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Mean Cordial Labelling of Some Star-Related Graphs

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Abstract: Let f be a map from $V(G)$ to $\{0,1,2\}$. For each edge uv assign the label $f^*(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. f is called as a mean cordial labelling if $|v_f(i) - v_f(j)| \leqslant 1$ and $|e^*_f(i) - e^*_f(j)| \leqslant 1$, $i, j \in \{0,1,2\}$, where $v_f(x)$ and $e^*_f(x)$ denote the number of vertices and edges respectively labelled with x $(x = 0,1,2)$. A graph with mean cordial labelling is called mean cordial. In this paper we prove the graph $\langle K_{1,n} : 2\rangle$ and path union of n copies of star $K_{1,m}$ are mean cordial graphs.

Key Words: Mean cordial labelling, Smaarndachely mean cordial labelling, star, path, union graph.

AMS(2010): 05C78.

§1. Introduction

All graphs in this paper are finite, simple and undirected. The vertex set and edge set of a graph are denoted by $V(G)$ and $E(G)$ respectively. A graph labelling is an assignment of integers to the vertices or edges or both subject to certain conditions. A useful survey on graph labelling by J. A. Gallian(2014) can be found in [2].

The concept of cordial labelling was introduced by Cahit in the year 1987 in [1]. Here we introduce the notion of mean cordial labelling. We investigate the mean cordial labelling of the graph $\langle K_{1,n} : 2\rangle$ and path union of n copies of star $K_{1,m}$.

Definition 1.1 *Let f be a map from $V(G)$ to $\{0,1,2\}$. For each edge uv assign the label $f^*(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. f is called as a mean cordial labelling if $|v_f(i) - v_f(j)| \leq 1$ and $|e^*_f(i) - e^*_f(j)| \leq 1, i, j \in \{0,1,2\}$, otherwise, a Smarandachely mean labeling of G if $|v_f(i) - v_f(j)| \geq 2$ or $|e^*_f(i) - e^*_f(j)| \geq 2, i, j \in \{0,1,2\}$, where $v_f(x)$ and $e^*_f(x)$ denote the number of vertices and edges respectively labelled with $x(x = 0,1,2)$.*

A graph with mean cordial labelling is called a mean cordial graph.

Definition 1.2 *A complete bipartite graph $K_{1,n}$ is called a star graph. The vertex of degree n is called the apex vertex.*

Definition 1.3 *A bistar $B_{n,n}$ is the graph obtained by joining the apex vertices of two copies of star $K_{1,n}$ by an edge.*

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Definition 1.4 *Consider two copies of star $K_{1,n}$. Then $\langle K_{1,n} : 2\rangle$ is the graph obtained from $B_{n,n}$ by subdividing the middle edge with a new vertex. Such as those shown in Figure 1.*

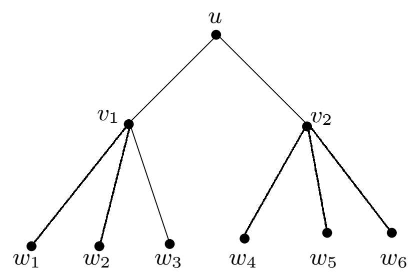

Figure 1

Definition 1.5 *Let G be a graph and $G_1, G_2, \ldots, G_n$ with $n \geqslant 2$ be n copies of graph G. Then the graph obtained by adding an edge from G_i to G_{i+1} for $i = 1, 2, \ldots, n-1$ is called path union of G.*

§2. Results

Theorem 2.1 *The graph $\langle K_{1,n} : 2\rangle$ admits a mean cordial labeling.*

Proof Let $G = \langle K_{1,n} : 2\rangle$ and let $V(G) = \{u, v_1, v_2, w_i : 1 \le i \le 2n\}$, $E(G) = \{uv_1, uv_2, v_1w_i : 1 \le i \le n, v_2w_j : n+1 \le j \le 2n\}$. Then, $|V(G)| = 2n+3$, $|E(G)| = 2n+2$.

Case 1. $n \equiv 0 \ (\ mod\ 3)$

Let $n = 3t, \quad t = 1, 2, \cdots$. Define $f : V(G) \to \{0, 1, 2\}$ as follows:

$f(u) = 2, \qquad f(v_1) = 0, \qquad f(v_2) = 1,$

$$f(w_i) = \begin{cases} 0, & 1 \leqslant i \leqslant 2t \\ 1, & 2t+1 \leqslant i \leqslant 4t \\ 2, & 4t+1 \leqslant i \leqslant 6t \end{cases}$$

The induced edge labelling $f^* : E(G) \to \{0, 1, 2\}$ is found as follows:

$f^*(uv_1) = 1,$
$f^*(uv_2) = 2,$
$f^*(v_1w_i) = 0, \qquad 1 \le i \le 2t,$
$f^*(v_1w_i) = 1, \qquad 2t+1 \le i \le 3t,$
$f^*(v_2w_i) = 1, \qquad 3t+1 \le i \le 4t,$
$f^*(v_2w_i) = 2, \qquad 4t+1 \le i \le 6t.$

Then,

$v_f(o) = 2t+1, \quad v_f(1) = 2t+1 \quad v_f(2) = 2t+1,$
$e_f^*(0) = 2t, \quad e_f^*(1) = 2t+1 \quad e_f^*(2) = 2t+1.$

Thus,

$|v_f(i) - v_f(j)| \le 1 \qquad \forall\, i, j \in \{0, 1, 2\},$
$|e_f^*(i) - e_f^*(j)| \le 1 \qquad \forall\, i, j \in \{0, 1, 2\}.$

Hence f is a mean cordial labeling.

