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Surfaces using Smarandache Asymptotic Curves in Galilean Space

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Abstract: In this paper, we consider the problem of constructing surfaces using special Smarandache curves in Galilean 3-space. We give the family of surfaces as a linear combination of the components of this frame, and derive the conditions for coefficients to satisfy both the asymptotic and isoparametric requirements. Finally, we present some examples to verify our method.

Key Words: Galilean space, Asymptotic curve, Smarandache curve.

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§1. Introduction

The classical theory of asymptotic curves is one of the most important and efficient methods that can be used to characterize surfaces. Asymptotic curves have a big importance in astronomy, astrophysics and CAD in architecture. A curve on a surface is said to be an asymptotic curve if the curve on a regular surface is given such that the normal curvature is zero in the asymptotic direction. This direction can only happen for non-positive (negative or zero) Gaussian curvature on a surface along the asymptotic curve [5,9]. The notion of the family of surfaces having a given characteristic curve was first investigated by Wang et al. [14] in Euclidean 3-space. Then Bayram et al. [3] extended the work of Wang to how to get a surface pencil from a given spatial asymptotic curve. Moreover, some more studies about the surface family with common asymptotic curves have been given in [12,13].

The classical curve theory is one of the most important research topics in differential geometry. There are many different studies on special curves. Smarandache curves are defined as a regular curve whose position vector is composed of Frenet frame vectors [10]. There has been a lot of researches about Special Smarandache curves and their characterizations in [1,2].

The starting point of our study is to investigate how to characterize parametric surfaces via a given curve as a common isoasymptotic and special Smarandache curves in Galilean 3-space. First, we give the surfaces as a linear combination of the Frenet frame of the given curve and derive the conditions on marching-scale functions to satisfy both isoasymptotic and Smarandache requirements. Finally, we illustrate some examples of these surfaces.

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§2. Preliminaries

The Galilean space $\mathbf{G}_3$ is a Cayley-Klein space equipped with the metric of signature $(0,0,+,+)$, given in [6]. The absolute figure of the Galilean space consists of an ordered triple $\{\omega, f, I\}$ in which ω is the ideal (absolute) plane, f is the line (absolute line) in ω and I is the fixed elliptic involution of f.

In the Galilean space there are just two types of vectors, non-isotropic $\mathbf{x} = (x_1, x_2, x_3)$ (for which holds $x_1 \neq 0$). Otherwise, it is said to be isotropic.

Definition 2.1([11]) *Let $x = (x_1, x_2, x_3)$ and $y = (y_1, y_2, y_3)$ be vectors in Galilean space G_3. The Galilean scalar product of x and y is given by*

$$\langle x, y\rangle = \begin{cases} x_1 y_1, & if\ x_1 \neq 0\ or\ y_1 \neq 0 \\ x_2 y_2 + x_3 y_3, & if\ x_1 = 0\ and\ y_1 = 0 \end{cases} . \tag{1}$$

Definition 2.2([8]) *Let $x = (x_1, x_2, x_3)$ and $y = (y_1, y_2, y_3)$ be vectors in Galilean space G_3. The Galilean vector product of x and y is given by*

$$x \wedge y = \begin{vmatrix} 0 & e_2 & e_3 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix} . \tag{2}$$

Let r be an admissible curve of the class C^∞ in G_3, parameterized by the invariant parameter u, given by

$$r(u) = (u, f(u), g(u)).$$

Then the curvature $\kappa(u)$ and the torsion $\tau(u)$ of the curve $r(u)$ can be given by

$$\kappa(u) = \sqrt{f''(u)^2 + g''(u)^2},$$

and

$$\tau(u) = \frac{\det(r'(u), r''(u), r'''(u))}{\kappa^2(u)}$$

and the associated moving trihedron satisfies

$$\begin{cases} T(u) = r'(u) = (1, f'(u), g'(u)) \\ N(u) = \frac{r''}{\kappa(u)} = \frac{1}{\kappa(u)}(0, f''(u), g''(u)) \\ B(u) = \frac{1}{\kappa(u)}(0, -g''(u), f''(u)) \end{cases} ,$$

where T, N and B are called the vectors of the tangent, principal normal and binormal of $r(u)$, respectively.

Frenet formulas are given by

$$\begin{cases} T' = \kappa N \\ N' = \tau B \\ B' = -\tau N \end{cases}, \tag{3}$$

for more information, we refer to [7].

