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Domination Number in 4-Regular Graphs

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Abstract: A set of vertices S in a graph G is said to be a *Smarandachely k-dominating set* if each vertex of G is dominated by at least k vertices of S. Particularly, if $k = 1$, such a set is called a dominating set of G. The *Smarandachely k-domination number* $\gamma_k(G)$ of G is the minimum cardinality of a Smarandachely k-dominating set of G. For abbreviation, we denote $\gamma_1(G)$ by $\gamma(G)$. In [9], Reed proved that the domination number $\gamma(G)$ of every $n-$vertex graph G with minimum degree at least 3 is at most $3n/8$. In this note, we present a sequence of Hamiltonian 4-regular graphs whose domination numbers are sharp. Here we state some results which will pave the way in characterization of domination number in regular graphs. Also, we determine independent, connected, total and forcing domination number of those graphs.

Key Words: Regular graph, Smarandachely k-dominating set, Hamiltonian graph.

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§1. Introduction

Throughout this paper, all graphs considered are finite, undirected, loopless and without multiple edges. We refer the reader to [11] for terminology in graph theory.

Let $G = (V, E)$ be a graph with vertex set V and edge set E, and let $v \in V$. The neighborhood of v, denoted by $N(v)$, is defined as the set of vertices adjacent to v, i.e., $N(v) = \{u \in V | uv \in E\}$. For $S \subseteq V$, the neighborhood of S, denoted by $N(S)$, is defined by $N(S) = \cup_{v \in S} N(v)$, and the closed neighborhood $N[S]$ of S is the set $N[S] = N(S) \cup S$ and the degree of x is $deg_G(x) = |N_G(x)|$.

A set of vertices S in a graph G is said to be a *Smarandachely k-dominating set*, if each vertex of G is dominated by at least k vertices of S. Particularly, if $k = 1$, such a set is called a *dominating set* of G. The *Smarandachely k-domination number* $\gamma_k(G)$ of G is the minimum

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cardinality of a Smarandachely k-dominating set of G. For abbreviation, we denote $\gamma_1(G)$ by $\gamma(G)$. The domination number has received considerable attention in the literature.

A dominating set S is called a *connected dominating set* if the subgraph $G[S]$ induced by S is connected. The *connected domination number* of G denoted by $\gamma_c(G)$ is the minimum cardinality of a connected dominating set of G. A dominating set S is called an *independent dominating set* if S is an independent set. The *independent domination number* of G denoted by $i(G)$ is the minimum cardinality of an independent dominating set of G. A dominating set S is a *total dominating set* of G if $G[S]$ has no isolated vertex and the *total domination number* of G, denoted by $\gamma_t(G)$, is the minimum cardinality of a total dominating set of G. A subset F of a minimum dominating set S is a *forcing subset* for S if S is the unique minimum dominating set containing F. The *forcing domination number* $f(G,\gamma)$ of S is the minimum cardinality among the forcing subsets of S, and the forcing domination number $f(G,\gamma)$ of G is the minimum forcing domination number among the minimum dominating sets of G ([1]-[7]). For every graph G, $f(G,\gamma) \le \gamma(G)$.

The problem of finding the domination number of a graph is NP-hard, even when restricted to 4-regular graphs. One simple heuristic is the greedy algorithm [10]. Let d_g be the size of the dominating set returned by the greedy algorithm. In 1991 Parekh [8] showed that $d_g \le n+1-\sqrt{2e+1}$. Reed [9] proved that $\gamma(G) \le \frac{3}{8}n$. Fisher et al. [3]-[4] repeated this result and showed that if G has girth at least 5 then $\gamma(G) \le \frac{5}{14}n$. In the light of these bounds on γ, in 2004 Seager considered bounds on d_g for r-regular graphs and showed that:

Theorem 1.1([10]) *For $r \ge 3$, $d_g \le \frac{r^2+4r+1}{(2r+1)^2} \times n$.*

Theorem 1.2([3]) *For any graph of order n, $\left\lceil \frac{n}{1+\Delta G} \right\rceil \le \gamma(G)$.*

The authors of [7] studied domination number in Hamiltonian cubic graphs, and stated in it the following problem.

Problem 1.3 *What are the domination numbers of the Hamiltonian 4-regular graphs?*

The aim of this article is to study the domination number $\gamma(G)$, independent domination number $i(G)$, connected domination number $\gamma_c(G)$, total domination number $\gamma_t(G)$ and forcing domination number $f(G,\gamma)$ for 4-regular graphs and give a sharp value for the domination numbers of these graphs.

§2. Domination Number

In this section we obtain a sharp value for the domination number of some 4-regular graph. In the following, we construct graphs G, G_1 and G_2 of which the graphs G and G_2 are 4-regular. The graph G_1 is not 4-regular but $deg_{G_1}(v_i) = 4$ where $2 \le i \le m-1$ and for the two remaining vertices, $deg_{G_1}(v_1) = deg_{G_1}(v_m) = 3$. Moreover, the graph G_2 will be obtained from the graphs G_1.

Remark 2.1 (i) Let G be a graph with $V(G) = \{v_1, v_2, ..., v_n\}$ and $E(G) = \{v_iv_j \mid |j-i| =$

$1\ or\ t\ or\ t+1\} \cup \{v_1v_t, v_{t+1}v_{2t}\}$ where $n = 2t$, $t \geq 3$;

(ii) Let G_1 be a graph with $V(G_1) = \{v_1, v_2, ..., v_m\}$ and $E(G_1) = \{v_iv_j \mid |j-i| = 1\ or\ s\ or\ s+1\}$ where $m = 2s+1$, $s \geq 2$;

(iii) Let $G_2 = \cup_{i=1}^{q} G_{m_i}$ where $G_{m_i} \cong G_1$, $|V(G_{m_i})| = m_i$ for all possible i and $|V(G_{m_1})| \leq |V(G_{m_2})| \leq ... \leq |V(G_{m_q})|$, such that $V(G_2) = \cup_{i=1}^{q} \cup_{j=1}^{m_i} \{v_iv_j\}$ and $E(G_2) = \cup_{i=1}^{q} E(G_{m_i}) \cup \{v_{im_i}v_{(i+1)1\ (mod\ q)} |\ i = 1, 2, ..., q\}$.

