International J.Math. Combin. Vol.3(2022), 99-106

4-Total Mean Cordial Labeling of Some Graphs Derived From H-Graph and Star

R.Ponraj[1], S.Subbulakshmi[2] and S.Somasundaram[3]

1. Department of Mathematics, Sri Paramakalyani College, Alwarkurichi-627412, Tamilnadu, India
2. Research Scholar, Department of Mathematics, Manonmaniam Sundarnar university, Tirunelveli-627012, Tamilnadu, India
3. Department of Mathematics, Manonmaniam sundarnar university, Tirunelveli-627012, Tamilnadu, India

E-mail: ponrajmaths@gmail.com, ssubbulakshmis@gmail.com, somutvl@gmail.com

Abstract: Let G be a graph. Let $f : V(G) \to \{0, 1, 2, \ldots, k-1\}$ be a function where $k \in \mathbb{N}$ and $k > 1$. For each edge uv, assign the label $f(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. f is called k-total mean cordial labeling of G if $|t_{mf}(i) - t_{mf}(j)| \leq 1$ for all $i, j \in \{0, 1, 2, \cdots, k-1\}$, where $t_{mf}(x)$ denotes the total number of vertices and edges labelled with x, $x \in \{0, 1, 2, \cdots, k-1\}$. A graph with admit a k-total mean cordial labeling is called k-total mean cordial graph.

Key Words: Total mean cordial labeling, Smarandachely total mean cordial labeling, path, complete graph, corona, star.

AMS(2010): 05C78.

§1. Introduction

In this paper we consider simple, finite and undirected graphs only. Cordial labeling was introduced by Cahit [3] and cordial relation labeling technique was studied in [1, 2, 4, 5, 6, 9, 17, 18, 19, 20]. The notation of k-total mean cordial labeling has been introduced in [10]. We investigate the 4-total mean cordial labeling behaviour of several graphs like cycle, complete graph, star, bistar, comb and crown in [10, 11, 12, 13, 14, 15, 16].. Let x be any real number. Then $\lceil x \rceil$ stands for the smallest integer greater than or equal to x. Terms are not defined here follow from Harary [8] and Gallian [7]. In this paper we investigate the 4-total mean cordial labeling of some graphs derived from H- graph and star.

§2. k-Total Mean Cordial Graph

Definition 2.1 *Let G be a graph. Let $f : V(G) \to \{0, 1, 2, \ldots, k-1\}$ be a function where $k \in \mathbb{N}$ and $k > 1$. For each edge uv, assign the label $f(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. f is called k-total mean cordial labeling of G if $|t_{mf}(i) - t_{mf}(j)| \leq 1$, for all $i, j \in \{0, 1, 2, \cdots, k-1\}$, where $t_{mf}(x)$ denotes the total number of vertices and edges labelled with x, $x \in \{0, 1, 2, \cdots, k-1\}$.*

[1]Received June 18, 2022, Accepted September 20, 2022.

A graph with admit a k-total mean cordial labeling is called a k-total mean cordial graph.

Such a labeling f is called a Smarandachely k-total mean cordial labeling of G if there are integers $i, j \in \{0, 1, 2, \cdots, k-1\}$ hold with $|t_{mf}(i) - t_{mf}(j)| \geq 2$ and G is called a Smarandachely k-total mean cordial graph.

§3. Preliminaries

Definition 3.1 *Let $P_n^{(1)} : u_1 u_2 \ldots u_n$ and $P_n^{(2)} : v_1 v_2 \cdots v_n$ be any two paths. We join the vertices $u_{\frac{n+1}{2}}$ and $v_{\frac{n+1}{2}}$ by an edge, if n is odd and join the vertices $u_{\frac{n}{2}}$ and $v_{\frac{n}{2}+1}$ by an edge, if n is even. Then the resulting graph is called a H-graph on $2n$ vertices. We denote it by $H(n)$.*

Definition 3.2 *If $e = uv$ is an edge of G then e is said to be subdivided when it is replaced by the edges uw and wv. The graph obtained by subdividing each edge of a graph G is called the subdivision graph of G and is denoted by $S(G)$.*

Definition 3.3 *The duplication of an edge $e = uv$ of a graph G is the graph G' obtained from G by adding a new vertex x to G such that x is adjacent to both u and v.*

