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Superior Edge Bimagic Labelling

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Abstract: A graph $G(p, q)$ is said to be edge bimagic total labeling with two common edge counts k_1 and k_2 if there exists a bijection $f : V \cup E \to \{1, 2, \cdots, p + q\}$ such that for each edge $uv \in E$, $f(u) + f(v) + f(e) = k_1$ or k_2. A total edge bimagic graph is called superior edge bimagic if $f(E(G)) = \{1, 2, \cdots, q\}$. In this paper we have proved superior edge bimagic labeling for certain class of graphs arising from graph operations.

Key Words: Graph, magic labeling, bijective function, edge bimagic, superior edge bimagic labeling.

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§1. Introduction

A labelling of a graph G is an assignment f of labels to either the vertices or the edges or both subject to certain conditions. Labeled graphs are becoming an increasingly useful family of mathematical Models from broad range of applications.Graph labelling was first introduced in the late 1960's. A useful survey on graph labelling by J.A. Gallian (2013) can be found in [1]. All the graphs considered here are finite, simple and undirected. We follow the notation and terminology of [2]. In most applications labels are positive (or nonnegative) integers, though in general real numbers could be used.

A (p, q)-graph $G = (V, E)$ with p vertices and q edges is called total edge magic if there is a bijection $f : V \cup E \to \{1, 2, \cdots, p + q\}$ such that there exists a constant k for any edge uv in E, $f(u) + f(uv) + f(v) = k$. The original concept of total edge-magic graph is due to Kotzig and Rosa [3]. They called it magic graph. A total edge-magic graph is called a superior edge-magic if $f(E(G)) = \{1, 2, \cdots, q\}$.

It becomes interesting when we arrive with magic type labeling summing to exactly two distinct constants say k_1 or k_2. Edge bimagic total labeling was introduced by J. Baskar Babujee [6]and studied in [7] as (1, 1) edge bimagic labeling. A graph $G(p, q)$ with p vertices and q edges is called total edge bimagic if there exists a bijection $f : V \cup E \to \{1, 2, \cdots, p + q\}$ such that

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for any edge $uv \in E$, we have two constants k_1 and k_2 with $f(u)+f(v)+f(uv)=k_1$ or k_2. A total edge-bimagic graph is called superior edge bimagic if $f(E(G))=\{1,2,\cdots,q\}$. Superior edge bimagic labelling was introduced and studied in [8].

Definition 1.1 *A pyramid graph $PY(n)$ is obtained from Prism graph $P_n \times C_3$ whose $V(P_n \times C_3)=\{v_{ij}: 1\leq i\leq n, 1\leq j\leq 3\}$ by adding a new vertex v_{00} adjacent to the three vertices v_{11}, v_{12}, v_{13} of $P_n\times C_3$. This graph has $3n+1$ vertices and $6n$ edges.*

Definition 1.2 *mK_n - Snake is a connected graph with m blocks whose block-cut point graph is a path and each of the m blocks is isomorphic to Complete graph K_n.*

Definition 1.3 *mW_n - Snake is a connected graph with m blocks whose block-cut point graph is a path and each of the m blocks is isomorphic to Wheel graph W_n.*

Definition 1.4 *A graph $G(p,q)$ is said to have an edge magic total labeling with common edge counts k_0 if there exists a bijection $f: V\cup E \to \{1,2,\cdots,p+q\}$ such that for each $e=(u,v)\in E$, $f(u)+f(v)+f(e)=k_0$. A total edge magic graph is called superior edge-magic if $f(E(G))=\{1,2,\cdots,q\}$.*

Definition 1.5 *A graph $G(p,q)$ is said to be edge bimagic total labeling with two common edge count k_1 and k_2 if there exists a bijection $f: V\cup E\to\{1,2,\cdots,p+q\}$ such that for each $e=(u,v)\in E$, $f(u)+f(v)+f(e)=k_1$ or k_2. A total edge-bimagic graph is called superior edge-bimagic if $f(E(G))=\{1,2,\cdots,q\}$.*