By Theorem 1.1, we have $d_g \leq (33/81)n$ for r-regular graphs where $r = 4$. In the following Theorems, we obtain the exact number for constructed 4-regular graphs.

In all following theorems, let m, n be odd and even respectively and $n \equiv l_1 \pmod 5$, $m \equiv l_2 \pmod 5$ then $m = 5p + l_2$ and $n = 5k + l_1$ where $0 \leq l_1, l_2 \leq 4$ and p, k are integers.

By Theorem 1.2, we have the following observation.

Observation 2.2 *$\gamma(G) \geq \lceil \frac{5k+l_1}{5} \rceil$ and $\gamma(G_1) \geq \lceil \frac{5p+l_2}{5} \rceil$.*

Theorem 2.3 *Let G be a graph of order n, then* $\gamma(G) = \begin{cases} k & if\ n \equiv 0\ (mod\ 5) \\ k+1 & otherwise \end{cases}$.

Proof We proceed by proving the series cases of following.

Case 1 If $n \equiv 0 \pmod 5$ then $n = 5k$. Let $S = \{v_3, v_8, v_{13}, \cdots, v_i, v_{i+5}, \cdots, v_{\frac{n}{2}-7}, v_{\frac{n}{2}-2}, v_{\frac{n}{2}+1}, v_{\frac{n}{2}+6}, \cdots, v_j, v_{j+5}, \cdots, v_{n-4}\}$. It is easy to verify that $|S| = (2 \times (\frac{n}{2} - 5)/5) + 2 = k$. Furthermore, every vertex in S dominates four vertices and itself and $N[x] \cap N[y] = \emptyset$ for any pair of vertices $x, y \in S$. It follows that S is a dominating set, so $\gamma(G) \leq k$. Using Observation 2.2 it is now straightforward to see that $\gamma(G) = k$.

Case 2 If $n \equiv 1 \pmod 5$ then $n = 5k+1$. Let $S = \{v_3, v_8, v_{13}, ..., v_i, v_{i+5}, ...v_{\frac{n}{2}-5}, v_{\frac{n}{2}-1}, v_{\frac{n}{2}+1}, v_{\frac{n}{2}+6}, \cdots, v_j, v_{j+5}, ..., v_{n-2}\}$ which implies $|S| = k+1$. Clearly, every vertex in $S - \{v_{\frac{n}{2}-1}\}$ dominates four vertices and itself. Then the non-dominated vertex $v_{\frac{n}{2}-1}$ is dominated by itself. Also, $N[x] \cap N[y] = \emptyset$ for every pair vertices $x, y \in S - \{v_{\frac{n}{2}-1}\}$. Thus S is a dominating set and $\gamma(G) \leq k+1$. Using Observation 2.2 it is now straightforward to see that $\gamma(G) = k+1$.

Case 3 If $n \equiv 2 \pmod 5$ so $n = 5k+2$. Assign $S = \{v_2, v_7, \cdots, v_i, v_{i+5}, \cdots, v_{\frac{n}{2}-9}, v_{\frac{n}{2}-4}, v_m, v_{\frac{n}{2}+5}, \cdots, v_j, v_{j+5}, ..., v_{n-1}\}$ and $m \in \{\frac{n}{2}, \frac{n}{2}+1\}$. One can see that any vertex in $S - \{v_m\}$ dominates four vertices and itself and the two non-dominated vertices $v_{\frac{n}{2}}$ and $v_{\frac{n}{2}+1}$ are dominated by vertex v_m. Obviously, $|S| = k+1$. Moreover, for every pair of vertices x and y from $S - \{v_m\}$, we have $N[x] \cap N[y] = \emptyset$. Therefore S is a dominating set for G that implies $\gamma(G) \leq k+1$. Using Observation 2.2 it is now straightforward to see that $\gamma(G) = k+1$.

Case 4 If $n \equiv 3 \pmod 5$ so $n = 5k+3$. Let $S = \{v_2, v_7, v_{12}, \cdots, v_i, v_{i+5}, \cdots, v_{\frac{n}{2}-2}, v_m, v_{\frac{n}{2}+5}, \cdots, v_j, v_{j+5}, \cdots, v_{n-4}\}$, where $m \in \{\frac{n}{2}, \frac{n}{2}+1, n\}$. By simple verification one can see that every vertex in $S - \{v_m\}$ dominates four vertices and itself and the three vertices $v_{\frac{n}{2}}$, $v_{\frac{n}{2}+1}$ and v_n are dominated by vertex v_m. Clearly, $|S| = k+1$ and $N[x] \cap N[y] = \emptyset$ for all possible vertices $x, y \in S - \{v_m\}$. Therefore S is a dominating set for G that implies $\gamma(G) \leq k+1$. Using Observation 2.2 it is now straightforward to see that $\gamma(G) = k+1$.

Case 5 If $n \equiv 4 \pmod 5$, so $n = 5k+4$. Let $S = \{v_2, v_7, ..., v_i, v_{i+5}, ..., v_{\frac{n}{2}-5}, v_{\frac{n}{2}}, v_{\frac{n}{2}+5}, ..., v_j,$

$v_{j+5}, \cdots, v_{n-2}\}$. We see every vertex in $S - \{v_{\frac{n}{2}}\}$ dominated four vertices and itself and the vertex $v_{\frac{n}{2}}$ dominates three vertices $\{v_{\frac{n}{2}-1}, v_{\frac{n}{2}+1}, v_n\}$ and itself. Since $|S| = k + 1$ and $N[x] \cap N[y] = \emptyset$ for all possible vertices $x, y \in S - \{v_{\frac{n}{2}}\}$. Then S is a dominating set for G that implies $\gamma(G) \leq k + 1$. By Observation 2.2 it is straightforward to see that $\gamma(G) = k + 1$. □

Theorem 2.4 *Let G_1 be a graph of order $m = 5p + l_2$ where $l_2 \in \{0, 1, 2, 3, 4\}$ and p is an integer, then* $\gamma(G_1) = \begin{cases} p \;\; if \; m \equiv 0 \; (mod \; 5); \\ p + 1 \;\; otherwise. \end{cases}$

Proof We consider the following sets such that $m \equiv l_2$ (mod 5) for $0 \leq l_2 \leq 4$.

