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Some Results on Relaxed Mean Labeling

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Abstract: In this paper, we investigate relaxed mean labeling of some standard graphs. We prove, any cycle is a relaxed mean graph; if n > 4, K_n is not a relaxed mean graph; $K_{2,n}$ is a relaxed mean graph for all n; any Triangular snake is a relaxed mean graph; any Quadrilateral snake is a relaxed mean graph; the graph P_n^2 is a relaxed mean graph; $L_n \Theta K_1$ is a relaxed mean graph. Also, we prove $K_n^c + 2K_2$ is a relaxed mean graph for all n; W_4 is a relaxed mean graph; $K_2 + mK_1$ is a relaxed mean graph for all m; if G_1 and G_2 are tree, then $G = G_1 \cup G_2$ is a relaxed mean graph; the planar grid $P_m \times P_n$ is a relaxed mean graph for $m \geqslant 2$, $n \geqslant 2$ and the prism $P_m \times C_n$ is a relaxed mean graph for $m \geqslant 2$ and for all $n \geqslant 3$.

Key Words: Smarandache relaxed k-mean graph, relaxed mean graph, cycle, path, star.

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§1. Introduction

All graphs in this paper are finite, simple an undirected. Terms not defined here are used in the sense of Harary [3]. In 1966, Rosa [5] introduced β-valuation of graph. Golomb subsequently called such a labeling graceful. In 1980, Graham and Sloane [2] introduced the harmonious labeling of a graph. Also, in 2003, Somasundaram and Ponraj [6] and [7] introduced the mean labeling of a graph. On similar lines, we define relaxed mean labeling. In [4], we proved any path is a relaxed mean graph and if $m = 5$, $K_{1,m}$ is not a relaxed mean graph. We proved the bistar $B_{m,n}$ is a relaxed mean graph if $|m-n| = 3$. Also, we proved that combs are relaxed mean graph and $C_3 \cup P_n$ is a relaxed mean graph for n = 2. In this paper, we prove any cycle is a relaxed mean graph; if n > 4, K_n is not a relaxed mean graph; $K_{2,n}$ is a relaxed mean graph for all n; any Triangular snake is a relaxed mean graph; any Quadrilateral snake is a relaxed mean graph; the graph P_n^2 is a relaxed mean graph; $L_n \Theta K_1$ is a relaxed mean graph and $K_n^c + 2K_2$ is a relaxed mean graph for all n. Also, we prove W_4 is a relaxed mean graph; $K_2 + mK_1$ is a relaxed mean graph for all m; If G_1 and G_2 are trees, then $G = G_1 \cup G_2$ is a relaxed mean graph; the planar grid $P_m \times P_n$ is a relaxed mean graph for $m \geqslant 2$, $n \geqslant 2$ and the prism $P_m \times C_n$ is a relaxed mean graph for $m \geqslant 2$ and for all $n \geqslant 3$. The condition for a graph to be relaxed mean is that $p = q + 1$ in [4].

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§2. Main Results

Definition 2.1 *A graph $G = (V, E)$ with p vertices and q edges is said to be a Smarandache relaxed k-mean graph if there exists a function f from the vertex set of G to $\{0, 1, 2, 3, \cdots, q+1\}$ such that in the induced map $f*$ from the edge set of G to $\{1, 2, 3, \cdots, q\}$ defined by*

$$f*(e = uv) = \begin{cases} \dfrac{f(u) + f(v)}{2} & if\ f(u) + f(v)\ is\ even \\ \dfrac{f(u) + f(v) + 1}{2} & if\ \mathrm{f(u)} + f(v)\ is\ odd,\ then \end{cases}$$

the resulting edge labels are distinct. Furthermore, such a graph is called a Smarandache relaxed k-mean graph if we replace 2 by k and $f*(uv)$ by

$$\left\lfloor \frac{f(u) + f(v)}{k} \right\rfloor.$$

Theorem 2.2 *Any cycle is a relaxed mean graph.*

Proof The proof is divided into two cases following.

Case 1. Let n be odd. Let C_n be a cycle $u_1u_2 \cdots u_nu_1$. Define f : $V(C_n) \to \{0, 1, 2, \cdots, q = n\}$ and $q+1 = n+1$ by $f(u_1) = 0$; $f(u_n) = n + 1$; $f(u_i) = i - 1$ for $2 \leqslant i \leqslant \frac{n-1}{2}$ and $f(u_j) = j$ for $\frac{n+1}{2} \leqslant j \leqslant n-1$.

