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Special Kinds of Colorable Complements in Graphs

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Abstract: Let $G = (V, E)$ be a graph and $C = \{C_1, C_2, \cdots, C_k\}$ be a partition of color classes of a vertex set $V(G)$. Then the graph G is a k-colorable complement graph G_k^C(with respect to C) if for all C_i and $C_j, i \neq j$, remove the edges between C_i and C_j, and add the edges which are not in G between C_i and C_j. Similarly, the $k(i)$- colorable complement graph $G_{k(i)}^C$ of a graph G is obtained by removing the edges in $\langle C_i \rangle$ and $\langle C_j \rangle$ and adding the missing edges in them. This paper aims at the study of Special kinds of colorable complements of a graph and its relationship with other graph theoretic parameters are explored.

Key Words: Graph, complement, k-complement, k(i)-complement, colorable complement.

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§1. Introduction

All the graphs considered here are finite, undirected and connected with no loops and multiple edges. As usual $n = |V|$ and $m = |E|$ denote the number of vertices and edges at a graph G, respectively. For the open neighborhood of a vertex $v \in V$ is $N(v) = \{u \in V/uv \in E\}$, the set of vertices adjacent to v. The closed neighborhood is $N[v] = N(v) \bigcup \{v\}$. In general, we use $\langle X \rangle$ to denote the sub graph induced by the set of vertices X. If $deg(v)$ is the degree of vertex v and usually, $\delta(G)$ is the minimum degree and $\Delta(G)$ is the maximum degree. The complement G_c of a graph G defined to be graph which has V as its sets of vertices and two vertices are adjacent in G_c if and only if they are not adjacent in G. Further, a graph G is said to be self-complementary (s.c), if $G \cong G_c$. For notation and graph theory terminology we generally follow [3], and [5].

Let $G = (V, E)$ be a graph and $P = \{V_1, V_2, \cdots, V_k\}$ be a partition of V. Then k-complement G_k^P and $k(i)$-complement $G_{k(i)}^P$(with respect to P) are defined as follows: For all V_i and $V_j, i \neq j$, remove the edges between V_i andV_j, and add the edges which are not in G between V_i and V_j. The graph G_k^P thus obtained is called the k-complement of a graph G with respect to P. Similarly, the $k(i)$-complement of $G_{k(i)}^P$ of a graph G is obtained by removing the edges in $\langle V_l \rangle$ and $\langle V_j \rangle$ and adding the missing edges in them for $l \neq j$. This concept was first

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introduced by Sampathkumar et al. [9] and [10]. For more detail on complement graphs, we refer [1], [2], [4], [8], [11] and [12].

A graph is said to be k-vertex colorable (or k-colorable) if it is possible to assign one color from a set of k colors to each vertex such that no two adjacent vertices have the same color. The set of all vertices with any one color is independent and is called a color class. An k-coloring of a graph G uses k colors: it there by partitions V into k color classes. The chromatic number $\chi(G)$ is defined as the minimum k for which G has an k-coloring. Hence, graph G is a k-colorable if and only if $\chi(G) \leq k$, [7].

We make use of the following results in sequel [6].

Theorem 1.1 *For any non-trivial graph G,*

$$\sum_{x_i \in V} deg(x_i) = 2m.$$

Theorem 1.2(Konig's [5]) *In a bipartite graph G, $\alpha_1(G) = \beta_0(G)$. Consequently, if a graph G has no vertex of degree 0, then $\alpha_0(G) = \beta_1(G)$.*

§2. k-Colorable Complement

Let $G = (V, E)$ be a graph. If there exists a k-coloring of a graph G if and only if $V(G)$ can be partitioned into k subsets $C_1, C_2, \cdots, C_k$ such that no two vertices in color classes of $C_i, i = 1, 2, \cdots, k$, are adjacent. Then, we have the following definitions.

