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Common Fixed Point Theorems on S-Metric Spaces Via C-Class Functions

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Abstract: In this work, we prove some common fixed point theorems on S-metric spaces via C-class functions and give some consequences of the main result. We also give some examples in support of the results. The results obtained in this article generalize, extend and improve several results from the existing literature regarding S-metric spaces.

Key Words: Common fixed point, S-metric space, C-class functions.

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§1. Introduction

The fixed point theory one of the most important research fields in nonlinear analysis. In the last decades, many number of authors have published papers and battened continuously. The application potential is the main cause for this involvement. Fixed point theory has an application in many areas such as chemistry, physics, biology, computer science and many branches of mathematics. The Banach contraction mapping principle ([3]) or the Banach fixed point theorem is the most celebrated and pioneer result in a complete metric space. The famous Banach contraction mapping principle states that every self mapping $\mathcal{Q}$ defined on a complete metric space (X, d) satisfying the condition:

$$d(\mathcal{Q}(x), \mathcal{Q}(y)) \leq r\, d(x, y) \tag{1.1}$$

for all $x, y \in X$, where $r \in (0, 1)$ is a constant, has a unique fixed point and for every $x_0 \in X$ a sequence $\{\mathcal{Q}^n x_0\}_{n\geq 1}$ is convergent to the fixed point.

Most of the works after this were basically generalizations of the work of Banach. These generalizations include more general metric spaces, or more general contractions etc. One of the generalizations of the metric space is the S-metric space.

In 2012, Sedghi et al. [28] introduced the concept of a S-metric space which is different from other spaces and proved fixed point theorems in such spaces. They also give some examples of a S-metric space which shows that the S-metric space is different from other spaces. They built up some topological properties in such spaces and proved some fixed point theorems in

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the framework of S-metric spaces. After this grateful beginning work of Sedghi et al. [28] many authors attracted to study the problems of the fixed point, common fixed point, coupled fixed point and common coupled fixed point by using various contractive conditions for mappings (see, for examples, [5, 6, 8, 13, 18, 29, 30, 31]).

Recently, a large number of authors have published many papers on S-metric spaces in different directions (see, e.g.,g.,[9, 10, 11, 12, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 32, 33] and many others).

In 2014, Ansari [1] introduced the notion of C-class function that is pivotal result in fixed point theory.

In this work, we prove some common fixed point theorems on S-metric spaces via C-class functions and give some consequences of the main result. We also give some examples to demonstrate the validity of the result. Our results generalize, extend and improve several results from the existing literature.

§2. **Preliminaries**

In this section, we recall some basic definitions, lemmas and auxiliary results to prove our main results.

Definition 2.1([28]) *Let X be a nonempty set and let $S\colon X^3 \to [0,\infty)$ be a function satisfying the following conditions for all u, v, w, $t \in X$ hold with*

$(S1)$ $S(u,v,w) = 0$ *if and only if* $u = v = w$*;*

$(S2)$ $S(u,v,w) \leq S(u,u,t) + S(v,v,t) + S(w,w,t)$.

Then, the function S is called an S-metric on X and the pair (X,S) is called an S-metric space or simply SMS.

Example 2.2([28]) Let $X = \mathbb{R}^n$ and $\|.\|$ a norm on X, then $S(u,v,w) = \|v+w-2u\|+\|v-w\|$ is an S-metric on X.

Example 2.3([28]) Let X be a nonempty set and d be an ordinary metric on X. Then $S(u,v,w) = d(u,w) + d(v,w)$ for all $u,v,w \in X$ is an S-metric on X.

Example 2.4([28]) Let $X = \mathbb{R}$ be the real line. Then $S(u,v,w) = |u-w| + |v-w|$ for all $u,v,w \in \mathbb{R}$ is an S-metric on X. This S-metric on X is called the usual S-metric on X.

Definition 2.5 *Let (X,S) be an S-metric space. For $\varepsilon > 0$ and $u \in X$ we define respectively the open ball $\mathcal{B}_S(u,\varepsilon)$ and closed ball $\mathcal{B}_S[u,\varepsilon]$ with center u and radius ε as follows:*

$$\mathcal{B}_S(u,\varepsilon) = \{v \in X : S(v,v,u) < \varepsilon\},$$

$$\mathcal{B}_S[u,\varepsilon] = \{v \in X : S(v,v,u) \leq \varepsilon\}.$$

Example 2.6([29]) Let $X = \mathbb{R}$. Denote $S(u,v,w) = |v+w-2u| + |v-w|$ for all $u,v,w \in \mathbb{R}$.

Then

$$\begin{aligned} \mathcal{B}_S(1,2) &= \{v \in \mathbb{R} : S(v,v,1) < 2\} = \{v \in \mathbb{R} : |v-1| < 1\} \\ &= \{v \in \mathbb{R} : 0 < v < 2\} = (0,2), \end{aligned}$$

and

$$\begin{aligned} \mathcal{B}_S[2,4] &= \{v \in \mathbb{R} : S(v,v,2) \le 4\} = \{v \in \mathbb{R} : |v-2| \le 2\} \\ &= \{v \in \mathbb{R} : 0 \le v \le 4\} = [0,4]. \end{aligned}$$

Definition 2.7([28],[29]) *Let (X,S) be an S-metric space and $A \subset X$.*

(Υ_1) *The subset A is said to be an open subset of X, if for every $x \in A$ there exists $c > 0$ such that $\mathcal{B}_S(x,c) \subset A$.*

(Υ_2) *A sequence $\{r_n\}$ in X converges to $r \in X$ if $S(r_n, r_n, r) \to 0$ as $n \to \infty$, that is, for each $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that for all $n \ge n_0$ we have $S(r_n, r_n, r) < \varepsilon$. We denote this by $\lim_{n\to\infty} r_n = r$ or $r_n \to r$ as $n \to \infty$.*

(Υ_3) *A sequence $\{r_n\}$ in X is called a Cauchy sequence if $S(r_n, r_n, r_m) \to 0$ as $n, m \to \infty$, that is, for each $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that for all $n, m \ge n_0$ we have $S(r_n, r_n, r_m) < \varepsilon$.*

