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Conformal Ricci Soliton in Almost $C(\lambda)$ Manifold

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Abstract: In this paper we have studied conformal curvature tensor, Ricci curvature tensor, projective curvature tensor in almost $C(\lambda)$ manifold admitting conformal Ricci soliton. We have studied conformally semi symmetric almost $C(\lambda)$ manifold admitting conformal Ricci soliton. We have found that a Ricci conharmonically symmetric almost $C(\lambda)$ manifold admitting conformal Ricci soliton is Einstein manifold. Similarly we have proved that a conformally symmetric almost $C(\lambda)$ manifold M with respect to projective curvature tensor admitting conformal Ricci soliton is η-Einstein manifold. We have studied Ricci projectively symmetric almost $C(\lambda)$ manifold also.

Key Words: Almost $C(\lambda)$ manifold, Ricci flow, conformal Ricci soliton, conformal curvature tensor, Ricci curvature tensor, projective curvature tensor.

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§1. Introduction

The concept of Ricci flow was first introduced by R. S. Hamilton [5] in 1982. This concept was developed to answer Thurston's geometric conjecture which says that each closed three manifold admits a geometric decomposition. Hamilton also [6]classified all compact manifolds with positive curvature operator in dimension four. The Ricci flow equation is given by

$$\frac{\partial g}{\partial t} = -2S \tag{1.1}$$

on a compact Riemannian manifold M with Riemannian metric g.

A self-similar solution to the Ricci flow [6], [10] is called a Ricci soliton [5] if it moves only by a one parameter family of diffeomorphism and scaling. The Ricci soliton equation is given by

$$\pounds_X g + 2S = 2\lambda g, \tag{1.2}$$

where $\pounds_X$ is the Lie derivative, S is Ricci tensor, g is Riemannian metric, X is a vector field and λ is a scalar. The Ricci soliton is said to be shrinking, steady, and expanding according as λ is positive, zero and negative respectively.

In 2004, A.E. Fischer [4] introduced the concept of conformal Ricci flow which is a variation

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of the classical Ricci flow equation. In classical Ricci flow equation the unit volume constraint plays an important role but in conformal Ricci flow equation scalar curvature r is considered as constraint. As the conformal geometry plays an important role to constrain the scalar curvature and the equations are the vector field sum of a conformal flow equation and a Ricci flow equation, the resulting equations are named as the conformal Ricci flow equations. The conformal Ricci flow equation on M is defined by the equation [4],

$$\frac{\partial g}{\partial t} + 2(S + \frac{g}{n}) = -pg \tag{1.3}$$

and $r = -1$, where p is a scalar non-dynamical field(time dependent scalar field), r is the scalar curvature of the manifold and n is the dimension of manifold.

The notion of conformal Ricci soliton was introduced by N. Basu and A. Bhattacharyya [1] in 2015 and the conformal Ricci soliton equation is given by

$$\pounds_X g + 2S = [2\sigma - (p + \frac{2}{n})]g. \tag{1.4}$$

The equation is the generalization of the Ricci soliton equation and it also satisfies the conformal Ricci flow equation.

In 1981, the notion of almost $C(\lambda)$ manifold was first introduced by D. Janssen and L. Vanhecke [7]. After that Z. Olszak and R. Rosca [9] have also studied such manifolds. Our present paper is motivated by this work.

In this paper we have studied conformal curvature tensor, conharmonic curvature tensor, Ricci curvature tensor, projective curvature tensor in almost $C(\lambda)$ manifold admitting conformal Ricci soliton. We have studied conformally semi symmetric almost $C(\lambda)$ manifold admitting conformal Ricci soliton. We have found that a Ricci conharmonically symmetric almost $C(\lambda)$ manifold admitting conformal Ricci soliton is Einstein manifold. Similarly we have proved that a conformally symmetric almost $C(\lambda)$ manifold M with respect to projective curvature tensor admitting conformal Ricci soliton is η-Einstein manifold. We have also studied Ricci projectively symmetric almost $C(\lambda)$ manifold.