Definition 2.3([1]) *Let $r(u)$ be an admissible curve in G_3 and $\{T, N, B\}$ be its moving Frenet frame. Smarandache TN, TB and TNB curves are respectively, defined by*

$$\begin{aligned} r_{TN} &= \frac{T+N}{\|T+N\|}, \\ r_{TB} &= \frac{T+B}{\|T+B\|}, \\ r_{TNB} &= \frac{T+N+B}{\|T+N+B\|}, \end{aligned}$$

The equation of a surface in G_3 can be given by the parametrization

$$\varphi(u,v) = (\varphi_1(u,v), \varphi_2(u,v), \varphi_3(u,v)),$$

where $\varphi_1(u,v), \varphi_2(u,v)$ and $\varphi_3(u,v) \in \mathbf{C}^3$, in [7].

Let $r(u)$ is an parametric curve on the surface $\varphi(u,v)$, there exists a parameter $v_0 \in [T_1, T_2]$ such that $r(u) = \varphi(u, v_0)$

$$x(u,v_0) = y(u,v_0) = z(u,v_0) = 0, \ L_1 \le u \le L_2 \text{ and } T_1 \le v \le T_2.$$

Given a curve $r(u)$ on the surface $\varphi(u,v)$ is an asymptotic iff the binormal $B(u)$ of the curve $r(u)$ and the normal $\eta(u,v_0)$ of the surface $\varphi(u,v)$ at any point on the curve $r(u)$ are parallel to each other [4].

§3. Surfaces with Common Smarandache Asymptotic Curves in Galilean Space G_3

Let $\varphi(u,v)$ be a parametric surface. The surface is defined by a given curve $r(u)$ as follows:

$$\varphi(u,v) = r(u) + [x(u,v)T(u) + y(u,v)N(u) + z(u,v)B(u)], \tag{4}$$

$$L_1 \le u \le L_2 \quad \text{and} \quad T_1 \le v \le T_2,$$

where $x(u,v_0)$, $y(u,v_0)$ and $z(u,v_0)$ which are the values of the marching-scale functions indicate, respectively, the extension-like, flexion-like and retortion-like by the point unit through time v, starting from $r(u)$, $\{T(u), N(u), B(u)\}$ is the frame associated with the curve $r(u)$ in G_3, and values of this functions are C^1 functions. Throughout this paper, we assume that $\kappa \neq 0$.

Our main aim is to get the conditions for which the some special Smarandache curves of the unit space curve $r(u)$ is a parametric curve and an asymptotic curve on the surface $\varphi(u,v)$.

If the curve is both an asymptotic and a parameter curve on φ, then it is called isoasymptotic on a surface φ.

3.1 Surfaces with a Common Smarandache TN Asymptotic Curve in Galilean Space G_3

Theorem 3.1 *Smarandache TN curve of the curve $r(u)$ is an isoasympotic on a surface $\varphi(u,v)$ if and only if the following relations are satisfied*

$$x\left(u,v_0\right) = y\left(u,v_0\right) = z\left(u,v_0\right) = 0, \tag{5}$$

and

$$\tau = 0, \quad \frac{\partial x(u,v_0)}{\partial v} \neq 0. \tag{6}$$

Proof Taking account of [4], a parametric surface $\varphi(u,v)$ is given by a Smarandache TN curve of $r(u)$ as follows

$$\varphi(u,v) = \frac{T(u)+N(u)}{\|T(u)+N(u)\|} + \left[x(u,v)T(u) + y(u,v)N(u) + z(u,v)B(u)\right]. \tag{7}$$

Let $r(u)$ be a Smarandache TN curve on surface $\varphi(u,v)$. If Smarandache TN curve is a parametric curve on $\varphi(u,v)$, then there exists a parameter $v=v_0$ such that

$$\varphi(u,v) = \frac{T(u)+N(u)}{\|T(u)+N(u)\|},$$

that is

$$x(u,v_0) = y(u,v_0) = z(u,v_0) = 0.$$

The normal $\eta(u,v)$ of the surface is given by

$$\eta(u,v) = \varphi_u \times \varphi_v. \tag{8}$$

From (7),

$$\begin{aligned}\varphi_u \quad &= \quad \frac{\partial x(u,v)}{\partial u}T(u) + \left(\kappa + x(u,v)\kappa + \frac{\partial y(u,v)}{\partial u} - z(u,v)\tau\right)N(u) \\ &\quad + \left(\tau + y(u,v)\tau + \frac{\partial z(u,v)}{\partial u}\right)B(u)\end{aligned}$$

and

$$\varphi_v = \frac{\partial x(u,v)}{\partial v}T(u) + \frac{\partial y(u,v)}{\partial v}N(u) + \frac{\partial z(u,v)}{\partial v}B(u).$$

