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Skolem Difference Odd Mean Labeling For Some Simple Graphs

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Abstract: A graph G with p vertices and q edges is said to have skolem difference odd mean labeling if there exists an injective function $f : V(G) \to \{1, 2, 3, \cdots, 4q-1\}$ such that the induced map $f^* : E(G) \to \{1, 3, 5, \cdots, 2q-1\}$ defined by

$$f^*(uv) = \begin{cases} \dfrac{|f(u)-f(v)|}{2} & \text{if } |f(u)-f(v)| \text{ is even} \\ \dfrac{|f(u)-f(v)|+1}{2} & \text{if } |f(u)-f(v)| \text{ is odd} \end{cases}$$

is a bijection. A graph that admits skolem difference odd mean labeling is called a skolem difference odd mean graph. Here we investigate skolem difference odd mean behaviour of some standard graphs.

Key Words: Labeling, skolem difference odd mean graph, Smarandache k-mean graph.

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§1. Introduction

Throughout this paper, by a graph we mean a finite, undirected and simple graph. Let $G(V, E)$ be a graph with p vertices and q edges. For notations and terminology we follow [1].

Path on n vertices is denoted by P_n and a cycle on n vertices is denoted by C_n. A graph $G = (V, E)$ is called bipartite if $V = V_1 \cup V_2$ with $\phi = V_1 \cap V_2$, and every edge of G is of the form $\{u, v\}$ with $u \in V_1$ and $v \in V_2$. If each vertex in V_1 is joined with every vertex in V_2, we have a complete bipartite graph. In this case $|V_1| = m$ and $|V_2| = n$, the graph is denoted by $K_{m,n}$. The complete bipartite graph $K_{1,n}$ is called a star graph and it is denoted by S_m. The bistar $B_{m,n}$ is the graph obtained from K_2 by identifying the center vertices of $K_{1,m}$ and $K_{1,n}$ at the end vertices of K_2 respectively. $B_{m,m}$ is often denoted by $B(m)$.

A quadrilateral snake Q_n is obtained from a path $u_1, u_2, \cdots, u_{n+1}$ by joining u_i and u_{i+1} to new vertices v_i and w_i respectively and joining v_i and w_i, $1 \le i \le n$, that is, every edge of a path is replaced by a cycle C_4. The corona of a graph G on p vertices $v_1, v_2, \cdots, v_p$ is the graph obtained from G by adding p new vertices $u_1, u_2, \cdots, u_p$ and the new edges u_iv_i for $1 \le i \le p$. The corona of G is denoted by $G \odot K_1$. The graph $P_n \odot K_1$ is called a comb. Let G_1 and G_2 be any two graphs with p_1 and p_2 vertices respectively. Then the cartesian product $G_1 \times G_2$ has p_1p_2 vertices which are $\{(u, v)/u \in G_1, v \in G_2\}$. The edge set of $G_1 \times G_2$ is obtained as follows: (u_1, v_1) and (u_2, v_2) are adjacent in $G_1 \times G_2$ if either

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$u_1 = u_2$ and v_1 and v_2 are adjacent in G_2 or u_1 and u_2 are adjacent in G_1 and $v_1 = v_2$. The product $P_m \times P_n$ is called a planar grid and $P_n \times P_2$ is called a ladder, denoted by L_n. The graph $P_2 \times P_2 \times P_2$ is called a cube and is denoted by Q_3. A dragon is a graph formed by joining the end vertex of a path to a vertex of the cycle.

The concept of mean labeling was introduced and studied by S. Somasundaram and R. Ponraj [5]. Some new families of mean graphs are studied by S.K. Vaidya et al. [10]. Further some more results on mean graphs are discussed in [4,6,7]. A graph G is said to be a mean graph if there exists an injective function f from $V(G)$ to $\{0, 1, 2, \cdots, q\}$ such that the induced map f^* from $E(G)$ to $\{1, 2, 3, \cdots, q\}$ defined by $f^*(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ is a bijection. Furthermore, if f^* is defined by $f^*(uv) = \left\lceil \frac{f(u)+f(v)}{k} \right\rceil$ for an integer $k \geq 2$ hold with previous properties, then G is called a *Smarandache k-mean graph*.