Definition 1.6 *If $G_1(p_1,q_1)$ and $G_2(p_2,q_2)$ are two connected graphs, $G_1\hat{O}G_2$ is obtained by superimposing any selected vertex of G_2 on any selected vertex of G_1. The resultant graph G belongs to the class $G_1\hat{O}G_2$ consists of p_1+p_2-1 vertices and q_1+q_2 edges. In general, we can construct p_1p_2 possible combination of graphs from G_1 and G_2.*

§2. Superior Edge Bimagic Labeling for Special Class of Graphs

Theorem 2.1 *A pyramid graph $PY(n)$ is superior edge bimagic for $n\geq 3$.*

Proof Let $f: V\cup E\to\{1,2,3,\cdots,9n+1\}$ be a bijection defined by

(i) $f(v_{00})=6n+1$, $f(v_{00}v_{11})=6n$, $f(v_{00}v_{12})=6n-1$, $f(v_{00}v_{13})=6n-2$, and

(ii) $f(v_{3i-2,1})=9i+6n-7$, $f(v_{3i-2,2})=9i+6n-6$, $f(v_{3i-2,3})=9i+6n-5$, $f(v_{3j-1,1})=9j+6n-2$, $f(v_{3j-1,2})=9j+6n-4$, $f(v_{3j-1,3})=9j+6n-3$, $f(v_{3k,1})=9k+6n$, $f(v_{3k,2})=9k+6n+1$, $f(v_{3k,3})=9k+6n-1$, $f(v_{3i-2,1}v_{3i-2,2})=6n-18i+15$, $f(v_{3i-2,2}v_{3i-2,3})=6n-18i+13$, $f(v_{3i-2,3}v_{3i-2,1})=6n-18i+14$, $f(v_{3j-1,1}v_{3j-1,2})=6n-18j+8$, $f(v_{3j-1,2}v_{3j-1,3})=6n-18j+9$, $f(v_{3j-1,3}v_{3j-1,1})=6n-18j+7$, $f(v_{3k,1}v_{3k,2})=6n-18k+1$, $f(v_{3k,2}v_{3k,3})=6n-18k+2$, $f(v_{3k,3}v_{3k,1})=6n-18k+3$, where

(a) $i,j,k=1,2,3,\cdots,[n/3]$, when $n\equiv 0 \pmod 3$ and

(b) $i=1,2,3,\cdots,[n/3]+1$; $j,k=1,2,3,\cdots,[n/3]$, when $n\equiv 1 \pmod 3$,

(c) $i, j = 1, 2, 3, \cdots, [n/3] + 1$; $k = 1, 2, 3, \cdots, [n/3]$, when $n \equiv 2 \pmod 3$,