(Υ_4) *The S-metric space (X,S) is called complete if every Cauchy sequence in X is convergent.*

(Υ_5) *Let τ be the set of all $A \subset X$ having the property that for every $x \in A$, A contains an open ball centered in x. Then τ is a topology on X (induced by the S-metric space).*

(Υ_6) *A nonempty subset A of X is S-closed if closure of A is equal to A.*

Definition 2.8 *Let X be a non-empty set and let $A, B\colon X \to X$ be two self mappings of X. Then a point $u \in X$ is called a (Ω_1) fixed point of operator A if $A(u) = u$ and a (Ω_2) common fixed point of A and B if $A(u) = B(u) = u$.*

Definition 2.9([28]) *Let (X,S) be an S-metric space. A mapping $\mathcal{A}\colon X \to X$ is said to be a contraction if there exists a constant $0 \le k < 1$ such that*

$$S(\mathcal{A}u, \mathcal{A}v, \mathcal{A}w) \le k\, S(u,v,w) \tag{2.1}$$

for all $u, v, w \in X$.

Remark 2.10([28]) *If the S-metric space (X,S) is complete and $\mathcal{A}\colon X \to X$ is a contraction mapping, then $\mathcal{A}$ has a unique fixed point in X.*

Definition 2.11([28]) *Let (X,S) and (X',S') be two S-metric spaces. A function $R\colon X \to X'$ is said to be continuous at a point $x_0 \in X$ if for every sequence $\{r_n\}$ in X with $S(r_n, r_n, x_0) \to 0$, $S'(R(r_n), R(r_n), R(x_0)) \to 0$ as $n \to \infty$. We say that R is continuous on X if R is continuous at every point $x_0 \in X$.*

Definition 2.12([1]) *A mapping $F\colon [0,\infty) \times [0,\infty) \to R$ is called a C-class function if it is continuous and satisfies the following axioms:*

(*i*) $F(s,t) \le s$;

(*ii*) $F(s,t) = s$ *implies that either* $s = 0$ *or* $t = 0$, *for all* $s,t \in [0,\infty)$.

Note that for some F, we have that $F(0,0) = 0$. The letter $\mathcal{C}$ denotes the set of all C-class functions. The following example shows that $\mathcal{C}$ is nonempty.

Example 2.13([1]) Each of the functions $F\colon [0,\infty) \times [0,\infty) \to R$ defined below are elements of $\mathcal{C}$.

(*i*) $F(s,t) = s - t$;

(*ii*) $F(s,t) = m\,s$, $0 < m < 1$;

(*iii*) $F(s,t) = \frac{s}{(1+t)^r}$, $r \in (0,\infty)$;

(*iv*) $F(s,t) = \frac{log(t+a^s)}{1+t}$, $a > 1$;

(*v*) $F(s,t) = \frac{ln(1+a^s)}{2}$, $a > e$;

(*vi*) $F(s,t) = (s+l)^{(1/(1+t)^r)} - l$, $l > 1$, $r \in (0,\infty)$;

(*vii*) $F(s,t) = s\,log_{t+a}\,a$, $a > 1$;

(*viii*) $F(s,t) = s - \big(\frac{1+s}{2+s}\big)\big(\frac{t}{1+t}\big)$;

(*ix*) $F(s,t) = s\beta(s)$, where $\beta\colon [0,\infty) \to [0,\infty)$ and is continuous;

(*x*) $F(s,t) = s - \big(\frac{t}{k+t}\big)$;

(*xi*) $F(s,t) = \frac{s}{(1+s)^r}$, $r \in (0,\infty)$.

Remark 2.14 *The items* (*i*), (*ii*) *and* (*ix*) *in Example* 2.13 *are pivotal results in fixed point theory ([1]). Also see [2] and [7].*

Definition 2.15([1]) *A function* $\psi\colon [0,\infty) \to [0,\infty)$ *is called an altering distance function if the following properties are satisfied:*

(ψ_1) ψ *is non-decreasing and continuous function;*

(ψ_2) $\psi(t) = 0$ *if and only if* $t = 0$.

Remark 2.16 *We denote* Ψ *the class of all altering distance functions.*

Definition 2.17([1]) *A function* $\varphi\colon [0,\infty) \to [0,\infty)$ *is said to be an ultra altering distance function, if it is continuous, non-decreasing such that* $\varphi(t) > 0$ *for* $t > 0$.

We denote by Φ_u the class of all ultra altering distance functions.

Lemma 2.18([28], Lemma 2.5) *Let* (X,S) *be an* S*-metric space. Then,* $S(u,u,v) = S(v,v,u)$ *for all* $u,v \in X$.

Lemma 2.19([28], Lemma 2.12) *Let* (X,S) *be an* S*-metric space. If* $r_n \to r$ *and* $p_n \to p$ *as* $n \to \infty$ *then* $S(r_n,r_n,p_n) \to S(r,r,p)$ *as* $n \to \infty$.

Lemma 2.20([6], Lemma 8) *Let* (X,S) *be an* S*-metric space and* A *be a nonempty subset of* X. *Then* A *is* S*-closed if and only if for any sequence* $\{r_n\}$ *in* A *such that* $r_n \to r$ *as* $n \to \infty$, *then* $r \in A$.

Lemma 2.21([28]) *Let* (X,S) *be an* S*-metric space. If* $c > 0$ *and* $x \in X$, *then the ball* $\mathcal{B}_S(x,c)$ *is a subset of* X.

Lemma 2.22([29]) *The limit of a convergent sequence in a S-metric space (X, S) is unique.*

Lemma 2.23([28]) *In a S-metric space (X, S), any convergent sequence is Cauchy.*

§3. Main Results

In this section, we shall prove some common fixed point theorems on S-metric spaces via C-class functions.