Using (8), the normal $\eta(u,v)$ can be written as

$$\begin{aligned}\eta(u,v) &= \left[-\frac{\partial x(u,v)}{\partial u}\frac{\partial z(u,v)}{\partial v} + \left(\tau + y(u,v)\tau + \frac{\partial z(u,v)}{\partial u}\right)\frac{\partial x(u,v)}{\partial v}\right] N(u) \\ &+ \left[\frac{\partial x(u,v)}{\partial u}\frac{\partial y(u,v)}{\partial v} - \left(\kappa + x(u,v)\kappa + \frac{\partial y(u,v)}{\partial u} - z(u,v)\tau\right)\frac{\partial x(u,v)}{\partial v}\right] B(u)\end{aligned}$$

and from (5), we have

$$\eta(u,v_0) = \left[\tau\frac{\partial x(u,v_0)}{\partial v}\right] N(u) + \left[-\kappa\frac{\partial x(u,v_0)}{\partial v}\right] B(u), \tag{9}$$

which gives that $r(u)$ is an asymptotic curve if and if $B(u) \parallel \eta(u,v_0)$, we obtain

$$\tau\frac{\partial x(u,v_0)}{\partial v} = 0$$

and

$$\kappa\frac{\partial x(u,v_0)}{\partial v} \neq 0.$$

Since $\frac{\partial x(u,v_0)}{\partial v} \neq 0$, we have

$$\begin{aligned}\tau &= 0, \\ \frac{\partial x(u,v_0)}{\partial v} &\neq 0.\end{aligned}$$

This completes the proof. □

Corollary 3.2 *Smarandache TN curve of the curve $r(u)$ is an isoasympotic if and only if $r(u)$ is a plane curve.*

The set of surfaces given by (7) and satisfying (5) and (6) is called the family of surfaces with common Smarandache TN asymptotic curve in Galilean space G_3. The functions $x(u,v_0)$, $y(u,v_0)$ and $z(u,v_0)$ can be given in two different forms:

Case 1. If we take

$$\begin{aligned}x(u,v) &= a(u)X(v), \\ y(u,v) &= b(u)Y(v), \\ z(u,v) &= c(u)Z(v),\end{aligned}$$

then the sufficient condition for which Smarandache TN curve of the curve $r(u)$ is an isoasymptotic on $\varphi(u,v)$ can be given by

$$\begin{aligned}X(v_0) &= Y(v_0) = Z(v_0) = 0, \\ a(u) &\neq 0 \quad \text{and} \quad \frac{dX(v_0)}{dv} \neq 0,\end{aligned} \tag{10}$$

where $a(u), b(u), c(u), X(v), Y(v)$ and $Z(v)$ are C^1 functions and $a(u), b(u)$ and $c(u)$ are not identically zero. Also $r(u)$ should be a plane curve.

Case 2. If we take

$$\begin{aligned} x(u,v) &= f(a(u)X(v)), \\ y(u,v) &= g(b(u)Y(v)), \\ z(u,v) &= h(c(u)Z(v)), \end{aligned}$$

then the sufficient condition for which Smarandache TN curve of the curve $r(u)$ is an isoasymptotic and a plane curve on $\varphi(u,v)$ can be expressed as

$$\begin{aligned} X(v_0) &= Y(v_0) = Z(v_0) = 0, \\ f(0) &= g(0) = h(0) = 0, \\ a(u) &\neq 0, \ \frac{dX(v_0)}{dv} \neq 0 \text{ and } f'(0) \neq 0, \end{aligned} \tag{11}$$

where $a(u), b(u), c(u), X(v), Y(v)$ and $Z(v)$ are C^1functions and $a(u), b(u)$ and $c(u)$ are not identically zero. Also, $r(u)$ should be a plane curve.