Case 2 $n \equiv 1\ (mod)\ 3)$

Let $n = 3t + 1$, $t = 1, 2 \cdots$. Define $f : V(G) \to \{0, 1, 2\}$ as follows:

$f(u) = 2$, $f(v_1) = 0$, $f(v_2) = 1$,

$$f(w_i) = \begin{cases} 0, & 1 \le i \le 2t+1 \\ 1, & 2t+2 \le i \le 4t+2 \\ 2, & 4t+3 \le i \le 6t+2 \end{cases}$$

The induced edge labelling $f^* : E(G) \to \{0, 1, 2\}$ is defined as follows:

$f^*(uv_1) = 1$,
$f^*(uv_2) = 2$,
$f^*(v_1w_i) = 0$, $1 \le i \le 2t+1$,
$f^*(v_1w_i) = 1$, $2t+2 \le i \le 3t+1$,
$f^*(v_2w_i) = 1$, $3t+2 \le i \le 4t+2$,
$f^*(v_2w_i) = 2$, $4t+3 \leqslant i \leqslant 6t+2$.

Then,

$v_f(0) = 2t + 2$, $v_f(1) = 2t + 2$, $v_f(2) = 2t + 1$,
$e^*_f(0) = 2t + 1$, $e^*_f(1) = 2t + 2$, $e^*_f(2) = 2t + 1$.

Thus,

$|v_f(i) - v_f(j)| \le 1 \quad \forall \quad i, j \in \{0, 1, 2\}$,
$|e^*_f(i) - e^*_f(j)| \le 1 \quad \forall \quad i, j \in \{0, 1, 2\}$

Hence f is a mean cordial labeling.

Case 3. $n \equiv 2(mod3)$

Let $n = 3t + 2$, $t = 1, 2 \cdots$. Define $f : V(G) \to \{0, 1, 2\}$ as follows:

$f(u) = 2$, $f(v_1) = 0$, $f(v_2) = 1$,

$$f(w_i) = \begin{cases} 0, & 1 \leqslant i \leqslant 2t+2 \\ 1, & 2t+3 \le i \le 4t+3 \\ 2, & 4t+4 \le i \le 6t+4 \end{cases}$$

The induced edge labelling $f^* : E(G) \to \{0, 1, 2\}$ is found as follows:

$f^*(uv_1) = 1$,
$f^*(uv_2) = 2$,
$f^*(v_1w_i) = 0$, $1 \le i \le 2t+2$,
$f^*(v_1w_i) = 1$, $2t+3 \le i \le 3t+2$,
$f^*(v_2w_i) = 1$, $3t+3 \le i \le 4t+3$,
$f^*(v_2w_i) = 2$, $4t+4 \le i \le 6t+4$.

Then,

$v_f(0) = 2t + 3$, $v_f(1) = 2t + 2$, $v_f(2) = 2t + 2$,
$e^*_f(0) = 2t + 2$, $e^*_f(1) = 2t + 2$, $e^*_f(2) = 2t + 2$.

Thus,

$|v_f(i) - v_f(j)| \le 1 \quad \forall\, i, j \in \{0, 1, 2\}$,
$|e^*_f(i) - e^*_f(j)| \le 1 \quad \forall \ i, j \in \{0, 1, 2\}$.

Thus, f is a mean cordial labelling. Hence, $\langle K_{1,n} : 2\rangle$ is a mean cordial graph. □

Illustration 2.2 The mean cordial labelling of $\langle K_{1,6} : 2\rangle$ is shown in Figure 2.

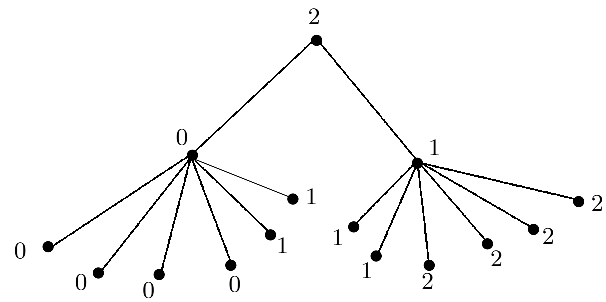

Figure 2

Illustration 2.3 The mean cordial labelling of $\langle K_{1,7} : 2\rangle$ is shown in Figure 3.

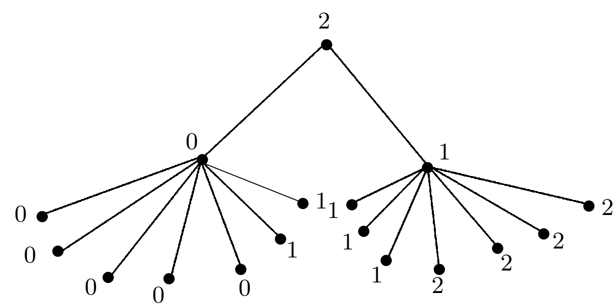

Figure 3

Theorem 2.4 *The path union of n copies of star $K_{1,m}$ is a mean cordial graph.*

Proof Let G be the path union of $'n'$ copies of star$K_{1,m}$ and let $V(G) = \{v_i : i = 1, \cdots n; w_{ij} : 1 \leqslant i \leqslant n, 1 \leqslant j \leqslant m\}$, $E(G) = \{v_iv_{i+1} : 1 \leqslant i \leqslant n-1\} \cup \{v_iw_{ij} : 1 \leqslant i \leqslant n, 1 \leqslant j \leqslant m\}$. Then, $|V(G)| = n(m+1)$ *and* $|E(G)| = n(m+1) - 1$.

