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Almost Pseudo Ricci Symmetric Viscous Fluid Spacetime

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Abstract: The object of the present paper is to investigate the application of almost pseudo Ricci symmetric manifolds to the General Relativity and Cosmology. Also we study the space time when the anisotropic pressure tensor in energy momentum tensor of type (0,2) takes the different form.

Key Words: Pseudo Ricci symmetric manifold, almost pseudo Ricci symmetric manifold, scalar curvature, Viscous fluid space time, energy momentum tensor.

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§1. Introduction

The study of Riemannian symmetric manifolds began with the work of Cartan [1]. A Riemannian manifold (M^n, g) is said to be locally symmetric due to Cartan if its curvature tensor R satisfy the relation $\nabla R = 0$, where ∇ denotes the operator of covariant differentiation with respect to the metric tensor g. The notion of locally symmetric manifolds has been weakened by many authors in several ways to a different extent such as recurrent manifolds by Walker [10], semi symmetric manifold by Szab*ó* [8], pseudo symmetric manifold by Chaki [2], generalized pseudo symmetric manifold by Chaki [3], and weakly symmetric manifold by T*á*massy and Binh [9]. In 1988 Chaki [4] introduced the notion of pseudo Ricci symmetric manifolds. A Riemannian manifold (M^n, g) $(n > 2)$ is said to be pseudo Ricci symmetric if its Ricci tensor S of type (0, 2) is not identically zero and satisfy the relation

$$(\nabla_X S)(Y,Z) = 2\alpha(X)S(Y,Z) + \alpha(Y)S(X,Z) + \alpha(Z)S(Y,X),$$

where α is a non zero 1-form such that $g(X, \rho) = \alpha(X)$ for every vector field X. Such an n - dimensional manifold is denoted by $(PRS)_n$.

Again, M. C. Chaki and T. Kawaguchi [5] introduced the notion of almost pseudo Ricci symmetric manifolds. A Riemannian manifold (M^n, g) is called an almost pseudo Ricci symmetric manifolds if its Ricci tensor S of type (0, 2) is not identically zero and satisfies the condition

$$(\nabla_X S)(Y,Z) = [\alpha(X) + \beta(X)]S(Y,Z) + \alpha(Y)S(X,Z) + \alpha(Z)S(Y,X), \tag{1.1}$$

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where α and β are nowhere vanishing 1-forms such that $g(X, \rho) = \alpha(X)$ and $g(X, \mu) = \beta(X)$ for all X and ρ, μ are called the basic vector fields of the manifold. The 1-forms α and β are called associated 1-forms and an n-dimensional manifold of this kind is denoted by $A(PRS)_n$. If, in particular, $\beta = \alpha$ then it reduces a pseudo Ricci symmetric manifolds.

In general relativity the matter content of the spacetime is described by the energy momentum tensor T which is determined from the physical considerations dealing with the distribution of matter and energy. Since the matter content of the universe is assumed to behave like a perfect fluid in the standard cosmological models, the physical motivation for studying Lorentzian manifolds is the assumption that a gravitational field may be effectively modelled by some Lorenzian metric defined on a suitable four dimensional manifold M. The Einstein equations are fundamental in the construction of cosmological models which imply that the matter determine the geometry of the spacetime and conversely the motion of matter is determined by the metric tensor of the space which is non flat.

The physical motivation for studying various types of spacetime models in cosmology is to obtain the information of different phases in the evolution of the universe, which may be classified into three different phases, namely, the initial phase, the intermediate phase, and the final phase. The initial phase is just after the Big Bang when the effects of both viscosity and heat flux were quite pronounced. The intermediate phase is that when the effect of viscosity was no longer significant but the heat flux was still not negligible. The final phase extends to the present state of the universe when both the effects of viscosity and heat flux have become negligible and the matter content of the universe may be assumed to be a perfect fluid. The study of $A(PRS)_4$ is important because such spacetime represents the intermediate phase in the evolution of the universe. Consequently the investigations of $A(PRS)_4$ help us to have a deeper understanding of the global character of the universe including the topology, because the nature of the singularities can be defined from a differential geometric standpoint.

