International J.Math. Combin. Vol.4(2013), 37-48

Difference Cordiality of Some Derived Graphs

R.Ponraj and S.Sathish Narayanan

(Department of Mathematics, Sri Paramakalyani College, Alwarkurichi-627412, India)

E-mail: ponrajmaths@gmail.com, sathishrvss@gmail.com

Abstract: Let G be a (p,q) graph. Let $f : V(G) \to \{1, 2 \ldots, p\}$ be a function. For each edge uv, assign the label $|f(u) - f(v)|$. f is called a difference cordial labeling if f is a one to one map and $|e_f(0) - e_f(1)| \leq 1$ where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively. A graph with a difference cordial labeling is called a difference cordial graph. In this paper we investigate the difference cordial labeling behavior of Splitting, Degree splitting and Shadow graph of some standard graphs.

Key Words: Splitting graph, degree splitting graph, shadow graph, corona.

AMS(2010): 05C78

§1. Introduction

Let $G = (V, E)$ be (p, q) graph. Throughout this paper we have considered only simple and undirected graphs. The number of vertices of G is called the order of G and the number of edges of G is called the size G. Graph labeling is an assignment of integers to the vertices or edges, or both, subject to certain conditions. Graph labeling plays an important role of various fields of science and few of them are astronomy, coding theory, x-ray crystallography, radar, circuit design, communication network addressing, database management, secret sharing schemes, and models for constraint programming over finite domains [4]. The graph labeling problem was introduced by Rosa and he has introduced graceful labeling of graphs [21] in the year 1967. In 1980, Cahit [2] introduced the Cordial labeling of graphs. Kuo, Chang, and Kwong [8], Youssef [25], Liu and Zhu [10], Kirchherr [7], Ho, Lee, and Shee [6], Riskin [20], Seoud and Abdel Maqusoud [23], Diab [3], Lee and Liu [9], Andar, Boxwala, and Limaye [1], Vaidya, Ghodasara, Srivastav, and Kaneria [24] were worked in cordial labeling. Ponraj et al. introduced k-product cordial labeling [17], k-total product cordial labeling [18] recently. Inspiration of the above work, R. Ponraj, S. Sathish Narayanan and R. Kala introduced difference cordial labeling of graphs [11]. Let G be a (p, q) graph. Let f be a map from $V(G)$ to $\{1, 2 \ldots p\}$. For each edge uv, assign the label $|f(u) - f(v)|$. f is called difference cordial labeling if f is $1-1$ and $|e_f(0) - e_f(1)| \leq 1$ where $e_f(1)$ and $e_f(0)$ denote the number of edges labeled with 1 and not labeled with 1 respectively. A graph with a difference cordial labeling is called a

[1]Received June 28, 2013, Accepted November 20, 2013.

difference cordial graph. In [11]-[16] difference cordial labeling behavior of several graphs like path, cycle, complete graph, complete bipartite graph, bistar, wheel, web, grid, prism, book and some more standard graphs have been investigated. In this paper, we investigate the difference cordial labeling behavior of some derived graphs like splitting graph, Degree splitting graph and shadow graphs. Let x be any real number. Then the symbol $\lfloor x \rfloor$ stands for the largest integer less than or equal to x and $\lceil x \rceil$ stands for the smallest integer greater than or equal to x. Terms and definitions not defined here are used in the sense of Harary [5].

§2. Splitting Graphs

A splitting graph of a graph was introduced by E.Sampath Kumar and H.B.Waliker [22]. For a graph G, the splitting graph of G, $S^{'}(G)$, is obtained from G by adding for each vertex v of G a new vertex $v^{'}$ so that $v^{'}$ is adjacent to every vertex that is adjacent to v. Note that if G is a (p,q) graph then $S^{'}(G)$ is a $(2p,3q)$ graph.

Theorem 2.1 *$S^{'}(P_n)$ is difference cordial.*

Proof Let $P_n : u_1u_2\ldots u_n$ be the path. Let $V\left(S^{'}(P_n)\right) = \{v_i : 1 \le i \le n\} \cup V(P_n)$ and $E\left(S^{'}(P_n)\right) = E(P_n) \cup \{u_iv_{i+1}, v_iu_{i+1} : 1 \le i \le n-1\}$.

Case 1 n is odd.