Definition 2.1 *The k-colorable complement graph G_k^C (with respect to C) of a graph G is obtained by for every C_i and C_j, $i \neq j$, remove the edges between C_i and C_j in G, and add the edges which are not in a graph G.*

Definition 2.2 *The graph G is k-self colorable complement graph, if $G \cong G_k^C$.*

Definition 2.3 *The graph G is k-co-self colorable complement graph, if $G_c \cong G_k^C$.*

Lemma 2.1 *Let G be a k-colorable graph. Then in any k-coloring of G, the subgraph induced by the union of any two color classes is connected.*

Proof If possible, let C_1 and C_2 be two color classes of vertex set $V(G)$ such that the subgraph induced by $C_1 \cup C_2$ is disconnected. Let G_1 be a component of the subgraph induced by $C_1 \cup C_2$. Obviously, no vertex of G_1 is adjacent to a vertex in $V(G) - V(G_1)$, which is assign the color either C_1 or C_2. Thus interchanging the colors of the vertices in G_1 and retaining the original colors for all other vertices, we gets a different k-coloring of a graph G, which is a contradiction. □

Theorem 2.1 *Let G be a (n, m)-graph. If for every C_l and C_j, $l \neq j$, and each vertex of C_l is adjacent to each vertex of C_j, then $m(G_k^C) = \varnothing$.*

Proof If for every C_l and C_j, $l \neq j$ in a (n, m)- graph with $\langle C_k \rangle$ is totally disconnected, where C_k is the partition of color classes of vertex set $V(G)$, then by the definition of k-colorable complement, $m(G_k^C) = \varnothing$ follows. Conversely, suppose the given condition is not satisfied, then there exist at least two vertices u and v such that $u \in C_l$ is not adjacent to vertex $u \in C_j$ with $l \neq j$. Thus by above lemma, this implies that $m(G_k^C) \geq 1$, which is a contradiction. □

A graph that can be decomposed into two partite sets but not fewer is bipartite; three sets but not fewer, tripartite; k sets but not fewer, k-partite; and an unknown number of sets, multipartite. An 1-partite graph is the same as an independent set, or an empty graph. A 2-partite graph is the same as a bipartite graph. A graph that can be decomposed into k partite sets is also said to be k-colorable. That is $\chi(K_n) = n$, but the chromatic number of complete k- partite graph $\chi(K_{r_1,r_2,r_3,\cdots,r_k}) = k < n$ for $r_i > 2$, where $i = 1, 2, \cdots, k$. By virtue of the facts, we have following corollaries.

Corollary 2.1 *Let G be a complete graph K_n; $n \geq 1$ vertices and $m = \dfrac{n(n-1)}{2}$ edges with $\chi(K_n) = n$. Then $m(G_n^C) = \varnothing$.*

Corollary 2.2 *Let G be a complete bipartite graph K_{r_1,r_2}; $1 \leq r_1 \leq r_2$, with $\chi(K_{r_1,r_2}) = 2$ for $n = (r_1 + r_2)$- vertices and m =$(r_1.r_2)$ edges. Then $m(G_2^C) = \varnothing$.*

Theorem 2.2 *Let G be a path P_n with $\chi(P_n) = 2$; $n \geq 2$ vertices. Then*

$$m(G_2^C) = \begin{cases} \frac{1}{4}(n-2)^2 & \text{if } n \text{ is even} \\ \frac{1}{4}(n-1)(n-3) & \text{if } n \text{ is odd.} \end{cases}$$

Proof Let G be a path P_n with $\chi(P_n) = 2$; $n \geq 2$ vertices, and $C = \{C_1, C_2\}$ be a partition of colorable class of vertex set of P_n. We have the following cases.

Case 1 If $\{u_1, u_2, \cdots, u_{t-1}, u_t\} \in C_1$ and $\{v_1, v_2, \cdots, v_{t-1}, v_t\} \in C_2$ with $v_1 - v_t$ is path of even length. Then $u_1, u_2, \cdots, u_{t-1}$ are adjacent $(t-2)$-vertices, that is $deg(u_i) = (t-2)$ if $1 \leq i \leq t-1$. Similarly, $v_1, v_2, \cdots, v_t$ are adjacent to $(t-2)$- vertices that is $deg(u_i) = (t-2)$ if $2 \leq i \leq t-1$, and v_1 and u_t are adjacent to $(t-1)$- vertices in G_2^C. Thus, $2(t-1) + (n-2)(t-2) = 2m(G_2^C)$. By Theorem 1.1, with the fact that $n = 2t$ and $m(G) = n-1$. Hence $m(G_2^C) = \dfrac{1}{4}(n-2)^2$.

