aligned}$$

$$\begin{aligned} d_{(2)}\mu_1(v_n) &= \{\mu_2(e_1) \wedge \mu_2(e_n)\} + \{\mu_2(e_{n-1}) \wedge \mu_2(e_{n-2})\} \\ &= \{c_1 \wedge c_n\} + \{c_{n-1} \wedge c_{n-2}\} \\ &= c_n + c_{n-1}. \end{aligned}$$

Now

$$\begin{aligned} d_{(2)}\gamma_1(v_1) &= \{\gamma_2(e_1) \vee \gamma_2(e_2)\} + \{\gamma_2(e_n) \vee \gamma_2(e_{n-1})\} \\ &= \{k_1 \vee k_2\} + \{k_n \vee k_{n-1}\} \\ &= k_2 + k_n. \\ d_{(2)}\gamma_1(v_2) &= \{\gamma_2(e_1) \vee \gamma_2(e_n)\} + \{\gamma_2(e_2) \vee \gamma_2(e_3)\} \\ &= \{k_1 \vee k_n\} + \{k_2 \vee k_3\} \\ &= k_n + k_3. \end{aligned}$$

For $3 \leq i \leq n-1$,

$$\begin{aligned} d_{(2)}\gamma_1(v_i) &= \{\gamma_2(e_{i-1}) \vee \gamma_2(e_{i-2})\} + \{\gamma_2(e_{i+1}) \vee \gamma_2(e_i)\} \\ &= \{k_{i-1} \vee k_{i-2}\} + \{k_i \vee k_{i+1}\} \\ &= k_{i-1} + k_{i+1}. \\ d_{(2)}\gamma_1(v_n) &= \{\gamma_2(e_1) \vee \gamma_2(e_n)\} + \{\gamma_2(e_{n-1}) \vee \gamma_2(e_{n-2})\} \\ &= \{k_1 \vee k_n\} + \{k_{n-1} \vee k_{n-2}\} \\ &= k_n + k_{n-1}. \end{aligned}$$

Here, Every two adjacent vertices have distinct d_2- pseudo degrees. Hence G is 2-pseudo neighbourly irregular intuitionistic fuzzy graph. □

Remark 4.4 Even if the values of the edges $e_1, e_2, e_3, \cdots, e_n$ are respectively $(c_1, k_1), (c_2, k_2), (c_3, k_3), \cdots, (c_n, k_n)$ such that $c_i > c_{i+1}$ and $k_i < k_{i+1}$, for $i = 1, 2, \cdots, n-1$, then then G need not be 2-pseudo neighbourly totally irregular intuitionistic fuzzy graph.

Remark 4.5 Let $G : (A, B)$ be an intuitionistic fuzzy graph on a cycle $G^* : (V, E)$ of length n. If the values of the edges $e_1, e_2, e_3, \cdots, e_n$ are respectively $(c_1, k_1), (c_2, k_2), (c_3, k_3), \cdots, (c_n, k_n)$ are all distinct, then G need not be 2-pseudo neighbourly irregular intuitionistic fuzzy graph.

§5. 2-Pseudo Neighbourly Irregular Intuitionistic Fuzzy Graph on a Bi-star $B_{n,m}(m \neq n)$ with Specific Membership Functions

In this section, Theorems 5.1 and 5.6 provide 2-pseudo neighbourly irregularity on intuitionistic fuzzy graph $G : (A, B)$ on $G^* : (V, E)$ which is a Bistar $B_{n,m}(m \neq n)$.

Theorem 5.1 *Let $G : (A, B)$ be an intuitionistic fuzzy graph on $G^* : (V, E)$ which is a Bi-star $B_{n,m}(m \neq n)$. If B is a constant function, then G is 2-pseudo neighbourly irregular intuitionistic* *fuzzy graph.*

