*International J.Math. Combin. Vol.4(2015), 66-73*

# Edge Antimagic Total Labeling of Graphs

A.Raheem

Department of Mathematics

COMSATS Institute of information Technology, Islamabad, Pakistan

E-mail: rahimciit7@gmail

**Abstract**: The definition of $(a,d)$-edge-antimagic total labeling was introduced by Simanjuntak, Bertault and Miller as a natural extension of edge-magic labeling defined by Kotzig and Rosa. The present paper deals with a class of subdivided star for all possible values of the parameter $d \in \{0,1,2,3\}$.

**Key Words**: Smarandachely super $(a,d)$-edge-antimagic total labeling, super $(a,d)$-EAT labeling, subdivision of stars.

**AMS(2010)**: 05C78.

## §1. Introduction

All graphs in this paper are finite, undirected and simple. For a graph $G$ we denote the vertex-set and edge-set by $V(G)$ and $E(G)$, respectively. A $(v,e)$-graph $G$ is a graph such that $v = |V(G)|$ and $e = |E(G)|$. A general reference for graph-theoretic ideas can be seen [21]. There are many types of labeling, for example magic, antimagic, gracegraceful, odd gracefu, cordial, radio, sum and mean labeling. This paper deals with different results on super $(a,d)$-edge-antimagic total $((a,d)-EAT)$ labelings for a subclass of a subdivided star trees. The more details on antimagic total labeling can be seen in [5,9]. The subject of edge-magic total labeling of graphs has its origin in the works of Kotzig and Rosa [12, 13] on what they called magic valuations of graphs. The definition of $(a,d)$-edge-antimagic total labeling was introduced by Simanjuntak, Bertault and Miller in [18] as a natural extension of edge-magic labeling defined by Kotzig and Rosa.

**Conjecture** 1.1([6]) *Every tree admits a super edge-magic total labeling.*

For supporting this conjecture, many authors have considered super edge-magic total labeling for many particular classes of trees for example [1, 2, 3, 4, 5, 7, 8, 10, 11, 16, 17, 18, 19]. Lee and Shah [14] verified this conjecture by a computer search for trees with at most 17 vertices. However, this conjecture is still as an open problem.

A star is a particular type of tree graph and many authors have proved the magicness for subdivided stars. Ngurah et. al. [15] proved that $T(m,n,k)$ is also super edge-magic if $k = n+3$ or $n+4$. In [20], Salman et. al. found the super edge-magic total labeling of a

[1]Received April 17, 2015, Accepted November 30, 2015.

subdivision of a star $S_n^m$ for $m = 1, 2$. Javaid et. al. [11] proved super edge-magic total labeling on subdivided star $K_{1,4}$ and w-trees.

However, super $(a, d)$-edge-antimagic total labeling of $G \cong T(n_1, n_2, n_3, \cdots, n_r)$ for different $\{n_i : 1 \leq i \leq r\}$ is still open.

**Definition** 1.1 *A graph $G$ is called $(a, d)$-edge-antimagic total ($(a, d) - EAT$) if there exist integers $a > 0$, $d \geq 0$ and a bijection*

$$\lambda : V(G) \cup E(G) \to \{1, 2, 3, \cdots, v + e\}$$

*such that $W = \{w(xy) : xy \in E(G)\}$ forms an arithmetic sequence starting from $a$ with the common difference $d$, where $w(xy) = \lambda(x) + \lambda(y) + \lambda(xy)$ for every $xy \in E(G)$. $W$ is called the set of edge-weights of the graph $G$.*

*Furthermore, let $H \leq G$. If there is a bijective function $\lambda : V(H) \to \{1, 2, \cdots, |H|\}$ such that the set of edge-sums of all edges in $H$ forms an arithmetic progression $\{a, a + d, a + 2d, \cdots, a + (|E(H)| - 1)d\}$ but for all edges not in $H$ is a constant, such a labeling is called a Smarandachely $(a, d)$-edge-antimagic labeling of $G$ respect to $H$. Clearly, an $(a, d)$-EAV labeling of $G$ is a Smarandachely $(s, d)$-EAV labeling of $G$ respect to $G$ itself.*