In [2], K. Manickam and M. Marudai introduced odd mean labeling of a graph. A graph G is said to be odd mean if there exists an injective function f from $V(G)$ to $\{0, 1, 2, 3, \cdots, 2q-1\}$ such that the induced map f^* from $E(G)$ to $\{1, 3, 5, \cdots, 2q-1\}$ defined by $f^*(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ is a bijection. Some more results on odd mean graphs are discussed in [8,9].

The concept of skolem difference mean labeling was introduced and studied by K. Murugan and A. Subramanian [3]. A graph $G = (V, E)$ with p vertices and q edges is said to have skolem difference mean labeling if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $1, 2, 3 \cdots, p+q$ in such a way that for each edge $e = uv$, let $f^*(e) = \left\lceil \frac{|f(u)-f(v)|}{2} \right\rceil$ and the resulting labels of the edges are distinct and are from $1, 2, 3, \cdots, q$. A graph that admits a skolem difference mean labeling is called a skolem difference mean graph. It motivates us to define a new concept called skolem difference odd mean labeling.

A graph with p vertices and q edges is said to have a skolem difference odd mean labeling if there exists an injective function $f : V(G) \to \{1, 2, 3, \cdots, 4q-1\}$ such that the induced map $f^* : E(G) \to \{1, 3, 5, \cdots, 2q-1\}$ defined by $f^*(uv) = \left\lceil \frac{|f(u)-f(v)|}{2} \right\rceil$ is a bijection. A graph that admits a skolem difference odd mean labeling is called a skolem difference odd mean graph.

For example, a skolem difference odd mean labeling of cube Q_3 shown in Fig.1.

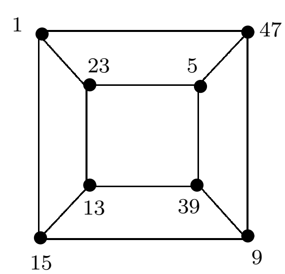

Fig.1

In this paper, we prove that the path P_n, the cycle C_n for $n \geq 4$, $K_{m,n} (m \geq 1, n \geq 1)$, the bistar $B_{m,n}$ for $m \geq 1, n \geq 1$, the quadrilateral snake Q_n, the ladder $L_n, L_n \odot K_1$ and $K_{1,n} \odot K_1$ for $n \geq 1$ are skolem difference odd mean graphs.

§2. Skolem Difference Odd Mean Graphs

Theorem 2.1 *Any path is a skolem difference odd mean graph.*

Proof Let $u_1, u_2, \ldots, u_n$ be the vertices of the path P_n. Define $f : V(P_n) \to \{1, 2, 3, \cdots, 4q-1 =$

$4n-5\}$ as follows:

$$f(u_{2i-1}) = 4i-3, \qquad 1 \le i \le \left\lceil \frac{n}{2} \right\rceil$$
$$f(u_{2i}) = 4n-4i-1, \qquad 1 \le i \le \left\lfloor \frac{n}{2} \right\rfloor$$

The label of the edge u_iu_{i+1} is $2n-2i-1$, $1 \le i \le n-1$. Hence, P_n is a skolem difference odd mean graph. □

For example, a skolem difference odd mean labeling of P_8 and P_{11} are shown in Fig.2.

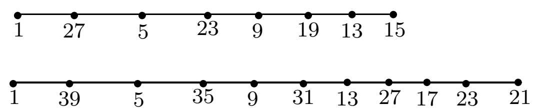

Fig.2

Theorem 2.2 *Cycle C_n is a skolem difference odd mean graph for $n \ge 4$.*

Proof Let $u_1, u_2, \cdots, u_n$ be the vertices of the cycle C_n. Define $f : V(C_n) \to \{1, 2, 3, \cdots, 4q-1 = 4n-1\}$ as follows:

Case 1. $n \equiv 0(mod\ 4)$

$$f(u_i) = \begin{cases} 2i-1, & 1 \le i \le n \text{ and } i \text{ is odd,} \\ 4n-2i+3, & 1 \le i \le \frac{n}{2} \text{ and } i \text{ is even,} \\ 4n-2i-1, & \frac{n+4}{2} \le i \le n \text{ and } i \text{ is even.} \end{cases}$$