Case 2. Let n be even. Let C_n be a cycle $u_1u_2 \cdots {}_nu_1$. Define f : $V(C_n) \to \{0, 1, 2, \cdots, q = n\}$ and $q+1 = n+1$ by $f(u_1) = 0$; $f(u_n) = n + 1$; $f(u_i) = i - 1$ for $2 \leqslant i \leqslant \frac{n}{2}$ and $f(u_j) = j$ for $\frac{n}{2} + 1 \leqslant j \leqslant n-1$.

Therefore, the set of labels of the edges of C_n is $\{1, 2, \ldots, n\}$. Hence C_n is a relaxed mean graph. □

Theorem 2.3 *If $n > 4$, K_n is not a relaxed mean graph.*

Proof Suppose $n > 4$, K_n is a relaxed mean graph. To get the edge label $q+1 = \frac{n(n-1)}{2} + 1$, we must have $q+1$ and $q-2$ as the vertex labels. Let u and v be the vertices whose vertex labels are $q+1$ and $q-2$ respectively.

To get the edge label 1 we must have 0 and 1 as the vertex label (or) 0 and 2 as the vertex label. In either case 0 must be a label of some vertex. Let w be the vertex whose vertex label is 0.

If $q+1$ is even, the edges uw and vw get the same label $\frac{q+2}{2}$ which should not happen. If $q+1$ is odd and 0,1 are the vertex labels with labels w_1 having vertex label 1, then the edges uw and uw_1 get the same label $\frac{q+2}{2}$; if q is odd and 0, 2 are the vertex labels with w_1 having vertex label 2, then the edges uw and vw_1 get the same label $\frac{q+2}{2}$ which again should not happen. Hence K_n is not a relaxed mean graph for $n > 4$. □

Theorem 2.4 *$K_{2,n}$ is a relaxed mean graph for all n.*

Proof Let (V_1, V_2) be the bipartition of $K_{2,n}$ with $V_1 = \{u, v\}$, $V_2 = \{u_1, u_2, \cdots, u_n\}$. Define f: $V(K_{2,n}) \to \{0, 1, 2, \ldots, q = 2n\}$ and $q+1 = 2n+1$ by $f(u) = 1$; $f(v) = 2n+1$; $f(u_1) = 0$ and $f(u_{i+1}) = 2i$ for $1 \leqslant i \leqslant n-1$.

The corresponding edge labels are as follows:

The label of the edge uu_1 is 1. The label of the edge uu_{i+1} is $i + 1$ for $1 \leqslant i \leqslant n - 1$. The label of the edge vu_{i+1} is $n + i + 1$ for $1 \leqslant i \leqslant n - 1$. The label of the edge vu_1 is $n + 1$. Hence $K_{2,n}$ is a relaxed mean graph for all n. □

Definition 2.5 *A triangular snake is obtained from a path $v_1v_2 \cdots v_n$ by joining v_i and v_{i+1} to a new vertex w_i for $1 \leqslant i \leqslant n - 1$. That is, every edge of a path is replaced by a triangle C_3.*

Theorem 2.6 *Any triangular snake is a relaxed mean graph.*

Proof Let T_n be a triangular snake. Define $f : V(T_n) \rightarrow \{0, 1, 2, \ldots, q = 3n - 3\}$ and $q + 1 = 3n - 2$ by $f(v_i) = 3i - 3$ for $1 \leqslant i \leqslant n - 1$ and $f(v_n) = 3n - 2$; $f(w_i) = 3i - 1$ for $\leqslant i \leqslant n - 2$ and $f(v_{n-1}) = 3n - 5$.

