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# Computing Smarandachely Scattering Number of Total Graphs

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**Abstract**: Let $S$ be a set consist of chosen components in $G-X$. The *Smarandachely scattering number* of a graph $G$ is defined by

$$\gamma_S(G) = \max\{w(G-X) - |X| - \sum_{H\in S} |H| : X \subset V(G), w(G-X) > 1\}.$$

Particularly, if $S = \emptyset$ or $S = \{the\ \ largest\ \ component\ \ in\ \ G-X\}$, then $\gamma_S(G)$ is the scattering number or rupture degree of a graph $G$. In this paper, general results on the Smarandachely scattering number of a graph are considered. Firstly the relationships between the Smarandachely scattering number and some vulnerability parameters, namely scattering, integrity and toughness are given. Further, we calculate the Smarandachely scattering number of total graphs. Also several results are given about total graphs and graph operations.

**Key Words**: Smarandachely scattering number, connectivity, network design and communication, graph operations, rupture degree.

**AMS(2000)**: 05C40, 05C76, 68M10, 68R10.

## §1. Introduction

In a communication network, the vulnerability measures the resistance of network to disruption of operation after the failure of certain stations or communication links. The stability of communication networks is of prime importance to network designers.In analysis of vulnerability of a communication network to disruption, two quantities that come to mind are:

(1)the size of the largest remaining group within which mutual communication can still occur,

(2)the number of elements that are not functioning.

If we think of the graph as a model of a communication network, many graph theoreti-

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cal parameters have been used to describe the stability of communication networks including connectivity, integrity, toughness, tenacity, binding number and scattering number (see [2]-[3], [7]-[9] and [13]).

A graph $G$ is denoted by $G = (V(G), E(G))$, where $V(G)$ is the vertex set of $G$ and $E(G)$ is the edges set of $G$. The number of vertices and the number of edges of the graph $G$ are denoted by $|V| = n$, $|E| = q$ respectively.

In this paper we will deal with the Smarandachely scattering number. But first we will give some basic definitions and notation. After that we give the the Smarandachely scattering number of total graph of specific families of graphs (see [4],[6],[10] and [12]).

•$t(G)$ : The toughness of a graph $G$ is defined by

$$t(G) = \min_{X\subseteq V(G)} \frac{|X|}{w\,(G-X)},$$

where X is a vertex cut of $G$ and $w(G-X)$ is the number of the components of $G-X$.

•$I(G)$ : The integrity of a graph is given by

$$I(G) = \min_{X\subseteq V(G)} \{|X| + m\,(G-X)\}\,,$$

where $m(G-X)$ is the maximum number of vertices in a component of $G-X$.

•$s(G)$ : The scattering number of a graph is defined by

$$s(G) = \max\{w(G-X) - |X| : X \subseteq V(G),\, w(G-X) \geqslant 2\}\,,$$

where $w(G-X)$ denotes the number of components of the graph $G-X$.

•$\gamma_S(G)$: The *Smarandachely scattering number* of a graph $G$ is defined by

$$\gamma_S(G) = \max\{w(G-X) - |X| - \sum_{H\in S} |H| : X \subset V(G), w(G-X) > 1\}.$$

Particularly, if $S = \emptyset$ or $S = \{$the largest component in $G-X\}$, then $\gamma_S(G)$ is the scattering number or rupture degree of a graph $G$ (see [11]).

**Definition 1.1** *Two vertices are said to cover each other in a graph $G$ if they are incident in $G$. A vertex cover in $G$ is a set of vertices that covers all edges of $G$. The minimum cardinality of a vertex cover in a graph $G$ is called the vertex covering number of $G$ and is denoted by $\alpha(G)$(see [4],[6],[10] and [12]).*

**Definition 1.2**([4],[6],[10] and [12]) *An independent set of vertices of a graph $G$ is a set of vertices of $G$ whose elements are pairwise nonadjacent. The independence number $\beta(G)$ of $G$ is the maximum cardinality among all independent sets of vertices of $G$.*