For the vertex labeling f, the induced edge labeling f^* is given as follows:

$$f^*(u_iu_{i+1}) = \begin{cases} 2n-2i+1, & 1 \le i \le \frac{n}{2} \\ 2n-2i-1, & \frac{n+2}{2} \le i \le n-1 \end{cases}$$

and
$$f^*(u_nu_1) = n-1.$$

Case 2. $n \equiv 1(mod\ 4)$

$$f(u_i) = \begin{cases} 2i-1, & 1 \le i \le n-2 \text{ and } i \text{ is odd} \\ 4n-2i+3, & 1 \le i \le \frac{n-1}{2} \text{ and } i \text{ is even} \\ 4n-2i-1, & \frac{n+3}{2} \le i \le n-1 \text{ and } i \text{ is even} \\ 2n, & i = n. \end{cases}$$

For the vertex labeling f, the induced edge labeling f^* is obtained as follows:

$$f^*(u_iu_{i+1}) = \begin{cases} 2n-2i+1, & 1 \le i \le \frac{n-1}{2} \\ 2n-2i-1, & \frac{n+1}{2} \le i \le n-1 \end{cases}$$

and
$$f^*(u_nu_1) = n.$$

Case 3. $n \equiv 2(mod\ 4)$

$$f(u_i) = \begin{cases} 2i-1, & 1 \leq i \leq \frac{n}{2} \text{ and } i \text{ is odd} \\ 2i+3, & \frac{n+4}{2} \leq i \leq n-1 \text{ and } i \text{ is odd,} \\ 4n-2i+3, & 1 \leq i \leq n-2 \text{ and } i \text{ is even,} \\ 2n-1, & i = n. \end{cases}$$

For the vertex labeling f, the induced edge labeling f^* is given as follows:

$$f^*(u_iu_{i+1}) = \begin{cases} 2n-2i+1, & 1 \leq i \leq \frac{n}{2} \\ 2n-2i-1, & \frac{n+2}{2} \leq i \leq n-1 \end{cases}$$

and
$$f^*(u_nu_1) = n-1.$$

Case 4. $n \equiv 3(mod\ 4)$

$$f(u_i) = \begin{cases} 2i-1, & 1 \leq i \leq n-4 \text{ and } i \text{ is odd} \\ 2n-4, & i = n-2 \\ 4n-2, & i = n \\ 4n-2i-1, & 1 \leq i \leq \frac{n-3}{2} \text{ and } i \text{ is even} \\ 4n-2i-5, & \frac{n+1}{2} \leq i \leq n-3 \text{ and } i \text{ is even} \\ 2n-2, & i = n-1. \end{cases}$$

For the vertex labeling f, the induced edge labeling f^* is obtained as follows:

$$f^*(u_iu_{i+1}) = \begin{cases} 2n-2i-1, & 1 \leq i \leq \frac{n-3}{2} \\ 2n-2i-3, & \frac{n-1}{2} \leq i \leq n-2 \\ n, & i = n-1. \end{cases}$$

and
$$f^*(u_nu_1) = 2n-1.$$

Then, f is a skolem difference odd mean labeling. Thus, C_n for $n \geq 4$ is a skolem difference odd mean graph. □

For example, a skolem difference odd mean labeling of C_{12} and C_{11} are shown in Fig.3.

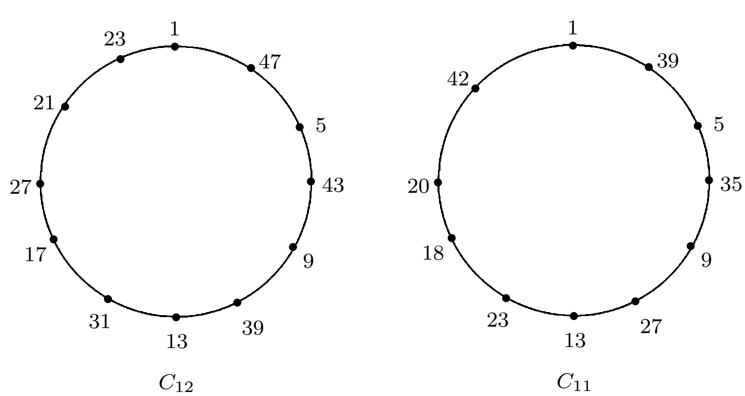

Fig.3

Theorem 2.3 *Every complete bipartite graph $K_{m,n}(m \geq 1, n \geq 1)$ is a skolem difference odd mean graph.*