Case 2 If $\{u_1, u_2, \cdots, u_{t-1}, u_t\} \in C_1$ and $\{v_1, v_2, \cdots, v_t, v_{t+1}\} \in C_2$ with $v_1 - v_{t+1}$ is path of even length. Then $u_1, u_2, \cdots, u_t$ are adjacent $(t-1)$-vertices, $v_2, v_3, \cdots, v_t$ are adjacent to $(t-2)$- vertices and, v_1 and u_{t-1} are adjacent to $(t-1)$ - vertices in G_2^C. Thus, $t(t-1) + (t-1)(t-2) + 2(t-1) = 2m(G_2^C)$. By theorem 1.1, with the fact that $n = 2t+1$ and $m(G) = n-1$. Hence $m(G_2^C) = \dfrac{1}{4}(n-1)(n-3)$. □

Theorem 2.3 *Let G be a cycle C_n; $n \geq 3$ vertices. Then*

(i) $m(G_2^C) = \dfrac{(n-4)n}{4}$, *if $\chi(C_n) = 2$ and n is even.*

(ii) $m(G_3^C) = \dfrac{(n+1)(n-3)}{4}$, *if $\chi(C_n) = 3$ and exactly one vertex is contain in any one colorable class of a vertex partition set of an odd cycle C_n.*

Proof The proof follows from Theorem 2.2, with even cycle of C_n and exactly one vertex is contain in any one colorable class of a vertex partition set of an odd cycle C_n. □

Theorem 2.4 *Let G be a Wheel W_n; $n \geq 4$ vertices and $m = 2(n-1)$ edges. Then*

(i) $m(G_4^C) = \frac{(n-4)n}{4}$*, if $\chi(C_n) = 4$ and n is even.*

(ii) $m(G_3^C) = \frac{(n+1)(n-3)}{4}$ *, if $\chi(W_n) = 3$ and exactly one vertex is contain in any one colorable class of a vertex partition set of an odd cycle C_{n-1} of W_n.*

Proof By Theorem 2.3 and $m(K_1) = 0$ due to the fact of $W_n = K_1 + C_{n-1}$, the result follows. □

Theorem 2.5 *Let T be a nontrivial tree with $\chi(T) = 2$. Then*

$$m(G_2^C) = (r_1.r_2) - n(T) + 1.$$

Proof Let $C = \{C_1, C_2\}$ be a partition of colorable class of a tree T with $n \geq 2$ vertices and $m(T) = n(T) - 1$. If every vertex in C_1 is adjacent to every vertex in C_2, that is K_{r_1,r_2} with $m(K_{r_1,r_2}) = r_1.r_2$. By definition of G_k^C with $\chi(T) = 2$, we have $m(G_2^C) = m(K_{r_1,r_2}) - m(T)$. Thus the results follows. □

Theorem 2.6 *For any non trivial graph G is k - self colorable complement if and only if $G \cong P_7$ or $2K_2$.*

Proof By definition of k-self colorable complement. It is clear that both G and G_2^C are isomorphic to P_7 or $2K_2$ with $\chi(P_7) = \chi(2K_2) = 2$. On the other hand, suppose G is k-self colorable complement, when G is not isomorphic with P_7 or $2K_2$. Then there exist at least two adjacent vertices u and v in G such that $u \in C_1$ and $v \in C_2$ are in disjoint color classes of $C = \{C_1, C_2\}$ with $\chi(P_7) = \chi(2K_2) = 2$. This implies that, u and v are not adjacent in G_2^C or they are in one color classes in G_1^C, that is totally disconnected graph. Thus the graph G and its colorable complements G_k^C are not isomorphic to each other, which is a contradiction. Hence the results follows. □

Theorem 2.7 *Let G be a k-self colorable complement graph. Then G has a vertex of degree at least* $\frac{n(\chi(G)-1)}{2\chi(G)}$.

Proof Let G be a (n,m)- graph with $G \cong G_k^C$ and $C = \{C_1, C_2, \cdots, C_k\}$ be a partition of color classes of a vertex set $V(G)$. Suppose, if $\chi(G) = k$ and $V(G)$ is partitioned into k independent sets $C_1, C_2, \cdots, C_k$. Thus, $n = |V(G)| = |C_1, C_2, \cdots, C_k| = \sum_{i=1}^{k} |V(G)| \leq k\beta(G)$, where $\beta(G)$ is the independence number of a graph G. There fore $\chi(G) = k = n/\beta(G)$. Also, suppose $v \in C_i$, where C_i is a colorable set in C with at most $n/\chi(G)$. Then the sum of the degree of v in G and G_k^C is greater than $\frac{n(\chi(G)-1)}{\chi(G)}$. This implies that the degree of v is at least $\frac{1}{2}(n - \frac{n}{\chi(G)})$. Hence the result follows. □

