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Super F-Centroidal Mean Graphs

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Abstract: Let G be a graph and $f : V(G) \rightarrow \{1, 2, 3, \ldots, p + q\}$ be an injection. For each uv, the induced edge labeling f^* is defined as

$$f^*(uv) = \left\lfloor \frac{2\left[f(u)^2 + f(u)f(v) + f(v)^2\right]}{3\left[f(u) + f(v)\right]} \right\rfloor.$$

Then f is called a super F-centroidal mean labeling if $f(V(G)) \cup \{f^*(uv) : uv \in E(G)\} = \{1, 2, 3, \ldots, p+q\}$. A graph that admits a super F-centroidal mean labeling is called a super F-centroidal mean graph. In this paper, the super F-centroidal meanness of some standard graphs have been studied.

Key Words: F-centroidal mean graph, super F-centroidal mean labeling, Smarandachely super F-centroidal mean labeling, super F-centroidal mean graph.

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§1. Introduction

Throughout this paper, by a graph we mean a finite, undirected and simple graph. Let $G(V, E)$ be a graph with p vertices and q edges. For notations and terminology, we follow [7]. For a detailed survey on graph labeling, we refer [6].

Path on n vertices is denoted by P_n and a cycle on n vertices is denoted by C_n. A star graph S_n is the complete bipartite graph $K_{1,n}$. The union $G_1 \cup G_2$ of any two graphs G_1 and G_2 with disjoint vertex sets, has vertex set $V(G_1) \cup V(G_2)$ and edge set $E(G_1) \cup E(G_2)$. The middle graph $M(G)$ of a graph G is the graph whose vertex set is $\{v : v \in V(G)\} \cup \{e : e \in E(G)\}$ and the edge set is $\{e_1e_2 : e_1, e_2 \in E(G)$ and e_1 and e_2 are adjacent edges of $G\} \cup \{ve : v \in V(G), e \in E(G)$ and e is incident with $v\}$. The graph $G \circ S_m$ is obtained from G by attaching m pendant vertices to each vertex of G. A Twig $TW(P_n), n \geq 3$ is a graph obtained from a path by attaching exactly two pendant vertices to each internal vertices of the path P_n. A subdivision of a graph G, denoted by $S(G)$, is a graph obtained by subdividing edge of G by a vertex. An arbitrary subdivision of a graph G is a graph obtained from G by a sequence of elementary subdivisions forming edges into paths through new vertices of degree 2. Square of a graph G,

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denoted by G^2, has the vertex set as in G and two vertices are adjacent in G^2 if they are at a distance either 1 or 2 apart in G. The baloon of a graph G, $P_n(G)$ is the graph obtained from G by identifying an end vertex of P_n at a vertex of G. The graph $P_n(C_m)$ is called a dragon.

The concept of geometric mean labeling [1] and super geometric mean labeling [2] were introduced by Durai Baskar et al. and studied for some standard graphs. Arockiaraj et al. introduced the concept of F-root square labeling [3] and super F-root square labeling [4]. The concept of F-centroidal mean labeling [5] was introduced and developed its meanness for some standard graphs.

Arockiaraj et al. [5], defined the F-centroidal mean labeling as follows:

A function f is called an F-centroidal mean labeling of a graph $G(V,E)$ with p vertices and q edges if $f : V(G) \rightarrow \{1, 2, 3, \cdots, q+1\}$ is injective and the induced function $f^* : E(G) \rightarrow \{1, 2, 3, \cdots, q\}$ defined as

$$f^*(uv) = \left\lfloor \frac{2\left[f(u)^2 + f(u)f(v) + f(v)^2\right]}{3\left[f(u) + f(v)\right]} \right\rfloor$$

is bijective for all $uv \in E(G)$. A graph that admits an F-centroidal mean labeling is called an F-centroidal mean graph. Motivated by the works of so many authors in the area of graph labeling, we introduced a new type of labeling called a super F-centroidal mean labeling.

Let G be a graph and $f : V(G) \rightarrow \{1, 2, 3, \cdots, p+q\}$ be an injection. For each uv, the induced edge labeling f^* is defined as

$$f^*(uv) = \left\lfloor \frac{2\left[f(u)^2 + f(u)f(v) + f(v)^2\right]}{3\left[f(u) + f(v)\right]} \right\rfloor .$$

Then f is called a super F-centroidal mean labeling if $f(V(G)) \cup \{f^*(uv) : uv \in E(G)\} = \{1, 2, 3, \cdots, p+q\}$. A graph that admits a super F-centroidal mean labeling is called a super F-centroidal mean graph. Generally, let $C \subset \{1, 2, 3, \cdots, p+q\}$. If $f(V(G)) \bigcup \{f^*(uv) : uv \in E(G)\} = \{1, 2, 3, \cdots, p+q\} \setminus C$, such a f is called a Smarandachely super F-centroidal mean labeling on C. Clearly, if $C = \emptyset$, a Smarandachely super F-centroidal mean labeling on C is nothing else but the super F-centroidal mean labeling on G.

A super F-centroidal mean labeling of the graph C_4 is shown in Figure 1.

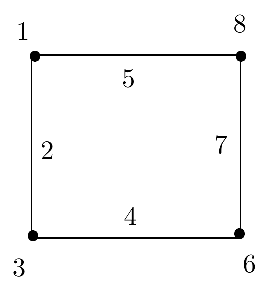

Figure 1 A super F-centroidal mean labeling of C_4

In this paper, we have studied the super F-centroidal meanness of some standard graphs.