Proof Let $V = V_1 \cup V_2$ where $V_1 = \{u_1, u_2, \cdots, u_m\}$ and $V_2 = \{v_1, v_2, \cdots, v_n\}$. The graph $K_{m,n}$ has $m+n$ vertices and mn edges. Define $f : V(K_{m,n}) \to \{1, 2, 3, \cdots, 4q-1 = 4mn-1\}$ as follows:

$$f(u_i) = 4i - 3, \qquad 1 \leq i \leq m$$
$$f(v_j) = 4mn - 4m(j-1) - 1, \qquad 1 \leq j \leq n$$

For the vertex labeling f, the induced edge label f^* is obtained as $f^*(u_iv_j) = 2mn - 2i + 1 - 2m(j-1), 1 \leq i \leq m, 1 \leq j \leq n$. Then, f gives a skolem difference odd mean labeling. Hence, $K_{m,n}$ is a skolem difference odd mean graph for all $m \geq 1, n \geq 1$. □

For example, a skolem difference odd mean labeling of $K_{4,5}$ is shown in Fig.4.

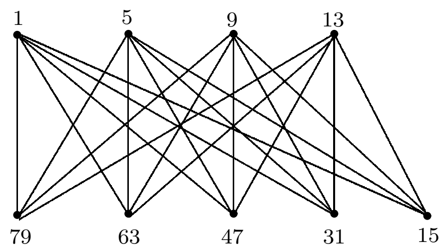

Fig.4

Corollary 2.4 *By taking $m = 1$, in the proof of the above theorem, we get a star graph $K_{1,n}$ and it is a skolem difference odd mean graph.*

Theorem 2.5 *The bistar $B_{m,n}$ is a skolem difference odd mean graph for $m \geq 1, n \geq 1$.*

Proof Let $V(K_2) = \{u, v\}$ and $u_i(1 \leq i \leq m)$, $v_j(1 \leq j \leq n)$ be the vertices adjacent to u and v respectively. Define $f : V(B_{m,n}) \to \{1, 2, \cdots, 4q-1 = 4(m+n)+3\}$ by

$$f(u) = 1,$$
$$f(v) = 4(m+n) + 3,$$
$$f(u_i) = 4i - 1, \quad 1 \leq i \leq m,$$
$$f(v_j) = 4j + 1, \quad 1 \leq j \leq n.$$

The induced edge labels are given as follows:

$$f^*(uv) = 2m + 2n + 1,$$
$$f^*(uu_i) = 2i - 1, \quad 1 \leq i \leq m$$
$$f^*(vv_j) = 2m + 2n - 2j + 1, \quad 1 \leq j \leq n.$$

Then, f is a skolem difference odd mean labeling and hence $B_{m,n}$ is a skolem difference odd mean graph for all $m \geq 1, n \geq 1$. □

For example, a skolem difference odd mean labeling of $B_{4,7}$ is shown in Fig.5.

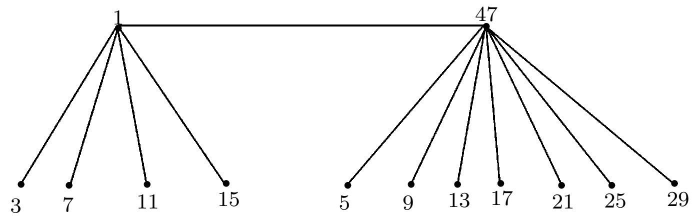

Fig.5

Theorem 2.6 *A quadrilateral snake is a skolem difference odd mean graph.*

Proof Let Q_n denote the quadrilateral snake obtained from $u_1, u_2, \ldots, u_{n+1}$ by joining u_i, u_{i+1} to new vertices v_i, w_i respectively and joining v_i and $w_i, 1 \leq i \leq n$. The graph Q_n has $3n+1$ vertices and $4n$ edges. We define $f : V(Q_n) \rightarrow \{1, 2, 3, \cdots, 4q-1 = 16n-1\}$ as follows:

$$f(u_i) = \begin{cases} 6i-5, & 1 \leq i \leq n+1 \text{ and } i \text{ is odd} \\ 16n-10i+11, & 1 \leq i \leq n+1 \text{ and } i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} 16n-10i+9, & 1 \leq i \leq n \text{ and } i \text{ is odd} \\ 6i-3, & 1 \leq i \leq n \text{ and } i \text{ is even} \end{cases}$$

$$f(w_i) = \begin{cases} 6i-1, & 1 \leq i \leq n \text{ and } i \text{ is odd} \\ 16n-10i+3, & 1 \leq i \leq n \text{ and } i \text{ is even.} \end{cases}$$