**Theorem 1.1**([10],[12]) *For any graph $G$ of order $n$,*

$$\alpha(G) + \beta(G) = n.$$

**Definition 1.3** *The vertex-connectivity or simply connectivity $k(G)$ of a graph $G$ is the minimum number of vertices whose removal from $G$ result in a disconnected or trivial graph. The complete graph $K_n$ cannot be disconnected by the removal of vertices, but the deletion of any $n-1$ vertices result in $K_n$; thus $k(K_n) = n-1$.*

$$k(G) = min\{|X| : X \subset V(G), \omega(G-X) > 1\},$$

*where $\lceil x \rceil$ is the smallest integer greater than or equal to $x$. $\lfloor x \rfloor$ is the greatest integer less than or equal to $x$.*

## §2. Some Results

We use $P_n$ and $C_n$ to denote the path and cycle with $n$ vertices, respectively. A comet $C_{t,r}$ is defined as the graph obtained by identifying one end of the path $P_t, (t \geqslant 2)$ with the center of the star $K_{1,r}$. In this section we review the Smarandachely scattering number of $P_n$, $C_n$, $C_{t,r}$ and the $k$-complete partite graph $K_{n_1,n_2,\ldots,n_k}$.

**Theorem 2.1**([11]) *The Smarandachely scattering number of the comet $C_{t,r}$ the path $P_n, (n \geqslant 3)$, the star $K_{1,n-1}, (n \geqslant 3)$ and the cycle $C_n$ are given in the following.*

a) *The Smarandachely scattering number of the comet $C_{t,r}$ is*

$$\gamma_S\left(C_{t,r}\right) = \begin{cases} r-1, & \text{if t is even} \\ r-2, & \text{if t is odd} \end{cases}$$

b) *The Smarandachely scattering number of the path $P_n$ $(n \geqslant 3)$ is*

$$\gamma_S\left(P_n\right) = \begin{cases} -1, & \text{if } n \text{ is even} \\ 0, & \text{if } n \text{ is odd} \end{cases}$$

c) *The Smarandachely scattering number of the star $K_{1,n-1}$ $(n \geqslant 3)$ is n-3.*

d) *The Smarandachely scattering number of the cycle $C_n$ is*

$$\gamma_S\left(C_n\right) = \begin{cases} -1, & \text{if } n \text{ is even} \\ -2, & \text{if } n \text{ is odd} \end{cases}$$

**Theorem 2.2**([11]) *The Smarandachely scattering number of the complete k-partite graph $K_{n_1,n_2,\ldots,n_k}$ is $2\max\{n_1, n_2, ..., n_k\} - \sum\limits_{i=1}^{k} n_i - 1$.*

**Theorem 2.3**([11]) *Let $G_1$ and $G_2$ be two connected graphs of order $n_1$ and $n_2$, respectively. Then $\gamma_S(G_1 + G_2) = max\{\gamma_S(G_1) - n_2, \gamma_S(G_2) - n_1\}$.*

**Theorem 2.4**([11]) *Let $G$ be an incomplete connected graph of order $n$. Then*

a) $2\alpha(G) - n - 1 \leqslant \gamma_S(G) \leqslant \frac{[\alpha(G)]^2 - \kappa(G)[\alpha(G)-1]-n}{\alpha(G)}$.

b) $3 - n \leqslant \gamma_S(G) \leqslant n - 3$.

c) $\gamma_S(G) \leqslant 2\delta(G) - 1$.

## §3. Bounds for Smarandachely Scattering Number

In this section, we consider the relations between the Smarandachely scattering number and toughness, integrity and scattering number. Parameters that will be used in this paper are as the following:

$\alpha(G)$, the covering number;

$\beta(G)$, the independence number;

$k(G)$, the connectivity number;

$\delta(G)$, the minimum vertex degree and

$\Delta(G)$, the maximum vertex degree.

