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On Super (a, d)-Edge-Antimagic Total Labeling of a Class of Trees

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Abstract: The concept of labeling has its origin in the works of Stewart (1966), Kotzig and Rosa (1970). Later on Enomoto, Llado, Nakamigawa and Ringel (1998) defined a super $(a, 0)$-edge-antimagic total labeling and proposed the conjecture that every tree is a super $(a, 0)$-edge-antimagic total graph. In the favour of this conjecture, the present paper deals with different results on antimagicness of a class of trees, which is called subdivided stars.

Key Words: Smarandachely super (a, d)-edge-antimagic total labeling, super (a, d)-edge-antimagic total labeling, stars and subdivision of stars.

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§1. Introduction

All graphs in this paper are finite, undirected and simple. For a graph G, $V(G)$ and $E(G)$ denote the vertex-set and the edge-set, respectively. A (v, e)-graph G is a graph such that $|V(G)| = v$ and $|E(G)| = e$. A general reference for graph-theoretic ideas can be seen in [28]. A *labeling* (or *valuation*) of a graph is a map that carries graph elements to numbers (usually to positive or non-negative integers). In this paper, the domain will be the set of all vertices and edges and such a labeling is called *a total labeling*. Some labelings use the vertex-set only or the edge-set only and we shall call them *vertex-labelings* or *edge-labelings*, respectively.

Definition 1.1 *An (s, d)-edge-antimagic vertex (abbreviated to (s, d)-EAV) labeling of a (v, e)-graph G is a bijective function $\lambda : V(G) \rightarrow \{1, 2, \cdots, v\}$ such that the set of edge-sums of all edges in G, $\{w(xy) = \lambda(x) + \lambda(y) : xy \in E(G)\}$, forms an arithmetic progression $\{s, s+d, s+2d, \cdots, s+(e-1)d\}$, where $s > 0$ and $d \geqslant 0$ are two fixed integers.*

Furthermore, let $H \leq G$. If there is a bijective function $\lambda : V(H) \rightarrow \{1, 2, \cdots, |H|\}$

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such that the set of edge-sums of all edges in H forms an arithmetic progression $\{s, s+d, s+2d, \cdots, s+(|E(H)|-1)d\}$ but for all edges not in H is a constant, such a labeling is called a Smarandachely (s,d)-edge-antimagic labeling of G respect to H. Clearly, an (s,d)-EAV labeling of G is a Smarandachely (s,d)-EAV labeling of G respect to G itself.

Definition 1.2 *A bijection $\lambda : V(G)\cup E(G) \to \{1,2,\cdots,v+e\}$ is called an (a,d)-edge-antimagic total ((a,d)-EAT) labeling of a (v,e)-graph G if the set of edge-weights $\{\lambda(x)+\lambda(xy)+\lambda(y) : xy \in E(G)\}$ forms an arithmetic progression starting from a and having common difference d, where $a > 0$ and $d \geq 0$ are two chosen integers. A graph that admits an (a,d)-EAT labeling is called an (a,d)-EAT graph.*

Definition 1.3 *If λ is an (a,d)-EAT labeling such that $\lambda(V(G)) = \{1,2,\cdots,v\}$ then λ is called a super (a,d)-EAT labeling and G is known as a super (a,d)-EAT graph.*

In Definitions 1.2 and 1.3, if $d = 0$ then an $(a,0)$-EAT labeling is called an edge-magic total (EMT) labeling and a super $(a,0)$-EAT labeling is called a super edge magic total (SEMT) labeling. Moreover, in general a is called minimum edge-weight but particularly magic constant when $d = 0$. The definition of an (a,d)-EAT labeling was introduced by Simanjuntak, Bertault and Miller in [23] as a natural extension of *magic valuation* defined by Kotzig and Rosa [17-18]. A super (a,d)-EAT labeling is a natural extension of the notion of *super edge-magic labeling* defined by Enomoto, Llado, Nakamigawa and Ringel. Moreover, Enomoto et al. [8] proposed the following conjecture.