Define a map $f : V\left(S^{'}(P_n)\right) \to \{1, 2\ldots 2n\}$ by

$$\begin{aligned} f(u_{2i-1}) &= 4i-1, & 1 \le i \le \left\lfloor \frac{n-1}{2} \right\rfloor, \\ f(u_{2i}) &= 4i-2, & 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor, \\ f(v_{2i-1}) &= 4i-3, & 1 \le i \le \left\lceil \frac{n}{2} \right\rceil, \\ f(v_{2i-1}) &= 4i, & 1 \le i \le \left\lfloor \frac{n-1}{2} \right\rfloor, \\ f(u_n) &= 2n. \end{aligned}$$

Obviously, the above labeling is a difference cordial labeling of $S^{'}(P_n)$.

Case 2 n is even.

Assign the label to vertices u_{2i-1}, $v_{2i}, 1 \le i \le \left\lfloor \frac{n-1}{2} \right\rfloor$, u_{2i}, $1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$, v_{2i-1}, $1 \le i \le \left\lceil \frac{n}{2} \right\rceil$) as in Case 1 and then define $f(u_{n-1}) = 2n$ and $f(v_n) = 2n-1$. Since $e_f(0) = \frac{3n-4}{2}$ and $e_f(1) = \frac{3n-2}{2}$, f is a difference cordial labeling of $S^{'}(P_n)$. □

Theorem 2.2 *$S^{'}(C_n)$ is difference cordial.*

Proof Let $C_n : u_1u_2\cdots u_nu_1$ be a cycle. Let $V\left(S^{'}(C_n)\right) = V(C_n) \cup \{v_i : 1 \le i \le n\}$ and

$E\left(S^{'}\left(C_n\right)\right) = \left\{u_i v_{i+1(\text{mod } n)}, v_i u_{i+1(\text{mod } n)} : 1 \le i \le n\right\} \cup E\left(C_n\right)$.

Case 1 n is odd.

Let f be a difference cordial defined in Case 1 of Theorem 2.1. Define a map $g : V\left(S^{'}\left(C_n\right)\right) \to \{1, 2 \ldots 2n\}$ by

$$\begin{aligned} g\left(u_i\right) &= f\left(u_i\right), \quad 1 \le i \le n-1, \\ g\left(v_i\right) &= f\left(v_i\right), \quad 1 \le i \le n-1, \\ g\left(v_n\right) &= f\left(u_n\right), \\ g\left(u_n\right) &= f\left(v_n\right). \end{aligned}$$

Since $e_f(0) = \dfrac{3n+1}{2}$ and $e_f(1) = \dfrac{3n-1}{2}$, g is a difference cordial labeling of $S^{'}\left(C_n\right)$.

Case 2 n is even.

Let f be a difference cordial labeling defined in Case 2 of Theorem 2.1. Define a map $h : V\left(S^{'}\left(C_n\right)\right) \to \{1, 2 \ldots 2n\}$ by

$$\begin{aligned} h\left(u_i\right) &= f\left(u_i\right) \quad \forall i \ne n-1 \\ h\left(v_i\right) &= f\left(v_i\right) \quad \forall i \ne n-1 \end{aligned}$$

$h\left(u_{n-1}\right) = f\left(u_n\right)$, $h\left(v_n\right) = f\left(u_{n-1}\right)$. Since $e_f(0) = e_f(1) = \frac{3n}{2}$, h is a difference cordial labeling of $S^{'}\left(C_n\right)$. □

The corona of G with H, $G \odot H$ is the graph obtained by taking one copy of G and p copies of H and joining the i^{th} vertex of G with an edge to every vertex in the i^{th} copy of H. The graph $P_n \odot K_1$ is called a comb.

Theorem 2.3 *$S^{'}\left(P_n \odot K_1\right)$ is difference cordial.*

Proof Let $u_i^{'}$ $(1 \le i \le n)$ be the vertex corresponding to u_i $(1 \le i \le n)$ and $v_i^{'}$ $(1 \le i \le n)$ be the vertex corresponding to v_i $(1 \le i \le n)$. Define a map $f : V\left(S^{'}\left(P_n \odot K_1\right)\right) \to \{1, 2 \ldots 4n\}$ by $f\left(u_1\right) = 3$, $f\left(u_1^{'}\right) = 1$,

$$\begin{aligned} f\left(u_{i+1}\right) &= 3i+1 \quad 1 \le i \le n-1 \\ f\left(u_{i+1}^{'}\right) &= 3i+3 \quad 1 \le i \le n-1 \\ f\left(v_i\right) &= 3i-1 \quad 1 \le i \le n \\ f\left(v_i^{'}\right) &= 3n+1 \quad 1 \le i \le n \end{aligned}$$

Since $e_f(0) = 3n-2$ and $e_f(1) = 3n-1$, f is a difference cordial labeling of $S^{'}\left(P_n \odot K_1\right)$. □

Theorem 2.4([11]) *If G is a (p, q) difference cordial graph, then $q \le 2p-1$.*

The graph $W_n = C_n + K_1$ is called a wheel.