The present paper is concerned with certain investigations in general relativity by the coordinate free method of differential geometry. In this method of study the spacetime of general relativity is regarded as a connected four-dimensional semi-Riemannian manifold (M^4, g) with Lorenz metric g with signature (-, +, +, +). The geometry of the Lorentz manifold begins with the study of the causal character of vectors of the manifold. It is due to this casuality that the Lorentz manifold becomes a convenient choice for the study of general relativity. Also we study the space time when the anisotropic pressure tensor in energy momentum tensor of type (0, 2) takes the different form. Here we consider a special type of spacetime which is called almost pseudo Ricci symmetric spacetime.

§2. **Preliminaries**

Let L be the symmetric endomorphism of the tangent space at each point of (M^n, g) corresponding to the Ricci tensor S. Then $g(LX, Y) = S(X, Y)$ for all vector fields X, Y. Let $\{e_i : i = 1, 2, \cdots, n\}$ be an orthonormal basis of the tangent space at any point of the manifold.

Then setting $Y = Z = e_i$ in (1.1) and then taking summation over i, $1 \leq i \leq n$, we obtain

$$dr(X) = r[\alpha(X) + \beta(X)] + 2\alpha(LX), \tag{2.1}$$

where r is the scalar curvature of the manifold. Again from (1.1) we get

$$(\nabla_X S)(Y, Z) - (\nabla_Y S)(X, Z) = \beta(X)S(Y, Z) - \beta(Y)S(X, Z). \tag{2.2}$$

Setting $Y = Z = e_i$ in (2.2) then taking summation over i, $1 \leq i \leq n$, we obtain

$$dr(X) = 2r\beta(X) - 2\beta(LX). \tag{2.3}$$

If the scalar curvature r is constant then

$$dr(X) = 0 \quad for \quad all \quad X. \tag{2.4}$$

By virtue of (2.4), (2.3) yields

$$\beta(LX) = r\beta(X), \tag{2.5}$$

i.e.,

$$S(X, \mu) = rg(X, \mu). \tag{2.6}$$

These formula will be used in the sequel.

§3. Almost Pseudo Ricci Symmetric Spacetime with Viscous Fluid Matter Content

A viscous fluid spacetime is a connected semi-Riemannian manifold (M^4, g) with signature $(-, +, +, +)$. In general relativity the key role is played by Einstein equation

$$S(X, Y) - \frac{r}{2}g(X, Y) + \lambda g(X, Y) = kT(X, Y) \tag{3.1}$$

for all vector fields X, Y, where S is the Ricci tensor of type (0, 2), r is the scalar curvature, λ is the cosmological constant, k is the gravitational constant and T is the energy-momentum tensor of type (0, 2). The matter content of the spacetime is described by the energy-momentum tensor T which is to be determined from physical considerations dealing with distribution of matter and energy. Let us consider the energy-momentum tensor T of a viscous fluid spacetime of the following form [7]

$$T(X, Y) = pg(X, Y) + (\sigma + p)\gamma(X)\gamma(Y) + P(X, Y), \tag{3.2}$$

where σ, p are the energy density and isotropic pressure, respectively, and P denotes the anisotropic pressure tensor of the fluid, μ is the unit timelike vector field, called flow vector field of the fluid associated with the 1-form γ given by $g(X, \mu) = \gamma(X)$ for all X. Then by virtue of (3.2), (3.1) can be written as

$$S(X,Y) = (\frac{r}{2} + kp - \lambda)g(X,Y) + k(\sigma + p)\gamma(X)\gamma(Y) + kP(X,Y), \tag{3.3}$$

