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On m-Neighbourly Irregular Instuitionistic Fuzzy Graphs

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Abstract: In this paper, m-neighbourly irregular intuitionistic fuzzy graphs and m-neighbourly totally irregular intuitionistic fuzzy graphs are defined. Relation between m-neighbourly irregular intuitionistic fuzzy graph and m-neighbourly totally irregular intuitionistic fuzzy graph are discussed. An m-neighbourly irregularity on intuitionistic fuzzy graphs whose underlying crisp graphs are cycle C_n, a path P_n are studied.

Key Words: d_m-degree and total d_m-degree of a vertex in intuitionistic fuzzy graph, irregular intuitionistic fuzzy graph, neighbourly irregular intuitionistic fuzzy graph, neighbourly totally irregular intuitionistic fuzzy graph.

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§1. Introduction

In 1965, Lofti A. Zadeh [20] introduced the concept of fuzzy subset of a set as method of representing the phenomena of uncertainty in real life situation. K.T.Attanassov [1] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. K.T.Atanassov added a new component(which determines the degree of non-membership) in the definition of fuzzy set. The fuzzy sets give the degree of membership of an element in a given set(and the non-membership degree equals one minus the degree of membership), while intuitionistic fuzzy sets give both a degree of membership and a degree of non-membership which are more-or-less independent from each other, the only requirement is that the sum of these two degrees is not greater than one. Intuitionistic fuzzy sets have been applied in a wide variety of fields including computer science, engineering, mathematics, medicine, chemistry and economics [1,2].

Azriel Rosenfeld introduced the concept of fuzzy graph in 1975 ([12]). It has been growing fast and has numerous application in various fields. Bhattacharya [5] gave some remarks on fuzzy graphs, and some operations on fuzzy graphs were introduced by Morderson and Peng [11]. Krassimir T Atanassov [2] introduced the intuitionistic fuzzy graph theory. R.Parvathi and M.G.Karunambigai [10] introduced intuitionistic fuzzy graphs as a special case of Atanassov's

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IFG and discussed some properties of regular intuitionistic fuzzy graphs [7]. M. G. Karunambigai and R. Parvathi and R. Buvaneswari introduced constant intuitionistic fuzzy graphs [8].

M. Akram, W. Dudek [3] introduced the regular intuitionistic fuzzy graphs. M.Akram and Bijan Davvaz [4] introduced the notion of strong intuitionistic fuzzy graphs and discussed some of their properties. R.Jahirhussain and S.Yahyu Mohammed discussed Properties on intuitionistic fuzzy graphs [6]. A.Nagoorgani and S.Shajitha Begum introduced the degree, order and size in intuitionistic fuzzy graphs [9].

N.R.Santhi Maheswari and C.Sekar introduced d_2- degree of a vertex in fuzzy graphs and introduced 2-neighbourly irregular fuzzy graphs and 2-neighbourly totally irregular fuzzy graphs [13]. Also, they introduced d_m-degree, total d_m-degree, of a vertex in fuzzy graphs and introduced an m-neighbourly irregular fuzzy graphs [14, 17]. S.Ravinarayanan and N.R.Santhi Maheswari introduced m-neighbourly irreular bipolar fuzzy graphs [15].

N.R.Santhi Maheswari and C.Sekar introduced d_m- degree of a vertex in intuitionistic fuzzy graphs and introduced $(m,(c_1,c_2))$-regular fuzzy graphs and totally $(m,(c_1,c_2))$-regular fuzzy graphs [19]. These motivates us to introduce m-neighbourly irregular intuitionistic fuzzy graphs and totally m-neighbourly irregular intuitionistic fuzzy graphs.

§2. Preliminaries

We present some known definitions related to fuzzy graphs and intuitionistic fuzzy graphs for ready reference to go through the work presented in this paper.