Theorem 3.1 *Let (X, S) be a complete S-metric space and $f, g \colon X \to X$ be two self-mappings satisfying the inequality:*

$$\psi(S(fx, fy, gz)) \leq F\Big(\psi(\Theta(x, y, z)), \varphi(\Theta(x, y, z))\Big), \tag{3.1}$$

where

$$\begin{aligned} \Theta(x, y, z) \quad &= \quad a_1\, S(x, y, z) + a_2\, S(x, x, fx) + a_3\, S(z, z, gz) \\ &\quad +a_4[S(z, z, fx) + S(x, x, gz)] + a_5 \Big(\frac{S(z, z, gz)}{[1 + S(x, y, z)]}\Big) \end{aligned}$$

for all $x, y, z \in X$, where $a_1, a_2, a_3, a_4, a_5 > 0$ are nonnegative reals with $a_1 + a_2 + a_3 + 3a_4 + a_5 < 1$, $\psi \in \Psi$, $\varphi \in \Phi_u$ and $F \in \mathcal{C}$. Then f and g have a unique common fixed point in X.

Proof For each $x_0 \in X$. Let $x_{2n+1} = fx_{2n}$ and $x_{2n+2} = gx_{2n+1}$ for $n = 0, 1, 2, \ldots$. We prove that $\{x_n\}$ is a Cauchy sequence in (X, S). It follows from (3.1) for $x = y = x_{2n}$, $z = x_{2n-1}$ and using $(S1)$, $(S2)$ and Lemma 2.18, we have

$$\begin{aligned} \psi(S(x_{2n+1}, x_{2n+1}, x_{2n})) \quad &= \quad \psi(S(fx_{2n}, fx_{2n}, gx_{2n-1})) \\ &\leq \quad F\Big(\psi(\Theta(x_{2n}, x_{2n}, x_{2n-1})), \varphi(\Theta(x_{2n}, x_{2n}, x_{2n-1}))\Big), \end{aligned} \tag{3.2}$$

where

$$\begin{aligned} \Theta(x_{2n}, x_{2n}, x_{2n-1}) \quad &= \quad a_1\, S(x_{2n}, x_{2n}, x_{2n-1}) + a_2\, S(x_{2n}, x_{2n}, fx_{2n}) \\ &\quad +a_3\, S(x_{2n-1}, x_{2n-1}, gx_{2n-1}) \\ &\quad +a_4\, [S(x_{2n-1}, x_{2n-1}, fx_{2n}) + S(x_{2n}, x_{2n}, gx_{2n-1})] \\ &\quad +a_5 \Big(\frac{S(x_{2n-1}, x_{2n-1}, gx_{2n-1})}{[1 + S(x_{2n}, x_{2n}, x_{2n-1})]}\Big) \\ &= \quad a_1\, S(x_{2n}, x_{2n}, x_{2n-1}) + a_2\, S(x_{2n}, x_{2n}, x_{2n+1}) \\ &\quad +a_3\, S(x_{2n-1}, x_{2n-1}, x_{2n}) \end{aligned}$$

$$
\begin{aligned}
& +a_4\,[S(x_{2n-1},x_{2n-1},x_{2n+1})+S(x_{2n},x_{2n},x_{2n})] \\
& +a_5\,\Big(\frac{S(x_{2n-1},x_{2n-1},x_{2n})}{[1+S(x_{2n},x_{2n},x_{2n-1})]}\Big) \\
\leq\ & a_1\,S(x_{2n},x_{2n},x_{2n-1})+a_2\,S(x_{2n+1},x_{2n+1},x_{2n}) \\
& +a_3\,S(x_{2n},x_{2n},x_{2n-1}) \\
& +a_4\,[2S(x_{2n-1},x_{2n-1},x_{2n})+S(x_{2n+1},x_{2n+1},x_{2n})] \\
& +a_5\,\Big(\frac{S(x_{2n},x_{2n},x_{2n-1})}{[1+S(x_{2n},x_{2n},x_{2n-1})]}\Big) \\
\leq\ & a_1\,S(x_{2n},x_{2n},x_{2n-1})+a_2\,S(x_{2n+1},x_{2n+1},x_{2n}) \\
& +a_3\,S(x_{2n},x_{2n},x_{2n-1}) \\
& +a_4\,[2S(x_{2n-1},x_{2n-1},x_{2n})+S(x_{2n+1},x_{2n+1},x_{2n})] \\
& +a_5\,S(x_{2n},x_{2n},x_{2n-1})\Big) \\
=\ & (a_1+a_3+2a_4+a_5)S(x_{2n},x_{2n},x_{2n-1}) \\
& +(a_2+a_4)S(x_{2n+1},x_{2n+1},x_{2n}). \qquad (3.3)
\end{aligned}
$$

Using equation (3.3) in equation (3.2) and using the property of F, we get

$$
\begin{aligned}
\psi(S(x_{2n+1},x_{2n+1},x_{2n})) \leq\ & F\Big(\psi\Big((a_1+a_3+2a_4+a_5)S(x_{2n},x_{2n},x_{2n-1}) \\
& +(a_2+a_4)S(x_{2n+1},x_{2n+1},x_{2n})\Big), \\
& \varphi\Big((a_1+a_3+2a_4+a_5)S(x_{2n},x_{2n},x_{2n-1}) \\
& +(a_2+a_4)S(x_{2n+1},x_{2n+1},x_{2n})\Big)\Big) \\
\leq\ & \psi\Big((a_1+a_3+2a_4+a_5)S(x_{2n},x_{2n},x_{2n-1}) \\
& +(a_2+a_4)S(x_{2n+1},x_{2n+1},x_{2n})\Big). \qquad (3.4)
\end{aligned}
$$

Since $\psi \in \Psi$, so using the property of ψ, we deduce that

$$
\begin{aligned}
S(x_{2n+1},x_{2n+1},x_{2n}) \leq\ & (a_1+a_3+a_4+a_5)S(x_{2n},x_{2n},x_{2n-1}) \\
& +(a_2+a_4)S(x_{2n+1},x_{2n+1},x_{2n}),
\end{aligned}
$$

or

$$
\begin{aligned}
S(x_{2n+1},x_{2n+1},x_{2n}) \leq\ & \Big(\frac{a_1+a_3+2a_4+a_5}{1-a_2-a_4}\Big)S(x_{2n},x_{2n},x_{2n-1}) \\
=\ & t\,S(x_{2n},x_{2n},x_{2n-1}), \qquad (3.5)
\end{aligned}
$$

where

$$t=\Big(\frac{a_1+a_3+2a_4+a_5}{1-a_2-a_4}\Big)<1,$$

since $a_1+a_2+a_3+3a_4+a_5<1$. This implies that

$$S(x_{n+1},x_{n+1},x_n) \leq t\,S(x_n,x_n,x_{n-1}) \qquad (3.6)$$

for $n = 0, 1, 2, \cdots$.