Proof Let $v_1, v_2, v_3, \cdots, v_n$ be the vertices adjacent to the vertex x and $u_1, u_2, u_3, \cdots, u_m$ be the vertices adjacent to the vertex y and xy is the middle edge of K_2. Since B is a constant function, then $B(uv) = (c_1, c_2)$, a constant for all $uv \in E$. So, $d_{(2)}(v_i) = n(c_1, c_2)$, $(1 \leq i \leq n-1)$, $d_{(2)}(x) = m(c_1, c_2)$, $d_{(2)}(y) = n(c_1, c_2)$ and $d_{(2)}(u_i) = m(c_1, c_2)$, $(1 \leq i \leq m)$. Then, $d_{(a)(2)}(v_i) = m(c_1, c_2)$, $(1 \leq i \leq n-1)$, $d_{(a)(2)}(x) = n(c_1, c_2)$, $d_{(a)(2)}(y) = m(c_1, c_2)$ and $d_{(a)(2)}(u_i) = n(c_1, c_2)$, $(1 \leq i \leq m)$. Hence $d_{(a)(2)}(v_i) \neq d_{(a)(2)}(x)$, $(1 \leq i \leq n)$ and $d_{(a)(2)}(x) \neq d_{(a)(2)}(y)$ and $d_{(a)(2)}(u_i) \neq d_{(a)(2)}(y)$, $(1 \leq i \leq m)$. Hence G is 2-pseudo neighbourly irregular intuitionistic fuzzy graph. □

Remark 5.2 Even if B is a constant function, then G need not be 2-pseudo neighbourly totally irregular intuitionistic fuzzy graph.

Remark 5.3 Converse of Theorem 5.1 need not be true.

Theorem 5.4 *Let $G : (A, B)$ be an intuitionistic fuzzy graph on $G^* : (V, E)$ which is a Bi-star $B_{n,m}(m \neq n)$. If the pendant edges have the same membership values less than or equal to membership value of the middle edge and same non-membership values greater than or equal to non-membership value of the middle edge, then G is a 2-pseudo neighbourly irregular intuitionistic fuzzy graph.*

Proof Let $v_1, v_2, v_3, \cdots, v_n$ be the vertices adjacent to the vertex x and $u_1, u_2, u_3, \cdots, u_m$ be the vertices adjacent to the vertex y and xy is the middle edge of K_2. If the pendant edges have the same membership value then

$$\mu_2(e_i) = \begin{cases} c_1, & \text{if } e_i \text{ is an pendant edge.} \\ c_2, & \text{if } e_i \text{ is an middle edge.} \end{cases} \quad \gamma_2(e_i) = \begin{cases} k_1, & \text{if } e_i \text{ is an pendant edge.} \\ k_2, & \text{if } e_i \text{ is an middle edge.} \end{cases}$$

If $c_1 = c_2$ and $k_1 = k_2$ then B is a constant function. By Theorem 5.1, G is a 2-pseudo neighbourly irregular intuitionistic fuzzy graph.

If $c_1 < c_2$, and $k_1 > k_2$, then $d_{(2)}(v_i) = n(c_1, k_1)$, $(1 \leq i \leq n)$, $d_{(2)}(x) = m(c_1, k_1)$, $d_{(2)}(y) = n(c_1, k_1)$, and $d_{(2)}(u_i) = m(c_1, k_1)$, $(1 \leq i \leq m)$.

Also, $d_{(a)(2)}(v_i) = m(c_1, k_1)$, $(1 \leq i \leq n)$, $d_{(a)(2)}(x) = n(c_1, k_1)$, $d_{(a)(2)}(y) = m(c_1, k_1)$, and $d_{(a)(2)}(u_i) = n(c_1, k_1)$, $(1 \leq i \leq m)$.

Hence $d_{(a)(2)}(v_i) \neq d_{(a)(2)}(x)$,$(1 \leq i \leq n)$, $d_{(a)(2)}(x) \neq d_{(a)(2)}(y)$, $d_{(a)(2)}(u_i) \neq d_{(a)(2)}(y)$, $(1 \leq i \leq m)$ and G is a 2-pseudo neighbourly irregular intuitionistic fuzzy graph. □

Remark 5.5 Even if the pendant edges have the same membership values less than or equal to membership value of the middle edge and same non-membership values greater than or equal to membership value of the middle edge, then G need not be 2-pseudo neighbourly totally irregular intuitionistic fuzzy graph.

§6. 2-Pseudo Neighbourly Irregular Intuitionistic Fuzzy Graph on Sub$(B_{n,m})$ with Specific Membership Functions

In this section, Theorem 6.1 provides a condition for 2-pseudo neighbourly irregularity on intuitionistic fuzzy graph $G:(A,B)$ on $G^*:(V,E)$, Sub$(B_{n,m}), n, m \geq 3$.

