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On Radio Mean Number of Some Graphs

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Abstract: A radio mean labeling of a connected graph G is a one to one map f from the vertex set $V(G)$ to the set of natural numbers N such that for each distinct vertices u and v of G, $d(u,v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + \text{diam }(G)$. The radio mean number of f, $rmn(f)$, is the maximum number assigned to any vertex of G.The radio mean number of G, $rmn(G)$ is the minimum value of $rmn(f)$ taken over all radio mean labeling f of G. In this paper we find the radio mean number of some graphs which are related to complete bipartite graph and cycles.

Key Words: Carona, path, complete bipartite graph, cycle, Smarandache radio mean number, radio mean number.

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§1. Introduction

We considered finite, simple undirected and connected graphs only. Let $V(G)$ and $E(G)$ respectively denote the vertex set and edge set of G. Chatrand et al.[1] defined the concept of radio labeling of G in 2001. Radio labeling of graphs is applied in channel assignment problem [1]. Radio number of several graphs determined [2,7,5,9]. In this sequal Ponraj et al. [8] introduced the radio mean labeling in G. A radio mean labeling is a one to one mapping f from $V(G)$ to N satisfying the condition

$$d(u,v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + \text{diam }(G) \qquad (1.1)$$

for every $u, v \in V(G)$. The span of a labeling f is the maximum integer that f maps to a vertex of Graph G. For any subgraph $H \leq G$, a *Smarandache radio mean number* of G on H is the lowest span taken over al such labelings of the graph G that its constraint on H is a radio mean labeling. Particularly, if $H = G$, such a Smarandache radio mean number is called the *radio mean number* of G, denoted by $rmn(G)$. The condition (1.1) is called radio mean condition. In [8] we determined the radio mean number of some graphs like graphs with diameter three,

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lotus inside a circle, gear graph, Helms and Sunflower graphs. In this paper we determine radio mean number of subdivision of complete bipartite, corona complete graph with path and one point union of cycle C_6. The subdivision graph $S(G)$ of a graph G is obtained by replacing each edge uv by a path uwv. The corona of G with H, $G \odot H$ is the graph obtained by taking one copy of G and p copies of H and joining the i^{th} vertex of G with an edge to every vertex in the i^{th} copy of H. Let x be any real number. Then $\lceil x \rceil$ stands for smallest integer greater than or equal to x. Terms and definitions not defined here are follow from Harary [6].

§2. Main Results

Theorem 2.1 *$rmn(S(K_{m,n})) = (m+1)(n+1) - 1$, $m > 1, n > 1$.*

Proof Let $V(S(K_{m,n})) = \{u_i, v_j : 1 \leq i \leq m, 1 \leq j \leq n\} \cup \{w_{i,j} : 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E(S(K_{m,n})) = \{u_i w_{i,j}, w_{i,j} v_j : 1 \leq i \leq m, 1 \leq j \leq n\}$. Note that $diam(S(K_{m,n})) = 4$. Here we display $S(K_{2,2})$ with a vertex labeling in Figure 1.

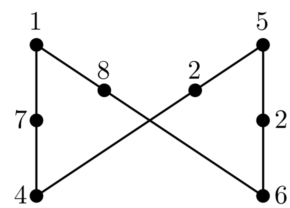

Figure 1

One can easily verify that the above vertex labeling satisfies the radio mean condition. We now explain a method for labeling the vertices of $S(K_{m,n})$ where $n \geq 3$. Consider the vertex $w_{i,j}$. Assign the label 2 to the vertex $w_{m,n}$. Put the label 3 to $w_{m,(n-1)}$. Similarly for $w_{m,(n-2)}$ we can label it by by 4. Proceeding like this $w_{m,1}$ is labeled by $n+1$. Next we label the neighbours of u_{m-1}. Allocate the labels $2n+3-j$ to the vertices $w_{(m-1),j}$ $(1 \leq j \leq n)$. Then we move to the vertices which are adjacent to w_{m-2}. Put the labels $3n+4-j$ to the vertices $w_{(m-2),j}$ $(1 \leq j \leq n)$. Proceeding like this the labels of the neighbours of u_1 are $mn+m+1-j$, $1 \leq j \leq n$. Now consider the vertices u_i $(1 \leq i \leq m)$. Put the label 1 to u_1. Then the vertices u_i $(2 \leq i \leq m)$ are labeled by $n+2+(n+1)(m-i)$. Then the integers from $\{mn+m+1, mn+m+2, \ldots, mn+m+n\}$ are assigned to the remaining vertices in any order.

