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Some Results on 4-Total Prime Cordial Graphs

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Abstract: Let G be a (p, q) graph. Let $f : V(G) \to \{1, 2, \cdots, k\}$ be a map where $k \in \mathbb{N}$ and $k > 1$. For each edge uv, assign the label $\gcd(f(u), f(v))$. f is called k-total prime cordial labeling of G if $|t_f(i) - t_f(j)| \leq 1$, $i, j \in \{1, 2, \cdots, k\}$ where $t_f(x)$ denotes the total number of vertices and the edges labeled with x. A graph with a k-total prime cordial labeling is called k-total prime cordial graph. In this paper we investigate the 4-total prime cordial labeling of certain graphs like shadow graph, P_n^2, $T_n \odot K_2$ and subdivision of $T_n \odot K_1$.

Key Words: k-Total prime cordial labeling, Smarandachely k-total prime cordial labeling, corona, P_n^2, shadow graph.

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§1. Introduction

Graphs considered here are finite, simple and undirected. Ponraj et al. [4], have been introduced the concept of k-total prime cordial labeling and investigate the k-total prime cordial labeling of certain graphs. Also in [4, 5, 6, 7, 8, 9, 10, 12], the 4-total prime cordial labeling behavior of path, cycle, star, bistar, some complete graphs, comb, double comb, triangular snake, double triangular snake, ladder, friendship graph, flower graph, gear graph, Jelly fish, book, irregular triangular snake, prism, helm, dumbbell graph, sunflower graph, corona of irregular triangular snake, dragon, Möbius ladder, corona of some graphs and subdivision of some graphs. 3-total prime cordial labeling behavior of some graphs have been investigated [11]. In this paper we investigate the 4-total prime cordial labeling of certain graphs like shadow graph, P_n^2, $T_n \odot K_2$ and subdivision of $T_n \odot K_1$.

§2. Preliminary Results

Definition 2.1 *Let G_1, G_2 respectively be (p_1, q_1), (p_2, q_2) graphs. A corona of G_1 with G_2 is the graph $G_1 \odot G_2$ obtained by taking one copy of G_1 , p_1 copies of G_2 and joining the i^{th}*

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vertex of G_1 by an edge to every vertex in the i^{th} copy of G_2 where $1 \leq i \leq p_1$.

Definition 2.2 *A shadow graph $D_2(G)$ of a connected graph G is constructed by taking two copies of G, G' and G'' and joining each vertex u' in G' to the neighbors of the corresponding vertex u'' in G''.*

Definition 2.3 *If $e = uv$ is an edge of G then e is said to be subdivided when it is replaced by the edges uw and wv. The graph obtained by subdividing each edge of a graph G is called the subdivision graph of G and is denoted by $S(G)$.*

Definition 2.4 *For a simple connected graph G the square of graph G is denoted by G^2 and defined as the graph with the same vertex set as of G and two vertices are adjacent in G^2 if they are at a distance 1 or 2 apart in G.*

Theorem 2.5([4]) *A cycle C_n is 4-total prime cordial iff $n \notin \{4, 6, 8\}$.*

Remark 2.6 *A 2-total prime cordial graph is 2-total product cordial graph.*

§3. k-Total Prime Cordial Labeling

Definition 3.1 *Let G be a (p, q) graph. Let $f : V(G) \to \{1, 2, \cdots, k\}$ be a function where $k \in \mathbb{N}$ and $k > 1$. For each edge uv, assign the label $\gcd(f(u), f(v))$. f is called k-total prime cordial labeling of G if $|t_f(i) - t_f(j)| \leq 1$, $i, j \in \{1, 2, \cdots, k\}$ where $t_f(x)$ denotes the total number of vertices and the edges labeled with x. Conversely, a non-k-total prime cordial labeling of G is called a Smarandachely k-total prime cordial labeling f, i.e., $|t_f(i) - t_f(j)| \geq 2$ for an integer pair $\{i, j\}$, where $i, j \in \{1, 2, \cdots, k\}$.*

A graph with a k-total prime cordial labeling is called k-total prime cordial graph.

