International J.Math. Combin. Vol.3(2018), 42-54

Blaschke Approach to the Motion of a Robot End-Effector

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Abstract: In this paper, we examine the motion of a robot end-effector by using the Blaschke approach of a ruled surface generated by a line fixed in the robot end-effector. In this way, we determine time dependent linear and angular differential properties of motion such as velocity and acceleration which play important roles in robot trajectory planning. Moreover, motion of a robot end-effector which can be represented by a right conoid and an additional parameter called spin angle is investigated as a practical example.

Key Words: Blaschke frame, curvature theory, robot end-effector, robot trajectory planning, ruled surface.

AMS(2010): 53A05, 53A17, 53A25.

§1. Introduction

In robotics, a robot end-effector is a device at the end of a robotic arm. Robot end-effectors are widely used in transportation, welding industry, medical science, military and many other areas. Recently, they can be used in the research areas which have critical importance of accurate motion such as surgical operations and bomb disposal. So accurate trajectory planning of a robot end-effector becomes an important research area of robotics. In this area, one of the most interesting problems is determining time dependent differential properties of motion of a robot end-effector which are linear and angular velocities and accelerations. These differential properties play important roles in robot trajectory planning.

As a robot end-effector moves on a specified trajectory in space, a line fixed in the end-effector generates a ruled surface [13]. There is an important relationship between time dependent properties of motion of the robot end-effector and differential geometry of the ruled surface. By using this relationship, Ryuh and Pennock proposed a method based on the curvature theory of a ruled surface generated by a line fixed in the end-effector to determine linear and angular properties of motion [12, 13, 14]. After that, this research area was also studied in Lorentzian space. Ekici et al. examined motion of a robot end-effector in Lorentzian space by using the curvature theory of timelike ruled surface with timelike ruling [7]. Ayyıldız and Turhan also

[1]Received January 29, 2018, Accepted August 8, 2018.

determined differential properties of motion of a robot end-effector whose trajectory is a null curve [3].

On the other hand, there is also an efficient relationship between directed lines and dual unit vectors. This relationship known as "E. Study mapping" or "transference principle" which can be stated as: "there exists one-to-one correspondence between the directed lines in line space and dual unit vectors in dual space" [11, 16]. By the aid of this correspondence, W. Blaschke defined a frame called Blaschke frame on a ruled surface by taking directed lines pass through striction curve of the ruled surface instead of real unit vectors used in Frenet frame of ruled surface. He also gave some invariants which characterize the shape of a ruled surface. Several authors used Blaschke frame in their researches concerning with kinematics, spatial mechanisms and many other areas [1, 2, 18].

In this paper, we use the relationships between kinematic, ruled surfaces and dual vector algebra. First, we represent motion of a robot end-effector on a specified trajectory in space as a ruled surface generated by a line fixed in the end-effector and an additional parameter called spin angle. We define a dual frame called dual tool frame on robot end-effector in order to obtain a relationship between Blaschke frame of the ruled surface, which is used to study the differential geometry of a ruled surface by means of dual quantities, and time dependent differential properties of robot end-effector. By using this relation, we determine time dependent differential properties of motion of a robot end-effector which are linear (translational) and angular (rotational) velocities and accelerations. These differential properties have important roles in robot trajectory planning. In this method, we use just a dual vector called dual instantaneous rotation vector of dual tool frame to determine the differential properties. So, this method has more advantages than traditional methods which based on matrix representations in terms of being simple and systematic.

§2. Preliminaries

In this section, we give a brief summary of basic concepts for the reader who is not familiar with dual numbers, dual vectors and dual space.

As introduced by W. Clifford, a dual number can be defined as $\bar{a} = a + \varepsilon a^*$, where a and a^* are real numbers and called real part and dual part of dual number $\bar{a}$, respectively, and ε is dual unit which satisfies the condition $\varepsilon^2 = 0$, [17]. The set of all dual numbers can be denoted by $\mathbb{D}$. Addition and multiplication of two dual numbers $\bar{a} = a + \varepsilon a^*$ and $\bar{b} = b + \varepsilon b^*$ can be defined as

$$\bar{a} + \bar{b} = (a + b) + \varepsilon(a^* + b^*)$$

and

$$\bar{a}\,\bar{b} = ab + \varepsilon(ab^* + a^*b)$$

respectively [4, 10]. The set $\mathbb{D}$ is a commutative ring, not a field. A function of a dual number $f(\bar{a})$ can be expanded in a Maclaurin series as

$$f(\bar{a}) = f(a + \varepsilon a^*) = f(a) + \varepsilon a^* f'(a),$$

where the prime indicates derivation of $f(a)$ with respect to a [5].

