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Reciprocal Transmission Hosoya Polynomial of Graphs

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Abstract: In this paper we define reciprocal transmission Hosoya polynomial of graphs and obtain general formula for some graphs. Also calculate reciprocal transmission Hosoya polynomial of cluster graphs and of reciprocal transmission distance balanced graphs.

Key Words: Distance, reciprocal transmission of a vertex, reciprocal transmission distance balanced graphs.

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§1. Introduction

The concept of counting polynomial was first introduced in chemistry by Polya [5] in 1936. However the subject received attention from chemists for several decades even though the spectra of the characteristic polynomial of graphs were studied extensively by numerical means in order to obtain the molecular orbitals of unsaturated hydrocarbons.

The Hosoya polynomial of a graph was introduced in the Hosoya's seminal paper [4] in 1988 and received a lot of attention afterwards. The polynomial was later independently introduced and considered by Sagan, Yeh and Zhang [7] under the name Wiener polynomial of a graph. Both names are still used for the polynomial but the term Hosoya polynomial is nowadays used by majority of researchers. The main advantage of the Hosoya polynomial is that it contains a wealth of information about distance based graph invarients. For instance, knowing the Hosoya polynomial of graph, it is straightforward to determine the Wiener index of a graph as the first derivative of the polynomial at variable $x = 1$. Cash [1] noticed that the hyper-Wiener index can be obtained from the Hosoya polynomial in a similar simple manner. Also, Estrada et al. [2] studied several chemical applications of the Hosoya polynomial.

Let G be a connected graph on n vertices with vertex set $V(G)$ and edge set $E(G)$. If $d(G,k)$ is the number of unordered pairs of its vertices that are at distance k, then the Hosoya polynomial is defined as

$$H(G,x) = \sum_{k\geq 0} d(G,k)x^k. \tag{1}$$

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The reciprocal transmission (status) of a vertex u of a graph G is defined as [6]

$$rs(u) = \sum_{v \in V(G), u \neq v} \frac{1}{d(u,v)}.$$

The first reciprocal transmission (status) connectivity index of a graph G is defined as [?]

$$RS_1(G) = \sum_{uv \in E(G)} \left[rs(u) + rs(v) \right].$$

The reciprocal transmission Hosoya polynomial of a graph G is defined as

$$H_{rs}(G,x) = \sum_{uv \in E(G)} x^{rs(u)+rs(v)}. \tag{2}$$

where $rs(u)$ is the reciprocal transmission of a vertex u.

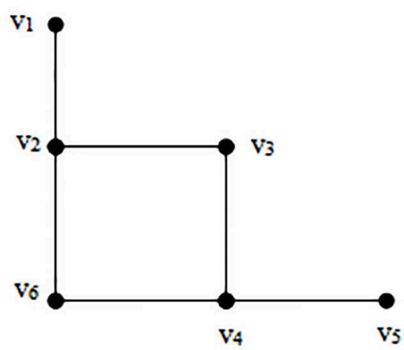

Figure 1

For a graph given in Figure 1, $rs(v_1) = 2.58$, $rs(v_2) = 3.83$, $rs(v_3) = 3.5$, $rs(v_4) = 3.83$, $rs(v_5) = 2.58$ and $rs(v_6) = 3.5$. Therefore

$$H_{rs}(G,x) = 2x^{6.41} + 4x^{7.33}.$$

§2. Reciprocal Transmission Hosoya Polynomial of Some Class of Graphs

Proposition 2.1 *Let G be a connected graph with n vertices and m edges. Let $diam(G) \leq 2$ and $d(u)$ be the degree of a vertex u in G. Then*

$$H_{rs}(G,x) = x^{n-1} \sum_{uv \in E(G)} x^{\frac{1}{2}(d(u)+d(v))}. \tag{3}$$

Proof If $diam(G) \leq 2$, then $d(u)$ number of vertices are at distance 1 from the vertex u and the remaining $n-1-d(u)$ vertices are at distance 2. Hence $rs(u) = d(u) + \frac{1}{2}\left(n-1-d(u)\right)$.