Theorem 3.2 *If $n \equiv 1 \pmod 4$, then P_n^2 is 4-total prime cordial.*

Proof Let $u_1 u_2 \cdots u_n$ be the path. Let u_i is adjacent to u_{i+2}, $(1 \leq i \leq n-2)$. Clearly $\left|V(P_n^2)\right| + \left|E(P_n^2)\right| = 3n - 3$.

Let $n = 4r + 1$, $r \in \mathbb{N}$. Assign the label 4 to the vertices $u_1, u_2, \cdots, u_{r+1}$ and assign the label 2 to the vertices $u_{r+2}, u_{r+3}, \cdots, u_{2r+1}$. Next we assign the label 3 to the vertices $u_{2r+2}, u_{2r+3}, \cdots, u_{3r+2}$. Finally we assign the label 1 to the vertices $u_{3r+3}, u_{3r+4}, \cdots, u_{4r}$. It is easy to verify that $t_f(1) = t_f(2) = t_f(3) = t_f(4) = 3r$. □

Theorem 3.3 *The shadow graph of P_n, $D_2(P_n)$ is 4-total prime cordial iff $n \notin \{2, 4\}$.*

Proof Let $u_1 u_2 \cdots u_n$ and $v_1 v_2 \cdots v_n$ be the two copies of the path P_n. Let u_i is adjacent to v_{i+1} and v_i is adjacent to u_{i+1}, $(1 \leq i \leq n-1)$. Clearly $|V(D_2(P_n))| + |E(D_2(P_n))| = 6n - 4$.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4r$, $r > 1$ and $r \in \mathbb{N}$. Assign the label 4 to the vertices $u_1, u_2, \cdots, u_r$ and assign the label 2 to the vertices $u_{r+1}, u_{r+2}, \cdots, u_{2r}$. Next we assign the label 3 to the vertices

$u_{2r+1}, u_{2r+2}, \cdots, u_{3r}$ then we assign the label 1 to the vertices $u_{3r+1}, u_{3r+2}, \cdots, u_{4r-2}$. Finally we assign the labels 4, 2 to the vertices u_{4r-1} and u_{4r} respectively. Now we move to the vertices v_i $(1 \leq i \leq n)$. Assign the label 4 to the vertices $v_1, v_2, \cdots, v_r$ and assign the label 2 to the vertices $v_{r+1}, v_{r+2}, \cdots, v_{2r-1}$. Next we assign the label 3 to the vertices $v_{2r}, v_{2r+1}, \cdots, v_{3r}$. Next we assign the label 1 to the vertices $v_{3r+1}, v_{3r+2}, \cdots, v_{4r-1}$. Finally we assign the label 4 to the vertex v_{4r}. Here $t_f(1) = t_f(2) = t_f(3) = t_f(4) = 6r - 1$.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4r + 1$, $r > 1$ and $r \in \mathbb{N}$. As in Case 1, assign the label to the vertices u_i, v_i $(1 \leq i \leq 4r - 1)$. Finally we assign the labels 4, 3 respectively to the vertices u_{4r} and v_{4r}. Clearly $t_f(1) = t_f(4) = 6r + 1$ and $t_f(2) = t_f(3) = 6r$.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4r + 2$, $r > 1$ and $r \in \mathbb{N}$. Assign the label to the vertices u_i $(1 \leq i \leq 4r - 3)$, v_i $(1 \leq i \leq 4r - 4)$ by in Case 1. Finally we assign the labels 4, 3, 2, 4, 3, 2, 4 respectively to the vertices u_{4r-2}, u_{4r-1}, u_{4r}, v_{4r-3}, v_{4r-2}, v_{4r-1} and v_{4r}. It is easy to verify that $t_f(1) = t_f(2) = t_f(3) = t_f(4) = 6r + 2$.

Case 4. $n \equiv 3 \pmod 4$.