Case 1. Let us consider $P(X,Y) = tg(X,Y)$, where t is any real number, then equation (3.3) become

$$S(X,Y) = (\frac{r}{2} + k(p+t) - \lambda)g(X,Y) + k(\sigma + p)\gamma(X)\gamma(Y), \tag{3.4}$$

which shows that the spacetime under consideration is a $(QE)_4$ with $\varrho = \frac{r}{2} + k(p+t) - \lambda$ and $\zeta = k(\sigma + p)$ as associated scalars; λ as the associated 1-form with generator μ and the anisotropic pressure tensor P which is of the form $P(X,Y) = tg(X,Y)$. Hence we can state

Theorem 3.1 *A viscous fluid spacetime satisfying Einstein's equation with a cosmological constant is a 4-dimensional quasi-Einstein manifold with generator μ as the flow vector field and p as the isotropic pressure tensor when the anisotropic pressure tensor P is of the form $P(X,Y) = tg(X,Y)$.*

In this section we consider a relativistic spacetime as a Lorentzian $A(PRS)_4$ with associate 1-form γ and basic vector field μ whose matter content is the viscous fluid with the velocity vector field μ. Then μ is a timelike unit vector field. Hence $g(\mu,\mu) = -1$ and $g(X,\mu) = \gamma(X)$ for all X. Taking a frame field and contracting (3.5) over X and Y we get

$$r = -k(3p - \sigma) + 4\lambda - 4kt, \tag{3.5}$$

Now putting $Y = \mu$, it follows from (3.4) that

$$S(X,\mu) = (\frac{r}{2} - \lambda - k\sigma + kt)g(X,\mu). \tag{3.6}$$

So, it follows from (3.6) that $\frac{r}{2} - \lambda - k\sigma + kt$ is an eigenvalue of Ricci tensor S and μ is an eigenvector corresponding to the eigenvalue.

Let ρ be another eigenvector os S different from μ. Then ρ must be orthogonal to μ. Hence $g(\mu,\rho) = 0$, i.e., $\gamma(\rho) = 0$.

By putting $Y = \rho$, it follows from (3.4) that

$$S(X,\rho) = (\frac{r}{2} - \lambda + kp + kt)g(X,\rho). \tag{3.7}$$

So from (3.7), $\frac{r}{2} - \lambda + kp + kt$ is another eigenvalue of S corresponding to the eigenvector ρ. Here we see that two eigenvalues are different and for a given eigenvector there is a only one eigenvalue. So, it follows that the Ricci tensor S has only two different eigenvalues, namely $\frac{r}{2} - \lambda - k\sigma + kt$ and $\frac{r}{2} - \lambda - k\sigma + kt$.

Let the multiplicity of $\frac{r}{2} - \lambda - k\sigma + kt$ be m and therefore the multiplicity of $\frac{r}{2} - \lambda - k\sigma + kt$ be $(4-m)$ because the dimension of the spacetime is 4.

Hence

$$m(\frac{r}{2} - \lambda - k\sigma + kt) + (4-m)(\frac{r}{2} - \lambda - k\sigma + kt) = r.$$

Then after some calculations and using (3.5), we get

$$(\sigma + p)(m - 1) = 0. \tag{3.8}$$

Since $(\sigma + p) \neq 0$ it follows from (3.8) $m = 1$. Thus the multiplicity of the eigenvalue $\frac{r}{2} - \lambda - k\sigma + kt$ is 1 and the the multiplicity of the eigenvalue $\frac{r}{2} - \lambda - k\sigma + kt$ is 3. Therefore the segre characteristic [6] of S is [(111),1]. Hence we can state

Theorem 3.2 *If in an almost pseudo Ricci symmetric spacetime of basic vector field μ, the matter content is a viscous fluid with μ as the velocity vector field and when the anisotropic pressure tensor P is of the form $P(X,Y) = tg(X,Y)$, then the Ricci tensor of the spacetime is segre characteristic [(111),1].*