Therefore,

$$rs(u)+rs(v)=(n-1)+\frac{1}{2}(d(u)+d(v)).$$

Hence, from Eq.(2) we get

$$\begin{aligned} H_{rs}(G,x) &= \sum_{uv\in E(G)} x^{rs(u)+rs(v)} \\ &= \sum_{uv\in E(G)} x^{\left((n-1)+\frac{1}{2}(d(u)+d(v))\right)} = x^{(n-1)} \sum_{uv\in E(G)} x^{\frac{1}{2}(d(u)+d(v))}. \end{aligned}$$ □

Proposition 2.2 *Let G be a connected graph on n vertices and m edges. Then the first reciprocal transmission connectivity index $RS_1(G)=\frac{d}{dx}H_{rs}(G,x)|_{x=1}$.*

Corollary 2.3 *Let G be a connected r-regular graph on n vertices and m edges. Let $diam(G)\leq 2$. Then*

$$H_{rs}(G,x)=mx^{r+n-1}. \tag{4}$$

Proof Since degree of each vertex is r, then by Proposition 2.1 we have,

$$H_{rs}(G,x)=x^{n-1}\sum_{uv\in E(G)} x^{r}=mx^{r+n-1}.$$ □

Corollary 2.4 *For a complete bipartite graph $K_{p,q}$ on $n=p+q$ vertices,*

$$H_{rs}(K_{p,q},x)=pqx^{\frac{3}{2}(p+q)-1}. \tag{5}$$

Proof The graph $K_{p,q}$ has $n=p+q$ vertices and $m=pq$ edges. Also $diam(K_{p,q})\leq 2$. The vertex set $V(K_{p,q})$ can be partitioned into two sets V_1 and V_2 such that for every edge uv of $K_{p,q}$, the vertex $u\in V_1$ and $v\in V_2$, where $|V_1|=p$ and $|V_2|=q$. Therefore $d(u)=q$ and $d(v)=p$. Therefore, by Proposition 2.1 we have

$$\begin{aligned} H_{rs}(K_{p,q},x) &= x^{n-1}\sum_{uv\in E(K_{p,q})} x^{\frac{1}{2}(d(u)+d(v))} \\ &= x^{p+q-1}\sum_{uv\in E(K_{p,q})} x^{\frac{1}{2}(p+q)} = pqx^{\frac{3}{2}(p+q)-1}. \end{aligned}$$ □

Proposition 2.5 *For a cycle C_n on $n\geq 3$ vertices,*

$$H_{rs}(C_n,x)=\begin{cases} nx^{4\left(\frac{1}{n}+\sum_{i=1}^{\frac{n-2}{2}}\frac{1}{i}\right)}, & \text{if } n \text{ is even} \\ nx^{4\sum_{i=1}^{\frac{n-1}{2}}\frac{1}{i}}, & \text{if } n \text{ is odd.} \end{cases} \tag{6}$$

Proof If n is even number, then for every vertex u of C_n,

$$rs(u) = \frac{2}{n} + 2\sum_{i=1}^{\frac{n-2}{2}} \frac{1}{i}.$$

Therefore, from Eq.(2) we have

$$\begin{aligned} H_{rs}(C_n, x) &= \sum_{uv \in E(C_n)} x^{rs(u)+rs(v)} \\ &= \sum_{uv \in E(C_n)} x^{4\left(\frac{1}{n}+\sum_{i=1}^{\frac{n-2}{2}}\frac{1}{i}\right)} = nx^{4\left(\frac{1}{n}+\sum_{i=1}^{\frac{n-2}{2}}\frac{1}{i}\right)}. \end{aligned}$$

If n is odd number, then for every vertex u of C_n,

$$rs(u) = 2\sum_{i=1}^{\frac{n-1}{2}} \frac{1}{i}.$$

Therefore from Eq.(2) we have

$$\begin{aligned} H_{rs}(C_n, x) &= \sum_{uv \in E(C_n)} x^{rs(u)+rs(v)} \\ &= \sum_{uv \in E(C_n)} x^{4\sum_{i=1}^{\frac{n-1}{2}}\frac{1}{i}} = nx^{4\sum_{i=1}^{\frac{n-1}{2}}\frac{1}{i}}. \end{aligned}$$ □

Proposition 2.6 *For a wheel* W_{n+1}, $n \geq 3$,

$$H_{rs}(W_{n+1}, x) = n\left[x^{\frac{3}{2}(n+1)} + x^{n+3}\right]. \qquad (7)$$