Let $\mathcal{D}_n = S(x_{n+1}, x_{n+1}, x_n)$ and $\mathcal{D}_{n-1} = S(x_n, x_n, x_{n-1})$. Then from equation (3.6), we conclude that

$$\mathcal{D}_n \leq t\,\mathcal{D}_{n-1} \leq t^2\,\mathcal{D}_{n-2} \leq \cdots \leq t^n\,\mathcal{D}_0. \tag{3.7}$$

Therefore, since $0 \leq t < 1$, taking the limit as $n \to \infty$, we have

$$\lim_{n\to\infty} S(x_{n+1}, x_{n+1}, x_n) = 0. \tag{3.8}$$

Now, we shall show that $\{x_n\}$ is a Cauchy sequence in (X, S).

Thus for any $n, m \in \mathbb{N}$ with $m > n$ and using Lemma 2.18, then we have

$$\begin{aligned}
S(x_n, x_n, x_m) &\leq 2S(x_n, x_n, x_{n+1}) + S(x_m, x_m, x_{n+1}) \\
&= 2S(x_n, x_n, x_{n+1}) + S(x_{n+1}, x_{n+1}, x_m) \\
&\leq 2S(x_n, x_n, x_{n+1}) + 2S(x_{n+1}, x_{n+1}, x_2) \\
&\quad + S(x_{n+2}, x_{n+2}, x_m) \\
&\leq 2S(x_n, x_n, x_{n+1}) + 2S(x_{n+1}, x_{n+1}, x_2) \\
&\quad + 2S(x_{n+2}, x_{n+2}, x_{n+3}) + \cdots + S(x_{m-1}, x_{m-1}, x_m) \\
&\leq 2\big(t^n + t^{n+1} + t^{n+2} + \cdots + t^{m-1}\big) S(x_0, x_0, x_1) \\
&= 2\big(t^n + t^{n+1} + t^{n+2} + \cdots + t^{m-1}\big) \mathcal{D}_0 \\
&\leq \Big(\frac{2t^n}{1-t}\Big) \mathcal{D}_0 \to 0 \text{ as } n, m \to \infty
\end{aligned}$$

since $0 \leq t < 1$. Thus, the sequence $\{x_n\}$ is a Cauchy sequence in the space (X, S). By the completeness of the space, there exists $u \in X$ such that $\lim_{n\to\infty} x_n = u$.

Now, we shall show that u is a fixed point of g. For this, using the given inequality (3.1) for $x = y = x_{2n}$ and $z = u$, we have

$$\begin{aligned}
\psi(S(x_{2n+1}, x_{2n+1}, gu)) &= \psi(S(fx_{2n}, fx_{2n}, gu)) \\
&\leq F\Big(\psi(\Theta(x_{2n}, x_{2n}, u)), \varphi(\Theta(x_{2n}, x_{2n}, u))\Big),
\end{aligned} \tag{3.9}$$

where

$$\begin{aligned}
\Theta(x_{2n}, x_{2n}, u) &= a_1\, S(x_{2n}, x_{2n}, u) + a_2\, S(x_{2n}, x_{2n}, fx_{2n}) + a_3\, S(u, u, gu) \\
&\quad + a_4[S(u, u, fx_{2n}) + S(x_{2n}, x_{2n}, gu)] \\
&\quad + a_5\Big(\frac{S(u, u, gu)}{[1 + S(x_{2n}, x_{2n}, u)]}\Big) \\
&= a_1\, S(x_{2n}, x_{2n}, u) + a_2\, S(x_{2n}, x_{2n}, x_{2n+1}) + a_3\, S(u, u, gu) \\
&\quad + a_4[S(u, u, x_{2n+1}) + S(x_{2n}, x_{2n}, gu)] \\
&\quad + a_5\Big(\frac{S(u, u, gu)}{[1 + S(x_{2n}, x_{2n}, u)]}\Big).
\end{aligned}$$

Letting $n \to \infty$ in the above inequality and using $(S1)$, we get

$$\Theta(x_{2n}, x_{2n}, u) = (a_3 + a_4 + a_5)S(u, u, gu). \tag{3.10}$$

Using equation (3.10) in equation (3.9) and using the property of F, we have

$$\begin{aligned} \psi(S(x_{2n+1}, x_{2n+1}, gu)) &\leq F\Big(\psi((a_3 + a_4 + a_5)S(u, u, gu))), \varphi((a_3 + a_4 + a_5)S(u, u, gu))\Big) \\ &\leq \psi((a_3 + a_4 + a_5)S(u, u, gu)). \end{aligned} \tag{3.11}$$

Letting $n \to \infty$ in equation (3.11), we obtain

$$\psi(S(u, u, gu)) \leq \psi((a_3 + a_4 + a_5)S(u, u, gu)). \tag{3.12}$$

Since $\psi \in \Psi$, so using the property of ψ in equation (3.12), we deduce that

$$\begin{aligned} S(u, u, gu) &\leq (a_3 + a_4 + a_5)S(u, u, gu) \\ &\leq (a_1 + a_2 + a_3 + 3a_4 + a_5)S(u, u, gu) \\ &< S(u, u, gu), \ \text{ since } \ a_1 + a_2 + a_3 + 2a_4 + a_5 < 1, \end{aligned}$$

which is a contradiction. Hence $S(u, u, gu) = 0$, that is, $gu = u$. This shows that u is a fixed point of g. By similar fashion, we can show that $fu = u$. Consequently, u is a common fixed point of f and g.