A dual vector can also be defined as $\tilde{a}= a+\varepsilon a^*$, where a and a^* are three dimensional vectors in real space and $\varepsilon^2 = 0$. The set of all dual vectors is a module over the ring $\mathbb{D}$ and is called dual space or $\mathbb{D}$-module, denoted by $\mathbb{D}^3$, [15]. Dual scalar and vector products of two dual vectors $\tilde{a}= a+\varepsilon a^*$ and $\tilde{b}= b+\varepsilon b^*$ can be defined as

$$\left\langle \tilde{a}, \tilde{b} \right\rangle = \langle a, b\rangle + \varepsilon \left(\langle a, b^*\rangle + \langle a^*, b\rangle\right)$$

and

$$\tilde{a} \times \tilde{b} = a \times b + \varepsilon \left(a \times b^* + a^* \times b\right)$$

respectively [16]. The norm of a dual vector $\tilde{a}$ can also be given by [10, 17]

$$\|\tilde{a}\| = \|a\| + \varepsilon \frac{\langle a, a^*\rangle}{\|a\|}, \quad (a \neq 0).$$

If $\|\tilde{a}\| = 1$, then $\tilde{a}$ is called a dual unit vector. The set

$$S^2 = \left\{\tilde{a}= a+\varepsilon a^* \mid \ \|\tilde{a}\| = 1; \ \ a, a^* \in \mathbb{R}^3\right\}$$

is called dual unit sphere.

Theorem 2.1([8]) *The set of all directed straight lines in $\mathbb{R}^3$ are in one-to-one correspondence with the set of all points of the dual unit sphere in $\mathbb{D}^3$.*

A dual angle between two oriented lines in three dimensional real space can be defined as $\bar{\theta} = \theta + \varepsilon\ \theta^*$, where θ and θ^* are the real angle and the shortest distance between these lines, respectively, [4].

§3. A Robot End-Effector and its Dual Tool Frame

In this section, we introduce tool frame of a robot end-effector which consists of three mutually perpendicular unit vectors described by Ryuh and Pennock [13] in detail. Then, we represent motion of a robot end-effector by using a ruled surface generated by a line fixed in the end-effector and an additional parameter called spin angle. By taking three lines instead of three unit vectors, we define a dual frame called dual tool frame on robot end-effector which will be used to study the motion.

The tool frame consists of three orthogonal unit vectors strictly attached to robot end-effector. These are; orientation vector O which is a unit vector in the direction of the gripper motion as it opens and closes, approach vector A which is a unit vector in the direction normal to the palm of robot end-effector, and normal vector N which is a unit vector in the direction perpendicular to the plane of the gripper (see Figure 1), [12]. The origin of the tool frame is called tool center point and denoted by TCP. By using tool frame and tool center point, location and orientation of a robot end-effector can be described completely.

As a robot end-effector moves on a specified trajectory in space, a line called tool line fixed in the end-effector which passes through TCP and whose direction vector is parallel to the orientation vector O generates a ruled surface [12]. This ruled surface can be expressed as

$$X(t, v) = \alpha(t) + v\, u(t),$$

where α is the specified trajectory which robot end-effector follows (directrix of the ruled surface), u is a unit vector called ruling parallel to the orientation vector O, t is the parameter of time, and v is an arbitrary parameter.

During motion, the approach vector A may not be always perpendicular to the ruled surface. As seen in Figure 1, there may be an angle between the approach vector A and the surface normal vector on the directrix which is denoted by S_n. This angle is called spin angle and denoted by η [12]. Thus, a robot end-effector motion which has six degrees of freedom in space can be completely described by a ruled surface generated by a line in robot end-effector which provides five independent parameters and a spin angle.