Proof A wheel graph W_{n+1} has $n+1$ vertices and $2n$ edges. Also $diam(W_{n+1}) \leq 2$. The edge set $E(W_{n+1})$ can be partitioned into two sets E_1, E_2, such that $E_1 = \{uv \mid d(u) = n \text{ and } d(v) = 3\}$ and $E_2 = \{uv \mid d(u) = 3 \text{ and } d(v) = 3\}$. It is easy to check that $|E_1| = n$ and $|E_2| = n$ and $diam(W_{n+1}) \leq 2$. Therefore from Proposition 2.1 we get

$$\begin{aligned} H_{rs}(W_{n+1}, x) &= x^{n+1-1} \sum_{uv \in E(W_{n+1})} x^{\frac{1}{2}(d(u)+d(v))} \\ &= x^n \left[\sum_{uv \in E_1(W_{n+1})} x^{\frac{1}{2}(d(u)+d(v))} + \sum_{uv \in E_2(W_{n+1})} x^{\frac{1}{2}(d(u)+d(v))}\right] \\ &= x^n \left[nx^{\frac{1}{2}(n+3)} + nx^{\frac{1}{2}(3+3)}\right] \\ &= x^n n \left[x^{\frac{1}{2}(n+3)} + x^3\right] \\ &= n\left[x^{\frac{3}{2}(n+1)} + x^{n+3}\right]. \end{aligned}$$ □

Proposition 2.7 *For a friendship graph* F_n, $n \geq 2$,

$$H_{rs}(F_n, x) = n\left[2x^{3n+1} + x^{2(n+1)}\right]. \tag{8}$$

Proof The edge set $E(F_n)$ can be partitioned into two sets E_1 and E_2, such that $E_1 = \{uv \mid d(u) = 2n \quad \text{and} \quad d(v) = 2\}$ and $E_2 = \{uv \mid d(u) = 2 \text{ and } d(v) = 2\}$. It is easy to check that $|E_1| = 2n$ and $|E_2| = n$ and $diam(F_n) = 2$. Therefore by Proposition 2.1, we have

$$\begin{aligned} H_{rs}(F_n, x) &= x^{2n+1-1} \sum_{uv \in E(F_n)} x^{\frac{1}{2}(d(u)+d(v))} \\ &= x^{2n}\left[\sum_{uv \in E_1(F_n)} x^{\frac{1}{2}(d(u)+d(v))} + \sum_{uv \in E_2(F_n)} x^{\frac{1}{2}(d(u)+d(v))}\right] \\ &= x^{2n}\left[\sum_{uv \in E_1(F_n)} x^{\frac{1}{2}(2n+2)} + \sum_{uv \in E_2(F_n)} x^{\frac{1}{2}(2+2)}\right] \\ &= x^{2n}\left[2nx^{n+1} + nx^2\right] \\ &= n\left[2x^{3n+1} + x^{2(n+1)}\right]. \end{aligned}$$

□

§3. Reciprocal Transmission Hosoya Polynomial of Cluster Graphs

Graphs with large number of edges are referred as cluster graphs [3].

Definition 3.1([3]) *Let* e_i, $i = 1, 2, \cdots, k$, $1 \leq k \leq n-2$, *be the distinct edges of a complete graph* K_n, $n \geq 3$, *all being incident to a single vertex. The graph* $Ka_n(k)$ *is obtained by deleting* $e_i, i = 1, 2, \cdots, k$ *from* K_n. *In addition* $Ka_n(0) \cong K_n$.

Definition 3.2([3]) *Let* $f_i, i = 1, 2, \cdots, k, 1 \leq k \leq \lfloor \frac{n}{2} \rfloor$ *be independent edges of the complete graph* $K_n, n \geq 3$. *The graph* $Kb_n(k)$ *is obtained by deleting* f_i, $i = 1, 2, \cdots, k$ *from* K_n. *In addition* $Kb_n(0) \cong K_n$.

Definition 3.3([3]) *Let* V_k *be a* k*-element subset of the vertex set of the complete graph* K_n, $2 \leq k \leq n-1, n \geq 3$. *The graph* $Kc_n(k)$ *is obtained by deleting from* K_n *all the edges connecting pairs of vertices from* V_k. *In addition* $Kc_n(0) \cong Kc_n(1) \cong K_n$.