For $l_2 = 0$. We say $S = \{v_2, v_7, ..., v_i, v_{i+5}, ..., v_s, v_{s+5}, ..., v_j, v_{j+5}, ..., v_{m-3}\}$.
For $l_2 = 1$. We say $S = \{v_2, v_7, ..., v_i, v_{i+5}, ..., v_{s-3}, v_s, v_{s+5}, v_{s+10}, ..., v_j, v_{j+5}, ..., v_{m-1}\}$.
For $l_2 = 2$. We say $S = \{v_2, v_7, .., v_i, v_{i+5}, .., v_{s-1}, v_{s+1}, v_{s+5}, v_{s+10}, .., v_j, v_{j+5}, ..., v_{m-4}\}$.
For $l_2 = 3$. We say $S = \{v_2, v_7, ..., v_i, v_{i+5}, ..., v_{s-4}, v_s, v_{s+5}, v_{s+10}, ..., v_j, v_{j+5}, ..., v_{m-2}\}$.
For $l_2 = 4$. We say $S = \{v_2, v_7, ..., v_i, v_{i+5}, ..., v_{s-2}, v_s, v_{s+5}, v_{s+10}, ..., v_j, v_{j+5}, ..., v_{m-5}\}$.

A method similar to that described in proof of Theorem 2.3 can be applied for proof of this Theorem. From this, one can see that all of the considered sets are dominating sets. Using Observation 2.2 it is now straightforward to obtain the stated results in this Theorem. □

Now we are ready to study domination number of more 4-regular graphs which are stated in Theorem 2.3.

Remark 2.5 We construct graph $G' = G_{n_1} \cup G_{n_2} \cup ... \cup G_{n_r}$ in which between every two 4-regular graphs we add an edge such that $d(v) = 5$ to each of first and end vertices of G_{n_i} for all possible i and $G_{n_i} \cong G$, $|V(G_{n_1})| \leq |V(G_{n_2})| \leq ... \leq |V(G_{n_r})|$.

Theorem 2.6 $\gamma(G') = \sum_{i=1}^{r} \gamma(G_{n_i})$ *such that there exists a G with $G_{n_i} \cong G$ for each i.*

Proof The result follows by Theorem 2.3. □

Let $G_1^{'}$ and $G_1^{''}$ be the graphs in which these are two induced subgraphs of G_1 such that $V(G_1^{'}) = V(G_1) - \{v_1, v_m\}$ and $V(G_1^{''}) = V(G_1) - \{v_1\}$ (or $V(G_1^{''}) = V(G_1) - \{v_m\}$).

Proposition 2.7 *(i)* $\gamma(G_1^{'}) = \gamma(G_1^{''}) = \gamma(G_1)$ *where* $V(G_1) \equiv l$ *to modulo 5 and* $l \in \{0, 2, 3, 4\}$*;*

(ii)Let $V(G_1) \equiv 1$ *to modulo 5. Then (a):* $\gamma(G_1^{'}) = \gamma(G_1) - 1$ *(b):* $\gamma(G_1^{''}) = \gamma(G_1)$ *where* $V(G_1^{''}) = V(G_1) - \{v_1\}$ *(c):* $\gamma(G_1^{''}) = \gamma(G_1) - 1$ *where* $V(G_1^{''}) = V(G_1) - \{v_m\}$.

Proof (i) The result follows by Observation 2.2 and Theorem 2.4.

(ii) Let $V(G_1) \equiv 1$ (mod 5). We say $S = \{v_4, v_9, ..., v_{s-1}, v_{s+2}, v_{s+7}, ..., v_{m-4}\}$. Clearly S is a dominating set for $G_1^{'}$ and $G_1^{''}$ where $V(G_1^{''}) = V(G_1) - \{v_m\}$. Therefore $\gamma(G_1^{'}) = \gamma(G_1^{''}) = \gamma(G_1) - 1$ because $|S| = \frac{m-1}{5}$. Finally if $V(G_1^{''}) = V(G_1) - \{v_1\}$, one can check by simple verification that $\gamma(G_1^{''}) = \gamma(G_1)$. □

Proposition 2.8 *Let G_2 be the graph with $V(G_{m_i}) \equiv 1$ (mod 5) for all i. Then* $\gamma(G_2) = \sum_{i=1}^{q} \gamma(G_{m_i}) - \lfloor \frac{q}{2} \rfloor$.

Proof The result follows by Proposition 2.7 (ii)(c). Moreover, It is sufficient to show the truth of the statement when $q = 2$ ($G_2 = G_{m_1} \cup G_{m_2}$).

Let $S = \{v_{14}, v_{19}, \cdots, v_{1i}, v_{1(i+5)}, \cdots, v_{1(s_{m_1}-1)}, v_{1(s_{m_1}+2)}, v_{1(s_{m_1}+7)}, \cdots, v_{1j}, v_{1(j+5)}, \cdots, v_{1(m_1-4)}, v_{21}, v_{26}, \cdots, v_{2i'}, v_{2(i'+5)}, \cdots, v_{2(s_{m_2}-4)}, v_{2(s_{m_2}+1)}, \cdots, v_{2j'}, v_{2(j'+5)}, \cdots, v_{2(m_2-2)}\}$.

Obviously, $\gamma(G_2) = \gamma(G_{m_1}) + \gamma(G_{m_2}) - 1$. It is now straightforward to prove the result for $q > 2$, by Proposition 2.7(ii) and a method similar to that described for $q = 2$. Thus $\gamma(G_2) = \sum_{i=1}^{q} \gamma(G_{m_i}) - \lfloor \frac{q}{2} \rfloor$ with $V(G_{m_i}) \equiv 1 \pmod 5$ for all i. □

Let l be the number of occurrences of consecutive G_1's with $V(G_1) \equiv 1 \pmod 5$. For $1 \le i' \le l$, let $H_{i'} = \{G_2 - e \mid G_2 = \cup_{j=1}^{r_{i'}} G_{m_j},\ r_{i'}$ is the number of consecutive $\mathrm{G_{m_j}}$s with $\mathrm{V(G_{m_j})} \equiv 1 \pmod 5$ for all j, $e (= v_{11}^{i'} v_{r_{i'} m_{r_{i'}}}^{i'}) \notin G_{m_j}\}$.