**Theorem 3.1** *Let $G$ be a connected graph of order $n$ such that $t(G) = t$, $\gamma_S(G) = \gamma_S$ and $\delta(G) = \delta$. Then $\gamma_S \leqslant \frac{n}{t+1} - (\delta + 1)$.*

*Proof* Let $X$ be cut set of vertices of $G$. From the definition of $t(G)$, we know that $t \leqslant \frac{|X|}{w(G-X)}$. Therefore,

$$w(G - X) \leqslant \frac{|X|}{t}.$$

We also have $w(G - X) + |X| \leq n$. In this inequality, $|X| \leq n - w(G - X)$and get $w(G - X) + |X| \leqslant n$. In this inequality, $|X| \leqslant n - w(G - X)$. Therefore,

| | |
|---|---|
| $w(G - X)$ | $\leqslant \frac{\lvert X\rvert}{t}$ |
| $w(G - X)$ | $\leqslant \frac{n-w(G-X)}{t}$ |
| $w(G - X)$ | $\leqslant \frac{n}{t+1}$. |

On the other hand, for every graph $G$, it's known that

$$\delta(G) + 1 \leqslant I(G) \leqslant \alpha(G) + 1$$

and

$$I(G) = |X| + m(G - X) \geqslant \delta(G) + 1.$$

Then, we have $m(G - X) \geqslant \delta(G - X) + 1 \geqslant \delta(G) - |X| + 1$.
Therefore, we have $m(G - X) \geqslant \delta(G) + 1 - |X|$.

Let's construct the definition of the Smarandachely scattering number.

| | |
|---|---|
| $w(G-X)-\|X\|-m(G-X)$ | $\leqslant w(G-X)-\|X\|-\delta(G)-1+\|X\|$ |
| $\gamma_S(G)$ | $\leqslant w(G-X)-\delta(G)-1$ |
| $\gamma_S(G)$ | $\leqslant \frac{n}{t+1}-\delta(G)-1$ |

The proof is completed. □

**Theorem 3.2** *Let $G$ be a connected graph of order $n$ such that $t(G)=t$, $\gamma_S(G)=\gamma_S$, $\alpha(G)=\alpha$ and $k(G)=k$. Then $\gamma_S(G)\geqslant \frac{k}{t}-(\alpha+1)$.*

*Proof* Let $X$ be a cut set of vertices of $G$. From the definition of $\gamma_S(G)$, we know that $w(G-X)-|X|-m(G-X)\leqslant \gamma_S$. Moreover, for every graph G, it is known that $I(G)\leqslant \alpha(G)+1$. So, we have $I(G)=|X|+m(G-X)\leqslant \alpha(G)+1$. We have the following inequality:

$$w(G-X)\leqslant \gamma_S(G)+\alpha(G)+1.$$

| | |
|---|---|
| $\frac{1}{w(G-X)}$ | $\geqslant \frac{1}{\gamma_S+\alpha+1}$ |
| $\frac{\|X\|}{w(G-X)}$ | $\geqslant \frac{\|X\|}{\gamma_S+\alpha+1},\quad \|X\|\geqslant k(G)$ |
| $\frac{\|X\|}{w(G-X)}$ | $\geqslant \frac{k}{\gamma_S+\alpha+1}$ |
| $min\left\{\frac{\|X\|}{w(G-X)}\right\}$ | $\geqslant min\left\{\frac{k}{\gamma_S+\alpha+1}\right\}$ |
| $t$ | $\geqslant \frac{k}{\gamma_S+\alpha+1}$ |
| $\gamma_S$ | $\geqslant \frac{k}{t}-(\alpha+1)$ |

The proof is completed. □

**Theorem 3.3** *LetG be a non-complete connected graph such that $s(G)=s$, $\gamma_S(G)=\gamma_S$, $I(G)=I$ and $\alpha(G)=\alpha$ is the covering number of graph $G$. Then $\gamma_S \leqslant s$ - $I+\alpha$.*

*Proof* Let $X$ be a vertex cut of $G$, then from the definition of $s(G)$ we know that $w(G-X)-|X| \leqslant s$.