Conjecture 1.1 *Every tree admits a super $(a,0)$-EAT labeling.*

In the favor of this conjecture, many authors have considered a super $(a,0)$-EAT labeling for different particular classes of trees. Lee and Shah [19] verified this conjecture by a computer search for trees with at most 17 vertices. For different values of d, the results related to a super (a,d)-EAT labeling can be found for w-trees [13], stars [20], subdivided stars [14, 15, 21, 22, 29, 30], path-like trees [3], caterpillars [17, 18, 25], disjoint union of stars and books [10] and wheels, fans and friendship graphs [24], paths and cycles [23] and complete bipartite graphs [1]. For detail studies of a super (a,d)-EAT labeling reader can see [2, 4, 5, 7, 9-12].

Definition 1.4 *Let $n_i \geq 1$, $1 \leq i \leq r$, and $r \geq 2$. A subdivided star $T(n_1, n_2, \cdots, n_r)$ is a tree obtained by inserting $n_i - 1$ vertices to each of the ith edge of the star $K_{1,r}$. Moreover suppose that $V(G) = \{c\} \cup \{x_i^{l_i} | 1 \leq i \leq r;\ 1 \leq l_i \leq n_i\}$ is the vertex-set and $E(G) = \{cx_i^1 | 1 \leq i \leq r\} \cup \{x_i^{l_i} x_i^{l_i+1} | 1 \leq i \leq r;\ 1 \leq l_i \leq n_i - 1\}$ is the edge-set of the subdivided star $G \cong T(n_1, n_2, \cdots, n_r)$ then $v = \sum\limits_{i=1}^{r} n_i + 1$ and $e = \sum\limits_{i=1}^{r} n_i$.*

Lu [29,30] called the subdivided star $T(n_1, n_2, n_3)$ as a three-path tree and proved that it is a super $(a,0)$-EAT graph if n_1 and n_2 are odd with $n_3 = n_2 + 1$ or $n_3 = n_2 + 2$. Ngurah et al. [21] proved that the subdivided star $T(n_1, n_2, n_3)$ is also a super $(a,0)$-EAT graph if $n_3 = n_2 + 3$ or $n_3 = n_2 + 4$. Salman et al. [22] found a super $(a,0)$-EAT labeling on the subdivided stars $T\underbrace{(n, n, n, \cdots, n)}_{r-times}$, where $n \in \{2,3\}$.

Moreover, Javaid et al. [14,15] proved the following results related to a super (a,d)-EAT labeling on different subclasses of subdivided stars for different values of d:

• For any odd $n \geq 3$, $G \cong T(n, n-1, n, n)$ admits a super $(a,0)$-EAT labeling with $a = 10n + 2$;

• For any odd $n \geq 3$ and $m \geq 3$, $G \cong T(n, n, m, m)$ admits a super $(a,0)$-EAT labeling with $a = 6n + 5m + 2$;

• For any odd $n \geq 3$ and $p \geq 5$, $G \cong T(n, n, n+2, n+2, n_5, \cdots, n_p)$ admits a super $(a,0)$-EAT labeling with $a = 2v + s - 1$, a super $(a,1)$-EAT labeling with $a = s + \frac{3}{2}v$ and a super $(a,2)$-EAT labeling with $a = v + s + 1$ where $v = |V(G)|$, $s = (2n+6) + \sum_{m=5}^{p} [(n+1)2^{m-5} + 1]$ and $n_r = 1 + (n+1)2^{r-4}$ for $5 \leq r \leq p$.