Theorem 2.9 *Let $G_3 = \cup_{i=1}^{q} G_{m_i}$ which contains the induced subgraph $H_{i'}$ for $1 \le i' \le l$ and $G_3 \cong G_2$. Then $\gamma(G_3) = \sum_{i=1}^{q} \gamma(G_{m_i}) - (\lfloor \frac{r_1}{2} \rfloor + \lfloor \frac{r_2}{2} \rfloor + ... + \lfloor \frac{r_l}{2} \rfloor)$.*

Proof The result follows by Theorem 2.4 and Propositions 2.7 and 2.8. □

§3. Independent Domination Number of Some Graphs

Theorem 3.1 *If $n \equiv l$ (mod 5) where $0 \le l \le 4$, then $i(G) = \gamma(G)$.*

Proof We Suppose that $n \equiv 0 \pmod 5$ and n is even. Since $i(G) \ge \gamma(G)$, Theorem 2.3 implies that $i(G) \ge k$. Let $S = S_1 \cup S_2 = \{v_3, ..., v_i, v_{i+5}, ..., v_{\frac{n}{2}-7}, v_{\frac{n}{2}-2}\} \cup \{v_{\frac{n}{2}+1}, v_{\frac{n}{2}+6}, ..., v_j, v_{j+5}, ..., v_{n-4}\}$. It is sufficient to prove that there exists no pair of vertices (x, y) with $xy \in E(G)$ in S. Because, on the one hand $d_{P_n}(x, y) = 5$ (Let $P_n = v_1 v_2 ... v_n$) for any two consecutive vertices with $x, y \in S_1$ (or $x, y \in S_2$). On the other hand each $v_i \in S$ is adjacent to vertices v_{i-1}, v_{i+1}, $v_{i+\frac{n}{2}}$ and $v_{i+\frac{n}{2}+1}$. So by simple verification one can see that there exists no vertex in S from $\{v_{i-1}, v_{i+1}, v_{i+\frac{n}{2}}, v_{i+\frac{n}{2}+1}\}$. Hence S is an independent set of G, then $i(G) = \gamma(G)$.

Similar argument settles proof of cases $n \equiv l$ where $1 \le l \le 4$. □

Theorem 3.2 *If $m \equiv l$ (mod 5) where $0 \le l \le 4$, then $i(G_1) = \gamma(G_1)$.*

Proof Similar to that of Theorem 3.1, we settle the proof of this Theorem. □

Theorem 3.3 *$i(G_x) = \gamma(G_x)$ where $x = 2, 3$.*

Proof The result follows by Theorems 2.4 and 2.9. □

§4. Connected Domination Number of Some Graphs

Let $N_p[v_i] = N[v_i] - (N(v_i) \cap S)$ where S is an arbitrary set.

Theorem 4.1 *If $n \equiv l$ (mod 5), where $0 \le l \le 4$, then $\gamma_c(G) = \frac{n}{2} - 1$.*

Proof Let $n \equiv 0 \pmod 5$. Since $\gamma_c(G) \geq \gamma(G)$, Theorem 2.3 implies $\gamma_c(G) \geq k$. We introduce $S_0 = \{v_2, v_3, ..., v_{\frac{n}{2}}\}$. Obviously, S_0 is a connected dominating set for G, then $\gamma_c(G) \leq \frac{n}{2} - 1$. Now we suppose that S is an arbitrary connected dominating set for G with $|S| = l \leq \frac{n}{2} - 2$. Clearly, $\langle S \rangle$ is containing a path of length $l \leq \frac{n}{2} - 2$, and $|N_p[x]|$, $|N_p[y]| \leq 4$ and $|N_p[z]| = 3$ where x, y are pendant vertices of path and $z \in S - \{x, y\}$. Furthermore $|N_p[u] \cap N_p[v]| = 1$ where u, v are two consecutive vertices from S. By the assumptions we have $|\cup_{x \in S} N_p[x]| \leq (2 \times 4) + (\frac{n}{2} - 4) \times 3 - (\frac{n}{2} - 3) = n - 1$. Then S cannot dominate all vertices of G. This implies that S_0 is minimum connected dominating set of G, hence $\gamma_c(G) = \frac{n}{2} - 1$.

Similar argument settles proof of cases $n \equiv l$ where $1 \leq l \leq 4$. □

Theorem 4.2 *If $m \equiv l$ (mod 5) where $0 \leq l \leq 4$ then $\gamma_c(G_1) = s - 1$.*

Proof In a manner similar to Theorem 4.1 we can prove the Theorem. □

Theorem 4.3 $\gamma_c(G_2) = \sum_{i=1}^{q}(s_{m_i} + 1) - 2$.

Proof Theorem 4.2 implies that $\gamma_c(G_2) \geq \sum_{i=1}^{q}(s_{m_i} - 1)$. Because if S_1 and S_2 are arbitrary γ_c-sets for G_{m_1} and G_{m_2} with $|S_1| = s_{m_1} - 1$, $|S_2| = s_{m_2} - 1$ then $\langle S_1 \cup S_2 \rangle$ is disconnected. Furthermore, any γ_c-set for G_{m_i} does not contain first or endvertex of G_{m_i}. Therefore, to obtain a γ_c-set for G_2, we must add all of the end and first vertices of the graph G_{m_i} except for two graphs. For the first graph, say (G_{m_1}), we can add the endvertex and the last graph, say (G_{m_q}), we may add its first vertex (note that we may choose in a similar manner for two other graphs). Then $\gamma_c(G_2) = \sum_{i=1}^{q}(s_{m_i} + 1) - 2$. □

§5. Total Domination Number of Some Graphs

Let S be a minimum total dominating set, then we have the following Observations.

Observation 5.1 For any vertex $x \in S$, there exists at least one vertex $y \in S$ such that $xy \in E(G)$.

Observation 5.2 Let G be a 4-regular graph then $|N[x] \cup N[y]| \leq 8$, where $x, y \in S$ and $xy \in E(G)$.

Immediately we have the following lemma.