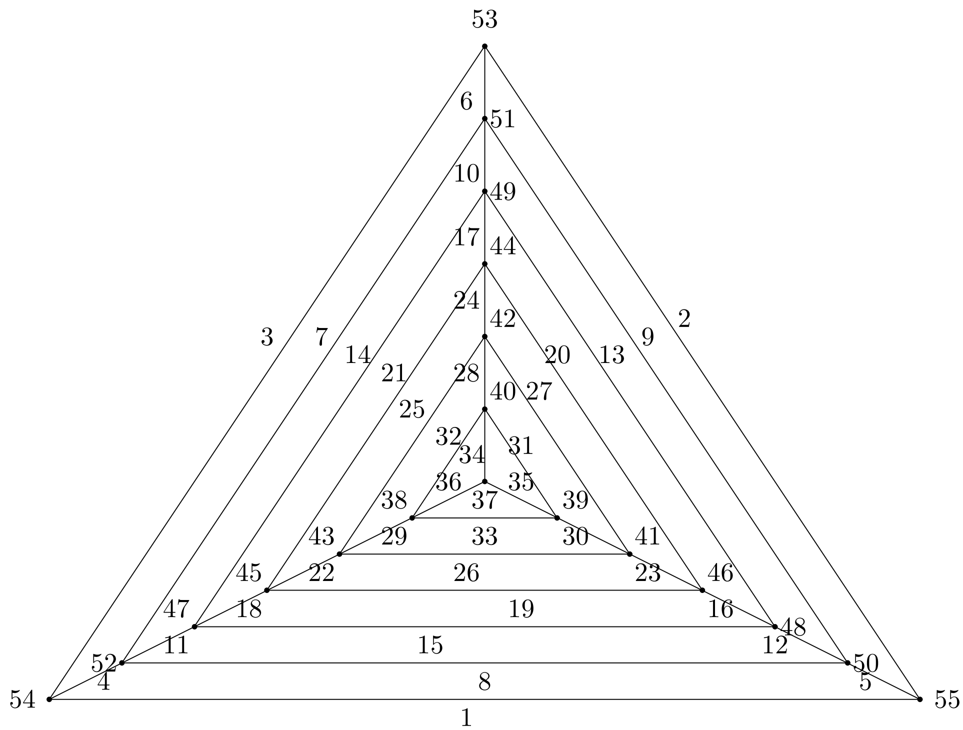


Figure 1 Superior bimagic of $PY(6)$ with $k_1 = 111, k_2 = 110$

(iii) $f(v_{3i-2,1}v_{3i-1,1}) = 6n-18i+11$, $f(v_{3i-2,2}v_{3i-1,2}) = 6n-18i+12$, $f(v_{3i-2,3}v_{3i-1,3}) = 6n-18i+10$, $f(v_{3j-1,1}v_{3j,1}) = 6n-18j+4$, $f(v_{3j-1,2}v_{3j,2}) = 6n-18j+5$, $f(v_{3j-1,3}v_{3j,3}) = 6n-18j+6$, $f(v_{3k,1}v_{3(k+1)-2,1}) = 6n-18k$, $f(v_{3k,2}v_{3(k+1)-2,2}) = 6n-18k-2$, $f(v_{3k,3}v_{3(k+1)-2,3}) = 6n-18k-1$, where

(a) $i, j = 1, 2, 3, \cdots, [n/3]$; $k = 1, 2, 3, \cdots, [n/3] - 1$ when $n \equiv 0 \pmod 3$,

(b) $i, j, k = 1, 2, 3, \cdots, [n/3]$, when $n \equiv 1 \pmod 3$,

(c) $i = 1, 2, 3, \cdots, [n/3] + 1$; $j, k = 1, 2, 3, \cdots [n/3]$, when $n \equiv 2 \pmod 3$.

We prove this labelling is superior edge bimagic. Now

$$f(v_{00}) + f(v_{11}) + f(v_{00}v_{11}) = 6n + 1 + 6n + 2 + 6n = 18n + 3,$$
$$f(v_{00}) + f(v_{12}) + f(v_{00}v_{12}) = 6n + 1 + 6n + 3 + 6n - 1 = 18n + 3,$$
$$f(v_{00}) + f(v_{13}) + f(v_{00}v_{13}) = 6n + 1 + 6n + 4 + 6n - 2 = 18n + 3.$$

Given n, considering appropriate values for i, j and k, we have

$$f(v_{3i-2,1}) + f(v_{3i-2,2}) + f(v_{3i-2,1}v_{3i-2,2}) = 9i + 6n - 7 + 9i + 6n - 6 + 6n - 18i + 15 = 18n + 2,$$

$$f(v_{3i-2,2})+f(v_{3i-2,3})+f(v_{3i-2,2}v_{3i-2,3}) = 9i+6n-6+9i+6n-5+6n-18i+13 = 18n+2,$$

$$f(v_{3i-2,3})+f(v_{3i-2,1})+f(v_{3i-2,3}v_{3i-2,1}) = 9i+6n-5+9i+6n-7+6n-18i+14 = 18n+2,$$

$$f(v_{3j-1,1})+f(v_{3j-1,2})+f(v_{3j-1,1}v_{3j-1,2}) = 9j+6n-2+9j+6n-4+6n-18j+8 = 18n+2,$$

$$f(v_{3k,1}) + f(v_{3k,2}) + f(v_{3k,1}v_{3k,2}) = 9k + 6n + 9k + 6n + 1 + 6n - 18k + 1 = 18n + 2.$$

Also for the remaining any edge uv, the sums $f(u) + f(v) + f(uv) = 18n + 2$. Hence the graph $PY(n)$ admits superior edge bimagic labelling. □

The superior edge bimagic labelling of $PY(6)$ shown in the Figure 1.