Theorem 2.5 *$S^{'}\left(W_n\right)$ is not difference cordial.*

Proof Clearly, the number of vertices and edges in $S^{'}(W_n)$ are $2n+2$ and $6n$ respectively. By Theorem 2.4, $6n \leq 2(2n+2) - 1 \leq 4n - 3$. This is impossible. □

Theorem 2.6([11]) *Any Path is a difference cordial graph.*

Theorem 2.7 *$S^{'}(K_{1,n})$ is difference cordial iff $n \leq 3$.*

Proof Since $S^{'}(K_{1,1}) \cong P_4$, by theorem 2.6, $S^{'}(K_{1,1})$ is difference cordial. The difference cordial labeling of $S^{'}(K_{1,2})$ and $S^{'}(K_{1,3})$ is shown in Figure 1.

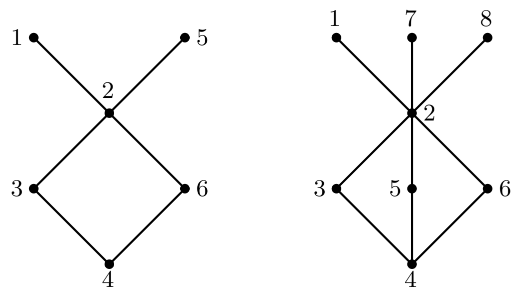

Figure 1

Suppose f is a difference cordial labeling of $K_{1,n}$ with $n > 3$. Clearly, $e_f(1) \leq 4$. Then $e_f(0) \geq q - 4 \geq 3n - 4$. This implies, $e_f(0) - e_f(1) \geq 3n - 8 > 1$, a contradiction. □

Theorem 2.8 *$S^{'}(K_n)$ is not difference cordial.*

Proof The order and size of $S^{'}(K_n)$ are $2n$ and $\frac{3n(n-1)}{2}$ respectively. By Theorem 2.4, $\frac{3n(n-1)}{2} \leq 2(2n) - 1$. This implies $2 \leq 5n^2 + 3n$. Hence, $S^{'}(K_n)$ is not difference cordial. □

The graph $C_n \times P_2$ is called prism.

Theorem 2.9 *$S^{'}(C_n \times P_2)$ is not difference cordial.*

Proof The order and size of $S^{'}(C_n \times P_2)$ are $4n$ and $9n$ respectively. By Theorem 2.4, $9n \leq 2(4n) - 1$. $\Rightarrow n \leq -1$. This is impossible. □

The helm H_n is the graph obtained from a wheel by attaching a pendant edge at each vertex of the n-cycle. A flower Fl_n is the graph obtained from a helm by joining each pendant vertex to the central vertex of the helm.

Theorem 2.10 *A splitting graph of a flower graph is not difference cordial.*

Proof The number of vertices and edges in the splitting graph of a flower graph are $4n+2$ and $12n$ respectively. By theorem 2.4, $12n \leq 2(4n+2) - 1$. This implies $4n \leq 3$. This is impossible. □

§3. Degree Splitting Graphs

The concept of Degree splitting graph was introduced by R.Ponraj and S.Somasundaram in [19]. Let $G = (V, E)$ be a graph with $V = S_1 \cup S_2 \cup \cdots \cup S_t \cup T$ where each S_i is a set of vertices having at least two vertices and having the same degree and $T = V - \bigcup_{i=1}^{t} S_i$. The Degree Splitting graph of G denoted by $DS(G)$ is obtained from G by adding vertices $w_1, w_2 \ldots, w_t$ and joining w_i to each vertex of Si $(1 \le i \le t)$.

Theorem 3.1 *$DS(P_n)$ is difference cordial.*

Proof Let P_n be the path $u_1u_2\ldots u_n$. Let $V(DS(P_n)) = V(P_n) \cup \{u, v\}$ and $E(DS(P_n)) = \{uu_i : 2 \le i \le n-1\} \cup \{uu_1, vu_n\}$. Define $f : V(DS(P_n)) \to \{1, 2 \ldots n+2\}$ by $f(u_i) = i$, $1 \le i \le n$, $f(v) = n+1$, $f(u) = n+2$. Since $e_f(1) = n$, $e_f(0) = n-1$, f is a difference cordial labeling of $DS(P_n)$. □