Lemma 5.3 *Let G and G_1 be the graphs defined in Remark* 2.1. *For any $x, y \in S$ with $xy \in S$ then $|N[x] \cup N[y]| \leq 7$.*

Proof Let $x = v_i$ and $y = v_{i+1}$ (or $y = v_{i-1}$) then $|N[x] \cup N[y]| = 7$. Now suppose that $x = v_i$ and $y = v_{i+\frac{n}{2}}$ (or $y = v_{i+\frac{n}{2}+1}$) then $|N[x] \cup N[y]| = 6$. Hence $|N[x] \cup N[y]| \leq 7$. □

We consider the following Theorem.

Theorem 5.4 $$\gamma_t(G) = \begin{cases} 2\lceil \frac{n}{7} \rceil & n \equiv l \ (mod\ 7) \ where\ l \in \{0, 3, 4, 5, 6\} \\ 2\lfloor \frac{n}{7} \rfloor + 1 & n \equiv 1 or\ 2 \ (mod\ 7) \end{cases}.$$

Proof The proof is divided into the following cases by considering $n \equiv$ (mod7).

Case 1 $n \equiv 0 \pmod 7$

Let $S = \{v_1, v_2, v_8, v_9, \cdots, v_i, v_{i+1}, \cdots, v_{\frac{n}{2}-6}, v_{\frac{n}{2}-5}, v_{\frac{n}{2}+5}, v_{\frac{n}{2}+6}, v_{\frac{n}{2}+12}, v_{\frac{n}{2}+13}, \cdots, v_j, v_{j+1}, \cdots, v_{n-2}, v_{n-1}\}$. It is easy to verify that S is a γ_t-set for G where $n \equiv 0 \pmod 7$. Moreover any two adjacent vertices from S have 7 vertices as neighbors, so by Lemma 5.3, S is minimum total dominating set for G and $\gamma_t(G) = |S| = 2\lceil \frac{n}{7} \rceil$ where $n \equiv 0 \pmod 7$.

Case 2 $n \equiv 1 \pmod 7$

Let $S_1 = \{v_1, v_2, v_8, v_9, \cdots, v_i, v_{i+1}, \cdots, v_{\frac{n}{2}-3}, v_{\frac{n}{2}-2}, v_{\frac{n}{2}+5}, v_{\frac{n}{2}+6}, v_{\frac{n}{2}+12}, v_{\frac{n}{2}+13}, \cdots, v_j, v_{j+1}, \cdots, v_{n-6}, v_{n-5}\}$. It is easy to verify that $(N[x] \cup N[y]) \cap (N[z] \cup N[t]) = \emptyset$ for each two pairs of vertices (x, y) and (z, t) and $xy, zt \in E(G)$ and $x, y, z, t \in S_1$. Also, $|N[r] \cup N[s]| = 7$ for all possible $r, s \in S_1$ and $rs \in E(G)$. Meanwhile, Lemma 5.3 implies that the set S_1 is a minimum γ_t-set for $G - M_1$ where $M_1 = \{v_n\}$. Now, we give $S_2 = \{v_{\frac{n}{2}-1}\}$. Clearly $S = S_1 \cup S_2$ is a γ_t-set of G where $n = 7k + 1$. Then $\gamma(G) = |S| = 2\lfloor \frac{n}{7} \rfloor + 1$ where $n \equiv 1 \pmod 7$.

Case 3 $n \equiv 2 \pmod 7$

Let $S_1 = \{v_1, v_2, v_8, v_9, \cdots, v_i, v_{i+1}, \cdots, v_{\frac{n}{2}-7}, v_{\frac{n}{2}-6}, v_{\frac{n}{2}+5}, v_{\frac{n}{2}+6}, v_{\frac{n}{2}+12}, v_{\frac{n}{2}+13}, \cdots, v_j, v_{j+1}, \cdots, v_{n-3}, v_{n-2}\}$. It is easy to verify that $(N[x] \cup N[y]) \cap (N[z] \cup N[t]) = \emptyset$ for each two pairs of vertices (x, y) and (z, t) with $xy, zt \in E(G)$ and $x, y, z, t \in S_1$. Also, $|N[r] \cup N[s]| = 7$ for all possible $r, s \in S_1$ and $rs \in E(G)$. Hence, Lemma 5.3 implies that the set S_1 is a minimum γ_t-set for $G - M_2$ where $M_2 = \{v_{\frac{n}{2}-1}, v_n\}$. Now, let $S_2 = \{v_{n-1}\}$. Clearly $S = S_1 \cup S_2$ is a γ_t-set of G where $n = 7k + 2$. Then $\gamma(G) = |S| = 2\lfloor \frac{n}{7} \rfloor + 1$ where $n \equiv 2 \pmod 7$.

Case 4 $n \equiv 3 \pmod 7$

Let $S_1 = \{v_1, v_2, v_8, v_9, ..., v_i, v_{i+1}, ..., v_{\frac{n}{2}-4}, v_{\frac{n}{2}-3}, v_{\frac{n}{2}+5}, v_{\frac{n}{2}+6}, v_{\frac{n}{2}+12}, v_{\frac{n}{2}+13}, \cdots, v_j, v_{j+1}, \cdots, v_{n-7}, v_{n-6}\}$. It is easy to verify that $(N[x] \cup N[y]) \cap (N[z] \cup N[t]) = \emptyset$ for each two pairs of vertices (x, y) and (z, t) with $xy, zt \in E(G)$ and $x, y, z, t \in S_1$. Furthermore, $|N[r] \cup N[s]| = 7$ for all possible $r, s \in S_1$ and $rs \in E(G)$. Clearly, Lemma 5.3 implies that the set S_1 is a minimum γ_t-set for $G - M_3$ where $M_3 = \{v_{\frac{n}{2}-1}, v_{n-1}, v_n\}$. Now, let S_2 be 2-subset from M_3 which are adjacent in G. Clearly $S = S_1 \cup S_2$ is a γ_t-set of G where $n = 7k + 3$. Then $\gamma(G) = |S| = 2\lceil \frac{n}{7} \rceil$ where $n \equiv 3 \pmod 7$.