Now, we shall show the uniqueness. Let u_1 be another common fixed point of f and g such that $fu_1 = u_1 = gu_1$ with $u_1 \neq u$. Using given contractive condition (3.1) for $x = y = u$, $z = u_1$ and using $(S1)$ and Lemma 2.18, we obtain

$$\begin{aligned} \psi(S(u, u, u_1)) &= \psi(S(fu, fu, gu_1)) \\ &\leq F\Big(\psi(\Theta(u, u, u_1)), \varphi(\Theta(u, u, u_1))\Big), \end{aligned} \tag{3.13}$$

where

$$\begin{aligned} \Theta(u, u, u_1) &= a_1\, S(u, u, u_1) + a_2\, S(u, u, fu) + a_3\, S(u_1, u_1, gu_1) \\ &\quad + a_4[S(u_1, u_1, fu) + S(u, u, gu_1)] + a_5 \left(\frac{S(u_1, u_1, gu_1)}{[1 + S(u, u, u_1)]}\right) \\ &= a_1\, S(u, u, u_1) + a_2\, S(u, u, u) + a_3\, S(u_1, u_1, u_1) \\ &\quad + a_4[S(u_1, u_1, u) + S(u, u, u_1)] + a_5 \left(\frac{S(u_1, u_1, u_1)}{[1 + S(u, u, u_1)]}\right) \\ &= (a_1 + 2a_4)S(u, u, u_1). \end{aligned}$$

Substituting in equation (3.13) and using the property of F, we have

$$\begin{aligned} \psi(S(u, u, u_1)) &\leq F\Big(\psi((a_1 + 2a_4)S(u, u, u_1)), \varphi((a_1 + 2a_4)S(u, u, u_1))\Big) \\ &\leq \psi((a_1 + 2a_4)S(u, u, u_1)). \end{aligned} \tag{3.14}$$

Since $\psi \in \Psi$, so using the property of ψ in equation (3.14), we deduce that

$$\begin{aligned} S(u,u,u_1) &\leq (a_1+2a_4)S(u,u,u_1) \\ &\leq (a_1+a_2+a_3+3a_4+a_5)S(u,u,u_1) \\ &< S(u,u,u_1), \text{ since } a_1+a_2+a_3+2a_4+a_5<1, \end{aligned} \tag{3.15}$$

which is a contradiction. Hence $S(u,u,u_1)=0$, that is, $u=u_1$. This shows the uniqueness of the common fixed point of f and g. This completes the proof. □

If we take $F(s,t)=ms$ for some $m \in [0,1)$ and $\psi(t)=t$ for all $t \geq 0$ in Theorem 3.1, then we have the following result (with $ma_1 \to a_1$, $ma_2 \to a_2$, $ma_3 \to a_3$, $ma_4 \to a_4$, $ma_5 \to a_5$).

Corollary 3.2 *Let (X,S) be a complete S-metric space and $f,g\colon X \to X$ be two self-mappings satisfying the inequality:*

$$\begin{aligned} S(fx,fy,gz) \leq\ & a_1\,S(x,y,z)+a_2\,S(x,x,fx)+a_3\,S(z,z,gz) \\ & +a_4[S(z,z,fx)+S(x,x,gz)]+a_5\left(\frac{S(z,z,gz)}{[1+S(x,y,z)]}\right) \end{aligned} \tag{3.16}$$

for all $x,y,z \in X$, where $a_1,a_2,a_3,a_4,a_5>0$ are nonnegative reals with $a_1+a_2+a_3+3a_4+a_5<1$. Then f and g have a unique common fixed point in X.

Proof Follows from Theorem 3.1 by taking $F(s,t)=ms$ for some $m \in [0,1)$ and $\psi(t)=t$ for all $t \geq 0$ with $ma_1 \to a_1$, $ma_2 \to a_2$, $ma_3 \to a_3$, $ma_4 \to a_4$, $ma_5 \to a_5$. □

Putting $g=f$ in Theorem 3.1, then we obtain the following result.

Corollary 3.3 *Let (X,S) be a complete S-metric space and $f\colon X \to X$ be a self-mapping satisfying the following inequality:*

$$\psi(S(fx,fy,fz)) \leq F\Big(\psi(\Lambda(x,y,z)),\varphi(\Lambda(x,y,z))\Big), \tag{3.17}$$

where

$$\begin{aligned} \Lambda(x,y,z) =\ & a_1\,S(x,y,z)+a_2\,S(x,x,fx)+a_3\,S(z,z,fz) \\ & +a_4[S(z,z,fx)+S(x,x,fz)]+a_5\left(\frac{S(z,z,fz)}{[1+S(x,y,z)]}\right) \end{aligned}$$

for all $x,y,z \in X$, where $a_1,a_2,a_3,a_4,a_5>0$ are nonnegative reals with $a_1+a_2+a_3+3a_4+a_5<1$, $\psi \in \Psi$, $\varphi \in \Phi_u$ and $F \in \mathcal{C}$. Then f has a unique fixed point in X.

Proof This result immediately follows from Theorem 3.1 by taking $g=f$. □

Corollary 3.4 *Let (X,S) be a complete S-metric space such that for some positive integer n, f^n satisfies the contraction condition* (3.17) *for all $x,y,z \in X$, where $\Lambda(x,y,z)$, $\psi \in \Psi$, $\varphi \in \Phi_u$ and $F \in \mathcal{C}$ are as in Corollary* 3.3*. Then f has a unique fixed point in X.*

Proof From Corollary 3.3, let z_0 be the unique fixed point of f^n, that is, $f^n(z_0) = z_0$. Then

$$f(f^n z_0) = f z_0 \quad \text{or} \quad f^n(f z_0) = f z_0.$$

This gives $f z_0 = z_0$. This shows that z_0 is a unique fixed point of f and completes the proof.□

If we take $F(s,t) = ms$ for some $m \in [0,1)$, $\psi(t) = t$ for all $t \geq 0$ and putting $a_1 = k$, where $k \in [0,1)$ and $a_2 = a_3 = a_4 = a_5 = 0$ in Corollary 3.3, then we have the following result (with $mk \to k$).

