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On 4-Total Product Cordiality of Some Corona Graphs

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Abstract: Let f be a map from $V(G)$ to $\{0, 1, \cdots, k-1\}$ where k is an integer, $2 \leq k \leq |V(G)|$. For each edge uv, assign the label $f(u)f(v)$ (mod k). f is called a k-total product cordial labeling of G if $|ev_f(i) - ev_f(j)| \leq 1$, $i, j \in \{0, 1, \cdots, k-1\}$ where $ev_f(x)$ denotes the total number of vertices and edges labelled with x ($x = 0, 1, 2, \cdots, k-1$). We investigate the 4-Product cordial labeling behaviour of comb, double comb and subdivision of some corona graphs.

Key Words: Labelling, k-total product cordial labeling, Smarandachely k-total product cordial labeling, comb, double comb, crown.

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§1. Introduction

Throughout this paper we have considered finite, undirected and simple graphs only. The vertex set and edge set of a graph G are denoted by $V(G)$ and $E(G)$ respectively. The graph obtained by subdividing each edge of a graph G by a new vertex is denoted by $S(G)$. The corona $G_1 \odot G_2$ of two graphs G_1 and G_2 is obtained by taking one copy of G_1(which has p_1 vertices) and p_1 copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex in the i^{th} copy G_2 The notion of k-Total Product cordial labeling of graphs was introduced in [2]. In this paper we investigate the 4-Total Product cordial labeling behaviour of $P_n \odot K_1$, $P_n \odot 2K_1$, $S(P_n \odot K_1)$, $S(P_n \odot 2K_1)$, $S(C_n \odot K_1)$ and $S(C_n \odot 2K_1)$ Terms not defined here are used in the sense of Harary [1].

§2. k-Total Product Cordial Labeling

Definition 2.1 *Let f be a map from $V(G)$ to $\{0, 1, \cdots, k-1\}$ where k is an integer, $2 \leq k \leq |V(G)|$. For each edge uv, assign the label $f(u)f(v)$ (mod k). f is called a k-total product cordial labeling of G if $|ev_f(i) - ev_f(j)| \leq 1$, otherwise, a Smarandachely k-total product cordial labeling of G if $|ev_f(i) - ev_f(j)| \geq 2$ for $i, j \in \{0, 1, \cdots, k-1\}$, where $ev_f(x)$ denotes the total number of vertices and edges labelled with x ($x = 0, 1, 2, \cdots, k-1$).*

A graph with k-total product cordial labeling is called k-total product cordial graph.

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Now we investigate the 4-Total product cordiality of $P_n \odot K_1$ and $P_n \odot 2K_1$.

Theorem 2.2 *$P_n \odot K_1$ is 4-total product cordial.*

Proof Let $u_1 u_2 \cdots u_n$ be the path P_n and let v_i be the pendant vertices adjacent to u_i $(1 \leq i \leq n)$.

Case 1. n is even.

Define $f : V(P_n \odot K_1) \to \{0, 1, 2, 3\}$ by $f(u_1) = 0$,

$$\begin{array}{lcll} f(u_i) & = & 2, & 2 \leq i \leq \frac{n-2}{2} \\ f(u_{\frac{n-2}{2}+i}) & = & 3, & 1 \leq i \leq \frac{n}{2} \\ f(v_i) & = & 2 & 1 \leq i \leq \frac{n}{2} \\ f(v_{\frac{n}{2}+i}) & = & 3, & 1 \leq i \leq \frac{n}{2}. \end{array}$$

Clearly $ev_f(0) = ev_f(2) = ev_f(3) = n$ and $ev_f(1) = n - 1$. Hence f is a 4-total product cordial labeling.

Case 2. n is odd.

Define $f : V(P_n \odot K_1) \to \{0, 1, 2, 3\}$ by $f(u_1) = f(u_2) = 0$,

$$\begin{array}{lcll} f(u_i) & = & 2, & 3 \leq i \leq \frac{n-1}{2} \\ f(u_{\frac{n-1}{2}+i}) & = & 3, & 1 \leq i \leq \frac{n-1}{2} \\ f(v_i) & = & 2, & 1 \leq i \leq \frac{n+1}{2} \\ f(v_{\frac{n+1}{2}+i}) & = & 3, & 1 \leq i \leq \frac{n-1}{2}. \end{array}$$

Values of i	$ev_f(i)$
0	n
1	$n-1$
2	n
3	n

Table 1

Table 1 establish that f is a 4-total product cordial labeling. □

Theorem 2.3 *$P_n \odot 2K_1$is 4-total product cordial.*

Proof Let $u_1 u_2 \ldots u_n$ be the path P_n and let v_i and w_i be the pendant vertices adjacent to u_i $(1 \leq i \leq n)$.