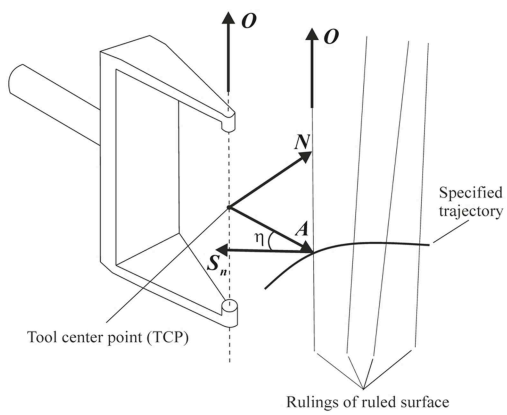

Figure 1 Robot end-effector and spin angle

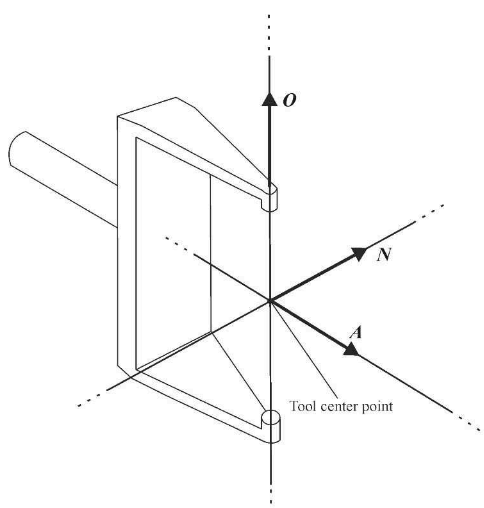

Figure 2 Dual tool frame of a robot end-effector

Now, we define dual tool frame by taking three directed lines instead of three unit vectors of the tool frame. These lines pass through the TCP and their direction vectors are the orientation vector O, the approach vector A and the normal vector N, respectively. From Theorem 2.1, these lines correspond to three dual unit vector which can be called dual orientation vector, dual approach vector and dual normal vector and can be denoted by $\tilde{O}$, $\tilde{A}$ and $\tilde{N}$, respectively (see Figure 2).

§4. Blaschke Approach to the Motion

In this section, we give Blaschke frame of a ruled surface generated by a line fixed in the robot end-effector. By relating Blaschke frame and dual tool frame, we determine linear and angular differential properties of motion. Furthermore, we give corollaries for some special cases of motion.

From Theorem 2.1, it can be said that a ruled surface can be represented by a dual unit vector based on a real parameter. So, we can consider the ruled surface generated by motion of robot end-effector as a dual unit vector $\tilde{u}(t) = u(t) + \varepsilon u^*(t)$, where u is ruling of the ruled surface, u^* is moment vector of u about the origin, t is the parameter of time, and $\varepsilon^2 = 0$. The moment vector can be found as $u^* = c \times u$, where c is striction curve of the ruled surface satisfies the condition that $\langle c', u' \rangle = 0$, [6]. In this paper, we consider the case without $u(t) = c_1$ which means ruled surface is a cylinder and $u^*(t) = c_1$ which means ruled surface is a cone, where c_1 is a constant. In order to simplify formulations, arc-length parameter of the striction curve denoted by s can be used instead of the parameter of time t and it can be obtained as

$$s(t) = \int_0^t \left\| \frac{dc}{dt} \right\| \, dt.$$

The Blaschke frame of a ruled surface is defined on striction curve and it consists of three orthogonal dual unit vectors given as follows [4]:

$$\tilde{u}_1 = \tilde{u} \ , \ \ \tilde{u}_2 = \frac{\tilde{u}_1'}{\bar{p}} \ , \ \ \tilde{u}_3 = \tilde{u}_1 \times \tilde{u}_2,$$

where $\bar{p} = p + \varepsilon p^* = \|\tilde{u}_1'\|$, $\tilde{u}_2$ and $\tilde{u}_3$ are normal line and tangent line of the ruled surface on the striction curve, respectively, and the prime indicates the derivation with respect to s, [4]. The derivative formulae of Blaschke frame can be given as