Definition 3.4([3]) *Let* $3 \leq k \leq n, n \geq 3$. *The graph* $Kd_n(k)$ *is obtained by deleting from the complete graph* K_n, *the edges belonging to a* k*-membered cycle.*

Proposition 3.5 *For* $n \geq 3$ *and* $1 \leq k \leq n-2$,

$$\begin{aligned} H_{rs}(Ka_n(k), x) &= x^{n-1}\left[(n-k-1)x^{\frac{1}{2}(2n-k-2)} + \frac{k(k-1)}{2}x^{n-2}\right. \\ &\quad \left. +(n-k-1)kx^{\frac{1}{2}(2n-3)} + \frac{(n-k-1)(n-k-2)}{2}x^{n-1}\right]. \end{aligned}$$

Proof The graph $Ka_n(k)$ has n vertices, $\left(\frac{n(n-1)}{2}-k\right)$ edges. The edge set $E(Ka_n(k))$ can be partitioned into four sets E_1, E_2, E_3 and E_4, where $E_1 = \{uv \mid d(u) = n-1-k \text{ and } d(v) = n-1\}$, $E_2 = \{uv \mid d(u) = n-2 \text{ and } d(v) = n-2\}$, $E_3 = \{uv \mid d(u) = n-2 \text{ and } d(v) = n-1\}$ and $E_4 = \{uv \mid d(u) = n-1 \text{ and } d(v) = n-1\}$. It is easy to check that $|E_1| = n-k-1$, $|E_2| = \frac{k(k-1)}{2}$, $|E_3| = (n-k-1)k$ and $|E_4| = \frac{(n-k-1)(n-k-2)}{2}$. Also $diam(Ka_n(k)) \leq 2$. Therefore, from Proposition 2.1 we have

$$\begin{aligned}
H_{rs}(Ka_n(k),x) &= x^{n-1}\sum_{uv\in E(Ka_n(k))} x^{\frac{1}{2}(d(u)+d(v))}\\
&= x^{n-1}\Bigg[\sum_{uv\in E_1(Ka_n(k))} x^{\frac{1}{2}(d(u)+d(v))} + \sum_{uv\in E_2(Ka_n(k))} x^{\frac{1}{2}(d(u)+d(v))}\\
&\quad + \sum_{uv\in E_3(Ka_n(k))} x^{\frac{1}{2}(d(u)+d(v))} + \sum_{uv\in E_4(Ka_n(k))} x^{\frac{1}{2}(d(u)+d(v))}\Bigg]\\
&= x^{n-1}\Bigg[\sum_{uv\in E_1(Ka_n(k))} x^{\frac{1}{2}(2n-k-2)} + \sum_{uv\in E_2(Ka_n(k))} x^{n-2}\\
&\quad + \sum_{uv\in E_3(Ka_n(k))} x^{\frac{1}{2}(2n-3)} + \sum_{uv\in E_4(Ka_n(k))} x^{n-1}\Bigg]\\
&= x^{n-1}\Bigg[(n-k-1)x^{\frac{1}{2}(2n-k-2)} + \frac{k(k-1)}{2}x^{n-2}\\
&\quad +(n-k-1)kx^{\frac{1}{2}(2n-3)} + \frac{(n-k-1)(n-k-2)}{2}x^{n-1}\Bigg]. \qquad \square
\end{aligned}$$

Proposition 3.6 *For $n \geq 3$ and $1 \leq k \leq \lfloor \frac{n}{2} \rfloor$,*

$$\begin{aligned}
H_{rs}(Kb_n(k),x) &= x^{n-1}\Bigg[2k(n-2k)x^{\frac{1}{2}(2n-3)} + \frac{(n-2k)(n-2k-1)}{2}x^{n-1}\\
&\quad + \left(\frac{2k(2k-1)}{2}-k\right)x^{n-2}\Bigg].
\end{aligned}$$

Proof The graph $Kb_n(k)$ has n vertices and $\left(\frac{n(n-1)}{2}-k\right)$ edges and $diam(Kb_n(k)) = 2$. The edge set $E(Kb_n(k))$ can be partitioned into three sets E_1, E_2 and E_3, where $E_1 = \{uv \mid d(u) = n-2 \text{ and } d(v) = n-1\}$, $E_2 = \{uv \mid d(u) = n-1 \text{ and } d(v) = n-1\}$ and $E_3 = \{uv \mid d(u) = n-2 \text{ and } d(v) = n-2\}$. It is easy to check that $|E_1| = 2k(n-2k)$, $|E_2| = (n-2k)(n-2k-1)/2$ and $|E_3| = (2k(2k-1)/2) - k$.