Case 1. n is even.

Define $f : V(P_n \odot 2K_1) \to \{0,1,2,3\}$ by $f(u_1) = 0$,

$$\begin{aligned} f(u_i) &= 2, \quad 2 \le i \le \tfrac{n-2}{2} \\ f(u_{\frac{n-2}{2}+i}) &= 3 \quad 1 \le i \le \tfrac{n+2}{2} \\ f(v_i) &= 2, \quad 1 \le i \le \tfrac{n}{2} \\ f(v_{\frac{n}{2}+i}) &= 3 \quad 1 \le i \le \tfrac{n}{2} \\ f(w_i) &= 2, \quad 1 \le i \le \tfrac{n}{2} \\ f(w_{\frac{n}{2}+i}) &= 3, \quad 1 \le i \le \tfrac{n}{2}. \end{aligned}$$

Clearly $ev_f(0) = ev_f(2) = ev_f(3) = \frac{3n}{2}$ and $ev_f(1) = \frac{3n}{2} - 1$. Hence f is a 4-total product cordial labeling.

Case 2. n is odd.

Define $f : V(P_n \odot 2K_1) \to \{0,1,2,3\}$ by $f(u_1) = f(u_2) = 0$,

$$\begin{aligned} f(u_i) &= 2, \quad 3 \le i \le \tfrac{n-1}{2} \\ f(u_{\frac{n-1}{2}+i}) &= 3, \quad 1 \le i \le \tfrac{n}{2} \\ f(v_i) &= 2, \quad 1 \le i \le \tfrac{n+1}{2} \\ f(v_{\frac{n-1}{2}+i}) &= 3, \quad 1 \le i \le \tfrac{n-1}{2} \\ f(w_i) &= 2, \quad 1 \le i \le \tfrac{n-1}{2} \\ f(w_{\frac{n-1}{2}+i}) &= 3, \quad 1 \le i \le \tfrac{n+1}{2}. \end{aligned}$$

Values of i	$ev_f(i)$
0	$\frac{3n-1}{2}$
1	$\frac{3n-1}{2}$
2	$\frac{3n-1}{2}$
3	$\frac{3n+1}{2}$

Table 2

Table 2 shows that f is a 4-total product cordial labeling. □

Now we look in to the subdivision graphs.

Theorem 2.4 *$S(P_n \odot K_1)$ is 4-total product cordial.*

Proof Let $V(S(P_n \odot K_1)) = \{u_i, v_i, w_i, z_j : 1 \le i \le n,\ 1 \le j \le n-1\}$ and $E(S(P_n \odot K_1)) = \{u_iv_i, v_iw_i, u_iz_j, z_ju_{j+1} : 1 \le i \le n,\ 1 \le j \le n-1\}$.

Case 1. $n \equiv 0 (\mathrm{mod}\ 4)$.

Let $n = 4t$. Define $f(u_1) = 0$,

$$\begin{array}{lll} f(u_i) & = & 2, \quad 2 \le i \le \frac{n-2}{2} \\ f(u_{\frac{n-2}{2}+i}) & = & 3, \quad 1 \le i \le \frac{n}{2} \\ f(v_i) & = & 2 \quad 1 \le i \le \frac{n}{2} \\ f(v_{\frac{n}{2}+i}) & = & 3, \quad 1 \le i \le \frac{n}{2} \\ f(w_i) & = & 2 \quad 1 \le i \le \frac{n}{2} \\ f(w_{\frac{n}{2}+i}) & = & 3, \quad 1 \le i \le \frac{n}{2} \\ f(z_i) & = & 2, \quad 1 \le j \le \frac{n-2}{2} \\ f(z_{\frac{n-2}{2}+i}) & = & 3, \quad 1 \le j \le \frac{n}{2}. \end{array}$$

Clearly $ev_f(0) = ev_f(1) = ev_f(2) = 4t - 1$ and $ev_f(3) = 4t$. Hence f is a 4-total product cordial labeling.

Case 2. $n \equiv 1 (\text{mod } 4)$.