$2. **Preliminaries**

Let M be a $(2n+1)$ dimensional connected almost contact metric manifold with an almost contact metric structure(ϕ, ξ, η, g) where ϕ is a $(1,1)$ tensor field, ξ is a covariant vector field, η is a 1-form and g is compatible Riemannian metric such that

$$\phi^2 X = -X + \eta(X)\xi, \eta(\xi) = 1, \phi\xi = 0, \eta o\phi = 0, \tag{2.1}$$

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y), \tag{2.2}$$

$$g(X, \phi Y) = -g(\phi X, Y), \tag{2.3}$$

$$g(X, \xi) = \eta(X), \tag{2.4}$$

$$(\nabla_X \phi)Y = g(X,Y)\xi - \eta(Y)X, \tag{2.5}$$

$$\nabla_X \xi = -\phi X, \tag{2.6}$$

for all $X, Y \in \chi(M)$.

If an almost contact Riemannian manifold M satisfies the condition

$$S = ag + b\eta \otimes \eta,$$

for some functions $a, b \in C^\infty(M)$ and S is the Ricci tensor, then M is said to be an η- Einstein manifold.

An almost contact manifold is called an almost $C(\lambda)$ manifold if the Riemann curvature R satisfies the following relations [8]

$$R(X,Y)Z = R(\phi X, \phi Y)Z - \lambda[Xg(Y,Z) - g(X,Z)Y - \phi X g(\phi Y, Z) + g(\phi X, Z)\phi Y], \tag{2.7}$$

where $X, Y, Z \in TM$ and λ is a real number.

From (2.7) we have

$$\left.\begin{aligned} R(X,Y)\xi &= R(\phi X, \phi Y)\xi - \lambda[X\eta(Y) - Y\eta(X)], \\ R(\xi, X)Y &= \lambda[\eta(Y)X - g(X,Y)\xi]. \end{aligned}\right\} \tag{2.8}$$

Now from definition of Lie derivative we have

$$(\pounds_\xi g)(X,Y) = (\nabla_\xi g)(X,Y) + g(-\phi X, Y) + g(X, -\phi Y) = 0 \tag{2.9}$$

$$(\because g(X, \phi Y) = -g(\phi X, Y)).$$

Now applying (2.9) in conformal Ricci soliton equation (1.4) we get

$$S(X,Y) = Ag(X,Y), \tag{2.10}$$

where $A = \frac{1}{2}[2\sigma - (p + \frac{2}{n})]$. Hence the manifold becomes an Einstein manifold.

Also we have,

$$QX = AX. \tag{2.11}$$

If we put $Y = \xi$ in (2.10) we get

$$S(X,\xi) = A\eta(X). \tag{2.12}$$

Again if we put $X = \xi$ in (2.12) we get

$$S(\xi,\xi) = A. \tag{2.13}$$

Using these results we shall prove some important results of almost $C(\lambda)$ manifold in the following sections.

§3. Almost $C(\lambda)$ Manifold Admitting Conformal Ricci Soliton and $R(\xi, X).C = 0$

Let M be a $(2n+1)$ dimensional almost $C(\lambda)$ manifold admitting conformal Ricci soliton (g, V, σ). Conformal curvature tensor C on M is defined by

$$C(X,Y)Z = R(X,Y)Z - \frac{1}{2n-1}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY]$$
$$+[\frac{r}{2n(2n-1)}][g(Y,Z)X - g(X,Z)Y], \tag{3.1}$$

where r is scalar curvature.

Since the manifold satisfies conformal Ricci soliton so we have $r = -1$ ([4]).