Claim 1 The labeling f is a valid radio mean labeling. We must show that the condition

$$d(u,v) + \left\lceil \frac{f(u)+f(v)}{2} \right\rceil \geq 1 + diam(S(K_{m,n})) = 5,$$

holds for all pairs of vertices (u,v) where $u \neq v$.

Case 1. Check the pair (u_i, v_j).

$$d(u_i, v_j) + \left\lceil \frac{f(u_i)+f(v_j)}{2} \right\rceil \geq 2 + \left\lceil \frac{1+mn+m+1}{2} \right\rceil \geq 6.$$

Case 2. Verify the pair (u_i, u_j).

$$d(u_i, u_j) + \left\lceil \frac{f(u_i) + f(u_j)}{2} \right\rceil \geq 4 + \left\lceil \frac{1+n+2}{2} \right\rceil \geq 7.$$

Case 3. Consider the pair $(u_1, w_{1,j})$, $n \geq 3$.

$$d(u_1, w_{1,j}) + \left\lceil \frac{f(u_1) + f(w_{1,j})}{2} \right\rceil \geq 1 + \left\lceil \frac{1+mn+m-n+1}{2} \right\rceil \geq 5.$$

Case 4. Examine the pair $(u_1, w_{i,j})$, $i \neq 1$.

$$d(u_1, w_{i,j}) + \left\lceil \frac{f(u_1) + f(w_{i,j})}{2} \right\rceil \geq 3 + \left\lceil \frac{1+2}{2} \right\rceil \geq 5.$$

Case 5. Examine the pair $(u_i, w_{i,j})$, $u_i \neq u_1$, $n \geq 3$.

$$d(u_i, w_{i,j}) + \left\lceil \frac{f(u_i) + f(w_{i,j})}{2} \right\rceil \geq 1 + \left\lceil \frac{n+2+2}{2} \right\rceil \geq 5.$$

Case 6. Check the pair $(v_i, w_{i,j})$.

$$d(v_i, w_{i,j}) + \left\lceil \frac{f(v_i) + f(w_{i,j})}{2} \right\rceil \geq 1 + \left\lceil \frac{mn+m+1+2}{2} \right\rceil \geq 6.$$

Case 7. Consider the pair $(w_{i,j}, w_{r,t})$.

$$d(w_{i,j}, w_{r,t}) + \left\lceil \frac{f(w_{i,j}) + f(w_{r,t})}{2} \right\rceil \geq 2 + \left\lceil \frac{2+3}{2} \right\rceil \geq 5.$$

Case 8. Verify the pair (v_i, v_j).

$$d(v_i, v_j) + \left\lceil \frac{f(v_i) + f(v_j)}{2} \right\rceil \geq 4 + \left\lceil \frac{mn+m+1+mn+m+2}{2} \right\rceil \geq 12.$$

Hence $rmn(S(K_{m,n})) = (m+1)(n+1) - 1$. □

Theorem 2.2 *$rmn(K_{m,n} \odot P_t) = (m+n)(t+1)$, $m \geq 2$, $n \geq 2$, $t \geq 2$.*

Proof Let $V(K_{m,n}) = \{x_i, y_i : 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E(K_{m,n}) = \{x_i y_j : 1 \leq i \leq m, 1 \leq j \leq n\}$. Let $u_1^i u_2^i \cdots u_t^i$ be the path P_t^i and $v_1^j v_2^j \cdots v_t^j$ be the path P_t^{*j}, where $1 \leq i \leq m$, $1 \leq j \leq n$. The vertex set and edge set of the corona graph $K_{m,n} \odot P_t$ is given below. Let $V(K_{m,n} \odot P_t) = V(K_{m,n}) \cup (\bigcup_{i=1}^{m} V(P_t^i)) \cup (\bigcup_{j=1}^{n} V(P_t^{*j}))$ and $E(K_{m,n} \odot P_t) = E(K_{m,n}) \cup (\bigcup_{i=1}^{m} E(P_t^i)) \cup (\bigcup_{j=1}^{n} E(P_t^{*j})) \cup \{x_i u_j^i : 1 \leq i \leq m, 1 \leq j \leq t\} \cup \{y_i v_j^i : 1 \leq i \leq n, 1 \leq j \leq t\}$. Assign the label $1, 2, \cdots, m$ to the vertices $u_1^1, u_1^2, \cdots, u_1^m$ respectively. Then we move to the path vertices of P_t^{*j}. Assign the label $m+1, m+2, \cdots, m+t$ to the vertices $v_1^1, v_2^1, \cdots, v_t^1$ respectively. Then assign $m+t+1, m+t+2, \cdots, m+2t$ to the vertices $v_1^2, v_2^2, \cdots, v_t^2$ respectively. Proceeding like this until we reach the vertices of P_t^{*n}. Note that $v_1^n, v_2^n, \cdots, v_t^n$ received the labels $m+(n-1)t+1, m+(n-1)t+2, \cdots, m+nt$. Again we move to the vertices of the path P_t^i. Assign the label $m+nt+1, m+nt+2, \cdots, m+nt+t-1$ to the vertices $u_2^1, u_3^1, \cdots, u_t^1$ respectively.