Definition 2.1([11]) *A fuzzy graph $G:(\sigma,\mu)$ is a pair of functions (σ,μ), where $\sigma:V\to$[0,1] is a fuzzy subset of a non empty set V and $\mu:VXV\to$[0, 1] is a symmetric fuzzy relation on σ such that for all u,v in V, the relation $\mu(u,v)\le\sigma(u)\wedge\sigma(v)$ is satisfied. A fuzzy graph G is called complete fuzzy graph if the relation $\mu(u,v)=\sigma(u)\wedge\sigma(v)$ is satisfied.*

Definition 2.2([14]) *Let $G:(\sigma,\mu)$ be a fuzzy graph. The d_m-degree of a vertex u in G is $d_m(u)=\sum\mu^m(uv)$, where $\mu^m(uv)=sup\{\mu(uu_1)\wedge\mu(u_1u_2)\wedge\ldots,\mu(u_{m-1}v):u,u_1,u_2,\ldots,u_{m-1},v$ is the shortest path connecting u and v of length $m\}$. Also, $\mu(uv)=0$, for uv not in E.*

Definition 2.3([14]) *Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*:(V,E)$. The total d_m-degree of a vertex $u\in V$ is defined as $td_m(u)=\sum\mu^m(uv)+\sigma(u)=d_m(u)+\sigma(u)$.*

Definition 2.4([14]) *Let $G:(V,E)$ be a fuzzy graph on $G^*(V,E)$. Then G is said to be an m-neighbourly irregular fuzzy graph if every two adjacent vertices in G have distinct d_m-degrees.*

Definition 2.5([14]) *Let $G:(V,E)$ be a bipolar fuzzy graph on $G^*(V,E)$. Then G is said to be an m-neighbourly totally irregular fuzzy graph if every two adjacent vertices in G have distinct total d_m-degrees.*

Definition 2.6([8]) *An intuitionistic fuzzy graph with underlying set V is defined to be a pair $G=(V,E)$ where*

(i) $V=\{v_1,v_2,v_3,\cdots,v_n\}$ such that $\mu_1:V\to[0,1]$ and $\gamma_1:V\to[0,1]$ denote the

degree of membership and nonmembership of the element $v_i \in V$*,* $(i = 1, 2, 3, \cdots, n)$ *such that* $0 \le \mu_1(v_i) + \gamma_1(v_i) \le 1$*;*

(ii) $E \subseteq V \times V$*, where* $\mu_2 : V \times V \to [0, 1]$ *and* $\gamma_2 : V \times V \to [0, 1]$ *are such that* $\mu_2(v_i, v_j) \le min\{\mu_1(v_i), \mu_1(v_j)\}$ *and* $\gamma_2(v_i, v_j) \le \max\{\gamma_1(v_i), \gamma_1(v_j)\}$ *and* $0 \le \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \le 1$ *for every* $(v_i, v_j) \in E$*,* $(i, j = 1, 2, \cdots, n)$*.*

Definition 2.7([8]) *If* $v_i, v_j \in V \subseteq G$*, the* μ*-strength of connectedness between two vertices* v_i *and* v_j *is defined as* $\mu_2^{\infty}(v_i, v_j) = \sup\{\mu_2^k(v_i, v_j) : k = 1, 2, \cdots, n\}$ *and* γ*-strength of connectedness between two vertices* v_i *and* v_j *is defined as* $\gamma_2^{\infty}(v_i, v_j) = \inf\{\gamma_2^k(v_i, v_j) : k = 1, 2, \cdots, n\}$*.*

If u *and* v *are connected by means of paths of length* k *then* $\mu_2^k(u, v)$ *is defined as* $\sup\{\mu_2(u, v_1) \wedge \mu_2(v_1, v_2) \wedge \cdots \wedge \mu_2(v_{k-1}, v) \colon (u, v_1, v_2, \cdots, v_{k-1}, v) \in V\}$ *and* $\gamma_2^k(u, v)$ *is defined as* $\inf\{\gamma_2(u, v_1) \vee \gamma_2(v_1, v_2) \vee \cdots \vee \gamma_2(v_{k-1}, v) \colon (u, v_1, v_2, \cdots, v_{k-1}, v) \in V\}$*.*