§2. Main Results

Theorem 2.1 *A union of any number of paths is a super F-centroidal mean graph.*

Proof Let the graph G be the union of k paths. Let $\{v_j^{(i)} : 1 \leq j \leq p_i\}$ be the vertices of the i^{th} path P_{p_i} with $p_i \geq 2$ and $1 \leq i \leq k$. Define $f : V(G) \to \left\{1, 2, 3, \cdots, \sum\limits_{i=1}^{k} 2p_i - k\right\}$ as follows:

$$f(v_j^{(1)})) = 2j-1, \text{ for } 1 \leq j \leq p_1 \text{ and}$$
$$f\left(v_j^{(i)}\right) = f(v_{p_{i-1}}^{(i-1)}) + 2j - 1, \text{ for } 2 \leq i \leq k \text{ and } 1 \leq j \leq p_i.$$

Then the induced edge labeling f^* is obtained as follows:

$$f^*(v_j^{(1)} v_{j+1}^{(1)}) = 2j, \text{ for } 1 \leq j \leq p_1 - 1 \text{ and}$$
$$f^*\left(v_j^{(i)} v_{j+1}^{(i)}\right) = f(v_{p_{i-1}}^{(i-1)}) + 2j, \text{ for } 2 \leq i \leq k \text{ and } 1 \leq j \leq p_1 - 1.$$

Hence, f is a super F-centroidal mean labeling of G. Thus the graph G is a super F-centroidal mean graph. □

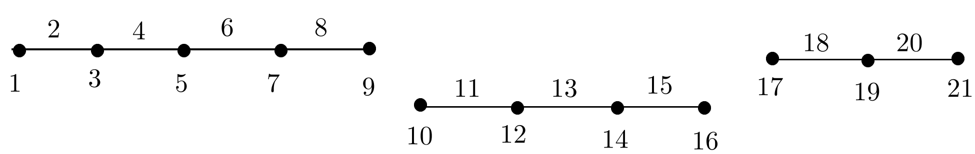

Figure 2 A super F-centroidal mean labeling of union of P_5, P_4 and P_3

Corollary 2.2 *Every path P_n is a super F-centroidal mean graph, for $n \geq 1$.*

Theorem 2.3 *The middle graph $M(P_n)$ of a path P_n is a super F-centroidal mean graph, for $n \geq 4$.*

Proof Let $V(P_n) = \{v_1, v_2, v_3, \cdots, v_n\}$ and $E(P_n) = \{e_i = v_i v_{i+1} : 1 \leq i \leq n-1\}$ be the vertex set and edge set of the path P_n. Then,

$$V(M(P_n)) = \{v_1, v_2, v_3, \ldots, v_n, e_1, e_2, e_3, \ldots, e_{n-1}\} \text{ and}$$
$$E(M(P_n)) = \{v_i e_i, e_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{e_i e_{i+1} : 1 \leq i \leq n-2\}.$$

Define $f : V(M(P_n)) \to \{1, 2, 3, \cdots, 5n-5\}$ as follows:

$$f(v_i) = 5i - 4, \text{ for } 1 \leq i \leq n-1,$$
$$f(v_n) = 5n - 5 \text{ and}$$
$$f(e_i) = 5i - 2, \text{ for } 1 \leq i \leq n-1.$$

Then, the induced edge labeling f^* is obtained as follows:

$$f^*(v_ie_i) = 5i - 3, \text{ for } 1 \leq i \leq n-1$$
$$f^*(e_iv_{i+1}) = 5i - 1, \text{ for } 1 \leq i \leq n-1$$
$$f^*(e_ie_{i+1}) = 5i, \text{ for } 1 \leq i \leq n-2.$$

Hence f is a super F-centroidal mean labeling of $M(P_n)$. Thus the middle graph $M(P_n)$ of a path P_n is a super F-centroidal mean graph, for $n \geq 4$. □