The induced edge labels are given by

$$f^*(u_iu_{i+1}) = \begin{cases} 8n-8i+3, & 1 \leq i \leq n \text{ and } i \text{ is odd} \\ 8n-8i+5, & 1 \leq i \leq n \text{ and } i \text{ is even} \end{cases}$$

$$f^*(v_iw_i) = \begin{cases} 8n-8i+5, & 1 \leq i \leq n \text{ and } i \text{ is odd} \\ 8n-8i+3, & 1 \leq i \leq n \text{ and } i \text{ is even} \end{cases}$$

$$f^*(u_iv_i) = 8n-8i+7, \quad 1 \leq i \leq n \text{ and}$$

$$f^*(u_iw_{i-1}) = 8n-8i+9, \quad 2 \leq i \leq n+1.$$

Thus, f is a skolem difference odd mean labeling and hence Q_n is a skolem difference odd mean graph. □

For example, a skolem difference odd mean labeling of Q_5 is shown in Fig.6.

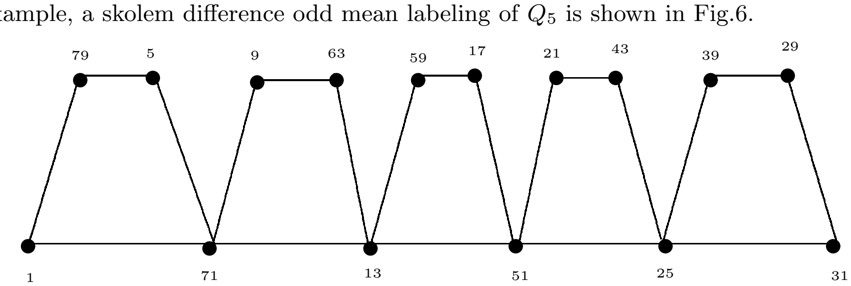

Fig.6

Theorem 2.7 *The ladder $L_n = P_n \times K_2$ is a skolem difference odd mean graph.*

Proof Let $u_1, u_2, \cdots, u_n$ and $v_1, v_2, \cdots, v_n$ be the vertices of L_n and $E(L_n) = \{u_i v_i : 1 \leq i \leq n\} \cup \{u_i u_{i+1} : 1 \leq i \leq n-1\} \cup \{v_i v_{i+1} : 1 \leq i \leq n-1\}$. Define $f : V(L_n) \to \{1, 2, 3, \cdots, 4q-1 = 12n-9\}$ as follows:

$$f(u_i) = \begin{cases} 4i-3, & 1 \leq i \leq n \text{ and } i \text{ is odd} \\ 12n-8i-1, & 1 \leq i \leq n \text{ and } i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} 12n-8i-1, & 1 \leq i \leq n \text{ and } i \text{ is odd} \\ 4i-3, & 1 \leq i \leq n \text{ and } i \text{ is even.} \end{cases}$$

For the vertex labeling f, the induced edge labeling f^* is given as follows:

$$f^*(u_i v_i) = 6n - 6i + 1, \quad 1 \leq i \leq n$$

$$f^*(u_i u_{i+1}) = \begin{cases} 6n-6i-3, & 1 \leq i \leq n-1 \text{ and } i \text{ is odd} \\ 6n-6i-1, & 1 \leq i \leq n-1 \text{ and } i \text{ is even} \end{cases}$$

$$f^*(v_i v_{i+1}) = \begin{cases} 6n-6i-1, & 1 \leq i \leq n-1 \text{ and } i \text{ is odd} \\ 6n-6i-3, & 1 \leq i \leq n-1 \text{ and } i \text{ is even.} \end{cases}$$

Then, f is a skolem difference odd mean labeling and hence L_n is a skolem difference odd mean graph. □

For example, a skolem difference odd mean labeling of $L_8 = P_8 \times K_2$ is shown in Fig.7.