Let $n = 4r + 3$, $r > 1$ and $r \in \mathbb{N}$. In this case, assign the label to the vertices u_i, v_i $(1 \leq i \leq 4r - 1)$ by in Case 3. Finally we assign the labels 3, 4 to the vertices u_{4r} and v_{4r} respectively. Here $t_f(1) = t_f(4) = 6r + 4$ and $t_f(2) = t_f(3) = 6r + 3$.

Case 5. $n = 2$.

Theorem 2.5 gives $n = 2$ is not a 4-total prime cordial.

Case 6. $n = 4$.

Suppose f is a 4-total prime cordial labeling of $D_2(P_4)$. Then $t_f(1) = t_f(2) = t_f(3) = t_f(4) = 5$. Under the labeling f, we have $t_f(4) = 5$. For this, it is easy to verify that 4 must be labeled to 3 consecutive vertices of $D_2(P_4)$. That is, 4 must be labeled to all the three vertices of an induced subpath P_3 of $D_2(P_4)$. Similarly for $t_f(3) = 5$, 3 must be labeled to all the three vertices of another induced subpath P_3' of $D_2(P_4)$ which is disjoint from P_3. Now, we have only two vertices are remaining in $D_2(P_4)$. If the two vertices are labeled by 2, then $t_f(2) > 5$ or $t_f(2) < 5$, according as 2 is labels of adjacent vertices (or) 2 is labels of non-adjacent vertices, a contradiction.

Case 7. $n = 4, 5, 6, 7$.

A 4-total prime cordial labeling follows from Table 1.

n	4	5	6	7
u_1	4	2	4	4
u_2	2	4	4	4

u_3	3	2	2	2
u_4		4	3	3
u_5		3	3	3
u_6			2	1
u_7				4
v_1	4	4	4	4
v_2	3	4	4	4
v_3	4	3	2	2
v_4		3	3	2
v_5		3	3	3
v_6			2	3
v_7				3

Table 1

This completes the proof. □

Theorem 3.4 *The corona of T_n with K_2, $T_n \odot K_2$ is 4-total prime cordial for all $n \geq 2$.*

Proof Let $u_1u_2\cdots u_n$ be the path and v_i is adjacent to u_i, u_{i+1}. Let x_i, y_i be the vertices adjacent to v_i and x_i, y_i be adjacent. Let z_i, w_i be the vertices adjacent to u_i and z_i, w_i be adjacent. Clearly $|V(T_n \odot K_2)| + |E(T_n \odot K_2)| = 15n - 9$.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4r$, $r > 1$ and $r \in \mathbb{N}$. Assign the label 4 to the vertices $u_1, u_2, \cdots, u_r$ and assign the label 2 to the vertices $u_{r+1}, u_{r+2}, \cdots, u_{2r}$. Next we assign the label 3 to the vertices u_{2r+1}, u_{2r+2},$\cdots$, u_{3r} then we assign the label 1 to the vertices $u_{3r+1}, u_{3r+2}, \cdots, u_{4r-1}$. Finally, we assign the label 2 to the vertices u_{4r}. Next we consider the vertices v_i $(1 \leq i \leq n-1)$. Assign the label 4 to the vertices $v_1, v_2, \cdots, v_r$ and assign the label 2 to the vertices $v_{r+1}, v_{r+2}, \cdots, v_{2r-1}$. Next we assign the label 3 to the vertices $v_{2r}, v_{2r+1}, \cdots, v_{3r-1}$. Then we assign the label 1 to the vertices $v_{3r}, v_{3r+1}, \cdots, v_{4r-2}$. Finally, we assign the label 2 to the vertices v_{4r-1}. Now we move to the vertices x_i, y_i $(1 \leq i \leq n-1)$. Assign the label 4 to the vertices $x_1, x_2, \cdots, x_r$ and $y_1, y_2, \cdots, y_r$ and assign the label 2 to the vertices $x_{r+1}, x_{r+2}, \cdots, x_{2r-1}$ and $y_{r+1}, y_{r+2}, \cdots, y_{2r-1}$. Next we assign the label 3 to the vertices $x_{2r}, x_{2r+1}, \cdots, x_{3r-1}$ and $y_{2r}, y_{2r+1}, \cdots, y_{3r-1}$. Finally we assign the label 1 to the vertices $x_{3r}, x_{3r+1}, \cdots, x_{4r-1}$ and $y_{3r}, y_{3r+1}, \cdots, y_{4r-1}$. Next we consider the vertices z_i, w_i $(1 \leq i \leq n)$. Assign the label 4 to the vertices $z_1, z_2, \cdots, z_r$ and $w_1, w_2, \cdots, w_r$ and assign the label 2 to the vertices $z_{r+1}, z_{r+2}, \cdots, z_{2r}$ and $w_{r+1}, w_{r+2}, \cdots, w_{2r}$. Next we assign the label 3 to the vertices $z_{2r+1}, z_{2r+2}, \cdots, z_{3r}$ and $w_{2r+1}, w_{2r+2}, \cdots, w_{3r}$ then we assign the label 1 to the vertices $z_{3r+1}, z_{3r+2}, \cdots, z_{4r-1}$ and $w_{3r+1}, w_{3r+2}, \cdots, w_{4r-1}$. Finally we assign the labels 1, 2 respectively to the vertices z_{4r} and