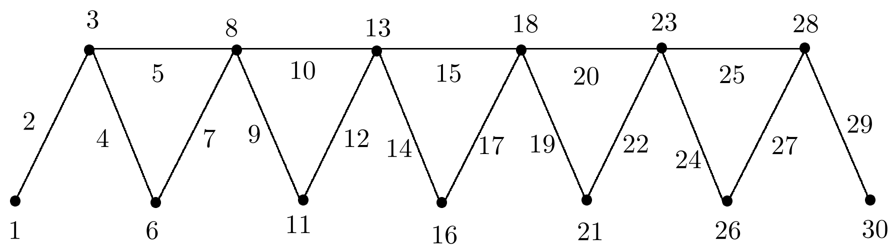

Figure 3 A super F-centroidal mean labeling of $M(P_7)$

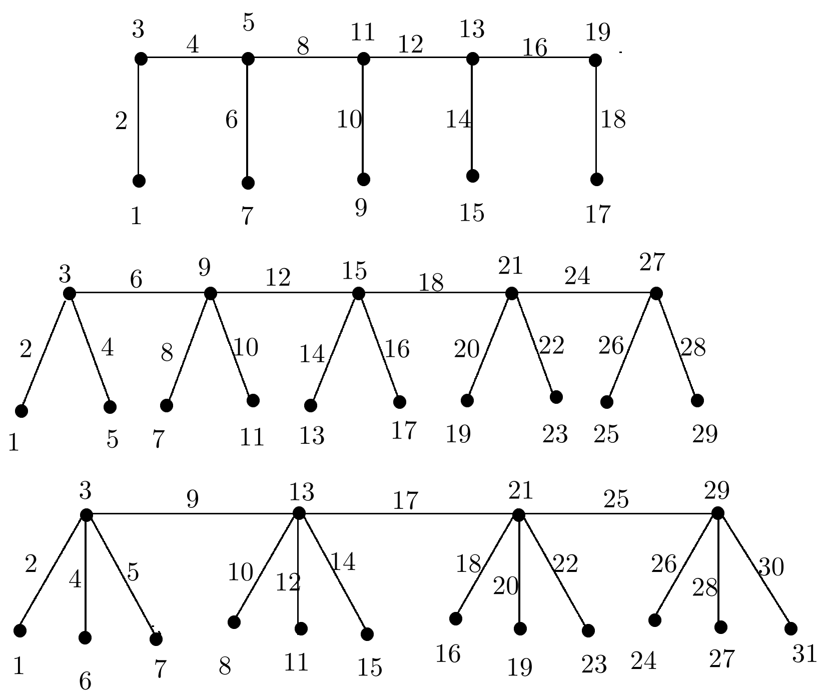

Figure 4. A super F-centroidal mean labeling of $P_5 \circ S_1$, $P_6 \circ S_2$ and $P_4 \circ S_3$

Theorem 2.4 *The graph $P_n \circ S_m$ is a super F-centroidal mean graph, for $n \geq 1$ and $m \leq 3$.*

Proof Let $u_1, u_2, \cdots, u_n$ be the vertices of the path P_n and $v_1^{(i)}, v_2^{(i)}, \ldots, v_m^{(i)}$ be the pendant vertices attached at each vertex u_i of the path P_n, for $1 \leq i \leq n$.

Case 1. $m = 1$.

Define $f : V(P_n \circ S_1) \rightarrow \{1, 2, 3, \cdots, 4n-1\}$ as follows:

$$f(u_i) = \begin{cases} 4i-1, & 1 \leq i \leq n \text{ and } i \text{ is odd} \\ 4i-3, & 2 \leq i \leq n \text{ and } i \text{ is even and} \end{cases}$$

$$f(v_1^{(i)}) = \begin{cases} 4i-3, & 1 \leq i \leq n \text{ and } i \text{ is odd} \\ 4i-1, & 2 \leq i \leq n \text{ and } i \text{ is even.} \end{cases}$$

Then, the induced edge labeling f^* is obtained as follows:

$$f^*(u_i u_{i+1}) = 4i, \text{ for } 1 \leq i \leq n-1 \text{ and}$$
$$f^*(v_1^{(i)} u_i) = 4i-2, \text{ for } 1 \leq i \leq n.$$

Case 2. $m = 2$.

Define $f : V(P_n \circ S_2) \rightarrow \{1, 2, 3, \cdots, 6n-1\}$ as follows:

$$f(u_i) = 6i-3, \text{ for } 1 \leq i \leq n,$$
$$f(v_1^{(i)}) = 6i-5, \text{ for } 1 \leq i \leq n \text{ and}$$
$$f(v_2^{(i)}) = 6i-1, \text{ for } 1 \leq i \leq n.$$

Then, the induced edge labeling f^* is obtained as follows:

$$f^*(u_i u_{i+1}) = 6i, \text{ for } 1 \leq i \leq n-1,$$
$$f^*(v_1^{(i)} u_i) = 6i-4, \text{ for } 1 \leq i \leq n \text{ and}$$

$$f^*(v_2^{(i)} u_i) = 6i-2, \text{ for } 1 \leq i \leq n.$$

Case 3. $m = 3$.

Define $f : V(P_n \circ S_3) \rightarrow \{1, 2, 3, \cdots, 8n-1\}$ as follows:

$$f(u_i) = \begin{cases} 3, & i = 1 \\ 8i-3, & 2 \leq i \leq n, \end{cases}$$

$$f(v_1^{(i)}) = \begin{cases} 1, & i = 1 \\ 8i-8, & 2 \leq i \leq n, \end{cases}$$

$$f(v_2^{(i)}) = \begin{cases} 6, & i = 1 \\ 8i-5, & 2 \leq i \leq n \text{ and} \end{cases}$$

$$f(v_3^{(i)}) = 8i-1, \text{ for } 1 \leq i \leq n.$$

Then, the induced edge labeling f^* is obtained as follows:

$$f^*(u_iu_{i+1}) = 8i+1, \text{ for } 1 \le i \le n-1,$$
$$f^*(v_1^{(i)}u_i) = 8i-6, \text{ for } 1 \le i \le n,$$
$$f^*(v_2^{(i)}u_i) = 8i-4, \text{ for } 1 \le i \le n \text{ and}$$
$$f^*(v_3^{(i)}u_i) = \begin{cases} 5, & i=1 \\ 8i-2, & 2 \le i \le n. \end{cases}$$

In each case, f is a super F-centroidal mean labeling of $P_n \circ S_m$. Thus the graph $P_n \circ S_m$ is a super F-centroidal mean graph, for $n \ge 1$ and $m \le 3$. □

Theorem 2.5 *The twig graph $TW(P_n)$ of the path P_n is a super F-centroidal mean graph, only when $n \ge 4$.*

Proof Let $v_1, v_2, v_3, \cdots, v_n$ be the vertices of the path P_n and $u_1^{(i)}, u_2^{(i)}$ be the pendant vertices at each vertex v_i, for $2 \le i \le n-1$.