Theorem 3.2([11]) *The wheel W_n is difference cordial.*

Theorem 3.3 *$DS(C_n)$ is difference cordial.*

Proof Since $DS(C_n) \cong W_n$, the proof follows from Theorem 3.2. □

Theorem 3.4([11]) *K_n is difference cordial iff $n \le 4$.*

Theorem 3.5 *$DS(K_n)$ is difference cordial iff $n \le 3$.*

Proof Since $DS(K_n) \cong K_{n+1}$, the proof follows from Theorem 3.4. □

Theorem 3.6([11]) *$K_{2,n}$ is difference cordial iff $n \le 4$.*

Theorem 3.7 *$DS(K_{1,n})$ is difference cordial iff $n \le 4$.*

Proof Since $DS(K_{1,n}) \cong K_{2,n}$, the proof follows from Theorem 3.6. □

Theorem 3.8 *$DS(W_n)$ is difference cordial iff $n = 3$.*

Proof The difference cordial labeling of $DS(W_3)$ is given in figure 2.

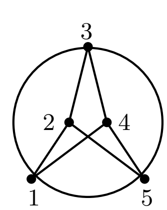

Figure 2

Suppose $DS(W_n)$ is difference cordial, then by Theorem 2.4, $3n \le 2(n+2) - 1$. This implies $n = 3$. □

Theorem 3.9 *$DS(K_n^c + 2K_2)$ is difference cordial iff $n = 1$.*

Proof The order and size of $DS(K_n^c + 2K_2)$ are $n + 6$ and $5n + 6$ respectively. Suppose $DS(K_n^c + 2K_2)$ is difference cordial, then by theorem 2.4, $5n + 6 \leq 2(n + 6) - 1$. This is true when $n = 1$. The difference cordial labeling of $DS(K_1^c + 2K_2)$ is given in Figure 3.

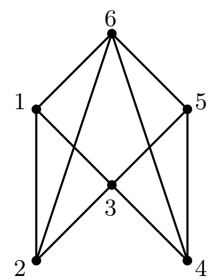

Figure 3

This completes the proof. □

Theorem 3.10 *$DS(K_2 + mK_1)$ is difference cordial iff $n \leq 3$.*

Proof The graph $DS(K_2 + mK_1)$ consists of $m + 4$ vertices and $3m + 3$ edges. Since $DS(K_2 + K_1) \cong W_3$, using Theorem 3.2, $DS(K_2 + K_1)$ is difference cordial. the difference cordial labeling of $DS(K_2 + 2K_1)$ and $DS(K_2 + 3K_1)$ are given in Figure 4.

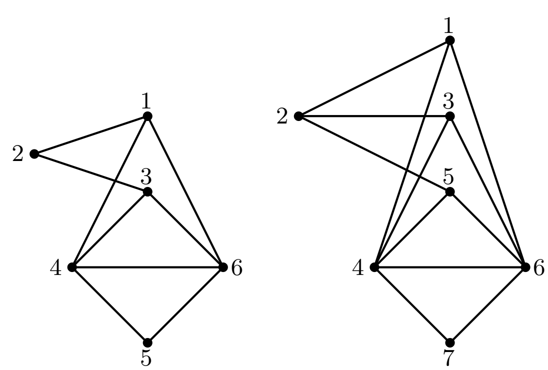

Figure 4

Suppose $DS(K_2 + mK_1)$ is difference cordial, then by theorem 2.4, $3m + 3 \leq 2(m + 4) - 1$. $\Rightarrow m \leq 4$. When $m = 4$, $e_f(0) \geq 4+3+2 \geq 9$. Obviously, $e_f(1) \leq 7$. Hence $e_f(0) - e_f(1) \geq 2$. This implies $DS(K_2 + mK_1)$ is not difference cordial. □

Theorem 3.11 *$DS(K_{n,n})$ is difference cordial iff $n \leq 2$.*

Proof The order and size of $DS(K_{n,n})$ are $2n + 1$ and $n^2 + 2n$ respectively. Suppose $DS(K_{n,n})$ is difference cordial, then by theorem 2.4, $n^2 + 2n \leq 2(2n + 1) - 1$, $\Rightarrow n^2 - 2n - 1 \leq 0$. $\Rightarrow n \leq 2$. Since $DS(K_{1,1}) \cong K_3$, $DS(K_{2,2}) \cong W_4$, using Theorems 3.2 and 3.4, $DS(K_{1,1})$ and $DS(K_{2,2})$ are difference cordial. □

The triangular snake T_n is obtained from the path P_n by replacing each edge of the path by a triangle C_3.