**Definition** 1.2 *A $(a, d)$-edge-antimagic total labeling $\lambda$ is called super $(a, d)$-edge-antimagic total labeling if $\lambda(V(G)) = \{1, 2, 3, \cdots, v\}$.*

**Lemma** 1.1([3]) *If $f$ is a super edge-magic total labeling of $G$ with the magic constant $c$, then the function $f_1 : V(G) \cup E(G) \to \{1, 2, 3, \cdots, v + e\}$ defined by*

$$f_1(x) = \begin{cases} v + 1 - f(x), & \text{for } x \in V(G), \\ 2v + e + 1 - f(x), & \text{for } x \in E(G). \end{cases}$$

*is also a super edge-magic total labeling of $G$ with the magic constant $c_1 = 4v + e + 3 - c$.*

**Definition** 1.3 *For $n_i \geq 1$ and $r \geq 3$, let $G \cong T(n_1, n_2, n_3, \cdots, n_p)$ be a graph obtained by inserting $n_i - 1$ vertices to each of the $i$-th edge of the star $K_{1,p}$, where $1 \leq i \leq p$.*

## §2. Main Results

We consider the following proposition which we will use frequently in the main results.

**Proposition** 2.1([4]) *If a $(v, e)$-graph $G$ has a $(s, d)$-EAV labeling then*

(1) *$G$ has a super $(s + v + 1, d + 1)$-EAT labeling;*
(2) *$G$ has a super $(s + v + e, d - 1)$-EAT labeling.*

**Theorem** 2.1 *For all positive integers $n$, $G \cong T(n_1, n_2, n_3, n_4)$ admits super $(a, 0)$-EAT labeling with $a = v + e + s$ and admits super $(a, 2)$-EAT labeling with $a = v + s + 1$, where $n_1 = n, n_2 = n + 1, n_3 = 2n + 1, n_4 = 4n + 2, s = 4n + 5$.*

*Proof* Suppose that the vertex-set and edge-set of $G$ are as follows:

$V(G) = \{c\} \cup \{x_i^{l_i} \mid 1 \le i \le 4\ ;\ 1 \le l_i \le n_i\}$,
$E(G) = \{c\ x_i^1 \mid 1 \le i \le 4\} \cup \{x_i^{l_i} x_i^{l_i+1} \mid 1 \le i \le 4\ ;\ 1 \le l_i \le n_i - 1\}$.

If $v = |V(G)|$ and $e = |E(G)|$ then $v = 8n + 5$ and $e = v - 1$.

We define the labeling $\eta : V(G) \to \{1, 2, \cdots, |V(G)|\}$ as follows:

$\eta(c) = 4(n + 1)$. For $l_i$ odd $1 \le l_i \le n_i$, where $i = 1, 2, 3, 4$, we define

$$\eta(u) = \begin{cases} \frac{l_1+1}{2}, & \text{for } u = x_1^{l_1}, \\ (n+2) - \frac{l_2+1}{2}, & \text{for } u = x_2^{l_2}, \\ (2n+3) - \frac{l_3+1}{2}, & \text{for } u = x_3^{l_3}, \\ 4(n+1) - \frac{l_4+1}{2}, & \text{for } u = x_4^{l_4}. \end{cases}$$

For $l_i$ even $1 \le l_i \le n_i$, where $i = 1, 2, 3, 4$, we define

$$\eta(u) = \begin{cases} 4(n+1) + \frac{l_1}{2}, & \text{for } u = x_1^{l_1}, \\ 5(n+1) - \frac{l_2}{2}, & \text{for } u = x_2^{l_2}, \\ (6n+5) - \frac{l_3}{2}, & \text{for } u = x_3^{l_3}, \\ (8n+6) - \frac{l_4}{2}, & \text{for } u = x_4^{l_4}. \end{cases}$$