Definition 2.8([8]) *Let* $G = (V, E)$ *be an Intuitionistic fuzzy graph on* $G^*(V, E)$*. Then the degree of a vertex* $v_i \in G$ *is defined by* $d(v_i) = (d_{\mu_1}(v_i), d_{\gamma_1}(v_i))$*, where* $d_{\mu_1}(v_i) = \sum \mu_2(v_i, v_j)$ *and* $d_{\gamma_1}(v_i) = \sum \gamma_2(v_i, v_j)$ *for* $v_i, v_j \in E$ *and* $\mu_2(v_i, v_j) = 0$ *and* $\gamma_2(v_i, v_j) = 0$ *for* $v_i, v_j \notin E$*.*

Definition 2.9([8]) *Let* $G = (V, E)$ *be an Intuitionistic fuzzy graph on* $G^*(V, E)$*. Then the total degree of a vertex* $v_i \in G$ *is defined by* $td(v_i) = (td_{\mu_1}(v_i), td_{\gamma_1}(v_i))$*, where* $td_{\mu_1}(v_i) = d\mu_1(v_i) + \mu_1(v_i)$ *and* $td_{\gamma_1}(v_i) = d\gamma_1(v_i) + \gamma_1(v_i)$*.*

Definition 2.10([19]) *Let* $G = (V, E)$ *be an intuitionistic fuzzy graph on* $G^*(V, E)$*. Then the* d_m *- degree of a vertex* $v \in G$ *is defined by* $d_{(m)}(v) = (d_{(m)\mu_1}(v), d_{(m)\gamma_1}(v))$*, where* $d_{(m)\mu_1}(v) = \sum \mu_2^{(m)}(u, v)$ *and* $\mu_2^{(m)}(u, v) = \sup\{\mu_2(u, u_1) \wedge \mu_2(u_1, u_2) \wedge \cdots \wedge \mu_2(u_{m-1}, v) : u, u_1, u_2, \cdots, u_{m-1}v$ *is the shortest path connecting* u *and* v *of length* $m\}$ *and* $d_{(m)\gamma_1}(v) = \sum \gamma_2^{(m)}(u, v)$*, where* $\gamma_2^{(m)}(u, v) = \inf\{\gamma_2(u, u_1) \vee \gamma_2(u_1, u_2) \vee \cdots \vee \gamma_2(u_{m-1}, v) : u, u_1, u_2, \cdots, u_{m-1}v$ *is the shortest path connecting* u *and* v *of length* $m\}$*. The minimum* d_m*-degree of* G *is* $\delta_m(G) = \wedge\{(d_{(m)\mu_1}(v), d_{(m)\gamma_1}(v)) : v \in V\}$ *and the maximum* d_m*-degree of* G *is* $\Delta_m(G) = \vee\{(d_{(m)\mu_1}(v), d_{(m)\gamma_1}(v)) : v \in V\}$*.*

Definition 2.11([19]) *Let* $G : (V, E)$ *be an intuitionistic fuzzy graph on* $G^*(V, E)$*. If all the vertices of* G *have same* d_m*- degree then* G *is said to be an* $(m, (c_1, c_2))$*- regular intuitionistic fuzzy graph.*

Definition 2.12([19]) *Let* $G = (V, E)$ *be an intuitionistic fuzzy graph on* $G^*(V, E)$*. Then the total* d_m*-degree of a vertex* $v \in G$ *is defined by* $td_{(m)}(v) = (td_{(m)\mu_1}(v), td_{(m)\gamma_1}(v))$*, where* $td_{(m)\mu_1}(v) = d_{(m)\mu_1}(v) + \mu_1(v)$ *and* $td_{(m)\gamma_1}(v) = d_{(m)\gamma_1}(v) + \gamma_1(v)$*. The minimum* td_m*-degree of* G *is* $t\delta_m(G) = \wedge\{(td_{(m)\mu_1}(v),\ td_{(\gamma_1}(v)) : v \in V\}$*. The maximum* td_m*-degree of* G *is* $t\Delta_m(G) = \vee\{(td_{(m)\mu_1}(v),\ td_{(m)\gamma_1}(v)) : v \in V\}$*.*