However, the investigation of the different results related to a super (a,d)-EAT labeling of the subdivided star $T(n_1, n_2, n_3, \cdots, n_r)$ for $n_1 \neq n_2 \neq n_2, \cdots, \neq n_r$ is still open. In this paper, for $d \in \{0,1,2\}$, we formulate a super (a,d)-EAT labeling on the subclasses of subdivided stars denoted by $T(kn, kn, kn, kn, 2kn, n_6, \cdots, n_r)$ and $T(kn, kn, 2n, 2n+2, n_5, \cdots, n_r)$ under certain conditions.

§2. Basic Results

In this section, we present some basic results which will be used frequently in the main results. Ngurah et al. [21] found lower and upper bounds of the minimum edge-weight a for a subclass of the subdivided stars, which is stated as follows:

Lemma 2.1 *If $T(n_1, n_2, n_3)$ is a super $(a,0)$-EAT graph, then $\frac{1}{2l}(5l^2 + 3l + 6) \leq a \leq \frac{1}{2l}(5l^2 + 11l - 6)$, where $l = \sum_{i=1}^{3} n_i$.*

The lower and upper bounds of the minimum edge-weight a for another subclass of subdivided stats established by Salman et al. [22] are given below:

Lemma 2.2 *If $T\underbrace{(n, n, \cdots, n)}_{n-times}$ is a super $(a,0)$-EAT graph, then $\frac{1}{2l}(5l^2 + (9-2n)l + n^2 - n) \leq a \leq \frac{1}{2l}(5l^2 + (2n+5)l + n - n^2)$, where $l = n^2$.*

Moreover, the following lemma presents the lower and upper bound of the minimum edge-weight a for the most generalized subclass of subdivided stars proved by Javaid and Akhlaq:

Lemma 2.3([16]) *If $T(n_1, n_2, n_3, \cdots, n_r)$ has a super (a,d)-EAT labeling, then $\frac{1}{2l}(5l^2 + r^2 - 2lr + 9l - r - (l-1)ld) \leq a \leq \frac{1}{2l}(5l^2 - r^2 + 2lr + 5l + r - (l-1)ld)$, where $l = \sum_{i=1}^{r} n_i$ and $d \in \{0,1,2,3\}$.*

Bača and Miller [4] state a necessary condition far a graph to be super (a,d)-EAT, which provides an upper bound on the parameter d. Let a (v,e)-graph G be a super (a,d)-EAT. The minimum possible edge-weight is at least $v+4$. The maximum possible edge-weight is no more than $3v+e-1$. Thus $a+(e-1)d \leq 3v+e-1$ or $d \leq \frac{2v+e-5}{e-1}$. For any subdivided star, where $v=e+1$, it follows that $d \leq 3$.

Let us consider the following proposition which we will use frequently in the main results.

Proposition 2.1([3]) *If a (v,e)-graph G has a (s,d)-EAV labeling then*

(1) *G has a super $(s+v+1, d+1)$-EAT labeling;*

(2) *G has a super $(s+v+e, d-1)$-EAT labeling.*

§3. Super (a,d)-EAT Labeling of Subdivided Stars

Theorem 3.1 *For any even $n \geq 2$ and $r \geq 6$, $G \cong T(n+3, n+2, n, n+1, 2n+1, n_6, \cdots, n_r)$ admits a super $(a,0)$-edge-antimagic total labeling with $a = 2v+s-1$ and a super $(a,2)$-edge-antimagic total labeling with $a = v+s+1$ where $v = |V(G)|$, $s = (3n+7) + \sum\limits_{m=6}^{r}[2^{m-5}n+1]$ and $n_m = 2^{m-4}n+1$ for $6 \leq m \leq r$.*

Proof Let us denote the vertices and edges of G, as follows:

$V(G) = \{c\} \cup \{x_i^{l_i} | 1 \leq i \leq r;\ 1 \leq l_i \leq n_i\}$,

$E(G) = \{cx_i^1 | 1 \leq i \leq r\} \cup \{x_i^{l_i} x_i^{l_i+1} | 1 \leq i \leq r; 1 \leq l_i \leq n_i - 1\}$.

