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Bicoset of an $(\in v\ q)$-Fuzzy Bigroup

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Abstract: In this paper, we define $(\in, \in v\ q)$- fuzzy normal bigroup and $(\in, \in v\ q)$- fuzzy bicoset of a bigroup and discuss their properties as an extension of our work in [2]. We show that an $(\in, \in v\ q)$- fuzzy bigroup of a bigroup G is an $(\in, \in v\ q)$- fuzzy normal bigroup of G if and only if $(\in, \in v\ q)$- fuzzy left bicosets and $(\in, \in v\ q)$- fuzzy right bicosets of G are equal. We also define appropriate algebraic operation on the set of all $(\in, \in v\ q)$- fuzzy normal bigroup of a bigroup G and show that it forms a group.

Key Words: bigroup, fuzzy bigroup, $(\in vq)$-fuzzy bigroup, $(\in, \in vq)$-fuzzy bicosets.

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§1. Introduction

Zadeh [14]introduced fuzzy set in 1965. Rosenfeld [9] introduced the notion of fuzzy subgroups in 1971. Ming and Ming [8] gave a condition for fuzzy subset of a set to be a fuzzy point, and used the idea to introduce and characterize the notions of quasi coincidence of a fuzzy point with a fuzzy set. Bhakat and Das [3] used these notions by Ming and Ming to introduce and characterize another class of fuzzy subgroup known as $(\in vq)$- fuzzy subgroups. These authors in [4] extended these concepts to $(\in vq)$-fuzzy normal subgroups.

The notion of bigroup was first introduced by P.L.Maggu [5] in 1994. This idea was extended in 1997 by Vasantha and Meiyappan [11]. Meiyappan [7] introduced and characterized fuzzy sub-bigroup of a bigroup in 1998. Akinola and Agboola in [2] introduced the idea of fuzzy singleton to fuzzy bigroup and used it to introduce restricted fuzzy bigroup. These authors also studied the properties of $(\in, \in v\ q)$ fuzzy bigroup.

In this paper, we define $(\in, \in v\ q)$- fuzzy normal bigroup and $(\in, \in v\ q)$- fuzzy bicoset of a bigroup and discuss their properties as an extension of our work in [2]. We show that an $(\in, \in v\ q)$- fuzzy bigroup of a bigroup G is an $(\in, \in v\ q)$- fuzzy normal bigroup of G if and only if $(\in, \in v\ q)$- fuzzy left bicosets and $(\in, \in v\ q)$- fuzzy right bicosets of G are equal. We also show that the set all $(\in, \in v\ q)$- fuzzy normal bigroup of G forms a group under a well defined operation.

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§2. Preliminary Results

Definition 2.1([5,6]) *A set $(G,+,\cdot)$ with two binary operations $+$ and $\cdot$ is called a bi-group if there exist two proper subsets G_1 and G_2 of G such that*

(i) $G = G_1 \cup G_2$;
(ii) $(G_1,+)$ is a group;
(iii) $(G_2,\cdot)$ is a group.

Definition 2.2)([5]) *A subset $H(\neq 0)$ of a bi-group $(G,+,\cdot)$ is called a sub bi-group of G if H itself is a bi-group under the operations of $+$ and $\cdot$ defined on G.*

Theorem 2.3([5]) *Let $(G,+,\cdot)$ be a bigroup. If the subset $H \neq 0$ of a bigroup G is a sub bigroup of G, then $(H,+)$ and $(H,\cdot)$ are generally not groups.*

Definition 2.4([12]) *Let G be a non empty set. A mapping $\mu : G \to [0,1]$ is called a fuzzy subset of G.*

Definition 2.5([12]) *Let μ be a fuzzy set in a set G. Then, the level subset μ_t is defined as $\mu_t = \{x \in G : \mu(x) \geq t\}$ for $t \in [0,1]$.*

Definition 2.6([9]) *Let μ be a fuzzy set in a group G. Then, μ is said to be a fuzzy subgroup of G, if the following hold:*

(i) $\mu(xy) \geq \min\{\mu(x),\mu(y)\}\ \ \forall\, x,y \in G$;
(ii) $\mu(x^{-1}) = \mu(x)\ \forall\, x \in G$.

