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Total Mean Cordial Labeling of Graphs

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Abstract: In this paper, we introduce a new type of graph labeling known as total mean cordial labeling. A total mean cordial labeling of a graph $G = (V, E)$ is a mapping $f : V(G) \to \{0, 1, 2\}$ such that $f(xy) = \left\lceil \frac{f(x)+f(y)}{2} \right\rceil$ where $x, y \in V(G)$, $xy \in G$, and the total number of 0, 1 and 2 are balanced. That is $|ev_f(i) - ev_f(j)| \leq 1$, $i, j \in \{0, 1, 2\}$ where $ev_f(x)$ denotes the total number of vertices and edges labeled with x $(x = 0, 1, 2)$. If there exists a total mean cordial labeling on a graph G, we will call G is Total Mean Cordial. In this paper, we study some classes of graphs and their Total Mean Cordial behaviour.

Key Words: Smarandachely total mean cordial labeling, total mean cordial labeling, path, cycle, wheel, complete graph, complete bipartite graph.

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§1. Introduction

Unless mentioned otherwise, a graph in this paper shall mean a simple finite and undirected. For all terminology and notations in graph theory, we follow Harary [3]. The vertex and edge set of a graph G are denoted by $V(G)$ and $E(G)$ so that the order and size of G are respectively $|V(G)|$ and $|E(G)|$. Graph labeling is an assignment of integers to the vertices or edges, or both, subject to certain conditions. Graph labeling plays an important role of various fields of science and few of them are astronomy, coding theory, x-ray crystallography, radar, circuit design, communication network addressing, database management, secret sharing schemes, and models for constraint programming over finite domains [2]. The graph labeling problem was introduced by Rosa and he has introduced graceful labeling of graphs [5] in the year 1967. In 1980, Cahit [1] introduced the cordial labeling of graphs. In 2012, Ponraj et al. [6] introduced the notion of mean cordial labeling. Motivated by these labelings, we introduce a new type of labeling, called total mean cordial labeling. In this paper, we investigate the total mean cordial labeling behaviour of some graphs like path, cycle, wheel, complete graph etc. Let x be any real number. Then the symbol $\lfloor x \rfloor$ stands for the largest integer less than or equal to x and $\lceil x \rceil$ stands for the smallest integer greater than or equal to x.

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§2. Main Results

Definition 2.1 *Let f be a function f from $V(G) \to \{0,1,2\}$. For each edge uv, assign the label $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. Then, f is called a total mean cordial labeling if $|ev_f(i) - ev_f(j)| \leq 1$ where $ev_f(x)$ denote the total number of vertices and edges labeled with $x_(x = 0,1,2)$. A graph with a total mean cordial labeling is called total mean cordial graph.*

Furthermore, let $H \leq G$ be a subgraph of G. If there is a function f from $V(G) \to \{0,1,2\}$ such that $f|_H$ is a total mean cordial labeling but $\left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ is a constant for all edges in $G \setminus H$, such a labeling and G are then respectively called Smarandachely total mean cordial labeling and Smarandachely total mean cordial labeling graph respect to H.

Theorem 2.2 *Any Path P_n is total mean cordial.*

Proof Let $P_n : u_1 u_2 \cdots u_n$ be the path.

Case 1. $n \equiv 0\ (mod\ 3)$.

Let $n = 3t$. Define a map $f : V(P_n) \to \{0,1,2\}$ by

$$\begin{cases} f(u_i) & = & 0 & 1 \leq i \leq t \\ f(u_{t+i}) & = & 1 & 1 \leq i \leq t \\ f(u_{2t+i}) & = & 2 & 1 \leq i \leq t. \end{cases}$$

Case 2. $n \equiv 1\ (mod\ 3)$.

Let $n = 3t+1$. Define a function $f : V(P_n) \to \{0,1,2\}$ by

$$\begin{cases} f(u_i) & = & 0 & 1 \leq i \leq t+1 \\ f(u_{t+1+i}) & = & 1 & 1 \leq i \leq t \\ f(u_{2t+1+i}) & = & 2 & 1 \leq i \leq t. \end{cases}$$

Case 3. $n \equiv 2\ (mod\ 3)$.