Theorem 6.1 *Let $G:(A,B)$ be an intuitionistic fuzzy graph on $G^*:(V,E)$ which is a Sub$(B_{n,m}), n, m \geq 3$. If B is a constant function, then G is 2-pseudo neighbourly irregular intuitionistic fuzzy graph.*

Proof Let $v_1, v_2, v_3, \cdots, v_n$ be the vertices adjacent to the vertex x and $u_1, u_2, u_3, \cdots, u_m$ be the vertices adjacent to the vertex y and xy is the middle edge of K_2. Subdivide each edge of $B_{n,m}$.

Then the additional edges are xw_i, w_iv_i $(1 \leq i \leq n)$ and yt_i, t_iu_i $(1 \leq i \leq n)$ and two more edges xs, sy.

If B is a constant function say $B(uv) = (c_1, c_2)$, for $uv \in E$.

Case 1. If $n \neq m$, then $d_{(2)}(v_i) = (c_1, c_2)$, $(1 \leq i \leq n)$, $d_{(2)}(w_i) = n(c_1, c_2)$, $(1 \leq i \leq n)$, $d_{(2)}(x) = (n+1)(c_1, c_2), d_{(2)}(s) = (m+n)(c_1, c_2), d_{(2)}(y) = (m+1)(c_1, c_2), d_{(2)}(t_i) = m(c_1, c_2)$, $(1 \leq i \leq m)$, and $d_{(2)}(u_i) = (c_1, c_2)$, $(1 \leq i \leq m)$.

Hence we have, $d_{(a)(2)}(v_i) \neq d_{(a)(2)}(w_i)$, $(1 \leq i \leq n)$ and $d_{(a)(2)}(w_i) \neq d_{(a)(2)}(x)$, $(1 \leq i \leq n)$, $d_{(a)(2)}(x) \neq d_{(a)(2)}(s)$, $d_{(a)(2)}(s) \neq d_{(a)(2)}(y)$, $d_{(a)(2)}(t_i) \neq d_{(a)(2)}(y)$, $(1 \leq i \leq m)$, and $d_{(a)(2)}(t_i) \neq d_{(a)(2)}(u_i)$, $(1 \leq i \leq m)$.

Hence G is a 2-pseudo neighbourly irregular intuitionistic fuzzy graph.

Case 2. If $n = m$, then $d_{(2)}(v_i) = (c_1, c_2)$, $(1 \leq i \leq n)$, $d_{(2)}(w_i) = n(c_1, c_2)$, $(1 \leq i \leq n)$, $d_{(2)}(x) = (n+1)(c_1, c_2)$, $d_{(2)}(s) = (2n)(c_1, c_2), d_{(2)}(y) = (n+1)(c_1, c_2), d_{(2)}(t_i) = n(c_1, c_2)$, $(1 \leq i \leq n)$, and $d_{(2)}(u_i) = (c_1, c_2)$, $(1 \leq i \leq n)$.

Hence we have, $d_{(a)(2)}(v_i) \neq d_{(a)(2)}(w_i)$, $(1 \leq i \leq n)$, $d_{(a)(2)}(w_i) \neq d_{(a)(2)}(x)$, $(1 \leq i \leq n)$, $d_{(a)(2)}(x) \neq d_{(a)(2)}(s)$, $d_{(a)(2)}(s) \neq d_{(a)(2)}(y)$, $d_{(a)(2)}(t_i) \neq d_{(a)(2)}(y)$, $(1 \leq i \leq m)$, $d_{(a)(2)}(t_i) \neq d_{(a)(2)}(u_i)$, $(1 \leq i \leq m)$.

Hence G is a 2-pseudo neighbourly irregular intuitionistic fuzzy graph. □

Remark 6.2 Even if B is a constant function, then G need not be 2-pseudo neighbourly totally irregular intuitionistic fuzzy graph.

Remark 6.3 Converse of the Theorem 6.1 need not be true.

§7. 2-Pseudo Neighbourly Irregular Intuitionistic Fuzzy Graph on a Path of n Vertices with Specific Membership Functions

In this section, Theorems 7.1 and 7.4 provides a condition for 2-pseudo neighbourly irregularity on intuitionistic fuzzy graph $G:(A,B)$ on a path $G^*:(V,E)$ on n vertices.