Theorem 2.8 *Let G be a k-self colorable complement graph. Then*

$$\frac{(k-1)(2n-k)}{4} \leq m(G) \leq \frac{2n(n-k)+k(k-1)}{4}.$$

Proof Let G be a k-self colorable complement graph and $C = \{C_1, C_2, \cdots, C_k\}$ be a partition of color classes of a vertex set $V(G)$. If $|C_t| = n_t$ for $1 \leq t \leq k$, then the total number of edges between C_l and C_j in C, $l \neq j$, in both the graph G and its colorable complement graph G_k^C is $\sum\limits_{l\neq j} n_l n_j$. Since the graph G is k-self colorable complement graph G_k^C, half of these edges are not there in G. Hence $m(G) \leq \begin{pmatrix} n \\ 2 \end{pmatrix} - \sum\limits_{l\neq j} n_l n_j$. Clearly, $\sum\limits_{l\neq j} n_l n_j$ is minimum, when $n_t = 1$ for $k-1$ of the indices. Thus, we have

$$m(G) \leq \begin{pmatrix} n \\ 2 \end{pmatrix} - \frac{1}{2}[\begin{pmatrix} k-1 \\ 2 \end{pmatrix} + (k-1)(n-k+1)].$$

Hence the upper bound follow. To establish the lower bound, the graph G being k-self colorable complement has at least $\sum\limits_{l\neq j} n_l n_j$- edges. So, $\frac{1}{2}[\begin{pmatrix} k-1 \\ 2 \end{pmatrix} + (k-1)(n-k+1)] \leq m(G)$ and the result follows. $\square$

Theorem 2.9 *For any non trivial graph G is k - co - self colorable complement if and only if $G \cong K_n$.*

Proof On contrary, suppose given condition is not satisfied, then there exists at least three vertices u, v and w such that v is adjacent to both u and w, and u is not adjacent to w. This implies that an edge $e = uw \in G_c$ and induced subgraph $\langle u, v, w\rangle$ in G_2^C is totally disconnected. Thus $E(G_2^C) \subset E(G_c)$, which is a contradiction to the fact of $G_c \cong G_n^C$ with $\chi(K_n) = n$. Converse is obvious. $\square$

§3. $k(i)$-Colorable Complement

Let $G = (V, E)$ be a graph and $C = \{C_1, C_2, \cdots, C_k\}$ be a partition of color classes of a vertex set $V(G)$. Then, we have the following definitions.

Definition 3.1 *The $k(i)$ - colorable complement graph $G_{k(i)}^C$(with respect to C) of a graph G is obtained by removing the edges in $\langle C_l\rangle$ and $\langle C_j\rangle$ and adding the missing edges in them for $l \neq j$.*

Definition 3.2 *The graph G is $k(i)$-self colorable complement graph, if $G \cong G_{k(i)}^C$.*

Definition 3.3 *The graph G is $k(i)$-co-self colorable complement graph, if $G_c \cong G_{k(i)}^C$.*

Theorem 3.1 *For any graph G, $m(G_{k(i)}^C) = \dfrac{n(n-1)}{2}$ if and only if the graph G is isomorphic with complete n- partite graph $K_{r_1,r_2,r_3,\cdots,r_n}$ or $(K_n)_c$.*

Proof To prove the necessity, we use the mathematical induction. Let G be a graph with $n = 1$ vertex. Then $\chi(G) = 1$ and $m(G^C_{1(i)}) = \varnothing$. Hence the result follows. Suppose the graph G with $n > 1$ vertices. Then the following cases are arises.

Case 1 If the graph G is totally disconnected, that is $(K_n)_c$, complement of a complete graph K_n, then G has a only one color class C_1 with $\chi((K_n)_c) = 1$. By the definition of $G^C_{1(i)}$, the induced subgraph of $\langle C_1 \rangle$ is complete, which form a $\frac{n(n-1)}{2}$- edges.