Corollary 3.5 *Let (X,S) be a complete S-metric space and $f\colon X \to X$ be a self-mapping satisfying the inequality:*

$$S(fx, fy, fz)) \leq k\, S(x,y,z) \tag{3.18}$$

for all $x,y,z \in X$, where $k \in [0,1)$ is a constant. Then f has a unique fixed point in X.

Remark 3.6 *Corollary* 3.5 *extends the well-known Banach fixed point theorem [3] from complete metric space to the setting of complete S-metric space.*

If we take $F(s,t) = s - t$ in Theorem 3.1, then we obtain the following result.

Corollary 3.7 *Let (X,S) be a complete S-metric space and $f,g\colon X \to X$ be two self-mappings of X satisfying the inequality:*

$$\psi(S(fx, fy, gz)) \leq \psi(\Theta(x,y,z)) - \varphi(\Theta(x,y,z)) \tag{3.19}$$

for all $x,y,z \in X$, where $\Theta(x,y,z)$, ψ, φ and $a_1, a_2, a_3, a_4, a_5 > 0$ are as in Theorem 3.1*. Then f and g have a unique common fixed point in X.*

Proof This result follows from Theorem 3.1. □

If we take $F(s,t) = s$ in Theorem 3.1, then we obtain the following result.

Corollary 3.8 *Let (X,S) be a complete S-metric space and $f,g\colon X \to X$ be two self-mappings of X satisfying the inequality:*

$$\psi(S(fx, fy, gz)) \leq \psi(\Theta(x,y,z)) \tag{3.20}$$

for all $x,y,z \in X$, where $\Theta(x,y,z)$, ψ and $a_1, a_2, a_3, a_4, a_5 > 0$ are as in Theorem **??***. Then f and g have a unique common fixed point in X.*

Proof This result follows from Theorem 3.1. □

If we take $\psi(t) = t$ for all $t \geq 0$ in Corollary 3.8, then we obtain the following result.

Corollary 3.9 *Let (X,S) be a complete S-metric space and $f,g\colon X \to X$ be two self-mappings of X satisfying the inequality:*

$$S(fx, fy, gz) \leq \Theta(x,y,z) \tag{3.21}$$

for all $x, y, z \in X$*, where* $\Theta(x, y, z)$ *and* $a_1, a_2, a_3, a_4, a_5 > 0$ *are as in Theorem* 3.1*. Then* f *and* g *have a unique common fixed point in* X*.*

Proof It follows from Theorem 3.1. □

If we take $g = f$ in Corollary 3.2, then we have the following result.

Corollary 3.10 *Let* (X, S) *be a complete* S*-metric space and* $f : X \to X$ *be a self-mapping satisfying the inequality:*

$$\begin{aligned} S(fx, fy, fz)) \quad \leq \quad & a_1\, S(x, y, z) + a_2\, S(x, x, fx) + a_3\, S(z, z, fz) \\ & + a_4[S(z, z, fx) + S(x, x, fz)] + a_5 \Big(\frac{S(z, z, fz)}{[1 + S(x, y, z)]} \Big) \end{aligned} \tag{3.22}$$

for all $x, y, z \in X$*, where* $a_1, a_2, a_3, a_4, a_5 > 0$ *are nonnegative reals with* $a_1 + a_2 + a_3 + 3a_4 + a_5 < 1$*. Then* f *has a unique fixed point in* X*.*

Other consequences of our results are the following.

Denote $\mathbf{\Gamma}$ the set of functions $\varphi : [0, \infty) \to [0, \infty)$ satisfying the following hypothesis:

$(h1)$ *φ is a Lebesgue-integrable mapping on each compact subset of* $[0, \infty)$*;*
$(h2)$ *for any* $\varepsilon > 0$ *we have* $\int_0^\varepsilon \varphi(t)dt > 0$*.*

Then, we get the result following.

Theorem 3.11 *Let* (X, S) *be a complete* S*-metric space. Suppose that the mappings* f, $g : X \to X$ *satisfy the following inequality:*

$$\int_0^{\psi(S(fx, fy, gz))} \phi(t)dt \quad \leq \quad F\Big(\psi\big(\int_0^{\Theta(x,y,z)} \phi(t)dt\big), \varphi\big(\int_0^{\Theta(x,y,z)} \phi(t)dt\big)\Big)$$

for all $x, y, z \in X$*, where* φ*,* ψ*,* F*,* $\Theta(x, y, z)$*,* a_1*,* a_2*,* a_3, a_4*,* $a_5 > 0$ *are as in Theorem* 3.1 *and* $\phi \in \mathbf{\Gamma}$*. Then,* f *and* g *have a unique common fixed point in* X*.*

If we take $F(s, t) = ms$ for some $m \in [0, 1)$, $g = f$ and $\psi(t) = t$ for all $t \geq 0$ in Theorem 3.11, then we have the following result (with $ma_1 \to a_1$, $ma_2 \to a_2$, $ma_3 \to a_3$, $ma_4 \to a_4$, $ma_5 \to a_5$).

Corollary 3.12 *Let* (X, S) *be a complete* S*-metric space. Suppose that the mapping* $f : X \to X$ *satisfying the following inequality:*

$$\begin{aligned} \int_0^{S(fx, fy, fz)} \phi(t)dt \quad \leq \quad & a_1 \int_0^{S(x,y,z)} \phi(t)dt + a_2 \int_0^{S(x,x,fx)} \phi(t)dt \\ & + a_3 \int_0^{S(z,z,fz)} \phi(t)dt + a_4 \int_0^{[S(z,z,fx)+S(x,x,fz)]} \phi(t)dt \\ & + a_5 \int_0^{\left(\frac{S(z,z,fz)}{[1+S(x,y,z)]}\right)} \phi(t)dt \end{aligned}$$

for all $x, y, z \in X$, where a_1, a_2, a_3, a_4, $a_5 > 0$ are as in Theorem 3.1 and $\phi \in \mathbf{\Gamma}$. Then f has a unique fixed point in X.