Definition 2.7([11]) *Let μ_1 be a fuzzy subset of a set X_1 and μ_2 be a fuzzy subset of a set X_2, then the fuzzy union of the sets μ_1 and μ_2 is defined as a function $\mu_1 \cup \mu_2 : X_1 \cup X_2 \longrightarrow [0,1]$ given by:*

$$(\mu_1 \cup \mu_2)(x) = \begin{cases} max(\mu_1(x),\mu_2(x)) & if\ x \in X_1 \bigcap X_2, \\ \mu_1(x) & if\ x \in X_1\ \&\ x \notin X_2, \\ \mu_2(x) & if\ x \in X_2\ \&\ x \notin X_1. \end{cases}$$

Definition 2.8([2]) *Let $G = G_1 \cup G_2$ be a bi-group. Let $\mu = \mu_1 \cup \mu_2$ be a fuzzy bigroup. A fuzzy subset $\mu = \mu_1 \cup \mu_2$ of the form:*

$$\mu(x) = \begin{cases} M(t,s) \neq 0 & if\ x = y \in G, \\ 0 & if\ x \neq y. \end{cases}$$

where $t, s \in [0,1]$ such that

$$\mu_1(x) = \begin{cases} t \neq 0 & if\ x = y \in G_1, \\ 0 & if\ x \neq y. \end{cases}$$

and

$$\mu_2(x) = \begin{cases} s \neq 0 & if \ x = y \ \in G_2, \\ 0 & if \ x \neq y. \end{cases}$$

is said to be a fuzzy point of the bi-group G with support x and value $M(t,s)$, and denoted by $x_{M(t,s)}$.

Definition 2.9([2]) *A fuzzy point $x_{M(t,s)}$ of the bigroup $G = G_1 \cup G_2$, is said to belong to (resp. be quasi coincident with) a fuzzy subset $\mu = \mu_1 \cup \mu_2$ of G, written as $x_{M(t,s)} \in \mu$[resp. $x_{M(t,s)} q\mu$] if $\mu(x) \geq M(t,s)$(resp. $\mu(x) + M(t,s) > 1$). $x_{M(t,s)} \in \mu$ or $x_{M(t,s)}\ q\mu$ will be denoted by $x_M(t,s) \in vq\mu$.*

Definition 2.10([2]) *A fuzzy bisubset μ of a bigroup G is said to be an (∈ v q)- fuzzy sub bigroup of G if for every $x,\ y \in G$ and $t_1,\ t_2,\ s_1,\ s_2,\ t,\ s, \in [0,1]$,*

(i) $x_{M(t_1,t_2)} \in \mu,\ y_{M(s_1,s_2)} \in \mu \Rightarrow (xy)_{M(t,s)} \in vq\mu$;
(ii) $x_{M(t_1,t_2)} \in \mu \Rightarrow (x^{-1})_{M(t_1,t_2)} \in vq\mu$, *where* $t = M(t_1,t_2)$ *and* $s = M(s_1,s_2)$.

Theorem 2.11([2]) *Let $\mu = \mu_1 \cup \mu_2 :\ G = G_1 \cup G_2 \to [0,1]$ be a fuzzy subset of G. Suppose that μ_1 is an $(\in vq)$-fuzzy subgroup of G_1 and μ_2 is an $(\in vq)$-fuzzy subgroup of G_2, then μ is an $(\in vq)$-fuzzy subgroup of G.*

§3. Main Results

Definition 3.1 *An (∈ v q)- fuzzy bigroup μ of a bigroup G is said to be an (∈ v q)- fuzzy normal bigroup of G if for any $x,\ y \in G$ and $t_1,\ t_2, \in [0,1]$, $x_{M(t_1,t_2)} \in \mu \Rightarrow (yxy^{-1})_{M(t_1,t_2)} \in vq\mu$.*