Definition 3.4 *Let G_1, G_2 respectively be (p_1, q_1), (p_2, q_2) graphs. The corona of G_1 with G_2 is the graph $G_1 \odot G_2$ obtained by taking one copy of G_1 , p_1 copies of G_2 and joining the i^{th} vertex of G_1 by an edge to every vertex in the i^{th} copy of G_2 where $1 \leq i \leq p_1$.*

Definition 3.5 *The complement $\overline{G}$ of a graph G also has $V(G)$ as its vertex set, but two vertices are adjacent in $\overline{G}$ if and only if they are not adjacent in G.*

Definition 3.6 *The complete bipartite graph $K_{1,n}$ is called a star.*

Definition 3.7 *$K_{1,3} * K_{1,n}$ is the graph obtained from $K_{1,3}$ by attaching root of a star $K_{1,n}$ at each pendent vertex of $K_{1,3}$.*

Definition 3.8 *Consider two copies of graph G namely G_1 and G_2. Then the graph $G' = \langle G_1 \Delta G_2 \rangle$ is the graph obtained by joining the apex vertices of G_1 and G_2 by an edge as well as to a new vertex x.*

Definition 3.9 *A sparkler denoted as P_m^{+n} is a graph obtained from the path P_m and appending n edges to an end point. This is a special case of a caterpillar. We refer to the hub of P_m^{+n}, the sparkler as the vertex of degree $n+1$.*

Definition 3.10 *Let $u_i^{(k)}$ and $v_i^{(k)}$ be the vertices in the k^{th} copy of H-graph, where $i = 1, 2, 3, \cdots, n$ and $k = 1, 2, 3, \cdots, r$. Join the vertices v_1^k and v_1^{k+1} for $k = 1, 2, 3, \ldots, r-1$. The resulting graph is denoted by $P(r, H(n))$.*

Theorem 3.1([10]) *Any path is k-total mean cordial.*

§4. Main Results

4.1 Graphs Derived From H-Graph

Theorem 4.1 *The graph $H(n)$ is a 4-total mean cordial for all values of $n \geq 2$.*

Proof Take the vertex set and edge set of $H(n)$ as in Definition 3.1. Clearly $|V(H(n))| + |E(H(n))| = 4n-1$. Obviously $H(2) \cong P_4$. Therefore $H(2)$ is 4-total mean cordial follow from Theorem 3.1.

Case 1. $n \equiv 1 \pmod 2$.

Let $n = 2r+1$, $r \in \mathbb{N}$. Assign the label 2 to the $r+1$ vertices $u_1, u_2, \ldots, u_{r+1}$. Next we assign the label 3 to the r vertices $u_{r+2}, u_{r+3}, \cdots, u_{2r+1}$. Now we assign the label 0 to the $r+1$ vertices $v_1, v_2, \cdots, v_{r+1}$. Finally we assign the label 1 to the r vertices $v_{r+2}, v_{r+3}, \cdots, v_{2r+1}$.

Case 2. $n \equiv 0 \pmod 2$.

Let $n = 2r$, $r \geq 2$. We assign the label 2 to the r vertices $u_1, u_2, \cdots, u_r$. Now we assign the label 3 to the r vertices $u_{r+1}, u_{r+2}, \cdots, u_{2r}$. Next we assign the label 0 to the r vertices $v_1, v_2, \cdots, v_r$. Finally we assign the label 1 to the r vertices $v_{r+1}, v_{r+2}, \cdots, v_{2r}$.

This shows that vertex labeling f is a 4-total mean cordial labeling follows from the Table 1.This completes the proof. □

n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n = 2r+1$	$2r+1$	$2r+1$	$2r+1$	$2r$
$n = 2r$	$2r-1$	$2r$	$2r$	$2r$

Table 1.

Theorem 4.2 *The subdivision of $H(n)$, $S(H(n))$ is a 4-total mean cordial for all values of $n \geq 2$.*

Proof Take the vertex set and edge set as in Definition 3.1. Let x_i $(1 \leq i \leq n-1)$ be the vertex which subdivide the edge u_iu_{i+1} $(1 \leq i \leq n-1)$ and y_i $(1 \leq i \leq n-1)$ be the vertex which subdivide the edge v_iv_{i+1} $(1 \leq i \leq n-1)$. Let w be the vertex which subdivide the edge $u_{\frac{n+1}{2}}v_{\frac{n+1}{2}}$, if n is odd and w be the vertex which subdivide the edge $u_{\frac{n}{2}}v_{\frac{n}{2}+1}$, if n is even.