§3. m-Neighbourly Irregular intuitionistic Fuzzy Graphs

Definition 3.1 *let* G *be an intuitionistic fuzzy graph on* $G^*(V, E)$*. Then* G *is said to be an* m*-neighbourly irregular intuitionistic fuzzy graph if every two adjacent vertices in* G *have distinct*

d_m- degrees.

Example 3.2 Consider an intuitionistic fuzzy graph on $G^* : (V, E)$.

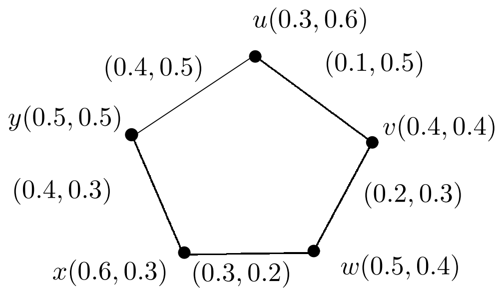

Figure 1

Here, $d(u) = (0.5, 1)$; $d(v) = (0.3, 0.8)$; $d(w) = (0.5, 0.5)$; $d(x) = (0.7, .5)$; $d(y) = (0.8, 0.8)$; $d_{(2)\mu_1}(u) = (0.1 \wedge 0.2) + (0.4 \wedge 0.4) = 0.1 + 0.4 = 0.5$; $d_{(2)\gamma_1}(u) = (0.5 \vee 0.2) + (0.4 \vee 0.4) = (0.5) + (0.4) = 0.9$; $d_{(2)}(u) = (0.5, 0.9)$; $d_{(2)}(v) = (0.3, 0.8)$; $d_{(2)}(w) = (0.4, 0.8)$; $d_{(2)}(x) = (0.6, 0.8)$; $d_{(2)}(y) = (0.4, 0.8)$.

Every pair of adjacent vertices in G have distinct degrees and distinct d_2- degrees. Hence G is m -neighbourly irregular intuitionistic fuzzy graph for m=1,2.

§4. m-Neighbourly Totally Irregular Intuitionistic Fuzzy Graphs

Definition 4.1 *Let G be a intuitionistic fuzzy graph on $G^*(V, E)$. Then G is said to be m-neighbourly totally irregular intuitionistic fuzzy graph if every two adjacent vertices in G have distinct total d_m- degrees.*

Example 4.2 Consider a intuitionistic fuzzy graph on $G^* : (V, E)$ in Figure 1, $td_2(u) = (0.8, 1.5), td_2(v) = (0.7, 1.2), td_2(w) = (0.9, 1.2), td_2(x) = (1.2, 1.1), td_2(y) = (0.9, 1.3)$. Every Pair of adjacent vertices in G have distinct total degrees and distinct total d_2- degrees. Hence G is m-neighbourly totally irregular intuitionistic fuzzy graph for m=1,2.

Remark 4.3 An m-neighbourly irregular instuitionistic fuzzy graph need not be m- neighbourly totally irregular intuitionistic fuzzy graph.

Example 4.4 For example consider $G = (V, E)$ be an intuitionistic fuzzy graph such that $G^*(V, E)$ is path on 6 vertices.