When we subtract $m(G-X)$ from both sides of this inequality, we have the following.

$$w(G-X)-|X|-m(G-X)\leqslant s-m(G-X).$$

From the definition of $I(G)$ we know that $I(G)\leqslant |X|+m(G-X)$.

$$I(G)\leqslant |X|+m(G-X)\Rightarrow \quad m(G-X)\geqslant I-|X|$$
$$-m(G-X)\leqslant -I+|X|.$$

Then we have,

$$w(G-X)-|X|-m(G-X)\leqslant s-I+|X|$$

since $X$ is a cut set of vertices, $|X|\leqslant\alpha$ is always satisfied,

$$w(G-X)-|X|-m(G-X)\leqslant s-I+\alpha$$

$$\max \{w(G-X) - |X| - m(G-X)\} \leqslant \max\{s - I + \alpha\}$$
$$r \leqslant s - I + \alpha$$

The proof is completed. □

## §4. The Smarandachely Scattering Number of of Total Graphs Some Graph Types and Cartesian Product of Graphs

In this section, firstly, we will give definition of total graph of a graph and Cartesian product operation on graphs. After that we will give some results about the The Smarandachely scattering number of $T(P_n)$, $T(C_n), T(S_{1,n}), T(K_2xP_n)$ and $T(K_2xC_n)$.

**Definition 4.1** *The vertices and edges of a graph are called its elements. Two elements of a graph are neighbors if they are either incident or adjacent. The total graph $T(G)$ of the graph $G = (V(G), E(G))$, has vertex set $V(G) \cup E(G)$, and two vertices of $T(G)$ are adjacent whenever they are neighbors in $G$. It is easy to see that $T(G)$always contains both $G$ and Line graph $L(G)$ as a induced subgraphs. The total graph is the largest graph that is formed by the adjacent relations of elements of a graph.*

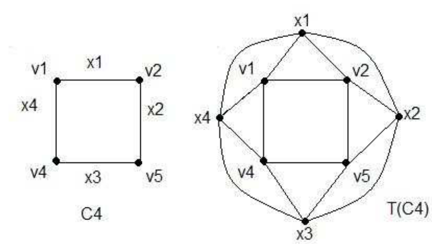


**Fig.**1

**Definition 4.2** *The Cartesian product of two graphs $G_1$ and $G_2$, denoted by $G_1xG_2$, is defined as follows:*

*$V(G_1xG_2) = V(G_1)xV(G_2)$, two vertices $(u_1, u_2)$ and $(v_1, v_2)$ are adjacent if and only if $u_1 = v_1$ and $u_2v_2 \in E(G_2)$ or $u_1v_1 \in E(G_1)$ and $u_2 = v_2$. The Cartesian product of $n$ graphs $G_1, G_2, \cdots, G_n$ denoted by $G_1xG_2x\cdots xG_n$ is defined inductively as the Cartesian product $G_1xG_2x\cdots xG_{n-1}$ and $G_n$.*

**Theorem 4.1** *The Smarandachely scattering number of $T(P_n)$ order of 2n-1 is*

$$\gamma_S(T(P_n)) = 2 - \left\lfloor \sqrt{1+2n} \right\rfloor - \left\lceil \frac{1+2n-2\left\lfloor \sqrt{1+2n} \right\rfloor}{\left\lfloor \sqrt{1+2n} \right\rfloor} \right\rceil.$$

*Proof* If we remove $p$ vertices from graph $T(P_n)$, then the number of the remaining connected components is at most $\lfloor \frac{p}{2} \rfloor + 1$. In this case the order of the largest remaining component is $m(G-X) \geqslant \frac{2n-1-p}{\lfloor \frac{p}{2} \rfloor + 1}$. So,

$$\gamma_S(T(P_n)) \quad \geqslant max_p \left\{ \frac{p}{2} + 1 - p - \frac{2n-1-p}{\frac{p}{2}+1} \right\}.$$