Let $n = 4t + 1$. Assign the label to the vertices u_i, v_i, w_i, z_j $1 \le i \le n-1,\ 1 \le j \le n-1$ as in case 1. Then label $3, 3, 2, 0$ to the vertices z_n, u_n, v_n, w_n respectively. Here $ev_f(0) = ev_f(1) = ev_f(2) = 4t + 1$ and $ev_f(3) = 4t + 2$. Hence f is a 4-total product cordial labeling.

Case 3. $n \equiv 2 (\text{mod } 4)$.

Let $n = 4t + 2$. Assign the label to the vertices u_i, v_i, w_i, z_j $1 \le i \le n-1,\ 1 \le j \le n-1$ as in case 2. Then label $3, 3, 2, 0$ to the vertices z_n, u_n, v_n, w_n respectively. Here $ev_f(0) = ev_f(1) = ev_f(2) = 4t + 3$ and $ev_f(3) = 4t + 4$. Hence f is a 4-total product cordial labeling.

Case 4. $n \equiv 3 (\text{mod } 4)$.

Let $n = 4t + 3$. Assign the label to the vertices u_i, v_i, w_i, z_j $1 \le i \le n-1,\ 1 \le j \le n-1$ as in case 3. Then label $3, 3, 2, 0$ to the vertices z_n, u_n, v_n, w_n respectively. Here $ev_f(0) = ev_f(1) = ev_f(2) = 4t + 5$ and $ev_f(3) = 4t + 6$. Hence f is a 4-total product cordial labeling. □

Theorem 2.5 *$S(P_n \odot 2K_1)$ is 4-total product cordial.*

Proof Let $V(S(P_n \odot 2K_1)) = \{u_i, v_i, w_i, a_j, b_i, c_i : 1 \le i \le n,\ 1 \le j \le n-1\}$ and $E(S(P_n \odot 2K_1)) = \{u_i a_j, u_i b_i, u_i c_i, b_i v_i, c_i w_i, a_j u_{j+1} : 1 \le i \le n,\ 1 \le j \le n-1\}$.

Case 1. $n \equiv 0 (\text{mod } 4)$.

Let $n = 4t$ and let $f(u_1) = 0$,

$$\begin{array}{lll} f(u_i) & = & 2, \quad 2 \le i \le \frac{n-2}{2} \\ f(u_{\frac{n-2}{2}+i}) & = & 3, \quad 1 \le i \le \frac{n}{2} \\ f(v_i) & = & 2, \quad 1 \le i \le \frac{n}{2} \\ f(v_{\frac{n}{2}+i}) & = & 3, \quad 1 \le i \le \frac{n}{2} \end{array}$$

$$\begin{array}{lcl} f(w_i) & = & 2, \quad 1 \le i \le \frac{n}{2} \\ f(w_{\frac{n}{2}+i}) & = & 3, \quad 1 \le i \le \frac{n}{2} \\ f(a_j) & = & 2, \quad 1 \le j \le \frac{n-2}{2} \\ f(a_{\frac{n-2}{2}+1}) & = & 1 \\ f(a_{\frac{n-2}{2}+1+j}) & = & 1, \quad 1 \le j \le \frac{n-2}{2} \\ f(b_i) & = & 2, \quad 1 \le i \le \frac{n}{2} \\ f(b_{\frac{n}{2}+i}) & = & 3, \quad 1 \le i \le \frac{n}{2} \\ f(c_i) & = & 2, \quad 1 \le i \le \frac{n}{2} \\ f(c_{\frac{n}{2}+i}) & = & 3, \quad 1 \le i \le \frac{n}{2}. \end{array}$$

Clearly $ev_f(0) = ev_f(1) = ev_f(2) = 4t+7$ and $ev_f(3) = 4t+8$. Hence f is a 4-total product cordial labeling.

Case 2. $n \equiv 1 (\text{mod } 4)$.

Let $n = 4t+1$ and assign the label to the vertices $u_i, v_i, w_i, a_j, b_i, c_i$ $1 \le i \le n-1$, $1 \le j \le n-2$ as in case 1. Then label $3, 3, 2, 2, 1, 0$ to the vertices a_n, u_n, b_n, v_n, c_n, w_n respectively. Here $ev_f(0) = ev_f(1) = ev_f(2) = 4t+10$ and $ev_f(3) = 4t+11$. Hence f is a 4-total product cordial labeling.

Case 3. $n \equiv 2 (\text{mod } 4)$.