Then assign the label $m+nt+t, m+nt+t+1, \cdots, m+nt+2t-2$ to the vertices $u_2^2, u_3^2, \cdots, u_t^2$ respectively. Proceed in the same way, assign the labels to the remaining vertices. Clearly the vertices $u_2^m, u_3^m, \cdots, u_t^m$ respectively received the labels $nt+mt-t+1, nt+mt-t+2, \cdots, nt+mt$. Finally assign the labels $nt+mt+1, nt+mt+2, \ldots, nt+mt+m$ to the vertices $x_1, x_2, \cdots, x_m$ and $nt+mt+m+1, nt+mt+2, \cdots, nt+mt+m+n$ to the vertices $y_1, y_2, \ldots, y_n$ respectively. We now check the radio mean condition for every pair of vertices.

Case 1. Consider the pair (u_i^j, u_s^r).

Subcase 1.1 $j \neq r$.

$$d(u_i^j, u_s^r) + \left\lceil \frac{f(u_i^j) + f(u_s^r)}{2} \right\rceil \geq 4 + \left\lceil \frac{1+2}{2} \right\rceil \geq 6$$

Subcase 1.2 $j = r$.

$$d(u_i^j, u_s^j) + \left\lceil \frac{f(u_i^j) + f(u_s^j)}{2} \right\rceil \geq 1 + \left\lceil \frac{1+m+nt+1}{2} \right\rceil \geq 5$$

Case 2 Check the pair (u_i^j, x_r).

$$d(u_i^j, x_r) + \left\lceil \frac{f(u_i^j) + f(x_r)}{2} \right\rceil \geq 1 + \left\lceil \frac{1+nt+mt+1}{2} \right\rceil \geq 6$$

Case 3 Verify the pair (u_i^j, y_r).

$$d(u_i^j, y_r) + \left\lceil \frac{f(u_i^j) + f(y_r)}{2} \right\rceil \geq 2 + \left\lceil \frac{1+nt+m}{2} \right\rceil \geq 6$$

Case 4 Examine the pair (u_i^j, v_r^s).

$$d(u_i^j, v_r^s) + \left\lceil \frac{f(u_i^j) + f(v_r^s)}{2} \right\rceil \geq 3 + \left\lceil \frac{1+m+t+1}{2} \right\rceil \geq 6$$

Case 5 Consider the pair (v_i^j, v_r^s).

$$d(v_i^j, v_r^s) + \left\lceil \frac{f(v_i^j) + f(v_r^s)}{2} \right\rceil \geq 1 + \left\lceil \frac{m+t+1+m+t+2}{2} \right\rceil \geq 7$$

Case 6 Verify the pair (v_i^j, x_r).

$$d(v_i^j, x_r) + \left\lceil \frac{f(v_i^j) + f(x_r)}{2} \right\rceil \geq 2 + \left\lceil \frac{m+t+1+nt+mt+1}{2} \right\rceil \geq 9$$

Case 7 Verify the pair (v_i^j, y_r).

$$d(v_i^j, y_r) + \left\lceil \frac{f(v_i^j) + f(y_r)}{2} \right\rceil \geq 1 + \left\lceil \frac{m+t+1+nt+mt+m+1}{2} \right\rceil \geq 9$$

Case 8 Consider the pair (x_i, x_j).