Now from (3.6), $S(X,\mu) = (\frac{r}{2} - \lambda - k\sigma + kt)g(X,\mu)$ and on the other hand $S(X,\mu) = rg(X,\mu)$, using this two equations we can write

$$[r - \frac{r}{2} + \lambda + k\sigma - kt]g(X,\mu) = 0. \tag{3.9}$$

Further setting $X = \mu$ in equation (3.9), we get

$$\sigma = \frac{2kt - 2\lambda - r}{2k}. \tag{3.10}$$

Again from (3.5) and using the result of (3.10), we can write

$$p = \frac{2\lambda - r - 2kt}{2k}. \tag{3.11}$$

Here we see that σ and p are constant. Hence we can state the following

Theorem 3.3 *If a viscous fluid $A(PRS)_4$ spacetime obeys Einstein's equation with a cosmological constant and when the anisotropic pressure tensor P is of the form $P(X,Y) = tg(X,Y)$, then energy density and isotropic pressure are constants.*

Let us consider in $A(PRS)_4$ spacetime $p > 0$. Then since $\sigma > 0$, we have from (3.10) and (3.11) that

$$\lambda < kt - \frac{r}{2} \text{ and } \lambda > \frac{r}{2} + kt.$$

Therefore we can state the following

Theorem 3.4 *If a viscous fluid $A(PRS)_4$ spacetime with positive isotropic pressure obeys Einstein's equation with a cosmological constant λ and when the anisotropic pressure tensor P is of the form $P(X,Y) = tg(X,Y)$, then λ satisfies either $\lambda < kt - \frac{r}{2}$ or $\lambda > \frac{r}{2} + kt$.*

Next we discuss whether a viscous fluid $A(PRS)_4$ spacetime with generator μ as unit timelike flow vector field can admit heat flux or not. Let us consider the energy momentum tensor T is of the following form [10]

$$T(X,Y) = pg(X,Y) + (\sigma + p)\gamma(X)\gamma(Y) + \gamma(X)\eta(Y) + \eta(X)\gamma(Y) + P(X,Y),$$

where $\eta(X) = g(X,\rho)$ for all vector field X, ρ being the heat flux vector field, σ, p are energy density and isotropic pressure tensor respectively and P denotes the anisotropic pressure tensor of the fluid. Thus we have $g(\mu,\rho) = 0$ that is $\eta(\mu) = 0$.

Now using this energy momentum tensor in equation (3.1) and then setting $Y = \mu$, we get

$$(\frac{r}{2} + \lambda + k\sigma - kt)g(X,\mu) = -k\eta(X). \tag{3.12}$$

Then putting $X = \mu$ in (3.12), we obtain

$$\frac{r}{2} + \lambda + k\sigma - kt = 0$$

Therefore we can write that from (3.12), $\eta(X) = 0$ for all X, since $k \neq 0$. Thus we have the following

Theorem 3.5 *A viscous fluid $A(PRS)_4$ can not admit heat flux when the anisotropic pressure tensor P is of the form $P(X,Y) = tg(X,Y)$.*

Case 2. Next we consider $P(X,Y) = D(X,Y)$, where $D(X,Y) = D(Y,X)$, $trace(D) = 0$ and $D(X,\mu) = 0$ for all vector field X. Then equation (3.3) become

$$S(X,Y) = (\frac{r}{2} + kp - \lambda)g(X,Y) + k(\sigma + p)\gamma(X)\gamma(Y) + kD(X,Y), \tag{3.13}$$

From (3.13) it follows that the spacetime under consideration is a pseudo quasi-Einstein manifold with $\varrho_1 = \frac{r}{2} + kp - \lambda$, $\zeta_1 = k(\sigma + p)$ and $\xi_1 = k$ as associate scalars; γ as the associate 1-form with generator μ and the anisotropic pressure function D as the structure tensor. Hence we can sate the following

Theorem 3.6 *A viscous fluid spacetime satisfying Einstein equation with cosmological constant and when the anisotropic pressure tensor P is of the form $P(X,Y) = D(X,Y)$ is a 4-dimensional connected pseudo quasi-Einstein manifold with generator μ is the flow vector field and the structure tensor D as the anisotropic pressure tensor.*

Next we discuss about the segre characteristic of S, for that after some calculations we get two different eigenvalues of S namely, $\frac{r}{2} - \lambda - k\sigma$ and $\frac{r}{2} - \lambda + kp$. Here we see that two eigenvalues are different and for a given eigenvector there is a only one eigenvalue.