Therefore, from Proposition 2.1 we have

$$\begin{aligned}
H_{rs}(Kb_n(k),x) &= x^{n-1}\sum_{uv\in E(Kb_n(k))} x^{\frac{1}{2}(d(u)+d(v))}\\
&= x^{n-1}\left[\sum_{uv\in E_1(Kb_n(k))} x^{\frac{1}{2}(d(u)+d(v))}+\sum_{uv\in E_2(Kb_n(k))} x^{\frac{1}{2}(d(u)+d(v))}\right.\\
&\quad\left.+\sum_{uv\in E_3(Kb_n(k))} x^{\frac{1}{2}(d(u)+d(v))}\right]\\
&= x^{n-1}\left[\sum_{uv\in E_1(Kb_n(k))} x^{\frac{1}{2}(2n-3)}+\sum_{uv\in E_2(Kb_n(k))} x^{n-1}\right.\\
&\quad\left.+\sum_{uv\in E_3(Kb_n(k))} x^{(n-2)}\right]\\
&= x^{n-1}\left[2k(n-2k)x^{\frac{1}{2}(2n-3)}+\frac{(n-2k)(n-2k-1)}{2}x^{n-1}\right.\\
&\quad\left.+\left(\frac{2k(2k-1)}{2}-k\right)x^{n-2}\right]. \qquad \square
\end{aligned}$$

Proposition 3.7 *For $n \geq 3$ and $2 \leq k \leq n-1$,*

$$H_{rs}(Kc_n(k),x) = x^{n-1}\left[(n-k)kx^{\frac{1}{2}(2n-k-1)}+\frac{(n-k)(n-k-1)}{2}x^{n-1}\right].$$

Proof The graph $Kc_n(k)$ has n vertices and $\frac{1}{2}(n-k)(n+k-1)$ edges. Also $diam(Kc_n(k)) = 2$. The edge set $E(Kc_n(k))$ ca be partitioned into two sets E_1 and E_2, where $E_1 = \{uv \mid d(u) = n-k \text{ and } d(v) = n-1\}$ and $E_2 = \{uv \mid d(u) = n-1 \text{ and } d(v) = n-1\}$. It is easy to check that $|E_1| = (n-k)k$ and $|E_2| = (n-k)(n-k-1)/2$. Therefore, from Proposition 2.1 we have

$$\begin{aligned}
H_{rs}(Kc_n(k),x) &= x^{n-1}\sum_{uv\in E(Kc_n(k))} x^{\frac{1}{2}(d(u)+d(v))}\\
&= x^{n-1}\left[\sum_{uv\in E_1(Kc_n(k))} x^{\frac{1}{2}(d(u)+d(v))}+\sum_{uv\in E_2(Kc_n(k))} x^{\frac{1}{2}(d(u)+d(v))}\right]\\
&= x^{n-1}\left[\sum_{uv\in E_1(Kc_n(k))} x^{\frac{2}{2}(n-k-+n-1)}+\sum_{uv\in E_2(Kc_n(k))} x^{\frac{1}{2}(2(n-1))}\right]\\
&= x^{n-1}\left[(n-k)kx^{\frac{1}{2}(2n-k-1)}+\frac{(n-k)(n-k-1)}{2}x^{n-1}\right]. \qquad \square
\end{aligned}$$

Proposition 3.8 *For $3 \leq k \leq n$ and $n \geq 5$,*

$$\begin{aligned} H_{rs}(Kd_n(k),x) &= x^{n-1}\left[((k(k-1)/2)-k)\,x^{n-3}+(n-k)kx^{n-2}\right.\\ &\quad \left.+((n-k)(n-k-1)/2)\,x^{n-1}\right]. \end{aligned}$$