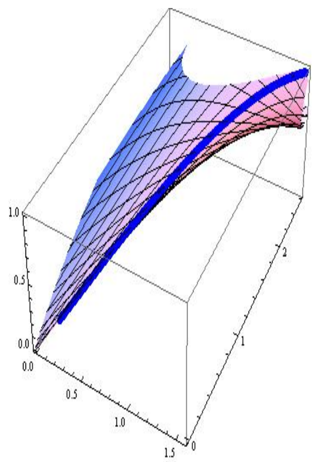

Figure 1 $\varphi(u,v)$ surface with curve $r(u)$.

Example 3.3 Let $r(u) = (u, u + \sin u, \sin u)$ be a curve and if we take $0 < u < \pi$, It is easy to show that

$$\begin{aligned} T(u) &= (1, 1 + \cos u, \cos u), \\ N(u) &= (0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}), \\ B(u) &= (0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}), \end{aligned}$$

where $\tau = 0$ is the torsion and $\kappa = \sqrt{2}\sin u$ and $u \neq \frac{k\pi}{2}$ $(k = 0, 2, \cdots, 2n.)$ is the curvature of the curve in G_3. We obtain the family of surfaces with this isoasymptotic curve. If we choose

$$x(u,v)=v,\ y(u,v)=\sin(uv),\ z(u,v)=u(1-\cos v),$$

and $v_0=0$ such that equation (10) is satisfied, a member of this family in G_3 is presented by (see Figure 1 for details)

$$\varphi(u,v)=\begin{pmatrix} u+v, u+v+\sin u+v\cos u+\dfrac{u-\sin(uv)-u\cos v}{\sqrt{2}}, \\ \sin u+v\cos u+\dfrac{u\cos v-u-\sin(uv)}{\sqrt{2}} \end{pmatrix}. \qquad (12)$$

If we take

$$x(u,v)=v,\ y(u,v)=\sin(uv),\ z(u,v)=u(1-\cos v)$$

and $v_0=0$, then (5) and (6) are satisfied. Thus, we obtain a member of the surfaces with this common Smarandache TN isoasymptotic curve as (see Figure 2 for details)

$$\varphi_{TN}(u,v)=\begin{pmatrix} 1+v, (1+v)(\cos u+1)+\frac{u-1-\sin(uv)-u\cos v}{\sqrt{2}}, \\ (1+v)\cos u+\frac{u\cos v-\sin(uv)-u-1}{\sqrt{2}} \end{pmatrix} \qquad (13)$$

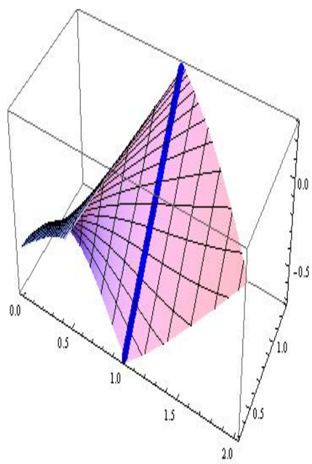

Figure 2 Surfaces $\varphi_{TN}(u,v)$ and its Smarandache TN asymptotic curve of $r(u)$.

3.2 Surfaces with a Common Smarandache TB AsymptoticCurve in Galilean Space G_3

Theorem 3.4 *Smarandache TB curve of the curve $r(u)$ is an isoasympotic on a surface $\varphi(u,v)$ if and only if the following relations are satisfied*

$$x(u,v_0)=y(u,v_0)=z(u,v_0)=0, \qquad (14)$$

$$\kappa\neq\tau \quad \text{and} \quad \frac{\partial x(u,v_0)}{\partial v}\neq 0 \qquad (15)$$

Proof By using (4), the parametric surface $\varphi(u,v)$ is defined by a given Smarandache TB curve of $r(u)$ as follows

$$\varphi(u,v) = \frac{T(u)+B(u)}{\|T(u)+B(u)\|} + \left[x(u,v)T(u) + y(u,v)N(u) + z(u,v)B(u)\right]. \tag{16}$$

Let $r(u)$ be a Smarandache TB curve on surface $\varphi(u,v)$. If Smarandache TB curve is a parametric curve on this surface, then there exists a parameter $v = v_0$ such that

$$r(u) = \frac{T(u)+B(u)}{\|T(u)+B(u)\|},$$

that is

$$x(u,v_0) = y(u,v_0) = z(u,v_0) = 0.$$

By using (16),

$$\begin{aligned}\varphi_u &= \frac{\partial x(u,v)}{\partial u}T(u) + \left(\kappa - \tau + x(u,v)\kappa + \frac{\partial y(u,v)}{\partial u} - z(u,v)\tau\right)N(u) \\ &\quad + \left(y(u,v)\tau + \frac{\partial z(u,v)}{\partial u}\right)B(u)\end{aligned}$$

and

$$\varphi_v = \frac{\partial x(u,v)}{\partial v}T(u) + \frac{\partial y(u,v)}{\partial v}N(u) + \tau + \frac{\partial z(u,v)}{\partial v}B(u).$$