Assume that $n \ge 4$.

Define $f : V(TW(P_n)) \to \{1, 2, 3, \cdots, 6n-9\}$ as follows:

$$f(v_i) = \begin{cases} 2i-1, & 1 \le i \le 2 \\ 6i-7, & 3 \le i \le n-1 \\ 6i-9, & i = n, \end{cases}$$
$$f(u_1^{(i)}) = \begin{cases} 6, & i=2 \\ 6i-9, & 3 \le i \le n-1 \end{cases} \text{ and}$$
$$f(u_2^{(i)}) = \begin{cases} 8, & i=2 \\ 6i-5, & 3 \le i \le n-2 \\ 6i-4, & i = n-1. \end{cases}$$

Then, the induced edge labeling f^* is obtained as follows:

$$f^*(v_iv_{i+1}) = \begin{cases} 5i-3, & 1 \le i \le 2 \\ 6i-4, & 3 \le i \le n-2, \end{cases}$$
$$f^*(v_{n-1}v_n) = 6n-11,$$
$$f^*(v_iu_1^{(i)}) = 6i-8, \text{ for } 2 \le i \le n-1 \text{ and}$$
$$f^*(v_iu_2^{(i)}) = \begin{cases} 5, & i=2 \\ 6i-6, & 3 \le i \le n-1. \end{cases}$$

Hence f is a super F-centroidal mean labeling of $TW(P_n)$. Thus the twig graph $TW(P_n)$ is a super F-centroidal mean graph, for $n \ge 4$. □

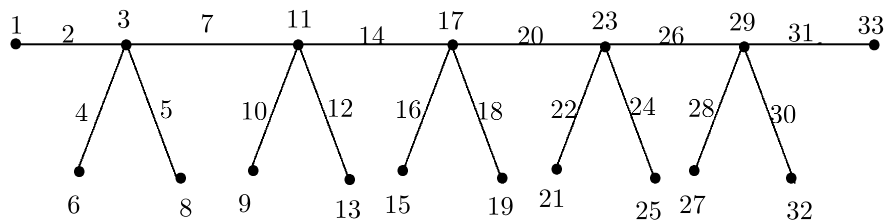

Figure 5. A super F-centroidal mean labeling of $TW(P_7)$

Theorem 2.6 *The graph $[P_n; S_1]$ is a super F-centroidal mean graph, for $n \geq 1$.*

Proof Let $u_1, u_2, u_3, \cdots, u_n$ be the vertices of the path P_n and $v_1^{(i)}, v_2^{(i)}, v_3^{(i)}, \cdots, v_{m+1}^{(i)}$ be the vertices of the star graph S_m such that $v_1^{(i)}$ is the central vertex of the star graph $S_m, 1 \leq i \leq n$.

Assume that $m = 1$. Define $f : V([P_n; S_1]) \to \{1, 2, 3, \cdots, 6n-1\}$ as follows:

$$f(u_i) = 6i - 1, \text{ for } 1 \leq i \leq n,$$
$$f(v_1^{(i)}) = 6i - 3, \text{ for } 1 \leq i \leq n \text{ and}$$
$$f(v_2^{(i)}) = \begin{cases} 1, & i = 1 \\ 6i - 6, & 2 \leq i \leq n. \end{cases}$$

Then, the induced edge labeling f^* is obtained as follows:

$$f^*(u_i u_{i+1}) = 6i + 2, \text{ for } 1 \leq i \leq n-1,$$
$$f^*(u_i v_1^{(i)}) = 6i - 2, \text{ for } 1 \leq i \leq n \text{ and}$$
$$f^*(v_1^{(i)} v_2^{(i)}) = \begin{cases} 2, & i = 1 \\ 6i - 5, & 2 \leq i \leq n. \end{cases}$$

Hence f is a super F-centroidal mean labeling of $[P_n; S_1]$. Thus the graph $[P_n; S_1]$ is a super F-centroidal mean graph, for $n \geq 1$. □

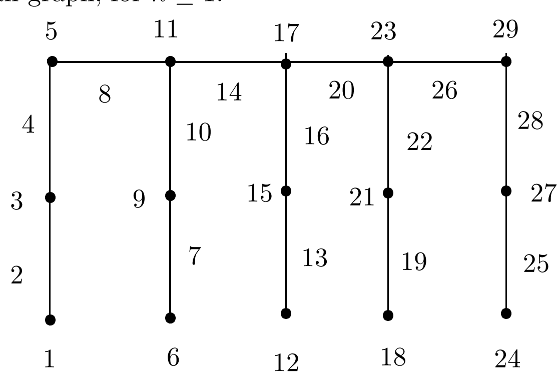

Figure 6 A super F-centroidal mean labeling of $[P_5; S_1]$

Theorem 2.7 *Arbitrary subdivision of $K_{1,3}$ is a super F-centroidal mean graph.*

Proof Let G be the graph of arbitrary subdivision of $K_{1,3}$. Let v_0, v_1, v_2 and v_3 be the vertices of $K_{1,3}$ in which v_0 is the central vertex and v_1, v_2 and v_3 are the pendent vertices of $K_{1,3}$. Let the edges v_0v_1, v_0v_2 and v_0v_3 of S_3 be subdivided by p_1, p_2 and p_3 number of vertices respectively.