Clearly, $\left|V\left(S\left(H_{(n)}\right)\right)\right| + \left|E\left(S\left(H_{(n)}\right)\right)\right| = 8n-3$.

Case 1. $n \equiv 0 \pmod 2$.

Let $n = 2r$, $r \in \mathbb{N}$. Assign the label 2 to the vertex w. We now assign the label 0 to the r vertices $u_1, u_2, \cdots, u_r$. Now we assign the label 1 to the r vertices $u_{r+1}, u_{r+2}, \cdots, u_{2r}$. Next we assign the label 0 to the r vertices $x_1, x_2, \cdots, x_r$. Now we assign the label 1 to the $r-1$ vertices $x_{r+1}, x_{r+2}, \cdots, x_{2r-1}$. Next we assign the label 3 to the r vertices $v_1, v_2, \ldots, v_r$. Now we assign the label 2 to the r vertices $v_{r+1}, v_{r+2}, \cdots, v_{2r}$. Next we assign the label 3 to the r vertices $y_1, y_2, \cdots, y_r$. Finally we assign the label 2 to the $r-1$ vertices $y_{r+1}, y_{r+2}, \cdots, y_{2r-1}$.

Case 2. $n \equiv 1 \pmod 2$.

Let $n = 2r+1$, $r \in \mathbb{N}$. Now we assign the label 1 to the vertex w. We now assign the label 0 to the $r+1$ vertices $u_1, u_2, \cdots, u_{r+1}$. Next we assign the label 2 to the r vertices $u_{r+2}, u_{r+3}, \cdots, u_{2r+1}$. We now assign the label 0 to the r vertices $x_1, x_2, \ldots, x_r$. Now we assign the label

2 to the r vertices x_{r+1}, x_{r+2}, $\cdots$, x_{2r}. Now we assign the label 3 to the $r+1$ vertices v_1, v_2, $\cdots$, v_{r+1}. Next we assign the label 1 to the r vertices v_{r+2}, v_{r+3}, $\cdots$, v_{2r+1}. Now we assign the label 3 to the r vertices y_1, y_2, $\cdots$, y_r. Finally we assign the label 1 to the r vertices y_{r+1}, y_{r+2}, $\cdots$, y_{2r}. This shows that the vertex labeling f is a 4-total mean cordial labeling follows from the Table 2.

n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n=2r$	$4r-1$	$4r-1$	$4r-1$	$4r$
$n=2r+1$	$4r+1$	$4r+2$	$4r+1$	$4r+1$

Table 2.

Theorem 4.3 *Duplication of all edges of H-graph $H(n)$ is a 4-total mean cordial labeling, if n is odd.*

Proof Take the vertex set and edge set of $H(n)$ as in Definition 3.1. Let $H^*(n)$ be the graph obtained by duplication of all edges u_1u_2,u_2u_3,$\cdots$,$u_{n-1}u_n$ and v_1v_2,v_2v_3,$\cdots$,$v_{n-1}v_n$ by a new vertices x_1,x_2,$\cdots$,x_{n-1} and y_1,y_2,$\cdots$,y_{n-1} respectively. Let w be a new vertex obtained by duplicating the edge $u_{\frac{n+1}{2}}v_{\frac{n+1}{2}}$. In graph $H^*(n)$, $|V(H^*(n))|+|E(H^*(n))|=10n-4$.