Let $n = 3t+2$. Define a function $f : V(P_n) \to \{0,1,2\}$ by

$$\begin{cases} f(u_i) & = & 0 & 1 \leq i \leq t+1 \\ f(u_{t+1+i}) & = & 1 & 1 \leq i \leq t \\ f(u_{2t+1+i}) & = & 2 & 1 \leq i \leq t \end{cases}$$

and $f(u_{3t+2}) = 1$. The following table Table 1 shows that the above vertex labeling f is a total mean cordial labeling.

Nature of n	$ev_f(0)$	$ev_f(1)$	$ev_f(2)$
$n \equiv 0\ (mod\ 3)$	$2t-1$	$2t$	$2t$
$n \equiv 1\ (mod\ 3)$	$2t+1$	$2t$	$2t$
$n \equiv 2\ (mod\ 3)$	$2t+1$	$2t+1$	$2t+1$

Table 1

This completes the proof. □

Theorem 2.3 *The cycle C_n is total mean cordial if and only if $n \neq 3$.*

Proof Let $C_n : u_1u_2\ldots u_nu_1$ be the cycle. If $n = 3$, then we have $ev_f(0) = ev_f(1) = ev_f(2) = 2$. But this is an impossible one. Assume $n > 3$.

Case 1. $n \equiv 0\ (mod\ 3)$.

Let $n = 3t$, $t > 1$. The labeling given in Figure 1 shows that C_6 is total mean cordial.

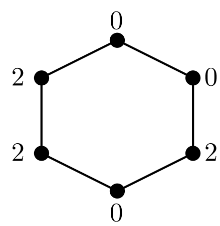

Figure 1

Take $t \geq 3$. Define $f : V(C_n) \to \{0, 1, 2\}$ by

$$\begin{cases} f(u_i) = 0 & 1 \leq i \leq t \\ f(u_{t+i}) = 2 & 1 \leq i \leq t \\ f(u_{2t+i}) = 1 & 1 \leq i \leq t-2. \end{cases}$$

and $f(u_{3t-1}) = 0$, $f(u_{3t}) = 1$. In this case $ev_f(0) = ev_f(1) = ev_f(2) = 2t$.

Case 2. $n \equiv 1\ (mod\ 3)$.

The labeling f defined in case 2 of Theorem 2.1 is a total mean cordial labeling of here also. In this case, $ev_f(0) = ev_f(1) = 2t + 1$, $ev_f(2) = 2t$.

Case 3. $n \equiv 2\ (mod\ 3)$.

The labeling f defined in case 3 of Theorem 2.1 is a total mean cordial labeling. Here, $ev_f(0) = ev_f(2) = 2t + 1$, $ev_f(1) = 2t + 2$. □

The following three lemmas 2.4−2.6 are used for investigation of total mean cordial labeling of complete graphs.

Lemma 2.4 *There are infinitely many values of n for which $12n^2 + 12n + 9$ is not a perfect square.*

Proof Suppose $12n^2+12n+9$ is a square, α^2, say. Then $3/\alpha$. So $\alpha=3\beta$. This implies $12n^2+12n+9=9\beta^2$. Hence we obtain $4n^2+4n+3=3\beta^2$. On rewriting, we have $(2n+1)^2-3\beta^2=-2$. Substituting $2n+1=U$, $\beta=V$, we get the Pell's equation $U^2-3V^2=-2$. The fundamental solutions of the equations $U^2-3V^2=-2$ and $A^2-3B^2=1$ are $1+\sqrt{3}$ and $2+\sqrt{3}$, respectively. Therefore, all the integral solutions $u_k+\sqrt{3}v_k$ of the equation $U^2-3V^2=-2$ are given by $(1+\sqrt{3})(2+\sqrt{3})^k$, where $k=0,\pm1,\pm2,\cdots$ Applying the result of Mohanty and Ramasamy [4] on Pell's equation, it is seen that the solutions $u_k+\sqrt{3}v_k$ of the equation $U^2-3V^2=-2$ are proved by the recurrence relationships $u_0=-1$, $u_1=1$, $u_{k+2}=4u_{k+1}-u_k$ and $v_0=1$, $v_1=1$, $v_{k+2}=4v_{k+1}-v_k$. Hence the square values of $12n^2+12n+9$ are given by the sequence $\{n_k\}$ where $n_1=0$, $n_2=2$, $n_{k+2}=4n_{k+1}-n_k+1$. It follows that such of those integers of the form $12m^2+12m+9$ which are not in the sequence $\{n_k\}$ are not perfect squares. □