Theorem 3.12 *$DS(T_n)$ is difference cordial iff $n \leq 5$.*

Proof Clearly, the order and size of $DS(T_n)$ $(n > 3)$ are $2n+1$ and $5n-4$ respectively. By Theorem 2.4, $5n - 4 \leq 2(2n+1) - 1$. This implies $n \leq 5$. Since $DS(T_2) \cong W_3$, using theorem 3.2, $DS(T_2)$ is difference cordial. The difference cordial labeling of $DS(T_3)$, $DS(T_4)$ and $DS(T_5)$ are given in Figure 5.

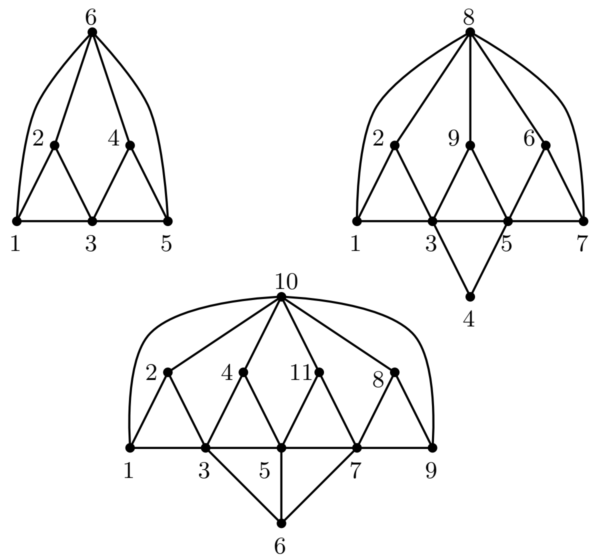

Figure 5

This completes the proof. □

The Quadrilateral snake Q_n is obtained from the path P_n by replacing each edge of the path by a cycle C_4.

Theorem 3.13 *$DS(Q_n)$ is difference cordial iff $n \leq 5$.*

Proof The difference cordial labeling of $DS(Q_n)$ $(n \leq 5)$ is given in Figure 6.

The number of vertices and edges in $DS(Q_n)$ are $3n$ and $7n - 6$ respectively. Suppose $DS(Q_n)$ is difference cordial, then by Theorem 2.4, $7n - 6 \leq 2(3n) - 1$. This implies $n \leq 5$. □

The sunflower graph S_n is obtained by taking a wheel with central vertex v_0 and the cycle $C_n : v_1 v_2 \ldots v_n v_1$ and new vertices $w_1 w_2 \ldots w_n$ where w_i is joined by vertices $v_i, v_{i+1 (\bmod\ n)}$.

The Lotus inside a circle LC_n is a graph obtained from the cycle $C_n : u_1 u_2 \ldots u_n u_1$ and a star $K_{1,n}$ with central vertex v_0 and the end vertices $v_1 v_2 \ldots v_n$ by joining each v_i to u_i and $u_{i+1 (\bmod\ n)}$.

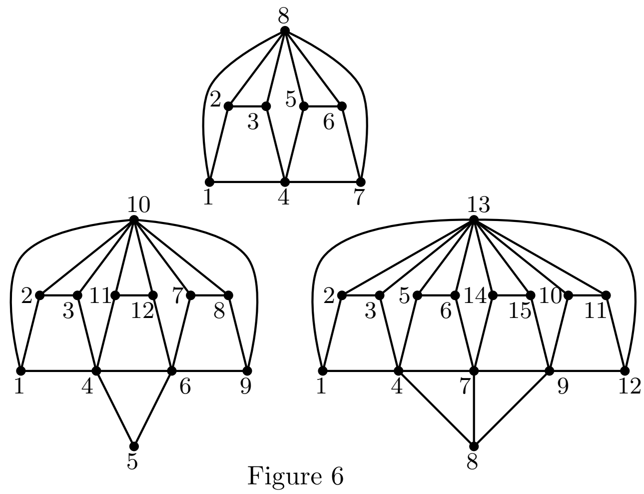

Figure 6

Theorem 3.14 *The following are not difference cordial:* $DS(S_n)$, $DS(LC_n)$ *and* $DS(Fl_n)$.

Proof Since the order and size of the graphs given above are $2n+3$ and $6n$ respectively. Suppose the graphs given above are difference cordial, then using theorem 2.4, $6n \leq 2(2n+3)-1$. This implies $2n \leq 5$, a contradiction. □

The graph $L_n = P_n \times P_2$ is called ladder.

Theorem 3.15 $DS(L_n)$ *is difference cordial iff* $n \leq 5$.