Case 1. $m \equiv 0(mod3)$

Subcase 1.1 $n \equiv 0(mod3)$

Define $f : V(G) \to \{0, 1, 2\}$ as follows:

$$f(v_i) = \begin{cases} 0, & 1 \leq i \leq \frac{n}{3} \\ 1, & \frac{n}{3} + 1 \leqslant i \leqslant \frac{2n}{3} \\ 2, & \frac{2n}{3} + 1 \leqslant i \leqslant n \end{cases}$$

$$f(w_{ij}) = \begin{cases} 0, & 1 \leqslant i \leqslant \frac{n}{3} & 1 \leqslant j \leqslant m \\ 1, & \frac{n}{3} + 1 \leqslant i \leqslant \frac{2n}{3} & 1 \leqslant j \leqslant m \\ 2, & \frac{2n}{3} + 1 \leqslant i \leqslant n & 1 \leqslant j \leqslant m \end{cases}$$

The induced edge labelling $f^* : E(G) \to \{0, 1, 2\}$ is known as follows:

$$f^*(v_iv_{i+1}) = \begin{cases} 0, & 1 \leqslant i \leqslant \frac{n}{3} - 1 \\ 1, & \frac{n}{3} \leqslant i \leqslant \frac{2n}{3} - 1 \\ 2, & \frac{2n}{3} \leqslant i \leqslant n - 1 \end{cases}$$

$$f^*(v_iw_{ij}) = \begin{cases} 0, & 1 \leqslant i \leqslant \frac{n}{3} & 1 \leqslant j \leqslant m \\ 1, & \frac{n}{3} + 1 \leqslant i \leqslant \frac{2n}{3} & 1 \leqslant j \leqslant m \\ 2, & \frac{2n}{3} + 1 \leqslant i \leqslant n & 1 \leqslant j \leqslant m \end{cases}$$

Then,

$$v_f(0) = \frac{nm+n}{3}, \quad v_f(1) = \frac{nm+n}{3}, \quad v_f(2) = \frac{nm+n}{3} \text{ and}$$
$$e^*_f(0) = \frac{mn+n-3}{3}, \quad e^*_f(1) = \frac{mn+n}{3}, \quad e^*_f(2) = \frac{mn+n}{3}.$$

Thus,

$$|v_f(i) - v_f(j)| \leqslant 1 \text{ and } |e^*_f(i) - e^*_f(j)| \leqslant 1 \quad \forall \quad i, j \in \{0, 1, 2\}.$$

Hence f is a mean cordial labelling.

Subcase 1.2 $n \equiv 1(mod3)$

Define $f : V(G) \to \{0, 1, 2\}$ as follows:

$$f(v_i) = \begin{cases} 0, & 1 \leqslant i \leqslant \frac{n+2}{3} \\ 1, & \frac{n+2}{3} + 1 \leqslant i \leqslant \frac{2n+1}{3} \\ 2, & \frac{2n+1}{3} + 1 \leqslant i \leqslant n \end{cases}$$

$$f(w_{ij}) = \begin{cases} 0, & 1 \leqslant i \leqslant \frac{n+2}{3} - 1 & 1 \leqslant j \leqslant m \\ 0, & i = \frac{n+2}{3} & 1 \leqslant j \leqslant \frac{m}{3} \\ 1, & i = \frac{n+2}{3} & \frac{m}{3} + 1 \leqslant j \leqslant m \\ 1, & \frac{n+5}{3} \leqslant i \leqslant \frac{2n-2}{3} & 1 \leqslant j \leqslant m \\ 1, & i = \frac{2n+1}{3} & 1 \leqslant j \leqslant \frac{2m}{3} \\ 2, & i = \frac{2n+1}{3} & \frac{2m}{3} + 1 \leqslant j \leqslant m \\ 2, & \frac{2n+4}{3} \leqslant i \leqslant n & 1 \leqslant j \leqslant m \end{cases}$$

We get the induced edge labelling $f^* : E(G) \longrightarrow \{0, 1, 2\}$ as follows:

$$f^*(v_iv_{i+1}) = \begin{cases} 0, & 1 \leqslant i \leqslant \frac{n+2}{3} - 1 \\ 1, & \frac{n+2}{3} \leqslant i \leqslant \frac{2n-2}{3} \\ 2, & \frac{2n-2}{3} + 1 \leqslant i \leqslant n - 1 \end{cases}$$

$$f^*(v_iw_{ij}) = \begin{cases} 0, & 1 \leqslant i \leqslant \frac{n-1}{3} & 1 \leqslant j \leqslant m \\ 0, & i = \frac{n+2}{3} & 1 \leqslant j \leqslant \frac{m}{3} \\ 1, & i = \frac{n+2}{3} & \frac{m+3}{3} \leqslant j \leqslant m \\ 1, & \frac{n+5}{3} \leqslant i \leqslant \frac{2n-2}{3} & 1 \leqslant j \leqslant m \\ 1, & i = \frac{2n+1}{3} & 1 \leqslant j \leqslant \frac{2m}{3} \\ 2, & i = \frac{2n+1}{3} & \frac{2m+3}{3} \leqslant j \leqslant m \\ 2, & \frac{2n+4}{3} \leqslant i \leqslant n, & 1 \leqslant j \leqslant m \end{cases}$$