The set of all edge-sums $\{\eta(x) + \eta(y) : xy \in E(G)\} = \{4n + 4 + j : 1 \le j \le e\}$ form an arithmatic sequence starting with minimum edge-sum $4(n + 1)$. Therefore, by Proposition 2.1, $\eta$ can be extended to a super $(a, 0)$-EAT labeling and we obtain the magic constant $a = v + e + s = 20n + 14$. Similarly, by Proposition 2.1, $\eta$ can be extended to a super $(a, 2)$-EAT labeling and we obtain the minimum edge weight is $a = v + 1 + s = 12n + 11$. □

**Theorem** 2.2 *For all positive integers* $n$*,* $G \cong T(n_1, n_2, n_3, n_4)$ *admits super* $(a, 1)$*-EAT labeling with* $a = 2v + s - 1$ *and admits super* $(a, 3)$*-EAT labeling with* $a = v + s + 1$*, where* $v = |V(G)|$*,* $s = 3$*,* $n_1 = n$*,* $n_2 = n + 1$*,* $n_3 = 2n + 1$*,* $n_4 = 4n + 2$*.*

*Proof* Suppose that the $V(G)$ and $E(G)$ are defined as in the proof of the Theorem 2.1. Let us consider $v = |V(G)|$ and $e = |E(G)|$ then $v = (8n + 5)$ and $e = 4(2n + 1)$. We define the vertex labeling $\eta : V(G) \to \{1, 2, \cdots, V(G)\}$ as follows:

$\eta(c) = 2$. For $1 \le l_i \le n_i$ with $i = 1, 2, 3, 4$, we define

$$\eta(u) = \begin{cases} l_1, & \text{for } u = x_1^{l_1} \text{ and } l_1 \equiv 1(\text{mod}2), \\ 2 + l_1, & \text{for } u = x_1^{l_1} \text{ and } l_1 \equiv 0(\text{mod}2), \\ (2n+1) - (l_2 - 1), & \text{for } u = x_2^{l_2} \text{ and } l_2 \equiv 1(\text{mod}2), \\ (2n+2) - (l_2 - 2), & \text{for } u = x_2^{l_2} \text{ and } l_2 \equiv 0(\text{mod}2), \\ 4(n+1) - l_3, & \text{for } u = x_3^{l_3}, \\ (8n+6) - l_4, & \text{for } u = x_4^{l_4}, \end{cases}$$

The set of edge-sums $\{\eta(x)+\eta(y) : xy \in E(G)\}$ generated by the above scheme forms a integer sequence $3, 3+2, \cdots, 3+2(e-1)$ with difference difference 2. Therefore, by Proposition 2.1, $\eta$ can be extended to a super $(a,1)$-EAT labeling and the minimum edge weight is $a = v+e+s = 2(v+1)$ Similarly, by Proposition 2.1, $\eta$ can be extended to a super $(a,3)$-EAT labeling and the minimum edge weight is $a = v+1+s = v+4$. □

**Theorem** 2.3 *For all positive integers n, $G \cong T(n_1, n_2, n_3, n_4, n_5)$ admits super $(a,0)$-EAT labeling with $a = v+e+s$ and admits super $(a,2)$-EAT labeling with $a = v+s+1$, where $n_1 = n$, $n_2 = n+1$, $n_3 = 2n+1$, $n_4 = 4n+2$,$n_5 = 8n+4$ and $s = 8(n+1)$.*

*Proof* Suppose that the vertex-set and edge-set of the graph $G$ are as follows:

$V(G) = \{c\} \cup \{x_i^{l_i} \mid 1 \le i \le 5\ ;\ 1 \le l_i \le n_i\}$,
$E(G) = \{c\ x_i^1 \mid 1 \le i \le 5\} \cup \{x_i^{l_i} x_i^{l_i+1} \mid 1 \le i \le 5\ ;\ 1 \le l_i \le n_i - 1\}$.