Let the multiplicity of $\frac{r}{2} - \lambda - k\sigma$ be m and therefore the multiplicity of $\frac{r}{2} - \lambda + kp$ be $(4 - m)$ because the dimension of the spacetime is 4.

Hence

$$m(\frac{r}{2} - \lambda - k\sigma) + (4 - m)(\frac{r}{2} - \lambda + kp) = r.$$

Then after some calculations, we get $m = 1$. Thus the multiplicity of the eigenvalue $\frac{r}{2} - \lambda - k\sigma$

is 1 and the the multiplicity of the eigenvalue $\frac{r}{2} - \lambda + kp$ is 3. Therefore the segre characteristic of S is [(111),1]. Hence we can state

Theorem 3.7 *If in an almost pseudo Ricci symmetric spacetime of basic vector field μ, the matter content is a viscous fluid with μ as the velocity vector field and when the anisotropic pressure tensor P is of the form $P(X,Y) = D(X,Y)$, then the Ricci tensor of the spacetime is segre characteristic [(111),1].*

Setting $Y = \mu$ in equation (3.12), we get $S(X,\mu) = (\frac{r}{2} - \lambda - k\sigma)g(X,\mu)$ and on the other hand $S(X,\mu) = rg(X,\mu)$, using this two equations we can write

$$[r - \frac{r}{2} + \lambda + k\sigma]g(X,\mu) = 0. \tag{3.14}$$

Further setting $X = \mu$ in equation (3.14), we get

$$\sigma = \frac{-2\lambda - r}{2k}. \tag{3.15}$$

Taking contraction on (3.13), we get

$$r = 4(\frac{r}{2} - \lambda + kp) - k(\sigma + p).$$

Then using (3.15) in the above equation, we can write

$$p = \frac{2\lambda - r}{2k}. \tag{3.16}$$

Here we see that σ and p are constant. Hence we can state the following

Theorem 3.8 *If a viscous fluid $A(PRS)_4$ spacetime obeys Einstein's equation with a cosmological constant and the anisotropic pressure tensor P is of the form $P(X,Y) = D(X,Y)$, then energy function and isotropic pressure of the fluid are constants.*

Let us consider in $A(PRS)_4$ spacetime $p > 0$. Then since $\sigma > 0$, we have from (3.15) and (3.16) that

$$\lambda < -\frac{r}{2} \text{ and } \lambda > \frac{r}{2}.$$

Therefore we can state the following

Theorem 3.9 *If a viscous fluid $A(PRS)_4$ spacetime with positive isotropic pressure obeys Einstein's equation with a cosmological constant λ and when the anisotropic pressure tensor P is of the form $P(X,Y) = D(X,Y)$, then λ satisfies either $\lambda < -\frac{r}{2}$ or $\lambda > \frac{r}{2}$.*

Next we now discuss whether a viscous fluid $A(PRS)_4$ spacetime with generator μ as unit timelike flow vector field can admit heat flux or not. Let us consider the energy momentum tensor T is of the following form

$$T(X,Y) = pg(X,Y) + (\sigma + p)\gamma(X)\gamma(Y) + \gamma(X)\eta(Y) + \eta(X)\gamma(Y) + D(X,Y),$$

where $\eta(X)=g(X,\rho)$ for all vector field X, ρ being the heat flux vector field, σ, p are energy density and isotropic pressure tensor respectively and D denotes the anisotropic pressure tensor of the fluid. Thus we have $g(\mu,\rho)=0$ that is $\eta(\mu)=0$.

