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On the Involute D̃-scroll in Euclidean 3-Space

Şeyda Kılıçoğlu

(Baskent University, Ankara, Turkey)

Süleyman ŞENYURT

(Ordu University, Ordu, Turkey)

E-mail: seyda@baskent.edu.tr, senyurtsuleyman@hotmail.com

Abstract: In this paper we consider two special ruled surfaces associated to a space curve α with curvature $k_1 \neq 0$ and its involute curve β. We will define and work on $\tilde{D}-scroll$, which is known as the rectifying developable surface, of any curve α and the *involute* $\tilde{D}-scroll$ of the curve α. Also we have examined the normal vectors of these special ruled surfaces $\tilde{D}-scroll$ and *involute* $\tilde{D}-scroll$, associated to each other. Further, as an example, we examined the positions of the $\tilde{D}-scroll$ and the *involute* $\tilde{D}-scroll$ relative to each other of a cylindrical helix.

Key Words: Darboux vector, involute curve, ruled surface, helix.

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§1. Introduction and Preliminaries

Deriving curves based on the other curves is a subject in geometry. Involute-evolute curves, Bertrand curves are this kind of curves. By using the similar method we produce a new ruled surface based on the other ruled surface. It is well-known that, if a curve is differentiable in an open interval, at each point, a set of mutually orthogonal unit vectors can be constructed. And these vectors are called Frenet frame or moving frame vectors. The rates of these frame vectors along the curve define curvatures of the curves. The set, whose elements are frame vectors and curvatures of a curve α, is called Frenet-Serret apparatus of the curves. Involvents play a part in the construction of gears. The evolute is the locus of the centers of tangent circles of the given planar curve, [12]. Let Frenet vector fields be V_1, V_2, V_3 of the curve α and let the first and second curvatures of the curve α be k_1 and k_2, respectively. The quantities $\{V_1, V_2, V_3, k_1, k_2\}$ are collectively Frenet-Serret apparatus of the curves. Also the Darboux vector provides a concise way of interpreting curvature k_1 and torsion k_2 geometrically; curvature is the measure of the rotation of the Frenet frame about the binormal unit vector, and torsion is the measure of the rotation of the Frenet frame about the tangent unit vector. For any unit speed curve α,

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in terms of the Frenet-Serret apparatus, the Darboux vector D can be expressed as, [10]

$$D(s) = k_2(s)V_1(s) + k_1(s)V_3(s). \tag{1.1}$$

Let a vector field be

$$\tilde{D}(s) = \frac{k_2}{k_1}(s)V_1(s) + V_3(s) \tag{1.2}$$

along $\alpha(s)$ under the condition that $k_1(s) \neq 0$ and it is called the modified Darboux vector field of α [6]. We will work on the special ruled surface $\tilde{D} - scroll$ which is also the rectifying developable surface, of the curves *evolute* α, and *involute* β. Further we will define and introduce *involute* $\tilde{D} - scroll$ of α. Also *involute* $\tilde{D} - scroll$ of α will be examined in terms of the Frenet-Serret apparatus of the curve α.

A ruled surface can always be described (at least locally) as the set of points swept by a moving straight line. A ruled surface is one which can be generated by the motion of a straight line ([2],[3]). Choosing a directrix on the surface, i.e. a smooth unit speed curve $\alpha(s)$ orthogonal to the straight lines, and then choosing $v(s)$ to be unit vectors along the curve in the direction of the lines, the velocity vector α_s and v satisfy

$$\left\langle \alpha', v \right\rangle = 0.$$

To illustrate the current situation, we bring here the famous example of L. K. Graves, [4], so called the $B - scroll$. The special ruled surfaces $B - scroll$ over null curves with null rulings in 3-dimensional Lorentzian space form has been introduced by L. K. Graves. The Gauss map of B-scrolls has been examined in [1]. The properties of the $B - scroll$ are also examined and $n - space$ and in Lorentzian $3 - space$ and $n - space$ with time-like directrix curve and null rulings ([7], [8], [9]). Also *involutive* $B - scroll$ (binormal scroll) of the curve α is defined as in the following definition and examined in [13]. In [14] the Differential geometric elements of the Involute D̃-scroll is examined too.