After putting $r = -1$ and $Z = \xi$ in (3.1) we have

$$C(X,Y)\xi = R(X,Y)\xi - \frac{1}{2n-1}[S(Y,\xi)X - S(X,\xi)Y + g(Y,\xi)QX - g(X,\xi)QY]$$
$$-\frac{1}{2n(2n-1)}[g(Y,\xi)X - g(X,\xi)Y]. \tag{3.2}$$

Using $(2.4), (2.8), (2.11), (2.12)$ in (3.2) we get

$$C(X,Y)\xi = R(\phi X, \phi Y)\xi - \lambda[X\eta(Y) - Y\eta(X)] - \frac{1}{2n-1}[A\eta(Y)X - A\eta(X)Y$$
$$+\eta(Y)AX - \eta(X)AY] - \frac{1}{2n(2n-1)}[\eta(Y)X - \eta(X)Y].$$

After a brief simplification we get

$$C(X,Y)\xi = R(\phi X, \phi Y)\xi - B(\eta(Y)X - \eta(X)Y), \tag{3.3}$$

where $B = \lambda + \frac{2A}{2n-1} + \frac{1}{2n(2n-1)}$, and

$$\eta(C(X,Y)Z) = \eta(R(\phi X, \phi Y)Z) + B[\eta(Y)g(X,Z) - \eta(X)g(Y,Z)]. \tag{3.4}$$

Now we assume that $R(\xi, X).C = 0$ holds in M i.e. the manifold is locally isometric to the hyperbolic space $H^{n+1}(-\alpha^2)$ ([11]), which implies

$$R(\xi, X)(C(Y,Z)W) - C(R(\xi, X)Y, Z)W - C(Y, R(\xi, X)Z)W$$
$$-C(Y,Z)R(\xi, X)W = 0, \tag{3.5}$$

for all vector fields X, Y, Z, W on M.

Using (2.8) in (3.5) and putting $W = \xi$ we get

$$\eta(C(Y,Z)\xi)X - g(X, C(Y,Z)\xi)\xi - \eta(Y)C(X,Z)\xi + g(X,Y)C(\xi,Z)\xi - \eta(Z)C(Y,X)\xi$$
$$+g(X,Z)C(Y,\xi)\xi - \eta(\xi)C(Y,Z)X + g(X,\xi)C(Y,Z)\xi = 0. \tag{3.6}$$

Using $(2.1), (3.3), (3.4)$ in (3.6) we have

$$\eta(R(\phi Y, \phi Z)\xi))X - g(X, R(\phi Y, \phi Z)\xi - B[Y\eta(Z) - Z\eta(Y)])\xi - \eta(Y)C(X,Z)\xi$$
$$+g(X,Y)C(\xi,Z)\xi - \eta(Z)C(Y,X)\xi + g(X,Z)C(Y,\xi)\xi - C(Y,Z)X$$
$$+\eta(X)C(Y,Z)\xi = 0. \tag{3.7}$$

Operating with η and putting $Z = \xi$ in (3.7) we get

$$Bg(X,Y) - B\eta(X)\eta(Y) - \eta(C(Y,\xi)X) - \eta(R(\phi Y, \phi X)\xi) = 0. \tag{3.8}$$

Now,

$$C(Y,\xi)X = R(Y,\xi)X - \frac{1}{2n-1}[S(\xi,X)Y - S(Y,X)\xi + g(\xi,X)QY - g(Y,X)Q\xi]$$
$$-\frac{1}{2n(2n-1)}[g(\xi,X)Y - g(Y,X)\xi]. \tag{3.9}$$

Using $(2.1), (2.8), (2.12)$ in (3.9) and operating with η we get

$$\eta(C(Y,\xi)X) = (\lambda + \frac{A}{2n-1} + \frac{1}{2n(2n-1)})g(X,Y) - (\lambda + \frac{2A}{2n-1}$$
$$+\frac{1}{2n(2n-1)})\eta(X)\eta(Y) + \frac{1}{2n-1}S(X,Y). \tag{3.10}$$

Putting (3.10) in (3.8) we obtain

$$\frac{A}{2n-1}g(X,Y) + \eta(R(\phi X, \phi Y)\xi) - \frac{1}{2n-1}S(X,Y) = 0. \tag{3.11}$$