Now using this energy momentum tensor in equation (3.1) and then setting $Y=\mu$, we get

$$(\frac{r}{2}+\lambda+k\sigma)g(X,\mu)=-k\eta(X). \tag{3.17}$$

Then putting $X=\mu$ in (3.17), we obtain

$$\frac{r}{2}+\lambda+k\sigma=0$$

Therefore we can write that from (3.16), $\eta(X)=0$ for all X, since $k\neq 0$. Thus we have the following

Theorem 3.10 *A viscous fluid $A(PRS)_4$ can not admit heat flux when the anisotropic pressure tensor P is of the form $P(X,Y)=D(X,Y)$.*

Case 3. In this case, we consider $P(X,Y)=\gamma(X)\gamma(Y)$. Then equation (3.3) becomes

$$S(X,Y)=(\frac{r}{2}+kp-\lambda)g(X,Y)+k(\sigma+p)\gamma(X)\gamma(Y)+k\gamma(X)\gamma(Y).$$

Therefore,

$$S(X,Y)=(\frac{r}{2}+kp-\lambda)g(X,Y)+k(\sigma+p+1)\gamma(X)\gamma(Y). \tag{3.18}$$

From (3.18) it follows that the spacetime under consideration is a quasi-Einstein manifold with $\varrho_2=\frac{r}{2}+kp-\lambda$ and $\zeta_2=k(\sigma+p+1)$ as associate scalars; λ as the associated 1-form with generator μ and the anisotropic pressure tensor P which is of the form $P(X,Y)=\gamma(X)\gamma(Y)$. Hence we can state the following

Theorem 3.11 *A viscous fluid spacetime satisfying Einstein's equation with a cosmological constant is a 4-dimensional quasi-Einstein manifold with generator μ as the flow vector field and p as the isotropic pressure tensor when the anisotropic pressure tensor P is of the form $P(X,Y)=\gamma(X)\gamma(Y)$.*

Next we discuss about the segre characteristic of S, for that after some calculations we get two different eigenvalues of S namely, $\frac{r}{2}-\lambda-k\sigma-k$ and $\frac{r}{2}-\lambda+kp$. Here we see that two eigenvalues are different and for a given eigenvector there is a only one eigenvalue.

Let the multiplicity of $\frac{r}{2}-\lambda-k\sigma-k$ be m and therefore the multiplicity of $\frac{r}{2}-\lambda+kp$ be $(4-m)$ because the dimension of the spacetime is 4.

Hence

$$m(\frac{r}{2}-\lambda-k\sigma-k)+(4-m)(\frac{r}{2}-\lambda+kp)=r.$$

Then after some calculations, we get $m=1$. Thus the multiplicity of the eigenvalue $\frac{r}{2}-\lambda-k\sigma-k$ is 1 and the the multiplicity of the eigenvalue $\frac{r}{2}-\lambda+kp$ is 3. Therefore the segre characteristic of S is [(111),1]. Hence we can state

Theorem 3.12 *If in an almost pseudo Ricci symmetric spacetime of basic vector field μ, the matter content is a viscous fluid with μ as the velocity vector field and when the anisotropic pressure tensor P is of the form $P(X,Y)=\gamma(X)\gamma(Y)$, then the Ricci tensor of the spacetime is segre characteristic [(111),1].*