Then,

$$v_f(0)=\frac{n+nm-2}{3},\ v_f(1)=\frac{n+nm-1}{3},\ v_f(2)=\frac{n+nm-1}{3},$$
$$e_f^*(0)=\frac{mn+n-1}{3},\ e_f^*(1)=\frac{mn+n-1}{3},\ e_f^*(2)=\frac{mn+n-1}{3}.$$

Thus,

$$|v_f(i)-v_f(j)|\leqslant 1\ \forall\, i,j\in\{0,1,2\}\ \text{ and }\ |e_f^*(i)-e_f^*(j)|\leqslant 1\ \forall\, i,j\in\{0,1,2\}.$$

Hence f is a mean cordial labelling.

Subcase 1.3 $n\equiv 2(mod 3)$

Define $f: V(G)\to\{0,1,2\}$ as follows:

$$f(v_i)=\begin{cases}0, & 1\leqslant i\leqslant \frac{n+1}{3}\\ 1, & \frac{n+1}{3}+1\leqslant i\leqslant \frac{2n+2}{3}\\ 2, & \frac{2n+2}{3}+1\leqslant i\leqslant n\end{cases}$$

$$f(w_{ij})=\begin{cases}0, & 1\leqslant i\leqslant \frac{n+1}{3}-1 & 1\leqslant j\leqslant m\\ 0, & i=\frac{n+1}{3} & 1\leqslant j\leqslant \frac{2m}{3}\\ 1, & i=\frac{n+1}{3} & \frac{2m}{3}+1\leqslant j\leqslant m\\ 1, & \frac{n+4}{3}\leqslant i\leqslant \frac{2n+2}{3}-1 & 1\leqslant j\leqslant m\\ 1, & i=\frac{2n+2}{3} & 1\leqslant j\leqslant \frac{m}{3}\\ 2, & i=\frac{2n+2}{3} & \frac{m}{3}+1\leqslant j\leqslant m\\ 2, & \frac{2n+2}{3}+1\leqslant i\leqslant n & 1\leqslant j\leqslant m\end{cases}$$

We get the induced edge labelling $f^*: E(G)\longrightarrow\{0,1,2\}$ as follows:

$$f^*(v_iv_{i+1})=\begin{cases}0, & 1\leqslant i\leqslant \frac{n+1}{3}-1\\ 1, & \frac{n+1}{3}\leqslant i\leqslant \frac{2n+2}{3}-1\\ 2, & \frac{2n+2}{3}\leqslant i\leqslant n-1\end{cases}$$

$$f^*(v_iw_{ij})=\begin{cases}0, & 1\leqslant i\leqslant \frac{n-2}{3} & 1\leqslant j\leqslant m\\ 0, & i=\frac{n-2}{3}+1 & 1\leqslant j\leqslant \frac{2m}{3}\\ 1, & i=\frac{n-2}{3}+1 & \frac{2m}{3}+1\leqslant j\leqslant m\\ 1, & \frac{n+4}{3}\leqslant i\leqslant \frac{2n+2}{3}-1 & 1\leqslant j\leqslant m\\ 1, & i=\frac{2n+2}{3} & 1\leqslant j\leqslant \frac{m}{3}\\ 2, & i=\frac{2n+2}{3} & \frac{m}{3}+1\leqslant j\leqslant m\\ 2, & \frac{2n+2}{3}+1\leqslant i\leqslant n & 1\leqslant j\leqslant m\end{cases}$$

Then,

$$v_f(0)=\frac{n+nm+1}{3},\ v_f(1)=\frac{n+nm+1}{3},\ v_f(2)=\frac{n+nm-2}{3},$$
$$e_f^*(0)=\frac{mn+n-2}{3},\ e_f^*(1)=\frac{mn+n+1}{3},\ e_f^*(2)=\frac{mn+n-2}{3}.$$

Thus,

$$|v_f(i)-v_f(j)|\leqslant 1\ \text{ and }\ |e_f^*(i)-e_f^*(j)|\leqslant 1\ \forall\, i,j\in\{0,1,2\}.$$

Hence f is a mean cordial labelling.

Case 2. $m \equiv 1(mod3)$

Subcase 2.1 $n \equiv 0(mod3)$

Define $f : V(G) \to \{0, 1, 2\}$ as follows:

$$f(v_i) = \begin{cases} 0, & 1 \leq i \leq \frac{n}{3} \\ 1, & \frac{n}{3} + 1 \leqslant i \leqslant \frac{2n}{3} \\ 2, & \frac{2n}{3} + 1 \leqslant i \leqslant n \end{cases}$$

$$f(w_{ij}) = \begin{cases} 0, & 1 \leqslant i \leqslant \frac{n}{3} & 1 \leqslant j \leqslant m \\ 1, & \frac{n}{3} + 1 \leqslant i \leqslant \frac{2n}{3} & 1 \leqslant j \leqslant m \\ 2, & \frac{2n}{3} + 1 \leqslant i \leqslant n & 1 \leqslant j \leqslant m \end{cases}$$