Let $v_0, v_1^{(1)}, v_2^{(1)}, v_3^{(1)}, \cdots, v_{p_1+1}^{(1)} = v_1, v_0, v_1^{(2)}, v_2^{(2)}, v_3^{(2)}, \cdots, v_{p_2+1}^{(2)} = v_2$ and $v_0, v_1^{(3)}, v_2^{(3)}, v_3^{(3)}, \cdots, v_{p_3+1}^{(3)} (= v_3)$ be the vertices of G and $v_0 = v_0^{(i)}$ for $1 \leq i \leq 3$.

Let $e_j^{(i)} = v_{j-1}^{(i)} v_j^{(i)}$ for $1 \leq j \leq p_i + 1$ and $1 \leq i \leq 3$ be the edges with G and it has $p_1 + p_2 + p_3 + 4$ vertices and $p_1 + p_2 + p_3 + 3$ edges with $p_1 \leq p_2 \leq p_3$.

Case 1. $p_1 = p_2,\ p_1 \geq 1$ and $p_3 \geq 3$.

Define $f : V(G) \to \{1, 2, 3, \cdots, 2(p_1 + p_2 + p_3) + 7\}$ as follows:

$$f(v_0) = 2(p_1 + p_2) + 5,$$
$$f(v_j^{(1)}) = \begin{cases} 2(p_1 + p_2), & j = 1 \\ 2(p_1 + p_2) + 5 - 4j, & 2 \leq j \leq p_1 + 1, \end{cases}$$
$$f(v_j^{(2)}) = \begin{cases} 2(p_1 + p_2) + 7 - 4j, & 1 \leq j \leq 2 \\ 2(p_1 + p_2) + 6 - 4j, & 3 \leq j \leq p_2 + 1 \text{ and} \end{cases}$$
$$f^*(v_j^{(3)}) = 2(p_1 + p_2) + 5 + 2j \text{ for } 1 \leq j \leq p_3 + 1.$$

Then, the induced edge labeling f^* is obtained as follows:

$$f^*(v_0v_1^{(i)}) = 2(p_1 + p_2) + 2i, \text{ for } 1 \leq i \leq 3,$$
$$f^*(v_j^{(1)}v_{j+1}^{(1)}) = \begin{cases} 2(p_1 + p_2) - 2, & j = 1 \\ 2(p_1 + p_2) + 3 - 4j, & 2 \leq j \leq p_1, \end{cases}$$
$$f^*(v_j^{(2)}v_{j+1}^{(2)}) = \begin{cases} 2(p_1 + p_2) + 1, & j = 1 \\ 2(p_1 + p_2) + 4 - 4j, & 2 \leq j \leq p_2 \ \textit{and} \end{cases}$$
$$f^*(v_j^{(3)}v_{j+1}^{(3)}) = 2(p_1 + p_2) + 6 + 2j, \text{ for } 1 \leq j \leq p_3.$$

Case 2. $p_1 < p_2$.

Define $f : V(G) \to \{1, 2, 3, \cdots, 2(p_1 + p_2 + p_3) + 7\}$ as follows:

$$f(v_0) = 2(p_1 + p_2) + 5,$$

$$f(v_j^{(1)}) = \begin{cases} 2(p_1+p_2)+3, & j=1 \\ 2(p_1+p_2)+8-4j, & 2 \le j \le p_1+1, \end{cases}$$

$$f(v_j^{(2)}) = \begin{cases} 2(p_1+p_2)+3-4j, & 1 \le j \le p_1 \\ 2p_2+3-2j, & p_1+1 \le j \le p_2+1 \text{ and} \end{cases}$$

$$f^*\left(v_j^{(3)}\right) = 2(p_1+p_2)+5+2j \text{ for } 1 \le j \le p_3+1.$$

Then, the induced edge labeling f^* is obtained as follows:

$$f^*(v_0v_1^{(i)}) = 2(p_1+p_2)+2i, \text{ for } 1 \le i \le 3,$$

$$f^*(v_j^{(1)}v_{j+1}^{(1)}) = \begin{cases} 2(p_1+p_2)+5-4j, & j=1 \\ 2(p_1+p_2)+6-4j, & 2 \le j \le p_1, \end{cases}$$

$$f^*(v_j^{(2)}v_{j+1}^{(2)}) = \begin{cases} 2(p_1+p_2)+1-4j, & 1 \le j \le p_1-1 \\ 2p_2+2-2j, & p_1 \le j \le p_2 \quad \text{and} \end{cases}$$

$$f^*(v_j^{(3)}v_{j+1}^{(3)}) = 2(p_1+p_2)+6+2j, \text{ for } 1 \le j \le p_3.$$

In both cases, f is a super F-centroidal mean labeling of the arbitrary subdivision of S_3.