Definition 1.1 *Let α and β be the curves . The tangent lines to a curve α generate a surface called the tores of α. If the curve β which lies on the tores intersect the tangent lines orthogonally is called an involute of α. If a curve β is an involute of α, then by definition α is an evolute of β. Hence given α, its evolutes are the curves whose tangent lines intersect α orthogonally.*

If the curve $\beta(s)$ is the involute of $\alpha(s)$, then we have that

$$\beta(s) = \alpha(s) + (c - s)V_1(s) \tag{1.3}$$

and $d(\alpha(s), \beta(s)) = |c - s|$, where $\forall s \in I$, $c = constant$, [5].

Theorem 1.1([5]) *α, $\beta \subset \boldsymbol{E}^3$, α and β are the arclengthed curves with the arcparametres. Let β be the involute of the curve α. The quantities $\{V_1, V_2, V_3, k_1, k_2\}$ and $\{V_1^*, V_2^*, V_3^*, k_1^*, k_2^*\}$ are collectively Frenet-Serret apparatus of the curve α and the involute β, respectively. The Frenet-Serret apparatus of the involute β, in terms of the Frenet-Serret apparatus of the its* *evolute curve α are*

$$\begin{cases} V_1^* = V_2, \\ V_2^* = -\frac{k_1}{\sqrt{k_1^2+k_2^2}}V_1 + \frac{k_2}{\sqrt{k_1^2+k_2^2}}V_3, \\ V_3^* = \frac{k_2}{\sqrt{k_1^2+k_2^2}}V_1 + \frac{k_1}{\sqrt{k_1^2+k_2^2}}V_3 \end{cases} \tag{1.4}$$

$$k_1^* = \frac{\sqrt{k_1^2+k_2^2}}{(c-s)k_1}, \quad k_2^* = -\frac{k_2^2\left(\frac{k_1}{k_2}\right)'}{(c-s)k_1\left(k_1^2+k_2^2\right)}. \tag{1.5}$$

Corollary 1.1 *If the second curvature k_2 of the curve $\alpha(s)$ is a nonzero constant, i.e. $k_2' = 0$, then second curvature of involute β is*

$$k_2^* = \frac{-k_1' k_2}{(c-s)\, k_1\left(k_1^2+k_2^2\right)}. \tag{1.6}$$

Theorem 1.2 *Let β be the involute of the curve α. Let the first and second curvatures of the curve α be k_1 and k_2, respectively. The modified Darboux vector field of the involute β is*

$$\tilde{D}^* = \frac{k_2}{\sqrt{k_1^2+k_2^2}}V_1 - \frac{k_2^2\left(\frac{k_1}{k_2}\right)'}{\left(k_1^2+k_2^2\right)^{\frac{3}{2}}}V_2 + \frac{k_1}{\sqrt{k_1^2+k_2^2}}V_3. \tag{1.7}$$

Proof Since the definition of the modified Darboux vector field $\tilde{D}^* = \frac{k_2^*}{k_1^*}V_1^* + V_3^*$ and Theorem 1.2 it is trivial. □

Corollary 1.2 *If the second curvature k_2 of the curve α is constant but not equal to zero, then $k_2' = 0$. Hence, we have that the modified Darboux vector field of the involute β is*

$$\tilde{D}^* = \frac{k_2}{\sqrt{k_1^2+k_2^2}}V_1 - \frac{k_1' k_2}{\left(k_1^2+k_2^2\right)^{\frac{3}{2}}}V_2 + \frac{k_1 V_3}{\sqrt{k_1^2+k_2^2}}. \tag{1.8}$$

Definition 1.2([13]) *Let α and β be the arclengthed curves. Let $\beta(s)$ be the involute of the curve $\alpha(s)$. The equation*

$$\varphi^*(s,v) = \beta(s) + vV_3^*(s) \tag{1.9}$$

is the parametrization of the ruled surface which is called involutive $V_3^* -$ scroll (binormal scroll) of the curve β.