Using (8), the normal $\eta(u,v)$ can be expressed as

$$\begin{aligned}\eta(u,v) &= \left[-\frac{\partial x(u,v)}{\partial u}\frac{\partial z(u,v)}{\partial v} + \left(y(u,v)\tau + \frac{\partial z(u,v)}{\partial u}\right)\frac{\partial x(u,v)}{\partial v}\right]N(u) \\ &\quad + \left[\frac{\partial x(u,v)}{\partial u}\frac{\partial y(u,v)}{\partial v} - \left(\kappa - \tau + x(u,v)\kappa + \frac{\partial y(u,v)}{\partial u} - z(u,v)\tau\right)\frac{\partial x(u,v)}{\partial v}\right]B(u)\end{aligned}$$

and from (15), we have

$$\eta(u,v_0) = \left[(\tau - \kappa)\frac{\partial x(u,v_0)}{\partial v}\right]B(u). \tag{17}$$

We know that $r(u)$ is an asymptotic curve if and only if

$$(\tau - \kappa)\frac{\partial x(u,v_0)}{\partial v} \neq 0.$$

That is

$$\kappa \neq \tau \quad \text{and} \quad \frac{\partial x(u,v_0)}{\partial v} \neq 0. \tag{18}$$

This completes the proof. □

The set of surfaces given by (16) and satisfying (14) and (15) is called the family of surfaces with common Smarandache TB asymptotic curve in Galilean space G_3. The marching-scale functions can be given in two different forms:

Case 1. If we take

$$\begin{aligned} x(u,v) &= a(u)X(v), \\ y(u,v) &= b(u)Y(v), \\ z(u,v) &= c(u)Z(v), \end{aligned}$$

then the sufficient condition for which Smarandache TB curve of the curve $r(u)$ is an isoasymptotic on the surface $\varphi(u,v)$ can be expressed as

$$\begin{aligned} X(v_0) &= Y(v_0) = Z(v_0) = 0 \text{ and } \kappa \neq \tau, \\ a(u) &\neq 0 \quad \text{and} \quad \frac{dX(v_0)}{dv} \neq 0, \end{aligned} \tag{19}$$

where $a(u), b(u), c(u), X(v), Y(v)$ and $Z(v)$ are C^1functions and $a(u), b(u)$ and $c(u)$ are not identically zero.

Case 2. If we take

$$\begin{aligned} x(u,v) &= f(a(u)X(v)), \\ y(u,v) &= g(b(u)Y(v)), \\ z(u,v) &= h(c(u)Z(v)), \end{aligned}$$

then the sufficient condition for which Smarandache TB curve of the curve $r(u)$ is an isoasymptotic on the surface $\varphi(u,v)$ can be expressed as

$$\begin{aligned} X(v_0) &= Y(v_0) = Z(v_0) = 0 \quad \text{and} \quad \kappa \neq \tau, \\ f(0) &= g(0) = h(0) = 0, \\ a(u) &\neq 0, \ \frac{dX(v_0)}{dv} \neq 0, \ f'(0) \neq 0, \end{aligned} \tag{20}$$

where $a(u), b(u), c(u), X(v), Y(v)$ and $Z(v)$ are C^1 functions and $a(u), b(u)$ and $c(u)$ are not identically zero.