Let $n = 4t+2$. Assign the label to the vertices $u_i, v_i, w_i, a_j, b_i, c_i$ $1 \le i \le n-2$, $1 \le j \le n-3$ as in case 2. Then label $3, 3, 2, 2, 2, 2, 3, 3, 2, 3, 0, 3$ to the vertices $a_{n-2}, u_{n-1}, b_{n-1}, v_{n-1}, c_{n-1}, w_{n-1}$, a_{n-1}, u_n, b_n, v_n, c_n, w_n respectively. Here $ev_f(0) = ev_f(1) = ev_f(2) = 4t+13$ and $ev_f(3) = 4t+14$. Hence f is a 4-total product cordial labeling.

Case 4. $n \equiv 3 (\text{mod } 4)$.

Let $n = 4t+3$. We assign the label to the vertices $u_i, v_i, w_i, a_j, b_i, c_i$ $1 \le i \le n-3$, $1 \le j \le n-4$ as in case 3. Then label $3, 3, 2, 0, 2, 2, 3, 3, 2, 2, 2, 2, 3, 3, 3, 2, 3, 3$ to the vertices $a_{n-3}, u_{n-2}, b_{n-2}, v_{n-2}, c_{n-2}, w_{n-2}$, a_{n-2}, u_{n-1}, b_{n-1}, v_{n-1}, c_{n-1}, w_{n-1}, a_{n-1}, u_n, b_n, v_n, c_n, w_n respectively. Here $ev_f(0) = ev_f(1) = ev_f(2) = 4t+22$ and $ev_f(3) = 4t+23$. Hence f is a 4-total product cordial labeling. □

Theorem 2.6 *$S(C_n \odot K_1)$ is 4-total product cordial.*

Proof Let $V(S(C_n \odot K_1)) = \{u_i, v_i, w_i, z_i : 1 \le i \le n\}$ and $E(S(C_n \odot K_1)) = \{u_i z_i, u_i w_i, w_i v_j, z_i u_{i+1} : 1 \le i \le n\}$.

Case 1. $n \equiv 0, 2 (\text{mod } 4)$.

Let $n = 4t$ and let $f(u_1) = 0$,

$$\begin{array}{lllll} f(u_i) & = & f(z_i) & = & 3 \quad 1 \leq i \leq n \\ f(v_i) & = & 2 & & 1 \leq i \leq n \\ f(w_i) & = & 2 & & 1 \leq i \leq \frac{n}{2} \\ f(v_{\frac{n}{2}+i}) & = & 0 & & 1 \leq i \leq \frac{n}{2}. \end{array}$$

In this case, $ev_f(0) = ev_f(1) = ev_f(2) = ev_f(3) = 2n$. Hence f is a 4-total product cordial labeling.

Case 2. $n \equiv 1 (\text{mod } 4)$.

Let $n = 4t + 1$. We assign the label to the vertices u_i, v_i, w_i, z_i, $1 \leq i \leq n - 1$ as in case 1. Then label $3, 3, 2, 0$ to the vertices u_n, z_n, w_n, v_n respectively. Hence $ev_f(0) = ev_f(1) = ev_f(2) = ev_f(3) = 2n$. Hence f is a 4-total product cordial labeling.

Case 3. $n \equiv 3 (\text{mod } 4)$.

Let $n = 4t + 3$ and assign the label to the vertices u_i, v_i, w_i, z_i, $1 \leq i \leq n - 1$ as in case 1. Then label $3, 3, 2, 0$ to the vertices u_n, z_n, w_n, v_n respectively. Hence $ev_f(0) = ev_f(1) = ev_f(2) = ev_f(3) = 2n$. Therefore f is a 4-total product cordial labeling. □

Theorem 2.7 *$S(C_n \odot 2K_1)$ is 4-total product cordial.*

Proof Let $V(S(C_n \odot 2K_1)) = \{u_i, v_i, w_i, a_i, b_i, c_i : 1 \leq i \leq n,\}$ and $E(S(C_n \odot 2K_1)) = \{u_i u_{i+1 \ (\text{mod } n)}, u_i a_i, u_i b_i, b_i v_i, u_i c_i, c_j w_i : 1 \leq i \leq n\}$.