Proof The graph $Kd_n(k)$ has n vertices and $n(n-1)/2-k$ edges. Also $diam(Kd_n(k)) = 2$. The edge set $E(Kd_n(k))$ can be partitioned into three sets E_1, E_2 and E_3, where $E_1 = \{uv \mid d(u) = n-3 \text{ and } d(v) = n-3\}$, $E_2 = \{uv \mid d(u) = n-3 \text{ and } d(v) = n-1\}$ and $E_3 = \{uv \mid d(u) = n-1 \text{ and } d(v) = n-1\}$. It is easy to check that $|E_1| = (k(k-1)/2) - k$, $|E_2| = (n-k)k$ and $|E_3| = (n-k)(n-k-1)/2$. Therefore, from Proposition 2.1 we have,

$$\begin{aligned} H_{rs}(Kd_n(k),x) &= x^{n-1}\sum_{uv\in E(Kd_n(k))} x^{\frac{1}{2}(d(u)+d(v))}\\ &= x^{n-1}\left[\sum_{uv\in E_1(Kd_n(k))} x^{\frac{1}{2}(d(u)+d(v))}+\sum_{uv\in E_2(Kd_n(k))} x^{\frac{1}{2}(d(u)+d(v))}\right.\\ &\quad \left.+\sum_{uv\in E_3(Kd_n(k))} x^{\frac{1}{2}(d(u)+d(v))}\right]\\ &= x^{n-1}\left[\sum_{uv\in E_1(Kd_n(k))} x^{\frac{1}{2}(2(n-3))}+\sum_{uv\in E_2(Kd_n(k))} x^{\frac{1}{2}(2n-4)}\right.\\ &\quad \left.+\sum_{uv\in E_3(Kd_n(k))} x^{\frac{1}{2}(2(n-1))}\right]\\ &= x^{n-1}\left[((k(k-1)/2)-k)\,x^{n-3}+(n-k)kx^{n-2}\right.\\ &\quad \left.+((n-k)(n-k-1)/2)\,x^{n-1}\right]. \qquad \square \end{aligned}$$

§4. Reciprocal Transmission Hosoya Polynomial of Some Reciprocal Transmission Distance Balanced Graphs

A bijection $\propto$ on $V(G)$ is called automorphism of G if it preserves $E(G)$. In other words, $\propto$ is an automorphism if for each $u, v \in V(G)$, $e = uv \in E(G)$ if and only if

$$\propto(e) = \propto(u)\propto(v) \in E(G).$$

Let $Aut(G) = \{\propto \mid \propto : V(G) \to V(G)$ is a bijection, which preserves the adjacency$\}$.

It is known that $Aut(G)$ forms a group under the composition of mappings. A graph G is called vertex-transitive if for every two vertices u and v of G, there exists an automorphism $\propto$ of G such that $\propto(u) = \propto(v)$.

Theorem 4.1([6]) *Let G be a connected graph on n vertices with the automorphism group $Aut(G)$ and the vertex set $V(G)$. Let $V_1, V_2, \cdots, V_t$ be all orbits of the action $Aut(G)$ on $V(G)$. Suppose that for each $1 \leq i \leq t$, k_i are the reciprocal transmission of vertices in the orbit V_i, respectively. Then*

$$H(G) = \frac{1}{2}\sum_{i=1}^{t}|V_i|k_i.$$

Specially if G is vertex-transitive (i.e., $t = 1$), then

$$H(G) = \frac{1}{2}nk,$$

where k is the reciprocal transmission of each vertex of G.

Analogous to Theorem 4.1 and as a consequence of Proposition 2.1, we have the following.

Lemma 4.2 *Let G be a connected k-reciprocal transmission regular graph with m edges and $diam(G) \leq 2$. Then*

$$H_{rs}(G, x) = mx^{n+k-1}.$$

Proof For any k-reciprocal transmission distance balanced graph, $rs(u) = k$ for every vertex $u \in V(G)$. Therefore, from Eq.(2) we have,

$$\begin{aligned} H_{rs}(G, x) &= x^{n-1}\sum_{uv\in E(G)} x^{\frac{1}{2}(rs(u)+rs(v))} \\ &= x^{n-1}\sum_{uv\in E(G)} x^{\frac{1}{2}(2k)} = x^{n-1}mx^k = mx^{k+n-1}. \end{aligned}$$

□

Theorem 4.3 *Let G be a connected graph on n vertices with automorphism group $Aut(G)$ and the vertex set $V(G)$. Let $V_1, V_2, \cdots, V_t$ be all orbits of the action $Aut(G)$ on $V(G)$. Suppose that for each $1 \leq i \leq t$, d_i and k_i are the vertex degree and the reciprocal transmission of vertices in the orbit V_i, respectively. Then*

$$H_{rs}(G, x) = \frac{nd}{2}x^{n+k-1},$$

where d and k are the degree and the reciprocal transmission of each vertex of G respectively.

Proof Applying Theorem 4.1 and Lemma 4.2, we get the result. □

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