Example 3.5 Let $r(u) = (u, \ 2\sin u, 2\cos u)$ be a curve. It is easy to show that

$$\begin{aligned} T(u) &= (1, 2\cos u, -2\sin u), \\ N(u) &= (0, -\sin u, -\cos u), \\ B(u) &= (0, \cos u, -\sin u), \end{aligned}$$

where $\tau = -1$ is the torsion and $\kappa = 2$ is the curvature of the curve in G_3. We will give the family of surfaces with this isoasymptotic curve. If we choose

$$x(u,v) = v, \ y(u,v) = \cos v - 1, \ z(u,v) = \sin(uv)$$

and $v_0 = 0$ such that equation (19) is satisfied, a member of this family in G_3 is obtained by

(See Figure 3 for details)

$$\begin{aligned}\varphi(u,v) &= (u+v, 2v\cos u+\sin u(3-\cos v)+\cos u\sin(uv),\\ &\quad \cos u(3-\cos v)-2v\sin u-\sin u\sin(uv)). \qquad (21)\end{aligned}$$

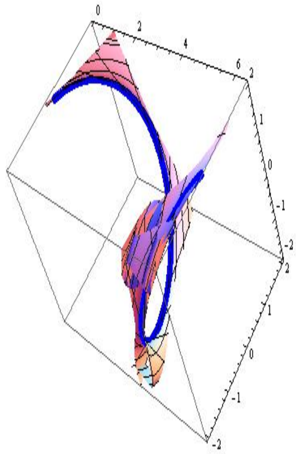

Figure 3 $\varphi(u,v)$ surface and the curve $r(u)$.

If we take

$$x(u,v)=v,\ \ y(u,v)=\cos v-1,\ \ z(u,v)=\sin(uv)$$

and $v_0=0$ then the equality (14) and (15) are satisfied. Thus, we obtain a member of the surfaces with this common Smarandache TB isoasymptotic curve as (Figure 4)

$$\begin{aligned}\varphi_{TB}(u,v) &= (v+1, \cos u(2v+\sin(uv)+3)+\sin u(1-\cos v),\\ &\quad \cos u(1-\cos v)-\sin u(2v+\sin(uv)+3)). \qquad (22)\end{aligned}$$

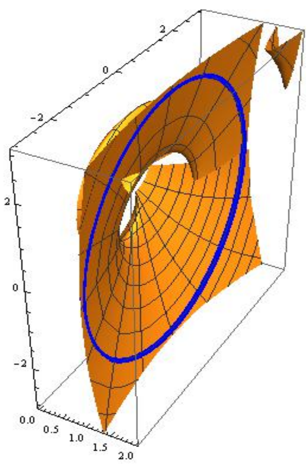

Figure 4 $\varphi_{TB}(u,v)$ surface and its Smarandache TB asymptotic curve of $r(u)$.

3.3 Surfaces with Common Smarandache TNB Asymptotic Curve in Galilean Space G_3

Theorem 3.6 *Smarandache TNB curve of the curve $r(u)$ is an isoasympotic on a surface $\varphi(u,v)$ if and only if the following conditions are satisfied:*

$$x(u,v_0) = y(u,v_0) = z(u,v_0) = 0, \tag{23}$$

$$\tau = 0 \text{ and } \frac{\partial x(u,v_0)}{\partial v} \neq 0 \tag{24}$$

Proof From (4), a parametric surface $\varphi(u,v)$ is defined by a given Smarandache TNB curve of $r(u)$ as follows

$$\varphi(u,v) = \frac{T(u)+N(u)+B(u)}{\|T(u)+N(u)+B(u)\|} + \left[x(u,v)T(u) + y(u,v)N(u) + z(u,v)B(u)\right]. \tag{25}$$

Let $r(u)$ be a Smarandache TNB curve on surface $\varphi(u,v)$. If Smarandache TNB curve is an parametric curve on this surface, then there exists a parameter $v = v_0$ such that

$$r(u) = \frac{T(u)+N(u)+B(u)}{\|T(u)+N(u)+B(u)\|},$$

that is

$$x(u,v_0) = y(u,v_0) = z(u,v_0) = 0.$$

From

$$\begin{aligned}\varphi_u &= \frac{\partial x(u,v)}{\partial u}T(u) + \left(\kappa - \tau + x(u,v)\kappa + \frac{\partial y(u,v)}{\partial u} - z(u,v)\tau\right)N(u)\\ &\quad + \left(y(u,v)\tau + \frac{\partial z(u,v)}{\partial u} + \tau\right)B(u)\end{aligned}$$

and

$$\varphi_v = \frac{\partial x(u,v)}{\partial v}T(u) + \frac{\partial y(u,v)}{\partial v}N(u) + \frac{\partial z(u,v)}{\partial v}B(u).$$