Case 1. $n \equiv 0 (\text{mod } 4)$

Define

$$\begin{array}{lllll} f(u_i) & = & f(a_i) & = & 3 \quad 1 \leq i \leq n \\ f(v_i) & = & f(b_i) & = & 2 \quad 1 \leq i \leq \frac{n}{2} \\ f(b_{\frac{n}{2}+i}) & = & f(v_{\frac{n}{2}+i}) & = & 0 \quad 1 \leq i \leq \frac{n}{4} \\ f(b_{\frac{3n}{4}+i}) & = & f(v_{\frac{3n}{4}+i}) & = & 0 \quad 1 \leq i \leq \frac{n}{4} \\ f(w_i) & = & f(c_i) & = & 2 \quad 1 \leq i \leq \frac{n}{2} \\ f(c_{\frac{n}{2}+i}) & = & f(w_{\frac{n}{2}+i}) & = & 0 \quad 1 \leq i \leq \frac{n}{4} \\ f(c_{\frac{3n}{4}+i}) & = & f(w_{\frac{3n}{4}+i}) & = & 0 \quad 1 \leq i \leq \frac{n}{4} \end{array}$$

Therefore $ev_f(0) = ev_f(1) = ev_f(2) = ev_f(3) = 3n$. Hence f is a 4-total product cordial labeling.

Case 2. $n \equiv 1 (\text{mod } 4)$

Let $n = 4t + 1$. We assign the label to the vertices $u_i, v_i, w_i, a_i, b_i, c_i$ $1 \leq i \leq n - 1$ as in case 1. Then label $3, 3, 3, 2, 2, 2$ to the vertices u_n, a_n, b_n, v_n, w_n, c_n respectively. Hence $ev_f(0) = ev_f(1) = ev_f(2) = ev_f(3) = 3n$. Hence f is a 4-total product cordial labeling.

Case 3. $n \equiv 2 (\text{mod } 4)$

Let $n = 4t + 2$. Define

$$\begin{array}{lllll} f(u_i) & = & f(a_i) & = & 3 \quad 1 \leq i \leq n \\ f(v_i) & = & f(b_i) & = & 2 \quad 1 \leq i \leq \frac{n}{2} \\ f(b_{\frac{n}{2}+i}) & = & f(v_{\frac{n}{2}+i}) & = & 0 \quad 1 \leq i \leq \frac{n}{2} - 2 \\ f(b_{n-2+i}) & = & f(v_{n-2+i}) & = & 3 \quad 1 \leq i \leq \frac{n}{2} - 3 \\ f(w_i) & = & f(c_i) & = & 2 \quad 1 \leq i \leq \frac{n}{2} \\ f(c_{\frac{n}{2}+i}) & = & f(w_{\frac{n}{2}+i}) & = & 0 \quad 1 \leq i \leq \frac{n}{2} - 3 \\ f(c_{n-3+i}) & = & f(w_{n-3+i}) & = & 3 \quad 1 \leq i \leq \frac{n}{2} - 2. \end{array}$$

Therefore $ev_f(0) = ev_f(1) = ev_f(2) = ev_f(3) = 3n$. Hence f is a 4-total product cordial labeling.

Case 4. $n \equiv 3 (\text{mod } 4)$

Let $n = 4t + 3$ and let

$$\begin{array}{lllll} f(u_i) & = & f(a_i) & = & 3 \quad 1 \leq i \leq n \\ f(v_i) & = & f(b_i) & = & 2 \quad 1 \leq i \leq \frac{n-1}{2} \\ f(v_{\frac{n+1}{2}}) & = & 2 & & \\ f(w_{\frac{n+1}{2}}) & = & 0 & & \\ f(v_n) & = & 3 & & \\ f(w_n) & = & 2 & & \\ f(v_{\frac{n+1}{2}+i}) & = & f(w_{\frac{n+1}{2}+i}) & = & 0 \quad 1 \leq i \leq \frac{n-3}{4} \\ f(v_{\frac{3n-1}{4}+i}) & = & f(w_{\frac{3n-1}{4}+i}) & = & 3 \quad 1 \leq i \leq \frac{n-3}{4} \\ f(b_i) & = & f(c_i) & = & 2 \quad 1 \leq i \leq \frac{n-1}{2} \\ f(b_{\frac{n-1}{2}+i}) & = & f(c_{\frac{n-1}{2}+i}) & = & 0 \quad 1 \leq i \leq \frac{n+1}{4} \\ f(b_{\frac{3n-1}{4}+i}) & = & f(c_{\frac{3n-1}{4}+i}) & = & 3 \quad 1 \leq i \leq \frac{n+1}{4} \end{array}$$

Therefore $ev_f(0) = ev_f(1) = ev_f(2) = ev_f(3) = 3n$. Hence f is a 4-total product cordial labeling. □

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