Theorem 3.2 *Let $\mu = \mu_1 \cup \mu_2 :\ G = G_1 \cup G_2 \to [0,1]$ be a fuzzy bi-subset of G. Suppose that μ_1 is an $(\in vq)$-fuzzy normal subgroup of G_1 and μ_2 is an $(\in vq)$-fuzzy normal subgroup of G_2, then μ is an $(\in vq)$-fuzzy normal bi-group of G.*

Proof That μ is an $(\in vq)$-fuzzy bigroup is clear from Theorem 2.11. To now show that it is normal. Suppose that μ_1 and μ_2 are $(\in vq)$-fuzzy normal subgroups of G_1 and G_2 respectively. For $x,\ y \in G$ and $t_1,\ t_2, \in [0,1]$,

$$x_{t_1} \in \mu_1 \Rightarrow (yxy^{-1})_{t_1} \in vq\mu_1$$

and

$$x_{t_2} \in \mu_2 \Rightarrow (yxy^{-1})_{t_2} \in vq\mu_2.$$

So that

$$\mu_1(yxy^{-1}) \geq t_1 \ or \ \mu_1(yxy^{-1}) + t_1 > 1$$

and

$$\mu_2(yxy^{-1}) \geq t_2 \ or \ \mu_2(yxy^{-1}) + t_2 > 1,$$

which shows that

$$max\{\mu_1(yxy^{-1}),\ \mu_2(yxy^{-1})\} \geq M(t_1,t_2)\ or\ max\{\mu_1(yxy^{-1}),\ \mu_2(yxy^{-1})\} + M(t_1,t_2) > 1.$$

Thus

$$\mu_1 \cup \mu_2(yxy^{-1}) \geq M(t_1,t_2)\ or\ \mu_1 \cup \mu_2(yxy^{-1}) + M(t_1,t_2) > 1.$$

So

$$\mu(yxy^{-1}) \geq M[t_1,t_2]\ or\ \mu(yxy^{-1}) + M[t_1,t_2]\ >\ 1,$$

which concludes that $(yxy^{-1})_{M(t_1,t_2)}\ \in vq\mu$. $\square$

Definition 3.3 *Let* $\mu\ =\ \mu_1 \cup \mu_2 :\ G = G_1 \cup G_2\ \rightarrow [0,1]$ *be a fuzzy bigroup of a bigroup* G. *For* $x \in G$, μ^l_x *(res* μ^r_x*)* $:\ G \rightarrow [0,1]$ *defined as*

$$\mu^l_x(g) = \mu(gx^{-1})\ (res\ \mu^r_x(g) = \mu(x^{-1}g)$$

is called an $(\in,\ \in)$*- fuzzy left(resp. fuzzy right) cosets of* G *determined by* x *and* μ.

Remark 3.4 Let μ be a fuzzy bigroup of a bigroup G, then μ is an $(\in,\ \in)$- fuzzy normal bigroup of a G if and only if $\mu^l_x(g)\ =\ \mu^r_x(g)$.

$(\in,\ \in)$- fuzzy bigroup here refers to fuzzy bigroup that satisfy Meiyappian's fuzzy bigroup conditions.

Example 3.5 Let $G = \{e,\ a,\ b,\ c,\ d,\ f,\ x,\ y,\ z,\ w\}$ be a bigroup where $G_1 = \{e,\ a,\ b,\ c,\ d,\ f,\}$ with the cayley table

×	e	a	b	c	d	f
e	e	a	b	c	d	f
a	a	b	c	f	e	d
b	b	e	a	d	f	c
c	c	d	f	e	a	b
d	d	f	c	b	e	a
f	f	c	d	a	b	e

and $G_2 = \{x,\ y,\ z,\ w,\}$ with Cayley table given below

∘	x	y	z	w
x	x	y	z	w
y	y	x	w	z
z	z	w	x	y
w	w	z	y	x

be the constituting subgroups. Define $\mu = \mu_1 \cup \mu_2 : G = G_1 \cup G_2 \to [0,1]$ as $\{0.6,\ 0.75,\ 0.8,\ 0.4,\ 0.4,\ 0.4\}$ for $\{e,\ a,\ b,\ c,\ d,\ f,\}$ respectively, and $\{0.6,\ 0.3,\ 0.3,\ 0.5\}$ for $\{x,\ y,\ z,\ w,\}$ respectively. It is also easy to see that fuzzy bisubset μ so defined on the bigroup G is an $(\in, \in vq)$ fuzzy bigroup. Now consider