Putting $Y=\mu$ in equation (3.18), we get $S(X,\mu)=(\frac{r}{2}-\lambda-k\sigma-k)g(X,\mu)$ and on the other hand $S(X,\mu)=rg(X,\mu)$, using this two equations we can write

$$[r-\frac{r}{2}+\lambda+k\sigma+k]g(X,\mu)=0. \tag{3.19}$$

Further setting $X=\mu$ in equation (3.19), we get

$$\sigma=\frac{-2\lambda-r-2k}{2k}. \tag{3.20}$$

Taking contraction on (3.18), we get

$$r=4(\frac{r}{2}-\lambda+kp)-k(\sigma+p+1).$$

Then using (3.20) in the above equation, we can write

$$p=\frac{2\lambda-r}{2k}. \tag{3.21}$$

Here we see that σ and p are constant. Hence we can state the following

Theorem 3.13 *If a viscous fluid $A(PRS)_4$ spacetime obeys Einstein's equation with a cosmological constant and the anisotropic pressure tensor P is of the form $P(X,Y)=\gamma(X)\gamma(Y)$, then energy function and isotropic pressure of the fluid are constants.*

Let us consider in $A(PRS)_4$ spacetime $p>0$. Then since $\sigma>0$, we have from (3.20) and (3.21) that

$$\lambda<-(\frac{r}{2}+k) \text{ and } \lambda>\frac{r}{2}.$$

Therefore we can state the following

Theorem 3.14 *If a viscous fluid $A(PRS)_4$ spacetime with positive isotropic pressure obeys Einstein's equation with a cosmological constant λ and the anisotropic pressure tensor P is of the form $P(X,Y)=\gamma(X)\gamma(Y)$, then λ satisfies either $\lambda<-(\frac{r}{2}+k)$ or $\lambda>\frac{r}{2}$.*

Next we now discuss whether a viscous fluid $A(PRS)_4$ spacetime with generator μ as unit timelike flow vector field can admit heat flux or not. Let us consider the energy momentum tensor T is of the following form

$$T(X,Y)=pg(X,Y)+(\sigma+p)\gamma(X)\gamma(Y)+\gamma(X)\eta(Y)+\eta(X)\gamma(Y)+\gamma(X)\gamma(Y),$$

where $\eta(X)=g(X,\rho)$ for all vector field X, ρ being the heat flux vector field, σ, p are energy density and isotropic pressure tensor respectively and P denotes the anisotropic pressure tensor of the fluid which is of the form $P(X,Y) = \gamma(X)\gamma(Y)$. Thus we have $g(\mu,\rho) = 0$ that is $\eta(\mu) = 0$.

Now using this energy momentum tensor in equation (3.1) and then setting $Y = \mu$, we get

$$(\frac{r}{2} + \lambda + k\sigma + k)g(X,\mu) = -k\eta(X). \tag{3.22}$$

Then putting $X = \mu$ in (3.22), we obtain

$$\frac{r}{2} + \lambda + k\sigma + k = 0$$

Therefore we can write that from (3.22), $\eta(X) = 0$ for all X, since $k \neq 0$. Thus we have the following

Theorem 3.15 *A viscous fluid $A(PRS)_4$ can not admit heat flux when the anisotropic pressure tensor P is of the form $P(X,Y) = \gamma(X)\gamma(Y)$.*

Case 4. Here we consider $P(X,Y) = \gamma(X)\eta(Y) + \eta(X)\gamma(Y)$, where $\eta(X) = g(X,\rho)$ then equation (3.3) become

$$S(X,Y) = (\frac{r}{2} + kp - \lambda)g(X,Y) + k(\sigma + p)\gamma(X)\gamma(Y) + k[\gamma(X)\eta(Y) + \eta(X)\gamma(Y)]. \tag{3.23}$$

From (3.23) it follows that the spacetime under consideration is a generalized quasi-Einstein manifold with $\varrho_3 = \frac{r}{2} + kp - \lambda$, $\zeta_3 = k(\sigma + p)$ and $\xi_2 = k$ as associate scalars; λ as the associated 1-form with generator μ and the anisotropic pressure tensor P which is of the form $P(X,Y) = \gamma(X)\eta(Y) + \eta(X)\gamma(Y)$. Hence we can state the following

Theorem 3.16 *A viscous fluid spacetime satisfying Einstein's equation with a cosmological constant is a 4-dimensional generalized quasi-Einstein manifold with generator μ as the flow vector field and p as the isotropic pressure tensor when the anisotropic pressure tensor P is of the form $P(X,Y) = \gamma(X)\eta(Y) + \eta(X)\gamma(Y)$.*

Next we discuss about the segre characteristic of S, for that after some calculations we does not get two different eigenvalues of S. So, in that case we can't say anything about the segre characteristic of S.