Theorem 2.2 *A nK_4-Snake graph admits superior edge bimagic labelling for $n \geq 1$.*

Proof The vertex set of nK_4-Snake graph is given by $V = \{x_i : 1 \leq i \leq n+1\} \cup \{y_i, w_i : 1 \leq i \leq n\}$ and edge set is given by $E = \{x_ix_{i+1}, x_iy_i, y_ix_{i+1}, x_iw_i, w_ix_{i+1} : 1 \leq i \leq n\}$.

Consider the k^{th} block of nK_4-Snake graph label the vertices and edges as follows

$$f(x_k) = 3k + 6n - 2,\ f(x_{k+1}) = 3k + 6n + 1,\ f(y_k) = 3k + 6n,\ f(w_k) = 3k + 6n - 1,$$

$$f(x_kx_{k+1}) = 6n - 6k + 4,\ f(x_ky_k) = 6n - 6k + 5,\ f(y_kx_{k+1}) = 6n - 6k + 1,$$

$$f(x_kw_k) = 6n - 6k + 6,\ f(w_kx_{k+1}) = 6n - 6k + 2 \text{ and } f(w_ky_k) = 6n - 6k + 3.$$

A superior edge bimagic labelling of $4K_4$-Snake graph shown in the Figure 2.

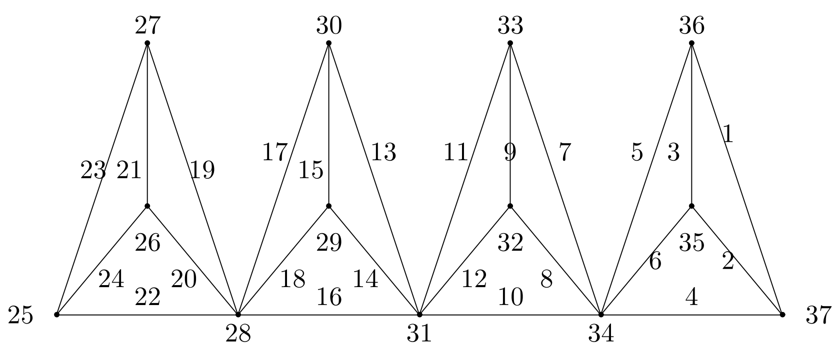


Figure 2 Superior edge bimagic of $4K_4$-Snake graph with $k_1 = 75$ and $k_2 = 74$

It is sufficient to prove that the k^{th} block of $4K_4$-Snake graph is superior edge bimagic where $1 \leq k \leq n$

$$f(x_k) + f(x_{k+1}) + f(x_kx_{k+1}) = 3k + 6n - 2 + 3k + 6n + 1 + 6n - 6k + 4 = 18n + 3,$$

$$f(x_k) + f(y_k) + f(x_ky_k) = 3k + 6n - 2 + 3k + 6n + 6n - 6k + 5 = 18n + 3,$$

$$f(y_k) + f(x_{k+1}) + f(y_kx_{k+1}) = 3k + 6n + 3k + 6n + 1 + 6n - 6k + 1 = 18n + 2,$$

$$f(x_k) + f(w_k) + f(x_kw_k) = 3k + 6n - 2 + 3k + 6n - 1 + 6n - 6k + 6 = 18n + 3,$$