Definition 1.3 *The ruled surface*

$$\varphi(s,u) = \alpha(s) + u\tilde{D}(s)$$
$$\varphi(s,u) = \alpha(s) + u\frac{k_2}{k_1}(s)V_1(s) + uV_3(s)$$

is the parametrization of the ruled surface which is called rectifying developable surface of the *curve α in [6]. Here, it is referred to as $\tilde{D}-$scroll cause of generator vector is modified Darboux vector field $\tilde{D}$.*

Definition 1.4 *Let the curve β be involute of α, hence*

$$\varphi^{*}(s,v)=\beta(s)+v\left(\frac{k_2^*}{k_1^*}(s)V_1^*\left(s\right)+V_3^*\left(s\right)\right) \tag{1.4}$$

is the parametrization of the $\tilde{D}-$scroll of involute β. Further this rectifying developable surface is called involute $\tilde{D}-$ scroll of α.

We can write the parametrization of the $\tilde{D}-$ *scroll* of *involute* β, in terms of the Frenet-Serret apparatus of the curve α, as in the following theorem. Hence it can be called *involute* $\tilde{D}-$ *scroll* of the curve α.

§2. On the Involute $\tilde{\mathbf{D}}$-scroll in Euclidean 3-Space

In this section to determine the positions of the $\tilde{D}-$*scroll* and *involute* $\tilde{D}-$*scroll*, we questioned their normal vector vectors.

Theorem 2.1 *If β is the involute curve of the curve α, then the parametrization of the involute $\tilde{D}-$ scroll of the curve α in terms of the Frenet-Serret apparatus of the curve α is*

$$\varphi^{*}\left(s,v\right)=\alpha+\left(\lambda+\frac{vk_2}{\sqrt{k_1^2+k_2^2}}\right)V_1-\frac{k_2^2\left(\frac{k_1}{k_2}\right)'}{(k_1^2+k_2^2)^{\frac{3}{2}}}V_2+\frac{vk_1}{\sqrt{k_1^2+k_2^2}}V_3. \tag{2.1}$$

Proof Substituting equation (2.1) into equations (1.3) and (1.8), the proof is complete. □

Corollary 2.1 *If the second curvature k_2 of the curve α is constant but not equal to zero, then $k_2'=0$. Hence, the parametrization of involute $\tilde{D}-$ scroll is*

$$\varphi^{*}\left(s,v\right)=\alpha+\left(c-s+\frac{vk_2}{\sqrt{k_1^2+k_2^2}}\right)V_1-\frac{vk_1'k_2}{(k_1^2+k_2^2)^{\frac{3}{2}}}V_2+\frac{vk_1}{\sqrt{k_1^2+k_2^2}}V_3. \tag{2.2}$$

Theorem 2.2 *The equation $\varphi\left(s,u\right)=\alpha\left(s\right)+u\tilde{D}\left(s\right)$ is the parametrization of the ruled surfaces which is called $\tilde{D}-$ scroll. Then the normal vector field N of ruled surface $\tilde{D}-$ scroll is*

$$N=V_2. \tag{2.3}$$

Proof We can calculate that

$$\tilde{D}'=\left(\frac{k_2}{k_1}\right)'V_1.$$

For the surface

$$\varphi(s,u) = \alpha(s) + u\tilde{D}(s)$$

the vectors

$$\begin{aligned}\varphi_s &= \left(1 + u\left(\frac{k_2}{k_1}\right)'\right)V_1,\\ \varphi_u &= \tilde{D}(s) = \left(\frac{k_2}{k_1}(s)V_1(s) + V_3(s)\right)\end{aligned}$$

are not a system of orthogonal vectors. Hence we will use the Gram–Schmidt orthogonalization. Let us take

$$e_1 = \frac{\varphi_s}{\|\varphi_s\|} = \mp V_1,\ \ e_2 = \frac{\varphi_u - \frac{\langle\varphi_s,\varphi_u\rangle}{\langle\varphi_s,\varphi_s\rangle}\varphi_s}{\|\varphi_u - \frac{\langle\varphi_s,\varphi_u\rangle}{\langle\varphi_s,\varphi_s\rangle}\varphi_s\|} = V_3$$