Assign the label 3 to the vertex w. We now assign the label 2 to the $\frac{n+1}{2}$ vertices u_1, u_2, $\cdots$, $u_{\frac{n+1}{2}}$. Now we assign the label 3 to the $\frac{n-1}{2}$ vertices $u_{\frac{n+3}{2}}$, $u_{\frac{n+5}{2}}$, $\cdots$, u_n. Next we assign the label 2 to the $\frac{n-1}{2}$ vertices x_1, x_2, $\cdots$, $x_{\frac{n-1}{2}}$. Now we assign the label 3 to the $\frac{n-1}{2}$ vertices $x_{\frac{n+1}{2}}$, $x_{\frac{n+3}{2}}$, $\cdots$, x_{n-1}. We now assign the label 0 to the $\frac{n+1}{2}$ vertices v_1, v_2, $\cdots$, $v_{\frac{n+1}{2}}$. Next we assign the label 1 to the $\frac{n-1}{2}$ vertices $v_{\frac{n+3}{2}}$, $v_{\frac{n+5}{2}}$, $\cdots$, v_n. Now we assign the label 0 to the $\frac{n-1}{2}$ vertices y_1, y_2, $\cdots$, $y_{\frac{n-1}{2}}$. Finally we assign the label 1 to the $\frac{n-1}{2}$ vertices $y_{\frac{n+1}{2}}$, $y_{\frac{n+3}{2}}$, $\ldots$, y_{n-1}.

Clearly, $t_{mf}(0)=t_{mf}(1)=\frac{5n-3}{2}$; $t_{mf}(2)=t_{mf}(3)=\frac{5n-1}{2}$. This completes the proof. □

Theorem 4.4 *The graph $H(n)\odot K_1$ is a 4-total mean cordial for all values of $n\geq 2$.*

Proof Let $V(H(n))=\{u_i,v_i:1\leq i\leq n\}$ and let x_1,x_2,$\cdots$, x_n be the pendent vertices connected to u_1,u_2,$\cdots$, u_n and y_1,y_2,$\cdots$, y_n be the pendent vertices connected to v_1,v_2,$\cdots$, v_n. Clearly, $|V(H(n)\odot K_1)|+|E(H(n)\odot K_1)|=8n-1$.

Case 1. $n\equiv 1 \pmod 2$.

Let $n=2r+1$, $r\in\mathbb{N}$. Assign the label 2 to the $2r+1$ vertices u_1, u_2, $\cdots$, u_{2r+1}. Now we assign the label 3 to the $2r+1$ vertices x_1,x_2,$\cdots$, x_{2r+1}. Next we assign the label 0 to the r vertices v_1, v_2, $\cdots$, v_r. We now assign the label 1 to the $r+1$ vertices v_{r+1},v_{r+2},$\cdots$, v_{2r+1}. Now we assign the label 0 to the $r+2$ vertices y_1, y_2, $\cdots$, y_{r+2}. Finally we assign the label 1 to the $r-1$ vertices y_{r+3},y_{r+4},$\cdots$, y_{2r+1}.

Case 2. $n\equiv 0 \pmod 2$.

Let $n=2r$, $r\in\mathbb{N}$. We assign the label 2 to the $2r$ vertices u_1, u_2, $\cdots$, u_{2r}. Next we assign the label 3 to the $2r$ vertices x_1,x_2,$\cdots$, x_{2r}. Now we assign the label 0 to the r vertices v_1, v_2, $\cdots$, v_r. Next we assign the label 1 to the r vertices v_{r+1}, v_{r+2}, $\cdots$, v_{2r}. Now we assign the label 0 to the $r+1$ vertices y_1,y_2, $\cdots$, y_{r+1}. Finally we assign the label 1 to the $r-1$ vertices y_{r+2},y_{r+3},$\cdots$, y_{2r}. Thus, this vertex labeling f is a 4-total mean cordial labeling follows from the Table 3. □

n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n=2r+1$	$4r+1$	$4r+2$	$4r+2$	$4r+2$
$n=2r$	$4r$	$4r-1$	$4r$	$4r$

Table 3.

Theorem 4.5 *The graph $H(n)\odot\overline{K_2}$ is a 4-total mean cordial for all values of $n\geq 2$.*

Proof Let

$$V\left(H(n)\odot\overline{K_2}\right)=\{u_i,v_i,x_i,y_i,p_i,q_i:1\leq i\leq n\},$$
$$E\left(H(n)\odot\overline{K_2}\right)=\{u_iu_{i-1},v_iv_{i-1}:1\leq i\leq n-1\}\bigcup\{u_ix_i,u_iy_i,v_ip_i,v_iq_i:1\leq i\leq n\}.$$