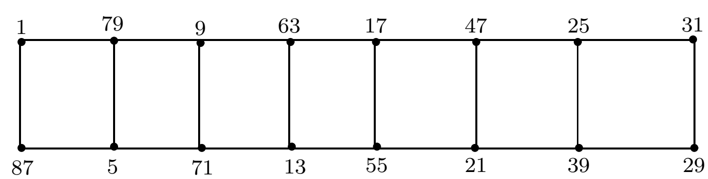

Fig.7

Theorem 2.8 *$L_n \odot K_1$ is a skolem difference odd mean graph.*

Proof Let L_n be the ladder. Let G be the graph obtained by joining a pendant edge to each vertex of the ladder. let $u_1, u_2, \cdots, u_n$ and $v_1, v_2, \cdots, v_n$ be the vertices of the ladder. For $1 \leq i \leq n$, let u_i' and v_i' be the new vertices made adjacent with u_i and v_i respectively. The graph G has $4n$ vertices and $5n-2$ edges.

Define $f : V(G) \to \{1, 2, \cdots, 4q-1 = 20n-9\}$ by

$$f(u_i) = \begin{cases} 20n-8i-1, & 1 \leq i \leq n \text{ and } i \text{ is odd} \\ 12i-7, & 1 \leq i \leq n \text{ and } i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} 12i-7, & 1 \leq i \leq n \text{ and } i \text{ is odd} \\ 20n-8i-1, & 1 \leq i \leq n \text{ and } i \text{ is even} \end{cases}$$

$$f(u_i') = \begin{cases} 12i-11, & 1 \leq i \leq n \text{ and } i \text{ is odd} \\ 20n-8i-5, & 1 \leq i \leq n \text{ and } i \text{ is even} \end{cases}$$

$$f(v_i') = \begin{cases} 20n-8i-5, & 1 \leq i \leq n \text{ and } i \text{ is odd} \\ 12i-11 & 1 \leq i \leq n \text{ and } i \text{ is even} \end{cases}$$

For the vertex labeling f, the induced edge labeling f^* is obtained as follows:

$$f^*(u_iu_{i+1}) = \begin{cases} 10n-10i-3, & 1 \leq i \leq n-1 \text{ and } i \text{ is odd} \\ 10n-10i-1, & 1 \leq i \leq n-1 \text{ and } i \text{ is even} \end{cases}$$

$$f^*(v_iv_{i+1}) = \begin{cases} 10n-10i-1, & 1 \leq i \leq n-1 \text{ and } i \text{ is odd} \\ 10n-10i-3, & 1 \leq i \leq n-1 \text{ and } i \text{ is even} \end{cases}$$

$$f^*(u_iu_i') = \begin{cases} 10n-10i+5, & 1 \leq i \leq n \text{ and } i \text{ is odd} \\ 10n-10i+1, & 1 \leq i \leq n \text{ and } i \text{ is even} \end{cases}$$

$$f^*(v_iv_i') = \begin{cases} 10n-10i+1, & 1 \leq i \leq n \text{ and } i \text{ is odd} \\ 10n-10i+5, & 1 \leq i \leq n \text{ and } i \text{ is even.} \end{cases}$$

Thus, $L_n \odot K_1$ is a skolem difference odd mean labeling and hence $L_n \odot K_1$ is a skolem difference odd mean graph. □

For example, a skolem difference odd mean labeling of $L_7 \odot K_1$ is shown in Fig.8.