Proof Since $DS(L_2) \cong W_4$, by theorem 3.2, $DS(L_2)$ is difference cordial. For $n \geq 3$, $V(DS(L_n)) = \{u_i, v_i : 1 \leq i \leq n\} \cup \{u, v\}$ and $E(DS(L_n)) = \{u_iu_{i+1}, v_iv_{i+1} : 1 \leq i \leq n-1\} \cup \{u_iv_i : 1 \leq i \leq n\} \cup \{uu_i, uv_i : 2 \leq i \leq n-1\} \cup \{vu_1, vv_1, vu_n, vv_n\}$. The difference cordial labeling of $DS(L_n)$ is given in Table 1.

n	u	v	u_1	u_2	u_3	u_4	u_5	v_1	v_2	v_3	v_4	v_5
3	4	8	1	5	6			2	3	7		
4	5	10	1	4	7	8		2	3	6	9	
5	7	12	1	4	5	8	11	2	3	6	9	10

Table 1:

Conversely, Suppose f is a difference cordial labeling, then by Theorem 2.4, $5n-2 \leq 2(2n+2)-1$, $\Rightarrow n \leq 5$. □

Theorem 3.16 *If* $m+n > 8$ *then* $DS(B_{m,n})$ $(m \neq n)$ *is not difference cordial.*

Proof Clearly, the order and size of $DS(B_{m,n})$ are $m+n+3$ and $2m+2n+1$ respectively. Obviously, $e_f(1) \leq m+m+2$. Also we observe that $e_f(0) \geq m+n-2+m-1+n-2 \geq 2m+2n-5$.

$\Rightarrow e_f(0) - e_f(1) \geq m+n-7 \longrightarrow (1)$. Suppose $DS(B_{m,n})$ is difference cordial with $m+n > 8$, then a contradiction arises to (1). □

Theorem 3.17 *$DS(B_{n,n})$ is difference cordial iff $n \leq 2$.*

Proof The order and size of $DS(B_{n,n})$ are $2n+4$ and $4n+3$ respectively. Clearly $e_f(0) \geq 2n-2+n+n-1 \geq 4n-3$. Also $e_f(1) \leq 2n+3$. Then $e_f(0) - e_f(1) \geq 2n-6$. It follows that $n \leq 3$. when $n=3$, $e_f(1) \leq 6$. This implies $DS(B_{3,3})$ is not difference cordial. $DS(B_{1,1}) \cong DS(P_4)$, using Theorem 3.1, $DS(B_{1,1})$ is difference cordial. The difference cordial labeling of $DS(B_{2,2})$ is given in Figure 7.

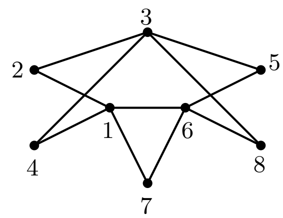

Figure 7

This completes the proof. □

Theorem 3.18 *$DS(B_{1,n})$ is difference cordial iff $n \leq 4$.*

Proof $DS(B_{1,n})$ consists of $n+4$ vertices and $2n+3$ edges. Note that $e_f(1) \leq 5$. Then $e_f(0) \geq q-5 \geq 2n-2$. $\Rightarrow e_f(0) - e_f(1) \geq 2n-7$. It follows that $n \leq 4$. By Theorem 3.17, $DS(B_{1,1})$ is difference cordial. The difference cordial labeling of $DS(B_{1,2})$, $DS(B_{1,3})$ and $DS(B_{1,4})$ are given in Figure 8.

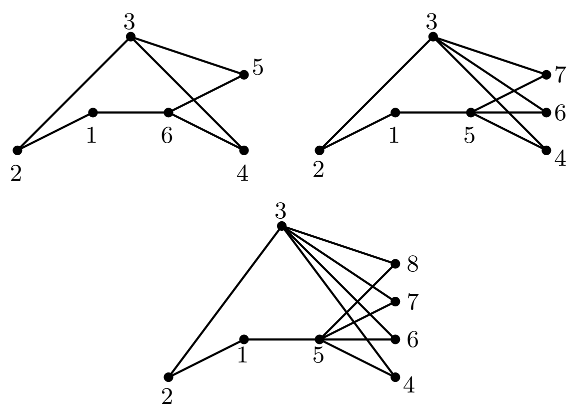

Figure 8

This completes the proof. □

Theorem 3.19 *$DS(B_{2,n})$ is difference cordial iff $n \leq 4$.*

Proof The order and size of $DS(B_{2,n})$ are $n+5$ and $2n+5$ respectively. It is clear that $e_f(1) \leq 6$. Then $e_f(0) \geq q-6 \geq 2n-1$. Hence $e_f(0) - e_f(1) \geq 2n-7$. This implies $n \leq 4$.