$f(w_k)+f(x_{k+1})+f(w_kx_{k+1}) = 3k+6n-1+3k+6n+1+6n-6k+2 = 18n+2,$

$f(w_k)+f(y_k)+f(w_ky_k) = 3k+6n-1+3k+6n+6n-6k+3 = 18n+2.$

Therefore for any edge uv, $f(u)+f(v)+f(uv)$ yields either $18n+3$ or $18n+2$. Hence the nK_4-Snake graph admits superior edge bimagic labelling. □

Theorem 2.3 *A nW_4-Snake graph admits superior edge bimagic labelling.*

Proof The vertex set of nW_4 is given by $V = \{x_i : 1 \le i \le n+1\} \cup \{y_i, z_i, w_i : 1 \le i \le n\}$ and edge set is given by $E = \{x_iy_i, x_iz_i, x_iw_i, z_ix_{i+1}, y_ix_{i+1}, w_ix_{i+1} : 1 \le i \le n\}$.

Consider the k^{th} block of nW_4 and label the vertices and edges as follows

$f(x_k) = 4k+8n-3,\ f(y_k) = 4k+8n-1,\ f(x_{k+1}) = 4k+8n+1,$

$f(z_k) = 4k+8n-2,\ f(w_k) = 4k+8n,\ f(x_ky_k) = 8n-8k+7,$

$f(y_kx_{k+1}) = 8n-8k+2,\ f(x_kz_k) = 8n-8k+8,\ f(x_kw_k) = 8n-8k+5,$

$f(z_kx_{k+1}) = 8n-8k+4,\ f(w_kx_{k+1}) = 8n-8k+1,$

$f(z_ky_k) = 8n-8k+6$, and $f(y_kw_k) = 8n-8k+3.$

It is sufficient to prove that the k^{th} block of nW_4 is superior edge bimagic where $1 \le k \le n$

$f(x_k)+f(y_k)+f(x_ky_k) = 4k+8n-3+4k+8n-1+8n-8k+7 = 24n+3,$

$f(y_k)+f(x_{k+1})+f(y_kx_{k+1}) = 4k+8n-1+4k+8n+1+8n-8k+2 = 24n+2.$

Similarly for any remaining edge uv, $f(u)+f(v)+f(uv)$ equals $24n+3$ or $24n+2$. Hence the graph nW_4 admits superior edge bimagic labelling for $n \ge 1$. □

Theorem 2.4 *There exist a graph from the class $P_n\hat{o}K_{1,m}$ that admits superior edge magic labeling if n is odd and superior edge bimagic labelling if n is even.*

Proof Consider the path P_n with vertex set $\{x_1, x_2, \cdots, x_n\}$ and edge set $\{x_ix_{i+1}; 1 \le i \le n-1\}$. Let $K_{1,m}$ have vertex set $\{y_1, y_2, \cdots, y_{m+1}\}$ and edge set $\{y_1y_j; 2 \le j \le m+1\}$.

Let G be a one of the graph from the class $P_n\hat{O}K_{1,m}$ where we superimpose on the vertex say y_1 of $K_{1,m}$ on the selected vertex x_n in P_n. The vertex set and edge set of graph G is given by $V = \{x_i, y_j; 1 \le i \le n, 2 \le j \le m+1\}$ and $E = E_1 \cup E_2$, where $E_1 = \{x_ix_{i+1}; 1 \le i \le n-1\}$ and $E_2 = \{x_ny_j; 2 \le j \le m-1\}$.

Case 1. n is odd.

Let $f : V \cup E \to \{1, 2, 3, \cdots, 2m+2n-1\}$ be the bijective function defined by

$$f(x_i) = \begin{cases} (4m+4n-1-i)/2; & i = 1, 3, \cdots, n \\ (4m+3n-1-i)/2; & i = 2, 4, \cdots, n-1 \end{cases},$$

$f(y_j) = (2m+n+1-j);\ 2 \le j \le m+1,$

$f(x_ix_{i+1}) = i;\ 1 \le i \le n-1,$

$f(x_n y_j) = (n - 2 + j)$; $2 \leq j \leq m + 1$.