$$d(x_i, x_j) + \left\lceil \frac{f(x_i) + f(x_j)}{2} \right\rceil \geq 2 + \left\lceil \frac{nt + mt + 1 + nt + mt + 2}{2} \right\rceil \geq 12$$

Case 9 Examine the pair (y_i, y_j).

$$d(y_i, y_j) + \left\lceil \frac{f(y_i) + f(y_j)}{2} \right\rceil \geq 2 + \left\lceil \frac{nt + mt + m + 1 + nt + mt + m + 2}{2} \right\rceil \geq 14$$

Case 10 Check the pair (x_i, y_j).

$$d(x_i, y_j) + \left\lceil \frac{f(x_i) + f(y_j)}{2} \right\rceil \geq 1 + \left\lceil \frac{nt + mt + 1 + nt + mt + m + 1}{2} \right\rceil \geq 11$$

Hence $rmn\,(K_{m,n} \odot P_t) = (m+n)\,(t+1)$. □

The one point union of t cycles of length n is called the friendship graph and it is denoted by $C_n^{(t)}$.

Theorem 2.3 *For any integer $t \geq 2$,*

$$rmn\left(C_6^{(t)}\right) = \begin{cases} 5t+3 & \text{if} \quad t = 2 \\ 5t+2 & \text{if} \quad t = 3 \\ 5t+1 & \text{otherwise} \end{cases}$$

Proof Let $u_1^i u_2^i u_3^i u_4^i u_4^i u_5^i u_1^i$ be the i^{th} copy of the cycle $C_6^{(i)}$. Identify the vertex u_1^i $(1 \leq i \leq t)$. It is easy to verify that

$$diam\left(C_6^{(t)}\right) = \begin{cases} 3 & \text{if} \quad t = 1 \\ 6 & \text{otherwise} \end{cases}$$

Case 1 $t = 2$.

Claim 1 $rmn(C_6^{(2)}) \neq 5t+1$.

Suppose $rmn(C_6^2) = 5t+1$. Let f be the radio mean labeling of $C_6^{(2)}$ for which $rmn(f) = 5t+1$. Then the vertices are labeled from the set $\{1, 2, \cdots, 5t+1\}$. Clearly 1 and 2 should be labeled to the vertices with a distance at least 5. The possible vertices with label 1 and 2 are indicated in Figures 2 and 3.

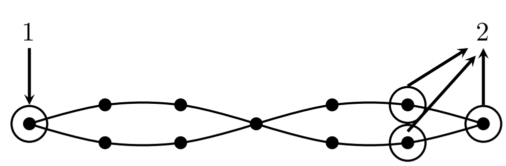

Figure 2

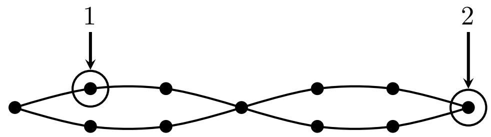

Figure 3

Clearly 2 and 3 are labeled at a distance at least 4 and 3 and 1 are labeled at a distance at least 5. There is no such vertex. Hence $rmn(C_6^{(2)}) \neq 5t+1$.

Claim 2 $rmn(C_6^{(2)}) \neq 5t+2$.

Suppose $rmn(C_6^{(2)}) = 5t+2$ then the vertices are labeled from the set $\{1, 2, \cdots, 5t+2\}$. If 1 is a label of a vertex then 3 and 4 are not labels of any vertices. Therefore the vertices are labeled from the set $\{2, 3, \cdots, 5t+2\}$. Note that 2 and 3 should be labeled to the vertices which are at a distance at least 4. Therefore 2 can not be a label of the identified vertex u_1^i. Suppose 2 is a label of the vertex u_2^i. This implies 3 should be a label of the vertex u_4^2. Then 4 can not be a label of any of the remaining vertices. If we put the label 2 to the vertex u_3^i, then 3 should be a label of either of the vertices u_3^2, u_4^2, u_5^2. In this case also 4 can not be a label of the remaining vertices. The same fact arises when 2 is a label of the vertex u_4^1. By symmetry, this is true for the other cases also. Hence we can not label the vertices of $C_6^{(2)}$ with the labels from the set $\{2, 3, \cdots, 5t+2\}$. Therefore $rmn(C_6^{(2)}) \neq 5t+2$.

Claim 3 $rmn(C_6^{(2)}) = 5t+3$.

The Figure 4 given below shows that the vertex labels are satisfies the radio mean condition.