Using (8), the normal $\eta(u,v)$ can be written as

$$\begin{aligned}\eta(u,v) &= \left[-\frac{\partial x(u,v)}{\partial u}\frac{\partial z(u,v)}{\partial v} + \left(y(u,v)\tau + \frac{\partial z(u,v)}{\partial u} + \tau\right)\frac{\partial x(u,v)}{\partial v}\right]N(u)\\ &\quad + \left[\frac{\partial x(u,v)}{\partial u}\frac{\partial y(u,v)}{\partial v} - \left(\kappa - \tau + x(u,v)\kappa + \frac{\partial y(u,v)}{\partial u} - z(u,v)\tau\right)\frac{\partial x(u,v)}{\partial v}\right]B(u)\end{aligned}$$

and from (23), we have

$$\eta(u,v_0) = \left[\frac{\partial x(u,v_0)}{\partial v}\tau\right]N(u) + \left[(\tau-\kappa)\frac{\partial x(u,v_0)}{\partial v}\right]B(u). \tag{26}$$

We know that $r(u)$ is an asymptotic curve if and only if

$$\frac{\partial x(u,v_0)}{\partial v}\tau = 0$$

and

$$(\tau-\kappa)\frac{\partial x(u,v_0)}{\partial v} \neq 0.$$

Consequently, we have

$$\frac{\partial x(u,v_0)}{\partial v} \neq 0, \tau = 0. \tag{27}$$

This completes the proof. □

Corollary 3.7 *Smarandache TNB curve of the curve $r(u)$ is an isoasympotic if and only if $r(u)$ is a plane curve.*

The set of surfaces given by (25) and satisfying (23) and (24) is called the family of surfaces with common Smarandache TNB asymptotic curve in Galilean space G_3. The marching-scale functions $x(u,v_0)$, $y(u,v_0)$ and $z(u,v_0)$ can be given two different forms:

Case 1. If we take

$$\begin{aligned} x(u,v) &= a(u)X(v), \\ y(u,v) &= b(u)Y(v), \\ z(u,v) &= c(u)Z(v), \end{aligned}$$

then the sufficient condition for which Smarandache TNB curve of the curve $r(u)$ is isoasymptotic on the surface $\varphi(u,v)$ can be expressed as

$$\begin{aligned} X(v_0) &= Y(v_0) = Z(v_0) = 0, \\ a(u) &\neq 0, \ \frac{dX(v_0)}{dv} \neq 0, \end{aligned} \tag{28}$$

where $a(u), b(u), c(u), X(v), Y(v)$ and $Z(v)$ are C^1 functions and $a(u), b(u)$ and $c(u)$ are not identically zero. Also $r(u)$ should be a plane curve.

Case 2. If we take

$$\begin{aligned} x(u,v) &= f(a(u)X(v)), \\ y(u,v) &= g(b(u)Y(v)), \\ z(u,v) &= h(c(u)Z(v)), \end{aligned}$$

then the sufficient condition for which Smarandache TNB curve of the curve $r(u)$ is isoasymptotic on the surface $\varphi(u,v)$ can be expressed as

$$\begin{aligned} X(v_0) &= Y(v_0) = Z(v_0) = 0, \\ f(0) &= g(0) = h(0) = 0, \\ a(u) &\neq 0, \ \frac{dX(v_0)}{dv} \neq 0, \ f'(0) \neq 0, \end{aligned} \tag{29}$$

where $a(u), b(u), c(u), X(v), Y(v)$ and $Z(v)$ are C^1 functions and $a(u), b(u)$ and $c(u)$ are not identically zero. Also $r(u)$ should be a plane curve.

Example 3.8 Let $r(u) = (u, \ \cos u, u + \cos u)$ be a curve and if we take $0 \leq u \leq \pi$, It is easy to show that

$$\begin{aligned} T(u) &= (1, -\sin u, 1 - \sin u), \\ N(u) &= (0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}), \\ B(u) &= (0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}), \end{aligned}$$

where $\tau = 0$, $\kappa = \sqrt{2}\cos u$ and $u \neq \frac{k\pi}{2}$ $(k = 1, 3, \cdots, 2n-1.)$ in G_3. We will give the family of surfaces with this isoasymptotic curve. If we choose

$$x(u, v) = v, \ y(u, v) = \sin v, \ z(u, v) = e^{uv} - 1.$$

and $v_0 = 0$ such that equation (28) is satisfied, a member of this family in G_3 is obtained by (see Figure 5 for details)

$$\varphi(u, v) = \left(\begin{array}{c} u + v, \cos u - v \sin u + \dfrac{e^{uv} - 1 - \sin v}{\sqrt{2}}, \\ u + v + \cos u - v \sin u + \dfrac{1 - e^{uv} - \sin v}{\sqrt{2}} \end{array} \right).$$