$$\frac{d}{ds}\begin{bmatrix} \tilde{u}_1 \\ \tilde{u}_2 \\ \tilde{u}_3 \end{bmatrix} = \begin{bmatrix} 0 & \bar{p} & 0 \\ -\bar{p} & 0 & \bar{q} \\ 0 & -\bar{q} & 0 \end{bmatrix} \begin{bmatrix} \tilde{u}_1 \\ \tilde{u}_2 \\ \tilde{u}_3 \end{bmatrix}, \qquad (1)$$

where $\bar{q} = q + \varepsilon q^* = \dfrac{\det(\tilde{u}_1, \tilde{u'}_1, \tilde{u''}_1)}{\|\tilde{u}_1'\|^2}$. $\bar{p}$ and $\bar{q}$ which are called the Blaschke's invariants characterize the shape of a ruled surface. If $p = 0$, ruled surface is a cylinder; if $p^* = 0$, ruled surface is a developable ruled surface which is a surface that can be flattened onto a plane without distortion; if $q = 0$, all rulings of ruled surface are parallel to a plane; if $\bar{q} = 0$, ruled surface consists of binormal vectors of a curve, [4].

Let $\bar{\varphi} = \varphi + \varepsilon\, \varphi^*$ be a dual angle between dual unit vectors $\tilde{A}$ and $\tilde{u}_2$, where $\varphi = \eta + \sigma$ is real angle, where η is the spin angle mentioned in Section 3 and σ is an angle between two normal vectors of ruled surface, one is on the directrix and other is on the striction curve, and φ^* is the shortest distance from striction curve to directrix, i.e., $\varphi^* = \dfrac{\langle \alpha', u' \rangle}{\|u'\|^2}$ (see Figure 3).

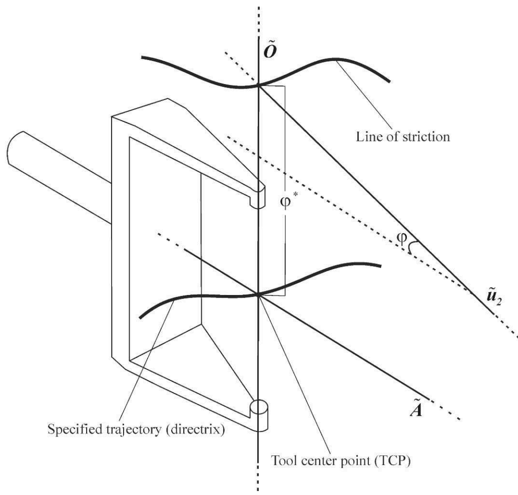

Figure 3 Dual angle between the dual unit vectors $\tilde{A}$ and $\tilde{u}_2$

By the aid of dual angle $\bar{\varphi}$, we can give dual tool frame relative to Blaschke frame in matrix form as

$$\begin{bmatrix} \tilde{O} \\ \tilde{A} \\ \tilde{N} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\bar{\varphi} & \sin\bar{\varphi} \\ 0 & -\sin\bar{\varphi} & \cos\bar{\varphi} \end{bmatrix} \begin{bmatrix} \tilde{u}_1 \\ \tilde{u}_2 \\ \tilde{u}_3 \end{bmatrix}. \tag{2}$$

By differentiating equation (2) and substituting equation (1) into the result, we have

$$\begin{bmatrix} \tilde{O}' \\ \tilde{A}' \\ \tilde{N}' \end{bmatrix} = \begin{bmatrix} 0 & \bar{p} & 0 \\ -\bar{p}\cos\bar{\varphi} & -\bar{\delta}\sin\bar{\varphi} & \bar{\delta}\cos\bar{\varphi} \\ \bar{p}\sin\bar{\varphi} & -\bar{\delta}\cos\bar{\varphi} & -\bar{\delta}\sin\bar{\varphi} \end{bmatrix} \begin{bmatrix} \tilde{u}_1 \\ \tilde{u}_2 \\ \tilde{u}_3 \end{bmatrix},$$

where $\bar{\delta} = \bar{\varphi}' + \bar{q}$. By using equation (2), derivative formulas of the dual tool frame can be obtained in terms of itself in matrix form as