We then know the induced edge labelling $f^* : E(G) \to \{0, 1, 2\}$ as follows:

$$f^*(v_i v_{i+1}) = \begin{cases} 0, & 1 \leqslant i \leqslant \frac{n}{3} - 1 \\ 1, & \frac{n}{3} \leqslant i \leqslant \frac{2n}{3} - 1 \\ 2, & \frac{2n}{3} \leqslant i \leqslant n - 1 \end{cases}$$

$$f^*(v_i w_{ij}) = \begin{cases} 0, & 1 \leqslant i \leqslant \frac{n}{3} & 1 \leqslant j \leqslant m \\ 1, & \frac{n}{3} + 1 \leqslant i \leqslant \frac{2n}{3} & 1 \leqslant j \leqslant m \\ 2, & \frac{2n}{3} + 1 \leqslant i \leqslant n & 1 \leqslant j \leqslant m \end{cases}$$

Then,

$$v_f(0) = \frac{nm + n}{3}, \quad v_f(1) = \frac{nm + n}{3}, \quad v_f(2) = \frac{nm + n}{3},$$
$$e^*_f(0) = \frac{mn + n - 3}{3}, \quad e^*_f(1) = \frac{mn + n}{3}, \quad e^*_f(2) = \frac{mn + n}{3}.$$

Thus,

$$|v_f(i) - v_f(j)| \leqslant 1 \text{ and } |e^*_f(i) - e^*_f(j)| \leqslant 1 \qquad \forall \quad i, j \in \{0, 1, 2\}.$$

Hence f is a mean cordial labelling.

Subcase 2.2 $n \equiv 1(mod3)$

Define $f : V(G) \to \{0, 1, 2\}$ as follows:

$$f(v_i) = \begin{cases} 0, & 1 \leqslant i \leqslant \frac{n+2}{3} \\ 1, & \frac{n+2}{3} + 1 \leqslant i \leqslant \frac{2n+1}{3} \\ 2, & \frac{2n+1}{3} + 1 \leqslant i \leqslant n \end{cases}$$

$$f(w_{ij}) = \begin{cases} 0, & 1 \leqslant i \leqslant \frac{n+2}{3} - 1 & 1 \leqslant j \leqslant m \\ 0, & i = \frac{n+2}{3} & 1 \leqslant j \leqslant \frac{m-1}{3} \\ 1, & i = \frac{n+2}{3} & \frac{m-1}{3} + 1 \leqslant j \leqslant m \\ 1, & \frac{n+5}{3} \leqslant i \leqslant \frac{2n+1}{3} & 1 \leqslant j \leqslant m \\ 1, & i = \frac{2n+1}{3} & 1 \leqslant j \leqslant \frac{2m+1}{3} \\ 2, & i = \frac{2n+1}{3} & \frac{2m+1}{3} + 1 \leqslant j \leqslant m \\ 2, & \frac{2n+4}{3} \leqslant i \leqslant n & 1 \leqslant j \leqslant m \end{cases}$$

The induced edge labelling$f^*: E(G) \longrightarrow \{0,1,2\}$ is known as follows:

$$f^*(v_iv_{i+1}) = \begin{cases} 0, & 1 \leqslant i \leqslant \frac{n+2}{3} - 1 \\ 1, & \frac{n+2}{3} \leqslant i \leqslant \frac{2n+1}{3} - 1 \\ 2, & \frac{2n+1}{3} \leqslant i \leqslant n-1 \end{cases}$$

$$f^*(v_iw_{ij}) = \begin{cases} 0, & 1 \leqslant i \leqslant \frac{n+2}{3} - 1 & 1 \leqslant j \leqslant m \\ 0, & i = \frac{n+2}{3} & 1 \leqslant j \leqslant \frac{m-1}{3} \\ 1, & i = \frac{n+2}{3} & \frac{m-1}{3} + 1 \leqslant j \leqslant m \\ 1, & \frac{n+5}{3} \leqslant i \leqslant \frac{2n-2}{3} & 1 \leqslant j \leqslant m \\ 1, & i = \frac{2n+1}{3} & 1 \leqslant j \leqslant \frac{2m+1}{3} \\ 2, & i = \frac{2n+1}{3} & \frac{2m+1}{3} + 1 \leqslant j \leqslant m \\ 2, & \frac{2n+1}{3} + 1 \leqslant i \leqslant n & 1 \leqslant j \leqslant m \end{cases}$$

Then,

$$v_f(0) = \frac{n+nm+1}{3},\ v_f(1) = \frac{n+nm+1}{3},\ v_f(2) = \frac{n+nm-2}{3},$$
$$e_f^*(0) = \frac{mn+n-2}{3},\ e_f^*(1) = \frac{mn+n+1}{3},\ e_f^*(2) = \frac{mn+n-2}{3}.$$

Thus,

$$|v_f(i) - v_f(j)| \leqslant 1 \text{ and } |e_f^*(i) - e_f^*(j)| \leqslant 1 \quad \forall\, i,j \in \{0,1,2\}.$$

Hence f is a mean cordial labelling.