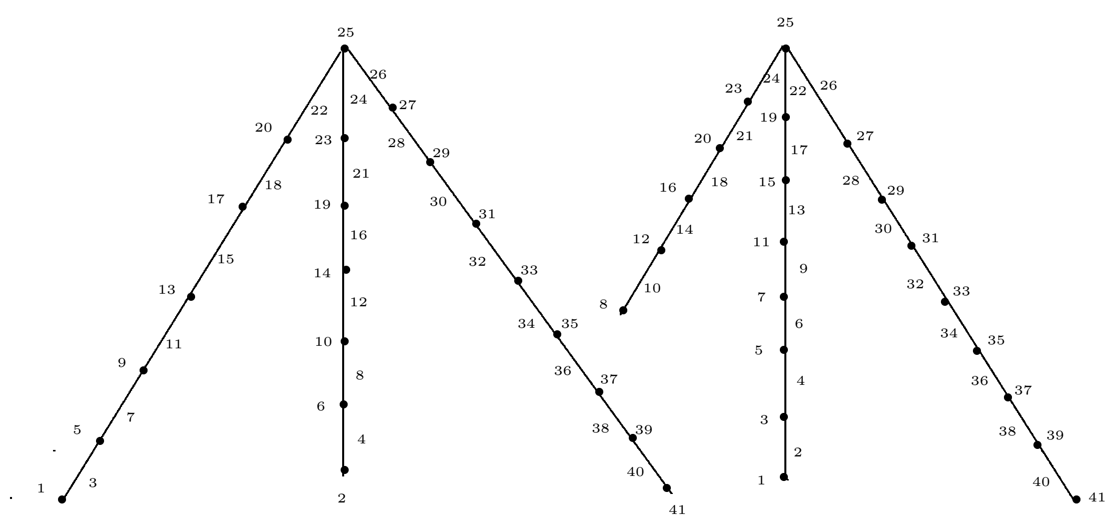

Figure 7. A super F-centroidal mean labeling of G with $p_1 = p_2 = 5, p_3 = 7$ and $p_1 = 4, p_2 = 6, p_3 = 7$

The graphs does not fall on the Case 1 are found to be a super F-centroidal mean graphs whose super F-centroidal mean labeling is shown in Figure 8. □

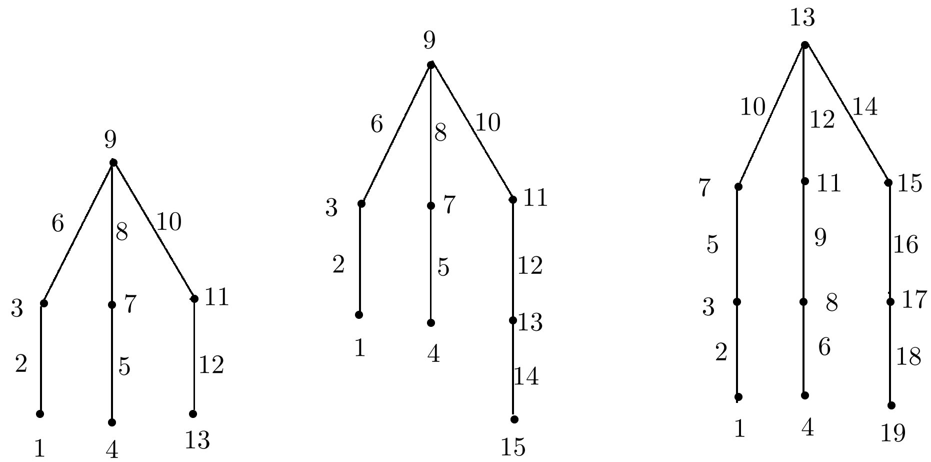

Figure 8 A super F-centroidal mean labeling of G with $p_1 = p_2 = p_3 = 1$, $p_1 = p_2 = 1, p_3 = 2$ and $p_1 = p_2 = p_3 = 2$

Theorem 2.8 *Every cycle C_n is a super F-centroidal mean graph, for $n \geq 4$.*

Proof Let $u_1, u_2, \cdots, u_n$ be the vertices of the cycle C_n. Assume that $n \geq 5$.

A vertex labeling $f : V(C_n) \to \{1, 2, 3, \cdots, 2n\}$ is defined as

$$f(u_i) = \begin{cases} 1, & i = 1 \\ 4i - 4, & 2 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor + 1 \text{ and } n \text{ is odd} \\ 4i - 6, & i = \left\lfloor \frac{n}{2} \right\rfloor + 2 \text{ and } n \text{ is odd} \\ 4n - 4i + 5, & \left\lfloor \frac{n}{2} \right\rfloor + 3 \leq i \leq n \text{ and } n \text{ is odd} \\ 4i - 5, & 2 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \text{ and } n \text{ is even} \\ 4i - 4, & i = \left\lfloor \frac{n}{2} \right\rfloor + 1 \text{ and } n \text{ is even} \\ 4n - 4i + 6, & \left\lfloor \frac{n}{2} \right\rfloor + 2 \leq i \leq n \text{ and } n \text{ is even.} \end{cases}$$

Then, the induced edge labeling f^* is obtained as follows:

$$f^*(u_i u_{i+1}) = \begin{cases} 2, & i = 1 \text{ and } n \text{ is odd} \\ 4i - 2, & 2 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \text{ and } n \text{ is odd} \\ 4i - 3, & i = \left\lfloor \frac{n}{2} \right\rfloor + 1 \text{ and } n \text{ is odd} \\ 4n - 4i + 3, & \left\lfloor \frac{n}{2} \right\rfloor + 2 \leq i \leq n - 1 \text{ and } n \text{ is odd} \\ 3i - 1, & 1 \leq i \leq 2 \text{ and } n \text{ is even} \\ 4i - 3, & 3 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor \text{ and } n \text{ is even} \\ 4i - 5, & i = \left\lfloor \frac{n}{2} \right\rfloor + 1 \text{ and } n \text{ is even} \\ 4n - 4i + 4, & \left\lfloor \frac{n}{2} \right\rfloor + 2 \leq i \leq n - 1 \text{ and } n \text{ is even and} \end{cases}$$

$$f^*(u_nu_1) = \begin{cases} 3, & n \text{ is odd} \\ 4, & n \text{ is even.} \end{cases}$$

Hence f is a super F-centroidal mean labeling of C_n, for $n \geq 5$.
Thus the graph C_n is a super F-centroidal mean graph, for $n \geq 5$.