If $v = |V(G)|$ and $e = |E(G)|$, then

$$v = (6n+8) + \sum_{m=6}^{r}[2^{m-6}4n+1] \quad \text{and} \quad e = v-1.$$

Now, we define the labeling $\lambda : V(G) \to \{1, 2, \cdots, v\}$ as follows:

$$\lambda(c) = (4n+8) + \sum_{m=6}^{r}[2^{m-6}2n+1].$$

For odd $1 \leq l_i \leq n_i$, where $i = 1,2,3,4,5$ and $6 \leq i \leq r$, we define

$$\lambda(u) = \begin{cases} \frac{l_1+1}{2}, & \text{for } u = x_1^{l_1}, \\ n+3-\frac{l_2-1}{2}, & \text{for } u = x_2^{l_2}, \\ (n+4)+\frac{l_3-1}{2}, & \text{for } u = x_3^{l_3}, \\ (2n+4)-\frac{l_4-1}{2}, & \text{for } u = x_4^{l_4}, \\ (3n+5)-\frac{l_5-1}{2}, & \text{for } u = x_5^{l_5}. \end{cases}$$

and

$$\lambda(x_i^{l_i}) = (3n+5) + \sum_{m=6}^{i}[2^{m-6}2n+1] - \frac{l_i - 1}{2},$$

respectively. For even $1 \le l_i \le n_i$, $\alpha = (3n+5) + \sum_{m=6}^{r}[2^{m-6}2n+1]$, $i = 1, 2, 3, 4, 5$ and $6 \le i \le r$, we define

$$\lambda(u) = \begin{cases} (\alpha+1) + \dfrac{l_1 - 2}{2}, & \text{for } u = x_1^{l_1}, \\ (\alpha+n+2) - \dfrac{l_2 - 2}{2}, & \text{for } u = x_2^{l_2}, \\ (\alpha+n+4) + \dfrac{l_3 - 2}{2}, & \text{for } u = x_3^{l_3}, \\ (\alpha+2n+3) - \dfrac{l_4 - 2}{2}, & \text{for } u = x_4^{l_4}, \\ (\alpha+3n+3) - \dfrac{l_5 - 2}{2}, & \text{for } u = x_5^{l_5} \end{cases}$$

and

$$\lambda(x_i^{l_i}) = (\alpha+3n+3) + \sum_{m=6}^{i}[2^{m-6}2n] - \frac{l_i - 2}{2},$$

respectively.

The set of all edge-sums generated by the above formula forms a consecutive integer sequence $s = \alpha+2, \alpha+3, \cdots, \alpha+1+e$. Therefore, by Proposition 2.1, λ can be extended to a super $(a, 0)$-edge-antimagic total labeling and we obtain the magic constant $a = v + e + s = 2v + (3n+6) + \sum_{m=6}^{r}[2^{m-6}2n+1]$.

Similarly by Proposition 2.2, λ can be extended to a super $(a, 2)$-edge-antimagic total labeling and we obtain the magic constant $a = v + 1 + s = v + (3n+8) + \sum_{m=6}^{r}[2^{m-6}2n+1]$. □

Theorem 3.2 *For any odd $n \ge 3$ and $r \ge 6$, $G \cong T(n+3, n+2, n, n+1, 2n+1, n_6, \cdots, n_r)$ admits a super $(a, 1)$-edge-antimagic total labeling with $a = s + \frac{3v}{2}$ if v is even, where $v = |V(G)|$, $s = (3n+7) + \sum_{m=6}^{r}[2^{m-5}n+1]$ and $n_m = 2^{m-4}n+1$ for $6 \le m \le r$.*