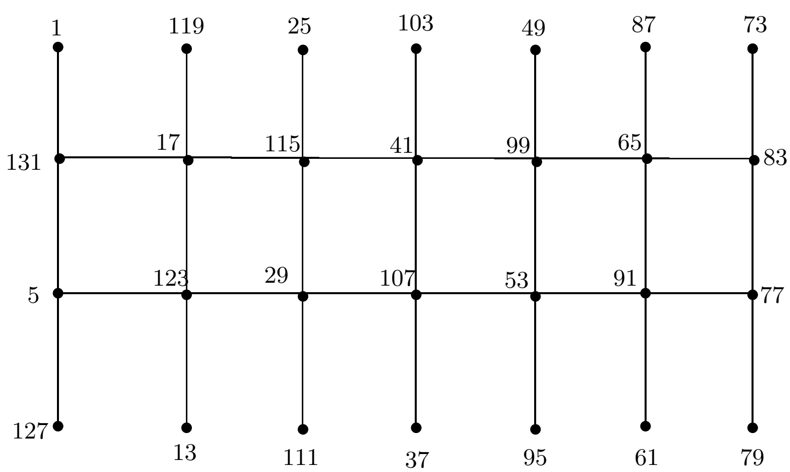

Fig.8

Theorem 2.9 *Let the path $G_1 = (p_1, q_1)$ and the star $G_2 = (p_2, q_2)$ have skolem difference odd mean* *labeling f and g respectively. Let u be the end vertex of G_1 and v be the central vertex of G_2 such that $f(u) = 1$ and $g(v) = 1$. Then the graph $(G_1)_f * (G_2)_g$ obtained from G_1 and G_2 by identifying the vertices u and v is also skolem difference odd mean.*

Proof Let $V(G_1) = \{u, u_i : 1 \le i \le p_1 - 1\}$ and $V(G_2) = \{v, v_i : 1 \le i \le p_2 - 1\}$. Then the graph $(G_1)_f * (G_2)_g$ has $p_1 + p_2 - 1$ vertices and $q_1 + q_2$ edges.

Define $h : V((G_1)_f * (G_2)_g) \to \{1, 2, 3, \cdots, 4(q_1 + q_2) - 1\}$ as follows:

$$h(u_i) = f(u_i), 1 \le i \le p_1 - 1$$
$$h(u) = f(u) = g(v) \text{ and}$$
$$h(v_i) = g(v_i) + 2(p_1 + q_1 - 1), 1 \le i \le p_2 - 1.$$

Then the induced edge labels of G_1 are $1, 3, 5, \ldots, 2q_1 - 1$ and that of G_2 are

$$2q_1 + 1, 2q_1 + 3, \cdots, 2(q_1 + q_2) - 1.$$

Hence, the graph $(G_1)_f * (G_2)_g$ obtained from G_1 and G_2 by identifying the vertices u and v is a skolem difference odd mean graph. □

For example, a skolem difference odd mean labeling of G_1, G_2 and $G_1 * G_2$ are shown in Fig.9.

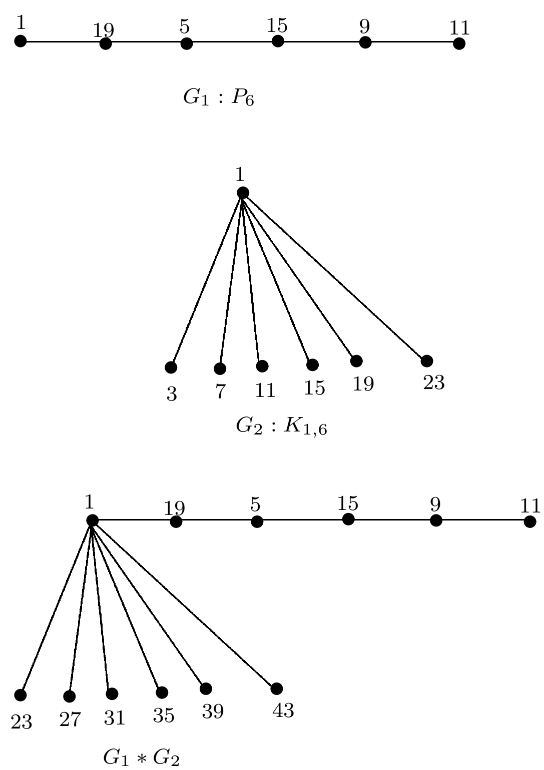

Fig.9

Theorem 2.10 *The graph $K_{1,n} \odot K_1$ is skolem difference odd mean for all $n \ge 1$.*

Proof Let G be the graph $K_{1,n} \odot K_1$ obtained from the star $K_{1,n}$ with vertices $u_0, u_1, u_2, \cdots, u_n$ by joining a vertex v_i to $u_i, 0 \le i \le n$.