The corresponding edge labels are as follows:

The labels of the edge $v_{i-1}v_i$ is $3i - 4$ for $2 \leqslant i \leqslant n - 1$. The labels of the edge $v_{n-1}v_n$ is $3n - 4$. The labels of the edge w_iv_i is $3i - 2$ for $1 \leqslant i \leqslant n - 2$. The labels of the edge $w_{n-1}v_{n-1}$ is $3n - 5$. The labels of the edge $w_{i-1}v_i$ is $3i - 3$ for $2 \leqslant i \leqslant n - 1$. The labels of the edge $w_{n-1}v_n$ is $3n - 3$. Hence T_n is a relaxed mean graph. □

Definition 2.7 *A quadrilateral snake is obtained from a path $u_1u_2 \ldots u_n$ by joining u_i, u_{i+1} to new vertices v_i, w_i respectively and joining v_i and w_i. That is, every edge of a path is replaced by a cycle C_4.*

Theorem 2.8 *Any quadrilateral snake is a relaxed mean graph.*

Proof Let Q_n denote a quadrilateral snake. Define $f : V(Q_n) \rightarrow \{0, 1, 2, \ldots, q = 4n - 4\}$ and $q + 1 = 4n - 3$ by $f(u_i) = 4i - 4$ for $1 \leqslant i \leqslant n - 1$ and $f(u_n) = 4n - 3$. $f(v_i) = 4i - 2$ for $1 \leqslant i \leqslant n - 2$ and $f(v_{n-1}) = 4n - 7$. $f(w_i) = 4i - 1$ for $1 \leqslant i \leqslant n - 2$ and $f(w_{n-1}) = 4n - 6$.

The corresponding edge labels are as follows:

The labels of the edge $u_{i-1}u_i$ is $4i - 6$ for $2 \leqslant i \leqslant n-1$ and $u_{n-1}u_n$ is $4n-5$. The labels of the edge u_iv_i is $4i - 3$ for $1 \leqslant i \leqslant n - 2$ and $u_{n-1}v_{n-1}$ is $4n - 7$. The labels of the edge $u_{i+1}w_i$ is $4i$ for $1 \leqslant i \leqslant n - 2$ and u_nw_{n-1} is $4n - 4$. The labels of the edge v_iw_i is $4i-1$ for $1 \leqslant i \leqslant n - 2$ and $v_{n-1}w_{n-1}$ is $4n - 6$. Hence Q_n is a relaxed mean graph. □

Definition 2.9 *The square G^2 of a graph G has $V(G^2) = V(G)$ with u,v is adjacent in G^2 whenever $d(u, v) \leqslant 2$ in G. The powers G^3, G^4, ... of G are similarly defined.*

Theorem 2.10 *The graph P_n^2 is a relaxed mean graph.*

Proof Let $u_1u_2 \ldots u_n$ be the path P_n. Clearly, P_n^2 has n vertices and $2n-3$ edges. Define $f : V(P_n^2) \rightarrow \{0, 1, 2, \ldots, q = 2n - 3\}$ and $q + 1 = 2n - 2$ by $f(u_i) = 2i - 2$ for $1 \leqslant i \leqslant n - 2$; $f(u_{n-1}) = 2n - 5$ and $f(u_n) = 2n - 2$.

The corresponding edge labels are as follows:

The labels of the edge u_iu_{i+1} is $2i - 1$ for $1 \leqslant i \leqslant n - 2$ and $u_{n-1}u_n$ is $2n - 3$. The labels of the edge u_iu_{i+2} is $2i$ for $1 \leqslant i \leqslant n - 2$. Hence P_n^2 is a relaxed mean graph. □

Definition 2.11 *The graph $C_3^{(t)}$ denotes that the one point union of t copies of cycle C_n. The graph $C_3^{(t)}$ is called a friendship graph or Dutch t-windmill.*

The graph $C_3^{(t)}$ is a relaxed mean graph. For instance, $C_3^{(4)}$ is shown in Fig.1.