w_{4r}. Clearly $t_f(1) = 15r - 3$ and $t_f(2) = t_f(3) = t_f(4) = 15r - 2$.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4r + 1$, $r > 1$ and $r \in \mathbb{N}$. As in Case 1, assign the label to the vertices u_i $(1 \le i \le n-1)$, v_i $(1 \le i \le n-2)$, x_i $(1 \le i \le n-2)$, y_i $(1 \le i \le n-2)$, z_i $(1 \le i \le n-2)$ and w_i $(1 \le i \le n-1)$. Finally we assign the labels 4, 4, 2, 1, 2, 3, 3 respectively to the vertices u_{4r}, v_{4r-1}, x_{4r-1}, y_{4r-1}, z_{4r-1}, z_{4r} and w_{4r}. Here $t_f(1) = t_f(2) = 15r + 2$ and $t_f(3) = t_f(4) = 15r + 1$.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4r + 2$, $r > 1$ and $r \in \mathbb{N}$. As in Case 2, assign the label to the vertices u_i $(1 \le i \le n-1)$, v_i $(1 \le i \le n-2)$, x_i $(1 \le i \le n-2)$, y_i $(1 \le i \le n-2)$, z_i $(1 \le i \le n-1)$ and w_i $(1 \le i \le n-1)$. Finally we assign the labels 2, 4, 4, 3, 3, 3 to the vertices u_{4r}, v_{4r-1}, x_{4r-1}, y_{4r-1}, z_{4r} and w_{4r} respectively. It is easy to verify that $t_f(1) = 15r + 6$ and $t_f(2) = t_f(3) = t_f(4) = 15r + 5$.

Case 4. $n \equiv 3 \pmod 4$.

Let $n = 4r + 3$, $r > 1$ and $r \in \mathbb{N}$. As in Case 3, assign the label to the vertices u_i $(1 \le i \le n-4)$, v_i $(1 \le i \le n-4)$, x_i $(1 \le i \le n-4)$, y_i $(1 \le i \le n-4)$, z_i $(1 \le i \le n-4)$ and w_i $(1 \le i \le n-4)$. Now we assign the labels 2, 4, 3, 3, 2, 4, 3 respectively to the vertices u_{4r-3}, u_{4r-2}, u_{4r-1}, u_{4r}, v_{4r-3}, v_{4r-2}, v_{4r-1}. Next we assign the labels 2, 4, 1, 4, 4, 3 to the vertices x_{4r-3}, x_{4r-2}, x_{4r-1}, y_{4r-3}, y_{4r-2} and y_{4r-1} respectively. Finally we assign the labels 2, 3, 1, 4, 2, 3, 4, 1 respectively to the vertices z_{4r-3}, z_{4r-2}, z_{4r-1}, z_{4r}, w_{4r-3}, w_{4r-2}, w_{4r-1} and w_{4r}. Here $t_f(1) = t_f(2) = t_f(3) = t_f(4) = 15r + 9$.