Proof Let us consider the vertices and edges of G, as defined in Theorem 3.1. Now, we define the labeling $\lambda : V(G) \to \{1, 2, \cdots, v\}$ as in same theorem. It follows that the edge-weights of all edges of G constitute an arithmetic sequence $s = \alpha+2, \alpha+3, \cdots, \alpha+1+e$ with common difference 1, where

$$\alpha = (3n+5) + \sum_{m=6}^{r}[2^{m-6}2n+1].$$

We denote it by $A = \{a_i;\ 1 \le i \le e\}$. Now for G we complete the edge labeling λ for super $(a, 1)$-edge-antimagic total labeling with values in the arithmetic sequence $v+1$, $v+2$, $\cdots$, $v+e$ with common difference 1. Let us denote it by $B = \{b_j\ ;\ 1 \le j \le e\}$. Define $C = \{a_{2i-1} + b_{e-i+1}\ ;\ 1 \le i \le \frac{e+1}{2}\} \cup \{a_{2j} + b_{\frac{e-1}{2}-j+1}\ ;\ 1 \le j \le \frac{e+1}{2} - 1\}$. It is easy to see that C constitutes an arithmetic sequence with $d = 1$ and

$$a = s + \frac{3v}{2} = (12n + 19) + \frac{1}{2}\sum_{m=6}^{r}[2^{m-3}2n + 5].$$

Since all vertices receive the smallest labels, λ is a super $(a, 1)$-edge-antimagic total labeling.□

Theorem 3.3 *For any even $n \geq 2$ and $r \geq 6$, $G \cong T(n+2, n, n, n+1, 2(n+1), n_6, \cdots, n_r)$ admits a super $(a, 0)$-edge-antimagic total labeling with $a = 2v + s - 1$ and a super $(a, 2)$-edge-antimagic total labeling with $a = v + s + 1$ where $v = |V(G)|$, $s = (3n + 5) + \sum\limits_{m=6}^{r}[2^{m-5}n + 2]$ and $n_m = 2^{m-4}n + 2$ for $6 \leq m \leq r$.*

Proof Let us denote the vertices and edges of G as follows:

$V(G) = \{c\} \cup \{x_i^{l_i} | 1 \leq i \leq r; 1 \leq l_i \leq n_i\}$;

$E(G) = \{cx_i^1 | 1 \leq i \leq r\} \cup \{x_i^{l_i}x_i^{l_i+1} | 1 \leq i \leq r;\ 1 \leq l_i \leq n_i - 1\}$.

If $v = |V(G)|$ and $e = |E(G)|$, then

$$v = (6n + 6) + \sum_{m=6}^{r}[2^{m-6}4(n+)] \ \text{ and } \ e = v - 1.$$

Now, we define the labeling $\lambda : V(G) \to \{1, 2, \cdots, v\}$ as follows:

$$\lambda(c) = (4n + 5) + \sum_{m=6}^{r}[2^{m-6}2n + 2].$$

For odd $1 \leq l_i \leq n_i$, where $i = 1, 2, 3, 4, 5$ and $6 \leq i \leq r$, we define

$$\lambda(u) = \begin{cases} \frac{l_1 + 1}{2}, & \text{for } u = x_1^{l_1}, \\ n + 1 - \frac{l_2 - 1}{2}, & \text{for } u = x_2^{l_2}, \\ (n + 2) - \frac{l_3 + 1}{2}, & \text{for } u = x_3^{l_3}, \\ (2n + 2) - \frac{l_4 - 1}{2}, & \text{for } u = x_4^{l_4}, \\ (3n + 3) - \frac{l_5 - 1}{2}, & \text{for } u = x_5^{l_5}. \end{cases}$$

$$\lambda(x_i^{l_i}) = (3n + 3) + \sum_{m=6}^{i}[2^{m-6}2n + 2] - \frac{l_i - 1}{2},$$

respectively. For even $1 \leq l_i \leq n_i$, $\alpha = (3n + 43) + \sum\limits_{m=6}^{r}[2^{m-6}2n + 2]$, $i = 1, 2, 3, 4, 5$ and