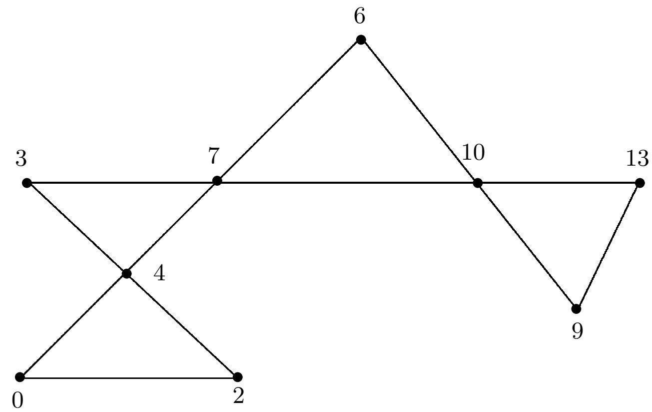

Fig.1

Theorem 2.12 *Let C_n be the cycle $u_1u_2\ldots u_nu_1$. Let G be a graph with $V(G) = V(C_n) \cup \{w_i : 1 \leqslant i \leqslant n\}$ and $E(G) = E(C_n) \cup \{u_iw_i\ ,\ u_{i+1}w_i : 1 \leqslant i \leqslant n\}$. Then G is a relaxed mean graph.*

Proof The proof is divided into two cases following.

Case 1. n is odd.

Define $f : V(G) \to \{0, 1, 2, \cdots, q = 3n\}$ and $q+1 = 3n+1$ by $f(u_i) = 3i-3$ for $1 \leqslant i \leqslant (n-1)/2$; $f(w_i) = 3i-1$ for $1 \leqslant i \leqslant (n-1)/2$; $f(u_{(n+1)/2}) = (3n-1)/2$; $f(u_{(n+3)/2}) = (3n+9)/2$; $f(u_{(n+3)/2+i}) = (3n+9)/2 + 3i + 1$ for $1 \leqslant i \leqslant (n-3)/2$; $f(w_{(n+1)/2}) = (3n+7)/2$; $f(w_{(n+3)/2}) = (3n+5)/2$ and $f(w_{(n+3)/2+i}) = (3n+7)/2 + 3i - 1$ for $1 \leqslant i \leqslant (n-3)/2$. Clearly, f is a relaxed mean labeling of G.

Case 2. n is even and $n \geqslant 8$.

Define $f : V(G) \to \{0, 1, 2, \ldots, q = 3n\}$ and $q + 1 = 3n + 1$ by $f(u_1) = 3$; $f(u_i) = 3i - 4$ for $2 \leqslant i \leqslant n/2$; $f(u_{(n/2)+1}) = (3n/2) + 1$; $f(u_{(n/2)+i}) = (3n/2) - 2 + 3i$ for $2 \leqslant i \leqslant (n-4)/2$; $f(u_{n-1}) = 3n - 3$; $f(u_n) = 3n + 1$; $f(w_1) = 0$; $f(w_2) = 7$; $f(w_i) = 3i + 1$ for $3 \leqslant i \leqslant (n-2)/2$; $f(w_{(n/2)}) = (3n-2)/2$; $f(w_{(n/2)+1}) = (3n+12)/2$; $f(w_{(n/2)+i+1}) = (3n+12)/2 + 3i$ for $1 \leqslant i \leqslant (n-8)/2$; $f(w_{n-2}) = 3n - 4$; $f(w_{n-1}) = 3n - 5$ and $f(w_n) = 3n - 2$.

Clearly, f is a relaxed mean labeling of G. Hence G is a relaxed mean graph. □

Theorem 2.13 *Let C_n be the cycle $u_1u_2\ldots u_nu_1$. Let G be a graph with $V(G) = V(C_n)$ and $E(G) = E(C_n) \cup \{u_1u_3\}$. Then G is a relaxed mean graph.*

Proof The proof is divided into two cases.

Case 1. n is odd.

Define $f : V(C_n) \to \{0, 1, 2, \ldots, q = n + 1\}$ and $q + 1 = n + 2$ by $f(u_1) = 0$ and $f(u_n) = n + 2$. Also, $f(u_i) = i$ for $i = 2, 3$; $f(u_j) = j + 1$ for $\frac{n+1}{2} \leqslant j \leqslant n-1$ and $f(u_k) = k$ for $k \neq i, j$.

Case 2. n is even.

Define f : $V(C_n) \rightarrow \{0, 1, 2, \ldots, q = n + 1\}$ and $q + 1 = n + 2$ by $f(u_1) = 0$ and $f(u_n) = n + 2$. Also, $f(u_i) = i$ for $i = 2, 3$; $f(u_j) = j + 1$ for $\frac{n}{2} \leqslant j \leqslant n - 1$and $f(u_k) = k$ for $k \neq i, j$.
Clearly, f is a relaxed mean labeling of G. □

Theorem 2.14 *Let C_n be the cycle $u_1u_2\ldots u_nu_1$. Let G be a graph with $V(G) = V(C_n)$ and $E(G) = E(C_n) \cup \{u_3u_6\}$. Then G is a relaxed mean graph.*

Proof The proof is divided into two cases.