Let $V(G) = \{u_0, u_i, v_0, v_i : 1 \leq i \leq n\}$ and $E(G) = \{u_0v_0, u_0u_i, u_iv_i : 1 \leq i \leq n\}$. The graph G has $2n+2$ vertices and $2n+1$ edges. Define $f : V(G) \to \{1, 2, \cdots, 4q-1 = 8n+3\}$ as follows:

$$f(u_0) = 1$$
$$f(u_i) = 4n + 4i - 1, 1 \leq i \leq n$$
$$f(v_0) = 8n + 3$$
$$f(v_i) = 8i - 3, 1 \leq i \leq n.$$

For the vertex labeling f, the induced edge labeling f^* in given as follows:

$$f^*(u_0v_0) = 4n + 1$$
$$f^*(u_0u_i) = 2n + 2i - 1, 1 \leq i \leq n$$
$$f^*(u_iv_i) = 2n - 2i + 1, 1 \leq i \leq n.$$

Then, f is a skolem difference odd mean labeling and hence G is a skolem difference odd mean graph. □

For example, a skolem difference odd mean labeling of $K_{1,7} \odot K_1$ is shown in Fig.10.

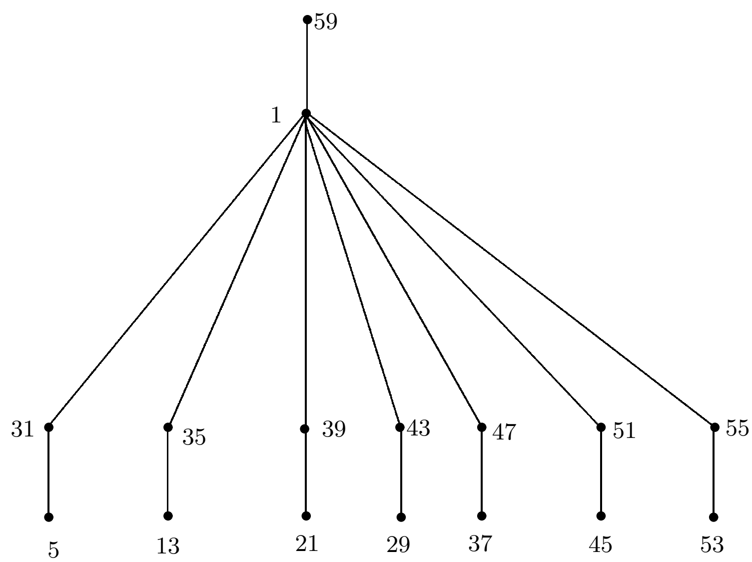

Fig.10

References

[1] F.Harary, *Graph Theory*, Addison-Wesley, Reading Mass., 1972.

[2] K.Manikam and M.Marudai, Odd mean labeling of graphs, *Bulletin of Pure and Applied Sciences*, **25E**(1) (2006), 149–153.

[3] K.Murugan and A.Subramanian, Skolem difference mean labeling of H-graphs, *International Journal of Mathematics and Soft Computing*, **1**(1) (2011), 115–129.

[4] A.Nagarajan and R.Vasuki, On the meanness of arbitrary path super subdivision of paths, *Australian J. Combin.*, **51** (2011), 41–48.

[5] S.Somasundaram and R.Ponraj, Mean labelings of graphs, *National Academy Science Letters*, **26** (2003), 210–213.

[6] R.Vasuki and A.Nagarajan, Meanness of the graphs $P_{a,b}$ and P_a^b, *International Journal of Applied Mathematics*, **22**(4) (2009), 663–675.

[7] R.Vasuki and S.Arokiaraj, On mean graphs, *International Journal of Mathematical Combinatorics*, **3** (2013), 22–34.

[8] R.Vasuki and A.Nagarajan, Odd mean labeling of the graphs $P_{a,b}, P_a^b$ and $P_{\langle 2a \rangle}^b$, *Kragujevac Journal of Mathematics*, **36**(1) (2012), 125–134.

[9] R.Vasuki and S.Arokiaraj, On odd mean graphs, *Journal of Discrete Mathematical Sciences and Cryptography*, (To appear).

[10] S.K.Vaidya and Lekha Bijukumar, Some new families of mean graphs, *Journal of Mathematics Research*, **2**(3) (2010), 169–176.