If we take $a_1 = k$ and $a_2 = a_3 = a_4 = a_5 = 0$ in Corollary 3.12, then we have the following result.

Corollary 3.13 *Let (X, S) be a complete S-metric space. Suppose that the mapping $f \colon X \to X$ satisfying the following inequality:*

$$\int_0^{S(fx,fy,fz)} \phi(t)dt \leq k \int_0^{S(x,y,z)} \phi(t)dt$$

for all $x, y, z \in X$, where $k \in [0, 1)$ is a constant and $\phi \in \mathbf{\Gamma}$. Then f has a unique fixed point in X.

Remark 3.14 *Corollary* 3.13 *extends Theorem 2.1 of Branciari [4] from complete metric space to that setting of complete S-metric space.*

Remark 3.15 *Corollary* 3.13 *also extends Banach contraction mapping principle [3] from complete metric space to that setting of complete S-metric space for integral type contraction.*

Now, we give some examples in support of our results.

Example 3.16 Let $X = [0, 1]$ and f, $g \colon X \to X$ be given by $f(x) = \frac{x}{2}$ and $g(x) = \frac{x}{4}$for all $x \in X$. Define the function $S \colon X^3 \to [0, \infty)$ by $S(x, y, z) = \max\{x, y, z\}$ for all $x, y, z \in X$, then S is an S-metric on X. Let $x, y, z \in X$ such that $x \geq y \geq z$.

(1) We have

$$\begin{aligned} S(fx, fy, gz) &= \max\left\{\frac{x}{2}, \frac{y}{2}, \frac{z}{4}\right\} = \frac{x}{2}, \\ S(x, y, z) &= \max\{x, y, z\} = x, \\ S(x, x, fx) &= \max\{x, x, \frac{x}{2}\} = x, \\ S(z, z, gz) &= \max\{z, z, \frac{z}{4}\} = z, \\ S(z, z, fx) &= \max\{z, z, \frac{x}{2}\} = \frac{x}{2}, \\ S(x, x, gz) &= \max\{x, x, \frac{z}{4}\} = x, \\ S(x, x, fz) &= \max\{x, x, \frac{z}{2}\} = x, \\ S(z, z, fz) &= \max\{z, z, \frac{z}{2}\} = z. \end{aligned}$$

Consider the inequality (3.16) of Corollary 3.2,we have

$$\frac{x}{2} \leq a_1.x + a_2.x + a_3.z + \frac{3a_4}{2}.x + a_5.\frac{z}{1+x},$$

Putting $x = 1$, $y = \frac{1}{2}$ and $z = \frac{1}{3}$, then we have

$$\frac{1}{2} \leq a_1 + a_2 + a_3.\frac{1}{3} + a_4.\frac{1}{2} + a_5.\frac{1}{6} \quad \text{or} \quad 3 \leq 6a_1 + 6a_2 + 2a_3 + 9a_4 + a_5.$$

The above inequality is satisfied for: (1) $a_1 = \frac{1}{3}$, $a_2 = \frac{1}{4}$ and $a_3 = a_4 = a_5 = 0$; (2) $a_1 = \frac{1}{3}$, $a_3 = \frac{1}{4}$, $a_4 = \frac{1}{8}$ and $a_2 = a_5 = 0$; (3) $a_2 = \frac{1}{3}$, $a_3 = \frac{1}{2}$ and $a_1 = a_4 = a_5 = 0$ with $a_1 + a_2 + a_3 + 3a_4 + a_5 < 1$. Thus all the conditions of Corollary 3.2 are satisfied. Hence by applying Corollary 3.2, f and g have a unique common fixed point in X. Indeed, $0 \in X$ is the unique common fixed point of f and g in this case.

(2) Now, consider the inequality (3.18) of Corollary 3.5, we have

$$\frac{x}{2} \leq k\,x, \quad \text{or} \quad k \geq \frac{1}{2}.$$

If we take $0 < k < 1$, then all the conditions of Corollary 3.5 are satisfied and $0 \in X$ is the unique fixed point of f.

Example 3.17 Let $X = [0, 1]$ and $S\colon X^3 \to \mathbb{R}_+$ be given by

$$S(x, y, z) = \begin{cases} |x - z| + |y - z|, & \text{if } x, y, z \in [0, 1), \\ 1, & \text{if } x = 1 \text{ or } y = 1 \text{ or } z = 1, \end{cases}$$

for all $x, y, z \in X$. Then (X, S) is a complete S-metric space.

Let the mapping $f\colon X \to X$ be given by

$$f(x) = \begin{cases} \frac{1}{2}, & \text{if } x, y, z \in [0, 1), \\ \\ \frac{1}{6}, & \text{if } x = y = z = 1. \end{cases}$$

Now, we consider the following cases for verification of inequality (3.22) of Corollary 3.10.

Case 1. If $x, y \in [0, \frac{1}{2}]$, $z \in [\frac{1}{2}, 1)$ or $z \in [0, \frac{1}{2}]$, $x, y \in [\frac{1}{2}, 1)$. Then

$$\begin{aligned} S(fx, fy, fz) &= S\Big(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\Big) = 0 \\ &\leq a_1\, S(x, y, z) + a_2\, S(x, x, fx) + a_3\, S(z, z, fz) \\ &\quad + a_4[S(z, z, fx) + S(x, x, fz)] + a_5 \left(\frac{S(z, z, fz)}{[1 + S(x, y, z)]}\right). \end{aligned}$$

Thus, the inequality (3.22) of Corollary 3.10 is trivially satisfied.