Since $\{e_1, e_2\}$ is a system of orthogonal vectors, normal vector field N is

$$N = e_1 \wedge e_2 = V_2. \qquad \square$$

Theorem 2.3 *The normal vector field of involute $\tilde{D} - scroll$ of the curve $\alpha$is*

$$N^* = \frac{-k_1V_1 + k_2V_3}{(k_1^2 + k_2^2)^{\frac{1}{2}}} \tag{2.5}$$

Proof We have already get the equation of the *involute* $\tilde{D} - scroll$ of the curve α. Also we know that the normal vector field N^* of any $\tilde{D} - scroll$ is

$$N^* = V_2^*.$$

So normal vector field N^* of the *involute* $\tilde{D} - scroll$ is

$$N^* = \frac{-k_1V_1 + k_2V_3}{(k_1^2 + k_2^2)^{\frac{1}{2}}}. \qquad \square$$

Lets examine the positions of the ruled surface $\tilde{D} - scroll$ and the *involute* $\tilde{D} - scroll$. Based on their normal vector fields.

Theorem 2.4 *The ruled surface $\tilde{D} - scroll$ and the involute $\tilde{D} - scroll$ of the curve α are perpendicular surfaces.*

Proof Using the orthogonality condition; $\langle N, N^*\rangle = 0$,

$$\left\langle V_2, \frac{-k_1V_1 + k_2V_3}{(k_1^2 + k_2^2)^{\frac{1}{2}}}\right\rangle = 0$$

it is easy to say that, the normal vector field N of $\tilde{D} - scroll$ of the curve α and the normal vector field $N^*$of *involute* $\tilde{D}-scroll$ of the curve α are perpendicular. then

$$N^* \perp N. \qquad \square$$

Theorem 2.5 *The tangent vector fields* $V_1(s)$ *and* $V_1^*(s)$ *are perpendicular, then the ruled surface* $\tilde{D}-scrolls$ *along to the curves* α *and the* β *are perpendicular surfaces.*

Helix is one of the fascinating curve in science and nature. A helix which lies on the cylinder is called cylindrical helix or general helix. A curve α with $k_2(s) \neq 0$ is called a cylindrical helix if the tangent lines of make a constant angle with a fixed direction. If the curve is a general helix, the ratio of the first curvature of the curve to the torsion of the curve must be constant. Further we call a curve a circular helix if both $k_2(s) \neq 0$ and $k_1(s)$ are constant.

Corollary 2.2 *If the curve* α *is a cylindrical helix, then the involute* β *of the curve* α *is a planar curve.*

Proof It has been known that the curve $\alpha(s)$ is a cylindrical helix if and only if $\left(\frac{k_1}{k_2}\right)=d$ is constant, then $\left(\frac{k_1}{k_2}\right)'=0$, also $k_2^*=0$. □

Lemma 2.1([6]) *For a the ruled surface* $\varphi(s,u)=\alpha(s)+u\eta(s)$ *and its unit speed curve* α*, with* $k_1 \neq 0$*, the following are equivalent:*

(1) *The ruled surface is a non-singular surface;*
(2) α *is a cylindrical helix;*
(3) *The ruled surface of* α *is a cylindrical surface.*

Theorem 2.6 *The involute* $\tilde{D}-scroll$ *of the curve of* α *is a cylindrical surface, if*

$$\frac{\left(k_1^2+k_2^2\right)^{\frac{3}{2}}}{k_2^2\left(\frac{k_1}{k_2}\right)'} = constant. \tag{2.5}$$