Case 1. *n* is odd.

Define f : $V(C_n) \rightarrow \{0, 1, 2, \ldots, q = n + 1\}$ and $q + 1 = n + 2$ by $f(u_1) = 0$; $f(u_2) = 2$; $f(u_3) = 1$ and $f(u_n) = n + 2$.
Also, $f(u_i) = i + 1$ for $\frac{n+1}{2} \leqslant i \leqslant n - 1$ and $f(u_j) = j + 1$ for $i \neq j$.

Case 2. *n* is even.

Define f : $V(C_n) \rightarrow \{0, 1, 2, \ldots, q = n + 1\}$ and $q + 1 = n + 2$ by $f(u_1) = 0$; $f(u_2) = 2$; $f(u_3) = 1$ and $f(u_n) = n + 2$.
Also, $f(u_i) = i + 1$ for $\frac{n}{2} \leqslant i \leqslant n - 1$ and $f(u_j) = j + 1$ for $i \neq j$.
Clearly, f is a relaxed mean labeling of G. Hence G is a relaxed mean graph. □

Definition 2.15 *The graph* $L_n = P_n \times P_1$ *is called the ladder.*

We proceed to corona with ladder.

Theorem 2.16 *$L_n \Theta K_1$ is a relaxed mean graph.*

Proof Let $V(L_n) = \{a_i, b_i : 1 \leqslant i \leqslant n\}$ and $E(L_n) = \{a_ib_i : 1 \leqslant i \leqslant n - 1\} \cup \{a_ia_{i+1} : 1 \leqslant i \leqslant n - 1\} \cup \{b_ib_{i+1} : 1 \leqslant i \leqslant n - 1\}$.

Let c_i be the pendent vertex adjacent to a_i and let d_i be the pendent vertex adjacent to b_i. Define f: $V(L_n \ \Theta \ K_1) \rightarrow \{0, 1, 2, \ldots, q = 5n - 2\}$ and $q + 1 = 5n - 1$ by $f(a_i) = 5i - 4$ for $1 \leqslant i \leqslant n$; $f(b_i) = 5i - 3$ for $1 \leqslant i \leqslant n$; $f(c_i) = 5i - 5$ for $1 \leqslant i \leqslant n$; $f(d_i) = 5i - 2$ for $1 \leqslant i \leqslant n - 1$ and $f(d_n) = 5n - 1$.

The corresponding edge labels are as follows:

The labels of the edge c_ia_i is $5i - 4$ for $1 \leqslant i \leqslant n$. The labels of the edge a_ib_i is $5i - 3$ for $1 \leqslant i \leqslant n$. The labels of the edge b_id_i is $5i - 2$ for $1 \leqslant i \leqslant n$. The labels of the edge a_ia_{i+1} is $5i - 1$ for $1 \leqslant i \leqslant n - 1$. The labels of the edge b_ib_{i+1} is $5i$ for $1 \leqslant i \leqslant n - 1$.
Clearly, f is a relaxed mean labeling of G. Hence $L_n \ \Theta \ K_1$is a relaxed mean graph. □

Definition 2.17 *The graph* $K_n^c + 2K_2$ *is the join of complement of the complete graph on n vertices and two disjoint copies of* K_2. *First we prove that* $K_n^c + 2K_2$ *is a relaxed mean graph.*

Theorem 2.18 $K_n^c + 2K_2$ *is a relaxed mean graph for all* n.

Proof Let $V(K_n) = \{u_1, u_2, ..., u_n\}$, $V(2K_2) = \{u, v, w, z\}$ and $E(2K_2) = \{uv, wz\}$.

Define f : $V(K_n^c + 2K_2) \rightarrow \{0, 1, 2, ..., q = 4n + 2\}$ and $q + 1 = 4n + 3$ by $f(u) = 2$, $f(v) = 0$, $f(w) = 4n + 3$, $f(z) = 4n$ and $f(u_i) = 4i - 1$ for $1 \leqslant i \leqslant n$.