If $v = |V(G)|$ and $e = |E(G)|$ then $v = 16n+9$ and $e = 8(2n+1)$. We define the labeling $\eta : V(G) \to \{1, 2, \cdots, |V(G)|\}$ as follows:

$\eta(c) = 8n+6$. For $l_i$ odd $1 \le l_i \le n_i$, where $i = 1,2,3,4,5$, we define

$$\eta(u) = \begin{cases} \frac{l_1+1}{2}, & \text{for } u = x_1^{l_1}, \\ (n+2) - \frac{l_2+1}{2}, & \text{for } u = x_2^{l_2}, \\ (2n+3) - \frac{l_3+1}{2}, & \text{for } u = x_3^{l_3}, \\ 4(n+1) - \frac{l_4+1}{2}, & \text{for } u = x_4^{l_4}, \\ (8n+6) - \frac{l_5+1}{2}, & \text{for } u = x_5^{l_5}. \end{cases}$$

For $l_i$ even $1 \le l_i \le n_i$, where $i = 1,2,3,4,5$, we define

$$\eta(u) = \begin{cases} (8n+6) + \frac{l_1}{2}, & \text{for } u = x_1^{l_1}, \\ (9n+7) - \frac{l_2}{2}, & \text{for } u = x_2^{l_2}, \\ (10n+6) - \frac{l_3}{2}, & \text{for } u = x_3^{l_3}, \\ (12n+8) - \frac{l_4}{2}, & \text{for } u = x_4^{l_4} \\ (16n+9) - \frac{l_5}{2}, & \text{for } u = x_5^{l_5}. \end{cases}$$

The set of all edge-sums $\{\eta(x)+\eta(y) : xy \in E(G)\}$=$\{8n+6+j : 1 \le j \le e\}$ form an arithmatic sequence starting with minimum edge-sum $(8n+7)$. Therefore, by Proposition 2.1, $\eta$ can be extended to a super $(a,0)$-EAT labeling and we obtain the magic constant $a = v+e+s = 40n+25$. Similarly, by Proposition 2.1, $\eta$ can be extended to a super $(a,2)$-EAT labeling and the minimum edge weight is $a = v+1+s = 24n+18$. □

**Theorem** 2.4 *For all positive integers n , $G \cong T(n_1, n_2, n_3, n_4, n_5)$ admits super $(a,1)$-EAT labeling with $a = 2v+s-1$ and admits super $(a,3)$-EAT labeling with $a = v+s+1$, where $v = |V(G)|$, $s = 3$,$n_1 = n$, $n_2 = n+1$, $n_3 = 2n+1$, $n_4 = 4n+2$.*

*Proof* Suppose that the $V(G)$ and $E(G)$ are defined as in the proof of the Theorem 2.3.

Let us consider $v = |V(G)|$ and $e = |E(G)|$ then $v = 16n + 9$ and $e = 8(2n + 1)$. We define the vertex labeling $\eta : V(G) \to \{1, 2, \cdots, V(G)\}$ as follows:

$\eta(c) = 2$. For $1 \le l_i \le n_i$, where $i = 1, 2, 3, 4, 5$, we define

$$\eta(u) = \begin{cases} l_1, & \text{for } u = x_1^{l_1} \text{ and } l_1 \equiv 1(\text{mod}2), \\ 2 + l_1, & \text{for } u = x_1^{l_1} \text{ and } l_1 \equiv 0(\text{mod}2), \\ (2n+1) - (l_2 - 1), & \text{for } u = x_2^{l_2} \text{ and } l_2 \equiv 1(\text{mod}2), \\ 2(n+1) - (l_2 - 2), & \text{for } u = x_2^{l_2} \text{ and } l_2 \equiv 0(\text{mod}2), \\ 4(n+1) - l_3, & \text{for } u = x_3^{l_3}, \\ 2(4n+3) - l_4, & \text{for } u = x_4^{l_4}, \\ 2(8n+5) - l_5, & \text{for } u = x_5^{l_5}. \end{cases}$$

The set of edge-sums $\{\eta(x) + \eta(y) : xy \in E(G)\}$ generated by the above scheme forms a integer sequence $3, 3+2, \cdots, 3+2(e-1)$ with difference difference 2. Therefore, by Proposition 2.1, $\eta$ can be extended to a super $(a, 1)$-EAT labeling and the minimum edge weight is $a = v + e + s = 2(v + 1)$. Similarly, by Proposition 2.1, $\eta$ can be extended to a super $(a, 3)$-EAT labeling and the minimum edge weight is $a = v + 1 + s = v + 4$.