Lemma 2.5 *There are infinitely many values of n for which* $12n^2+12n-15$ *is not a perfect square.*

Proof As in Lemma 2.4 the square values of $12n^2+12n-15$ are given by the sequence $\{n_k\}$ where $n_1=1$, $n_2=4$, $n_{k+2}=4n_{k+1}-n_k+1$. It follows that such of those integers of the form $12m^2+12m-15$ which are not in the sequence $\{n_k\}$ are not perfect squares. □

Lemma 2.6 *There are infinitely many values of n for which* $12n^2+12n+57$ *is not a perfect square.*

Proof As in Lemma 2.4 the square values of $12n^2+12n+57$ are given by the sequence $\{n_k\}$ where $n_1=1$, $n_2=7$, $n_{k+2}=4n_{k+1}-n_k+1$. It follows that such of those integers of the form $12m^2+12m-15$ which are not in the sequence $\{n_k\}$ are not perfect squares. □

Theorem 2.7 *If* $n\equiv 0,2\ (mod\ 3)$ *and* $12n^2+12n+9$ *is not a perfect square then the complete graph* K_n *is not total mean cordial.*

Proof Suppose f is a total mean cordial labeling of K_n. Clearly $|V(K_n)|+|E(K_n)|=\frac{n(n+1)}{2}$. If $n\equiv 0,2\ (\mathrm{mod}\ 3)$ then 3 divides $\frac{n(n+1)}{2}$. Clearly $ev_f(0)=m+\binom{m}{2}$ where $m\in\mathbb{N}$. Then

$$\frac{m(m+1)}{2}=\frac{n(n+1)}{6}$$

$$\Longrightarrow\qquad m=\frac{-3\pm\sqrt{12n^2+12n+9}}{2},$$

a contradiction since $12n^2+12n+9$ is not a perfect square. □

Theorem 2.8 *If* $n\equiv 1\ (mod\ 3)$, $12n^2+12n-15$ *and* $12n^2+12n+57$ *are not perfect squares then the complete graph* K_n *is not total mean cordial.*

Proof Suppose there exists a total mean cordial labeling of K_n, say f. It is clear that $ev_f(0)=\frac{n^2+n-2}{6}$ or $ev_f(0)=\frac{n^2+n+4}{6}$.

Case 1. $ev_f(0) = \dfrac{n^2+n-2}{6} = m.$

Suppose k zeros are used in the vertices. Then $k + \binom{k}{2} = m$ where $k \in \mathbb{N}$.

$$\begin{aligned} \Longrightarrow \quad & k(k+1) = \frac{n^2+n-2}{3} \\ \Longrightarrow \quad & 3k^2+3k-(n^2+n-2) = 0 \\ \Longrightarrow \quad & k = \frac{-3 \pm \sqrt{12n^2+12n-15}}{6}. \end{aligned}$$

a contradiction since $12n^2+12n-15$ is not a perfect square.

Case 2. $ev_f(0) = \dfrac{n^2+n+4}{6} = m.$

Suppose k zeros are used in the vertices. Then $k + \binom{k}{2} = m$ where $k \in \mathbb{N}$.

$$\begin{aligned} \Longrightarrow \quad & k(k+1) = \frac{n^2+n+4}{3} \\ \Longrightarrow \quad & 3k^2+3k-(n^2+n+4) = 0 \\ \Longrightarrow \quad & k = \frac{-3 \pm \sqrt{12n^2+12n+57}}{6}. \end{aligned}$$

a contradiction since $12n^2+12n+57$ is not a perfect square. □

Theorem 2.9 *The complete graph K_n is not total mean cordial for infinitely many values of n.*

Proof Proof follow from Lemmas 2.4 – 2.6 and Theorems 2.7 – 2.8. □

Theorem 2.10 *The star $K_{1,n}$ is total mean cordial.*

Proof Let $V(K_{1,n}) = \{u, u_i : 1 \le i \le n\}$ and $E(K_{1,n}) = \{uu_i : 1 \le i \le n\}$. Define a map $f : V(K_{1,n}) \to \{0,1,2\}$ by $f(u) = 0$,

$$\begin{cases} f(u_i) & = & 0 & 1 \le i \le \lfloor \frac{n}{3} \rfloor \\ f(u_{\lfloor \frac{n}{3} \rfloor + i}) & = & 2 & 1 \le i \le \lceil \frac{2n}{3} \rceil \end{cases}$$

The Table 2 shows that f is a total mean cordial labeling.