$$\begin{bmatrix} \tilde{O}' \\ \tilde{A}' \\ \tilde{N}' \end{bmatrix} = \begin{bmatrix} 0 & \bar{p}\cos\bar{\varphi} & -\bar{p}\sin\bar{\varphi} \\ -\bar{p}\cos\bar{\varphi} & 0 & \bar{\delta} \\ \bar{p}\sin\bar{\varphi} & -\bar{\delta} & 0 \end{bmatrix} \begin{bmatrix} \tilde{O} \\ \tilde{A} \\ \tilde{N} \end{bmatrix}.$$

From the above matrix equality, dual instantaneous rotation vector of the dual tool frame which plays an important role to determine both linear and angular differential properties of motion of a robot end-effector can be obtained as

$$\tilde{w}_O = \bar{\delta}\,\tilde{O} + \bar{p}\sin\bar{\varphi}\,\tilde{A} + \bar{p}\cos\bar{\varphi}\,\tilde{N}.$$

By using equation (2), the dual instantaneous rotation vector can also be expressed in terms of the Blaschke frame as

$$\tilde{w}_O = \bar{\delta}\,\tilde{u}_1 + \bar{p}\,\tilde{u}_3. \tag{3}$$

This dual vector is similar to dual Pfaff vector in terms of playing role in motion. The dual Pfaff vector is considered as dual velocity vector in dual spherical motion (see ref. [9]). So, we can consider the dual instantaneous rotation vector of dual tool frame $\tilde{w}_O$ as dual velocity vector of the motion of robot end-effector.

The dual tool frame attached to robot end-effector moves along unit direction $\dfrac{\tilde{w}_O}{\|\tilde{w}_O\|}$ with dual angle $\|\tilde{w}_O\|$. This dual motion contains both rotational and translational motion in real space. The real and dual parts of the dual vector $\tilde{w}_O$ correspond to instantaneous angular velocity and instantaneous linear velocity, respectively. By separating equation (3) into the real and dual parts, these velocity vectors can be found as follows

$$w_O = \delta\,u_1 + p\,u_3, \tag{4}$$

and

$$w_O^* = \delta u_1^* + \delta^* u_1 + p u_3^* + p^* u_3. \tag{5}$$

In order to find dual acceleration vector of the motion, we should differentiate dual velocity vector. By differentiating equation (3) and using equation (1), the dual acceleration vector can be obtained in terms of the Blaschke frame as

$$\tilde{w}_O' = \bar{\delta}'\,\tilde{u}_1 + \bar{\varphi}'\bar{p}\,\tilde{u}_2 + \bar{p}'\tilde{u}_3, \tag{6}$$

where the prime indicates differentiation with respect to s. By separating equation (6) into the real and dual parts, instantaneous angular acceleration vector and instantaneous linear acceleration vector can be found as

$$w_O' = \delta'\,u_1 + \varphi' p\,u_2 + p' u_3 \tag{7}$$

and

$$w_O^{*'} = \delta'\,u_1^* + \delta^{*'} u_1 + \varphi' p\,u_2^* + (\varphi' p^* + \varphi^{*'} p)u_2 + p' u_3^* + p^{*'} u_3, \tag{8}$$

respectively. Thus, linear and angular velocities and accelerations which are important differential properties of motion of a robot end-effector are found in terms of the parameter s which is the arc-length parameter of striction curve of the generating ruled surface. In order to determine time dependent differential properties, the vectors given in equations (4), (5), (7) and (8) should be related to the parameter of time. Now, we give time dependent linear and angular differential properties of motion of a robot end-effector as corollaries.

Corollary 4.1 *Let the motion of a robot end-effector be represented by a ruled surface* $X(t,v) = \alpha(t) + v\ u(t)$ *and a spin angle* η*, where* α *is specified trajectory of robot end-effector,* u *is a unit vector parallel to the orientation vector* O*, and* t *is the parameter of time. Angular and linear velocities of robot end-effector can be given, respectively, as*

$$v_A = w_O\ \dot{s} \tag{9}$$

and

$$v_L = w_O^*\ \dot{s}, \tag{10}$$

where w_O *and* w_O^* *are given by equations (4) and (5), respectively, and the dot indicates differentiation with respect to time, i.e.,* $\dot{s} = \dfrac{ds}{dt}$.