Clearly, $\left|V\left(H(n)\odot\overline{K_2}\right)\right|+\left|E\left(H(n)\odot\overline{K_2}\right)\right|=12n-1$. Assign the label 1 to the n vertices u_1, u_2, $\ldots$, u_n. Now we assign the label 3 to the n vertices x_1,x_2,$\ldots$, x_n. We now assign the label 0 to the n vertices y_1,y_2,$\ldots$, y_n. Next we assign the label 0 to the n vertices v_1, v_2, $\ldots$, v_n. We now assign the label 3 to the n vertices p_1,p_2,$\ldots$, p_n. Finally we assign the label 3 to the n vertices q_1, q_2, $\ldots$, q_n. Thus $t_{mf}(0)=3n-1$; $t_{mf}(1)=t_{mf}(2)=t_{mf}(3)=3n$. □

Theorem 4.6 *The graph $P(r,H(n))$ is a 4-total mean cordial for all values of $n\geq 2$.*

Proof Take the vertex set and edge set of $P(r,H(n))$ as in Definition 3.10. In the graph $P(r,H(n))$, $|V(P(r,H(n)))|+|E(P(r,H(n)))|=4nr-1$.

Case 1. $n\equiv 1 \pmod 2$.

Let $n=2t+1$, $t\in\mathbb{N}$. Assign the label 1 to the t vertices u_1^k, u_2^k, $\cdots$, u_t^k. Now we assign the label 0 to the $t+1$ vertices u_{t+1}^k, u_{t+2}^k, $\cdots$, u_{2t+1}^k. Next we assign the label 3 to the t vertices v_1^k, v_2^k, $\cdots$, v_t^k. Finally we assign the label 2 to the $t+1$ vertices v_{t+1}^k, v_{t+2}^k, $\cdots$, v_{2t+1}^k.

Case 2. $n\equiv 0 \pmod 2$.

Let $n=2t$, $t\in\mathbb{N}$. We assign the label 2 to the t vertices u_1^k, u_2^k, $\cdots$, u_t^k. Next we assign the label 3 to the t vertices u_{t+1}^k, u_{t+2}^k, $\cdots$, u_{2t}^k. Now we assign the label 0 to the t vertices v_1^k, v_2^k, $\cdots$, v_t^k. Finally we assign the label 1 to the t vertices v_{t+1}^k, v_{t+2}^k, $\cdots$, v_{2t}^k. This shows that vertex labeling f is a 4-total mean cordial labeling follows from the Table 4. □

n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
n is odd	$(2t+1)r$	$(2t+1)r$	$(2t+1)r$	$(2t+1)r-1$
n is even	$2tr$	$2tr$	$2tr$	$2tr-1$

Table 4.

4.2 Graphs Derived From Stars

Theorem 4.7 *The graph $K_{1,3} * K_{1,n}$ is 4-total mean cordial for all values of n.*

Proof Let $V(K_{1,3} * K_{1,n}) = \{u, u_1, u_2, u_3, x_i, y_i, z_i : 1 \le i \le n\}$ and $E(K_{1,3} * K_{1,n}) = \{uu_1, uu_2, uu_3\} \cup \{u_1x_i, u_2y_i, u_3z_i : 1 \le i \le n\}$. Note that $|V(K_{1,3} * K_{1,n})| + |E(K_{1,3} * K_{1,n})| = 6n + 7$. Assign the labels 0,0,3,1 to the vertices u,u_1,u_2,u_3.

Case 1. $n \equiv 0 \pmod 2$.

Let $n = 2r$, $r \in \mathbb{N}$. Assign the label 0 to the $r-1$ vertices $x_1, x_2, \cdots, x_{r-1}$. Now we assign the label 1 to the vertex x_r. Now we assign the label 2 to the r vertices $x_{r+1}, x_{r+2}, \cdots, x_{2r}$. Next we assign the label 0 to the r vertices $y_1, y_2, \cdots, y_r$. We now assign the label 3 to the r vertices $y_{r+1}, y_{r+2}, \cdots, y_{2r}$. Now we assign the label 1 to the $r-1$ vertices $z_1, z_2, \cdots, z_{r-1}$. Finally we assign the label 3 to the $r+1$ vertices $z_r, z_{r+1}, \cdots, z_{2r}$.

Case 2. $n \equiv 1 \pmod 2$.