Using theorems 3.18, 3.17, $DS\left(B_{2,1}\right)$ and $DS\left(B_{2,2}\right)$ are difference cordial. The difference cordial labeling of $DS\left(B_{2,3}\right)$ and $DS\left(B_{2,4}\right)$ are given in Figure 9.

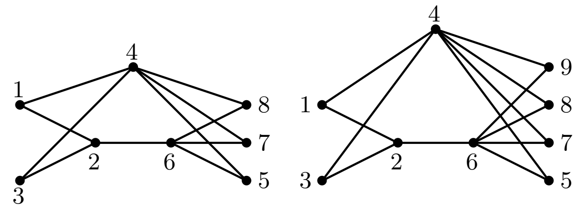

Figure 9

This completes the proof. □

Theorem 3.20 *$DS\left(B_{3,n}\right)$ is difference cordial iff $n \leq 2$.*

Proof The number of vertices and edges in $DS\left(B_{3,n}\right)$ are $n+6$ and $2n+7$ respectively. obviously $e_f(1) \leq 6$. $\Rightarrow e_f(0) \geq q-6 \geq 2n+1$. Therefore $e_f(0) - e_f(1) \geq 2n-5 \longrightarrow (1)$. Suppose $n > 3$ then a contradiction arises to (1). By Theorem 3.17, $DS\left(B_{3,3}\right)$ is not difference cordial. Using theorems 3.18, 3.19, $DS\left(B_{3,1}\right)$ and $DS\left(B_{3,2}\right)$ are difference cordial. □

§4. Shadow Graphs

The shadow graph $D_2(G)$ of a connected graph G is constructed by taking two copies of G, $G^{'}$ and $G^{''}$ and joining each vertex $u^{'}$ in $G^{'}$ to the neighbors of the corresponding vertex $u^{''}$ in $G^{''}$.

Theorem 4.1 *Let G be a (p,q) graph with $q \geq p$. Then $D_2(G)$ is not difference cordial.*

Proof Suppose G is a difference cordial graph with $q \geq p$. Clearly, $D_2(G)$ consists of $2p$ vertices and $4q$ edges. By Theorem 2.4, $4q \leq 2(2p) - 1$. This implies $4q \leq 4q - 1$, a contradiction. □

Theorem 4.2 *$D_2(P_n)$ is difference cordial.*

Proof Let $V(D_2(P_n)) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(D_2(P_n)) = \{u_iu_{i+1}, v_iv_{i+1} : 1 \leq i \leq n-1\} \cup \{u_iv_{i+1}, v_iu_{i+1} : 1 \leq i \leq n-1\}$. Define a map $f : V(D_2(P_n)) \rightarrow \{1, 2 \ldots 2n\}$ by

$$\begin{aligned} f(u_i) &= i & 1 \leq i \leq n \\ f(v_i) &= n+i & 1 \leq i \leq n. \end{aligned}$$

Since $e_f(0) = e_f(1) = 2n-2$, f is a difference cordial labeling of $D_2(P_n)$. □

Theorem 4.3([11]) *Any Cycle is a difference cordial graph.*

Theorem 4.4 *$D_2(K_n)$ is difference cordial iff $n \leq 2$.*

Proof The order and size of $D_2(K_n)$ are $2n$ and $2\binom{n}{2} + n(n-1)$ respectively. Suppose $D_2(K_n)$ is difference cordial. By Theorem 2.4, $2\binom{n}{2} + n(n-1) \leq 2(2n) - 1$. This implies,

$2n^2-6n+1\leq 0$. It follows that, $n\leq 2$. Since $D_2(K_2)\cong C_4$, using Theorem 4.3, $D_2(K_2)$ is difference cordial. □

Theorem 4.5 *$D_2(K_{1,m})$ is difference cordial iff $m\leq 2$.*

Proof The order and size of $D_2(K_{1,m})$ are $2m+2$ and $4m$ respectively. Clearly, $e_f(1)\leq 2m+1$. let v be the central vertex of $K_{1,m}$ and $v^{'}$ be the corresponding shadow vertex. Note that the degree of v and $v^{'}$ in $D_2(K_{1,m})$ are $2m$. Therefore, $e_f(0)\geq(2m-2)+(2m-2)\geq 4m-4$. Hence, $e_f(0)-e_f(1)\geq 2m-3$. This implies, $m\leq 2$. Since $D_2(K_{1,1})\cong C_4$, by Theorem 4.3, $D_2(K_{1,1})$ is difference cordial. A difference cordial labeling of $D_2(K_{1,2})$ is given in Figure 10.