Case 5 $n \equiv 4 \pmod 7$

We assign $S_1 = \{v_1, v_2, v_8, v_9, \cdots, v_i, v_{i+1}, \cdots, v_{\frac{n}{2}-8}, v_{\frac{n}{2}-7}, v_{\frac{n}{2}+5}, v_{\frac{n}{2}+6}, v_{\frac{n}{2}+12}, v_{\frac{n}{2}+13}, \cdots, v_j, v_{j+1}, ..., v_{n-4}, v_{n-3}\}$. It is easy to verify that $(N[x] \cup N[y]) \cap (N[z] \cup N[t]) = \emptyset$ for each two pairs of vertices (x, y) and (z, t) with $xy, zt \in E(G)$. Also, $|N[r] \cup N[s]| = 7$ for all possible $r, s \in S_1$ and $rs \in E(G)$. Hence, Lemma 5.3 implies that the set S_1 is a minimum γ_t-set for $G - M_4$, where $M_4 = \{v_{\frac{n}{2}-2}, v_{\frac{n}{2}-1}, v_{n-1}, v_n\}$. Now, let S_2 be a 2-subset from M_4 which are adjacent in G. Clearly $S = S_1 \cup S_2$ is a γ_t-set of G where $n = 7k + 4$. Then $\gamma(G) = |S| = 2\lceil \frac{n}{7} \rceil$ where $n \equiv 4 \pmod 7$.

Case 6 $n \equiv 5 \pmod 7$

Say $S_1 = \{v_1, v_2, v_8, v_9, \cdots, v_i, v_{i+1}, \cdots, v_{\frac{n}{2}-5}, v_{\frac{n}{2}-4}, v_{\frac{n}{2}+5}, v_{\frac{n}{2}+6}, v_{\frac{n}{2}+12}, v_{\frac{n}{2}+13}, \cdots, v_j, v_{j+1},$

$\cdots, v_{n-8}, v_{n-7}\}$. It is easy to verify that $(N[x] \cup N[y]) \cap (N[z] \cup N[t]) = \emptyset$ for each two pairs of vertices (x, y) and (z, t) with $xy, zt \in E(G)$ and $x, y, z, t \in S_1$. Also, $|N[r] \cup N[s]| = 7$ for all possible $r, s \in S_1$ and $rs \in E(G)$. Hence, Lemma 5.3 implies that the set S_1 is a minimum γ_t-set for $G - M_5$, where $M_5 = \{v_{\frac{n}{2}-2}, v_{\frac{n}{2}-1}, v_{n-2}, v_{n-1}, v_n\}$. Now, let $S_2 = \{v_{\frac{n}{2}-2}, v_{\frac{n}{2}-1}\}$. Clearly $S = S_1 \cup S_2$ is a γ_t-set of G where $n = 7k + 5$. Then $\gamma(G) = |S| = 2\lceil \frac{n}{7} \rceil$ where $n \equiv 5 \pmod 7$.

Case 7 $n \equiv 6 \pmod 7$

Let $S_1 = \{v_1, v_2, v_8, v_9, ..., v_i, v_{i+1}, \cdots, v_{\frac{n}{2}-9}, v_{\frac{n}{2}-8}, v_{\frac{n}{2}+5}, v_{\frac{n}{2}+6}, v_{\frac{n}{2}+12}, v_{\frac{n}{2}+13}, \cdots, v_j, v_{j+1}, \cdots, v_{n-5}, v_{n-4}\}$. It is easy to verify that $(N[x] \cup N[y]) \cap (N[z] \cup N[t]) = \emptyset$ for each two pairs of vertices (x, y) and (z, t) with $xy, zt \in E(G)$ and $x, y, z, t \in S_1$. Also, $|N[r] \cup N[s]| = 7$ for all possible $r, s \in S_1$ and $rs \in E(G)$. Hence, Lemma 5.3 implies that the set S_1 is a minimum γ_t-set for $G - M_6$, where $M_6 = \{v_{\frac{n}{2}-3}, v_{\frac{n}{2}-2}, v_{\frac{n}{2}-1}, v_{n-2}, v_{n-1}, v_n\}$. Now, let $S_2 = \{v_{\frac{n}{2}-2}, v_{\frac{n}{2}-1}\}$. Clearly $S = S_1 \cup S_2$ is a γ_t-set of G where $n = 7k + 6$. Then $\gamma(G) = |S| = 2\lceil \frac{n}{7} \rceil$ where $n \equiv 6 \pmod 7$. □

Theorem 5.5 $\gamma_t(G_1) = \begin{cases} 2\lceil \frac{m}{7} \rceil & if\ m \equiv l\ (mod\ 7)\ where\ l \in \{0, 3, 4, 5, 6\} \\ 2\lfloor \frac{m}{7} \rfloor + 1 & if\ m \equiv 1\ or\ 2\ (mod\ 7) \end{cases}$.

Proof Lemma 5.3 implies that $\gamma_t(G_1) \geq 2\lceil \frac{m}{7} \rceil$. Now we consider the following cases.

Case 1 $m \equiv 0 \pmod 7$

We assign $S_{t_0} = \{v_5, v_6, v_{12}, v_{13}, \cdots, v_i, v_{i+1}, \cdots, v_{s-5}, v_{s-4}, v_{s+2}, v_{s+3}, \cdots, v_j, v_{j+1}, \cdots, v_{m-2}, v_{m-1}\}$. It is easy to see that S_{t_0} is a γ_t-set for G_1. Hence $\gamma_t(G_1) \leq 2\lceil \frac{m}{7} \rceil$. Moreover Lemma 5.3 implies $\gamma_t(G_1) \geq 2\lceil \frac{m}{7} \rceil$. It follows that $\gamma_t(G_1) = 2\lceil \frac{m}{7} \rceil$ with $m \equiv 0 \pmod 7$.

Case 2 $m \equiv l \pmod 7$ where $l \in \{1, 2, 3, 4, 5, 6\}$.

We assign S_{t_l} to each l as follows:

$S_{t_1} = \{v_1, v_2, v_3, v_9, v_{10}, ..., v_i, v_{i+1}, v_{i+7}, ..., v_{s-5}, v_{s-4}, v_{s+6}, v_{s+7}, ..., v_j, v_{j+1}, ..., v_{m-2}, v_{m-1}\}$.

$S_{t_2} = \{v_2, v_3, v_9, v_{10}, ..., v_i, v_{i+1}, v_{i+7}, ..., v_{s-2}, v_{s-1}, v_s, v_{s+6}, v_{s+7}, ..., v_j, v_{j+1}, ..., v_{m-6}, v_{m-5}\}$.