**Theorem** 2.5 *For all positive integers $n$, $G \cong T(n_1, n_2, n_3, n_4, n_5, n_6, ..., n_p)$ admits super $(a, 0)$-EAT labeling with $a = v + e + s$ and admits super $(a, 2)$-EAT labeling with $a = v + s + 1$, where $n_1 = n, n_2 = n + 1, n_3 = 2n + 1, n_4 = 4n + 2, n_5 = 8n + 4$,*

$$s = 8(n+1) + \sum_{m=6}^{p} [2^{m-3}(2n+1)] \quad and n_m = 2^{m-3}(2n+1)$$

*for $6 \le m \le p$.*

*Proof* Suppose that the vertex-set and edge-set of $G$ are as follows:

$V(G) = \{c\} \cup \{x_i^{l_i} \mid 1 \le i \le p \ ; \ 1 \le l_i \le n_i\}$,

$E(G) = \{c\ x_i^1 \mid 1 \le i \le p\} \cup \{x_i^{l_i} x_i^{l_i+1} \mid 1 \le i \le p \ ; \ 1 \le l_i \le n_i - 1\}$.

If $v = |V(G)|$ and $e = |E(G)|$ then

$$v = (16n+9) + \sum_{m=6}^{p} [2^{m-2}(2n+1)] \quad \text{and} \quad e = (16n+8) + \sum_{m=6}^{p} [2^{m-2}(2n+1)].$$

We define the labeling $\eta : V(G) \to \{1, 2, \cdots, |V(G)|\}$ as follows:

$$\eta(c) = (8n+6) + \sum_{m=6}^{p} [2^{m-3}(2n+1)].$$

For $l_i$ odd $1 \leq l_i \leq n_i$, where $i = 1, 2, 3, 4, 5$ and $6 \leq m \leq p$ we define

$$\eta(u) = \begin{cases} \frac{l_1+1}{2}, & \text{for } u = x_1^{l_1}, \\ (n+2) - \frac{l_2+1}{2}, & \text{for } u = x_2^{l_2}, \\ (2n+3) - \frac{l_3+1}{2}, & \text{for } u = x_3^{l_3}, \\ 4(n+1) - \frac{l_4+1}{2}, & \text{for } u = x_4^{l_4}. \\ (8n+6) - \frac{l_5+1}{2}, & \text{for } u = x_5^{l_5}. \end{cases}$$

and

$$\eta(x_i^{l_i}) = (8n+6) + \sum_{m=6}^{i} [2^{m-3}(2n+1)] - \frac{l_i+1}{2}$$

respectively. For $l_i$ even $1 \leq l_i \leq n_i$, where $i = 1, 2, 3, 4, 5$ and $\eta(c) = \beta$ we define

$$\eta(u) = \begin{cases} \beta + \frac{l_1}{2}, & \text{for } u = x_1^{l_1}, \\ (\beta+n) - \frac{l_2}{2}, & \text{for } u = x_2^{l_2}, \\ (\beta+2n-1) - \frac{l_3}{2}, & \text{for } u = x_3^{l_3}, \\ (\beta+4n+1) - \frac{l_4}{2}, & \text{for } u = x_4^{l_4}, \\ (\beta+8n+3) - \frac{l_5}{2}, & \text{for } u = x_5^{l_5}. \end{cases}$$

and

$$\eta(x_i^{l_i}) = (\beta+8n+3) + \sum_{m=6}^{i} [2^{m-3}(2n+1)] - \frac{l_i}{2}$$