Case 5. $n = 2, 3, 4, 5, 6, 7$.

A 4-total prime cordial labeling follows from Table 2.

n	2	3	4	5	6	7
u_1	4	4	4	4	4	4
u_2	3	2	2	2	4	4
u_3		3	3	3	2	2
u_4			1	2	3	2
u_5				2	3	3
u_6					3	3
u_7						1
v_1	4	4	4	4	4	4
v_2		2	2	2	4	4
v_3			3	3	2	2
v_4				1	3	3

v_5					1	3
v_6						1
x_1	2	4	4	4	4	4
x_2		3	2	2	4	4
x_3			3	3	2	2
x_4				1	3	3
x_5					1	3
x_6						1
y_1	1	4	4	4	4	4
y_2		3	1	1	2	2
y_3			3	3	2	2
y_4				1	3	3
y_5					1	3
y_6						1
z_1	4	4	4	4	4	4
z_2	3	1	2	2	2	4
z_3		3	3	3	2	2
z_4			1	4	3	3
z_5				3	3	3
z_6					1	3
z_7						1
w_1	2	2	4	4	4	4
w_2	3	1	2	2	2	2
w_3		3	3	3	1	2
w_4			1	4	3	3
w_5				3	1	4
w_6					1	4
w_7						1

Table 2

This completes the proof. □

Theorem 3.5 *The subdivision of $T_n \odot K_1$, $S(T_n \odot K_1)$ is 4-total prime cordial for all $n \geq 2$.*

Proof Let P_n be the path $u_1u_2\cdots u_n$. Let $v_1, v_2, \cdots, v_n$ be the vertices such that v_i is adjacent to both u_i and u_{i+1} $(1 \leq i \leq n-1)$. Let w_i be the pendent vertices adjacent to v_i $(1 \leq i \leq n-1)$. Let p_i be the pendent vertices adjacent to u_i $(1 \leq i \leq n)$. Let s_i, x_i, y_i, z_i, q_i be the vertices which subdivide the edge u_iu_{i+1}, u_iv_i, v_iu_{i+1}, v_iw_i, u_ip_i respectively. It is easy to show that $|V(S(T_n \odot K_1))| + |E(S(T_n \odot K_1))| = 19n - 14$.

Case 1. $n \equiv 0 \pmod 4$.