$S_{t_3} = \{v_3, v_4, v_{10}, v_{11}, ..., v_i, v_{i+1}, v_{i+7}, ..., v_{s-5}, v_{s-4}, v_s, v_{s+1}, v_{s+7}, v_{s+8}, ..., v_j, v_{j+1}, ..., v_{m-2}, v_{m-1}\}$.

$S_{t_4} = \{v_1, v_2, v_7, v_8, ..., v_i, v_{i+1}, v_{i+7}, .., v_{s-5}, v_{s-4}, v_{s+4}, v_{s+5}, .., v_j, v_{j+1}, ..., v_{m-2}, v_{m-1}\}$.

$S_{t_5} = \{v_4, v_5, v_{11}, v_{12}, v_{18}, .., v_i, v_{i+1}, v_{i+7}, .., v_{s-5}, v_{s-4}, v_{s+1}, v_{s+2}, v_{s+8}, v_{s+9}, ..., v_j, v_{j+1}, ..., v_{m-2}, v_{m-1}\}$.

$S_{t_6} = \{v_1, v_2, v_8, v_9, .., v_i, v_{i+1}, v_{i+7}, .., v_{s-5}, v_{s-4}, v_{s+5}, v_{s+6}, .., v_j, v_{j+1} ..., v_{m-2}, v_{m-1}\}$.

In the same manner as in Case 1 we settle this Case. Hence $\gamma_t(G_1) = 2\lceil \frac{m}{7} \rceil$ where $m \equiv 3$ or 4 or 5 or 6 (mod 7) and $\gamma_t(G_1) = 2\lfloor \frac{m}{7} \rfloor + 1$ where $m \equiv 1$ or 2 (mod 7). □

Motivated by Theorem 5.5, we are now really ready to state of following Theorem.

Theorem 5.6 $\gamma_t(G_2) = \sum_{i=1}^{q} \gamma_t(G_{m_i})$.

§6. Forcing Domination Number of Some Graphs

Observation 6.1 $f(H,\gamma) \geq 1$ where $H \in \{G, G_1, G_2\}$.

Proof It is easy to see that the graphs G, G_1 and G_2 have at least two γ-sets. Then it immediately implies that $f(H,\gamma) \geq 1$ where $H \in \{G, G_1, G_2\}$. □

Observation 6.2 $f(G,\gamma), f(G_1,\gamma) \geq 2$ *where* $|V(G)|, |V(G_1)| \equiv l$ *with* $l \in \{1,2,3,4\}$ *(mod 5).*

Proof It is straightforward to see that with any 1-subset, say T from any arbitrary dominating set, we can obtain at least two different γ-sets for G containing T. Then $f(G,\gamma) \geq 2$.

Similar argument settles that $f(G_1,\gamma) \geq 2$ too. □

Theorem 6.3 *(i) If* $n \equiv 0$ *(mod 5) then* $f(G,\gamma) = 1$*;*
(ii)If $m \equiv 0$ *(mod 5) then* $f(G_1,\gamma) = 1$*;*
(iii) $f(G_2,\gamma) = q$ *where* $V(G_{m_i}) \equiv 0$ *(mod 5) for all* i*.*

Proof (i) We apply Observation 6.1 with $H = G$, so $f(G,\gamma) \geq 1$. Now let $S = \{v_3, v_8, ..., v_i, v_{i+5}, \cdots, v_{\frac{n}{2}-2}, v_{\frac{n}{2}+1}, v_{\frac{n}{2}+6}, \cdots, v_j, v_{j+5}, \cdots, v_{n-4}\}$. It is easy to see that $F = \{v_3\} \subset S$ is a forcing subset for G which implies $f(G,\gamma) \leq 1$. It is now straightforward to give $f(G,\gamma) = 1$.

(ii) By Observation 6.1 with $H = G_1$, it implies that $f(G_1,\gamma) \geq 1$. Let $F = \{v_2\}$. Obviously, F is a forcing subset for G_1. From this and by Theorem 2.4, it follows that $f(G_1,\gamma) = 1$.

(iii) The Case(ii) settles this case. Moreover, let $F = \{v_{12}, v_{22}, v_{32}, ..., v_{i2}, ..., v_{q2}\}$ then it implies that $f(G_2,\gamma) = q$. □

Theorem 6.4 *(i) If* $n \equiv 1$ *(mod 5), then* $f(G,\gamma) = 2$*;*
(ii)If $m \equiv 1$ *(mod 5), then* $f(G_1,\gamma) = 2$*;*
(iii) $f(G_2,\gamma) = 2\lceil \frac{q}{2} \rceil$ *where* $V(G_{m_i}) \equiv 1$ *(mod 5) for all* i*.*

Proof (i) Observation 6.2 implies that $f(G,\gamma) \geq 2$. Say $S = \{v_1, v_6, ..., v_i, v_{i+5}, \cdots, v_{\frac{n}{2}-2}, v_{\frac{n}{2}+1}, v_{\frac{n}{2}+6}, \cdots, v_j, v_{j+5}, \cdots, v_{n-4}\}$. Suppose that $F = \{v_1, v_{\frac{n}{2}+1}\} \subset S$, clearly F is a forcing subset for G and it follows that $f(G,\gamma) \leq 2$. This implies that $f(G,\gamma) = 2$.

(ii)Using Observation 6.2 we have $f(G_1,\gamma) \geq 2$. Now we define $F = \{v_s, v_{m-1}\}$. Clearly, $|N[v_s] \cup N[v_{m-1}]| = 6$. On the other hand, since $m \equiv 1$ (mod 5) then cardinality of the set of remaining vertices is multiple of 5. It immediately follows that the set $\{v_2, v_7, ..., v_i, v_{i+5}, ..., v_{s-3}, v_{s+5}, v_{s+10}, ..., v_j, v_{j+5}, ..., v_{m-6}\} \cup F$ is the unique γ-set containing F. Thus $f(G_1,\gamma) = 2$.