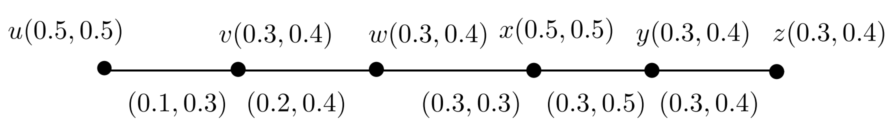

Figure 2

Here, $d_{(3)}(u) = (0.1, 0.4)$; $d_{(3)}(v) = (0.2, 0.5)$; $d_{(3)}(w) = (0.3, 0.5)$;$d_{(3)}(x) = (0.1, 0.4)$;$d_{(3)}(y) = (0.2, 0.5)$;$d_{(3)}(z) = (0.3, 0.5)$. Hence G is m- neighbourly irregular intuitionistic fuzzy graph. Here, $td_{(3)}(w) = td_{(3)}(x) = td_{(3)}(y) = (0.6, 0.9)$. Hence G is not m-neighbourly totally irregular intuitionistic fuzzy graph.

Remark 4.5 An m- neighbourly totally irregular intuitionistic fuzzy graph need not be m-neighbourly irregular intuitionistic fuzzy graph.

Example 4.6 Consider a intuitionistic fuzzy graph on $G^*(V, E)$.

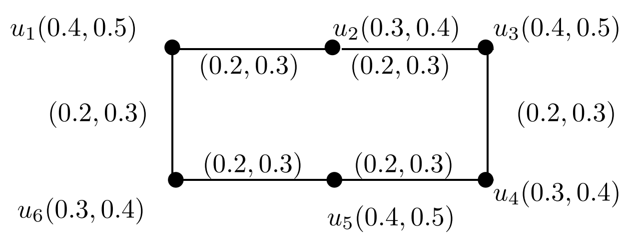

Figure.3

Here, $d_{(2)}(u_1) = d_{(2)}(u_2) = d_{(2)}(u_3) = d_{(2)}(u_4) = d_{(2)}(u_5) = d_{(2)}(u_6) = (0.4, 0.6)$. Hence G is not an m- neighbourly irregular intuitionistic fuzzy graph.

$td_{(2)}(u_1) = (0.8, 1.1)$; $td_{(2)}(u_2) = (0.7, 1)$; $td_{(2)}(u_3) = (0.8, 1.1)$; $td_{(2)}(u_4) = (0.7, 1)$; $td_{(2)}(u_5) = (0.8, 1.1)$; $td_{(2)}(u_6) = (0.7, 1)$.

Here, all adjacent vertices have distinct total d_m-degrees. Hence G is m-neighbourly totally irregular intuitionistic fuzzy graph.

Theorem 4.7 *If the membership values of adjacent vertices are distinct then $(m, (c_1, c_2))$-regular intuitionistic fuzzy graph is an m- neighbourly totally irregular intuitionistic fuzzy graph.*

Proof Let $G : (V, E)$ be an intuitionistic fuzzy graph on $G^*(V, E)$. If $(m, (c_1, c_2))$- regular intuitionistic fuzzy graph and the membership values of adjacent vertices are distinct, then d_m-degree of all vertices are the same $\Rightarrow d_m(v) = (c_1, c_2)$ for all $v \in G \Rightarrow$ total degrees of adjacent vertices are distinct. So G is an m-neighbourly totally irregular intuitionistic fuzzy graph. □

Theorem 4.8 *Let $G : (V, E)$ be an intuitionistic fuzzy graph on $G^*(V, E)$. If G is an m-neighbourly irregular intuitionistic fuzzy graph and A is a constant function then G is an m-neighbourly totally irregular intuitionistic fuzzy graph.*

Proof Let G be m- neighbourly irregular intuitionistic fuzzy graph. Then the d_m degree of every two adjacent vertices are distinct. Let u and v be two adjacent vertices of G with distinct degrees. Then $d_m(u) = (k_1, k_2)$ and $d_m(v) = (c_1, c_2)$, where $k_1 \neq c_1, k_2 \neq c_2$. Assume that $A(u) = A(v) = (r_1, r_2)$. Suppose $td_m(u) = td_m(v) \Rightarrow d_m(u) + A(u) = d_m(v) + A(v)$ $\Rightarrow (k_1, k_2 + (r_1, r_2) = (c_1, c_2) + (r_1, r_2) \Rightarrow (k_1 + r_1, k_2 + r_2 = (c_1 + r_1, c_2 + r_2) \Rightarrow k_1 + r_1 = c_1 + r_1$

and $k_2 + r_2 = c_2 + r_2 \Rightarrow k_1 = c_1$ and $k_2 = c_2$, which is a contradiction. So $td_m(u) \neq td_m(v)$. Thus every pair of adjacent vertices have distinct total $d_m$degree provided A is a constant function. This is true for every pair of adjacent vertices in G. Hence G is an m- neighbourly totally irregular intuitionistic fuzzy graph. □