$$0.6 = \mu(e) = \mu_1(e) = \mu_1(bb^{-1}) = \mu_1(ba)$$
$$\leq \min\{\mu_1(b), \mu_1(a)\}$$
$$= \min\{0.75, 0.7\} = 0.7$$

even though

$$0.6 = \mu_2(x) = \mu_2(zz^{-1}) = \mu_2(zz)$$
$$\geq max\{\mu_2(z), \mu_2(z)\} = 0.3.$$

Hence, μ so defined on the bigroup G is an $(\in, \in)$ fuzzy bigroup.

Also, consider

$$\mu^l_d(c) = \mu^l_{1d}(c) = \mu_1(cd^{-1}) = \mu_1(cd) = \mu_1(a) = 0.75$$

since $\mu^l_{2d}(c) = 0$ and

$$\mu^r_d(c) = \mu^r_{1d}(c) = \mu_1(d^{-1}c) = \mu_1(dc) = \mu_1(b) = 0.8$$

also since $\mu^r_{2d}(c) = 0$. Even though,

$$\mu^l_y(z) = \mu^l_{2y}(z) = \mu_2(yz^{-1}) = \mu_2(yz) = \mu_2(w) = 0.5$$

and

$$\mu^r_y(z) = \mu^r_{2y}(z) = \mu_2(z^{-1}y) = \mu_2(zy) = \mu_2(w) = 0.5$$

where $\mu^l_{1r}(z) = 0$ and $\mu^r_{1r}(z) = 0$. It is clear that $\mu^l_x \neq \mu^r_x$ generally.

Definition 3.6 *Let μ be a fuzzy bigroup of a bigroup G. For any $x \in G$, $\hat{\mu}_x$(resp.$\check{\mu}_x$) $: G \to [0,1]$ defined by*

$$\hat{\mu}_x(g) = M[\mu(gx^{-1}), 0.5]\ (resp.\ \check{\mu}_x(g) = M[\mu(gx^{-1}), 0.5]$$

for every $g \in G$ is called $(\in, \in vq)$-fuzzy left bicoset (resp.$(\in, \in vq)$-fuzzy right bicoset) of G determined by x and μ.

Theorem 3.7 *Let μ be a fuzzy bigroup of a bigroup G. Then μ is an $(\in, \in vq)$-fuzzy normal bigroup of G if*

$$\hat{\mu}_x = \check{\mu}_x\ \forall\ x \in G.$$

Proof Let μ be an $(\in, \in vq)$-fuzzy normal bigroup of G. Let $x \in G$, then $\forall\ g \in G$, if $x,\ g \in G \setminus G_2$,

$$\begin{aligned} \hat{\mu}_x(g) &= (\hat{\mu_1} \cup \hat{\mu_2})_x(g) = \hat{\mu}_{1x}(g) = M[\mu_1(gx^{-1}), 0.5] \\ &\geq M[\mu_1(x^{-1}g), 0.5] = \check{\mu}_{1x}(g) = \check{\mu}_x(g) \end{aligned}$$

Therefore, $\hat{\mu}_x(g) \geq \check{\mu}_x(g)$. By similar argument, we can show that $\check{\mu}_x(g) \geq \hat{\mu}_x(g)$ for all $x,\ g \in G \setminus G_2$.

If $x,\ g \in G \setminus G_1$, then,

$$\hat{\mu}_x(g) = (\hat{\mu_1} \cup \hat{\mu_2})_x(g) = \hat{\mu}_{2x}(g) = M[\mu_2(gx^{-1)}, 0.5]$$

$$\geq M[\mu_2(x^{-1}g), 0.5] = \check{\mu}_{2x}(g) = \check{\mu}_x(g)$$

Therefore, $\hat{\mu}_x(g) \geq \check{\mu}_x(g)$. By similar argument, we can show that $\check{\mu}_x(g) \geq \hat{\mu}_x(g)$ for all $x,\ g \in G \setminus G_1$.