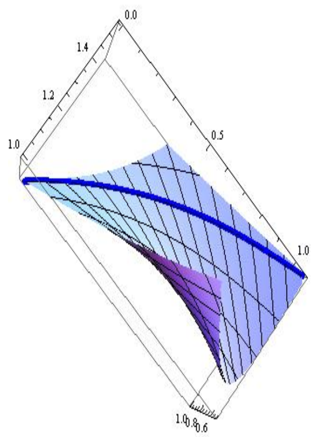

Figure 5 $\varphi(u, v)$ surface with the curve $r(u)$.

If we take

$$x(u,v) = v,\ \ y(u,v) = \sin v,\ \ z(u,v) = e^{uv} - 1$$

and $v_0 = 0$ then (23) and (24) are satisfied. Thus, we obtain a member of the surfaces with this common Smarandache TNB isoasymptotic curve as (see Figure 6 for details)

$$\varphi_{TNB}(u,v) = \begin{pmatrix} v+1, \dfrac{e^{uv} - 1 - \sin v}{\sqrt{2}} - v\sin u - \sin u, \\ v - v\sin u + \dfrac{1 - e^{uv} - \sin v}{\sqrt{2}} + 1 - \sqrt{2} - \sin u \end{pmatrix}.$$

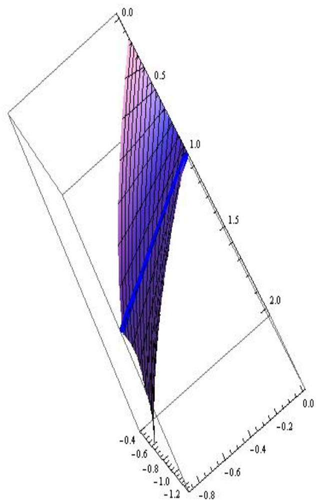

Figure 6 $\varphi_{TNB}(u,v)$ surface and its Smarandache TNB asymptotic curve of $r(u)$.

References

[1] H.S. Abdel-Aziz, M. K. Saad, Smarandache curves of some special curves in the Galilean, *Infinite Study*,(2015).

[2] G.S. Atalay and E. Kasap, Surfaces family with common Smarandache asymptotic curve, *Bolyai Soc. Para. Math.*, Vol.34, 1(2016), 9-20.

[3] E. Bayram, F. Güler and E. Kasap, Parametric representation of a surface pencil with a common asymptotic curve, *Comput. Aided Des.*, Vol.44,(7)(2012), 637-643.

[4] G. E. Farin, Curves and surfaces for CAGD: A practical guide, *Computer Graphics and Geometric Modeling*, Morgan Kaufmann, 2002.

[5] W. Klingenberg, *A Course in Differential Geometry*, Springer, New York, 1978.

[6] E. Molnar, The projective interpretation of the eight 3-dimensional homogeneous geometries, *Beitr. Algebra Geom.*, 38(1997), 261–288.

[7] O. Röschel, *Die Geometrie des Galileischen Raumes*, Habilitationsschrift, Leoben, 1984.

[8] Z. M. Sipus, Ruled Weingarten surfaces in Galilean space, Period. Math. Hungvol, Vol.56, 2(2008), 213–225.

[9] D. J. Struik, *Lectures on Classical Differential Geometry*, Dover, New York, 1988.

[10] M. Turgut and S. Yilmaz, Smarandache curves in Minkowski space-time, *Int. J. Math. Combin.*, 3(2008), 51-55.

[11] I. M. Yaglom, *A Simple non-Euclidean Geometry and Its Physical Basis*, Springer-Verlag: New York Inc, 1979.

[12] D. W. Yoon, Z. K. Yüzbasi and M. Bektas, An approach for surfaces using an asymptotic curve in Lie group, *J. Adv. Phys.*, Vol.6, 4(2017), 586-590.

[13] Z. K. Yüzbasi, On a family of surfaces with common asymptotic curve in the Galilean space G^3, *J. Nonlinear Sci. Appl.*, 9(2016), 518-523.

[14] G. J. Wang, K. Tang, C. L. Tai, Parametric representation of a surface pencil with a common spatial geodesic, *Comput. Aided Des.*, 36(2004), 447–459.