Corollary 4.2 *Let the motion of a robot end-effector be represented by a ruled surface* $X(t,v) = \alpha(t) + v\ u(t)$ *and a spin angle* η*, where* α *is specified trajectory of robot end-effector,* u *is a unit vector parallel to the orientation vector* O*, and* t *is the parameter of time. Angular and linear accelerations of the robot end-effector can be given, respectively, as*

$$a_A = w_O\ \ddot{s} + w_O^{'}\dot{s}^2 \tag{11}$$

and

$$a_L = w_O^*\ \ddot{s} + w_O^{*'}\dot{s}^2, \tag{12}$$

where $w_O^{'}$ *and* $w_O^{*'}$ *are as given by equations (7) and (8), respectively.*

Now, we consider some special cases of motion of a robot end-effector and give some corollaries about these cases.

Case 1. As a robot end-effector moves on a specified trajectory in real space, spin angle η may be constant. Then, the derivative of the spin angle is equal to zero. For this case, by substituting the value of spin angle into equations (4), (5), (7), and (8), and by rearranging these equations, we can give time dependent linear and angular differential properties of the motion of a robot end-effector as in the following corollaries.

Corollary 4.3 *Let the motion of a robot end-effector be represented by a ruled surface* $X(t,v) = \alpha(t) + v\ u(t)$ *and a spin angle* η*, where* α *is specified trajectory of the robot end-effector,* u *is a unit vector parallel to the orientation vector* O*, and* t *is the parameter of time. If the spin angle* η *is a constant, then angular and linear velocities of robot end-effector can be given as*

$$v_A = ((\sigma' + q)\ u_1 + p\ u_3)\ \dot{s}$$

and

$$v_L = ((\sigma' + q)u_1^* + \delta^* u_1 + p u_3^* + p^* u_3)\ \dot{s},$$

respectively.

Corollary 4.4 *Let the motion of a robot end-effector be represented by a ruled surface* $X(t,v) =$

$\alpha(t) + v\ u(t)$ and a spin angle η, where α is the specified trajectory of the robot end-effector, u is a unit vector parallel to the orientation vector O, and t is the parameter of time. If the spin angle η is constant, then angular and linear accelerations of robot end-effector can be respectively given as

$$\begin{aligned} a_A &= ((\sigma' + q)\ u_1 + p\ u_3)\ \ddot{s} + ((\sigma'' + q')\ u_1 + \sigma' p u_2 + p' u_3)\ \dot{s}^2, \\ a_L &= ((\sigma' + q) u_1^* + \delta^* u_1 + p u_3^* + p^* u_3)\ \ddot{s} \\ &\quad + ((\sigma'' + q')\ u_1^* + \delta^{*'} u_1 + \sigma' p\ u_2^* + (\sigma' p^* + \varphi^{*'} p) u_2 + p' u_3^* + p^{*'} u_3) \dot{s}^2. \end{aligned}$$

Case 2. A specified trajectory which robot end-effector follows may be striction curve of ruled surface generated by a line fixed in the robot end-effector. Namely, directrix and striction curve of generating ruled surface may be the same curve. Then, the angle σ which is the angle between two normal vectors on directrix and striction curve and the distance between these curves are equal to zero. For this case, by rearranging equations (4), (5), (7), and (8), we can give time dependent linear and angular differential properties of the motion of a robot end-effector as in the following corollaries.

Corollary 4.5 *Let the motion of a robot end-effector be represented by a ruled surface $X(t, v) = \alpha(t) + v\ u(t)$ and a spin angle η, where α is specified trajectory of the robot end-effector, u is a unit vector parallel to the orientation vector O, and t is the parameter of time. If the specified trajectory is also the striction curve of the ruled surface, then angular and linear velocities of robot end-effector can be given as*

$$v_A = ((\eta' + q)\ u_1 + p\ u_3)\ \dot{s}$$

and

$$v_L = ((\eta' + q) u_1^* + q^* u_1 + p u_3^* + p^* u_3)\ \dot{s},$$

respectively.