Theorem 4.9 *Let G be an intuitionistic fuzzy graph on $G^*(V,E)$. If G is an m- neighbourly totally irregular intuitionistic fuzzy graph and A is constant function then G is an m- neighbourly irregular intuitionistic fuzzy graph.*

Proof The proof is similar to above theorem 4.8. □

Remark 4.10 The above two theorems jointly yield the following result. let $G : (V,E)$ be a intuitionistic fuzzy graph on $G^*(V,E)$. If A is constant function then G is m- neighbourly irregular intuitionistic fuzzy graph if and only if G is m- neighbourly totally irregular intuitionistic fuzzy graph.

Remark 4.11 Let $G : (V,E)$ be an intuitionistic fuzzy graph on $G^*(V,E)$. If G is both m-neighbourly irregular intuitionistic fuzzy graph and m- neighbourly totally irregular intuitionistic fuzzy graph then A need not be a constant function.

Theorem 4.12 *Let G be intuitionistic fuzzy graph on $G^*(V,E)$, a cycle of length n. If the edges takes positive membership values $c_1, c_2, \cdots, c_n$ and negative membership values $k_1, k_2, \cdots, k_n$ such that $c_1 < c_2 < \cdots < c_n$ and $k_1 > k_2 > \cdots > k_n$ then G is m- neighbourly irregular intuitionistic fuzzy graph.*

Proof Let the edges take membership values $c_1, c_2, \cdots, c_n$ and $k_1, k_2, \cdots, k_n$ such that $c_1 < c_2 < \cdots < c_n$ and $k_1 > k_2 > \cdots > k_n$. Then,

$$\begin{aligned}
d_{m\mu_1}(v_1) &= \mu_2(e_1) \wedge \mu_2(e_2) \wedge \cdots \mu_2(e_m) + \mu_2(e_{m+1}) \wedge \mu_2(e_{n-1})) \wedge \cdots \wedge \mu_2(e_{n-(m-1)}) \\
&= (c_1 \wedge c_2 \wedge \cdots \wedge c_m) + (c_n \wedge c_{n-1} \wedge \cdots \wedge c_{n-(m-1)}) \\
&= c_1 + c_{n-(m-1)}, \\
d_{(m)\gamma_1}(v_1) &= \gamma_2(e_1) \vee \gamma_2(e_2) \vee \cdots \vee \gamma_2(e_m) + \gamma_2(e_n) \vee \gamma_2(e_{n-1}) \vee \cdots \vee \gamma_2(e_{n-(m-1)}) \\
&= k_1 + k_{n-(m-1)}, \\
d_m(v_1) &= (c_1 + c_{n-(m-1)}, k_1 + k_{n-(m-1)}).
\end{aligned}$$

Similarly, $d_m(v_2) = (c_1 + c_2, k_1 + k_2)$. For $i = 3, 4, \cdots, n-1$, $d_{(m)\mu_1}(v_i) = \mu_2(e_i) \wedge \mu_2(e_{i+1}) \wedge \cdots \wedge \mu_2(e_{i+m}) + \mu_2(e_{i-1}) \wedge \mu_2(e_{i-2}) \wedge \cdots \mu_2(e_{n-(m-3)})$