respectively. The set of all edge-sums $\{\eta(x) + \eta(y) : xy \in E(G)\}$=$\{\beta + j : 1 \leq j \leq e\}$ form an arithmatic sequence starting with minimum edge-sum $\beta$. Therefore, by Proposition 2.1, $\eta$ can be extended to a super $(a, 0)$-EAT labeling and we obtain the magic constant

$$a = v + e + s = (2v + 8n + 7) + \sum_{m=6}^{p} [2^{m-3}(2n+1)]$$

Similarly, by Proposition 2.1, $\eta$ can be extended to a super $(a, 2)$-EAT labeling and obtain the minimum edge weight is

$$a = v + 1 + s = (v + 8n + 9) + \sum_{m=6}^{p} [2^{m-3}(2n+1)]. \qquad \square$$

**Theorem** 2.6 *For all positive integers $n$, $G \cong T(n_1, n_2, n_3, n_4, n_5, n_6, \cdots, n_p)$ admits super $(a, 1)$-EAT labeling with $a = 2v + s - 1$ and admits super $(a, 3)$-EAT labeling with $a = v + s + 1$, where $v = |V(G)|$, $s = 3$, $n_1 = n$, $n_2 = n + 1$, $n_3 = 2n + 1$, $n_4 = 4n + 2$, $n_5 = 8n + 4$,*

$$s = 8(n+1) + \sum_{m=6}^{p} [2^{m-3}(2n+1)] \quad \text{and} \quad n_m = 2^{m-3}(2n+1)$$

*for* $6 \leq m \leq p$.

*Proof* Suppose that the $V(G)$ and $E(G)$ are defined as in the proof of the Theorem 2.5. Let us consider $v = |V(G)|$ and $e = |E(G)|$ then

$$v = (16n+9) + \sum_{m=6}^{p}[2^{m-2}(2n+1)] \quad \text{and} \quad e = 8(2n+1) + \sum_{m=6}^{p}[2^{m-2}(2n+1)].$$

We define the vertex labeling $\eta : V(G) \to \{1, 2, \cdots, V(G)\}$ as follows:

$\eta(c) = 2$. For $1 \leq l_i \leq n_i$, where $i = 1, 2, 3, 4, 5$ and $6 \leq m \leq p$, we define

$$\eta(u) = \begin{cases} l_1, & \text{for } \ u = x_1^{l_1} \text{ and } l_1 \equiv 1(\text{mod}2), \\ (2+l_1), & \text{for } \ u = x_1^{l_1} \text{ and } l_1 \equiv 0(\text{mod}2), \\ (2n+1)-(l_2-1), & \text{for } \ u = x_2^{l_2} \text{ and } l_2 \equiv 1(\text{mod}2), \\ 2(n+1)-(l_2-2), & \text{for } \ u = x_2^{l_2} \text{ and } l_2 \equiv 0(\text{mod}2), \\ 4(n+1)-l_3, & \text{for } \ u = x_3^{l_3}, \\ (8n+6)-l_4, & \text{for } \ u = x_4^{l_4}, \\ (16n+10)-l_5, & \text{for } \ u = x_5^{l_5}. \end{cases}$$

and

$$\eta(x_i^{l_i}) = (16n+10) + \sum_{m=6}^{i}[2^{m-3}(2n+1)] - l_i$$

respectively. The set of edge-sums $\{\eta(x) + \eta(y) : xy \in E(G)\}$ generated by the above scheme forms a integer sequence $3, 3+2, \cdots, 3+2(e-1)$ with difference difference 2. Therefore, by Proposition 2.1, $\eta$ can be extended to a super $(a,1)$-EAT labeling and obtain the minimum edge weight is $a = v + e + s = 2(v+1)$ Similarly, by Proposition 2.1, $\eta$ can be extended to a super $(a,3)$-EAT labeling and obtain the minimum edge weight is $a = v + 1 + s = v + 4$. □

## §3. Conclusion

In this paper, we show that a subclass of a tree, namely a subdivision of a star tree denoted by $G \cong T(n_1, n_2, n_3, ..., n_p)$ admits super $(a,d)$-EAT labeling for all positive integers $n$ and all possible values of the parameter $d$. However, for different values of the minimum edge-weight $a$ and $n_i$, problem is still open.