In view of (2.8) we get from (3.11)

$$\frac{A}{2n-1}g(X,Y) + \eta(R(X,Y)\xi) - \frac{1}{2n-1}S(X,Y) = 0,$$

which can be written as

$$\frac{A}{2n-1}g(X,Y) - \frac{1}{2n-1}S(X,Y) = -g(R(X,Y)\xi,\xi). \tag{3.12}$$

Then we have

$$S(X,Y) = Ag(X,Y),$$

since $g(R(X,Y)\xi,\xi)=0$, where $A=\frac{1}{2}[2\sigma-(p+\frac{2}{n})]$.

Theorem 3.1 *If an almost $C(\lambda)$ manifold admitting conformal Ricci soliton is conformally semi symmetric i.e. $R(\xi,X).C=0$, then the manifold is Einstein manifold where C is Conformal curvature tensor and $R(\xi,X)$ is derivation of tensor algebra of the tangent space of the manifold.*

§4. Almost $C(\lambda)$ Manifold Admitting Conformal Ricci Soliton and $K(\xi,X).S=0$

Let M be a $(2n+1)$ dimensional almost $C(\lambda)$ manifold admitting conformal Ricci soliton (g,V,σ). The conharmonic curvature tensor K on M is defined by [3]

$$K(X,Y)Z=R(X,Y)Z-\frac{1}{2n-1}[S(Y,Z)X-S(X,Z)Y$$
$$+g(Y,Z)QX-g(X,Z)QY], \tag{4.1}$$

for all $X,Y,Z\in\chi(M)$, R is the curvature tensor and Q is the Ricci operator.

Also the equation (4.1) can be written in the form

$$K(\xi,X)Y=R(\xi,X)Y-\frac{1}{2n-1}[S(X,Y)\xi-S(\xi,Y)X+g(X,Y)Q\xi$$
$$-g(\xi,Y)QX]. \tag{4.2}$$

Using (2.8), (2.11), (2.12) in (4.2) we have

$$K(\xi,X)Y=\lambda[\eta(Y)X-g(X,Y)\xi]-\frac{1}{2n-1}[S(X,Y)\xi-A\eta(Y)X-\eta(Y)AX$$
$$+g(X,Y)A\xi]. \tag{4.3}$$

Similarly from (4.2) we get

$$K(\xi,X)Z=\lambda[\eta(Z)X-g(X,Z)\xi]-\frac{1}{2n-1}[S(X,Z)\xi-A\eta(Z)X-\eta(Z)AX$$
$$+g(X,Z)A\xi]. \tag{4.4}$$

Now we assume that the tensor derivative of S by $K(\xi,X)$ is zero i.e. $K(\xi,X).S=0$ (the manifold is locally isometric to the hyperbolic space $H^{n+1}(-\alpha^2)$ ([11]). It follows that

$$S(K(\xi,X)Y,Z)+S(Y,K(\xi,X)Z)=0, \tag{4.5}$$

which implies

$$S(\lambda\eta(Y)X-\lambda g(X,Y)\xi-\frac{1}{2n-1}S(X,Y)\xi+\frac{A}{2n-1}\eta(Y)X$$
$$-\frac{A}{2n-1}g(X,Y)\xi+\frac{A}{2n-1}\eta(Y)X,Z)+S(Y,\lambda\eta(Z)X-\lambda g(X,Z)\xi$$

$$-\frac{1}{2n-1}S(X,Z)\xi+\frac{A}{2n-1}\eta(Z)X-\frac{A}{2n-1}g(X,Z)\xi+\frac{A}{2n-1}\eta(Z)X)=0. \qquad (4.6)$$

Putting $Z=\xi$ in (4.6), using (2.1), (2.4), (2.12), (2.13) and after a long calculation we obtain

$$S(X,Y)=Ag(X,Y),$$

where $A=\frac{1}{2}[2\sigma-(p+\frac{2}{n})]$.