Case 2. If $x, y \in [0, \frac{1}{2}]$ and $z = 1$. Then,

$$S(fx, fy, fz) = S\Big(\frac{1}{2}, \frac{1}{2}, \frac{1}{6}\Big) = \frac{2}{3}.$$

Taking $x = y = \frac{1}{2}$,

$$S(x,y,z) = 1,\ S(x,x,fx) = 0,\ S(z,z,fz) = \frac{5}{3},$$

$$S(z,z,fx) = 1,\ S(x,x,fz) = \frac{2}{3}.$$

Now

$$\begin{aligned}
\frac{2}{3} &\leq a_1\, S(x,y,z) + a_2\, S(x,x,fx) + a_3\, S(z,z,fz) \\
&\quad + a_4[S(z,z,fx) + S(x,x,fz)] \\
&\quad + a_5 \Big(\frac{S(z,z,fz)}{[1 + S(x,y,z)]}\Big) \\
&= a_1.1 + a_2.0 + a_3.\frac{5}{3} + a_4.\frac{5}{3} + a_5.\frac{5}{6},
\end{aligned}$$

or

$$4 \leq 6a_1 + 10a_3 + 10a_4 + 5a_5.$$

The above inequality is satisfied for: (1) $a_1 = \frac{1}{3}$, $a_3 = \frac{1}{5}$ and $a_2 = a_4 = a_5 = 0$, (2) $a_1 = \frac{1}{3}$, $a_4 = \frac{1}{5}$ and $a_2 = a_3 = a_5 = 0$ and (3) $a_3 = \frac{1}{10}$, $a_4 = \frac{1}{5}$; $a_5 = \frac{1}{5}$ and $a_1 = a_2 = 0$ with $a_1 + a_2 + a_3 + 3a_4 + a_5 < 1$. Thus all the conditions of Corollary 3.10 are satisfied. Hence by applying Corollary 3.10, f has a unique fixed point in X. Indeed, $\frac{1}{2} \in X$ is the unique fixed point of f in this case.

Case 3. If $x, z \in [0, \frac{1}{2}]$ and $y = 1$. Then

$$S(fx, fy, fz) = S\Big(\frac{1}{2}, \frac{1}{6}, \frac{1}{2}\Big) = \frac{1}{3}.$$

Taking $x = z = \frac{1}{2}$,

$$S(x,y,z) = \frac{1}{2},\ S(x,x,fx) = 0,\ S(z,z,fz) = 0,\ S(z,z,fx) = 0,$$

$$S(x,x,fz) = 0.$$

Now

$$\begin{aligned}
\frac{1}{3} &\leq a_1\, S(x,y,z) + a_2\, S(x,x,fx) + a_3\, S(z,z,fz) \\
&\quad + a_4[S(z,z,fx) + S(x,x,fz)] + a_5 \Big(\frac{S(z,z,fz)}{[1 + S(x,y,z)]}\Big) \\
&= \frac{a_1}{2} + a_2.0 + a_3.0 + a_4.0 + a_5.0 \\
&= \frac{a_1}{2} \quad \text{or} \quad \frac{2}{3} \leq a_1.
\end{aligned}$$

The above inequality is satisfied for $a_1 = \frac{2}{3}$ and $a_2 = a_3 = a_4 = a_5 = 0$ with $a_1 + a_2 + a_3 + 3a_4 + a_5 < 1$. Thus all the conditions of Corollary 3.10 are satisfied. Hence by applying Corollary 3.10, f has a unique fixed point in X. Indeed, $\frac{1}{2} \in X$ is the unique fixed point of f in this case.

Case 4. If $y, z \in [0, \frac{1}{2}]$ and $x = 1$. Then

$$S(fx, fy, fz) \quad = \quad S\Big(\frac{1}{6}, \frac{1}{2}, \frac{1}{2}\Big) = \frac{1}{3}.$$

Taking $y = z = \frac{1}{2}$,

$$S(x, y, z) = \frac{1}{2},\ S(x, x, fx) = \frac{5}{3},\ S(z, z, fz) = 0,$$

$$S(z, z, fx) = \frac{2}{3},\ S(x, x, fz) = 1.$$

Now

$$\begin{aligned}
\frac{1}{3} \quad &\leq \quad a_1\, S(x, y, z) + a_2\, S(x, x, fx) + a_3\, S(z, z, fz) \\
&\quad +a_4[S(z, z, fx) + S(x, x, fz)] + a_5 \Big(\frac{S(z, z, fz)}{[1 + S(x, y, z)]}\Big) \\
&= \quad \frac{a_1}{2} + a_2.\frac{5}{3} + a_3.0 + a_4.\frac{5}{3} + a_5.0 \\
&= \quad \frac{a_1}{2} + \frac{5a_2}{3} + \frac{5a_4}{3}
\end{aligned}$$

or

$$2 \leq 3a_1 + 10a_2 + 10a_4.$$

The above inequality is satisfied for: (1) $a_1 = \frac{1}{3}$, $a_2 = \frac{1}{10}$, $a_3 = a_4 = a_5 = 0$; (2) $a_1 = \frac{1}{3}$, $a_4 = \frac{1}{10}$, $a_2 = a_3 = a_5 = 0$ and (3) $a_2 = a_4 = \frac{1}{10}$ and $a_1 = a_3 = a_5 = 0$ with $a_1 + a_2 + a_3 + 3a_4 + a_5 < 1$. Thus all the conditions of Corollary 3.10 are satisfied. Hence by applying Corollary 3.10, f has a unique fixed point in X. Indeed, $\frac{1}{2} \in X$ is the unique fixed point of f in this case.

Considering all the above cases, we conclude that the inequality used in Corollary 3.10 remains valid for mapping f constructed in the above example and consequently by applying Corollary 3.10, f has a unique fixed point. One can easily see that $u = \frac{1}{2} \in X$ is the unique fixed point of f.

§4. **Conclusion**

In this paper, we prove some common fixed point theorems in the setting of complete S-metric spaces via C-class functions and we give some examples in support of our results. Also, we give some consequences as corollaries of the established results. The results obtained in this paper extend, generalize and enrich several results from the existing literature regarding complete S-metric spaces.

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