Theorem 7.1 *Let $G:(A,B)$ be an intuitionistic fuzzy graph on a path $G^*:(V,E)$ on n vertices.*

If the membership values of the edges $e_1, e_2, e_3, \cdots, e_{n-1}$ *are respectively* $c_1, c_2, c_3, \cdots, c_{n-1}$ *such that* $c_1 < c_2 < c_3 < \cdots < c_{n-1}$, *and non-membership values of the edges* $e_1, e_2, e_3, \ldots, e_{n-1}$ *are respectively* $k_1, k_2, k_3, \cdots, k_{n-1}$ *such that* $k_1 > k_2 > k_3 > \cdots > k_{n-1}$, *then* G *is a 2-pseudo neighbourly irregular intuitionistic fuzzy graph.*

Proof Let $G : (A, B)$ be an intuitionistic fuzzy graph on a path $G^* : (V, E)$ on n vertices. Let $e_1, e_2, e_3, \cdots, e_{n-1}$ be the edges of the path G^* in that order. Let membership value of the edges $e_1, e_2, e_3, \cdots, e_{n-1}$ be respectively $c_1, c_2, c_3, \cdots, c_{n-1}$ such that $c_1 < c_2 < c_3, \cdots, < c_{n-1}$ and non-membership values of the edges $e_1, e_2, e_3, \cdots, e_{n-1}$ are respectively $k_1, k_2, k_3, \cdots, k_{n-1}$ such that $k_1 > k_2 > k_3 > \cdots > k_{n-1}$.

$$d_{(2)}(v_1) = \{(\mu_2(e_1) \wedge \mu_2(e_2), \gamma_2(e_1) \vee \gamma_2(e_2)\} = \{c_1 \wedge c_2, k_1 \vee k_2\} = (c_1, k_1).$$
$$d_{(2)}(v_2) = \{(\mu_2(e_2) \wedge \mu_2(e_3), \gamma_2(e_2) \vee \gamma_3(e_2)\} = \{c_2 \wedge c_3, k_2 \vee k_3\} = (c_2, k_2).$$

For $3 \leq i \leq n-2$,

$$\begin{aligned}
d_{(2)}(v_i) &= \{\{\mu_2(e_{i-1}) \wedge \mu_2(e_{i-2})\} + \{\mu_2(e_i) \wedge \mu_2(e_{i+1})\}, \{\gamma_2(e_{i-1}) \wedge \gamma_2(e_{i-2})\} \\
&\quad + \{\gamma_2(e_i) \wedge \gamma_2(e_{i+1})\}\} = (c_{i-2} + c_i, k_{i-2} + k_i). \\
d_{(2)}(v_{n-1}) &= \{\mu_2(e_{n-3}) \wedge \mu_2(e_{n-2})\}, \{\gamma_2(e_{n-3}) \wedge \gamma_2(e_{n-2})\} \\
&= \{c_{n-3} \wedge c_{n-2}, k_{n-3} \wedge k_{n-2}\} = (c_{n-3}, k_{n-3}). \\
d_{(2)}(v_n) &= \{\mu_2(e_{n-1}) \wedge \mu_2(e_{n-2})\}, \{\gamma_2(e_{n-1}) \wedge \gamma_2(e_{n-2})\} \\
&= \{c_{n-1} \wedge c_{n-2}, k_{n-1} \wedge k_{n-2}\} = (c_{n-2}, k_{n-2}).
\end{aligned}$$

So, every two adjacent vertices have distinct d_2- pseudo degrees. Hence G is a 2-pseudo neighbourly irregular intuitionistic fuzzy graph. □

Remark 7.2 Even if the membership values of the edges $e_1, e_2, e_3, \cdots, e_{n-1}$ are respectively $c_1, c_2, c_3, \cdots, c_{n-1}$ such that $c_1 < c_2 < c_3 < \cdots < c_{n-1}$ and non-membership values of the edges $e_1, e_2, e_3, \cdots, e_{n-1}$ are respectively $k_1, k_2, k_3, \cdots, k_{n-1}$ such that $k_1 > k_2 > k_3 > \cdots > k_{n-1}$. then G need not be 2-pseudo neighbourly totally irregular intuitionistic fuzzy graph.