Proof Let α not be a cylindrical helix , $k_1 \neq 0$, *involute* β is a helix. If *involute* β is a cylindrical helix, then $\frac{k_1^*}{k_2^*}$ is constant. Hence

$$\frac{k_1^*}{k_2^*} = -\frac{\left(k_1^2+k_2^2\right)^{\frac{3}{2}}}{k_2^2\left(\frac{k_1}{k_2}\right)'}$$

is constant. Where $\dfrac{k_1}{k_2} \neq constant$. Cause of the Lemma 2.1; if $-\frac{\left(k_1^2+k_2^2\right)^{\frac{3}{2}}}{k_2^2\left(\frac{k_1}{k_2}\right)'} = constant$, then *involute* $\tilde{D}-scroll$ is a cylindrical surface. □

Theorem 2.7 *Let* β *be involute of* α*, if the curve* α *is a cylindrical helix, the angle between the modified Darboux vector field of the involute* $-$ *evolute pair* (α,β) *is a nonzero constant. It* *is the function of* $d = \frac{k_1}{k_2}$ *as in the following equality*

$$\left\langle \tilde{D}^{*}, \tilde{D} \right\rangle = \frac{\sqrt{d^2+1}}{d}; \qquad d = \frac{k_1}{k_2}. \tag{2.6}$$

Proof Since

$$\left\langle \tilde{D}^{*}, \tilde{D} \right\rangle = \frac{\sqrt{k_1^2+k_2^2}}{k_1}$$

it is trivial. □

Theorem 2.8 *Let the involute curve of a cylindrical helix* α *be* $\beta(s) = \alpha(s) + (c-s)V_1(s)$, *then the involute* $\tilde{D} - scroll$ *of a cylindrical helix* α *with* $\frac{k_1}{k_2} = d$, *is*

$$\varphi^{*}(s,v) = \alpha + \left(c - s + \frac{v}{\sqrt{d^2+1}} \right) V_1 + \frac{vd}{\sqrt{d^2+1}} V_3. \tag{2.7}$$

Proof If $\alpha(s)$ is a cylindrical helix, then $\left(\frac{k_1}{k_2}\right)' = 0$. Using the equation $\tilde{D}^{*}$,

$$\varphi^{*}(s,v) = \alpha + \left(c - s + \frac{vk_2}{\sqrt{k_1^2+k_2^2}} \right) V_1 + \frac{vk_1}{\sqrt{k_1^2+k_2^2}} V_3$$

is the *involute* $\tilde{D} - scroll$ of a cylindrical helix α. □

Example 2.1 Lets examine the $\tilde{D} - scroll$ of the a cylindrical helix

$$\alpha(s) = (a\cos ws, a\sin ws, bws) \ , a > 0,$$

with curvatures $k_1 = w^2 a$ and $k_2 = w^2 b$. Here $\frac{k_1}{k_2} = \frac{a}{b} = d$ and $w^2 = \frac{1}{a^2+b^2}$ we have the parametrization of the $\tilde{D} - scroll$ of the cylindrical helix α

$$\varphi(s,u) = \alpha(s) + u\tilde{D}(s)$$
$$\varphi(s,u) = \alpha(s) + u\frac{k_2}{k_1} V_1(s) + uV_3(s)$$

$$\varphi(s,u) = \begin{pmatrix} a\cos ws - \sin ws \left(ubw - abuw^3\right), a\sin ws + \cos ws \left(buw - abuw^3\right), \\ bws + u\left(\frac{b^2}{a} w + a^2 w^3\right) \end{pmatrix}$$

where

$$V_3 = \left(abw^3 \sin ws, -abw^3 \cos ws, a^2 w^3\right).$$

Example 2.2 The $\tilde{D} - scroll$ of the a cylindrical helix α,

$$\alpha(s) = \left(\cos\frac{s}{\sqrt{2}}, \sin\frac{s}{\sqrt{2}}, \frac{s}{\sqrt{2}} \right) \ , a = 1 > 0$$