The corresponding edge labels are as follows:

The label of the edge uv is 1. The label of the edge wz is $4n+2$. The label of the edge uu_i is $2i+1$ for $1 \leqslant i \leqslant n$. The label of the edge vu_i is $2i$ for $1 \leqslant i \leqslant n$. The label of the edge wu_i is $2n+2i+1$ for $1 \leqslant i \leqslant n$. The label of the edge zu_i is $2n+2i$ for $1 \leqslant i \leqslant n$.

Hence $K_n^c + 2K_2$ is a relaxed mean graph for all n. □

The wheel W_n is the join of the graphs C_n and K_1. Next we investigate the relaxed mean labeling of the wheel $W_n = C_n + K_1$. The wheel $W_3 = K_4$ is a relaxed mean graph. We investigate W_n for any n, we take the case $n = 4$.

Theorem 2.19 it W_4 is a relaxed mean graph.

Proof Suppose W_4 is a relaxed mean graph with labeling f. Let $W_4 = C_4 + K_1$, where C_4 is the cycle $u_1u_2u_3u_4u_1$ and $V(K_1) = \{u\}$. To get the edge label 1 either 0 and 1 or 0 and 2 are the vertex labels of adjacent vertices. To get the edge label 8, 9 and 6 must be the vertex label of adjacent vertices. Let 0 and 2 are the vertex labels of adjacent vertices.

Then $f(u)=6$; $f(u_i)=0$; $f(u_{i+1})=2$; $f(u_{i+2})=9$ and $f(u_{i+3})=4$ for some i, $1 \leqslant i \leqslant 4$. Therefore, the induced edge labels are distinct.

Clearly, f is a relaxed mean labeling of G. Hence W_4 is a relaxed mean graph. □

Definition 2.20 $K_2 + mK_1$ *is the join of the graph* K_2 *and* m *disjoint copies of* K_1. *Some authors call this graph a Book with triangular pages. We now investigate the relaxed mean labeling of* $K_2 + mK_1$.

Theorem 2.21 $K_2 + mK_1$ *is a relaxed mean graph for all* m.

Proof Let u, v be the vertices of K_2 and $u_1, u_2, \ldots, u_m$ be the remaining vertices of $K_2 + mK_1$. Define $f : V(K_2 + mK_1) \to \{0, 1, 2, \ldots, q = 2m+1\}$ and $q+1 = 2m+2$ by $f(u) = 0$, $f(v) = 2m+2$, $f(u_i) = 2i$ for $1 \leqslant i \leqslant m-1$; $f(u_m) = 2m-1$. The label of the edge uu_i is i for $1 \leqslant i \leqslant m-1$. The label of the edge uv is $m+1$. The label of the edge vu_i is $m+1+i$ for $1 \leqslant i \leqslant m-1$. The label of the edge uu_m is m. The label of the edge vu_m is $2m+1$. Therefore the induced edge labels are distinct.

Clearly, f is a relaxed mean labeling of G. Hence $K_2 + mK_1$ is a relaxed mean graph. □

Theorem 2.22 *If* G_1 *and* G_2 *are trees, then* $G = G_1 \cup G_2$ *is a relaxed mean graph.*

Proof Let $G_1 = (p_1, q_1)$, $G_2 = (p_2, q_2)$ be the given trees and let G be a (p, q) graph.

Therefore, $p = p_1 + p_2$ and $q = q_1 + q_2$. Since G_1 and G_2 are trees, $q_1 = p_1 - 1$ and $q_2 = p_2 - 1$.

Now, $q+1 = q_1 + q_2 + 1 = p_1 - 1 + p_2 - 1 + 1 = p_1 + p_2 - 1 = p - 1$. Whence, $G = G_1 \cup G_2$ is a relaxed mean graph. □

Theorem 2.23 *The planar grid* $P_m \times P_n$ *is a relaxed mean graph for* $m \geqslant 2$ *and* $n \geqslant 2$.