Let $n = 2r+1$, $r \in \mathbb{N}$. We now assign the label 0 to the r vertices $x_1, x_2, \cdots, x_r$. Now we assign the label 1 to the two vertices x_{r+1}, x_{r+2}. Next we assign the label 2 to the $r-1$ vertices $x_{r+3}, x_{r+4}, \cdots, x_{2r+1}$. Now we assign the label 0 to the $r+1$ vertices $y_1, y_2, \cdots, y_{r+1}$. We now assign the label 3 to the r vertices $y_{r+2}, y_{r+3}, \cdots, y_{2r+1}$. Now we assign the label 1 to the $r-1$ vertices $z_1, z_2, \cdots, z_{r-1}$. Finally we assign the label 3 to the $r+2$ vertices $z_r, z_{r+1}, \cdots, z_{2r+1}$. Thus, this vertex labeling f is a 4-total mean cordial labeling follows from the Table 5. This completes the proof. □

n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
$n = 2r+1$	$3r+4$	$3r+3$	$3r+3$	$3r+3$
$n = 2r$	$3r+1$	$3r+2$	$3r+2$	$3r+2$

Table 5.

Example 4.1 A 4-total mean cordial labeling of $K_{1,3} * K_{1,5}$ is given in Figure 1.

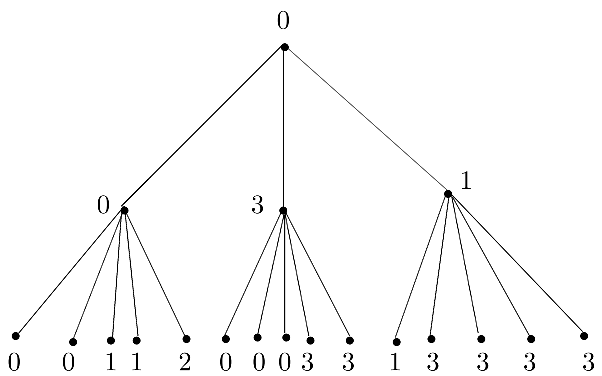

Figure 1

Theorem 4.8 *The graph $\left\langle K_{1,n}^{(1)} \Delta K_{1,n}^{(2)} \right\rangle$ is 4-total mean cordial for all values of n.*

Proof Let u_1,u_2,$\cdots$, u_n be the pendent vertices of $K_{1,n}^{(1)}$ and v_1,v_2,$\cdots$, v_n be the pendent vertices of $K_{1,n}^{(2)}$. Let u and v be the vertices of $K_{1,n}^{(1)}$ and $K_{1,n}^{(2)}$ which adjacent to u_i $(1 \leq i \leq n)$ and v_i $(1 \leq i \leq n)$ respectively. Let u and v are adjacent to a new common vertex x. Note that $\left|V\left(\left\langle K_{1,n}^{(1)}\Delta K_{1,n}^{(2)}\right\rangle\right)\right| + \left|E\left(\left\langle K_{1,n}^{(1)}\Delta K_{1,n}^{(2)}\right\rangle\right)\right| = 4n+6$. Assign the labels 0,1,3 to the vertices x,u,v. Consider the vertices u_1, u_2, $\cdots$, u_n. Now we assign the label 0 to the n vertices u_1, u_2, $\cdots$, u_n. We now consider the vertices v_1, v_2, $\cdots$, v_n. Finally we assign the label 2 to the n vertices v_1, v_2, $\cdots$, v_n. Obviously $t_{mf}(0) = t_{mf}(3) = n+1$; $t_{mf}(1) = t_{mf}(2) = n+2$. □

Theorem 4.8 *The graph P_n^{+n} is a 4-total mean cordial for all values of n.*

Proof Let u_1 u_2 $\cdots$ u_n be the path P_n. Then $V(P_n^{+n}) = V(P_n) \cup \{v_j : 1 \leq j \leq n\}$ and $E(P_n^{+n}) = E(P_n) \cup \{u_iv_j : 1 \leq j \leq n\}$. Note that $|V(P_n^{+n})| + |E(P_n^{+n})| = 4n-1$.

Case 1. $n \equiv 1 \pmod 2$.

Let $n = 2r+1$, $r \in \mathbb{N}$. Assign the label 0 to the $r+1$ vertices u_1, u_2, $\cdots$, u_{r+1}. Next we assign the label 1 to the r vertices u_{r+2}, u_{r+3}, $\cdots$, u_{2r+1}. Now we assign the label 3 to the $2r+1$ vertices v_1, v_2, $\cdots$, v_{2r+1}.