Let $n = 4r$, $r > 1$ and $r \in \mathbb{N}$. Assign the label 4 to the vertices $u_1, u_2, \cdots, u_r$ and assign the label 2 to the vertices $u_{r+1}, u_{r+2}, \cdots, u_{2r}$. Next we assign the label 3 to the vertices u_{2r+1}, $u_{2r+2}, \cdots$, u_{3r} then we assign the label 1 to the vertices $u_{3r+1}, u_{3r+2}, \cdots, u_{4r-1}$. Finally, we assign the label 4 to the vertices u_{4r}. Next we consider the vertices v_i $(1 \leq i \leq n-1)$. Assign the label 4 to the vertices $v_1, v_2, \cdots, v_r$ and assign the label 2 to the vertices $v_{r+1}, v_{r+2}, \cdots, v_{2r-1}$. Next we assign the label 3 to the vertices $v_{2r}, v_{2r+1}, \cdots, v_{3r-1}$. Then we assign the label 1 to the vertices $v_{3r}, v_{3r+1}, \cdots, v_{4r-2}$. Finally we assign the label 3 to the vertices v_{4r-1}. Now we move to the vertices s_i $(1 \leq i \leq n-1)$. Assign the label 4 to the vertices $s_1, s_2, \cdots, s_r$ and assign the label 2 to the vertices $s_{r+1}, s_{r+2}, \cdots, s_{2r}$. Next we assign the label 3 to the vertices s_{2r+1}, $s_{2r+2}, \ldots$, s_{3r} then we assign the label 1 to the vertices $s_{3r+1}, s_{3r+2}, \cdots, s_{4r-1}$. Next we consider the vertices x_i, y_i $(1 \leq i \leq n-1)$. Assign the label 4 to the vertices $x_1, x_2, \ldots, x_r$ and $y_1, y_2, \cdots, y_{r-1}$ and assign the label 2 to the vertices $x_{r+1}, x_{r+2}, \cdots, x_{2r}$ and $y_r, y_{r+1}, \cdots, y_{2r-1}$. Next we assign the label 3 to the vertices $x_{2r+1}, x_{2r+2}, \cdots, x_{3r-1}$ and $y_{2r}, y_{2r+1}, \cdots, y_{3r-1}$. Finally we assign the label 1 to the vertices $x_{3r}, x_{3r+1}, \ldots, x_{4r-1}$ and $y_{3r}, y_{3r+1}, \ldots, y_{4r-1}$. Now we move to the vertices z_i, w_i $(1 \leq i \leq n-1)$. Assign the label 4 to the vertices $z_1, z_2, \cdots, z_r$ and $w_1, w_2, \cdots, w_r$ and assign the label 2 to the vertices $z_{r+1}, z_{r+2}, \cdots, z_{2r-1}$ and $w_{r+1}, w_{r+2}, \cdots, w_{2r-1}$. Next we assign the label 3 to the vertices $z_{2r}, z_{2r+1}, \cdots, z_{3r-1}$ and $w_{2r}, w_{2r+1}, \cdots, w_{3r-1}$ then we assign the label 1 to the vertices $z_{3r}, z_{3r+1}, \ldots, z_{4r-1}$ and w_{3r}, $w_{3r+1}, \ldots, w_{4r-1}$. Next we consider the vertices p_i, q_i $(1 \leq i \leq n)$. Assign the label 4 to the vertices $p_1, p_2, \ldots, p_r$ and $q_1, q_2, \cdots, q_r$ and assign the label 2 to the vertices $p_{r+1}, p_{r+2}, \cdots, p_{2r}$ and $q_{r+1}, q_{r+2}, \cdots, q_{2r}$. Next we assign the label 3 to the vertices $p_{2r+1}, p_{2r+2}, \cdots, p_{3r}$ and $q_{2r+1}, q_{2r+2}, \cdots, q_{3r}$ then we assign the label 1 to the vertices p_{3r+1} ,$p_{3r+2}, \cdots, p_{4r}$ and $q_{3r+1}, q_{3r+2}, \cdots, q_{4r-1}$. Finally we assign the label 2 to the vertex q_{4r}. Clearly $t_f(1) = t_f(2) = 19r - 4$ and $t_f(3) = t_f(4) = 19r - 3$.

Case 2. $n \equiv 1 \pmod 4$.

Let $n = 4r + 1$, $r > 1$ and $r \in \mathbb{N}$. As in Case 1, assign the label to the vertices u_i $(1 \leq i \leq n-1)$, v_i $(1 \leq i \leq n-2)$, s_i $(1 \leq i \leq n-2)$, x_i $(1 \leq i \leq n-2)$, y_i $(1 \leq i \leq n-2)$, z_i $(1 \leq i \leq n-2)$, w_i $(1 \leq i \leq n-2)$, p_i $(1 \leq i \leq n-1)$, and q_i $(1 \leq i \leq n-1)$. Finally we assign the labels 1, 3, 4, 4, 1, 3, 3, 2, 2 respectively to the vertices u_{4r}, v_{4r-1}, s_{4r-1}, x_{4r-1}, y_{4r-1}, z_{4r-1}, w_{4r-1}, p_{4r} and q_{4r}. Here $t_f(1) = t_f(2) = t_f(4) = 19r + 1$ and $t_f(3) = 19r + 2$.