Subcase 2.3 $n \equiv 2(mod3)$

Define $f: V(G) \to \{0,1,2\}$ as follows:

$$f(v_i) = \begin{cases} 0, & 1 \leqslant i \leqslant \frac{n+1}{3} \\ 1, & \frac{n+1}{3} + 1 \leqslant i \leqslant \frac{2n+2}{3} \\ 2, & \frac{2n+2}{3} + 1 \leqslant i \leqslant n \end{cases}$$

$$f(w_{ij}) = \begin{cases} 0, & 1 \leqslant i \leqslant \frac{n+1}{3} - 1 & 1 \leqslant j \leqslant m \\ 0, & i = \frac{n+1}{3} & 1 \leqslant j \leqslant \frac{2m+1}{3} \\ 1, & i = \frac{n+1}{3} & \frac{2m+1}{3} + 1 \leqslant j \leqslant m \\ 1, & \frac{n+4}{3} \leqslant i \leqslant \frac{2n+2}{3} - 1 & 1 \leqslant j \leqslant m \\ 1, & i = \frac{2n+2}{3} & 1 \leqslant j \leqslant \frac{m-1}{3} \\ 2, & i = \frac{2n+2}{3} & \frac{m-1}{3} + 1 \leqslant j \leqslant m \\ 2, & \frac{2n+2}{3} + 1 \leqslant i \leqslant n & 1 \leqslant j \leqslant m \end{cases}$$

The induced edge labelling $f^*: E(G) \longrightarrow \{0,1,2\}$ is known as follows:

$$f^*(v_iv_{i+1}) = \begin{cases} 0, & 1 \leqslant i \leqslant \frac{n+1}{3} - 1 \\ 1, & \frac{n+1}{3} \leqslant i \leqslant \frac{2n+2}{3} - 1 \\ 2, & \frac{2n+2}{3} \leqslant i \leqslant n-1 \end{cases}$$

$$f^*(v_iw_{ij})=\begin{cases}0, & 1\leqslant i\leqslant \frac{n+1}{3}-1 \quad 1\leqslant j\leqslant m\\ 0, & i=\frac{n+1}{3} \quad 1\leqslant j\leqslant \frac{2m+1}{3}\\ 1, & i=\frac{n+1}{3} \quad \frac{2m+1}{3}+1\leqslant j\leqslant m\\ 1, & \frac{n+1}{3}+1\leqslant i\leqslant \frac{2n+2}{3}-1 \quad 1\leqslant j\leqslant m\\ 1, & i=\frac{2n+2}{3} \quad 1\leqslant j\leqslant \frac{m-1}{3}\\ 2, & i=\frac{2n+2}{3} \quad \frac{m-1}{3}+1\leqslant j\leqslant m\\ 2, & \frac{2n+2}{3}+1\leqslant i\leqslant n \quad 1\leqslant j\leqslant m\end{cases}$$

Then,

$$v_f(0)=\frac{n+nm+2}{3},\ v_f(1)=\frac{n+nm-1}{3},\ v_f(2)=\frac{n+nm-1}{3},$$
$$e_f^*(0)=\frac{mn+n-1}{3},\ e_f^*(1)=\frac{mn+n-1}{3},\ e_f^*(2)=\frac{mn+n-1}{3}.$$

Thus,

$$|v_f(i)-v_f(j)|\leqslant 1 \text{ and } |e_f^*(i)-e_f^*(j)|\leqslant 1\ \forall\, i,j\in\{0,1,2\}.$$

Hence f is a mean cordial labelling.

Case 3. $m\equiv 2(mod3)$

Subcase 3.1 $n\equiv 0(mod3)$

Define $f:V(G)\to\{0,1,2\}$ as follows:

$$f(v_i)=\begin{cases}0, & 1\leq i\leq \frac{n}{3}\\ 1, & \frac{n}{3}+1\leqslant i\leqslant \frac{2n}{3}\\ 2, & \frac{2n}{3}+1\leqslant i\leqslant n\end{cases}$$

$$f(w_{ij})=\begin{cases}0, & 1\leqslant i\leqslant \frac{n}{3} \quad 1\leqslant j\leqslant m\\ 1, & \frac{n}{3}+1\leqslant i\leqslant \frac{2n}{3} \quad 1\leqslant j\leqslant m\\ 2, & \frac{2n}{3}+1\leqslant i\leqslant n \quad 1\leqslant j\leqslant m\end{cases}$$

We get the induced edge labelling $f^*:E(G)\to\{0,1,2\}$ as follows:

$$f^*(v_iv_{i+1})=\begin{cases}0, & 1\leqslant i\leqslant \frac{n}{3}-1\\ 1, & \frac{n}{3}\leqslant i\leqslant \frac{2n}{3}-1\\ 2, & \frac{2n}{3}\leqslant i\leqslant n-1\end{cases}$$

$$f^*(v_iw_{ij})=\begin{cases}0, & 1\leqslant i\leqslant \frac{n}{3} \quad 1\leqslant j\leqslant m\\ 1, & \frac{n}{3}+1\leqslant i\leqslant \frac{2n}{3} \quad 1\leqslant j\leqslant m\\ 2, & \frac{2n}{3}+1\leqslant i\leqslant n \quad 1\leqslant j\leqslant m\end{cases}$$

Then,

$$v_f(0)=\frac{nm+n}{3},\quad v_f(1)=\frac{nm+n}{3},\quad v_f(2)=\frac{nm+n}{3},$$
$$e_f^*(0)=\frac{mn+n-3}{3},\quad e_f^*(1)=\frac{mn+n}{3},\quad e_f^*(2)=\frac{mn+n}{3}.$$

Thus,

$$|v_f(i)-v_f(j)|\leqslant 1 \text{ and } |e_f^*(i)-e_f^*(j)|\leqslant 1\quad \forall\quad i,j\in\{0,1,2\}.$$

Hence f is a mean cordial labelling.