If $x,\ g \in G_1 \cap G_2$, then,

$$\begin{aligned}
\hat{\mu}_x(g) &= (\hat{\mu_1} \cup \hat{\mu_2})_x(g)) = \max\{\hat{\mu_1}(g), \hat{\mu_2}(g) \\
&= \max\{M[\mu_1(gx^{-1}), 0.5], M[\mu_2(gx^{-1}), 0.5]\} \\
&\geq \max\{M[\mu_1(x^{-1}g), 0.5], M[\mu_2(x^{-1}g), 0.5]\} \\
&= \max\{\check{\mu}_{1x}(g), \check{\mu}_{2x}(g) = (\check{\mu}_1 \cup \check{\mu}_2)_x(g) = \check{\mu}_x(g).
\end{aligned}$$

Therefore, $\hat{\mu}_x(g) \geq \check{\mu}_x(g)$ for all $x,\ g \in G_1 \cap G_2$. Similar argument shows that $\check{\mu}_x(g) \geq \hat{\mu}_x(g)$. Hence, $\hat{\mu}_x = \check{\mu}_x\ \forall\ x \in G$. □

Theorem 3.8 *Let μ be a fuzzy bigroup of a bigroup G. If $\hat{\mu}_x = \check{\mu}_x\ \forall\ x \in G$, then μ is an $(\in, \in vq)$-fuzzy normal bigroup of G.*

Proof The theorem is a direct converse of Theorem 3.1.6. Let $\hat{\mu}_x = \check{\mu}_x\ \forall\ x \in G$, then, for all $g \in G$,

$$(\hat{\mu_1} \cup \hat{\mu_2})_x(g) = (\check{\mu}_1 \cup \check{\mu}_2)_x(g)$$

which implies that

$$\max\{M[\mu_1(gx^{-1}), 0.5], M[\mu_2(gx^{-1}), 0.5]\} = \max\{M[\mu_1(x^{-1}g), 0.5], M[\mu_2(x^{-1}g), 0.5]\}.$$

if we replace g by xyx, it follows that

$$\max\{M[\mu_1(xy), 0.5], M[\mu_2(xy), 0.5]\} = \max\{M[\mu_1(yx), 0.5], M[\mu_2(yx), 0.5]\},$$

which shows that μ, which is a fuzzy bigroup of the bigroup G is normal. That μ is an $(\in, \in vq)$-fuzzy normal bigroup of G is a direct consequence of equivalent conditions of Proposition 2.4.1. Hence the proof. □

What can we say about the properties of a set that contains all the $(\in, \in vq)$- fuzzy normal bigroup of a bigroup G? Can an appropriate operation be defined on this set to form a group or a normal subgroup of that set? The following observations have been made to give an insight into the answers:

In a bigroup $G = G_1 \cup G_2$, if we let μ_1 be a normal fuzzy subgroup of G_1 and S, the set of all fuzzy cosets $\hat{\mu}_1$ of μ_1 in G_1. If we follow the approach used for similar concept in [4], define composition on S as:

$$\hat{\mu}_{1x} \cdot \hat{\mu}_{1y} = \hat{\mu}_{1xy} \qquad \forall\ x,\ y\ \in\ G.$$

For any $g \in G_1$, if we let

$$\hat{\mu}_{1x}(g) = \hat{\mu}_{1y}(g) \ \ and \ \ \hat{\mu}_{1z}(g) = \hat{\mu}_{1w}(g),$$

then

$$M[\mu_1(gx^{-1}), 0.5] = M[\mu_1(gy^{-1}), 0.5] \qquad (\star)$$

and

$$M[\mu_1(gz^{-1}), 0.5] = M[\mu_1(gw^{-1}), 0.5], \qquad (\star\star)$$

so that

$$\hat{\mu}_{1xz}(g) = M[\mu_1(gz^{-1}x^{-1}), 0.5] = M[\mu_1(gz^{-1}x^{-1}), 0.5].$$

By replacing g by gz^{-1} in $(\star)$.