Case 3. $n \equiv 2 \pmod 4$.

Let $n = 4r + 2$, $r > 1$ and $r \in \mathbb{N}$. Assign the label to the vertices u_i $(1 \leq i \leq n-1)$, v_i $(1 \leq i \leq n-2)$, s_i $(1 \leq i \leq n-2)$, x_i $(1 \leq i \leq n-2)$, y_i $(1 \leq i \leq n-2)$, z_i $(1 \leq i \leq n-2)$, w_i $(1 \leq i \leq n-2)$, p_i $(1 \leq i \leq n-1)$, and q_i $(1 \leq i \leq n-1)$ by in Case 2. Finally we assign the labels 1, 4, 3, 4, 4, 2, 2, 3, 2 to the vertices u_{4r}, v_{4r-1}, s_{4r-1}, x_{4r-1}, y_{4r-1}, z_{4r-1}, w_{4r-1},

p_{4r} and q_{4r} respectively. It is easy to verify that $t_f(1) = t_f(2) = t_f(3) = t_f(4) = 19r + 6$.

Case 4. $n \equiv 3 \pmod 4$.

Let $n = 4r + 3$, $r > 1$ and $r \in \mathbb{N}$. As in Case 3, assign the label to the vertices u_i $(1 \leq i \leq n-1)$, v_i $(1 \leq i \leq n-2)$, s_i $(1 \leq i \leq n-2)$, x_i $(1 \leq i \leq n-2)$, y_i $(1 \leq i \leq n-2)$, z_i $(1 \leq i \leq n-2)$, w_i $(1 \leq i \leq n-2)$, p_i $(1 \leq i \leq n-1)$, and q_i $(1 \leq i \leq n-1)$. Finally we assign the labels 2, 4, 3, 3, 2, 4, 4, 2, 1 respectively to the vertices u_{4r}, v_{4r-1}, s_{4r-1}, x_{4r-1}, y_{4r-1}, z_{4r-1}, w_{4r-1}, p_{4r} and q_{4r}. Here $t_f(1) = t_f(2) = t_f(4) = 19r + 11$ and $t_f(3) = 19r + 10$.

Case 5. $n = 2, 3, 4, 5, 6, 7$.

A 4-total prime cordial labeling follows from Table 3.

n	2	3	4	5	6	7
u_1	4	4	4	4	4	4
u_2	3	2	2	4	4	4
u_3		3	3	2	2	2
u_4			1	3	3	2
u_5				1	3	3
u_6					1	1
u_7						1
v_1	4	4	4	4	4	4
v_2		3	2	2	2	4
v_3			3	3	2	2
v_4				1	3	3
v_5					1	3
v_6						1
s_1	2	4	4	4	4	4
s_2		3	3	2	2	4
s_3			1	3	2	2
s_4				3	3	3
s_5					1	3
s_6						1
x_1	4	4	4	4	4	4
x_2		2	2	2	2	4
x_3			3	3	2	2
x_4				1	3	3
x_5					1	3
x_6						1

y_1	3	2	2	4	4	4
y_2		3	3	3	2	2
y_3			1	3	1	2
y_4				1	3	3
y_5					1	3
y_6						1
z_1	2	4	4	4	4	4
z_2		1	2	2	2	2
z_3			3	3	1	2
z_4				1	3	3
z_5					1	3
z_6						1
w_1	2	4	4	4	4	4
w_2		1	3	2	2	2
w_3			2	3	4	2
w_4				1	3	3
w_5					3	3
w_6						1
p_1	1	2	4	4	4	4
p_2	4	1	2	2	4	4
p_3		3	3	2	2	2
p_4			1	3	3	3
p_5				1	3	3
p_6					1	1
p_7						1
q_1	3	2	4	4	4	4
q_2	3	1	2	2	4	4
q_3		3	3	2	2	2
q_4			1	3	3	2
q_5				1	3	3
q_6					1	1
q_7						1

Table 3

This completes the proof. □

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