Theorem 7.3 *Let* $G : (A, B)$ *be an intuitionistic fuzzy graph on* $G^* : (V, E)$, *a path on* n *vertices. If the membership values of the edges* $e_1, e_2, e_3, \cdots, e_{n-1}$ *are respectively* $c_1, c_2, c_3, \cdots, c_{n-1}$ *such that* $c_1 > c_2 > c_3 > \cdots, > c_{n-1}$ *and non-membership values of the edges* $e_1, e_2, e_3, \cdots, e_{n-1}$ *are respectively* $k_1, k_2, k_3, \cdots, k_{n-1}$ *such that* $k_1 < k_2 < k_3 < \cdots < k_{n-1}$. *then* G *is a 2-pseudo neighbourly irregular intuitionistic fuzzy graph.*

Proof Let $G : (A, B)$ be an intuitionistic fuzzy graph on $G^* : (V, E)$ is a path on n vertices. Let $e_1, e_2, e_3, \cdots, e_{n-1}$ be the edges of the path G^* in that order. Let membership values of the edges $e_1, e_2, e_3, \cdots, e_{n-1}$ are respectively $c_1, c_2, c_3, \ldots, c_{n-1}$ such that $c_1 > c_2 > c_3 > \cdots > c_{n-1}$ and non-membership values of the edges $e_1, e_2, e_3, \cdots, e_{n-1}$ are respectively $k_1, k_2, k_3, \cdots, k_{n-1}$

such that $k_1 < k_2 < k_3 < \cdots < k_{n-1}$.

$$\begin{aligned} d_{(2)}(v_1) &= \{(\mu_2(e_1) \wedge \mu_2(e_2), \gamma_2(e_1) \wedge \gamma_2(e_2))\} = \{c_1 \wedge c_2, k_1 \vee k_2\} = (c_2, k_2). \\ d_{(2)}(v_2) &= \{(\mu_2(e_2) \wedge \mu_2(e_3), \gamma_2(e_2) \wedge \gamma_2(e_3))\} = \{c_2 \wedge c_3, k_2 \vee k_3\} = (c_3, k_3). \end{aligned}$$

For $3 \leq i \leq n-2$,

$$\begin{aligned} d_{(2)}(v_i) &= \{\mu_2(e_{i-1}) \wedge \mu_2(e_{i-2})\} + \{\mu_2(e_i) \wedge \mu_2(e_{i+1})\} + \{\gamma_2(e_{i-1}) \wedge \gamma_2(e_{i-2})\} \\ &\quad + \{\gamma_2(e_i) \wedge \gamma_2(e_{i+1})\} = (c_{i-1} + c_{i+1}, k_{i-1} + k_{i+1}) \\ d_{(2)}(v_{n-1}) &= \{\mu_2(e_{n-3}) \wedge \mu_2(e_{n-2}), \gamma_2(e_{n-3}) \wedge \gamma_2(e_{n-2})\} = \{c_{n-3} \wedge c_{n-2}, k_{n-3} \wedge k_{n-2}\} \\ &= (c_{n-2}, k_{n-2}). \\ d_{(2)}(v_n) &= \{\mu_2(e_{n-1}) \wedge \mu_2(e_{n-2}), \gamma_2(e_{n-1}) \wedge \gamma_2(e_{n-2})\} = \{c_{n-1} \wedge c_{n-2}, k_{n-1} \wedge k_{n-2}\} \\ &= (c_{n-1}, k_{n-1}). \end{aligned}$$

Every two adjacent vertices have distinct d_2-pseudo degrees. Hence G is a 2-pseudo neighbourly irregular intuitionistic fuzzy graph. $\square$

Remark 7.4 Even if the membership values of the edges $e_1, e_2, e_3, \cdots, e_{n-1}$ are respectively $c_1, c_2, c_3, \cdots, c_{n-1}$ such that $c_1 > c_2 > c_3 > \cdots, > c_{n-1}$ and non-membership values of the edges $e_1, e_2, e_3, \cdots, e_{n-1}$ are respectively $k_1, k_2, k_3, \cdots, k_{n-1}$ such that $k_1 < k_2 < k_3 < \cdots < k_{n-1}$. then G need not be 2-pseudo neighbourly totally irregular intuitionistic fuzzy graph.

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