Case 2. If the graph G is complete n- partite graph $K_{r_1,r_2,r_3,\ldots,r_n}$, then for every two color classes C_l and C_j for $l \neq j$, and each vertex C_l adjacent to each vertex of C_j in complete n-partite graph $K_{r_1,r_2,r_3,\ldots,r_n}$ with $m(K_{r_1,r_2,r_3,\ldots,r_n}) = r_1 r_2 r_3 \ldots r_n$. By the definition of $G^C_{n(i)}$ with $G = K_{r_1,r_2,r_3,\ldots,r_n}$, we have

$$m(G^C_{n(i)}) = \binom{r_1}{2} + \binom{r_2}{2} + \ldots + \binom{r_n}{2} + r_1 r_2 r_3 \ldots r_n,$$

where $\binom{r_t}{2}$ is the maximum number edges of induced subgraph $\langle C_t \rangle$ if $t = 1, 2, \ldots, n$, which are complete. This forms $\frac{n(n-1)}{2}$- edges.

Conversely, suppose the graph G is not isomorphic to complete n- partite graph $K_{r_1,r_2,r_3,\ldots,r_n}$ or $(K_n)_c$. Then there exist at least three vertices $\{a, b, c\}$ such that at least two adjacent vertices a and b are not adjacent to isolated vertex c. By the definition of $G^C_{k(i)}$ with $\chi(G) = k \geq 2$, which form a path $(a - b - c)$ or $(b - a - c)$ of length 2, which is not a complete, a contradiction. This proves the sufficiency. □

Theorem 3.2 *Let G be a path P_n with $\chi(P_n) = 2$ and $n \geq 2$ vertices. Then*

$$m(G^C_{2(i)}) = \begin{cases} \frac{1}{4}[n^2 + 2n - 4]^2 & \text{if } n \text{ is even} \\ \frac{1}{4}(n-1)(n+3) & \text{if } n \text{ is odd} \end{cases}$$

Proof Let G be a path P_n with $\chi(P_n) = 2$; $n \geq 2$ vertices, and $C = \{C_1, C_2\}$ be a partition of colorable class of vertex set of P_n. We have the following cases.

Case 1 Let $C = \{C_1, C_2\}$ be a partition of colorable class of P_n. If $\{u_1, u_2, \cdots, u_{t-1}, u_t\} \in C_1$ and $\{v_1, v_2, \cdots, v_{t-1}, v_t\} \in C_2$ with $v_1 - u_t$ is path of even length. Then $\langle C_1 \rangle$ and $\langle C_2 \rangle$ are complete in $G^C_{2(i)}$ and also $v_1 - u_t$ path have $(n-1)$ - edges in both the graph G and its $k(i)$-colorable complement graph $G^C_{2(i)}$. Thus, $m(G) + t(t-1) = (n-1) + n(n-2)/4 = m(G^C_{2(i)})$ and this implies $m(G^C_{2(i)}) = \frac{1}{4}[n^2 + 2n - 4]^2$.

Case 2 Let $C = \{C_1, C_2\}$ be a partition of colorable class of P_n. If $\{u_1, u_2, \cdots, u_{t-1}, u_t\} \in C_1$ and $\{v_1, v_2, \cdots, v_{t-1}, v_t\} \in C_2$ with $v_1 - u_{t+1}$ is path of odd length. Then $\langle C_1 \rangle$ and $\langle C_2 \rangle$ are complete in $G^C_{2(i)}$ and also $v_1 - u_{t+1}$ path have $(n-1)$ - edges in both the graph G and its $2(i)$-colorable complement graph $G^C_{2(i)}$. Thus, $m(G) + t(t-1)/2 + t(t+1)/2 = (n-1)[1 + (n - 3)/8 + (n+1)/8] = m(G^C_{2(i)})$ and this implies $m(G^C_{2(i)}) = \frac{1}{4}(n-1)(n+3)$. □

Theorem 3.3 *Let G be a cycle C_n; $n \geq 3$ vertices. Then*

(*i*) $m(G^C_{2(i)}) = \frac{1}{4}[n(n+2)]$, *if* $\chi(C_n) = 2$ *and n is even.*

(*ii*) $m(G^C_{3(i)}) = \frac{1}{4}(n^2+3)$, *if* $\chi(C_n) = 3$ *and exactly one vertex is contain in any one colorable class of a vertex partition set of an odd cycle* C_n.

Proof The proof follows from Theorem 3.2, with even cycle of C_n and exactly one vertex is contain in any one colorable class of a vertex partition set of an odd cycle C_n. □

Theorem 3.4 *Let T be a nontrivial tree with $\chi(T) = 2$. If $C = \{C_1, C_2\}$ be a partition of colorable class of a tree T, then*

$$m(G^C_{2(i)}) = \frac{1}{2}[r^2 + s^2 + n - 2],$$

where $|C_1| = r$ *and* $|C_2| = s$.