Putting $Y = \mu$ in equation (3.23), we get $S(X,\mu) = (\frac{r}{2} - \lambda - k\sigma)g(X,\mu)$ and on the other hand $S(X,\mu) = rg(X,\mu)$, using this two equations we can write

$$[r - \frac{r}{2} + \lambda + k\sigma]g(X,\mu) = 0. \tag{3.24}$$

Further setting $X = \mu$ in equation (3.24), we get

$$\sigma = \frac{-2\lambda - r}{2k}. \tag{3.25}$$

Taking contraction on (3.23), we get

$$r = 4\lambda - k(3p - \sigma)$$

. Then using (3.25) in the above equation, we can write

$$p = \frac{2\lambda - r}{2k}. \tag{3.26}$$

Here we see that σ and p are constant. Hence we can state the following

Theorem 3.17 *If a viscous fluid $A(PRS)_4$ spacetime obeys Einstein's equation with a cosmological constant and the anisotropic pressure tensor P is of the form $P(X,Y) = \gamma(X)\eta(Y) + \eta(X)\gamma(Y)$, then energy function and isotropic pressure of the fluid are constants.*

Next we consider in $A(PRS)_4$ spacetime $p > 0$. Then since $\sigma > 0$, we have from (3.25) and (3.26) that

$$\lambda < -\frac{r}{2} \text{ and } \lambda > \frac{r}{2}.$$

Therefore we can state the following

Theorem 3.18 *If a viscous fluid $A(PRS)_4$ spacetime with positive isotropic pressure obeys Einstein's equation with a cosmological constant λ and when the anisotropic pressure tensor P is of the form $P(X,Y) = \gamma(X)\eta(Y) + \eta(X)\gamma(Y)$, then λ satisfies either $\lambda < -\frac{r}{2}$ or $\lambda > \frac{r}{2}$.*

Next we now discuss whether a viscous fluid $A(PRS)_4$ spacetime with generator μ as unit timelike flow vector field can admit heat flux or not. Let us consider the energy momentum tensor T is of the following form

$$T(X,Y) = pg(X,Y) + (\sigma + p)\gamma(X)\gamma(Y) + \gamma(X)\psi(Y) + \psi(X)\gamma(Y) + [\gamma(X)\eta(Y) + \eta(X)\gamma(Y)],$$

where $\psi(X) = g(X, \rho)$ for all vector field X, ρ being the heat flux vector field, σ, p are energy density and isotropic pressure tensor respectively and P denotes the anisotropic pressure tensor of the fluid which is of the form $P(X,Y) = \gamma(X)\eta(Y) + \eta(X)\gamma(Y)$. Thus we have $g(\mu, \rho) = 0$ that is $\psi(\mu) = 0$.

Now using this energy momentum tensor in equation (3.1) and then setting $Y = \mu$, we get

$$\left(\frac{r}{2} + \lambda + k\sigma\right) g(X, \mu) = -k(\eta(X) + \psi(X)). \tag{3.27}$$

Then putting $X = \mu$ in (3.27), we obtain

$$\frac{r}{2} + \lambda + k\sigma = 0.$$

So we can write that from (3.27), $\psi(X) = 0$ for all X since $k \neq 0$ and obtain the following

Theorem 3.19 *A viscous fluid $A(PRS)_4$ can not admit heat flux when the anisotropic pressure tensor P is of the form $P(X,Y) = \gamma(X)\eta(Y) + \eta(X)\gamma(Y)$.*

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