Values of n	$ev_f(0)$	$ev_f(1)$	$ev_f(2)$
$n \equiv 0\ (mod\ 3)$	$\frac{2n+3}{3}$	$\frac{2n}{3}$	$\frac{2n}{3}$
$n \equiv 1\ (mod\ 3)$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$	$\frac{2n+1}{3}$
$n \equiv 2\ (mod\ 3)$	$\frac{2n-1}{3}$	$\frac{2n+2}{3}$	$\frac{2n+2}{3}$

Table 2

This completes the proof. □

The join of two graphs G_1 and G_2 is denoted by $G_1 + G_2$ with

$$V(G_1+G_2) = V(G_1) \cup V(G_2),$$
$$E(G_1+G_2) = E(G_1) \cup E(G_2) \cup \{uv : u \in V(G_1), v \in V(G_2)\}.$$

Theorem 2.11 *The wheel $W_n = C_n + K_1$ is total mean cordial if and only if $n \neq 4$.*

Proof Let $C_n : u_1u_2 \ldots u_nu_1$ be the cycle. Let $V(W_n) = V(C_n) \cup \{u\}$ and $E(W_n) = E(C_n) \cup \{uu_i : 1 \leq i \leq n\}$. Here $|V(W_n)| = n+1$ and $|E(W_n)| = 2n$.

Case 1. $n \equiv 0\ (mod\ 6)$.

Let $n = 6k$ where $k \in \mathbb{N}$. Define a map $f : V(W_n) \to \{0,1,2\}$ by $f(u) = 0$,

$$\begin{cases} f(u_i) & = & 0 & 1 \leq i \leq 2k \\ f(u_{5k+i}) & = & 1 & 1 \leq i \leq k \\ f(u_{2k+i}) & = & 2 & 1 \leq i \leq 3k. \end{cases}$$

In this case, $ev_f(0) = ev_f(2) = 6k$, $ev_f(1) = 6k+1$.

Case 2. $n \equiv 1\ (mod\ 6)$.

Let $n = 6k-5$ where $k \in \mathbb{N}$ and $k > 1$. Suppose $k = 2$ then the Figure 2 shows that W_7 is total mean cordial.

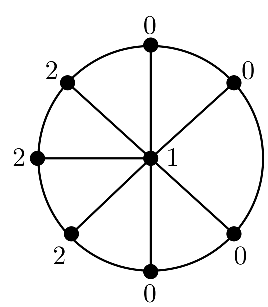

Figure 2

Assume $k > 2$. Define a function $f : V(W_n) \to \{0,1,2\}$ by $f(u) = 0$ and

$$f(u_i) = \begin{cases} 0 & \text{if} & 1 \leq i \leq 2k-2\ \ \&\ \ i = 5k-3 \\ 1 & \text{if} & 5k-2 \leq i \leq 6k-5 \\ 2 & \text{if} & 2k-1 \leq i \leq 5k-4. \end{cases}$$

It is clear that $ev_f(0) = 6k-4$, $ev_f(1) = ev_f(2) = 6k-5$.

Case 3. $n \equiv 2\ (mod\ 6)$.

Let $n = 6k - 4$ where $k \in \mathbb{N}$ and $k > 1$. Define $f : V(W_n) \to \{0, 1, 2\}$ by $f(u) = 0$ and

$$f(u_i) = \begin{cases} 0 & \text{if} \quad 1 \leq i \leq 2k-1 \\ 1 & \text{if} \quad 5k-2 \leq i \leq 6k-4 \\ 2 & \text{if} \quad 2k \leq i \leq 5k-3. \end{cases}$$

Note that $ev_f(0) = 6k - 3$, $ev_f(1) = ev_f(2) = 6k - 4$.

Case 4. $n \equiv 3$ $(mod\ 6)$.

Let $n = 6k - 3$ where $k \in \mathbb{N}$. Define a function $f : V(W_n) \to \{0, 1, 2\}$ by $f(u) = 0$,

$$\begin{cases} f(u_i) & = & 0 & 1 \leq i \leq 2k-1 \\ f(u_{5k-2+i}) & = & 1 & 1 \leq i \leq k-1 \\ f(u_{2k-1+i}) & = & 2 & 1 \leq i \leq 3k-1. \end{cases}$$

In this case $ev_f(0) = ev_f(1) = 6k - 3$, $ev_f(2) = 6k - 2$.