The function $\frac{p}{2}+1-p-\frac{2n-1-p}{\frac{p}{2}+1}$ takes its maximum value at $p = -2+2\sqrt{1+2n}$. Then we write this $p$ value in the definitions of $w$ and $m$ to calculate the Smarandachely scattering number as follows:

$$w = \lfloor \sqrt{1+2n} \rfloor, \quad m = \left\lceil \frac{1+2n-2\lfloor \sqrt{1+2n} \rfloor}{\lfloor \sqrt{1+2n} \rfloor} \right\rceil,$$

$$\gamma_S = \lfloor \sqrt{1+2n} \rfloor - \left(-2+2\lfloor \sqrt{1+2n} \rfloor\right) - \left\lceil \frac{1+2n-2\lfloor \sqrt{1+2n} \rfloor}{\lfloor \sqrt{1+2n} \rfloor} \right\rceil$$

and

$$\gamma_S(T(P_n)) = 2 - \lfloor \sqrt{1+2n} \rfloor - \left\lceil \frac{1+2n-2\lfloor \sqrt{1+2n} \rfloor}{\lfloor \sqrt{1+2n} \rfloor} \right\rceil$$

This completes the proof. □

**Theorem 4.2** *The Smarandachely scattering number of $T(C_n)$ order of $2n$ is*

$$\gamma_S(T(C_n)) = -\left\lfloor \sqrt{2n} \right\rfloor + 2 - \left\lceil \frac{2n}{\lfloor \sqrt{2n} \rfloor} \right\rceil.$$

*Proof* If we remove $p$ vertices from graph $T(C_n)$, then the number of the remaining connected components is at most $\lfloor \frac{p}{2} \rfloor$. In this case the order of the largest remaining component is $m(T(C_n)-X) \geqslant \frac{2n-p}{\lfloor \frac{p}{2} \rfloor}$. So,

$$\gamma_S(T(C_n)) \quad \geqslant max_p \left\{ \frac{p}{2} - p - \frac{2n-p}{\frac{p}{2}} \right\}.$$

The function $\frac{p}{2} - p - \frac{2n-p}{\frac{p}{2}}$ takes its maximum value at $p = 2\sqrt{2n}$. Then we write this $p$ value in the definitions of $w$ and $m$ to calculate the Smarandachely scattering number.

$$\gamma_S(T(C_n)) = \frac{2\left\lfloor \sqrt{2}\sqrt{n} \right\rfloor}{2} - 2\left\lfloor \sqrt{2}\sqrt{n} \right\rfloor - \frac{2n-2\left\lfloor \sqrt{2}\sqrt{n} \right\rfloor}{\frac{2\lfloor \sqrt{2}\sqrt{n} \rfloor}{2}},$$

$$\gamma_S(T(C_n)) = -2\left\lfloor \sqrt{2}\sqrt{n} \right\rfloor - \frac{2n-2\left\lfloor \sqrt{2}\sqrt{n} \right\rfloor}{\left\lfloor \sqrt{2}\sqrt{n} \right\rfloor},$$

then

$$\gamma_S(T(C_n)) = -\left\lfloor \sqrt{2n} \right\rfloor + 2 - \left\lceil \frac{2n}{\lfloor \sqrt{2n} \rfloor} \right\rceil.$$

□

**Theorem 4.3** *The Smarandachely scattering number of $T(S_{1,n})$ order of $2n+1$ is*

$$r(T(S_{1,n})) = -2.$$

*Proof* Our proof is divided into two cases following.

**Case 1** Let $|X| \leqslant \alpha(S_{1,n}) + \alpha(K_n) = 1 + (n-1) = n$ be a cut set of vertices of $T(S_{1,n})$. The number of the components in$T(S_{1,n})$is at most $p$, after removing p vertices. If $|X| = n$, then $w(T(S_{1,n}) - X = n$. In this case the order of the largest remaining component is

$$m(T(S_{1,n}) - X) \geqslant \left\lceil \frac{2n+1-n}{n} \right\rceil \geqslant \left\lceil \frac{n+1}{n} \right\rceil \geqslant 2.$$

Hence

$$w(T(S_{1,n}) - X) - |X| - m(T(S_{1,n}) - X) \leqslant n - n - 2 \leqslant -2.$$