For any edge $x_i x_{i+1}$ in E_1,

$$f(x_i) + f(x_{i+1}) + f(x_i x_{i+1}) = \frac{4m + 4n - 1 - i + 4m + 3n - 1 - i - 1 + 2i}{2} = \frac{8m + 7n - 3}{2} = k.$$

For any edge $x_n y_j$ in E_2,

$$f(x_n) + f(y_j) + f(x_n y_j) = \frac{4m + 4n - 1 - n + 4m + 2n + 2 - 2j + 2n - 4 + 2j}{2} = \frac{8m + 7n - 3}{2} = k.$$

Hence the graph G from the class $P_n \hat{O} K_{1,m}$ admits superior edge magic labeling if n is odd.

Case 2. n is even.

Let $f : V \cup E \to \{1, 2, 3, \cdots, 2m + 2n - 1\}$ be the bijective function defined by

$$f(x_i) = \begin{cases} (4m + 4n - 1 - i)/2; & i = 1, 3, \cdots, n - 1 \\ (4m + 3n - i)/2; & i = 2, 4, \ldots, n \end{cases},$$

$f(y_j) = (2m + n + 1 - j)$; $2 \leq j \leq m + 1$,

$f(x_i x_{i+1}) = i$; $1 \leq i \leq n - 1$,

$f(x_n y_j) = (n - 2 + j)$; $2 \leq j \leq m + 1$.

For any edge $x_i x_{i+1}$ in E_1,

$$f(x_i) + f(x_{i+1}) + f(x_i x_{i+1}) = \frac{4m + 4n - 1 - i + 4m + 3n - i - 1 + 2i}{2} = \frac{8m + 7n - 2}{2} = k_1.$$

For any edge $x_n y_j$ in E_2,

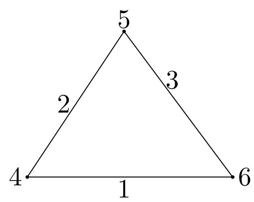
$$f(x_n) + f(y_j) + f(x_n y_j) = \frac{4m + 3n - n + 4m + 2n + 2 - 2j + 2n - 4 + 2j}{2} = \frac{8m + 6n - 2}{2} = k_2.$$

Hence the graph G from the class $P_n \hat{O} K_{1,m}$ admits superior edge bimagic labeling if n is even. □

§3. Superior Edge Bimagic Labeling for Some General Graphs

Theorem 3.1 *A complete graph K_n ($n \geq 6$) is not superior edge bimagic.*

Proof The superior edge bimagic labeling for the complete graph K_3, K_4 and K_5 are in the Figures 3-5 following.



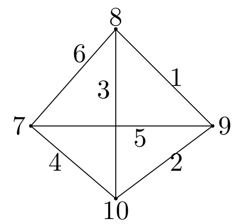


Figure 3 $k_1 = 11$, $k_2 = 14$ **Figure 4** $k_1 = 18$, $k_2 = 21$

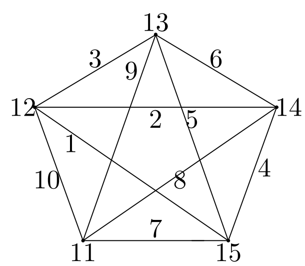


Figure 5 $k_1 = 28$, $k_2 = 33$

In case $n \geq 6$ we show that it is not to do super edge bimagic labeling. There are n vertices and nC_2 edges in a complete graph K_n. By labeling the n vertices from $(n^2-n+2)/2$, $(n^2-n+4)/2$, $\ldots$, $(n^2+n)/2$ and adding for all edges uv in the complete graph K_n we find that vertex sum of edges

$$f(u)+f(v) = \begin{cases} n^2-n+3, n^2-n+4, \cdots, n^2-n+7, \cdots, n^2+1; \\ n^2-n+5, n^2-n+6, n^2-n+7, \cdots, n^2+2; \\ n^2-n+7, n^2-n+8, n^2-n+9, \cdots, n^2+3; \\ \cdots\cdots\cdots \\ n^2-n-3, n^2-n-2, n^2-n-1. \end{cases}$$