Case 5. $n \equiv 4$ $(mod\ 6)$.

When $n = 4$ it is easy to verify that the total mean cordiality condition is not satisfied. Let $n = 6k - 2$ where $k \in \mathbb{N}$ and $k > 1$. From Figure 3, it is clear that $ev_f(0) = 11$, $ev_f(1) = ev_f(2) = 10$ and hence W_{10} is total mean cordial.

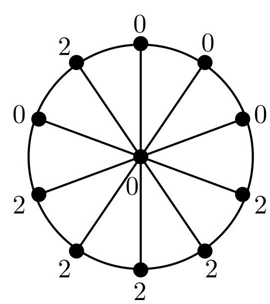

Figure 3

Let $k \geq 3$. Define a function $f : V(W_n) \to \{0, 1, 2\}$ by $f(u) = 0$, $f(u_{6k-3}) = 0$, $f(u_{6k-2}) = 1$ and

$$\begin{cases} f(u_i) & = & 0 & 1 \leq i \leq 2k-1 \\ f(u_{5k-2+i}) & = & 1 & 1 \leq i \leq k-2 \\ f(u_{2k-1+i}) & = & 2 & 1 \leq i \leq 3k-1. \end{cases}$$

In this case $ev_f(0) = 6k - 1$, $ev_f(1) = ev_f(2) = 6k - 2$.

Case 6. $n \equiv 5$ $(mod\ 6)$.

Let $n = 6k - 1$ where $k \in \mathbb{N}$. For $k = 1$ the Figure 4 shows that W_5 is total mean cordial.

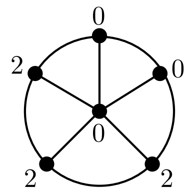

Figure 4

Assume $k \geq 2$. Define a function $f : V(W_n) \to \{0,1,2\}$ by $f(u) = 0$ and

$$f(u_i) = \begin{cases} 0 & \text{if} \quad 1 \leq i \leq 2k \\ 1 & \text{if} \quad 5k+1 \leq i \leq 6k-1 \;\; \& \;\; i = 2k+1 \\ 2 & \text{if} \quad 2k+2 \leq i \leq 5k. \end{cases}$$

It is clear that $ev_f(0) = 6k$, $ev_f(1) = ev_f(2) = 6k - 1$. □

Theorem 2.12 *$K_2 + mK_1$ is total mean cordial if and only if m is even.*

Proof Clearly $|V(K_2 + mK_1)| = 3m + 3$. Let $V(K_2 + mK_1) = \{u, v, u_i : 1 \leq i \leq m\}$ and $E(K_2 + mK_1) = \{uv, uu_i, vu_i : 1 \leq i \leq m\}$.

Case 1. m is even.

Let $m = 2t$. Define $f : V(K_2 + mK_1) \to \{0,1,2\}$ by $f(u) = 0$, $f(v) = 2$

$$\begin{cases} f(u_i) & = & 0 & 1 \leq i \leq t \\ f(u_{t+i}) & = & 2 & 1 \leq i \leq t. \end{cases}$$

Then $ev_f(0) = ev_f(1) = ev_f(2) = 2t + 1$ and hence f is a total mean cordial labeling.

Case 2. m is odd.

Let $m = 2t + 1$. Suppose f is a total mean cordial labeling, then $ev_f(0) = ev_f(1) = ev_f(2) = 2t + 2$.

Subcase 1. $f(u) = 0$ and $f(v) = 0$.

Then $ev_f(2) \leq 2t + 1$, a contradiction.

Subcase 2. $f(u) = 0$ and $f(v) \neq 0$.

Since the vertex u has label 0, we have only $2t + 1$ zeros. While counting the total number of zeros each vertices u_i along with the edges uu_i contributes 2 zeros. This implies $ev_f(0)$ is an odd number, a contradiction.

Subcase 3. $f(u) \neq 0$ and $f(v) \neq 0$.

Then $ev_f(0) \leq 2t + 1$, a contradiction. □

The corona of G with H, $G \odot H$ is the graph obtained by taking one copy of G and p copies of H and joining the i^{th} vertex of G with an edge to every vertex in the i^{th} copy of H. $C_n \odot K_1$ is called the crown, $P_n \odot K_1$ is called the comb and $P_n \odot 2K_1$ is called the double comb.