with curvatures $k_1 = k_2 = \frac{1}{2}$, we have the parametrization of the $\tilde{D}-scroll$ along the cylindrical helix α

$$\begin{aligned}\varphi(s,u) &= \alpha(s) + u\frac{k_2}{k_1}(s)V_1(s) + uV_3(s)\\ &= \left(\cos\frac{s}{\sqrt{2}} - \frac{u}{2\sqrt{2}}\sin\frac{s}{\sqrt{2}}, \sin\frac{s}{\sqrt{2}} + \frac{u}{2\sqrt{2}}\cos\frac{s}{\sqrt{2}}, \frac{s}{\sqrt{2}} + u\frac{3}{2\sqrt{2}}\right).\end{aligned}$$

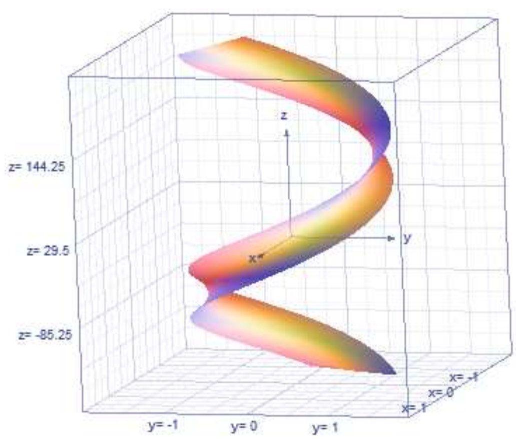

FIGURE 1 $\tilde{D}-$ scroll cylindrical helix

Example 2.3 The parametrization of the *involute* $\tilde{D} - scroll$ along the cylindrical helix $\alpha(s) = (a\cos ws, a\sin ws, bws)$, $a > 0$,with curvatures $k_1 = w^2a$ and $k_2 = w^2b$. Let find the *involute* $\tilde{D} - scroll$ along the cylindrical helix α have parametrization as with $\frac{k_1}{k_2} = \frac{a}{b} = d$, is

$$\varphi^*(s,v) = \alpha + \left(c - s + \frac{v}{\sqrt{d^2+1}}\right)V_1 + \frac{vd}{\sqrt{d^2+1}}V_3$$

$$\begin{aligned}\varphi^*(s,v) &= \alpha + \left(\lambda + \frac{vk_2}{\sqrt{k_1^2+k_2^2}}\right)V_1 + \frac{vk_1}{\sqrt{k_1^2+k_2^2}}V_3\\ &= \begin{bmatrix}\cos ws & -\sin ws & 0\\ \sin ws & \cos ws & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix} a \\ \left[\lambda aw + vabw^2\left(1-aw^2\right)\right] \\ 0\end{bmatrix}\\ &+ \begin{bmatrix}0\\0\\cbw + vw^2\left(b^2+a^3w^2\right)\end{bmatrix}.\end{aligned}$$

Corollary 2.3 *The involute $\tilde{D}-scroll$ of the a cylindrical helix α can be produced by rigid motion.*

Example 2.4 The *involute* $\tilde{D}-scroll$ of the a cylindrical helix α,

$$\alpha(s)=\left(\cos\frac{s}{\sqrt{2}},\sin\frac{s}{\sqrt{2}},\frac{s}{\sqrt{2}}\right)\ ,a=1>0$$

with curvatures $k_1=k_2=\frac{1}{2}$, we have the parametrization of the *involute* $\tilde{D}-scroll$ along the cylindrical helix α,

$$\varphi^*(s,v)=\left(\cos\frac{s}{\sqrt{2}}-\left[\frac{c-s}{\sqrt{2}}+\frac{v}{4}\right]\sin\frac{s}{\sqrt{2}},\sin\frac{s}{\sqrt{2}}+\left[\frac{c-s}{\sqrt{2}}+\frac{v}{4}\right]\cos\frac{s}{\sqrt{2}},\frac{c}{\sqrt{2}}+\frac{3v}{4}\right).$$