On observing the above sequence of vertex sums of the edges we find that starting from n^2-n+7 onwards each integers occurs at least three times. Hence on adding the label of edge $f(uv)$ to each sum $f(u)+f(v)$, it is impossible to obtain to common edge count k_1 or k_2. Hence the complete graph K_n for $n \geq 6$ is not superior edge bimagic labeling. □

Theorem 3.2 *If G has superior edge magic then $G+K_1$ admits edge bimagic total labelling.*

Proof Let $G(p,q)$ be a superior edge magic graph with bijective function $f : V \cup E \rightarrow$

$\{1, 2, 3, \cdots, p + q\}$ such that $f(u) + f(v) + f(uv) = k_1$ for all $uv \in E(G)$ where $f(E(G)) = \{1, 2, 3, \cdots, q\}$. Now we define a new graph $G_1 = G + K_1$ with vertex set $V_1 = V \cup \{x_1\}$ and edge set $E_1 = E \cup \{v_i x_1;\ 1 \le i \le p\}$.

Now define the bijective function $g : V_1 \cup E_1 \to \{1, 2, 3, \cdots, p+q, p+q+1, \cdots, 2p+q+1\}$ as $g(v) = f(v)$ for all $v \in V(G)$ with $f(v_i) = q + i$, $g(x_1) = p + q + 1$, $g(uv) = f(uv)$ for all $uv \in E(G)$ and $g(v_i x_1) = (2p + q + 2) - i$; $1 \le i \le p$.

The edge set of G_1 consisting edges of G and remaining edges $\{v_i x_1; 1 \le i \le p\}$. Since G is superior edge magic, the edges of G have common count k_1. Now we need to prove edges $\{v_i x_1; 1 \le i \le p\}$ will have a common count k_2.

For edge $v_i x_1$, $g(v_i)+g(x_1)+g(v_i x_1) = q+i+p+q+1+(2p+q+2)-i = 3p+3q+3 = k_2$. Hence $G + K_1$ admit edge bimagic total labelling. □

Theorem 3.3 *If G has superior edge magic then there exist a graph from the class $G\hat{O}P_n$ admits edge bimagic total labelling.*

Proof Let $G(p, q)$ be a superior edge magic graph with bijective function $f : V \cup E \to \{1, 2, 3, \cdots, p + q\}$ such that $f(u) + f(v) + f(uv) = k_1$ for all $uv \in E(G)$ where $f(E(G)) = \{1, 2, 3, \cdots, q\}$. Consider the graph P_n with Vertex set $\{x_1, x_2, \cdots, x_n\}$ and Edge set $\{x_i x_{i+1}; 1 \le i \le n - 1\}$. We superimpose one of the vertex say x_1 of P_n on selected vertex v_p in G. Now we define new graph $G_1 = G\hat{O}P_n$ with vertex set $V_1 = V \cup \{x_i; 2 \le i \le n\}$ and edge set $E_1 = E \cup \{x_i x_{i+1}; 1 \le i \le n - 1\}$.

Now define the bijective function $g : V_1 \cup E_1 \to \{1, 2, 3, \cdots, p+q, p+q+1, \cdots, p+q+2n-2\}$ as $g(v) = f(v)$ for all $v \in V(G)$ with $f(v_i) = q + i$, $g(uv) = f(uv)$ for all $uv \in E(G)$ if n is odd,

$$g(x_i) = \begin{cases} (2p + 2q + i - 1)/2; & i = 1, 3, 5, 7, \cdots, n \\ (2p + 2q + n - 1 + i)/2; & i = 2, 4, 6, \cdots, n - 1 \end{cases},$$

$$g(x_i x_{i+1}) = (p + q + 2n - 1) - i; \quad 1 \le i \le n - 1$$

if n is even and

$$g(x_i) = \begin{cases} (2p + 2q + i - 1)/2; & i = 1, 3, 5, 7, \cdots, n - 1 \\ (2p + 2q + n - 2 + i)/2; & i = 2, 4, 6, \cdots, n \end{cases},$$

$$g(x_i x_{i+1}) = (p + q + 2n - 1) - i; \quad 1 \le i \le n - 1.$$

The edge set of G_1 consisting edges of G and remaining edges $\{x_i x_{i+1}; 1 \le i \le n - 1\}$. Since G is superior edge magic, the edges of G will have common count k_1. Now we need to prove edges $\{x_i x_{i+1}; 1 \le i \le n - 1\}$ have a common count k_2. We prove it in two cases.