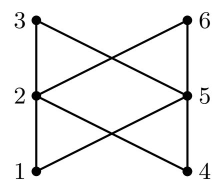

Figure 10

This completes the proof. □

References

[1] M.Andar, S.Boxwala, and N.Limaye, Cordial labelings of some wheel related graphs, *J. Combin. Math. Combin. Comput.,* **41** (2002) 203-208.

[2] I.Cahit, Cordial graphs: a weaker version of graceful and harmonious graphs, *Ars combin.,* **23** (1987) 201-207.

[3] A.Diab, Study of some problems of cordial graphs, *Ars Combin.,* **92** (2009) 255-261.

[4] J.A.Gallian, *A Dynamic survey of graph labeling*, *The Electronic Journal of Combinatorics*, **18** (2012) Ds6.

[5] F.Harary, *Graph theory*, Addision wesley, New Delhi (1969).

[6] Y.S.Ho, S.M.Lee and S.C.Shee, Cordial labellings of the cartesian product and composition of graphs, *Ars Combin.,* **29** (1990) 169-180.

[7] W.W.Kirchherr, On the cordiality of some specific graphs, *Ars Combin.,* **31** (1991) 127-138.

[8] D.Kuo, G.Chang, and Y.-H.Kwong, Cordial labeling of mKn, *Discrete Math.,* **169** (1997) 121-131.

[9] S.M.Lee and A.Liu, A construction of cordial graphs from smaller cordial graphs, *Ars Combin.,* **32** (1991) 209-214.

[10] Z.Liu and B.Zhu, A necessary and suficient condition for a 3-regular graph to be cordial, *Ars Combin.,* **84** (2007) 225-230.

[11] R.Ponraj, S.Sathish Narayanan and R.Kala, Difference cordial labeling of graphs, *Global Journal of Mathematical Sicences: Theory and Practical,* **5**(2013), 185-196.

[12] R.Ponraj, S.Sathish Narayanan, R.Kala, Difference cordial labeling of graphs obtained from Double snakes, *International Journal of Mathematics Research,* **5**(2013), 317-322.

[13] R.Ponraj, S.Sathish Narayanan, Difference cordiality of some graphs obtained from Double Alternate Snake Graphs, *Global Journal of Mathematical Sciences: Theory and Practical,* **5**(2013), 167-175.

[14] R.Ponraj, S.Sathish Narayanan and R.Kala, Difference cordial labeling of corona graphs,*J. Math. Comput. Sci.*, **3**(2013), 1237-1251.

[15] R.Ponraj and S.Sathish Narayanan, Further results on difference cordial labeling of corona graphs,*The Journal of the Indian Academy of Mathematics*, **35**(2013).

[16] R.Ponraj, S.Sathish Narayanan and R.Kala, Difference cordiality of Some Product related graphs (communicated).

[17] R.Ponraj, M.Sivakuamr and M.Sundaram, k-Product cordial labeling of graphs, *Int. J. Contemp. Math. Sciences,* Vol. 7, 2012, no. 15, 733 - 742.

[18] R.Ronraj, M.Sivakumar, M.Sundaram, k- Total product cordial labelings of graphs, *Applications and Applied Mathematics,*Vol. 7, Issue 2 (December 2012), pp. 708- 716

[19] R.Ponraj and S.Somasundaram, On the Degree Splitting graph of a graph, *National Academy Science Letter*, **27**(2004) 275-278.

[20] A.Riskin, Cordial deficiency, *Bull. Malays. Math. Sci. Soc.,* **30** (2007) 201-204. arXiv:math /0610760v1 [math.CO] 25 Oct 2006.

[21] A. Rosa, On certain valuations of the vertices of a graph, *Theory of Graphs (Internat. Symposium, Rome, July 1966)*, Gordon and Breach, N. Y. and Dunod Paris (1967) 349-355.

[22] E.Sampathkumar and H.B.Waliker, On the Splitting graph of a graph, *J. Karnatak Uni., Sci,* **25** (1980)13-16.

[23] M.A.Seoud and A.E.I.Abdel Maqsoud, On cordial and balanced labelings of graphs, *J. Egyptian Math. Soc.,* **7**(1999) 127-135.

[24] S.K.Vaidya, G.Ghodasara, S.Srivastav, and V.Kaneria, Cordial labeling for two cycle related graphs, *Math. Student,* **76** (2007) 237-246.

[25] M.Z.Youssef, On Skolem-graceful and cordial graphs, *Ars Combin.,* **78** (2006) 167-177.