$5 \le i \le r$, we define

$$\lambda(u) = \begin{cases} (\alpha+1) + \dfrac{l_1-2}{2}, & \text{for } u = x_1^{l_1}, \\ (\alpha + n(\alpha+n+1) - \dfrac{l_2-2}{2}, & \text{for } u = x_2^{l_2}, \\ (\alpha+n+3) - \dfrac{l_3-2}{2}, & \text{for } u = x_3^{l_3}, \\ (\alpha+2n+2) - \dfrac{l_4-2}{2}, & \text{for } u = x_4^{l_4}, \\ (\alpha+3n+3) - \dfrac{l_5-2}{2}, & \text{for } u = x_5^{l_5} \end{cases}$$

and

$$\lambda(x_i^{l_i}) = (\alpha+3n+3) + \sum_{m=6}^{i}[2^{m-6}4(n+1)] - \frac{l_i-2}{2},$$

respectively.

The set of all edge-sums generated by the above formula forms a consecutive integer sequence $s = \alpha+2, \alpha+3, \cdots, \alpha+1+e$. Therefore, by Proposition 2.1, λ can be extended to a super $(a,0)$-edge-antimagic total labeling and we obtain the magic constant

$$a = v+e+s = 2v + (3n+4) + \sum_{m=6}^{r}[2^{m-6}2n+2].$$

Similarly by Proposition 2.2, λ can be extended to a super $(a,2)$-edge-antimagic total labeling and we obtain the magic constant $a = v+1+s = v + (3n+6) + \sum\limits_{m=6}^{r}[2^{m-6}2n+2]$. □

Theorem 3.4 *For any odd $n \ge 3$ and $r \ge 6$, $G \cong T(n+2, n, n, n+1, 2(n+1), n_6, \cdots, n_r)$ admits a super $(a,1)$-edge-antimagic total labeling with $a = s + \dfrac{3v}{2}$ if v is even, where $v = |V(G)|$, $s = (3n+5) + \sum\limits_{m=6}^{r}[2^{m-5}n+2]$ and $n_m = 2^{m-4}n+2$ for $6 \le m \le r$.*

Proof Let us consider the vertices and edges of G, as defined in Theorem 3.3. Now, we define the labeling $\lambda : V(G) \to \{1, 2, \cdots, v\}$ as in same theorem. It follows that the edge-weights of all edges of G constitute an arithmetic sequence $s = \alpha+2, \alpha+3, \cdots, \alpha+1+e$ with common difference 1, where

$$\alpha = (3n+3) + \sum_{m=6}^{r}[2^{m-6}2(n+1)].$$

We denote it by $A = \{a_i; 1 \le i \le e\}$. Now for G we complete the edge labeling λ for super $(a,1)$-edge-antimagic total labeling with values in the arithmetic sequence $v+1$, $v+2$, $\cdots$, $v+e$ with common difference 1. Let us denote it by $B = \{b_j\ ;\ 1 \le j \le e\}$. Define $C = \{a_{2i-1} + b_{e-i+1}\ ;\ 1 \le i \le \dfrac{e+1}{2}\} \cup \{a_{2j} + b_{\frac{e-1}{2}-j+1}\ ;\ 1 \le j \le \dfrac{e+1}{2} - 1\}$. It is easy to see that C constitutes an arithmetic sequence with $d = 1$ and

$$a = s + \frac{3v}{2} = (12n + 14) + \sum_{m=6}^{r} [2^{m-5}(4n + 3) + 2].$$

Since all vertices receive the smallest labels, λ is a super $(a, 1)$-edge-antimagic total labeling.□

§4. Conclusion

In this paper, we have shown that two different subclasses of subdivided stars admit a super (a, d)-EAT labeling for $d \in \{0, 1, 2\}$. However, the problem is still open for the magicness of $T(n_1, n_2, n_3, \cdots, n_r)$, where $n_i = n$ and $1 \leq i \leq r$.

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