Proof Let $C = \{C_1, C_2\}$ be a partition of colorable class of a tree T with $\chi(T) = 2$ and $m(T) = n(T) - 1 = r + s + 1$. Then by definition of $G^C_{k(i)}$, we have $\langle C_1\rangle$ and $\langle C_2\rangle$ are complete.

There fore, $m(C_1) = \begin{pmatrix} r \\ 2 \end{pmatrix}$ and $m(C_2) = \begin{pmatrix} s \\ 2 \end{pmatrix}$.

Thus, we have

$$m(G^C_{2(i)}) = \begin{pmatrix} r \\ 2 \end{pmatrix} + \begin{pmatrix} s \\ 2 \end{pmatrix} + m(T) = \frac{1}{2}[r(r+1) + s(s+1) - 2].$$

Hence the result follows. □

Theorem 3.5 *For any non trivial graph G is $k(i)$ - self colorable complement if and only if G is isomorphic with K_n.*

Proof Let $G = K_n$ be a complete graph with $\chi(G) = n$. Then by the definition of $G^C_{k(i)}$, the induced subgraph $\langle C_t\rangle$ for $t = 1, 2, \ldots, n$ are connected and $|C_t| = 1$ for $t = 1, 2, \ldots, n$. Thus $G^C_{n(i)} \cong K_n$ and the result follows. Conversely, suppose given condition is not satisfied, then there exists at least two non adjacent vertices u and v in a graph G such that $\chi(G) = 1$ and $m(G) = \varnothing$. By the definition of $G^C_{k(i)}$, we have $\chi(G^C_{1(i)}) = 2$ with an induced subgraph $\langle u, v\rangle$ in $G^C_{1(i)}$ is connected. Thus $m(G) < m(G^C_{1(i)})$, which is a contradiction to the fact of $G \cong G^C_{k(i)}$. □

§4. $\{G, G^p_k, G^p_{k(i)}\}$ - Realizability

Here, we show the $G, G^p_k, G^p_{k(i)}$ - Realizability for some graph theoretic parameter.

Let G be a graph. Then $S \subseteq V(G)$ is a separating set if $G - S$ has more than one component. The connectivity $\kappa(G)$ of G is the minimum size of $S \subseteq V(G)$ such that $G - S$ is disconnected or a single vertex. For any $k \leq \kappa(G)$, we say that G is k-connected. Then, we have

Theorem 4.1 *Let G be a graph with $C = \{C_1, C_2\}$ be a partition of colorable class of a vertex set V. If $\langle C_1\rangle$ and $\langle C_2\rangle$ are $(t-1)$ -colorable with Max.$\{\chi(G_k^C), \chi(G_{k_i}^C)\} \geq t$, then Min.$\{k(G), k(G_k^C), k(G_{k_i}^C)\}$ has at least $(t-1)$ -edges.*

Theorem 4.2 *Let G be a (n,m)- graph. Then*

(i) $\chi(G_k^C) = 1$ *if and only if G is isomorphic with K_n or $(K_n)_c$ or $K_{r_1,r_2,r_3,\cdots,r_k}$.*
(ii) $\chi(G_{k(i)}^C) = n$ *if and only if G is isomorphic with K_n or $(K_n)_c$ or $K_{r_1,r_2,r_3,\cdots,r_k}$.*

Proof By the definition of G_k^C and Theorem 2.1, (i) follows. Also by the definition of $G_{k(i)}^C$ and Theorem 3.1, (ii) follows. □

A set M of vertices in a graph G is independent if no two vertices of M are adjacent. The number of vertices in a maximum independent set of G is denoted by $\beta(G)$. Opposite to an independent set of vertices in a graph is a clique. A clique in a graph G is a complete subgraph of G. The order of the largest clique in a graph G and its clique number, which is denoted by $\omega(G)$. In fact $\beta(G) = k$ if and only if $\omega(\overline{G}) = k$. Then, we have

Theorem 4.3 *Let G be a nontrivial (n,m)- graph. Then*

(i) $\beta(G_{k(i)}^C) \leq \beta(G) \leq \beta(G_k^C)$.
(ii) $\omega(G_k^C) \leq \omega(G) \leq \omega(G_{k_i}^C)$.

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