Theorem 2.13 *The comb $P_n \odot K_1$ admits a total mean cordial labeling.*

Proof Let $P_n : u_1u_2 \ldots u_n$ be the path. Let $V(P_n \odot K_1) = \{V(P_n) \cup \{v_i : 1 \leq i \leq n\}$ and $E(P_n \odot K_1) = E(P_n) \cup \{u_iv_i : 1 \leq i \leq n\}$. Note that $|V(P_n \odot K_1)| + |E(P_n \odot K_1)| = 4n - 1$.

Case 1. $n \equiv 0\ (mod\ 3)$.

Let $n = 3t$. Define a map $f : V(P_n \odot K_1) \to \{0, 1, 2\}$ by

$$\begin{cases} f(u_i) & = & 0 & 1 \leq i \leq 2t \\ f(u_{2t+i}) & = & 1 & 1 \leq i \leq t \\ f(v_i) & = & 2 & 1 \leq i \leq 3t. \end{cases}$$

Case 2. $n \equiv 1\ (mod\ 3)$.

Let $n = 3t + 1$. Define a function $f : V(P_n \odot K_1) \to \{0, 1, 2\}$ by

$$\begin{cases} f(u_i) & = & 0 & 1 \leq i \leq 2t+1 \\ f(u_{2t+1+i}) & = & 1 & 1 \leq i \leq t \\ f(v_i) & = & 2 & 1 \leq i \leq 3t+1. \end{cases}$$

Case 3. $n \equiv 2\ (mod\ 3)$.

Let $n = 3t + 2$. Define a function $f : V(P_n \odot K_1) \to \{0, 1, 2\}$ by

$$\begin{cases} f(u_i) & = & 0 & 1 \leq i \leq 2t+2 \\ f(u_{2t+2+i}) & = & 1 & 1 \leq i \leq t \\ f(v_i) & = & 2 & 1 \leq i \leq 3t+2. \end{cases}$$

From Table 3 it is easy that the labeling f is a total mean cordial labeling.

Nature of n	$ev_f(0)$	$ev_f(1)$	$ev_f(2)$
$n \equiv 0\ (mod\ 3)$	$4t - 1$	$4t$	$4t$
$n \equiv 1\ (mod\ 3)$	$4t + 1$	$4t + 1$	$4t + 1$
$n \equiv 2\ (mod\ 3)$	$4t + 3$	$4t + 2$	$4t + 2$

Table 3

This completes the proof. □

Theorem 2.14 *The double comb $P_n \odot 2K_1$ is total mean cordial.*

Proof Let $P_n : u_1u_2 \ldots u_n$ be the path. Let $V(P_n \odot 2K_1) = \{V(P_n) \cup \{v_i, w_i : 1 \leq$

$i \leq n\}$ and $E(P_n \odot 2K_1) = E(P_n) \cup \{u_i v_i, u_i w_i : 1 \leq i \leq n\}$. Note that $|V(P_n \odot 2K_1)| + |E(P_n \odot 2K_1)| = 6n - 1$.

Case 1. $n \equiv 0\ (mod\ 3)$.

Let $n = 3t$. Define a map $f : V(P_n \odot 2K_1) \to \{0, 1, 2\}$ by

$$\begin{cases} f(u_i) = f(v_i) = f(w_i) = 0 & 1 \leq i \leq t \\ f(u_{t+i}) = f(v_{t+i}) = f(w_{t+i}) = 1 & 1 \leq i \leq t \\ f(u_{2t+i}) = f(v_{2t+i}) = f(w_{2t+i}) = 2 & 1 \leq i \leq t \end{cases}$$

Case 2. $n \equiv 1\ (mod\ 3)$.

Let $n = 3t + 1$. Define a function $f : V(P_n \odot 2K_1) \to \{0, 1, 2\}$ by

$$\begin{cases} f(u_i) = 0 & 1 \leq i \leq t+1 \\ f(u_{t+1+i}) = 1 & 1 \leq i \leq t \\ f(u_{2t+1+i}) = 2 & 1 \leq i \leq t \\ f(v_i) = 0 & 1 \leq i \leq t \\ f(v_{t+i}) = 1 & 1 \leq i \leq t+1 \\ f(v_{2t+1+i}) = 2 & 1 \leq i \leq t \\ f(w_i) = 0 & 1 \leq i \leq t \\ f(w_{t+i}) = 1 & 1 \leq i \leq t \\ f(w_{2t+i}) = 2 & 1 \leq i \leq t+1 \end{cases}$$

Case 3. $n \equiv 2\ (mod\ 3)$.