$$M[\mu_1(gz^{-1}x^{-1}), 0.5] \geq M\{M[\mu_1(y^{-1}gz^{-1}),\ 0.5\ ], 0.5\}$$

and since μ_1 is fuzzy normal, it follows that

$$M\{M[\mu_1(y^{-1}gz^{-1}),\ 0.5\ ], 0.5\} \geq M[\mu_1(y^{-1}gw^{-1}),\ 0.5]$$

replacing g by $y^{-1}g$ in $(\star\star)$.

$$M[\mu_1(y^{-1}gw^{-1}),\ 0.5] \geq M[\mu_1(gw^{-1}y^{-1}),\ 0.5]$$

and since μ_1 is fuzzy normal, it follows that

$$M[\mu_1(gw^{-1}y^{-1}),\ 0.5] \geq \hat{\mu}_{1yw}(g).$$

By a similar argument, it can be shown that $\hat{\mu}_{1yw}(g) \geq \hat{\mu}_{1xz}(g) \quad \forall \quad g \in G$, so that $\hat{\mu}_{1xz} = \hat{\mu}_{1yw}$, which shows that the composition defined on S is well defined.

It is easy to see that S is a group with the identity element $\hat{\mu}_{1e}$, and $\hat{\mu}_{1x^{-1}}$ as the inverse of $\hat{\mu}_{1x}$ for every $x \in G_1$. Let $\bar{\mu} : S \to [0, 1]$ be defined by

$$\bar{\mu}(\hat{\mu}_{1x}) = \mu_1(x) \qquad \forall x \in G_1,$$

it is observed that

$$\begin{aligned} \bar{\mu}(\hat{\mu}_{1x} \cdot \hat{\mu}_{1y^{-1}}) = \mu(\hat{\mu}_{1xy^{-1}}) &= \mu_1(xy^{-1}) \\ &= M[\mu_1(x),\ \mu_1(y),\ \ 0.5] \\ &= M[\bar{\mu}(\hat{\mu}_{1x}),\ \bar{\mu}(\hat{\mu}_{1y}),\ \ 0.5] \quad \forall \quad \hat{\mu}_{1x},\ \hat{\mu}_{1y}\ \in\ S. \end{aligned}$$

Also,

$$\begin{aligned} \bar{\mu}(\hat{\mu}_{1x}\hat{\mu}_{1a}\hat{\mu}_{1x^{-1}}) = \bar{\mu}(\hat{\mu}_{1xax^{-1}}) &= \mu_1(xax^{-1}) \\ &= M[\mu_1(a),\ \ 0.5] \end{aligned}$$

since μ is fuzzy normal,

$$M[\mu_1(a),\ \ 0.5] = M[\bar{\mu}(\hat{\mu}_{1x},\ \ 0.5].$$

which shows that $\bar{\mu}$ is a fuzzy normal subgroup of S.

Now that it has been established that in a bigroup $G = G_1 \cup G_2$, if μ_1 is a normal fuzzy subgroup of G_1 and S_1, the set of all fuzzy cosets $\hat{\mu}_1$ of μ_1 in G_1, is a normal subgroup with respect to a well defined operation.

By extended implication, we can say that in a bigroup $G = G_1 \cup G_2$, if μ_2 is a normal fuzzy subgroup of G_2 and S_2, the set of all fuzzy cosets $\hat{\mu}_2$ of μ_2 in G_2, is a normal subgroup with respect to a well defined operation, so that we can then conclude that in a bigroup $G = G_1 \cup G_2$, if $\mu = \mu_1 \cup \mu_2$ is a normal fuzzy subgroup of G and S, the set of all fuzzy cosets $\hat{\mu}$ of μ in G is a normal subgroup with respect to a well defined operations S.

This result is summarized below:

Corollary 3.9 *Let $G = G_1 \cup G_2$ be a bigroup. If $\mu = \mu_1 \cup \mu_2$ is a normal fuzzy subgroup of G, the set S, of all fuzzy cosets $\hat{\mu}$ of μ in G is a normal subgroup with respect to a well defined operations on S.*

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