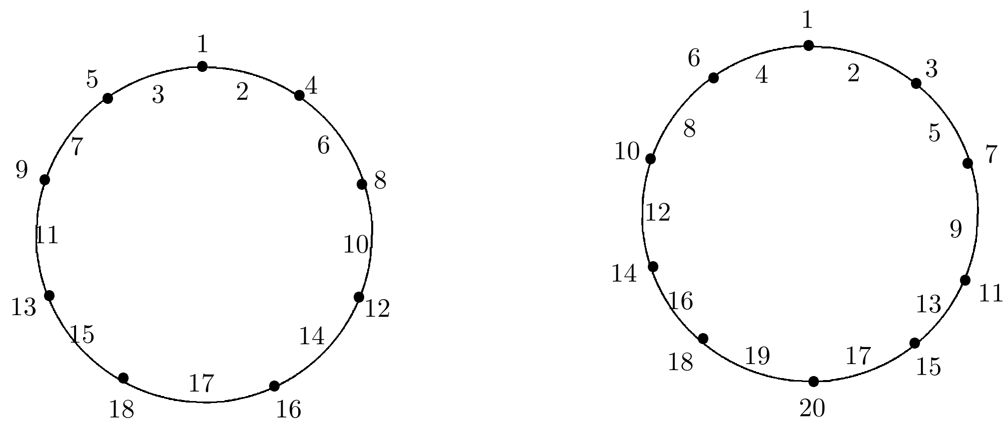

Figure 9 A super F-centroidal mean labeling of C_9 and C_{10}

For $n = 4$, a super F-centroidal mean labeling of C_4, is shown in Figure 1. But, the graph C_3 is not a super F-centroidal mean graph. □

Theorem 2.9 *$P_n \cup C_m$ is a super F-centroidal mean graph, for $n \geq 1$ and $m \geq 3$.*

Proof Let $u_1, u_2, \cdots, u_m$ and $v_1, v_2, \cdots, v_n$ be the vertices of the cycle C_m and the path P_n respectively.

Case 1. $m \geq 4$.

Define $f : V(P_n \cup C_m) \rightarrow \{1, 2, 3, \cdots, 2m + 2n - 1\}$ as follows:

$$f(u_i) = \begin{cases} 1, & i = 1 \\ 4i - 4, & 2 \leq i \leq \left\lfloor \frac{m}{2} \right\rfloor \\ 2m - 3, & i = \left\lfloor \frac{m}{2} \right\rfloor + 1 \text{ and } m \text{ is odd} \\ 2m, & i = \left\lfloor \frac{m}{2} \right\rfloor + 1 \text{ and } m \text{ is even} \\ 2m, & i = \left\lfloor \frac{m}{2} \right\rfloor + 2 \text{ and } m \text{ is odd} \\ 2m - 3, & i = \left\lfloor \frac{m}{2} \right\rfloor + 2 \text{ and } m \text{ is even} \\ 4m + 5 - 4i, & \left\lfloor \frac{m}{2} \right\rfloor + 3 \leq i \leq n \text{ and } n \text{ is even and} \end{cases}$$

$$f(v_i) = 2m + 2i - 1, \text{ for } 1 \leq i \leq n.$$

Then, the induced edge labeling f^* is obtained as follows:

$$f^*(u_iu_{i+1}) = \begin{cases} 4i-2, & 1 \le i \le \lfloor \frac{m}{2} \rfloor \\ 2m-1, & i = \lfloor \frac{m}{2} \rfloor + 1 \\ 2m-2, & i = \lfloor \frac{m}{2} \rfloor + 2 \text{ and } m \text{ is odd} \\ 2m-5, & i = \lfloor \frac{m}{2} \rfloor + 2 \text{ and } m \text{ is even} \\ 4m+3-4i, & \lfloor \frac{m}{2} \rfloor + 3 \le i \le m-1, \end{cases}$$

$$f^*(u_1u_m) = 3 \text{ and}$$
$$f^*(v_iv_{i+1}) = 2m+2i, \text{ for } 1 \le i \le n-1.$$

Case 2. $m = 3$.

Define $f : V(P_n \cup C_3) \to \{1, 2, 3, \cdots, 2n+5\}$ as follows:

$$f(v_i) = 2i-1, \text{ for } 1 \le i \le n,$$
$$f(u_1) = 2n,$$
$$f(u_2) = 2n+3 \text{ and}$$
$$f(u_3) = 2n+5.$$

Then, the induced edge labeling f^* is obtained as follows:

$$f^*(v_iv_{i+1}) = 2i, \text{ for } 1 \le i \le n-1,$$
$$f^*(u_1u_2) = 2n+1,$$
$$f^*(u_2u_3) = 2n+4 \text{ and}$$
$$f^*(u_1u_3) = 2n+2.$$

Hence f is a super F-centroidal mean labeling of $P_n \cup C_m$. Thus the graph $P_n \cup C_m$ is a super F-centroidal mean graph for $n \ge 1$ and $m \ge 3$. □