Let $n = 3t + 2$. Define a function $f : V(P_n \odot 2K_1) \to \{0, 1, 2\}$ by

$$\begin{cases} f(u_i) = 0 & 1 \leq i \leq t+1 \\ f(u_{t+1+i}) = 1 & 1 \leq i \leq t+1 \\ f(u_{2t+2+i}) = 2 & 1 \leq i \leq t \\ f(v_i) = 0 & 1 \leq i \leq t+1 \\ f(v_{t+1+i}) = 1 & 1 \leq i \leq t \\ f(v_{2t+1+i}) = 2 & 1 \leq i \leq t+1 \\ f(w_i) = 0 & 1 \leq i \leq t \\ f(w_{t+i}) = 1 & 1 \leq i \leq t+1 \\ f(w_{2t+1+i}) = 2 & 1 \leq i \leq t+1 \end{cases}$$

The Table 4 shows that the labeling f is a total mean cordial labeling.

Nature of n	$ev_f(0)$	$ev_f(1)$	$ev_f(2)$
$n \equiv 0\ (mod\ 3)$	$6t-1$	$6t$	$6t$
$n \equiv 1\ (mod\ 3)$	$6t+1$	$6t+2$	$6t+2$
$n \equiv 2\ (mod\ 3)$	$6t+3$	$6t+4$	$6t+4$

Table 4

This completes the proof. □

Theorem 2.15 *The crown $C_n \odot K_1$ is total mean cordial.*

Proof Let $C_n : u_1u_2 \ldots u_nu_1$ be the cycle. Let $V(C_n \odot K_1) = \{V(C_n) \cup \{v_i : 1 \leq i \leq n\}$ and $E(C_n \odot K_1) = E(C_n) \cup \{u_iv_i : 1 \leq i \leq n\}$. Note that $|V(C_n \odot K_1)| + |E(C_n \odot K_1)| = 4n$.

Case 1. $n \equiv 0\ (mod\ 3)$.

Let $n = 3t$. For $t = 1$ we refer Figure 5.

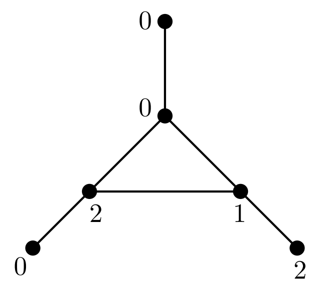

Figure 5

Let $t > 1$. Define a map $f : V(C_n \odot K_1) \to \{0, 1, 2\}$ by

$$\begin{cases} f(u_i) & = & f(v_i) & = & 0 & 1 \leq i \leq t \\ f(u_{t+i}) & = & f(v_{t+i}) & = & 1 & 1 \leq i \leq t-1 \\ f(u_{2t-1+i}) & = & f(u_{2t-1+i}) & = & 2 & 1 \leq i \leq t-1 \end{cases}$$

and $f(u_{3t-1}) = 2$, $f(u_{3t}) = 1$, $f(v_{3t-1}) = 1$, $f(v_{3t}) = 0$. Here $ev_f(0) = ev_f(1) = ev_f(2) = 4t$.

Case 2. $n \equiv 1\ (mod\ 3)$.

The labeling f defined in case 2 of Theorem 2.13 is a total mean cordial labeling. Here, $ev_f(0) = 4t + 1$, $ev_f(1) = 4t + 2$, $ev_f(2) = 4t + 1$.

Case 3. $n \equiv 2\ (mod\ 3)$.

The labeling f defined in case 3 of Theorem 2.13 is a total mean cordial labeling. Here, $ev_f(0) = ev_f(1) = 4t + 3$, $ev_f(2) = 4t + 2$. □

The triangular snake T_n is obtained from the path P_n by replacing every edge of the path by a triangle.

Theorem 2.16 *The triangular snake T_n is total mean cordial if and only if $n > 2$.*

Proof Let $P_n : u_1u_2 \ldots u_n$ be the path and $V(T_n) = V(P_n) \cup \{v_i : 1 \leq i \leq n-1\}$. Let $E(T_n) = E(P_n) \cup \{u_iv_i, v_iu_{i+1} : 1 \leq i \leq n-1\}$. If $n = 2$, $T_2 \cong C_3$, by Theorem 2.3, T_2 is not total mean cordial. Let $n \geq 3$. Here $|V(T_n)| + |E(T_n)| = 5n - 4$.