Proof Let $V(P_m \times P_n) = \{a_{ij} : 1 \leqslant i \leqslant m, 1 \leqslant j \leqslant n\}$ and

$$E(P_m \times P_n) = \left\{a_{i(j-1)}a_{ij} : 1 \leqslant i \leqslant m,\ 2 \leqslant j \leqslant n\right\} \cup \left\{a_{(i-1)j}a_{ij} : 2 \leqslant i \leqslant m,\ 1 \leqslant j \leqslant n\right\}.$$

Define $f : V(P_m \times P_n) \to \{0, 1, 2, \ldots, q = 2mn - (m+n)\}$ and $q+1 = 2mn - (m+n-1)$ by

$f(a_{1j}) = j-1$, $1 \leqslant j \leqslant n$ and

$$f(a_{ij}) = \begin{cases} f\left(a_{(i-1)n}\right) + (n-1) + j, & 2 \leqslant i \leqslant m, 1 \leqslant j \leqslant n. \\ f\left(a_{(i-1)n}\right) + (n-1) + j + 1 & if\ m \ and\ n\ are\ maximum \end{cases}$$

The label of the edge $a_{ij}a_{i(j+1)}$ is $(i-1)(2n-1)+j$ for $1 \leqslant i \leqslant m$, $1 \leqslant j \leqslant n-1$.

The label of the edge $a_{ij}a_{(i+1)j}$ is $(n-1)+(i-1)(2n-1)+j$ for $1 \leqslant i \leqslant m-1$, $1 \leqslant j \leqslant n$.

Clearly, f is a relaxed mean labeling of G Hence $P_m \times P_n$ is a relaxed mean graph for $m \geqslant 2$ and $n \geqslant 2$. □

Theorem 2.24 *The prism $P_m \times C_n$ is a relaxed mean graph for $m \geqslant 2$ and for all $n \geqslant 3$.*

Proof Let $V(P_m \times C_n) = \{a_{ij} : 1 \leqslant i \leqslant m, 1 \leqslant j \leqslant n\}$ and

$$E(P_m \times C_n) = \left\{a_{i\,(j-1)}\ a_{i\,j} : 1 \leqslant i \leqslant m,\ 2 \leqslant j \leqslant n\right\} \bigcup \left\{a_{(i-1)\,j}\ a_{ij} : 2 \leqslant i \leqslant m, 1 \leqslant j \leqslant n\right\} \bigcup \{a_{i1}\ a_{in} : 1 \leqslant i \leqslant m\}.$$

Take

$$n = \begin{cases} 2r & if\ n\ is\ even \\ 2r+1 & if\ n\ is\ odd \end{cases}$$

Define $f : V(P_m \times C_n) \rightarrow \{0, 1, 2, ..., q+1 = (2mn-n)+1\}$ by $f(a_{1j}) = 2j$ for $1 \leqslant j \leqslant r$ and $f(a_{1n}) = 0$, Also,

$$f(a_{1\ r+j}) = \begin{cases} n-2j+2 & \text{if } n \text{ is odd for } 1 \leqslant j \leqslant r \\ n-2j+1 & \text{if } n \text{ is even for } 1 \leqslant j \leqslant r-1 \end{cases}$$

$$f(a_{2j}) = 2n + 2(j-1) \text{ for } 1 \leqslant j \leqslant r+1$$

$$f(a_{2\ r+1+j}) = \begin{cases} 3n-2j+2 & \text{if } n \text{ is odd for } 1 \leqslant j \leqslant r \\ 3n-2j+1 & \text{if } n \text{ is even for } 1 \leqslant j \leqslant r-1 \end{cases}$$

$$f\left(a_{(2i-1)\ j}\right) = f(a_{1j}) + 4n(i-1) \text{ for } 2 \leqslant i \leqslant \frac{m-1}{2}, 1 \leqslant j \leqslant n;$$

Also, $f(a_{(2i-1)1}) = f(a_{11}) + 5m$ for $i = \frac{m+1}{2}$;

$f(a_{(2i-1)2}) = f(a_{12}) + 5m + 2$ for $i = \frac{m+1}{2}$ and f $(a_{(2i-1)3})$ = f (a_{13}) + 5m + 1 for $i = \frac{m+1}{2}$.

$f\left(a_{(2i-2)\ j}\right) = f(a_{2j}) + 4n(i-2)$ for $3 \leqslant i \leqslant m$, $1 \leqslant j \leqslant n$.

Clearly, f is a relaxed mean labeling of $P_m \times C_n$ for $m \geqslant 2$, $n \geqslant 3$. Hence G is a relaxed mean graph.

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