Subcase 3.2 $n \equiv 1(mod3)$

Define $f : V(G) \to \{0,1,2\}$ as follows:

$$f(v_i) = \begin{cases} 0, & 1 \leqslant i \leqslant \frac{n+2}{3} \\ 1, & \frac{n+2}{3}+1 \leqslant i \leqslant \frac{2n+1}{3} \\ 2, & \frac{2n+1}{3}+1 \leqslant i \leqslant n \end{cases}$$

$$f(w_{ij}) = \begin{cases} 0, & 1 \leqslant i \leqslant \frac{n+2}{3}-1 & 1 \leqslant j \leqslant m \\ 0, & i = \frac{n+2}{3} & 1 \leqslant j \leqslant \frac{m-2}{3} \\ 1, & i = \frac{n+2}{3} & \frac{m-2}{3}+1 \leqslant j \leqslant m \\ 1, & \frac{n+2}{3}+1 \leqslant i \leqslant \frac{2n+1}{3}-1 & 1 \leqslant j \leqslant m \\ 1, & i = \frac{2n+1}{3} & 1 \leqslant j \leqslant \frac{2m-1}{3} \\ 2, & i = \frac{2n+1}{3} & \frac{2m-1}{3}+1 \leqslant j \leqslant m \\ 2, & \frac{2n+1}{3}+1 \leqslant i \leqslant n & 1 \leqslant j \leqslant m \end{cases}$$

The induced edge labelling$f^* : E(G) \longrightarrow \{0,1,2\}$ is calculated as follows:

$$f^*(v_iv_{i+1}) = \begin{cases} 0, & 1 \leqslant i \leqslant \frac{n+2}{3}-1 \\ 1, & \frac{n+2}{3} \leqslant i \leqslant \frac{2n+1}{3}-1 \\ 2, & \frac{2n+1}{3} \leqslant i \leqslant n-1 \end{cases}$$

$$f^*(v_iw_{ij}) = \begin{cases} 0, & 1 \leqslant i \leqslant \frac{n+2}{3}-1 & 1 \leqslant j \leqslant m \\ 0, & i = \frac{n+2}{3} & 1 \leqslant j \leqslant \frac{m-2}{3} \\ 1, & i = \frac{n+2}{3} & \frac{m-2}{3}+1 \leqslant j \leqslant m \\ 1, & \frac{n+2}{3}+1 \leqslant i \leqslant \frac{2n+1}{3}-1 & 1 \leqslant j \leqslant m \\ 1, & i = \frac{2n+1}{3} & 1 \leqslant j \leqslant \frac{2m-1}{3} \\ 2, & i = \frac{2n+1}{3} & \frac{2m-1}{3}+1 \leqslant j \leqslant m \\ 2, & \frac{2n+1}{3}+1 \leqslant i \leqslant n & 1 \leqslant j \leqslant m \end{cases}$$

Then,

$$v_f(0) = \frac{n+nm}{3},\ v_f(1) = \frac{n+nm}{3},\ v_f(2) = \frac{n+nm}{3},$$
$$e_f^*(0) = \frac{mn+n-3}{3},\ e_f^*(1) = \frac{mn+n}{3},\ e_f^*(2) = \frac{mn+n}{3}.$$

Thus,

$$|v_f(i) - v_f(j)| \leqslant 1 \text{ and } |e_f^*(i) - e_f^*(j)| \leqslant 1\ \forall\, i,j \in \{0,1,2\}.$$

Hence f is a mean cordial labelling.

Subcase 3.3 $n \equiv 2(mod3)$

Define $f : V(G) \to \{0,1,2\}$ as follows:

$$f(v_i) = \begin{cases} 0, & 1 \leqslant i \leqslant \frac{n+1}{3} \\ 1, & \frac{n+1}{3}+1 \leqslant i \leqslant \frac{2n+2}{3} \\ 2, & \frac{2n+2}{3}+1 \leqslant i \leqslant n \end{cases}$$

$$f(w_{ij}) = \begin{cases} 0, & 1 \leqslant i \leqslant \frac{n+1}{3} - 1 & 1 \leqslant j \leqslant m \\ 0, & i = \frac{n+1}{3} & 1 \leqslant j \leqslant \frac{2m-1}{3} \\ 1, & i = \frac{n+1}{3} & \frac{2m-1}{3} + 1 \leqslant j \leqslant m \\ 1, & \frac{n+1}{3} + 1 \leqslant i \leqslant \frac{2n+2}{3} - 1 & 1 \leqslant j \leqslant m \\ 1, & i = \frac{2n+2}{3} & 1 \leqslant j \leqslant \frac{m-2}{3} \\ 2, & i = \frac{2n+2}{3} & \frac{m-2}{3} + 1 \leqslant j \leqslant m \\ 2, & \frac{2n+2}{3} + 1 \leqslant i \leqslant n & 1 \leqslant j \leqslant m \end{cases}$$