## References


[1] A.Ahmed, A. Q.Baig and M.Imran, On super edge-magicness of graphs, *Utilitas Math.*, to appear.

[2] E.T.Baskoro and A.A.G.Ngurah, On super edge-magic total labelings, *Bull. Inst. Combin. Appil.*, 37 (2003), 82-87.

[3] E.T.Baskoro, I.W.Sudarsana and Y.M.Cholily, How to construct new super edge-magic graphs from some old ones, *J. Indones. Math. Soc. (MIHIM)*, 11:2 (2005), 155-162.

[4] M.Bača, Y.Lin and F.A.Muntaner-Batle, Super edge-antimagic labelings of the path-like trees, *Utilitas Math.*, 73 (2007), 117–128.

[5] M.Bača and M.Miller, Super Edge-Antimagic Graphs, Brown Walker Press, Boca Raton, Florida USA, (2008).

[6] H.Enomoto, A.S.Llado, T.Nakamigawa and G.Ringle, Super edge-magic graphs, *SUT J. Math.*, 34 (1980), 105-109.

[7] R.M.Figueroa-Centeno, R.Ichishima, and F.A.Muntaner-Batle, The place of super edge-magic labeling among other classes of labeling, *Discrete Math.*, 231 (2001), 153–168.

[8] R.M.Figueroa-Centeno, R.Ichishima and F.A.Muntaner-Batle, On super edge-magic graph, *Ars Combin.*, 64 (2002), 81–95.

[9] J.A.Gallian, A dynamic survey of graph labeling, *Elec. J. Combin.*, 17 (2014).

[10] M.Hussain, E.T.Baskoro and Slamin, On super edge-magic total labeling of banana trees, *Utilitas Math.,* 79 (2009), 243-251.

[11] M.Javaid, M.Hussain, K.Ali and H. Shaker, Super edge-magic total labeling on subdivision of trees, *Utilitas Math.*, to appear.

[12] A.Kotzig and A.Rosa, Magic valuations of finite graphs, *Canad. Math. Bull.*, 13 (1970), 451-461.

[13] A.Kotzig and A.Rosa, Magic valuation of complete graphs, *Centre de Recherches Mathematiques, Universite de Montreal*, (1972), CRM-175.

[14] S.M.Lee and Q.X.Shah, All trees with at most 17 vertices are super edge-magic, *16th MCCCC Conference, Carbondale*, University Southern Illinois, November (2002).

[15] A.A.G.Ngurah, R.Simanjuntak and E.T.Baskoro, On (super) edge-magic total labeling of subdivision of $K_{1,3}$, *SUT J. Math.*, 43 (2007), 127-136.

[16] A.Raheem, A.Q.Baig, Antimagic total labeling of subdivision of caterpillar, *Science International Journal*, to appear.

[17] A.Raheem, On super $(a, d)$-edge-antimagic total labeling of subdivided stars, *Ars Combinatoria*, to appear.

[18] R.Simanjuntak, F.Bertault and M.Miller, Two new $(a, d)$-antimagic graph labelings, *Proc. of Eleventh Australasian Workshop on Combinatorial Algorithms* (2000), 179–189.

[19] K.A.Sugeng, M.Miller, Slamin, and M. Bača, $(a, d)$-edge-antimagic total labelings of caterpillars, *Lecture Notes Comput. Sci.*, 3330 (2005), 169–180.

[20] A.N. M.Salman, A.A.G.Ngurah and N.Izzati, On Super Edge-Magic Total Labeling of a Subdivision of a Star $S_n$, *Utilitas Mthematica*, 81 (2010), 275–284.

[21] D. B. West, *An Introduction to Graph Theory*, Prentice-Hall, (1996).