$$\begin{aligned}
&\Rightarrow d_{(m)\mu_1}(v_i) = (c_i \wedge c_{i+1} \wedge \cdots \wedge c_{i+m}) + (c_{i-1} \wedge c_{i-2} \wedge \cdots \wedge c_{n-(m-3)}) \\
&\Rightarrow d_{(m)\mu_1}(v_i) = c_i + c_{n-(m-3)} \\
&\Rightarrow d_{(m)\gamma_1}(v_i) = \gamma_2(e_i) \vee \gamma_2(e_{i+1}) \vee \cdots \vee \gamma_2(e_{i+m}) + \gamma_2(e_{i-1}) \vee \gamma_2(e_{i-2}) \vee \cdots \vee \gamma_2(e_{n-(m-3)}) \\
&\Rightarrow d_{(m)\gamma_1}(v_i) = k_i \vee k_{i+1} \vee \cdots \vee k_{i+m} + k_{i-1} \vee k_{i-2} \vee \cdots \vee k_{n-(m-3)} \\
&\Rightarrow d_{(m)\gamma_1}(v_i) = k_i + k_{n-(m-3)} \\
&\Rightarrow d_m(v_i) = (c_i + c_{n-(m-3)}, (k_i + k_{n-(m-3)}) \\
&\Rightarrow d_{(m)\mu_1}(v_n) = \mu_2(e_n) \wedge \mu_2(e_1) \wedge \cdots \wedge \mu_2(e_{m-1}) + \mu_2(e_{n-1}) \wedge \mu_2(e_{n-2}) \wedge \cdots \wedge \mu_2(e_{n-m}) \\
&\Rightarrow d_{(m)\mu_1}(v_n) = (c_n \wedge c_1 \wedge \cdots \wedge c_{m-1}) + (c_{n-1} \wedge c_{n-2} \wedge \cdots \wedge c_{n-m}) \\
&\Rightarrow d_{(m)\mu_1}(v_n) = c_1 + c_{n-1}
\end{aligned}$$

$\Rightarrow d_{(m)\gamma_1}(v_n) = \gamma_2(e_n) \vee \gamma_2(e_1) \vee \cdots \vee \gamma_2(e_{m-1}) + \gamma_2(e_{n-1}) \vee \gamma_2(e_{n-2}) \vee \cdots \vee \gamma_2(e_{n-m})$
$\Rightarrow d_{(m)\gamma_1}(v_n) = k_n \vee k_1 \vee \cdots \vee k_{m-1} + k_{n-1} \vee k_{n-2} \vee \cdots \vee k_{n-m}$
$\Rightarrow d_{(m)\gamma_1}(v_n) = k_1 + k_{n-1}$
$\Rightarrow d_m(v_n) = (c_1 + c_{n-1}, k_1 + k_{n-1})$.

Hence G is m- neighbourly irregular intuitionistic fuzzy graph. □

Remark 4.13 The above theorem 4.12 does not hold for m- neighbourly totally irregular intuitionistic fuzzy graph.

Theorem 4.14 *Let G be an intuitionistic fuzzy Graph on $G^*(V,E)$, a path on n vertices. If the edges takes positive membership values $c_1, c_2, \ldots, c_n$ and negative membership values $k_1, k_2, \ldots, k_n$ such that $c_1 < c_2 < \cdots < c_n$ and $k_1 > k_2 > \cdots > k_n$ then G is m - neighbourly irregular intuitionistic fuzzy graph.*

Proof Let the edges take membership values $c_1, c_2, \cdots, c_n$ and $k_1, k_2, \cdots, k_n$ such that $c_1 < c_2 < \cdots < c_n$ and $k_1 > k_2 > \cdots > k_n$. Then,

$d_{(m)\mu_1}(v_1) = \mu_2(e_1) \wedge \mu_2(e_2) \wedge \cdots \wedge \mu_2(e_m) = c_1 \wedge c_2 \wedge \cdots \wedge c_m = c_1$.
$d_{(m)\gamma_1}(v_1) = \gamma_2(e_1) \vee \gamma_2(e_2) \vee \cdots \vee \gamma_2(e_m) = k_1 \vee k_2 \vee \cdots \vee k_m = k_1$.
$d_m(v_1) = (c_1, k_1)$.