The induced edge labelling $f^* : E(G) \longrightarrow \{0, 1, 2\}$ is found as follows:

$$f^*(v_i v_{i+1}) = \begin{cases} 0, & 1 \leqslant i \leqslant \frac{n+1}{3} - 1 \\ 1, & \frac{n+1}{3} \leqslant i \leqslant \frac{2n+2}{3} - 1 \\ 2, & \frac{2n+2}{3} \leqslant i \leqslant n - 1 \end{cases}$$

$$f^*(v_i w_{ij}) = \begin{cases} 0, & 1 \leqslant i \leqslant \frac{n+1}{3} - 1 & 1 \leqslant j \leqslant m \\ 0, & i = \frac{n+1}{3} & 1 \leqslant j \leqslant \frac{2m-1}{3} \\ 1, & i = \frac{n+1}{3} & \frac{2m-1}{3} + 1 \leqslant j \leqslant m \\ 1, & \frac{n+1}{3} + 1 \leqslant i \leqslant \frac{2n+2}{3} - 1 & 1 \leqslant j \leqslant m \\ 1, & i = \frac{2n+2}{3} & 1 \leqslant j \leqslant \frac{m-2}{3} \\ 2, & i = \frac{2n+2}{3} & \frac{m-2}{3} + 1 \leqslant j \leqslant m \\ 2, & \frac{2n+2}{3} + 1 \leqslant i \leqslant n & 1 \leqslant j \leqslant m \end{cases}$$

Then,

$$v_f(0) = \frac{n + nm}{3},\ v_f(1) = \frac{n + nm}{3},\ v_f(2) = \frac{n + nm}{3},$$
$$e^*_f(0) = \frac{mn + n - 3}{3},\ e^*_f(1) = \frac{mn + n}{3},\ e^*_f(2) = \frac{mn + n}{3}.$$

Thus,

$$|v_f(i) - v_f(j)| \leqslant 1 \text{ and } |e^*_f(i) - e^*_f(j)| \leqslant 1 \ \forall\ i, j \in \{0, 1, 2\},$$

Hence f is a mean cordial labelling. Thus, from all these above cases we conclude that the path union of n copies of star $K_{1,m}$ is a mean cordial graph. □

Illustration 2.5 A mean cordial labelling of four copies of star $K_{1,6}$ is shown in Figure 4.

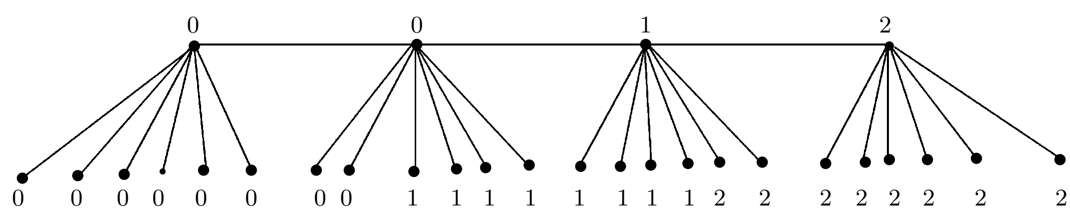

Figure 4

Illustration 2.6 A mean cordial labelling of two copies of star $K_{1,6}$ is shown in Figure 5.

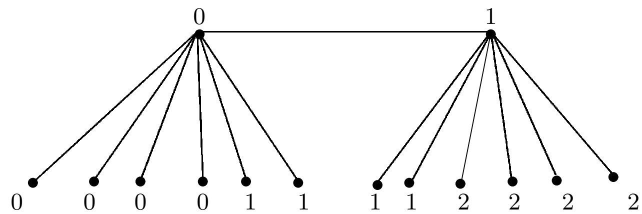

Figure 5

Illustration 2.7 A mean cordial labelling of six copies of star $K_{1,3}$ is shown in Figure 6.

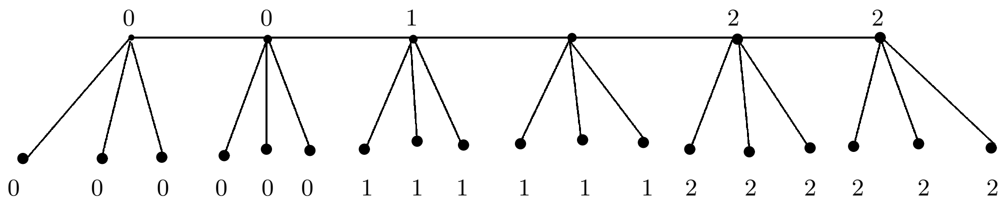

Figure 6

References

[1] I.Cahit, Cordial graph: A weaker version of graceful and harmonious graph, *Art Combinatorial*,23(1987), 201-208.

[2] J.A.Galian, A dynamic survey of Graph Labelling, *Electronic Journal of Combinatorics*, 19(2012).

[3] R.Ponraj, M.Shivkumar and M.Sundaram, Mean cordial libeling of graphs, *Open Journal of Discrete Mathematics*,2(2012), 145-148.

[4] M.Sundaram,R.Ponraj and S.Somosundaram, Product- cordial labeling of graphs, *Bulletin of Pure and Applied Science*,Vol.1(2004), 155-162.

[5] A.William, I.Raja Singh and S.Roy, Mean cordial labeling of certain graphs, *J.Comp and Math.Sci.*, Vol.4(2013), 274-281.