**Case 2** Let us take $|X| \geqslant n$. We assume $|X| = n+1$. In this case,

$$w(T(S_{1,n}) - X) \leqslant 2n + 1 - |X| = 2n + 1 - n - 1 = n,$$

$$w(T(S_{1,n}) - X) \leqslant n.$$

The order of the largest remaining component is

$$m(T(S_{1,n}) - X) \geqslant \left\lceil \frac{2n+1-|X|}{2n+1-|X|} \right\rceil = 1,$$

$$m(T(S_{1,n}) - X) \geqslant 1.$$

Hence

$$w(T(S_{1,n}) - X) - |X| - m(T(S_{1,n}) - X) \leqslant n - (n+1) - 1$$

$$w(T(S_{1,n}) - X) - |X| - m(T(S_{1,n}) - X) \leqslant -2$$

From the choice of $X$ and the definition of the Smarandachely scattering number, we obtain $\gamma_S(T(S_{1,n})) = -2$.

It is easy to see that there is a vertex cut set $X^*$ of $T(S_{1,n})$ such that $| \quad X^*| = n, w(T(S_{1,n}) - X^*) = n$and $m(T(S_{1,n}) - X^*) = 2$. From the definition of the Smarandachely scattering number, we have $r(T(S_{1,n})) \geqslant w(T(S_{1,n}) - X^*) - |X^*| - m(T(S_{1,n}) - X^*) = -2$. This implies that $r(T(S_{1,\ n})) = -2$. $\square$

**Theorem 4.4** *For $n \geq 3$, the Smarandachely scattering number of $T(K_2xP_n)$ of order $5n-2$ is*

$$\gamma_S(T(K_2xP_n)) = -2\left\lceil \sqrt{6+15n} \right\rceil + 8.$$

*Proof* There exist at most $\lfloor \frac{p}{4} \rfloor + 1$ components when $p$ vertices are removed from the graph. The order of the largest remaining component is $m(T(K_2xP_n) - |X|) \geqslant \frac{5n-2-p}{\lfloor \frac{p}{4} \rfloor + 1}$. So,

$$\gamma_S(T(K_2xP_n)) \geqslant max_p \left\{ \frac{p}{4} + 1 - p - \frac{5n-2-p}{\frac{p}{4}+1} \right\}.$$

The function $\frac{p}{4} + 1 - p - \frac{5n-2-p}{\frac{p}{4}+1}$ takes its maximum value at $p = -4 + \frac{4}{3}\sqrt{(6+15n)}$. Then we obtain

$$\gamma_S(T(K_2xP_n)) = -2\left\lceil \sqrt{6+15n} \right\rceil + 8.$$

This completes the proof. □

**Theorem 4.5** *The Smarandachely scattering number of $T(K_2xC_n)$ order of $5n$ is*

$$\gamma_S\left(T\left(K_2\, x\, C_n\right)\right) \geqslant 6 - \left\lceil \sqrt{60n+24} \right\rceil.$$

*Proof* The number of the components is at most $\lfloor \frac{p+6}{4} \rfloor - 1$ when $p$ vertices are removed. The number of vertices in each component is at least $m(T(K_2xC_n)$ - $|X|$ $) \geqslant \frac{5n-p}{\lfloor \frac{p+6}{4} \rfloor - 1}$. So,

$$\gamma_S(T(K_2xC_n)) \geqslant max_p \left\{ \frac{p+6}{4} - 1 - p - \frac{20n-4p}{p+2} \right\}.$$

The function $\frac{p+6}{4} - 1 - p - \frac{20n-4p}{p+2}$ takes its maximum value at $p = -2 + \frac{2}{3}\sqrt{9+3\left(20n+5\right)}$. Hence we obtain

$$\gamma_S\left(T\left(K_2\, x\, C_n\right)\right) \geqslant 6 - \left\lceil \sqrt{60n+24} \right\rceil.$$

This completes the proof. □

## §5. Conclusion

If we want to design a communications network, we wish it as stable as possible. Any communication network can be modeled by a connected graph. In graph theory, we have many stability measures such as connectivity, toughness, integrity and tenacity. The Smarandachely scattering number is the new parameter which measures the vulnerability of a graph $G$. When we design two networks which have the same number of processors, if we want to choose the more stable one from two graphs with the same number of vertices, it is enough to choose the one whose The Smarandachely scattering number is greater.

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