Theorem 4.1 *If an almost $C(\lambda)$ manifold admitting conformal Ricci soliton and the manifold is Ricci conharmonically symmetric i.e. $K(\xi,X).S=0$, then the manifold is Einstein manifold where K is conharmonic curvature tensor and S is a Ricci tensor.*

§5. Almost $C(\lambda)$ Manifold Admitting Conformal Ricci Soliton and $P(\xi,X).C=0$

Let M be a $(2n+1)$ dimensional almost $C(\lambda)$ manifold admitting conformal Ricci soliton (g,V,σ). The Weyl projective curvature tensor P on M is given by [2]

$$P(X,Y)Z=R(X,Y)Z-\frac{1}{2n}[S(Y,Z)X-S(X,Z)Y]. \qquad (5.1)$$

(5.1) can be written as

$$P(\xi,X)Y=R(\xi,X)Y-\frac{1}{2n}[S(X,Y)\xi-S(\xi,Y)X].$$

Using (2.8), (2.12) in the above equation we get

$$P(\xi,X)Y=\lambda[\eta(Y)X-g(X,Y)\xi]-\frac{1}{2n}[S(X,Y)\xi-A\eta(Y)X]. \qquad (5.2)$$

Now we consider that the tensor derivative of C by $P(\xi,X)$ is zero i.e. $P(\xi,X).C=0$ holds in M (the manifold is locally isometric to the hyperbolic space $H^{n+1}(-\alpha^2)$ [11]). So

$$P(\xi,X)C(Y,Z)W-C(P(\xi,X)Y,Z)W-C(Y,P(\xi,X)Z)W$$
$$-C(Y,Z)P(\xi,X)W=0 \qquad (5.3)$$

for all vector fields X,Y,Z,W on M.

Using (5.2) in (5.3) and putting $W=\xi$ we get

$$\lambda\eta(C(Y,Z)\xi)X-\lambda g(X,C(Y,Z)\xi)\xi-\frac{1}{2n}S(X,C(Y,Z)\xi)\xi+\frac{A}{2n}\eta(C(Y,Z)\xi)X$$
$$-\lambda\eta(Y)C(X,Z)\xi-\frac{A}{2n}\eta(Y)C(X,Z)\xi-\lambda\eta(Z)C(Y,X)\xi-\frac{A}{2n}\eta(Z)C(Y,X)\xi$$
$$-\lambda\eta(\xi)C(Y,Z)X+\lambda g(X,\xi)C(Y,Z)\xi+\frac{1}{2n}S(X,\xi)C(Y,Z)\xi-\frac{A}{2n}\eta(\xi)C(Y,Z)X=0. \qquad (5.4)$$

Operating with η, using (2.4), (2.12), (3.3) and putting $Z=\xi$ we get after a lengthy calculation that

$$(\lambda B-(\lambda+\frac{A}{2n})(\lambda+\frac{A}{2n-1}+\frac{1}{2n(n-1)}))g(X,Y)$$
$$+((\lambda+\frac{A}{2n})(\lambda+\frac{A}{2n-1}+\frac{1}{2n(n-1)})-\lambda B-\frac{AB}{2n})\eta(X)\eta(Y)$$
$$=((\lambda+\frac{A}{2n})(\frac{1}{2n-1})-\frac{B}{2n})S(X,Y),$$

which clearly shows that the manifold is η-Einstein.

Theorem 5.1 *If an almost $C(\lambda)$ manifold admitting conformal Ricci soliton and $P(\xi,X).C=0$ holds i.e. the manifold is conformally symmetric with respect to projective curvature tensor, then the manifold becomes η-Einstein manifold, where P is projective curvature tensor and C is conformal curvature tensor.*

§6. Almost $C(\lambda)$ Manifold Admitting Conformal Ricci Soliton and $R(\xi,X).P=0$

Let M be a $(2n+1)$ dimensional almost $C(\lambda)$ manifold admitting conformal Ricci soliton (g,V,σ). We assume that the manifold is projectively semi-symmetric i.e. $R(\xi,X).P=0$ holds in M, which implies

$$R(\xi,X)(P(Y,Z)W)-P(R(\xi,X)Y,Z)W-P(Y,R(\xi,X)Z)W$$
$$-P(Y,Z)R(\xi,X)W=0 \tag{6.1}$$

for all vector fields X,Y,Z,W on M.