Case 2. $n \equiv 0 \pmod 2$.

Let $n = 2r$, $r \in \mathbb{N}$. We now assign the label 0 to the r vertices u_1, u_2, $\cdots$, u_r. Now we assign the label 1 to the r vertices u_{r+1}, u_{r+2}, $\cdots$, u_{2r}. Finally we assign the label 3 to the $2r$ vertices v_1, v_2, $\cdots$, v_{2r}.

Thus this vertex labeling f is a 4-total mean cordial labeling follows from the Table 6. □

n	$t_{mf}(0)$	$t_{mf}(1)$	$t_{mf}(2)$	$t_{mf}(3)$
n is odd	$2r+1$	$2r$	$2r+1$	$2r+1$
n is even	$2r-1$	$2r$	$2r$	$2r$

Table 6.

References

[1] M.Andar, S.Boxwala and N.Limaye, New families of cordial graphs, *J. Combin. Math. Combin. comput.*, 53 (2005), 117-154.

[2] M.Andar, S.Boxwala and N.Limaye, On the cordinality of corona graphs, *Ars. Combin.*, 78 (2006), 179-199.

[3] I.Cahit, Cordial Graphs: A weaker version of Graceful and Harmonious graphs, *Ars Combin.*, 23 (1987), 201-207.

[4] I.Cahit, H-cordial graphs, *Bull.Inst Combin. Appl.,* 18 (1996), 87-101.

[5] G.Chartrand, S.M.Lee and P.Zhang, Uniformly cordial graphs, *Discrete Math.*, 306 (2006), 726-737.

[6] A.T.Diab, Generalization of some result on cordial graphs, *Ars Combin.*, 99 (2011), 161-173.

[7] J.A.Gallian, A dynamic survey of graph labeling, *The Electronic Journal of Combinatorics*, 19 (2016), #Ds6.

[8] Harary, *Graph Theory*, Addision wesley, New Delhi (1969).

[9] N.Khan, Cordial labelling of cycles, *Annals Pure Appl. Math.*, 1, No.2 (2012), 117-130.

[10] R.Ponraj, S.Subbulakshmi, S.Somasundaram, k-Total mean cordial graphs, *J.Math. Comput. Sci.*, 10(2020), No.5, 1697-1711.

[11] R.Ponraj, S.Subbulakshmi, S.Somasundaram, 4-Total mean cordial graphs derived from paths, *J.Appl and Pure Math.*, Vol 2(2020), 319-329.

[12] R.Ponraj, S.Subbulakshmi, S.Somasundaram, 4-Total mean cordial labeling in subdivision graphs, *Journal of Algorithms and Computation*, 52(2020), 1-11.

[13] R.Ponraj, S.Subbulakshmi, S.Somasundaram, Some 4-total mean cordial graphs derived from wheel, *J. Math. Comput. Sci.*, 11(2021), 467-476.

[14] R.Ponraj, S.Subbulakshmi, S.Somasundaram, 4-Total mean cordial graphs derived from star and bistar, *Turkish Journal of Computer and Mathematics Education*, 12(2021), 951-956.

[15] R.Ponraj, S.Subbulakshmi, S.Somasundaram, On 4-total mean cordial graphs, *J. Appl. Math and Informatics,* Vol 39(2021), 497-506.

[16] R.Ponraj, S.Subbulakshmi, S.Somasundaram, 4-Total mean cordial labelling of special graphs, *Journal of Algorithms and Computation,* 53(2021), 13-22.

[17] M.A.Seoud and A.E.I.Abdul, Magsoud on cordial and balanced labeling of graphs, *J. Egyptian Math. Soc.*, 7 (1999) 127-135.

[18] A.Sugumaran and K.Rajesh, Some graph operations on sum divisor cordial labeling related to H-graph, *International Journal of Statistics and Applied Mathematics*, (2018), 389-395.

[19] S.K.Vaidya and M.C.Barasara, Product cordial labeling of line graph of some graphs, *Kragujevac. J. Math.*, 40(2) (2016), 290-297.

[20] S.K.Vaidya and N.H.Shah, Cordial labeling for some bistar related graphs, *Internat. J. Mathematics Soft Comput.*, 4(2) (2014), 33-39.