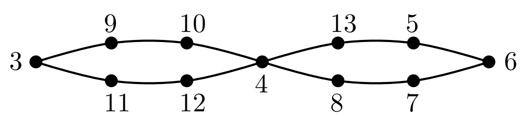

Figure 4

This implies $rmn(C_6^{(2)}) = 5t+3$.

Case 2. $t = 3$.

Claim 4 $rmn(C_6^{(3)}) \geq 5t+1$.

We observe that, for satisfying the radio mean condition, the labels 1, 2 and 3 are labels of the vertices of different cycles. Without loss of generality assume that 1 is a vertex label of the first copy of C_6, 2 is a vertex label of the second copy of C_6 and 3 is a vertex label of the third copy of C_6. Note that if 1 is a label of u_1^1 or u_2^1 then 24 can not be a label. If $f(u_3^1) = 1$ then 2 should be a label of u_4^3 . This implies 4 can not be a label of any of the remaining vertices. Suppose u_4^1 is labeled by 1. Then 2 is labeled by either one of the vertices u_3^2, u_5^2 or u_4^2. It follows that 3 should be a label of either u_3^3, u_5^3 or u_5^2 according as 2 is labeled. In either case 4 can not be a label of any of the vertices. Thus $rmn(C_6^{(3)}) \geq 5t+1$.

Claim 5 $rmn(C_6^{(3)}) \leq 5t+2$.

The vertex labeling given in figure 5 establish that it satisfies the radio mean condition and hence $rmn(C_6^{(3)}) \leq 5t+2$.

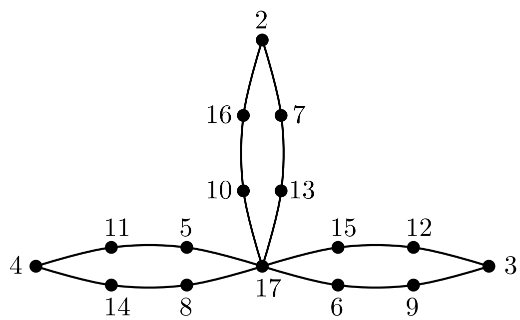

Figure 5

Therefore $rmn(C_6^{(3)}) = 5t+2$.

Case 3. $t \neq 2, 3$.

When $t = 1$, the vertex labels given in Figure 6 satisfies the requirements.

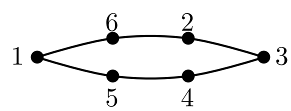

Figure 6

Hence $rmn(C_6^1) = 8$. Assume $t \geq 4$. Here we describe a labeling f as follows.

$$\begin{array}{lcll} f\left(u_4^i\right) & = & i & 1 \leq i \leq t \\ f\left(u_2^{t-i+1}\right) & = & t+i & 1 \leq i \leq t \\ f\left(u_6^{t-i+1}\right) & = & 2t+i & 1 \leq i \leq t \\ f\left(u_3^{t-i+1}\right) & = & 3t+i & 1 \leq i \leq t \\ f\left(u_5^{t-i+1}\right) & = & 4t+i & 1 \leq i \leq t \\ f\left(u_1^i\right) & = & 5t+1. & \end{array}$$

We now check whether the vertex labeling f is a valid labeling.

Case 3.1 Consider the pair (u_1^i, u_j^r).

$$d(u_1^i, u_j^r) + \left\lceil \frac{f(u_1^i) + f(u_j^r)}{2} \right\rceil \geq 2 + \left\lceil \frac{5t+1+1}{2} \right\rceil \geq 12$$

Case 3.2 Consider the pair (u_4^i, u_4^j).

$$d(u_4^i, u_4^j) + \left\lceil \frac{f(u_4^i) + f(u_4^j)}{2} \right\rceil \geq 6 + \left\lceil \frac{1+2}{2} \right\rceil \geq 8$$

Case 3.3 Consider the pair (u_4^i, u_2^i).

$$d(u_4^i, u_2^i) + \left\lceil \frac{f(u_4^i) + f(u_2^i)}{2} \right\rceil \geq 2 + \left\lceil \frac{1+2t}{2} \right\rceil \geq 7$$

It is easy to verify that all the other pair of distinct vertices are also satisfies the radio mean condition. Hence $rmn(C_6^{(t)}) = 5t + 1$ where $t \neq 2, 3$. □

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