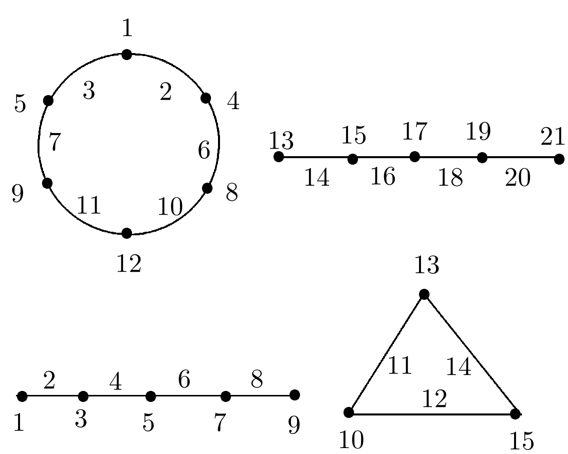

Figure 10 A super F-centroidal mean labeling of $P_5 \cup C_6$ and $P_4 \cup C_3$

Theorem 2.10 *P_n^2 is a super F-centroidal mean graph, for $n \geq 3$.*

Proof Let $v_1, v_2, \cdots, v_n$ be the vertices of the path P_n. Assume that $n \neq 5$. Define $f : V(P_n^2) \rightarrow \{1, 2, 3, \cdots, 3n-3\}$ as follows:

$$f(v_i) = \begin{cases} 3i-2, & 1 \leq i \leq n-2 \text{ and } i \text{ is odd} \\ 3i-3, & 1 \leq i \leq n-2 \text{ and } i \text{ is even,} \end{cases}$$

$$f(v_{n-1}) = 3n-5 \text{ and}$$

$$f(v_n) = 3n-3.$$

Then, the induced edge labeling f^* is obtained as follows:

$$f^*(v_i v_{i+1}) = 3i-1, \text{ for } 1 \leq i \leq n-1,$$

$$f^*(v_i v_{i+2}) = \begin{cases} 3i+1, & 1 \leq i \leq n-4 \text{ and } i \text{ is odd} \\ 3i, & 2 \leq i \leq n-4 \text{ and } i \text{ is even,} \end{cases}$$

$$f^*(v_{n-3} v_{n-1}) = \begin{cases} 3n-9, & n \text{ is odd} \\ 3n-8, & n \text{ is even and} \end{cases}$$

$$f^*(v_{n-2} v_n) = 3n-6.$$

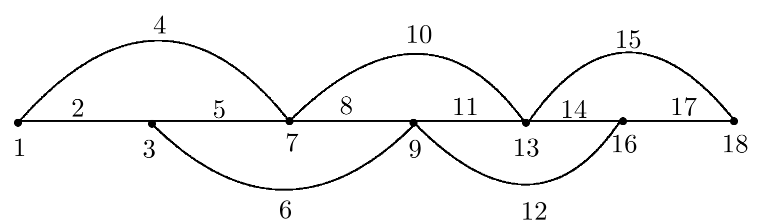

Figure 11 A super F-centroidal mean labeling of P_7^2

For $n = 5$, a super F-centroidal mean labeling of P_n^5 is shown the Figure 12.

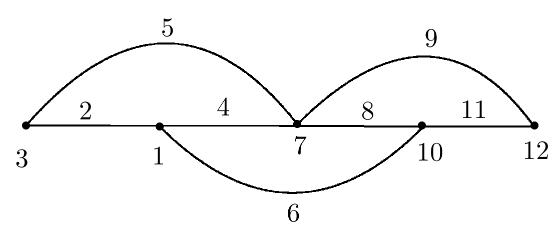

Figure 12 A super F-centroidal mean labeling of P_5^2

Hence f is a super F-centroidal mean labeling of P_n^2. Thus the graph P_n^2 is a super F-centroidal mean graph, for $n \geq 3$. □

Theorem 2.11 *If the graph G is a super F-centroidal mean graph, then $P_n(G)$ is also a super F-centroidal mean graph.*

Proof Let f be a super F-centroidal mean graph of G. Let $v_1, v_2, v_3, \cdots, v_p$ be the vertices and $e_1, e_2, e_3, \cdots, e_q$ be the edges of G so that the vertex having maximum vertex label is taken as v_p. Let $u_1, u_2, u_3, \cdots, u_n$ and $E_1, E_2, E_3, \cdots, E_{n-1}$ be the vertices and edges of P_n respectively and v_p is identified with u_1 in $P_n(G)$.

Define $g : V(P_n(G)) \rightarrow \{1, 2, 3, \cdots, p + q + 2j - 2\}$ as follows:

$$g(v_i) = f(v_i), \text{ for } 1 \leq i \leq p \text{ and}$$
$$g(u_j) = p + q + 2j - 2, \text{ for } 1 \leq j \leq n.$$

Then, the induced edge labeling g^* is obtained as follows:

$$g^*(e_i) = f(e_i), \text{ for } 1 \leq i \leq p \text{ and}$$
$$g^*(E_j) = p + q + 2j - 1, \text{ for } 1 \leq j \leq n - 1.$$

Hence $P_n(G)$ is a super F-centroidal mean graph. Thus the graph G is a super F-centroidal mean graph then $P_n(G)$ is also a super F-centroidal mean graph. □

Corollary 2.12 *A dragon $P_n(C_m)$ is a super F-centroidal mean graph, for $m \geq 4$ and $n \geq 2$.*

§3. Conclusion

In this paper, the super F-centroidal meanness of some standard graphs have been studied. It is possible to investigate the super F-centroidal meanness for other graphs.

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