Case 1. n is odd.

For edge $x_i x_{i+1}$

$$\begin{aligned} g(x_i) + g(x_{i+1}) + g(x_i x_{i+1}) &= (2p + 2q + i - 1)/2 + (2p + 2q + n + i)/2 \\ &\quad + (p + q + 2n - 1) - i \\ &= (6p + 6q + 5n - 3)/2 = k_2 \end{aligned}$$

Thus we have $G\hat{O}P_n$ has two common count k_1 and k_2 if n is odd.

Case 2. n is even.

For edge $x_i x_{i+1}$

$$\begin{aligned} g(x_i) + g(x_{i+1}) + g(x_i x_{i+1}) &= (2p + 2q + i - 1)/2 + (2p + 2q + n - 1 + i)/2 \\ &\quad + (p + q + 2n - 1) - i \\ &= (6p + 6q + 5n - 4)/2 = k_2 \end{aligned}$$

Thus we have $G\hat{O}P_n$ has two common count k_1 and k_2 if n is even.

Hence there exist a graph from the class $G\hat{O}P_n$ admits edge bimagic total labelling. □

Theorem 3.4 *If G has superior edge magic then there exist a graph from the class $G\hat{O}K_{1,n}$ admits edge bimagic total labelling.*

Proof Let $G(p,q)$ be a superior edge magic graph with bijective function $f : V \cup E \to \{1, 2, 3, \cdots, p + q\}$ such that $f(u) + f(v) + f(uv) = k_1$ for all $uv \in E(G)$ where $f(E(G)) = \{1, 2, 3, \cdots, q\}$. Consider the graph $K_{1,n}$ with Vertex set $\{x_1, x_2, \cdots, x_{n+1}\}$ and Edge set $\{x_1 x_i; 2 \leq i \leq n + 1\}$. We superimpose one of the vertex say x_1 of $K_{1,n}$ on selected vertex v_p in G. Now we define new graph $G_1 = G\hat{O}K_{1,n}$ with vertex set $V_1 = V \cup \{x_i; 2 \leq i \leq n + 1\}$ and edge set $E_1 = E \cup \{x_1 x_i; 2 \leq i \leq n + 1\}$.

Now define the bijective function $g : V_1 \cup E_1 \to \{1, 2, 3, \cdots, p+q, p+q+1, \ldots, p+q+2n\}$ as $g(v) = f(v)$ for all $v \in V(G)$ with $f(v_i) = q + i$, $g(uv) = f(uv)$ for all $uv \in E(G)$, $g(x_i) = p + q - 1 + i$; $1 \leq i \leq n + 1$ and $g(x_1 x_i) = (p + q + 2n + 2) - i$; $2 \leq i \leq n + 1$.

The edge set of G_1 consisting edges of G and remaining edges $\{x_1 x_i; 2 \leq i \leq n+1\}$. Since G is superior edge magic, the edges of G will have common count k_1. Now we need to prove edges $\{x_1 x_i; 2 \leq i \leq n + 1\}$ will have a common count k_2. For edge $x_1 x_i$,

$$\begin{aligned} g(x_1) + g(x_i) + g(x_1 x_i) &= p + q + p + q - 1 + i + (p + q + 2n + 2) - i \\ &= 3p + 3q + 2n + 1 = k_2 \end{aligned}$$

Thus we have $G\hat{O}K_{1,n}$ has two common count k_1 and k_2. Hence there exist a graph from the class $G\hat{O}K_{1,n}$ admits edge bimagic total labelling. □

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