(iii) We consider the following cases. (a): If q is even, let $F_1 = \cup_{i=2}^{q} \{v_{i1}, v_{i(s_i+1)}\}$ where i is even. (b): If q is odd let $F_2 = \cup_{i=2}^{q-1} \{v_{i1}, v_{i(s_i+1)}\} \cup \{v_{(q)1}, v_{q(s_q+1)}\}$ where i is even. By simple verification one can check that F_1 and F_2 are forcing subsets for G_2 in two stated cases. Hence, it follows that $f(G_2,\gamma) = 2\lceil \frac{q}{2} \rceil$. □

Theorem 6.5 *(i) If* $n \equiv 2$ *(mod 5), then* $f(G,\gamma) = 2$*;*
(ii) If $m \equiv 2$ *(mod 5), then* $f(G_1,\gamma) = 2$*;*
(iii) $f(G_2,\gamma) = 2q$ *where* $V(G_{m_i}) \equiv 2$ *(mod 5) for all* i*.*

Proof (i)Using Observation 6.2 we have $f(G,\gamma) \geq 2$. Now we define $F = \{v_{\frac{n}{2}-1}, v_{\frac{n}{2}}\} \subset S$. Clearly, $|N[v_{\frac{n}{2}-1}] \cup N[v_{\frac{n}{2}}]| = 7$. Moreover, since $m \equiv 2 \pmod 5$ then cardinality of the set of remaining vertices is a multiple of 5. It immediately follows that the set $\{v_5, v_{10}, ..., v_i, v_{i+5}, ..., v_{\frac{n}{2}-6}, v_{\frac{n}{2}+3}, v_{\frac{n}{2}+8}, ..., v_j, v_{j+5}, ..., v_{n-3}\} \cup F$ is the unique γ-set containing F. Thus $f(G,\gamma) = 2$.

(ii) Using Observation 6.2 we have $f(G_1,\gamma) \geq 2$. Now we define $F = \{v_{s+1}, v_{s+2}\}$. It immediately follows that the set $\{v_4, v_9, ..., v_i, v_{i+5}, ..., v_{s-4}, v_{s+7}, v_{s+12}, ..., v_j, v_{j+5}, ..., v_{m-2}\} \cup F$ is the unique γ-set containing F. Thus $f(G_1,\gamma) = 2$.

(iii): Clearly, the obtained forcing subset in the case (ii) is extendible to G_2. Therefore, we can assert that $f(G_2,\gamma) = 2q$. □

Theorem 6.6 *(i) If $n \equiv 3$ (mod 5), then $f(G,\gamma) = 2$;*

(ii) If $m \equiv 3$ (mod 5), then $f(G_1,\gamma) = 2$;

(iii) $f(G_2,\gamma) = 2q$ where $V(G_{m_i}) \equiv 3$ (mod 5) for all i.

Proof (i) Using Observation 6.2 we have $f(G,\gamma) \geq 2$. Now we define $F = \{v_1, v_{\frac{n}{2}+3}\} \subset S$. Clearly, $|N[v_1] \cup N[v_{\frac{n}{2}+3}]| = 8$. On the other hand, since $m \equiv 2 \pmod 5$ then cardinality of the set of remaining vertices is a multiple of 5. It immediately follows that the set $\{v_5, v_{10}, ..., v_i, v_{i+5}, ..., v_{\frac{n}{2}-4}, v_{\frac{n}{2}+8}, v_{\frac{n}{2}+13}, ..., v_j, v_{j+5}, ..., v_{n-1}\} \cup F$ is the unique γ-set containing F. Thus $f(G,\gamma) = 2$.

(ii) Using Observation 6.2 we have $f(G_1,\gamma) \geq 2$. Let $F = \{v_1, v_3\}$. Since $|N[v_1] \cup V[v_3]| = 8$, cardinality of the set of non-dominated vertices is a multiple of 5. From this it immediately follows that S consists of v_{s+6}, v_8, v_{s+11}, v_{13},...,v_{m-1}, v_{s-3}. Thus $f(G_1,\gamma) = 2$.

(iii) Clearly, the obtained forcing subset in Case (ii) is extendible to G_2. Therefore, it implies that $f(G_2,\gamma) = 2q$. □

Theorem 6.7 *(i) If $n \equiv 4$ (mod 5), then $f(G,\gamma) = 2$;*

(ii) If $m \equiv 4$ (mod 5), then $f(G_1,\gamma) = 2$;

(iii) $f(G_2,\gamma) = 2q$ where $V(G_{m_i}) \equiv 4$ (mod 5) for all i.

Proof (i) Using Observation 6.2 we have $f(G,\gamma) \geq 2$. Now we define $F = \{v_{\frac{n}{2}-2}, v_{\frac{n}{2}}\} \subset S$. Clearly, $|N[v_{\frac{n}{2}-2}] \cup N[v_{\frac{n}{2}}]| = 9$. Furthermore, since $m \equiv 2 \pmod 5$ then cardinality of the set of remaining vertices is a multiple of 5. It immediately follows that the set $\{v_5, v_{10}, \cdots, v_i, v_{i+5}, \cdots, v_{\frac{n}{2}-7}, v_{\frac{n}{2}+3}, v_{\frac{n}{2}+8}, \cdots, v_j, v_{j+5}, \cdots, v_{n-4}\} \cup F$ is the unique γ-set containing F. Thus $f(G,\gamma) = 2$.

(ii) By Observation 6.2 we have $f(G_1,\gamma) \geq 2$. Let $F = \{v_s, v_{s+2}\}$. It immediately follows that the set $\{v_4, v_9, \cdots, v_i, v_{i+5}, \cdots, v_{s-5}, v_{s+7}, v_{s+12}, \cdots, v_j, v_{j+5}, \cdots, v_{m-3}\} \cup F$ is the unique γ-set containing F. Thus $f(G_1,\gamma) = 2$.

(iii) Clearly, the obtained forcing subset in Case (ii) is extendible to G_2. Therefore, it implies that $f(G_2,\gamma) = 2q$. □

We close this section by the following Theorem for which we are motivated by the results of this section.

Theorem 6.8 *Let G_3 be the graph defined in Section 2. Then* $f(G_3,\gamma)=\sum_{i=1}^{q} f(G_{m_i},\gamma)-(\lfloor\frac{r_1}{2}\rfloor+\lfloor\frac{r_2}{2}\rfloor+...+\lfloor\frac{r_l}{2}\rfloor)$.

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