Case 1. $n \equiv 0\ (mod\ 3)$.

Let $n = 3t$. For T_3, the vertex labeling in Figure 6 is a total mean cordial labeling.

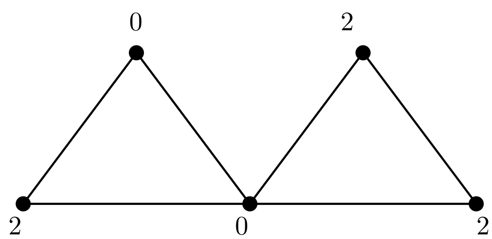

Figure 6

Let $t \geq 2$. Define a map $f : V(T_n) \to \{0, 1, 2\}$ by

$$\begin{cases} f(u_i) & = & 0 & 1 \leq i \leq t \\ f(u_{t+i}) & = & 1 & 1 \leq i \leq t \\ f(u_{2t+i}) & = & 2 & 1 \leq i \leq t-1 \\ f(v_i) & = & 0 & 1 \leq i \leq t \\ f(v_{t+i}) & = & 1 & 1 \leq i \leq t-1 \\ f(v_{2t-1+i}) & = & 2 & 1 \leq i \leq t \end{cases}$$

and $f(u_{3t}) = 1$.

Case 2. $n \equiv 1\ (mod\ 3)$.

Let $n = 3t + 1$. Define $f : V(T_n) \to \{0, 1, 2\}$ by

$$\begin{cases} f(u_i) & = & 0 & 1 \leq i \leq t+1 \\ f(u_{t+1+i}) & = & 1 & 1 \leq i \leq t \\ f(u_{2t+1+i}) & = & 2 & 1 \leq i \leq t \\ f(v_i) & = & 0 & 1 \leq i \leq t \\ f(v_{t+i}) & = & 1 & 1 \leq i \leq t \\ f(v_{2t+i}) & = & 2 & 1 \leq i \leq t \end{cases}$$

Case 3. $n \equiv 2\ (mod\ 3)$.

Let $n = 3t + 2$. Define $f : V(T_n) \to \{0, 1, 2\}$ by

$$\begin{cases} f(u_i) & = & 0 & 1 \le i \le t+1 \\ f(u_{t+1+i}) & = & 2 & 1 \le i \le t \\ f(u_{2t+1+i}) & = & 1 & 1 \le i \le t \\ f(v_i) & = & 0 & 1 \le i \le t \\ f(v_{t+i}) & = & 2 & 1 \le i \le t+1 \\ f(v_{2t+1+i}) & = & 1 & 1 \le i \le t \end{cases}$$

and $f(u_{3t+2}) = 0$. The Table 5 shows that T_n is total mean cordial.

Nature of n	$ev_f(0)$	$ev_f(1)$	$ev_f(2)$
$n \equiv 0\ (mod\ 3)$	$5t-2$	$5t-1$	$5t-1$
$n \equiv 1\ (mod\ 3)$	$5t+1$	$5t$	$5t$
$n \equiv 2\ (mod\ 3)$	$5t+2$	$5t+2$	$5t+2$

Table 5

This completes the proof. □

References

[1] I.Cahit, Cordial graphs: a weaker version of graceful and harmonious graphs, *Ars combin.*, 23(1987) 201-207.

[2] J.A.Gallian, A dynamic survey of graph labeling, *The Electronic Journal of Combinatorics*, 16(2013) # Ds6.

[3] F.Harary, *Graph theory*, Addision wesley, New Delhi, 1969.

[4] S.P.Mohanty and A.M.S.Ramasamy, The characteristic number of two simultaneous Pell's equations and its applications, *Simon Stevin*, 59(1985), 203-214.

[5] A.Rosa, On certain valuations of the vertices of a graph, *Theory of Graphs (Internat. Symposium, Rome, July 1966)*, Gordon and Breach N.Y. and Dunod Paris (1967) 349-355.

[6] R.Ponraj, M.Sivakumar and M.Sundaram, Mean Cordial labeling of graphs, *Open Journal of Discrete Mathematics*, 2(2012), 145-148.