Using (2.8) in (6.1) and putting $W=\xi$ we get

$$\lambda\eta(R(Y,Z)\xi-\frac{1}{2n}S(Z,\xi)Y+\frac{1}{2n}S(Y,\xi)Z)X-\lambda g(X,R(Y,Z)\xi-\frac{1}{2n}S(Z,\xi)Y$$
$$+\frac{1}{2n}S(Y,\xi)Z)\xi-\lambda\eta(Y)P(X,Z)\xi+\lambda g(X,Y)P(\xi,Z)\xi-\lambda\eta(Z)P(Y,X)\xi$$
$$+\lambda g(X,Z)P(Y,\xi)\xi-\lambda P(Y,Z)X+\lambda\eta(X)P(Y,Z)\xi=0.$$

Using $(2.4),(2.8),(2.12)$ and operating with η in the above equation we get

$$-\lambda^2\eta(Y)g(X,Z)+\frac{\lambda A}{2n}\eta(Z)g(X,Y)-\frac{\lambda A}{2n}\eta(Y)g(X,Z)$$
$$+\lambda^2 g(X,Y)\eta(Z)-\lambda\eta(P(Y,Z)X)=0. \tag{6.2}$$

Putting $Z=\xi$ in (6.2) we get

$$S(X,Y)=Ag(X,Y),$$

which implies that the manifold is an Einstein manifold.

Theorem 6.1 *If an almost $C(\lambda)$ manifold admitting conformal Ricci soliton and $R(\xi, X).P = 0$ holds i.e. the manifold is projectively semi-symmetric, then the manifold is an Einstein manifold, where P is projective curvature tensor and $R(\xi, X)$ is derivation of tensor algebra of the tangent space of the manifold.*

§7. Almost $C(\lambda)$ Manifold Admitting Conformal Ricci Soliton and $P(\xi, X).S = 0$

Let M be a $(2n+1)$ dimensional almost $C(\lambda)$ manifold admitting conformal Ricci soliton (g, V, σ). Now the equation (5.1) can be written as

$$P(\xi, X)Y = R(\xi, X)Y - \frac{1}{2n}[S(X, Y)\xi - S(\xi, Y)X] \tag{7.1}$$

and

$$P(\xi, X)Z = R(\xi, X)Z - \frac{1}{2n}[S(X, Z)\xi - S(\xi, Z)X]. \tag{7.2}$$

Now we assume that the manifold is Ricci projectively symmetric i.e. $P(\xi, X).S = 0$ holds in M, which gives

$$S(P(\xi, X)Y, Z) + S(Y, P(\xi, X)Z) = 0. \tag{7.3}$$

Using $(2.10), (2.12), (7.1), (7.2)$ in (7.3) we have

$$\begin{aligned} &Ag(R(\xi, X)Y - \frac{1}{2n}S(X, Y)\xi + \frac{A}{2n}\eta(Y)X, Z) + Ag(Y, R(\xi, X)Z \\ &- \frac{1}{2n}S(X, Z)\xi + \frac{A}{2n}\eta(Z)X) = 0. \end{aligned} \tag{7.4}$$

Using $(2.4), (2.8)$ in (7.4) and putting $Z = \xi$ we get

$$S(X, Y) = Ag(X, Y),$$

which proves that the manifold is an Einstein manifold.

Theorem 7.1 *If an almost $C(\lambda)$ manifold admitting conformal Ricci soliton and $P(\xi, X).S = 0$ holds i.e. the manifold is Ricci projectively symmetric, then the manifold is an Einstein manifold, where P is projective curvature tensor and S is the Ricci tensor.*

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