Corollary 4.6 *Let the motion of a robot end-effector be represented by a ruled surface $X(t, v) = \alpha(t) + v\ u(t)$ and a spin angle η, where α is specified trajectory of robot end-effector, u is a unit vector parallel to the orientation vector O, and t is the parameter of time. If the specified trajectory is also the striction curve of the ruled surface, then angular and linear accelerations of robot end-effector can be given as*

$$a_A = ((\eta' + q)\ u_1 + p\ u_3)\ \ddot{s} + ((\eta'' + q')\ u_1 + \eta' p u_2 + p' u_3)\ \dot{s}^2$$

and

$$\begin{aligned} a_L &= ((\eta' + q) u_1^* + q^* u_1 + p u_3^* + p^* u_3)\ \ddot{s} \\ &\quad + ((\eta'' + q')\ u_1^* + q^{*'} u_1 + \eta' p\ u_2^* + \eta' p^* u_2 + p' u_3^* + p^{*'} u_3) \dot{s}^2, \end{aligned}$$

respectively.

Case 3. Ruled surface generated by a line fixed in a robot end-effector may be a developable ruled surface (except a cylinder and a cone). So, the dual part of Blaschke's invariant $\bar{p}$ is equal to zero, i.e., $p^* = 0$. For this case, by making the necessary arrangement in equations (4), (5), (7), and (8), we can give time dependent linear and angular differential properties of the motion of a robot end-effector as in the following corollaries.

Corollary 4.7 *Let the motion of a robot end-effector be represented by a ruled surface $X(t, v) = \alpha(t) + v\ u(t)$ and a spin angle η, where α is specified trajectory of robot end-effector, u is a unit vector parallel to the orientation vector O, and t is the parameter of time. If the ruled surface is developable, then angular and linear velocities of robot end-effector can be given as*

$$v_A = ((\eta' + q)\ u_1 + p\ u_3)\ \dot{s}$$

and

$$v_L = ((\eta' + q)u_1^* + \delta^* u_1 + p u_3^*)\ \dot{s},$$

respectively.

Corollary 4.8 *Let the motion of a robot end-effector be represented by a ruled surface $X(t, v) = \alpha(t) + v\ u(t)$ and a spin angle η, where α is specified trajectory of robot end-effector, u is a unit vector parallel to the orientation vector O, and t is the parameter of time. If the ruled surface is a developable, then angular and linear accelerations of the robot end-effector can be given as*

$$a_A = ((\eta' + q)\ u_1 + p\ u_3)\ \ddot{s} + ((\eta'' + q')\ u_1 + \eta' p u_2 + p' u_3)\ \dot{s}^2$$

and

$$a_L = ((\eta' + q)u_1^* + \delta^* u_1 + p u_3^*)\ \ddot{s} + ((\eta'' + q')\ u_1^* + {\delta^*}' u_1 + \eta' p\ u_2^* + {\varphi^*}' p\ u_2 + p' u_3^*)\dot{s}^2,$$

respectively.

§5. An Example

Let the motion of a robot end-effector be represented a right conoid given by the equation $X(t, v) = (v\cos t,\ v\sin t,\ 2\sin t)$ and a spin angle η, where t is the parameter of time (see Figure 4). Directrix and ruling of the right conoid are $\alpha(t) = (0,\ 0,\ 2\sin t)$ and $u(t) = (\cos t,\ \sin t,\ 0)$, respectively. Since $\langle \alpha', u' \rangle = 0$, directrix and striction curve of the ruled surface are the same curve, i.e., $c = \alpha$. The right conoid can be expressed as a dual unit vector

$$\tilde{u}(s) = u(s) + \varepsilon u^*(s) = (\cos t,\ \sin t,\ 0) + \varepsilon(-2\sin^2 t,\ \sin 2t,\ 0)$$

where s is the arc-length parameter of striction curve. The first dual unit vector of Blaschke frame is $\tilde{u}_1(s) = \tilde{u}(s)$. The second and third dual unit vectors of Blaschke frame can be found as