Similarly, $d_m(v_2) = (c_2, k_2)$. For $i = 3, 4, \cdots, n-2$ $d_{(m)\mu_1}(v_i) = \mu_2(e_{i-1}) \wedge \mu_2(e_{i-2}) \wedge \cdots \wedge \mu_2(e_{i-m}) + \mu_2(e_i) \wedge \mu_2(e_{i+1}) \wedge \cdots \wedge \mu_2(e_{i+(m-1)}$

$\Rightarrow d_{(m)\mu_1}(v_i) = c_{i-1} \wedge c_{i-2} \wedge \cdots \wedge c_{i-m} + c_i \wedge c_{i+1} \wedge \cdots \wedge c_{i+m-1}$
$\Rightarrow d_{(m)\mu_1}(v_i) = c_{i-m} + c_i$
$\Rightarrow d_{(m)\gamma_1}(v_i) = \gamma_2(e_{i-1}) \vee \gamma_2(e_{i-2}) \vee \cdots \vee \gamma_2(e_{i-m}) + \gamma_2(e_i) \vee \gamma_2(e_{i+1}) \vee \cdots \vee \gamma_2(e_{i+(m-1)})$
$\Rightarrow d_{(m)\gamma_1}(v_i) = k_{i-1} \vee k_{i-2} \vee \cdots \vee k_{i-m} + k_i \vee k_{i+1} \vee \cdots \vee k_{i+m-1}$
$\Rightarrow d_{(m)\gamma_1}(v_i) = k_{i-m} + k_i$
$\Rightarrow d_m(v_i) = (c_{i-m} + c_i, k_{i-m} + k_i)$
$\Rightarrow d_{(m)\mu_1}(v_{n-1}) = \mu_2(e_{n-2}) \wedge \mu_2(e_{n-3}) \wedge \cdots \wedge \mu_2(e_{n-(m+1)})$
$\Rightarrow d_{(m)\mu_1}(v_{n-1}) = c_{n-2} \wedge c_{n-3} \wedge \cdots \wedge c_{n-(m+1)}$
$d_{(m)\mu_1}(v_{n-1}) = c_{n-(m+1)}$
$\Rightarrow d_{(m)\gamma_1}(v_{n-1}) = \gamma_2(e_{n-2}) \vee \gamma_2(e_{n-3}) \vee \cdots \vee \gamma_2(e_{n-(m+1)})$
$\Rightarrow d_{(m)\gamma_1}(v_{n-1}) = k_{n-2} \vee k_{n-3} \vee \cdots \vee k_{n-(m+1)} = k_{n-(m+1)}$
$\Rightarrow d_m(v_{n-1}) = (c_{n-(m+1)}, k_{n-(m+1)})$
$\Rightarrow d_{(m)\mu_1}(v_n) = \mu_2(e_{n-1}) \wedge \mu_2(e_{n-2}) \wedge \cdots \wedge \mu_2(e_{n-m})$
$d_{(m)\mu_1}(v_n) = c_{n-1} \wedge c_{n-2} \wedge \cdots \wedge c_{n-m}$
$\Rightarrow d_{(m)\mu_1}(v_n) = c_{n-m}$
$\Rightarrow d_{(m)\gamma_1}(v_n) = \gamma_2(e_{n-1}) \vee \gamma_2(e_{n-2}) \vee \cdots \vee \gamma_2(e_{n-m})$
$\Rightarrow d_{(m)\gamma_1}(v_n) = k_{n-1} \vee k_{n-2} \vee \cdots \vee k_{n-m}$
$\Rightarrow d_{(m)\gamma_1}(v_n) = k_{n-m}$
$\Rightarrow d_m(v_n) = (c - n - m, k_{n-m})$.

Hence G is m - neighbourly irregular intuitionistic fuzzy graph. □

Remark 4.15 Theorem 4.14 does not hold for m-neighbourly totally irregular intuitionistic fuzzy graph.

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