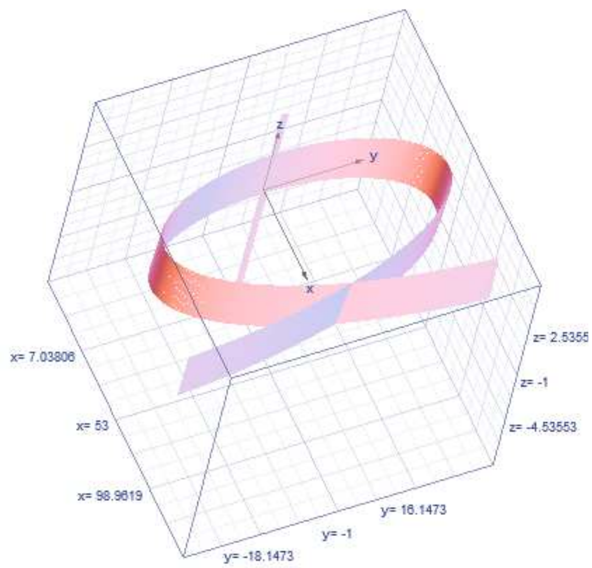

FIGURE 2 $\tilde{D}-$ scroll cylindrical helix

References

[1] Alias L.J., Ferrandez A., Lucas P. and Merono, M. A., On the Gauss map of B-scrolls, *Tsukuba J. Math.*, 22,371-377, 1998.

[2] Do Carmo M. P., *Differential Geometry of Curves and Surfaces*, Prentice-Hall, 1976.

[3] Eisenhart, Luther P., *A Treatise on the Differential Geometry of Curves and Surfaces*, Dover, 2004.

[4] Graves L.K., Codimension one isometric immersions between Lorentz spaces, *Trans. Amer. Math. Soc.*, 252, 367-392, 1979.

[5] Hacısalihoğlu H.H., *Diferensiyel Geometri*, Cilt 1, Ínönü Üniversitesi Press, 1994.

[6] Izumiya S. and Takeuchi N., *Special curves and Ruled surfaces*, Beitr¨age zur Algebra und Geometrie Contributions to Algebra and Geometry, 44(1),203-212, 2003.

[7] Kılıçoğlu Ş., *n-Boyutlu Lorentz uzayinda B-scrollar,* Doktora tezi, Ankara Üniversitesi Fen Bilimleri Enstitüsü, Ankara, 2006.

[8] Kılıçoğlu Ş., On the B-scrolls with time-like directrix in 3-dimensional Minkowski Space, *Beykent University Journal of Science and Technology,*2(2), 206-217, 2008.

[9] Kılıçoğlu Ş., On the generalized B-scrolls with p th degree in n- dimensional Minkowski space and striction (central spaces), *Sakarya Üniversitesi Fen Edebiyat Dergisi,* 10(2), 15-29 (ISSN: 1301-3769).,10(2), 15-29, 2008.

[10] Koenderink, J., *Solid Shape*, MIT Press, Cambridge, MA, 1990.

[11] Lipschutz M.M., *Differential Geometry*, Schaum's Outlines, 1969.

[12] Springerlink, *Encyclopaedia of Mathematics*, Springer-Verlag, Berlin Heidelberg New York 2002.

[13] Şenyurt S., On involute $B-$ scroll a new view, *University of Ordu Journal of Science and Techonology* ,4(1), 59-74, 2014.

[14] Şenyurt S., Kılıçoğlu Ş., On the Differential geometric elements of the Involute $\tilde{\text{D}}$-scroll in Euclidean 3-space, *Advances in Applied Clifford Algebras*, doi:10.1007/s00006-015-0535-z, 25(4), 977-988, 2015.