$$\tilde{u}_2(s) = (-\sin t,\ \cos t,\ 0) + \varepsilon(-\sin 2t,\ -2\sin^2 t,\ 0)$$

and

$$\tilde{u}_3(s) = (0,\ 0,\ 1)\,,$$

respectively. The Blaschke's invariants can be obtained as $\bar{p} = p + \varepsilon p^* = 1 + \varepsilon 2\cos t$ and $\bar{q} = q + \varepsilon q^* = 0 + \varepsilon 0$. Let $\bar{\varphi} = \varphi + \varepsilon\varphi^*$ be a dual angle between dual unit vectors $\tilde{A}$ and $\tilde{u}_2$, where φ and φ^* are the real angle and the shortest distance between the lines correspond to the dual vectors $\tilde{A}$ and $\tilde{u}_2$, respectively. Since directrix is also striction curve, the distance between these curves equals to zero, i.e., $\varphi^* = 0$, and the angle between two normal vectors on directrix and on striction curve equals to zero, i.e., $\sigma = 0$. Thus, we have $\bar{\varphi} = \eta + \varepsilon 0$. Dual instantaneous rotation vector of dual tool frame can be found as

$$\tilde{w}_O = w_O + \varepsilon w_O^* = (\eta' \cos s,\ \eta' \sin s,\ 1) + \varepsilon(-2\eta' \sin^2 s,\ \eta' \sin 2s,\ 2\cos s).$$

Angular and linear velocities of the robot end-effector can be obtained by substituting w_O and w_O^* into equations (9) and (10), respectively. By differentiating the dual instantaneous rotation vector, we get

$$\begin{aligned} \tilde{w}_O' = w_O^{'} + \varepsilon w_O^{*'} \quad &= \quad (\eta'' \cos s - \eta' \sin s,\ \eta'' \sin s + \eta' \cos s) \\ &+\varepsilon(-2\eta'' \sin^2 s - 2\eta' \sin 2s,\ \eta'' \sin 2s + 2\eta' \cos 2s,\ -2\sin s). \end{aligned}$$

Angular and linear accelerations of the robot end-effector can also be obtained by substituting $w_O^{'}$ and $w_O^{*'}$ into equations (11) and (12), respectively.

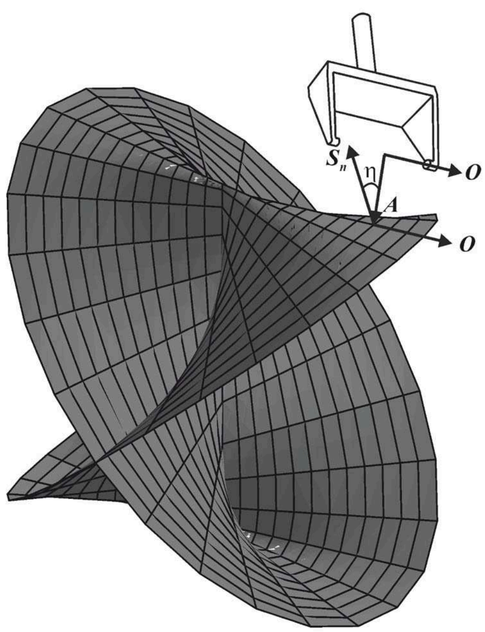

Figure 4 Motion of a robot end-effector
which can be represented by a right conoid and a spin angle η

§6. Conclusions

In this paper, time dependent differential properties which are linear and angular velocities and accelerations of the motion of a robot end-effector are determined by using Blaschke approach of a ruled surface generated by a line fixed in the end-effector. These differential properties are important information in robot trajectory planning. By the aid of Blaschke approach which uses dual numbers and dual vectors as basic tool, both linear and angular differential properties can be determined. This is achieved only by using a dual vector which is dual instantaneous rotation vector of dual tool frame. Thus, Blaschke approach presents a simple and systematic method to study motion of a robot end-effector without redundant parameter. This paper does not contain a computer program which compares Blaschke approach and conventional method of scalar curvature theory of ruled surfaces in real space. This is the subject of ongoing research works. However, it is believed that the presented method based on Blaschke approach will reduce computation time in computer programming for determining differential properties of motion and contribute to research area of robot trajectory planning.

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